ON THE TREATMENT OF NONSIGNIFICANT PARAMETERS
IN ARIMA IDENTIFIED MODELS WHEN THE GENERATING
PROCESS IS FIRST ORDER AUTOREGRESSIVE

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Summary:

This study investigates the treatment of insignificant parameters in an identified
autoregressive moving average model when the underlying process is generated by a
first order autoregressive process. The findings indicated that insignificant
parameters should be dropped, and that the parameters in the reduced model should
be reestimated. Such a procedure, as compared to using insignificant parameters
in the forecast model, produces a marked increase in forecast accuracy.

INTRODUCTION

In recent years there has been an increased focus on the use of Box-Jenkins[1] modeling procedures for purposes of forecasting univariate time series. One problem that users of this methodology must face is the proper treatment of insignificant parameters in the identified auto-regressive integrated moving average (ARIMA) model. In particular the decision maker is faced with at least three alternatives including: (A) retain and use the model with insignificant parameters for fore-casting, (B) drop insignificant parameters and use the resulting reduced model for forecasting, (C) drop insignificant parameters, reestimate the parameters in the reduced model and then forecast. The present study investigates the use of these alternatives in the case where the. gener-ating process is first order autoregressive. It is demonstrated that the highest degree of accuracy is achieved by alternative (C) and the least degree of accuracy is achieved by alternative (A). Furthermore it is shown that there is little difference between alternatives (B) and (C). The following section discusses the problem in detail. Subsequent sections develop the methodology, findings and conclusions.

THE PROBLEM OF DEALING WITH INSIGNIFICANT PARAMETERS IN ARIMA MODELS

When an ARIMA model is identified containing insignificant parame-ters, it is not clear which of the above mentioned three alternatives is best for the decision maker. If alternative (A) is selected there is a reasonable possibility that a parameter will be used in the forecast model which is actually zero in the population model. On the other hand if (B) is selected, and insignificant parameters are dropped before

forecasting, then the resulting model will not in general be one that minimizes the sum of squares for the residuals. In addition the residuals for the reduced model are likely to be autocorrelated, since any insignificant parameters were probably included in the model for purposes of removing autocorrelation in the residuals. Finally if alternative (C) is selected, the problem of obtaining a minimum sum of squares is resolved. However it is very probable that the resultant residual series will be autocorrelated, and possibly more so than in case B.

METHODOLOGY

Simulation of the Time Series

Simulation was used to obtain time series for study. The procedure was to generate series from a first order autoregressive (AR1) population and select those for analysis which contained at least one insignificant parameter after modeling. The AR1 process was selected because it often occurs in business decision making contexts.[2]

Several types of AR1 processes were generated via simulation.[3] In particular, series of length 50 and 300 were generated. For each of these two lengths, first order parameter values of .2 and .7 were used. Both series length and parameter size were considered as factors because they have an effect on the identifiability of the model. "Long" series are easier to identify than "short" series. Similarly series with "large" parameters are easier to identify than series with "small" parameters. The latter is true because the expectation of the first autocorrelation coefficient is equal to the first order autoregressive parameter in the population process. If the parameter is small, then the first

autocorrelation is likely to be buried in noise (i.e., found to be insignificant). The former has been empirically demonstrated to be true in the cases studied [5].

Modeling of the Time Series and Computation of the Forecast Errors

The simulated series were modeled until 100 models for each of the 4 series types were obtained such that each model contained at least 1 insignificant parameter (with $\alpha = .05$). These models were then used to forecast from 1 to 30 steps ahead.

The next step was to generate the theoretical forecast for each model. This was done by applying the theoretical generating model to the individual time series and then forecasting 1 to 30 steps ahead. Note that another alternative for obtaining the theoretical forecasts would be to extend the original simulated series an additional 30 steps via simulation. However the use of the former method is preferred because it produces conditional expectation forecasts given the known model and observed series.

Finally absolute and quadratic forecast errors were computed for all forecasts. Since the empirical findings were essentially the same for both metrics, only the absolute error findings are presented.

EMPIRICAL FINDINGS

Notationally, let the 4 series types be represented by S_1, S_2, S_3 and S_4 such that the following definitions hold:

Notation	Parameter Size in Population Model	Series Length
S_1	.2	50
S_2	.2	300
S_3	.7	50
S_4	.7	300

Mean error profiles were plotted for S_1, S_2, S_3 and S_4 (Tables 1, 2, 3 and 4 respectively). These contain the mean forecast error, for a given series type, plotted for each of the thirty steps on the forecast horizon.

