The value of dynamic information device to the individual user and to the traffic: a probabilistic model with economic analysis

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THE VALUE OF DYNAMIC INFORMATION DEVICE TO THE INDIVIDUAL USER AND TO THE TRAFFIC: A PROBABILISTIC MODEL WITH ECONOMIC ANALYSIS

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SUMMARY ABSTRACT

Purported to analyze the value of a dynamic information device to an equipped network user, a model of route choice with two classes of users respectively equipped or not is worked out with assumptions about congestion, dynamic disturbances, user perception and behaviour.

ABSTRACT

The user of a road network chooses his route and/or departure time on the basis of his knowledge about the network state. Owing to the development of traffic measurement, data processing and telecommunication techniques, road information has kept improving both in scope, quality and availability. The disposal of sharp information enables the road user to make choices in a more opportunistic way, by adapting himself to the particular circumstances on the network during his trip.

The objective of the paper is to model the value of a road information device to a user, and more widely to the society. Put in other words, which profit may be derived from using a traffic information system based on individual devices, and how would this profit compare to the equipment cost? To gain insight into these issues, we work out a model of route choice with two classes of users respectively equipped and better informed with a dynamic component, or non-equipped thus less informed. We model the behaviour of each class and also their interaction under specific assumptions of congestion, disruption and perception: the effective cost taking into account the congestion and the random disturbances; the subjective cost as perceived by a user on the basis of the information which is available to him. We simulate the model sensitivity with respect to two parameters: the rate of equipment and the total volume of demand, which determines the level of congestion and the population in each class. The individual (resp. collective) profit of using information is addressed on the basis of specific indicators: the average cost per user class and the average cost to all network users.

Much emphasis is put on the physical features and behavioural effects of the random disturbances in traffic conditions, thus inspiring our assumptions of stochasticity. This sharpness of interpretation distinguishes our model from previous analytical approaches: the model of Maher and Hughes (1995) is shown to be irrelevant, as would be any "naive" model designed under commercial software for static traffic assignment.

KEYWORDS: *Traffic information. Dynamic information. Multi-class traffic assignment. Stochastic assignment*

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1. INTRODUCTION

1.1. Background: dynamic information and its profits to the individual user and to the traffic

Within a road transportation network, the traffic is made up of the moves of many vehicles. Subject to the network topology, the operating rules and the traffic conditions, every trip maker decides on which route to use and/or at which time to depart. His choices are determined by what he knows about the travel context and the traffic state, on the basis of his experience and the information sources that are available to him ([2]).

Traveller information services enjoy constant development in a variety of forms: these include network maps, fixed road signs and variable message signs (VMS), traffic radio etc. Moreover, traffic information services by the telephone or the internet, as well as guidance services based on onboard devices provide advanced information which is customized to the individual needs. Customized information enables the user to make his travel decision in a more opportunistic way: making en route or pre-trip adaptation to any particular circumstance ([3]).

Traffic information gives its user an individual profit by allowing him to optimize his choice. More precisely, in a disrupted situation dynamic information allows the user to react and to reduce his exposure to delay or uncertainty: the user can save time, maybe not with respect to the “target” traffic pattern that he expected, but with respect to the disrupted pattern that he would bear if he did not react. This paper deals with the travel time saving – we do not take into account any comfort or safety saving. Thus, the user can derive individual profit from dynamic information, all the more so with improved quality of information and prediction: this pushes him into getting an individual information device, thus making an investment that he will amortize over a series of uses.

In a more global vision, the collective profit may well exceed the total of individual profits. As the whole traffic behaviour is made up by the aggregation of individual behaviours, traffic information may induce aggregate effects of regulation and guidance that can reduce significantly the scope and the magnitude of a disturbance: this justifies the collective investment in traffic information equipment, from traffic sensors to information provision throughout the media.

1.2. Objective: a rigorous model for dynamic information and equipment

This paper studies the effective performance of a dynamic information system with respect to its diffusion rate across the trip-makers, subject to a given structure of congestion and disruptions. We consider both the performance to the individual user and the collective value of such an information and equipment system. Rather than trying to quantify the costs and profits of a particular system, our focus is on a crucial assumption which has yet been mis-modelled in the evaluation of traffic information systems: namely, the probabilistic structure of dynamic disruptions, and the influence of the rate of individual equipment onto the resilience of the flow to the disruptions.

[2] See e.g. the PIARC ITS manual (2000); Leurent (2004)

[3] see e.g. Nguyen (2006)

Precisely, our objective is to develop a model of disturbances, of individual equipment in information device and of the reaction of users to disruptions, which is explicit and realistic enough in these respects, yet as simple as possible without distorting the physic and behavioural features. The model is applied to a classroom network of two parallel links, with two classes of users respectively equipped or not, in order to simulate the response of travellers to disruptions and to evaluate the profit to either the individual traveller or the society – the profit being restricted to travel time savings.

1.3. Method

Our method here is of probabilistic modelling, aimed at economic analysis. Our model makes explicit assumptions about: (i) the network; (ii) the traffic conditions subject to the congestion phenomena; (iii) the statistical distribution of disruptions; (iv) the composition of traffic out of equipped and non-equipped users, every of whom chooses their route on the basis of their information. The information device enables the equipped user to know the disrupted times, while a non-equipped user perceives only the average time out of trip reiteration.

With this model, we simulate the effect of the demand volume on the congestion, and the influence of equipment rate on the resilience of flow to disruptions: arguably, a certain level of equipment may be sufficient to smooth the effects of disruptions.

The physical and economic issues are kept under control by way of a formal, analytical treatment which enables us to trace out the effects of each assumption.

1.4. Structure of paper

Section 2 sets up the modelling assumptions. These are brought together in Section 3 to make up a framework for equilibrium analysis: formulae are provided for arc flows by user class at equilibrium, and also for the utility of being equipped. Section 4 addresses the case of Gaussian disturbances: simple formulae are obtained that enable us to discuss the magnitude of the equipment utility and its variations with respect to the demand volume, the sensitivity to congestion and the equipment rate.

