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FACULTY WORKING PAPER NO. 1270

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August 1986

On the Nonoptimality of Three-Level Goal Programming Composition Models

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Abstract

The purpose of this research is to investigate a set of multiple criteria, composition models for hierarchical organizations. It presents a generalized decomposition approach to an overall organizational resource allocation problem. This approach generates in a three-level, decision-making hierarchy applicable to the composition models. For each model, the basic decisions and coordinative mechanisms used at every level within this decision-making hierarchy are detailed. Potential shortcomings of the models are cited. Through the use of a simple example the potential for the nonoptimality of the models' solutions is demonstrated. The results of the study indicate that these models may offer little assistance in allocating resources in real world hierarchical organizations.

Keywords: Composition Models, Decomposition Models, Goal Programming,
Resource Allocation, Hierarchical Decision Making

Abstract

The purpose of this research is to investigate [illegible] multiple-criteria, decomposition models for hierarchical organizations [illegible] generalized decomposition approach to a central [illegible] allocation problem. This approach [illegible] hierarchy applicable to the example [illegible] decisions and coordinating [illegible] decision-making hierarchy [illegible] are cited. Through the [illegible] nonoptimality of the [illegible] study indicate that [illegible] resources in [illegible]

Keywords: [illegible]

I. Introduction

This paper analyzes a group of three-level resource allocation models. The models include:

Ruefli's (1971a, 1971b)	Generalized Goal Decomposition (GGD) Model
Freeland's (1973, 1976) and Freeland and Baker's (1975)	Modified GGD (MGGD) Model,
Davis (1975) and Davis and Talavage's (1977)	Centralized Goal Decomposition (CGD) and Hybrid Goal Decomposition (HGD) Models
Davis (1978) and Whitford and Davis' (1983)	Generalized Hierarchical Model (GHM), and
Davis and Whitford's (1985)	Reformulation of Freeland's MGGD (MGGD II) Model

Sweeney, et al. (1978) have characterized the formulation and structure of these models as a "composition approach" to organizational decision-making. Unlike the decomposition approach which begins with an overall problem and derives an ensemble of decisions to effect its solution, the composition approach begins with an ensemble of subproblems (or decisions) that emulate the organization's actual decision-making structure. In discussing these models, Sweeney et al. focus attention on two formulations: (1) the Ideal Organizational Problem (IOP) which is the problem that the organization would like to solve and (2) the Decision Process Model (DPM) which consists of i) a mathematical statement of the subproblem's solved by each of the separate units of the organization and ii) an algorithm for solving the subproblems. Sweeney, et al. suggest that an analysis of the IOP and DPM's formulations and solutions can provide a basis for assessing the efficacy of the current or proposed organizational structure and its coordinating mechanisms. If the solutions to both problems are the same, Sweeney, et al. define the DPM to be "coordinable."

This paper will develop a framework for comparing the IOP and DPM's of these "composition models". In deriving this framework; two specific tasks will be undertaken. The first task requires the derivation of a generalized decomposition approach for an organization's overall resource allocation problem. This derivation, which is presented in Section 2, generates a three-level, decision-making hierarchy which is applicable to each of the models. The second task, given in Section 3, is to specify and compare the basic decisions, the coordinative mechanisms and solution algorithm used by each composition model. Section 3 also analyzes the relationship between these composition models and mathematical decomposition procedures and identifies potential difficulties for each model. Section 4 presents a simple but straightforward problem in its IOP format. Next it is shown that each models' solution procedure (i.e., its DPM) has a potential path towards a nonoptimal solution of the IOP. This nonoptimality, however, does not arise from the organizational architecture of the model, but rather from nonunique solutions and/or nonunique coordinative information that arise during the solution process.

Because the source of these nonoptimal solutions arises from the mathematical structure of the composition models and not the structure of the organization, the precise definition of the DPM is shown to be at best ambiguous. These findings suggest that one should be extremely cautious in allocating an organization's resources or in recommending the restructuring of an organization based upon the results of any of these models as they are currently formulated.

2. DEVELOPMENT OF THE DECOMPOSITION APPROACH

This section focuses upon an overall resource allocation problem faced by a three-level, hierarchical organization. This overall problem (which is assumed to be an IOP) possesses a structure which can be decomposed into a three-level decision-making hierarchy.

The overall organizational problem is defined in equations (1) through (4).[1]

$$\text{Min} \quad \Sigma_{k=1}^{M}[\Sigma_{i=r_{k-1}+1}^{r_k} C_iX_i + W_k^+Y_k^+ + w_k^+y_k^+ + W_k^-Y_k^- + w_k^-y_k^- + C_{G_k}G_k \tag{1}$$

$$\text{s.t.} \quad \Sigma_{i=r_{k-1}+1}^{r_k} B_iX_i - I_{m_k}Y_k^+ + I_{m_k}Y_k^- - G_k = 0$$

$$\Sigma_{i=r_{k-1}+1}^{r_k} B_i'X_i - I_{m_k'}y_k^+ + I_{m_k'}y_k^- = g_k \tag{2.k}$$

for k=1, ..., M

$$D_iX_i \quad \left\{\begin{matrix}\leq\\=\\\geq\end{matrix}\right\} F_i \tag{3.i}$$

for i=1, ..., N

$$\Sigma_{k-1}^{M} P_kG_k \left\{\begin{matrix}\leq\\=\\\geq\end{matrix}\right\} G_o \tag{4}$$

with all variables ≥ 0, and where I_{m_k} and $I_{m_k'}$ are $(m_k \times m_k)$ and $(m_k' \times m_k')$ identity matrices, respectively. At this point, the mathematical formulation

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[1]Omission of all cost vectors, C, creates a pure goal programming structure. In certain situations, non-zero cost vectors may be desirable; see Davis (1978).

