Vulnerability and Resilience Assessment of Power Systems: from Deterioration to Recovery via a Topological Model Based on Graph Theory

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***Abstract*—Traditionally, vulnerability is the level of degradation caused by failures or disturbances, and resilience is the ability to recover after a high-impact event. This paper presents a topological procedure based on graph theory to evaluate the vulnerability and resilience of power grids. A cascading failures model is developed by eliminating lines both deliberately and randomly, and four restoration strategies inspired by the network approach are proposed. In the two cases, the degradation and recovery of the electrical infrastructure are quantified through four centrality measures. Here, an index called flow-capacity is proposed to measure the level of network overload during the iterative processes. The developed sequential framework was tested on a graph of 600 nodes and 1196 edges built from the 400 kV high-voltage power system in Spain. The conclusions obtained show that the statistical graph indices measure different topological aspects of the network, so it is essential to combine the results to obtain a broader view of the structural behaviour of the infrastructure.**

***Index Terms*—Cascading failures, graph theory, power systems, resilience, vulnerability.**

I. Introduction

The complexity of power systems has increased in recent years due to the process of energy transition towards an increasingly electrified society. This energy transition has required significant effort to analyse and prevent disruptions, to improve power quality and continuity indicators, and to protect critical energy infrastructures [1].

Reliability and vulnerability have become the watchwords for electric utilities in terms of managing the above risks. Reliability is the ability of the electrical network to meet demand continuously and with an acceptable level of quality [2]. Vulnerability is the degradation of the system after assets fail due to malfunctions, attacks, or disruptions [3].

More recently, resilience is a concept that has attracted attention as planners began to think about the increase in natural disasters produced by climate change, human-made interference due to malicious cyber-attacks, and instability caused by the addition of large quantities of renewable energy [4]. Resilience does not yet have a clear definition; however, some define it as the ability of infrastructure to adapt, absorb, and recover from a disturbance [5].

The restoration of a power system depends on both its robustness and the speed of load recovery. An electrical infrastructure can operate in stable conditions until an asset fails, which could trigger adverse effects and degrade a significant part of the network. On the latter condition, operators execute iterative actions to recover the demand for electricity.

This is an area of current research. In the scientific literature, most of investigations focus on the resilience of the power network to climate events, proposing metrics and quantifying the operational state during such episodes [6]. Other approaches consider modifying the network topology to minimise the impact of disturbances and reduce the probability of blackouts. The resulting procedures are of interest because of their direct applicability to other available techniques [7].

Some models quantify the phases experienced by the power grid during high-impact events in order to increase resilience [8]. Other studies consider not only the above aspect but also incorporate the effects of contingencies [9].

Experts point out that resilience is an intrinsic property which implies a process of detection, anticipation, learning, and adaptation [10]. They define and quantify it in terms of criticality, frequency, impact, and recovery. Under these assumptions, the most relevant works employ the techniques of modularity, graph theory, multi-criteria, fuzzy theory, and information gap decision theory to address the above problem [11]–[15].

Reliability, vulnerability, and resilience assess the operation

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and performance of the power grid, so it is necessary to study them together to minimise the interruption of the power supply in the event of contingencies. Here, it is important to note that there is only related work on the joint consideration of reliability and vulnerability, and documents on vulnerability and resilience are barely receiving any attention [16].

Because of the above, this paper presents an original procedure based on complex network theory to sequentially study the vulnerability and resilience of electrical infrastructures. The network approach is already a widely validated and used technique in the scientific literature to carry out this kind of study. The resulting methodology only needs the topology of the network under study as an input, so it is suitable for those analyses where technical information is not available or is not open access.

