MULTI-TARGET TRACKING FOR SMARTNET: MULTI-LAYER PROBABILITY HYPOTHESIS FILTER FOR NEAR-EARTH OBJECT TRACKING

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ABSTRACT

In this paper, a modified version of the finite set statistics-based Probability Hypothesis Density (PHD) filter is developed specifically for the optical multi-target tracking of objects in the near-Earth realm for Space Situational Awareness (SAA). A two-step PHD filter is proposed in a modified version. One labeled PHD filter is used on the orthogonal image plane, in which linear dynamics in a four-parameter state is employed, forming so-called tracklets. Tracklets are associated sets of a few closely-spaced observations covering a negligible part of the overall orbit. Furthermore, tracklets are fed into a second PHD filter in a modified measurement update version, utilizing the full near-Earth astrodynamics with a six parameter state. In the modification, each tracklet leads to only one update in the PHD, but all observations within the tracklet are processed in the single target Markov transition process within the filter. In this case, the single target filter is an Extended Kalman Filter. In addition, the birth process that has been usually in typical SSA applications shifted to the birth step, forcing a data-driven birth with the disadvantage of a severe model mismatch, back to the propagation step, as in the original PHD filter formulation, avoiding the mismatch. In order to overcome the lack of probabilistic description availability (one of the triggers of the shift to the data-driven update step of previous authors), the data is preprocessed. This has the advantage that birth can employ traditional initial orbit determination methods and does not have to rely on the initialization with an incomplete state using, e.g., an admissible regions approach. The results are generated using the optical data of the DLR SMARTnet telescope network and are compared to the DLR BACARDI data processing.

Keywords: Multi-Target Tracking, Space Situational Awareness, Optical Observations and Tracking, Probability Hypothesis Density Filter, FISST.

1. INTRODUCTION

SMARTnet is a ground-based optical sensor network founded in 2017 by DLR with the Astronomical Institute of the University of Bern (AIUB) as its founding member [8, 7]. SMARTnet aims to track all human-made objects in high altitude orbits, namely in the geosynchronous (GEO) and geostationary transfer (GTO) orbital regions. High altitude objects are non-resolved in ground-based observations. Charged particles impinging on the detector, among other sources, trigger the release of photo-electrons and, therefore, clutter that does not correspond to the image of an actual object. Objects in the field of view (FOV) might not be detected because of the non-sufficient reflectivity of the object, illumination geometry, or when in front of a star.

As such, the high altitude space object tracking can be understood as a multi-target tracking (MTT) problem. In MTT, the states alongside the cardinality, the number of observed objects, is estimated, solving, either explicitly or implicitly, the data-to-(new or existing)-object association problem.

Historically, two primary research approaches are used to explore the MTT regime: the track-based approaches and population-based approaches. The track-based approaches associate the measurements explicitly with the single targets to form a track. Decisions are made only with the measurements that are present in the scene. The

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most well-known representation of this approach is the Multi-Hypothesis filter.

Population-based approaches model all the objects in the scene as a single random entity and formulate the filtering problem using Finite Set Statistics (FISST) [17]. The random entity also exists in the absence of measurements and can provide a fully probabilistic description of the entire scene. Both track-based approaches and the population-based approaches have been researched extensively in the context of tracking for Space Situational Awareness, especially for high-altitude objects [10, 11, 20, 3, 12, 13, 14, 1, 2].

One of the challenges is the absence of exact knowledge on many of the probabilistic input parameters needed in order to run the filter properly. Modeling misfits lead to severely impacted filter performance [9]. One of the problems is the uncertainty present in the probability of detection, which has been addressed for SSA in reformulating in the Probability Hypothesis Density (PHD) filter framework [19]. A second problem is finding the probabilities of birth and their associated probability density functions (pdf), which is especially crucial as Space Situational Awareness tracking is a data-deprived regime. As such, ill-fitting pdfs are not quickly corrected with new data. In addition, in optical tracking, only a small subset (two angles) of the entire six-parameter state is measured. As a result, the FISST filters are applied in a data-driven birth, [1, 20, 12, 13, 14, 2, 12, 3, 6, 5, 19] in combination with an admissible regions formulation [18, 4]. Data-driven birth is not in the original PHD formulation and overuses observations, necessitating the need for gating. Even when additional constraints are applied, admissible regions grow large very quickly, significantly limiting the FISST framework's ability to pick up and maintain new objects, which were not previously cataloged. For the admissible regions, a single *detection* needs to be assumed to be both angle and angle rates. This, however, requires single measurements (two angles) that are already associated, i.e., as described in [10]. Using a different process in the pre-association leads to inconsistencies at best and significant performance impacts at worst. Furthermore, processing all single measurements of a short observation series, which are from the operational context closely spaced and do only cover a small fraction of the object's orbit at a time, leads to a very high computational load within the filter, even when using the first-order approximation of a PHD filter only.

