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Abstract. In this paper we test the Unified Multifractal model of atmospheric dynamics in the tropics. In the first part, we empirically investigate the scaling behaviour along the horizontal, in the second part along the vertical. Here we concentrate on the presentation of basic multifractal notions and techniques and on how they give rise to self-organized critical structures. Indeed, we point out a rather simple and clear characterisation of these structures which may help to clarify both the nature of the oft-cited coherent structures and the generation of cyclones. Using 30 aircraft series of horizontal wind and temperature, we find rather remarkable constancy of the three universal multifractal indices H, C_1 and α as well as the value of critical exponents q_D, γ_D associated with multifractal phase transitions and self-organized critical structures. This constancy extends not only from wind tunnel and midlatitude to the tropics, but also to multifractals generated by Navier-Stokes like equations.

1 Introduction

During the last few decades, numerous investigations have been performed on the tropical atmospheric structures: boundary layer coherent structures, cloud bands, typhoons, tropical storms and depressions (see for review Mikhailova and Ordanovich (1991)). Indeed, in spite of its strong mixing, there are identifiable structures at all scales inside of the tropical boundary layer. Of particular importance are the large convective rolls with their axes approximately in the direction of the mean flow (geostrophic wind), and whose cross-sections are comparable in scale to the height of the boundary layer. A visual confirmation of the occurrence of such structures is given by the ordered "cloud streets" structures observed, for instance on satellite images of the earth's cloud cover. Until now there exists no satisfactory theory of the appearance of these structures and the mechanism of their generation remains rather mysterious. However, it has been recently pointed out that inhomogeneity plays a central role by increasing the stability of these structures (Ordanovich and Chigirinskaya, 1993). In the present paper, we develop considerably this idea by considering inhomogeneity as intervening at all scales.

Indeed, contrary to classical approaches, we investigate the inhomogeneity over a large range of scales and intensities and we try to understand the crucial relationships between extremes events (such as extreme wind shears) and the mean events (more quiescent flow regions) including how the latter can build up to the appearance of the former. The simplest and most natural framework for considering extreme nonlinear variability over a wide range of scales is multifractals since the variability simply results from an elementary scale invariant process, the generator of the field, which reproduces itself from scale to scale. Indeed, a unified multifractal model of atmospheric dynamics (also called the "Unified Scaling model") has been developed (Schertzer and Lovejoy, 1983, 1985; Lovejoy et al., 1993) involving a unique multifractal and anisotropic regime in opposition to the classical model (e.g. Monin, 1972), which involves two distinct (quasi-) isotropic and rather homogeneous regimes. These regimes are separated by "meso-scale gap" or a "dimensional transition" (Schertzer and Lovejoy, 1985): isotropic two dimensional turbulence[3] for large scales and isotropic three dimensional turbulence for small scales. In the Unified Scaling model, at a given scale, the generator creates structures of all intensities while simultaneously creating structures over a wide range of scales in anisotropic manner. The model therefore unifies both intense and weak events as well as events with different degrees of stratification. Different typhoon expeditions give us an unique opportunity to test and improve this model since data were collected along both the

[3] The same comment applies to quasi two dimensional, quasi geostrophic variants.

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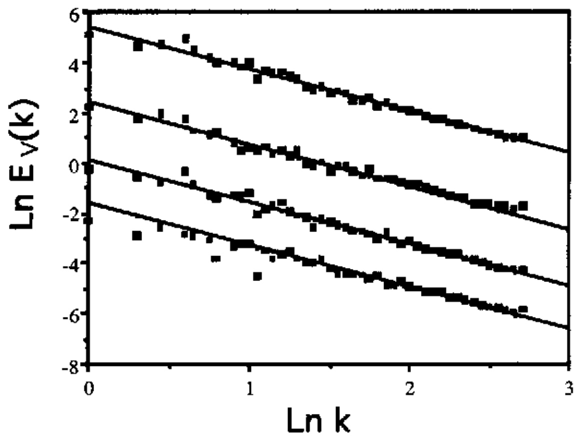

Fig. 1-a. The spectrum (open squares) of horizontal wind velocity fluctuations, averaged over the 3 data sets taken roughly at one year intervals (each contains 10 samples) and also the 3 individual spectra (closed squares) obtained by averaging over the 10 samples of each expedition. As all these spectra were very similar, in order to improve the display, their *Log* was respectively vertically shifted by -3, -5, -7. The absolute slopes are close to Kolmogorov-Obukhov value 5/3 : $\beta_V = 1.68 \pm 0.05$ over the frequencies range $\omega_0/20 - \omega_0/20480$ ($\omega_0 = 8\ Hz$).

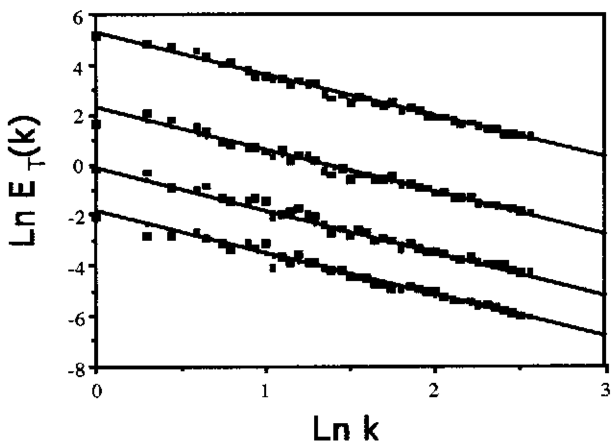

Fig. 1-b. The spectrum (open squares) of temperature fluctuations, averaged over the 3 data sets taken roughly at one year intervals (each contains 10 samples) and also the 3 individual spectra (closed squares) obtained by averaging over the 10 samples of each expedition. As all these spectra were very similar, in order to improve the display, their *Log* was respectively vertically shifted by -3, -5, -7. The absolute slopes are close to Corrsin-Obukhov value 5/3 : $\beta_V = 1.7 \pm 0.05$ over the frequencies range $\omega_0/20 – \omega_0/20480$ ($\omega_0 = 8\ Hz$).

horizontal (by plane) as well as along the vertical (by balloon soundings). The identification of the multifractal generator in both directions is particularly straightforward in the framework of universal multifractals (Schertzer and Lovejoy, 1987, 1989; Schertzer et al., 1991; Lovejoy and Schertzer, 1990, 1991; Schmitt et al., 1992a; see the discussion in Sect. 2 below).