The vulnerability study is carried out by degrading the performance of the electrical network through a cascading failures procedure, and the resilience study is performed by executing the iterative reconnection of the disconnected power lines. The cascading failures model consists of removing the links, one by one, according to two removal strategies: deliberately and randomly. In the meantime, the reconnection model, which also works on a one-by-one basis, considers four restoration strategies for the lines: random, subgraph, shortest paths, and degree. In both cases, the quantification of vulnerability and resilience is done by measuring the largest connected component, the connection rate, the weighted efficiency, and the flow-capacity indices. The first three indicators are widely known in the scientific literature, while the fourth indicator is proposed to more precisely measure the level of infrastructure overload. These procedures and strategies follow the guidelines of graph theory. The case study corresponds to a graph composed of 600 nodes and 1196 edges, based on the structure and topology of the 400 kV high-voltage electric power system in Spain. This research shows that resilience is a legitimate field of study involving not only industry but also academia and research centres.

The rest of this paper is organised as follows: Section II describes the disintegration and recovery strategies proposed to perform the two studies under consideration. Section III details the case study based on the Spanish high-voltage power grid. Section IV presents and discusses the vulnerability and resilience outcomes obtained in this manuscript. Lastly, Section V draws the main conclusions and details some future research directions.

II. Disintegration and Recovery Strategies

Reliability and vulnerability studies are well-documented approaches in the scientific literature. On the one hand, reliability indices analyse the continuity of the operations in case of failure of one or two assets (*n*-1 or *n*-2 contingencies), measuring metrics such as the frequency and duration of power outages, the expected energy not supplied, among others. On the other hand, vulnerability indices study the weakness and measure the performance under cascading failures (n-*k* contingencies), using steady-state power flow models.

Graph theory is an efficient approach to study cascading failures in power grids and, consequently, to evaluate both the disintegration and the recovery of the infrastructures. This technique involves mapping the network topology and converting each of the assets to nodes and edges in a graph [17].

A. Network performance indices

The development procedure requires statistical indicators to quantify the levels of vulnerability and resilience of the electrical infrastructure iteratively. In this article, the largest connected component, connection rate, and weighted efficiency indices are used, and the flow-capacity index is proposed. This last indicator is not intended to replace the traditional indices, but rather to complement them in order to obtain a broader view of the operational state of the system. Each index is described below.

- Largest connected component or LCC: this topological index quantifies the largest connected subgraph during the cascading failures and recovery processes;
- Connection rate or CR: this is an indicator that measures not only the largest subgraph but also all islands or subnetworks. Unlike the LCC measure, the CR index quantifies all operational networks in both disintegration and recovery;
- Weighted efficiency or WEFF: this centrality measure quantifies the effectiveness with which information can be exchanged within the network. In both an unweighted and a weighted graph, the flow between two nodes travels over the shortest geodesic distance. This distance is, in the first type of network, the minimum count of edges that must be travelled to join a pair of nodes, and in the second type of network, the minimum sum of the weights of each edge that must be passed to join two nodes. Thus, in a power system where the weights represent the capacity of the links, a higher weight implies a higher capacity route, so the reciprocal of the weight must be taken to calculate the shortest weighted route. A more detailed description of this index can be found in [18];
- Flow-capacity or FC: this is an index which measures the overload of the network compared to its base case, i.e. before the occurrence of contingencies. This measure serves as a complement to the WEFF indicator. The FC index is calculated as follows:

$$FC_s = \frac{\sum_{i=1}^{n} \frac{C_i}{\frac{EB_{i_s}}{EB_{i_0}}}}{n} \tag{1}$$

where FC is the level of network overload in the iteration *s*, C_i is the capacity of the line *i*, EB is the betweenness score of the line *i* in the iteration *s*, and *n* is the total number of links in the network.

Note that a high FC score indicates that the lines are operating below capacity; on the contrary, a low FC score indicates that the lines are operating above capacity.

B. Network attack and recovery strategies

In this paper, the vulnerability and resilience assessment of power systems is carried out through the sequential execution of a cascading failures model and an iterative reconnection model of disconnected electrical lines.

