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DISTRIBUTION OF LOADS TO GIRDERS IN SLAB-AND-GIRDER BRIDGES: THEORETICAL ANALYSES AND THEIR RELATION TO FIELD TESTS

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Distribution of Loads to Girders in Slab-and-Girder Bridges: Theoretical Analyses and Their Relation to Field Tests

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SYNOPSIS

THE object of this paper is to present a picture, based on theoretical analyses, of the manner in which loads on slab-and-girder highway bridges are distributed to the supporting girders. The discussion is restricted to simple-span, right bridges consisting of a slab of constant thickness supported on five girders, spaced equidistantly, and having equal flexural stiffnesses but no torsional stiffness.

The numerous variables influencing the behavior of this type of structure are listed, and the effects of the following are considered in detail: (1) the relative stiffness of girders and slab, H; (2) the ratio of girder spacing to span of bridge, b/a; (3) the number and arrangement of the loads on the bridge; and (4) the effect of diaphragms, their stiffness, number, and location on the structure. Particular emphasis is placed on the relative magnitudes of the maximum moments in interior and exterior girders.

It is shown that when the slab is fairly flexible in comparison to the girders, the maximum moment in an interior girder will usually be larger than the corresponding maximum moment in an exterior girder, if the loads in each case are arranged so as to produce maximum effects in the girder considered. This condition of maximum moment in an interior girder is found to be typical for reinforced-concrete T-beam brides having no diaphragms. However, if the transverse stiffness of the structure is fairly large in comparison with the stiffness of the girders, then the maximum moment in the exterior girder will generally be the greatest. Such conditions will usually be encountered for typical I-beam bridges and for concrete-girder bridges having adequate transverse diaphragms.

For those arrangements of loads which are critical in design, an increase in relative stiffness of the slab and the girders (decrease in H) will generally reduce the maximum moment in the interior girders. For exterior girders, a corresponding decrease in H may either increase or decrease the maximum moment.

A change in the ratio b/a affects the distribution of loads to the girders in much the same way as a change in H, since both of these quantities are measures of the relative stiffness of the slab and girders. Thus, a decrease in b/a improves the load distribution in about the same manner as a decrease in H.

The behavior of a slab-and-girder bridge under a single wheel load is found to be different from the behavior of the same structure under multiple wheel loads. Unless the performance of the structure and the effects of the numerous variables affecting its behavior are investigated for all possible conditions of loading to which the bridge may be subjected, certain aspects of the action of the structure may be overlooked.

The addition of diaphragms in slab-and-girder bridges supplements the capacity of the roadway slab to distribute loads to the supporting girders. The manner and extent to which diaphragms modify the distribution of load depends on such factors as the stiffness of the diaphragm, the number employed, their longitudinal location, and also on all those parameters influencing the behavior of slab-and-girder bridges without diaphragms. Diaphragms will almost always reduce the maximum moment in an interior girder but they will usually increase the maximum moment in an exterior girder. These effects, which are a function of the many variables referred to above, may be beneficial or harmful depending on whether the moment controlling design occurs in an interior or exterior girder. The conditions under which diaphragms will increase or decrease the controlling design moments are described in the body of the report.

The simplifying assumptions involved in the analyses and the limitations imposed by these assumptions are discussed in detail, and consideration is given to the probable effects of the neglected variables.

The relationship between thoretical analyses and the behavior of actual structures is also considered, and the paper concludes with a discussion of the manner in which theoretical analyses can best be used in planning field tests on slab-and-girder bridges, and in interpreting the results obtained.

The slab-and-girder highway bridge is a structure for which neither theoretical analyses nor laboratory or field tests alone can be expected to yield a complete and trustworthy description of its action. Only by considering together the results of both analyses and tests can we hope to understand a type of structure whose behavior depends on so many variables.

● THE slab-and-girder highway bridge as considered in this paper consists essentially of a reinforced-concrete slab supported by a number of parallel steel or concrete girders extending in the direction of traffic. The wide use of such bridges, together with an increasing awareness of their inherent complexity, has emphasized the need for a better understanding of the way in which they function. Of particular interest has been the manner in which wheel loads from vehicles are distributed to the supporting beams.

Studies of slab-and-girder bridges were begun in 1936 at the University of Illinois in cooperation with the Illinois Division of Highways and the U. S. Bureau of Public Roads. The results of these studies have been presented in several publications (*1, 2, 3, 4, 5, 6*). Included in this program were extensive theoretical analyses in which the effects of several important variables were studied, and a rather complete picture of the behavior of such structures was obtained. In addition, numerous laboratory tests on scale-model I-beam bridges were made to determine the accuracy of certain assumptions in the analyses and to study the behavior of the bridges at ultimate loads.

The object of this paper is to present a picture, based on theoretical analyses, of the manner in which loads are distributed to the girders in slab-and-girder bridges. The scope of these analyses, and thus also the scope of this paper, has been limited to the behavior of the bridge under working loads. This is an important limitation, since both the ultimate strength of the structure and its behavior at loads producing yielding are factors which should be given great weight in the selection of design methods.

A second purpose of this paper is to consider the relationship between the results obtained from theoretical analyses and those obtained from tests of actual structures. This is a two-way relationship; neither approach to the problem can be considered alone and each can benefit from a study of the other. The theoretical approach cannot be accepted with entire confidence until its predictions have been verified by comparison with the behavior of real bridges. On the other hand, no field test can give the full picture, since the number of variables that can be considered is necessarily quite limited. Only by considering the two together can we obtain a complete and generally applicable solution to the problem.

Analyses of Slab-and-Girder Bridges

Variables

The slab-and-girder bridge is a complex structure, and an exact analysis can be made only by relatively complex means. In essence, this structure consists of a slab continuous in one direction over a series of flexible girders. The presence of the slab as a major element of the structure is, of course, one complicating factor. However, the complexity of the structure is further increased by the continuity of the slab and by the deflections of the supporting girders.

The problem of studing analytically the slab-and-girder bridge is further complicated by the larger number of variables that may conceivably affect its behavior. The more significant variables may be listed as follows:

Variables relating to the geometry of the structure: (1) Whether girders are simply supported, continuous, or cantilevered; (2) whether the bridge is right or skewed; (3) the number of girders; (4) the span length of the girders; (5) the spacing of the girders, and whether or not it is uniform; and (6) the number and locations of diaphragms.

