Review of risk and uncertainty concepts for climate change assessments including human dimensions

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04/02/12

Abstract

This paper discusses aspects of risk and uncertainty relevant in an interdisciplinary assessment of climate change policy. It opposes not only the objective approach versus the subjective approach, but also situations when precise probabilities are well founded versus situations of broader forms of error such as Knightian or deep uncertainty, incompleteness, vagueness. Additional human and social dimensions of ignorance: strategic uncertainties, surprises, values diversity, and taboos, are discussed. We argue that the broader forms of error affect all sciences, including those studying Nature. For these aspects the IPCC guidance notes provides an interdisciplinary unified approach on risk and uncertainty. This is a significant advance from a simple multidisciplinary justaposition of approaches. However, these guidance notes are not universal, they mostly omit the human and social dimensions of ignorance.

Acknowledgments

Detailed comments of Kirsten Halsnaes, Priyadarshi R. Shukla, Roger Beale, colleagues at CIRED and several anonymous reviewers are gratefully acknowledged. Errors and opinions remain solely mine.

Keywords

risk; uncertainty; climate change; integrated assessment

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The role of the IPCC is to assess on a comprehensive, objective, open and transparent basis the scientific, technical and socio-economic information relevant to understanding the scientific basis of risk of human-induced climate change, its potential impacts and options for adaptation and mitigation.
(Principles governing IPCC work, 2003)

1. Introduction

Debates on risk and uncertainty often mingle two deep but distinct questions:

The first is the classical distinction between objective versus subjective probabilities. The social sciences literature relevant to climate policy uses both notions, so none can be dismissed out of hand. The example of weather risk management will show that knowing what the market believe can be as useful as knowing objective historical frequencies.

The second question is about the difference between risk and uncertainty. It is often agreed that risk refers to situations in which probabilities are well defined, while uncertainty refers to a broader form of ignorance. This means that probability theory is only one paradigm among others, which is appropriate to describe risk but (by definition, so there should be no controversy here) not uncertainty. This paper exhibits a few mathematical tools for situations in which probabilities are not well defined, but not totally unknown either. It discusses examples of contributions to the climate change literature using imprecise probabilities.

In this paper I add a third dimension to the debate: passive error versus actively constructed ignorance. In a global environmental issue such as climate change, risk and uncertainty issues do not solely come from imperfection of knowledge about the state of the world. This paper expands the discussion beyond mere `errors' to aspects of ignorance caused by human will. The relevance of these human and social dimensions to climate policy assessment is discussed, starting with the strategic use of information, and then extending to the notion of surprise, values uncertainty and taboos.

It might be tempting to segregate the subjective, deep and human dimensions of uncertainty as relevant only to the so-called soft or social sciences --i.e. those of Intergovernmental Panel on Climate Change (IPCC) Working Group III-- with the idea that the mandate of `hard' sciences (i.e. those of Working Group I) is essentially to provide precise and objective probabilities. This paper's rejects this idea. With limited data (and there is only one Earth's climate change experiment), even hard science produce imprecise results.

The argument is organized as follows. Sections 2, 3 and 4 explains the objective/subjective and the risk/uncertainty dimensions, and explain the methods used to deal with them in the sciences related to climate change and climate policy. Section 5 describes human and social aspects of ignorance. Section 6 argues that the broader forms of error affect all sciences, including those studying Nature. Section 7 examines how this is accounted for in the IPCC guidance notes on risk and uncertainty. Section 8 discusses the significance of that for precautionary decision-making. Finally, section 9 concludes that IPCC guidance notes provide a significant advance from a simple multidisciplinary justaposition of approaches, but including the human and social dimensions of ignorance would make them more comprehensive.

2. Epistemic differences : objective vs. subjective probabilities

The classical starting point of the discussion about risk and uncertainty is the foundation of mathematical probability theory. This foundation is the notion of equiprobability, a mathematical idealization of physical situations of perfect symmetry.

Mathematical randomness, historically, was based on the self-evident intuitive notion of equiprobability , in the same way as points and lines can be considered self-evident intuitive notions in Euclidean geometry. Algorithmic information theory provides contemporary definitions of randomness in terms of data compression, statistical testing and betting.

But in practical applications involving probabilities, the numbers rarely come from the axioms. When assessing the likelihood that the oil price is above 80$/barrel in 2010, dividing the world in equally probable events makes little practical sense. So where do the probability numbers come from ? There is a variety of procedures to measure levels of uncertainty. So in addition to the probabilities based on symmetry, this paper will distinguish subjective probabilities, frequentist probabilities and personal probabilities. Given the historical depth and disciplinary width of the literature, there is no unambiguous choice of words, but hopefully these three adjectives will sound familiar to contemporary economists.

Frequentist approaches determine levels of probability by observation of relative frequencies. It works best when a statistically significant body of observations is available. On the contrary, when there is a low amount of evidence (a small number of observations, missing data, or correlation between experiments), the accuracy of numbers determined by relative frequencies is low.

