Faculty Working Papers

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Transportation Research Paper No. 12

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There is a substantial mileage of light traffic railroad lines in the United States and yet little is known of the cost behavior of these roads. Knowledge of their cost functions is necessary to determine to what extent these light density lines can be made more economically viable by a traffic increase. The economic viability of a light traffic road depends to a large extent upon the manner in which costs react to a change in the volume of traffic. If total costs change very little with an increase in volume (that is, marginal cost is very low), then improvement of traffic will have a substantial impact upon average cost and therefore upon profitability.

This paper seeks to determine the short run responsiveness of various cost categories to volume changes. Specifically, this paper is concerned with the short run cost functions of a sample of ten Class II[1] railroads for the period 1963-1973. Other work has previously been reported on the long-run cost functions for this group.[2]

The first part of this paper describes the statistical work used to estimate the short run cost functions of these railroads and the estimates of the average costs derived therefrom. Then, elasticities are derived to

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[1] Class II railroads are those with annual gross revenues of less than $5 million.

[2] Nancy D. Sidhu, Alberta H. Charney, and John F. Due, "Cost Functions for Class II Railroads," Faculty Working Papers, No. 262, College of Commerce and Business Administration, University of Illinois at Urbana-Champaign, 1975.

indicate the magnitude of the reductions in average costs that result from an increase in traffic density.

I

The model. Traditionally, economists divide a firm's total costs in the short run into a fixed component that does not vary with changes in output and a variable component that does. This classification is, however, inadequate when applied to railroads. The usual examples of fixed costs do exist in railroading: depreciation of equipment, interest charges, some taxes (e.g., property taxes), some minimal administrative costs (necessary to hold the firm together), and a normal rate of return on capital. However, the variable cost component requires disaggregation into three groups. The first, or constant variable costs, are costs that can be eliminated if the railroad is not operating (their variable dimension) but change very little if at all when railroad output changes (their constant dimension). A major item in this group is minimum track maintenance cost. The second group of variable costs are more or less constant, in total, per train and, therefore, vary only with increments of output large enough to require operation of more trains. For example, the size of a train crew does not vary with the number of cars but the man-hours vary with the number of trains operated per time period. The third group of variable costs includes the usual variety that vary directly with the level of traffic. These include part of the fuel costs, any taxes related to traffic levels, per diem charges and switching costs, if additional traffic comes from different sidings.

This brief classification scheme implies that, aside from the fixed costs, total railroad costs can be expected to vary with volume but less

than proportionately. More tonnage requires more assembly time, etc., and if great enough, more trains. Because some costs are related to terminal operations, independent of the length of haul, it would also be expected that increased length of haul would not increase costs proportionately.

Let us, therefore, begin with a function determining total costs (TC):

(1) $TC = f$ (tons, average length of haul)

Dividing both sides by the unit of output ton-miles (T-M):

$$(2) \quad AC = \frac{TC}{T\text{-}M} = \frac{f(\text{tons, average length of haul})}{T\text{-}M}.$$

We choose a linear specification of (2) to get:

$$(3) \quad AC = a + b_1 \frac{\text{average length of haul}}{T\text{-}M} + b_2 \frac{\text{tons}}{T\text{-}M}$$

$$= a + b_1 \frac{1}{V} + b_2 \frac{1}{D},$$

where V = volume, or ton-miles divided by average length of haul, and

D = distance, or ton-miles divided by total tons.

Equation (3) represents one of the functional forms used in estimation of short run statistical average cost functions in this paper.

The use of equation (3) assumes that average costs approach an asymptote, a, as V and D increase. To avoid this restriction, we also ran each regression presented in this paper in a semi-log form:

$$(3') \quad AC = a + b_1 \ln V + b_2 \ln D.$$

Results of these regressions are summarized below. It will become evident that differences between the two models are not great over the observed ranges of output.

It was expected that the signs of b_1 and b_2 would be positive for the inverse model and negative for the semi-log model, indicating that average costs per ton-mile decline as volume or distance or both increase.

The data. Data for this study were compiled from annual reports filed by ten railroads (listed on Table 1) with the Interstate Commerce Commission for the years 1963 through 1973.[3] We collected data and calculated figures for total tons, ton-miles, average length of haul, and the eighteen cost items that are listed in Table 2 for each year for each of the ten roads.

To avoid biasing the results on account of inflation during the sample time period, all items that required it were deflated. We tried to match the index used as a deflator as closely as possible to the variable to be deflated. For example, all cost components involving wages were deflated by an index of "wages--excluding supplements," train fuel by an index of "fuel costs," property taxes by a weighted average of relevant property tax deflators.[4] Further details on the deflating procedures are available from the authors upon request.

Because of the number of observations on each road is relatively small, we had hoped to pool the data to increase the efficiency of our estimates.

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[3] Our heartfelt thanks go to Mr. Robert Byrne, formerly with the Rail Services Planning Office, I.C.C., who arranged to make copies of the annual reports available to us.

[4] Association of American Railroads, Yearbook of Railroad Facts, 1974 edition, was the source of the wage, fuel, and "other materials and supplies" deflators. Where appropriate we also used as deflators the wholesale price index, the federal unemployment index, and the federal retirement index, and property tax indices of the various states assessing railroad property belonging to roads in our sample.

TABLE 1

RAILROADS IN THE SAMPLE

Line No.	Railroad Name	Abbr.	Median V	Median D
1	Arcade & Attica	A&A	17.193	4.837
2	Amador	AMAD	102.253	11.790
3	Apache	APAC	515.560	43.258
4	Bellefonte Central	BELC	59.303	5.611
5	Cadiz	CAD	27.147	10.000
6	City of Prineville	CoP	359.799	18.340
7	Corinth & Counce	C&C	876.424	10.000
8	Hillsboro & Northeastern	HNE	19.160	5.000
9	Mississippi Export	MISS	795.121	38.413
10	Pecos Valley Southern	PVS	48.736	13.925

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*Distance is measured in miles; volume is measured in thousands of tons.