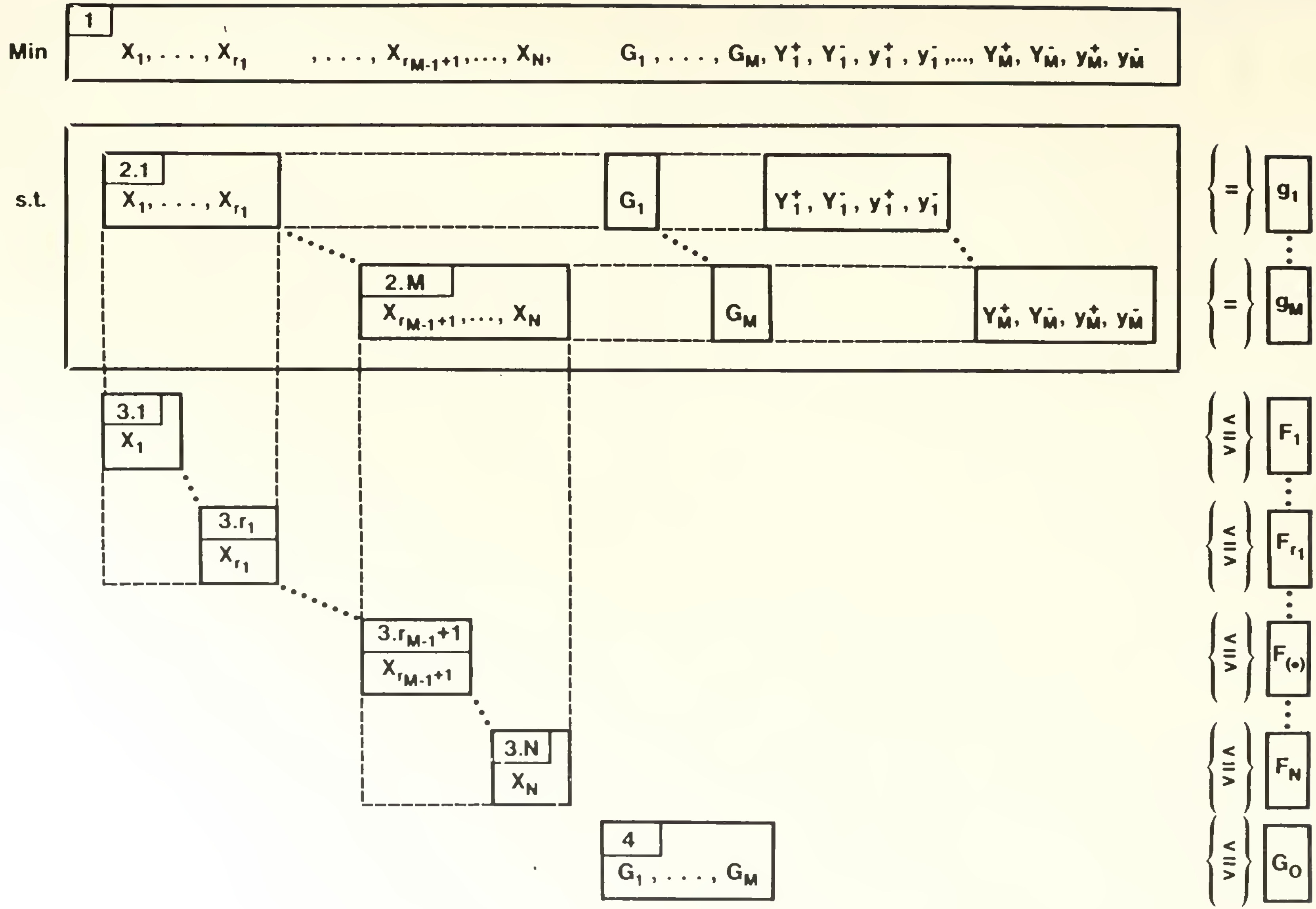
given in equations (1) through (4) will be treated as a problem statement only. Specific definitions will begin shortly.

It should be noted that each of the composition models' cited in the introduction use a different set of variable definitions and, in some cases, slightly different constraints in the original statement of his (their) models. Although the IOP's variables and constraints may differ slightly from the original formulations, they are applicable to all of the models. As will become apparent, the essential differences among the models are not their original variable/constraint definitions, but rather the implicit approach that each model employs to "decompose" and solve the overall problem.

Figure 1 gives a variable/ constraint diagram for this problem. In Figure 1, each row of boxes represents a specific equation of the overall problem. The defining equation is given in the upper left-hand corner of the leftmost box while the boxes contain the decision variables for each equation. By grouping equations (2.1) through (2.M), the classic block-angular structure of the overall problem is apparent. Thus, a two-level decomposition approach to the problem can easily be applied where equations (2.1) through (2.M) define the restricted master program, and equations (3.1) through (3.N) and (4) define the appropriate column generators.[2]

The structure of this overall problem permits the consideration of a more sophisticated decomposition approach. It is evident that the overall problem is nearly separable into M subproblems; only constraint (4) prevents this separation. Therefore, to permit separation, constraint (4) will be initially neglected, and the vectors, $(G_1,\ldots,G_M)$, will be assumed to be constant. The

[2]The terms, block-angular structure, restricted master program, and column generators are standard terminology in decomposition theory. An excellent presentation of this theory is given in Lasdon (1970).