Subjective approaches are based on the idea that the beliefs of a rational agent can be discovered by observing its choices. For example, if people buy shares in oil companies, it is generally a sign that they expect higher oil prices. This has led over the last decade to the creation of prediction markets (see Wolfers et al., 2004), that is speculative markets designed for the purpose of making predictions. Participants bet by trading assets whose final cash value is tied to a particular event or parameter. The current market prices can then be interpreted as predictions of the probability of the event or of the expected value of the parameter. Real-money prediction markets have been set up with some success to reveal market beliefs on political, financial, and technology-related questions. Regarding energy and environmental questions, there are several public play-money prediction markets but there is certainly an incentive problem with claims that are to be adjudicated in the distant future[1].

[1] Consider for example the claim CO2LVL - CO2 Level 2030 at the foresight exchange prediction market (www.ideosphere.com) : *This claim is based on the ambient CO2 level in December of 2030. The claim pays $0.01 for each PPM by volume (PPMV) of CO2 in excess of 400 PPMV, up to 500 PPMV. For instance, 0.0 for <400.5 PPMV, 0.5 for 450 PPMV, and 1.0 for >499.5 PPMV. If available, data from the Mauna Loa Observatory will be used to judge the claim*. This claim opened in may 2002 at around $0.40 (corresponding to 440 ppmv),

Objective probabilities (degrees of Truth)
Mathematical (axioms of randomness)
Frequentist (statistics from data)

Bayesian probabilities (degrees of Certainty)
Personal (directly stated or elicited)
Subjective (from observed choices)

Figure 1: Kinds of probabilities

Personal approaches directly ask people to quantify their strength of opinion or level of confidence. There is a psychophysiological basis: studying electroencephalograms. Sutton et al. (1965) were able to find a measure of brain activity that increases with unpredictable, unlikely, or highly significant stimuli, the P300 event-related brain potential. There is ample evidence that the P300 increases in amplitude as the target's probability decreases (Polich 2007, 2.3).

A direct method to elicit a probability distribution from experts is to ask them to dispatch a stake of 100 chips over the alternative outcomes considered. Formal expert surveys are difficult and expensive to conduct rigorously. When led according to the best practices in the field they may provide the best information available in some situations. They have been used to inform policy-making and risk management, for example in the nuclear industry. In a setting such as the IPCC writing teams, experts agree verbally.

In contrast to the mathematical approach, for physical applications the operational procedure used to measure a variable is fundamental to the definition of what the variable is. Even if frequentist, personal and subjective probability distributions are defined by the same mathematical properties, they describe different variables. They should be viewed as having different units.

There are different viewpoints on the relative merits of these ways to measure probabilities. The most important division line lies between the **objective** and the Bayesian views of probabilities[2], see Figure 1. Objective probabilities are seen as a physical propensity. They are defined using a frequentist approach or using "physical laws" models based on symmetry. Bayesian probabilities are seen as degrees of beliefs. They are defined either directly, using the personal approaches, or indirectly, using subjective probabilities. To summarize, objective probabilities are degrees of Truth, while Bayesian probabilities are degrees of Certainty.

VanderMarck (2003) provides an example of the dilemma between subjective and frequentist methods in Weather risk management. Weather derivatives are contingent financial goods whose value depends on the future weather, such as the number of heating degree days through the winter season. Weather is the major source of income variability in the energy sector. These financial instruments can be used to alleviate that risk. Their market has been developing rapidly since the late nineties, along with deregulation in the power industry. To evaluate a portfolio of these derivatives, one can use models based on historical weather

increased, stabilized around \$0.70 between mid 2003 to mid 2005, and dropped to around \$0.56 in early 2060, showing an expected value of 456 ppmv.

2 The same remark as above applies : The precise technical meanings of the words *subjective*, *personal* and *bayesian* have varied with time and place.

data: that is an objective frequentist method. But there is also the possibility of valuing positions based on current market price levels. Well-known finance and econometrics techniques allow to infer the risk-neutral probability distribution of an asset from the prices of options on this asset (Hull 1997, chapter 9.2). These market-based probability distributions are in essence subjective.

VanderMarck concludes that both techniques can be used, along with hybrid approaches. The market-based approach works better when the market is sufficiently liquid, i.e. large. Financial institutions tend to be more familiar with market valuation than the frequentist model approach, since most other products they trade are solely based on supply/demand dynamics. Marking to market also ensures a more accurate reflection of a portfolio's value should it need to be liquidated.

This example shows that the subjective methods are necessary when human beliefs and expectations are significant variables. This is a sufficient reason to allow for Bayesian probabilities in an interdisciplinary assessment of the climate change issue, at least in the working group interested in social issues. We will argue that there are deeper reasons to use Bayesian approaches even in climate sciences, but that discussion is deferred to section 5.

3. Imprecision can be objective

The next important dimension of the debates on risk and uncertainty is the distinction between *risk* and *uncertainty*. Classically, this paper will use the word *risk* to refer to situations in which precise probabilities are well defined, while *uncertainty* refers to a broader form of ignorance.

Our point is that the two dimensions are orthogonal, so that both risk and uncertainty can be either objective or Bayesian To emphasize this uncertainty will be discussed first using a subjective approach to uncertainty, and then using an objective approach.

As discussed above, the subjective approach suggests to observe betting behavior, and use the rationality assumption to infer believed probabilities levels (Ramsey, 1926, Bruno de Finetti 1937). Indeed if in a football game someone states that betting 1:1 on the home team is fair, that can be construed as a statement of equiprobability. But stating a "fair" odd is not the same as actually making a bet. If a person is observed betting at 1:1 on an event, it might be because he is certain that the event will occur, but nobody offered a better rate.