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Tables 1, 2, 3 and 4
About Here
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Note that in all cases alternative (A) consistently produces the largest forecast errors. In addition, for all cases except S_2, alternative (C) produces the smallest forecast errors. In the case of S_2 alternatives (B) and (C) are virtually identical. Note also that in all cases the difference between alternatives decreases as the number of steps increases.

Since the graphical profile analysis does not provide formal hypothesis testing, multivariate analysis of variance (MANOVA) was used to assess differences in mean vectors.[4] The design involved the use of orthogonal polynomial one sample tests for S_1, S_2, S_3 and S_4.[5] Since the tests were one sample, no homogeneity assumptions for pooling of covariance matrices were needed. This left the assumption of multivariate normality which has been proven to hold for large samples via the multivariate central limit theorem [3].

Letting $E_{i,j}$ equal the mean absolute forecast error for step i
(i = 1, 15) under modeling alternative j (j = A,B,C) the following null
hypothesis was tested for S_1, S_2, S_3 and S_4:

$$H_1: \begin{bmatrix} E_{1,A} \\ E_{2,A} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,A} \end{bmatrix} = \begin{bmatrix} E_{1,B} \\ E_{2,B} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,B} \end{bmatrix} = \begin{bmatrix} E_{1,C} \\ E_{2,C} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,C} \end{bmatrix}$$

Also, in the event that H_1 was rejected, the following tests were made:

$$H_2: \begin{bmatrix} E_{1,A} \\ E_{2,A} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,A} \end{bmatrix} = \begin{bmatrix} E_{1,B} \\ E_{2,B} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,B} \end{bmatrix} \qquad H_3: \begin{bmatrix} E_{1,A} \\ E_{2,A} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,A} \end{bmatrix} = \begin{bmatrix} E_{1,C} \\ E_{2,C} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,C} \end{bmatrix}$$

$$H_4: \begin{bmatrix} E_{1,B} \\ E_{2,B} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,C} \end{bmatrix} = \begin{bmatrix} E_{1,C} \\ E_{2,C} \\ \cdot \\ \cdot \\ \cdot \\ E_{15,C} \end{bmatrix}$$

The results of the tests are presented in Table 5.

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Table 5 About Here
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Note that in general the MANOVA test statistics confirm the graphical
profile analysis. This is demonstrated by the significant difference the

overall tests for S_1, S_2 and S_3. Also note that for these 3 series types that alternative (A) is significantly different from alternatives (B) and (C). The nonsignificance of the S_4 overall test and of the individual (B)-(C) difference tests must be viewed in light of the fact that, for all 4 series types, the profiles demonstrated a consistent (C), (B), (A) ranking. However where the null is not rejected, the between alternative differences are at a minimum. This tends to imply that the nonrejection is a result of the population differences being too small for the multivariate test to measure.

CONCLUSION

The results indicate that insignificant parameters in identified ARIMA models should be dropped, and that the remaining parameters in the reduced model should be reestimated. Such a procedure produces a marked increase in forecast accuracy over the procedure of retaining insignificant parameters. Furthermore most of this increased accuracy can be achieved by simply dropping the insignificant parameter before forecasting. The question of which of these two procedures should be used in practice must be addressed in terms of the required accuracy of the decision maker. However it seems that in no case would it be desirable to retain nonsignificant parameters in the forecast model.

