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Carbon price and optimal extraction of a polluting fossil fuel with restricted carbon capture

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Carbon Price and Optimal Extraction of a Polluting Fossil Fuel with Restricted Carbon Capture.

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Abstract

Among technological options to mitigate greenhouse gas (GHG) emissions, Carbon Capture and Storage technology (CCS) seems particularly promising. This technology allows to keep on extracting polluting fossil fuels without drastically increasing CO_2 atmospheric concentration. We examine here a two-sector model with two primary energy resources, a polluting exhaustible resource and an expensive carbon-free renewable resource, in which an environmental regulation is imposed through a cap on the atmospheric carbon stock. We assume that only the emissions from one sector can be captured. Previous literature, based on one-sector models in which all emissions are capturable, finds that CCS technology should not be used before the threshold has been reached. We find that, when technical constraints make it impossible to capture emissions from both sectors, this result does not always hold. CCS technology should be used before the ceiling is reached if non capturable emissions are large enough. In that case, we find that energy prices paths must differ between sectors reflecting the difference of social cost of the resource according to its use. Numerical exercise shows that, when the ceiling is set at 450ppm CO_2, the initial carbon tax should equal 52\$/tCO2 and that using CCS before the ceiling is optimal.

Keywords : Dynamic Models, Global Warming, Externalities, Nonrenewable Resources, Carbon Capture, Energy Markets.

JEL Classification : Q31, Q38, Q41, Q54, Q55

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1 Introduction

Interactions between carbon regulation and fossil fuels markets must be carefully analyzed since an important part of anthropogenic greenhouse gas (GHG) emissions comes from energy production based on fossil fuels burning [1]. A growing literature uses the Hotelling textbook model of exhaustible resources (Hotelling (1931)) in various settings to determine the optimal path of extraction of a carbon-emitting exhaustible resource (fossil fuel). The fact that carbon-emitting resources are non renewable is indeed determining for the design of the optimal time path of a carbon tax. This topic has already received some attention (see for instance Tahvonen (1997), Ulph & Ulph (1994) and Chakravorty et al. (2006b)). The optimal extraction path depends on the technologies that are available to reduce atmospheric CO_2. For instance, Chakravorty et al. (2006b) examine the optimal fossil fuel price path, when it is possible to abate CO_2 directly from the stock in a Hotelling model with a ceiling on the pollution stock, considering stationary and non-stationary energy demands, with rare or abundant solar energy, and low or high abatement costs.

Some technological options allow keeping on using fossil fuels without drastically increasing carbon concentration. Among these options, Carbon Capture and Storage (CCS) technology appears as a promising tool to reduce CO_2 emissions. This technology represents *"an important part of the lowest-cost greenhouse gas mitigation portfolio"* for IEA (International Energy Agency) experts who suggest that *"without CCS, overall costs to reduce emissions to 2005 levels by 2050 increase by 70%."* According to them, it will contribute *"one-fifth of the necessary emissions reductions to achieve stabilization of GHG concentration in the most cost-effective manner"* IEA (2009a). This technology consists in a three-phase process : capturing CO_2 directly at the source of emissions in large-scale fossil fuel-fired plants, transporting it (via trains or ships, mainly via pipeline in the future), and then injecting it into large geological storage sites.

Lafforgue et al. (2007) focus on carbon capture technology. They determine the optimal extraction and sequestration paths when all emissions are capturable thanks to CCS and when large or small carbon reservoirs can be used to store carbon emissions. The main result of these two articles, in which all emissions are capturable, is that it is never optimal to abate or to capture CO_2 before the ceiling is binding. Abating later is preferred in order to benefit from the natural free decay of the pollution stock and because the discounted unitary cost of CCS is decreasing over time. It is always possible in their model to substitute later capture (once the ceiling has been reached) for early capture (before the ceiling) because the optimal extraction and sequestration paths are such that emissions at the ceiling are only partially captured. Indeed, a fraction of the emissions at the

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[1]82% of the global total energy supply comes from fossil fuels in 2007 and CO_2 from energy production represents 60% of GHG emissions(IEA (2009a)).

ceiling need not be captured thanks to the natural decay. As later capture is cheaper, it is always optimal to wait for the ceiling to start capturing emissions [2]. Obviously, this result is conditioned by the implicit binary damage function : carbon concentration has no effect on welfare as long as its concentration is under the ceiling and has catastrophic effect if its concentration is over the ceiling. This feature implies that the pollution path to reach the ceiling has no effect on welfare and as a result sequestration is useless as long as the pollution stock is below this ceiling. We would expect different results if we assumed that the pollution is harmful even under the ceiling ; but imposing a ceiling on carbon concentration is widespread in the literature and it can be seen as an exogenous constraint (for example from the Kyoto protocol). The main contribution of our model[3], compared to Lafforgue et al. (2007), is to examine the change in this result when only a part of total emissions can be captured.

We consider a two-sector economy in a stationary environment with two primary energy resources, a polluting exhaustible resource (oil) and a carbon-free renewable resource (solar energy), two energy demands (for electricity production and for transports) and a CCS technology available only in the electricity sector. Note that we consider that capturing CO_2 directly from the stock, enhancing CO_2 absorption by natural carbon sinks (forest, oceans) or filtering CO_2 from air using chemical process for example, is too expensive here. We impose a regulation through a ceiling on the carbon stock. The originality of our model lays in the realistic assumption that CCS technology is only available for a part of global emissions, that we assume here to be the emissions from the electricity production sector. Emissions from transports cannot be captured. Indeed, this technology is actually dedicated to capture CO_2 emissions from large and fix plants, like fossil fuels power plants, and cannot be applied to small or mobile sources of emissions at a reasonable cost [4]. How does the introduction of a CCS technology dedicated to a specific sector affect the optimal path and in particular the timing of its use ? We find that it may be optimal to start capturing before the ceiling . If energy demand for transports is large enough, later capture cannot replace anymore earlier capture, because at the ceiling emissions from the electricity sector are entirely captured. Abating before the ceiling delays the date of arrival of the ceiling, and relaxes the environmental constraint on energy demand for transports. In that case, energy prices paths must differ between sectors reflecting the difference of social cost of the resource according to its use.

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[2] Because of free natural decay of CO_2 stock at rate α, abating one unit of CO_2 at time t is more expensive than abating the *equivalent* of this unit at time $t + 1$. Abating one unit of CO_2 at time t costs c_{cs} and abating $1 - \alpha$ unit in $t + 1$ costs $\frac{1-\alpha}{1+r} c_{cs}$ (in value of time t). Considering two identical consumption paths with the same date of arrival at the ceiling, one with later abatement (before the ceiling) and the other with earlier abatement (before the ceiling), the first one is strictly preferred to the second one. With upward reasoning, it becomes quite obvious that there is no reason to capture CO_2 before the ceiling.

[3] At the same time this article was written, a similar result was found independently by Amigues et al. (2010)

[4] According to IEA experts, *"the CCS is the only technology available to mitigate greenhouse gas (GHG) emissions from large-scale fossil fuel usage in fuel transformation, industry and power generation"*. In industry, it can be applied to *"emissions-intensive industrial sectors like cement, iron and steel, chemicals, and pulp and paper"* IEA (2009 b).

In section 2, we present our two-sector model when CCS technology is only available for capturing emissions from electricity production. In section 3, we define the optimal paths of extraction and capture and determine in which case it is optimal to start capturing before the ceiling. We describe optimal energy prices paths in both sectors as well as the optimal use of CCS technology along the extraction path. Section 4 is a numerical exercise and section 5 concludes.

2   The Model

2.1   Assumptions and Notations

Energy demands in both sectors can be supplied by two energy sources : a polluting exhaustible resource (fossil fuel) and a carbon-free renewable resource (solar energy). They are perfect substitutes. Energy demand in electricity sector (resp. in transports sector) is written D_e (resp. D_v). Energy demands in both sectors are stationary. The stock of exhaustible resource under the ground at time t is denoted Q_t. The quantity of exhaustible resource extracted for transports (resp. for electricity production) is described by x_v (resp. x_e). The carbon-free energy is denoted s and is available in unlimited quantity at constant unitary price q. The assumption that solar energy is abundant comes from the fact that, given a unitary price for photovoltaic cells, solar energy that can be collected is only limited by the size of areas where solar cells can be used, which is supposed to be large enough.

The pollution coefficient, i.e the amount of CO_2 emitted for one unit of the fossil fuel, is set to one regardless of the final use. We note Z_t the stock of CO_2 in the atmosphere at time t and α the natural constant instantaneous rate of decay of CO_2 in the atmosphere. We write δ_t the fraction of emissions from the electricity sector that are captured at time t. Thus the carbon concentration follows the law of motion:

$$\dot{Z}_t = -\alpha Z_t + x_e(t)(1 - \delta_t) + x_v(t)$$

The variation of carbon concentration equals anthropogenic emissions minus natural dilution. The maximal stock of CO_2 in the atmosphere allowed by the environmental regulation is $\overline{Z}$. The CCS technology can only be used to capture emissions from the electricity sector and its unit cost, written c_{cs}, is constant. We assume that fossil fuel extraction costs and energy delivery costs are null. We also assume that geological storage sites are very large, so that the cumulative amount of captured emissions is not constrained by their size [5]. We note r the social discount rate.

