Residual-based tests for cointegration and multiple deterministic structural breaks: A Monte Carlo study

Matteo Mogliani

PARIS SCHOOL OF ECONOMICS
ECOLE D'ECONOMIE DE PARIS

WORKING PAPER N° 2010 - 22

Residual-based tests for cointegration and multiple deterministic structural breaks: A Monte Carlo study

Matteo Mogliani

JEL Codes: C12, C13, C15, C22
Keywords: Cointegration, single-equation, structural breaks, Monte Carlo simulations

PARIS-JOURDAN SCIENCES ECONOMIQUES
48, BD JOURDAN – E.N.S. – 75014 PARIS
TÉL. : 33(0) 1 43 13 63 00 – FAX : 33 (0) 1 43 13 63 10
www.pse.ens.fr

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE – ECOLE DES HAUTES ETUDES EN SCIENCES SOCIALES
ÉCOLE DES PONTS PARISTECH – ECOLE NORMALE SUPÉRIEURE – INSTITUT NATIONAL DE LA RECHERCHE AGRONOMIQUE

Residual-Based Tests for Cointegration and Multiple Deterministic Structural Breaks: A Monte Carlo Study*

Matteo Mogliani†

Paris School of Economics, France

First Version: November 11, 2009
This Version: August 31, 2010

Abstract

The aim of this paper is to study the performance of residual-based tests for cointegration in the presence of multiple deterministic structural breaks via Monte Carlo simulations. We consider the KPSS-type LM tests proposed in Carrion-i-Silvestre and Sansò (2006) and in Bartley, Lee, and Strazicich (2001), as well as the Schmidt and Phillips-type LM tests proposed in Westerlund and Edgerton (2007). This exercise allow us to cover a wide set of single-equation cointegration estimators. Monte Carlo experiments reveal a trade-off between size and power distortions across tests and models. KPSS-type tests display large size distortions under multiple breaks scenarios, while Schmidt and Phillips-type tests appear well-sized across all simulations. However, when regressors are endogenous, the former group of tests displays quite high power against the alternative hypothesis, while the latter shows severe low power.

Keywords: Cointegration; single-equation; structural breaks; Monte Carlo simulations.

JEL classification: C12, C13, C15, C22.

*The author is indebted to Giovanni Urga and Eduardo Rossi for helpful comments and discussions. He also wish to thank participants in the CEA reading group seminar at Cass Business School for useful discussions. However, the usual disclaimer applies.

† *Corresponding address*: Paris School of Economics, Paris-Jourdan Sciences Economiques, 48 Boulevard Jourdan, 75014 Paris (France). Tel: +33 (0)1 43 13 63 22. Fax: +33 (0)1 43 13 63 10. Email: mogliani@pse.ens.fr.

1 Introduction

Cointegration has been at the heart of a vast macroeconomic and econometric research since the seminal contribution of Engle and Granger (1987). This concept, *i.e.*, the hypothesis that one stationary linear combination of individually non-stationary variables exists, has been widely used for empirical purposes in many areas of economics. Indeed, the development of cointegrating and error-correction models allowed applied economists to shed light on long-run and short-run theoretical economic relationships, such as, for instances, money demand (e.g., Hendry and Ericsson, 1991, and Stock and Watson, 1993), balanced growth (e.g., King et al., 1991) and purchase power parity (e.g., Taylor and McMahon, 1988, and Cheung and Lai, 1993).

Many cointegration tests have been proposed in the econometric literature. Among them, the class of residual-based tests is the most popular, thanks to the simple computation and the straight interpretation in terms of economic theory. Following the unit-root testing approach, the literature has proposed tests for the null hypothesis of non-cointegration (Engle and Granger, 1987; Phillips and Ouliaris, 1990), as well as tests for the null hypothesis of cointegration (Hansen, 1992; Shin, 1994).

These tests show nevertheless serious size distortions when specific features of data are neglected. Indeed, one potential feature of long-run economic relationships is structural breaks, *i.e.*, the significant change of one or more parameters affecting persistently the data generating process (DGP) of the underlying economic model. This issue is addressed in Gregory and Hansen (1996), who extend the general framework of Engle and Granger (1987) and Phillips and Ouliaris (1990) to account for the presence of one structural break. However, as pointed out in Carrion-i-Silvestre and Sansò (2006), the statistical tests proposed in Gregory and Hansen (1996) are not able to discern between the situation of unstable cointegrating relationship and that of stability with regime-shifts, the null hypothesis of non-cointegration being tested against the alternative of cointegration with break.

Residual-based tests recently proposed in the literature address this issue through the inclusion of structural breaks under both the null and the alternative hypothesis. The generalization of the break hypothesis makes the latter tests independent (stand alone tests for the hypothesis of cointegration or non-cointegration), compared to the complementarity role of the former (auxiliary tests for the hypothesis of spurious cointegration led by a neglected break). However, these recent contributions have only explored the "one structural break" hypothesis. This is mainly due to the well-known econometric circular problem of, on the one hand, estimating and testing for multiple (deterministic or stochastic) breaks in the presence of non-stationary variables (unit-root) or cointegrated systems, and, on the other hand, assessing non-stationarity or cointegration when breaks are neglected or their actual number is misspecified. This issue has then attracted increasing attention in the econometric literature during the last decade. To deal with the circular problem in the unit-root testing, various approaches have been recently proposed to check for the presence of breaks (in trend and level) in univariate I(1) or I(0) processes (Perron and Zhu, 2005; Harvey et al., 2009a,b; Perron and Yabu, 2009; Kejriwal and Perron, 2009a), as well as to embed the hypothesis of multiple breaks in a large class of standard unit-root tests (Carrion-i-Silvestre et al., 2009). Based on these theoretical developments, Kejriwal and Lopez (2010) propose a sequential testing strategy designed to help applied economists to minimize the model specification error.

As to the circular problem in the cointegration testing, more emphasis has been put on the selection of the actual number of breaks in long-run regressions. To tackle this issue in a single-equation cointegration framework, approaches based on global minimizers algorithms (Bai and Perron, 1998, 2003; Qu, 2007; Kejriwal and Perron, 2009b), as well as sequential bootstrap procedures (De Peretti and Urga, 2004), have been so far proposed in the literature. Indeed, as pointed out by Mogliani, Urga, and Winograd (2009), accounting for multiple breaks in economic relationships can be a crucial issue when dealing, for instances, with emerging economies and/or long span datasets. However, to our knowledge, too little has been said in the literature about the behaviour of residual-based tests for cointegration when multiple breaks affect the long-run relationship of non-stationary series.

The main aim of this paper is to compare the size and power distortions of residual-based tests for cointegration in the case of multiple breaks. For this purpose, we run Monte Carlo simulations involving several single-equation cointegration estimators (OLS, DOLS, DGLS, FM-OLS and CCR) and breaks scenarios. For the latter issue, we follow Perron (1989, 1990) and Hao (1996) and we only consider deterministic structural breaks (constant and trend). We also account for endogenous regressors and potential misspecification of model residuals. The results of the study should lead to specific recommendations for applied economists in terms of the best performing estimator/test pair to use for cointegrating regression models with multiple breaks.

We consider the residual-based tests for the null hypothesis of cointegration proposed in Bartley, Lee, and Strazicich (2001) and Carrion-i-Silvestre and Sansò (2006). These contributions deal with the generalization of the univariate LM test of Kwiatkowski, Phillips, Schmidt, and Shin (1992) - henceforth KPSS -, as in Shin (1994), Hao (1996) and Lee (1999), to the case of cointegration with one structural break, while efficient estimates of the cointegrating relationship are carried out through the Canonical Cointegration Regression (Park, 1992), the dynamic OLS (Saikkonen, 1991; Stock and Watson, 1993) and the Fully-Modified approach (Phillips and Hansen, 1990). We also consider testing procedures proposed in Westerlund and Edgerton (2007) and involving instead the null hypothesis of non-cointegration. This work extends the univariate LM test of Schmidt and Phillips (1992) - henceforth SP - to the cointegration with a single break framework. The proposed statistical tests are built upon the OLS estimate of the cointegrating relationship (Engle and Granger, 1987; Phillips and Ouliaris, 1990), and they are thus mainly designed for strictly exogenous regressors.

Our main findings show that KPSS-based tests display severe size distortions when more deterministic breaks are included in the cointegration model, in particular when both level and trend breaks are considered. The opposite is true for the tests proposed in Westerlund and Edgerton (2007), which appear quite correctly sized across all our simulation exercises. However, these results are reverted in the power analysis: KPSS-based tests show quite high power against the alternative hypothesis in all simulations, while SP-based tests show very low power which tends to be close to the nominal size. Simulations reveal that the latter result is mainly driven by the presence of endogenous regressors. Overall, tests based on the DOLS and, in particular, on the DGLS estimators display the best size-power performance.

The remainder of the paper is as follows. In Section 2, we introduce a general model of cointegration with structural breaks and we briefly describe the estimators and the residual-based tests of cointegration studied in this paper. In Section 3 we define the DGP used for simulation purposes and we explain the Monte Carlo design. In Section 4 we discuss simulation results. Section 5 concludes.

2 Estimators and Tests for Cointegration with Structural Breaks

In this Section we briefly describe the general single-equation cointegration model with structural breaks and six alternative residual-based tests for cointegration used in our Monte Carlo experiments. Four of these test statistics (CS_{DOLS}, CS_{DGLS}, CS_{FM} and BLS_{CCR}) are based on the null hypothesis of cointegration (Bartley, Lee, and Strazicich, 2001; Carrion-i-Silvestre and Sansò, 2006), while the remaining two (WE_{Φ} and $WE_{t\text{-stat}}$) are based on the null of non-cointegration (Westerlund and Edgerton, 2007). For ease of exposition, statistical tests are presented along with their related estimators of cointegrating relationships.

2.1 The Cointegrated Regression Model

Let's assume that the data generating process (DGP) is of the form:

$$y_t = \alpha + g(t) + x_t'\beta + e_t, \tag{1}$$

with

$$\begin{aligned} e_t &= \rho e_{t-1} + \varepsilon_t \\ x_t &= x_{t-1} + \mu_t, \end{aligned}$$

where $t = 1, \ldots, T$ is the time series index, x_t is the K-dimensional vector of I(1) regressors and ε_t and μ_t are i.i.d. processes with distribution $N(0, \Sigma)$. We define $g(t)$ as the function collecting the deterministic components of the model, except for the constant. Following Perron (1989, 1990), Hao (1996), Bartley, Lee, and Strazicich (2001) and Carrion-i-Silvestre and Sansò (2006), we choose to study an empirically relevant set of deterministic functions:

$$g(t) = \begin{cases} \theta_1 DU_t & \text{Model A} \\ \tau t + \theta_1 DU_t & \text{Model B} \\ \tau t + \theta_1 DU_t + \theta_2 DT_t & \text{Model C} \end{cases} \tag{2}$$

where $DU_t = (DU_{1,t}, \ldots, DU_{m,t})'$ and $DT_t = (DT_{1,t}, \ldots, DT_{m,t})'$ are the vectors of deterministic breaks and

$$DU_{j,t} = \begin{cases} 1, & \text{for } t > T_{jb} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad DT_{j,t} = \begin{cases} (t - T_{jb}), & \text{for } t > T_{jb} \\ 0, & \text{otherwise} \end{cases}$$

is the structure of deterministic breaks at dates $T_{jb} = \lambda_j T$, with $\lambda_j \in (0, 1)$, for $j = 1, \ldots, m$, where m is the number of breaks. Model A allows for multiple level breaks without a linear trend. Model B allows for a linear trend and multiple level breaks. Finally, Model C allows for both multiple level and trend breaks, which are assumed for simplicity to pairwise occur at the same date.

2.2 A Test Based on the OLS Estimator

A test based on the standard OLS estimator of the cointegrating relationship in (1) (Engle and Granger, 1987; Phillips and Ouliaris, 1990) is proposed in Westerlund and Edgerton (2007) - henceforth WE. Following Schmidt and Phillips (1992), WE propose an LM-type test for the null hypothesis of non-cointegration against the alternative of cointegration, with a structural break under both the null and the alternative.

According to the LM (score) principle, the cointegration test is obtained from the following regression:

$$\Delta \hat{S}_t = \vartheta + \Phi \hat{S}_{t-1} + \epsilon_t, \tag{3}$$

where ϑ is a constant, ϵ_t is the error term, $\hat{S}_t = y_t - \hat{\alpha} - \hat{g}_i(t) - x_t'\hat{\beta}$ and $\hat{\alpha}$ is the restricted maximum likelihood estimate of $\tilde{\alpha} = \alpha + e_0$, given by $\hat{\alpha} = y_1 - \hat{g}_i(1) - x_1'\hat{\beta}$. Estimates of $\hat{\beta}$ and parameters in $\hat{g}_i(t)$, for $i = \{\text{A, B, C}\}$, are obtained from the OLS regression of Δy_t over $\Delta g_i(t)$ and $\Delta x_t'$. It is worth noticing that the expression $\Delta g_i(t)$ involves one-period jumps (ΔDU_t) and changes in drift (ΔDT_t), rather than constant (DU_t) and trend (DT_t) breaks. From Equation (3), the hypothesis of non-cointegration can be formulated as a test of $\Phi = 0$ against $\Phi < 0$, which can be verified through the OLS estimate of Φ or its LM t-statistic. WE then propose the following two statistical tests:

$$WE_{\Phi} = T \times \hat{\Phi} \qquad \text{and} \qquad WE_{t\text{-stat}} = \frac{\hat{\Phi}}{\hat{\sigma}} \times \sqrt{\sum_{t=2}^{T} (\hat{S}_{t-1})_p^2}, \tag{4}$$

where $\hat{\sigma}$ is the estimated standard error from regression (3) and $(\hat{S}_{t-1})_p$ is the error from projecting $\hat{S}_{t-1}$ onto its mean value. To account for autocorrelated and heteroskedastic errors, WE follow the parametric correction proposed in Ahn (1993) and include augmented terms in Equation (3) :

$$\Delta\hat{S}_t = \vartheta + \Phi\hat{S}_{t-1} + \sum_{j=1}^{p} \psi_j \Delta\hat{S}_{t-j} + \epsilon_t, \tag{5}$$

where the optimal lag order p is chosen by following the "general to specific" procedure suggested by Perron (1989), Campbell and Perron (1991) and Ng and Perron (1995). In our Monte Carlo simulations we allow for a maximum number of 6 lags.[1] WE show that only the statistic WE_{Φ} is affected by the presence of autocorrelated errors. This requires the following correction:

$$WE_{\Phi} = T \times \hat{\Phi} \times \sqrt{\frac{\hat{\omega}}{\hat{\sigma}^2}}, \tag{6}$$

where $\hat{\sigma}^2$ is the residual variance from the augmented test regression (5) and $\hat{\omega}$ is the long-run variance of $\Delta\hat{S}_t$ evaluated at frequency zero:

$$\hat{\omega} = \frac{1}{T}\sum_{t=1}^{T} \Delta\hat{S}_t \Delta\hat{S}_t' + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \Delta\hat{S}_t \Delta\hat{S}_{t-j}',$$

where $w(\cdot)$ and M are the kernel function and the bandwidth parameter, respectively. We follow WE and we use a Bartlett kernel with bandwidth parameter $M = p$ (the optimal lag order in the auxiliary regression (5)).

For the case of Model B, it can be shown that both WE_{Φ} and $WE_{t\text{-stat}}$ statistics follow the asymptotic distributions derived in Schmidt and Phillips (1992). In addition, distributions are unaffected by the presence of multiple mean breaks, the number of regressors (K) and the breaks fraction (λ_j). For the case of Model A, our simulations show that the exclusion of the

[1] We sequentially test at 5% level the significance of the last term in the augmented test regression (5), until either the optimal lag is found or $p = 0$.

linear trend from the cointegrating equation does affect the asymptotic distribution of both statistics. Nevertheless, distributions are unaltered by the presence of multiple mean breaks. Differently, for the case of Model C, our simulations show that the statistics under consideration follow asymptotic distributions which depend on the number of breaks and their location in the sample (λ_j).

It is worth noticing that the testing procedure proposed in WE is valid until regressors x_t are strictly exogenous. Relaxing this assumption would imply a potential bias arising from the OLS estimate of $\hat{\beta}$ for the computation of $\hat{S}_t$. To correct for endogeneity bias, WE propose to estimate $\hat{\beta}$ by IV. In practice, finding out consistent instruments for endogenous regressors can be difficult in the context of cointegrated macroeconomic time series. For this reason, in our simulations we prefer studying the performance of WE_Φ and $WE_{t\text{-stat}}$ statistics under endogeneity bias.

2.3 A Test Based on the Dynamic Leads-and-Lags Estimator

A test based on the leads-and-lags correction of the cointegrating regression (Saikkonen, 1991; Stock and Watson, 1993) is developed in Carrion-i-Silvestre and Sansò (2006) - henceforth CS. Following Shin (1994), CS propose a LM-type test for the null hypothesis of cointegration against the alternative of non-cointegration, with a structural break under both the null and the alternative. Let's define $v_t = \Delta x_t$ and $\eta_t = (e_t, v_t')$ and assume that η_t satisfies the multivariate invariance principle (Herrndorf, 1984; Phillips and Durlauf, 1986):

$$T^{-1/2}\Omega \sum_{t=1}^{[Tr]} \eta_t \Rightarrow W(r), \qquad 0 \leq r \leq 1,$$

where $\Rightarrow$ denotes weak convergence in probability and $W(r) = (W_1(r), W_{2K}(r)')'$ is a $(K+1)$-dimensional Wiener process. Ω is the long-run covariance matrix, which can be written (partitioned in conformity with η_t) as:

$$\Omega = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{T} E(\eta_j \eta_t') = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \Omega_{22} \end{bmatrix} = \Sigma + \Lambda + \Lambda',$$

where long-run variances ω_{11} and Ω_{22} of processes $W_1(r)$ and $W_{2K}(r)$ are positive definite to rule out multicointegration (Granger and Lee, 1990) and

$$\Sigma = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E(\eta_t \eta_t') = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \Sigma_{22} \end{bmatrix}$$

$$\Lambda = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{t} E(\eta_j \eta_t') = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{12} & \Lambda_{22} \end{bmatrix}.$$

Standard asymptotics cannot apply here because of the presence of correlation between disturbance terms. This means that regressors x_t are not strictly exogenous and the OLS estimator of the cointegrating regression (1) is inefficient. To overcome this problem, CS propose to estimate (1) through the following Dynamic OLS regression:

$$y_t = \alpha_0 + g_i(t) + x_t'\beta + \sum_{j=-k}^{k} \Delta x_{t-j}'\xi_j + e_t^*, \tag{7}$$

where k is the (finite truncated) number of leads and lags for first-differenced non-stationary regressors.

Since errors e_t^* can be serially correlated and uncorrelated with the regressors at all leads and lags, we follow Stock and Watson (1993) and we introduce the Dynamic GLS estimator. A feasible DGLS estimator is constructed by transforming regressors in (7) as $\tilde{x}_t = x_t'\hat{\varphi}(L)$, where $\hat{\varphi}(L)$ is an estimate of the lag polynomial of residuals $\varphi(L)$.[2]

In our Monte Carlo experiments, we construct $\varphi(L)$ as an AR(1) model of residuals. We follow the Cochrane-Orcutt iterative procedure and we allow the AR(1) parameter to converge across the sequential estimation. Finally, we allow the number of leads and lags to be selected by the SBC criterion, starting with a maximum number of 4.[3]

The multivariate LM-type test proposed in CS is then given by:

$$CS_{\text{DOLS}} = \frac{T^{-2}}{\hat{\omega}_{11\cdot 2}^*} \times \sum_{t=1}^{T} (S_t^*)^2 \qquad \text{and} \qquad CS_{\text{DGLS}} = \frac{T^{-2}}{\hat{\omega}_{11\cdot 2}^*} \times \sum_{t=1}^{T} (S_t^*)^2, \tag{8}$$

[2]It is worth noticing that the DGLS estimator is not considered in the original work of CS, but it is expressly introduced by the author of the present paper.