The results are limited to the case where the underlying population process is first order autoregressive. However since in practice the true generating process is typically unknown, the findings should be of interest to the decision maker who suspects that the population process is first order autoregressive.
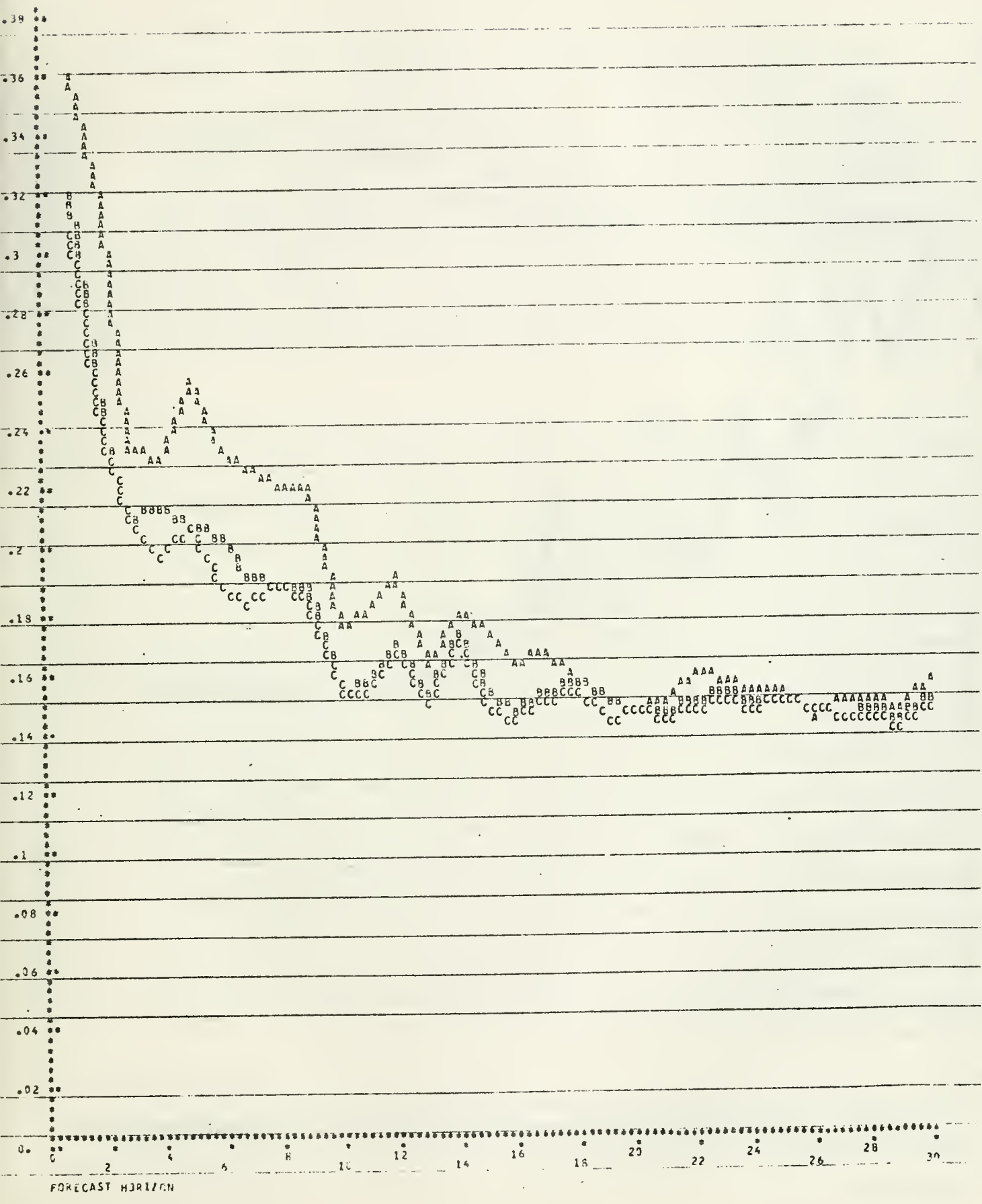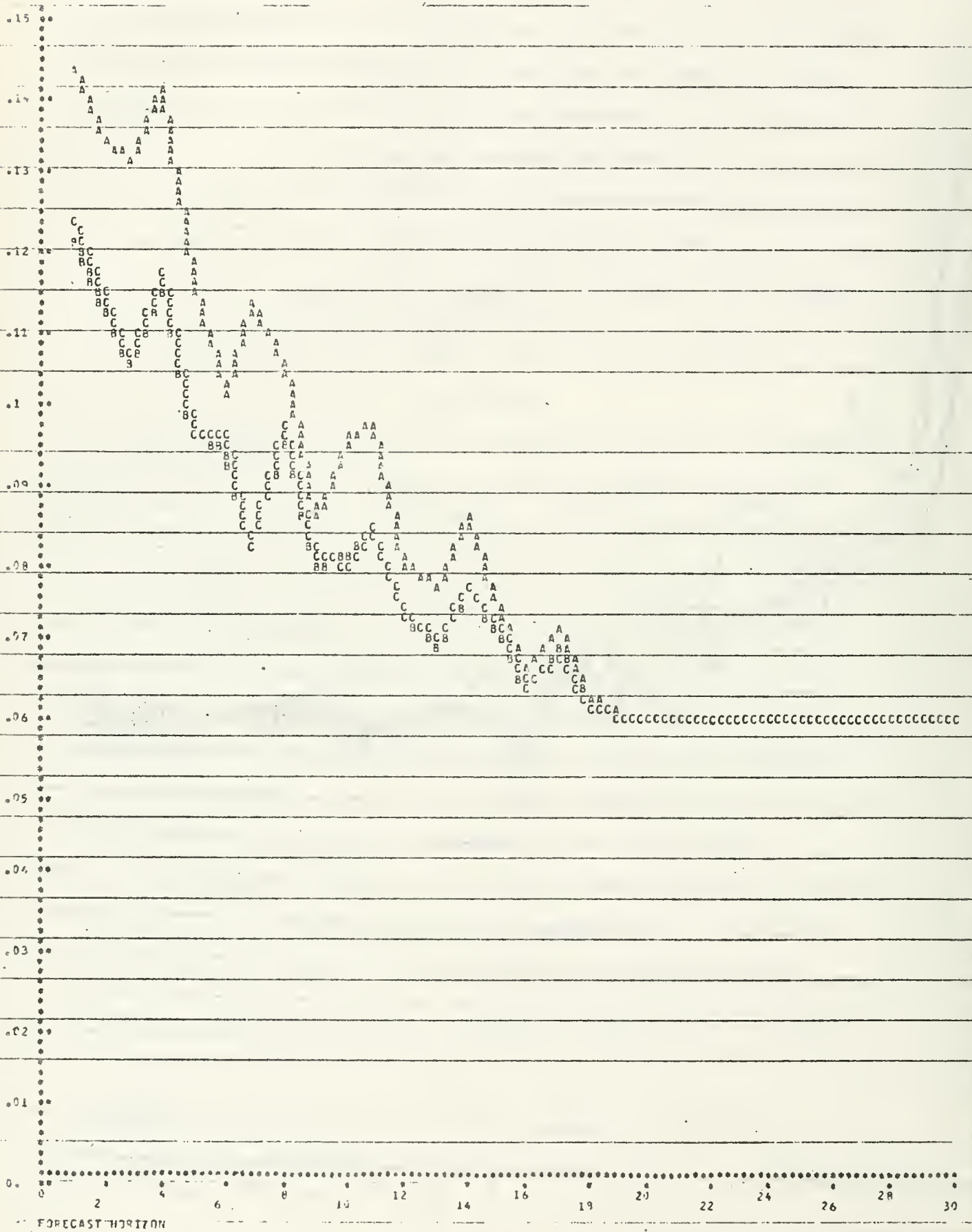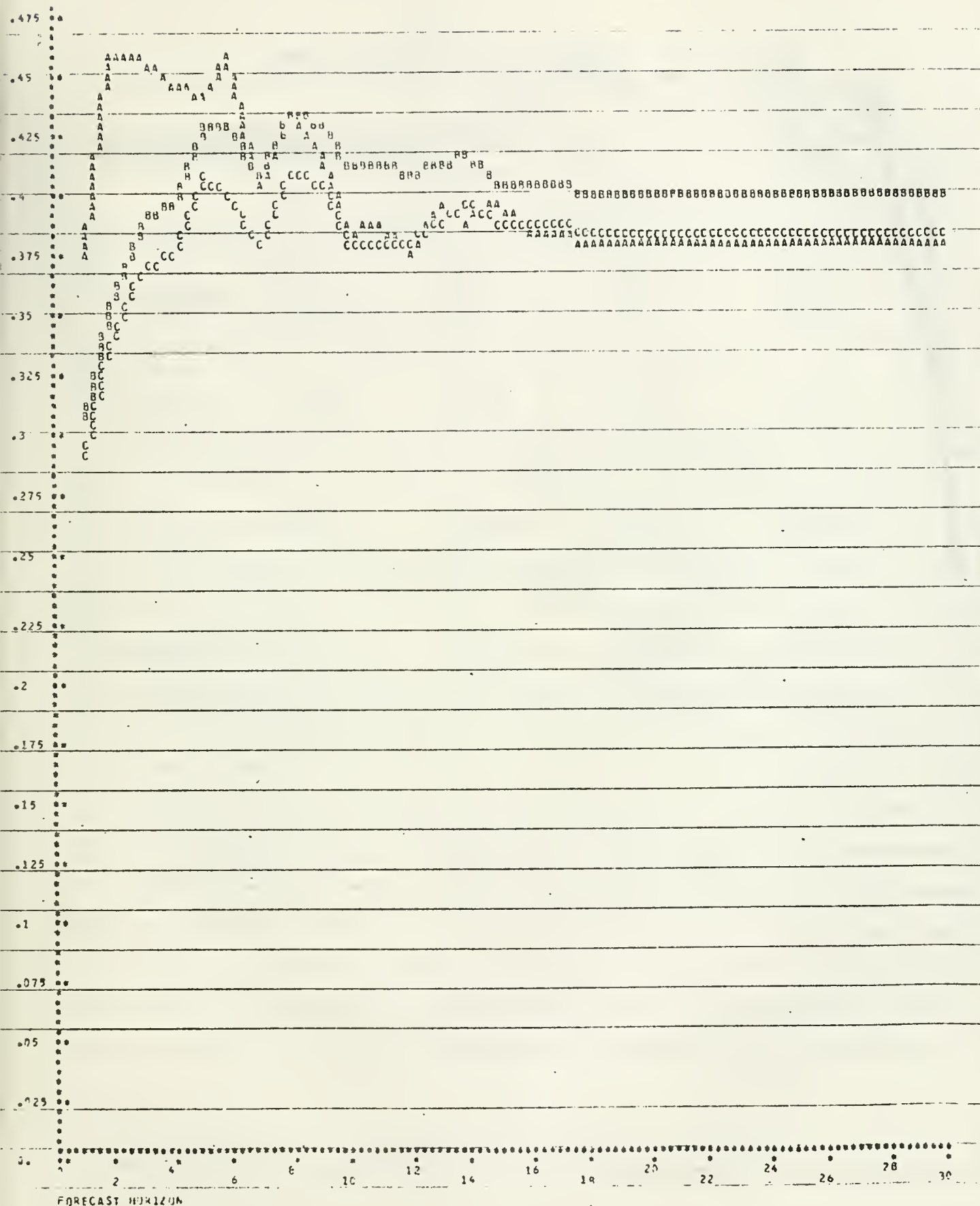
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Table 1