Variables relating to the stiffness of the bridge elements: (7) The flexural stiffness of the girders (this

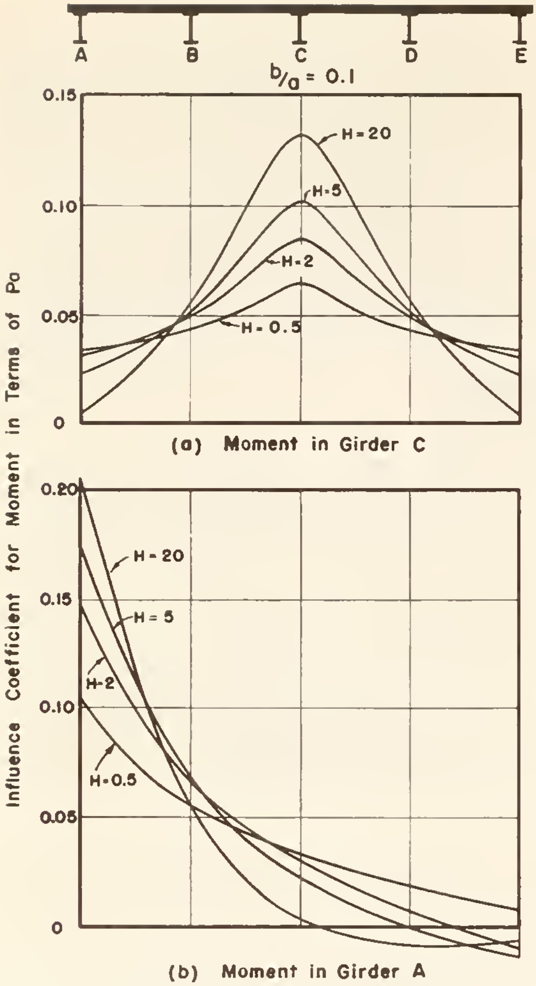


Figure 2. Influence lines for moment in girders at midspan for load moving transversely across bridge at midspan.

The effects of these variables are discussed in the following sections of this paper.

Effect of Relative Stiffness H

The relative stiffness of the girders and the slab, as expressed by the ratio H, is one of the most important variables affecting the load distribution to the girders. The effectiveness of the slab in distributing loads will increase as its stiffness increases. Moreover, a slab of a given stiffness will be more effective when the potential relative deflections of the girders are large; that is, when the girder stiffness is small. Thus the distribution of load will generally become greater as the value of H decreases, whether the change is due to a decrease in girder stiffness or to an increase in slab stiffness.

The effects of variations in H can best be illustrated by means of examples taken from the analyses of five-girder bridges. Typical influence lines for moment at midspan of the girders are shown in Figure 2 for a structure with $b/a=0.1$ and for various values of H.

Figure 2(a) shows the influence lines for the center girder. For small values of H, corresponding to a relatively stiff slab, the curves are rather flat, indicating that the slab is quite effective in distributing the moment among the girders. As the value of H increases, the moment becomes more and more concentrated in the loaded girder, and for $H=$infinity, would theoretically be carried entirely by that girder.

Figure 2(b) shows influence lines for an edge girder. Although the shape of these curves is quite different, owing to the location of the girder, the trends with changes in H are similar to those for Figure 2(a).

It may also be seen from the influence lines in Figure 2 that the effects of a concentrated load on the more distant girders is relatively small. Thus, the addition of more girders on either side in Figure 2(a), or on the side opposite the load in Figure 2(b), would obviously have little effect on the character or magnitudes of the influence lines. Although this

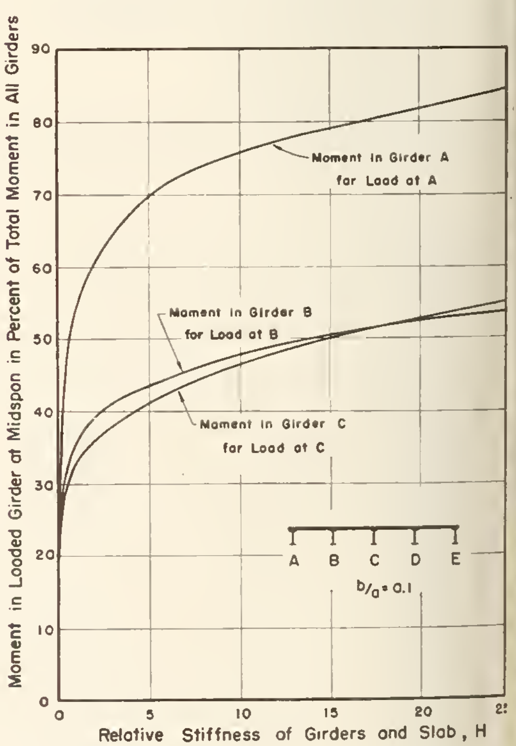


Figure 3. Variation of moment in loaded girder as a function of H for concentrated load at midspan.

conclusion does not apply without reservation for all possible values of H and b/a, it is reasonably valid for practically all structures having the proportions considered in the analyses. This observation then provides justification for extending the results of the analyses to bridges having more than five girders, and possibly also in some cases to bridges having only four girders.

The effects of changes in the relative stiffness H may be shown more directly by the curves of Figure 3 for a bridge having $b/a=0.1$. Relative moments at midspan of girders A, B, and C for a single, concentrated load directly over the girder at midspan are shown as a function of H. The moments are given in percent of the total moment in all the girders; that is, neglecting the portion of the static moment carried directly by the slab.[1]

The close agreement between the curves for Girders B and C suggests that the behavior of all interior girders is much the same regardless of their location. It also provides further justification for extending the results of these analyses to bridges having more than five girders or to bridges having only four girders.

It can also be seen from Figure 3 that relatively much less distribution of moment occurs for a concentrated load over an edge beam than for a load over an interior beam. When a load is applied over Beam A, the slab, no matter how stiff, cannot transfer the load effectively to the more distant girders, which are relatively farther away for this loading than for a load over Beam C. Such a reduction in the degree of distribution is evident also from Figure 2(b).

A further illustration of the way in which the moments resulting from a single, concentrated load are distributed among the beams is provided by Figure 4 for a bridge having five girders and $b/a=0.1$. Relative moments in all girders for a load over Girder B are plotted as a function of H in this figure. The curve for moment in Girder B is the same as that on Figure 3. For this girder the moment increases continuously as the value of H increases. For an infinitely stiff slab, corresponding to $H=0$, all girders participate equally in carrying the load, while for $H=$infinity all of the moment is carried by the loaded girder. A study of the variation of moment in the remaining girders as H decreases from near infinity to zero in Figure 4 gives further insight into the behavior of this type of structure. Consider first the moments in Girder A. At H equals infinity this