On the one hand, the cascading failures model is performed by iteratively removing the edges of the graph, which implies n-1 contingencies on a power grid that continually changes its topology after each removal. Each event is associated with a contingency, and therefore, with an iteration in the process of infrastructure disintegration.

The removal of the links is conducted using the following two strategies:

- Deliberate removals: power lines are deleted, in descending order, according to their edge betweenness score. Links with high scores are removed first. This strategy represents those events carried out by malicious agents, e.g. acts of vandalism, terrorism, cyber-attacks, among others;
- Random removals: power lines are randomly disconnected. These events correspond to malfunction of the protection equipment, technical failures, human errors, among others. Here, three simulation samples are run to avoid randomness in the results.

On the other hand, the reconnection model is developed by reconnecting the transmission lines removed by the cascading effects, that is, those calculated in the previous model. The following four strategies, inspired by complex network theory, are considered to determine the order of edge reconnection:

- Shortest paths: electrical lines are reconnected based on their EB score in the fully connected graph. That is, if the shortest weighted path between a pair of nodes is the longest in the network, this edge is reconnected first;
- Subgraph: electrical lines are reconnected according to the size of the subnets. In other words, the disconnected lines of the largest subgraphs are re-established first;
- Degree: electrical lines from the hub nodes are first reconnected in descending order;
- Random: electrical lines are randomly reconnected at each step.

III. Current topology of the Spanish high-voltage power system

In this document, the 400 kV power grid in Spain is used to test the disintegration and recovery procedures developed. This electrical network is made up of more than 21,000 km of high-voltage power lines, more than 1000 substations, and more than 80,000 MVA of transformation capacity [16].

Using the above information, a synthetic network composed of 600 nodes and 1196 edges was generated (see Fig. 1). This network only considered substations, generators, loads, power lines, and transformers, without taking into account the technical parameters of the assets. The resulting graph is a scale-free network that follows a power-law distribution, $P(k){\sim}k^{-\gamma}$, and has the growth and preferential attachment characteristics of real complex systems [16]. These types of graphs are useful for evaluating the structural robustness and vulnerability of networks and characterising their topological properties through centrality measures [19].

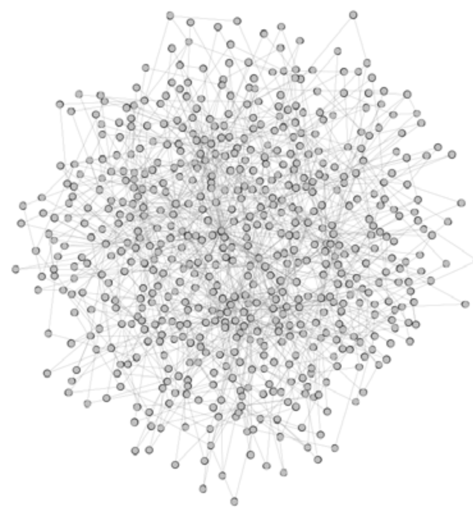

Fig. 1. Scale-free network of the Spanish high-voltage power system.

The objective of this research is to propose a useful and straightforward procedure that can be applied in studies where technical information on the electrical infrastructure is not readily available. The resulting framework should attempt to emulate the structural behaviour of a real power system when it is subject to multiple contingencies. Of course, this is a challenging task to perform when only the network topology is provided. Because of that, three simulation assumptions are made following the provision of complex network theory to try to address these shortcomings.