That intuition was seen as weakening the case for the subjective approach by Keynes (1921, ch. III par. 4), who explained:

> It might perhaps be held that a presumption in favor of the numerical valuation of all probabilities can be based on the practice of underwriters and the willingness of Lloyd's to insure against practically any risk. Underwriters are actually willing, it might be urged, to name a numerical measure in every case, and to back their opinion with money. But this practice shows no more than many probabilities are greater or less than some numerical measure, not that they are themselves practically definite. It is sufficient for the underwriter if the premium he names *exceeds* the probable risk.

Generally, observed betting behavior only implies upper or lower bounds on the subjective probabilities. More precisely, observing an economic transaction on a contingent good only allows one to infer that the expected value to the buyer was greater than the transaction price, and was lower for the seller[3]. If a rational actor buys 30 euros a gambling ticket giving the chance of a 100 euros prize, and further sell that ticket for 50 euros, one only learns that to the rational actor, the expected value of the ticket was between 30 and 50, so that the probability of winning is between 0.3 and 0.5. The mathematics of intervals are more complicated than those of point numbers, but Walley (1991)'s seminal book on imprecise probabilities demonstrated that Keyne's critique is not fatal to De Finetti's subjective view.

Uncertainty that can be represented by interval probability can also arise in a purely objective situation. Consider the Ellesberg's urn. This urn is a classical image of statistics: drawing a colored marble from a bag containing 100 such marbles. Suppose that the bag contains between 30 and 50 black marbles, and the other marbles are white. With that information, one can only say that p(black) is between 0.3 and 0.5.

Poorly defined probabilities are also an issue in the elicitation of expert knowledge. A practical way to elicit probabilities from an expert is to hand out a stake of 100 chips and ask the expert to distribute the chips among each alternative outcome. The facts that experts often feel uncomfortable doing this procedure, and that when one asks an expert a certainty level for each outcome separately, numbers do not necessarily add up to unity, suggests that there may also be uncertainties about personal probabilities.

4. Beyond the probabilistic model of uncertainty

Having shown that the dimension of precision is orthogonal to the objective vs. Bayesian debate, we now discuss five aspects of imprecision. We will call these randomness, possibility, deep uncertainty, incompleteness and fuzziness[4]. To support the point made in the previous section, these aspects will be discussed using an objective example: the bag with 100 colored marbles introduced above.

Randomness: The composition of the bag is known, so there is a well founded probability distribution. For example, assuming an unchanged climate, the potential annual supply of wind, sun or hydro power in a given area is a statistically known variable. Climate is the average weather in a location over a long period of time, so climate predictions are statistical in essence. This example shows that scientific predictions are not always deterministic.

Beyond the fundamental indeterminacy in quantum theory, an important reason for randomness in science is the problem of scale and chaos. Deterministic systems can follow chaotic dynamics, when the imperfect knowledge about the

[3] Only part of the gap between the buyer and the seller's valuation come from the different beliefs about the probabilities, another part comes from the different levels of utility provided by that good. Still, given the buyer's utility function, observing the transaction provides only a lower bound on the buyer's expected value.

[4] We do not consider that these five aspects of uncertainty can be ordered from the "least uncertain" to the "most uncertain" kind of ignorance. Possibility and deep uncertainty are about probabilities, while incompleteness and fuzziness are about the states of the world.

present state of the world limits to the ability of science to provide predictions at the relevant timescale given. For example, the best available socio-economic description of the consequences of most mitigation measures are very likely not a deterministic model, because the global society is a complex system that may be very sensitive to initial conditions. In other words, small perturbations possibly lead to large changes in the human response to the climate issue.

Possibility: The list of outcomes is known, and there is an upper bound on the number of some colors. For example, the bag contains three colors, less than 30 black, less than 60 red and less than 100 white marbles. Stating a possibility level amounts to state an upper bound on the admissible probability of a future, knowing on the other hand that the lower bound is infinitesimal (there is an infinity of futures that could happen). Ha-Duong (2003) argued that possibility theory (Dubois et al. 1998) is more relevant than probability to quantify the plausibility of far-distant futures.

Knightian or Deep Uncertainty: Knight (1921) seminal work describes a class of situations where the list of outcomes is known, but the probabilities are imprecise. This generalizes both kinds of uncertainty above. An extreme case would be that nothing is known about the proportion of each color in the bag. However, less unspecific statements could be made that still leave deep uncertainty about the drawing's outcome. The situation of interval probability presented above, that p(black) is between 0.3 and 0.5 if it is known that the bag contains at least 30 black marbles and 50 white ones, is a simple example of deep uncertainty. More generally, imprecise probability theory suggests to represent such deep uncertainty using a set of equally admissible probabilities (sets being more general than intervals when there is more than two outcomes).

Ha-Duong (2003), Kriegler et al. (2003) , Borsuk et al (2004) and Hall et al. (2005) argued that the kind of ignorance about the long-term future of climate change is a situation of Knightian uncertainty that should be treated with imprecise probabilities. Kriegler (2005) made an integrated assessment of climate change using imprecise probabilities and concluded that it was very unlikely that the warming in the 21st century would remain below 2 Kelvin in the absence of policy intervention. Moreover, he found that it would require a very stringent stabilization level of around 450 ppm CO_2 equivalent in the atmosphere to obtain a non-negligible value for the lower probability of limiting the warming to 2 Kelvin.