In Section 5 we relate our model to previous research work: the assignment model of Maher and Hughes (1995) targeted at the same issue is shown to be deceptive; correcting its interpretation of dynamic information yields a mixed deterministic-stochastic assignment model, which is a coarse version of ours. Lastly, Section 6 provides concluding comments together with some topics for further research.

2. MODELLING ASSUMPTIONS

Let us come to the modelling assumptions that pertain to, respectively: (i) the supply side of the network and its routes, with the local travel times, congestion effects and random disturbances; (ii) the demand side of the trip-makers whether or not equipped with a device for dynamic information; (iii) the interaction of supply and demand.

2.1 Supply side assumptions

Let us consider a transport network made up of arcs $a \in A$ the arc set, with endpoints $n \in N$ the node set. Our application here is restricted to a classroom case of two parallel arcs linking an origin node to a destination node, as in Figure 1: hence $a \in \{1,2\}$.

On each network arc a, the arc flow x_a induces an individual travel time of T_a which is subject to congestion effects on the basis of a travel time function $T_a = \tilde{t}_a(x_a)$, an increasing function of the arc flow. For instance let us take a linear affine function as follows, $\tilde{t}_a(x_a) = \alpha_a + \gamma_a x_a$ in which α_a denotes a free-flow travel time and γ_a the sensitivity of the individual time to the flow (see Figure 2). This assumption is consistent with the unqueued state of traffic, not with the queued state in which the flow is constrained by a flowing capacity.

Despite our flow-based model is basically a stationary model, we shall consider dynamic effects from period to period, i.e. interperiod variability to be modelled by a random variable. To that end, we assume that there is a set Ω of circumstances (or periods) ω, each of which has arc flow $x_a(\omega)$ and arc travel time

$$T_{a\omega} = \tilde{t}_a(x_{a\omega}) + \zeta_a(\omega).$$

The random variable $\zeta_a(\omega)$ models the eventual variation in travel time that may arise due to exogenous disturbances. For instance we take $\zeta_a \approx \mathrm{N}(0, \sigma_a^2)$ a Gaussian variable with null mean and variance of σ_a^2. The assumption of null mean is consistent with the no bias assumption on the travel time function, that is $\mathrm{E}_\omega[T_{a\omega} : x_{a\omega} = x] = \tilde{t}_a(x)$.

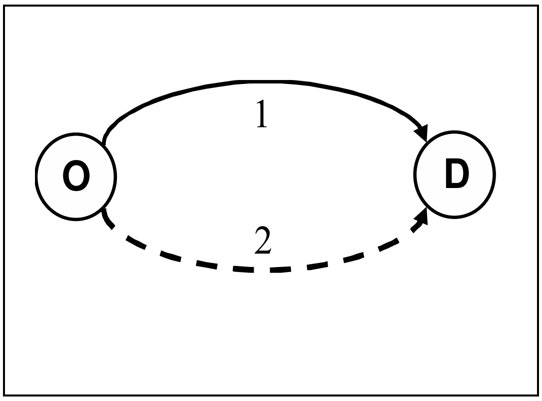

Fig. 1 The two parallel link network

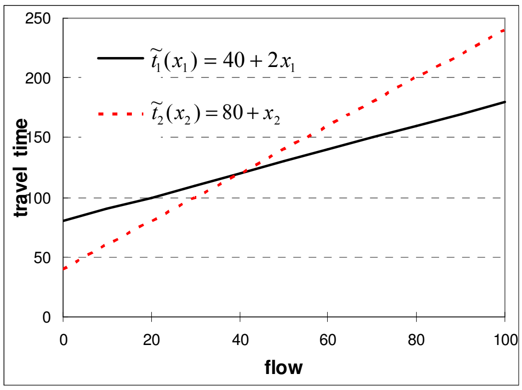

Fig. 2 Arc travel time function

2.2 Demand side assumptions

Let us analyze the network trips by origin-destination (OD) pair, e.g. from node O to node D in Figure 1. The trips are made on an OD pair by a population of network users who fall into one out of two classes: either class I of equipped users who have got a device to receive dynamic information, or class N of the non-equipped users that do not receive dynamic information. Every user is assumed to choose his network route from origin to destination under a rational behaviour of cost minimization, subject to his knowledge of the costs. An equipped user is assumed to derive perfect knowledge from dynamic information whatever the circumstance:

$$t_a^I(\omega) = t_a(\omega), \tag{1a}$$

while a non-equipped user is assumed to possess only coarse knowledge on the basis of the average travel time:

$$t_a^N = \mathrm{E}_\omega[t_a(\omega)]. \tag{1b}$$

Here the user cost is restricted to the travel time component – neglecting tolls, comfort and other quality criteria in order to focus on the disturbances which make our primary concern.

Let us denote by $Q = x_1 + x_2$ the total demand volume, assumed constant whatever the circumstance. Let $\beta = q^I / Q$ denote the equipment rate i.e. the ratio of the number of equipped users, q^I, to Q, and let $q^N = Q - q^I$ be the number of unequipped users.

By arc a, occurrence ω and user class $u \in \{I, N\}$, $x_a^u(\omega)$ denotes the flow on arc a: it holds that

$$x_a(\omega) = x_a^I(\omega) + x_a^N(\omega) \tag{2a}$$

$$x_1^I(\omega) + x_2^I(\omega) = q^I \tag{2b}$$

$$x_1^N(\omega) + x_2^N(\omega) = q^N \tag{2c}$$

2.3 The interaction of supply and demand

Let us chain the assumptions about supply and demand in the following statement:

- in any occurrence ω, every network user chooses his route: his travel along that route makes a piece of flow.
- Considering the population of trip-makers, their individual choices induce the arc flows throughout the network.
- The arc flows determine the arc travel times on the basis of the congestion functions.
- The travel times determine the individual route choice.