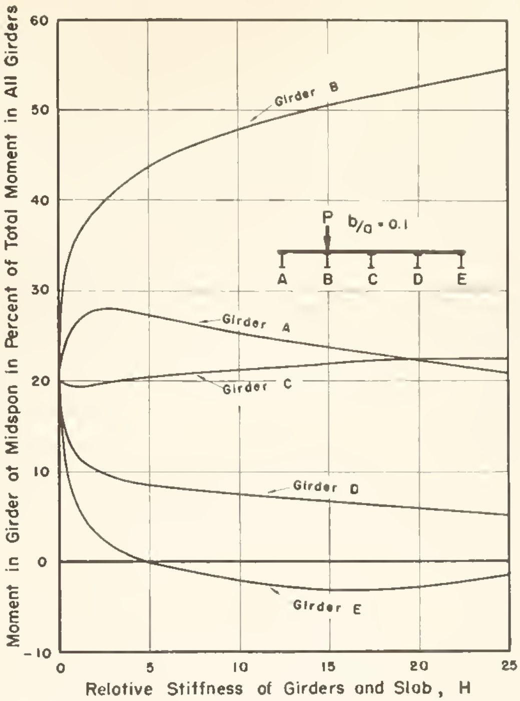
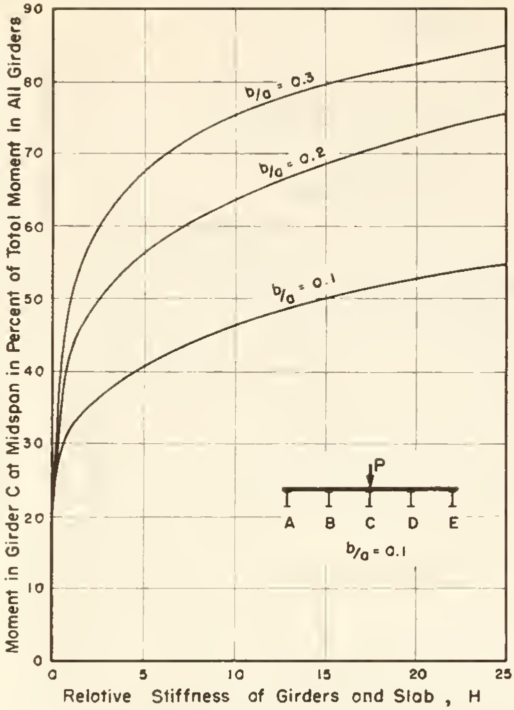


Figure 4. Variation of moment in girders as a function of H for a concentrated load over Girder B at midspan.

moment is zero. As the slab becomes stiffer and H decreases, this moment gradually increases until a value of $H=2$ or 3 is reached. At this point, the moment in Girder A begins to decrease with further decrease in H and finally reaches a value of 20 percent at $H=0$. This rather interesting behavior can be explained in terms of the increasing ability of the slab to distribute moment to the more distant girders as its stiffness increases. Note first that the moment in Girder C changes very little for the range of H on the figure. For values of H greater than about 5, the moments in Girders D and E are relatively small and do not change rapidly with H, indicating that in this range the stiffness of the slab is not sufficient to transfer an appreciable portion of the load to these more distant girders. Consequently, most of the decrease in moment in Girder B as H decreases is accomplished by transfer of moment to Girder A. However, for values of H less than 5 in Figure 4 the stiffness of the slab becomes great enough to increase appreciably the participation of girders D and E, and the moment in these girders begin to increase more rapidly as H decreases. In this stage the load applied over Girder B is more widely distributed and the adjacent Girder A is no

[1] The portion of the longitudinal moment carried by the slab is usually quite small. An approximate expression for determining this moment is given on pp. 24-25 of Reference 2.



'igure 5. Effect of b/a on midspan moment in loaded Girder C for concentrated load at midspan.

onger required to resist as much moment as before. Thus the moment in Girder A ceases to increase and actually decreases to its final value of 20 percent at $H=0$. The nature of the curve for Girder A in this igure is generally typical of those for this loading condition and for other values of b/a. However, as b/a ncreases, the maximum moment in girder A occurs for smaller values of H than that shown in Figure 4 for $b/a=0.1$.

Effect of Ratio b/a

The second major variable included in the analyses s the ratio of girder spacing to span, b/a. A change n the relative span lengths of the slab and the girders, as represented by a change in b/a, causes a corresponding change in the relative stiffnesses of these wo elements; that is, an increase in b/a corresponds to a decrease in the transverse stiffness of the bridge. Thus, in general, the effect of increasing b/a is similar to that of increasing H. This is illustrated in Figure 5 which contains curves of relative moments at midspan of Girder C for a concentrated load over Girder C. The variation of moment with H is shown for structures having $b/a=0.1$, 0.2, and 0.3. The relative effects of changing b/a and H are easily seen from this figure. For example, an increase of b/a from 0.1 to 0.2 produces an increase in moment in Girder C approximately equal to that resulting from about a sixfold increase in H. That is, a change from $b/a=0.1$, $H=4$ to $b/a=0.2$, $H=4$ is equivalent to a change from $b/a=0.1$, $H=4$ to $b/a=0.1$, $H=25$. Similar relations hold for an increase in b/a from 0.2 to 0.3 but the equivalent change in H in this case is less than threefold.

Although an increase in b/a will always result in less distribution of load, the effect for an actual slab-and-girder bridge will usually be less than indicated in Figure 5 because of changes in H that occur as a result of changes in b/a. For example, if b/a is increased by shortening the span a, the change in span results in smaller and less stiff girders and thus causes a decrease in H which partially offsets the effects of increasing b/a. Similarly, if b/a is increased by making the girder spacing b larger, changes in H are again produced, chiefly because of increase in slab thickness which usually results from the changed span of the slab. Although the girder stiffness may also be increased as a result of the wider spacing, the net result is usually a decrease in H, since the slab stiffness varies as the cube of the thickness and may be increased a fairly large amount.

Effect of Loading

The preceding discussions of the manner in which load distribution depends on H and b/a have been confined to the case of a single, concentrated load on the structure. This loading condition was chosen partly for its simplicity but also because all of the effects discussed are greater for a single, concentrated load than for multiple loads. For this reason it is necessary to discuss also the behavior of the structure for the case of more than one load applied at a given section, since highway bridges are always subjected to multiple loads. In some cases, two loads corresponding to a single truck may be considered, but more commonly the loading will consist of four loads representative of two trucks.

The curves in Figure 6 show the variation with H of the maximum moments in Girders A and C of a five-girder bridge having $b/a=0.1$. In each case the loads are placed transversely in the position to produce maximum moment in the girder considered. The spacing of the loads corresponds to the spacing of truck wheels on a bridge having a girder spacing of 6 ft.

Consider first the curve for Girder C in Figure 6. This curve is very similar to that for the same girder in Figure 3, except that the decrease in moment with a decrease in H is much less. For a concentrated load (Fig. 3), the moment decreases from 54 percent of the total moment at $H=25$ to only 20 percent at $H=0$. However, for four loads (Fig. 6), the moment in Girder C for $H=25$ is only about 30.3 percent of the total, since the application of four loads provides in itself a better distribution of total moment among the girders. Since this girder must resist 20 percent of the moment at $H=0$, it is evident that a decrease in H can produce much less reduction in moment for multiple loads than for a single load.

The curve for Girder A in Figure 6 is quite different from that for Girder C, in that there is a range of H in which the moment increases as H decreases. This phenomenon was observed also in the curve for moment in Girder A for a single load over Girder B (Fig. 4). The similarity between these two curves is to be expected since the center of gravity of the four loads in Figure 6 is very close to Girder B. Thus, the explanation for the peculiarities of this curve are the same as those given in the discussion of Figure 4.

It can be seen from Figure 6 that for H less than about 10 the moment in the edge girder is the greater while for H greater than 10 the opposite is true. This condition is fairly typical for other structures with a load over the edge girder as shown in Figure 6, but the value of H at which the two curves cross will depend on the values of other variables, such as b/a and the spacing of the wheel loads relative to the spacing of the girders. Obviously, the magnitude of the moment in an edge girder will be decreased if the loads are shifted away from it. If conditions are such that the outer wheel load cannot be placed directly over the edge girder or sufficiently close to it, the moment in the edge girder may be less than that in an interior girder for all values of H.

