Faculty Working Papers

A SPECIFICATION ANALYSIS OF THE DEMAND FOR CRUDE OIL

Albert L. Danielsen and
Cheng F. Lee

#366

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

January 4, 1977

A SPECIFICATION ANALYSIS OF THE DEMAND FOR CRUDE OIL

Albert L. Danielsen and
Cheng F. Lee

#366

A SPECIFICATION ANALYSIS OF THE DEMAND FOR CRUDE OIL

Albert L. Danielsen*

and

Cheng F. Lee**

Assistant Professor, Finance Department

*Associate Professor

*University of Georgia and

Market Structure & Regulations Impact Division,
Office of Microeconomic Impact
Office of Energy Information and Analysis

FEDERAL ENERGY ADMINISTRATION

**University of Illinois

A SPECIFICATION ANALYSIS OF THE DEMAND FOR CRUDE OIL

1. INTRODUCTION

It has long been recognized that price elasticities of demand are dependent upon the level of a consumer's income and that income elasticities are a function of price. However, the formal introduction of interactions among income and prices into econometric analyses is a recent phenomena. Frisch was perhaps the first to formally recognize the importance of interaction effects [11, 180], but Cramer has been the more instrumental in its adoption in recent years [8; 9]. Cramer selected a log linear functional form to investigate the interaction effect but it is well known that the interaction effect can be reduced by the generalized functional form developed by Box and Cox [5].

The approach in this study is to test for the proper functional form and then to determine if interaction effects add important new information to the model from both theoretical and empirical viewpoints. Using annual United States data on crude oil, it is found that the log linear specification yields unbiased estimates only when the functional form parameter does not differ significantly from zero. The inclusion of an interaction term improves the specification on theoretical grounds but the problem of multicollinearity is apparent in the empirical results. Both theoretical and empirical criteria are established and used to select the most appropriate specification.

The study is presented in five sections. The following section specifies the functional form of models normally used to investigate demand relations, shows how they are special cases of the generalized functional form, explains why interaction effects should be included and extends the functional form analysis to test the appropriateness of a first difference of logarithm specification. Section 3 outlines the models and empirical criteria employed to select among alternative specifications and presents the basic empirical results. Section 4 compares the models and empirical estimates with previous studies. And Section 5 is the conclusion.

2. FUNCTIONAL FORM AND THE INTERACTION EFFECT

In the work of Houthakker and Taylor [13], Klein [17], Sato [22], and in most other studies the functional form used to investigate demand relations is assumed to be either linear or log linear. The following specifications are typical:

[A] $q_t = a_o + a_1 y_t + a_2 p_t$ (1)

[B] $q_t = b_o + b_1 y_t + b_2 p_t, b_3 q_{t-1}$ or

[A] $\log q_t = a'_o + a'_1 \log y_t + a'_2 \log p_t$ (2)

[B] $\log q_t = b'_o + b'_1 \log y_t + b'_2 \log p_t + b'_3 \log q_{t-1}$

where q_t represents per capita consumption of crude oil in the t^{th} time period; y_t represents real per capita income in the t^{th} time period, and p_t denotes the real price of crude oil in the t^{th} time period.

Equations (1) and (2) are special cases of the general transformation of variables form of Box and Cox [5] in which

$$[A] \quad (q_t^{\lambda} - 1)/\lambda = c_0 + c_1(y_t^{\lambda} - 1)/\lambda + c_2(p_t^{\lambda} - 1)/\lambda \qquad (3)$$

$$[B] \quad (q_t^{\lambda'} - 1)/\lambda' = d_0 + d_1(y_t^{\lambda'} - 1)/\lambda' + d_2(p_t^{\lambda'} - 1)/\lambda'$$
$$+ d_3\,(q_t^{\lambda'} - 1)/\lambda'$$

where λ and λ' are the functional form parameters to be estimated. Equation (3) becomes equation (1) when λ and λ' equal one, reduces to equation 2 when λ and λ' approach zero, and therefore includes both the linear and the logarithmic forms as special cases. Thus, equation (3) provides a generalized test to determine the functional form using a multiple regression technique [5, 211].

