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Abstract

This paper develops an equilibrium model of vertical mergers. We show that competition on an upstream market between integrated firms only is less intense than in the presence of unintegrated upstream firms. Indeed, when an integrated firm supplies the upstream market, it becomes a soft downstream competitor to preserve its upstream profits. This benefits other integrated firms, which may therefore choose not to cut prices on the upstream market. This mechanism generates waves of vertical mergers in which every upstream firm integrates with a downstream firm, and the remaining unintegrated downstream firms obtain the input at a high upstream price. We show that these anticompetitive vertical mergers waves are more likely when downstream competition is fiercer.

1   Introduction

The anticompetitive effects of vertical mergers have long been a hotly debated issue among economists. Until the end of the 1960s, the traditional vertical foreclosure theory was widely accepted by antitrust practitioners. According to this theory, vertical mergers were harmful

to competition, since vertically integrated firms had incentives to raise their rivals' costs. This view was seriously challenged by Chicago school authors in the 1970s, notably Bork (1978) and Posner (1976), on the ground that firms cannot leverage market power from one market to another. A more recent strategic approach of the subject, initiated by Ordover, Saloner and Salop (1990) and Hart and Tirole (1990), shows how vertical integration might relax competition. A number of papers has established that vertical mergers have anticompetitive effects under some specific assumptions including extra commitment power for vertically integrated firms (Ordover, Saloner and Salop, 1990), choice of input specification (Choi and Yi, 2000), switching costs (Chen, 2001), tacit collusion (Nocke and White, 2007), exclusive dealing (Chen and Riordan, 2007).

In this paper we show that vertical mergers can be harmful to competition even in the absence of such assumptions. More precisely, a wave of vertical mergers that eliminates all the unintegrated upstream firms may lead to a monopoly-like outcome on the upstream market. This is because competition on the upstream market between vertically integrated firms only – a market structure the literature has surprisingly overlooked – can be ineffective.

In our model there are initially two upstream firms and three downstream firms. First, the downstream firms bid to integrate backward with the first upstream firm. Then, if a merger has taken place, the remaining unintegrated downstream firms bid to acquire the second upstream firm. Upstream firms (integrated or not) then compete in prices to sell the intermediate input to the remaining unintegrated downstream firms. Finally, downstream firms (integrated or not) compete in prices with differentiated products. The upstream market exhibits the usual ingredients of tough competition: upstream firms compete in prices, produce a perfectly homogeneous upstream good and incur the same constant marginal cost. When there has been zero or one vertical merger, the standard Bertrand logic applies and upstream competition drives the upstream price to the marginal cost.

When two mergers have taken place, however, the Bertrand logic can collapse. The intuition is following. There are now two vertically integrated firms, called $U1 - D1$ and $U2 - D2$, and one unintegrated downstream firm, called $D3$. Assume that $U1 - D1$ sells the intermediate input to $D3$ at a strictly positive price-cost margin, and consider the incentives of its integrated rival $U2 - D2$ to corner the upstream market. Notice first that, when $U1 - D1$ increases its downstream price, it recognizes that some of the final consumers it loses will eventually purchase from $D3$, thereby increasing upstream demand and revenues. Therefore, supplying the upstream market strengthens an integrated firm's incentives to be a soft competitor on the downstream market; we refer to this effect as the *softening effect.*

The softening effect benefits $U2 - D2$, which faces a less aggressive competitor on the final market. Now, if $U2 - D2$ undercuts $U1 - D1$ on the upstream market and becomes the upstream supplier, then $U1 - D1$ stops being a softening competitor on the downstream market. To sum up, integrated firm $U2 - D2$ faces the following trade-off when deciding whether to undercut: on the one hand, undercutting yields upstream profits; on the other hand, it makes integrated firm $U1 - D1$ more aggressive on the downstream market. When the latter effect dominates, the incentives to undercut vanish and the Bertrand logic collapses.

We exhibit equilibria in which there are two vertical mergers, one of integrated firm charges its monopoly upstream price, and its integrated rival decides rationally to make no upstream offer. These monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria from the integrated firms' point of view. They are also the only equilibria which do not involve weakly dominated strategies. Besides, we show that partial foreclosure equilibria degrade both social welfare and consumers' surplus.

Using linear demand functions, we show that downstream competition is fiercer, the weaker competition on the upstream market. Intuitively, when downstream products are good substitutes, the softening effect is strong since a firm's downstream price has a large impact on its rivals' demands. Thus, undercutting on the upstream market is not profitable and the monopoly-like outcome is an equilibrium. Conversely, when downstream products are strongly differentiated, the softening effect is weak and undercutting on the upstream market is always profitable.

Our analysis can shed light on the recent wave of vertical mergers in the satellite navigation industry. The only two (upstream) firms that provide navigable digital maps, Tele Atlas and Navteq, have been acquired by, respectively, (downstream) TomTom and Nokia. TomTom embed digital maps in its portable navigation devices, Nokia in its mobile handsets with navigation possibilities. Our model suggests that, as long as mobile phones and portable navigation devices remain rather imperfect substitutes, competition between digital map providers should not be harmed. However, when these products become increasingly substitutes over time, as it is envisaged by the European Commission, the softening effect should strengthen and upstream competition might weaken.

Our paper contributes to the literature on the competitive effects of vertical mergers. A strand of the literature can be summarized in a common framework with two upstream firms, two downstream firms, and price competition on both markets.[1] Ordover, Saloner and Salop

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[1]Exceptions include an early contribution by Salinger (1988) who considers Cournot competition on both markets, and the strand of the literature initiated by Hart and Tirole (1990) which analyzes the consequences of upstream secret offers and focuses mainly on the commitment problem faced by an upstream monopolist.

(1990) show that a vertical integration raises the upstream price when the integrated firm can commit to exiting the upstream market and letting the remaining unintegrated upstream firm monopolize the upstream market. As pointed out by Reiffen (1992), their analysis relies crucially on the assumption that the merged entity has a strong commitment power. Choi and Yi (2000) provide foundations for this commitment power through the choice of input specification. Nocke and White (2007) and Normann (2009) provide another justification for the commitment assumption; they show that the commitment can be enforced in an infinitely repeated game through tacit collusion, and that a vertical merger facilitates the enforcement of the commitment. Chen and Riordan (2007) argue that vertical integration and exclusive contracts complement each other to implement partial foreclosure.

The softening effect that shows up in our model has been unveiled by Chen (2001).[2,3] He shows that when there is one vertical merger, the remaining downstream firm prefers purchasing the input from the integrated firm than from the unintegrated upstream firm in order to benefit from the softening effect. If there are upstream cost asymmetries and upstream switching costs, then the unintegrated upstream firm is unable to undercut the integrated firm on the upstream market and there is partial foreclosure in equilibrium. Our result is different. We show that in the two-merger situation the integrated rival is *able* to undercut since we assume away any cost differential or switching cost, but it is not *willing* to do so. Our result supports the classical analysis of Ordover, Saloner and Salop (1990), for we show that no commitment is actually necessary to sustain the monopoly outcome when the softening effect is strong enough.

