A Three Echelon Location Routing Problem with Multiple Commodities and Courier Deliveries

Sara Ahmed

A Thesis

in

The Department

of

Mechanical and Industrial Engineering

Presented in Partial Fulfillment of the Requirements

for the Degree of

Master of Applied Science (Industrial Engineering) at

Concordia University

Montréal, Québec, Canada

September 2021

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

By: **Sara Ahmed**

Entitled: **A Three Echelon Location Routing Problem with Multiple Commodities and Courier Deliveries**

and submitted in partial fulfillment of the requirements for the degree of

Master of Applied Science (Industrial Engineering)

complies with the regulations of this University and meets the accepted standards with respect to originality and quality.

Signed by the Final Examining Committee:

______________________ Chair
Dr. Ali Akgunduz

______________________ Examiner
Dr. Ali Akgunduz

______________________ Examiner
Dr. Mingyuan Chen

______________________ Supervisor
Dr. Ivan Contreras

______________________ Co-supervisor
Dr. Ana María Anaya-Arenas

Approved by ______________________
Martin D. Pugh, Chair
Department of Mechanical and Industrial Engineering

__________ 2021

____
Mourad Debbabi, Dean
Faculty of Engineering and Computer Science

Abstract

A Three Echelon Location Routing Problem with Multiple Commodities and Courier Deliveries

Sara Ahmed

Research on location-routing is extensive, providing new variants for numerous applications. It is worth mentioning that most contributions present in the literature of freight distribution are limited to two-echelon, single commodity networks imposing one delivery strategy for all flows. While there are recent variants that consider extensions to this problem, they rarely consider modifying all the features simultaneously. To address this gap, we define a multi-echelon location and routing problem with multi-commodity pick-up-and-delivery (3E-LRP-MC-PD). We further consider the choice between three different transportation strategies for each commodity. The problem is formulated as an integer program and three additional variants of the formulation are considered. A slope scaling decomposition matheuristic is proposed to solve the problem in reasonable computational time. Computational experiments are performed to asses the different formulations and to evaluate the performance of the matheuristic. Finally, additional experiments are conducted to investigate the impact of the unique attributes of the 3E-LRP-MC-PD.

Keywords: *Location-routing (LR), Multi-echelon, Multi-commodity, Hybrid network, Decomposition, Matheuristic*

Acknowledgments

I would first like to express my deepest gratitude to my supervisors **Dr. Ivan Contreras** and **Dr. Ana María Anaya-Arenas** for their patience and guidance. I could not have asked for more understanding or knowledgeable mentors. Your support has been truly appreciated.

I would also like to thank my amazing parents for their endless encouragement. I am especially grateful to my brother **Ahmed Ahmed**, my sister **Rehab Ahmed** and my dearest friend **Reem Osama**. This work would have never been possible without your support.

Last but by no means least, I would like to thank my grandfather **Ismael Ahmed Ali**, who although is oceans away, never fails to be my biggest supporter.

Contents

List of Figures viii

List of Tables ix

1 Introduction 1

1.1 Definition & Scope 1

1.2 Industrial Scenarios 3

1.3 The Optimization Approach 4

1.4 Contribution of the Research 5

1.5 Thesis Structure 8

2 Literature Review 9

2.1 Location-Routing 9

2.2 Variants and Extensions of LRP 10

2.2.1 Multi-echelon LRP 11

2.3 Hybrid Nerworks 13

2.4 Multi-commodity Networks 14

2.5 Pick-up & Delivery 15

2.6 Matheuristics in LRP 16

3 Problem Definition and Formulation 18

3.1 Problem Definition 18

3.2 The 3E-LRP-MC-PD Formulation 19

3.3 Modifications 21
3.3.1 Equality Constraints 21
3.3.2 Sub-tour elimination Constraints 23
3.3.3 Resulting Formulations 24

4 Methodology **25**
4.1 Decomposition Scheme 25
4.2 Sub-problem 1: Min-cost flow problem 31
4.3 Sub-problem 2: Multi-commodity arc routing problem 34
4.4 A Heuristic for MCARP 36
4.4.1 Initialization & Selection 37
4.4.2 Commodity Assignment IP 38
4.4.3 Updates 38
4.4.4 Assignment costs 40
4.4.5 Vehicle Size Change 41
4.4.6 Feasible Route IP 41
4.5 Updating Cost Coefficient 43
4.6 Randomizing the search 44
4.6.1 Removing a vehicle from the solution 44
4.6.2 Iterative MCFP 44
4.6.3 Cost Coefficients Update 44
4.7 New Solution Space 45
4.7.1 Adding Commodity-Courier Assignment 45
4.7.2 Removing Commodity-Courier Delivery 45

5 Instance Generation **47**
5.1 Network 47
5.1.1 Non-Courier Nodes 47
5.1.2 Courier Nodes 49
5.1.3 Transshipment Nodes 50

5.1.4 Travel time 50

5.2 Commodities 51

5.3 Vehicles 52

5.4 Courier costs 53

6 Computational Experiments 55

6.1 CPLEX Parameters 56

6.2 Categorizing Instances 56

6.3 Session 1: Comparing Formulations 57

6.4 Session 2: Matheuristic Performance 62

6.5 Session 3: Complexity of the 3E-LRP-MC-PD 67

6.5.1 Bundling & Transit Time 67

6.5.2 Courier Collection Nodes 70

6.6 Session 4: Importance of the 3E-LRP-MC-PD 73

6.6.1 Bundling & Transit Time 73

6.6.2 Courier Collection Nodes 76

7 Conclusion and Future Research 77

Bibliography 79

List of Figures

Figure 1.1 Key Components of distribution and logistics (Rushton, Croucher, & Baker, 2017) . . . 2
Figure 1.2 Echelons present in the 3E-LRP-MC-PD . . . 6
Figure 1.3 An example of a vehicle route including arcs in different echelons . . . 6
Figure 2.1 The standard single-echelon location routing problem (Min, Jayaraman, & Srivastava, 1998) . . . 10
Figure 2.2 Two-echelon location-routing problem representation (Boccia, Crainic, Sforza, & Sterle, 2011) . . . 11
Figure 2.3 Pure vs Hybrid two-echelon distribution network (Guastaroba, Speranza, & Vigo, 2016) . . . 13
Figure 4.1 Feasible commodity paths . . . 27
Figure 4.2 Feasible Vehicle Assignment . . . 27
Figure 4.3 Feasible Vehicle Assignment for restricted bundling . . . 28
Figure 4.4 A Feasible Commodity Path Split Between Two Vehicles . . . 29
Figure 4.5 Examples of vehicle routing infeasibility . . . 39
Figure 4.6 Example of commodity path infeasibility . . . 39
Figure 4.7 Examples of feasible vehicle routes . . . 42
Figure 5.1 Cluster representation . . . 48
Figure 5.2 Example Nodes Generated . . . 48
Figure 5.3 Example Courier Nodes Generated . . . 49

List of Tables

Table 3.1 Summary of the 3EIRP-MC-PD Formulations 24

Table 6.1 Formulation Comparison for Set 1 59

Table 6.2 Formulation Comparison for Set 2 61

Table 6.3 Formulation Comparison for Set 3 61

Table 6.4 Matheuristic Performance: Set 1 63

Table 6.5 Matheuristic Performance: Set 2 64

Table 6.6 Matheuristic Performance: Set 2 continued 66

Table 6.7 Relating Bundling and Transit Time Requirements to Complexity 69

Table 6.8 Relating Bundling and Transit Time Requirements to Complexity (2) 69

Table 6.9 Relating Courier Transportation Strategy to Complexity 70

Table 6.10 Relating Number of Courier Nodes $|C|$ to Complexity 72

Table 6.11 The Impact of Relaxing Bundling Restrictions on Costs 73

Table 6.12 The Impact of Transit Time Restrictions on Costs 75

Table 6.13 The Cost Saving Potential of Considering Courier Delivery 76

Chapter 1

Introduction

1.1 Definition & Scope

Distribution logistics plays a pivotal role in all industries and economies. In 2017 the growth of the logistics industry in Canada was 1.5 times faster than any other industry (Transport Canada, 2018). More recently, Chopra (2019) highlighted that freight distribution costs make up about 20% of the cost of manufacturing. Finally, Armstrong & Associates (2020) reported that distribution logistics expenditures in 2019, were greater than 8% percent of the sales revenue for North America, Europe and Asia.

This rapid growth is indicative of the new challenging freight distribution landscape. Nowadays, companies are operating in a more global marketplace (Brandimarte & Zotteri, 2007; Rushton et al., 2017). This translates to geographical dispersion between the supply and demand points. Additionally, most industries have a diverse product portfolio, where products can differ in terms of size and transportation requirements. Lastly, customers costly expectations of faster deliveries, result in very restrictive service level requirements.