---

[5] The theoretical storage capacity ranges from 8 090 to 15 500 GtCO2 (IEA (2009b)). The Blue Map scenario requires a use of only 1% of this theoretical capacity in 2050. The case where capture is constrained by the size of

2.2 The Social Planner Problem

We assume that the utility function is separable in transport and energy. We assume that this utility function satisfies the standard regularity conditions ($u \epsilon C^2$, $u' > 0$, $u'' < 0$, and satisfies Inada conditions $\lim_{z \to \infty} u'(z) = 0$ and $\lim_{z \to 0} u'(z) = +\infty$). The social planner seeks to find the extraction path of the resource which maximizes the net discounted social surplus under the environmental constraint. Thus the social planner chooses carbon capture use and extraction paths $\{\delta_t^*, x_e^*(t), x_v^*(t), s_e^*(t), s_v^*(t)\}$ that maximize:

$$\int_0^\infty e^{-rt}\left(u^e\big(x_e(t) + s_e(t)\big) + u^v\big(x_v(t) + s_v(t)\big) - c_{cs}\delta_t x_e(t) - q(s_e(t) + s_v(t))\right)dt$$

subject to : $\forall t$,

$$
\begin{aligned}
\dot{Q}_t &= -x_e(t) - x_v(t) \\
Q_t &\geq 0 \\
\dot{Z}_t &= -\alpha Z_t + x_e(t)(1 - \delta_t) + x_v(t) \\
Z_t &\leq \overline{Z} \\
x_v(t), x_e(t), s_e(t), s_v(t) &\geq 0 \\
0 \leq \delta_t &\leq 1
\end{aligned}
$$

where $u^e\big(x_e(t) + s_e(t)\big) + u^v\big(x_v(t) + s_v(t)\big)$ is the current social surplus, $c_{cs}\delta_t x_e(t)$ represents the cost of capturing a fraction δ_t of emissions from electricity production and $q(s_e(t) + s_v(t))$ represents solar energy production cost and with Z_0, $\overline{Z}$, α, and Q_0 given.

The Hamiltonian in current value is :

$$
\begin{aligned}
\mathbb{H} &= u^e\big(x_e(t) + s_e(t)\big) + u^v\big(x_v(t) + s_v(t)\big) - q(s_v(t) + s_e(t)) - c_{cs}\delta_t x_e(t) \\
&\quad -\lambda_t(x_e(t) + x_v(t)) + \mu_t(-\alpha Z_t + x_e(t)(1 - \delta_t) + x_v(t)) \\
&\quad +\nu_t(\overline{Z} - Z_t) + \beta_t \delta_t + \gamma_t(1 - \delta_t)
\end{aligned}
$$

the reservoirs in a one-sector economy is analyzed by Lafforgue, Magné and Moreaux (2008). This problem includes a supplementary exhaustible resource : the remaining space in the carbon sinks.

- The First Order Conditions are :

$$\frac{\partial H(t)}{\partial x_v(t)} = 0 \iff p_{x,v}(t) = \lambda_t + \mu_t \tag{1}$$

$$\frac{\partial H(t)}{\partial x_e(t)} = 0 \iff p_{x,e}(t) = \lambda_t + \mu_t(1 - \delta_t) + c_{cs}\delta_t \tag{2}$$

$$\frac{\partial H(t)}{\partial s_e(t)} = 0 \iff p_{s,e}(t) = q \tag{3}$$

$$\frac{\partial H(t)}{\partial s_v(t)} = 0 \iff p_{s,v}(t) = q \tag{4}$$

$$\frac{\partial H(t)}{\partial \delta_t} = 0 \iff c_{cs}x_e(t) = (\beta_t - \gamma_t) + \mu_t x_e(t) \tag{5}$$

Where $p_{i,j}$ represents the optimal price of resource i in sector j. $p_{i,j}$ is the gross marginal surplus denoted $\frac{du^j}{di}$. Optimal price must equal the full marginal cost, including scarcity rent and the shadow cost of pollution[6]. Because both energy resource are perfect substitutes, demands are supplied by the cheaper energy resource. Writing p_e and p_v inverse energy demands for electricity production and transports, we get $p_e(t) = \min(p_{s,e}(t), p_{x,e}(t))$ and $p_v(t) = \min(p_{s,v}(t), p_{x,v}(t))$.

- The dynamics of the co-state variables, λ_t and μ_t, are determined by :

$$\dot{\lambda}_t = r\lambda_t - \frac{\partial H(t)}{\partial Q_t} \iff \dot{\lambda}_t = r\lambda_t \iff \dot{\lambda}_t = \lambda_0 e^{rt}$$

$$\dot{\mu}_t = r\mu_t - \frac{\partial H(t)}{\partial Z_t} \iff \dot{\mu}_t = (r + \alpha)\mu_t + \nu_t$$

The co-state variable λ_t represents the value at time t of the scarcity rent of the exhaustible resource, it increases at rate r. Indeed at each moment of time along the optimal path of extraction, the planner is indifferent between extracting a supplementary unit or saving it for a latter use. So, at the optimum, the discounted value of net marginal surplus of an extracted unit of fossil fuel must be constant when there is no environmental externality.

The co-state variable μ_t represents the marginal shadow cost of pollution at time t. Optimal

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[6]In our model, we do not address the question of who receives the rent and how to implement optimal prices (via a carbon tax, a carbon market), a tax that would be higher than μ_t by a constant everywhere would have no effect of global welfare but would redistribute the rent from resource owners to the social planner. This question is not addressed here and we consider that μ_t is the carbon price.

taxation requires to tax non-captured emissions of time t with a unitary tax equaling this marginal cost μ_t[7]. Along the optimal path, the carbon tax before the ceiling must be such that we are indifferent between emitting a unit of CO_2 now or later : the discounted marginal cost of a unit of pollution must be constant. Because of natural absorption at rate α, the shadow cost of pollution must increase before the ceiling at rate $r + \alpha$. We must notice that even if pollution under the ceiling is not harmful by assumption, emissions must be taxed before the ceiling is reached as they contribute to reaching the ceiling later. Under perfect competition between fossil fuel sellers, the social planner can decentralize the problem by setting the carbon tax μ_t over non-captured emissions.

- With the following slackness conditions :

$$\nu_t \geq 0, \quad \text{and} \quad \nu_t(\overline{Z} - Z_t) = 0 \tag{6}$$

$$\eta_t \geq 0 \quad \text{and} \quad \eta_t Q_t = 0 \tag{7}$$

$$a_t \geq 0, \quad \text{and} \quad a_t x_v(t) = 0 \tag{8}$$

$$b_t \geq 0, \quad \text{and} \quad b_t x_e(t) = 0 \tag{9}$$

$$\beta_t \geq 0, \quad \text{and} \quad \beta_t \delta_t = 0 \tag{10}$$

$$\gamma_t \geq 0, \quad \text{and} \quad \gamma_t(1 - \delta_t) = 0 \tag{11}$$

$$f_t \geq 0, \quad \text{and} \quad f_t s_e(t) = 0 \tag{12}$$

$$g_t \geq 0, \quad \text{and} \quad g_t s_v(t) = 0 \tag{13}$$

From equations 5 and slackness conditions 10 and 11, we get :

$$\mu_t < c_{cs} \Longleftrightarrow (\beta_t - \gamma_t) > 0 \Longleftrightarrow \beta_t > 0; \gamma = 0$$

$$\mu_t > c_{cs} \Longleftrightarrow (\beta_t - \gamma_t) < 0 \Longleftrightarrow \beta_t = 0; \gamma > 0$$

$$\mu_t = c_{cs} \Longleftrightarrow (\beta_t - \gamma_t) = 0 \Longleftrightarrow \beta_t = 0; \gamma = 0$$

Along the optimal path, all the emissions from the electricity sector are captured if and only if bearing pollution is more costly than capturing emissions ($\mu_t > c_{cs}$). Capture in electricity sector is incomplete if and only if the society is indifferent between capturing emissions and bearing pollution ($\mu_t = c_{cs}$) and there is no capture if and only if carbon capture costs more than pollution ($\mu_t < c_{cs}$).

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[7]In this partial equilibrium model, the tax is not redistributed.

- Transversality conditions are given by :

$$\lim_{t \to \infty} \lambda_t e^{-rt} Q_t = \lim_{t \to \infty} \lambda_0 Q_t = 0 \tag{14}$$
$$\lim_{t \to \infty} \mu_t e^{-rt} Z_t = 0$$

Equation 14 simply says that the fossil fuel must be exhausted in the long run.