Incompleteness (absence, unknown unknown, black swans, structural uncertainty) relates to things that can not be talked about in a given frame of reference: concepts missing from a language or variables not included in a model. There would be this kind of uncertainty in the urn example if the list of possible colors was not completely known. This is dealt with by using a variety of models and by being explicit that any results given are only conditional to the frame of reference used.

Probabilities distributions are normally given over a universal set Ω . Randomness corresponds to situations when it can be assumed that Ω is exhaustive, and incompleteness corresponds to situations when it cannot. Transferable Belief Theory suggests to deal with absence by giving some probability weight to the empty subset {} (sometimes noted as $\emptyset$). It is straightforward to interpret $p(\{\})$ as the probability of an event not described in Ω . Curiously, most other theories of uncertainty make the assumption that Ω is exhaustive, and rarely discuss the idea that the empty subset is after all a subset of Ω .

Climate policy assessments cannot get rid of incompleteness: it is impossible to consider any and all the technologies and physical processes potentially involved. To be precise despite this issue, IPCC guidelines stress the need to explicit as much as possible the frame of reference by asking writing teams to explain the conditions and the assumptions leading to the conclusions. There are limits to this, since no proposition can make sense without a context, but the context can never be completely explicit. In any case, it is also important to keep in mind the meaning of statements in the global context, because they will be quoted as such by the media.

Incompleteness is acute when dealing with scenarios, since a set of scenarios does not make an exhaustive frame of reference Ω to describe the alternative futures of the energy-economy-climate system. Worst, if scenarios are given with enough precision, the probability of the scenario set itself is infinitesimal, since so many alternatives are possible. Consequently, while it could be mathematically meaningful to assign absolute probabilities level to each scenario within a set, these are probabilities conditional to a set of probability zero[5].

Fuzziness or vagueness describes the nature of things that don't fall sharply in one category or another. This kind of uncertainty is prevalent in natural language. In the urn's example, given the full spectrum of colors the number of 'dark' marbles would better be represented using a fuzzy number. From a modeling point of view, fuzziness contradicts the assumption that states of the world are mutually exclusive. It relaxes the modeling assumption that one and only one element of the universal set Ω will obtain.

While fuzzy modeling could potentially be used to integrate experts' knowledge with precise quantitative information, major integrated assessment models of energy and climate problems have not used much these techniques so far. Informally, IPCC experts do not ignore the fact that there is vagueness in the natural language. For example, the guidance note on uncertainty is explicit that categories should be considered as having “fuzzy” boundaries. In the previous report, the « burning embers » diagram (TAR WG II fig. SPM 2) used a fuzzy graphical representation of « Reasons for concern » to assign a fuzzy quantitative meaning to the word `dangerous’ of the UNFCC article 2.

5. Social and human dimensions of uncertainty

Seeing ignorance as a passive condition of an human-therefore-imperfect mind caused by missing information about the state of the world misses half of the picture. Information is a product. Ignorance can be actively caused by human volition. It can even be intentional. This section discusses four social and human

[5] The problem of conditioning with events of probability zero can be dealt with mathematically. Instead of defining conditional probability from the notion of unconditional probability: P(A|B) = P(A and B) / P (B), some advanced courses in Probabilities simply assume that unconditional probabilities do not exist, and consider conditional probabilities as the basic building block of the theory.

aspects of ignorance: strategic ignorance; surprise; values undecidabillity; and taboos. These aspectss are relevant to the study of climate change, mitigation and adaptation, because these activities are the outcome of social interactions.

Strategic ignorance involves the fact that rational agents, who are aware of information can use uncertainty as a strategic tool. Strategic uncertainties are an important human dimension of the response to climate change, since this response requires coordination at the international and national level.

Action in the context of strategic ignorance is usually formalized with game theory using the hypothesis of information asymmetry, that is assuming that one party in a transaction has more or better information than the other party. The informed party may therefore be able to extract a rent from this advantage. The following aspects of strategic ignorance have been recognized as important in the literature:

Adverse selection is a consequence of uncertainty that degrades the quality of the participants in a market. Adverse buyer selection occurs in insurance markets : agents who know they have a higher risk will buy more insurance than those who have a below-average risk. The classical example of adverse seller selection is the used cars market described by Akerlof (1970) : owners of good cars will be more likely to keep them for themselves. This leads to a vicious situation in which buyers presume that most used cars are bad (“lemons”), which may depress the price to the point where good car owners are not interested to sell at all.

Moral hazard occurs when the presence of a contract can affect the behavior of one or more parties (Mirrlees 1999). For example in the insurance industry, coverage of a loss may increase the risk-taking of the insured.

Free riders are actors who consume more than their fair share of a resource, or shoulder less than a fair share of the costs of its production. This issue is compounded when it is difficult to monitor the behavior of other actors. Even the possibility of free riding is likely to affect collective actions.

Information asymmetry is an important issue for the regulation of firms by governments and for international agreement. Adverse selection, free riding and moral hazard are key factors in the design of mechanisms to mitigate climate change.