Another difference in the behavior of edge and interior girders is the way in which the moments vary with H. For an interior girder, the maximum moment always decreases as H becomes smaller and this trend is independent of the type or number of loads. However, the moment in an edge girder first increases and then decreases as H is made smaller. The value of H at which this change takes place depends somewhat on the other variables not shown in Figure 6.

Another characteristic of the structure loaded with several loads is worthy of mention although it is not illustrated in Figure 6. As the number of loads increases, the distribution of load *along* the girders becomes more nearly alike for the several girders. Consequently, the differences between relative loads, moments, and deflections become less. For example, consider a structure having $b/a=0.1$ and $H=5$. For a concentrated load over Girder C the moment in that girder is 2.05 times the average moment for all the girders, while the deflection of Girder C is only 1.55 times the average. However, for four loads placed as in Figure 6, the corresponding ratios of maximum to average are 1.28 for moment and 1.23 for deflection. This relatively close agreement between the distribution of moment and deflection for a practical case of loading is quite convenient in that it makes it possible to use the same assumptions for the computation of moments and deflections in the design of slab-and-girder bridges.

Action of Diaphragms in Distributing Loads

Diaphragms or other kinds of transverse bracing between the girders are often used in slab-and-girder bridges, in an attempt to improve the distribution of loads among the girders. The results of analyses show, however, that the addition of diaphragms does not always accomplish this aim since in certain cases it may actually increase the maximum moment in a girder. The conditions which determine whether diaphragms will decrease or increase the moment in a particuler girder can best be described by considering two typical examples.

First, consider a five-girder bridge with four loads

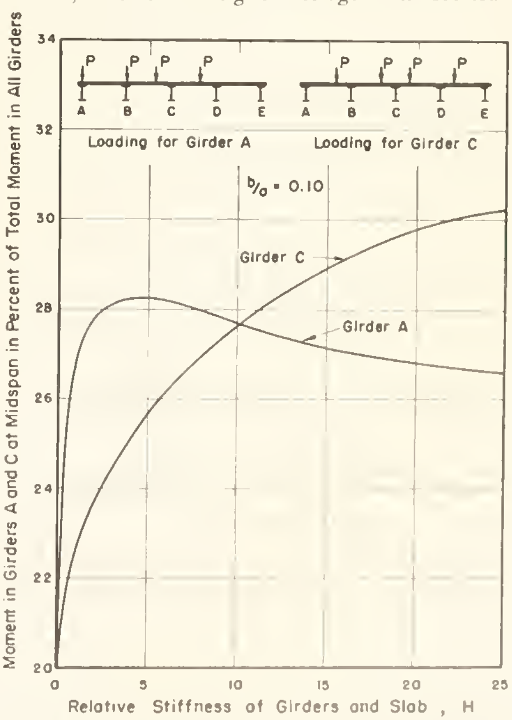


Figure 6. Variation with H of maximum moment in exterior and interior girders for four wheel loads at midspan.

placed to produce maximum moment in the center girder. The moments in this girder as a function of H are shown in Figure 6. Note that the loads are located symmetrically about the longitudinal centerline of the structure, and that it is the moment in Girder C that is being considered. If no diaphragms are present, the effect of increasing the transverse stiffness by increasing the stiffness of the slab causes a continuous decrease in moment as illustrated by the curve in Figure 6 for decreasing values of H. When the slab becomes infinitely stiff ($H=0$), the load and moment is distributed equally to all of the girders, and the maximum distribution is thus obtained. Now consider the same structure, having a slab with a stiffness corresponding to say $H=20$, but having a diaphragm added at midspan. If the diaphragm is assumed to be infinitely stiff, the load and moment will be distributed uniformly among the girders, since the applied loads are placed symmetrically about the longitudinal centerline of the bridge. The effect of providing infinite transverse stiffness is therefore the same whether the added stiffness is provided in the slab or by means of a diaphragm. It is reasonable to assume, therefore, that this equivalence in effect of slab and diaphragm will hold also for intermediate diaphragm stiffnesses, and analysis has shown this to be true. Thus, for a symmetrically loaded bridge, the addition of transverse stiffness by means of diaphragms produces a reduction in the maximum girder moments in much the same manner as would an increase in slab stiffness (decrease in H).

Consider next the other loading condition illustrated in Figure 6 with loads placed eccentrically in the transverse direction so as to produce maximum moments in an exterior girder. In the structure without diaphragms, the effect of increasing the slab stiffness is shown by the curve in Figure 6 as H decreases. At first, the moment in the edge girder increases. Then, as the stiffness becomes very great (H small), the moment begins to decrease. And finally, for infinite slab stiffness ($H=0$), the load and moment is again distributed uniformly to all of the girders just as it was for symmetrically placed loads. This ability of an infinitely stiff slab to provide uniform distribution of load for any arrangement of the loads results from the torsional stiffness of the slab which, in theory, becomes infinite when the transverse stiffness does. This property of the slab is not possessed by a diaphragm. Thus, if the transverse stiffness is increased by the addition of a diaphragm at midspan the behavior of the bridge is quite different from that produced by an increase in slab stiffness. Consider the limiting case of an infinitely stiff diaphragm. For this condition, the deflection of the girders, and thus the distribution of load to equally stiff girders, becomes linear, but not uniform. In other words, the structure tilts because of the eccentricity of the loading, and the moment in Girder A becomes something greater than 20 percent. Actually, for the loading arrangement shown in Figure 6, the moment in Girder A for an infinitely stiff diaphragm is theoretically equal to 33.3 percent. Thus, if the load is eccentrically located on the bridge, the addition of diaphragms may result in an appreciable increase in the edge-girder moment.

Magnitude of Effects

The foregoing discussion has shown clearly that beneficial effects are not always produced by the addition of diaphragms. It is important, therefore, to know under which conditions a diaphragm is able to exert its greatest effects and to have some idea of how great these effects might be. Since a diaphragm, like the slab, derives its effectiveness in transferring load from its ability to resist relative deflections of the girders, any condition leading to large relative deflections, or to more nonuniform distribution of load or moment, will provide the diaphragm with a better opportunity to transfer loads. Thus, the following conditions should lead to the greatest effects of diaphragms: large values of H; large values of b/a; or a decrease in the number of loads. The effects of these variables, as well as others, are discussed in the sections following.

Effect of H *and Diaphragm Stiffness*

The relative stiffnesses of the slab, the diaphragms, and the girders are all related in their effect on the load distribution. It is convenient to combine these three stiffnesses in two dimensionless ratios. One of these is, of course, H, which relates the stiffness of the girders to the stiffness of the slab. The other is defined as

$$k = \frac{E_d I_d}{E_g I_g}$$

where $E_d I_d$ and $E_g I_g$ are the moduli of elasticity and moments of inertia of a diaphragm and a girder, respectively.

