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A STUDY ON THE 3D INERTIAL INSTABILITY MECHANISM IN THE SUB-MESOSCALE OCEAN

Ayah LAZAR[1,2], Alexandre STEGNER[2] & Eyal HEIFETZ[1]

[1] *Department of Geophysics and Planetary Science, University of Tel Aviv, Tel Aviv 69978, Israel.*

[2] *Laboratoire de Météorologie Dynamique, CNRS, ENS, 24 Rue Lhomond, 75005 Paris, France.*

In the sub-mesoscale ocean vortices tend to be predominantly cyclonic [4]. Inertial instability (hereafter II), which is a centrifugal instability mechanism in the presence of the Coriolis force, is a destructive mechanism that acts only on anti-cyclones, and therefore is hypothesized to cause this asymmetry. Furthermore, since II is in fact the growth of the overturning vorticity, thus creating vertical mixing, it is expected to contribute to nutrient enrichment from the deep, affecting primary production and the oceanic carbon cycle.

Linear stability analysis [6] shows that three-dimensional unstable modes of parallel shear flow (without curvature) may have stronger growth rates than the standard two-dimensional barotropic modes when the absolute vorticity is negative, thus II is more significant than shear instability in the destruction of strong anticyclonic shear. For circular vortices (with curvature), vortex columns are unstable to 3D perturbations where the generalized Rayleigh discriment is negative $\chi(r) \equiv \left(\frac{1}{r}\partial_r(rV) + f\right)(2V/r + f) < 0$, where $V(r)$ is the azimuthal velocity and f the Coriolis parameter, which implies that the region of instability is in the anticyclone periphery [2]. Axis-symmetry seems to be a proper simplification, as it was shown by [1] that the growth rates of axis-symmetric disturbances are larger than non axis-symmetric ones, and the latter are completely stable above a relatively low cut-off azimuthal wave number. Recent fully non-linear numerical simulations show that the area of the instability is breached [5] into the anticyclone core from the outside [3] in the nonlinear phase.

However, for surface oceanic vortices, which are among the most energetic structures of the oceans, one should also take into account the thickness and the stratification of the thermocline. Both the shallow-water constraint and the stratification stabilize these intense anticyclones. The stratification induces a low vertical wave-number cutoff (smaller wavelength) [3, 5]. The small scale perturbations are more sensitive to the vertical dissipation and the growth rate of the unstable modes could be strongly reduced or vanish completely. We apply linear stability analysis to examine 3D II in a vertically confined and stratified, axis-symmetric, mean-flow. We define the dynamical parameters that govern the stability, and find the parameters, which are both insensitive to different vorticity profiles and are relatively easy to estimate from laboratory experiments and oceanic in situ measurements. These are the eddy Rossby number, characterized by the maximal azimuthal velocity and the radius at which this velocity is reached, $Ro = V_{max}/r_{max}f$, which turns out to be a more suitable parameter than, say, the normalized vorticity of the eddy; the aspect ratio, $\delta = h/R$ (which is small $\delta << 1$); the normalized stratification, $S = N/f$; and the vertical Ekman number, $E_k = \nu/fh^2$. In cases of strong stratification, we use the Burger number to combine both the aspect ratio and the stratification. We solve the problem analytically for a Rankine vortex (see figure 1a). For other vorticity profiles an eigenvalue decomposition is used, which we confirmed by recalculating for the Rankine vortex. We map the parameter space (see figure 1b), and corroborate our findings (see figure 2) with large-scale laboratory experiments studies, performed at the LEGI-Coriolis platform.

References

[1] P. Billant and F. Gallaire. generalized rayleigh criterion for non-axisymmetric centrifugal instabilities. *Journal of Fluid Mechanics*, 542:365–379, 2005.

[2] R. C. Kloosterziel, G. F. Carnevale, and P. Orlandi. Inertial instability in rotating and stratified fluids: barotropic vortices. *Journal of Fluid Mechanics*, 583:379, July 2007.

[3] R. C. Kloosterziel, G. F. Carnevale, and P. Orlandi. Saturation of inertial instability in rotating planar shear flows. *Journal of Fluid Mechanics*, 583:413, July 2007.

[4] Walter Munk, Laurence Armi, Kenneth Fische, and F. Zachariasen. Spirals on the sea. *Proceedings: Mathematical, Physical and Engineering Sciences*, 456(1997):1217–1280, May 2000.

[5] Riwal Plougonven and Vladimir Zeitlin. Nonlinear development of inertial instability in a barotropic shear. *Physics of Fluids*, 21(10):106601, 2009.

[6] Shinichiro Yanase, Carlos Flores, O. Métais, and J.J. Riley. Rotating free-shear flows. I. Linear stability analysis. *Physics of Fluids A: Fluid Dynamics*, 5(November):2725, 1993.