Fortunately, the rapidly changing landscape also facilitates new cost-saving opportunities. Research in city logistics (Cleophas, Cottrill, Ehmke, & Tierney, 2019; Ranieri, Digiesi, Silvestri, & Roccotelli, 2018; Savelsbergh & Woensel, 2016) and e-commerce (Houde, Newberry, & Seim, 2017; Rodrigue, 2020) highlighted these opportunities. The more commonly utilized strategy is the use of consolidation facilities to improve efficiency and reduce costs. These facilities allows

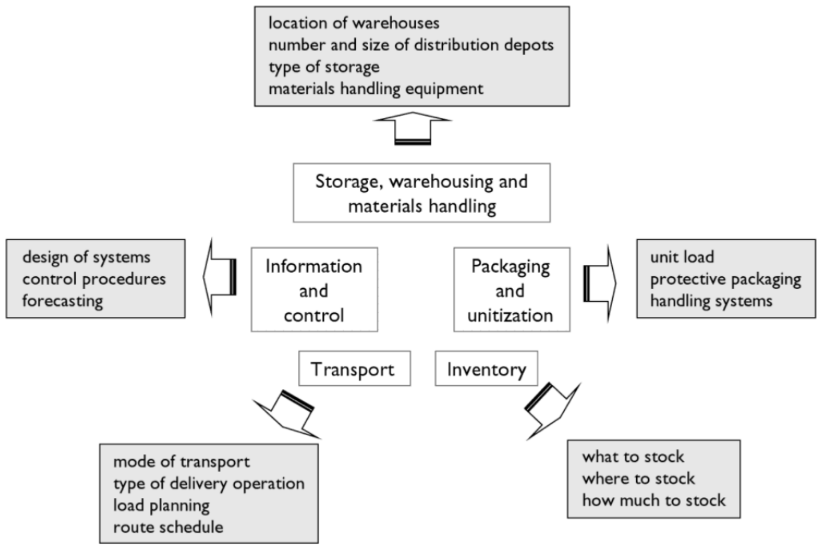


Figure 1.1: Key Components of distribution and logistics (Rushton et al., 2017)

exploitation of economies of scale from load and route consolidations. Distribution costs can also be reduced through collaboration initiatives. In essence, collaboration allows companies to benefit from the established infrastructure of other supply chains (Ranieri et al., 2018). A common example is when companies complete only part of the delivery and use couriers to perform last mile deliveries to consumers (Cleophas et al., 2019; Ranieri et al., 2018).

To cope with today's competitive distribution environment, the academic and business world redefined the scope of distribution logistics. Originally distribution was only seen as the physical movement of goods. Nowadays, the more acceptable definition of distribution logistics has a much wider scope (Rushton et al., 2017). As explained by Rushton et al. (2017), logistics is a part of the supply chain ensuring profitable transfer of goods while maintaining acceptable service level requirements. Hence, logistics management covers a wide range of activities summarized in Figure 1.1.

Rushton et al. (2017) explained that logistics activities can be further categorized as material handling logistics and distribution logistics. Although the areas sometimes overlap, distribution logistics broadly covers the flow of goods from production sites to consumers. These logistics activities usually include network design and transportation management (mode, route and load planning).

This new extensive scope acknowledges that distribution management requires making both strategic and everyday operational decisions. Strategic decisions imply a long planning horizon. These decisions usually cover locating facilities and determining their capacities. The strategic level provides the conditions in which the operational decisions are implemented (Schmidt & Wilhelm, 2000). Operational decisions then determine how the distribution network is utilized to respond to customer demands (Schmidt & Wilhelm, 2000). Operational decisions include everyday scheduling and routing operations.

1.2 Industrial Scenarios

To design and utilize a competitive distribution network, it is important to: (1) optimize the interaction between strategic and operational decisions (Schmidt & Wilhelm, 2000); and (2) exploit the opportunities of the new distribution landscape (Savelsbergh & Woensel, 2016). This is best exemplified by Amazon's distribution strategy. Amazon is a key player in the e-commerce industry. The company's unique selling proposition comes from making a wide range of products accessible in very short delivery time (Rodrigue, 2020). This unique selling advantage creates very challenging distribution requirements. Firstly, Amazon's geographical expansion means that supply and delivery points are sparsely located. Additionally, the wide range of products require tailored transportation strategies because of the different products' size and transportation requirements. Finally, offering consumers same day or next day deliveries translate to very short allowed transit time.

To achieve a competitive strategy, Amazon divides distribution to two main stages (Rodrigue, 2020). The first stage involves the delivery of orders to consolidation facilities near delivery points. This allows the company to exploit economies of scale. According to Rodrigue (2020), the ideal location and size of each facility was determined by considering a trade-off between economies of scale, fixed costs, operational requirements and lead time. The second stage is last mile delivery, where individualized deliveries are made to final consumers. The nature of last mile delivery can excessively increase routing costs. To overcome this, some deliveries are routed between a network of local post offices. While the remaining parcels are delivered using other third party logistics (3PL) providers . Collaborating with couriers and 3PLs allows Amazon to use a unique transportation strategy for each parcel, which can significantly reduce costs.

The healthcare industry also highlights the importance of optimizing distribution logistics. Beaulieu, Roy, and Landry (2018) and Dooley (2009) showed that healthcare logistics costs are 8 to 20 times greater than other industries. This clear logistic inefficiency is often associated with the complexity of the healthcare supply chains (Abdulsalam, Gopalakrishnan, Maltz, & Schneller, 2015; Beaulieu et al., 2018; Landry, Beaulieu, & Roy, 2016). According to Beaulieu et al. (2018), the wide product range is a major contributor to the complexity. Hospitals' inventories include medical supplies, drugs, food products and house keeping products (Landry & Beaulieu, 2013). In addition to product range, hospital logistics often require transportation between geographically dispersed locations. A closer look, however, shows that the distribution requirements of the healthcare industry are very similar to that of e-commerce. Both in terms of the diverse product range with corresponding unique transportation requirements; and in terms of geographical dispersion. However, as identified by Beaulieu et al. (2018), healthcare organizations do not usually apply distribution strategies that are often utilized by other industries which results in excessive logistics costs.

1.3 The Optimization Approach

As highlighted, a good distribution strategy is invaluable. In addition to excessive expenditures, poor distribution logistics can also affect a business's competitiveness, quality of service and customer satisfaction. It is therefore only natural that extensive research has been done in planning problems for freight transport and distribution. One popular academic approach is modelling distribution decisions as location-routing optimization problems. Then developing efficient solution strategies to make cost-effective strategic (location) and operational (routing) decisions.

From an modelling perspective, a distribution network connects supply and delivery points. At the very least, when all freight is transported directly from origin (suppliers) to destination (consumers), the network will include a supply stage and a delivery stage. Often, they also include one or more intermediary stages involving storage, merging or consolidation of freight. Transportation occurs between each pair of stages and each pair is represented by one echelon (level) of the distribution network (Cuda, Guastaroba, & Speranza, 2015).

Distribution networks can be broadly categorized based on the number of echelons present (Cuda et al., 2015). A single echelon network is utilized if all the commodities (freight) are *directly* delivered from their origin to their destination (i.e. without intermediate facilities). Direct deliveries should be used if there is a short distance between origins and destinations, the shipment fills the capacity of the vehicle, or there are tight time restrictions (Guastaroba et al., 2016). Conversely, freight can be *indirectly* shipped to its destination through a multi-echelon distribution network. An indirect delivery involves commodities being transported from their origin to one or more intermediate facilities before reaching their destination. Guastaroba et al. (2016) provided a detailed review of the different intermediate facilities, their impact on freight transportation, and the relevant problems in the literature.

It is challenging to realistically model the distribution landscape without excessively increasing complexity, ultimately hindering good solutions. Hence, optimization problems addressing distribution networks are usually simplified and do not accurately reflect real industrial scenarios (Azizi & Hu, 2020). More recent variants are addressing one or more realistic industrial aspect. This includes more realistic modeling of multi-commodity attributes and allowing unique transportation strategy for each commodity. More realistic variants may also consider more flexible routing decisions. Finally, few variants account for possible collaboration strategies along the distribution network. It is fair to note that while the recent variants consider more realistic aspects, they rarely consider all the limitations simultaneously.

1.4 Contribution of the Research

This research considers how to strategically design and route a range of commodities through a multi-echelon distribution network represented in Figure1.2. For each commodity there is a choice between: (1) direct delivery, (2) consolidated (indirect) delivery, and (3) courier delivery. For the commodity flows that utilize consolidated delivery, the locations of the consolidation points are selected from a set of possible transshipment locations. Finally vehicle routes are determined allowing both pick-up and delivery of commodities by the same vehicle and also allowing vehicles to

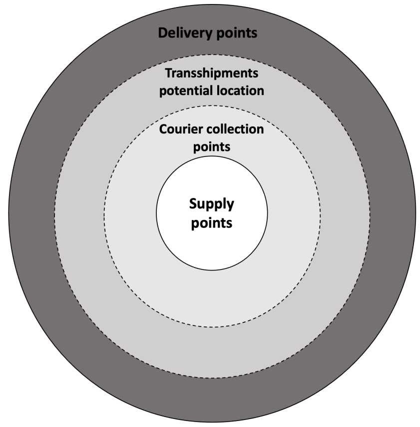


Figure 1.2: Echelons present in the 3E-LRP-MC-PD

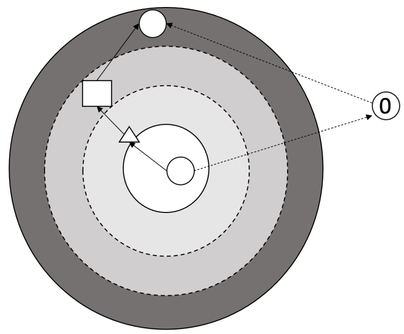


Figure 1.3: An example of a vehicle route including arcs in different echelons

transverse any arc on the network. An example of the flexible routing is seen in Figure 1.3. The aim is to generate a routing plan that reduces logistic costs while respecting the quality restrictions of the products being transported. This is formulated as a three-echelon location and routing problem with multi-commodity pick-up-and-delivery (3E LRP-MC-PD) considering both direct and courier shipments. This thesis aims to model this new more realistic variant of the location-routing problem and design a matheuristic to efficiently solve challenging instances.

The motivation behind considering three different transportation strategies (*hybrid network*) is to: (i) more accurately represent real industrial scenarios, (ii) account for the difference between commodities' attributes and hence the different transportation requirements, and (iii) explore the potential cost-saving opportunities of allowing flows on the same network to be transported differently. Direct and consolidated delivery are often studied in the literature. Direct delivery from a commodity's origin to its destination is often ideal for large or time-sensitive shipments and for shipments with close origin and destination. Consolidating the delivery of multiple shipments can be utilized when transporting less restrictive commodities to improve vehicle utilization and benefit from economies of scale.