It is obvious that the effectiveness of the diaphragm is a function of its stiffness, and that it increases with an increase in k. However, the change in moment produced by the addition of a diaphragm of given stiffness depends on the stiffness of the slab already present. This can best be illustrated by reference to the moment curve for Girder C in Figure 6. The structure considered in this figure is representative

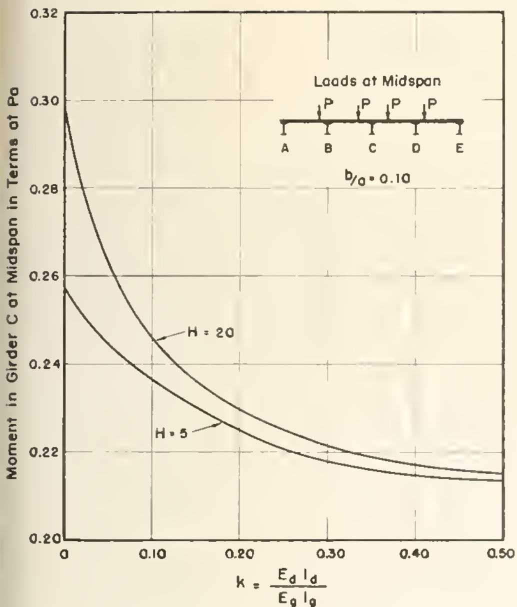


Figure 7. Effect of adding diaphragm at midspan of bridge on moments at midspan.

of a bridge having a girder spacing of 6 ft. and a span of 60 ft. A concrete-girder bridge of these dimensions would have a value of H in the neighborhood of 20 to 50, while a noncomposite I-beam bridge would have an H of about 5. Since results of analyses are available for values of $H=5$ and 20, these will be used for comparisons; they can be considered roughly typical of the two types of bridges mentioned. First consider the larger value of H. The moment in Girder C for no diaphragm is found to be 0.298 *Pa*. If a diaphragm is now added at midspan with a stiffness corresponding to $k=0.40$, a fairly large value, the moment in Girder C at midspan is reduced to 0.217. The reduction in this case is 27 percent. Now consider a bridge having $H=5$, and add the same diaphragm. For no diaphragm the moment in C is 0.256 *Pa*, and with a diaphragm having $k=0.40$ it becomes 0.215. The reduction in this case is only 16 percent, or a little more than half as much as for the other bridge. The reason for this becomes evident if it is noted that the moment after the diaphragm was added was approximately the same in both structures, 0.217 and 0.215. This means that the action of a diaphragm of this stiffness dominates the action of the slab and leads to about the same result in the two cases. However, since the bridge with $H=5$ initially has a somewhat smaller moment than the bridge with $H=20$, the change produced by the diaphragm is correspondingly less. The relations just discussed are illustrated better in Figure 7 which gives moments for the same structure and loading as in Figure 6. The moment in Girder C for symmetrical loading is shown as a function of k for the two values of H. It is easily seen from this figure that a given diaphragm stiffness provides a much greater reduction of moment if $H=20$ than if $H=5$.

Figure 8 is similar to Figure 7, except that the moment given is that in Girder A for the eccentric load arrangement shown. Again, the bridge and loading are the same as in Figure 6. In Figure 8, the maximum moment in an edge girder increases as the diaphragm stiffness increases, for the reasons given previously. Comparisons can be made as before for structures having values of $H=5$ and 20. For $H=20$, the addition of a diaphragm with $k=0.4$ increases the moment from 0.268 *Pa* to 0.319 *Pa*, an increase of 19 percent. For $H=5$, the corresponding increase is from 0.283 to 0.302, or only 7 percent. Thus in this case also, the effect of adding a diaphragm is greater for the larger value of H.

Figures 7 and 8 show also that the diaphragm has a diminishing effect as its stiffness increases; that is the moment curves tend to flatten out as k increases. For example, for Girder C and $H=20$ in Figure 7, an increase in k from 0 to 0.40 reduces the moment 27 percent, while a further increase in k from 0.40 to infinity would produce an additional decrease of only about 6 percent in terms of the moment for $k=0$.

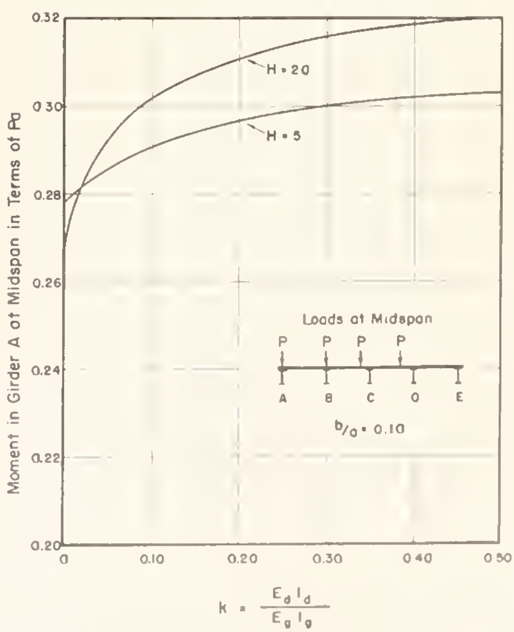


Figure 8. Effect of adding diaphragm at midspan of bridge on moments at midspan.

The comparisons in the preceding paragraphs have been presented only to give a picture of the relative effects of adding diaphragms to structures having different values of H. The numerical values are applicable only to the particular structures considered and no general conclusions regarding the absolute effects of diaphragms can be drawn from them, since there are several other variables whose effects have not yet been considered.

It is also important to note that the theoretical analyses on which the foregoing discussions are based involve the assumption that the longitudinal girders have no torsional stiffness. If such stiffness is present, the action of a diaphragm for eccentric loading approaches more nearly that of the slab. However, a relatively high degree of torsional stiffness and a fairly stiff connection between diaphragms and girders is required before this effect becomes appreciable. These conditions are more likely to be present in bridges with concrete girders and diaphragms than in the I-beam-type of bridge.

Effect of b/a

The relative deflections of the girders in a bridge without diaphragms become greater as the value of b/a increases. Therefore, the effects of the diaphragms, which are dependent on the relative deflections, will tend to be greater for larger values of b/a. The actual effects will be similar to those discussed in the preceding sections; that is, the moment in an interior girder for symmetrical loading will be decreased, while the moment in an exterior girder will be increased if the loads are placed eccentrically with respect to the longitudinal centerline of the bridge. In either case, the changes in moment will be greater for larger values of b/a.

Effect of Number of Loads

The effects produced by adding diaphragms will depend on the number of loads considered to act on the structure at a given transverse section. The choices in either analyses or test programs are normally three: (1) a single concentrated load; (2) two loads, representing a single truck; or (3) four loads, representing two trucks. Data have been presented previously to show that the distribution of load and the deflections of the girders tend to become more uniform as the number of loads is increased. Obviously then, added diaphragms will be more effective for a single load than for two or four loads.

Effect of Transverse Location of Loads

If the loads are placed symmetrically with respect to the longitudinal centerline of the bridge, the addition of diaphragms will always produce a more uniform distribution of load, and the largest girder moment, occurring for this case in an interior girder will be decreased. However, if the loads are shifted transversely toward one side of the bridge, the largest moment may occur in the edge girder, and will be increased by the addition of diaphragms.