In order to recast the rather different scaling behaviors along the horizontal and the vertical within the Unified Scaling model, in Part II (Lazarev et al., 1994) we will consider the rather general framework of Generalized Scale Invariance (GSI) (Schertzer and Lovejoy, 1983, 1984, 1985; Lovejoy et al., 1993), displaying scaling anisotropy involving rather more complex scale transformations than self-similar (isotropic) dilations.

At the same time, the determination of the underlying multifractal processes allows us to discuss the origin of the appearance of the ordered tropical structures in terms of non classical Self-Organized Criticality. Indeed, as we discuss in Sect. 3, whereas classical self-organized criticality (Bak et al., 1987, 1988) is related to cellular automata and a deterministic dynamics, an alternative stochastic route has been discussed in a series of papers (Schertzer and Lovejoy, 1992, 1993; Schertzer et al.,1993). Building on the earlier closely related notion of "hyperbolic intermittency" (Schertzer and Lovejoy, 1983, 1985), these papers show how Self-Organized Criticality is generically reached in scaling processes via a first order multifractal phase transition at a critical order of singularity and order of statistical moment both of which are empirically determined.

After the presentation of the data sets (Sect. 4) we proceed (Sect. 5) to the empirical determination of the corresponding universal exponents and compare them to those determined in rather different meteorological conditions as well as in time rather than in space (Schmitt et al., 1992a, 1993, 1994). In Sect. 6 we determine the critical order of singularity and order of statistical moment of the multifractal phase transition. In Sect. 7, we discuss the features of dynamics which should theoretically determine the values of the universal exponents. In conclusion, we summarize and discuss our findings and their importance for the understanding of structures of the tropical atmosphere as well as for the unified multifractal model.

2 Universal multifractals and their statistical analysis

In the case of a stochastic multifractal field, – for example the turbulent energy flux density (ε) – observed at different scale ratios λ ($= L/l$, where L is the outer scale and l is the scale of observation), the statistics of the field can be described in the framework of the codimension multifractal formalism (Schertzer and Lovejoy, 1987, 1989, 1992; Schertzer et al., 1991; Mandelbrot, 1991) either in terms of probability distributions or statistical moments, involving respectively the codimension function ($c(\gamma)$) of the order of singularities (γ) and scaling function ($K(q)$) of the moments of order q:

$$\Pr\left(\varepsilon_\lambda \geq \lambda^\gamma\right) \approx \lambda^{-c(\gamma)} \qquad (1)$$

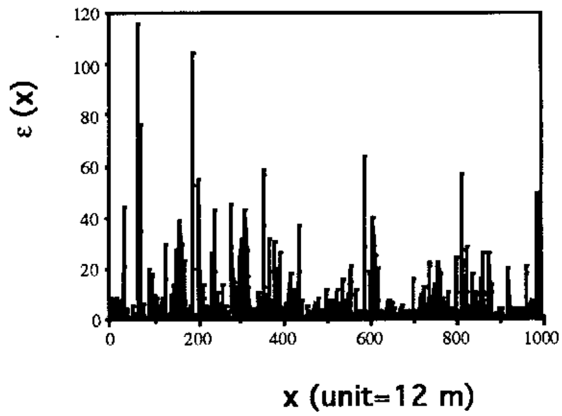

Fig. 2-a. One of the time series of the estimate of the energy flux density ε. It displays rather strong intermittency: most of the time the values are less than 1 but there are very extreme values. The normalization $<\varepsilon>=1$ has been performed over the 30 realizations.

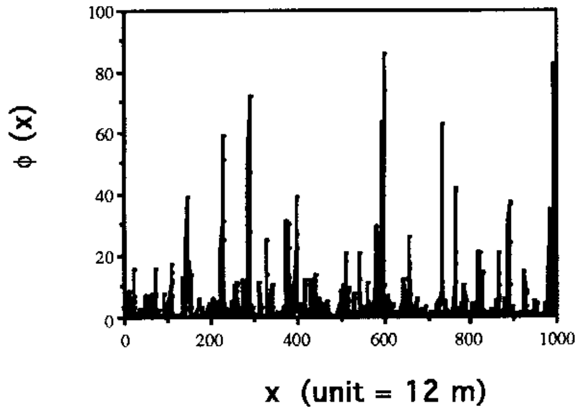

Fig. 2-b. One of the time series of the estimate of the energy flux density φ. It displays rather strong intermittency: most of the time the values are less than 1 but there are very extreme values. The normalization $<\varphi>=1$has been performed over the 30 realizations.

$$\left\langle \varepsilon_\lambda{}^q \right\rangle \approx \lambda^{K(q)} \tag{2}$$

$c(\gamma)$ and $K(q)$ are dual for the (involutive) Legendre transform (Parisi and Frisch, 1985):

$$\begin{aligned} c(\gamma) &= \max_q (q\gamma - K(q)); \\ K(q) &= \max_\gamma (q\gamma - c(\gamma)) \end{aligned} \tag{3}$$

the codimensions and the order of singularity of the density are related in the following manner to the dimension formalism (Halsey et al., 1986) of deterministic chaos[4] (see Schertzer et al. (1991); Schertzer and Lovejoy (1992) for more discussion especially concerning the limitations of the dimension formalism when considering stochastic processes):

$$f_D(\alpha_D) = D - c(\gamma); \qquad \alpha_D = D - \gamma \tag{4}$$

The only constraints that must be respected by the two functions $K(q)$ and $c(\gamma)$ are that they should both be convex, and $c(\gamma)$ should be an increasing function[5]. Therefore, the determination of these functions generally corresponds to the determination of an infinity of parameters, which would be prohibitive both at the empirical and theoretical level.