1) In a power system, transmission lines are the assets responsible for carrying the electrical energy produced in the generation centres to the consumption centres, where it is used for residential, commercial, or industrial purposes. The power transmission capacity is limited by the operational and security characteristics of the links. In the scale-free network used in this study, the edges are transmission lines, and each one has a pre-selected transfer threshold;
2) The objective of the load flow study is to obtain the voltage and angle magnitudes in each bus for specified load and power conditions. Once this information is known, the power flows in the lines and generators are calculated. In the developed topological framework, the flows are transmitted over the shortest geodesic distance between two nodes, i.e. the minimum count of edges that must be travelled to join them. Here, the edge betweenness (EB) index is used to calculate the analogous flows in the network links [20]. Theoretically, the higher the index score, the higher the flow through the edges. Geodesic distances are calculated using the

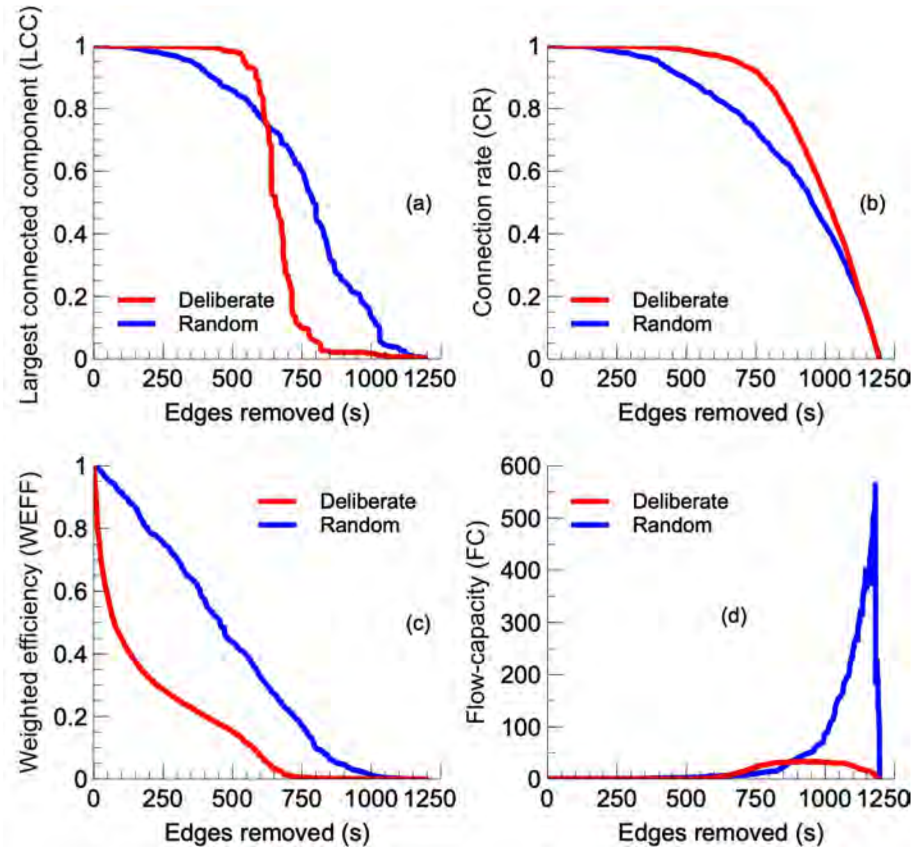

Fig. 2. Disintegration curves of the Spanish power grid.

Bellman-Ford algorithm [21];

3) The transfer capacities (C) of the links are calculated by normalising the EB score of each edge to the largest value in the graph. This can be expressed as follows:

$$C_i = \frac{EB_{i_0}}{max(EB_{1_0}, EB_{2_0}, \ldots, EB_{n_0})} \quad (2)$$

where C_i is the threshold of the link i and EB is the edge betweenness score of line i at the zero stage, i.e. when the graph is entirely connected.

Note that flows and capacities are balanced at the beginning of the simulation and that they do not change during decomposition and recovery stages. However, both the structure and the topology do change as the edges are removed, which influences the EB scores. This unbalances operating conditions, induces failures in the assets, breaks the balance in the load distribution, and degrades the performance of the network.

IV. Simulation Results

In this section, the simulation results obtained when applying the cascading failures and recovery models are presented and described. Here, the Spanish high-voltage power grid is used as a case study and the performance indicators proposed above are measured.