The practical significance of an increase in edge-girder moment depends on the relative magnitudes of the moments in edge and interior girders, the loads being placed in each case to produce maximum moments in the girder being considered. If truck loads can be placed on the bridge with one wheel load directly over or very close to an edge girder and if the value of H is relatively small, the moment in an edge girder will usually be greater than that in an interior girder when each is loaded for maximum effect (see Fig. 6). In this case, the addition of diaphragms will increase the moment in the edge girder, while decreasing the moment in the interior girder. The governing moment is thus increased and the effect of adding diaphragms may be considered to be harmful for these conditions. On the other hand, if the layout of the bridge and the locations of the curbs are such that a large transverse eccentricity of load is not possible, or if H is large, the governing moment will usually be that in an interior girder. The addition of diaphragms will again cause a decrease in moment in the interior girder and an increase in moment in the exterior girder. If the final result is equal moments in the two girders, each for its own loading condition, the effect of diaphragms is beneficial, since the governing moment has been reduced. However, the diaphragms may change the moments so much that the edge-girder moment is the greater, and may even produce the condition in which the edge-girder moment with diaphragms is greater than the interior-girder moment without them. In this case, the effect of the diaphragms is again harmful.

It is evident from the foregoing discussion that the transverse location of the loads has an important bearing on whether the effect of adding diaphragms is to increase or decrease the governing moment in the girders. However, the effects of the other variables affecting the behavior of the structure should not be ignored. Whether the governing moments in a given bridge will be increased or decreased, and to what degree, will depend also on the values of H, b/a, k, and on the longitudinal location of the diaphragms as discussed in the following sections. This phase of the action of bridges with diaphragms is quite complex and the theoretical studies are still too limited in scope to state, in terms of all the variables, the conditions under which added diaphragms will be beneficial or harmful.

Effect of Longitudinal Location of Diaphragms Relative to Load

It is almost obvious that a diaphragm will be most effective when it is located in the structure at the same longitudinal location as the loads being considered. However, in a highway bridge the loads may be applied at any point along the girders, while diaphragms can be placed at only a few locations. Since maximum moments in a bridge will usually be produced by loads applied in the neighborhood of midspan, a diaphragm or diaphragms located at or near midspan should be most effective. Consider the examples given previously for the structures and loadings shown in Figures 6, 7, and 8. In this case, the loads and moments are at midspan, and the effects of adding a single diaphragm at midspan have been discussed. If, instead, two diaphragms had been added at the third points, each having a stiffness corresponding to $k=0.40$, the results would have been somewhat different. For example, for the interior girder, the addition of *two* diaphragms at the third points would decrease the moment by 9 and 23 percent, respectively, for $H=5$ and 20, as compared to reductions of 16 and 27 percent for a single diaphragm at midspan. Similarly, the moment in Girder A would be increased 3 and 13 percent, respectively, for $H=5$ and 20, by the addition of diaphragms at the third points, as compared to increases of 7 and 19 percent for a diaphragm at midspan. It should be noted that although the total diaphragm stiffness is twice as great in one case as in the other, the effect is still reduced significantly because of the less advantageous location with respect to the load. Of course, if loads were applied at a third point of the span the diaphragm at this location would be quite effective, but the girder moments produced for this location of the load would not be significant in design.

Analyses have shown also that if a diaphragm has been added at midspan, the addition of other diaphragms, say at the quarter points, will have little effect for loads at or near midspan. This can be explained by the fact that the relative deflections of the girders at the quarter points have been decreased by the addition of a diaphragm at midspan.

It has been shown that if the loads are applied at midspan, the effectiveness of diaphragms will decrease the more distant they are from the loads. Conversely, if a diaphragm is located at midspan, its effectiveness will decrease as the loads move away from midspan. Analyses have shown that the maximum girder moments in a bridge with a diaphragm at midspan will be obtained for loads placed a short distance from midspan. The exact location of the loads for maximum moment will depend on the values of H, k, b/a, and the number of loads on the structure. For the bridges and loading of Figures 6, 7 and 8, and for a single diaphragm at midspan having $k=0.40$, the maximum moments in Girder C for loads off midspan are 2 and 6 percent greater, respectively for $H=5$ and 20, than the moments for loads at midspan. The magnitude of this increase depends on a number of factors and the above values should be considered only illustrative. Since the moment in Girder A is increased by the addition of a diaphragm, it will be a maximum for loads applied at the location of the diaphragm.

The foregoing remarks may be summarized as follows: Diaphragms, unlike the slab (which acts at all points along the girders), can be added only at discrete points; their effectiveness is therefore not equal at all locations but extends only for some distance either side of the diaphragm. Consequently, for greatest effectiveness, diaphragms should be placed near the locations at which loads will be placed for maximum moments, usually near midspan. Furthermore, since maximum moments do not decrease greatly as the loads are moved away from midspan, analyses have shown that in many cases the optimum arrangement will consist of two diaphragms placed a short distance either side of midspan.

Flexibility of Diaphragm Connections

All of the analyses used as a basis for the foregoing discussions of the effects of diaphragms involve the assumption that the diaphragms are continuous members extending across the full width of the bridge. However diaphragms in I-beam bridges commonly consist of short sections of rolled beams or of transverse frames spanning between adjacent girders. In such cases, the continuity of the diaphragm is derived solely from the rigidity of its connections to the girders. If these connections are not sufficiently rigid to provide flexural stiffness equal to that of the diaphragms proper, the effective stiffness of the diaphragm, and thus its ability to distribute load, will be decreased.

It seems reasonable to assume that the condition of a fully continuous diaphragm is approached more closely where reinforced-concrete beams are used for diaphragms, as is the case in concrete-girder bridges and in some I-beam bridges.

The problem of determining the effective rigidity of a diaphragm, taking into account the flexibility of the connections, and the problem of evaluating the stiffness of framed bracing are outside the scope of this paper. Nevertheless, it is one of the most important problems confronting the designer who wishes to use diaphragms as an aid to load distribution.

Another problem of similar nature is represented by the skew bridge in which the diaphragms are frequently staggered longitudinally and thus depend on the torsional rigidity of the girders as well as on the rigidity of the connection to provide continuity across the bridge. This problem is also outside the scope of this paper.

Limitations of Analyses

The applicability of the analyses described in this paper is necessarily limited by the simplifying assumptions that have been made and by the fact that not all of the variables affecting the behavior of slab-and-girder bridges have been considered. Consequently, close agreement between the predictions of the analyses and the real behavior of actual bridges should not be expected unless the properties and characteristics of the structure are reasonably similar to those assumed in the analyses. It becomes desirable, therefore, to consider the assumptions of the analyses and the limitations imposed by those assumptions, and to consider so far as possible the effects of the neglected variables.

Properties of Materials

A basic assumption in the analyses is that the slab is homogeneous, elastic, and isotropic. Although a reinforced-concrete slab satisfies none of these conditions, especially after cracking has occurred, the results of tests on scale-model I-beam bridges have shown that the distribution of load to the girders is predicted very closely by an elastic analysis. This conclusion, of course, does not apply after extensive yielding of the slab reinforcement has occurred.