Courier deliveries are often used for last mile delivery of e-commerce freight where freight needs to be transported from warehouses to scattered individual consumers. A commodity is transported from its origin to an intermediary courier collection point and the courier would be responsible for the delivery to the destination. Courier deliveries can reduce routing costs and provide flexibility because courier collection points are numerous, already established and more conveniently located compared to consumers. As highlighted by Amazon's last-mile distribution, using courier deliveries provides the flexibility needed for frequent collection and timely deliveries of special commodities without adding complexity to the vehicle route planning. They can also be beneficial for transporting small shipments with distant origin and destination if the courier delivery cost is less than the vehicle routing cost.

Finally unique commodity attributes are considered to more realistically convey product variety and transportation restrictions. The literature often distinguishes multiple commodities in terms of the amount of each commodity and the service level requirements. A new bundling attribute is defined between commodity pairs, such that some commodity pairs can not be transported together.

The main contributions of this research are:

- Introducing a new multi-commodity three-echelon location-routing problem in a hybrid network with pick-up and delivery.
- Distinguishing the different commodities based on multiple criteria including bundling restrictions and service level constraints.
- Proposing courier deliveries as a transportation strategy, in addition to the typical direct and consolidated delivery options.
- Allowing vehicle routes to include pick-up and delivery requests and to transverse all echelons.

- Proposing a new decomposition scheme for a matheuristic algorithm to solve the 3E-LRP-MC-PD problem.

1.5 Thesis Structure

The structure of the thesis is organized as follows. Chapter 2 provides a detailed literature review of the relevant variants of the location-routing problem and the matheuristic approaches used to solve them. Chapter 3 presents an integer programming (IP) formulation for the 3E-LRP-MC-PD and proposes additional variants that can help improve the performance of commercial solvers. In Chapter 4 we present a new decomposition scheme and present a slope-scaling matheuristic algorithm to solve the 3E-LRP-MC-PD in reasonable computational time. Chapter 5 proposes a procedure to generate instances of the 3E-LRP-MC-PD such that all attributes are represented without inducing bias. In Chapter 6, we present the computational studies to assess the formulations; evaluate the matheuristic and analyze the impact of the unique attributes of the studied problem. Finally, we summarize our research and present a direction for future research in Chapter 7.

.

Chapter 2

Literature Review

In this chapter, first we define the standard location-routing problem. Then we discuss the recent relevant variants and extensions of the problem. Finally, we review the decomposition matheuristic approaches proposed to solve large instances of the LRP.

2.1 Location-Routing

Distribution network design encompasses both strategic decisions related to facility location and operational routing decisions. In the past the two problems were addressed separately by determining locations first and then the routes (Mara, Kuo, & Asih, 2021). The traditional approach failed to account for the interdependency of the two decisions which led to sub-optimal results (Min et al., 1998; Salhi & Rand, 1989).

As explained by Mara et al. (2021), early work was done by Boventer (1961), Maranzana (1964) and Webb (1968) to highlight the ”inter-dependency” of the location and routing decisions. However, the computational power at the time was not developed enough to solve both problems concurrently. Since then, the constant development in optimization research, along with the technological advancements allowed extensive research to address the integrated approach to the location-routing problem (Mara et al., 2021).

In their earlier work, Nagy and Salhi (2007) defined the standard location-routing problem as

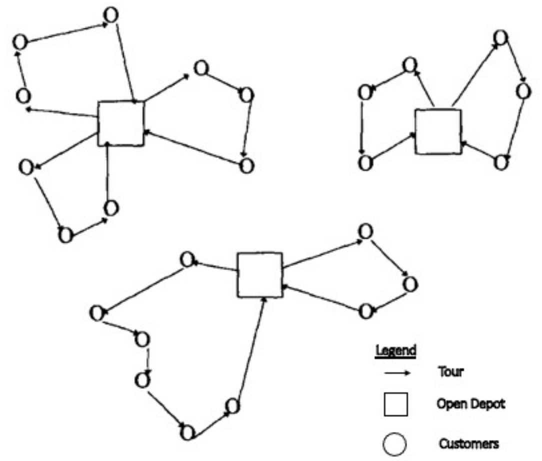


Figure 2.1: The standard single-echelon location routing problem (Min et al., 1998)

”location planning with tour planning aspect taken into account.” Following this Drexl and Schneider (2015) defined LRPs to be a problem where at least two interdependent decisions must be made: (i) the location of facilities to open from a finite or infinite set of potential locations, and (ii) the vehicle routes to build, such that customer demand is satisfied. Schneider and Drexl (2017) further distinguished the standard LRP to be discrete, static, single objective problem with no inventory decision in a single-echelon system where a customer is visited only once for delivery. Figure 2.1 shows a feasible solution to the standard LRP highlighting its key characteristics. For extensive reviews on the standard LRP we refer the interested readers to the surveys by Prodhon and Prins (2014) and Schneider and Drexl (2017).

2.2 Variants and Extensions of LRP

According to Drexl and Schneider (2015), varaints of LRPs change one or more characteristic of the standard problem (defined in section 2.1). Mara et al. (2021) and Drexl and Schneider (2015) provided a detailed taxonomy for classifying variants of LRPs studied in the literature.

The scope of this thesis is limited to extensions of the standard LRP that consider multiple echelons, commodities or transportation strategies for a single planning horizon. As well as LRP

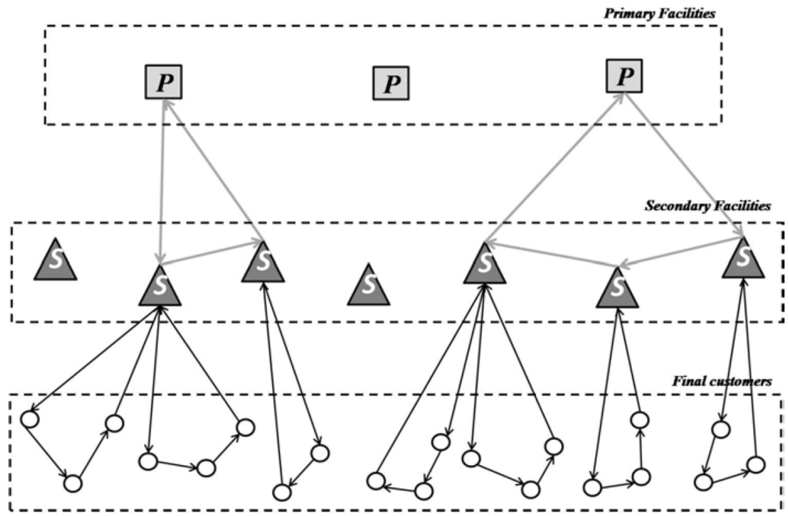


Figure 2.2: Two-echelon location-routing problem representation (Boccia et al., 2011)

variants that allow pick-up & delivery along the vehicle route. This thesis addresses exclusively distribution logistics and accordingly variants of the LRP that consider material handling logistics including inventory and production planning decisions are outside of its scope.

2.2.1 Multi-echelon LRP

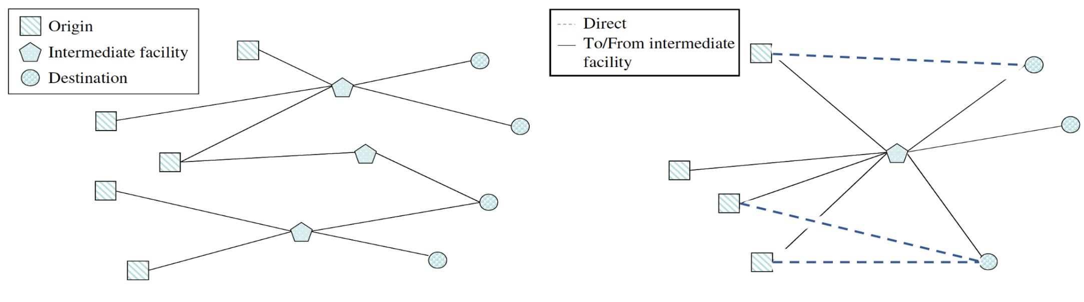
Multi-echelon LRPs are extensions of the standard LRP considering one or more intermediary stages between primary facilities and final customers. Instead of being served directly from a central facility (Figure 2.1), in a multi-echelon LRP a commodity passes two or more stages in the network before reaching the final customer. Figure 2.2 present a two echelon LRP. In a typical multi-echelon network vehicle routes are restricted to a single echelon. This can be seen in Figure 2.2 where a vehicle route in the first echelon starts at a primary facility and visits multiple secondary facilities. The second echelon vehicle routes start at a secondary facility and visits multiple customer locations. It is worth noting that vehicles do not typically transverse arcs in more than one echelon (i.e. no vehicle visits all primary, secondary and customer locations).

Distribution systems in industrial applications are often modeled as two-echelon networks.

Therefore, the two echelon LRP (2E-LRP) is the most frequently addressed extension of the standard LRP in the freight distribution literature (Cuda et al., 2015). The two echelon network is a special case of multi-echelon networks, where commodities are transported from origin to destination through one type (stage) of intermediate facilities (Mirhedayatian, Crainic, Guajardo, & Wallace, 2019). The most recent contributions tackled variants of the same basic problem that rise from different industrial scenarios, or proposed new solution algorithms. For example, Abbassi, Kharraja, El Hilali Alaoui, Boukachour, and Paras (2020) optimized the distribution of non-medical products in a two-echelon healthcare network. They proposed a multi-objective mathematical formulation. Then applied two heuristics: a swarm optimization algorithm and a genetic algorithm, both improved by variable neighborhood search, to solve it. Mirhedayatian et al. (2019) and Winkenbach, Kleindorfer, and Spinler (2016) considered the postal delivery distribution system. Mirhedayatian et al. (2019) studied an uncapacitated 2E-LRP with synchronization of flows at the intermediate facilities and designed a decomposition-based heuristic to address this problem. While Winkenbach et al. (2016) modeled the postal delivery problem in France as a capacited 2E-LRP, and they also proposed a two stage heuristic to solve the model.