3 Optimal Extraction Paths and Carbon Tax

Several types of extraction path can be distinguished depending on some key parameters. We assume here that Q_0 is sufficiently large so that the environmental constraint is binding[8]. In other words, in the pure Hotelling path of extraction, the stock of pollution exceeds the ceiling at some date so that $\exists t, \mu_t > 0$. We examine the case when c_{cs} is low enough to use CCS at a moment of time [9].

We define $\bar{p}$ the price at which global emissions exactly equal natural absorption at the ceiling, i.e. the price such that $D_e(\bar{p}) + D_v(\bar{p}) = \alpha \bar{Z}$. We assume that $q > \bar{p}$, so that solar energy is only used once the fossil fuel is exhausted, see Chakravorty et al. (2006b). During the first phase, energy demands in both sectors are supplied by fossil fuel and, without CCS, lead to CO_2 emissions. We do not examine the case where solar is cheap enough to be used at the ceiling, $q < \bar{p}$. We assume that the initial CO_2 atmospheric stock is under the ceiling ($Z_0 < \bar{Z}$) and that $Q_0 > 0$. From slackness conditions 6 and 7, we know that, at date 0, $\nu_0 = 0$ and $\eta_0 = 0$.

---

[8] The carbon concentration always reaches a maximum at which emissions just equal natural absorption. For low demand, this maximum can occur at the beginning if initially emissions are lower than the initial amount of naturally absorbed CO_2. Let us consider 2 models, one with a ceiling, the other without. If optimal paths are the same in both models, the ceiling is an inactive constraint in the model with a ceiling. If the carbon concentration along the optimal path in a model without ceiling is always under the carbon concentration equivalent to the ceiling, the constraint is never binding. In a model without ceiling, the larger the initial amount of resource is, the larger is the energy demand and as a consequence the larger is the maximal carbon concentration. The maximal stock of fossil fuel $\overline{Q}$ that allows to get an unconstrained optimal path in a model with a ceiling ($\overline{Z}$) is such that: $\max_t Z(t) = \overline{Z}$ with $Z(t) = e^{-\alpha t} Z_0 + \int_0^T e^{-\alpha t} D(\lambda_0 e^{rt}) dt$ with T the time at which the resource price equals the price of the backstop technology (q) and λ_0 the correct initial scarcity rent, such that $\int_0^T D(\lambda_0 e^{rt}) = \overline{Q}$. T is defined by $\lambda_0 e^{rT} = q$ so $T = ln(\frac{q}{\lambda_0})/r$. For a specific Z_0 and $\overline{Z}$ there is a unique $\overline{Q}$ such that $\max_t Z(t) = \overline{Z}$. This represents the maximal stock of fossil fuel to get an unconstrained Hotelling path of extraction. We assume in our model that $Q_0 > \overline{Q}$.

[9] CCS is used if and only if $\exists t^*$ such that $c_{cs} + \lambda_0 e^{rt^*} \geq \lambda_0 e^{rt^*} - \mu_0 e^{(r+\alpha)t^*}$ with $c_{cs} + \lambda_0 e^{rt^*} < \bar{p}$ and $\bar{p}$ such that $D_e(\bar{p}) + D_v(\bar{p}) = \alpha \bar{Z}$

The First Order Conditions become before the ceiling is binding :

$$\mu_t = \mu_0 e^{(r+\alpha)t} \tag{15}$$

$$\lambda_t = \lambda_0 e^{rt} \tag{16}$$

$$p_v(t) = \lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t} \tag{17}$$

$$p_e(t) = c_{cs}\delta_t + \lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t}(1 - \delta_t) \tag{18}$$

$$c_{cs}x_e(t) = \mu_0 e^{(r+\alpha)t}x_e(t) + (\beta_t - \gamma_t) \tag{19}$$

3.1 CCS technology available in both sectors: no capture before the ceiling.

As a benchmark model, we assume that CCS technology is available in both sectors. The total energy demand is the aggregate energy demand composed by our two sectoral demands. This case is studied by Lafforgue et al. (2007) except that they consider just one sector. Their result holds: it is never optimal to capture before the ceiling. Indeed, abating later costs less than abating now, pollutionn is not harmful before the ceiling and it is always possible at the ceiling to abate more emissions (because of the natural decay, capture of capturable emissions is incomplete at the ceiling). Along this optimal path, it is possible to compute energy demand of transports sector $D_v^*(t)$. We call $\underline{t}$ the date at which the ceiling starts to bind in this benchmark model and $\bar{t}$ the date at which the ceiling ceases to bind, i.e the date the scarcity rent becomes higher than $\bar{p}$. The amount of CO_2 that is naturally absorbed at the ceiling is $\alpha\bar{Z}$.

If during the ceiling, in this benchmark model, emissions from transports sector remain below the maximal natural decay $\alpha\bar{Z}$, i.e. $\forall t$ such that $\underline{t} \leq t \leq \bar{t}$, $D_v^*(t) \leq \alpha\bar{Z}$, then it is never optimal to capture CO_2 before the ceiling even if the emissions from transports sector cannot be technically captured. Indeed, in such a case, the extraction and sequestration paths are still sustainable when only the emissions from the electricity sector can be captured. In other words, introducing the constraint that CCS can be used in only one sector rather than in both, has no effect in this case: the optimal solution of the benchmark model is still optimal.

On the other hand, if $\exists t$, $\underline{t} \leq t \leq \bar{t}$ such that $D_v^*(t) > \alpha\bar{Z}$ in the benchmark optimal extraction path, then this solution is not feasible anymore when CCS can only be used in electricity sector. Indeed, in this case, even if all emissions from electricity sector are captured, non-capturable emissions, i.e. emissions from transports sector, are large enough for the carbon concentration to exceed the

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ceiling. The size of the transports sector energy demand is then a key element to determine whether the optimal path in our model is the same as in the benchmark case. We distinguish two cases according to the size of energy demand for transports. We assume that emissions from transports sector are non-capturable. When we refer to the benchmark model below, we refer to the virtual case where CCS can be applied to both sectors.

3.2 Small energy demand for transports and restricted CCS field of application : no capture before the ceiling.

Proposition 1: If the energy demand related to non capturable emissions is small enough, the CCS technology must not be used before the ceiling.

As explained above, this case is equivalent to the case where CCS can be used in both sectors. We say that energy demand for transports is small if, in the benchmark case, $\forall t$ such that $\underline{t} \leq t \leq \bar{t}$, we have $D_v^*(t) \leq \alpha \bar{Z}$. In the benchmark case, energy demand for transports is decreasing with time, so that it is sufficient that $D_v^*(\underline{t}) \leq \alpha \bar{Z}$. In this case, the benchmark solution can still be reached and it is never optimal to abate before the ceiling and the optimal path is as described in previous literature with one-sector (See Figure 1).

- Phase 1: for $t \in [0, \theta_1[$ the ceiling is not binding and there is no capture. During this initial phase, $\mu_0 e^{(r+\alpha)t} < c_{cs}$, so $\beta_t > 0$ and $\gamma_t = 0$, we get $\delta_t = 0$: CCS is not used. The price of the exhaustible resource in both sectors equals the sum of the scarcity rent and the marginal cost of pollution $\lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t}$. The carbon tax equals $\mu_0 e^{(r+\alpha)t}$. Energy demands in both sectors are decreasing. Energy price is under $\bar{p}$, the carbon concentration increases. This phase comes to an end at time θ_1 when $Z_{\theta_1} = \bar{Z}$.

- Phase 2: for $t \in [\theta_1, \theta_2[$ the ceiling is binding and capture is incomplete. During this phase, the ceiling is binding. Because emissions from transports sector are lower than natural absorption at the ceiling, it is sufficient to partially capture emissions from electricity sector to remain at the ceiling. It follows that the carbon tax is constant and equals c_{cs}. So the price of the exhaustible resource equals $\lambda_0 e^{rt} + c_{cs}$ in both sectors. Both energy demands are decreasing, so the part of captured emissions in electricity sector is decreasing. CCS is no more used when the price is sufficiently high to keep the pollution stock exactly at the ceiling without using CCS. This occurs at time θ_2 when $\lambda_0 e^{r\theta_2} + ccs = \bar{p}$.