It is important to note that the developed procedures only take into account the connectivity of the topology and do not incorporate the electrical parameters of the infrastructure under study. Also, under real conditions, power system disintegration and recovery occur in seconds and minutes, respectively. In the restoration process, there is no consensus in the industry on the maximum number of lines that can be operated at each stage, as this will depend on the physical characteristics and procedures applied by each control centre, to ensure the safe operation of the system. The impacts of time could be considered by establishing a reconnection time and quantifying the number of safe manoeuvres in each strategy.

Both models were programmed and executed on the MatlabR2019 platform, using a computer with an Intel® Core^{TM} i7 processor, 3.40 GHz CPU and 16 GB of RAM.

A. Vulnerability assessment

Fig. 2 reports the disintegration of the infrastructure studied for both deliberate and random edge removal. In this simulation, the 1196 links are eliminated, and the indices described in Section II.A are quantified. Here, the LCC, CR, and WEFF measures take a value equal to 1 when all links are connected. Then, they decrease progressively, as the graph decomposes, until they reach a value equal to 0, i.e. when all links are removed. The latter represents the theoretical interruption of the power supply. In contrast, the FC measure increases when links operate below their capacity and decreases when they

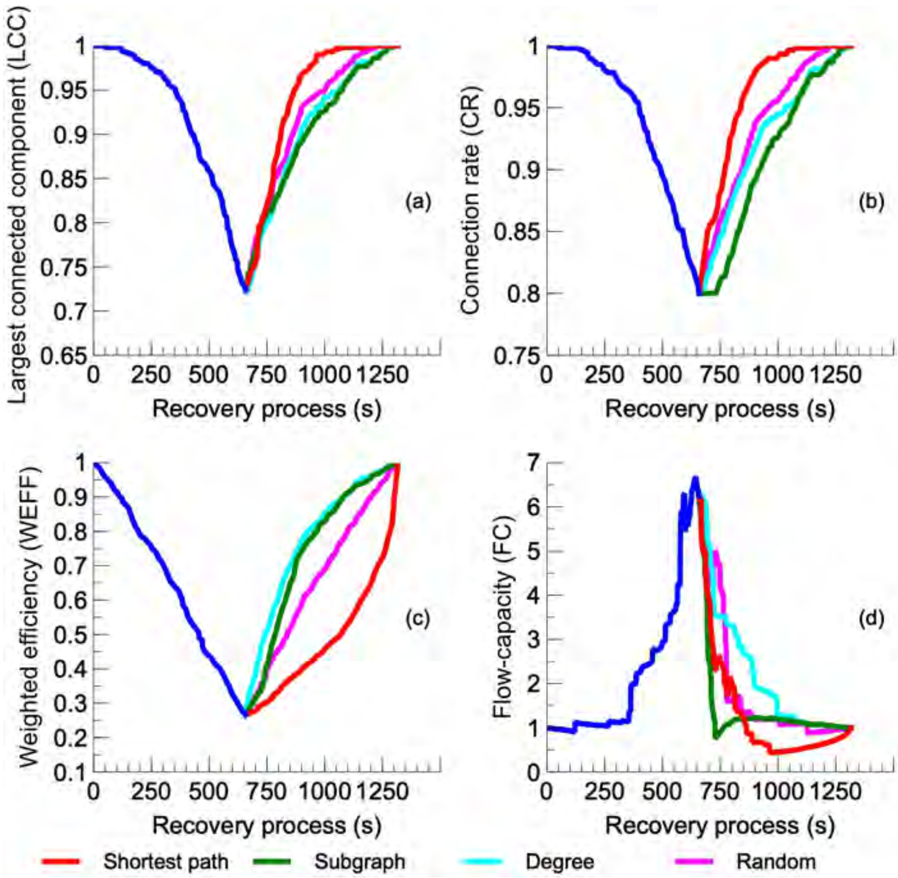

Fig. 3. Recovery curves of the Spanish power grid.

operate above their overload threshold. This index has a value equal to 0 when the graph is completely disconnected.