The rest of the paper is organized as follows. Section 2 describes the model. We solve the upstream-downstream competition subgames in Section 3 and the vertical integration game in Section 4. We present several extensions and robustness checks in Section 5 and conclude in Section 6 by discussing the recent wave of vertical mergers in the satellite navigation industry.

2   Model

We consider a vertically related industry with two identical upstream firms, $U1$ and $U2$, and three symmetric downstream firms, $D1$, $D2$ and $D3$. The upstream firms produce an homogeneous input at constant marginal cost m and supply it to the downstream firms. The

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[2]Chen (2001) refers to it as the collusive effect. We adopt a different terminology to make it clear that the softening effect does not involve any form of tacit or overt collusion.

[3]See also Fauli-Oller and Sandonis (2002) for an application of the softening effect in a licensing context.

downstream firms transform the intermediate input into a differentiated final product on a one-to-one basis at zero cost. The input can also be obtained from an alternative source at a constant marginal cost $\overline{m} > m$.[4]

The demand for downstream firm Di's product, $i = 1, 2, 3$, is $q_i(p_1, p_2, p_3)$, where p_j denotes Dj's price. The demand addressed to a firm is decreasing in its own price and increasing in its competitors' prices: $\partial q_i/\partial p_i \leq 0$ (with a strict inequality if $q_i > 0$) and $\partial q_i/\partial p_j \geq 0$ (with a strict inequality if $q_i > 0$ and $q_j > 0$), for $i \neq j$ in $\{1, 2, 3\}$. We assume that these demand functions are twice continuously differentiable. Symmetry between downstream firms implies that Di's demand can be written as $q_i = q(p_i, p_{-i})$, where p_{-i} denotes the set of prices charged by Di's rivals and $q(., .)$ is the same for all downstream firms.

We now describe the four-stage game played by the firms. In the first stage, the three downstream firms can bid to acquire upstream firm $U1$. In the second stage, if a merger has occurred, the remaining unintegrated downstream firms can counter it by bidding to integrate backward with $U2$. In the third stage, each upstream firm (integrated or not) Ui, $i = 1, 2$, announces the price w_i at which it is ready to supply any unintegrated downstream firm.[5,6] Unintegrated downstream firms then choose from which upstream producer to purchase. Downstream prices are set in the fourth stage. Unintegrated downstream firms are allowed to switch to another upstream supplier at zero cost once downstream prices are set, if this is strictly profitable.[7] To avoid trivial situations, we also consider that a firm decides to merge if it is strictly preferred. This would obviously be the case whenever mergers involve transaction costs. We look for subgame-perfect pure strategy Nash equilibria.

For all the market structures studied in this article and for i in $\{1, 2, 3\}$, we denote by π_i the total profit made by Di (including the profit made by its upstream subsidiary if it is integrated). We make the following assumptions:

(i) Firms' best responses on the downstream market are unique and defined by the first order conditions $\partial \pi_i/\partial p_i = 0$.

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[4]This assumption is also made, e.g., by Ordover, Saloner and Salop (1990) and Hart and Tirole (1990). We relax it in Section 5. The alternative source of supply can come from a competitive fringe of inefficient upstream firms.

[5]The internal transfer price for a vertically integrated firm is irrelevant since it cancels out in the expression of its total profit.

[6]Notice that discrimination is not possible on the upstream market and that only linear tariffs are used. We relax these assumptions in Section 5.

[7]As we explain in Section 5, this assumption simplifies the analysis by ensuring that downstream firms always buy the input from the cheapest supplier. This is in contrast to Chen (2001), in which upstream switching costs, together with an upstream cost asymmetry, generate anticompetitive vertical mergers.

(ii) There exists a unique Nash equilibrium on the downstream market.

(iii) Prices are strategic complements: for all $i \neq j$ in $\{1, 2, 3\}$, $\partial^2 \pi_i / \partial p_i \partial p_j \geq 0$.

Assumption (i) together with (iii) implies that the best response function of a firm is increasing in its rivals' prices. Combining (ii) with (iii), we also get that the unique downstream equilibrium is stable.[8] Finally, we assume that $\overline{m}$ is a relevant outside option: whatever the market structure, an unintegrated downstream firm earns strictly positive profits if it buys the intermediate input at a price lower than or equal to $\overline{m}$.

3 Upstream-Downstream Equilibrium

3.1 Upstream-Downstream Equilibrium With No Merger

Consider that no merger has taken place. Since downstream firms can switch to another supplier at zero cost after downstream prices are set, they always eventually buy from the cheapest supplier. Consequently the upstream equilibrium features both unintegrated upstream firms charging m and making no profit. The three downstream firms compete on a level playing field and earn the same profit, denoted by π^*.[9]

Lemma 1. *When no merger has taken place, the unique equilibrium outcome on the upstream market is the Bertrand outcome: $w_1 = w_2 = m$.*

3.2 Upstream-Downstream Equilibrium With One Merger

We now consider that exactly one vertical merger has taken place. Without loss of generality, we assume that $D1$ has merged with $U1$ to form integrated firm $U1 - D1$. We establish that the upstream market is supplied at marginal cost in equilibrium.

First, it cannot be that both unintegrated downstream firms purchase the upstream good from $U1 - D1$ at price $w_1 > m$, or from the alternative source of input at $\overline{m}$. Otherwise $U2$ would obviously undercut since the upstream market is its sole source of profit.

Conversely, if $U2$ serves the upstream market at price $w_2 > m$, then $U1 - D1$ is willing to undercut. First, this brings in upstream profits. Second, becoming the upstream supplier modifies the downstream outcome in a way that is, as we now show, profitable to $U1 - D1$. Consider indeed that downstream firms $D2$ and $D3$ buy the input at an upstream price

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[8]See Vives (1999), p.54.
[9]Formally, $\pi^* = (p^* - m)q(p^*, p^*, p^*)$, where $p^* = \arg\max_p(p - m)q(p, p^*, p^*)$.

$w > m$. If $U2$ is their upstream supplier, the profits are given by $\pi_1 = (p_1 - m)q_1$ and $\pi_j = (p_j - w)q_j$, for $j \in \{2, 3\}$. If they purchase from $U1 - D1$, the profit of $U1 - D1$ becomes $\pi_1 = (p_1 - m)q_1 + (w - m)q_2 + (w - m)q_3$ and the profits of $D2$ and $D3$ are unchanged. Moving from the former to the latter situation, $U1 - D1$'s first order condition shifts from

$$q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} = 0$$

to

$$q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} + (w - m)\frac{\partial q_2}{\partial p_1} + (w - m)\frac{\partial q_3}{\partial p_1} = 0.$$

Then its best response function moves upwards. As already pointed out by Chen (2001), when $U1 - D1$ supplies the upstream market, it realizes that any customer lost on the downstream market may be recovered *via* the upstream market. This provides it with additional incentives to raise its price. Strategic complementarity, in turn, leads $D2$ and $D3$ to charge higher prices as well. At the end of the day, all downstream prices are higher when $U1 - D1$ becomes the upstream supplier, and in particular those of $D2$ and $D3$. This makes $U1 - D1$ better off on the downstream market.