Thus there is a cyclical chaining in the interaction of supply and demand. Figure 3 makes the statement more precise by addressing each user class in a specific way: a class I user reacts to any particular occurrence ω by adapting his route choice to the dynamic context – thus leading to $x_a^I(\omega)$ flows that vary with ω hence in the short run, while a class N user makes his route choice only in the long run on the basis of the route performance averaged over the distribution Ω of cases ω, leading to $\bar{x}_a^N$ flows that do not vary with respect to ω.

Our model should not be mistaken into a two-class assignment model of a deterministic user class together with a probit user class: prior to the comparative analysis provided in Section 5, notice that in a probit model the arc time is $\tilde{t}_a(\bar{x}_a)+\zeta_{a\omega}$ i.e. the travel time function has an average flow $\bar{x}_a$ as argument, instead of an occurrence flow $x_a(\omega)$.

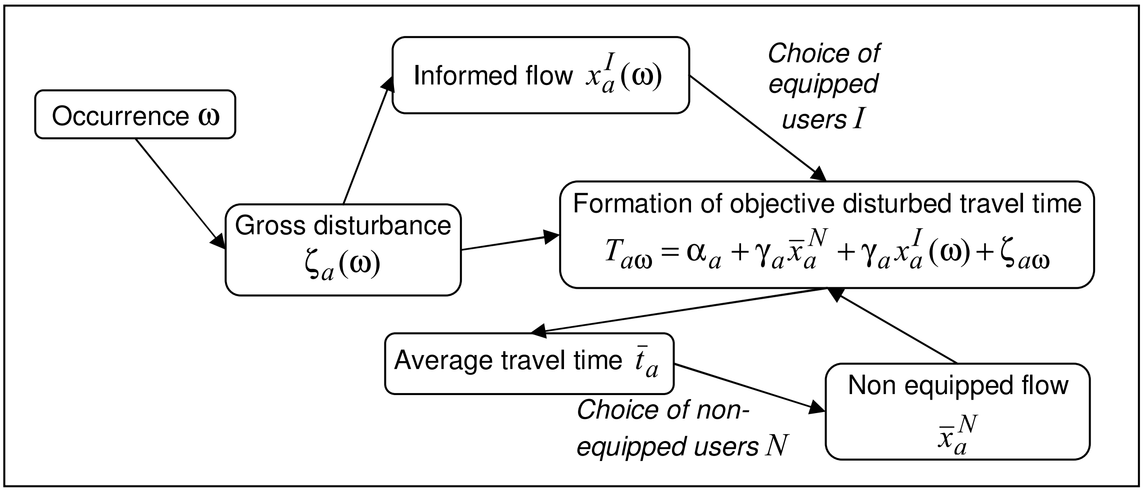

Fig. 3. Model composition by chaining the assumptions

3. EQUILIBRIUM ANALYSIS

Following the logical structure in Figure 3, we shall first assume a given assignment of class *N* and focus on the assignment of class I in any occurrence ω (Subsection 3.1); then we will average the informed flows over all cases (Subsection 3.2). Next the class *N* users will be assigned conditional on the class I average assignment (Subsection 3.3). Lastly we shall characterize the equilibrium as a fixed point problem (Subsection 3.4), and provide the indicators of utility to either the individual or the traffic (Subsection 3.5).

3.1 Occurrence assignment of equipped users

Let $\alpha_a^I=\alpha_a+\gamma_a\bar{x}_a^N$: then $T_{a\omega}=\alpha_a^I+\gamma_a x_a^I(\omega)+\zeta_{a\omega}$ is the case travel time of arc a under occurrence ω. If there were only one equipped user, he would choose the route of minimum $T_{a\omega}(0)=\alpha_a^I+\zeta_{a\omega}$. However the dynamic reassignment of informed users will tend to increase the travel time of that route due to its congestion function: this effect may yield partial compensation i.e. $T_{a\omega}$ remains less than $T_{b\omega}$, or total compensation i.e.

$$\alpha_a^I+\gamma_a x_a^I(\omega)+\zeta_{a\omega}=\alpha_b^I+\gamma_b x_b^I(\omega)+\zeta_{b\omega} \tag{3a}$$

In the latter case, since $x_a^I(\omega)+x_b^I(\omega)=q^I$ it holds that

$$x_a^I(\omega)=\frac{\gamma_b q^I+\alpha_b^I+\zeta_b-\alpha_a^I-\zeta_a}{\gamma_a+\gamma_b}, \tag{3b}$$

$$T_{a\omega}=\frac{\gamma_a\gamma_b q^I+\gamma_a\alpha_b^I+\gamma_b\alpha_a^I}{\gamma_a+\gamma_b}+\frac{\gamma_b\zeta_a+\gamma_a\zeta_b}{\gamma_a+\gamma_b}. \tag{3c}$$

In the former case, $x^I_{a\omega} = q^I$ and $x^I_{b\omega} = 0$, hence $T_{a\omega} = \alpha^I_a + \gamma_a q^I + \zeta_{a\omega}$ and $T_{b\omega} = \alpha^I_b + \zeta_{b\omega}$, with $T_{a\omega} \le T_{b\omega}$ hence

$$\zeta_{b\omega} - \zeta_{a\omega} \ge \alpha^I_a - \alpha^I_b + \gamma_a q^I . \quad (4)$$

3.2 Conditional and average assignment of equipped users

Let $\mathrm{B} \equiv \alpha^I_1 - \alpha^I_2 + \gamma_1 q^I$ and $\mathrm{A} \equiv \alpha^I_1 - \alpha^I_2 - \gamma_2 q^I$: conditional on $z = \zeta_2 - \zeta_1$ it holds that