[3]The use of this information criterion is supported by simulation results reported in Kejriwal and Perron (2008).

where $S_t^* = \sum_{j=1}^t \hat{e}_j^*$, $\hat{e}_t^*$ are estimated residuals from DOLS/DGLS regression (7) and $\hat{\omega}_{11\cdot 2}^*$ is any consistent estimate of $\omega_{11\cdot 2} = \omega_{11} - \omega_{12}\Omega_{22}^{-1}\omega_{21}$, *i.e.*, the endogeneity-corrected long-run variance of residuals e_t. In practice, a consistent estimate of $\omega_{11\cdot 2}$ can be obtained as follows:

$$\hat{\omega}_{11\cdot 2}^* = \frac{1}{T}\sum_{t=1}^{T} \hat{e}_t^* \hat{e}_t^{*\prime} + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \hat{e}_t^* \hat{e}_{t-j}^{*\prime},$$

with $w(\cdot)$ and M being the kernel function and the bandwidth parameter, respectively. To avoid the inconsistency on the estimate of the long-run variance $\hat{\omega}_{11\cdot 2}^*$, we follow CS and we use the kernel and the bandwidth parameter proposed in Kurozumi (2002). This issue will be discussed in Section 3.3.

For the case of a single break, CS show that the asymptotic distribution of the test statistic depends on the number of regressors (K), the break fraction (λ) and the deterministic model considered ($g_i(t)$). This result can be readily generalized to the case of multiple structural breaks. In this case, the number of breaks (m) and their location in the sample (λ_j) also affect the asymptotic distribution.

2.4 A Test Based on the Fully-Modified Estimator

Carrion-i-Silvestre and Sansò (2006) also extend the test presented above to the Fully-Modified estimator of cointegrating relationships (Phillips and Hansen, 1990), *i.e.*, solving non-parametrically the issue of the OLS inefficiency when regressors are non-strictly exogenous.

Consider the set of asymptotic assumptions illustrated in the first part of paragraph 2.3. We exploit here the long-run correlation properties of the innovations vector $\eta_t = (e_t, v_t')$ to rule out the bias due to the endogeneity of regressors x_t. Preliminary simulations suggest that cointegration tests based on the pre-whitened Fully-Modified estimator lead to improved results in terms of size and power. We then follow Andrews and Monahan (1992) and Hansen (1992) and we build the Fully-Modified correction by firstly fitting a VAR(1) to η_t and then consistently estimating the long-run covariance matrix from whitened residuals $\hat{\varepsilon}_t = \hat{\eta}_t - \hat{\eta}_{t-1}'\hat{\zeta}$:

$$\Omega_\varepsilon = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t' + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right) \sum_{t=j+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-j}',$$

with partitions

$$\Sigma_\varepsilon = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_t'$$

$$\Lambda_\varepsilon = \frac{1}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_{t-j}',$$

where the kernel function $w(\cdot)$ used for our simulations is the Quadratic Spectral and its associated plug-in bandwidth estimator (Andrews, 1991).[4] The long-run covariance matrix used for the Fully-Modified estimation is then recolored: $\Omega = (I-\hat{\zeta})^{-1}\Omega_\varepsilon(I-\hat{\zeta}')^{-1}$ and $\Lambda = (I-\hat{\zeta})^{-1}\Lambda_\varepsilon(I-\hat{\zeta}')^{-1} - (I-\hat{\zeta})^{-1}\hat{\zeta}\Sigma$, where $\Sigma = 1/T\sum_t^T \hat{\eta}_t\hat{\eta}_t'$.

Fully-Modified estimation is then computed by partitioning Ω and Λ, setting $\omega_{11\cdot 2} = \omega_{11} - \omega_{12}\Omega_{22}^{-1}\omega_{21}$ and $\lambda_{21}^{+} = \lambda_{21} - \Lambda_{22}\Omega_{22}^{-1}\omega_{21}$ and transforming the dependent variable $y_t^{+} = y_t - \omega_{12}\Omega_{22}^{-1}v_t'$. The Fully-Modified estimator of cointegrating parameters is obtained through the following OLS regression:

$$\beta_X^{+} = (X_t'X_t)^{-1}\left(X_t'y_t^{+} - \kappa\lambda_{21}^{+}\right),$$

where X_t is the vector of regressors (deterministic and stochastic) included in (1) and $\kappa = [\mathbf{0}, I]$ is a matrix of dimension $(d+K)\times K$, with first $d\times K$ zero elements followed by a $K\times K$ identity matrix (d being the number of deterministic regressors in the model).

Fully-Modified residuals $\hat{e}_t^{+} = y_t^{+} - X_t'\hat{\beta}_X^{+}$ are then used to compute the LM-type statistic:

$$CS_{\text{FM}} = \frac{T^{-2}}{\hat{\omega}_{11\cdot 2}^{+}}\times\sum_{t=1}^{T}\left(S_t^{+}\right)^2, \tag{9}$$

where $S_t^{+} = \sum_{j=1}^{t}\hat{e}_j^{+}$ and the consistent estimate of the long-run variance of residuals e_t^{+} is obtained as follows:

$$\hat{\omega}_{11\cdot 2}^{+} = \frac{1}{T}\sum_{t=1}^{T}\hat{e}_t^{+}\hat{e}_t^{+\prime} + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{e}_t^{+}\hat{e}_{t-j}^{+\prime},$$

[4] The Quadratic Spectral kernel is defined as $w(x) = \frac{25}{12\pi^2x^2}\left(\frac{\sin(6\pi x/5)}{(6\pi x/5)} - \cos(6\pi x/5)\right)$ and its optimal bandwidth parameter is $M = 1.3221(\hat{\alpha}(2)T)^{1/5}$, where $\hat{\alpha}(2) = \sum_{a=1}^{p}\frac{4\rho_a^2\sigma_a^2}{(1-\rho_a)^8}\Big/\sum_{a=1}^{p}\frac{\sigma_a^2}{(1-\rho_a)^4}$ is obtained from an AR(1) model of each element $\varepsilon_{a,t}$, for $a = 1,\ldots,p$, of ε_t.

with $w(\cdot)$ and M being, respectively, the kernel function and the bandwidth parameter proposed in Kurozumi (2002) (see Section 3.3).

For the case of a single break, CS show that the asymptotic distribution of the test statistic based on the Fully-Modified correction is the same as assuming x_t strictly exogenous. Again, the asymptotic distribution depends on the number of regressors (K), the break fraction (λ) and the deterministic structure ($g_i(t)$). In the multiple breaks framework considered here, the asymptotic distribution also depends on the number of breaks (m) and their location in the sample (λ_j).

2.5 A Test Based on the Canonical Cointegration Estimator

A test based on the *feasible* Canonical Cointegration Regression estimator (Park, 1992) is developed in Bartley, Lee, and Strazicich (2001) - henceforth BLS. The authors propose a LM-type test for the null hypothesis of cointegration against the alternative of non-cointegration, with a structural break under both the null and the alternative.

As for the Fully-Modified estimator, preliminary simulations suggest that tests based on the pre-whitened CCR estimator lead to improved results in terms of size and power. We then fit a VAR(1) to η_t and we compute consistent estimate of the long-run covariance matrix from whitened residuals $\hat{\varepsilon}_t = \hat{\eta}_t - \hat{\eta}'_{t-1}\hat{\zeta}$:

$$\Omega_\varepsilon = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}'_t + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}'_{t-j},$$

with partitions

$$\begin{aligned}
\Sigma_\varepsilon &= \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}'_t \\
\Lambda_\varepsilon &= \frac{1}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}'_{t-j} \\
\Gamma_\varepsilon &= \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}'_t + \frac{1}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}'_{t-j},
\end{aligned}$$

where the kernel function $w(\cdot)$ used for our simulations is the Quadratic Spectral and its associated plug-in bandwidth estimator (Andrews, 1991) (see footnote 4). It is worth noticing that

$\Omega_\varepsilon = \Sigma_\varepsilon + \Lambda_\varepsilon + \Lambda'_\varepsilon = \Gamma_\varepsilon + \Lambda'_\varepsilon$. The long-run covariance matrix used for the CCR estimation is then recolored: $\Omega = (I - \hat{\zeta})^{-1}\Omega_\varepsilon(I - \hat{\zeta}')^{-1}$ and $\Lambda = (I - \hat{\zeta})^{-1}\Lambda_\varepsilon(I - \hat{\zeta}')^{-1} - (I - \hat{\zeta})^{-1}\hat{\zeta}\Sigma$, where $\Sigma = 1/T\sum_t^T \hat{\eta}_t\hat{\eta}'_t$.

CCR estimation is computed by first transforming the regressand and the stochastic regressors and then estimating by OLS the following corrected cointegration model:

$$y_t^\star = \alpha_0 + g_i(t) + x_t^{\star\prime}\beta^\star + e_t^\star, \tag{10}$$

where $y_t^\star = y_t - (\Sigma^{-1}\Gamma_2\hat{\beta} + (0, \omega_{12}\Omega_{22}^{-1})')'\hat{\eta}_t$, $x_t^\star = x_t - (\Sigma^{-1}\Gamma_2)'\hat{\eta}_t$, $\Gamma_2 = (\gamma_{12}, \Gamma_{22})$ and $\hat{\beta}$ is the vector of estimated parameters obtained from the auxiliary regression of the uncorrected model (1).

CCR residuals $\hat{e}_t^\star = y_t^\star - \hat{\alpha}_0 - g_i(t) - x_t^{\star\prime}\hat{\beta}^\star$ are then used to compute the LM-type statistic:

$$BLS_{\text{CCR}} = \frac{T^{-2}}{\hat{\omega}_{11\cdot 2}^\star} \times \sum_{t=1}^{T}\left(S_t^+\right)^2, \tag{11}$$

where $S_t^+ = \sum_{j=1}^t \hat{e}_j^\star$ and the consistent estimate of the long-run variance of residuals $e_t^\star$ is obtained as follows:

$$\hat{\omega}_{11\cdot 2}^\star = \frac{1}{T}\sum_{t=1}^{T}\hat{e}_t^\star\hat{e}_t^{\star\prime} + \frac{2}{T}\sum_{j=0}^{T} w\left(\frac{j}{M}\right)\sum_{t=j+1}^{T}\hat{e}_t^\star\hat{e}_{t-j}^{\star\prime},$$

with $w(\cdot)$ and M being, respectively, the kernel function and the bandwidth parameter proposed in Kurozumi (2002) (see Section 3.3).

For the case of a single break, BLS follow Choi and Ahn (1995) to derive the asymptotic distribution of the test statistic. It can be nevertheless shown that the statistic proposed in BLS has the same distribution as the statistic proposed in CS. For the case of multiple breaks, the asymptotic distribution depends on the number of regressors (K), the deterministic model considered ($g_i(t)$), the number of breaks (m) and their location in the sample (λ_j).

3 The Design of Monte Carlo Experiments

3.1 Data Generating Process

In this Section we describe the design of Monte Carlo experiments used to study the finite sample properties (size and power) of the statistical tests discussed in Section 2. For this purpose, we simulate 20,000 series of dimension $T = \{100, 200\}$ using the following triangular system representation of the DGP (Gregory and Hansen, 1996; Haug, 1996; McCabe et al., 1997; Carrion-i-Silvestre and Sansò, 2006):

$$y_t = \alpha_0 + g_i(t) + \beta x_t + e_t \tag{12}$$

$$e_t = \rho e_{t-1} + \varepsilon_t \tag{13}$$

$$\varepsilon_t = \phi \varepsilon_{t-1} + u_t - \gamma u_{t-1} \tag{14}$$

$$\alpha_1 y_t - \alpha_2 x_t = w_t \tag{15}$$

$$w_t = w_{t-1} + \mu_t \tag{16}$$

where $g_i(t)$, for $i = \{\text{A, B, C}\}$, is the deterministic function as defined in (2). The error-correction term (e_t) is assumed to be autocorrelated with coefficient $|\rho| \leq 1$, depending on the null hypothesis involved by the selected statistical test. We account for potential misspecification of residuals by allowing the error term ε_t to follow an ARMA(1,1) process, with AR parameter ϕ and MA parameter γ. Simple AR(1) and MA(1) processes can be simulated by setting either $\gamma = 0$ or $\phi = 0$, respectively. Finally, μ_t is the vector of innovations. The system also accounts for endogenous ($\alpha_1 = 1$) or exogenous ($\alpha_1 = 0$) regressors x_t.
In this general specification, u_t and μ_t are i.i.d. with distribution:

$$\begin{pmatrix} u_t \\ \mu_t \end{pmatrix} \sim \text{i.i.d.}N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \delta\sigma_\mu \\ \delta\sigma_\mu & \sigma_\mu^2 \end{pmatrix} \right],$$

where δ controls for the correlation between u_t and μ_t. To avoid data dependence on initial conditions, the actual Monte Carlo sample dimension is $T_{\text{MC}} = T + T_0$, where $T_0 = 100$ is the number of initial observations to be discarded.

To compare the size and power performance of the tests discussed in Section 2, we consider a reasonable and computationally feasible number of breaks m. We then provide simulation results for $m = \{1, 3, 5\}$.

3.2 Parameter Space

We consider two sets of parameter space, a first one common to all simulations and a second one dependent on each specific Monte Carlo exercise.

In the first set, we consider the parameter space $(\alpha_0, \tau, \beta, \alpha_1, \alpha_2, \rho, \sigma_\mu^2, \delta, \phi, \gamma)$, where $\alpha_0 = 1$, $\tau = \{0, 0.2\}$, $\beta = 1$, $\alpha_1 = \{0, 1\}$, $\alpha_2 = -1$, $\rho = \{0, 0.1, 0.9, 1\}$, $\sigma_\mu^2 = \{0.5, 1, 2\}$, $\delta = \{0, 0.5\}$, $\phi = \{0, 0.4\}$ and $\gamma = \{0, 0.4\}$.

In the second set, we consider the parameter space $(\theta_1, \theta_2, m, \lambda)$. For each Model $i = \{\text{A, B, C}\}$, the value of these parameters is defined as follows:

- $m = 1$, $\lambda = 50\%$, $\theta_1 = 0.5$, $\theta_2 = \{0, 0.2\}$.
- $m = 3$, $\lambda = (30\%, 50\%, 70\%)$, $\theta_1 = (0.5, -0.8, 0.5)$, $\theta_2 = \{(0, 0, 0), (0.2, -0.5, 0.2)\}$.
- $m = 5$, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $\theta_1 = (0.5, -0.8, 0.5, -0.2, 0.5)$, $\theta_2 = \{(0, 0, 0, 0, 0), (0.2, -0.5, 0.2, -0.3, 0.4)\}$.

3.3 Long-run Variance Estimator

Some of the statistical tests reported in this paper require a consistent estimate of the long-run variance (ω_{11}) of cointegration residuals. For this purpose, Andrews (1991) and Andrews and Monahan (1992) recommend the use of the HAC estimator involving a Pre-Whitened Quadratic-Spectral kernel and an automatic data-dependent rule for the selection of the bandwidth parameter. Nevertheless, recent literature points out that a potential size distortion affecting statistical tests may arise from the small sample bias of pre-whitening coefficients (Kurozumi, 2002; Phillips and Sul, 2003; Sul et al., 2005).

To avoid finite sample inconsistency problems, we report experimental results involving the modified bandwidth selection rules recently proposed in Kurozumi (2002). This is mainly the standard Bartlett kernel function:

$$w(x) = \begin{cases} 1 - \frac{j}{M} & \text{if } \frac{j}{M} \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

with the bandwidth parameter M chosen following a modified automatic rule:

$$\tilde{M} = \min\left(1.1447\left\{\frac{4\hat{\rho}^2 T}{(1+\hat{\rho})^2(1-\hat{\rho})^2}\right\}^{1/3}, 1.1447\left\{\frac{4k^2 T}{(1+k)^2(1-k)^2}\right\}^{1/3}\right),$$

where $\hat{\rho}$ is the estimated AR(1) coefficient of $\hat{e}_t$, the estimated cointegration residual. The rule proposed in Kurozumi (2002) sets a boundary condition to the bandwidth parameter which depends on the predetermined value of k. Simulations show that $k = \{0.7, 0.8, 0.9\}$ is the best range of values for the power-size trade-off of the test. In this paper we follow CS and we fix $k = 0.8$.

4 Simulation Results

4.1 Asymptotic Densities

Figures 1 to 3 report asymptotic densities for CA, BLS and WE statistics under the breaks scenarios described in Section 3.2. CA and BLS densities are plotted together because, as expected by the theory, these test statistics show the same asymptotic distribution.

Many interesting features arise. First, all figures highlight the symmetry of distributions around the median break ($\lambda = 50\%$). This leads to distributions with fatter right tails for CA and BLS test statistics when breaks (in level and trend) take place asymmetrically around the middle-point of the sample. This feature is mainly displayed for Model A and C in Figure 1, while for Model B (multiple level breaks with trend) asymptotic densities appear mostly unaffected by the number and location of breaks in the sample. Second, looking at Figures 2 and 3, we do observe two key features of the WE test statistics. The first one, is the invariance of their asymptotic distribution, independently on the number and location of level breaks (Model A and B). This is consistent with the theoretical results presented by WE. However, this condition does not hold for Model C. In fact, when the DGP presents both level and trend breaks, asymptotic densities differ across simulations by the number and location of breaks. In addition, the symmetry of distributions around the median break arise again (as for the CS and BLS cases), but with a shift in the positive direction of the distribution as far as the breaks are distributed asymmetrically in the sample. This feature then leads to different asymptotic critical values for Model C, depending on the number and location of breaks.

4.2 Empirical Size

We report in Tables 1 to 6 rejection frequencies at 5% nominal confidence level. The null hypothesis is cointegration for CS and BLS tests and non-cointegration for WE tests. Results are based on a single endogenous regressor x_t (*i.e.*, $K = 1$ and $\alpha_1 = 1$ in Equation (15)). In Figures 4 to 6 we also report p-value plots of the empirical size of tests (Davidson and MacKinnon, 1998) for the case that x_t is endogenous (*i.e.*, $\alpha_1 = 1$ and $\delta = 0.5$) and strictly exogenous (*i.e.*, $\alpha_1 = 0$ and $\delta = 0$). For reasons of space. we only report graphical results for $T = 100$, $\phi = \gamma = 0$ and $\sigma^2_\mu = 2$.

Asymptotic critical values are computed by simulating $40,000$ series of dimension $T_\infty = 5,000$ and picking up the 95$^{\text{th}}$ percentile of the asymptotic distribution for CS and BLS tests and the 5$^{\text{th}}$ percentile for WE tests.

4.2.1 One break ($m = 1$)

Results from the single break case are reported in Tables 1 ($T = 100$) and 2 ($T = 200$).

For $\phi = \gamma = 0$, we do not observe strong size distortions for all tests and Models, except for some persistent under-rejection for the CS_{FM} and BLS_{CCR} tests when $\delta = 0.5$. As expected, tests display larger bias for lower signal-to-noise ratios. For large σ^2_μ, CS_{DOLS} and CS_{DGLS} tests show the strongest improvement in terms of rejection rates. When residuals are specified as an AR(1) process ($\phi \neq 0$), CS and BLS tests show the highest rates of rejection in all models. In particular, the CS_{DGLS} test shows the strongest over-size (between 15% and 40%) in Model A and C when σ^2_μ is low. However, the displayed high rejection rate (or the discrepancy between results for the CS_{DGLS} and the other tests) is reduced in larger samples (Table 2). On the other hand, the WE_Φ test is affected by a persistent under-rejection bias, which seems to exacerbate in larger samples. For the case of MA(1) residuals ($\gamma \neq 0$), actual size generally improves with respect to the AR(1) specification. However, CS_{DOLS} and CS_{DGLS} tests are affected by some under-rejection with large signal/noise ratios, while both $WE_{t\text{-stat}}$ and WE_Φ tests tend to over-reject instead.

P-value plots in Figure 4 show that actual rejection frequencies are very close to the nominal size when the regressor is exogenous. In Model C, however, CS and BLS tests tend to substantially over-reject the null hypothesis (Figure 4e). Strong differences with the case that

x_t is endogenous can be found in Model C, where the over-rejection bias exacerbates for CS_{FM} and BLS_{CCR} tests.