However, not any strategic use of uncertainty is negative, some are generalities aimed at building agreements. Na and Shin (1998) and others suggested that generally, reaching an agreement may be easier under a “veil of uncertainty”. Cooperation is more likely to emerge ex-ante, before uncertainty is resolved, than ex-post, because more agents potentially gain from the agreement before the uncertainty is resolved. In contrast, Bramoullé et al. (2004) have shown that, from an ex ante perspective, cooperation may be less likely under uncertainty. The reason is that the difference in social welfare between cooperation and non-cooperation, that is the collective gain to reach an agreement, may be lower under uncertainty. Finus and Pintassilgo (2010) found the veil of uncertainty generally help to stabilize international environmental agreements, but less so when there is uncertainty about the costs of joining the agreement in addition to uncertainty on the environmental risk.

Next we turn to social aspects of uncertainty of interest to other disciplines such as psychology or anthropology. These have a clear importance for the communication and implementation of climate policies, which require coordinated changes in people's perceptions and behaviors at all scales.

Surprise means a discrepancy between a stimulus and pre-established knowledge. Complex systems, both natural and human, exhibit behaviors that were not imagined by observers until they actually happened. Surprise is a subjective psychological state, it depends on the observer. It can occur in a situation of uncertainty, but also in a situation of randomness if an event with a small personal probability (see section 2) realizes.

Surprises arise because recognition of events that do not share many features with existing mental structures is difficult. Psychologists further distinguishes between two kinds of mental structures: schemata (the plural of the Greek word schema) and semantic networks[6]. `Global Warming' belongs to a schematic mental structure because it retains features of the event and relate to perception or visceral sensations. `Climatic change' is part of a semantic network, abstract and related to language and logic. Kagan (2000) argues that schematic discrepancy is distinct from semantic discrepancy, and calls Surprise only the former while he terms latter Uncertainty.

Marx et al. (2007) argue that the distinction between experiential versus analytic processing is central to understanding the problem of communicating uncertain climate information. "Better understanding of experiential processing may lead to more comprehensible risk communication products. Retranslation of statistical information into concrete (vicarious) experience facilitates intuitive understanding of probabilistic information and motivates contingency planning. Sharing vicarious experience in group discussions or simulations of forecasts, decisions, and outcomes provides a richer and more representative sample of relevant experience. The emotional impact of the concretization of abstract risks motivates action in ways not provided by an analytic understanding. "

Examples of surprise could include rapid technological breakthroughs, global social troubles affecting oil prices or greenhouse gases (GHG) emissions, or abrupt change to a cooler climatic trend.

In the IPCC Second Assessment Report, “surprise” were defined as rapid, non linear response. This is an incorrect definition for many readers. If no climate change at all occur over the next 50 years, that would be a surprise to the science community. IPCC guidance notes for the Third Assessment Report acknowledged surprise was ambiguously defined previously and that it is, strictly speaking, a surprise is an unanticipated outcome. The concept of surprise was not discussed in the guidance notes on uncertainty for assessement reports four and five (Mastrandrea et al. 2011).

By allowing decision makers to get familiar in advance with a number of diverse but plausible futures, scenarios are one way of reducing surprises. Scenarios do not only allow to test existing strategies against a wide range of futures, they also facilitate stakeholders participation and allow to plant signposts allowing to recognize early which future is happening.

[6] The difference between these two forms of knowledge is also known as the difference between Symbol and Sign in linguistics.

Values undecidability. Some things are not assigned a truth level because it is generally agreed that they cannot be verified, such as the mysteries of Faith, personal tastes or belief systems. While these cannot be judged to be true or false or given a mathematical distribution they can have bearing on both behavior and environmental policymaking.

Diversity is a source of resilience. In time, a society where a variety of decision makers have a diverse set of values may be more robust to adverse environmental change than a less internally diverse society.

Model-based decision analysis deals with this kind of uncertainty by isolating it in the parameters of a social welfare function. The social welfare function is supposed to be given by the decision-maker, the scientific model is only here to find how to maximize it. These parameters commonly include intergenerational equity parameters (the discount rate), attitudes towards risk (the risk aversion coefficient) and international equity parameters (Negishi weights).

Some administrations indeed prescribe the discount rate to use when evaluating the costs and benefits of public projects. This is not the case for climate policy analysis models. There is no single decision-making authority to decide on how welfare should be measured. Thus, the "exogenous social welfare function" approach to deal with values undecidability has limits:

- Empirical estimates have to be conducted to determine which values of the social welfare parameters are most consistent with the observed behaviors. This is fraught with the uncertainties inherent in abductive inference (see section 6 below). Moreover, there is no agreement that observed behavior coincide with the desirable social welfare function. For example, some have argued that a normative rather than descriptive discount rate should be used.
- Values uncertainty creep up in models outside the social welfare function. Tradeoffs between the three pillars of sustainable development are an example. How to balance progress on economic well-being, saved statistical lives and conserved ecosystems ?
- Some negative consequences of the arbitrariness in choosing the parameters of the social welfare function can be avoided, if sensitivity analysis are used and show the robustness of results. Unfortunately, important results --like the optimal level of effort against climate change in the near term-- are not robust. See the Stern-Nordhaus debates on the discount rate for example.
- For technical reasons, the commonly used utility functions restrict the range of values that can be represented. For example Ha-Duong and Treich (2004) have shown that standard intertemporal expected utility functions are less apt to represent precaution than the more general Kreps-Porteus recursive utility functions.