According to Cuda et al. (2015) city logistics is the most cited application of 2E-LRPs in freight distribution. Crainic, Ricciardi, and Storchi (2009) clarified that city logistics address freight distribution explicitly in urban areas, where it is critical to reduce nuisances related to freight transportation activities. Zhao, Wang, and De Souza (2018) studied the parcel delivery problem in a two echelon urban setting in China. They took into account a heterogeneous capacitated vehicle fleet, and joint delivery alliance practices. To solve the problem, the authors proposed a cooperative approximation heuristic algorithm which relies on a Lagrangian heuristic for locating facilities, and on improvement procedures for customer allocation. Yang and Zeng (2018) focused on a general 2E-LRP with time constraints in city logistics systems, and provided a probability based metaheuristic algorithm to solve the mixed integer model.

Fewer published studies involved three or more echelon LRPs. Wu, Nie, and Xu (2017) modeled rail distribution of perishable products as a three echelon LRP with time windows and budget constraints. The three echelons considered are between the following four stages: food suppliers, distribution centers, rail stations and trains. A relaxation model provided the lower bounds of the



(a) A pure network with multifacility

(b) A hybrid network with single facility

Figure 2.3: Pure vs Hybrid two-echelon distribution network (Guastaroba et al., 2016)

problem and a hybrid cross entropy algorithm is proposed to solve the problem. Janjevic, Winkenbach, and Merchán (2019) also addressed a three-echelon location-routing problem, but applied to urban last mile e-commerce distribution. The authors formulated the problem as a non-linear optimization model and used a heuristic approach to obtain good solutions for large instances of the problem. Dai, Aqlan, Gao, and Zhou (2019) investigated the potential of addressing three echelon and four echelon LRPs arising from applications in distribution systems and supply chains in city logistics. They implemented a two phase method based on the Clarke and Wright savings algorithm to solve the multi-echelons problems; proving that the algorithm can be utilized to solve the problem in a reasonable time.

2.3 Hybrid Nerworks

Multi-echelon LRP variants can also be classified based on the type of network. Guastaroba et al. (2016) identified two types of distribution networks in a multi-echelon system. A *pure network* (seen in Figures 2.2 and 2.3a) is where direct delivery is not considered, which means all shipments must be transferred through an intermediate facility. While a *hybrid network* allows the choice between direct and indirect transportation for each delivery (Figure 2.3b). LRPs in hybrid networks are not as frequently addressed in the literature.

Goodarzi and Zegordi (2016) condisered a 2E-LRP in a cross-dock network, allowing direct shipment. According to Goodarzi and Zegordi (2016), forcing all shipments to go through cross-docks could be economically unwise. Their analysis showed that direct delivery could be better for shipments near full truck-load and for some shipments with origins and destinations close together. Goodarzi and Zegordi (2016)'s computational experiment also proved that considering both distribution strategies simultaneously dramatically improves the objective function value, compared to considering either one of the strategies alone.

Hosseini, Shirazi, and Karimi (2014) also studied a cross-dock network; but considered three different transportation strategies. Similar to Goodarzi and Zegordi (2016), both direct deliveries and consolidated deliveries were allowed. Additionally, Hosseini et al. (2014) considered milk run deliveries that connect multiple suppliers with a single customer. Milk runs allowed consolidation of commodities within a vehicle, instead of cross-docks. Hosseini et al. (2014), concluded that it is worth investigating the role three transportation strategies could play in increasing flexibility and reducing costs and delivery time.

2.4 Multi-commodity Networks

Most contributions to LRP literature consider how a *single commodity* moves through the network. One variant of this problem, that is hardly studied in the literature, considers *multicommodity* transportation over the same network. Additional characteristics associated with multicommodity distribution include facility or vehicle capacity restrictions per commodity, time restrictions, or service level constraints (Guastaroba et al., 2016).

Hamidi, Farahmand, Sajjadi, and Nygard (2014), Gianessi, Alfandari, Létocart, and Calvo (2016) and Boccia, Crainic, Sforza, and Sterle (2018) all studied multi-commodity LRPs. The supply and demand at nodes were expressed per commodity type but there was no further distinction between the commodities' attributes. Hamidi et al. (2014) proposed a multi-commodity capacitated LRP in a 3-echelon distribution network. Their model allowed both consolidated delivery through capacitated depots and direct delivery between plants and customers. Gianessi et al. (2016) studied a specific variant of the multicommodity-ring LRP, where intermediate facilities need to be connected via a ring. Boccia et al. (2018) addressed a 2E-LRP with multi-commodity flows. The authors proposed a new flow intercepting formulation to model the problem.

Boudahri, Aggoune-Mtalaa, Bennekrouf, and Sari (2013), Asgari, Rajabi, Jamshidi, Khatami, and Farahani (2017) and Li, Lan, and Saldanha-Da-Gama (2019) addressed additional commodity attributes. Boudahri et al. (2013) used a multi-commodity LRP to optimize an agricultural (perishable) food supply chain. The authors considered the volume, weight and transit time requirements for each commodities. Additionally, the capacities of plants and vehicles were expressed per commodity type. Asgari et al. (2017) examined a waste LRP in a two-echelon system. The network contained generation nodes, treatment centers and disposal sites. Each type of waste was described in terms of recycling percentage at each node and risk of transporting along each arc. Finally, Li et al. (2019) studied a multi-objective LRP for perishable commodities. The authors used a penalty parameter to model the decreasing freshness of each commodity.

2.5 Pick-up & Delivery

Further extension of the multi-commodity LRP allows pick-up and delivery at consumer nodes. In a standard LRP, freight is delivered to consumers from one or more potential facilities. Pick-up and delivery LRPs allow picking up of goods at consumers and delivering them to facilities or other consumers. Rieck, Ehrenberg, and Zimmermann (2014) and Rahmani, Cherif-Khettaf, and Oulamara (2015) considered a multi-commodity 2E-LRP with pick-up and delivery. Rieck et al. (2014) studied a hub-and-spoke network, where a customer can have either a pick-up or a delivery demand. In the problem investigated by Rahmani et al. (2015), a single commodity is distributed from suppliers to intermediate facilities, it is then transformed to more than one commodity to be delivered to final consumers. Finally, Azizi and Hu (2020) considered a multi-commodity 2E-LRP with pick-up and delivery in a hybrid network (considering direct delivery and consolidated delivery options). The problem was formulated as a mixed integer linear programming model and solved using CPLEX.

2.6 Matheuristics in LRP

Over the last decade hybrid approaches have been used to tackle the LRPs to combine the power of different solution algorithms. Matheuristics are some of the most popular hybrid approaches combining mathematical programming and heuristics. A matheuristic approach enables researchers to benefit from the advancing computational powers of commercial solvers and lead to more flexible algorithms (Mara et al., 2021). According to Archetti and Speranza (2014) and Mara et al. (2021) decomposition matheuristics are best suited for integrated problems including LRPs. Archetti and Speranza (2014) defined a decomposition matheuristic approach to be a scheme where the problem is divided to several sub-problems, of which some are solved using mathematical programming.

The most common decomposition scheme addressing LRPs, divide the decisions to location/allocation decisions and routing decisions. Lam and Mittenthal (2013) and Escabor, Linfati, and Toth (2013) proposed a route first location second approach. Lam and Mittenthal (2013) considered a clustering-based heuristic for the multi-depot LRP. First customer-route allocation was determined by clustering customers based on geographical location such that each cluster did not violate the vehicle capacity. Each cluster was then treated as a single customer and a facility location problem was solved determining number, size and locations of facilities and the allocation of customers to facilities. A final descent heuristic was used to improve customer-route allocation and customer-facility allocation. Escabor et al. (2013) also used a clustering-based iterative heuristic. An initial tour was constructed containing all customers. Then a splitting strategy was utilized splitting the tour to smaller tours that respect vehicle capacity. Location/allocation decisions were determined by solving an integer linear program. Finally, local search and diversification strategies were employed to improve the search.

Schittekat et al. (2013) also separated location and routing decisions, but the location decisions were not solved for. The authors addressed a school bus location-routing problem where the bus stops are selected from a set of candidate stops. First the routing problem was solved using a greedy randomized search and improved using variable neighborhood descent. The stops selected in the route were automatically opened. The authors then used a a mathematical program to determine the allocation of students to stops such that the capacity of the buses were not exceeded.

Hemmelmayr, Doerner, Hartl, and Vigo (2014) solved a vehicle routing and bin allocation problem for a waste management system. Both a location first route second approach and a route first location second approach were tested. The location problem was solved using a commercial solver. A variable neighborhood search procedure was used to solve the routing part of the problem.

Winkenbach et al. (2016) utilized an iterative two stage procedure to solve a 2E-LRP with limited facility and vehicle capacity. Location and allocation decisions were determined first by solving a sub-problem with relaxed facility capacity constraints. The objective was to minimize daily operational costs. A second vehicle routing sub-problem, with the same objective, was then solved while considering the opened facilities' capacities. The second sub-problem could result in re-allocations of some customers to ensure the capacity of opened facilities are respected. When the re-allocations resulted in higher cost, the first problem was rendered infeasible and solved again to optimize the best facilities to open.

Pichka, Bajgiran, Petering, Jang, and Yue (2018) proposed a two phase procedure to address the 2E-LRP in city logistics. First location and allocation decisions were determined. Then feasible routing solutions for each echelon were generated using a modified Clarke and Wright algorithm. Finally the routing decisions in each echelon were improved using a simulated annealing approach.

Chapter 3

Problem Definition and Formulation

In this chapter we first formally define the 3E-LRP-MC-PD. We then propose an integer programming formulation to model this problem. Finally, we present two modifications to the formulation proposed which can potentially reduce the computational time of commercial solvers.

3.1 Problem Definition

Let $G = (N^0, A^0)$ be the complete graph, where N^0 is the set of nodes and A^0 is the set of arcs. Node 0 is a fictitious node where vehicles must start and end their routes. The sets N and A are the sets of nodes and arcs, respectively, without considering node 0. $H \subseteq N$ is defined as the set of nodes corresponding to the transshipment candidate set and for $i \in H$, c_i is the fixed cost of opening transshipment facility i. $C \subseteq N$ is defined as the set of intermediary collection points for couriers and and c_{cour} is the unit cost of using a courier for delivery. The set K denote the commodities. For every $i \in C$ and $k \in K$, tc_{ik} is defined as the time taken to deliver commodity k to its destination using courier i.