Fortunately, due to the existence of stable and attractive multifractal processes, under rather general circumstances, mixing of different multifractal processes may lead to universal processes which depend on very few aspects of the initial processes. Indeed – up to a critical order discussed below – these universal multifractal processes have codimension and moment scaling functions ruled by only three common exponents. The three basic universal exponent are:

-The Hurst exponent H measuring the degree of non conservation of the mean field[6],

$$<\varepsilon_\lambda> \approx \lambda^{-H} \tag{5}$$

-The mean singularity C_1, i.e. those contributing to the mean field, measures the fractality/sparseness of the mean field, it corresponds at the same time to the codimension of the mean field. Therefore (by Legendre transform) it corresponds to the following fixed point:

$$c(C_1-H) = C_1 \tag{6}$$

-The Lévy index α determines the extent of multifractality, it is indeed the Levy index α of the generator of the process. It is proportional to the radius of the curvature (R_C) of the codimension function around mean singularities:

$$R_C(C_1-H) = 2^{3/2} C_1 \alpha \tag{7}$$

The corresponding universal moment scaling and codimension functions have the following forms (Schertzer and Lovejoy, 1987; Brax and Peschanski, 1991; Kida, 1991; Schmitt et al., 1992a):

$$K(q) + Hq = \begin{cases} \dfrac{C_1}{\alpha - 1}\left(q^\alpha - q\right) & \alpha \neq 1 \\ C_1 q Ln(q) & \alpha = 1 \end{cases} \tag{8}$$

[4] We add a subscript D in order to render explicit the D dimensional dependency of the deterministic chaos notation when applied to a stochastic process observed on a D dimensional space. The α_D should not be confused with the Levy index discussed below.

[5] When defined as the exponent of the probability distribution (as in Eq. (1)) rather than the probability density.

[6] If ε represents the turbulent energy flux density, then $H=0$.

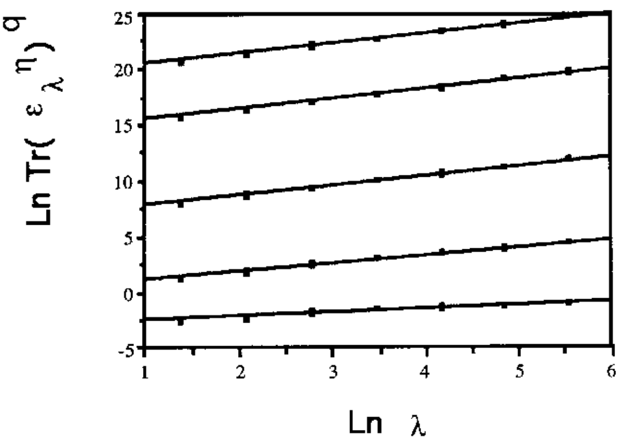

Fig. 3. A Log-Log plot, the double trace moment as a function of the scale ratio λ for different values of η and q=1.5. The straight lines confirm that scale invariance for moments of the wind are well respected.

$$c(\gamma)=\begin{cases} C_1\left(\dfrac{\gamma+H}{C_1\alpha'}+\dfrac{1}{\alpha}\right)^{\alpha'} & \alpha\neq 1 \\ C_1\exp\left(\dfrac{\gamma+H}{C_1}-1\right) & \alpha=1 \end{cases} \qquad \frac{1}{\alpha}+\frac{1}{\alpha'}=1 \tag{9}$$

where non conservative fields ($H\neq 0$) are obtained by fractional integration of order H over a conservative field. In the case of the wind shears Δv_λ at scale ratio λ (the amplitude of wind fluctuations: $\Delta v_\lambda = |\underline{v}(\underline{x}+\Delta x)-\underline{v}(\underline{x})|$; $|\Delta x| = L/\lambda$ in the isotropic case), this fractional integration ($H=1/3$) intervenes on the $1/3^{rd}$ power of the conservative energy flux ε_λ, this can be expressed *à la Kolmogorov* (Kolmogorov, 1941, 1962)[7]:

$$\Delta v_\lambda \approx (\varepsilon_\lambda)^{1/3}\lambda^{-H_v} \tag{10}$$

H_h can be estimated by the help of power spectrum, whose absolute slope β_h is related to H_h by[8]:

$$\beta_h = 2H_h + 1 - K(2/3)\ ;\ E_h(k)\approx k^{-\beta_h} \tag{11}$$

where the -K(2/3) term is the multifractal fractal intermittency correction. Using the values of C_1, α below, we obtain -K(2/3)≈0.08. Therefore, by power law filtering the wind velocity time series in Fourier space, we may obtain (Schmitt et al., 1992a) estimates of conservative turbulent energy flux densities (the scalar multifractal models of turbulence discussed by Schertzer and Lovejoy (1987), were explicitly based on this type of relationship between fields and fluxes). Using these estimates of the turbulent energy flux density, universal multifractal exponents C_1 and α can be estimated with the help of the double trace moment technique (DTM) (Lavallée, 1991; Lavallée et al., 1992, 1993). Indeed, we may first consider the normalized η powers of the field ε, $\varepsilon^{(\eta)}{}_\lambda$ defined in the following way:

$$\varepsilon_\lambda{}^{(\eta)} = (\varepsilon_\lambda)^\eta \Big/ \left\langle (\varepsilon_\lambda)^\eta \right\rangle \tag{12}$$