In this paper, a quick overview is given over the PHD filter in its classical understanding and the data-driven birth process as usually employed in optical SSA tracking. The first step is performed running a labeled PHD filter in the image plane using a reduced linear dynamics forming so-called tracklets. Those tracklets are fed into a second PHD filter operating on the full orbital dynamics in the second step. One update in the PHD filter is performed, while all measurements are processed on the single target Markov process underlying the filter. The new formulation for this step is shown explicitly in the paper. In both steps, birth is maintained in the propagation step. The probabilistic description is obtained via pre-processing of the data, which allows the use of classical, robust orbit determination methods with a much higher convergence date. The data inputs are actual optical measurements provided by SMARTnet.

2. A SHORT SUMMARY OF THE PHD FILTER AND ITS PREVIOUS USE FOR SSA

The PHD filter has been first proposed by Mahler [17] and is a first order approximation of the full FISST, and therefore the computationally leanest version of the FISST-based filters.

For the prediction step, the following assumptions are made: (1) the single objects are independent and their dynamics can be modeled by a Markov transition density on the single object level denoted by $f_{k+1|k}(\mathbf{x}|\mathbf{x}')$ from time step k to $k+1$ with the prior state $\mathbf{x}'$, and posterior state $\mathbf{x}$; (2) the survival of the existing objects can be modeled by a Bernoulli process with known probability p_s at each time step; (3) the new objects are being born independent of the existing targets, with a known birth process pdf $b_{k+1|k}(\mathbf{x})$. When neglecting spawning, which is not relevant for SSA, the PHD filter prediction equation for the multi-target probability hypothesis density $D_{k|k}(\mathbf{x})$ of the $k-th$ time step, to the $(k+1)-th$ time step in the state space denoted by $\mathbf{x}$ [17]:

$$D_{k+1|k}(\mathbf{x}) = b_{k+1|k}(\mathbf{x}) + \int p_S(\mathbf{x}') f_{k+1|k}(\mathbf{x}|\mathbf{x}') D_{k|k}(\mathbf{x}') d\mathbf{x}' \quad (1)$$

For the measurement update step, the assumptions are: (4) Each object produces maximally one measurement at any given time. The set of measurements $\mathbf{Z}$ of the single measurements $\mathbf{z}$ with $\mathbf{z} \in \mathbf{Z}$ at a given time $k+1$ is the union between the measurements produced by previous objects and the clutter process $\mathbf{Z} = \mathbf{Z}_{\text{obj}} \cup \mathbf{Z}_{\text{clut}}$; (5) there exists a known single target measurement likelihood function $f_{k+1}(\mathbf{z}|\mathbf{x})$ based on the object state $\mathbf{x}$ and a single measurements $\mathbf{z}$, respectively; (6) there is a known probability of detection, which may be state dependent and which can be modeled as a Bernoulli process, based on the object state and the sensor characteristics such as pointing direction and field of view, $\mathbf{x}_{\text{sensor}}$, respectively, $p_D(\mathbf{x}) := p_D(\mathbf{x}, \mathbf{x}_{\text{sensor}})$'(7) there is a false alarm clutter rate, which may depend upon the sensor characteristics, denoted by

$\mathbf{x}_{\text{sensor}}$, which can be modeled as a Poisson distribution with variance λ and spatial distribution $c(\mathbf{z}) = c(\mathbf{z}|\mathbf{x}_{\text{sensor}})$; (5) the multi-target prior is Poisson distributed with the variance λ_{prior}: $f_{k+1|k}(\mathbf{X}|\mathbf{Z}) = \exp(-\lambda_{\text{prior}})\Pi_{\mathbf{x}\in\mathbf{X}}\lambda_{\text{prior}} \cdot f_{k+1|k}(\mathbf{x}|\mathbf{Z})$.The measurement update step at time $k+1$ can then be formulated as the following [17]:

$$D_{k+1|k+1}(\mathbf{x}) = (1 - p_D(\mathbf{x})) \cdot D_{k+1|k}(\mathbf{x}) + p_D(\mathbf{x}) \sum_{\mathbf{z}\in\mathbf{Z}} \frac{f_{k+1}(\mathbf{z}|\mathbf{x}) D_{k+1|k}(\mathbf{x})}{\lambda c(\mathbf{z}) + \int p_D(\mathbf{x}) f_{k+1}(\mathbf{z}|\mathbf{x}') D_{k+1|k}(\mathbf{x}') d\mathbf{x}'} \quad (2)$$