Each commodity $k \in K$ is characterized by the origin node o_k; the destination node d_k; quantity in units q_k; the maximum allowed transit time (service level constraints) $tmax_k$ and the bundling restrictions $b_{(k,k')}$ between a pair of commodities.

A heterogeneous Vehicle fleet V is used to transport commodities along arcs. For $v \in V$: c_v is the cost of acquiring vehicle v and Γ^v is the capacity of vehicle v. c_{ij}^v and t_{ij}^v are the cost and time, respectively, of routing vehicle v along arc (i, j) such that $(i, j) \in A$.

The following set of binary decision values are defined:

- a_v *vehicle utilization variable*, is equal to 1 vehicle v is acquired and 0 otherwise.
- f^v_{kij} *flow variable*, is equal to 1 if commodity k passes through arc (i, j) in vehicle v and 0 otherwise.
- g_{ik} *courier delivery variable* is equal to 1 if commodity k is delivered through courier intermediate node $i \in C$ and 0 otherwise.
- x^v_{ij} *routing variable*, is equal to 1 if vehicle v passes through arc (i,j) and 0 otherwise.
- y_i *location variable*, is equal to 1 if transshipment facility $i \in H$ is open and 0 otherwise.
- Z^v_i *node routing variable*, is equal to 1 if vehicle v passes through node i and 0 otherwise.

3.2 The 3E-LRP-MC-PD Formulation

Next an integer programming formulation is introduced to address this problem.

$$\min \sum_{i \in H} y_i c_i + \sum_{v \in V} c_v a_v + \sum_{v \in V} \sum_{(i,j) \in A} c^v_{ij} x^v_{ij} + \sum_{k \in K} \sum_{i \in C} q_k g_{ik} c_{cour} \tag{1}$$

$$\sum_{v \in V} \sum_{(o_k, j) \in A} f^v_{k(o_k j)} = 1 \quad \forall k \in K \tag{2}$$

$$\sum_{v \in V} \sum_{(i, d_k) \in A} f^v_{k(i d_k)} + \sum_{i \in C} g_{ik} = 1 \quad \forall k \in K \tag{3}$$

$$\sum_{v \in V} \sum_{(i,j) \in A} f^v_{kij} = \sum_{v \in V} \sum_{(j,i) \in A} f^v_{kji} \quad \forall k \in K, i \notin C : i \neq d_k, i \neq o_k \tag{4}$$

$$\sum_{v \in V} \sum_{(i,j) \in A} f^v_{kij} + g_{ik} = \sum_{v \in V} \sum_{(j,i) \in A} f^v_{kji} \quad \forall k \in K, i \in C : i \neq d_k, i \neq o_k \tag{5}$$

$$f^v_{kij} \leq x^v_{ij} \quad \forall v \in V, (i, j) \in A, k \in K \tag{6}$$

$$\sum_{(i,j)\in A} f^v_{kij} \leq \sum_{(j,i)\in A} f^v_{kji} \quad \forall v \in V, k \in K, j \in H' \cap C' : j \neq o_k, j \neq d_k \tag{7}$$

$$\sum_{(i,j)\in A} f^v_{kij} - \sum_{(j,i)\in A} f^v_{kji} \leq y_j \quad \forall v \in V, k \in K, j \in H : j \neq o_k, j \neq d_k \tag{8}$$

$$\sum_{(i,j)\in A} f^v_{kij} - \sum_{(j,i)\in A} f^v_{kji} \leq g_{jk} \quad \forall v \in V, k \in K, j \in C : j \neq o_k, j \neq d_k \tag{9}$$

$$\sum_{k\in K} q_k f^v_{kij} \leq \Gamma^v x^v_{ij} \quad \forall (i,j) \in A, v \in V \tag{10}$$

$$f^v_{kij} + f^v_{k'ij} \leq 1 + b(k,k') \quad \forall v \in V, (i,j) \in A, k \in K, k' \in K : k \neq k' \tag{11}$$

$$\sum_{(i,j)\in A^0} x^v_{ij} = \sum_{(j,i)\in A^0} x^v_{ji} \quad \forall v \in V, i \in N^0 \tag{12}$$

$$\sum_{(0,j)\in A^0} x^v_{0j} = a^v \quad \forall v \in V \tag{13}$$

$$\sum_{(i,0)\in A^0} x^v_{i0} = a^v \quad \forall v \in V \tag{14}$$

$$x^v_{ij} \leq a^v \quad \forall v \in V, (i,j) \in A^0 \tag{15}$$

$$\sum_{(i,j)\in A^0} x^v_{ij} = Z^v_i \quad \forall i \in N^0, v \in V \tag{16}$$

$$\sum_{(i,j)\in A^0} x^v_{ij} = Z^v_j \quad \forall j \in N^0, v \in V \tag{17}$$

$$\sum_{i'\in S} \sum_{j'\in N^0/S} x^v_{i'j'} + \sum_{i'\in S} \sum_{j'\in N^0/S} x^v_{j'i'} \geq Z^v_i \quad \forall i \in S, S \subset N^0, |S| \geq 2, v \in V \tag{18}$$

$$\sum_{v\in V} \sum_{(i,j)\in A} t^v_{ij} f^v_{kij} + \sum_{i\in C} tc_{ik} g_{ik} \leq tmax_k \quad \forall k \in K \tag{19}$$

$$f^v_{kij} \in \{0,1\} \quad \forall k \in K, (i,j) \in A, v \in V \tag{20}$$

$$y_i \in \{0,1\} \quad \forall i \in H \tag{21}$$

$$x^v_{ij} \in \{0,1\} \quad \forall (i,j) \in A^0, v \in V \tag{22}$$

$$a^v \in \{0,1\} \quad \forall v \in V \tag{23}$$

$$Z_v^i \in \{0,1\} \quad \forall i \in N^0, v \in V \tag{24}$$

$$g_{ik} \in \{0,1\} \quad \forall i \in C, k \in K \tag{25}$$

The objective function (1) minimizes total costs. This comprises four main components: cost of selecting transshipment facilities, cost of acquiring vehicles, vehicle routing costs and courier delivery costs, respectively. Constraints (2) - (5) are flow balancing constraints for origin, destination, intermediary nodes and courier nodes. Constraint (6) ensures that if flow passes through arc in a vehicle, then the vehicle passes through this arc. Constraint (7)-(9) assures that the flow can only change vehicles at transshipment nodes or courier intermediary facilities. Constraint (10) is the vehicle capacity constraints and constraint (11) enforce bundling restrictions between products transported by the same vehicle. Constraints (12) - (18) are standard vehicle routing constraints. Constraint (12) balances vehicle flow assuring that if a vehicle arrives at a node it must leave it. Constraints (13) and (14) state that all vehicles should start and finish their route at node 0. Constraint (15) ensures that a vehicle passes through an arc only if the vehicle is acquired. Constraints (16) and (17) activate variable Z and constraint (18) is the standard sub-tour elimination constraint. The maximum allowed route length for every commodity is enforced by (19). Constraints (20)- (25) are the standard non-negativity and integrality constraints.

3.3 Modifications

It is hard to predict how difficult a formulation is to solve using commercial solvers. Accordingly, we propose two possible modifications to the 3E-LRP-MC-PD formulation proposed in section 3.2, which can help improve CPLEX's performance. The impact of these modifications on computational complexity and quality of CPLEX solutions is discussed in chapter 6.

3.3.1 Equality Constraints

The proposed formulation in section 3.2 has 9 sets of equality constraints. Our first proposition is to reformulate some of these constraints to inequality constraints.

Flow conservation constraints (2) - (5) can be rewritten as (26) - (29), respectively.

$$\sum_{v\in V}\sum_{(o_k,j)\in A} f^v_{k(o_kj)} - \sum_{v\in V}\sum_{(j,o_k)\in A} f^v_{k(jo_k)} \geq 1 \quad \forall k\in K \tag{26}$$

$$\sum_{v\in V}\sum_{(d_k,i)\in A} f^v_{k(d_k,i)} - \sum_{v\in V}\sum_{(i,d_k)\in A} f^v_{k(i,d_k)} - \sum_{i\in C} g_{ik} \leq -1 \quad \forall k\in K \tag{27}$$

$$\sum_{v\in V}\sum_{(i,j)\in A} f^v_{kij} - \sum_{v\in V}\sum_{(j,i)\in A} f^v_{kji} \leq 0 \quad \forall k\in K, i\notin C : i\neq d_k, i\neq o_k \tag{28}$$

$$\sum_{v\in V}\sum_{(i,j)\in A} f^v_{kij} + g_{ik} \leq \sum_{v\in V}\sum_{(j,i)\in A} f^v_{kji} \quad \forall k\in K, i\in C : i\neq d_k, i\neq o_k \tag{29}$$

Additional flow conservation constraints are required to ensure that the resulting new formulation is equivalent to the formulation proposed in section 3.2.

$$\sum_{v\in V}\sum_{(i,j)\in A} f^v_{kij} \leq 1 \quad \forall k\in K, i\in N\setminus G \tag{30}$$

$$\sum_{v\in V}\sum_{(i,j)\in A} f^v_{kji} \leq 1 \quad \forall k\in K, i\in N\setminus G \tag{31}$$

Constraints 30 and 31 ensure that a commodity can flow along a maximum of one arc and in one vehicle into (equation 30) or out of (equation 31) any non-courier node.