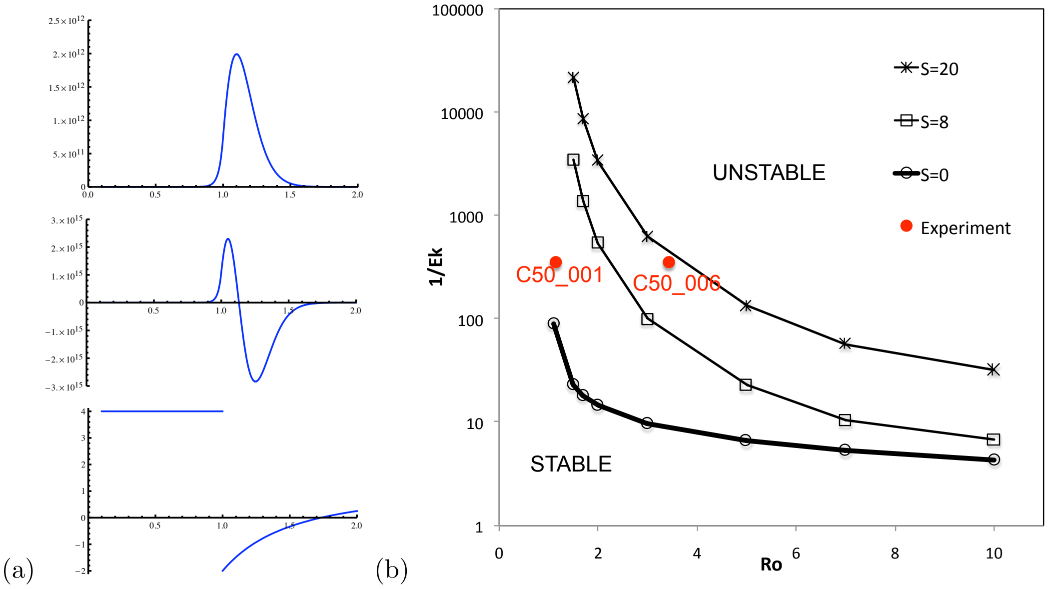

Figure 1: Panel (a) - First (top) and second (middle) horizontal modes for a sample Rankine vortex ($Ro = -3$). This is the solution of a modified bessel equation of order unity inside the vortex and of imaginary order outside. On the bottom is the χ parameter, and it is evident that the instability, in both modes, is localized to the area where $\chi < 0$. Panel (b) - Marginal stability curves ($\sigma = 0$) - on the ($1/E_k$, $|Ro|$) plane, for three stratification values $S = 0, 8, 20$ and $\delta = 0.2$. Not shown are curves for different types of non-isolated vortices, which on this parameter space, are very close to the ones shown for the Rankine vortex. The parameters of experiments $C50\_001$ and $C50\_006$ from the experiments performed at the LEGI-Coriolis platform are both marked with red dots. Both are with $\delta = 0.2$ and S 8, which means that $C50\_001$ should be stable and $C50\_006$ should be unstable as they are on either side of the marginal stability curve. See figure 2 for the results.

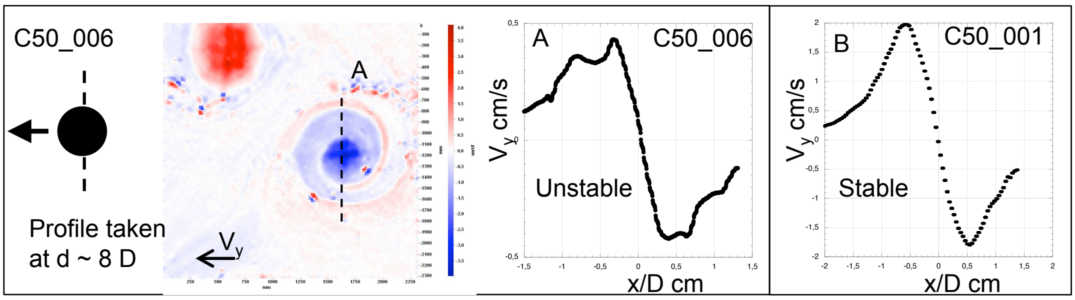

Figure 2: A good case study for II is expected in Island wakes, where the forcing that generates the anticyclones is strong. Then they are created, but destructed far more rapidly than their cyclonic counterparts in the wake. For that reason, we conducted experiments of island-wake vortices instability on the 14-meter diameter rotating platform at the LEGI-Coriolis in Grenoble with quantitative PIV measurements of the wake flow. On the left, vorticity field of experiment $C50\_006$ calculated from the PIV. The cylinder and its direction is shown schematically. Small scale patterns that can easily be seen with dye cannot be detected by 2D PIV measurements, therefore the signature of the II is extracted by the mesoscale evolution of individual eddies, in this case the velocity profiles of anticyclones taken when the vortex is at a distance of eight times the diameter of the cylindrical island model (in experiment $C50\_006$ the profile is taken where the line A is indicated). Panel (A) and (B) show the velocity profiles for experiments $C50\_006$ and $C50\_001$ respectively. Experiment $C50\_001$ is predicted by its place on the stability diagram (1b) to be stable and indeed, the anticyclone in this experiment is unperturbed. Experiment $C50\_006$ is predicted to be unstable and indeed the anticyclone is perturbed at the edges, where the parameter $\chi < 0$ (the cyclones, not shown, in all cases are unperturbed as the instability is asymmetric).