VARIABLE/CONSTRAINT DIAGRAM FOR OVERALL ORGANIZATIONAL PROBLEM

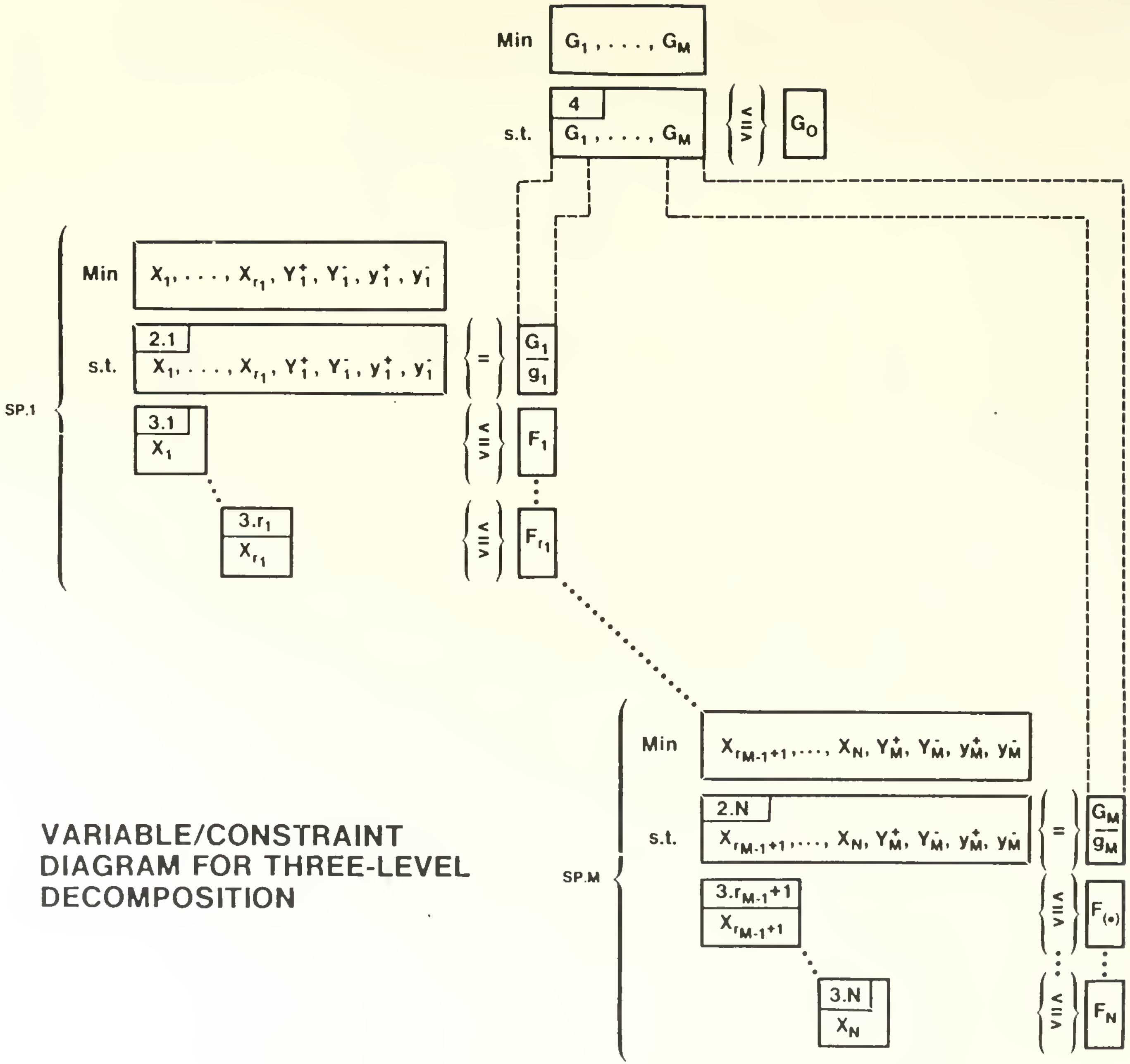
FIGURE 1

first of the M subproblems would then consist of constraints (2.1) and (3.1) through ($3.r_1$) with the first term of the summation (k=1) in equation (1) serving as its objective function.[3] Because G_1 is assumed to be a constant, it can be placed in the right-hand-side vector of equation (2.1). The resulting subproblem is shown graphically in Figure 2 as SP.1. In Figure 2, each of the M subproblems, SP.1 through SP.M, possesses a similar block-angular structure. For the subproblem SP.k, constraint (2.k) defines the restricted master program, while constraints ($3.r_{k-1}$ + 1) through ($3.r_k$) define column generators supporting k-th the restricted master.

In the overall problem, the subproblems are coupled through constraint (4). This coupling is illustrated in Figure 2 by the dashed lines. Hence, a mechanism through which constraint (4) can generate the composite goal vector (G_1,..., G_m) is required. The incorporation of such a mechanism represents the third level of decision-making employed by the decomposition procedure. The next section discusses the three procedures which have been employed in existing three-level composition models.

In contrasting the three-level decomposition approach with the two-level approach applicable to block-angular structure of Figure 1, several fundamental differences emerge. First, the single restricted master program resulting from constraints (2.1) through (2.M) in Figure 2 has now been replaced by M restricted master programs defined for each constraint (2.k) (k=1,...,M). For the two-level model in Figure 1, constraint (4) defines an appropriate column generator for the restricted master program. For the three-level approach in Figure 2, constraint (4) must be used to develop a

[3]Under the assumption that there are M restricted master programs, there exists a series of integers r_0, r_1, ..., r_M, such that column generators $r_{k-1}+1$, ..., r_k are associated with restricted master (2.k). Thus if there are in total N column generators r_0 must equal zero, and r_M must equal N.



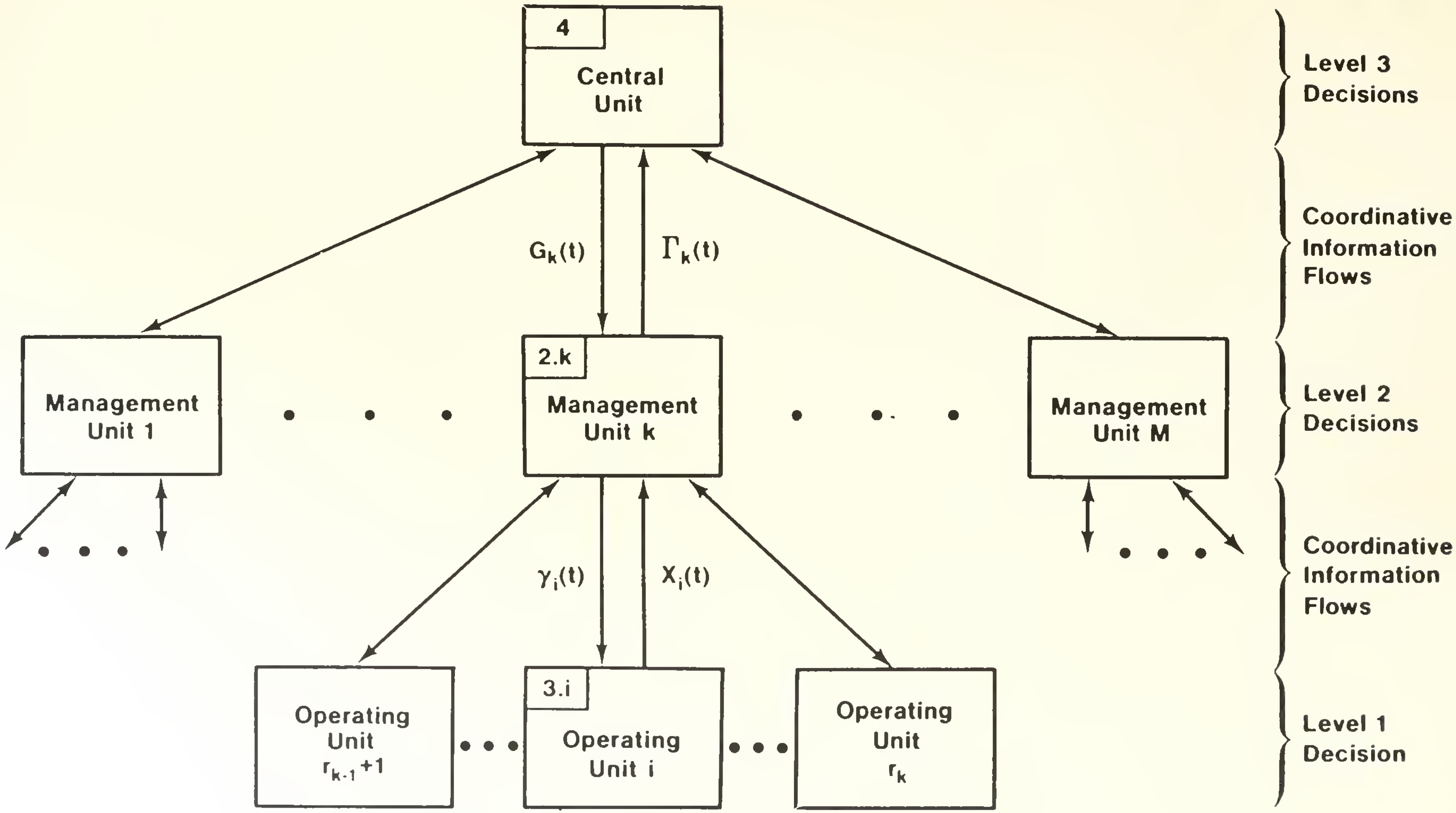
VARIABLE/CONSTRAINT DIAGRAM FOR THREE-LEVEL DECOMPOSITION

FIGURE 2

coordinative mechanism for the M separate restricted master programs. Perhaps the most fundamental difference, however, is that under the three-level decomposition procedure, no single decision-making subsystem has been assigned the task of optimizing the organization's overall objective function. That is, each of the M restricted master programs will consider only the k-th term of the summation given in equation (1). Both the two-level and three-level decomposition procedures will, however, use equations (3.1) through (3.N) to define column generators that support their respective restricted master programs.