TABLE 2

RAILROAD COSTS USED AS DEPENDENT VARIABLES

Type of Cost	Abbr.
I. Total Operating Costs	C_1
A. Maintenance of Way Costs	C_{1a}
1. Roadway maintenance	C_{1a_1}
2. Other maintenance of way costs	C_{1a_2}
B. Maintenance of Equipment Costs	C_{1b}
1. Locomotive repairs	C_{1b_1}
2. Equipment depreciation	C_{1b_2}
3. Other maintenance of equipment costs	C_{1b_3}
C. Transportation-Rail Line Costs	C_{1c}
1. Employee compensation (of train crews)	C_{1c_1}
2. Train fuel costs	C_{1c_2}
3. Costs of loss, damage, casualties, and personal injuries	C_{1c_3}
4. Other transportation costs	C_{1c_4}
D. Traffic, Administrative, and Miscellaneous Costs	C_{1d}
II. Other Expenses	E_{rtc}
A. Equipment Rentals	E_r
B. Rate of return calculated on railroad equipment	E_c
C. Tax Payments	E_t
III. Total Railroad Costs (I plus II)	ALL

Because we expected correlations to exist between the disturbances of equations for different railroads, an attempt was made to utilize a generalized least squares (GLS) estimator proposed by Zellner for situations involving what he calls seemingly unrelated regressions.[5] This estimator also has the desirable small sample properties of unbiasedness and efficiency relative to the ordinary least squares (OLS) estimator.[6]

However, an unexpected computational snag developed that made it impossible to use the GLS procedure. The many matrix manipulations required for GLS resulted in computer roundoff errors which were large enough to result in slightly negative numbers in the diagonal of the variance-covariance matrix--an impossibility. Rather than push the data farther than it could legitimately go, we therefore are presenting here only the ordinary least squares (OLS) results for each railroad. Comparisons with the few successful GLS runs indicate that the OLS parameter estimates are substantially the same, as is to be expected because the OLS and GLS estimators are both unbiased. However, the OLS standard errors are larger, and therefore the OLS estimators are less efficient.

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[5] The original article is A. Zellner, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," Journal of the American Statistical Association, LVII: 2 (June 1962), pp. 348-368; or see the discussion in Jan Kmenta, Elements of Econometrics (New York: The Macmillan Company, 1971), pp. 517-529.

[6] Small sample properties of these estimators are worked out in A. Zellner, "Estimators of Seemingly Unrelated Regressions: Some Exact Finite Sample Results," Journal of the American Statistical Association, LXIII: 4 (December 1968), pp. 1180-1200.

Three railroads in our sample--the City of Prineville, Corinth & Counce, and Hillsboro & Northeastern--showed little or no variation in their distance figures over the sample years; so we had to run their regressions in the following single variate form:

$$(4) \quad AC = a + b \left(\frac{1}{V}\right), \text{ and}$$

$$(4') \quad AC = a + b \ln V.$$

We also ran the other seven roads using equations (4) and (4') for purposes of comparison. We shall indicate where the differences between the two versions are large.

The results.

Examination of the results shows that the overall quality of the estimated cost functions varied widely. The best results of the group are those for the Mississippi Export and the Pecos Valley Southern, and the worst are for the Apache.

In the single variate analysis, most of the volume coefficients have the correct sign. Sign reversals tended to occur mostly in the smallest cost components like C_{1b_3} rather than in the larger divisions like C_{1b}, and only four of these are significantly different from zero. Model II (equation (4)) is preferred over Model I (equation (4')) in slightly more than one-half of the cost equations. This preference is based largely on sign reversals and significance of the b coefficients and the values of F since the estimated amount of Model I serial correlation, shown by the Durbin-Watson statistic in the last column, is almost always close to that of the Model II version.

The equations incorporating both volume and distance show the following differences from the single variable:

(1) Addition of the distance variable tends to increase slightly the number of sign reversals of the volume coefficient b_1;

(2) At the same time, it also decreases the number of significant volume coefficients. This phenomenon appears more or less at random except for equipment rentals (E_r); here, b_1 was reduced to insignificance in four of the seven roads we compared, and we can conclude that the bivariate specification may well be incorrect in this case. On the other hand,

(3) Well over one-half of the distance, or average length of haul, coefficients, the b_2s, are of the wrong sign, and

(4) Only about one-eighth of the b_2s are significantly different from zero. The significant b_2s also appeared at random, though about one-third of them occurred with the various maintenance of way costs. This finding leads to the contrary suspicion that for a few cost items, the bivariate specification is the correct one.

Of more interest are the descriptive uses to which these equations can be put. We calculated the average cost per ton-mile of each road, for a year in which it experienced its median volume level and median average length of haul. These costs are displayed in Tables 3 and 4, calculated from the tables for the individual road. Several items are of note:

(1) The costs estimated in Table 4 are remarkably close to those shown in Table 3 with the exception of the Arcade & Attica, for which the single variate model estimates average total costs that are about two cents per ton-mile less than those of the bivariate model. This difference is due to the significant distance variable for Arcade & Attica. As expected, most of the two cents difference is accounted for in the maintenance of way