The classical implementation as shown in Eqs.1 and 2 has the birth $b_{k+1|k}(\mathbf{x})$ process in the prediction step from time k to $k+1$. Overcoming the lack of a probabilistic description of birth in combination with a data-spare environment, prompted a shift of the birth process from the prediction step $b_{k+1|k}(\mathbf{x})$ to the update step $b_{k+1|k+1}(\mathbf{x}|\mathbf{Z})$, based on the measurements, a so-called data-driven birth [1, 20, 12, 13, 14, 2, 12, 3, 6, 5, 19]. While usually explicitly formulated this is leading to the modified equations:

$$D_{k+1|k}(\mathbf{x}) \overset{\text{mod}}{=} \int p_S(\mathbf{x}') f_{k+1|k}(\mathbf{x}|\mathbf{x}') D_{k|k}(\mathbf{x}') d\mathbf{x}' \quad (3)$$

$$D_{k+1|k+1}(\mathbf{x}) \overset{\text{mod}}{=} (1 - p_D(\mathbf{x})) \cdot D_{k+1|k}(\mathbf{x}) + p_D(\mathbf{x}) \sum_{\mathbf{z}\in\mathbf{Z}_\text{d}} \frac{f_{k+1}(\mathbf{z}|\mathbf{x}) D_{k+1|k}(\mathbf{x})}{\lambda c(\mathbf{z}) + \int p_D(\mathbf{x}) f_{k+1}(\mathbf{z}|\mathbf{x}') D_{k+1|k}(\mathbf{x}') d\mathbf{x}'} + \sum_{\mathbf{z}\in\mathbf{Z}_\text{b}} b_{k+1|k+1}(\mathbf{x}|\mathbf{z}) \quad (4)$$

In order to shift the birth to the update step, a violation of assumption (4) such that the measurements are generated either from clutter or from already known object. In order to not overuse the measurements, a split or gating has to introduced as a hard constraint, namely re-defining the set of all measurements $\mathbf{Z}$ as:

$$\mathbf{Z} \overset{\text{mod}}{=} \mathbf{Z}_\text{d} \cup \mathbf{Z}_\text{b} \quad (5)$$

$$\mathbf{Z}_\text{d} = \mathbf{Z}_\text{obj} \cup \mathbf{Z}_\text{clut} \quad (6)$$

$$\mathbf{Z}_\text{b} = \mathbf{Z}_\text{birth} \quad (7)$$

But as $\mathbf{Z}_\text{b}$ and which for $\mathbf{Z}_\text{d}$ are used seperately in Eq.4, the set of measurements has to be manually split. The decision of which measurements count for birth in $\mathbf{Z}_\text{b}$ and which for $\mathbf{Z}_\text{d}$ is often based on the measurement likelihood function of the known targets, on the other hand, that ill-represents the clutter process which can occur at any region on the image; furthermore not all known objects are detected and the probability of detection for the known objects is neglected. As such, the modified PHD equations in Eqs. 3 and 4 are not in alignment with the FISST filtering paradigms. A rederivation is necessary for a data-driven birth:

$$\mathbf{Z} \overset{\text{new}}{=} \mathbf{Z}_\text{obj} \cup \mathbf{Z}_\text{clut} \cup \mathbf{Z}_\text{birth} \quad (8)$$

(9)

This is future work for the authors and will be published seperately. For the Multi-Bernoulli filter a data-driven re-derivation is available [16].

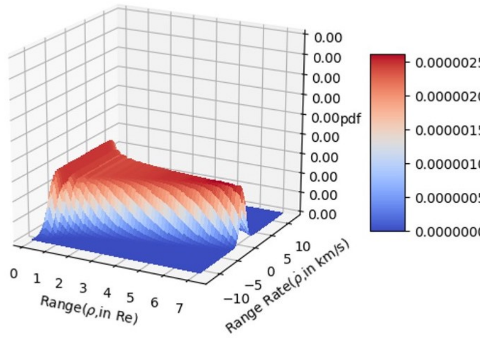


Figure 1: Gaussian Mixture Admissible Region representation, using 300 components, according to [4].