An organizational hierarchy based upon this three-level decomposition approach is depicted in Figure 3. The two lower levels of the organization result from the application of a given decomposition of subproblems SP.1 through SP.M. The restricted master program for each subproblem, SP.k ($k=1,\ldots,M$), will be called management unit or manager k. Manager k will coordinate the decision-making of the i-th operating unit, OU_i, ($i=r_{k-1}+1,\ldots,r_k$). Each subordinate operating unit will iteratively generate a proposal vector, $X_i(t)$, for its manager at iteration t. To coordinate the generation of this proposal vector, $X_i(t)$, manager k must generate the coordinative input vectors, $\gamma_i(t)$ ($i=r_{k-1}+1,\ldots,r_k$). OU_i will then incorporate its $\gamma_i(t)$ vector into the decision-making process. Furthermore, OU_i is responsible for assuring that the proposal vector, $X_i(t)$ satisfies constraint (3.i). In generating a composite proposal vector for OU_i, $X_i^*(t)$, manager k must also insure that constraint (2.k) is satisfied while simultaneously attempting to minimize the k-th term of the summation given in equation (1).

Finally, to coordinate the simultaneous solution of the M subproblems, SP.1 through SP.M, a third level of decision-making is introduced as the



DECISION-MAKING HIERARCHY FOR ORGANIZATIONAL MODELS

FIGURE 3

central unit (CU). The CU will interact with each management (k=1,...,M). This interaction again will be iterative in nature. To coordinate manager subsystem k's decision, the CU will generate the external goal vector, $G_k(t+1)$. In generating $G_k(t+1)$, the CU satisfies constraint (4) and ascertains the degree of success that manager k(k=1,...,M) has experienced in meeting its current goal vector, $G_k(t)$. To expedite the latter each manager must generate a feedback vector, $\Gamma_k(t)$ (k=1,...,M), that the CU can incorporate into its decision process at the next iteration. Once the CU generates the vector $G_k(t+1)$, manager k incorporates it into the right-hand-side of its constraint (2.k) on the next iteration.

3. DEFINITION OF THE COMPOSITION MODELS

As seen in Table 1 the composition models' CU utilize three basic decisions processes; these are given in equations (5) through (15). The managers all use a linear goal programming decision process, given in equations (16) through (21). Finally, the OUs utilize two basic decision processes which are given in equations (22) through (29).

Central Unit Decision Process (CUDP)

CUDP I (Generalized Linear Programming or the Dantzig and Wolfe Decomposition Algorithm

$$\text{Min} \quad \Sigma_{k=1}^{M} [C_{G_k} - \Pi_k(t)] G_k(t+1) \quad (5)$$

$$\text{s.t.} \quad \Sigma_{k=1}^{M} P_k G_k(t+1) \begin{Bmatrix} \leq \\ \geq \end{Bmatrix} G_0 \tag{6}$$

$$G_k(t+1) \geq 0 \quad \text{for } k=1, \ldots, M \tag{7}$$

Feedback Information: $\Gamma_k(t+1) = \Pi_k(t)$

CUDP II (Bender's Positioning Algorithm)

$$\text{Min} \quad \Sigma_{k+1}^{M} \zeta_k(t+1) \tag{8}$$

$$\text{s.t.} \quad \zeta_k(t+1) + [\Pi_k(s) - C_{G_k}] G_k(t+1) \geq$$

$$z_k^*(s) + [\Pi_k(s) - C_{G_k}] G_k(s) \tag{9}$$

$$\text{for } k=1, \ldots, M; \; s = 1, \ldots, t$$

$$\Sigma_{k=1}^{M} P_k G_k(t+1) \begin{Bmatrix} \leq \\ \geq \end{Bmatrix} G_0 \tag{10}$$

$$G_k(t+1) \geq 0 \quad \text{for } k=1, \ldots, M \tag{11}$$

Feedback Information: $\Gamma_k(t+1) = \Pi_k(t)$

Table 1

Decision Processes Utilized by the Composition Models

Level of the Hierarchy

Composition Model				
Author(s)	Model	Central Unit	Management Unit	Operation Unit
Ruefli	GGD	CUDP I	MUDP	OUDP I
Freeland	MGGD	CUDP II	MUDP	OUDP I
Davis and Talavage	CGD	CUDP III	MUDP	OUDP I
Davis and Talavage	HGD	CUDP I/III	MUDP	OUDP I
Davis'	GHM	CUDP III	MUDP	OUDP II
Davis and Whitford	MGGD-II	CUDP II	MUDP	OUDP I

In equation (9), $z_k^*(s)$ is the optimal value of manager k's objective function, i.e. the optimal value of equation (16) on iteration s for s=1, ..., t.

GUDP III (Goal Programming)

$$\text{Min} \quad \Sigma_{k+1}^{M} [C_{G_k} G_k(t+1) + W_k^+ S_k^+(t+1) + W_k^- S_k^-(t+1)] \qquad (12)$$

$$\text{s.t.} \quad G_K(t+1) + I_{m_k} S_k^+(t+1) - I_{m_k} S_k^-(t+1) =$$

$$G_k(t) + \Gamma_k(t+1) \qquad \text{for } k=1, \ldots, M \qquad (13)$$

$$\Sigma_{k=1}^{M} P_k G_k(t+1) \begin{Bmatrix} \leq \\ \geq \end{Bmatrix} G_0 \qquad (14)$$

$$G_k(t+1),\ S_k^+(t+1),\ S_k^-(t+1) \geq 0 \qquad \text{for } k=1, \ldots, M \qquad (15)$$