TABLE 3

Estimated Median Costs per Ton-Mile
for Sample Railroads--Volume Only *

	Type of Cost	Model	(1)** A&A	(6) AMAD	(8) APAC	(5) BELC	(3) CADIZ	(7) CoP
1	C_1	I	$.4935	$.0892	$.0201	$.1494	$.1786	$.0413
2		II	.4652	.0889	.0200	.1479	.1758	.0408
3	C_{1a}	I	.1275	.0442	.0084	.0270	.0826	.0102
4		II	.1192	.0441	.0083	.0267	.0840	.0101
5	C_{1a_1}	I	.0783	.0420	.0066	.0155	.0732	.0081
6		II	.0725	.0420	.0065	.0154	.0750	.0080
7	C_{1a_2}	I	.0492	.0022	.0018	.0114	.0095	.0021
8		II	.0467	.0022	.0018	.0113	.0090	.0021
9	C_{1b}	I	.0818	.0115	.0031	.0211	.0226	.0065
10		II	.0806	.0112	.0030	.0207	.0217	.0064
11	C_{1b_1}	I	.0398	.0051	.0013	.0035	.0053	.0049
12		II	.0389	.0050	.0013	.0034	.0054	.0049
13	C_{1b_2}	I	.0099	.0038	.0002	.0116	.0058	.0010
14		II	.0097	.0038	.0002	.0112	.0058	.0010
15	C_{1b_3}	I	.0308	.0024	.0015	.0060	.0115	.0005
16		II	.0308	.0024	.0015	.0060	.0105	.0005
17	C_{1c}	I	.1685	.0246	.0067	.0611	.0549	.0196
18		II	.1565	.0246	.0066	.0609	.0515	.0194
19	C_{1c_1}	I	.0993	.0122	.0028	.0293	.0252	.0112
20		II	.0916	.0122	.0028	,0290	.0238	.0111
21	C_{1c_2}	I	.0110	.0027	.0010	.0018	.0056	.0012
22		II	.0102	.0027	.0010	.0018	.0057	.0012
23	C_{1c_3}	I	.0092	.0013	.0012	.0033	.00004	.0012
24		II	.0089	.0013	.0012	.0033	.00003	.0012
25	C_{1c_4}	I	.0490	.0085	.0017	.0267	.0241	.0060
26		II	.0458	.0084	.0017	.0268	.0221	.0059
27	C_{1d}	I	.1157	.0089	.0020	.0403	.0184	.0050
28		II	.1089	.0089	.0020	.0396	.0186	.0049
29	E_r	I	.0212	.0153	.0051	.0146	.0027	.0039
30		II	.0180	.0153	.0050	.0150	.0024	.0039
31	E_c	I	.0268	.0058	.0011	.0091	.0205	.0043
32		II	.0252	.0058	.0011	.0089	.0191	.0043
33	E_t	I	.0368	.0081	.0032	.0175	.0166	.0028
34		II	.0358	.0081	.0032	.0171	.0157	.0027
35	ALL	I	.5763	.1184	.0294	.1905	.2185	.0523
36		II	.5424	.1181	.0292	.1888	.2131	.0518

(continued on the next page)

TABLE 3 CONTINUED

Type of Cost	Model	(10) C&O	(2) HNE	(9) MISS	(4) PVS
C_1	I	$.0220	$.1689	$.0217	$.0886
	II	.0220	.1677	.0212	.0853
C_{1a}	I	.0066	.0434	.0067	.0265
	II	.0066	.0447	.0065	.0253
C_{1a_1}	I	.0043	.0170	.0049	.0137
	II	.0043	.0181	.0047	.0129
C_{1a_2}	I	.0024	.0264	.0019	.0128
	II	.0024	.0266	.0018	.0125
C_{1b}	I	.0027	.0128	.0027	.0105
	II	.0023	.0125	.0026	.0100
C_{1b_1}	I	.0007	.0016	.0010	.0018
	II	.0007	.0017	.0009	.0018
C_{1b_2}	I	.0013	.0062	.0009	.0051
	II	.0013	.0055	.0008	.0048
C_{1b_3}	I	.0008	.0051	.0008	.0033
	II	.0008	.0053	.0008	.0032
C_{1c}	I	.0091	.0574	.0100	.0344
	II	.0090	.0572	.0098	.0332
C_{1c_1}	I	.0059	.0089	.0012	.0165
	II	.0058	.0093	.0011	.0160
C_{1c_2}	I	.0007	.0005	.0002	.0006
	II	.0007	.0006	.0002	.0006
C_{1c_3}	I	.0009	.0004	.0016	.0032
	II	.0009	.0004	.0016	.0032
C_{1c_4}	I	.0016	.0476	.0070	.0141
	II	.0016	.0470	.0069	.0135
C_{1d}	I	.0036	.0553	.0022	.0173
	II	.0035	.0533	.0022	.0167
E_r	I	.0075	.0315	.0075	.0039
	II	.0074	.0312	.0076	.0037
E_c	I	.0012	.0283	.0009	.0088
	II	.0012	.0267	.0009	.0086
E_t	I	.0037	.0123	.0022	.0087
	II	.0037	.0124	.0022	.0084
ALL	I	.0345	.2410	.0323	.1101
	II	.0343	.2379	.0319	.1061

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*The median costs were estimated using each road's median volume level as the value of the independent variable in the equations of Appendix Tables 1 through 11.