- if $z > \mathrm{B}$ then $T_{1\omega}(q^I) \le T_{2\omega}(0)$ hence $x^I_1(\omega) = q^I$ and $x^I_2(\omega) = 0$, (5a)
- if $z < \mathrm{A}$ then $T_{1\omega}(0) \ge T_{2\omega}(q^I)$ hence $x^I_1(\omega) = 0$ and $x^I_2(\omega) = q^I$, (5b)
- if $z \in [\mathrm{A},\mathrm{B}]$ then $T_{1\omega} = T_{2\omega}$ at flows $x^I_1(\omega) = \dfrac{z - \mathrm{A}}{\gamma_1 + \gamma_2}$ and $x^I_2(\omega) = \dfrac{\mathrm{B} - z}{\gamma_1 + \gamma_2}$. (5c)

Denoting by F the distribution function of $z = \zeta_2 - \zeta_1$ over the set Ω of cases ω, and by $\widetilde{\mathrm{F}}$ the truncated moment function $\widetilde{\mathrm{F}}(x) = \int^x z\,\mathrm{dF}(z)$, by aggregation

$$\bar{x}^I_1 = 0.\int_{-\infty}^{\mathrm{A}} \mathrm{dF}(z) + \int_{\mathrm{A}}^{\mathrm{B}} \frac{z - \mathrm{A}}{\gamma_1 + \gamma_2} \mathrm{dF}(z) + q^I . \int_{\mathrm{B}}^{+\infty} \mathrm{dF}(z) = q^I - \bar{x}^I_2 \text{, in which} \quad (6a)$$

$$\bar{x}^I_2 = \frac{\mathrm{G(B)} - \mathrm{G(A)}}{\gamma_1 + \gamma_2} \text{ where } \mathrm{G}(x) \equiv x\,\mathrm{F}(x) - \widetilde{\mathrm{F}}(x) . \quad (6b)$$

3.3 Assignment of unequipped users

Let $\alpha^N_a \equiv \alpha_a + \gamma_a \bar{x}^I_a$ and compare α^N_1 with α^N_2 :

- if $\alpha^N_1 \ge \alpha^N_2 + \gamma_2 q^N$ then $\bar{x}^N_2 = q^N$ and $\bar{x}^N_1 = 0$. (7a)
- if $\alpha^N_1 + \gamma_1 q^N \le \alpha^N_2$ then $\bar{x}^N_1 = q^N$ and $\bar{x}^N_2 = 0$. (7b)
- if $-\gamma_1 q^N \le \alpha^N_2 - \alpha^N_1 \le \gamma_2 q^N$ then $\bar{x}^N_a = \dfrac{\alpha^N_b - \alpha^N_a + \gamma_b q^N}{\gamma_a + \gamma_b}$. (7c)

The last condition stems from $\bar{t}_a = \bar{t}_b = \theta$ with $\bar{t}_a = \alpha_a + \gamma_a \bar{x}^I_a + \gamma_a \bar{x}^N_a$, when the assignment of unequipped users to the routes of minimal cost subject to the congestion functions yields a traffic equilibrium with equilibrium time of θ.

3.4 Fixed-point characterization of supply-demand equilibrium

Linking together the previous formulae, we obtain that the unequipped flows $(\bar{x}^N_a)$ induce the reference informed travel times (α^I_a), which in turn determine the average equipped flows $(\bar{x}^I_a)$, which in turn determine the reference unequipped travel times (α^N_a), which in turn determine the unequipped flows $(\bar{x}^N_a)$. This cycle states that any variable set out of $(\bar{x}^N_a)$, $(\bar{x}^I_a)$, (α^I_a), (α^N_a)

solves a specific problem of fixed point. In the two-link case that kind of problem is easy to solve since it involves only two real unknowns; a relaxation algorithm would be appropriate, for instance a convex combination algorithm in the unequipped flows. Section 4 provides an analytical solution in the case of Gaussian disruptions.

3.5 Indicators of utility

To an unequipped user of class N, the average cost is as follows

$$\overline{C}^N = \bar{t}_1^N \frac{x_1^N}{q^N} + \bar{t}_2^N \frac{x_2^N}{q^N}. \tag{8a}$$

To an equipped user of class I, the average cost stems from the aggregation of all occurrence costs:

$$\overline{C}^I = \mathrm{E}_\omega[\min\{c_1(\omega), c_2(\omega)\}] = \int_z \min\{\bar{c}_{1/z}^I, \bar{c}_{2/z}^I\}\mathrm{dF(z)} \tag{8b}$$

in which $\bar{c}_{a/z}^I \equiv \mathrm{E}_\omega[\alpha_a^I + \zeta_a(\omega) + \gamma_a x_a^I(\omega) \mid (\zeta_2 - \zeta_1)(\omega) = z]$ denotes the average cost of link a to a class I user conditional on $\zeta_2 - \zeta_1 = z$.

In line with the comparison of $\bar{c}_{1/z}^I$ to $\bar{c}_{2/z}^I$ in Subsection 3.2, the minimal cost is provided by one or two arcs in each of the three following cases:

$\bar{c}_{2/z}^I = \alpha_2^I + \mathrm{E}[\zeta_2(\omega)/\Delta\zeta = z] + \gamma_2 q^I$ if $z \in]-\infty, \mathrm{A}]$ hence arc 2 is optimal, (9a)

$\bar{c}_{1/z}^I = \alpha_1^I + \mathrm{E}[\zeta_1(\omega)/\Delta\zeta = z] + \gamma_1 q^I$ if $z \in]\mathrm{B}, +\infty]$ hence arc 1 is optimal, (9b)

$\bar{c}_{1/z}^I = \alpha_1^I + \mathrm{E}[\zeta_1(\omega)/\Delta\zeta = z] + \gamma_1 \frac{z - \mathrm{A}}{\gamma_1 + \gamma_2}$ if $z \in]\mathrm{A}, \mathrm{B}]$ hence both arcs are optimal. (9c)

Bringing together the three cases, we get that

$$\overline{C}^I = \bar{t}_1 - \mathrm{A.F(A)} + \int_{-\infty}^{\mathrm{A}} \overline{\zeta}_{2/z}\, \mathrm{dF}(z) + \int_{\mathrm{A}}^{+\infty} \overline{\zeta}_{1/z}\, \mathrm{dF}(z) \tag{10}$$

To a network user, the utility of holding an information device amounts to

$$\Gamma^{NI} = \overline{C}^N - \overline{C}^I \tag{11}$$

To the whole traffic, the average cost at equipment rate β amounts to

$$\overline{C}(\beta) = \beta\overline{C}^I + (1-\beta)\overline{C}^N = \overline{C}^N - \beta\Gamma^{NI} \tag{12}$$

to be compared to the average cost at another equipment rate β', for instance $\beta' = 0$.