Frisch and Cramer have emphasized the interaction between income and price; more specifically, they have argued that price and income elasticities (η and σ) are dependent on absolute levels of income and price respectively [11; 8]. The value of η obviously varies directly with income: as a consumer's income increases σ (which is generally negative) tends toward zero. This corresponds with traditional theories of a consumer's behavior which posit "saturation" levels of income or "bliss" points [8; 9; 10]. Perhaps less obviously, η also varies directly with prices: price increases have a negative income effect, η varies inversely with the level of income, and, it follows directly, that η varies directly with prices. The latter also corresponds with traditional theories of a consumer's behavior since price increases more the consumer away from a saturation level where his income elasticity is zero. Any movement away from the saturation point must increase the income elasticity.

Actually the transformation of variables is one way of reducing the importance of interaction effects among the independent variables. But merely transforming the variables does not ensure the elimination of the interaction term. The position taken in this study is that even though an acceptable functional form has been statistically ascertained, the presence or absence of interaction is a phenomena which should be tested rather than assumed. However, the problem of determining an appropriate specification of the interaction term remains.

Conceptually, there are a large number of interaction terms compatible with values of η and σ which vary directly with price (p) and income (y) respectively. Any of the family of multiplicative income - price interaction terms raised to a positive power have such properties. However, there are also a large number of specifications which will yield ambiguous results. The remainder of this section outlines some alternative specifications which conceptually satisfy the second order conditions outlined in the preceding paragraph. The only initial requirements are the Frisch-Cramer conditions

$$\text{(C:1)} \quad \frac{\partial \eta}{\partial p} > 0, \text{ and}$$

$$\text{(C:2)} \quad \frac{\partial \sigma}{\partial y} > 0.$$

These are not hypotheses but <u>theoretical</u> <u>criteria</u> which a model must satisfy in order to qualify for serious consideration.

The use of such theoretical criteria is based upon the practice of using economic theory as a guide in evaluating empirical results. They are comparable to requiring that a demand equation be specified such that it could have a negative slope. In section 3 the alternative models are confronted with actual data on crude oil to determine which specifications are acceptable on both theoretical and empirical grounds.

One approach is that of Richards [21] who investigated unexplained joint effects through an analysis of residuals. In his analysis six possible specifications were used to take interaction effects into account. Based upon equation [1], the possible interaction specifications between income and price are $y \cdot p$, p/y, $(y \cdot p)^2$, $y \cdot p^2$, $(y + p)$, and $(y - p)$, where the time subscript t on each variable is understood but omitted to avoid combrous notation. All of these interaction terms except the additive specifications meet criteria (C:1 and C:2); in the additive cases

$$\frac{\partial \eta}{\partial p} \equiv \frac{\partial \sigma}{\partial y} \equiv 0.$$

Integrating the interaction terms suggested by Richards into equation (3) we obtain:

$$[A]\ (q^{\lambda} - 1)/\lambda = e_0 + e_1(y^{\lambda} - 1)/\lambda + e_2(p^{\lambda} - 1)/\lambda + e_3(X^{\lambda} - 1)/\lambda \qquad (4)$$

$$[B]\ (q^{\lambda'} - 1)/\lambda' = f_0 + f_1(y^{\lambda'} - 1)/\lambda' + f_2(p^{\lambda'} - 1)/\lambda' + f_3(q_{t-1}^{\lambda'} - 1)/\lambda' + f_4(X^{\lambda'} - 1)/\lambda'$$

where X represents $[y \cdot p]$, p/y, $[y \cdot p_i]^2$, or $y \cdot (p)^2$. Again the time subscript t is understood but omitted. The specifications contained in equation (3) with $\lambda = 0$ have been used in a number of empirical studies [15; 24], the interaction effects have been investigated by Cramer but without testing the appropriate functional form, but to our knowledge the two approaches have not been explicitly integrated as in equation (4). Furthermore, equation (4) is, but equation (3) is not, consistent with the argument of Box and Cox that the transformation of variables may not entirely remove the interaction effect.

A second approach, strongly advocated by Granger [12], for example, is to express each variable in first differences of logarithms. However, this specification is a special case of the *current to one-period lag* ratio of each variable. The general case may be expressed as

$$\frac{\left(\frac{q_t}{q_{t-1}}\right)^{\lambda} - 1}{\lambda} = g_0 + g_1 \frac{\left(\frac{y_t}{y_{t-1}}\right)^{\lambda} - 1}{\lambda} + g_2 \frac{\left(\frac{p_t}{p_{t-1}}\right)^{\lambda} - 1}{\lambda} + g_3 \frac{\left(\frac{X_{it}}{X_{it-1}}\right)^{\lambda} - 1}{\lambda} . \tag{5}$$

Equation (5) is an equation of percentage changes when $\lambda = 1$, reduces to a first difference of logarithm specification as λ approaches zero, and therefore includes both the first difference and double logarithmic first difference equation as a special case.