This mechanism has a fat-cat flavor (Fudenberg and Tirole, 1984): being the upstream supplier and thus a soft downstream competitor, $U1 - D1$ relaxes downstream competition thanks to strategic complementarity. Moreover, the soft behavior of $U1 - D1$ benefits its downstream rivals. In the following, we shall refer to the consequences of $U1 - D1$'s soft behavior as the *softening effect.*

Finally, the upstream market cannot be shared between $U1 - D1$ and $U2$ at $w_1 = w_2 > m$ neither, since both unintegrated downstream firms would prefer purchasing the intermediate input from $U1 - D1$ to benefits from the softening effect. It is also straightforward to see that the upstream market cannot be supplied at a price $w < m$, and we obtain the following lemma.

Lemma 2. *When exactly one merger has taken place, the unique equilibrium outcome on the upstream market is the Bertrand outcome: $w_1 = w_2 = m$.*

Proof. See Appendix A.2 □

In equilibrium, all firms obtain the input at the same price m, and there are no upstream profits. Since downstream competition is not modified with respect to the no-merger case, the three downstream firms earn the same profits: $\pi_1 = \pi_2 = \pi_3 = \pi^*$.

7

3.3 Upstream-Downstream Equilibrium With Two Mergers

In this section we assume that two vertical mergers have taken place. Let us suppose, without loss of generality, that $D1$ has merged with $U1$, and $D2$ has merged with $U2$, giving birth to two integrated firms $U1 - D1$ and $U2 - D2$.

$U1 - D1$ and $U2 - D2$ compete to sell the input to unintegrated downstream firm $D3$. Since $D3$ can obtain the input at cost $\overline{m}$ and always chooses the cheapest supplier, any upstream offer strictly above $\overline{m}$ is equivalent to no offer. The strategy space on the upstream market can therefore be restricted to $[0, \overline{m}] \cup \{+\infty\}$, where an infinite price stands for no offer.

There is no equilibrium in which firm $D3$ obtains the input from the alternative supplier. Otherwise a vertically integrated firm would undercut to get upstream profits and relax competition on the downstream market through the softening effect. Therefore, the determination of the upstream equilibrium only requires to analyze the downstream equilibrium when firm $D3$ is supplied by a vertically integrated firm.

Downstream equilibrium. We assume, without loss of generality, that firm $D3$ purchases the input from firm $U1 - D1$ at price $w \in [0, \overline{m}]$. The profit functions can be written as

$$\pi_1 = (p_1 - m)q_1 + (w - m)q_3, \tag{1}$$

$$\pi_2 = (p_2 - m)q_2, \tag{2}$$

$$\pi_3 = (p_3 - w)q_3. \tag{3}$$

The equilibrium downstream prices, denoted by $p_i(w)$ for i in $\{1, 2, 3\}$, solve the set of first order conditions

$$q_1 + (p_1 - m)\frac{\partial q_1}{\partial p_1} + (w - m)\frac{\partial q_3}{\partial p_1} = 0, \tag{4}$$

$$q_2 + (p_2 - m)\frac{\partial q_2}{\partial p_2} = 0, \tag{5}$$

$$q_3 + (p_3 - w)\frac{\partial q_3}{\partial p_3} = 0. \tag{6}$$

We also denote by $\pi_i(w)$ the equilibrium profits. Last, define $\pi_i(+\infty)$ the equilibrium profits when $D3$ gets the input from the alternative source.

The comparison of the first order conditions of both vertically integrated firms indicates that the upstream supplier has more incentives to raise its downstream price. This is again

the softening effect. When $U1-D1$ charges a higher downstream price, some of the customers it loses will eventually purchase from $D3$, which increases its upstream revenues. Together with the stability of the downstream equilibrium, this implies that upstream supplier $U1-D1$ ends up charging a higher downstream price than $U2-D2$.

Firm $U2-D2$ benefits from firm $U1-D1$'s being a soft competitor on the downstream market. As a result, it earns larger downstream profits than the upstream supplier. These insights are summarized in the following lemma.

Lemma 3. *If $w > m$, then $U1-D1$ charges a strictly higher downstream price and earns strictly lower downstream profits than $U2-D2$,*

$$p_1(w) > p_2(w), \tag{7}$$
$$(p_1(w) - m)q_1(p_1(w), p_2(w), p_3(w)) < (p_2(w) - m)q_2(p_1(w), p_2(w), p_3(w)). \tag{8}$$

Proof. See Appendix A.3. $\qquad\qquad\square$

An important consequence of this result is that we cannot tell unambiguously which of the integrated firms earns more total profits. On the one hand, the upstream supplier extracts revenues from the upstream market, on the other hand, its integrated rival benefits from larger downstream profits thanks to the softening effect. It may well be that the latter effect is strong enough to outweigh the upstream profit effect and make $U2-D2$ earn more total profits than its rival.

Upstream equilibrium. There is an equilibrium in which firm $U1-D1$ offers $w_1 \leq \overline{m}$ and $U2-D2$ offers $w_2 \geq w_1$ if, and only if, the upstream supplier does not want to serve the upstream market at another price

$$\pi_1(w_1) \geq \max_{w < w_2, w \leq \overline{m}} \pi_1(w),$$

nor to exit the upstream market

$$\pi_1(w_1) \geq \pi_2(w_2),$$

and its vertically integrated rival is not willing to undercut

$$\pi_2(w_1) \geq \max_{w < w_1} \pi_1(w).$$

Note that w_2 can be infinite in the above expressions.

9

This set of necessary and sufficient conditions generally characterize multiple equilibria. As common sense suggests, there always exists an upstream equilibrium in which the input is priced at its marginal cost.

Lemma 4. *When two mergers have taken place, the Bertrand outcome on the upstream market is an equilibrium outcome.*

Proof. See Appendix A.4. □

However, partial foreclosure equilibria with an upstream price strictly above marginal cost can also exist. Consider that firm $U1 - D1$ supplies the upstream market at the monopoly upstream price $w_m = \arg\max_{w \leq \overline{m}} \pi_1(w).$[10] w_m is the price that a vertically integrated firm would charge if, for an exogenous reason, its integrated rival had made no upstream offer. As formally shown in Appendix A.5, it satisfies $w_m > m$ since charging an upstream price strictly above the marginal cost brings in upstream profits and relaxes downstream competition thanks to the softening effect and strategic complementarity.

Consider the incentives of $U2 - D2$ to corner the upstream market. On the one hand, this would generate upstream revenues. On the other hand, $U1 - D1$ would stop being a soft downstream competitor, which would lower $U2 - D2$'s downstream profits by Lemma 3. When the latter effect dominates the former, the proposed monopoly-like outcome on the upstream market is an equilibrium.