**The numbers above the columns indicate the ordering of the railroads by median volume.

TABLE 4

Estimated Median Costs per Ton-Mile for
Sample Railroads--Volume and Distance*

	Type of Cost	Model	(1)** A&A	(5) AMAD	(6) APAC	(4) BELC	(2) CADIZ	(7) MISS	(3) PVS
1	C_1	I	$.5027	$.0902	$.0188	$.1493	$.1787	$.0216	$.0900
2		II	.4848	.0889	.0190	.1480	.1754	.0211	.0867
3	C_{1a}	I	.1366	.0441	.0077	.0269	.0818	.0067	.0272
4		II	.1327	.0441	.0078	.0267	.0833	.0064	.0260
5	C_{1a_1}	I	.0860	.0418	.0060	.0155	.0721	.0048	.0145
6		II	.0830	.0419	.0060	.0154	.0741	.0046	.0137
7	C_{1a_2}	I	.0507	.0023	.0017	.0114	.0096	.0019	.0127
8		II	.0498	.0022	.0017	.0113	.0091	.0018	.0123
9	C_{1b}	I	.0853	.0123	.0029	.0211	.0227	.0027	.0104
10		II	.0853	.0113	.0029	.0207	.0217	.0026	.0100
11	C_{1b_1}	I	.0426	.0057	.0013	.0035	.0052	.0010	.0018
12		II	.0425	.0050	.0013	.0034	.0052	.0009	.0018
13	C_{1b_2}	I	.0098	.0039	.0002	.0116	.0055	.0009	.0052
14		II	.0098	.0038	.0002	.0112	.0055	.0008	.0049
15	C_{1b_3}	I	.0315	.0026	.0013	.0060	.0120	.0008	.0033
16		II	.0317	.0025	.0014	.0061	.0110	.0008	.0032
17	C_{1c}	I	.1670	.0246	.0063	.0611	.0562	.0100	.0346
18		II	.1578	.0246	.0064	.0609	.0524	.0099	.0335
19	C_{1c_1}	I	.0977	.0120	.0027	.0293	.0252	.0012	.0166
20		II	.0917	.0122	.0027	.0290	.0235	.0012	.0162
21	C_{1c_2}	I	.0107	.0026	.0010	.0018	.0055	.0002	.0006
22		II	.0101	.0027	.0010	.0018	.0056	.0002	.0006
23	C_{1c_3}	I	.0088	.0014	.0010	.0033	.00005	.0016	.0030
24		II	.0086	.0013	.0011	.0033	.00004	.0016	.0030
25	C_{1c_4}	I	.0497	.0087	.0015	.0267	.0254	.0070	.0143
26		II	.0475	.0085	.0016	.0268	.0232	.0069	.0137
27	C_{1d}	I	.1133	.0091	.0020	.0403	.0180	.0022	.0178
28		II	.1089	.0089	.0020	.0396	.0181	.0022	.0172
29	E_r	I	.0230	.0149	.0051	.0146	.0028	.0074	.0037
30		II	.0213	.0153	.0050	.0150	.0025	.0075	.0035
31	E_c	I	.0265	.0058	.0011	.0091	.0207	.0009	.0088
32		II	.0254	.0058	.0010	.0089	.0190	.0009	.0086
33	E_t	I	.0384	.0080	.0031	.0175	.0170	.0022	.0088
34		II	.0381	.0081	.0031	.0170	.0160	.0021	.0085
35	ALL	I	.5887	.1189	.0280	.1905	.2191	.0321	.1114
36		II	.5678	.1181	.0282	.1888	.2129	.0315	.1074

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*The median costs were estimated using each road's median levels of volume and distance as the values of the independent variables in the equations of Appendix Tables 12 through 18.

**The numbers above the columns indicate the ordering of the railroads by median volume.

category, C_{1a} (with about one cent due to different estimates of roadway maintenance (C_{1a_1}) alone), and most of the rest shows up in equipment maintenance, C_{1b}, particularly locomotive repairs, C_{1b_1}.

(2) If the roads are ordered according to median volume level, and their average total costs are compared, the expected pattern (AC falls as V rises) appears. We calculated the Spearman rank correlation coefficient[7] for these two variables and found it to be significantly different from zero even at the .001 level. This relation is also displayed graphically in Figures 3 and 4.

(3) If the ten roads are again ordered by median volume, a tendency exists for the ratio of average operating costs to average total costs (C_1/ALL) to fall as median volume rises. The Spearman rank correlation coefficient is significant at the .01 level for this comparison. However, no one component of operating costs displays this characteristic to any noticeable degree.

(4) Opposite tendencies exist for the ratios of average returns to capital to average total cost (E_c/ALL) and for average rentals to average total cost (E_r/ALL). The rental-to-cost ratio tends to rise as median volume rises, while the return-to-cost ratio tends to fall.

Another method of displaying these results is graphical. On Figures 1 and 2 we have plotted the average total cost curves for all ten railroads in the sample. The lengths of the curved segments indicate the range of volume observed for each railroad in the sample time period.

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[7] For a discussion of the uses and calculation of this statistic and for tables of significance for small samples, see Sidney Siegel, *Nonparametric Statistics for the Behavioral Sciences* (New York: McGraw-Hill Book Co., 1956), pp. 202-213, 284.