4. A GAUSSIAN MODEL

If the disruptions follow from a Gaussian distribution, then some specific statistical properties (recalled in Subsection 4.1) yield simple formulae for the average arc flows by user class and the average costs (Subsection 4.2). This enables us to study the sensitivity of both the individual and collective utility to the equipment rate (Subsection 4.3) and also to the total demand volume Q which sets the level of congestion (Subsection 4.4).

4.1 Assumptions and basic properties

In this Section it is assumed that $\zeta_a \approx \mathrm{N}(\mu_a, \sigma_a^2)$ and that the couple (ζ_a, ζ_b) makes a Gaussian vector, hence the difference $z \equiv \Delta\zeta = \zeta_2 - \zeta_1 \approx \mathrm{N}(\mu, \sigma^2)$ with $\mu = \mu_2 - \mu_1$ and $\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\,\mathrm{cov}(\zeta_a, \zeta_b)$ also has a Gaussian distribution. The variance is reduced to $\sigma^2 = \sigma_1^2 + \sigma_2^2$ if the local disruptions are independent.

The Gaussian variable z has:

- probability density function $\mathrm{f}(x) = \exp[-\frac{1}{2}(\frac{x-\mu}{\sigma})^2]/(\sigma\sqrt{2\pi})$ where $\phi(t)$ is the density function of a reduced Gaussian variable (with mean zero and variance one).

- cumulative distribution function $\mathrm{F}(x) = \Pr\{z \le x\} = \Phi(\frac{x-\mu}{\sigma})$ where $\Phi(t)$ $= \int_{-\infty}^{t} \phi(u)\mathrm{d}u$ is the distribution function of a reduced Gaussian variable.

- truncated moment function $\tilde{\mathrm{F}}(x) = \mu.\Phi(\frac{x-\mu}{\sigma}) - \sigma.\phi(\frac{x-\mu}{\sigma})$ which reduces to $\tilde{\mathrm{F}}(x) = -\,\sigma.\phi(x/\sigma)$ if $\mu = 0$.

This enables us to derive the mean of the minimum of two Gaussian variables:

$$\begin{aligned} \mathrm{E}[\min\{c_1(\omega), c_2(\omega)\}] &= \bar{c}_1 + \tilde{\mathrm{F}}(0) \\ &= \bar{c}_1.\Phi(\frac{\bar{c}_2 - \bar{c}_1}{\sigma}) + \bar{c}_2.\Phi(\frac{\bar{c}_1 - \bar{c}_2}{\sigma}) - \sigma.\phi(\frac{\bar{c}_1 - \bar{c}_2}{\sigma}) \end{aligned} \quad (13)$$

Lastly, let us define function $g(x) = \phi(x) + x\Phi(x)$ which is a positive, increasing function as depicted in Figure 4.

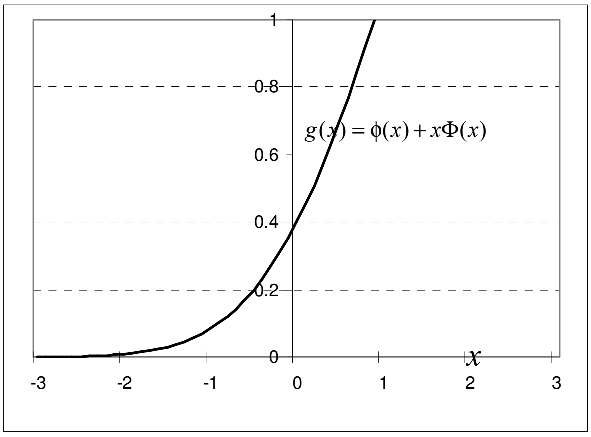

Fig. 4. The *g* function to measure the information gain

4.2 Formulae for equilibrium analysis

In the Gaussian model, it holds that $\mathrm{G}(x) = \sigma g(x/\sigma)$, and also that

$$\bar{x}_2^I = \frac{\sigma}{\gamma_1 + \gamma_2}[g(\frac{\mathrm{B}}{\sigma}) - g(\frac{\mathrm{A}}{\sigma})], \tag{14}$$

$$\overline{C}^I = \bar{t}_1 - \sigma.g(\frac{\mathrm{A}}{\sigma}). \tag{15}$$

Formula (15) stems from (10) and the fact that, in the Gaussian case, $\int_{-\infty}^{\mathrm{A}} \overline{\zeta}_{2/z}\, \mathrm{dF}(z) + \int_{\mathrm{A}}^{+\infty} \overline{\zeta}_{1/z}\, \mathrm{dF}(z) = \widetilde{\mathrm{F}}(\mathrm{A})$ (Leurent and Nguyen, 2008).

If the average travel times are equal, then

$$\bar{x}_2^I = \tfrac{1}{2} q^I = \bar{x}_1^I \tag{16}$$

$$\overline{C}^I = \bar{t}_1 - \sigma.g(-\frac{\gamma_1 + \gamma_2}{2\sigma} q^I) \tag{17}$$

$$\overline{C}^N = \bar{t}_1 = \frac{Q\gamma_1\gamma_2 + \alpha_1\gamma_2 + \alpha_2\gamma_1}{\gamma_1 + \gamma_2} \tag{18}$$

Under unequal average times such that $\bar{t}_1 \le \bar{t}_2$, then

$\overline{C}^I = \bar{t}_1 - \sigma.g(\frac{\mathrm{A}}{\sigma})$ where $\mathrm{A} = \alpha_1 + \gamma_1 x_1^N - \alpha_2 - \gamma_2 x_2^N - \gamma_2 q^I = \bar{t}_1 - \bar{t}_2 - (\gamma_1 + \gamma_2) x_1^I$.