Despite its inconsistencies, the data-driven birth Eq.4 solves the problem of birth when using optical measurements only partly. For the birth initialization, a pdf spanning the full state is required. Optical observations, however, measure only a subset of the state, two angles. One workaround is the use of at least a minimal preassociation of the data, assuming angle rates are available, in combination with admissible regions [**?** 12, 13, 14, 2, 12, 3, 6, 5, 19]. Fig.**??** shows the illustration of an admissible region represented via a Gaussian mixture using 300 components. This already illustrates that a large number of components are needed for a proper representation. Furthermore, even when using further constraints, the admissible regions, which can reduce the number of components, are fanning out quickly, requiring new observations of the same object soon not to lose the newly birthed objects again.

If the data is not assumed to be preassociated, the admissible region's approach cannot be used. However, then only a potentially wrongly associated subset of the observations is available, potentially relying on very different methods to form so-called tracklets [10]. Tracklets are observations that are determined to belong to the same object and span a very short part of the object's orbit. On the other hand, if all the single observations are inserted into the filter, many hypotheses, e.g., Gaussian components in a Gaussian mixture implementation, are generated with the result that very aggressive pruning and merging has to be applied, which degrades filter performance. A further complication is that in operational data, often small time corrections, e.g., for different shutter times, are applied, leading to slightly different times of the order of microseconds for detections on the same image. Such short propagation times are disadvantageous within a multi-target tracking filter with the full orbital dynamics employed.

3. TWO-STEP APPROACH OF SSA PHD FILTER

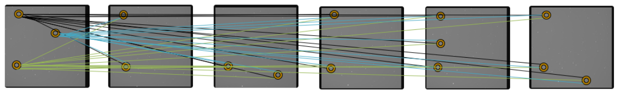
A new two-step modified implementation of the classical PHD filter is proposed to avoid some known shortcomings in the usual SSA implementations. In this new form, the birth process is shifted back to the prediction step, and a two-step approach is implemented.

3.1. Labeled Association in the Orthogonal Projection

As a first step, the PHD filter in its form of Eqs.1 and 2 are used in a Gaussian mixture implementation [23]. Labeling is applied for use in the subsequent filter step. The dynamics is a linear dynamics in the orthogonal image plane, constructing a state that consists of right ascension α, declination δ, right ascension rate $\dot{\alpha}$ and declination rates $\dot{\delta}$. The linear dynamics is justified because the method is applied to high altitude orbits spanning only a small fraction of the orbit. Adaptions are easily possible for objects in lower orbits. No catalog data is used at this step, but the filter is initialized cold on birth only without a priori information. Each series is processed separately.

The birth distribution is found via a pre-processing of the data. Each short observation series is pre-analyzed for determining from the data a Gaussian mixture distribution of birth distributions in the two-dimensional orthogonal image plane projection. For the velocity in angle rates, all detections are combined, excluding combinations on the same image. For positions, the locations within the image are used in a similar fashion. Again a Gaussian mixture distribution is assumed. The process is illustrated via pictograms in Fig. 2, where a series of six images are shown with detections, actual observations and clutter, marked by golden circles. The Gaussian distribution is, of course, not valid in spherical coordinates. But as a localized region in the orthogonal tangential plane is used, the inaccuracies in the projection are negligible. In order to limit the Gaussian mixture components, merging is applied within user-specified bounds using the Kullbach-Leibler divergence as criteria. For the observation epochs, rounding to one common epoch per image is applied in this first step.

(a) position probabilities



(b) velocity probabilities

Figure 2: Pictrographic illustration of finding position and velocity probabilities in the orthogonal image plane for detection on the image indecated by golden circles .

3.2. Full Orbital Dynamics Modified PHD Filter

The filter results within the orthogonal image plane are then transferred over into the full PHD filter employing a six-state vector of position and velocity in the Cartesian space. Again, birth is kept in the propagation step, as in the original definition of the PHD filter, Eqs.1. This requires a process again to determine a probabilistic birth process. This is done analogous to the previous step, only now in the full Cartesian space using proper orbit dynamics. Therefore classical methods of orbit determination can be used. In this paper, the authors decided to use the Gauss method, followed by the Herrick-Gibbs method [22] using combinations of three tracklets to determine orbital hypotheses. The process requires additional computations, however, it is very robust and allows leveraging classical initial orbit determination methods, including multi-revolution methods, avoiding the use of admissible regions. IOD for tracklets of different objects often is not successful, which in this context simply means they do not contribute to the Gaussian mixture birth population representation. For a proper representation of the birth process, a proper pdf representation is needed. Interpreting the initial orbit determination results as the means, the second moment can be approximated using an unscented transform of the expected measurement noise [15]. This introduces a slight bias, which is found to be overall negligible.

