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Numerical computation of stress relaxation of isotropic magnetorheological elastomer using fractional viscoelastic models

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Abstract

The paper presents numerical research on stress relaxation of isotropic magnetorheological elastomer (MRE) using fractional derivative viscoelastic models. The isotropic MRE has been fabricated by filling micro-sized carbonyl iron particles into silicone rubber. Effects of applied constant strain and magnetic field on short-term stress relaxation of isotropic MRE were carefully investigated by single- and multi-step relaxation shear tests using double-lap shear specimens. Three fractional viscoelastic models, namely, fractional Maxwell, fractional Kelvin–Voigt, and fractional Zener models were used to describe the stress relaxation response of isotropic MRE. Results showed that the fractional Kelvin-Voigt and Zener models were fitted well to the experimental data of isotropic MRE in both single- and multi-step stress relaxations. The calculated stress with long-term predictions for the isotropic MRE is in very good agreement with the measured one. Therefore, fractional Kelvin-Voigt and Zener models are applicable to predict the long-term stress relaxation of isotropic MRE.

1. Introduction

Magnetorheological elastomers (MREs) are regarded as smart composite materials because their mechanical and rheological properties can be controlled rapidly and reversibly by the application of an external magnetic field [1]. MREs have been prepared by filling micro-sized ferromagnetic particles into non-magnetic polymer matrices. The elastic and damping properties of MREs can be controlled in nearly real-time in response to changing external excitations [3]. With their controllable properties, MREs have been used in a broad range of engineering applications [5]. The viscoelastic properties of MREs have been extensively studied by experimental and analytical methods. The research on the stress relaxation of MREs has been carried out to determine their time-dependent rheological properties [7].

The stress relaxation behavior of polymer materials can be examined numerically using fractional viscoelastic models [4]. The fractional Maxwell, Kelvin–Voigt, and Zener models have been used to characterize viscoelastic behaviors of materials [2]. For this study, isotropic MRE was fabricated by dispersing micro-sized carbonyl iron particles (CIPs) into silicone rubber [6]. The stress relaxation behavior of isotropic MRE was examined by relaxation tests using double-lap shear specimens. Effects of constant strain level and magnetic field on short-term stress relaxation of isotropic MRE were investigated. The fractional Maxwell, Kelvin–Voigt, and Zener models were used to describe the stress relaxation response of the MRE.

2. Stress relaxation measurements

The stress relaxation of isotropic MRE was studied by single- and multi-step relaxation shear tests. The relaxation tests were conducted for double-lap shear samples with constant strains changing from 5 to 20% and magnetic flux density (MFD) from zero to 0.58 T using the Instron Electropuls testing system. The shear test, MRE double-lap specimen, and electromagnet system were depicted in our earlier paper [6]. In the single- and multi-step relaxation tests for isotropic MRE, a strain rate of 1.0/s was applied in loading and unloading paths. The strains of 5, 10, 15, and 20% were applied for the single-step relaxation test at room temperature. For the multi-step relaxation test, step strains from 0 to 100% with a 20% strain interval were applied in the loading path, and then applied strains were reduced to zero with a 20% strain interval in the unloading path. The stress relaxation was recorded for 1000 s in the single-step relaxation test and 600 s for each step of the multi-step relaxation test.

3. Fractional viscoelastic models and numerical simulation

3.1. Fractional derivative viscoelastic models

Fractional derivative viscoelastic models, namely, fractional Maxwell, Kelvin–Voigt, and Zener models were used to deal with the stress relaxation behavior of the MRE. These models were built by different combinations of elastic spring and fractional-order dashpot (Fig. 1).

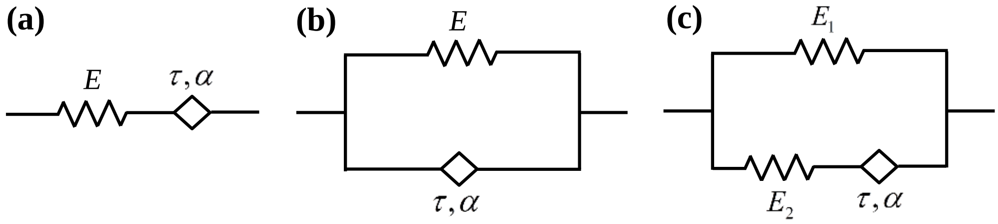

Fig. 1. Fractional derivative viscoelastic models: (a) fractional Maxwell model, (b) fractional Kelvin-Voigt model, and (c) fractional Zener model

The constitutive equations for the fractional Maxwell, Kelvin–Voigt, and Zener models [6] in the time domain are respectively written as follows:

$$\frac{d^\alpha \sigma}{dt^\alpha} + \frac{\sigma}{\tau^\alpha} = E\frac{d^\alpha \varepsilon}{dt^\alpha}, \tag{1}$$

$$\frac{d^\alpha \varepsilon}{dt^\alpha} + \frac{\varepsilon}{\tau^\alpha} = \frac{\sigma}{E\tau^\alpha}, \tag{2}$$

$$\frac{d^\alpha \sigma}{dt^\alpha} + \frac{\sigma}{\tau^\alpha} = \left(E_1 + E_2\right)\frac{d^\alpha \varepsilon}{dt^\alpha} + \frac{E_1\varepsilon}{\tau^\alpha}, \tag{3}$$

where E, E_1, and E_2 is the elastic moduli of the springs, and τ is the relaxation time of the fractional dashpot, and α is the fractional parameter with value varying between 0 and 1 [2,6].