Obviously, $\varepsilon^{(\eta)}{}_\lambda$ will have a moment scaling function $K(q,\eta)$:

$$\left\langle \left(\varepsilon^{(\eta)}{}_\lambda\right)^q \right\rangle \approx \lambda^{K(q,\eta)} \qquad K(q,\eta)=K(q\eta)-qK(\eta) \tag{13}$$

In the same way that we estimate the statistical moments $<\varepsilon_\lambda{}^q>$ by (simple) trace moments (Schertzer and Lovejoy, 1987) by combining spatial and statistical averages, we use their natural extension – the double trace moments – to estimate $<\varepsilon_\lambda{}^{(\eta)q}>$. More precisely, we are degrading the scale resolution Λ of the observations (the ratio of the outer or largest scale of interest to the smallest scale of measurement) by "dressing" (averaging) the η^{th} power of ε_Λ over larger and larger scales, i.e. over smaller and smaller scale ratio $\lambda\leq\Lambda$. We then study the scaling behavior of the various q^{th} trace moments at decreasing values of the scale ratios λ. As showed by Schertzer and Lovejoy (1993), this corresponds to analysing the scaling behaviour of the dressed counterpart of the "bare" η^{th} power observed ε_Λ at the scale ratio Λ defined as:

$$\varepsilon^{(\eta)}{}_{\lambda,\Lambda} = \varepsilon^{(\eta)}{}_\lambda \left\langle (\varepsilon_\Lambda)^\eta \right\rangle \tag{14}$$

which is simply proportional to $\varepsilon_\lambda{}^{(\eta)}$ and therefore has the same scaling behaviour. As a consequence, until a critical moment order $q_D(\eta)$ discussed below, the DTM indeed will be ruled by the scaling exponent $K(q,\eta)$ of Eq. (13). The real advantage of the DTM technique becomes apparent when it is applied to universal multifractals since $K(q,\eta)$ has a particularly simple dependence on η :

$$K(q,\eta)=\eta^\alpha K(q) \tag{15}$$

3 Self Organized Criticality and coherent structures

Many scaling phenomena display not only structures at all scales, but also at all intensities, i.e. for a fixed scale there is no characteristic intensity – at least for intensities greater than a critical intensity discussed below. This is contrary to for instance (fractional) brownian motion (which has gaussian probabilities). This absence of a characteristic intensity is expressed by an algebraic fall-off of the

[7] The subscript h refers to the scaling of the horizontal shears of the horizontal wind.

[8] Because the spectrum is the Fourier transform of a 2^{nd} order moment of v, $2/3^{rd}$ moment of ε.

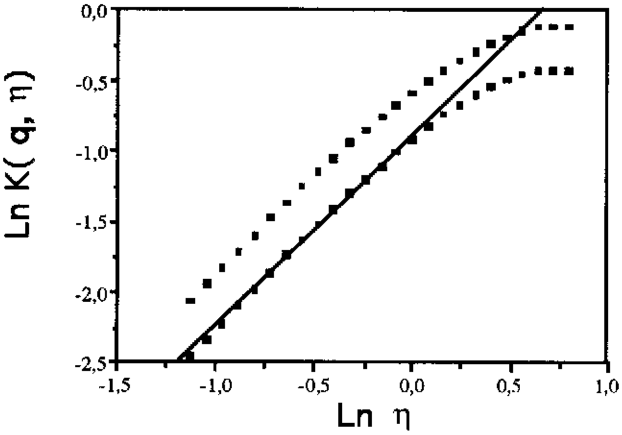

Fig. 4-a. A plot of Log$(K(q,\eta))$ vs. Log(η) for q=1.5 and q=2 (bottom to top) is shown with $K(q,\eta)$ estimated from the slopes of the straight lines of Fig.3. The value of α is then estimated as the slope of Log$(K(q,\eta))$ vs. Log(η), C_1 from the intercept with the vertical axis. We display the straight line with corresponding equation Log$(K(q,\eta))$=-0.91+1.35Log(η).

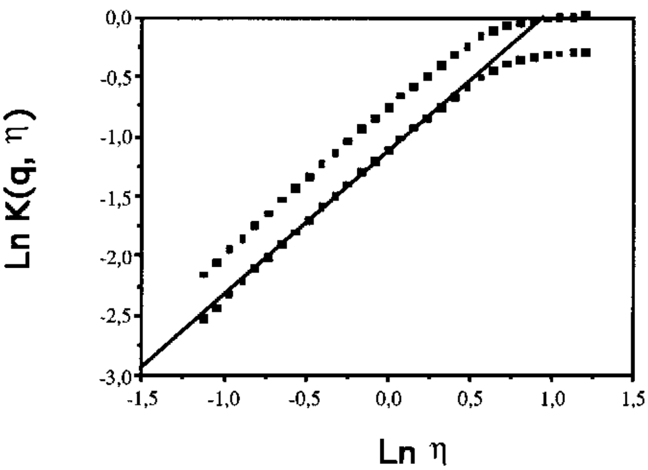

Fig. 4-b. Curves of Log$(K(q,\eta))$ vs. Log(η) for q=1.5 and q=2 (bottom to top). The value of α_φ is estimated as the slope of Log$(K(q,\eta))$ vs. Log(η); C_1 from the intercept with the vertical axis. We display the straight line with corresponding equation Log$(K(q,\eta))$=-0.94+1.23Log(η).