In the following section equations (3), (4), and (5) are used to investigate the demand functions for crude oil. Comparing these equations allows one to test for functional form and to determine if there exists an interaction effect. From these results one may select the model or models which meet specified theoretical and conventional statistical criteria.

3. EMPIRICAL ELABORATION AND TESTING

A stochastic disturbance term should be introduced into equations (3), (4), and (5) respectively in order to empirically estimate and test the functional form and other regression parameters. If the disturbance terms are normally and additively distributed then the maximum logarithmic likelihood method developed by Box and Cox [5] can be used to determine the optimal functional form for equations (3), (4), and (5). Omitting the constant term, they may be expressed as:

$$\text{Lmax } (\hat{\lambda}) = -n \log \hat{s} (\lambda) + (\lambda-1) \sum_{t=1}^{n} \log q_t \qquad (6)$$

where $\hat{\lambda}$ is the estimated value of λ, n the sample size, and $\hat{s} (\lambda)$ the estimated regression residual standard error of equations (3) and (4) with a disturbance term. Equation (6) is also applicable to equation (5) by replacing q_t with the ratio q_t/q_{t-1}.

Using the likelihood ratio method, an approximate 95% confidence region for λ can be obtained from

$$\text{Lmax } (\hat{\lambda}) - \text{Lmax } (\lambda) < \tfrac{1}{2}\chi^2_{(1)} (.05) = 1.92 \qquad (7)$$

The 95% confidence region for λ will be used to determine the true functional form in investigating the demand relations for crude oil.

Annual data on the per capita consumption and well-head (or mine) prices of natural gas, crude oil, and coal were used in this study. These are United States data for 1947-73 and are from standard trade or government sources [2; 3; 18; 19; 23].

The data were fitted to equation (3A) and evaluated at the maximum likelihood value of $\hat{\lambda}$. The most important finding from this preliminary investigation is that serial correlation is a problem at all values of λ. The results evaluated at $\hat{\lambda} = 0$ are shown in Table 1 under equation code number I. The high R^2 and low D.W. values confirm Granger's discussions regarding spurious regressions in econometrics [12].

A standard approach designed to eliminate serial correlation is to transform all variables into first differences of logarithms. However, it may be recalled that first differences and first differences of logarithms are actually special cases of equation (5). Therefore, the data were expressed as ratios and run to determine the maximum likelihood value of λ. It is -8.6 as shown in Table 1 under code II and it is significantly different from one but not significantly different from zero.

The very large absolute value of $\hat{\lambda}$ should not be viewed with alarm since the ratios $\approx$ = 1.0 and thus the values are fairly insensitive even to large values of λ. In addition $\hat{\lambda} \approx 0$ so that a first difference of logarithms appears to be appropriate. The results for $\lambda = 0$ are

recorded in Table 1 under Code III and are remarkably similar to the Code II results. The R^2 values are remarkably high for a first difference of logarithm specification. The signs of the income elasticity coefficients are acceptable on theoretical grounds for $\hat{\lambda}$ and for $\lambda = 0$ and are significant at the .05 level. The price elasticity coefficients are also the theoretically expected sign for $\hat{\lambda}$ and for $\lambda = 0$ and are significant at the .05 level.

The next question considered is whether the introduction of an interaction term improves the models in some important ways. Four interaction models were tried, including a multiplicative first difference of logarithms and those suggested by Richards. Furthermore, the study by Balestra and Nerlove [6] suggests that several variables should be investigated. Alternatively, coal and natural gas prices served as numeraire for the relative crude oil price variable. Thus there were a total of twelve alternative specifications. These alternative specifications were deemed necessary to determine the best possible model as judged on both theoretical and statistical grounds.

Cramer found the interaction effect was significant using cross sectional data but not when time series data were employed [9, 356]. The latter may have been the result of restricting the interaction term to a particular case or because he did not make ". . . a careful study of each item, taking into account specific factors and substitution effects" [9, 356]. The latter was probably not an important

consideration for Cramer but in the present case interfuel substitution is clearly relevant and a function of relative prices. Similarly, it seems preferable to test alternative interaction specifications rather than to simply select one arbitrarily.