Feedback Information: $\Gamma_k(t+1) = Y_k^+(t) - Y_k^-(t)$

<u>Manager k's Decision Process (MUDP)</u>

MUDP (Goal Programming)

$$\text{Min } \Sigma_{i=r_{k-1}+1}^{r_k} C_i X_i^*(t) + W_k^+ Y_k^+(t) + w_k^+ y_k^+(t) + W_k^- Y_k^-(t) + w_k^- y_k^-(t) \qquad (16)$$

$$\text{s.t. } \Sigma_{i=r_{k-1}+1}^{r_k} B_i X_i^*(t) - I_{m_k} Y_k^+(t) + I_{m_k} Y_k^-(t) = G_k(t) \qquad (17)$$

$$\Sigma_{i=r_{k-1}+1}^{r_k} B_i' X_i^*(t) - I_{m_k'} y_k^+(t) + I_{m_k'} y_k^-(t) = g_k \qquad (18)$$

$$X_i^*(t) = \Sigma_{j=1}^{t} \lambda_i(j) X_i(j) \qquad \text{for } i=r_{k-1}+1, \ldots, r_k \qquad (19)$$

$$\Sigma_{j=1}^{t} \lambda_i(j) = 1 \qquad \text{for } i=r_{k-1}+1, \ldots, r_k \qquad (20)$$

$$\lambda_i(j) \geq 0 \text{ for } i=r_{k-1}+1, \ldots r_k \text{ and } j=1, \ldots, t$$

$$(21)$$

$$Y_k^+(t),\ Y_k^-(t),\ y_k^+(t),\ y_k^-(t) \geq 0$$

Operating Unit i's Decision Processes (OUDP)

OUDP I (Generalized Linear Programming - the Dantzig and Wolfe Decomposition Algorithm)

$$\text{Min} \quad [C_i - \Pi_k(t)B_i - \Pi_k'(t)B_i']X_i(t+1) \tag{22}$$

$$\text{s.t.} \quad D_iX_i(t+1) \begin{Bmatrix}\leq\\ \geq\end{Bmatrix} F_i \tag{23}$$

$$X_i(t+1) \geq 0 \tag{24}$$

$$\text{Coordinative Input:} \quad \gamma_i(t+1) = \begin{bmatrix}\Pi_k(t)\\ \cdots\cdots\\ \Pi_k'(t)\end{bmatrix}$$

OUDP II (Goal Programming)

$$\text{Min } C_iX_i(t+1) + W_k^+\Psi_k^+(t+1) + w_k^+\psi_i^+(t+1) + W_k^-\Psi_i^-(t+1) + w_k^-\psi_i^-(t+1) \tag{25}$$

$$\text{s.t.} \begin{bmatrix}B_i\\ \cdots\\ B_i'\end{bmatrix}X_i(t+1) - I_{m_k+m_k'}\begin{bmatrix}\Psi_i^+(t+1)\\ \cdots\cdots\\ \psi_i^+(t+1)\end{bmatrix} + I_{m_k+m_k'}\begin{bmatrix}\Psi_i^-(t+1)\\ \cdots\cdots\\ \psi_i^-(t+1)\end{bmatrix} = \gamma_i(t+1) \tag{26}$$

$$D_iX_i(t+1) \begin{Bmatrix}\leq\\ \geq\end{Bmatrix} F_i \tag{27}$$

$$X_i(t+1),\ \Psi_i^+(t+1),\ \psi_i^+(t+1),\ \Psi_i^-(t+1),\ \psi_i^-(t+1) \geq 0 \tag{28}$$

$$\text{Coordinative Input:} \quad \gamma_i(t+1) = \begin{bmatrix}B_i\\ \cdots\\ B_i'\end{bmatrix}X_i^*(t) - \begin{bmatrix}Y_k^+(t)\\ \cdots\cdots\\ y_k^+(t)\end{bmatrix} + \begin{bmatrix}Y_K^-(t)\\ \cdots\cdots\\ y_k^-(t)\end{bmatrix} \tag{29}$$

Because the management units serve as the primary coordinators between the CU and the OUs, their decisions will be discussed first.

On iteration t, each manager (k=1,...,M) has an external goal vector, $G_k(t)$, which has been generated by the CU, and an internal goal vector, g_k, which is assumed to be constant throughout the iterative solution process. For each subordinate OU_i $(i=r_{k-1}+1,\ldots,r_k)$, manager k has a set of vectors, $\{X_i(1),\ldots,X_i(t)\}$, which OU_i has generated during the preceding iterations.

These vectors may be interpreted as a series of operating proposals. Using equations (19) and (20), manager k generates a composite proposal vector, $X_i^*(t)$, for each of its subordinate operating units as a convex combination of the previous vectors or operating proposals.[4] The B_i and B_i' matrices linearly relate these composite proposal vectors, $X_i^*(t)$, to the external goal vector, $G_k(t)$, and the internal goal vector, g_k, through equations (17) and (18), respectively. In these constraints, the deviation vectors, $Y_k^+(t)$ $Y_k^-(t)$, $y_k^+(t)$ and $y_k^-(t)$, are computed. The objective function of manager k minimizes the weighted sum of these deviation vectors in conjunction with the actual cost (if applicable) of the composite proposal vectors. This objective function corresponds to the k-th term of the summation in equation (1). Therefore, in solving its decision on iteration t, management unit k generates the optimal set of composite operating unit proposal vectors $X_i^*(t)$ for $i=r_{k-1}+1,\ldots,r_k$ and an optimal set of deviation vectors $Y_k^+(t)$, $Y_k^-(t)$, $y_k^+(t)$ and $y_k^-(t)$. Associated with this solution are two simplex multiplier vectors, $\Pi_k(t)$ and $\Pi_k'(t)$, for equations (17) and (18), respectively. From this solution, manager k extracts the coordinative inputs for the decisions at the other levels of the organization. These coordinative inputs include the feedback vector, $\Gamma_k(t)$, for the CU and the coordinative input vector, $\gamma_i(t)$, for OU_i $(i=r_{k+1}+1, \ldots, r_k)$. The formulation of these coordinative inputs depend upon the CU's and OU's decision processes.

The CU coordinates its managers. This is achieved through the generation of the set of external goal vectors, $(G_1(t+1),\ldots,G_M(t+1))$, which will be used by the managers on the next iteration. In the three decision structures used by the CU, equation (4) of the overall problem is considered during the generation of the external goal vectors. However, the basic strategy employed

[4]See Davis and Whitford (1985).

by the CU to generate the goal vectors differs for each decision process. In the CUDP I, the CU simply minimizes the reduced cost of the external goals with respect to managers' optimal solutions for the previous iteration. Thus, the CU behaves like a column generator for equation (4) in Figure 1. That is, the CU is acting as if the Dantzig-Wolfe (1960) decomposition procedure were applied to the block-angular structure displayed in Figure 1.

CUDP II uses the computational approach of Benders' (1960) decomposition procedure to generate partitioning constraints upon the feasible goal space given by equation (4). This approach allows the CU to generate any goal vector, $(G_1(t+1), \ldots, G_M(t+1))$, contained in the feasible region defined by equation (4). With CUDP I, the CU can only generate extreme points of this feasible region as potential goals, see Freeland (1976). Like the CUDP I, CUDP II uses the simplex multiplier associated with equation (17) as the primary feedback mechanism from the management subsystem k.