In all cases with $\bar{t}_1 \le \bar{t}_2$, it holds that $\Gamma^{NI} = \sigma.g(\frac{\mathrm{A}}{\sigma})$. (19)

If $\bar{t}_1 = \bar{t}_2$ then $\Gamma^{NI} = \sigma.g[-(\gamma_1 + \gamma_2).q^I/(2\sigma)]$: the individual utility of being equipped increases with σ i.e. with the magnitude of the disruptions. It decreases with $q^I = \beta Q$ hence with demand volume Q and equipment rate β. It also decreases with $\gamma_1 + \gamma_2$ which measures the sensitivity of travel times to arc flows – in other words the sensitivity to congestion.

Under unequal average times $\bar{t}_1 \le \bar{t}_2$, then $x_1^I \ge q^I/2$ so the gain Γ^{NI} of being equipped is less than in the case of equal travel times.

4.3 Sensitivity of gain to equipment rate

At a given demand volume Q, the information gain Γ^{NI} decreases with respect to the equipment rate β when the average travel times are equal on the two arcs. This is because the more equipped users there are, the more compensation they can induce between the links, which is beneficial to the unequipped users. This equalization process applies at any $\Delta\zeta \in [\mathrm{A}, \mathrm{B}]$ with $\mathrm{B} - \mathrm{A} = q^I(\gamma_1 + \gamma_2) = (\gamma_1 + \gamma_2)\beta Q$: an increase in β makes that interval larger.

Let us define a condition of saturation at the 1-η confidence level, as the coverage of at least a proportion 1-η of the occurrences ω by the compensated range B-A: it is satisfied when

$$\beta \geq \frac{2u_{\eta/2}\sigma}{(\gamma_1+\gamma_2).Q} \equiv \tilde{\beta}_\eta, \quad (20)$$

in which $u_{\eta/2}$ denotes the fractile at probability level $1-\eta/2$ of a reduced Gaussian variable.

The saturation effect is confirmed by numerical application, assuming that:

- $\tilde{t}_1(x_1)=40+2x_1$ and $\sigma_1^2=0.3\tilde{t}_1$.
- $\tilde{t}_2(x_2)=80+x_2$ and $\sigma_2^2=0.3\tilde{t}_2$.

Figure 5a drawn at $Q=20$ hence unequal average travel times $\bar{t}_1<\bar{t}_2$ shows that the equipped average cost $\overline{C}^I$ is lower than the unequipped cost $\overline{C}^N$, with the gap that decreases with respect to β and is near zero from $\beta=30\%$. The collective gain is obvious since $\overline{C}(\beta=0.3)<\overline{C}(\beta=0)$.

Figure 5b drawn at $Q=130$ leading to equal average travel times depicts the same effects but in an amortized way: the saturation effect starts from $\beta=10\%$; the equipment gain is certain at $\beta=0$ but of reduced size. Furthermore, the average travel cost $\overline{C}$ is almost constant whatever the equipment rate: this is because congestion is heavy under that set of numerical assumptions.

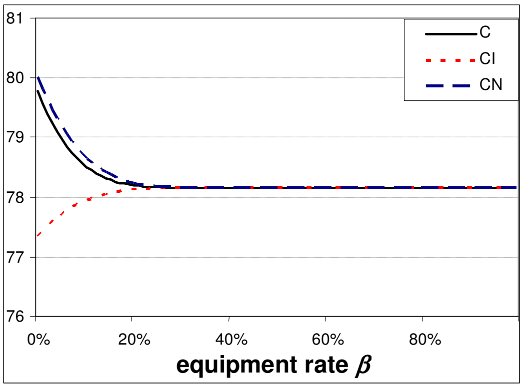

Fig.5a. Average travel times at *Q*=20

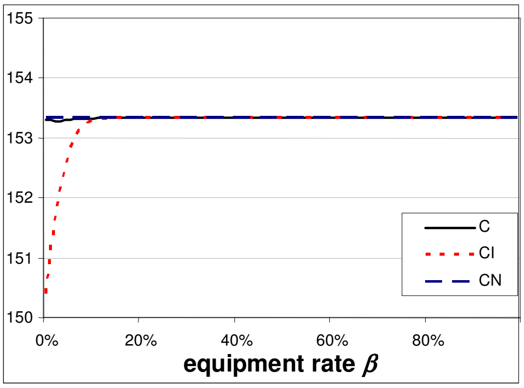

Fig.5b. Average travel times at *Q*=130

4.4 Sensitivity to demand volume

Let us now identify the domain of equal average travel times with respect to parameters Q and β: assuming that $\alpha_1 \leq \alpha_2$, it holds that if $Q<Q_0^* \equiv (\alpha_2-\alpha_1)/\gamma_1$ then whatever the value of β the average times are unequal with $\bar{t}_1 \leq \bar{t}_2$, since the assignment of all the demand volume to the link of minimal time would not make the other link competitive on the average. If $Q>Q_0^*$ then the assumption of equal average times would induce equal informed average flows, implying that the proportion π of unequipped users on arc 2 would satisfy

$$\pi = \frac{1}{2}+\frac{\alpha_1-\alpha_2+\frac{1}{2}(\gamma_1-\gamma_2)Q}{(\gamma_1+\gamma_2)(1-r)Q} \equiv \hat{\pi} \quad (21)$$

For $\hat{\pi}$ to belong to [0, 1] it must hold that β belongs to an interval that depends on γ_1 and γ_2; thus equal average travel times arise when

- if $\beta \le 2\min\{\gamma_1, \gamma_2\}/(\gamma_1 + \gamma_2)$ the equality is achieved if $Q \ge \frac{\alpha_2 - \alpha_1}{\gamma_1 - \beta(\gamma_1+\gamma_2)/2} \equiv Q^*_\beta$; under that threshold all of q^N is assigned to the link of minimal cost.
- $2\gamma_2 \le \beta(\gamma_1 + \gamma_2) \le 2\gamma_1$ equality holds if $\frac{\alpha_2 - \alpha_1}{\gamma_1 - \frac{1}{2}(\gamma_1+\gamma_2)\beta} \le Q \le \frac{\alpha_2 - \alpha_1}{\frac{1}{2}(\gamma_1+\gamma_2)\beta - \gamma_2}$.