The interpretation of the price elasticities is altered somewhat because relative rather than absolute prices are used to derive the estimates. The use of relative prices takes the substitution effect between two resources into account. This specification is a general case which includes the absolute price specification as a special case. The latter is based on the assumption that the prices of substitutes are constant. The relative price specification allows prices of substitutes to vary over time. The elasticity estimates derived from this specification could be appropriately termed relative price elasticities. In general, one would expect relative to be more highly inelastic than absolute price elasticities.

The criteria employed to select among the alternative specifications are conventional ones regarding the signs of the income and price elasticities. The income elasticity is assumed to be positive and its own price elasticity is negative; thus,

$$\eta = \frac{\partial \log q}{\partial \log y} > 0, \text{ and, } \sigma = \frac{\partial \log q}{\partial \log p} < 0, \qquad (8)$$

Two additional criteria refer to the interaction between income and price. An appropriate function of the genus described by the logarithmic version of equation (4) must satisfy the following conditions:

$$[A] \quad \frac{\partial \eta}{\partial \log p} = \frac{\partial^2 \log q}{\partial \log y \cdot \partial \log p} > 0 \qquad (9)$$

$$[B] \quad \frac{\partial \sigma}{\partial \log y} = \frac{\partial^2 \log q}{\partial \log p \cdot \partial \log y} > 0.$$

These are <u>empirical criteria</u> employed to select among theoretically plausible model specifications and are not to be confused with the more general C:1 and C:2.

In effect, we are selecting a model on the basis of the congruence of the empirical estimates with the theoretical criteria. This is comparable to rejecting a model specification if it yields a positive price elasticity, or, except in the case of an "inferior" good, if it yields a negative income elasticity. In the present case the specification must also satisfy the Frisch-Cramer conditions that the estimated income elasticity, $\hat{\eta}$, varies directly with price and the estimated price elasticity, $|\hat{\sigma}|$, varies inversely with income.

The empirical criteria actually used to select among models may be outlined more specifically as follows:

Criterion I: The model without an interaction term must have signs such that $\hat{\eta} > 0$ and $\hat{\sigma} < 0$. For the interaction models the interaction term must have the theoretically expected sign (generally positive but negative in the division case).

Criteria II: The model without an interaction must have $\hat{\eta}$ and $\hat{\sigma}$ values significant at the .10 level using a one-tail "t" test. For the interaction models the coefficient of the interaction terms must be significant at the .10 level, again using the one-tail "t" test.

Criteria III: For the interaction models the relation $\hat{\eta} > 0$ and $\hat{\sigma} < 0$ must obtain when evaluated at the mean.

Of the 12 specifications based on equations (3) and (4) only 4 survived Criterion I, and only 1 survived Criterion II. These results are summarized in Table 2. The model which survived Criterion II also survived Criterion III. The only model which survived was based on coal price as numeraire, and it contained a multiplicative interaction term. The results are recorded under equation code number IV in Table 1. The inclusion of an interaction term does not change the results very much. The elasticity coefficients are altered slightly but the overall performance of the code II, III, and IV variants are almost identical.

A more general observation concerns the zero-order correlation coefficients between the price and interaction variables which were greater than .90. A major conclusion is that although interaction effects can improve the specification of a model on theoretical grounds, they nevertheless complicate the empirical estimates and interpretation.

Comparing Code I with Code II, III, and IV results, it should be noted that the problem of serial correlation is eliminated as one moves from the Code I to the Code II specification. Code II and Code III are very similar as should be expected since the maximum logarithmic likelihood value of λ is not significantly different from zero. The Code III and Code IV models are also very similar. The best model on theoretical grounds is Code IV. On empirical grounds the story is mixed because the price and interaction term exhibit high multicollinearity. However, comparing the elasticity estimated based on Codes III and IV it is obvious that the parameter estimates were not affected very much. If multicollinearity has contaminated the results, it is not readily apparent in Table 1.

Forecasts based on the Code IV model perform well compared to Codes II and III. Consumption was underestimated for 1974 and overestimated for 1975. The forecasts are remarkably close for 1974 but consumption in 1975 did not increase by as much as one would predict on the basis of experience during 1948-73. The 1975 error may be the result of exogenous factors affecting crude oil consumption (e.g., 55 mph speed limit, coal conversion programs, and carpooling). The forecasts for 1974 are much closer to the actual figures, possibly because the structural changes noted above had not been fully accomplished during that year.