Because of values undecidability, model-based decision analysis can only help to agree on the disagreements. Dialogue is used to coordinate action in spite of differences which can not be erased.

Taboos matters are what people must not know or even inquire about (Smithson 1988, p. 8). Originally, the word was related to sacred matters and religious customs in South Pacific people. Here I will use the word in a technical sense, to mean "things that are off limit to discuss for social reasons". These actively created areas of uncertainty exist in any social group, they are part of what defines a social group.

IPCC authors work under the mission statement "be policy relevant but not policy-prescriptive". IPCC assessments are bound by a double set of limits which arise not only because of its scientific mandate, but also because of its political essence.

The scientific mandate means that art and culture works, however influencial, are off limits to discuss. While ideally the IPCC would assess all the relevant literature for climate change mitigation, it says little on population policies, for example. It publishes few economic scenarios where the less developing countries do not catch up fast. IPCC global scenarios for the XXIst century look unlike the XXth century from a financial and military stability point of view. Taboos on nuclear power; oil supply; or national security issues can impact climate policy assessments.

Most important IPCC productions, such as the Summary for Policymakers, are approved line-by-line in plenary with government experts. The diplomatic community has lots of 'non negotiable' points. Indeed B. Müller [p. 68] explains the pace of climate negotiations as a stand off between two taboos that happen to be the key issues for the opposite party : the refusal of developing countries to discuss commitments and the refusal by industrialized countries to discuss anything that could remotely be interpreted as an admission of climate impact liability, such as the question of primary entitlement of emissions rights.

Taboos biase scientific assessments because what is morally unjust is not necessarily unlikely. To deal with taboos in an organization, it is necessary to get input from outside the social group.

Concluding the first descriptive part of this paper, Figure 2 summarizes our review of risk and uncertainty concepts so far. In the second part of this paper, we examine how these concepts are handled in the scientific community working on climate change.

Error: ignorance that needs to be corrected
- Vagueness (Fuzzy theory)
- Incompleteness or absence (Logics, Transferable Belief Model)
- Knightian or deep uncertainty (Imprecise probability)
  - Possibility (Possibility theory)
  - Risk (Probability)

Social and human dimensions: actively constructed ignorance
- Strategic uncertainty (Game theory)
- Surprise (Psychology)
- Taboos (Sociology)
- Values undecidability (Cultural studies)

Figure 2: An ontology of kinds of ignorance (with relevant methods).

6. The nature of scientific knowledge

This section argues that the broader forms of error affect all sciences, including those studying Nature. Not only Science cannot be perfectly deterministic, it cannot be precisely probabilistic either. To see why, let us remind ourselves that reasoning is usually classified in three kinds of inference: deductive, inductive and abductive.

Deductive inference derives a specific result from general premises (A implies B, A holds, therefore B). It is the safest way to make a conclusion. Example: Increasing the CO_2 concentration increases radiative forcing, this planet's CO_2 concentration has increased, therefore this planet's radiative forcing has increased.

Inductive inference learns general rules from specific cases. Induction is less powerful that deduction, as the truth of the premises make it only likely that the conclusion is also true. Example: The historical rate of decrease in energy intensity per unit of value appears to have averaged about 1 percent per year since the mid-nineteenth century (Nakicenovic 1996), therefore it is reasonable to include an autonomous rate of energy efficiency improvement in energy policy models.

Abductive inference allows one to learn a general hypothesis by observing a particular case using a general rule (A causes B, observing B, therefore suspecting A). This is also known as « detective logic ». It is even weaker than the previous kinds of inference, as it runs contrary to deduction. Example: There is coral bleach all over the world. Global climatic change would explain that better than anything else. Therefore, there (probably) is global climatic change.

Deduction is the most convincing form of reasoning. Formal validity, that is the absence of contradiction in the reasoning, is straightforward to check. When the inference is logically valid, checking the truth of the premises ensures the truth of the result. In this case, the reasoning is sound. Deductive reasoning is mostly what numerical computer models do.

One source of uncertainty in science for policy is the difficulty in observing global systems. For example numeric models simulating the evolution of the ocean-atmosphere-ice-land earth system have to be calibrated with much less than one measurement per grid cell per variable. Energy economic models generally have data on trade flows, consumption patterns and technological costs, but not with the same precision for all regions of the world. In both classes of models, the quality of observations degrades considerably when looking more than forty years backwards -- a problem when looking more than ninety years forward.

This is not theoretically a problem with deduction. When the premises are uncertain, valid reasoning allows to propagate that uncertainty to the conclusions. Techniques like Monte Carlo simulations allow to propagate probabilistic uncertainty from the premises to the results. Computational limits may be an issue. Absent probabilities, for example when there is a set of scenarios, uncertainty propagation techniques are mostly ad hoc. This is an interesting active research frontier, but this introduces a methodological bias. In theory the outcome should not be probabilistic either.