probability distribution (which itself is often called "hyperbolic" or "fat tailed"):

$$\Pr(\varepsilon_\lambda > x) \approx x^{-q_D} \qquad x >> 1 \qquad (16)$$

the critical order q_D depends on the dimension D of the space-time integration and is the critical order of divergence of moments. This can be easily checked, we have equivalently to Eq. (16):

$$<\varepsilon_\lambda{}^q> = \infty \qquad q \geq q_D \qquad (17)$$

However, a critical singularity γ_D corresponds to q_D=$K'(q_D)$ and λ^{γ_D} is a lower bound on ε_λ which corresponds to the algebraic regime of the probability distributions. Therefore γ_D objectively discriminates the extreme behaviour of the field ε from the mean events, and the former are sensitive to the dimension of integration D whereas the latter are not (for universal multifractals they depend only on the exponents H, C_1, α). It is worthwhile emphasizing the nontrivial "hard behaviour" resulting from this divergence of moments. It entails the breakdown of the law of large numbers so that standard statistical estimators diverge and give spurious scaling estimates (Schertzer and Lovejoy, 1983). A single contribution can be of the same order as the sum of all the others. Furthermore due to the existence of rare singularities present in the process but almost surely absent in individual realizations, there is a loss of ergodicity which can be precisely quantified (Schertzer and Lovejoy, 1992).

Recently, scaling coupled with algebraic probability distributions has been considered as the defining features of self-organized critical (SOC) phenomena (Bak et al., 1987, 1988). However the classical origin of SOC is both deterministic and with vanishing input (vanishing flux of particles) and therefore could not apply to our problem since turbulence is maintained by a non zero flux of turbulent energy. Indeed this is one of the fundamental difficulties in directly linking turbulence to SOC (as speculated by Bak and Paczuski (1993)). Nevertheless, an alternative stochastic route to SOC with non zero flux has been more recently discussed in a series of papers (Schertzer and Lovejoy, 1992, 1993; Schertzer et al., 1993). Indeed the significance of this extreme multifractal behaviour (and the consequent necessity of using the general canonical rather than geometric or microcanonical multifractals) has been constantly emphasized (Schertzer and Lovejoy, 1983, 1985, 1987, 1989; Lovejoy and Schertzer, 1991) although the original term: "hyperbolic intermittency" has been dropped in favour of the more popular "Self-Organized Criticality".

Without relying on any specific model, one can consider a rather generic statistical mechanism for open dissipative nonequilibrium systems: the analogue of a *non-zero transition temperature* associated with a first order multifractal phase transition. The analogy (e.g. Tel, 1988; Schuster, 1988) between multifractal exponents and thermodynamic variables can be made using the following correspondences[9] (Schertzer and Lovejoy, 1991): (γ, $c(\gamma)$) description is the analogue of (energy, entropy), whereas (q, $K(q)$) description is the analogue of (inverse of temperature, thermodynamic potential), the scale ratio is the analogue of the correlation length. Indeed, the first order multifractal transition corresponds to the fact that for a finite q_D and corresponding γ_D, the effective scale ratio

[9] There are slight variations between authors over the exact analogies which are used.

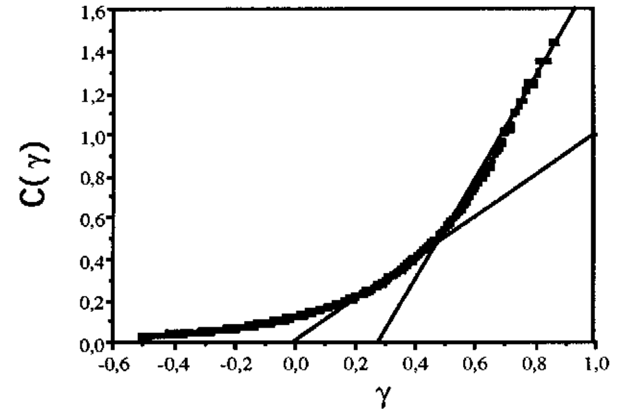

Fig. 5-a. The codimension function $c(\gamma)$ estimated by single scale PDMS on the whole data set (30 samples) at maximum resolution ($\Lambda=2^{10}$). One may note the tangency to the first bisectrix at the point (C_1,C_1); C_1=0.3±0.05 and the linear asymptote ($\gamma\geq\gamma_D$) with the slope q_D= 2.4±0.05.

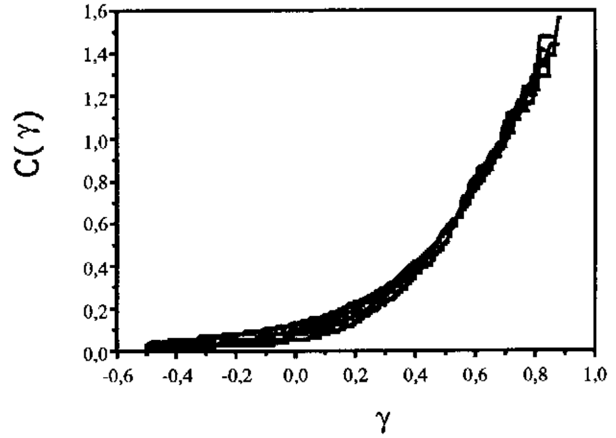

Fig. 5-b. Superposition of PDMS estimates of codimension function $c(\gamma)$ at ratio scales $\lambda_i = \Lambda / 2^i$, $\Lambda=2^{10}$ and i=0, 1, ..., 6 of (dressed) $\varepsilon_\Lambda / \lambda_i$ obtained by averaging ε_Λ by factor λ_i.

will diverge in analogy with the correlation length for thermodynamic phase transitions. Indeed, the scale of observation becomes irrelevant since the D - dimensional integration becomes unable to smooth singularities $\gamma\geq\gamma_D$, i.e. the small scale activity is dominant. Only the scale of homogeneity of the phenomena remains relevant and its corresponding ratio diverges for fully developed cascades.