4.2.2 Three breaks $(m = 3)$

Results from the three breaks case are reported in Tables 3 ($T = 100$) and 4 ($T = 200$). Simulations suggest that the inclusion of more breaks can significantly alter the size performance of tests. In particular, tests based on non-parametric endogeneity-bias corrections (CS_{FM} and BLS_{CCR}) display very large over-size when testing for cointegration in Model C.

As for the single break case, for $\phi = \gamma = 0$ we do not observe strong size distortions for all tests and Models. However, strong bias is displayed by CS_{DOLS}, CS_{FM} and BLS_{CCR} tests for Model C. In this case, the use of CS_{DGLS} and WE tests is recommended. When residuals are AR(1) ($\phi \neq 0$), best results are obtained by CS_{FM} and BLS_{CCR} tests in Model A and B, while the use of CS_{DOLS} and CS_{DGLS} tests is somewhat more recommended for Model C. Nevertheless, results for larger samples (Table 4) display similar rejection rates across all CS and BLS tests, in particular for higher signal-to-noise ratios. On the other hand, the $WE_{t\text{-stat}}$ test is high performant across Models and specifications. When residuals are MA(1) ($\gamma \neq 0$), CS_{DOLS} and CS_{DGLS} tests are generally well-sized in all Models, along with the WE tests.

P-value plots in Figure 5 highlight again the poor size performance of CS_{FM} and BLS_{CCR} when the regressor is endogenous and the DGP presents a broken trend (Figure 4f). However, a large oversize can be detected in Model C even when the regressor is exogenous (Figure 4e). In this case, CS_{DOLS}, CS_{FM} and BLS_{CCR} tests show the worst size distortion. When compared to the endogenous case, we nevertheless observe an improvement in terms of p-values for the CS_{DOLS} test, while the performance of CS_{FM} and BLS_{CCR} tests strongly deteriorates.

4.2.3 Five breaks $(m = 5)$

Results from the five breaks case are reported in Tables 5 ($T = 100$) and 6 ($T = 200$). Simulations remove any doubt about the evidence already reported above: the larger the number of breaks assumed in the DGP of the cointegrating process, the stronger the size bias affecting the tests under analysis. An exception arise again for the WE tests, for which the inclusion of multiple breaks does not seem to affect their finite sample performance overall. For $\phi = \gamma = 0$, the smallest over-rejection rates can be found for high signal-to-noise ratios in Model A and B. This is not the case in Model C, where CS and BLS tests perform very badly, in particular the CS_{FM} and BLS_{CCR} tests. However, strong size improvements can be obtained for larger samples (see Table 6). In addition, it is worth noticing that the empirical size of $WE_{t\text{-stat}}$ and WE_{Φ} lies between 5% and 10% in all Models. When residuals are AR(1) ($\phi \neq 0$), the smallest size distortions are instead reported for CS_{DOLS}, CS_{FM} and BLS_{CCR} statistics in Model A and B, mainly when $\delta > 0$. However, for small samples, these tests show very high over-rejection rates, which are exacerbated in Model C. When residuals are MA(1) ($\gamma \neq 0$), the use of CS_{DOLS} and CS_{DGLS}, along with the WE tests, is strongly recommended in all Models when T is low, although the reported evidence of some under-rejection. However, as highlighted in Table 6, CS_{FM} and BLS_{CCR} tests display strong size improvements in Model A and B when a larger sample is considered, while they show huge over-rejection in Model C for all considered sample sizes.

The p-value analysis (Figure 6) confirms the results discussed above. It is interesting to note that, as already observed in the 3 breaks case, the discrepancy arising from specifications involving either exogenous or endogenous regressors tends to widen with the number of breaks. However, over-rejection is high overall, whether the regressor is exogenous or not. In particular, Model C shows the strongest bias in terms of p-value rejection probabilities. An interesting feature is the diverging behaviour of CS_{DOLS}, CS_{FM} and BLS_{CCR} tests observed in the endogenous regressor specification: when compared to the case with exogenous regressors, for the first one the actual size improves, while for the last two tests it strongly deteriorates.

4.3 Empirical Power

We report in Tables 7 to 12 size-adjusted rejection frequencies at 5% actual confidence level. The alternative hypothesis is non-cointegration for CS and BLS tests and cointegration for WE tests. Critical values are computed by picking up the 95^{th} percentile from the actual distribution of CS and BLS tests and the 5^{th} percentile from the actual distribution of WE tests. For reasons of space, we only report power analysis for the case of correct specification of residuals ($\gamma = \phi = 0$). In Figures 7 to 9 we report power-size curves (Davidson and MacKinnon, 1998) for the case that x_t is endogenous (*i.e.*, $\alpha_1 = 1$ and $\delta = 0.5$) and strictly exogenous (*i.e.*,

$\alpha_1 = 0$ and $\delta = 0$).[5] For this exercise, we use again the following parameter space: $T = 100$, $\phi = \gamma = 0$ and $\sigma_\mu^2 = 2$.

4.3.1 One break ($m = 1$)

Results from the single break case are reported in Tables 7 ($T = 100$) and 8 ($T = 200$). Under the alternative hypothesis, CS and BLS tests show a quite high power in Model A and C. In particular, highest rejection rates are displayed by the CS_{DGLS} test, lying between 40% and 65% and growing with higher signal-to-noise ratios. Largest rejection rates in Model B are instead displayed by CS_{DOLS} and CS_{FM} tests. In addition, the former shows rejection rates decreasing faster than in other tests when we move away from the alternative hypothesis of non-cointegration. For larger samples, all tests display similar rejection power, although the CS_{DGLS} test still shows a slight better performance in Model A and C. A very important result is the serious low power across models and simulations for the WE tests. Rejection rates are overall close to the nominal size (and even their empirical size), which makes these tests unable to reject the alternative hypothesis of cointegration. A larger sample size does not seem to improve these results.

Size-power curves in Figure 7 show that the latter result is mainly driven by the endogeneity of regressors. When the regressor is strictly exogenous (Figure 7a, 7c and 7e), the WE_Φ test displays the highest power against the alternative hypothesis, while the $WE_{t\text{-stat}}$ is quite less performant above the 10% nominal size. However, the endogeneity of regressors dramatically alter their power (Figure 7b, 7d and 7f), while CS and BLS tests appear mostly unaffected.

4.3.2 Three breaks ($m = 3$)

Results from the three breaks case are reported in Tables 9 ($T = 100$) and 10 ($T = 200$). Results are somewhat different with respect to the single break case. The highest rejection rates in Model A and B are displayed by the CS_{DGLS} test, while in Model C the CS_{DOLS} test shows a slightly better power performance. However, rejection frequencies reported in Table 10 tend to be similar across tests and Models, except for the CS_{DOLS} test in Model A and B. Improved rejection power can be overall observed for higher signal-to-noise ratios and non-zero

[5] It is worth noticing that results reported in Tables 7 to 12 are size-adjusted rejection frequencies, while p-value curves in Figures 7 to 9 plot power against nominal size.

correlation between innovations ($\delta \neq 0$). It is worth noticing that the more the number of breaks in the cointegrating model, the larger the size-adjusted power of tests. This is at odds with the evidence reported for the actual size of tests. However, this finding doesn't hold for WE tests, which still display rejection rates close to the nominal size.

Size-power curves in Figure 8 reveal that, with strictly exogenous regressors (Figure 8a, 8c and 8e), the CS_{DGLS} test displays the highest power against the alternative hypothesis in Model A, while all tests show similar power in Model B and C, except for the $WE_{t\text{-stat}}$ test. When the regressor is endogenous (Figure 8b, 8d and 8f), WE tests, however, lack power. Size-power plots confirm results reported in Table 9, *i.e.*, multiple breaks appear to improve the overall power of CS and BLS tests when compared to the single break case.

4.3.3 Five breaks ($m = 5$)

Finally, results from the five breaks case are reported in Tables 11 ($T = 100$) and 12 ($T = 200$). As for the three breaks case, highest rejection rates in Model A and B are displayed by the CS_{DGLS} test, while in Model C the CS_{DOLS} is somewhat more performant. It is worth noticing that the CS_{FM} displays very low rejection rates in Model C when $\delta \neq 0$. However, as shown in Table 12, this high power distortion is partially absorbed in larger samples. Finally, WE tests show serious lack of power.

Size-power curves in Figure 9 reveal that, with strictly exogenous regressors (Figure 9a, 9c and 9e), all tests, except for the $WE_{t\text{-stat}}$ test, display high power against the alternative hypothesis in Model A, B and C. However, when the regressor is endogenous (Figure 9b, 9d and 9f), CS and BLS tests still display very high power, while WE tests show severe power distortions.

5 Concluding Remarks

In this paper we compare the size-power performance of residual-based tests for cointegration with structural breaks. In particular, we focus on statistical tests recently proposed in the literature by Bartley, Lee, and Strazicich (2001), Carrion-i-Silvestre and Sansò (2006) and Westerlund and Edgerton (2007). Through an extensive Monte Carlo study, we evaluate their performance in small samples when up to five (exogenous) deterministic breaks are included in the cointegrating equation. We consider several efficient estimators of single-equation cointegrating relationships (OLS, DOLS, DGLS, FM-OLS, CCR) and we design simulations to take into account for three deterministic breaks scenarios (breaks in constant, with and without trend, and breaks in both constant and trend), endogenous regressors and residuals misspecifications.

Results on the empirical size reveal many interesting features. First, the $WE_{t\text{-stat}}$ and WE_{Φ} tests show quite low size distortions across Models and break scenarios. Findings reported in this study strongly recommend the use of these tests when estimates of cointegrating relationships are conducted through the Engle-Granger OLS regression, *i.e.*, when potential endogeneity bias is *ex ante* ruled out by the researcher. Second, multiple breaks tend to severely deteriorate the size performance of the other tests under analysis. This finding appears even stronger in Model C (level and trend breaks). Nevertheless, results for CS and BLS tests appear overall mixed and can be briefly resumed in what follows.

For the single break case, when residuals are well-specified, CS_{DOLS} and CS_{DGLS} perform best in all Models. However, the CS_{FM} and BLS_{CCR} tests show a slight lower size distortion in Model C when residuals are misspecified. For the three breaks case, under white noise residuals, we recommend the use of the CS_{DGLS} test in Model C. When residuals are misspecified, CS_{FM} and BLS_{CCR} tests perform best in Model A and B, while we recommend the use CS_{DOLS} and CS_{DGLS} for Model C. For the five breaks case, we report large size distortions overall. Similar performances are found out across CS_{DOLS}, CS_{FM} and BLS_{CCR} tests in Model A and B, while the CS_{DGLS} test shows smaller (but still high) size distortions in Model C. With a sample size of $T = 100$ used in simulations, CS_{FM} and BLS_{CCR} tests display impressive size distortions in Model C. We then strongly advice against the use of these estimator/test pairs in a framework involving more then three level and trend breaks and less then 200 observations.

Despite the presence of strong size distortions, simulation results on the empirical (size-adjusted) power reveal that (under white noise residuals) CS and BLS tests have quite high power against the alternative hypothesis across all simulations and Models. In particular, the CS_{DGLS} displays overall best power performance in Model A and B, while the CS_{DOLS} test shows highest rejection rates in Model C. The severe lack of power of WE tests when regressors are endogenous (confirmed by size-power curves) should motivate their application for weak exogenous regression models only.

All in all, our results provide an important guideline for applied works involving cointegrating models and multiple deterministic structural breaks. Unless the researcher deals with weakly exogenous regressors, in which case the SP-type LM tests proposed in Westerlund and Edgerton (2007) show impressive size and power performances, the KPSS-type LM tests proposed by Carrion-i-Silvestre and Sansò (2006) based on DGLS and DOLS estimators should be used instead. This implies that the researcher should carefully select *ex ante* the estimator of cointegrating relationships leading, *ex post*, the most reliable test results.

References

Ahn, S. K., 1993. Some tests for unit roots in autoregressive-integrated-moving average models with deterministic trends. Biometrika 80 (4), 855–868.

Andrews, D. W. K., 1991. Heteroskedasticity and autocorrelation consistent covariance matrix estimation. Econometrica 59 (3), 817–858.

Andrews, D. W. K., Monahan, J. C., 1992. An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator. Econometrica 60 (4), 953–966.

Bai, J., Perron, P., 1998. Estimating and testing linear models with multiple structural changes. Econometrica 66 (1), 47–78.

Bai, J., Perron, P., 2003. Computation and analysis of multiple structural-change models. Journal of Applied Econometrics 18 (1), 1–22.

Bartley, W. A., Lee, J., Strazicich, M. C., 2001. Testing the null of cointegration in the presence of a structural break. Economics Letters 73 (3), 315–323.

Campbell, J., Perron, P., 1991. Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots. In: Blanchard, O. and Fisher, S. (eds.), NBER Macroeconomics Annual. MIT Press: Cambridge, MA (USA).

Carrion-i-Silvestre, J. L., Kim, D., Perron, P., 2009. GLS-based unit root tests with multiple structural breaks both under the null and the alternative hypothesis. Econometric Theory 25 (6), 1754–1792.

Carrion-i-Silvestre, J. L., Sansò, A., 2006. Testing the null of cointegration with structural breaks. Oxford Bulletin of Economics and Statistics 68 (5), 623–646.

Cheung, Y., Lai, K. S., 1993. Long-run purchasing power parity during the recent float. Journal of International Economics 34 (1-2), 181–192.

Choi, I., Ahn, B. C., 1995. Testing for cointegration in a system of equations. Econometric Theory 11 (5), 952–983.

Davidson, R., MacKinnon, J. G., 1998. Graphical methods for investigating the size and power of hypothesis tests. The Manchester School of Economic & Social Studies 66 (1), 1–26.

De Peretti, C., Urga, G., 2004. Stopping tests in the sequential estimation of multiple structural breaks. Econometric Society 2004 Latin American Meetings No. 320, Econometric Society.

Engle, R. F., Granger, C. W. J., 1987. Cointegration and error correction: Representation, estimation and testing. Econometrica 55 (2), 251–276.

Granger, C. W. J., Lee, T. H., 1990. Multicointegration. Advances in Econometrics 8, 71–84.

Gregory, A. W., Hansen, B. E., 1996. Residual-based tests for cointegration in models with regime shifts. Journal of Econometrics 70 (1), 99–126.

Hansen, B. E., 1992. Tests for parameter instability in regressions with I(1) processes. Journal of Business & Economic Statistics 10 (3), 321–335.

Hao, K., 1996. Testing for structural change in cointegrated regression models: Some comparisons and generalizations. Econometric Reviews 15 (4), 401–429.

Harvey, D. I., Leybourne, S. J., Taylor, A. M. R., 2009a. Robust methods for detecting multiple level breaks in autocorrelated time series. Granger Centre Discussion Paper No. 09/01, University of Nottingham (UK).

Harvey, D. I., Leybourne, S. J., Taylor, A. M. R., 2009b. Simple, robust and powerful tests of the breaking trend hypothesis. Econometric Theory 25 (4), 995–1029.

Haug, A. A., 1996. Tests for cointegration: a Monte Carlo comparison. Journal of Econometrics 71 (1-2), 89–115.

Hendry, D. F., Ericsson, N. R., 1991. An econometric analysis of the UK money demand in "Monetary trends in the United States and the United Kingdom" by Milton Friedman and Anna J. Schwartz. American Economic Review 81 (1), 8–38.

Herrndorf, N., 1984. A functional central limit theorem for weakly dependent sequences of random variables. Annals of Probability 12 (1), 141–153.

Kejriwal, M., Lopez, C., 2010. Unit roots, level shifts and trend breaks in per capita output: A robust evaluation. Economics Working Papers No. 2010-02, University of Cincinnati (USA).

Kejriwal, M., Perron, P., 2008. Data dependent rules for selection of the number of leads and lags in the dynamic OLS cointegrating regression. Econometric Theory 24 (5), 1425–1441.

Kejriwal, M., Perron, P., 2009a. A sequential procedure to determine the number of breaks in trend with an integrated or stationary noise component. Economics Working Papers No. 1217, Purdue University (USA).

Kejriwal, M., Perron, P., 2009b. Testing for multiple structural changes in cointegrated regression models. Journal of Business & Economic Statistics (forthcoming).

King, R. G., Plosser, C. I., Stock, J. H., Watson, M. W., 1991. Stochastic trends and economic fluctuations. American Economic Review 81 (4), 819–840.

Kurozumi, E., 2002. Testing for stationarity with a break. Journal of Econometrics 108 (1), 63–99.

Kwiatkowski, D., Phillips, P., Schmidt, P., Shin, Y., 1992. Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? Journal of Econometrics 54 (1-3), 159–178.

Lee, J., 1999. Stationarity Tests with Multiple Endogenized Breaks. In: Rothman, P. (ed.), Nonlinear Time Series Analysis of Economic and Financial Data. Kluwer Academic Publishers.

McCabe, B. P. M., Leybourne, S. J., Shin, Y., 1997. A parametric approach to testing the null of cointegration. Journal of Time Series Analysis 18 (4), 395–413.

Mogliani, M., Urga, G., Winograd, C., 2009. Monetary disorder and financial regimes: the demand for money in Argentina, 1900-2006. PSE Working Papers No. 2009-55, Paris School of Economics (France).

Ng, S., Perron, P., 1995. Unit root tests in ARMA models with data-dependent methods for the selection of the truncation lag. Journal of the American Statistical Association 90, 269–281.

Park, J. Y., 1992. Canonical cointegrating regressions. Econometrica 60 (1), 119–143.

Perron, P., 1989. The great crash, the oil price shock and the unit root hypothesis. Econometrica 57 (6), 1361–1401.

Perron, P., 1990. Testing for a unit root in a time series with a changing mean. Journal of Business & Economic Statistics 8 (2), 153–162.

Perron, P., Yabu, T., 2009. Testing for shifts in trend with an integrated or stationary noise component. Journal of Business & Economic Statistics 27 (3), 369–396.

Perron, P., Zhu, X., 2005. Structural breaks with deterministic and stochastic trends. Journal of Econometrics 129 (1-2), 65–119.

Phillips, P. C. B., Durlauf, S. N., 1986. Multiple time series regression with integrated processes. Review of Economic Studies 53 (4), 473–495.

Phillips, P. C. B., Hansen, B. E., 1990. Statistical inference in instrumental variables regression with I(1) processes. Review of Economic Studies 57 (1), 99–125.

Phillips, P. C. B., Ouliaris, S., 1990. Asymptotic properties of residual based tests for cointegration. Econometrica 58 (1), 165–193.

Phillips, P. C. B., Sul, D., 2003. Dynamic panel estimation and homogeneity testing under cross section dependence. The Econometrics Journal 6 (1), 217–260.

Qu, Z., 2007. Searching for cointegration in a dynamic system. Econometrics Journal 10 (3), 580–604.

Saikkonen, P., 1991. Asymptotically efficient estimation of cointegration regressions. Econometric Theory 7 (1), 1–21.

Schmidt, P., Phillips, P. C. B., 1992. LM tests for a unit root in the presence of deterministic trends. Oxford Bulletin of Economics and Statistics 54 (3), 257–287.

Shin, Y., 1994. A residual-based test of the null of cointegration against the alternative of non-cointegration. Econometric Theory 10 (1), 91–115.