Moreover, **vehicle routing constraints** (12) - (14) can be reformulated as (32) - (34), respectively.

$$\sum_{(i,j)\in A^0} x^v_{ij} \leq \sum_{(j,i)\in A^0} x^v_{ji} \quad \forall v\in V, i\in N^0 \tag{32}$$

$$\sum_{(0,j)\in A^0} x^v_{0j} \leq a^v \quad \forall v\in V \tag{33}$$

$$\sum_{(i,0)\in A^0} x^v_{i0} \leq a^v \quad \forall v\in V \tag{34}$$

Additional vehicle routing constraints are used to ensure feasibility of routes.

$$\sum_{(i,j)\in A^0} x_{ij}^v \leq 1 \quad \forall v \in V, i \in N^0 \tag{35}$$

$$\sum_{(j,i)\in A^0} x_{ji}^v \leq 1 \quad \forall v \in V, i \in N^0 \tag{36}$$

Equations 35 and 36 state that a vehicle can only transverse one arc to leave or enter a node, respectively.

3.3.2 Sub-tour elimination Constraints

We also propose replacing the subtour elimination constraints (16) - (18) with the Miller-Tucker-Zemlin (MTZ) formulation. To implement this, the variable Z_i^v (constraint 24) also needs to be redefined. The MTZ formulation is presented below.

$$a^v = Z_0^v \quad \forall v \in V \tag{37}$$

$$Z_i^v \geq 2a^v \quad \forall i \in N, v \in V \tag{38}$$

$$Z_i^v \leq |N|a^v \quad \forall i \in N, v \in V \tag{39}$$

$$Z_i^v - Z_j^v + (|N|-1)x_{ij}^v \leq |N| - 2 \quad \forall i \in N, j \in N, v \in V \tag{40}$$

$$Z_v^i \in \mathbb{Z}^+ \quad \forall i \in N^0, v \in V \tag{41}$$

Constraints (37) activate variable Z by labelling node 0 as 1 if a vehicle is acquired. Constraints (38) - (40) eliminate sub-tours by labeling the order in which the nodes are visited; such that the first non-zero node is labelled 2 and every subsequent node is consecutively labelled. Finally, constraint

(41) is the standard non-negativity and integrality constraint.

3.3.3 Resulting Formulations

Combining the modifications discussed in sections 3.3.1 and 3.3.2 result in four possible formulations summarized in the table 3.1.

Table 3.1: Summary of the 3ElRP-MC-PD Formulations

Formulation	**Modification**	
	Equality	Subtour
1	-	-
2	**x**	-
3	**x**	**x**
4	-	**x**

Formulation 1 is the one introduced in section 3.2. Formulation 2 consider the equality modifications and uses the same sub-tour prevention proposed in section 3.2. The third formulation uses both the equality modifications and the MTZ subtour elimination constraints. Finally, the fourth formulation uses the flow and routing constraints from section 3.2 and the MTZ subtour elimination constraints.

Chapter 4

Methodology

In this chapter we introduce an iterative slope scaling decomposition matheuristic to solve large instances of the 3E-LRP-PC-MD in a reasonable computational time. First we justify the choice of a decomposition matheuristic and propose a new decomposition scheme. Then we introduce the two corresponding sub-problems (min-cost flow and multi-commodity arc routing). We also introduce approximate cost-coefficients to guide the search. Furthermore, we present a heuristic to solve the vehicle assignment sub-problem. Finally, we discuss diversification strategies.

4.1 Decomposition Scheme

Medium to large instances of standard LRP are already hard to solve using exact methods because of: (1) the exponential number of variables and constraints; (2) the complexity and inter-dependency of the assignment, flow and routing constraints. The 3E-LRP-MC-PD introduces additional complexity in terms of increasing the number of decisions to be made and increasing the number of flow, routing and assignment constraints. The new variant of the LRP also introduces additional constraints to account for transit time and bundling limitations, making the flow and routing decisions more restrictive. Most importantly, the new characteristics of the 3E-LRP-MC-PD enforce additional inter-dependencies, making the constraints more strongly interconnected. Due to the additional complexity, for most medium sized instances it is even hard to find an initial feasible solutions in a reasonable computational time.

Three main reasons led us to choose a decomposition matheuristic approach to address the complexity of the 3E-LRP-MC-PD. First, decomposing the problem and focusing on a subset of decisions at a time, reduces both the problem size and the complexity arising from the interdependency of the decisions. Secondly, a decomposition approach allows us to consider the unique characteristics of the problem independent of the flow and routing constraints which provides flexibility. Finally, a matheuristic facilitates the use of the advanced computational power of solvers to solve some of the decomposed sub-problems to optimality which significantly improves the performance of the algorithm.

As discussed in Section 2.6, a typical decomposition scheme usually separates LRPs to location decisions and routing decisions. The two sub-problems are then solved iteratively to achieve good results. We propose a different decomposition accounting for the unique characteristics of the 3E-LRP-MC-PD problem, namely the multi-commodity attributes and the multiple transportation strategies.

We view the 3E-LRP-MC-PD as a hierarchy of three interdependent decisions:

(1) The path of each commodity through the network from origin to destination

(2) The number of vehicles and corresponding routes to allow the commodities to be transported along their selected paths

(3) The transshipment facilities to open

The commodity paths (first decision level) is constrained in terms of the transit time of each commodity. Figure 4.1 shows an example of three feasible paths for commodity k. Each arc is labelled with time taken to transverse the arc and $tmax_k$ is the maximum allowed transit time for k to reach its destination d_k.The first path is a direct path from o_k to d_k. The second path is from o_k to a transshipment center then to d_k. Finally, the third path is from o_k to a courier collection point and the courier will then deliver to d_k. All paths are feasible because the time taken to reach destination is less than $tmax_k$.

The characteristics of the second decision level is best illustrated using the example in Figure 4.2. Consider an instance where the paths of three commodities are as seen in Figure 4.2a). The

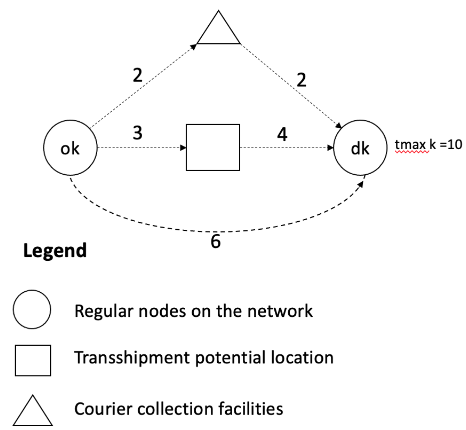


Figure 4.1: Feasible commodity paths

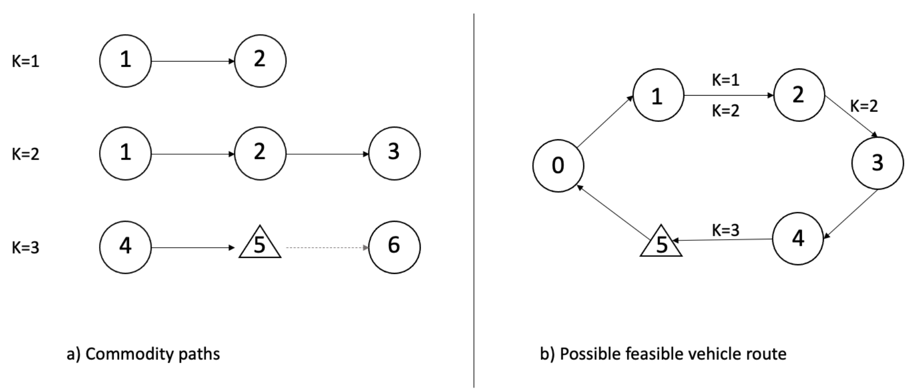


Figure 4.2: Feasible Vehicle Assignment

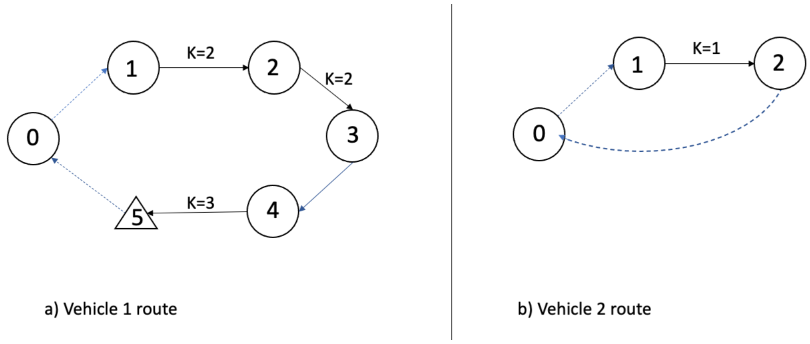


Figure 4.3: Feasible Vehicle Assignment for restricted bundling

vehicle assignment needs to ensure that every arc: (1,2) , (2,3) and (4,5) and corresponding commodities are assigned to a vehicle. For $k = 2$ all arcs (1,2) and (2,3) in the commodity paths must be assigned to the same vehicle because intermediate node (2) is not a transshipment node. If $q_1 + q_2 \leq \Gamma^v$ and $q_3 \leq \Gamma^v$ then assigning arcs (1,2) , (2,3) and (4,5) as seen in Figure 4.2b) is feasible in terms of vehicle capacity. Additionally if the bundling restrictions ($b_{12} = 1$) then transporting $k = 1andk = 2$ on the same arc (1,2) using the same vehicle is feasible. Finally, the vehicle assignment solution seen in 4.2b) is feasible because an additional arc (3,4) is added to ensure connectivity and arcs (0,1) and (5,0) are added to ensure that the vehicle starts and ends at node 0.