FORECAST HORIZON

Table 2

Table 3

FORECAST HORIZON

Table 4

FORECAST HORIZON
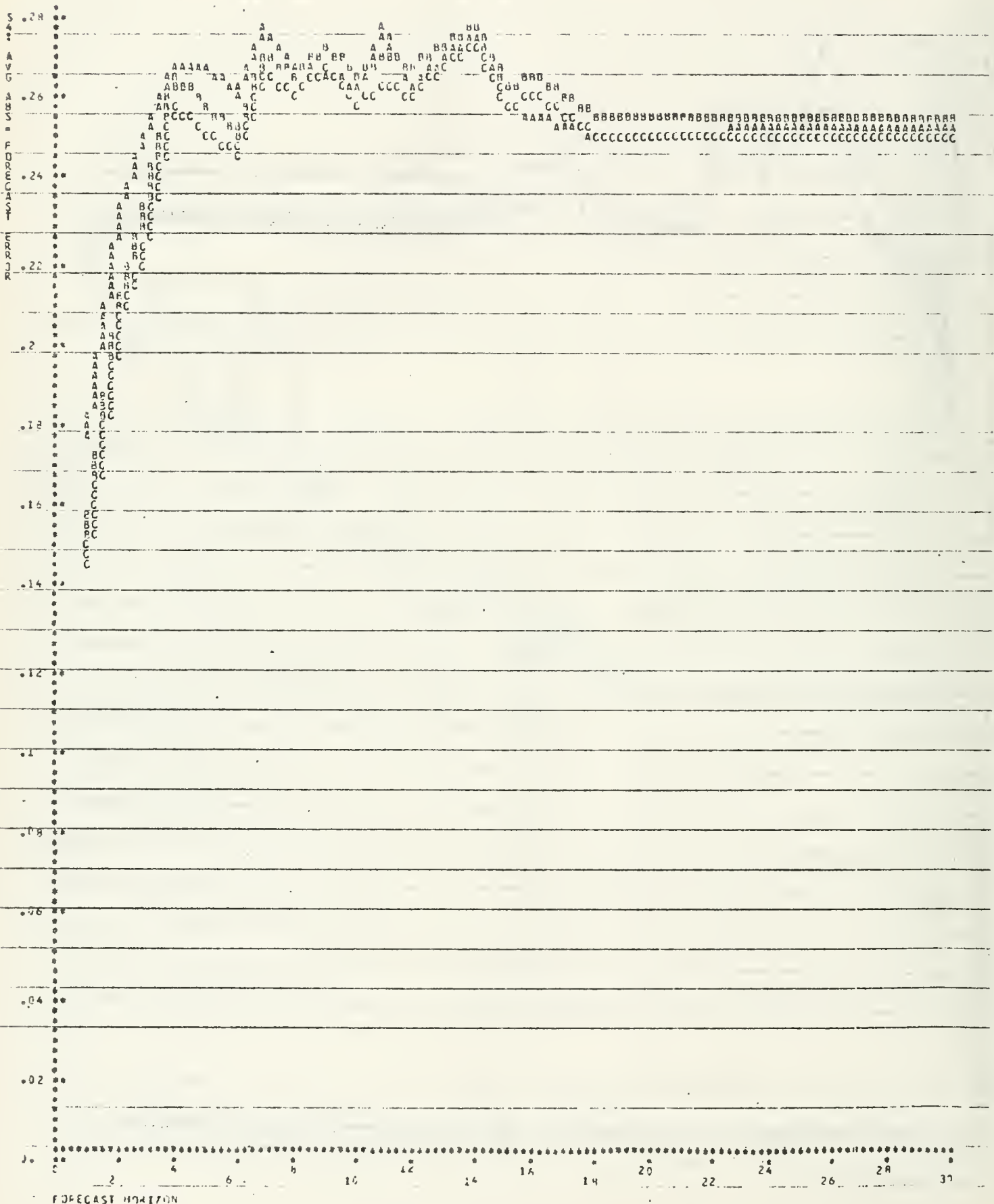
Table 5

Summary of the Manova Tests for the
Effect of Different Treatments of Insignificant Parameters

		S_1	S_2	S_3	S_4
H_1	F	2.26	2.64	3.08	1.1898
	Significance	.0028	.0005	.0001	.2717
	d.f.	30,70	30,70	30,70	30,70
H_2	F	2.029	3.4671	3.40	Not
	Significance	.0231	.0002	.0002	Tested
	d.f.	15,85	15,85	15,85	
H_3	F	2.838	2.74	3.067	Not
	Significance	.0014	.0018	.0006	Tested
	d.f.	15,85	15,85	15,85	
H_4	F	1.02	1.1989	1.4337	Not
	Significance	.44	.2887	.1506	Tested
	d.f.	15,85	15,85	15,85	

FOOTNOTES

[1] The Box-Jenkins methodology refers to a method (summarized by Box and Jenkins [2]) of model identification (selection), estimation, diagnosis, and forecasting for univariate time series. Recently this methodology has been used extensively in applications for a wide range of decision making contexts [2], [5], [6], [7], [9], [10]. The results of the above studies have consistently demonstrated the powerful nature of Box-Jenkins models.

[2] For examples of the use of AR1 processes in business decision making contexts see [3].

[3] The simulation involved superimposing the AR1 model on a normally distributed white noise (not autocorrelated) series with a mean of zero and variance of one. Since the models were of the form $zt = \phi z_{t-1} + u_t$, the random numbers were generated for the residual series u_t. In order to insure that the generated residuals were normal and not autocorrelated, the following procedure was used:

(A) 10,000 normal (with mean zero and unity variance) random numbers were generated to constitute a normal "population". Let x_i denote the individual element number in this population ($i = 1,10000$).

(B) The "population" was tested for normality. The null hypothesis of normality was not rejected.

(C) The individual series (of length 50 or 300) were sampled from the population. This was done, for a given series, by letting u_i equal x_ℓ where $\{\ell\}$ is a sequence of uniformly distributed random numbers. The latter procedure effectively shuffles x_i to insure independence.

(D) The simulated series were generated to contain an extra 30 unneeded observations. Then the first 50 observations were discarded. This was done in order to produce series that were reasonably independent of the starting value for z_t (a starting value of 0 was used since it is the unconditional expectation value for a given observation).

[4] For purposes of hypothesis testing, only the first 15 steps ahead were used. This was necessary because inclusion of steps 16 to 30 resulted in severe numerical problems due to a high degree of multicollinearity. In addition examination of the profiles revealed that often 15 steps ahead the three alternatives tended to converge together, indicating that the excluded data contained little, if any, information on differences between alternatives.

[5] The use of orthogonal polynomials for multivariate analysis is discussed in Bock [1] and McCall and Applebaum [7].

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