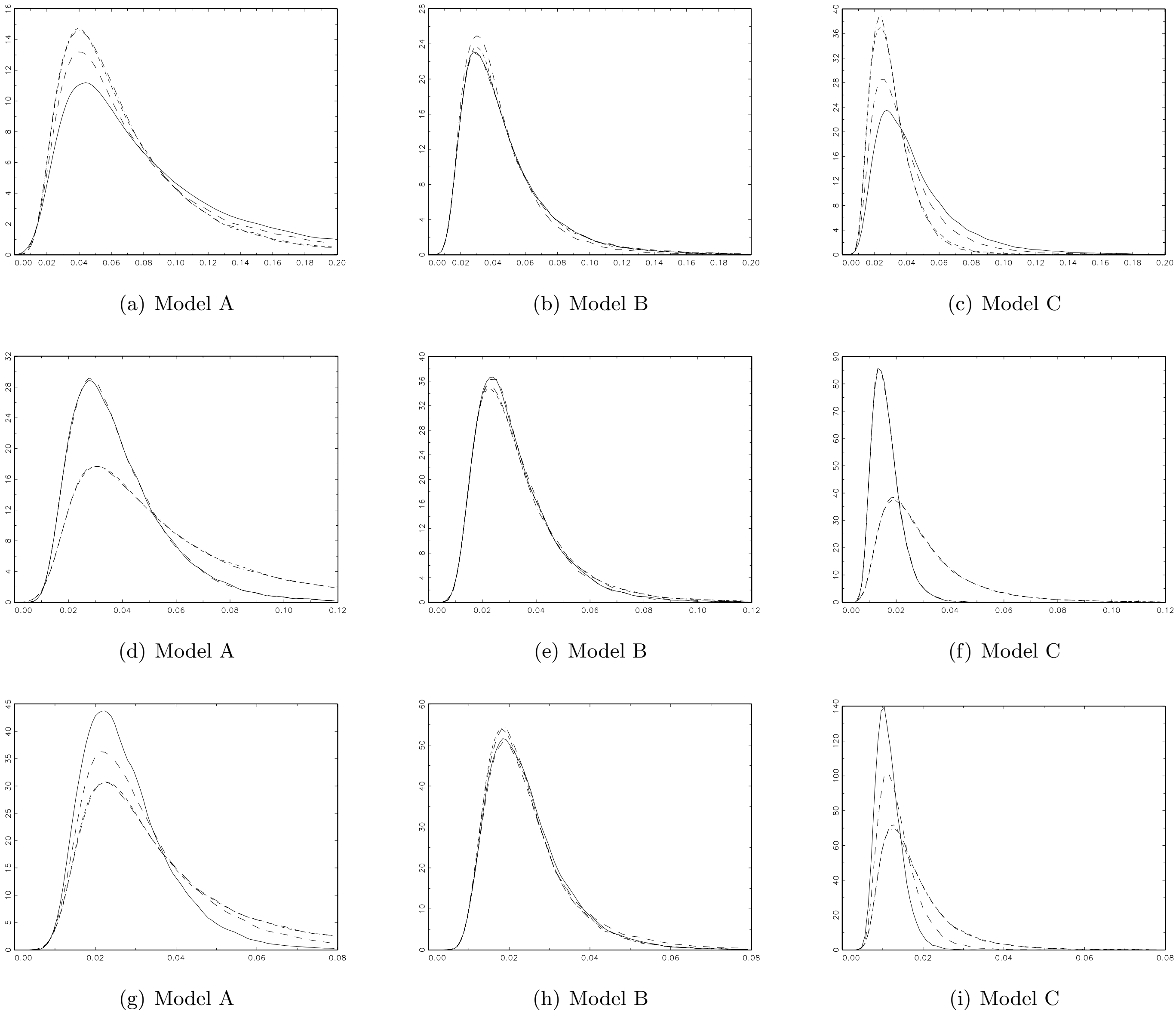
Stock, J. H., Watson, M. W., 1993. A simple estimator of cointegrating vectors in higher order integrated systems. Econometrica 61 (4), 783–820.

Sul, D., Phillips, P. C. B., Choi, C. Y., 2005. Prewhitening bias in HAC estimation. Oxford Bulletin of Economics and Statistics 67 (4), 517–546.

Taylor, M. P., McMahon, P. C., 1988. Long-run purchasing power parity in the 1920s. European Economic Review 32 (1), 179–197.

Westerlund, J., Edgerton, D. L., 2007. New improved tests for cointegration with structural breaks. Journal of Time Series Analysis 28 (2), 188–224.

Figure 1: Asymptotic Densities of CS and BLS Statistics.



(a) Model A (b) Model B (c) Model C

(d) Model A (e) Model B (f) Model C

(g) Model A (h) Model B (i) Model C

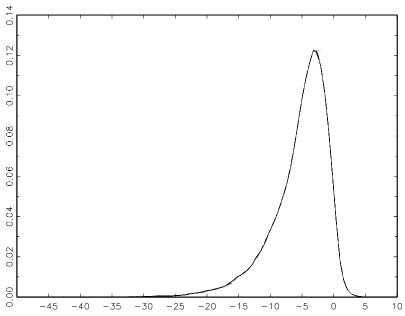
Notes: Kernel densities are obtained by simulating 40,000 series of dimension $T_{\infty} = 5,000$

Panels (a), (b) and (c) are the 1 break model. Solid line: $\lambda = 10\%$. Dashed line: $\lambda = 20\%$. Short dashed line: $\lambda = 40\%$. Dotted and dashed line: $\lambda = 50\%$.

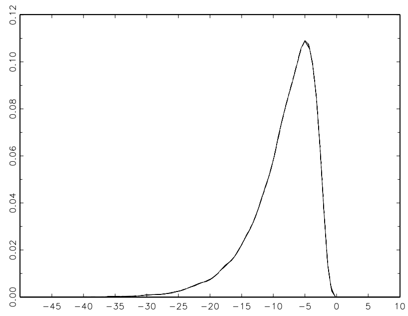
Panels (d), (e) and (f) are the 3 breaks model. Solid line: $\lambda = \{30\%, 50\%, 70\%\}$. Dashed line: $\lambda = \{20\%, 50\%, 80\%\}$. Short dashed line: $\lambda = \{10\%, 20\%, 30\%\}$. Dotted and dashed line: $\lambda = \{70\%, 80\%, 90\%\}$.

Panels (g), (h) and (i) are the 5 breaks model. Solid line: $\lambda = \{20\%, 30\%, 50\%, 70\%, 80\%\}$. Dashed line: $\lambda = \{30\%, 40\%, 50\%, 60\%, 70\%\}$. Short dashed line: $\lambda = \{10\%, 20\%, 30\%, 40\%, 50\%\}$. Dotted and dashed line: $\lambda = \{50\%, 60\%, 70\%, 80\%, 90\%\}$.

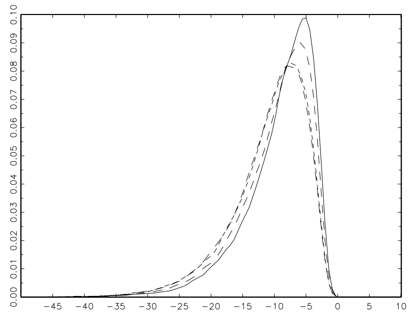
Figure 2: Asymptotic Densities of WE_{Φ} Statistic.



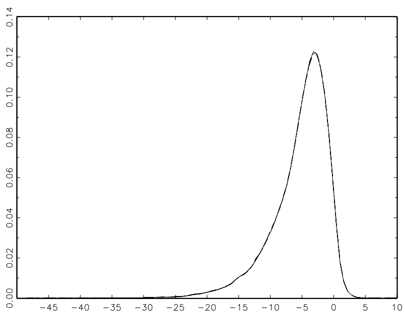
(a) Model A



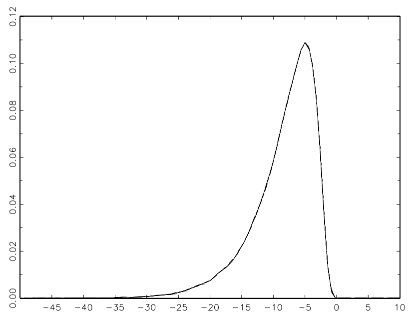
(b) Model B



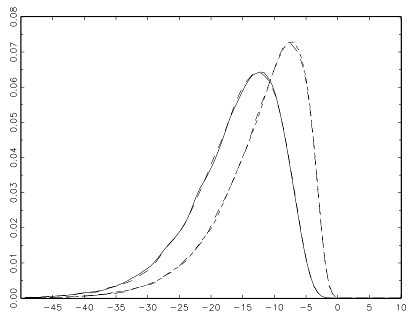
(c) Model C



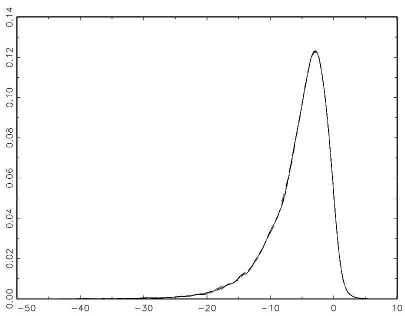
(d) Model A



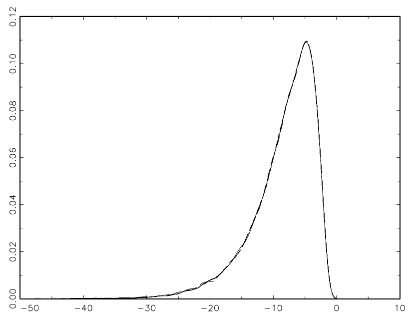
(e) Model B



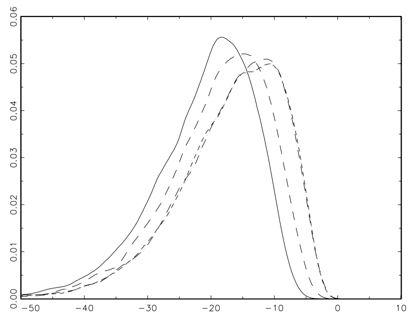
(f) Model C



(g) Model A



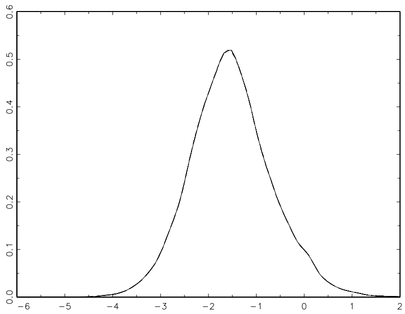
(h) Model B



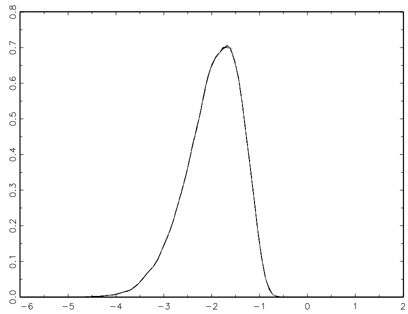
(i) Model C

Notes: Kernel densities are obtained by simulating 40,000 series of dimension $T_{\infty} = 2,000$
Panels (a), (b) and (c) are the 1 break model. Solid line: $\lambda = 10\%$. Dashed line: $\lambda = 20\%$. Short dashed line: $\lambda = 40\%$. Dotted and dashed line: $\lambda = 50\%$.
Panels (d), (e) and (f) are the 3 breaks model. Solid line: $\lambda = \{30\%, 50\%, 70\%\}$. Dashed line: $\lambda = \{20\%, 50\%, 80\%\}$. Short dashed line: $\lambda = \{10\%, 20\%, 30\%\}$. Dotted and dashed line: $\lambda = \{70\%, 80\%, 90\%\}$.
Panels (g), (h) and (i) are the 5 breaks model. Solid line: $\lambda = \{20\%, 30\%, 50\%, 70\%, 80\%\}$. Dashed line: $\lambda = \{30\%, 40\%, 50\%, 60\%, 70\%\}$. Short dashed line: $\lambda = \{10\%, 20\%, 30\%, 40\%, 50\%\}$. Dotted and dashed line: $\lambda = \{50\%, 60\%, 70\%, 80\%, 90\%\}$.

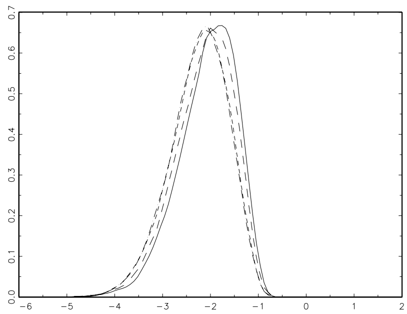
Figure 3: Asymptotic Densities of $WE_{t\text{-stat}}$ Statistic.



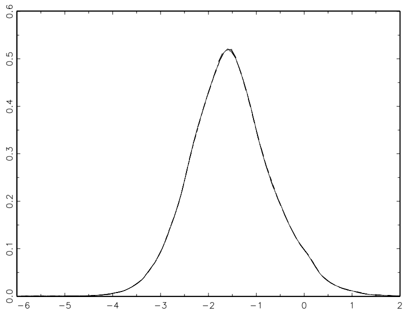
(a) Model A



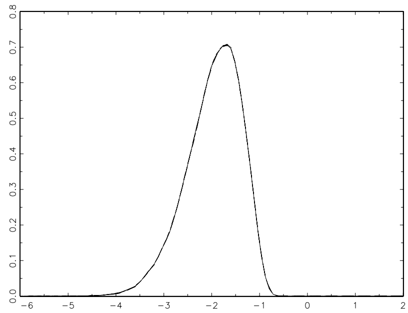
(b) Model B



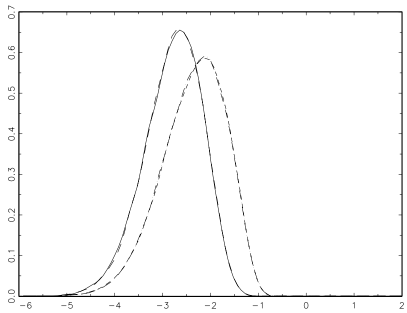
(c) Model C



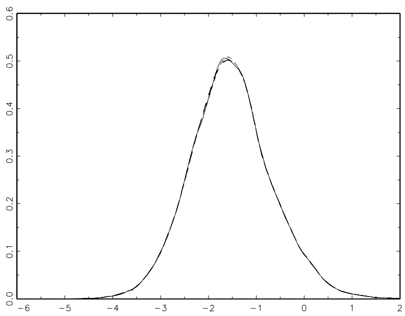
(d) Model A



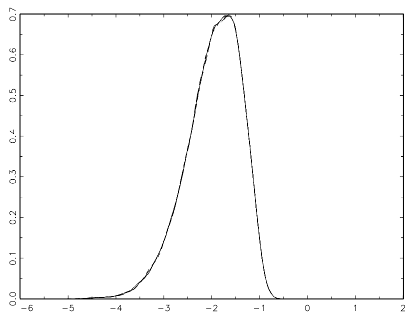
(e) Model B



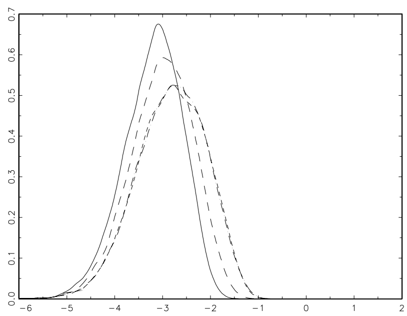
(f) Model C



(g) Model A



(h) Model B

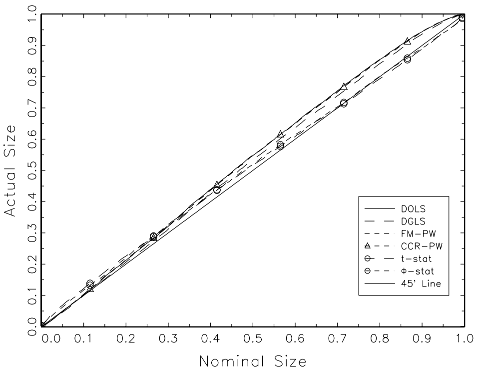


(i) Model C

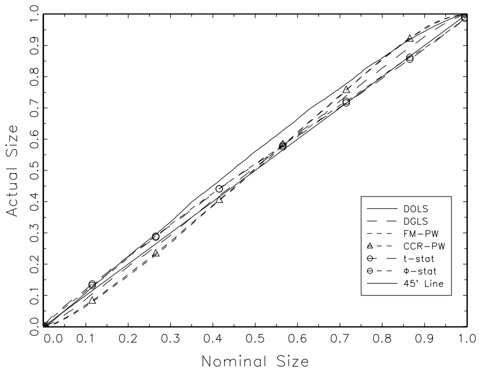
Notes: Kernel densities are obtained by simulating 40,000 series of dimension $T_\infty = 2,000$
Panels (a), (b) and (c) are the 1 break model. Solid line: $\lambda = 10\%$. Dashed line: $\lambda = 20\%$. Short dashed line: $\lambda = 40\%$. Dotted and dashed line: $\lambda = 50\%$.
Panels (d), (e) and (f) are the 3 breaks model. Solid line: $\lambda = \{30\%, 50\%, 70\%\}$. Dashed line: $\lambda = \{20\%, 50\%, 80\%\}$. Short dashed line: $\lambda = \{10\%, 20\%, 30\%\}$. Dotted and dashed line: $\lambda = \{70\%, 80\%, 90\%\}$.
Panels (g), (h) and (i) are the 5 breaks model. Solid line: $\lambda = \{20\%, 30\%, 50\%, 70\%, 80\%\}$. Dashed line: $\lambda = \{30\%, 40\%, 50\%, 60\%, 70\%\}$. Short dashed line: $\lambda = \{10\%, 20\%, 30\%, 40\%, 50\%\}$. Dotted and dashed line: $\lambda = \{50\%, 60\%, 70\%, 80\%, 90\%\}$.

Figure 4: P-value Plots: 1 break

MODEL A

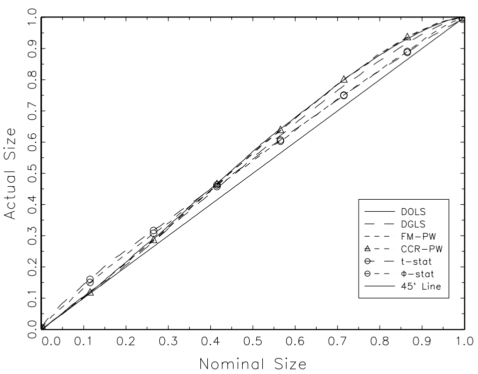


(a) $\alpha_1 = 0$, $\delta = 0$

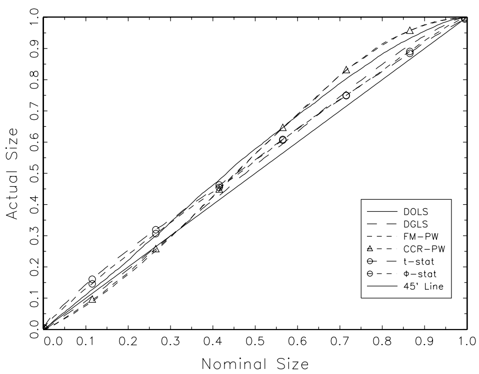


(b) $\alpha_1 = 1$, $\delta = 0.5$

MODEL B

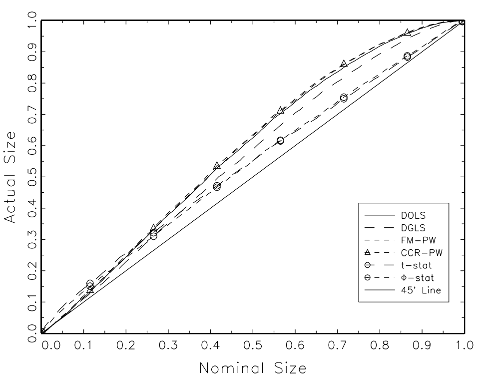


(c) $\alpha_1 = 0$, $\delta = 0$

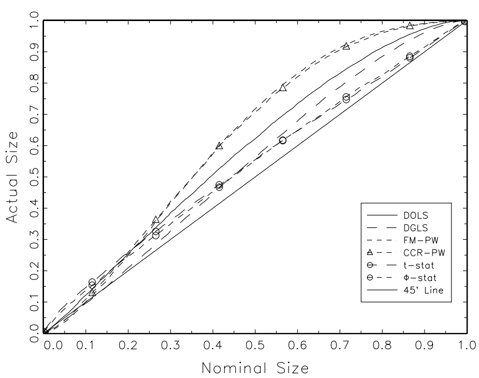


(d) $\alpha_1 = 1$, $\delta = 0.5$

MODEL C



(e) $\alpha_1 = 0$, $\delta = 0$

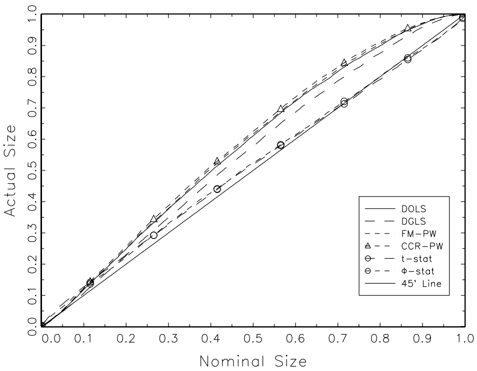


(f) $\alpha_1 = 1$, $\delta = 0.5$

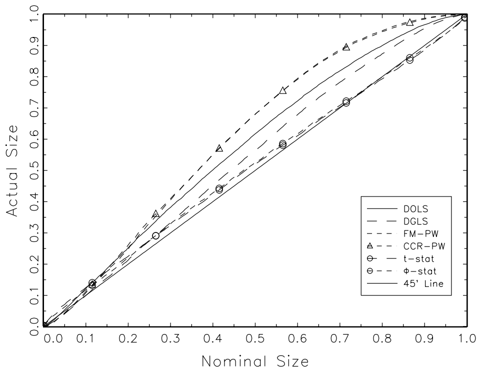
Notes: Asymptotic distributions are obtained by simulating 20,000 series of dimension $T_\infty = 2,000$. Montecarlo simulations are obtained by simulating 20,000 series of dimension $T = 100$. $\lambda = 50\%$, $\phi = \gamma = 0$, $\sigma_\mu^2 = 2$.