FIGURE 1

ESTIMATED AVERAGE TOTAL COST FUNCTIONS
FOR EACH SAMPLE RAILROAD--MODEL I

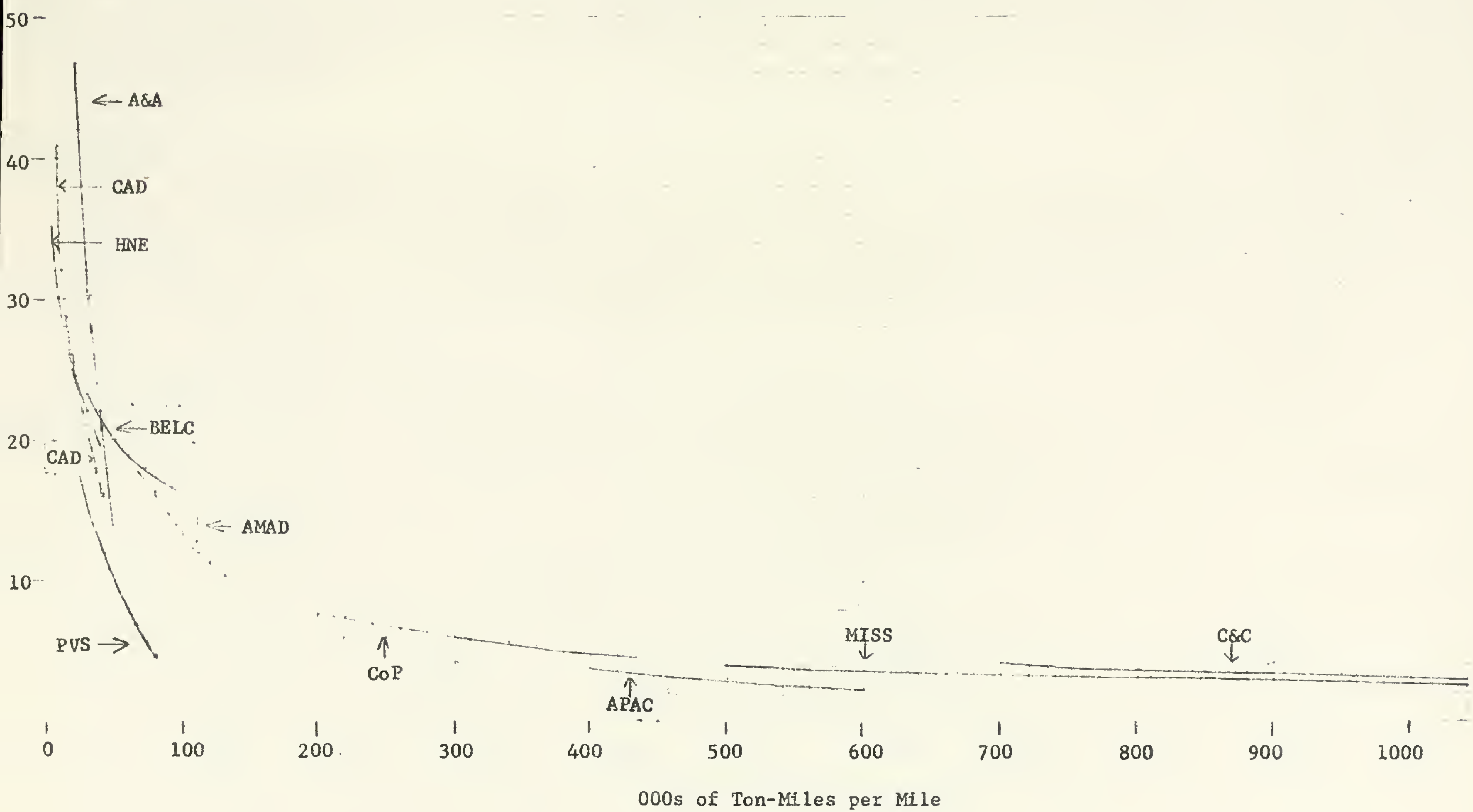
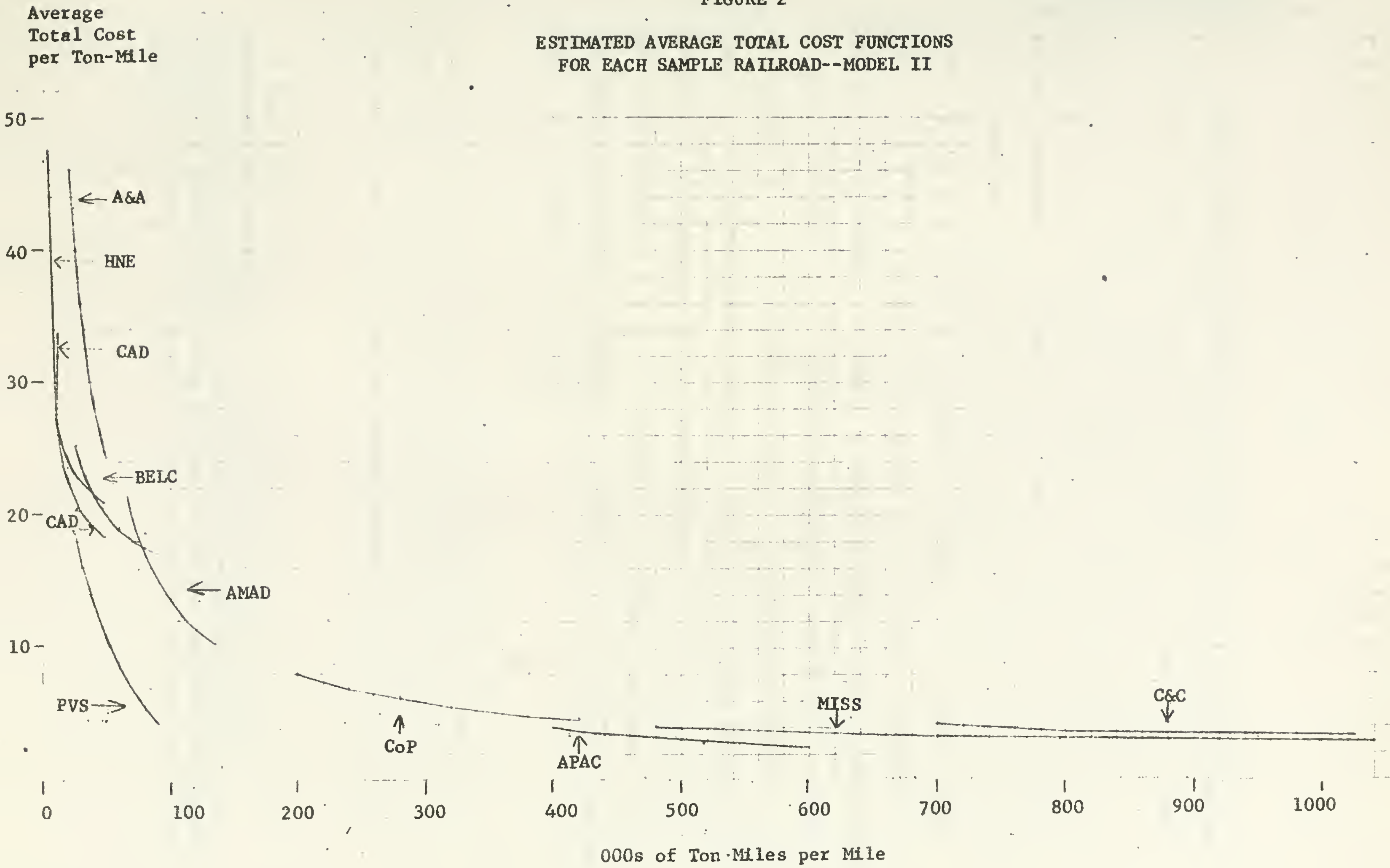