Ultimate Strength

Another basic assumption is that the entire structure—slab, girders, and diaphragms—behaves elastically; that is, deflections, moments, and shears are linear functions of load, and thus, superposition of effects is possible. Obviously, this condition is not satisfied after significant yielding has taken place in any element of the bridge, and these analyses are therefore not suitable for predicting ultimate capacities which are attained usually only after considerable inelastic acion.

Values of b/a

Of the several variables relating to the geometry of the structure, only the ratio of girder spacing to span, b/a, has been considered in the analysis, and this only for values of 0.1, 0.2, and 0.3. This range of values includes a majority of actual structures, and some extrapolation is possible, especially to lower values of b/a since the load distribution for $b/a=0$ is theoretically uniform.

Number of Girders

Although only bridges having five girders have been considered, it has been pointed out in a previous section that the influence lines for moments in the girders (Fig. 2) may be used for bridges with more than five girders and even, in some cases, for bridges with only four girders. Analyses have also been made for a three-girder structure; some of these have been published (*8*), while the others have not (*9*).

Continuous Bridges

A further limitation of the analyses is that only simple-span bridges have been considered. However, some analyses, and fairly extensive tests on scale models (not yet published), have shown that the distribution of moment to the girders in a continuous bridge is approximately the same as that in a simple-span structure having values of H and b/a corresponding to those for the continuous bridge using for a the span between points of contraflexure. This similarity extends also to the distribution of girder moments over an interior support.

Skew Bridges

Only right bridges have been considered, and no analyses for skew bridges are available. However, tests on scale models (*5*) have indicated that for angles of skew up to about 30 deg. the distribution of load is very similar to that for a right bridge. For larger angles of skew, the distribution of load is affected adversely; however, at the same time, the total moment in the girder is decreased in such a manner that the maximum girder moment is also decreased in spite of the changed distribution (*5*, *6*). The effects of diaphragms in skew bridges have not been studied.

Nonuniform Girder Spacing

It has been assumed in all of the analyses that the girder spacing b is uniform. If this spacing varies slightly it is probable that the use of an average value when computing b/a will be satisfactory. However, this approximation may not be valid if the variation in b is great; fortunately this condition is not common in slab-and-girder bridges.

Stiffness of Slab

Some uncertainty always exists regarding the absolute stiffness of a reinforced-concrete slab, since it is affected by the degree and extent of cracking. However, the tests of scale-model bridges (4) showed an excellent correlation between the results of analyses and tests when H was based on a slab stiffness computed for the gross concrete section, neglecting the reinforcement, and taking Poisson's ratio equal to zero. Whether a similar approximation will also be satisfactory when applied to actual structures can be determined only by studying the results of field tests.

Stiffness of Girders

The other quantity entering into the expression for H is the stiffness of the girders, and this too is subject to some uncertainty. For I-beam bridges the major problem is estimating the degree of composite action which exists between the slab and the girders of the bridge in question. If no composite action exists, the girder stiffness is easily determined. If composite action is provided by means of positive anchorage between the slab and girder, the stiffness of the composite T-beam may be computed easily by including a width of slab extending half the distance to the adjacent girder on each side. Tests in the laboratory as well as in the field have shown that some degree of interaction probably exists in most actual bridges, even if positive shear connection is not provided. The source of shear transfer in these structures is either bond or friction between the slab and I-beam, or perhaps both. Since the stiffness of an I-beam is increased markedly by the existence of even a small amount of interaction, the value of girder stiffness, and thus of H, may be quite indeterminate in a real bridge. For this reason, it is desirable that tests on such structures include strain measurements on both top and bottom flanges of the I-beams, so that the position of the neutral axis can be determined and the degree of interaction estimated.

The absolute stiffness of reinforced-concrete girders is also uncertain because of the indeterminate effects of cracking. It is customary in reinforced-concrete frames to compute relative stiffnesses on the basis of the gross concrete sections of the various members. This procedure may be used also for computing H when both the girder and the slab are reinforced concrete. However, the possibility should not be overlooked that the absolute stiffnesses of these two members may be affected differently by cracking and that their relative stiffnesses may be changed. Thus, again there may be some uncertainty regarding the real value of H for a particular bridge. However, the value of H will usually be fairly large for concrete-girder bridges and the moments in the girders are not especially sensitive to variations in H when H is large (Figs. 3 to 6).

Unequal Girder Stiffnesses

Only bridges in which all girders have the same stiffness have been considered in this paper. This condition, however, is frequently not satisfied in actual structures. In concrete-girder or composite I-beam bridges, the edge girders may have an increased stiffness because of the greater cross section of the curbs or sidewalks as compared to the slab proper. Also, some I-beam bridges have been designed with the edge beams smaller than the interior beams.

The effects of unequal girder stiffnesses have been studied analytically for one bridge having edge girders 20 percent stiffer than the interior girders (2, 9). These effects have also been observed in tests of scale-model I-beam bridges in which the edge beams were less stiff than the interior beams. In both cases the bridges had five girders. Although these data are not sufficient to permit precise statements regarding the behavior of bridges with girders of unequal stiffness, some idea can be given of how such a bridge will behave. Consider a structure in which the edge girders are stiffer than the interior girder, since this is a fairly common condition in actual highway bridges. In this case, the stiffer girders attract additional load, the amount of which depends on how much stiffer these girders are in comparison to the others, as well as on the transverse stiffness of the slab or diaphragms, through which loads reach the girders.

The limited data available indicate that the increase in load is not as great as the increase in stiffness. Thus, the deflections of the stiffer girder will not be increased. An increase in load produces also an increase in moment in about the same proportion; however, this does not necessarily lead to an increase in stress, since the section modulus is usually increased by the same factors which cause the increase in stiffness. Whether or not the stresses will be increased in any given case will depend on the relative magnitudes of the increases in moment and section modulus.

Torsional Stiffness of Girders

The torsional stiffness of the girders has been neglected in all of the analyses described herein. This is on the side of safety, since such stiffness always con tributes to a more-uniform distribution of load. The torsional stiffness of noncomposite I-beams is negligible compared to the flexural stiffness of the slab, and even for composite I-beams the effect may still be small. However, the torsional stiffness of concrete girders may be appreciable and may produce noticeable improvements in the load distribution, especially as it reduces the harmful effects of stiff diaphragms. If H is large and the diaphragm is relatively stiff, the contribution of the slab will be relatively small and the structure may be analyzed relatively easily, but with fairly good accuracy, by means of a crossing-beam or grid analysis, including the effects of torsion but neglecting the presence of the slab.

Stiffness of Diaphragms

A major uncertainty will always exist regarding the stiffness of the diaphragms. If rolled sections or framed bracing are used, the rigidity of the connections at the girders is the major problem. If reinforced-concrete diaphragms are used, the effect of cracking must be evaluated. This latter is particularly important where concrete diaphragms are used in a bridge with steel stringers, since the relative stiffness of diaphragms and girders, k, becomes quite uncertain, because of the two different materials involved. However, for these conditions the value of k is likely to be relatively large, and variations in k will consequently be less important (see Figs. 7 and 8).