Proposition 1. *When two mergers have taken place, there is an equilibrium in which one integrated firm proposes w_m and the other integrated firm makes no offer if, and only if,*

$$\pi_1(w_m) \leq \pi_2(w_m). \tag{9}$$

From the integrated firms' point of view, these monopoly-like equilibria, when they exist,

- *Pareto-dominate all other equilibria,*

- *are the only equilibria involving no weakly dominated strategies.*

Proof. See Appendix A.6. □

When the softening effect is large enough so that condition (9) holds, the hypothetical situation in which one of the integrated firm has exogenously exited the upstream market,

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[10]We assume, without loss of generality, that this price is unique. Defining w_m as $\max\{\arg\max_{w \leq \overline{m}} \pi_1(w)\}$, our results would still hold if $\pi_1(.)$ reached its maximum for several values of w.

granting a monopoly position to the other integrated firm, is an equilibrium. This might sound somewhat tautological. Yet, our contribution is to show that condition (9) may well be satisfied, because losers on the upstream market become winners on the downstream market. Note also that monopoly-like equilibria come by pairs since the upstream supplier can be either $U1 - D1$ or $U2 - D2$.

Proposition 1 gives foundations to the classical analysis of Ordover, Saloner and Salop (1990), in which a vertically integrated firm commits to exiting the upstream market in order to let the upstream rival charge the monopoly price. We show that no commitment is actually necessary when the upstream rival is integrated, provided that the softening effect is strong enough.

All other equilibria feature both vertically integrated firms setting the same upstream price (and only one of them actually supplying the market). Such an outcome is part of an equilibrium only if the softening effect and the upstream upstream profit effect exactly cancel out, so that the upstream supplier earns as much profits as the vertically integrated firm which does not supply the upstream market. Formally: $\pi_1(w) = \pi_2(w)$. The Bertrand outcome is one such symmetric equilibrium. Other symmetric equilibria can also feature an upstream price strictly above m, as well as strictly below m.[11]

This multiplicity of equilibria can be resolved using standard selection criteria. First, the monopoly-like equilibria Pareto-dominate all the symmetric equilibria from the integrated firms' standpoint. Indeed, the upstream price in a symmetric equilibrium is always smaller than w_m, otherwise one of the vertically integrated firm would set w_m. This implies that, in the symmetric equilibria, the vertically integrated firms earn less than $\pi_1(w_m)$, which is lower than $\pi_2(w_m)$.

Second, the monopoly-like equilibria are the only equilibria involving no weakly dominated strategies. In particular, any symmetric equilibrium strategy is weakly dominated by w_m. Therefore, it seems reasonable to think that integrated firms will coordinate on one of the monopoly-like equilibria.

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[11]When $w < m$, upstream profits are negative and the softening effect is reversed, with the upstream supplier adopting an aggressive stance on the downstream market to limit its upstream losses.

4 Equilibrium Vertical Mergers

4.1 Anticompetitive Mergers Wave

In monopoly-like equilibria the profits of the two merged entities are strictly larger than the profit of the remaining unintegrated firm. Therefore, when a monopoly-like outcome is expected to emerge after two mergers, downstream firms all bid to get a share of the profit created by the mergers. Then, a wave of vertical mergers occurs that eliminates all unintegrated upstream firms and implements the monopoly-like outcome.[12]

Although standard selection criteria indicate that the monopoly-like equilibria are likely to emerge when they exist, extra-model considerations may modify that presumption. For instance, a firm may have troubles with the competition authority, it may try to drive some of its rivals out of business, or it may be willing to develop a reputation of tough competitor. Besides, the Bertrand outcome is always an equilibrium of the upstream market by Lemma 4. Therefore, if we use no selection criterion, there always exists an anticipation scheme that leads to an equilibrium with no merger. This discussion is summarized in

Proposition 2. *There always exists an equilibrium with no merger.*
If $\pi_1(w_m) \leq \pi_2(w_m)$ and integrated firms

- *do not play weakly dominated strategies on the upstream market*

- *or do not play equilibria that are Pareto-dominated by another equilibrium,*

then, in equilibrium, there are two mergers and the upstream market is supplied at the monopoly price.

Proof. See Appendix A.7. □

We argue that a wave of vertical mergers that leads to partial foreclosure of the remaining downstream firm hurts consumers and lowers social welfare. Indeed, when the upstream price increases above the marginal cost in the two-merger subgame, the best response functions of $U1 - D1$ and $D3$ shift upwards, which increases all downstream prices by strategic complementarity. This is clearly detrimental to all consumers. This also degrades social welfare, since the total demand is already too low in the Bertrand outcome because of positive markups on the downstream market.

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[12]We do not discuss the equilibrium bids, since they depend strongly on the assumption that upstream firms have all the bargaining power.

Proposition 3. *Consumers' surplus and social welfare are strictly lower in an equilibrium with a mergers wave and partial foreclosure of the remaining downstream firm than in an equilibrium with no merger.*

Proof. See Appendix A.8.   □

4.2   Tension Between Upstream and Downstream Competition

We now present an example to illustrate when condition (9) is satisfied. The demand addressed to downstream firm $i \in \{1, 2, 3\}$ is given by $q_i(p_i, p_{-i}) = 1 - p_i - \gamma(p_i - (p_1 + p_2 + p_3)/3)$, where $\gamma \geq 0$ parameterizes the degree of differentiation between final products, which can be interpreted as the intensity of downstream competition. Perfect competition corresponds to γ approaching infinity and local monopolies to $\gamma = 0$. The upstream cost m is equal to zero, and the cost of the alternative source of input $\overline{m}$ is high enough not to constrain the monopoly upstream price of a vertically integrated firm. We solve the model with this specification and find that

Proposition 4. *In the linear case, there exists $\underline{\gamma} > 0$, such that, if $\gamma > \underline{\gamma}$, there exist exactly four upstream equilibria in the two-merger subgame*

- *two monopoly-like equilibria $(w_m, +\infty)$ and $(+\infty, w_m)$,*

- *one symmetric equilibrium (w_s, w_s) with $w_s > m$,*

- *the Bertrand equilibrium (m, m),*

and there are two mergers in equilibrium if integrated firms do not play weakly dominated strategies on the upstream market, or do not play equilibria that are Pareto-dominated by another equilibrium.

   If $\gamma < \underline{\gamma}$, the Bertrand outcome is the only upstream equilibrium in the two-merger subgame, and there is no merger in equilibrium.

Proof. See Appendix A.9.   □

   Proposition 4 unveils a tension between competition on the downstream market and competition on the upstream market. When the downstream market features fierce competition (high γ), there exist monopoly-like equilibria on the upstream market, while the upstream market is perfectly competitive when downstream competition is weak (low γ). To grasp the intuition, suppose that the upstream market is supplied at the monopoly upstream price.

13

When the substitutability between final products is strong, the integrated firm which supplies the upstream market is reluctant to set too low a downstream price since this would strongly contract its upstream profit. The other integrated firm benefits from a substantial softening effect and, as a result, is not willing to corner the upstream market. The reverse holds when downstream products are strongly differentiated.