Figure 5: P-value Plots: 3 breaks

MODEL A

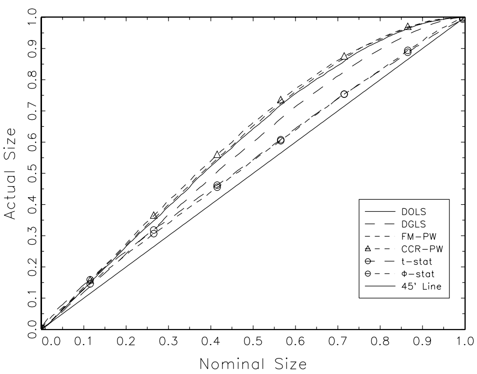


(a) $\alpha_1 = 0$, $\delta = 0$

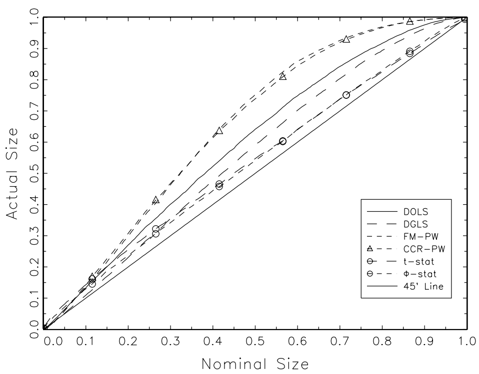


(b) $\alpha_1 = 1$, $\delta = 0.5$

MODEL B

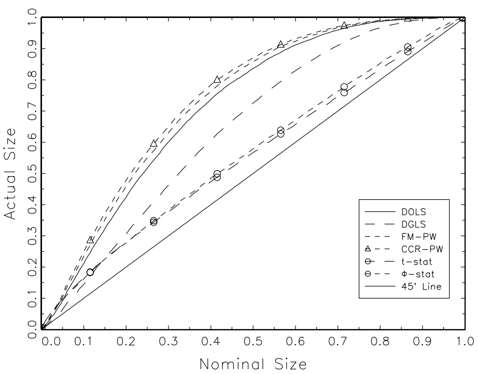


(c) $\alpha_1 = 0$, $\delta = 0$

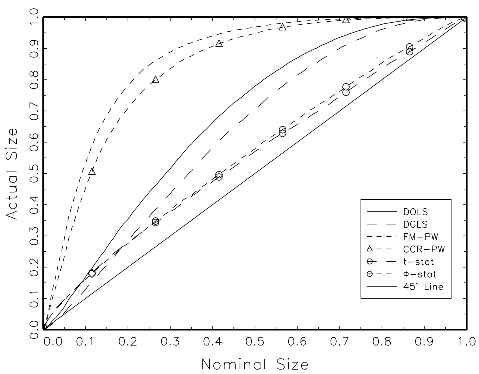


(d) $\alpha_1 = 1$, $\delta = 0.5$

MODEL C



(e) $\alpha_1 = 0$, $\delta = 0$

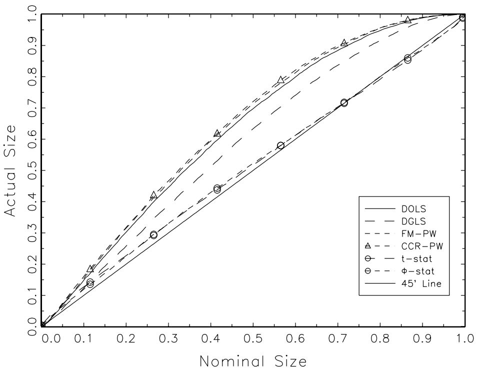


(f) $\alpha_1 = 1$, $\delta = 0.5$

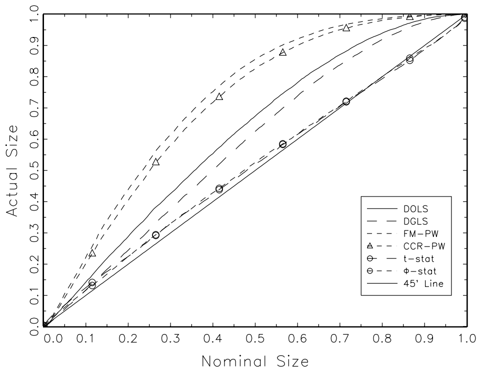
Notes: Asymptotic distributions are obtained by simulating 20,000 series of dimension $T_\infty = 2,000$. Montecarlo simulations are obtained by simulating 20,000 series of dimension $T = 100$. $\lambda = \{30\%, 50\%, 70\%\}$, $\phi = \gamma = 0$, $\sigma_\mu^2 = 2$.

Figure 6: P-value Plots: 5 breaks

MODEL A

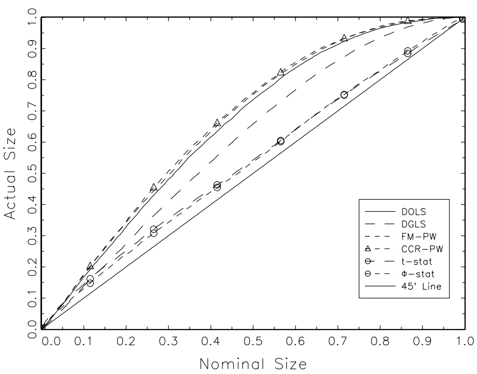


(a) $\alpha_1 = 0$, $\delta = 0$

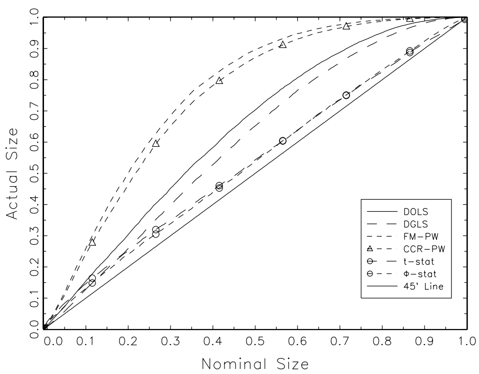


(b) $\alpha_1 = 1$, $\delta = 0.5$

MODEL B

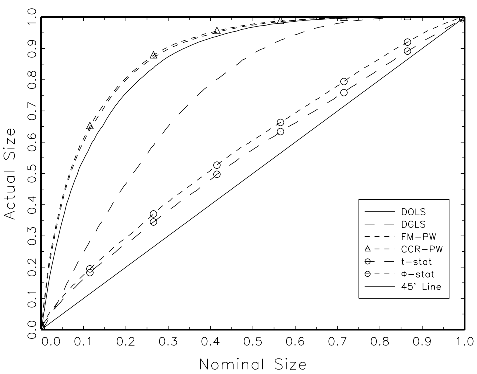


(c) $\alpha_1 = 0$, $\delta = 0$

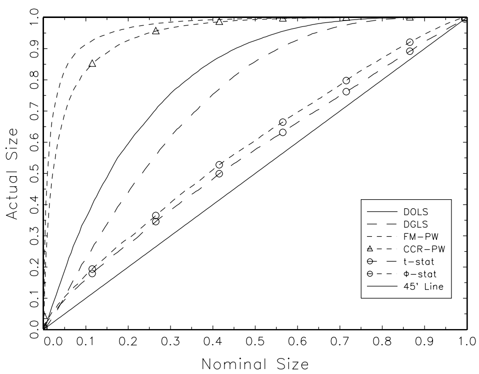


(d) $\alpha_1 = 1$, $\delta = 0.5$

MODEL C



(e) $\alpha_1 = 0$, $\delta = 0$

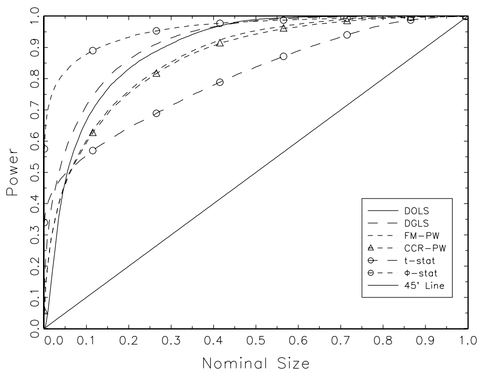


(f) $\alpha_1 = 1$, $\delta = 0.5$

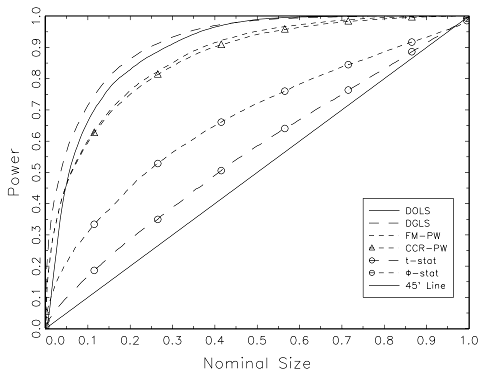
Notes: Asymptotic distributions are obtained by simulating 20,000 series of dimension $T_\infty = 2,000$. Montecarlo simulations are obtained by simulating 20,000 series of dimension $T = 100$. $\lambda = \{20\%, 30\%, 50\%, 70\%, 80\%\}$, $\phi = \gamma = 0$, $\sigma_\mu^2 = 2$.

Figure 7: Size-Power Curves: 1 break

MODEL A

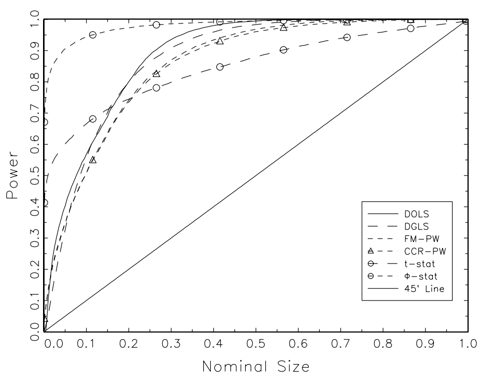


(a) $\alpha_1 = 0$, $\delta = 0$

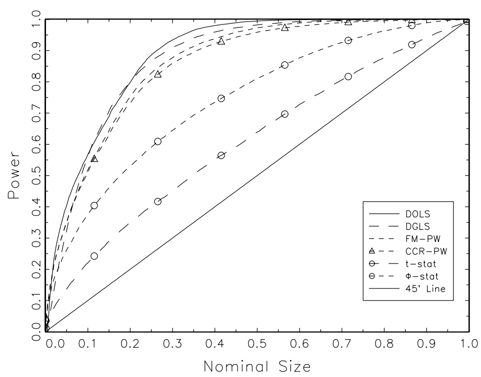


(b) $\alpha_1 = 1$, $\delta = 0.5$

MODEL B

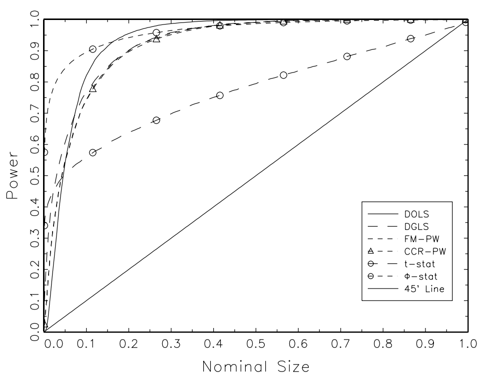


(c) $\alpha_1 = 0$, $\delta = 0$

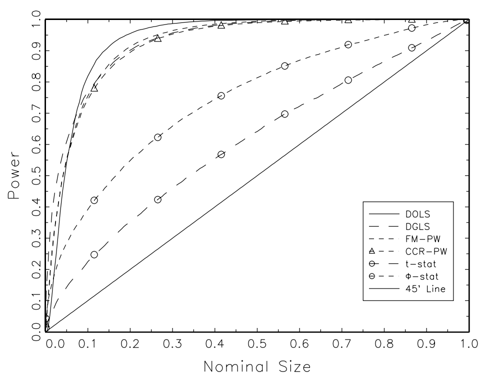


(d) $\alpha_1 = 1$, $\delta = 0.5$

MODEL C



(e) $\alpha_1 = 0$, $\delta = 0$

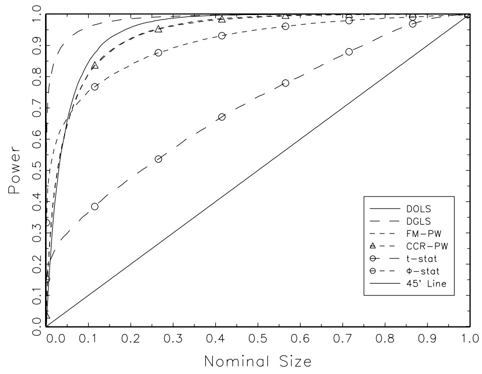


(f) $\alpha_1 = 1$, $\delta = 0.5$

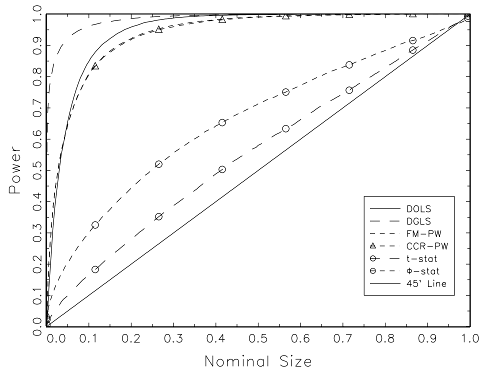
Notes: Asymptotic distributions are obtained by simulating 20,000 series of dimension $T_\infty = 2,000$. Montecarlo simulations are obtained by simulating 20,000 series of dimension $T = 100$. $\lambda = 50\%$, $\phi = \gamma = 0$, $\sigma_\mu^2 = 2$.

Figure 8: Size-Power Curves: 3 breaks

MODEL A

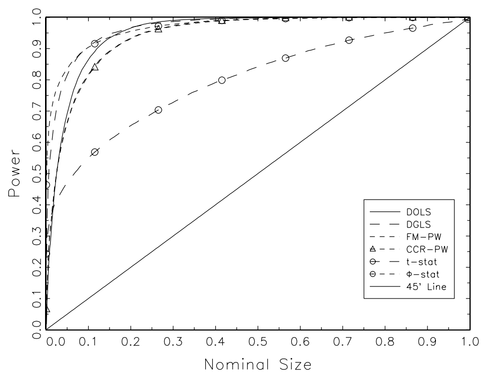


(a) $\alpha_1 = 0$, $\delta = 0$

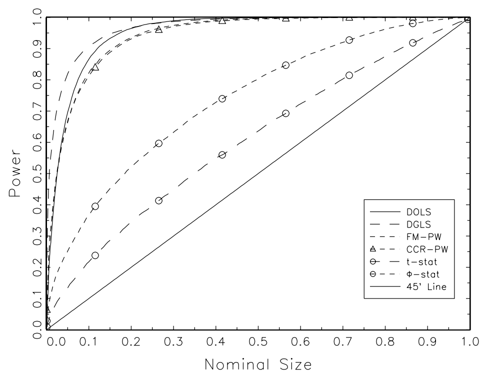


(b) $\alpha_1 = 1$, $\delta = 0.5$

MODEL B

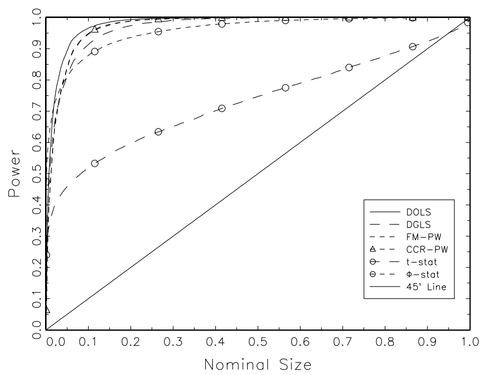


(c) $\alpha_1 = 0$, $\delta = 0$

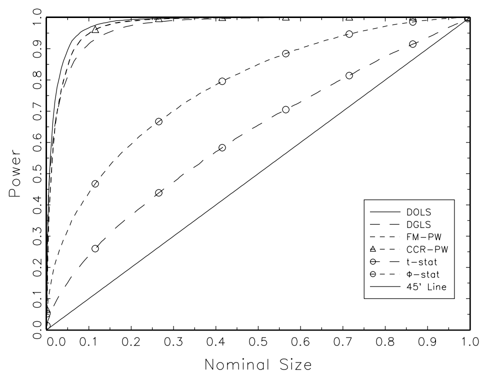


(d) $\alpha_1 = 1$, $\delta = 0.5$

MODEL C



(e) $\alpha_1 = 0$, $\delta = 0$

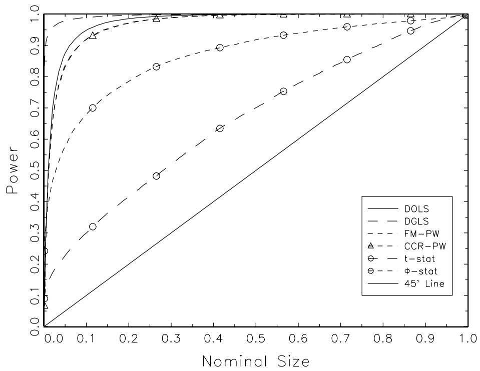


(f) $\alpha_1 = 1$, $\delta = 0.5$

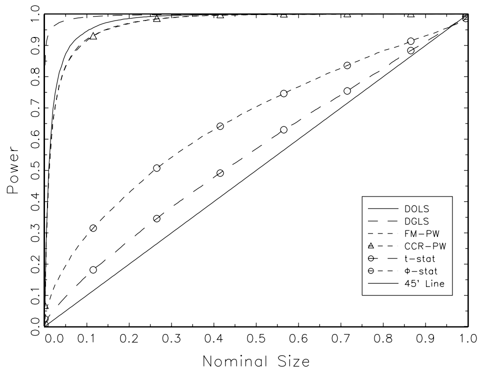
Notes: Asymptotic distributions are obtained by simulating $20,000$ series of dimension $T_\infty = 2,000$. Montecarlo simulations are obtained by simulating $20,000$ series of dimension $T = 100$. $\lambda = \{30\%, 50\%, 70\%\}$, $\phi = \gamma = 0$, $\sigma_\mu^2 = 2$.