Use of Analyses in Planning and Interpreting Field Tests

An important use of the results of analyses is in the planning of field tests to yield significant results, and in the interpretation of field tests to provide the greatest amount of useful information.

Load, Moment, and Deflection

Frequent reference has been made in this paper to the distribution of load. However, since the girders are designed for moment and shear, not load itself, a knowledge of the distribution of total load to the girders is of little value to the designer unless he knows also how the load is distributed along the length of each girder. For this reason, the measurement of load itself, for example, by measuring reactions, may provide little useful information except as a check on other measured quantities.

Since moments are of primary interest to the designer, it is certainly desirable that they be determined in field tests, if at all possible. Although moment cannot be measured directly, it can usually be computed from measured strains. In reinforced-concrete girders, the determination of moments from measured strains is usually a difficult problem because of the effects of cracking on the moment-strain relation. The calculation of moments from measured strains may be somewhat easier in the case of steel stringers, but even here the effective section modulus may not be known exactly, because of the existence of a partial interaction between the slab and girders in bridges without mechanical shear connectors. However, if strains are measured on both the top and bottom flange of the beam so as to locate the position of the neutral axis, the degree of interaction can be determined approximately and the effective section modulus and moment of inertia for the composite beam can be estimated from the theory of partial interaction presented in Reference *10*.

Measurements of deflection in tests of slab-and-girder bridges are always of value since the deflections are of interest in themselves. However, the assumption should not be made that the distribution of load or moment among the girders is the same as the distribution of deflection. Although these distributions may be nearly the same under certain conditions, they may be greatly different under others. Obviously, if the girders are of different stiffnesses, the distribution of deflection will depend on the relative stiffnesses of the girders as well as on the loads that they carry. Moreover, even if the girders are of equal stiffnesses, the distribution of deflection may not be the same as the distribution of moment, or even of total load, since the longitudinal distribution of load along the various girders may be quite different (Fig. 1). This difference will be especially pronounced if only a single concentrated load is used in the test, and comparisons of moments and deflections for this case have been given elsewhere in this paper. If several loads are applied to the bridge, the distribution of deflection and moment will become more nearly alike, and in many tests advantage may be taken of this relation if it is not possible or convenient to determine moments from measurements of strain.

Loading

The analyses have shown that the effects of variations in H, b/a, diaphragm stiffness, or diaphragm location will depend to a considerable extent on both the number and locations of the loads used in a test.

The loading considered in the design of a bridge usually consists of not less than two trucks for a two-lane bridge, the most common type, and it is the behavior of the bridge under this loading that is of greatest interest. Frequently, however, it is not possible to make field tests with two trucks, and only a single-truck loading is used. For this case, the maximum moments, the distribution of moment or deflection, and the effect of adding diaphragms will be different than for a two-truck loading. Moreover, the distribution of moment will be different from the distribution of deflection. These differences present certain difficulties in interpreting the results but they can be overcome partially by obtaining data for various transverse positions of the single truck and combining the results to simulate the effects of two trucks on the bridge. Such superposition of effects is valid only if all of the observed phenomena are linear functions of load; this condition will usually be satisfied, however, except possibly for concrete-girder bridges in which the degree and extent of cracking may increase as successive tests are made. In such bridges, it is usually desirable to load the structure at all of the test locations at least once before any measurements are made. A similar problem may be encountered in I-beam bridges in which the degree of composite action may change during the tests.

In some cases it may be more convenient to test the bridge under a single, concentrated load. The various phenomena observed for this loading will be greatly different from those corresponding to a load consisting of two trucks, and the results can be interpreted correctly only by obtaining influence lines, or an influence surface, for the desired quantity by placing the single load at several different transverse and longitudinal locations on the bridge. The problem of superposition is even more acute in this case than for single-truck loading, and special care should be taken to determine if the relation between load and moment or deflection is truly linear over the range necessary to permit addition of effects.

The transverse location of the loads at any section has been shown to have an appreciable effect on the maximum moments in the girder, especially if diaphragms are present. Consequently, an effort should be made in any field test to place the loads as eccentrically as permitted by the spacing and clearance requirements of the specifications. If this is not done, an erroneous concept of the action of diaphragms may be obtained.

The longitudinal location of the test loads will usually be that producing maximum moments in the bridge. If the bridge does not have diaphragms, the maximum moment in a simple span will occur under the rear axle of the truck or trucks when that axle is located a short distance from midspan. However, since the moment at midspan for the rear axle at midspan is only slightly less than the maximum, it is frequently more convenient to measure strain or deflection at midspan with the rear-axle loads at midspan. This procedure should prove entirely satisfactory if no diaphragms are present. However, if a diaphragm is present at midspan, the moments and deflections at midspan for load at midspan may be significantly less than those which may be found under a load placed a short distance away from the diaphragm. Obviously, such shifting of the locations at which the load is placed and measurements are made adds much to the complexity of the test. However, it is important to recognize that the effect of diaphragms depends on the longitudinal location of the load, and this variable should either be included in the test program or its effect should be evaluated theoretically.

Other factors influencing the results of tests are H and b/a. Although these quantities are not likely to vary in a single test structure, it is necessary to recognize that a concrete-girder bridge having a large value of H will not behave the same as an I-beam bridge having a small value of H. The same is true of bridges having different values of b/a. Obviously, then, tests made on a single bridge cannot be generalized to apply to all slab-and-girder bridges. Even tests on a number of bridges are not capable of giving a complete or general picture of the behavior of such bridges, since such a complex structure does not lend itself readily to a purely empirical study. The importance and usefulness of theory becomes evident at this point. If field tests can be planned and carried out so as to yield significant comparisons with the predictions of the analyses, and if these comparisons show reasonable agreement, the theory then becomes a tool which can be used with confidence to understand and predict the behavior of slab-and-girder bridges. Without verification from field tests, the theory is of limited value; and without the aid of the theory, field tests, unless very great in number, cannot give a general picture applicable to the full range of the variables.

Conclusion

The numerous variables affecting the distribution of load to girders in slab-and-girder bridges have been discussed solely on the basis of the results of theoretical analyses. The following major variables have been considered: (1) Relative stiffness of girders and slab, H; (2) ratio of girder spacing to span, b/a; (3) number and arrangement of loads; and (4) diaphragms, including effect of diaphragm stiffness and longitudinal location. The discussion has been limited throughout to simple-span, right bridges having five girders spaced equidistantly and all having the same stiffness. Torsional stiffness of the girders has been neglected.

The slab-and-girder bridge is a complex structure. Nevertheless, its behavior can be predicted and understood with the aid of theoretical analyses involving a number of the more important variables. The addition of diaphragms still further complicates the action of this type of bridge, but even here some insight into the effect of diaphragms can be obtained from analyses. This phase of the problem, however, has not yet been studied as fully as the action of the slab and girders alone.

Of course, an understanding of the theoretical behavior of this type of bridge is not enough. What we really desire is the ability to understand and predict the behavior of actual slab-and-girder bridges. To this end, the predictions of the analysis must be compared with the results of field tests; only in this way can we hope to understand a type of structure whose behavior depends on so many variables.

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