Why climate sensitivity may not be so unpredictable ?

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Different explanations have been proposed as to why the range of climate sensitivity predicted by GCMs have not lessened substantially in the last decades, and subsequently if it can be reduced. One such study (*Why is climate sensitivity so unpredictable?*, Roe and Baker, 2007 [@]) adressed these questions using rather simple theoretical considerations and reached the conclusion that reducing uncertainties on climate feedbacks and underlying climate processes will not yield a large reduction in the envelope of climate sensitivity. In this letter, we revisit the premises of this conclusion. We show that it results from a mathematical artefact caused by a peculiar definition of uncertainty used by these authors. Applying standard concepts and definitions of descriptive statistics to the exact same framework of analysis as Roe and Baker, we show that within this simple framework, reducing inter-model spread on feedbacks does in fact induce a reduction of uncertainty on climate sensitivity, almost proportionally. Therefore, following Roe and Baker assumptions, climate sensitivity is actually not so unpredictable.

1. Introduction

Uncertainties in projections of future climate change described in the last Assessment Report of the IPCC (IPCC, 2007 [@]) are high, as illustrated by the broad range of climate sensitivity − defined as the global mean temperature increase for a doubling of CO_2 − simulated by general circulation models (GCMs). Attempts to explain this fact have focused mainly on uncertainties in our understanding of the individual physical feedback processes (especially associated to clouds), difficulties to represent them faithfully in GCMs, nonlinearity of some processes and complex interactions among them giving rise to a chaotic behaviour of the climate system (Randall et al. [2007[a]]). A review of these explanations can be found in Bony et al., 2006 [@]. Nevertheless, in this letter, we leave aside all these considerations to focus our interest solely on the explanation proposed by Roe and Baker, 2007 [@] (RB07) which somewhat differ from the above-mentioned. This study uses the framework of feedback analysis, which has often been used to describe the relationship between physical processes involved in global warming and climate sensitivity (see for instance Lu and Cai, 2008 [@], Dufresne and Bony, 2008 [@], Soden and Held, 2006 [@]). The feedback analysis framework assumes a linear approximation of radiative feedbacks, resulting in a simple relationship between a global feedback gain f and climate sensitivity ΔT. In this classic setting, the main originality of RB07 approach consists in analyzing explicitly the way uncertainties on f, due to a limited understanding of their underlying physical processes, propagates into uncertainties on ΔT: assuming f is a random variable with mean $\bar{f}$ and standard deviation σ_f, RB07 uses this simple probabilistic model to highlight several fundamental properties of uncertainty propagation from feedbacks to climate sensitivity. The most prominent conclusion of this analysis is that reducing uncertainties on f does not yield a large reduction in the uncertainty of ΔT, and thus that improvements in the understanding of physical processes will not yield large reductions in the envelope of future climate projections. This conclusion, if true, would clearly have crucial implications for climate research and policy.

In section 2, we revisit the premises of RB07 conclusion. We highlight that it is the result of a peculiar way of defining uncertainty. Moreover, we show in section 5 that this conclusion is a pure mathematical artefact with no connection whatsoever to climate. Since the basic question of uncertainty definition appears to be at stake, section 3 briefly recalls widely used definitions and elementary results on uncertainty and its propagation as they can be found in Descriptive Statistics textbooks. In section 4, we apply these standard concepts and definitions to the exact same framework of analysis as RB07. We show that within this simple framework, reducing inter-model spread on feedbacks does in fact induce a reduction of uncertainty on climate sensitivity, almost proportionally. Finally, section 6 concludes.

2. Overview of RB07 approach

RB07 uses the feedback analysis framework. Denoting ΔT_0 the Planck temperature response to the radiative perturbation and f the feedback gain (referred to as feedback factor in RB07), they obtain:

$$\Delta T = \frac{\Delta T_0}{1 - f} \qquad (1)$$

RB07 then assumes uncertainty on Planck response to be neglictible so that the entire spread on ΔT results from the uncertainty on the global feedback gain f. To model this uncertainty, RB07 assumes that f follows a gaussian distribution with mean $\bar{f}$, standard deviation σ_f and implicit truncation for $f > 1$ (implications of this truncation are discussed in appendix 1). Then, they derive an exact mathematical expression of the distribution of ΔT through equation (1). This simple probabilistic climatic model is then used by RB07 to analyze the way uncertainties on f, due to a limited understanding of their underlying physical processes, propagates into uncertainties on ΔT. Their analysis highlights two fundamental properties of uncertainty propagation:

- Amplification: The term in $\frac{1}{1-f}$ in equation (1) amplifies uncertainty on feedbacks, all the more intensely as $\bar{f}$ is close to (though lower than) one. Small uncertainties on feedbacks are thus converted in large uncertainties on the rise of temperature.
- Insensitivity: Quoting RB07, *"reducing uncertainty on f has little effect in reducing uncertainty on ΔT"*, also stated as *"the breadth of the distribution of ΔT is relatively insensitive to decreases in σ_f."*

We fully subscribe to the first property and elaborate further on it in section 4. However, we are puzzled by the second property, that is, the claimed insensitivity of uncertainty on ΔT to uncertainty on feedbacks. The reason why one may find this second assertion a priori puzzling, is that it intuitively seems to be at a contradiction with the first property highlighted. Indeed, if small uncertainties on f are amplified into large uncertainties on ΔT, it suggests that a strong dependency exists between both uncertainties, rather than no or little dependency. We therefore dig into the details of RB07 argumentation regarding this assertion. To get to that conclusion, it appears that RB07 actually focus on the probability $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ that ΔT lies in the interval [4.5°C, 8°C] in response to a sustained doubling of the CO_2 concentration. This interval is defined as immediately above the range obtained with the CMIP3/AR4 GCMs (IPCC, 2007 [@]). They study graphically how this probability fluctuates with the level of uncertainty on feedbacks, by plotting for several values of σ_f the obtained cumulative distribution of ΔT. Doing this graphical analysis, they observe that the probability of large temperature increase $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ is insensitive to σ_f. This observation is easily verifiable: we replicated RB07 cumulative distribution chart in figure 1c, and we computed several values of $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ for $\bar{f} = 0.65$ and σ_f ranging from 0.10 to 0.20, finding it to fluctuate between 0.18 and 0.20. Therefore, in agreement with RB07, it is fair to say that the probability of large temperature increase (i.e. $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$) is quite insensitive to σ_f in this domain. However, concluding from this observation that *"the breadth of the distribution of ΔT is relatively insensitive to decreases in σ_f"* and that *"reducing uncertainty on f has little effect in reducing uncertainty on ΔT"* implicitly assumes two very different definitions of uncertainty: while on the side of feedback the uncertainty is measured by standard deviation σ_f, on the side of sensitivity the probability $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ is used as a metric of uncertainty. As will be developed in section 3, standard deviation is a standard, consensual uncertainty metric but the probability to lie in a fixed interval is not. While under this peculiar double definition of uncertainty RB07 conclusion holds, it is fair to ask whether it would still hold with a different uncertainty metric for ΔT; second, whether the probability to lie in a fixed interval can be considered an acceptable measure of distribution breadth; and third, what are the implications of using such an asymetric definition of uncertainty. The following sections attempt to answer these questions.

3. Standard measurement and propagation of distribution spread

To investigate the first question, which relates to the basic issue of uncertainty definition, we briefly recall a few standard definitions and concepts, as they can be found almost identically in most Descriptive Statistics textbooks. For details, the reader can refer for instance to Barlow, 1989 [@], Van der Vaart, 2000 [@], Reinard, 2006 [@], James and Eadie, 2006 [@] to mention but a few such textbooks.

Descriptive Statistics primary purpose is to provide metrics summarizing a sample of observations and similarly, in probabilistic terms, metrics summarizing the probability density function (pdf) underlying them. Technically, the correspondance between both is simply that a sample summary is an *estimator* (a function of the data) which estimates a distribution summary *estimand* (a parameter). In the present case, we study continuous random variables thus we are rather concerned about pdf metrics than sample metrics, even though these pdfs actually aim at fitting a sample of observations, in that case CMIP3/AR4 GCMs simulations (Meehl et al. [2007]). Descriptive Statistics usually group metrics under three categories: location, scale and shape parameters. The so-called location parameters are meant to identify the center of a distribution. Most common location measures are mean, mode and median. The so-called scale parameters, also referred to as dispersion, variability, variation, scatter or spread measures, describe how far from the above-defined center possible values covered by the distribution tend to be. This second group of metrics is the one we are interested in for our discussion, as it is concerned with the measurement of distribution spread. Most common measures are standard deviation, interquartile range (IQR), range or median absolute deviation (MAD), more rarely full width at half maximum (FWHM). Variance and coefficient of dispersion should also be mentioned though they are not expressed in the same unit as the variable. Above mentioned references give complete mathematical expressions, properties, strengths and limitations of these. We underline a property of particular interest to our discussion: above mentionned measures of spread are invariant in location and linear in scale. In other words, denoting S any particular measure of spread amongst those listed above, X a random variable and $Y = aX + b$ then:

$$S_Y = \mid a \mid \, . S_X \tag{2}$$

Further, in the general case of a dependency of the type $Y = \phi(X)$:

$$S_Y \simeq \mid \phi'(M_X) \mid \, . S_X \tag{3}$$

where ϕ' represents the first derivative of ϕ and M is a location parameter. This linear approximation is commonly used to combine errors on measurements, though generally in its multivariate formulation, and is thus sometimes referred to as the error propagation framework. It may also be used to study the way uncertainty on some input variable(s) propagates into uncertainty on an output obtained from a determinist function, as in section 4.

4. Standard uncertainty propagation in RB07 feedback model

We now analyse the dependency between uncertainty on feedbacks and uncertainty on climate sensitivity in RB07 model. Denoting $S_{\Delta T}$ a measure of climate sensitivity spread,

S_f a measure of feedback spread and M_f a measure of feedback location, the uncertainty propagation recalled in equation (3) can be applied straightforward to equation (1), leading to:

$$S_{\Delta T} \simeq \frac{\Delta T_0}{\left(1-M_f\right)^2} \cdot S_f \tag{4}$$

Note that Equation (4) holds for any choice of pdf for feedback factor f and thus applies more generally than in the particular case of a truncated gaussian pdf chosen by RB07. Equation (4) also provides a simple relationship between $S_{\Delta T}$, S_f and M_f which translates into the following two properties:

• Amplification: In agreement with RB07 first above recalled result, for a fixed level of feedback uncertainty S_f, the level of sensitivity uncertainty $S_{\Delta T}$ is amplified when feedback M_f approaches one. Since estimates of feedback parameters in CMIP3/AR4 models (Soden and Held, 2006 [@], Randall et al. [2007[a]]) suggest M_f is close enough to one ($M_f \simeq 0.65$) and hence yields subtantial amplification, it seems that *"the climate system is operating in a regime in which small uncertainties in feedbacks are amplified in the resulting climate sensitivity uncertainty"*, to quote RB07.

• Proportionality: In disagreement with RB07 second above recalled result, for a fixed level of average feedback M_f, the level of climate sensitivity uncertainty $S_{\Delta T}$ is proportional to the level of feedback uncertainty S_f ($S_{\Delta T} \simeq 9.8\ S_f$ for $M_f \simeq 0.65$). This simple relationship between both uncertainties is intuitive. Indeed, when $S_f = 0$, feedbacks are determinists and ΔT also is, considering no other source of uncertainty in the climate system, hence $S_{\Delta T} = 0$. As values of f get increasingly scattered, resulting values of climate sensitivity also get more scattered proportionally (figure 1a and 1b).

This proportionality has general validity in the sense that it holds for any above-recalled standard spread measure and for any distribution of f. However, it is an approximation for small values of S_f. We therefore find it relevant to investigate how this linear dependency is affected when S_f increases. To perform this analysis, we exhibit more precise results on uncertainty propagation in RB07 model. First, when spread is measured by IQR, an exact relationship holds for any value of S_f and any distribution of f (appendix 2):

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1-M_f\right)^2}\ S_f\ .\ \left\{1 - \frac{w_f}{1-M_f}\ S_f - \frac{1-w_f^2}{4(1-M_f)^2}\ S_f^2\right\}^{-1} \tag{5}$$

where w_f measures the asymetry of f distribution. Hence, when $S \equiv$ IQR, the dependency between $S_{\Delta T}$ and S_f is always overlinear when $w_f \geq 0$, eg when f has a symetric or right skewed distribution. When it is left skewed, the dependency is sublinear for small values of S_f but eventually becomes overlinear when S_f is large enough. Second, when spread is measured by standard deviation, a second order Taylor expansion of equation (1) leads to a more accurate approximation (appendix 3):

$$S_{\Delta T} \simeq \frac{\Delta T_0}{\left(1-M_f\right)^2}\ S_f\ .\ \left\{1 + \frac{2w_f}{1-M_f}\ S_f + \frac{k_f-1}{(1-M_f)^2}\ S_f^2\right\}^{\frac{1}{2}} \tag{6}$$