4.3   Pro-Competitive Single Merger

In this section we show that pro-competitive one-merger equilibria can arise. Assume that the monopoly-like outcome on the upstream market is an equilibrium of the two-merger subgame, and that it is played in some two-merger subgames only: the monopoly-like equilibrium is played if $D1$ merges with $U1$ and $D2$ merges with $U2$; the Bertrand equilibrium prevails in every other two-merger subgame. Then, $D3$ may merge in the first stage in order to avoid a wave of mergers involving $D1$ and $D2$ that would lead to its partial foreclosure. This one-merger situation is an equilibrium under the two following conditions. First, if $D1$ wins the first stage auction, then $D2$ wins the second stage auction. This occurs when $D2$'s gain from merging is larger than $D3$'s loss from not merging: $\pi_2(w_m) - \pi^* \geq \pi^* - \pi_3(w_m)$. Second, $D3$ wins the first stage auction, which occurs when its loss from not merging is larger than $D1$'s gain from merging: $\pi^* - \pi_3(w_m) \geq \pi_1(w_m) - \pi^*$.[13] These conditions hold in the linear example of Section 4.2 with concave downstream costs.[14]

In antitrust parlance, firm $D3$ is a maverick competitor: it will never accept to implement a non-competitive equilibrium. If the maverick is sufficiently harmed when its rivals merge and implement a partial foreclosure equilibrium, it can vertically integrate to ensure tough competition on the upstream market. In that case, the potential maverick becomes an effective maverick by preventing an anticompetitive wave of mergers.

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[13]There exist one-merger equilibria under weaker conditions, although they are supported by least natural anticipations schemes. For instance, if $D1$ wins the first auction, and if $D2$ or $D3$ wins the subsequent auction, then a monopoly-like outcome is implemented with $U1 - D1$ being the upstream supplier; in any other two-merger subgame, the Bertrand equilibrium is played. $D2$ and $D3$ have a lot to lose if they let $D1$ win the first auction, since they have to engage in a fierce bidding war in the subsequent auction to avoid partial foreclosure. As a result they have incentives to prevent the first merger. They can do so if the loss they incur following the first merger is higher than the gain captured by firm $D1$: $\pi^* - \pi_3(w_m) \geq \pi_1(w_m) - \pi^*$. The condition $\pi_2(w_m) - \pi^* \geq \pi^* - \pi_3(w_m)$ does need to be satisfied.

[14]The model is written with zero downstream cost, but all the results easily extend to the case of any twice differentiable downstream cost function.

5 Extensions and Robustness Checks

Two-part tariffs. We show that partial foreclosure equilibria with positive upstream profits still exist in the two-merger subgame when two-part tariffs are used on the upstream market. Denote by w_i (respectively, F_i) the variable (respectively, the fixed) part of the tariff. In a monopoly-like outcome, $U1 - D1$ sets the variable part which maximizes the sum of its profit and $D3$'s profit, i.e., $w_{tp} = \arg\max_w \pi_1(w) + \pi_3(w)$ which is strictly larger than m by strategic complementarity,[15] while $U2 - D2$ makes no upstream offer. The fixed fee captures $D3$'s profit, i.e., $F_1 = \pi_3(w_{tp})$. This is an equilibrium provided that $U2 - D2$ does not want to undercut: $\pi_1(w_{tp}) + \pi_3(w_{tp}) \leq \pi_2(w_{tp})$. When this inequality is not satisfied, there exists a symmetric equilibrium in which both integrated firms charge the variable part w_{tp} and a fixed fee equal to $\pi_2(w_{tp}) - \pi_1(w_{tp})$, which makes them indifferent between supplying the upstream demand or not.[16] In both cases, the upstream profit is strictly positive and integrated firms' profits are strictly higher than in the Bertrand outcome.

No alternative source of input. When there is no alternative source of input, the monopoly upstream price is defined as $w_m = \arg\max_{w \geq 0} \pi_1(w)$. If $w_m < +\infty$, then, our results are not affected at all: there exists an equilibrium with a wave of vertical mergers and a monopoly-like outcome on the upstream market when $\pi_1(w_m) \leq \pi_2(w_m)$. On the other hand, if $w_m = +\infty$, then, there exists an equilibrium in the two-merger subgame, in which both integrated firms choose to make no upstream offer to the unintegrated downstream firm. In that case, there is an equilibrium with an anticompetitive wave of mergers in which the remaining unintegrated downstream firm is completely foreclosed.

Upstream switching costs. In the model downstream firms can switch upstream supplier at no cost after downstream prices are set. This assumption makes clear that our results do not hinge on upstream switching costs as in Chen (2001). However, it leads to a unnatural timing since an unintegrated downstream firm chooses its input supplier after it sets its price. Consider now that unintegrated downstream firms elect their upstream suppliers before the downstream competition stage. Obviously, the analysis is not modified in the

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[15]See Bonanno and Vickers (1988).

[16]When $\pi_2(w_{tp}) < \pi_1(w_{tp})$ the equilibrium fixed fee is strictly negative. In that case the upstream contract features exclusive dealing, otherwise $D3$ accepts both contracts to pocket the fixed fees. If exclusive contracts are prohibited and $\pi_2(w_{tp}) < \pi_1(w_{tp})$, then the proposed equilibrium cannot be implemented, and the only equilibrium outcome becomes the Bertrand outcome. This is reminiscent to Chen and Riordan (2007) who show that vertical integration and exclusive contracts complement each other to achieve an anticompetitive effect.

zero-merger and two-merger subgames. In the one-merger subgame, we cannot exclude the following pathological equilibrium. Integrated firm $U1 - D1$ sets the upstream price w_1, and unintegrated upstream firm $U2$ sets the upstream price w_2, where w_1 and w_2 are their monopoly upstream prices when each one of them supplies one unintegrated downstream firm.[17] $w_1 > w_2$, which makes sense, since an integrated firm has more incentives than an unintegrated upstream firm to charge a high upstream price. $D2$ purchases from $U1 - D1$ to make the integrated firm less aggressive on the final market, while $D3$ buys from $U2$ to benefit from a lower upstream price. This equilibrium seems rather unlikely, and we have not been able to find one such example using standard specifications, then it seems safe to say that our results do not rely crucially on the timing of the game.

Discrimination on the upstream market. Consider that discrimination is allowed on the upstream market. Obviously, this does not change the outcome of the upstream price competition stage in the zero-merger and two-merger subgames. In the one-merger subgame, we cannot exclude the following pathological equilibrium. Integrated firm $U1 - D1$ offers its monopoly price w_1 to unintegrated downstream firm $D2$, and unintegrated upstream firm $U2$ offers its monopoly price w_2 to unintegrated downstream firm $D3$, where w_1 and w_2 are defined in footnote 17. $U2$ prefers not to make an acceptable offer to $D2$, since, if that offer were eventually accepted, integrated $U1 - D1$ would become more aggressive on the downstream market, which would erode the profit earned by $U2$ on $D3$. Similarly, $U1 - D1$ prefers not to make an acceptable offer to $D3$, since, if that offer were accepted, $U1 - D1$ would become less aggressive on the downstream market. By strategic complementarity, $D2$ would increase its downstream price as well, which could lower its demand, and hence, the upstream profit that $U1 - D1$ makes on $D2$. Since this equilibrium is rather unrealistic and does not show up in standard specifications, our analysis seems also robust to discrimination on the upstream market.