- Phase 3: for $t \in [\theta_2, \theta_3[$ the ceiling is binding with no capture. The ceiling remains binding but CCS technology is no more used. Demands are constant and globally amount to $\alpha \bar{Z}$.

price

q

Oil in electricity and transport

Solar in
electricity
and transport

$\bar{p}$

CCS in electricity

μ_t

$Z_t < \bar{Z}$
$p_t = \lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t}$

$Z_t = \bar{Z}$
$p_t = \lambda_0 e^{rt} + c_{cs}$

$\frac{\bar{Z}}{\bar{p}}$ $Z_t < \bar{Z}$
$p_t = \lambda_0 e^{rt}$ $Z_t < \bar{Z}$
$p_t = q$

μ_t

$\lambda_0 + c_{cs}$

μ_t

$\lambda_0 + \mu_0$
λ_0

pollution stock

$\bar{Z}$

Binding ceiling

pollution stock
from electricity

pollution stock
from transport

Z_0
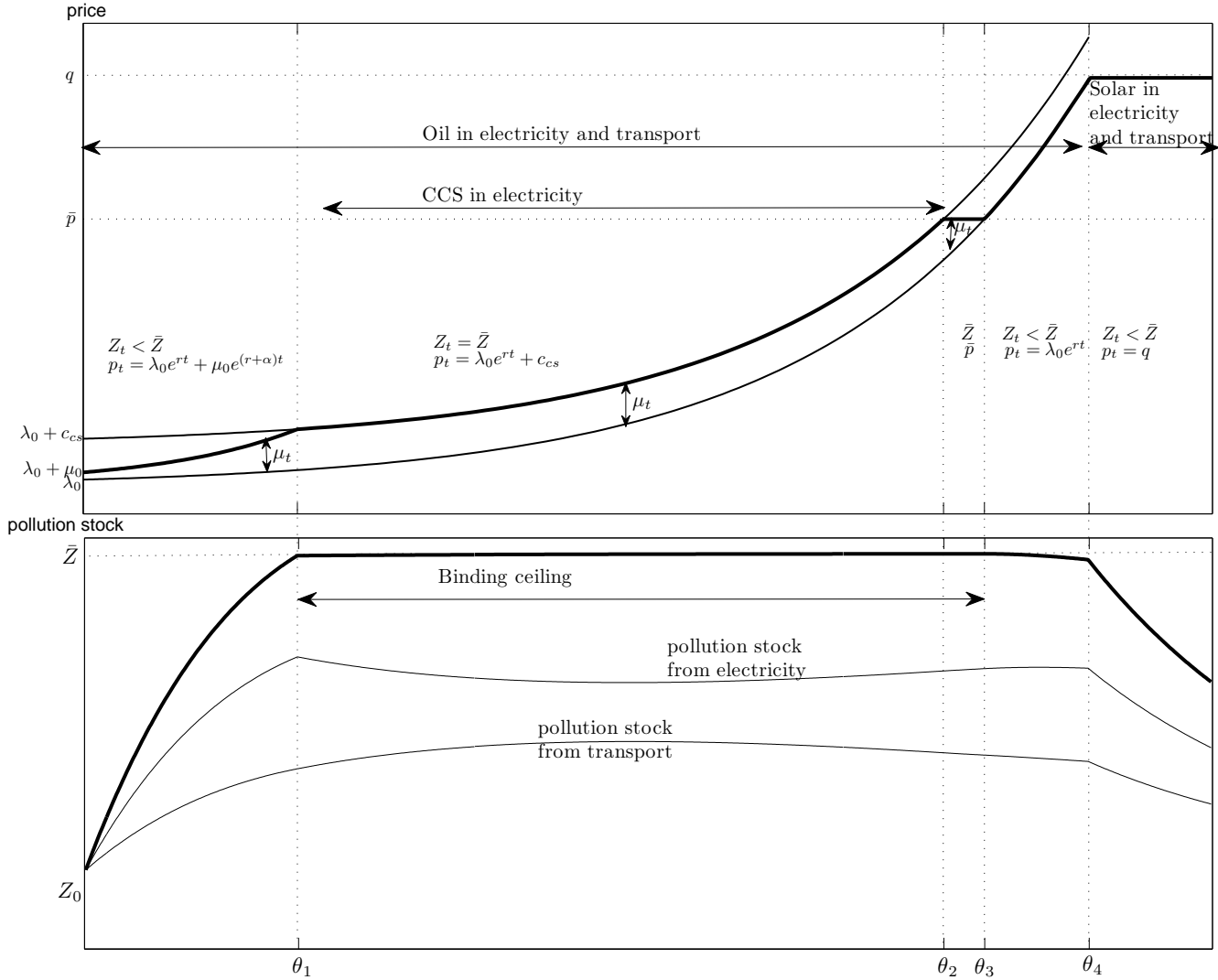
θ_1 θ_2 θ_3 θ_4

Figure 1: Price and pollution stock along the optimal path when the demand for transport is small ; CCS is not used before the ceiling.

Energy price is constant and equals $\bar{p}$ and the carbon tax is decreasing $(\mu_t = \bar{p} - \lambda_0 e^{rt})$. This phase comes to an end at time θ_3 when the scarcity rent equals $\bar{p}$: $\lambda_0 e^{r\theta_3} = \bar{p}$.

- Phase 4: for $t \in [\theta_3, \theta_4[$, the ceiling is no more binding, pure Hotelling path. The fossil fuel is scarce enough to get a pure Hotelling path. The environmental constraint will no more be binding ; the carbon tax equals 0. The price of the exhaustible resource in both sectors equals $\lambda_0 e^{rt}$. Energy demands in both sectors and carbon concentration are decreasing. This phase ends when the fossil fuel is exhausted at time θ_4 where $\lambda_0 e^{r\theta_4} = q$

- Phase 5: for $t \in [\theta_4; \infty[$, carbon-free energy phase. After θ_4 the fossil fuel is exhausted and solar energy begins to be used. In both sector, energy prices equal q and energy demands are constant. The carbon concentration keeps on decreasing.

Along the optimal path, the carbon tax exhibits the well known complex pattern described in previous literature. The carbon tax is increasing (Phase 1), constant (Phase 2) then decreasing (Phase 3) to fall to 0.

The initial scarcity rent, the initial carbon tax and the dates defined above $\{\lambda_0, \mu_0, \theta_1, \theta_2, \theta_3, \theta_4\}$ satisfy the following conditions:

- The continuity of energy prices between phases :

$$
\begin{aligned}
\mu_0 e^{(\alpha+r)\theta_1} &= c_{cs} \\
\lambda_0 e^{r\theta_2} + c_{cs} &= \bar{p} \\
\lambda_0 e^{r\theta_3} &= \bar{p} \\
\lambda_0 e^{r\theta_4} &= q
\end{aligned}
$$

- The carbon concentration reaches $\bar{Z}$ at the end of phase 1 :

$$
e^{-\alpha\theta_1}\left(Z_0 + \int_0^{\theta_1} e^{-\alpha(\theta_1-t)}\big(D_v((\lambda_0 + \mu_0 e^{\alpha t})e^{rt}) + D_e((\lambda_0 + \mu_0 e^{\alpha t})e^{rt})\big)\right)dt = \bar{Z}
$$

- The non renewable resource is finally exhausted:

$$\int_0^{\theta_1} \left(D_v((\lambda_0 + \mu_0 e^{\alpha t})e^{rt}) + D_e((\lambda_0 + \mu_0 e^{\alpha t})e^{rt}) \right) dt$$

$$+ \int_{\theta_1}^{\theta_2} \left(D_v(\lambda_0 e^{rt} + ccs) + D_e(\lambda_0 e^{rt} + ccs) \right) dt$$

$$+ (\theta_3 - \theta_2)\alpha \bar{Z} + \int_{\theta_3}^{\theta_4} \left(D_e(\lambda_0 e^{rt}) + D_v(\lambda_0 e^{rt}) \right) dt = Q_0$$

We note $\bar{p_v}$ the price such that $D_v(\bar{p_v}) = \alpha \bar{Z}$. It is the price such that, when the ceiling is binding, the carbon stock remains exactly at the ceiling if all emissions from electricity sector are captured. The solution described above holds if an only if, at the date θ_1 when CCS use starts, the price of the exhaustible resource that solves this six-equation system is over $\bar{p_v}$, i.e if an only if $\lambda_0 e^{r\theta_1} + c_{cs} \geq \bar{p_v}$. This condition means that when CCS use starts, the price of the resource is higher that the price that keeps the carbon stock exactly at the ceiling when all emissions from electricity are captured. In that case, CCS is incomplete and the ceiling is binding.

3.3 Large energy demand for transports and restricted CCS field of application: capture starts before the ceiling.

Proposition 2: If the energy demand related to non capturable emissions is large, then it is optimal to start capturing CO_2 before the ceiling has been reached.