The stress relaxation moduli of fractional Maxwell, Kelvin–Voigt, and Zener models based on a Mittag–Leffler function kernel respectively are expressed as follows [6]:

$$G(t) = EM_\alpha\left(-\left(\frac{t}{\tau}\right)^\alpha\right), \tag{4}$$

$$G(t) = E\left[1 + M_\alpha\left(-\left(\frac{t}{\tau}\right)^\alpha\right)\right], \tag{5}$$

$$G(t) = E_1 + E_2 M_\alpha\left(-\left(\frac{t}{\tau}\right)^\alpha\right). \tag{6}$$

3.2. Numerical simulation of MRE stress relaxation

The fractional Maxwell, Kelvin–Voigt, and Zener models were applied to study the stress relaxation behavior of isotropic MRE. Eqs. (4)–(6) were used to fit the relaxation modulus measured by single- and multi-step relaxation tests. The fitting was done using the nonlinear least-squares method in Matlab. The model parameters fitted with the experimental data of the single-step stress relaxation are given in Table 1. The fitted and measured relaxation modulus at different constant strain levels and MFDs are presented in Fig. 2. A comparison of calculated and measured shear stress with the long-term prediction for the isotropic MRE is depicted in Fig. 3. The fittings of the relaxation modulus to the experimental data under various MFDs for the isotropic MRE in the multi-step stress relaxation are presented in Fig. 4.

Table 1. Fitting parameters of models with experimental data of single-step relaxation tests for isotropic MRE

Strain level	Fractional Maxwell model			Fractional Kelvin-Voigt model			Fractional Zener model			
	E	α	τ	E	α	τ	E_1	E_2	α	τ
0.05	1.92	0.052	4.61E-09	0.375	0.348	3.62E-01	0.371	0.51	0.309	8.14E-02
0.10	1.79	0.051	6.73E-09	0.370	0.346	2.91E-01	0.365	0.56	0.300	3.76E-02
0.15	1.79	0.051	4.95E-09	0.365	0.354	3.10E-01	0.357	1.19	0.269	9.08E-04
0.20	1.73	0.050	6.01E-09	0.364	0.372	3.34E-01	0.355	2.64	0.270	3.85E-05

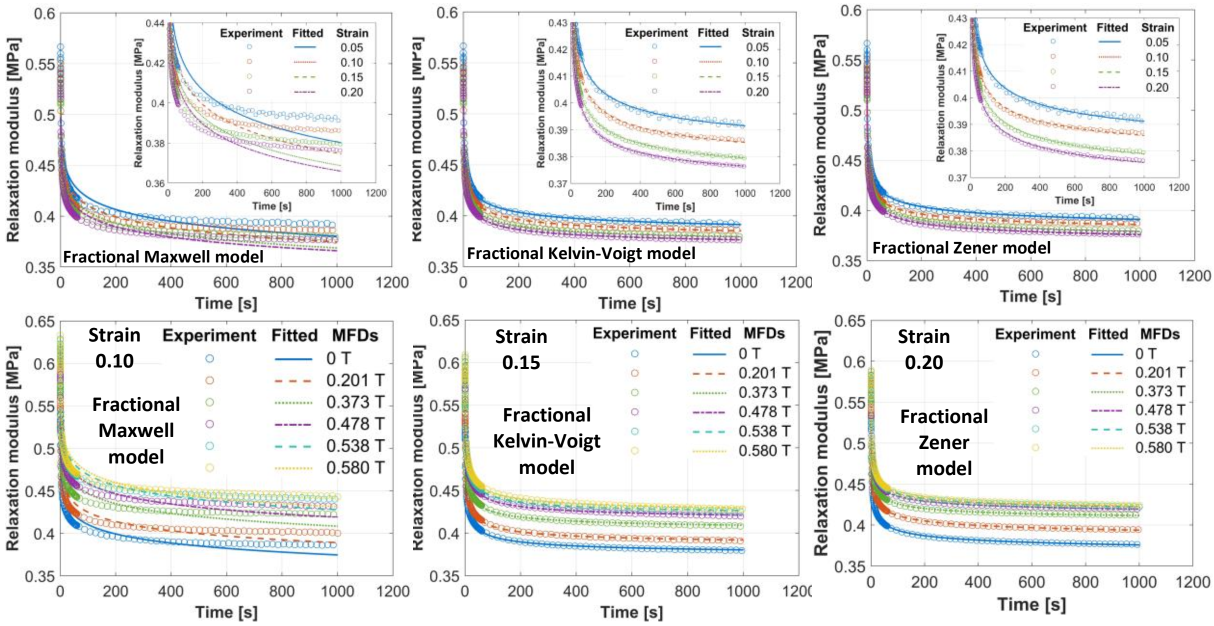

Fig. 2. Relaxation modulus (experimental and model fitted curves) of the isotropic MRE at different constant strain levels and various MFDs in the single-step stress relaxation