Quantity competition. The softening effect exists if the upstream supplier can enhance its upstream profits by behaving softly on the downstream market. One may wonder whether the softening effect hinges on the assumption of price competition on the downstream market, for if the downstream strategic variables are quantities and all firms play simultaneously, then the upstream supplier can no longer impact its upstream profit through its downstream

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[17]Formally, $w_1 = \arg\max_{\tilde{w}_1} \pi_{U1-D1}(\tilde{w}_1, w_2)$ and $w_2 = \arg\max_{\tilde{w}_2} \pi_{U2}(w_1, \tilde{w}_2)$, where $\pi_i(\tilde{w}_1, \tilde{w}_2)$ denotes the profit of firm i when firms $D2$ and $D3$ supplied by firms $U1 - D1$ and $U2$, respectively, at prices $\tilde{w}_1$ and $\tilde{w}_2$.

behavior. However, if for instance integrated firms are Stackelberg leaders on the downstream market, then the upstream supplier's quantity choice modifies its upstream profit, and the softening effect is still at work.[18] To summarize, the issue is not whether firms compete in prices or in quantities, but whether the strategic choice of a firm can strategically affect its rivals' quantities on the downstream market.

Strategic interactions. We have assumed that downstream prices are strategic complements, in line with the vertical mergers literature. This is however not a crucial assumption. On the contrary, we argue that strategic substitute prices would strengthen the softening effect and thus increase the scope for anticompetitive waves of mergers. Let us informally explain why, by considering the downstream competition stage when two mergers have occurred and the upstream price is strictly above the marginal cost. The upstream supplier has incentives to raise its downstream price to preserve its upstream profit. When prices are strategic complements, the integrated rival best responds by raising its downstream price as well, which reduces the gap between equilibrium downstream prices and weakens the softening effect. By contrast, when prices are strategic substitutes, the integrated rival lowers its downstream price, which enlarges the gap between equilibrium downstream prices and strengthens the softening effect.[19]

6   Conclusion

We show that upstream competition between integrated firms only is weaker than competition between integrated firms and unintegrated upstream firms, or between unintegrated firms only. This provides a rationale for the existence of anticompetitive waves of vertical mergers in which every unintegrated upstream firm is eliminated.

Our analysis can shed light on the recent wave of vertical mergers in the satellite navigation industry investigated by the European Commission.[20] The upstream market is the market for navigable digital map databases, where only Tele Atlas and Navteq are active, with pre-merger market shares of approximately 50% each. At the downstream level, firms embed digital maps in the devices they manufacture in order to provide their customers with

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[18]With quantity competition and a linear demand function, if integrated firms are Stackelberg leaders on the downstream market, then the monopoly-like outcome is an equilibrium and there is an anticompetitive wave of mergers.

[19]We have confirmed numerically this intuition using the linear demand functions of Section 4.2 and quadratic downstream costs to parameterize the sign of $\partial^2 \pi_i / \partial p_i \partial p_j$, $i \neq j$.

[20]See European Commission COMP M.4854 *TomTom/Tele Atlas* and COMP M.4942 *Nokia/Navteq*.

navigation solutions. Downstream firms include portable navigation device manufacturers TomTom, Garmin and Mio Tech & Navman, and manufacturers of mobile handsets that incorporate navigation possibilities, Nokia, Motorola, Samsung, Sony Ericsson. The European Commission does not consider portable navigation devices and mobile phones with navigation possibilities to be part of the same market yet, but envisages that, as technology evolves, both markets will increasingly converge.

In October 2007, TomTom announces its acquisition of Tele Atlas; four months later Nokia responds by announcing its acquisition of Navteq. The European Commission has given clearance for these two mergers, thereby allowing the vertical integration of every upstream firm. Our analysis suggests that, as long as mobile phones and portable navigation devices remain rather imperfect substitutes, competition between digital map providers should not be harmed. However, when the two markets converge as it is envisaged by the European Commission, the softening effect should strengthen and upstream competition might weaken.

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A   Appendix

A.1   A Preliminary Lemma

To ease the proofs of all lemmas and propositions, we begin by showing the following technical lemma.

Lemma 0. *If the best response function on the downstream market of at least one firm shifts upwards, then all equilibrium downstream prices increase strictly.*

Assume that firm i's best response shifts upwards. This happens if, and only if, the first derivative of its profit with respect to its price shifts upwards. For all j, let us denote by $\pi_j^{(0)}(.)$ (respectively $\pi_j^{(1)}(.)$) the profit of firm j before (resp. after) the marginal profit shift. The game $(\mathbb{R}; \pi_j^{(k)}(.), k = 0, 1; j = 1, 2, 3)$ is strictly supermodular. For all j, $\pi_j^{(k)}(p_j, p_{-j})$ has increasing differences in (p_j, k), and $\pi_i^{(k)}(p_i, p_{-i})$ has strictly increasing differences in (p_i, k). Since we assume that every configuration analyzed in this paper yields a unique downstream equilibrium, supermodularity theory (see Vives, 1999, p.35) tells us that this equilibrium is strictly increasing in k.

A.2   Proof of Lemma 2

First, if $U1 - D1$ supplies the market at $w > m$, then $U2$ clearly wants to undercut.

Conversely, assume that $U2$ supplies the market at $w > m$, and let us show that $U1 - D1$ wants to undercut. If it becomes the upstream supplier at price (arbitrarily close to) w, its first order condition on the downstream market shifts from $q_1 + (p_1 - m)\partial q_1/\partial p_1 = 0$ to $q_1 + (p_1 - m)\partial q_1/\partial p_1 + (w - m)(\partial q_2/\partial p_1 + \partial q_3/\partial p_1) = 0$. Its best response function shifts upwards, while all other best responses remain unaffected, then all downstream prices increase by Lemma 0. Therefore $U1 - D1$'s profit increases thanks to the upstream revenues and the softening of downstream competition. Formally, denoting with superscript 1 (respectively 2) the outcome variables when firm $U1 - D1$ undercuts (resp. does not undercut):

$$
\begin{aligned}
\pi_1^{(2)} \; &= \; p_1^{(2)} q_1(p_1^{(2)}, p_2^{(2)}, p_3^{(2)}) \\
&< \; p_1^{(2)} q_1(p_1^{(2)}, p_2^{(1)}, p_3^{(1)}) \quad \text{by Lemma 0} \\
&< \; p_1^{(2)} q_1(p_1^{(2)}, p_2^{(1)}, p_3^{(1)}) + (w - m)\left(q_2(p_1^{(2)}, p_2^{(1)}, p_3^{(1)}) + q_3(p_1^{(2)}, p_2^{(1)}, p_3^{(1)}) \right) \quad \text{since } w > m \\
&< \; p_1^{(1)} q_1(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) + (w - m)\left(q_2(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) + q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) \right) \quad \text{by revealed preference} \\
&= \; \pi_1^{(1)}.
\end{aligned}
$$