We say that energy demand for transports is large if, in the benchmark model, $\exists t$ such that $\underline{t} \leq t \leq \bar{t}$ with $D_v^*(t) > \alpha \bar{Z}$. Because energy demand is decreasing with time, it is sufficient that $D_v^*(\underline{t}) > \alpha \bar{Z}$. The benchmark solution is not feasible anymore because non-capturable emissions (emissions from transports) would be larger than natural absorption at the ceiling. In this case, we find that capture technology is used before the ceiling. We suppose that initially CCS is not used ($\mu_0 < c_{cs}$) (if it is used, Phase 1 does not exist and the extraction path is similar to the following path with Phase 2 the initial phase). The optimal path is described below, see Figure 2 :

- Phase 1: for t $\in [0, t_1[$ the ceiling is not binding and CCS is not used. During this initial phase, $\mu_0 e^{(r+\alpha)t} < c_{cs}$, so $\beta_t > 0$ and $\gamma_t = 0$, we get $\delta_t = 0$: CCS is not used. Energy prices in both sectors are equal : $p_e(t) = p_v(t) = \lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t}$. The carbon tax during this phase equals $\mu_0 e^{(r+\alpha)t}$. Energy demands in both sectors are decreasing. Because $p_e(t) = p_v(t) < \bar{p}$, the carbon concentration is increasing. Phase 1 lasts until time t_1 at which $\mu_0 e^{(r+\alpha)t} = c_{cs}$.

price

q

Solar in
electricity
and transport

Oil in electricity and transport

Complete CCS in electricity

Partial CCS
in electricity

$\bar{p}$

$p_t = \lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t}$ $p_{e,t} = \lambda_0 e^{rt} + c_{cs}$
$p_{v,t} = \lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t}$

$p_{e,t} = \lambda_0 e^{rt} + c_{cs}$
$p_{v,t} = \bar{p}_v$

$p_t = \lambda_0 e^{rt} + c_{cs}$

$\bar{p}$ $p_t = \lambda_0 e^{rt}$ $p_t = q$

μ_t

$\bar{p}_v$

transport

μ_t

electricity

μ_t

$\lambda_0 + c_{cs}$
$\lambda_0 + \mu_0$
λ_0

$Z_t < \bar{Z}$ $Z_t < \bar{Z}$
$\mu_t < c_{cs}$ $\mu_t > c_{cs}$

$Z_t = \bar{Z}$
$\mu_t > c_{cs}$

$Z_t = \bar{Z}$
$\mu_t = c_{cs}$

$Z_t = \bar{Z}$ $Z_t < \bar{Z}$
$\mu_t < c_{cs}$ $\mu_t = 0$

$Z_t < \bar{Z}$
$\mu_t = 0$

pollution stock

$\bar{Z}$

pollution stock
from transport

binding ceiling

Z_0

pollution stock
from electricity
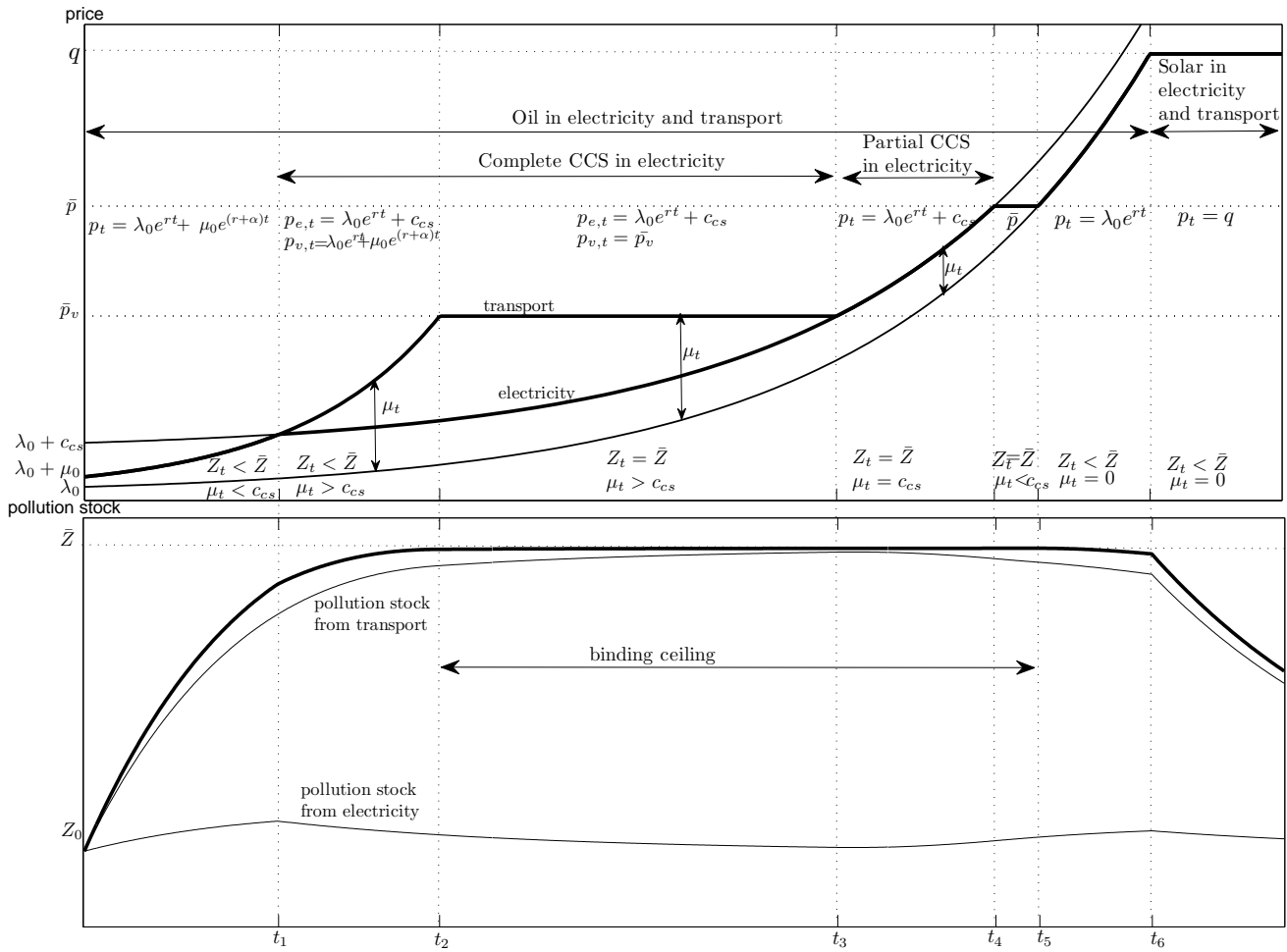
t_1 t_2 t_3 t_4 t_5 t_6

Figure 2: Price and pollution stock along the optimal path when the demand for transport is large
; CCS is used before the ceiling.

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- Phase 2: for t $\in [t_1, t_2[$, the ceiling is not binding and all the emissions from electricity production are captured. During this phase, $\mu_0 e^{(r+\alpha)t} > c_{cs}$, so $\beta_t = 0$ and $\gamma_t > 0$, we get $\delta_t = 1$: emissions from electricity production are totally captured. Implicit prices differ from one sector to another $p_v(t) = \lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t} > p_e(t) = c_{cs} + \lambda_0 e^{rt}$. The impossibility to capture emissions from transports sector makes energy more expensive for this sector. The carbon tax equals $\mu_0 e^{(r+\alpha)t}$ in transports sector. No tax is paid, properly speaking, in electricity sector, because all the emissions are captured. Energy demands are decreasing in both sectors. As long as $p_v(t) < \bar{p}_v$, the carbon concentration keeps on increasing. This phase ends at t_2 when the ceiling starts binding $Z_{t_2} = \bar{Z}$.

- Phase 3: for t $\in [t_2, t_3[$, the ceiling is binding and all the emissions from electricity production are captured. The ceiling is binding so $\nu_t > 0$ and $\mu_t > c_{cs}$, so $\delta_t = 1$: CCS is complete. The price of energy in transports sector is such that when all the emissions from electricity sector are captured the carbon concentration remains at the ceiling : $p_v(t) = \bar{p}_v > p_e(t) = c_{cs} + \lambda_0 e^{rt}$. The carbon tax is decreasing in transports sector $\mu_t = p_v(t) - \lambda_0 e^{rt} = \bar{p}_v - \lambda_0 e^{rt}$. Energy demand for transports is constant and energy demand for electricity sector is decreasing. This phase lasts until t_3 such that $c_{cs} + \lambda_0 e^{rt_3} = \bar{p}_v$: the scarcity rent of the resource is high enough so that the pollution stock does not exceed the ceiling when a fraction of electricity emissions are captured.

- Phase 4: for t $\in [t_3, t_4[$, the ceiling is binding and only a fraction of emissions from electricity production is captured. During this phase, energy demand in transports sector is sufficiently low to keep the pollution stock exactly at the the ceiling with partial capture in electricity sector, CCS is incomplete: the carbon tax is constant and equals c_{cs} in both sectors ($\mu_t = c_{cs}$). Prices in both sectors are the same ($p_v(t) = p_e(t) = c_{cs} + \lambda_0 e^{rt}$). Both energy demands are decreasing and the part of captured emissions from electricity production is decreasing. This phase ends at date t_4 when prices in both sectors reach $\bar{p}$. At this point, global emissions without capture just equal natural absorption.

- Phase 5: for t $\in [t_4, t_5[$, the ceiling is binding and CCS is no more used. Global emissions just equal natural absorption. Energy prices in both sectors equal $\bar{p}$. The carbon tax is decreasing in both sectors ($\mu_t = \bar{p} - \lambda_0 e^{rt}$). This phase lasts until t_5 when the scarcity rent reaches $\bar{p}$. At this point, the price is sufficiently high so that the ceiling is no more binding.