For the same instance of 3 commodities paths seen in Figure 4.2a), if ($b_{12} = 0$) then the vehicle assignment solution in Figure 4.2b) is no longer feasible. Alternatively, a feasible vehicle assignment is seen in Figure 4.3. Due to the bundling restriction commodities 1 and 2 can not be transported together along the same arc in the same vehicle. Hence, an additional vehicle is required to serve the path of commodity 1 (Figure 4.3b))

As seen in Figures 4.2 and 4.3 the second decision level is assigning arcs to vehicles such that each arc in a commodity paths is assigned to a vehicle. The vehicle assignment decisions must ensure feasibility of commodity paths. If a commodity path contains more than one arc then the arcs can be assigned to different vehicles only if they are connected by a transshipment facility. Additionally, the vehicle assignment decision must ensure connectivity of a vehicle route and that the vehicle capacities are respected. Finally, if the arc assigned to a vehicle is a part of more than

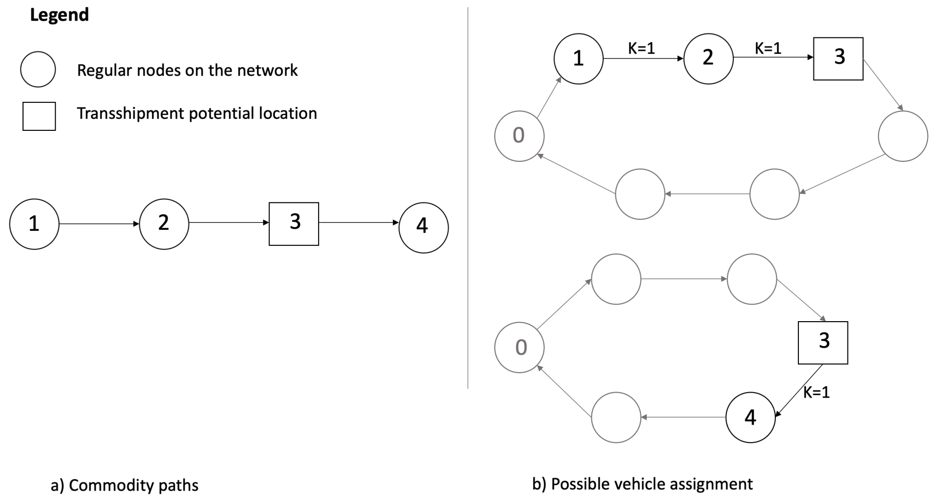


Figure 4.4: A Feasible Commodity Path Split Between Two Vehicles

one commodity path, then bundling restrictions need to be accounted for.

The third location decision assures feasibility of a commodity path and vehicle assignment. If a commodity path is split at a transshipment facility, and each segment is assigned to a vehicle, then the corresponding transshipment facility needs to be open. An example of this can be seen in Figure 4.4. The path of a commodity (seen in Figure 4.4a)) has an intermediate transshipment node (3). Accordingly, having arcs (1,2) and (2,3) assigned to one vehicle and arc (3,4) assigned to another (Figure 4.4 b)) is feasible if transshipment facility (3) is open.

With this decision hierarchy in mind the 3E-LRP-MC-PD can be decomposed to two sub-problems. The first sub-problem is a min-cost flow (MCFP) problem with transit time restrictions for each commodity. The second sub-problem is a multi-commodity arc routing problem (MCARP). The two sub-problems are solved as a part of the slope-scaling matheuristic. These steps are summarized in the algorithm 1 below, and explained in Sections 4.2 to 4.7.

Algorithm 1 Slope-Scaling Matheuristic Steps

$Initialize : K^c = \emptyset$
$Initialize : \overline{c}_{ij}^{k(0)} (eq\,(42))\, \overline{F}_{ij}^{k(0)} (eq\,(43))\ \forall (i,j) \in A, k \in K$
while $stopping\,criteria$ **do**
  while $2^{nd} stopping\,criteria$ **do**
    $Random\ Select : if\ Iterative\,MCFP\,should\,be\,used$
    if $true$ **then**
      $Solve : Iterative\,MCFP$
    else
      $Solve : MCFP$
    end if
    $Solve : MCARP\,using\,Heuristic$
    $Random\ Select : if\ a\,vehicle\,should\,be\,removed$
    if $true$ **then**
      $Remove : Vehicle$
      $Solve : MCARP\,using\,Heuristic$
    end if
    $Update\,Costs$
    $Random\ Select : if\ all\,arcs\,costs\,should\,be\,updated$
    if $False$ **then**
      $Reinitialize : Cost\,coefficients$
    end if
  end while
  $Randomly\,select\,if\ a\,commodity\,should\,be\,added\,to\,G - K\ space$
  if $true$ **then**
    $Random\ Select : strategy\,to\,use$
  end if
  if $|K^c| \geq 2$ **then**
    $Randomly\,select\,if\ a\,commodity\,should\,be\,removed\,from\,G - K\ space$
    if $true$ **then**
      $Random\ Select : strategy\,to\,usee$
    end if
  end if
end while

4.2 Sub-problem 1: Min-cost flow problem

For each commodity $k \in K$ an MCFP is solved to determine the minimum cost path such that the length of the path does not exceed the allowed transit time.

In principle the selection of a commodity path dictates the transportation strategy: direct, courier or consolidated delivery (seen in Figure 4.1). However, to be able to control the solution space being explored (see Section 4.7), we force a subset of commodities to use courier deliveries.

We define the subset $K^c \subseteq K$ to be the commodities that must utilize courier delivery and accordingly, we define the binary decision variable g_{ik} for $k \in K^c$ and $i \in C$, which is equal to 1 if courier collection point i is used to deliver commodity k and is 0 otherwise .

When solving for each commodity $k \in K$, we define a binary flow decision variable f_{ij}^k for every $(i, j) \in A^0$. f_{ij}^k is equal to 1 if arc (i, j) is in the path of commodity k and 0 otherwise. We define additional binary variables Z_i^k for $i \in N^0$ to ensure an elementary path for each commodity, such that Z_i^k is equal to 1 if node i is in the path of commodity k and 0 otherwise.

The objective function coefficients of the MCFP, at iteration t of the matheurisic, are approximated as **per commodity unit** routing cost ($\overline{c}_{ij}^{k(t)}$) and fixed costs ($\overline{F}_{ij}^{k(t)}$). At iteration 0, the cost coeffecients are initialized to small values using equations (42) and(43)respectively. The costs are updated using equations (100) and (99) after the MCARP is solved.

$$\overline{c}_{ij}^{k(0)} = 0.05 \frac{max\{c_{ij}^v : v \in V\}}{max\{\Gamma^v : v \in V\}} \tag{42}$$

$$\overline{F}_{ij}^{k(0)} = 0.0005 \frac{max\{c^v : v \in V\}}{max\{\Gamma^v : v \in V\}} \tag{43}$$

For commodities that are utilizing courier deliveries ($k \in K^c$), the MCFP is formulated as the integer problem below.

$$\min \sum_{k \in K} \sum_{(i,j) \in A^f} (\overline{c}_{ij}^{k(t)} + \overline{F}_{ij}^{k(t)}) q_k f_{ij}^k + \sum_{i \in C} q_k g_{ik} c_{cour} \tag{44}$$

$$\sum_{(o_k,j)\in A} f_{o_kj}^k - \sum_{(j,o_k)\in A^f} f_{jo_k}^k = 1 \tag{45}$$

$$\sum_{(i)\in C} g_{ik} - \sum_{(i,d_k)\in A} f_{id_k}^k = 1 \tag{46}$$

$$\sum_{(i,j)\in A} f_{ij}^k - \sum_{(i,j)\in A} f_{ji}^k = 0 \quad \forall i \in N : i \neq o_k, i \neq d_k \tag{47}$$

$$\sum_{(i,j)\in A} f_{ij}^k - g_{jk} = 0 \quad \forall j \in C \tag{48}$$

$$\sum_{(i,d_k)\in A} f_{id_k}^k = 0 \tag{49}$$

$$\sum_{(i,j)\in A} t_{ij} f_{ij}^k + \sum_{(i)\in C} tc_{ik} g_{ik} \leq tmax_k \tag{50}$$

$$\sum_{(i,j)\in A} f_{0j}^k = 1 \tag{51}$$

$$\sum_{(i,j)\in A} f_{i0}^k = 1 \tag{52}$$

$$\sum_{(i,j)\in A} f_{ij}^k - Z_i^k = 0 \quad \forall i \in N^0 \tag{53}$$

$$\sum_{(i,j)\in A} f_{ij}^k - Z_j^k = 0 \quad \forall j \in N^0 \tag{54}$$

$$\sum_{i'\in S} \sum_{j'\in N^0/S} f_{i'j'}^k + \sum_{i'\in S} \sum_{j'\in N^0/S} f_{j'i'}^k \geq Z_i^k \quad \forall i \in S, S \subseteq N^0, |S| \geq 2 \tag{55}$$

$$f_{ij}^k \in \{0,1\} \quad \forall (i,j) \in A^0 \tag{56}$$

$$g_{ik} \in \{0,1\} \quad \forall i \in G \tag{57}$$

$$Z_i^k \in \{0,1\} \quad \forall i \in N^0 \tag{58}$$

The objective function (44) minimizes the approximated path cost. Constraints (45) - (48) are flow balancing constraints at origin, destination, intermediate and courier nodes respectively. Constraint (49) ensures that the delivery of to destination is done through an intermediate courier facility. Constraint (50) is the transit time constraint. Constraint (51) and (52) ensures that a commodity path must start and end at a fictitious node. Constraints (53) - (55) are the standard subtour elimination constraints, used to ensure elementary paths. Constraints (56) - (58) are the standard non-negativity and integrality constraints.