On the one hand, deliberate attacks are those perpetrated by malicious agents, such as acts of terrorism, cyber-attacks, and acts of vandalism, among others. The red curves are obtained by removing the links according to its edge betweenness score in descending order. On the other hand, random events are those related to risks of a random nature, such as natural phenomena or technical failures in devices and hardware. The blue curves are obtained by randomly removing the edges and averaging the set of results from three independent samples.

Overall, deliberate attacks impact the performance of the infrastructure more dramatically compared to random events. However, their measurements are lower during the early stages when trends between the red and blue curves are analysed. For example, the LCC and CR indices decrease from iteration 44 in the case of random removals, while they decrease until iteration 227 in the case of deliberate attacks. Despite all of the above, decomposition is faster once the cascading failure takes place.

On the other hand, the WEFF and FC indicators again confirm the conclusion obtained previously, i.e. deliberate attacks are an efficient strategy to reduce the connectivity of the electrical network. Observe how the red curve of the WEFF index decreases abruptly from the first iterations, degrading the structural performance and reaching a value of zero before the other strategy. However, the FC measure proposed in this article reveals that the network operates most of the time with lower loads compared to its base case. This shows the possibility of implementing containment strategies for cascading failures in order to avoid the total collapse of the infrastructure. There are already studies addressing the above problem [22], [23].

B. Resilience assessment

The restoration process takes place immediately after 55% of the edges are randomly removed (blue curves in Fig. 2). This percentage is intended for simulation purposes only. In each iteration, one line is added according to the shortest paths, subgraph, degree, and random strategies. The largest connected component, connection rate, weighted efficiency, and flow-capacity indices are also measured during the recovery of the network. Here, the LCC, CR, and WEFF measures increase progressively, until they reach a value equal to 1, as the graph recovers its edges. The FC index maintains the same philosophy already mentioned, i.e. it increases when all lines operate below their capacity and decreases when all links operate above their capacity.

The curves in Fig. 3 show that the recovery of the network varies according to the four proposed strategies. By analysing

Figs. 3 (a) and (b), for example, the shortest path strategy (red curve) is more effective in reconnecting the graph because the two performance indices are above the other values obtained. On the contrary, the subgraph strategy (green curve) is less successful in recovering the meshing of the network.

However, Fig. 3 (c) shows that the degree strategy (cyan curve) is more suitable than the shortest approach to recover the operational state from the weighted efficiency point of view of the system. In this particular case, reconnecting the hub nodes increases structural robustness and facilitates the redistribution of flows within the system. This phenomenon is well studied in network science [16].

Fig. 3 (d), on the other hand, demonstrates that the FC index decreases until the fully connected network state is achieved. Here, the degree strategy is also an efficient reconnection approach because it causes a gradual and progressive increase in the load on the infrastructure.

V. Conclusions

In this manuscript, a topological procedure based on graph theory was proposed to assess both the vulnerability and the resilience of power systems. The vulnerability assessment considered the disintegration of the network using a cascading failures model, while the resilience assessment addressed the recovery of the infrastructure employing a planned and iterative reconnection model for power lines. The case study used the Spanish power grid, through which a graph composed of 600 nodes and 1196 edges was generated.

The results show that there are different strategies to recover the operating conditions of the electrical infrastructure after a collapse or blackout. In a sense, this favours the system operator by allowing her/him to select the best application approach for her/his network.

Power systems with high penetration of renewable energy operate close to their limits and face more uncertainty. Because of this, future research should be oriented to the current energy transition scenario, where these sources are expected to be the main producers of electrical energy, especially wind and solar. That is, the random nature of renewable energy technologies could condition the formulation of stochastic mathematical models to assess the robustness, reliability, and resilience of critical energy infrastructures.

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