FIGURE 2

ESTIMATED AVERAGE TOTAL COST FUNCTIONS
FOR EACH SAMPLE RAILROAD--MODEL II



These curves are plotted using the single variate models of equations (4) and (4') because we did not have a complete set of equations (3) and (3') for all ten roads. However, the role of distance is indirectly observable in the following way: Take any volume level experienced by two or more railroads. If X length of haul has any effect on costs, then the road(s) with the longer median length of haul (from Table 1) should exhibit lower average total costs per ton-mile on Figures 1 and 2. Indeed, with but a few possible exceptions, this proves to be the case. As examples, consider the following: The Arcade & Attica had the shortest median length of haul in the sample and the highest average total cost curve; the Pecos Valley Southern had a longer median distance and a lower average total cost curve per ton-mile than did the Amador; the same is true for the Apache compared with the City of Prineville or the Mississippi Export. The only exceptions to this general relation are those cases where the average total cost curves for two roads intersect, so that the conclusions to be drawn about X average length of haul cannot be clearcut.

II

For each cost item of each railroad, the elasticity of total cost with respect to ton-miles was calculated at the median volume levels. The elasticities were derived from the single variable equations[8] as follows:

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[8] For several reasons, only the single variate models were used for calculating the elasticities. Not only was the distance variable insignificant for many cost items, but it also caused some of the volume variables to change signs. Also, for some of the ten railroads the distance variable could not be used because there was little or no variation in that variable.

For the semi-log model (using equation 4'), we have average cost:

$$AC = \frac{TC}{T\text{-}M} = a = b \ln V = a + b \ln V$$

where AC = average cost

TC = total cost

T-M = ton-miles

M = miles of the road.

Then $$TC = a(T\text{-}M) + b(T\text{-}M)\ln(T\text{-}M/M)$$

$$MC = \frac{d\ TC}{d\ T\text{-}M} = a + b\left[\ln(T\text{-}M/M) + b\right]$$

$$= a + b \ln(T\text{-}M/M) + b$$

$$= AC + b$$

Therefore for Model I,

$$e_{TC} \cdot T\text{-}M = \frac{d\ TC}{d\ T\text{-}M} \cdot \frac{T\text{-}m}{TC}$$

$$= (AC + b) \circ \frac{1}{AC}$$

$$= \frac{MC}{AC}$$

For the inverse model (using equation 4), we have average cost:

$$AC = \frac{TC}{T\text{-}M} = a + b(1/V) = a + b\ (M/T\text{-}M).$$

Then $$TC = a(T\text{-}M) + bM$$

$$MC = \frac{d\ TC}{d\ T\text{-}M} = a.$$

Therefore for Model II,

$$e_{TC} \cdot T\text{-}M = \frac{d\ TC}{d\ T\text{-}M} \circ \frac{T\text{-}M}{TC}$$

$$= a \circ \frac{T\text{-}M}{TC}$$

$$= \frac{MC}{AC}.$$

Note that the elasticities are equal to marginal cost/average cost for both models. When marginal cost is very small compared to average cost, an increase in the traffic on that road should substantially reduce average cost and therefore substantially improve the profitability of the road. Therefore, the elasticities measure the extent to which roads can be made more economically viable by a traffic increase.

Table 5 contains the calculated elasticities for each railroad and each cost category. Observe that:

(1) The roads are ordered with respect to their median volume levels to see if the elasticities of light density roads are different from the elasticities of the heavy density roads. Examination of the elasticities indicates that the elasticities are independent of the volume of traffic of the road. The correlation between volume and the elasticities of the "ALL" variables is -.166 (almost zero). Low elasticities appear for some high volume roads as well as low volume roads.

(2) Some of the elasticities of certain cost items are greater than unity, implying that as volume increases, total cost increases more than proportionately. This occurred most frequently for the maintenance of way costs (C_{1a}) (including both subcategories and roadway maintenance (C_{1a_1}), "other" maintenance of way costs (C_{1a_3})),[9] and costs of loss, damage, casualties and personal injuries (C_{1c_3}).

(3) Some of the elasticities are negative. For model II, this occurs when the sign of the coefficient a is not of the expected sign. Most of

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[9] This effect would likely vanish if a period of years was used instead of a single year. When volume increases noticeably, roads try to catch up on badly deferred maintenance.

TABLE 5

ELASTICITIES, EACH COST CATEGORY, EACH RAILROAD

			Group 1	Group 2	Group 2	Group 1	Group 2
	Type of Cost	Model	A&A	HNE	CADIZ	PVS	BC
1	C_1	I	0.297	0.723	0.637	-0.154	0.678
2		II	0.170 a	0.809	0.780	-0.193 a	0.718
3	C_{1a}	I	0.213	1.352	1.007	-0.294	0.770
4		II	0.120 a	1.426	1.046	-0.339	0.755
5	C_{1a_1}	I	0.095	2.296	1.113	-0.754	0.769
6		II	-0.051 a	2.212	1.125	-0.876	0.750
7	C_{1a_2}	I	0.399	0.745	0.032	0.196	0.769
8		II	0.385 a	0.891 a	0.384	0.227 a	0.764
9	C_{1b}	I	0.836	0.636	0.280	-0.215	0.453
10		II	0.901	0.668 a	0.530	-0.221 a	0.499
11	C_{1b_1}	I	0.737	1.579	1.111	0.644	0.752
12		II	0.909	1.762 a	1.072 a	0.652 a	0.780 a
13	C_{1b_2}	I	0.796	-0.628	0.435	-0.792	-0.037
14		II	0.828	-0.995 a	0.711 a	-0.830	0.039 a
15	C_{1b_3}	I	0.989	1.883	-0.182	0.149	1.225
16		II	1.069	1.806 a	0.144 a	0.158 a	1.189
17	C_{1c}	I	0.117	0.795	-0.038	-0.053	0.903
18		II	-0.114	0.855	0.294	-0.104 a	0.923
19	C_{1c_1}	I	0.035	1.362	0.090	0.012	0.626
20		II	-0.239	1.495 a	0.378	-0.041 a	0.658
21	C_{1c_2}	I	0.089	4.260	1.248	0.027	0.881
22		II	-0.195 a	3.377	1.154	0.010 a	0.826
23	C_{1c_3}	I	0.601	3.770	-0.300	0.560	1.475
24		II	0.497	3.428	-0.256 a	0.649 a	1.466
25	C_{1c_4}	I	0.200	0.627	-0.469	-0.270	1.143
26		II	0.035 a	0.675 a	-0.014 a	-0.352 a	1.150
27	C_{1d}	I	0.270	0.358	1.431	-0.098	0.395
28		II	0.090 a	0.299 a	1.224	-0.139 a	0.455
29	E_r	I	-0.842	0.860	-0.471	-0.313	1.922
30		II	-1.356 a	0.771	-0.033 a	-0.362 a	1.789
31	E_c	I	0.296	-0.056	-0.200	0.062	0.205
32		II	0.131	-0.158 a	0.177	0.051	0.289
33	E_t	I	0.691	1.099	-0.091	-0.059	0.219
34		II	0.765 a	1.097	0.294 a	-0.104 a	0.264 a
35	ALL	I	0.281	0.698	0.490	-0.134	0.709
36		II	0.156 a	0.752	0.681	-0.171 a	0.734