Even when the reasoning is formally valid and specific facts are well observed, the general rule used for deductive inference may be uncertain. Rules are not given from Above, they have to be discovered by scientists' from observed facts. Thus, science cannot be solely based on deductive logic, scientists have to use induction. Induction is necessary, but can never be certain because there is no finite answer to the fundamental question : How many specific cases are needed to infer a general rule? Let us consider two polar cases:

- The scientific method bases induction on repeated verifiable experiments: a potentially infinite number of observations. Even in this ideal situation, inductive reasoning is open to revision if a counter-example is shown. Scientific laws are open to revision when new facts require it. The provisory nature of scientific knowledge is an instance of incompleteness (in reference to Figure 2), a kind of uncertainty not handled well by probabilities. The biggest scientific advances are also those that vastly increase, rather than decrease, the number of open questions.
- Many in the Humanities and Social Sciences have no use of the experimental method and do not see "laws" of nature as necessary in their research field. We can do with "stylized facts". Consider for example one very fundamental tenet of Economics: demand and offer. It says that a higher price leads to a lower demand for a normal good. If one forecasts a higher price of oil for the next decade, can one deduct that oil consumption will decline ? That conclusion is controversial, it is quite possible that the consumption increases as well. Factors such as the supply curve in other energy sources, energy policies, economic and demographic expansion will affect the equilibrium price level.

Most scientists are practical and compromise between these two extreme attitudes. When they find overwhelming evidence supporting a persistent pattern, they call it a scientific law, even if the evidence does not come from controlled experiments. The alternative would be to reserve the word "science" for a very restricted set of activities, the "hard sciences". Outside the laboratory, the conditions are never exactly the same.

Probability theory offers a powerful language to deal with uncertainty, and using probabilistic reasoning is certainly necessary to deal with induction. There is no space to review all the longstanding philosophical discussions here, some have even argued that probability is "the logic of science" (Jaynes et al. 2003). I argue that precise probability theory is not sufficient to express scientific knowledge. It works well only towards one end of the spectrum: wher there are a lot of data. I ague that when data is scarce, it is preferable to express knowledge imprecisely.

For example, economists often assume that the correct way of learning is to use Bayesian learning, limited to precise probabilities. This does not resolve the deeper kinds of uncertainties discussed in section 3 such as possibility and Knightian uncertainty. Imprecise probabilities would make the difference between an even chance (knowing only that $p = 0.5$) and no information (knowing only that $0 <= p <= 1$). If and when the mathematical models of learning are extended to imprecise probabilities, the problems of incompleteness and vagueness will remain to be solved, not to mention the human dimensions of science.

Ioannidis (2005) exposes the extend to which the problem of induction is poorly solved by our methods used today. He argued that most published research findings in medicine are false, simply because of technical problems with pre-study odds, insufficient statistical power of experiments and biases. He added that "for many current scientific fields, claimed research findings may often be simply accurate measures of the prevailing bias." To determine the effects of a new drug or therapy is an induction problem of great practical importance, but the officially reviewed methods do not seem to resolve it well.

The variety of accepted practical rules for scientific reasoning is another sign that the problem of is fundamentally not solved. Different scientific disciplines will have different methods of statistical testing: some would use parametric methods, other non-parametric methods, and others Bayesian methods. The 95% usual level of confidence for statistical testing is completely arbitrary and culturally determined. The theoretical physics community, for example, requires a 99.99994% surety (5 sigma confidence level) to declare that an elementary particle has been discovered. Within each discipline, culture and history have a role in determining the accepted way to establish scientific results.

Scientific inductive reasoning may be superior to informal inductive reasoning, as human minds tend to jump quickly to generalizations from a few observations. But there is no known general mathematical method for induction. It is not completely solved by probability theory and statistics, especially when the number of observations is low.

The methodological situation is even worse with respect to abductive reasoning. Abduction is necesary for the everyday conduct of science, for example to guess the hypothesis to be verified. Abduction is also important for practical policy questions. For example, the attribution of global warming to the use of fossil fuels by humans, as opposed to the natural factors like the variability of climate or the sun has been a popular scientific discussion. Pearl (2009) argued that causal analysis had produced many results in the past few decades, which were hardly known to researchers who could put them into practical use.

In summary, there are fundamental reasons why imprecision should be a problem for all scientific disciplines. The broader forms of error affect not only the sciences studying Humans and Societies, but also those studying Nature.

7. Risk and uncertainty in IPCC's guidance note

This section discusses Hassol et al. (1997) and Mastrandrea et al. (2 011), the IPCC guidance notes on risk and uncertainty. We show that they provide an interdisciplinary unified approach to risk and uncertainty which account for the conclusions of the previous section.

The climate change scientific community has been refining its guidelines on how to deal with risk and uncertainty over three assessment reports writing cycles. This effort's outcome is a short reference document (Mastrandrea et al. 2010), widely peer-reviewed and field tested multiple times with thousands of interdisciplinary scientists from a variety of cultural origins. The document "define s a common approach and calibrated language that can be used broadly for developing expert judgments and for evaluating and communicating the degree of certainty in findings of the assessment process."