Again, overlinearity prevails when $w_f \geq 0$ or S_f large enough, which is connected to the convexity of the dependency between ΔT and f. Third, when S is standard deviation and f distribution is log-normal, an exact formula holds for any S_f:

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1-M_f\right)^2}\ .\ S_f\ .\ \left\{1 + \left[\frac{S_f}{1-M_f}\right]^2\right\} \tag{7}$$

and is again overlinear. Finally, overlinear relationships can also be derived when the distribution of f is assumed to be gamma or beta (equations (12) and (14) in appendix 4).

To summarize the above discussion, its main outcome is rather intuitive and has actually few to do with climate: if the spread of feedback factor values decreases, the resulting spread of climate sensitivity values also decreases. Secondly, the dependency is as follows: it is linear for small feedback spreads and tends to get overlinear for larger values. Last, the proportionality coefficient in the dependency sharply increases as feedback intensifies.

5. Properties of the probability to lie in a fixed interval

We now focus on whether the probability to lie in a fixed interval can be considered an acceptable measure of distribution breadth, as implicitly done by RB07 to reach their main conclusion. We approach this question very generally: let X be a continuous random variable with location M_X, spread S_X and pdf p_X. Let $[a, b]$ be a fixed interval near but above the center ($M_X < a < b$). Then, when $S_X \to 0$ the variable becomes determinist ($X = M_X$) and it results that $\mathbb{P}(X \in [a, b])$ equals to zero since $M_X \notin [a, b]$. When $S_X \to +\infty$ the distribution covers such a wide range of values that the probability to exceed any given threshold slowly increases towards 0.5 (figure 2b). In particular $\mathbb{P}(X > a) \to 0.5$ and $\mathbb{P}(X > b) \to 0.5$, hence $\mathbb{P}(X \in [a, b]) = \mathbb{P}(X > a) - \mathbb{P}(X > b) \to 0$ (appendix 5). Hence the dependency between $\mathbb{P}(X \in [a, b])$ and S_X is characterized by a non monotonous function that increases, flattens and then decreases to zero (figure 2a). In light of this non monotonous dependency, it is difficult to hold $\mathbb{P}(X \in [a, b])$ as a valid measure for the width of X distribution. Further, the observed insensitivity of $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ to feedback spread S_f, which lead authors to their conclusion, happens to proceed directly from the above described dependency: this flattening of the dependency is a pure mathematical artefact which systematically manifests under these definitions, and has nothing to do with climate.

Finally, if one still wants to stick to this peculiar, asymetric definition of uncertainty, it has to be noted that in RB07 model, even though the dependency is flat in the domain $S_f \in [0.1, 0.2]$, the dependency is strong for $S_f < 0.1$ when $M_f \approx 0.65$ and subsequently leads to a steep decrease of $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ to zero (figure 1d). In fact, since feedback current estimates suggest $S_f \simeq 0.09$ and $M_f \approx 0.65$ (Soden and Held, 2006 [@], Randall et al. [2007[a]]), the domain of strong dependency may actually already be reached to date.

6. Conclusion

Developments in section 5 suggest that, while the probability $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ may be of interest practically, this metric is irrelevant to describe *"the breadth of the distribution of climate sensitivity"* which was RB07 explicit intent. To adress this question, any measure of distribution spread chosen amongst those clasically used in Descriptive Statistics and recalled in section 3, appear to us more appropriate. With such measures of spread, we showed in section 4 that in RB07 framework, when the spread of feedback parameter S_f decreases, the resulting spread of climate sensitivity $S_{\Delta T}$ values also decreases. Further, we also highlighted that in this framework, the decrease is approximately linear for S_f small and tends to be overlinear (i.e. to be steeper) for larger values of S_f owing to the convexity of the dependency between ΔT and f.