TABLE 5 continued

	Type of Cost	Model	Group 1 AMAD	Group 1 CoP	Group 1 APACHE	Group 2 MISS	Group 1 C&C
1	C_1	I	-0.009	0.066	-0.548	0.114	0.611
2		II	-0.031 a	0.163 a	-0.452 a	0.150 a	0.609
3	C_{1a}	I	0.401	-0.015	-0.740	-0.350	1.218
4		II	0.392 a	0.092 a	-0.648 a	-0.307 a	1.206
5	C_{1a_1}	I	0.484	-0.024	-1.103	-0.246	1.453
6		II	0.475 a	-0.072 a	-1.003 a	-0.233 a	1.441
7	C_{1a_2}	I	-0.992	0.713	0.589	-0.547	0.787
8		II	-0.978 a	0.719 a	0.637 a	-0.501	0.776
9	C_{1b}	I	-1.309	-0.118	-1.250	-0.258	1.887
10		II	-1.533 a	0.008 a	-1.176 a	-0.228 a	1.792
11	C_{1b_1}	I	-4.268	-0.322	-1.128	-0.397	0.221
12		II	-4.595 a	-0.149 a	-0.994 a	-0.478 a	0.193 a
13	C_{1b_2}	I	0.203	0.326	0.550	-0.350	1.207
14		II	0.129 a	0.401 a	0.560 a	-0.409 a	1.160 a
15	C_{1b_3}	I	2.489	0.774	-2.432	-0.011	2.339
16		II	2.281 a	0.756 a	-1.578 a	0.023 a	2.218
17	C_{1c}	I	-0.196	0.366	-0.254	0.444	0.041
18		II	-0.158 a	0.429	-0.175 a	0.472	0.045 a
19	C_{1c_1}	I	-0.663	0.493	-0.505	0.385	-0.021
20		II	-0.590 a	0.539	-0.408 a	0.370 a	-0.021 a
21	C_{1c_2}	I	0.295	0.744	-1.777	0.350	-0.011
22		II	0.338 a	0.768	-1.664 a	0.414 a	0.022 a
23	C_{1c_3}	I	-3.906	1.818	0.311	-0.065	0.686
24		II	-3.806	1.692	0.437 a	0.028 a	0.672 a
25	C_{1c_4}	I	0.901	-0.238	0.656	0.573	-0.073
26		II	0.875	-0.104 a	0.724 a	0.592	-0.060 a
27	C_{1d}	I	0.146	-0.708	0.437	0.447	-0.015
28		II	0.106 a	-0.542	0.457 a	0.476	-0.024 a
29	E_r	I	0.152	1.236	-0.498	1.829	-1.460
30		II	0.228 a	1.204 a	-0.449 a	1.742	-1.398 a
31	E_c	I	-0.137	0.361	0.305	0.173	0.302
32		II	-0.115 a	0.438	0.350	0.227	0.327 a
33	E_t	I	-0.609	0.488	0.942	-0.121	1.104
34		II	-0.508	0.532	0.980 a	-0.048 a	1.074
35	ALL	I	0.036	0.200	-0.351	0.498	0.204
36		II	0.079 a	0.285 a	-0.269	0.517	0.216 a

those coefficients with the wrong sign, however, were insignificant at the five percent level and were designated by an "a" on Table 3. For model I, the negative elasticities occur when the coefficient, b, of lnV is of the correct sign but large relative to AC. Usually these negative signs occur for cost items that are very small, and may be a result of unsatisfactory deflators.

(4) Six of the ten elasticities of "ALL" costs are very low (less than .3). Since the elasticity is the ratio of marginal to average cost, the small elasticities indicate that these firms were operating on the low-output downward-sloping portions of their short run cost curves, and, therefore, that substantial unused capacity existed for these firms during the sample time period. These roads are: Arcade & Attica, Pecos Valley Southern, Amador, City of Prineville, Apache, and Corinth and Counce.

(5) The other four roads have elasticities close to or above .5. Although en increase in traffic would reduce average cost for these roads, it would not have the substantial impact that it would for the other six roads. These four roads include: Hillsboro & Northeastern, Cadiz, Bellefonte Central, and Mississippi Export. These roads were likely operating closer to minimum short run average cost than those in the previous group--given their fixed inputs.

(6) Average costs, marginal costs, and the elasticities for the "ALL" cost items are reported in Table 6 for each railroad. They illustrate more clearly how the costs behave with a change in traffic of 1,000 ton-miles.