  Under the lower bound q^N is assigned only to the link of minimum free cost, whereas above the upper bound it is assigned all to the other link.
- If $\beta > 2\gamma_1/(\gamma_1 + \gamma_2)$ then equality cannot be achieved.

All three cases occur if $\gamma_2 \le \gamma_1$, whereas if $\gamma_2 > \gamma_1$ then the intermediary case cannot occur.

Figure 6 depicts the domain of equal average times, where the unequipped users are split between the two routes.

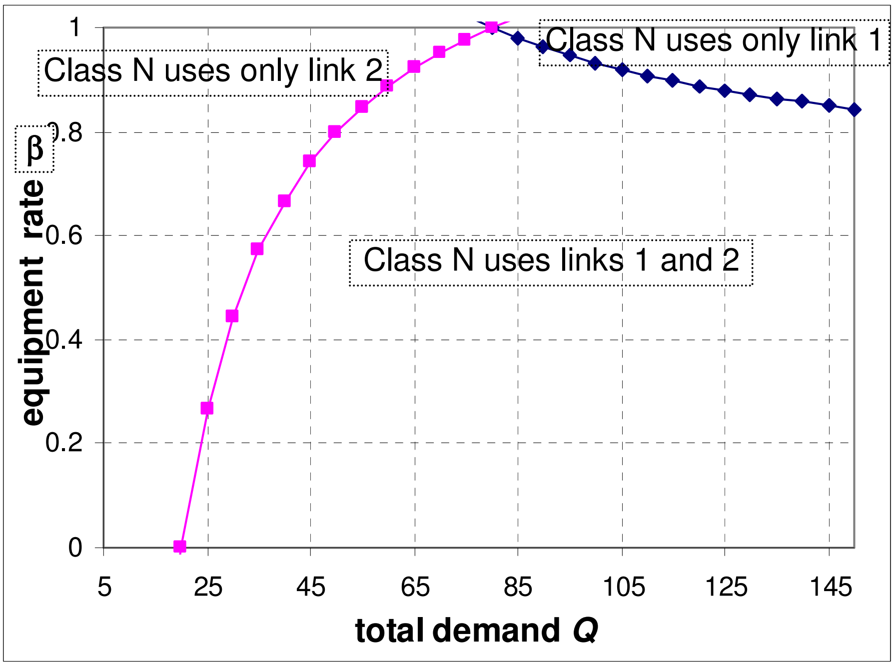

Fig. 6. Domains of equal and unequal average travel times subject to β and *Q*

5. DISCUSSION

Our model of dynamic information and route choice may be related to two streams of research. On one hand, Noland and Small (1995) and Leurent (2001, 2004) modelled the choice of departure time under dynamic information and random disturbances conditional on the instant of departure, in a way very close in spirit to our model here, though neither route choice nor equilibrium effects were considered. On the other hand, in the field of network assignment some stochastic models of route choice were applied to dynamic information by interpreting the random component of travel cost as dynamic uncertainty due to eventual disturbances: this is the line of attack taken by Maher and Hughes (1995).

The remainder of this Section is aimed at investigating the risks associated with such an attempt: first, we recall the assumptions of Maher and Hughes and we apply their model to our classroom case, yielding paradoxical results (Subsection 5.1). Second, we correct their interpretation of dynamic information and we solve the corrected model as a mixed assignment model with two classes of users acting deterministically or stochastically: that straightforward application of the classical toolbox for assignment also yields flawed results, because no account is taken of the real-time adaptation of informed users in the formation of flows and congestion (Subsection 5.2).

5.1 On the Maher-Hughes model

Maher and Hughes (1995) modelled the network effects of the diffusion of an information device across a population of trip-makers as an assignment problem with two classes of users: the informed users assumed to perceive the *average* cost taken as the single objective cost; and the unequipped users assumed to perceive a disturbed cost, the disturbance being subjective and coming from inaccurate knowledge and perception. These authors modelled a problem of network assignment at supply-demand equilibrium with two user classes respectively the informed users with deterministic behaviour denoted by *D*, and the unequipped users with stochastic behaviour denoted by *S*. Here we use the *D* and *S* notations to clearly distinguish their set of assumptions from ours.

The application of their model to the classroom case of two parallel links yields the following set of conditions:

- $x_a = x_a^D + x_a^S$ and $\bar{t}_a = \tilde{t}_a(x_a)$.
- to the unequipped class *S*, the perceived time on arc a is $t_a = \bar{t}_a + \zeta_{a\omega}$ and the probability to choose route 2 is $p_2^S \equiv \Pr\{t_2 \leq t_1 | S\}$ $= \Pr\{\zeta_2 - \zeta_1 \leq \bar{t}_1 - \bar{t}_2\}$ $= \Phi(\frac{\bar{t}_1 - \bar{t}_2}{\sigma})$ hence $x_2^S = p_2^S q^S$ and $x_1^S = (1 - p_2^S) q^S$.
- the informed class *D* is assigned to the quickest path on the basis of the average travel times, hence $q^D = x_1^D + x_2^D$, $x_a^D \geq 0$ and $x_a^D(\bar{t}_a - \min\{\bar{t}_1, \bar{t}_2\}) = 0$.

This set of conditions is reduced to only one equation in x_2^S (Leurent and Nguyen, 2008). It is easy to solve; numerical application with same parameters as in Section 4 yields Figures 7a-b for Q equal to 20 or 130, respectively.