Other than the definition issue discussed here, the relevance of RB07 simplified model to describe the dependency between climate sensitivity and feedbacks may also be discussed but this was beyond the scope of this article. In any case, if one holds this model to be accurate, a decrease of the spread on feedback will lead to a decrease of the uncertainty on climate sensitivity and a narrowing of the enveloppe of future climate projections. If enough studies are undertaken to better understand and assess the physical processes involved in the different feedbacks, neither are doomed to remain at their current level.

Appendix

1 – Implications of the truncation

Since the linear feedback model of RB07 implicitly assumes $f \leq 1$, the gaussian distribution $\mathcal{N}(\bar{f}, \sigma_f)$ proposed by RB07 is implicitly truncated for $f > 1$ – otherwise equation (1) would produce negative values of ΔT. This truncation has several implications. First, σ_f (resp. $\bar{f}$) does not exactly match standard deviation (resp. mean) of the truncated distribution. For instance, when $(\bar{f}, \sigma_f) = (0.75, 0.25)$ the standard deviation of f equals 0.18 and its mean equals 0.67. Second, it introduces some negative skewness in the distribution of f (-0.39 in the same example) which becomes more and more asymetric as σ_f and $\bar{f}$ increases. Finally, since the truncated gaussian pdf is finite and non zero in the vicinity of $f = 1$, the obtained pdf of climate sensitivity behave as a Pareto distribution in $\mathcal{O}(\Delta T^{-2})$ for high values, and hence does not have a finite mean, nor a finite variance. Hence, the truncated gaussian model of RB07 forbids the use of standard deviation as a measure of climate sensitivity spread, which explains the use of IQR in figure 1. For the purpose of RB07 which is to study climate sensitivity spread, assuming a parametric distribution of f – such as log-normal, gamma or beta – which leads to finite mean and deviation for sensitivity and exact mathematical expressions of the dependency between the deviation of ΔT and the deviation of f (appendix 3), would be in our view more convenient. However, the results on the dependency between $S_{\Delta T}$ and S_f presented in section 4 are general and also hold under RB07 gaussian assumption. Therefore, RB07 truncated gaussian is in our view mathematically unconvenient, but it does not affect uncertainty propagation: for a gaussian distribution just as for any other, the spread dependency is approximately linear for small spreads and overlinear otherwise, as equation (4) and (5) demonstrate and as figure 1b illustrates.

2 – Exact uncertainty propagation equation for IQR

If X is a continuous random variable X, we denote X_α its $\alpha-$quantile, $S_X = X_{0.75} - X_{0.25}$ its interquantile range, $M_X = X_{0.50}$ its median and $w_X = \frac{X_{0.75}+X_{0.25}-2\,X_{0.50}}{X_{0.75}-X_{0.25}}$ a dimensionless, quantile-based metric of asymetry. We thus have $X_{0.75} = M_X + \frac{1}{2}S_X(1+w_X)$ and $X_{0.25} = M_X - \frac{1}{2}S_X(1-w_X)$. Since when Φ is a diffeomorphism, we also have $[\Phi(X)]_\alpha = \Phi(X_\alpha)$, hence from (1):

$$\begin{aligned} S_{\Delta T} &= \Delta T_{0.75} - \Delta T_{0.25} = \tfrac{\Delta T_0}{(1-f_{0.75})} - \tfrac{\Delta T_0}{(1-f_{0.25})} = \tfrac{\Delta T_0}{(1-f_{0.75})(1-f_{0.25})}\; S_f \\ &= \tfrac{\Delta T_0}{\left(1-M_f\right)^2}\; S_f \,.\, \left\{1 - \tfrac{w_f}{1-M_f}\, S_f - \tfrac{1-w_f^2}{4(1-M_f)^2}\; S_f^2\right\}^{-1} \end{aligned}$$

3 – Second order term in uncertainty propagation equation

Assuming $Y = \phi(X)$, we analyse the way the approximation of the relationship between both spread measures S_Y and S_X is modified when a second order term is introduced in the Taylor development of ϕ about M_X:

$$Y \simeq \phi(M_X) + \phi'(M_X)(X - M_X) + \tfrac{1}{2}\phi''(M_X)(X - M_X)^2 \qquad (8)$$