We therefore have a clear framework in order to study the coherent or ordered tropical structures as (stochastic) self organized critical structures. Indeed, we first may define structures by the order of the singularity of their flux (scale by scale and intensity by intensity), i.e. filtering out the rest of the field having flux singularities smaller than a given order of singularity. Self organized critical structures are then those having avalanche-like fluxes, i.e. corresponding to singularities higher than the critical γ_D. We will estimate this critical singularity and the corresponding analogue of the critical temperature in Sect.6.

4 Data sets

We analyzed aircraft data sets on thermodynamic and wind fluctuation characteristics of three-dimensional convection in the tropical atmospheric boundary layer. Experimental data are obtained using the aircraft-laboratory IL-18D "Cyclone" during three Soviet-Vietnamese flying expeditions over the South China Sea in 1988, 1989 (Mikhailova et al, 1991) and 1990 equipped with special devices capable of measuring all the thermodynamic parameters as well as the component of the wind in the (horizontal) flight direction (Babrikin, 1981). Measurements were usually performed during the period from July to October on levels increasing from 50*m* up to 5*km* heights, along 20-40 *km* distances, every 0.125*s* (i.e. the frequency was ω_0= 8 *Hz* and corresponding spatial distance $\Delta x\approx 12\,m$ for a speed of ≈100 *m/s*) in the horizontal for each level across the largest clouds bands. During these expeditions, cyclones in various stages of their life history were studied.

For our preliminary study we selected one day per year corresponding to rather different meteorological situations. The first data set was taken during flight of 05/09/1988, in the central part of South China Sea where ordered cloud bands were observed. The synoptic situation in this region was determined by a continental monsoon depression and the influence of the Pacific Ocean subtropical anticyclone. This anticyclone came through the Philippines, reached the South China Sea and preserved this region from tropical disturbances (which took place only in central part of the Pacific Ocean from where they tended to travel northward). The flights trails were normal to the ordered cloud bands, whose base was at a height of ≈450*m* and top at ≈800*m*. The average length of the flights was 40*km* with speed ≈100*m/s* on the 11 vertical levels from 90 to 5000 *m*, the experiment lasted 2*hr*, 13 *min*.

On the contrary, on 20/10/1989 measurements were performed closer to cyclone Elsy, which was in a stage of growth. Cyclone Elsy was in the Eastern part of the South China Sea centred at 17°10' *N*, 117°20' *E*. The flights were over a region roughly 700 *km* from the center (15° *N*, 110° *E*), i.e. on the periphery of the tropical cyclone. The region with ordered cloud bands was chosen for study. The base of the cloud layer was at 750*m*, the top level at the height of 1270*m*. The measurements were carried out on 8 vertical levels: 50, 200, 300, 400, 500, 600, 750 and 1270 *m*. The average length of the flights was 20 *km* with speed 118*m/s*, the experiment lasted 1*hr*,10 *min*.

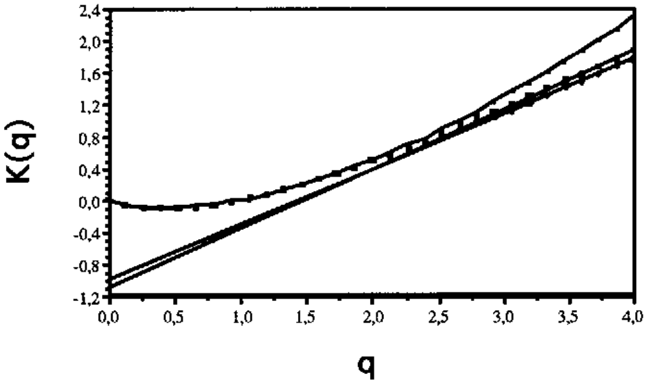

Fig. 6. The empirical K(q) function for the 10 series of the first experiment and for the combined 30 series of the three experiments (bottom to top) with theoretical bare curve corresponds to α =1.35 and C_1=0.32 (solid line). As expected, the results are independent of the sample size for $q<q_D$ = 2.4. The variation of the asymptotic slope ($\Delta\gamma_{d,s}$) is very close to that predicted theoretically (see Sect. 6).

The third day studied was 16/09/1990, the flight path was around typhoon Ed's center and it lasted 5*hr*, 38 *min* at an altitude of 3000 *m*. During the flight the cyclone was in the stage of strong tropical storm, with center coordinates of 18°50' *N*, 119° *E*. The pressure in the center of the cyclone was $p_{\min} = 970$, the velocity was $v_{\max} \geq 35 m/s$. For our study we chose the straight part of the flight which was as close as 7 *km* to the center of the typhoon. In Part II, we will study vertical soundings made in the same area and period during the years 1989 and 1990.

For each experiment, we studied 10 samples each of length 2^{10} at a fixed level. Study of individual samples shows that the height of the level does not seem to be relevant in the determination of the universal multifractal exponents (it simply changes the overall amplitude of the fluctuations), therefore in order to obtain more robust statistics we pooled the data from all the samples corresponding to different levels.