On iteration t+1, CUDP III uses the deviation vectors, $Y_k^+(t)$ and $Y_k^-(t)$, obtained from manager k's decision on the previous iteration as a source of feedback information. Using this information and the goal vector which the CU generated on the previous iteration for manager k, $G_k(t)$, the CU can generate the "effective goal vector" which manager k's current decision $(X^*_{r_{k-1}+1}(t), \ldots, X^*_{r_k}(t))$ would satisfy as an equality. This effective goal vector is given as the right-hand-side to equation (13). The deviation vectors $S_k^+(t+1)$ and $S_k^-(t+1)$ are then introduced to the left-hand-side of equation (13) in order to compute the deviations of $G_k(t+1)$ from this effective goal vector. Like CUDP II, equation (13) allows the CU to generate any composite goal vector $(G_1(t+1), \ldots, G_M(t+1))$ satisfying equation (4) for consideration by the management subsystems on the next iteration. Through this procedure, the CU attempts to adjust the composite goal vector so that the combination of the cost (if applicable) of the goals and the weighted sum of the resulting deviations from the effective goal vectors, $G_k(t) + \Gamma_k(t)$ $(k=1,\ldots,M)$, are minimized.

The operating units have two basic decision processes. The first is simply the basic column generator for equation (3.i) derived from the application of the Dantzig-Wolfe decomposition procedure to the subproblem (SP.k). In this approach OU_i attempts to minimize the relative cost of its proposal, $X_i(t+1)$, with respect to its manager's current solution while simultaneously insuring the feasibility of $X_i(t+1)$ with respect to equation (3.i). The coordinative inputs to OU_i are the simplex multipliers associated with its superordinate manager's equations (17) and (18).

The formulation of OUDP II is similar to the CUDP III. However, the formulations differ in that the CU subsystem must be concerned with the current solutions for all the management units, while OU_i is concerned only with the current solution of its manager's problem. The coordinative input for OU_i's decision is its goal vector, $\gamma_i(t+1)$, which its manager generates using equation (29). If OU_i $(i=r_{k-1}+1, \ldots, r_k)$ could generate a proposal vector, $X_i(t+1)$, that fulfills each of the goals contained in $\gamma_i(t+1)$ $(i=r_{k-1}+1, \ldots, r_k)$ then the management unit k could completely satisfy its current goals, $G_k(t)$ and g_k. Equation (26) allows OU_i to estimate the deviations from $G_k(t)$ and g_k that will result from its proposal, $X_i(t+1)$. The feasibility of $X_i(t+1)$ with respect to equation (3.i) is also insured. The selection of the optimum $X_i(t+1)$ is determined by the minimization of the cost of the proposal vector (if applicable) and the penalty weights for the deviations.

4. POTENTIAL PATHS TOWARD NONOPTIMALITY

A simple example has been formulated to show how all the three-level composition models can generate undesirable behavior. For brevity, this example will be discussed only for the GGD and the MGGD-II models as well as the GHM. These three models contain all the basic decision processes given in Table 1.

The example begins by assuming that there are OUs 1 and 2, subject to managers 1 and 2, respectively. The operating constraints for OU 1 and 2 are identical and defined as

$$50 \leq X_i \leq 100 \qquad (i=1,2)$$

while $C_i = 0$ $(i=1,2)$. The CU's single constraint is

$$G_1 + G_2 = 100$$

with G_1 and $G_2 \geq 0$, while $C_{G_1} = C_{G_2} = 0$. For Manager 1, $B_1 = 1$ and $W_1^+ = W_1^- = 10$. Similarly for Manager 2, $B_2 = 1$ and $W_2^+ = W_2^- = 10$. The resulting overall problem is given below:

$$
\begin{array}{llr}
\text{Min } Z = & 10Y_1^+ + 10Y_1^- + 10Y_2^+ + 10Y_2^- & (30) \\
\text{s.t.} & X_1 - Y_1^+ + Y_1^- - G_1 = 0 & (31) \\
& X_2 - Y_2^+ + Y_2^- - G_2 = 0 & (32) \\
& X_1 \geq 50 & (33) \\
& X_1 \leq 100 & (34) \\
& X_2 \geq 50 & (35) \\
& X_2 \leq 100 & (36) \\
& G_1 + G_2 = 100 & (37) \\
& Y_1^+,\ Y_1^-,\ Y_2^+,\ Y_2^-,\ G_1,\ G_2 \geq 0 & (38)
\end{array}
$$

The optimum solution for this problem is

$$X_1 = X_2 = G_1 = G_2 = 50 \text{ and } Y_1^+ = Y_1^- = Y_2^+ = Y_2^- = 0$$

with $Z^* = 0$.

4.1 Analysis of the Models' Solutions

All of the composition models begin by solving the OUs' problems on iteration 1 given as

$$\text{Min } 0\, X_i(1) \qquad (i=1,2) \tag{39}$$

$$\text{s.t. } 50 \leq X_i(1) \leq 100 \tag{40}$$

For the i-th OU's problem, there are multiple optimal basic feasible solutions with $X_i(1)$ equal either 50 or 100. Let us assume each OU returns $X_i(1) = 100$ (i=1,2). Also for each model an initial goal allocation is expected from the CU. Because no coordinative inputs have been generated by the managers, the CU problem is given as

$$\text{Min } 0G_1(1) + 0G_2(1) \tag{41}$$

$$\text{s.t. } G_1(1) + G_2(1) = 100 \tag{42}$$

$$G_1(1),\quad G_2(1) \geq 0 \tag{43}$$

There are two optimal basic feasible solutions to this problem with $\{G_1(1) = 100,\ G_2(1) = 0\}$ or $\{G_1(1) = 0,\ G_2(1) = 100\}$. Assume the first basic feasible solution is chosen.

At this point, Ruefli's GGD model can be eliminated from any further investigation. On every iteration using CUDP I, the CU's problem will have the form

$$\text{Min} - \Pi_1(t-1)G_1(t) - \Pi_2(t-1)G_2(t) \tag{44}$$

$$G_1(t) + G_2(t) = 100 \tag{45}$$

$$G_1(t),\ G_2(t) \geq 0 \tag{46}$$

Of the two basic feasible solutions for the CU defined on iteration one, one or the other must be optimum at every iteration. That is, on every iteration the CUDP I will set either $G_1(t)$ or $G_2(t)$ to 100 and the other goal to 0. The optimal assignment $G_1(t) = G_2(t) = 50$ can never be generated as a basic feasible solution. The GGD algorithm is destined to a suboptimal solution and is likely to demonstrate an oscillation in the overall objective Z(t).