At $Q = 20$, the average times are not equal and the diffusion of the equipment makes the mean travel costs increase, which is counterintuitive and erroneous.

At $Q = 130$ the average times are equal as in our model; the diffusion of the equipment has effects that make sense but of very little size, even much smaller than in our model.

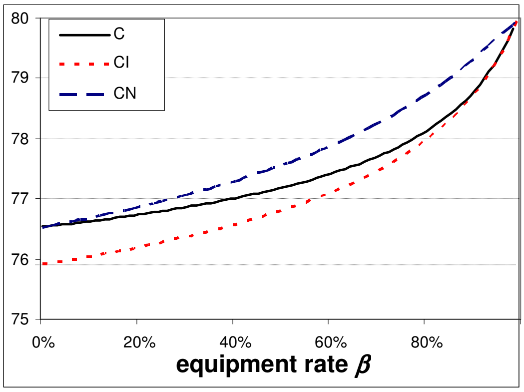

Fig.7a. Average travel times at Q=20 by the Maher-Hughes model

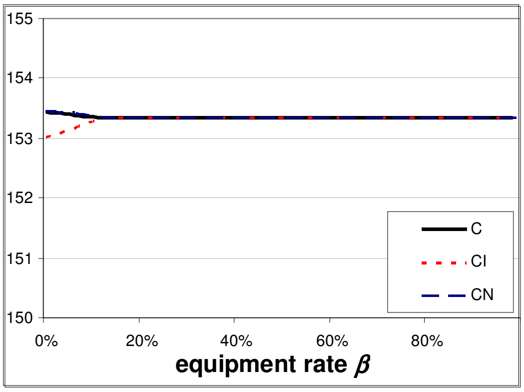

Fig.7b. Average travel times at Q=130 by the Maher-Hughes model

5.2 A corrected yet coarse model

The interpretation of dynamic information given by Maher and Hughes is faulty in that, according to them, dynamic information would enable its user to perceive a deterministic undisrupted time, while the unequipped user would perceive a disrupted time! In fact, dynamic information pertains to the real-time traffic state and disruptions to which the trip-makers are faced (Setra, 1996): every equipped user is informed about the actual conditions particular to their trip. Thus a correction to the Maher-Hughes model is obviously to interchange the *S* and *D* indices in the conditions associated to their user classes: this yields a corrected assignment model with two user classes, namely the equipped users denoted by *S* that are informed about the disruptions and take them into account in their route choice – in accordance to a stochastic assignment model, and the unequipped users denoted by *D* that know only the average travel time whatever the peculiarity of each occurrence – in accordance to a deterministic assignment model.

This obvious application of the classical assignment tools yields a coarse version of our model, in that the travel time at occurrence ω is modelled as $\tilde{t}_a(\bar{x}_a)+\zeta_{a\omega}$ with travel time function applied to average flow $\bar{x}_a$, while the dynamic adaptation of informed users should lead to travel time $\tilde{t}_a(x_{a\omega})+\zeta_{a\omega}$.

Numerical application with same parameters as previously yields Figures 8a-b for Q equal to 20 or 130, respectively. In both cases the coarse model yields results which mostly make sense, though being much far-fetched compared to our refined model. A troubling outcome, though, is that the unequipped users do not derive any indirect benefit at $Q=130$ from the equipment of the majority of trip-makers.

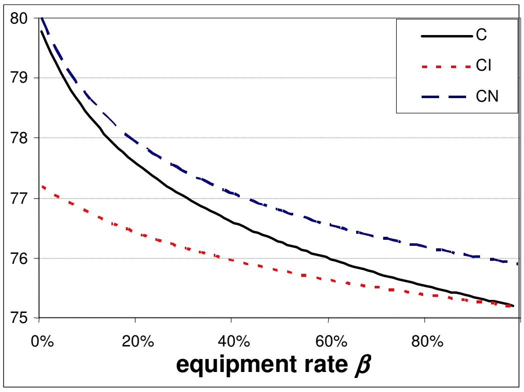

Fig.8a. Average travel times at Q=20 by the coarse model

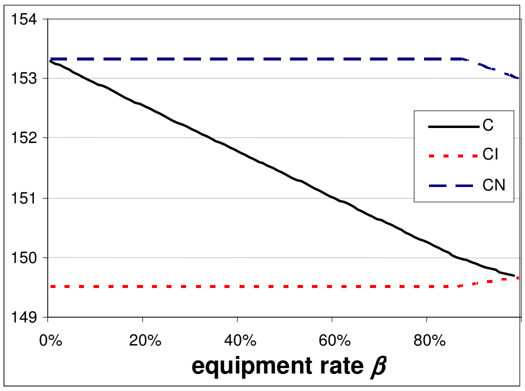

Fig.8b. Average travel times at Q=130 by the coarse model

6. CONCLUSION

The paper brought about a refined model of dynamic information and its effects on traffic subject to the structure of congestion and disturbances. A complete set of assumptions was provided, making a framework convenient to analyze the supply-demand equilibrium. Moreover, by taking Gaussian disturbances we obtained analytical formulae that allowed for both abstract sensitivity analysis and easy numerical application, of which all the results make sense.

The analytical approach was also useful to demonstrate the advantages of our model over the previous attempt by Maher and Hughes (1995) as well as a corrected attempt that would use classical assignment models (deterministic coupled with stochastic-probit).

The Gaussian assumption was instrumental to fulfil the analytical treatment; as some of the effects are of small magnitude, they would be hardly detectable in a numerical approach based e.g. on dynamic micro-simulation.

Further research is required about the following topics:

- to improve the model of disturbances, by identifying the probability of a noticeable delay and by relating it as well as the distribution of delay to the flow intensity.
- To consider nonlinear congestion, on the basis of nonlinear travel time functions.
- To design a network assignment model with several user classes distinguished by their disposal and usage of information. To that end, an intuitive approach is to transfer our refined model, by distinguishing several class assignment problems and coupling them through the definition of class perceived travel costs.

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