- Phase 6: for t $\in [t_5, t_6[$, the ceiling is no more binding, pure Hotelling path. During this phase, prices in both sectors are equal: $p_v(t) = p_e(t) = \lambda_0 e^{rt} > \bar{p}$. Energy demands are decreasing and the carbon concentration is decreasing. The carbon tax equals 0. Extraction of the exhaustible resource follows a Hotelling path until exhaustion. This phase ends when the fossil fuel is exhausted at date t_6 when $\lambda_0 e^{rt_6} = q$.

- Phase 7: for t $\in [t_6; \infty[$, carbon-free energy phase. After t_6, the fossil fuel is exhausted and solar energy starts to be used. In both sectors, energy prices equal q and energy demands are constant. The carbon concentration keeps on decreasing.

The carbon tax exhibits a complex pattern. In transports sector, the carbon tax is increasing (Phase 1 and 2), decreasing (Phase 3), then constant (Phase 4), and finally decreasing (Phase 5) to fall to 0. In the electricity sector, non-captured emissions are taxed as in transports sector. When CCS is complete in electricity sector (Phase 2 and 3), emissions from this sector are no more taxed because they are entirely captured. During this phase, in electricity sector, using CCS is cheaper than paying the tax over non-captured emissions, so final prices differ from a sector to another.

The initial scarcity rent, the initial carbon tax and the dates defined above $\{\lambda_0, \mu_0, \theta_1, \theta_2, \theta_3, \theta_4\}$ satisfy the following conditions:

- Continuity of energy prices between phases :

$$\mu_0 e^{(\alpha+r)t_1} = c_{cs} \tag{20}$$

$$(\lambda_0 + \mu_0 e^{\alpha t_2})e^{rt_2} = \bar{p_v} \tag{21}$$

$$\lambda_0 e^{rt_3} + c_{cs} = \bar{p_v} \tag{22}$$

$$\lambda_0 e^{rt_4} + c_{cs} = \bar{p} \tag{23}$$

$$\lambda_0 e^{rt_5} = \bar{p} \tag{24}$$

$$\lambda_0 e^{rt_6} = q \tag{25}$$

- The carbon concentration reaches $\bar{Z}$ at the end of phase 2:

$$
e^{-\alpha t_2}\Bigg(Z_0 + \int_0^{t_1} e^{-\alpha(t_2-t)}\bigg(D_v((\lambda_0 + \mu_0 e^{\alpha t})e^{rt}) + D_e((\lambda_0 + \mu_0 e^{\alpha t})e^{rt})\bigg)dt
$$
$$
+ \int_{t_1}^{t_2} e^{-\alpha(t_2-t)}D_v((\lambda_0 + \mu_0 e^{\alpha t})e^{rt})dt\Bigg) = \bar{Z}
$$

- The non renewable resource is finally exhausted:

$$\int_0^{t_1} \left(D_v((\lambda_0 + \mu_0 e^{\alpha t})e^{rt}) + D_e((\lambda_0 + \mu_0 e^{\alpha t})e^{rt}) \right) dt$$

$$+ \int_{t_1}^{t_2} D_v((\lambda_0 + \mu_0 e^{\alpha t})e^{rt}) dt$$

$$+ (t_3 - t_2)\alpha \bar{Z} + \int_{t_1}^{t_3} D_e(\lambda_0 e^{rt} + c_{cs}) dt$$

$$+ \int_{t_3}^{t_4} \left(D_v(\lambda_0 e^{rt} + c_{cs}) + D_e(\lambda_0 e^{rt} + c_{cs}) \right) dt$$

$$+ (t_5 - t_4)\alpha \bar{Z} + \int_{t_5}^{t_6} \left(D_v(\lambda_0 e^{rt}) + D_e(\lambda_0 e^{rt}) \right) dt \quad = \quad Q_0 \qquad (26)$$

Similarly to the previous case, the solution described above holds if an only if for λ_0 and t_1(when CCS begins to be used) solutions of the eight-equation system, we get $\bar{p_v} \geq c_{cs} + \lambda_0 e^{rt_1}$.[10]

4 Numerical exercise

4.1 Parametrization of the model.

We calibrate our model in order to get some evidence on the initial level of the optimal carbon tax, and to determine if it is optimal to use CCS before the ceiling is binding.

We divide the global energy demand into two energy demands D_A and D_B assuming that all D_A related emissions are capturable contrary to D_B related emissions. To compute these demands we use sectoral final sectoral energy demands and data on the parts of capturable emissions in each sector. Sectoral demands are assumed to be independent and take Cobb-Douglas form $D_j = A_j P_j^{\alpha_j} Y^{\beta_j}$ with α_j and β_j respectively price and income elasticities, with A_j the sector-specific coefficient, Y the world GDP and P_j the delivered energy price. We estimate these demands using elasticities and energy prices in 3 sectors (Transportation, Industry, Residential/Commercial, $j \epsilon \{T, I, RC\}$) from Chakravorty et al. (2006a). We assume P_j equal among sectors. Even if estimates of sectoral demands take into account fossil fuels conversion and extraction costs, we assume that these costs

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[10]If we consider the simple example of a one-sector economy in which it is not possible to capture at time t more than a fraction ϵ_t of $CO2$ emissions (intensity constraint), we get similar result than in our two-sector model. Considering the benchmark case, where CCS is not constrained, we note $D_S(t)^x$ demand at time t along the optimal path and $[\underline{t}; \overline{t}]$the period where the ceiling is binding. If $\forall t, \ \underline{t} \leq t \leq \overline{t}$, $(1 - \epsilon_t)D_S(t)^x \leq \alpha \bar{Z}$ the optimal solution of the benchmark case still holds : there is no carbon capture before the ceiling. If $\exists t, \ \underline{t} \leq t \leq \overline{t}$, $(1 - \epsilon_t)D_S(t)^x > \alpha \bar{Z}$, the benchmark solution no more holds, and CCS is used before the ceiling is binding. $(1 - \epsilon_t)D_S(t)^x$ plays the same role than $D_v(t)^x$ in the two-sector model.

Sector	Price Elasticity	Income Elasticity	Constant Parameter
Transportation	-0,6	0,7	141,4
Industry	-0,4	0,6	1318,5
Residential/Commercial	-0,5	0,5	36409,0

Sector	% Capturable emissions*		
	Low	Medium	High
Transportation	0	0	0
Industry	5	29	52
Residential/Commercial	2	15	28

Table 1: Estimates of sectoral Energy demands, and % of capturable emissions in each sector.
Constant parameters are estimated using Weighted Energy Prices, IEA (2001), Total final consumption from IEA (2002) and Price and income Elasticities from Chakravorty et al. (2006a) and 2000 World GDP in US-Dollars. Around 75% of electricity production is based on fossil fuels (IEA (2009c).), we exclude non fossil fuels based electricity from Energy consumption to estimate constant parameters.
* We assume that emissions content of delivered energy is similar in all sectors, see table 9 in annex (1 quad Btu $=72.10^6$ $CO2$ tons or 1EJ leads to 68.10^6 of tons of CO_2 in all sectors) .

are nil. We denote $f_{j,i}$ the part of emissions in sector j that are capturable in scenario i ; we simply get : $D_{A,i} = \sum_j f_{j,i} D_j$ and $D_{B,i} = \sum_j (1 - f_{j,i}) D_j$.

The parts of emissions that can be captured $f_{j,i}$ are assumed to be constant over time. Defining these parts is a key-point of our calibration because theoretical results of our model show that the size of non-capturable emissions impacts the optimal time path of sequestration. We assume that the marginal cost of CCS is constant and equals 70\$/tCO2 (2000 base Dollars)[11]. At this price only a part of global emissions can be captured but we do not have accurate data about world abatement costs using CCS technology (i.e how much emissions can be captured at a given price). We compute lower and upper bounds of the part of emissions that can be captured in each sector at this price, using data from IEA scenarios, see in annex.We define a medium part of capturable emissions in each sector as the mean of the upper bound and lower bound of these parts in each sector. The ratio of capturable emissions in all sectors are summarized in table 1.

In our model energy demands are stationary, so World GDP is assumed to be constant and equals 43 676 US-dollars (2008 World GDP in 2000 Dollars, source IMF). The base year of energy demands estimates is 2000.

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[11]"The BLUE Map scenario estimates that costs associated with large coal-fired power plants will represent the lowest cost opportunities within the power sector at around USD 35 to USD 50/tCO2 avoided, with capture from gasfired plants falling within the range of USD 53 to USD 66/tCO2 avoided. " IEA (2008b). In industry the cost of CCS is higher than in power generation and must amount to around 100\$/tCO2. (2005 base Dollars).

	Natural Gas (Cubic Feet)	Crude Oil (Barrels)
Estimated Reserves*	6,18939E+15	1,31662E+12
Gross Heat Content in mmBtu/Unit	0,00103	5,6
Potential Energy in mmBtu	6,37507E+12	7,37308E+12
	Recoverable Coal (Short Tons)	Total Potential Energy
Estimated Reserves	9,30423E+11	
Gross Heat Content in mmBtu/Unit	25	
Potential Energy in mmBtu	2,32606E+13	3,70087E+13

Table 2: Estimated reserves of fossil fuel.