However, the measurement update step is modified in its formulation. In order to curb the creation of new hypotheses, a single PHD filter step is employed per tracklet-object that has been found in the previous PHD filter step in the orthogonal plane. Nevertheless, the single labeled tracklet observations are processed in all steps through the Markov transition within the PHD filter with their exact respective epochs without rounding. This allows processing all measurements, which is advantageous for the sequential filtering of the Markov process to mitigate filter divergence, especially after longer observation gaps that frequently occur in SSA tracking. Mathematically, the update step can hence be formulated as the following:

$$D_{k+1|k+1}(\mathbf{x}) = (1 - p_D(\mathbf{x})) \cdot D_{k+1|k}(\mathbf{x}) + p_D(\mathbf{x}) \sum_{\mathbf{Z}\in\zeta} \frac{F(\mathbf{Z}|\mathbf{x}, D_{k|k}(\mathbf{x}))}{\lambda c(\mathbf{Z}) + \int p_D(\mathbf{x}) F(\mathbf{Z}|\mathbf{x}', D_{k|k}(\mathbf{x}'))d\mathbf{x}'} \tag{10}$$

Please note that the first term, representing the probability of non-detection uses the propagated multi-object density $D_{k+1|k}(\mathbf{x})$, whereas the newly formulated transition process still uses the non-propagated density $D_{k|k}(\mathbf{x})$, as several different propagation epochs are needed:

$$F(\mathbf{Z}|\mathbf{x}, D_{k|k}(\mathbf{x})) = f_{k_z+1}(\mathbf{z}|\mathbf{x}) \int f_{k_z+1|k}(\mathbf{x}|\mathbf{x}')D_{k|k}(\mathbf{x}')d\mathbf{x}' \quad \forall \mathbf{z} \in \mathbf{Z}_i \tag{11}$$

This requires a redefinition of the measurement set $\mathbf{Z}_i$, which now consists of the sets containing the measurements of a tracklet from the previous labelled PHD filter step in the orthogonal plane.

$$\mathbf{Z}_1 = \{\mathbf{z}_1, \mathbf{z}_3, ...\} \tag{12}$$

$$\mathbf{Z}_2 = \{\mathbf{z}_2, \mathbf{z}_5, ...\} \tag{13}$$

$$\mathbf{Z}_. = ... \tag{14}$$

To speed up computation no labelling is used in this step. A Gaussian mixture implementation is chosen again. In this step, the catalog of all known high altitude orbit is used as existing object population.

4. RESULTS

For the results, the data of a single observation night of March 3, 2021, collected by the SMARTnet sensor located in Sutherland, has been used. During that night, GEO surveys to detect new objects and catalog maintenance follow-up observations were collected.

The observation night consists of 1526 total detections, with a minimum of one detection and a maximum of 13 detections on a single image. Overall there were 81 observation series. A series spans on average 103 seconds, with a maximum time span of 220 seconds and a minimum one of 48 seconds. The average time from one image to the following (mid observation times) is 15 seconds; the average time from one observation series to the next is 231 seconds.

Unfortunately, the observation set was already formed into tracklets, is hence of a cleansed data set, so to speak. The previously formed associations were undone and not used in the processing.

On average, 40 velocity hypotheses and three position hypotheses per observation series were formed. A merging with standard deviations of 1 arcsecond per second in velocity and two arcseconds in position were performed. The latter is consistent with the measurement accuracy determined for the sensor. On average, three birth hypotheses remained after merging. The results for the labeled PHD filter in the orthogonal image plane were consistent with the BACARDI [21] processing; the previously undone associations were found again, with some outliers that BACARDI removes in a separate step, already removed in all but two cases by the PHD filter step.

For the orbit determination step, catalog objects have been confirmed, and new objects have been found. Because of technical problems, validation with the BACARDI system is currently still pending.

5. CONCLUSIONS

In conclusion, there is an established way of implementing a probability hypothesis density filter (PDH) for the use of tracking in space situational awareness. This standard implementation has several drawbacks, which limit performance and general use.

In this paper, a two-step approach has been derived and implemented, providing method consistency in all processing steps. Furthermore, traditional initial orbit determination methods can be used, avoiding a costly non-robust admissible regions approach for object birthing. Reformulating the filter update step allows curbing the number of needed hypotheses, allowing for more relaxed pruning and merging. At the same time, still, all measurements with their exact epochs are processed for the best possible orbit estimates in the sequential single object filter, which provides the PHD filter Markov transition process.

The first results are auspicious; however, more rigorous testing and validation are needed. For the birthing using multiple nights of data, better multi-revolution initial orbit determination methods need to be implemented. Future work includes the operational implementation of the method in the SMARTnet network. Unrelated to the implementation, future work includes the publication of the reformulation with a proper data-driven birth.

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