For the GHM on iteration 1, the manager 1's problem is given as

$$\text{Min} \quad 10Y_1^+(1) + 10Y_1^-(1) \tag{47}$$

$$100\lambda_1(1) - Y_1^+(1) + Y_1^-(1) = 100 \tag{48}$$

$$\lambda_1(1) = 1 \tag{49}$$

$$\lambda_1(1),\ Y_1^+(1),\ Y_1^-(1) \geq 0 \tag{50}$$

The optimal basic feasible solution has $\lambda_1(1) = 1$ with both $Y_1^+(1)$ and $Y_1^-(1)$ equal to 0. Manager 2's decision is given as

$$\text{Min} \quad 10Y_2^+(1) + 10Y_2^-(1) \tag{51}$$

$$100\lambda_2(1) - Y_2^+(1) + Y_2^-(1) = 0 \tag{52}$$

$$\lambda_2(1) = 1 \tag{53}$$

$$\lambda_2(1),\ Y_2^+(1),\ Y_2^-(1) \geq 0 \tag{54}$$

The optimal basic feasible solution is $\lambda_2(1) = 1$ and $Y_2^+(1) = 100$ with $Y_2^-(1) = 0$.

On iteration 2, OU_1 has the problem

$$\text{Min} \quad 10\Psi_1^+(2) + 10\Psi_2^-(2) \tag{55}$$

$$\text{s.t. } X_1(2) - \Psi_1^+(2) + \Psi_1^-(2) = 100 \tag{56}$$

$$\text{with } 50 \leq X_1(2) \leq 100;\ \Psi_1^+(2),\ \Psi_2^-(2) \geq 0 \tag{57}$$

The optimal basic feasible solution gives $X_1(2) = 100$ with $\Psi_2^-(2) = \Psi_2^+(2) = 0$. For OU_2, the problem is

$$\text{Min} \quad 10\Psi_2^+(2) + 10\Psi_2^-(2) \tag{58}$$

$$\text{s.t. } X_2(2) - \Psi_2^+(2) + \Psi_2^-(2) = 0 \tag{59}$$

$$\text{with } 50 \leq X_2(2) \leq 100;\ \Psi_2^+(2),\ \Psi_2^-(2) \geq 0 \tag{60}$$

to which the optimal basic feasible solution is $X_2(2) = 50$, $\Psi_2^+(2) = 50$ and $\Psi_2^-(2) = 0$.

Using CUDP-III, the CU on iteration 2 has the problem

$$\text{Min} \quad 10S_1^+(2) + 10S_2^+(2) + 10S_1^-(2) + 10S_2^-(2) \tag{61}$$

$$\text{s.t. } G_1(2) + S_1^+(2) - S_1^-(2) = 100 \tag{62}$$

$$G_2(2) + S_2^+(2) - S_2^-(2) = 100 \tag{63}$$

$$G_1(2) + G_2(2) = 100 \tag{64}$$

with all variables ≥ 0.

There are again multiple optimal basic solutions to solutions to this problem which assign either $G_1(2) = 100$ and $G_2(2) = 0$ or vice versa. Assume the basic feasible solution $G_1(2) = 100$, $G_2(2) = 0$ and $S_2^+(2) = 100$ is again chosen.

Manager 1's problem on iteration 2 is given as

$$\text{Min} \quad 10Y_1^+(2) + 10Y_1^-(2) \tag{65}$$

$$\text{s.t.} \quad 100\lambda_1(1) + 100\lambda_1(2) - Y_1^+(2) + Y_2^-(2) = 100 \tag{66}$$

$$\lambda_1(1) + \lambda_1(2) = 1 \tag{67}$$

with all variables ≥ 0. The obvious optimal solution gives either $\lambda_1(1)$ or $\lambda_2(1) = 1$ and all other variables equal to zero.

Manager 2 has the problem

$$\text{Min} \quad 10Y_2^+(2) + 10Y_2^-(2) \tag{68}$$

$$\text{s.t.} \quad 100\lambda_2(1) + 50\lambda_2(2) - Y_2^+(2) + Y_2^-(2) = 0 \tag{69}$$

$$\lambda_2(1) + \lambda_2(2) = 1 \tag{70}$$

with all variables ≥ 0. The optimum solution to this problem is $\lambda_2(2) = 1$ and $Y_2^+(2) = 50$ with all other variables equal to 0.

On iteration 3, the CU has the problem

$$\text{Min} \quad 10S_1^+(3) + 10S_2^+(3) + 10S_1^-(3) + 10S_2^-(3) \tag{71}$$

$$\text{s.t.} \quad G_1(3) + S_1^+(3) - S_1^-(3) = 100 \tag{72}$$

$$G_2(3) + S_2^+(3) + S_2^-(3) = 50 \tag{73}$$

$$G_1(3) + G_2(3) = 100 \tag{74}$$

with all variables ≥ 0. Again, there are multiple optimum basic solutions to this problem giving either $G_1(3) = 100$ with $G_2(3) = 0$ or $G_1(3) = G_2(3) = 50$. If the first basic solution is chosen, then working through the GHM algorithm the CU's decision on iteration 4 will be identical to that iteration 3. If the CU has identical solutions on successive iterations, the managers' and

OUs' problems on iteration 5 will be identical to their problems on iteration 4. Thus, the GHM converges to nonoptimal solution. If the CU on iteration 3, chooses the second basic solution with $G_1(3) = G_2(3) = 50$ then the GHM can proceed to generate the overall optimum solution. Hence the efficacy of the CHM is associated with which basic solution the computer code chooses.

Because the managers' problems are the last set of problems solved on iteration 1, the CU and OU problems are identical for the MGGD-II and the GGD models because no coordinative information is yet available. Hence the managers' problem will also be identical. Specifically for manager 1 we have

$$\text{Min} \quad 10Y_1^+(1) + 10Y_1^-(1) \qquad (75)$$

$$\text{s.t.} \; 100\lambda_1(1) - Y_1^+(1) + Y_1^-(1) = 100 \qquad (76)$$

$$\lambda_1(1) = 1 \qquad (77)$$

with all variables ≥ 0.

In the optimal basic solution $\lambda_1(1)$ must equal 1; however, either $Y_1^+(1)$ or $Y_2^-(1)$ must serve as the second degenerate basic variable. We note that further interactions with the OU will not eliminate the degeneracy as the deviations have been reduced to zero. Computationally the choice of which deviation is basic will likely be determined by numerical roundoff. Assume that $Y_1^-(1)$ is made basic. Then $\Pi_1(1) = 10$ and constraint (9) becomes

$$\zeta_1(t) - 10G_1(t) \geq 0 + 10(100) = 1000 \qquad (78)$$

Equation (78) will remain in the CU's constraint for all subsequent iterations.