4. A COMPARISON WITH PREVIOUS STUDIES

This section compares the models and empirical estimates derived in the last section with previous studies. A general conclusion which emerges is that most of the price elasticities derived from previous empirical studies are not reliable estimates of the aggregate relations which obtain. A corrolary is that the estimates presented in this study are more appropriate for use in simulation models and discussions of overall energy policy.

An examination of the literature reveals that the income elasticity estimates reported here are comparable to previous studies (at least they are in the ball park) but that the price elasticities are much lower. Which estimates should be considered reliable indicators of the phenomena they purport to measure? The answer is that a number of studies have presented reliable estimates of diverse phenomena. In recent years they have generally focused upon the individual consumer and attempted to describe his reaction to changes in income and prices under a variety of _ceteris paribus_ assumptions.

The demand for crude oil is usually considered derived from the demand for the principal derivative products, (1) gasoline, (2) kerosine, (3) distillate fuel oil, and (4) residual fuel oil. In addition, writers generally stress the desirability of investigating the products by demand sector, (a) residential, (b) industrial, and (c) transportation. The various demand relations are then integrated to derive the

demand for crude oil. The Houthakker-Kennedy model is a highly regarded one aiming in this direction [16]. Indeed this procedure would be appropriate if our knowledge of the interrelations among products, sectors, and other fuels were perfect or even fairly good. However, given our very imperfect perception of the world, the main problem is that demand elasticities for the derivative products are estimated under the assumption that prices of close substitutes are constant. This yields own price elasticities for gasoline -.82, kerosine -2.00, distillate -.76, and residual -1.58. Income elasticities exceed 1.0 except for kerosine which is classified an inferior good [16, 253-259]. These results are of course inappropriate for an analysis of aggregate crude oil demand. The estimate on the order of -.10 derived in this study appears the more reasonable.

In another study, using a different method, Verleger has presented a price elasticity estimate of -.40 for petroleum [25, 39]. The estimate of -.10 presented in this study appears to be superior for the following reasons: (1) aggregate demand is measured directly and the estimates are therefore not contaminated by interderivative product substitution; (2) the crude oil price variable is defined as relative to the price of its principal substitutes and thus inter-resource substitution is taken into account; and (3) the functional form has been subjected to extensive empirical testing.

5. SUMMARY AND CONCLUDING REMARKS

This paper has integrated the functional form and interaction specifications to investigate the demand for crude oil. It has also extended the functional form analysis to include a test for first differences of logarithms. Multicollinearity between the interaction and price variable suggests that one should examine the test statistics and coefficients carefully. For crude oil at least, the interaction term does not appear very important for estimating income and price elasticities. However, it does improve the forecasting power.

The empirical estimates derived from this study are felt to be appropriate for use in simulation models. The results are particularly useful for discussion of overall energy policy. They are felt to be all the more reliable since several price variables were tested and a relative price variable was used to derive the estimates. This simultaneously eliminated the problem of multicollinearity and took inter-resource substitution into account. Finally, the method of testing the appropriateness of a first difference of logarithm specification should be regarded as a methodological contribution.

Table 1. Comparison of Income and Price Elasticities for Crude Oil, Non-Interaction and Interaction Models Evaluated at the Mean

Equation Code Number	λ	Interaction Term	$\hat{\eta}$	$\hat{\sigma}$	Goodness of Fit Statistics R^2	D.W.
I	0	None	1.1178 (20.5752)*	-.0139 (-.2370)	.95	0.49
II	-8.6†	None	.6659 (4.1607)*	-.0984 (-2.1736)*	.48	2.31
III	0	None	.5815 (4.0487)*	-.1339 (-2.1819)*	.49	2.19
IV	0	$\Delta\log y \cdot \Delta\log p$	.6659 (4.1691)*	-.0997 (-2.1924)*	.51	2.05

*Significant at the .05 level; values in parentheses are "t" statistics.

†Significantly different from 1.0, but not 0.0 at the .05 level.

Table 2. Summary of Regression Models for Crude Oil, United States 1948-73.