Figure 9: Size-Power Curves: 5 breaks

MODEL A

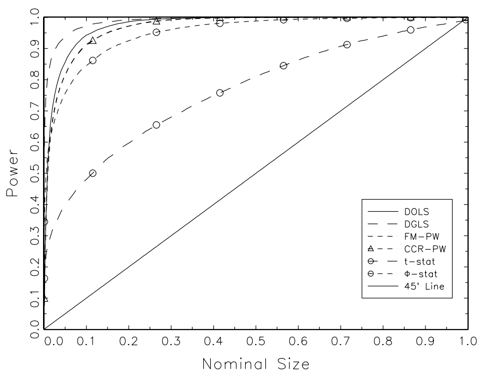


(a) $\alpha_1 = 0$, $\delta = 0$

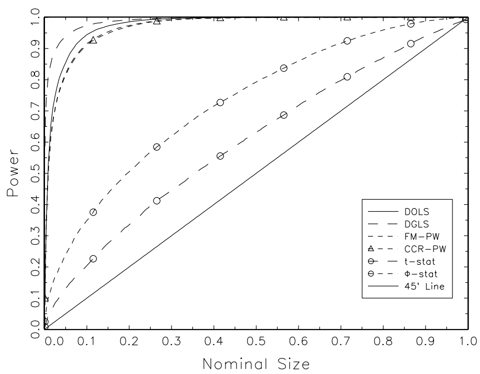


(b) $\alpha_1 = 1$, $\delta = 0.5$

MODEL B

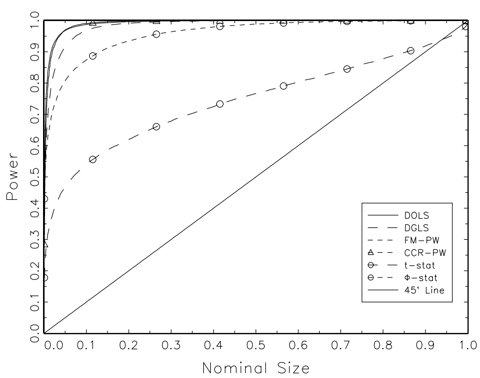


(c) $\alpha_1 = 0$, $\delta = 0$

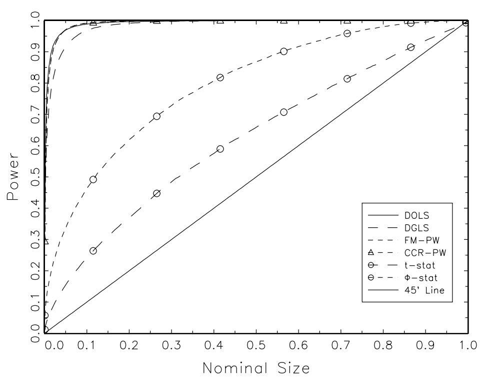


(d) $\alpha_1 = 1$, $\delta = 0.5$

MODEL C



(e) $\alpha_1 = 0$, $\delta = 0$



(f) $\alpha_1 = 1$, $\delta = 0.5$

Notes: Asymptotic distributions are obtained by simulating 20,000 series of dimension $T_\infty = 2,000$. Montecarlo simulations are obtained by simulating 20,000 series of dimension $T = 100$. $\lambda = \{20\%, 30\%, 50\%, 70\%, 80\%\}$, $\phi = \gamma = 0$, $\sigma^2_\mu = 2$.

Table 1: Empirical Size (5% nominal size), 1 Break, $\lambda = 50\%$, $T = 100$

MODEL A

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0822	0.0682	0.1012	0.0620	0.0706	0.0219	0.0636	0.0207	0.0738	0.0735	0.0642	0.0663
		1	0.0743	0.0649	0.0566	0.0462	0.0543	0.0221	0.0562	0.0252	0.0722	0.0718	0.0656	0.0659
		2	0.0660	0.0505	0.0467	0.0383	0.0520	0.0257	0.0554	0.0277	0.0721	0.0714	0.0648	0.0627
0.4	0	0.5	0.1047	0.0756	0.4026	0.1798	0.1142	0.0335	0.1098	0.0411	0.0655	0.0681	0.0364	0.0481
		1	0.0962	0.0756	0.2196	0.0817	0.0972	0.0434	0.0984	0.0516	0.0678	0.0668	0.0295	0.0307
		2	0.0966	0.0734	0.1372	0.0583	0.0948	0.0600	0.0950	0.0648	0.0697	0.0697	0.0205	0.0164
0	0.4	0.5	0.0501	0.0431	0.0304	0.0286	0.0513	0.0261	0.0221	0.0145	0.0734	0.0746	0.0876	0.0767
		1	0.0314	0.0262	0.0189	0.0207	0.0297	0.0184	0.0190	0.0155	0.0755	0.0773	0.1063	0.1063
		2	0.0174	0.0158	0.0147	0.0151	0.0213	0.0154	0.0193	0.0150	0.0801	0.0866	0.1327	0.1648

MODEL B

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1089	0.0894	0.1122	0.0734	0.1025	0.0397	0.0853	0.0331	0.0826	0.0812	0.0719	0.0710
		1	0.0907	0.0761	0.0694	0.0541	0.0721	0.0329	0.0699	0.0350	0.0813	0.0816	0.0742	0.0737
		2	0.0735	0.0517	0.0545	0.0412	0.0576	0.0325	0.0612	0.0350	0.0816	0.0807	0.0717	0.0702
0.4	0	0.5	0.1640	0.1141	0.2842	0.1572	0.1472	0.0527	0.1414	0.0606	0.0732	0.0756	0.0388	0.0509
		1	0.1379	0.0976	0.2047	0.0912	0.1213	0.0540	0.1190	0.0621	0.0723	0.0703	0.0293	0.0304
		2	0.1227	0.0821	0.1492	0.0657	0.1015	0.0639	0.1062	0.0680	0.0747	0.0721	0.0196	0.0160
0	0.4	0.5	0.0610	0.0557	0.0376	0.0386	0.1120	0.0639	0.0380	0.0217	0.0846	0.0815	0.1026	0.0866
		1	0.0377	0.0319	0.0261	0.0273	0.0618	0.0363	0.0288	0.0228	0.0936	0.0920	0.1278	0.1254
		2	0.0198	0.0187	0.0168	0.0174	0.0389	0.0255	0.0283	0.0234	0.1000	0.1130	0.1598	0.1977

MODEL C

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1135	0.0873	0.1347	0.0827	0.1413	0.0560	0.1254	0.0506	0.0840	0.0818	0.0741	0.0746
		1	0.0952	0.0811	0.0750	0.0557	0.0900	0.0418	0.0921	0.0430	0.0833	0.0816	0.0767	0.0757
		2	0.0818	0.0546	0.0571	0.0389	0.0664	0.0347	0.0730	0.0383	0.0891	0.0858	0.0789	0.0757
0.4	0	0.5	0.1905	0.1171	0.4069	0.2254	0.2204	0.0719	0.2219	0.0990	0.0724	0.0750	0.0358	0.0524
		1	0.1605	0.1023	0.2647	0.1084	0.1696	0.0735	0.1733	0.0896	0.0759	0.0736	0.0282	0.0295
		2	0.1504	0.0972	0.1777	0.0794	0.1482	0.0846	0.1534	0.0945	0.0783	0.0784	0.0209	0.0159
0	0.4	0.5	0.0552	0.0502	0.0313	0.0318	0.2059	0.1731	0.0637	0.0703	0.0864	0.0839	0.1093	0.0911
		1	0.0311	0.0257	0.0186	0.0196	0.1130	0.1105	0.0430	0.0601	0.0944	0.0910	0.1378	0.1322
		2	0.0151	0.0137	0.0134	0.0118	0.0635	0.0658	0.0357	0.0517	0.1031	0.1180	0.1761	0.2169

Notes: The DGP is given in equations (12)-(16). x_t is endogenous ($\alpha_1 = 1$), $\alpha_2 = -1$, $\rho = 0$ under H_0 for the CS and BLS tests, while $\rho = 1$ under H_0 for the WE tests. The LRV is computed as in Kurozumi (2002). Asymptotic critical values are obtained by simulating 40,000 series of dimension $T_\infty = 5,000$. Estimated critical values for Model A are: 95% cv $CS = BLS = 0.1552$; 5% cv t-stat = -2.871, 5% cv Φ = -14.206. Estimated critical values for Model B are: 95% cv $CS = BLS = 0.1057$; 5% cv t-stat = -3.019, 5% cv Φ = -18.150. Estimated critical values for Model C are: 95% cv $CS = BLS = 0.0557$; 5% cv t-stat = -3.333, 5% cv Φ = -22.084.

Table 2: Empirical Size (5% nominal size), 1 Break, $\lambda = 50\%$, $T = 200$

MODEL A

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0772	0.0732	0.0684	0.0549	0.0530	0.0180	0.0514	0.0184	0.0563	0.0565	0.0520	0.0534
		1	0.0753	0.0671	0.0553	0.0504	0.0503	0.0244	0.0514	0.0249	0.0588	0.0568	0.0571	0.0536
		2	0.0671	0.0566	0.0468	0.0435	0.0461	0.0256	0.0493	0.0259	0.0550	0.0599	0.0525	0.0558
0.4	0	0.5	0.0920	0.0794	0.2415	0.0946	0.0877	0.0248	0.0841	0.0276	0.0532	0.0540	0.0270	0.0374
		1	0.0872	0.0746	0.1107	0.0641	0.0846	0.0417	0.0822	0.0442	0.0532	0.0513	0.0199	0.0214
		2	0.0804	0.0733	0.0712	0.0532	0.0823	0.0600	0.0812	0.0610	0.0541	0.0568	0.0132	0.0133
0	0.4	0.5	0.0489	0.0490	0.0264	0.0305	0.0274	0.0107	0.0177	0.0087	0.0599	0.0558	0.0768	0.0649
		1	0.0365	0.0312	0.0220	0.0246	0.0214	0.0137	0.0185	0.0113	0.0644	0.0638	0.0999	0.0969
		2	0.0179	0.0168	0.0150	0.0156	0.0169	0.0115	0.0170	0.0111	0.0670	0.0752	0.1226	0.1506

MODEL B

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0898	0.0762	0.0825	0.0578	0.0618	0.0255	0.0591	0.0238	0.0632	0.0600	0.0582	0.0563
		1	0.0816	0.0723	0.0635	0.0549	0.0534	0.0303	0.0565	0.0294	0.0650	0.0643	0.0607	0.0604
		2	0.0697	0.0579	0.0489	0.0466	0.0491	0.0309	0.0534	0.0317	0.0678	0.0661	0.0632	0.0612
0.4	0	0.5	0.1170	0.0938	0.2400	0.1029	0.1014	0.0321	0.0988	0.0331	0.0527	0.0551	0.0268	0.0384
		1	0.1064	0.0857	0.1304	0.0711	0.0934	0.0477	0.0920	0.0496	0.0517	0.0545	0.0196	0.0218
		2	0.0943	0.0776	0.0881	0.0566	0.0868	0.0628	0.0866	0.0643	0.0581	0.0600	0.0126	0.0098
0	0.4	0.5	0.0538	0.0509	0.0301	0.0319	0.0512	0.0239	0.0238	0.0124	0.0683	0.0635	0.0882	0.0714
		1	0.0389	0.0334	0.0265	0.0281	0.0329	0.0215	0.0237	0.0165	0.0760	0.0742	0.1161	0.1144
		2	0.0198	0.0202	0.0169	0.0189	0.0250	0.0170	0.0220	0.0161	0.0872	0.0970	0.1547	0.1928

MODEL C

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0992	0.0843	0.0911	0.0664	0.0669	0.0221	0.0678	0.0215	0.0671	0.0662	0.0622	0.0625
		1	0.0914	0.0769	0.0638	0.0529	0.0559	0.0246	0.0632	0.0261	0.0659	0.0641	0.0623	0.0605
		2	0.0816	0.0664	0.0530	0.0491	0.0525	0.0269	0.0597	0.0297	0.0675	0.0682	0.0632	0.0627
0.4	0	0.5	0.1341	0.1023	0.3209	0.1374	0.1407	0.0369	0.1348	0.0402	0.0558	0.0585	0.0281	0.0403
		1	0.1178	0.0945	0.1619	0.0790	0.1211	0.0550	0.1170	0.0582	0.0557	0.0554	0.0187	0.0197
		2	0.1105	0.0923	0.1059	0.0649	0.1132	0.0794	0.1116	0.0807	0.0584	0.0588	0.0121	0.0089
0	0.4	0.5	0.0510	0.0497	0.0237	0.0275	0.0648	0.0344	0.0198	0.0134	0.0708	0.0672	0.0929	0.0770
		1	0.0325	0.0255	0.0166	0.0188	0.0357	0.0211	0.0179	0.0150	0.0757	0.0736	0.1237	0.1191
		2	0.0137	0.0133	0.0129	0.0124	0.0233	0.0154	0.0197	0.0150	0.0889	0.1024	0.1678	0.2139

Notes: See Table 1.

Table 3: Empirical Size (5% nominal size), 3 Breaks, $\lambda = (30\%, 50\%, 70\%)$, $T = 100$

MODEL A

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1008	0.0774	0.1816	0.1056	0.1329	0.0660	0.1043	0.0505	0.0733	0.0749	0.0667	0.0670
		1	0.0867	0.0726	0.0842	0.0560	0.0848	0.0480	0.0825	0.0478	0.0723	0.0741	0.0652	0.0680
		2	0.0746	0.0526	0.0589	0.0408	0.0628	0.0444	0.0704	0.0452	0.0737	0.0747	0.0675	0.0666
0.4	0	0.5	0.1655	0.0965	0.6140	0.3490	0.2018	0.0633	0.1898	0.0814	0.0687	0.0705	0.0365	0.0485
		1	0.1417	0.0927	0.3741	0.1471	0.1550	0.0639	0.1543	0.0813	0.0684	0.0675	0.0299	0.0306
		2	0.1292	0.0883	0.2328	0.0914	0.1342	0.0789	0.1387	0.0916	0.0712	0.0720	0.0211	0.0181
0	0.4	0.5	0.0527	0.0448	0.0397	0.0339	0.1906	0.1810	0.0595	0.0774	0.0726	0.0735	0.0868	0.0768
		1	0.0303	0.0259	0.0200	0.0189	0.1062	0.1162	0.0460	0.0671	0.0762	0.0768	0.1074	0.1073
		2	0.0165	0.0162	0.0126	0.0119	0.0643	0.0807	0.0406	0.0617	0.0786	0.0881	0.1338	0.1627

MODEL B

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1541	0.1170	0.2111	0.1316	0.1909	0.1058	0.1500	0.0805	0.0792	0.0789	0.0694	0.0694
		1	0.1188	0.0945	0.1066	0.0736	0.1151	0.0736	0.1092	0.0669	0.0795	0.0816	0.0716	0.0717
		2	0.0995	0.0637	0.0774	0.0504	0.0813	0.0564	0.0878	0.0625	0.0791	0.0792	0.0687	0.0707
0.4	0	0.5	0.2599	0.1660	0.5852	0.3549	0.2654	0.0934	0.2578	0.1197	0.0698	0.0724	0.0375	0.0496
		1	0.2124	0.1399	0.3871	0.1706	0.2006	0.0872	0.2041	0.1100	0.0708	0.0691	0.0295	0.0294
		2	0.1850	0.1207	0.2595	0.1138	0.1679	0.0934	0.1729	0.1081	0.0730	0.0712	0.0192	0.0165
0	0.4	0.5	0.0746	0.0636	0.0542	0.0502	0.3034	0.2823	0.1006	0.1275	0.0828	0.0777	0.0984	0.0812
		1	0.0423	0.0344	0.0291	0.0267	0.1788	0.1904	0.0698	0.1089	0.0924	0.0891	0.1260	0.1238
		2	0.0221	0.0234	0.0169	0.0156	0.1097	0.1225	0.0638	0.0981	0.0961	0.1077	0.1538	0.1899

MODEL C

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.3010	0.2088	0.2737	0.1897	0.4860	0.5895	0.3990	0.3576	0.0951	0.0964	0.0932	0.0932
		1	0.1921	0.1268	0.1505	0.0891	0.3133	0.4307	0.2598	0.2871	0.0987	0.0960	0.0967	0.0956
		2	0.1394	0.0736	0.0977	0.0523	0.2008	0.2944	0.1843	0.2368	0.0968	0.0959	0.0926	0.0925
0.4	0	0.5	0.6039	0.4018	0.5960	0.4327	0.5617	0.2297	0.6081	0.3547	0.0819	0.0866	0.0463	0.0637
		1	0.4804	0.2641	0.4428	0.2341	0.4332	0.1676	0.4802	0.2572	0.0870	0.0832	0.0350	0.0384
		2	0.4029	0.2051	0.3239	0.1558	0.3480	0.1575	0.3864	0.2107	0.0849	0.0881	0.0249	0.0200
0	0.4	0.5	0.1125	0.0909	0.0719	0.0693	0.8025	0.9646	0.5012	0.7850	0.0988	0.0958	0.1336	0.1105
		1	0.0476	0.0415	0.0302	0.0267	0.7373	0.9475	0.3970	0.7461	0.1051	0.1072	0.1657	0.1656
		2	0.0266	0.0604	0.0152	0.0122	0.6736	0.9055	0.3820	0.7469	0.1105	0.1279	0.2071	0.2509

Notes: See Table 1. Estimated critical values for Model A are: 95% cv $CS = BLS = 0.0746$; 5% cv t-stat = -2.873, 5% cv Φ = -14.154. Estimated critical values for Model B are: 95% cv $CS = BLS = 0.0604$; 5% cv t-stat = -3.026, 5% cv Φ = -18.235. Estimated critical values for Model C are: 95% cv $CS = BLS = 0.0266$; 5% cv t-stat = -3.849, 5% cv Φ = -29.467.

Table 4: Empirical Size (5% nominal size), 3 Breaks, $\lambda = (30\%, 50\%, 70\%)$, $T = 200$

MODEL A

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.0906	0.0787	0.1087	0.0753	0.0710	0.0270	0.0641	0.0245	0.0590	0.0591	0.0549	0.0556
		1	0.0851	0.0751	0.0703	0.0566	0.0581	0.0278	0.0581	0.0292	0.0591	0.0567	0.0568	0.0544
		2	0.0748	0.0614	0.0532	0.0462	0.0521	0.0298	0.0577	0.0308	0.0567	0.0591	0.0542	0.0551
0.4	0	0.5	0.1182	0.0924	0.4121	0.1865	0.1279	0.0298	0.1203	0.0365	0.0536	0.0558	0.0281	0.0387
		1	0.1082	0.0883	0.1940	0.0943	0.1145	0.0472	0.1095	0.0533	0.0527	0.0532	0.0203	0.0224
		2	0.1019	0.0867	0.1186	0.0691	0.1054	0.0693	0.1055	0.0759	0.0539	0.0570	0.0148	0.0130
0	0.4	0.5	0.0511	0.0479	0.0286	0.0292	0.0613	0.0393	0.0239	0.0204	0.0608	0.0577	0.0776	0.0657
		1	0.0335	0.0268	0.0187	0.0210	0.0362	0.0284	0.0223	0.0211	0.0654	0.0631	0.1004	0.0988
		2	0.0148	0.0154	0.0118	0.0129	0.0253	0.0216	0.0225	0.0217	0.0662	0.0717	0.1231	0.1477

MODEL B

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1160	0.1023	0.1312	0.0905	0.0869	0.0370	0.0791	0.0343	0.0614	0.0615	0.0575	0.0582
		1	0.1031	0.0854	0.0837	0.0622	0.0660	0.0348	0.0692	0.0355	0.0636	0.0640	0.0606	0.0589
		2	0.0889	0.0714	0.0627	0.0558	0.0592	0.0376	0.0657	0.0391	0.0661	0.0665	0.0616	0.0601
0.4	0	0.5	0.1700	0.1319	0.4389	0.2142	0.1581	0.0419	0.1538	0.0500	0.0534	0.0558	0.0276	0.0387
		1	0.1452	0.1113	0.2257	0.1070	0.1311	0.0546	0.1277	0.0630	0.0518	0.0538	0.0202	0.0207
		2	0.1309	0.1059	0.1439	0.0838	0.1244	0.0843	0.1228	0.0880	0.0578	0.0575	0.0126	0.0104
0	0.4	0.5	0.0620	0.0582	0.0361	0.0384	0.1043	0.0706	0.0330	0.0305	0.0661	0.0629	0.0870	0.0700
		1	0.0387	0.0297	0.0227	0.0230	0.0548	0.0432	0.0298	0.0317	0.0740	0.0731	0.1154	0.1100
		2	0.0174	0.0195	0.0151	0.0153	0.0381	0.0332	0.0316	0.0320	0.0859	0.0963	0.1520	0.1856

MODEL C

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1916	0.1465	0.1846	0.1411	0.2130	0.1190	0.1580	0.0899	0.0703	0.0718	0.0694	0.0698
		1	0.1400	0.1054	0.0949	0.0735	0.1123	0.0751	0.1128	0.0767	0.0744	0.0693	0.0716	0.0679
		2	0.1067	0.0722	0.0663	0.0537	0.0787	0.0570	0.0871	0.0650	0.0734	0.0749	0.0694	0.0722
0.4	0	0.5	0.3628	0.2388	0.5252	0.3192	0.3376	0.0715	0.3430	0.1290	0.0589	0.0621	0.0294	0.0430
		1	0.2701	0.1681	0.3030	0.1475	0.2486	0.0748	0.2642	0.1194	0.0602	0.0590	0.0192	0.0186
		2	0.2244	0.1390	0.2006	0.0929	0.2049	0.1016	0.2166	0.1275	0.0615	0.0634	0.0115	0.0080
0	0.4	0.5	0.0805	0.0700	0.0383	0.0425	0.4427	0.5125	0.1260	0.2119	0.0755	0.0728	0.1084	0.0887
		1	0.0373	0.0277	0.0183	0.0207	0.2563	0.3419	0.0814	0.1847	0.0852	0.0824	0.1475	0.1408
		2	0.0128	0.0184	0.0102	0.0117	0.1425	0.2186	0.0711	0.1698	0.0915	0.1113	0.1947	0.2505

Notes: See Table 3.