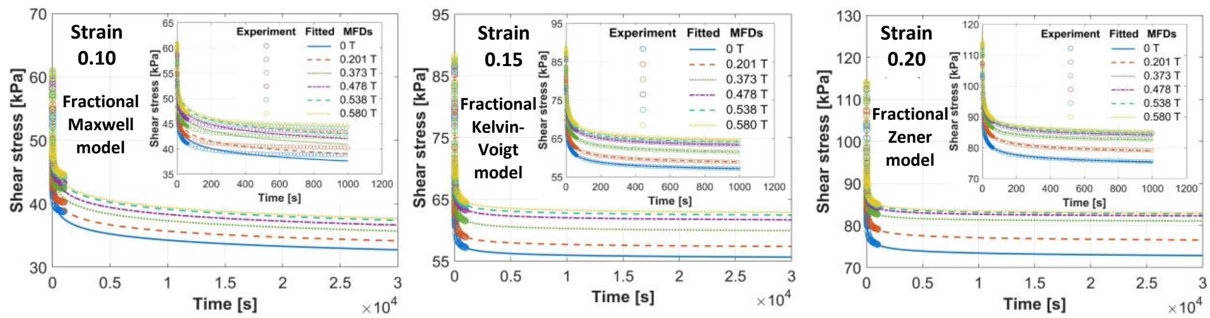

Fig. 3. Stress relaxation stress (experimental and model fitted curves) of the isotropic MRE under different MFDs in the single-step stress relaxation. The insert is a zoom of the first 1000 sec

The maximal differences between fitted curves of fractional Maxwell, Kelvin–Voigt, and Zener models and measured data for the relaxation modulus are 6.70%, 1.29%, and 1.02%, respectively. A very good agreement between experimental and fitted relaxation modulus of fractional Kelvin-Voigt and Zener models under different strain levels and MFDs is obtained.

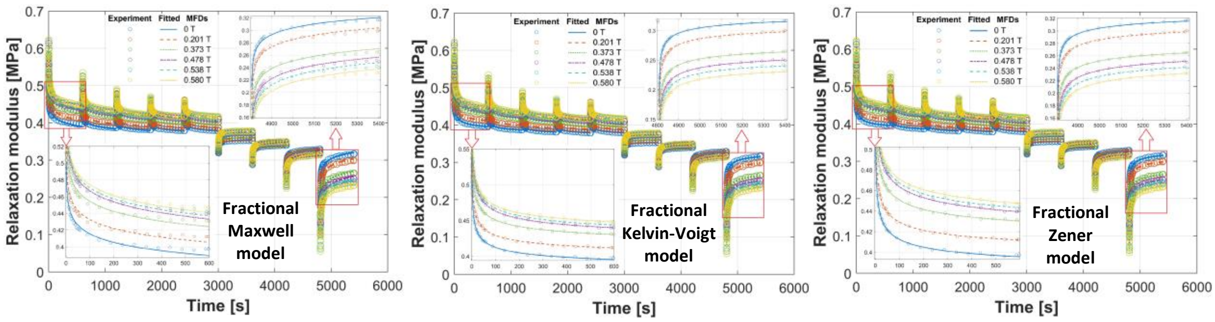

Fig. 4. Relaxation modulus (experimental and model fitted curves) of the isotropic MRE under different MFDs in the multi-step stress relaxation

4. Conclusions

The numerical studies on the shear stress relaxation of isotropic MRE made of silicone rubber and micro-sized CIPs using fractional derivative viscoelastic models have been conducted in this paper. The fractional Maxwell, Kelvin–Voigt, and Zener models were used to fit the experimental data of the isotropic MRE. The relaxation modulus and shear stress of the isotropic MRE were measured using single- and multi-step relaxation tests at different constant strains and magnetic fields. The simulated results showed that the fractional Kelvin-Voigt and Zener models were fitted well to the experimental data of the isotropic MRE under different strain constants and MFDs in both single- and multi-step stress relaxations. The maximal difference between measured and calculated relaxation modulus of the fractional Maxwell model is relatively large. The estimated stress with long-term predictions using fractional Kelvin-Voigt and Zener models for the isotropic MRE is in very good agreement with the measured one. In short, the fractional derivative Kelvin-Voigt and Zener models can be used to predict the long-term stress relaxation of the isotropic MRE.

Acknowledgments

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