Model Specifications	Total No. of Models Considered	Models Rejected on the Basis of Criterion I & II		Total No. of Models Accepted
		I	II	
A. Double logarithmic first differences				
1. $\Delta \log y \cdot \Delta \log p$	3	2		1
2. $(\Delta \log y \cdot \Delta \log p)^2$	3	2	1	
3. $\Delta \log y \cdot \Delta \log p)^2$	3	2	1	
4. $\Delta \log y/\Delta \log p$	3	2	1	

Table 3. Percentage Forecasting Error Using Alternative Models for Crude Oil, 1974 and 1975

Equation Code Number	1974 Actual	Forecast	Percent Error	1975 Actual	Forecast	Percent Error
II	28.6420	28.1205	-1.8	27.6449	29.3752	6.2
III	28.6420	28.1526	-1.6	27.6449	29.0842	5.2
IV	28.6420	28.2961	-1.2	27.6449	28.6079	3.5

REFERENCES

1. Allen, R.G.D., "The Foundations of a Mathematical Theory of Exchange," Economica, 12 (May, 1932), 197-226.

2. American Gas Association, Reserves of Crude Oil, Natural Gas Liquids, and Natural Gas in the United States and Canada and United States Productive Capacity as of December 31, 1973, 28 (June, 1974).

3. American Petroleum Institute, Petroleum Facts and Figures, New York: American Petroleum Institute, 1971.

4. Balestra, P. and M. Nerlove, "Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas," Econometrica 34 (July, 1966), 585-612.

5. Box, G.P.E. and D. R. Cox, "An Analysis of Transformations," Journal of the Royal Statistical Society, Series B, 26 (1964). 221-243.

6. Breyer, S. G. and P. W. MacAvoy, Energy Regulation by the Federal Power Commission. Washington, D.C.: The Brookings Institution, 1974.

7. Cochrane, D., and G. Orcutt, "Application of Least Squares Regression to Relationships Containing Auto-Correlated Error Terms, Journal of the American Statistical Association, 44 (March, 1949), 32-61.

8. Cramer, J. S., "Interaction of Price and Income in Consumer Demand," European Economic Review (Spring, 1970), 428-36.

9. _________., "Interaction of Income and Price in Consumer Demand," International Economic Review (June, 1973), 351-63.

10. Danielson, A. L., "Interdependent Utilities, Charity, and Pareto Optimality; Comment," Quarterly Journal of Economics 87 (August, 1975), 477-81.

11. Frisch, R., "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors," Econometrica, (April, 1959), 177-196.

12. Granger, C.W.J. and P. Newbold, "Spurious Regressions in Econometrics," Journal of Econometrics, 2(1974), 111-20.

13. Houthakker, H. and Taylor, L. Consumer Demand in the United States, Cambridge: Harvard University Press (1966).

14. Houthakker, H. S., "The Econometric of Family Budget," J. Royal Statistical Society, Series A, 115 (1952), 1-21.

15. Kau, James B. and Lee, Cheng F., "The Functional Form in Estimating the Density Gradient," *Journal of the American Statistical Association*, (June, 1976).

16. Kennedy, M., "An Economic Model of the World Oil Market." *The Bell Journal of Economics and Management Science*, 5 (Autumn, 1974), 540-77.

17. Klein, L. R., *A Textbook of Econometrics*. Evanston: Row, Peterson and Company, 1957.

18. *Metal Statistics*, 1973, Fairchild Publications, Inc. (1973).

19. National Coal Association, *Bituminous Coal Data* 24th edition (1974).

20. Nerlove, M. and W. Addison, "Statistical Estimation of Long-Run Elasticities of Supply and Demand," *Journal of Farm Economics* (May, 1958).

21. Richards, L. E., "Detection of Unexplained Joint Effects Through an Analysis of Residuals," *Decision Sciences* (1973), 40-43.

22. Sato, K. (1972), "Additive Utility Functions with Double-Log Consumer Demand Functions," *Journal of Political Economy*, 102-124.

23. U. S. Bureau of the Census, *Statistical Abstract of the United States*, various issues 1931-1947.

24. White, K. J., "Estimation of the Liquidity Trap with a Generalized Functional Form," *Econometrica*, 40 (February, 1972), 193-199.

25. Verleger, P. K., Jr., "The Relationship Between Energy Demand and Economic Activity," in M. S. Macrakis (ed.), *Energy: Demand, Conservation and Institutional Problems*. Cambridge, Mass.: The MIT Press, 1973.

26. Zarbembka, P., "Functional Form in the Demand for Money," *Journal of American Statistical Association*, 63 (August, 1968), 502-511.