The IPCC guidance note (Mastrandrea et al. 2010) relies on *two metrics for communicating the degree of certainty in key findings:*

- *Confidence in the validity of a finding, based on the type, amount, quality, and consistency of evidence (e.g., mechanistic understanding, theory, data, models, expert judgment) and the degree of agreement. Confidence is expressed qualitatively.*
- *Quantified measures of uncertainty in a finding expressed probabilistically (based on statistical analysis of observations or model results, or expert judgment).*

The guidance note leaves it to the scientists to assess, in terms appropriate for their discipline, what constitutes robust evidence (How many observations are needed to infer that a result ?) Thus, the guidance note does not seek to limit writing teams to a single approach. As Swart et al. (2008) discussed, agreeing to disagree allows the assessment to reflect the variety of risk and uncertainty analysis methods used in the vast climate change literature.

The confidence metric is explicitly qualitative. Using it is a prerequisite to using numerical assessment of uncertainties. Numbers are only justified for findings backed by robust evidence or high agreement, preferably both. This corroborates our claim that probability is not sufficient as the langage of science.

The guidance note calibrated langage for describing quantified uncertainty in the form of a likelihood scale. For example, "very likely" corresponds to "90-100% probability", "very unlikely" corresponds to "0-10% probability", and "virtually certain" to "99-100% probability"[7]. This is another use of imprecise probabilities.

The IPCC guidance note views uncertainty as a limit of scientific reasoning (deduction, induction and abduction) caused by the imperfection of empirical evidence. In terms of Figure 2's typology of kinds of ignorance, it is focused on the Error, as opposed to Social and human dimensions of uncertainty. Those are

[7] The guidance note adds that "in some cases, it may be appropriate to describe findings for which evidence and understanding are overwhelming as statements of fact without using uncertainty qualifiers."

largely not discussed in the guidance note.

8. Discussion: imprecision and decision-making

This section explains why it is critical to properly report imprecision in scientific assessments. It is because of the precautionary principle.

Decision-making when probabilities are well defined not the same as decisionmaking under deeper uncertainty. Uncertainty characterizes both the costs and benefits of climate policies, and under these conditions the standard decision-making criteria based on the maximization of expected utility, also called the Rational Actor Paradigm, is not robust. Most theories of decision making under uncertainty do not assume that it is always coherent or even possible to optimize expected utility. These include generalized expected utility by Ellesberg (2001), Knightian decision-making discussed in Bewley (2002) and Walley (2000), the rank-dependent expected utility and Prospect Theory. Some would argue that social decision making follows a system of procedural rules that are determined by evolution and selection rather than forward-looking rationality.

By this paper's definition, under uncertainty a probability distribution is not well defined, so that the expected value of a contingent good is described by an interval rather than a precise number. Knightian decision making remarks that intervals are not totally ordered as real numbers are, and brings forward the intuition that under uncertainty alternative acts may sometimes be incomparable. When there are large uncertainties (meaning expected value intervals are large) about a policy, it might not be possible to conclude clearly that the expected costs are less than the expected benefits. In this situation, the concept of a globally optimal choice is replaced by a set of equally admissible but incomparable choices.

Is there reason to believe that the issues with the Rational Actor Paradigm under uncertainty count for climate policy ? When the expected value of a good is an interval, the interval's lower bound can be interpreted as the maximum price acceptable to buy that good, namely the willingness to pay (WTP). Conversely, the upper bound can be viewed as the willingness to accept (WTA). This ties the degree of uncertainty regarding the value of the good with the WTA - WTP difference, a well studied anomaly of the Rational Actor Paradigm. Empirical research consistently finds that when people are asked to value non market goods, such as the quality of the environment, the gap between the two values can be significant. Moreover, Horowitz et al. (2002) meta-analysis found that the less the good is “like an ordinary market good”, the higher is the WTA/WTP ratio. Since climate stability and technical progress are not ordinary market goods at all, there is reason to believe that an evaluation of the climate policy would be subject indeed to the large WTA/WTP ratio effects.

9. Concluding summary

In the first part, this paper outlined a few philosophical dimensions of the risk and uncertainty debate in the context of climate change mitigation. It opposed not only the objective approach (viewing probabilities as degrees of truth) versus the Bayesian approach (viewing them as degrees of certainty), see Figure 1, but also situations of risk (related to situations where precise probabilities are well founded) versus situations of Knightian uncertainty (a broader form of ignorance). Then it added a the difference between error that must be corrected, and ignorance actively constructed by humans. Figure 2 summarizes this taxonomy.

In the second part, the paper argued that the broader forms of error affect all sciences, including those studying Nature. This was shown by reminding that scientific certainty is provisory at best, and that in most practical situations key scientific findings have to be formulated in an uncertainty framework because of the problem of induction. Not only Science cannot be perfectly deterministic, it cannot be precisely probabilistic either. That point is illustrated with the IPCC guidance note on risk and uncertainty. They provide an interdisciplinary framework severly bounding the applicability domain of probability numbers. It is critical to properly report imprecision in scientific assessments because it changes the decision-making situation.

We noted that the IPCC guidance notes use imprecise probabilities, but do not discuss the social and human dimensions of uncertainty: strategic uncertainties, surprises, metaphysics and taboos.

The main novelty of the 2010 version of IPCC guidance note is the introduction of the "Agreement" scale, which is meant to describe the extend to which information sources point in the same direction. That novelty could be developed by requiring the authors to reflect on the reasons for disagreement. Disagreements on oil reserves and peak oil, for example, are in a large part explained by the strategic manipulation of information. On the other hand, values undecidability may explain disagreements on the potential for nuclear or coal with carbon capture technologies.

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