When the chosen spread measure S is standard deviation, calculations can be performed explicitly:

$$S_Y \simeq \mid \phi'(M_X) \mid \, . S_X \, . \left\{1 + \left[\frac{\phi''(M_X)}{\phi'(M_X)} w_X\right] S_X + \left[\frac{\phi''(M_X)^2}{4\phi'(M_X)^2}(k_X - 1)\right] S_X^2 \right\}^{\frac{1}{2}} \qquad (9)$$

Equation (9) shows that non linear terms in the resulting relationship between S_Y and S_X depends on the shape of the distribution $p(x)$ through its skewness w_X (a dimensionless measure of assymetry) and kurtosis k_X (a dimensionless measure of peakedness), and on the shape of function ϕ through the curvature factor $\frac{\phi''(M_X)}{\phi'(M_X)}$ (the rate of increase of the slope in M_X). A remarkable consequence of equation (9) is that when X distribution is symetric ($w_X = 0$) and since kurtosis always exceeds one (Jensen inequality) hence the dependency of S_Y to S_X is always over linear. Actually, sublinearity would require quite special conditions: a distribution $p(x)$ with low kurtosis and high skewness, simultaneously with a function ϕ characterized by strong curvature with sign opposite to skewness.

Applying equation (9) to model (1), it follows:

$$S_{\Delta T} \simeq \frac{\Delta T_0}{\left(1 - M_f\right)^2} \, S_f \, . \left\{1 + \frac{2 w_f}{1 - M_f} \, S_f + \frac{k_f - 1}{(1 - M_f)^2} \, S_f^2 \right\}^{\frac{1}{2}} \qquad (10)$$

4 – Exact uncertainty propagation equations for standard deviation

Since the domain of value of f in RB07 model is $] - \infty, 1]$, we assume single tailed distributions defined on this support to avoid a truncation and make mathematical developments more convenients. For several usual distributions, the relationship between $S_{\Delta T}$ and S_f can thus be explicited. Assuming a log-normal distribution with pdf $\frac{1}{(1-f)\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln(1-f)-\mu)^2}{2\sigma^2}\right]$, mean $M_f = 1 - e^{\mu + \frac{\sigma^2}{2}}$ and variance $S_f^2 = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$ we obtain $S_{\Delta T}^2 = \Delta T_0^2 \,.\, e^{-2\mu+\sigma^2}(e^{\sigma^2} - 1)$. Recombining :

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1-M_f\right)^2} \,.\, S_f \,.\, \left\{1 + \left[\frac{S_f}{1-M_f}\right]^2\right\} \tag{11}$$

Assuming a gamma distribution with pdf $(1-f)^{k-1} \frac{\exp\left(-(1-f)/\theta\right)}{\Gamma(k)\,\theta^k}$, mean $M_f = 1 - \theta k$ and variance $S_f^2 = \theta^2 k$, we obtain $S_{\Delta T}^2 = \Delta T_0^2 \,.\, [\theta^2(k-1)(k-2)]^{-1}$. Recombining :

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1-M_f\right)^2} \,.\, S_f \,.\, \left\{1 - \left[\frac{S_f}{1-M_f}\right]^2\right\}^{-1} \,.\, \left\{1 + \left[\frac{S_f}{1-M_f}\right]^2\right\}^{-\frac{1}{2}} \tag{12}$$

Assuming a beta distribution with pdf $\frac{\Gamma(2k)}{\theta\Gamma(k)^2}\left(1 - \frac{1-f}{\theta}\right)^{k-1}\left(\frac{1-f}{\theta}\right)^{k-1}$ on $[1-\theta, 1]$, mean $M_f = 1 - \frac{\theta}{2}$ and variance $S_f^2 = \theta^2[8k+4]^{-1}$, we obtain $S_{\Delta T}^2 = \Delta T_0^2 \,.\, [k(2k-1)] \,.\, [\theta^2(k-1)^2(k-2)]^{-1}$. Recombining :

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1-M_f\right)^2} \,.\, S_f \,.\, \left\{1 - \left[\frac{S_f}{1-M_f}\right]^2\right\}^{\frac{1}{2}} \left\{1 - 2\left[\frac{S_f}{1-M_f}\right]^2\right\}^{\frac{1}{2}} \left\{1 - 3\left[\frac{S_f}{1-M_f}\right]^2\right\}^{-1} \left\{1 - 5\left[\frac{S_f}{1-M_f}\right]^2\right\}^{-\frac{1}{2}} \tag{13}$$