*IEA (2008*b*) (for Oil and Gas, reserves reported by *Oil and Gas Journal*).

The CO_2 concentration, according to the World Meteorological Organization GreenhouseGas Bulletin (2008) was in 2008 of 385 ppm[12]. The exogenous carbon ceiling is put at 450 ppm of CO_2. This maximal concentration is higher than the 450 ppm CO_2-eq target (that includes several greenhouse gas) leading to an increase of average temperature of 2° C (compared to pre-industrial level) according to IPCC (2007) (Intergovernmental Panel on Climate Change). Our carbon cap is equivalent, broadly speaking, to 550 ppm CO_2-eq, that is to say an increase of 3°C according to the IPCC (IPCC, p227).

We assume that the price of solar energy is constant and equals 0,30\$ per KWh or equivalently around 88\$/mmBtu in all sectors (in 2000 Dollars). These estimate is coherent with estimated costs of production ranging from 200 to 800 \$/MWh for Solar Photovoltaic power and from 130 to 230 \$/MWh for Concentrating solar power power (in 2005 Dollars, IEA (2008*b*)).

Our model includes only one fossil fuel resource, we simply consider the whole quantity of fossil fuel available and do not consider relative abundance among fossil fuels. Table 2 presents current reserves of crude oil, natural gas and recoverable coal and energy content of each fuel. The potential energy (based on fossil fuels use and average energy converts factor) amounts to around 37 008 billions of mmBtu.

We take a social discount rate $r = 4\%$. instantaneous rate of natural dilution of CO_2 in the atmosphere equals $0,4\%$

Parameters of the reference scenario are summarized in table :

[12]ppm = number of molecules of the gas per million molecules of dry air. A concentration of 1ppm is equivalent to 2.123 GtC tons in this atmosphere and 1 GtC is equivalent to 3,67 GtCO2

r	α	Z_0	$\bar{Z}$	Q_0	$D_{A,i}$
0.04	0.004	385ppm	450ppm	37E+12 mmBtu	$\sum_j x_{j,i}D_j$

$D_{B,i}$	D_T	D_I	D_{RC}	f_T
$\sum_j(1-x_{j,i})D_j$	$141.37P^{-0.6}Y^{0.7}$	$1.31E+3P^{-0.4}Y^{0.6}$	$3.64E+4P^{-0.5}Y^{0.5}$	0.00

f_I	f_{RC}	Y	c_{cs}	q
0.29	0.15	43.67E+12	88\$.mmBtu^{-1}	70\$.tCO$_2^{-1}$

Table 3: Parameters of the reference scenario.

4.2 Empirical Results[13]

First, we compute the initial carbon tax in the benchmark case where all the emissions are capturable by CCS. In that case, the carbon tax amounts to 26\$/tCO2. Detailed results are shown in Table 4. We find that the full initial optimal price of fossil fuel energy should be 3\$/mmBtu, or equivalently for coal, 75\$ per short ton of coal.

μ_0	λ_0	θ_1	θ_2	θ_3	θ_4
25,8	1,1	29,7	89,3	98,8	151,6

Table 4: Results when CCS is available to capture all the emissions.

From now, we assume that only a part of emissions are capturable using CCS. We have defined three scenarios (Low, Medium, High) according to the size of capturable emissions. For these three scenarios, the parameters value are those defined in Table 3. Table 5 presents results in these 3 scenarios.

In all cases, CCS is optimally used before the ceiling is reached ($t_1 < t_2$). So the case studied by previous literature where at the ceiling, emissions are sufficiently low to be only partially captured at the beginning of the ceiling, is not relevant compared with facts. The carbon tax is higher than in the benchmark case where all emissions are capturable: it ranges from 48\$/tCO2 to 57\$/tCO2. This result is not surprising as it is more difficult to stay below the ceiling when only a part of emissions are capturable: stronger regulation is required. The initial price of fossil fuel energy varies around 4 \$/mmBtu (equivalent to a price around 100\$ for a short ton of coal). We also find that the date the CCS is used for the first time is put forward compared to the case in which CCS is available for all use. Indeed, in order to postpone the date the ceiling will be binding, it is useful to start the capture early. The less emissions can be captured, the higher the shadow price of pollution μ_0, and the lower the scarcity rent : the fact that some emissions cannot be captured makes the optimal extraction slowed down and postpones the date of exhaustion (t_6) and then

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[13]The carbon tax μ_0 is expressed in \$/tCO2, λ_0 in \$/mmBtu and dates in years from 2008.

	μ_0	λ_0	t1	t2	t3	t4	t5	t6
Medium	52.3	0.6	8.9	38.4	97.0	109.4	119.0	167.6
Low	57.2	0.4	6.3	43.6	118.8	120.6	130.1	178.7
High	48.2	0.7	11.3	32.5	77.3	102.2	111.8	160.3

Table 5: Results in the Reference and extreme scenarios.

$\overline{Z}$	μ_0	λ_0	t1	t2	t3	t4	t5	t6
400.0	97.4	0.3	0.0	29.1	126.3	138.0	145.4	186.3
450.0	52.3	0.6	8.9	38.4	97.0	109.4	119.0	167.6
500.0	36.6	0.8	19.5	39.8	75.3	88.7	100.9	156.3
550.0	25.6	1.0	30.0	40.3	56.2	71.0	86.3	147.9

Table 6: Sensibility analysis of results when the carbon ceiling varies.

lowers the scarcity rent. The date when capture starts is all the more early since only a small part of emissions can be captured. Indeed, in order to postpone the date the ceiling will be binding and to continue consuming transport, it is useful to start the capture early when only a small fraction can be captured.

The ceiling level $\overline{Z}$ is a political exogenous parameter ; it is interesting to analyze how the initial carbon tax varies with the quantitative target. As shown in table 6, the carbon tax varies from 97 to 26 \$/tCO2 when $\overline{Z}$ ranges from 400ppm to 550ppm. As the ceiling becomes tighter, the date the CCS is used for the first time is brought forward: the carbon tax increases and the CCS is profitable sooner. The date when the ceiling is reached is also brought forward. If policy makers impose a quantitative target of 400 ppm, it becomes optimal to use CCS from the start. Indeed, with such a tight ceiling, the CCS cost is below the carbon tax from the start and it is optimal to capture CO_2 from the start. The initial price of fossil fuel energy varies from 7 to 3 \$/mmBtu (equivalent to a range of 180-70\$for a short ton of coal).

Because there are large uncertainties surrounding the values of some key-parameters (CCS and solar energy costs), we check the robustness of our results to these parameters. As shown in Annex (tables 10 and 11): the initial carbon tax is around 52 \$ /tCO2 for solar energy cost from 80 to 100 \$/mmBtu. The initial carbon tax goes from 45 to 59\$/tCO2 when the cost of CCS varies from 57 to 85\$/tCO2. The result that CCS should be used before the ceiling is robust to these changes in prices. The carbon tax increases with the cost of CCS. Indeed, more severe regulation is needed to stay below the ceiling when CCS is not available. The cheaper the CCS, the sooner it is used, and the later the ceiling is reached. As far as the backstop price q is concerned, the higher q is, the lower μ_0 and the later the CCS is used.

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5 Conclusions

In this article, we assume that Carbon Capture and Storage (CCS) technology can only be used in a specific sector (electricity sector). We examine how a restriction in the field of application of this technology affects the optimal timing of its use as well as the general pattern of extraction path. The result previously found in the literature (no Carbon Capture before the ceiling is reached) appears as a particular case of our model.

We find that for sufficiently small energy demand in transports sector, this restriction has no effect: CCS should not be used before the ceiling is binding and when CCS is used, only a part of capturable emissions are actually captured. In this case, the solution is similar to theoretical results found by Chakravorty et al. (2006b) andLafforgue et al. (2007). The carbon tax exhibits a familiar pattern (increasing, constant and then decreasing to 0). However, we find that for sufficiently large energy demand in transports sector, this restriction of CCS application leads to a new result : it is optimal to begin to *fully* capture emissions from electricity sector *before* the ceiling.

The underlying intuition is that when energy demands for transports is too large, abating before the ceiling allows to slack the environmental constraint for the transport sector by delaying the arrival of the ceiling. Prices in both sectors must differ when capture is complete in electricity sector : during this period, emissions of electricity sector are captured and optimal energy price includes the cost of CCS use ; in transports sector, the optimal price of the resource includes the carbon tax. When emissions from electricity production are captured, resource price in transports sector is higher than in electricity sector because non-capturable emissions have a higher social cost than captured emissions. When capture is not complete, optimal prices are similar in both sectors.

Calibrating our model, we find that the initial carbon tax must be put at a level of 52$/tCO2 and that using CCS before the ceiling is optimal. Comparing our empirical results with results from the benchmark model where all emissions are capturable indicates huge differences : our carbon tax is two or three times larger than the carbon tax of this benchmark case.