Table 5: Empirical Size (5% nominal size), 5 Breaks, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $T = 100$

MODEL A

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1887	0.1323	0.2891	0.1846	0.2729	0.2251	0.2037	0.1354	0.0728	0.0740	0.0640	0.0636
		1	0.1372	0.1023	0.1441	0.0893	0.1547	0.1461	0.1343	0.1080	0.0734	0.0726	0.0654	0.0674
		2	0.1102	0.0691	0.0943	0.0567	0.1064	0.1042	0.1035	0.0899	0.0745	0.0744	0.0661	0.0633
0.4	0	0.5	0.3695	0.2189	0.7246	0.4960	0.3774	0.1252	0.3712	0.1662	0.0690	0.0711	0.0381	0.0480
		1	0.2867	0.1666	0.5091	0.2497	0.2719	0.1069	0.2832	0.1398	0.0692	0.0692	0.0296	0.0318
		2	0.2442	0.1443	0.3590	0.1604	0.2207	0.1162	0.2340	0.1363	0.0719	0.0715	0.0222	0.0179
0	0.4	0.5	0.0830	0.0669	0.0716	0.0570	0.5055	0.6056	0.1905	0.3198	0.0741	0.0732	0.0883	0.0755
		1	0.0423	0.0329	0.0300	0.0241	0.3425	0.4729	0.1377	0.2700	0.0763	0.0761	0.1058	0.1052
		2	0.0232	0.0298	0.0166	0.0136	0.2407	0.3525	0.1213	0.2459	0.0781	0.0836	0.1272	0.1556

MODEL B

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.2290	0.1587	0.2782	0.1776	0.3202	0.2576	0.2448	0.1664	0.0800	0.0809	0.0704	0.0715
		1	0.1585	0.1122	0.1432	0.0902	0.1878	0.1714	0.1618	0.1335	0.0837	0.0811	0.0757	0.0724
		2	0.1224	0.0713	0.0973	0.0576	0.1236	0.1168	0.1161	0.1056	0.0826	0.0808	0.0742	0.0726
0.4	0	0.5	0.4305	0.2705	0.6876	0.4613	0.4182	0.1538	0.4286	0.2020	0.0710	0.0743	0.0397	0.0512
		1	0.3400	0.2017	0.4858	0.2352	0.3062	0.1245	0.3230	0.1616	0.0731	0.0711	0.0311	0.0314
		2	0.2877	0.1636	0.3399	0.1538	0.2468	0.1255	0.2597	0.1429	0.0739	0.0721	0.0221	0.0181
0	0.4	0.5	0.0964	0.0733	0.0732	0.0571	0.5846	0.6770	0.2474	0.3818	0.0829	0.0789	0.0994	0.0822
		1	0.0474	0.0374	0.0340	0.0276	0.4197	0.5450	0.1715	0.3213	0.0960	0.0910	0.1277	0.1249
		2	0.0257	0.0351	0.0187	0.0149	0.2997	0.4171	0.1456	0.2863	0.0987	0.1067	0.1549	0.1857

MODEL C

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.6479	0.4954	0.4933	0.3683	0.8245	0.9456	0.7569	0.7719	0.0964	0.0981	0.1067	0.1080
		1	0.4531	0.2743	0.3235	0.1954	0.6985	0.9071	0.5845	0.7232	0.0967	0.0980	0.1061	0.1089
		2	0.2998	0.1902	0.2212	0.1054	0.5754	0.8271	0.4559	0.6898	0.0970	0.0943	0.1043	0.1025
0.4	0	0.5	0.8852	0.7674	0.7737	0.6456	0.8516	0.5885	0.8958	0.7166	0.0837	0.0871	0.0537	0.0714
		1	0.8190	0.6015	0.6610	0.4454	0.7593	0.4435	0.8233	0.5773	0.0820	0.0805	0.0385	0.0410
		2	0.7481	0.4726	0.5509	0.3335	0.6636	0.3739	0.7204	0.4665	0.0827	0.0852	0.0285	0.0226
0	0.4	0.5	0.2906	0.2270	0.1880	0.1669	0.9626	0.9996	0.8344	0.9827	0.0967	0.0985	0.1454	0.1252
		1	0.1340	0.1654	0.0823	0.0625	0.9562	0.9998	0.7809	0.9849	0.1024	0.1061	0.1777	0.1843
		2	0.1516	0.4297	0.0368	0.0310	0.9636	0.9992	0.8164	0.9883	0.1156	0.1283	0.2279	0.2754

Notes: See Table 1. Estimated critical values for Model A are: 95% cv $CS = BLS = 0.0491$; 5% cv t-stat = -2.875, 5% cv Φ = -14.210. Estimated critical values for Model B are: 95% cv $CS = BLS = 0.0429$; 5% cv t-stat = -3.015, 5% cv Φ = -18.085. Estimated critical values for Model C are: 95% cv $CS = BLS = 0.0176$; 5% cv t-stat = -4.277, 5% cv Φ = -36.264.

Table 6: Empirical Size (5% nominal size), 5 Breaks, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $T = 200$

MODEL A

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1372	0.1113	0.1767	0.1109	0.1153	0.0554	0.0888	0.0425	0.0583	0.0583	0.0545	0.0557
		1	0.1177	0.0941	0.0990	0.0717	0.0781	0.0457	0.0779	0.0428	0.0600	0.0583	0.0565	0.0539
		2	0.0951	0.0732	0.0713	0.0572	0.0659	0.0431	0.0675	0.0424	0.0568	0.0604	0.0530	0.0554
0.4	0	0.5	0.2154	0.1522	0.5641	0.3084	0.2120	0.0452	0.2003	0.0566	0.0534	0.0548	0.0268	0.0389
		1	0.1784	0.1291	0.3175	0.1493	0.1711	0.0599	0.1669	0.0727	0.0533	0.0533	0.0203	0.0220
		2	0.1560	0.1188	0.1967	0.1008	0.1486	0.0862	0.1515	0.0990	0.0549	0.0566	0.0151	0.0130
0	0.4	0.5	0.0685	0.0576	0.0372	0.0361	0.1693	0.1535	0.0455	0.0644	0.0606	0.0571	0.0774	0.0650
		1	0.0379	0.0265	0.0212	0.0207	0.0853	0.0935	0.0378	0.0592	0.0662	0.0633	0.1007	0.0971
		2	0.0160	0.0197	0.0121	0.0137	0.0550	0.0674	0.0369	0.0543	0.0654	0.0708	0.1209	0.1446

MODEL B

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.1579	0.1222	0.1740	0.1103	0.1329	0.0675	0.1046	0.0523	0.0642	0.0649	0.0597	0.0608
		1	0.1284	0.0993	0.0985	0.0728	0.0838	0.0516	0.0852	0.0497	0.0650	0.0658	0.0614	0.0611
		2	0.1004	0.0723	0.0712	0.0567	0.0688	0.0485	0.0713	0.0501	0.0678	0.0670	0.0627	0.0613
0.4	0	0.5	0.2583	0.1789	0.5436	0.2923	0.2328	0.0573	0.2299	0.0709	0.0551	0.0576	0.0287	0.0412
		1	0.2105	0.1456	0.3027	0.1473	0.1825	0.0669	0.1875	0.0809	0.0532	0.0546	0.0216	0.0221
		2	0.1780	0.1294	0.1955	0.0998	0.1619	0.0937	0.1656	0.0995	0.0587	0.0584	0.0144	0.0108
0	0.4	0.5	0.0744	0.0633	0.0400	0.0379	0.2292	0.1917	0.0569	0.0722	0.0686	0.0646	0.0893	0.0725
		1	0.0399	0.0280	0.0233	0.0235	0.1162	0.1122	0.0439	0.0670	0.0764	0.0719	0.1153	0.1117
		2	0.0156	0.0202	0.0128	0.0138	0.0701	0.0781	0.0444	0.0658	0.0857	0.0960	0.1507	0.1875

MODEL C

			CS (2006)						BLS (2001)		WE (2007)			
			CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}		$WE_{t\text{-stat}}$		WE_{Φ}	
ϕ	γ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5	0	0.5
0	0	0.5	0.4780	0.3412	0.3236	0.2456	0.5125	0.6028	0.3839	0.3369	0.0727	0.0740	0.0753	0.0780
		1	0.3126	0.1967	0.1786	0.1263	0.2928	0.3971	0.2429	0.2606	0.0764	0.0748	0.0784	0.0781
		2	0.1976	0.1059	0.1147	0.0799	0.1746	0.2474	0.1685	0.2126	0.0774	0.0791	0.0790	0.0813
0.4	0	0.5	0.8336	0.6562	0.7156	0.5189	0.6755	0.1927	0.7483	0.3289	0.0600	0.0641	0.0297	0.0462
		1	0.7013	0.4459	0.5038	0.2792	0.5410	0.1532	0.6210	0.2549	0.0633	0.0624	0.0196	0.0212
		2	0.5749	0.3231	0.3716	0.1763	0.4491	0.1721	0.5063	0.2330	0.0640	0.0663	0.0118	0.0087
0	0.4	0.5	0.1693	0.1252	0.0770	0.0765	0.9238	0.9923	0.5527	0.8531	0.0759	0.0735	0.1203	0.0967
		1	0.0609	0.0388	0.0315	0.0287	0.8247	0.9738	0.3936	0.8002	0.0845	0.0844	0.1633	0.1577
		2	0.0205	0.0538	0.0137	0.0124	0.6972	0.9304	0.3578	0.7870	0.1002	0.1229	0.2215	0.2896

Notes: See Table 5.

Table 7: Empirical Size-Corrected Power (5% actual size), 1 Break, $\lambda = 50\%$, $T = 100$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.3248	0.3816	0.4174	0.5168	0.3981	0.5671	0.4177	0.5738	0	0.5	0.0492	0.0484	0.0648	0.0620
	1	0.3588	0.3845	0.5395	0.5707	0.4517	0.5756	0.4426	0.5624		1	0.0504	0.0560	0.0876	0.0940
	2	0.3857	0.4647	0.5649	0.6073	0.4559	0.5595	0.4419	0.5505		2	0.0604	0.0760	0.1273	0.1893
0.9	0.5	0.2115	0.2022	0.4335	0.5315	0.3056	0.4047	0.3340	0.4389	0.1	0.5	0.0509	0.0494	0.0687	0.0638
	1	0.1885	0.1478	0.5370	0.5322	0.3329	0.3963	0.3317	0.3978		1	0.0538	0.0601	0.0938	0.1038
	2	0.1724	0.1740	0.5333	0.4493	0.3166	0.3661	0.3134	0.3699		2	0.0658	0.0925	0.1398	0.2246
0.1	0.5	0.0507	0.0506	0.0763	0.0574	0.0562	0.0468	0.0570	0.0506	0.9	0.5	0.0628	0.0615	0.0733	0.0703
	1	0.0515	0.0520	0.0621	0.0533	0.0579	0.0496	0.0598	0.0507		1	0.0753	0.0953	0.0979	0.1306
	2	0.0518	0.0549	0.0574	0.0519	0.0586	0.0513	0.0592	0.0524		2	0.0980	0.1802	0.1348	0.2742

MODEL B

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.2460	0.2871	0.1479	0.2369	0.2309	0.3963	0.2564	0.4199	0	0.5	0.0551	0.0486	0.0749	0.0610
	1	0.2822	0.3206	0.2392	0.3051	0.2816	0.4314	0.2830	0.4181		1	0.0579	0.0599	0.0940	0.0993
	2	0.3294	0.4025	0.3043	0.3783	0.3216	0.4424	0.3176	0.4227		2	0.0683	0.0996	0.1459	0.2198
0.9	0.5	0.2407	0.2347	0.1273	0.2197	0.2502	0.3536	0.2794	0.3881	0.1	0.5	0.0568	0.0497	0.0782	0.0634
	1	0.2456	0.2188	0.2046	0.2479	0.2819	0.3561	0.2853	0.3512		1	0.0627	0.0666	0.1013	0.1106
	2	0.2497	0.2483	0.2467	0.2701	0.2941	0.3418	0.2918	0.3273		2	0.0788	0.1268	0.1600	0.2614
0.1	0.5	0.0537	0.0515	0.0658	0.0564	0.0559	0.0471	0.0580	0.0509	0.9	0.5	0.0690	0.0615	0.0735	0.0642
	1	0.0546	0.0526	0.0609	0.0546	0.0567	0.0483	0.0575	0.0497		1	0.0797	0.0985	0.0860	0.1057
	2	0.0567	0.0567	0.0610	0.0528	0.0578	0.0512	0.0594	0.0504		2	0.0997	0.1573	0.1142	0.1783

MODEL C

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.2800	0.3544	0.4063	0.5165	0.2751	0.5364	0.3094	0.5470	0	0.5	0.0528	0.0518	0.0743	0.0667
	1	0.3278	0.3573	0.5287	0.5896	0.4102	0.5925	0.4050	0.5801		1	0.0553	0.0614	0.0959	0.1057
	2	0.3527	0.5130	0.5857	0.6529	0.4911	0.6388	0.4601	0.6097		2	0.0684	0.0962	0.1492	0.2284
0.9	0.5	0.2552	0.2999	0.3660	0.4864	0.2504	0.4830	0.2857	0.4992	0.1	0.5	0.0565	0.0526	0.0796	0.0683
	1	0.2754	0.2602	0.4691	0.5012	0.3562	0.4989	0.3531	0.4904		1	0.0611	0.0684	0.1029	0.1177
	2	0.2853	0.3742	0.4920	0.5091	0.4180	0.5110	0.3878	0.4836		2	0.0818	0.1246	0.1645	0.2691
0.1	0.5	0.0536	0.0504	0.0785	0.0603	0.0554	0.0408	0.0583	0.0489	0.9	0.5	0.0641	0.0601	0.0658	0.0631
	1	0.0536	0.0525	0.0672	0.0550	0.0598	0.0440	0.0625	0.0477		1	0.0709	0.0823	0.0741	0.0903
	2	0.0564	0.0586	0.0627	0.0544	0.0638	0.0475	0.0640	0.0505		2	0.0842	0.1070	0.0921	0.1212

Notes: The DGP is given in equations (12)-(16). x_t is endogenous ($\alpha_1 = 1$), $\alpha_2 = -1$. The LRV is computed as in Kurozumi (2002).

Table 8: Empirical Size-Corrected Power (5% actual size), 1 Break, $\lambda = 50\%$, $T = 200$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.5573	0.5761	0.5511	0.5944	0.5872	0.7109	0.5857	0.6980	0	0.5	0.0543	0.0503	0.0735	0.0625
	1	0.5609	0.5767	0.5986	0.6163	0.5899	0.6808	0.5767	0.6741		1	0.0556	0.0599	0.0943	0.1047
	2	0.5921	0.6237	0.6257	0.6382	0.6096	0.6811	0.5894	0.6718		2	0.0669	0.0730	0.1424	0.1830
0.9	0.5	0.3219	0.2296	0.5644	0.6058	0.3877	0.4285	0.3925	0.4427	0.1	0.5	0.0559	0.0511	0.0774	0.0647
	1	0.2394	0.1663	0.6194	0.6191	0.3613	0.4101	0.3589	0.4265		1	0.0592	0.0627	0.1015	0.1154
	2	0.2033	0.1609	0.6372	0.4621	0.3568	0.4146	0.3453	0.4203		2	0.0755	0.0849	0.1555	0.2152
0.1	0.5	0.0517	0.0506	0.0591	0.0530	0.0581	0.0492	0.0579	0.0492	0.9	0.5	0.0778	0.0692	0.0957	0.0832
	1	0.0500	0.0504	0.0541	0.0518	0.0605	0.0508	0.0586	0.0504		1	0.1042	0.1518	0.1409	0.2137
	2	0.0525	0.0547	0.0519	0.0510	0.0615	0.0528	0.0606	0.0525		2	0.1595	0.3940	0.2245	0.5528

MODEL B

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.4002	0.4267	0.2265	0.3260	0.4604	0.6176	0.4614	0.6166	0	0.5	0.0547	0.0541	0.0778	0.0663
	1	0.4217	0.4408	0.3055	0.3472	0.4849	0.6031	0.4679	0.5850		1	0.0608	0.0609	0.1060	0.1105
	2	0.4431	0.4805	0.3759	0.4005	0.4963	0.5968	0.4807	0.5777		2	0.0683	0.0979	0.1507	0.2303
0.9	0.5	0.3224	0.2644	0.2045	0.3057	0.3575	0.4436	0.3719	0.4591	0.1	0.5	0.0569	0.0551	0.0817	0.0692
	1	0.2736	0.2008	0.2656	0.2866	0.3529	0.4181	0.3501	0.4112		1	0.0674	0.0692	0.1157	0.1248
	2	0.2467	0.1883	0.3226	0.2359	0.3347	0.4044	0.3246	0.3921		2	0.0787	0.1192	0.1698	0.2754
0.1	0.5	0.0523	0.0515	0.0597	0.0534	0.0570	0.0474	0.0591	0.0489	0.9	0.5	0.0860	0.0805	0.0938	0.0841
	1	0.0531	0.0512	0.0538	0.0498	0.0596	0.0502	0.0587	0.0508		1	0.1243	0.1788	0.1385	0.2027
	2	0.0535	0.0548	0.0547	0.0517	0.0612	0.0514	0.0595	0.0510		2	0.1763	0.4038	0.2017	0.4642

MODEL C

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_μ^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_μ^2/δ	0	0.5	0	0.5
1	0.5	0.3848	0.4168	0.5666	0.6308	0.6205	0.7803	0.6098	0.7816	0	0.5	0.0507	0.0523	0.0749	0.0633
	1	0.4006	0.4312	0.6370	0.6746	0.6475	0.7780	0.6301	0.7702		1	0.0578	0.0613	0.1078	0.1134
	2	0.4188	0.4728	0.6702	0.6858	0.6563	0.7636	0.6369	0.7503		2	0.0719	0.1013	0.1644	0.2523
0.9	0.5	0.2930	0.2642	0.5143	0.5903	0.4923	0.6734	0.5015	0.6688	0.1	0.5	0.0536	0.0540	0.0806	0.0659
	1	0.2565	0.2080	0.5385	0.5240	0.4809	0.6349	0.4789	0.6190		1	0.0658	0.0697	0.1186	0.1309
	2	0.2361	0.2155	0.5225	0.4137	0.4631	0.5994	0.4501	0.5814		2	0.0840	0.1281	0.1862	0.2980
0.1	0.5	0.0512	0.0509	0.0624	0.0561	0.0608	0.0462	0.0617	0.0479	0.9	0.5	0.0781	0.0733	0.0855	0.0781
	1	0.0538	0.0520	0.0568	0.0528	0.0620	0.0482	0.0624	0.0495		1	0.1108	0.1482	0.1216	0.1675
	2	0.0549	0.0568	0.0529	0.0522	0.0666	0.0515	0.0642	0.0516		2	0.1518	0.2856	0.1739	0.3306

Notes: See Table 7.