Then, we show that if firms $U1 - D1$ and $U2$ propose the same upstream price $w > m$, then both unintegrated downstream firms purchase from $U1 - D1$, which shall proves that this situation cannot

be an equilibrium since $U2$ would undercut. We have already seen that the best response function of $U1 - D1$ shifts upwards when $D3$ buys $U1 - D1$ rather than $U2$, which raises all downstream prices by Lemma 0 and makes $D3$ better off. Formally, denoting with superscript 1 (respectively 2) the outcome variables when firm $U1 - D1$ (resp. $U2$) supplies $D3$ at price w:

$$
\begin{aligned}
\pi_3^{(2)} &= (p_3^{(2)} - w)q_3(p_1^{(2)}, p_2^{(2)}, p_3^{(2)}) \\
&< (p_3^{(2)} - w)q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(2)}) \quad \text{since } p_i^{(1)} > p_i^{(2)} \text{ for all } i \\
&< (p_3^{(1)} - w)q_3(p_1^{(1)}, p_2^{(1)}, p_3^{(1)}) \quad \text{by revealed preference} \\
&= \pi_3^{(1)}.
\end{aligned}
$$

This implies that $D3$'s dominant strategy is to purchase from $U1 - D1$. By symmetry, this also holds for $D2$.

It remains to prove that the upstream market cannot be supplied at a price below the marginal cost. It is obvious that $U2$ never sells the input at $w < m$, otherwise it would be better off exiting the market. Assume now that $U1 - D1$ is the upstream supplier at $w < m$, and denote $U2$'s upstream offer by $w' \geq w$.[21] $U1 - D1$ is better off exiting the upstream market, since its shifts its best response upwards, which strictly increases all the downstream prices by 0.

A.3   Proof of Lemma 3

Let $w > m$. To show that $p_1(w) > p_2(w)$, we denote by $B_1(p_2, p_3, w)$ (respectively $B_2(p_1, p_3, w)$) firm $U1 - D1$ (resp. $U2 - D2$)'s best response when the upstream market is supplied by $U1 - D1$ at price w. The first order conditions (4) and (5) indicate that $B_1(., ., w) = B_2(., ., w)$. Then

$$
p_1(w) = B_1(p_2(w), p_3(w), w) > B_2(p_2(w), p_3(w), w).
$$

Besides

$$
p_2(w) = B_2(p_1(w), p_3(w), w).
$$

By strategic complementarity, B_2 is increasing in its first argument, therefore $p_1(w) > p_2(w)$.

A straightforward revealed preference argument shows that $U1 - D1$ earns a strictly lower downstream profit than $U2 - D2$.

A.4   Proof of Lemma 4

The only nontrivial point is that a downward deviation is not profitable: $\pi_1(w) < \pi_1(m =$ for $w < m$. The proof is along the line of the proof of Lemma 2. If an integrated firm sets $w < m$,

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[21] Actually $w' < w$. Indeed, if $w' = w$, unintegrated downstream firms strictly prefer purchasing from $D2$ since it shifts $U1 - D1$'s best response function upwards.

then its best response function and the one of $D3$ shifts downwards, and all downstream prices decreases by Lemma 0. Therefore, the downward deviation yields negative upstream profits and reduces downstream profits.

A.5 Proof of $w_m > m$

In the proof of Lemma 4, we have shown that $\pi_1(w) < \pi_1(m)$ for $w < m$, which implies that $w_m \geq m$. Moreover, taking the first derivative of $\pi_1(.)$ for $w = m$, we get, using the envelope theorem,

$$\frac{d\pi_1}{dw}(m) = (p_1 - m) \left(\frac{dp_2}{dw}(m)\frac{\partial q_1}{\partial p_2}(p_1(m), p_2(m), p_3(m)) + \frac{dp_3}{dw}(m)\frac{\partial q_1}{\partial p_3}(p_1(m), p_2(m), p_3(m)) \right)$$
$$+ q_3(p_1(m), p_2(m), p_3(m)) > 0,$$

since the downstream prices are strictly increasing in w by Lemma 0.

A.6 Proof of Proposition 1

Assume that $\pi_1(w_m) \leq \pi_2(w_m)$ and let us show that $(w_m, +\infty)$ is an equilibrium. Clearly, firm $U1-D1$ does not want to set another price, by definition of w_m. In addition, since $\pi_1(w_m) \leq \pi_2(w_m)$, and again by definition of w_m, firm $U2 - D2$ does not want to undercut its rival.

Conversely, if $\pi_1(w_m) > \pi_2(w_m)$, then monopoly-like outcomes cannot be equilibria, since the integrated firm which does not supply the upstream market would rather undercut its rival.

To show that monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria, we first show that all other equilibria are symmetric. Notice first that $\pi_2(.)$ is upward-sloping since, by the envelope theorem,

$$\frac{d\pi_2}{dw} = (p_2 - m) \left(\frac{\partial q_2}{\partial p_1}\frac{dp_1}{dw} + \frac{\partial q_2}{\partial p_3}\frac{dp_3}{dw} \right) > 0$$

as downstream prices are strictly increasing in w. Let $w_1 \neq w_m$ and $w_2 > w_1$, and assume, by contradiction, that (w_1, w_2) is an equilibrium. Then $w_2 \leq w_m$, otherwise the upstream supplier would rather set w_m. If $\pi_1(w_1) > \pi_2(w_1)$, then firm $U2 - D2$ has a strictly profitable deviation: setting $w_1 - \varepsilon$. If $\pi_1(w_1) \leq \pi_2(w_1)$, then $\pi_1(w_1) < \pi_2(w_2)$ since $\pi_2(.)$ is upward-sloping, and firm $U1 - D1$ has a strictly profitable deviation: setting $w_2 + \varepsilon$. In both cases we get a contradiction.

Consider now a monopoly-like equilibrium $(w_m, +\infty)$, and another equilibrium, which we know is symmetric, (w, w). Obviously, $\pi_1(w) = \pi_2(w)$, otherwise both firms would rather undercut or exit the upstream market. Then we have, by definition of w_m, $\pi_2(w) = \pi_1(w) < \pi_1(w_m) \leq \pi_2(w_m)$, which proves that the monopoly-like equilibria Pareto-dominate all other equilibria.