5 Empirical determination of universal exponents

The spectra of wind velocity and temperature fluctuations (Figs. 1 - a,b) were first computed in order to estimate the exponent H. This figure displays the spectra averaged over the 3 data sets taken roughly at one year interval (each contains 10 samples) and also 3 individual spectra obtained by averaging over 10 samples each. One may note the rather small dispersion around the average slope close to the Kolmogorov-Obukhov and Corrsin-Obukhov value of 5/3 (Kolmogorov, 1941; Obukhov, 1941, 1949; Corrsin, 1951): β_h = 1.68 ±0.05 and β_T = 1.70 ±0.05 over the frequency range $\omega_0/20$– $\omega_0/20480$ (ω_0= 8 *Hz*). Using the aircraft speed we converted the time series into a spatial series and then, following the development of Sect. 2, the velocity amplitude signal is then passed through a filter that weighs its Fourier components by $\omega^{1/3}$. This removes the $\lambda^{-1/3}$ scaling of the velocity (see Eq. (10)) yielding the conservative quantity $\varepsilon^{1/3}$. Fig. 2 - a,b confirm the very strong intermittency of the estimate of the energy flux densities: most of the time the values are lower than 1 but occasionally there are very high values. The normalisations $<\varepsilon>=1$ and $<\varphi>=1$ have been performed over the 30 realizations. *Mutadis mutandis*, the same technique is applied for the temperature (Schmitt et al., 1992b):

$$\Delta T_\lambda \approx \varphi_\lambda{}^{1/3}\lambda^{-H}; \quad \varphi_\lambda = \varepsilon_\lambda{}^{-1/2}\chi_\lambda{}^{3/2} \tag{18}$$

where φ is the flux density; χ is the temperature variance, which is conserved in the case of passive advection.

We may then proceed to estimate the C_1 and α values of the flux densities ε, φ. We first check that the corresponding DTM are indeed scaling for different orders of moments q and η (Fig. 3). $K(q,\eta)$, displayed in Fig.4, is then estimated by the slopes of the Log of the trace moments vs. Log(λ).

The exponent α is then estimated as the slope of Log$(K(q,\eta))$ vs. Log(η) (Fig. 4) and C_1 is estimated with the help of $K(q,1)=K(q)$ which is the intercept with the vertical axis (Logη =0). For the horizontal shears of velocity field we obtain: $\alpha = \alpha_h \approx 1.35 \pm 0.07$, $C_1 \approx 0.3 \pm 0.05$, $H_h \approx 0.33 \pm 0.03$ and for the temperature field: $\alpha_\varphi = \alpha_T \approx 1.25 \pm 0.06$, $C_{1,\chi} \approx 0.14 \pm 0.05$, $H_T \approx 0.33 \pm 0.03$. These values remain close to those obtained in mid-latitude boundary layers or wind tunnel experiments in time rather than in horizontal space (see Schmitt et al. (1992a, 1993) and Table 1, Part II).

6 Empirical analysis of multifractal transitions

Because the Lévy index α is greater than 1, we are in the case of unconditional hard turbulence: no matter what the dimension of the averaging space is, high enough order statistical moments will diverge leading to "hard" turbulence (Schertzer et Lovejoy, 1992). Therefore - at least for large enough sample sizes as discussed in Sect. 3 - we expect first order multifractal phase transitions, for singularities $\gamma>\gamma_D$ and corresponding moments of order $q>q_D$ (the analogue of the inverse of the critical temperature). For the different powers η we will have the same phenomenology with corresponding critical $\gamma_D(\eta)$ and $q_D(\eta)$. These phase transitions explain the departure from the (bare) theoretical $K(q,\eta)$ from the straight line behaviour for Log$(K(q,\eta))$ vs. Log(η) (Figs. 4 - a,b), for large η ($\eta \geq \eta_D(q) \equiv q_D{}^{-1}(\eta)$).

We thus studied the probability distribution of ε_{10}, ε_9, ε_8, ε_7, ε_6, i.e. with $\Lambda = 2^{10}$ (observation scale ratio) and $\lambda = 2^{10}$ - 2^6 respectively. Figs. 5 - a,b shows the corresponding estimates of $c(\gamma)$ obtained by:

$$c(\gamma) \approx - \text{Log} (\Pr(\varepsilon_\lambda > \lambda^\gamma)) / \text{Log}(\lambda) \tag{19}$$

The slope of the asymptote ($\gamma \geq \gamma_D$) of the resulting curves gives us q_D =2.4 ± 0.05 in close agreement (Table 1, Part II) with estimates of mid-latitude boundary layers or wind tunnel experiments. With the estimates of α and C_1 from the previous section we obtain for the critical singularity of the transition to the self-organized critical behaviour: γ_D=0.7 ± 0.05. Because of Eq. (10), the corresponding transition for the velocity field occurs for $q_{D,h} = 3q_D \approx 7 \pm 1$ and $\gamma_{D,h} = \gamma_D / 3 - H_h \approx -0.1 \pm 0.02$.

Figure 6 displays the theoretical bare moment scaling function $K(q)$ and the observed dressed $K_d(q)$ corresponding to a number of samples N_s of respectively 10 (a single expedition) and 30 (the full three expeditions). One may note that in agreement with the theory of multifractal phase transitions (Schertzer and Lovejoy 1992, 1993; Schertzer et al., 1993) the asymptotic linear behaviour of the dressed $K_d(q)$ has a steeper slope $\gamma_{d,s}$, contrary to a finite $D_\infty = d - \gamma_{d,s}$ as often hypothesised (e.g. Bershadskii and Tsinober, 1992; Bershadskii et al., 1993). More precisely its variation $\Delta\gamma_{d,s}$ follows:

$$\Delta\gamma_{d,s} = \Delta D_s / q_D \qquad (20)$$

where ΔD_s is the difference of the sampling dimension (D_s=Log(N_s)/Log(λ)) for the different sample sizes. Indeed, we have ΔD_s = Log(30/10)/Log(2^{10}) ≈ 0.16 and according to the estimate of q_D given above, we obtain $\Delta\gamma_{d,s}$ ≈ 0.066± 0.0015, in agreement with the variation of the slope estimated by linear regression: $\Delta\gamma_{d,s}$ ≈ 0.06.