5 – Dependency between spread and probability weight of an interval

Assume X_1 is a random real variable with pdf $p_1(x)$, cdf $P_1(x)$, center M_1 and spread $S_1 > 0$. Let $[a, b]$ be a fixed interval near but above the center (eg $M_1 < a$). For $\lambda > 0$, we introduce $X_\lambda = \lambda(X_1 - M_1) + M_1$, which has pdf $\frac{1}{\lambda}\, p(\frac{x-M_1}{\lambda} + M_1)$, cdf $P(\frac{x-M_1}{\lambda} + M_1)$, center M_1 and spread λS_1. To analyse the dependency between the probability of a real variable to fall in $[a, b]$ and the spread of its underlying distribution, we study $F(\lambda; a, b) = \mathbb{P}(X_\lambda \in [a, b])$. F can be expressed using the cdf of X_λ:

$$\begin{array}{ll} F(\lambda; a, b) = P(\frac{b-M_1}{\lambda} + M_1) - P(\frac{a-M_1}{\lambda} + M_1) & \\ F(0; a, b) = P(-\infty) - P(-\infty) = 0 & \text{since } M_1 < a < b \\ F(+\infty; a, b) = P(M_1) - P(M_1) = 0 & \end{array} \tag{14}$$

Since $F(0; a, b) = F(+\infty; a, b) = 0$, and $F \geq 0$, then F reaches a maximum, and it has the general pattern mentioned in the text. It is also straightforward to obtain that $F(\lambda; a, b) \sim \frac{(b-a)p_1(M_1)}{\lambda^2}$ for large λ.

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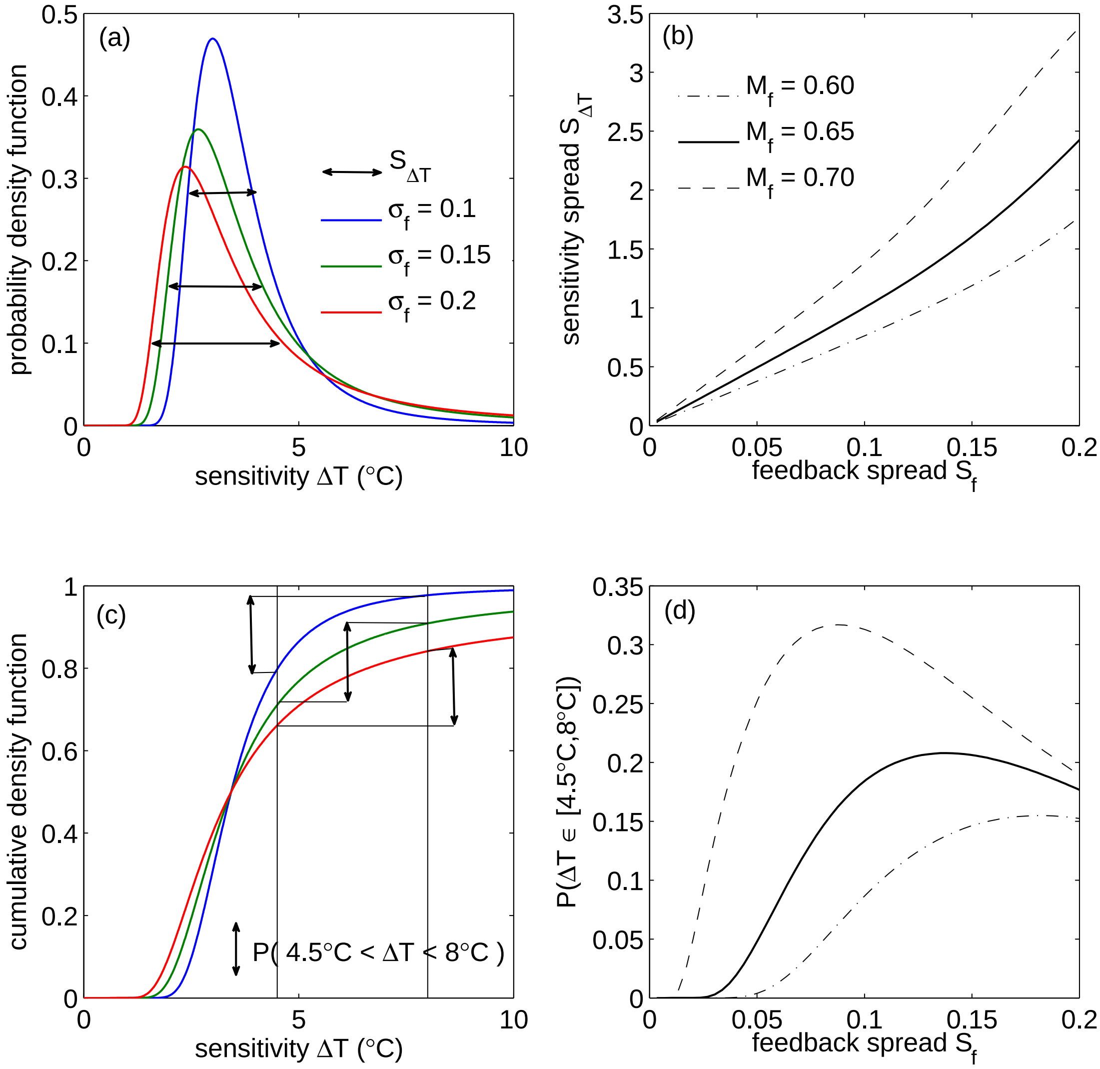
Figure 1 – In all charts, f is truncated gaussian $\mathcal{N}(M_f, \sigma_f)$ as in RB07. Upper left panel (a): pdf of ΔT with $M_f = 0.65$ and $\sigma_f = 0.20$, 0.15, 0.10. Arrows represent the decreasing sensitivity spread $S_{\Delta T}$ obtained for decreasing values of σ_f. Upper right panel (b): climate sensitivity spread $S_{\Delta T}$ as a function of feedback spread S_f, for $M_f = 0.60$, 0.65, 0.70. Feedback spread S_f is measured by standard deviation ($\simeq \sigma_f$) but climate sensitivity spread $S_{\Delta T}$ is measured by IQR (see appendix 1 for explanation). Lower left panel (c): cdf of ΔT. Arrows represent the stable probability $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ obtained for decreasing values of $\sigma_f = 0.20$, 0.15, 0.10. Lower right panel (d): probability $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ as a function of feedback spread S_f, spread measured with IQR.