(7) To get a better idea of the responsiveness of total cost to change in volume, average elasticities for each of the two groups of railroads

TABLE 6

COSTS AND ELASTICITIES FOR THE SAMPLE RAILROADS CALCULATED USING MODEL II AND THE MEDIAN VOLUME OF THE ROAD

	Railroad	AC/1000T-M	MC/1000T-M	$e_{TC \cdot T\text{-}M}$
(1)	A&A	$542.40	$ 84.82[a]	0.156
(6)	AMAD	118.10	9.29[a]	.079
(8)	APAC	29.20	- 7.86[a]	- .269
(5)	BELC	188.80	138.57	.734
(3)	CAD	213.10	145.10	.681
(7)	CoP	51.80	14.75[a]	.285
(10)	C&C	34.30	7.39[a]	.216
(2)	HNE	237.90	178.89	.752
(9)	MISS	31.90	16.48	.517
(4)	PVS	106.10	- 18.13[a]	- .171

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[a] Not significantly different from zero at the five percent level.

described above. These crude average elasticities appear in Table 7.[10] For most items, the elasticities for group I have substantially lower figures than group II. The cost items which have higher elasticity for group I than for group II are: Other maintenance of way (C_{1a_2}), Maintenance of equipment (C_{1b}), Equipment depreciation (C_{1b_2}), Other maintenance of equipment costs (C_{1b_3}), and Tax payments (E_t). Although C_{1a_2} and C_{1b_3} are relatively small cost components, maintenance of equipment, equipment depreciation, and tax payments represent a substantial proportion of this group's cost.

(8) There are three cost items which have elasticities of approximately unity for group II. These are roadway maintenance (C_{1a_1}), Locomotive repairs (C_{1b1}), and Equipment rentals (E_r). The average cost of these cost items did not change with an increase in traffic. The high elasticities of the first two presumably reflect "catch-up" maintenance as traffic rises; the third simply reflects proportionately greater car use. Two of the cost items for group II were much greater than unity: Train fuel costs (C_{1c2}) and Costs of loss, damage, casualties, and personal injuries (C_{1c_3}). The average cost of these items actually increased with an increase in traffic. The first must reflect running additional trains with less volume per train; the second is likely accidental.

(9) The most interesting observations which can be made about Table 7 concern those elasticities that are very low. The lowest two elasticities

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[10] When a negative elasticity appeared in Table 5, it was considered to be zero when these average elasticities were calculated.

TABLE 7

AVERAGE ELASTICITIES, VARIOUS COST CATEGORIES

	Type of Cost	Model	Group I	Group II
1	C_1	I	.162	.538
2		II	.157	.614
3	C_{1a}	I	.305	.782
4		II	.302	.807
5	C_{1a_1}	I	.339	1.045
6		II	.319	1.022
7	C_{1a_2}	I	.447	.387
8		II	.457	.510
9	C_{1b}	I	.454	.342
10		II	.450	.424
11	C_{1b_1}	I	.267	.861
12		II	.292	.9039
13	C_{1b_2}	I	.514	.109
14		II	.513	.188
15	C_{1b_3}	I	1.123	.777
16		II	1.080	.791
17	C_{1c}	I	.087	.536
18		II	.079	.636
19	C_{1c_1}	I	.090	.616
20		II	.090	.725
21	C_{1c_2}	I	.193	1.685
22		II	.190	1.443
23	C_{1c_3}	I	.663	1.311
24		II	.658	1.230
25	C_{1c_4}	I	.293	.586
26		II	.272	.604
27	C_{1d}	I	.142	.658
28		II	.109	.614
29	E_r	I	.231	1.128
30		II	.239	1.0755
31	E_c	I	.221	.095
32		II	.296	.173
33	E_t	I	.538	.330
34		II	.670	.414
35	ALL	I	.114	.599
36		II	.110	.661

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for group I are Transportation-Rail line costs (C_{1c}) and the subcategory of this, Employee Compensation (of train crews) (C_{1c_1}). The low elasticities (almost zero) imply that average labor costs will decrease substantially with an increase in traffic. Two other subcategories of Transportation-Rail line costs (C_{1c}) have low elasticities: Train fuel costs (C_{1c_2}) and Other transportation costs (C_{1c_4}). The very low elasticity for Train fuel costs (C_{1c_2}) indicates that an increase in volume will allow longer trains and proportionately less switching and therefore, more efficient fuel use. An inefficient use of labor for low levels of traffic is again indicated by the low elasticities of Locomotive repairs (C_{1b_1}), a subcategory of Maintenance of equipment costs (C_{1b}), and Traffic, administrative and miscellaneous costs (C_{1d}). The low elasticity of the Rate of return on railroad equipment (E_c) indicates an inefficient use of equipment at low volumes but the low elasticity of Equipment rentals (E_r) is difficult to explain. Total operating costs (C_1) has a very low elasticity as well. This is simply a reflection of the low elasticities of the cost components of Total operating costs.

Conclusion

The proposed abandonments of many light density lines and the reorganization of the Northeastern quadrant's railroads indicate a need for a better knowledge of railroad costs and revenues for evaluating the alternatives to abandonment, such as subsidies or converting marginal branch lines into either privately or municipally owned short lines.

We have attempted to shed some light on the responsiveness of the various cost components of low density railroads to a change in traffic.

Our results show that for all roads in the sample, MC is well below AC, and thus additional traffic will reduce AC. For six of the ten roads in the sample, MC is extremely low and additional traffic reduces AC dramatically; for the second group, the reduction is less, though substantial. The two major cost categories, train operating and wage costs and track maintenance costs, show very low elasticities for the first group. The primary difference between the two groups is in maintenance costs, suggesting that the higher elasticity of the second group is due to catching up deferred maintenance and, therefore, would be eliminated if a period of two or three years was used instead of one year.

The basic conclusion therefore is that, in the short run, additional traffic on light traffic lines will significantly lower train operating and maintenance of way costs, and therefore improve the financial viability of the lines. The roads are typically operating well below capacity. These results, however, are not conclusive about the ability of light traffic lines to adjust over time to changed volume levels.