Some refinements can be added to our theoretical model in order to fit more closely reality. The cost of solar must be different between sectors. In that case the substitution process will consist in gradual substitution, first in sectors where delivered solar energy price is the lowest (electricity production), see for instance Chakravorty et al. (1997) . A non-stationary environment for costs (Solar energy, fossil fuel extraction and CCS costs) and energy demands could be an interesting step to better understand interactions between fossil fuels extraction and carbon regulation.

A Annexes

A.1 Determining sectoral fractions of capturable emissions

We use IEA estimates from the World Energy Outlook 2009 to obtain lower bounds of these fractions IEA (2009*c*). Emissions from power generation amount to 17.8 GtCO2 in the IEA-reference scenario (business as usual scenario) in 2030. In the 450 IEA-scenario of the IEA, optimal reduction of emissions requires to capture around 1.1 GtCO2 in 2030 from power generation emissions by using CCS at a price close to 70\$/tCO2 (IEA (2009*c*)). So emissions from power generation that are captured using CCS in 2030 represents 6% of global emissions from power generation. It does not mean that the fraction of capturable emissions at this price is 6 %. Indeed, some emissions that are reduced in the 450 scenario are reduced using cheaper methods than CCS. It is plausible than a part of these emissions could be captured by CCS at a price close to 70\$/tCO2. We could expect that if CCS was the only option to reduce atmospheric emissions more than 1.1GtCO2 would be captured using CCS. So, 6% represents a lower bound of the part of capturable emissions in power generation. As an upper bound, we assume that 100% of emissions from power generation could be captured at a price close to 70\$/tCO2. We assimilate power generation sector and electricity used in the 3 final sectors (Transportation, Industry, Residential/Commercial).

In the IEA-reference scenario, direct emissions from Industry (global emissions from Industry minus indirect emissions from Electricity used in Industry) amount to 6.1 GtCO2 whose 2.5 GtCO2 in non-metallic minerals (Cement etc) and Iron and Steel sector. Captured direct emissions in the 450 scenario from Industry sector amounts to 0.3 GtCO2 (mainly in Iron, Steel and Cement production) ; it represents around 5% of global direct emissions from Industry sector. As an upper bound, we assume that 100% of direct emissions from Iron, Steel and Cement production or equivalently that 41% of direct emissions from Industry sector could be technically captured at a price close to 70\$/tCO2. In Residential/Commercial sector, we assume that direct emissions are non-capturable at a reasonable cost (70\$/tCO2).

We define a medium part of capturable emissions in each sector as the average of upper bound and lower bound of these parts in each sector. In this "Medium" scenario, we expect that 53% of emissions from power generation and 23% of direct emissions from Industry can be captured at a price close to 70\$/tCO2, and 0% of direct emissions from Residential/Commercial sector are capturable (Medium scenario). We assume that this ratio is constant over the time in particular it means that in Industry sector, the price elasticity of energy demand for Iron, Steel and Cement production is identical to the price elasticity of energy demand in the rest of this sector.

Some emissions from Industry and Residential/Commercial sectors come from power use, so emissions from power generation must be added to direct sectoral emissions. We assume that the part of capturable emissions from power generation is the same in all sectors where power is used. Taking into account global sectoral emissions (direct and indirect emissions linked with electricity use), we get the following fractions of capturable emissions for each sector for the 3 scenarios.[14]

A.2   Estimates of CO_2 content of fossil fuels.

From IEA (2008b), we get the following table, we conclude that 1 billion of mmBtu (1 Quad Btu) of delivered energy from fossils fuels leads in average to $72 * 10^6$ tons of CO_2 (1 tCO_2=13,9 mmBtu.). We assume that this average CO_2 content is the same among sectors. This assumption is not too far from reality as indicated in table 9:

A.3   Robustness check of the initial value of the carbon tax.

Values of initial carbon tax is robust to small changes in solar and CCS costs as shown in tables 10 and 11.

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[14]We assume that carbon content of delivered energy is similar between sectors. In real life, Electricity is produced by using emitting sources of primary energy or non emitting primary source of energy (Hydroelectricity, Wind, Solar, Uranium, Biomass.). We assume that all the electricity is produced using emitting fossil fuels and we assume that CO_2 content of delivered electricity is similar than CO_2 content of delivered energy using fossil fuels. Around 75% of power generation comes from fossil fuels.

	Energy Demand (EJ*)	Price Elasticity	Income Elasticity	Constant Parameter
Transportation	71.1	-0.6	0.7	141.4
Refined Petroleum	71.1			
Industry	81.4	-0.4	0.6	1318.5
Electricity	19.3			
Refined Petroleum	24.4			
Gas	20.6			
Coal	17.2			
Other	74.5	-0.5	0.5	36409.0
Electricity	25.5			
Petroleum Products	20.1			
Gas	23.9			
Coal	5.0			

	% Capturable emissions**			Average Emissions 10^6 tons of CO_2^{**}
	Low	**Medium**	**High**	
Transportation	**0.0**	**0.0**	**0.0**	4832.1
Refined Petroleum				4832.1
Industry	**5.2**	**28.7**	**52.1**	5209.0
Electricity	6.0	53.0	100.0	982.8
Refined Petroleum	5.0	23.0	41.0	1656.5
Gas				1398.1
Coal				1171.6
Other	**1.7**	**14.9**	**28.1**	4630.8
Electricity	6.0	53.0	100.0	1301.5
Petroleum Products	0.0	0.0	0.0	1367.5
Gas				1623.2
Coal				338.6

Table 7: Estimates of sectoral energy demands, and % of capturable emissions in each sector.
Constant parameters are estimated using weighted energy prices, IEA (2001), total final consumption from IEA (2002) and price and income elasticities from Chakravorty et al. (2006a), and 2000 world GDP in US-Dollars from IMF. Around 75% of electricity production is based on fossil fuels IEA (2009c), we exclude the rest of electricity from energy consumption to estimate constant parameters.
*1EJ=10^18Joules, 1.0 Quad = One quadrillion Btu (10^15 Btu) = 1.055 exajoules (EJ)

** We assume that emissions content of delivered energy is similar in all sectors (See table 8 , 1 Quad Btu leads to 72 millions of $CO2$ tons or 1EJ to 68 millions of tons of $CO2$ in all sectors) .

	Natural Gas	Crude Oil	Coal	Total
World consumption in Quad Btu (10^{15} Btu)	107.998	171.723	127.548	407.269
emissions* of CO_2 in Million of metric tons	5911.830	11218.940	12064.640	29195.410
CO_2 tons per mm Btu	0.055	0.065	0.095	0.072

Table 8: Fossil fuels world consumption, linked CO2 Emissions and CO2 content of fossil fuel based energy in 2006, IEA (2008a)
*from consumption and flaring of fossil fuels, 2006.

Fossil Fuel Energy	Transports	Electricity*	Industry	Buildings
Oil	100.00	8.30	23.51	38.47
gas	0.00	30.66	33.80	52.19
Coal	0.00	61.04	42.69	9.34
Total	100.00	100.00	100.00	100.00
CO_2 Millions of tons per billion of mmBtu	65.00	80.25	74.43	62.59

Table 9: Table 9: Decomposition of fossil fuel energy demands and CO2 content of fossil fuel based energy by sector in 2007, IEA (2009c)

*Fossil Fuel based Electricity Production.

c_{cs} in \$ per tCO2	μ_0	λ_0	t1	t2	t3	t4	t5	t6
57.0	45.3	0.6	6.8	41.6	96.1	107.6	115.1	163.7
59.8	46.8	0.6	7.2	40.9	96.4	108.1	115.9	164.5
62.6	48.2	0.6	7.7	40.3	96.6	108.5	116.8	165.3
65.3	49.6	0.6	8.1	39.6	96.8	108.8	117.5	166.1
68.1	51.0	0.6	8.5	39.0	96.9	109.2	118.3	166.8
70.9	52.3	0.6	8.9	38.4	97.0	109.4	119.0	167.6
73.7	53.6	0.6	9.3	37.8	97.0	109.7	119.7	168.2
76.5	54.9	0.6	9.7	37.2	97.0	109.9	120.3	168.9
79.2	56.2	0.5	10.1	36.7	97.0	110.1	120.9	169.5
82.0	57.4	0.5	10.5	36.1	96.9	110.2	121.6	170.1
84.8	58.7	0.5	10.8	35.6	96.8	110.4	122.1	170.7

Table 10: Sensibility analysis of results when CCS varies.

q	μ_0	λ_0	t1	t2	t3	t4	t5	t6
80.0	52.4	0.6	8.9	38.4	98.2	110.6	120.2	165.6
82.0	52.4	0.6	8.9	38.4	97.9	110.3	119.9	166.1
88.0	52.3	0.6	8.9	38.4	97.0	109.4	119.0	167.6
92.0	52.3	0.6	9.0	38.3	96.4	108.9	118.4	168.5
100.0	52.2	0.6	9.0	38.3	95.5	107.9	117.5	170.3

Table 11: Sensibility analysis of results when the cost of solar energy varies.

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