Figure 2 – X is centered gaussian with standard deviation S_X. Right panel: probability for X to exceed respectively 1 and 3, as functions of S_X. Left panel: probability for X to fall within interval $[1, 3]$ as a function of S_X.



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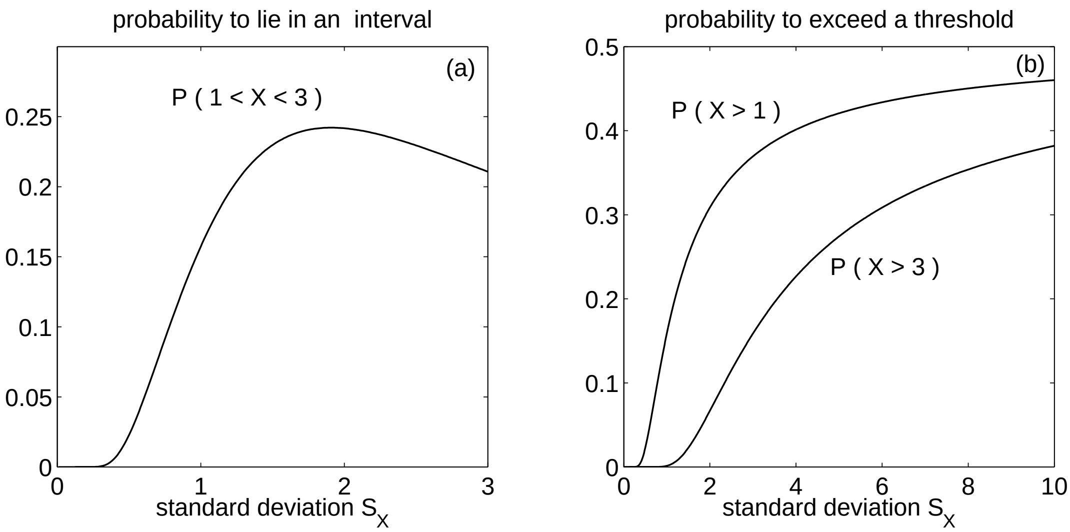

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