Finally, we may note that the dimension of integration (the "dressing dimension") leading to this phase transition, is the implicit solution of:

$$K(q_D) = (q_D - 1)D \qquad (21)$$

using the estimates of α and C_1 one obtains: $D \approx 0.51 \pm 0.1$.

7 Dynamics beyond multifractal statistics

The remarkable constancy of the universal multifractal exponents (H, C_1, α) obtained in tropical conditions compared with those of Schmitt et al. (1992a, 1993) suggests that they should be related to some fundamental structures of Navier-Stokes type equations, more or less independently of different boundary conditions and forcing. Therefore, one may suspect that these exponents might be recoverable in simplified Navier-Stokes like equations retaining just some of the fundamental aspects determining these exponents.

Indeed, Chigirinskaya et al. (1994a) reports very comparable estimates of (H, C_1, α) using a dynamical model of intermittency which is based on the Lie structure of the Navier-Stokes equations. We briefly summarize pertinent aspects below.

Obukhov (1973), Dolzhansky et al. (1974), following Arnold (1966) considered the similarities between Lie structures of hydrodynamic equations (e.g. the vorticity equation) and Euler's equations of the gyroscope. Obukhov proposed studying a hierarchical model of cascade of triplets (equivalent to gyroscopes). Gloukovsky (1975) pointed out that there should be a single most energetic path along which most of the energy flows. This observation lead subsequent workers to reduce the cascade of triplets to the "one path model". In fact this one path model may be obtained by some other direct phenomenological considerations (Gledzer, 1980) and is a predecessor of the "shell-models", obtained by averaging the flux energy over wave-vectors corresponding to octaves in Fourier space. These models are only able to study the flow of energy through different scales (wave numbers) and loses the important property of having an increasing number of spatial degrees of freedom as the resolution increases (i.e. with increasing Reynolds number). Despite this fundamental deficiency, shell-models became extremely popular (see, e.g. Gledzer et al., 1981), unfortunately the original full model was forgotten.

On the contrary, due to the fundamental role played by the spatial degrees of freedom, Chigirinskaya et al. (1994b) argued that the full model is indispensable to investigating intermittency. It is further argued that this model and refined versions of it can be derived by partial truncations of the direct interactions of Navier-Stokes equations in Fourier space, whereas "one path" models require some other steps involving oversimplifications. Indeed, the hierarchical structure of the cascade creating λ^d structures at resolution λ is broken in favour of a fixed number of eddies (N) at each scale (N=1 in the case of the derivation of the one-path models). The same criticisms were made about the model developed by Grossmann and Lohse (1993), which is rather similar to the one-path model and which – not too surprisingly – might generate vanishing intermittency corrections to the scaling of Kolmogorov (1941) with increasing Reynolds number.

For several tens of large eddy turn-over times and $Re \approx 10^5$, DTM analysis of simulations of the full model yields the Kolmogorov value H = 1/3 (due to the scaling structure of the model), $C_1 \approx 0.35 \pm 0.05$, $\alpha \approx 1.5 \pm 0.05$. It is even more remarkable that this model generates nearly the same critical order (q_D) for the first order phase transition to self-organized criticality : $q_D \approx 2.2 \pm 0.06$. This opens up new perspectives on the non classical SOC pointed out in the present paper.

Finally, due to the relative simplicity of these hierarchical dynamical turbulent cascade models, one may speculate, at least in the framework of these models, that the universal exponents (C_1, α) and q_D may be analytically computable.

8 Conclusion

A basic scientific goal in the study of the tropical atmosphere is the understanding of the generation of extreme events such as cyclones. We used the universal multifractal model to study the scale invariant horizontal variability of atmospheric cyclone velocity and temperature data. We focus this (preliminary) study on three very different stages of coherent structures and cyclone development. We showed that – in spite of the presence of fluctuations – the three universal multifractal exponents H, C_1 and α, directly obtained with the help of spectral analysis and the double trace moment technique, have a rather remarkable constancy. Furthermore, since their values are close to those obtained in very different situations (mid-latitude boundary layers or wind tunnel experiments) we may first conclude that these exponents are indeed universal for turbulence describing very general properties of turbulence and this validates the Unified Scaling model of atmospheric dynamics which unifies both weak and intense events as well as those at different scales.

On the other hand, the underlying dynamical multifractal processes undergo a first order phase transition, which explains the appearance of self-organized critical structures. Contrary to the usual deterministic models of self-organized criticality these arise from stochastic dynamics. We therefore propose to identify (scale by scale) the different types of structures by the order of singularities of their associated fluxes. In particular the critical singularity at which the phase transition occurs defines the self-organized critical structures. The dynamics of the structures – unlike the weaker ones – are dominated by the small scale interactions. The apparent constancy of γ_D values suggests that they are new universal exponents. In addition, the fact that the γ_D values for the horizontal, vertical and time are (to within experimental precision) the same (see Table 1, Part II: $\gamma_D \approx -0.10 \pm 0.02$) may be significant. This opens up an original way of understanding not only the generation of cyclones and other tropical structures, but more generally of coherent structures. These universal exponents also should be indispensable for modeling of critical phenomena such as "hot spots" of dispersion of chemical or radioactive pollutants (Salvadori et al., 1994).

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