We now show that all other equilibria than the monopoly-like equilibria involve weakly dominated strategies on the upstream market. We have just seen that these other equilibria are of the form (w, w), with $w < w_m$ and $\pi_1(w) = \pi_2(w)$. Let us show that offering $w_i = w_m$ weakly dominates offering $w_i = w$ for integrated firm i. If the integrated rival offers $w_j \leq w$, then both strategy are equivalent. If $w < w_j < w_m$, then offering w_m yields a payoff $\pi_2(w_j)$, which is larger than the payoff when offering w, $\pi_2(w)$, because $\pi_2(.)$ is increasing. If $w_j > w_m$, then offering w_m yields a payoff $\pi_1(w_m)$, which is larger than the payoff when offering w, $\pi_1(w)$, by definition of w_m. If $w_j = w_m$, the former two cases shows that it is also strictly preferable to offer w_m than w.

A.7 Proof of Proposition 2

When the Bertrand outcome arises in every two-merger subgames, firms are indifferent between merging or not. In that case, it is an equilibrium that downstream firms submit no bid.

We now show that there is two mergers in equilibrium if $\pi_1(w_m) \leq \pi_2(w_m)$, and integrated firms do not play weakly dominated strategies on the upstream market or do not play equilibria that are Pareto-dominated by another equilibrium. In that case, the only downstream equilibria in the two-merger subgames are the two monopoly-like equilibria. It cannot be that there is only zero or one merger in equilibrium, since any remaining downstream firm would rather bid a small amount to vertically integrate and increase its profit from π^* to $\pi_1(w_m)$ or $\pi_2(w_m)$. Therefore, in any equilibrium, there are two mergers and the input price is w_m.

A.8 Proof of Proposition 3

We compare an equilibrium with two mergers in which $D3$ purchases the from $U1 - D1$ at $w > m$, with an equilibrium with no merger in which all downstream firms access the input at marginal cost. First, all downstream prices are strictly higher in the partial foreclosure equilibrium by Lemma 0, therefore consumers are strictly worse off.

Second, we show that social welfare in also strictly lower in the partial foreclosure equilibrium. Assume that there exists a representative consumer with a quasi-linear, continuously differentiable and quasi-concave utility function $q_0 + u(q_1, q_2, q_3)$, where q_0 denotes consumption of the numeraire and q_k denotes consumption of product $k \in \{1, 2, 3\}$. We can then write the social welfare as

$$W(p_1, p_2, p_3) = u\left(q_1(p_1, p_2, p_3), q_2(p_1, p_2, p_3), q_3(p_1, p_2, p_3)\right) - m \sum_{k=1}^{3} q_k(p_1, p_2, p_3).$$

We have to show that $W(p_1(w), p_2(w), p_3(w)) - W(p_1(m), p_2(m), p_3(m))$, the variation in social welfare when one shifts from the Bertrand outcome from the partial foreclosure outcome, is strictly negative. Since the welfare function is symmetric in its arguments, we can relabel the downstream

prices in the partial foreclosure equilibrium by $\{\hat{p}_1, \hat{p}_2, \hat{p}_3\} = \{p_1(w), p_2(w), p_3(w)\}$ such as $\hat{p}_3 > \hat{p}_2 > \hat{p}_1 > p_1(m) = p_2(m) = p_3(m) \equiv p^*$.[22] The variation in welfare then writes as

$$\int_{p^*}^{\hat{p}_1} \sum_{k=1}^{3} \frac{\partial W}{\partial p_k}(r, r, r)dr + \int_{\hat{p}_1}^{\hat{p}_2} \sum_{k=2}^{3} \frac{\partial W}{\partial p_k}(\hat{p}_1, r, r)dr + \int_{\hat{p}_2}^{\hat{p}_3} \frac{\partial W}{\partial p_3}(\hat{p}_1, \hat{p}_2, r)dr.$$

All the integrands are strictly negative. Indeed, for instance,

$$\frac{\partial W}{\partial p_k}(p_1, p_2, p_3) = \sum_{k'=1}^{3} \left(\frac{\partial u}{\partial q_{k'}} - m \right) \frac{\partial q_{k'}}{\partial p_k} = \sum_{k'=1}^{3} (p_{k'} - m) \frac{\partial q_{k'}}{\partial p_k}$$

is strictly negative when $p_k \geq p_{k'}$, $k' \neq k$, since $\partial q_k / \partial p_k < -\sum_{k' \neq k} |\partial q_{k'} / \partial p_k|$. This concludes the proof.

A.9   Proof of Proposition 4

In the two-merger subgame, when the upstream market is supplied by $U1 - D1$ at price w, downstream prices are

$$p_1(w) = \frac{18 + \gamma(15 + w(9 + 5\gamma))}{2(3+\gamma)(6+5\gamma)} \quad p_2(w) = \frac{3(6 + \gamma(5 + w + w\gamma))}{2(3+\gamma)(6+5\gamma)} \quad p_3(w) = \frac{3(6+5\gamma) + w(18 + 7\gamma(3+\gamma))}{2(3+\gamma)(6+5\gamma)},$$

and profits

$$\pi_1(w) = \frac{3(3+\gamma)(6+5\gamma)^2 + 6w(1+\gamma)(6+5\gamma)(18 + \gamma(18+5\gamma)) - w^2(1+\gamma)(648 + 1296\gamma + 909\gamma^2 + 249\gamma^3 + 20\gamma^4)}{4(3+\gamma)^2(6+5\gamma)^2},$$

$$\pi_2(w) = \frac{3(3+2\gamma)(6 + \gamma(5 + w + w\gamma))^2}{4(3+\gamma)^2(6+5\gamma)^2},$$

$$\pi_3(w) = \frac{3(3+2\gamma)(6 + 5\gamma - w(1+\gamma)(6+\gamma))^2}{4(3+\gamma)^2(6+5\gamma)^2}.$$

$\pi_1(.)$ is strictly concave and reaches its maximum value for

$$w_m = \frac{3(6+5\gamma)(18 + \gamma(18+5\gamma))}{648 + 1296\gamma + 909\gamma^2 + 249\gamma^3 + 20\gamma^4}.$$

$\pi_3(.)$ is strictly decreasing and strictly positive if it purchases the input from the alternative source at price w_m. We assume that the price of the alternative source of input does not constrain the monopoly upstream price, $\overline{m} > w_m$, and provides $D3$ with positive profits.

Straightforward computations indicate that $\pi_2(w) \geq \pi_1(w)$ if, and only if, $w \in [m, w_s]$, where $w_s \geq w_m$ if, and only if, $\gamma \geq \underline{\gamma} \simeq 41$. The same holds with strict inequalities. As a result, the only equilibrium outcome is the Bertrand outcome when $\gamma < \overline{\gamma}$. When $\gamma > \overline{\gamma}$, there are also the

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[22]We do so since, in the general case, we cannot order the downstream prices of the unintegrated downstream firm and the upstream supplier.

two monopoly-like equilibria $(w_m, +\infty)$ and $(+\infty, w_m)$, and a symmetric equilibrium (w_s, w_s) with $w_s < w_m$. In that case, if integrated firms do not play weakly dominated strategies on the upstream market or do not play equilibria that are Pareto-dominated by another equilibrium, then there are two mergers in equilibrium and a monopoly-like outcome on the upstream market.