At this point the algorithm can be stopped; achievement of the optimum solution is no longer possible. To see this recall that in CUDP-II, (equations (8) through (11)), the objective function $\zeta_1(t) + \zeta_2(t)$, must equal the value of the overall objective function, i.e., equation (1), at optimality. Choosing $Y_1^+(1)$ as a basic variable introduced a constraint in the CU's problem that will require $\zeta_1(t) \geq 500$ whenever the optimal value of $G_1(t) = 50$ is assigned. Because this constraint will remain for all iterations. We can never generate the optimum solution requiring $\zeta_1(t) = 0$ with $G_1(t) = 50$. If $Y_1^+(t)$ had been chosen to be the degenerate basic variable, then constraint (9) becomes

$$\zeta_1(t) - 10G_1(t) \geq -1000 \qquad (79)$$

which would be introduced to the CU's problem on the next iteration. The optimal solution, $\zeta_1(t) = 0$ and $G_1(t) = 50$ satisfies this constraint. In conclusion, a simple choice of a degenerate basic variable determines whether the optimum solution can be achieved or not.

In Davis and Whitford (1985), it was argued that the manager and its OUs must be required to interact until an optimal solution for the ensemble of decision-makers is secured for the current $G_k(t)$. The necessity of this requirement can be demonstrated in studying manager 2's problem on iteration 1 given as

$$\text{Min} \quad 10Y_2^+(1) + 10Y_2^-(1) \qquad (80)$$

$$\text{s.t.} \quad 100\lambda_2(1) - Y_1^+(1) + Y_2^-(1) = 0 \qquad (81)$$

$$\lambda_2(1) = 1 \qquad (82)$$

The optimal solution for this problem is $\lambda_2(1) = 1$ and $Y_1^+(1) = 100$ with $\Pi_2(1) = -10$. If this simplex multiplier were passed to the CU, then constraint (9) gives

$$\zeta_2(t) - 10G_2(t) > 1000 - 10(100) \tag{83}$$

Obviously, the essential solution to generate the overall optimum solution, namely $\zeta_2(t) = 0$ and $G_2(t) = 50$ will not satisfy this constraint.

Suppose, however, OU_2 and MU_2 are allowed to interact on iteration 1 until the optimal solution to subproblem SP.2 for $G_2(1)$ is ascertained. Specifically, $\Pi_2(1) = -10$ was passed to the OU who generated the optimum solution to the following problem

$$\text{Min} \quad 10X_2(1') \tag{84}$$

$$\text{s.t.} \; 50 < X_2(1') < 100 \tag{85}$$

as $X_2(1') = 50$.

Manager 2 would then have the revised problem on iteration 1 given as

$$\text{Min} \qquad 10Y_2^+(1) + 10Y_2^-(1) \tag{86}$$

$$\text{s.t.} \; 100\lambda_2(1) + 50\lambda_2(1') - Y_2^+(1) + Y_2^1(1) = 0 \tag{87}$$

$$\lambda_2(1) + \lambda_2(1') = 1 \tag{88}$$

where both proposals generated by OU_2 would be considered. The optimal solution to this problem is $\lambda_2(1') = 1$ and $Y_2^+(1) = 50$ with $\Pi_2^2(1) = -10$. Associated with this solution, constraint (9) yields

$$\zeta_2(t) - 10G_2(t) > 500 - 10(100) = -500 \tag{89}$$

We quickly note that $\zeta_2(t) = 0$ and $G_2(t) = 50$ will satisfy this constraint.

4.2 Nonoptimal Paths in Two-Level Hierarchies

Even the two-level Freeland and Baker (1975) model is not immune from a path to nonoptimality. For the two-level model, the managers (the lowest level in this formulation) must have a feasible $G_1(1)$ and $G_2(1)$ to solve their respective problems. Because no coordinative information is available on the first iteration, the CU decision is again identical to that of the GGD model on iteration one. Let $G_1(1) = 100$ and $G_2(1) = 0$ be the initial goal allocation. Manager 1's problem is then given as

$$\text{Min} \quad 10Y_1^+(t) + 10Y_1^-(t) \tag{90}$$

$$\text{s.t. } X_1(t) - Y_1^+(t) + Y_1^-(t) = 100 \tag{91}$$

$$X_1(t) \geq 50 \tag{92}$$

$$X_1(t) \leq 100 \tag{93}$$

with $Y_1^+(t), Y_1^-(t) \geq 0$. Obviously $X_1(t) = 100$ is one basic variable for constraint (93) in the optimum solution. The slack variable to constraint (92) must serve as another basic variable. The third basic variable associated with constraint (91) will be degenerate and can be either $Y_1^+(t)$ or $Y_1^-(t) = 0$. If $Y_1^-(t)$ is chosen to be basic, then as discussed earlier, constraint (4) gives

$$\zeta_1(t) - 10G_1(t) \geq 1000 \tag{94}$$

which will be introduced to the CU's constraint set. At this point, the optimal solution $\zeta_1(t) = 0$ and $G_1(t) = 50$ can never be generated. This fact is extremely troubling since the two-level model represents a direct application of Benders, (1960) partitioning algorithm for which optimality has presumably been demonstrated.

5. CONCLUDING REMARKS

Our analysis in the previous section leads to two observations. First, the mathematical limitations of the potential feasible solutions for the CUDP-I technique are simply too restrictive to ever expect optimality of the model's limiting solution with respect to the overall problem. This corroborates Freeland's (1976) work.

However, the existence of a potential path for a nonoptimal solution for all the other decision processes at the various levels of the composition or decomposition models is alarming. One cannot be sure that these models will always generate a nonoptimal solution, but in many applications the potential for nonoptimality clearly exists. The primary source of the potential nonoptimality is a nonunique solution for the managers' goal programming problem. This situation arises when either there are multiple optimum solutions or a manager's optimal solution is degenerate. The complexity arising in the simple example considered earlier is significantly increased when larger problems with hundreds or thousands of decisions variables are considered. Computational experience with larger applications has shown that degeneracy is almost always present in the model's solutions, especially at the managerial level [see Whitford and Davis (1983)].

It is also interesting and disturbing that accepted decomposition methods, such as Benders' partitioning algorithm, also experience computational difficulties in solving linear goal programming formulations at the managers' level of hierarchy. (Although not discussed in this paper, the Dantzig-Wolfe decomposition algorithm also demonstrates computational difficulties with the linear goal programming formulations.) Thus one must conclude that composition models or any model which incorporate linear goal programming problems within the decision-making hierarchy should be used with great care unless the robustness of the algorithm can be demonstrated.

There is a positive result to this paper despite the previous pessimistic observations. Specifically, all the models are addressing the same overall problem. Thus, the potential for comparing the coordinative schemes for various solutions does exist. Further, recent work by BenAfia and Davis (1986) on a two-level hierarchical model indicates that the sources of nonoptimality can be removed simply by replacing the linear formulation of the penalty costs on the goal deviations with a quadratic formulation. Unfortunately, the introduction of this type of nonlinear formulation significantly increases the computational requirements and the complexity of computer codes used to solve the model.

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