Table 9: Empirical Size-Corrected Power (5% actual size), 3 Break, $\lambda = (30\%, 50\%, 70\%)$, $T = 100$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_{μ}^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_{μ}^2/δ	0	0.5	0	0.5
1	0.5	0.5081	0.5656	0.7796	0.8620	0.4186	0.6028	0.4742	0.6502	0	0.5	0.0498	0.0492	0.0646	0.0588
	1	0.5208	0.5582	0.8800	0.9098	0.5347	0.6520	0.5373	0.6549		1	0.0511	0.0530	0.0869	0.0927
	2	0.5442	0.6449	0.9085	0.9297	0.5966	0.6762	0.5866	0.6698		2	0.0568	0.0708	0.1203	0.1781
0.9	0.5	0.4063	0.4029	0.7739	0.8567	0.3136	0.4277	0.3713	0.4989	0.1	0.5	0.0514	0.0501	0.0674	0.0614
	1	0.3685	0.3210	0.8533	0.8535	0.3857	0.4417	0.3875	0.4614		1	0.0528	0.0571	0.0925	0.1009
	2	0.3401	0.3567	0.8443	0.7932	0.4052	0.4350	0.3991	0.4321		2	0.0617	0.0853	0.1336	0.2084
0.1	0.5	0.0540	0.0516	0.0919	0.0664	0.0533	0.0409	0.0586	0.0461	0.9	0.5	0.0600	0.0595	0.0704	0.0685
	1	0.0541	0.0513	0.0716	0.0573	0.0574	0.0430	0.0597	0.0450		1	0.0749	0.0899	0.0962	0.1258
	2	0.0550	0.0574	0.0628	0.0550	0.0590	0.0464	0.0628	0.0479		2	0.0951	0.1631	0.1301	0.2415

MODEL B

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_{μ}^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_{μ}^2/δ	0	0.5	0	0.5
1	0.5	0.4207	0.4928	0.5016	0.6607	0.3323	0.5226	0.3968	0.5717	0	0.5	0.0553	0.0494	0.0729	0.0601
	1	0.4785	0.5273	0.6882	0.7591	0.4791	0.6053	0.4810	0.6097		1	0.0553	0.0631	0.0905	0.1010
	2	0.5133	0.6437	0.7574	0.8166	0.5633	0.6502	0.5431	0.6240		2	0.0649	0.0894	0.1424	0.2115
0.9	0.5	0.3707	0.4050	0.4729	0.6438	0.2790	0.4155	0.3439	0.4832	0.1	0.5	0.0577	0.0503	0.0767	0.0621
	1	0.3868	0.3707	0.6422	0.6840	0.3872	0.4490	0.3901	0.4639		1	0.0613	0.0687	0.0988	0.1107
	2	0.3843	0.4335	0.6726	0.6783	0.4373	0.4614	0.4161	0.4410		2	0.0754	0.1162	0.1583	0.2529
0.1	0.5	0.0556	0.0528	0.0831	0.0647	0.0517	0.0390	0.0589	0.0477	0.9	0.5	0.0703	0.0611	0.0717	0.0640
	1	0.0554	0.0524	0.0717	0.0571	0.0564	0.0417	0.0583	0.0458		1	0.0789	0.0973	0.0853	0.1059
	2	0.0563	0.0599	0.0662	0.0556	0.0609	0.0445	0.0612	0.0463		2	0.0957	0.1477	0.1117	0.1739

MODEL C

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_{μ}^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_{μ}^2/δ	0	0.5	0	0.5
1	0.5	0.5866	0.6652	0.4279	0.5682	0.2084	0.2678	0.2722	0.3963	0	0.5	0.0510	0.0520	0.0752	0.0638
	1	0.6676	0.7516	0.6115	0.7228	0.3911	0.3788	0.4185	0.4798		1	0.0545	0.0591	0.0938	0.1003
	2	0.7279	0.8464	0.7100	0.8089	0.5404	0.5013	0.5385	0.5254		2	0.0677	0.0898	0.1446	0.2174
0.9	0.5	0.5697	0.6287	0.4002	0.5467	0.1925	0.2235	0.2498	0.3550	0.1	0.5	0.0528	0.0534	0.0783	0.0658
	1	0.6371	0.6974	0.5697	0.6701	0.3522	0.3041	0.3783	0.4023		1	0.0590	0.0645	0.1012	0.1114
	2	0.6913	0.7955	0.6541	0.7451	0.4849	0.3958	0.4911	0.4263		2	0.0772	0.1155	0.1588	0.2550
0.1	0.5	0.0678	0.0549	0.0782	0.0639	0.0483	0.0239	0.0559	0.0407	0.9	0.5	0.0573	0.0568	0.0619	0.0579
	1	0.0642	0.0566	0.0764	0.0597	0.0502	0.0227	0.0588	0.0348		1	0.0632	0.0658	0.0661	0.0686
	2	0.0661	0.0623	0.0711	0.0586	0.0512	0.0240	0.0610	0.0324		2	0.0684	0.0771	0.0701	0.0839

Notes: See Table 7.

Table 10: Empirical Size-Corrected Power (5% actual size), 3 Break, $\lambda = (30\%, 50\%, 70\%)$, $T = 200$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_{μ}^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_{μ}^2/δ	0	0.5	0	0.5
1	0.5	0.5553	0.5908	0.9082	0.9328	0.6868	0.8198	0.7048	0.8309	0	0.5	0.0519	0.0502	0.0702	0.0623
	1	0.5612	0.5915	0.9358	0.9487	0.7266	0.8186	0.7224	0.8127		1	0.0542	0.0610	0.0950	0.1037
	2	0.5916	0.6342	0.9517	0.9579	0.7369	0.8148	0.7259	0.8090		2	0.0642	0.0736	0.1386	0.1851
0.9	0.5	0.3637	0.2944	0.9102	0.9322	0.4620	0.5680	0.4906	0.6104	0.1	0.5	0.0540	0.0513	0.0733	0.0651
	1	0.2933	0.2216	0.9268	0.9074	0.4554	0.5377	0.4572	0.5519		1	0.0575	0.0655	0.1012	0.1137
	2	0.2630	0.2178	0.9115	0.7871	0.4337	0.5263	0.4173	0.5209		2	0.0706	0.0866	0.1522	0.2164
0.1	0.5	0.0516	0.0511	0.0664	0.0573	0.0576	0.0438	0.0598	0.0457	0.9	0.5	0.0745	0.0686	0.0897	0.0838
	1	0.0514	0.0509	0.0594	0.0543	0.0616	0.0481	0.0617	0.0488		1	0.1024	0.1469	0.1374	0.2092
	2	0.0536	0.0566	0.0548	0.0542	0.0643	0.0493	0.0619	0.0494		2	0.1528	0.3730	0.2193	0.5269

MODEL B

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_{μ}^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_{μ}^2/δ	0	0.5	0	0.5
1	0.5	0.4918	0.5233	0.7195	0.7933	0.6438	0.7894	0.6558	0.7954	0	0.5	0.0558	0.0505	0.0777	0.0628
	1	0.5301	0.5649	0.8075	0.8473	0.6954	0.8004	0.6881	0.7923		1	0.0601	0.0606	0.1032	0.1089
	2	0.5448	0.6101	0.8497	0.8591	0.7153	0.7852	0.6910	0.7810		2	0.0684	0.0929	0.1510	0.2248
0.9	0.5	0.3760	0.3275	0.6980	0.7762	0.4707	0.5989	0.4943	0.6259	0.1	0.5	0.0589	0.0516	0.0817	0.0657
	1	0.3346	0.2739	0.7610	0.7598	0.4740	0.5736	0.4771	0.5658		1	0.0661	0.0695	0.1137	0.1224
	2	0.2965	0.2643	0.7634	0.6349	0.4611	0.5310	0.4364	0.5234		2	0.0790	0.1158	0.1697	0.2702
0.1	0.5	0.0532	0.0508	0.0644	0.0592	0.0582	0.0435	0.0598	0.0462	0.9	0.5	0.0878	0.0770	0.0940	0.0831
	1	0.0541	0.0514	0.0604	0.0541	0.0603	0.0457	0.0614	0.0475		1	0.1230	0.1727	0.1361	0.1966
	2	0.0535	0.0564	0.0552	0.0532	0.0647	0.0472	0.0617	0.0482		2	0.1734	0.3845	0.1994	0.4431

MODEL C

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ_{μ}^2/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ_{μ}^2/δ	0	0.5	0	0.5
1	0.5	0.7453	0.8021	0.7065	0.7590	0.4935	0.7245	0.5703	0.7710	0	0.5	0.0498	0.0507	0.0794	0.0670
	1	0.8070	0.8619	0.8203	0.8551	0.6914	0.8065	0.6900	0.8053		1	0.0600	0.0640	0.1165	0.1271
	2	0.8598	0.9167	0.8692	0.8893	0.7823	0.8426	0.7474	0.8225		2	0.0713	0.0989	0.1817	0.2742
0.9	0.5	0.6959	0.6984	0.6505	0.7221	0.4314	0.6145	0.5119	0.6842	0.1	0.5	0.0537	0.0529	0.0838	0.0707
	1	0.7276	0.7125	0.7483	0.7607	0.5951	0.6478	0.5913	0.6646		1	0.0676	0.0741	0.1279	0.1461
	2	0.7662	0.7664	0.7777	0.7446	0.6779	0.6737	0.6367	0.6570		2	0.0874	0.1326	0.2047	0.3288
0.1	0.5	0.0564	0.0540	0.0693	0.0631	0.0577	0.0338	0.0627	0.0428	0.9	0.5	0.0699	0.0665	0.0725	0.0701
	1	0.0564	0.0536	0.0624	0.0557	0.0606	0.0354	0.0635	0.0402		1	0.0888	0.1143	0.0985	0.1286
	2	0.0566	0.0595	0.0594	0.0551	0.0656	0.0403	0.0649	0.0416		2	0.1064	0.1663	0.1232	0.1925

Notes: See Table 7.

Table 11: Empirical Size-Corrected Power (5% actual size), 5 Breaks, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $T = 100$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.6812	0.7526	0.8817	0.9397	0.4566	0.5728	0.5565	0.6910	0	0.5	0.0500	0.0482	0.0686	0.0606
	1	0.7379	0.7733	0.9528	0.9687	0.6296	0.6669	0.6642	0.7316		1	0.0516	0.0546	0.0882	0.0894
	2	0.7473	0.8338	0.9658	0.9777	0.7190	0.7351	0.7190	0.7582		2	0.0553	0.0676	0.1199	0.1683
0.9	0.5	0.5888	0.6230	0.8593	0.9238	0.3315	0.3960	0.4386	0.5406	0.1	0.5	0.0516	0.0495	0.0708	0.0627
	1	0.6033	0.5713	0.9189	0.9235	0.4637	0.4285	0.5041	0.5226		1	0.0547	0.0576	0.0955	0.0982
	2	0.5682	0.6209	0.9143	0.8938	0.5265	0.4767	0.5292	0.5146		2	0.0613	0.0837	0.1306	0.1954
0.1	0.5	0.0567	0.0524	0.0943	0.0708	0.0509	0.0316	0.0581	0.0411	0.9	0.5	0.0627	0.0579	0.0742	0.0673
	1	0.0575	0.0545	0.0774	0.0616	0.0528	0.0333	0.0622	0.0406		1	0.0762	0.0895	0.0971	0.1246
	2	0.0569	0.0619	0.0721	0.0587	0.0577	0.0367	0.0616	0.0427		2	0.0924	0.1538	0.1273	0.2325

MODEL B

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.6113	0.6897	0.7076	0.8329	0.3710	0.5389	0.4755	0.6338	0	0.5	0.0524	0.0500	0.0753	0.0596
	1	0.6689	0.7255	0.8505	0.8992	0.5635	0.6284	0.5828	0.6700		1	0.0556	0.0620	0.0869	0.1008
	2	0.7097	0.8127	0.8953	0.9319	0.6715	0.7019	0.6725	0.7095		2	0.0676	0.0888	0.1374	0.2024
0.9	0.5	0.5498	0.6017	0.6700	0.8160	0.2974	0.4235	0.4051	0.5395	0.1	0.5	0.0551	0.0511	0.0782	0.0623
	1	0.5853	0.5849	0.8068	0.8413	0.4518	0.4543	0.4802	0.5155		1	0.0600	0.0683	0.0936	0.1103
	2	0.5892	0.6596	0.8256	0.8366	0.5370	0.5075	0.5435	0.5238		2	0.0766	0.1128	0.1528	0.2381
0.1	0.5	0.0607	0.0546	0.0891	0.0689	0.0505	0.0322	0.0588	0.0431	0.9	0.5	0.0680	0.0626	0.0732	0.0642
	1	0.0592	0.0548	0.0751	0.0597	0.0529	0.0341	0.0620	0.0401		1	0.0780	0.0953	0.0825	0.1024
	2	0.0616	0.0609	0.0700	0.0575	0.0559	0.0375	0.0612	0.0410		2	0.0962	0.1411	0.1084	0.1642

MODEL C

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.6099	0.7339	0.4023	0.5673	0.2582	0.0559	0.3377	0.2754	0	0.5	0.0513	0.0507	0.0724	0.0594
	1	0.7414	0.8434	0.6174	0.7518	0.3687	0.0777	0.4937	0.3035		1	0.0515	0.0520	0.0929	0.1001
	2	0.8146	0.8943	0.7327	0.8498	0.4917	0.1454	0.6208	0.3031		2	0.0664	0.0941	0.1467	0.2166
0.9	0.5	0.5960	0.7107	0.3766	0.5505	0.2385	0.0390	0.3178	0.2339	0.1	0.5	0.0545	0.0519	0.0739	0.0611
	1	0.7236	0.8116	0.5833	0.7155	0.3341	0.0546	0.4591	0.2509		1	0.0564	0.0584	0.0980	0.1080
	2	0.7987	0.8685	0.6936	0.8073	0.4492	0.0991	0.5862	0.2390		2	0.0765	0.1193	0.1568	0.2454
0.1	0.5	0.0788	0.0671	0.0718	0.0673	0.0370	0.0154	0.0538	0.0279	0.9	0.5	0.0534	0.0535	0.0545	0.0531
	1	0.0782	0.0648	0.0777	0.0641	0.0324	0.0129	0.0563	0.0235		1	0.0574	0.0599	0.0602	0.0654
	2	0.0760	0.0522	0.0778	0.0651	0.0280	0.0126	0.0514	0.0194		2	0.0593	0.0686	0.0627	0.0714

Notes: See Table 7.

Table 12: Empirical Size-Corrected Power (5% actual size), 5 Break, $\lambda = (20\%, 30\%, 50\%, 70\%, 80\%)$, $T = 200$, $\gamma = \phi = 0$

MODEL A

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.7141	0.7674	0.9772	0.9870	0.7543	0.8606	0.7942	0.8812	0	0.5	0.0542	0.0513	0.0703	0.0609
	1	0.7374	0.7792	0.9865	0.9907	0.8203	0.8821	0.8146	0.8858		1	0.0531	0.0579	0.0921	0.1026
	2	0.7612	0.8166	0.9922	0.9934	0.8409	0.8833	0.8269	0.8841		2	0.0621	0.0700	0.1349	0.1790
0.9	0.5	0.5344	0.4945	0.9716	0.9846	0.5427	0.6331	0.6080	0.6906	0.1	0.5	0.0558	0.0520	0.0737	0.0633
	1	0.4820	0.4080	0.9779	0.9677	0.5779	0.6130	0.5761	0.6440		1	0.0570	0.0626	0.0978	0.1140
	2	0.4425	0.4022	0.9679	0.9123	0.5635	0.6015	0.5373	0.6074		2	0.0695	0.0808	0.1497	0.2082
0.1	0.5	0.0529	0.0518	0.0727	0.0637	0.0593	0.0377	0.0625	0.0423	0.9	0.5	0.0741	0.0682	0.0899	0.0809
	1	0.0540	0.0509	0.0641	0.0564	0.0636	0.0422	0.0643	0.0444		1	0.0993	0.1407	0.1332	0.2020
	2	0.0543	0.0576	0.0578	0.0538	0.0656	0.0450	0.0636	0.0474		2	0.1461	0.3509	0.2119	0.4993

MODEL B

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.6784	0.7398	0.9053	0.9421	0.6856	0.8187	0.7341	0.8426	0	0.5	0.0534	0.0505	0.0757	0.0636
	1	0.7195	0.7715	0.9436	0.9589	0.7779	0.8475	0.7757	0.8478		1	0.0573	0.0596	0.1005	0.1071
	2	0.7669	0.8228	0.9617	0.9710	0.8042	0.8509	0.7981	0.8439		2	0.0713	0.0916	0.1500	0.2210
0.9	0.5	0.5569	0.5476	0.8861	0.9310	0.5271	0.6328	0.5935	0.6866	0.1	0.5	0.0571	0.0514	0.0796	0.0656
	1	0.5342	0.4862	0.9100	0.9056	0.5818	0.6146	0.5905	0.6322		1	0.0639	0.0668	0.1094	0.1198
	2	0.5456	0.5113	0.9089	0.8388	0.5818	0.5962	0.5694	0.5889		2	0.0819	0.1134	0.1672	0.2630
0.1	0.5	0.0529	0.0534	0.0726	0.0618	0.0568	0.0392	0.0640	0.0449	0.9	0.5	0.0865	0.0748	0.0916	0.0807
	1	0.0551	0.0527	0.0614	0.0545	0.0623	0.0404	0.0659	0.0443		1	0.1181	0.1695	0.1356	0.1933
	2	0.0566	0.0582	0.0586	0.0549	0.0635	0.0442	0.0669	0.0447		2	0.1768	0.3674	0.2004	0.4274

MODEL C

		CS (2006)						BLS (2001)				WE (2007)			
		CS_{DOLS}		CS_{DGLS}		CS_{FM}		BLS_{CCR}				$WE_{t\text{-stat}}$		WE_{Φ}	
ρ	σ^2_μ/δ	0	0.5	0	0.5	0	0.5	0	0.5	ρ	σ^2_μ/δ	0	0.5	0	0.5
1	0.5	0.9125	0.9474	0.7671	0.8159	0.5379	0.6822	0.6492	0.8018	0	0.5	0.0555	0.0491	0.0794	0.0652
	1	0.9523	0.9738	0.8735	0.9164	0.7874	0.8027	0.8128	0.8550		1	0.0551	0.0610	0.1131	0.1215
	2	0.9738	0.9912	0.9182	0.9432	0.8901	0.8807	0.8848	0.8869		2	0.0729	0.1068	0.1870	0.2925
0.9	0.5	0.8892	0.9126	0.7161	0.7851	0.4794	0.5344	0.5960	0.7008	0.1	0.5	0.0593	0.0508	0.0847	0.0680
	1	0.9324	0.9426	0.8255	0.8596	0.7348	0.6436	0.7653	0.7373		1	0.0634	0.0708	0.1240	0.1388
	2	0.9539	0.9719	0.8631	0.8749	0.8336	0.7374	0.8329	0.7593		2	0.0915	0.1418	0.2097	0.3525
0.1	0.5	0.0651	0.0618	0.0767	0.0652	0.0556	0.0240	0.0618	0.0346	0.9	0.5	0.0669	0.0624	0.0670	0.0647
	1	0.0647	0.0595	0.0671	0.0600	0.0559	0.0220	0.0697	0.0324		1	0.0735	0.0928	0.0786	0.0975
	2	0.0640	0.0680	0.0637	0.0604	0.0577	0.0268	0.0709	0.0321		2	0.0931	0.1297	0.1003	0.1452

Notes: See Table 7.