For commodities not utilizing courier deliveries ($k \in K \setminus K^c$) the formulation is modified to exclude the courier options. The corresponding integer formulation is stated below.

$$\min \sum_{k \in K} \sum_{(i,j) \in A^f} (\overline{c}_{ij}^{k(t)} + \overline{F}_{ij}^{k(t)}) q_k f_{ij}^k \tag{59}$$

$$\sum_{(o_k,j) \in A} f_{o_k j}^k - \sum_{(j,o_k) \in A^f} f_{j o_k}^k = 1 \tag{60}$$

$$\sum_{(i,d_k) \in A} f_{i d_k}^k - \sum_{(d_k,i) \in A^f} f_{d_k i}^k = 1 \tag{61}$$

$$\sum_{(i,j) \in A} f_{ij}^k - \sum_{(i,j) \in A} f_{ji}^k = 0 \quad \forall i \in N : i \neq o_k, i \neq d_k \tag{62}$$

$$\sum_{(i,j) \in A} t_{ij} f_{ij}^k \leq tmax_k \tag{63}$$

$$\sum_{(i,j)\in A} f_{0j}^k = 1 \tag{64}$$

$$\sum_{(i,j)\in A} f_{i0}^k = 1 \tag{65}$$

$$\sum_{(i,j)\in A} f_{ij}^k - Z_i^k = 0 \quad \forall i \in N^0 \tag{66}$$

$$\sum_{(i,j)\in A} f_{ij}^k - Z_j^k = 0 \quad \forall j \in N^0 \tag{67}$$

$$\sum_{i'\in S}\sum_{j'\in N^0/S} f_{i'j'}^k + \sum_{i'\in S}\sum_{j'\in N^0/S} f_{j'i'}^k \geq Z_i^k \quad \forall i \in S, S \subseteq N^0, |S| \geq 2 \tag{68}$$

$$f_{ij}^k \in \{0,1\} \quad \forall (i,j) \in A^0 \tag{69}$$

$$Z_i^k \in \{0,1\} \quad \forall i \in N^0 \tag{70}$$

After solving the first sub-problem for all $k \in K$ any arc $(i,j) \in A$ that is not a part for any commodity path is closed. The subset $A^f \subseteq A$ defines the set of opened arcs.

4.3 Sub-problem 2: Multi-commodity arc routing problem

The aim of the MCARP is to select the minimum number of vehicles and determine the vehicle routes to serve the commodity paths while respecting vehicle capacity constraints and bundling restrictions.

The solution from the first sub-problem $\overline{f}_{ij}^k : (i,j) \in A^f$ and $k \in K$ (path per commodity) is used as an input parameter to the MCARP. To construct feasible vehicle routes further arcs might be needed to ensure connectivity of all nodes. One way to overcome this without adding excessive computational complexity is to define a small arc set A_v such that $A^f \subseteq A^v \subseteq A^0$. A^v can be determined by considering the arcs in A^f to be undirected and for every $(i,j) \in A^f$ additional arc (j,i) is added to A^v.

The MCARP is formulated as an integer program.

$$\min \sum_{v \in V} c^v a^v + \sum_{(i,j) \in A^v} c_{ij}^v x_{ij}^v \tag{71}$$

$$\sum_{v \in V} f_{kij}^v = \overline{f}_{ij}^k \quad \forall k \in K, (i,j) \in A^f \tag{72}$$

$$f_{kij}^v \leq x_{ij}^v \quad \forall v \in V, (i,j) \in A^f, k \in K \tag{73}$$

$$\sum_{k \in K} q_k f_{kij}^v \leq \Gamma^v x_{ij}^v \quad \forall (i,j) \in A^f, v \in V \tag{74}$$

$$f_{kij}^v + f_{k'ij}^v \leq 1 + b(k,k') \quad \forall v \in V, (i,j) \in A^f, k \in K, k' \in K : k \neq k' \tag{75}$$

$$\sum_{(i,j) \in A^f} x_{ij}^v = \sum_{(j,i) \in A^f} x_{ji}^v \quad \forall v \in V, i \in N^0 \tag{76}$$

$$\sum_{(0,j) \in A^f} x_{0j}^v = a^v \quad \forall v \in V \tag{77}$$

$$\sum_{(i,0) \in A^f} x_{i0}^v = a^v \quad \forall v \in V \tag{78}$$

$$x_{ij}^v \leq a^v \quad \forall v \in V, (i,j) \in A^v \tag{79}$$

$$\sum_{(i,j) \in A^f} x_{ij}^v = Z_i^v \quad \forall i \in N, v \in V \tag{80}$$

$$\sum_{(i,j) \in A^f} x_{ij}^v = Z_j^v \quad \forall j \in N, v \in V \tag{81}$$

$$\sum_{i' \in S} \sum_{j' \in N^0/S} x_{i'j'}^v + \sum_{i' \in S} \sum_{j' \in N^0/S} x_{j'i'}^v \geq Z_i^v \quad \forall i \in S, S \subseteq N, |S| \geq 2, v \in V \tag{82}$$

$$f_{kij}^{v} \in \{0,1\} \quad \forall k \in K, (i,j) \in A^{f}, v \in V \tag{83}$$

$$x_{ij}^{v} \in \{0,1\} \quad \forall (i,j) \in A^{v}, v \in V \tag{84}$$

The objective (71) is to reduce the cost of acquiring vehicles and the routing costs, respectively. Constraints (72) connects the first and second sub-problem, and enforces that one vehicle carries commodities along each arc in the commodity's path. Constraints (73) ensures that if flow passes through arc in a vehicle, then the vehicle passes through this arc. Constraints (74) - (82) are the same as (10) - (18) in the 3E-LRP-MC-PD formulation but for the subset of arcs A^{v}.

4.4 A Heuristic for MCARP

Algorithm 2 VAP Heuristic Steps

1: $set\, randomly\, H^* \subseteq H$
2: $Initialize: \forall (i,j) \in A^f : K_{ij},\ \nu_{ij},\ K_{ij}^{v}\, and\, \overline{c}_{ij}^{v}$
3: **while** $K_{ij} \neq \emptyset\ \forall (i,j) \in A^f$ **do**
4: $\quad (i^*, j^*), v^* \leftarrow Minimum\, Cost\, Selection$
5: $\quad k^* \leftarrow CommodityAssignmentIP((i^*, j^*), v^*, K_{i^*j^*}^{v^*})$
6: $\quad Update\ K_{i^*j^*},\ \nu_{i^*j^*},\ \overline{c}_{ij}^{v^*}\ \forall (i,j) \in A^f$
7: $\quad Update\, \forall (i,j) \in A^f : K_{ij}^{v},$
8: **end while**
9: $Vehicle\, Size\, Change$
10: $FeasibleRouteIP$

Despite the restricted number of arcs considered in the MCARP, it is still computationally taxing to solve using a commercial solver. Accordingly, a greedy MCARP heuristic is proposed to select vehicles and determine feasible vehicle routes. The steps of the MCARP heuristic is summarized in algorithm 2.

4.4.1 Initialization & Selection

Initialization

First a subset of transshipment centers to open $H^* \subseteq H$ is randomly chosen such that $h \in H^*$ is an intermediate node in one or more commodity paths.

Then, for every arc $(i, j) \in A^f$ we define:

- **A commodity set:** $K_{ij} = \{k \in K : \overline{f}_{ij}^k = 1\}$
- **A vehicle set:** ν_{ij} of vehicles that are allowed to transverse arc.
- **A feasible commodity set:** K_{ij}^v. For every $v \in \nu_{ij}$ a subset $k \in K_{ij}$ that are allowed to be assigned to the vehicle if arc (i, j) is assigned to the vehicle.

For all arcs $(i, j) \in A^f$, ν_{ij} is initialized to include all vehicles of the largest capacity: $\nu_{ij} = \{v \in V : \Gamma^v = max\{\Gamma^v : v \in V\}\}$. K_{ij}^v is initialized to be $K_{ij}^v = K_{ij}$ for all $v \in \nu_{ij}$. Finally, the cost $\overline{c}_{ij}^v$ of assigning arc (i, j) to vehicle v is initially estimated to be $c^v + c_{ij}^v$.

Minimum Cost Selection

The aim of this step is to determine the minimum cost arc (i^*, j^*) to vehicle v^* assignment. To do this we select the minimum $\overline{c}_{ij}^v$ from all feasible assignments.

Assigning arc (i, j) to vehicle v is feasible if all the following conditions apply:

- $K_{ij} \neq \emptyset$: The set of commodities to assign to arc (i, j) is not empty
- $v \in \nu_{ij}$: vehicle v is in the set of arcs allowed to transverse arc (i, j)
- $K_{ij}^v \neq \emptyset$: the subset of commodities allowed to be assigned to the v and (i, j) pair is not empty
- If there's at least one $k \in K_{ij}^v$ where $q_k \leq \Gamma^v$

4.4.2 Commodity Assignment IP

Given the minimum cost arc and vehicle pair $(i^*, j^*), v^*$ and the associated feasible commodity set $K^{v^*}_{i^*j^*}$: the commodity assignment IP is solved to maximize the $|K^*| : K^* \subseteq K^{v^*}_{i^*j^*}$. The commodity assignment IP is formulated as follows.

$$\max \sum_{k \in K^{v^*}_{i^*j^*}} f^{v^*}_{ki^*j^*} \tag{85}$$

$$\sum_{k \in K^{v^*}_{i^*j^*}} q_k f^{v^*}_{ki^*j^*} \leq \Gamma^v \tag{86}$$

$$f^{v^*}_{ki^*j^*} + f^{v^*}_{k'i^*j^*} = 1 + b_{(k,k')} \quad k, k' \in K^{v^*}_{i^*j^*} : k \neq k' \tag{87}$$

$$f^{v^*}_{ki^*j^*} \in \{0, 1\} \quad \forall k \in K^{v^*}_{i^*j^*} \tag{88}$$

The objective (85) is to maximize the number of commodities to be assigned to vehicle v^* on arc (i^*, j^*). Constraint (86) is the vehicle capacity constraint. Constraint (87) enforces bundling restrictions and constraint (88) is the non-negativity and integrality constraint.

4.4.3 Updates

Three main updating steps are required before the next minimum cost selection to improve and motivate feasible selections.

Vehicle Set Updates

After assigning an arc (i^*, j^*) to a vehicle v^*, the set ν_{ij} needs to be updated for all arcs $(i, j) \in A^f$ to prevent assigning any arc to v^* that would result in a routing infeasibility. The two possible sources of routing infeasibilities can be seen in the examples in Figure 4.5.

Figure 4.5a) shows an instance where arc (1,2) is assigned to v^* and arcs (1,3) and (4,2) are in the set A^f. Assigning either arcs to v^* would result in a routing infeasibility because it would mean

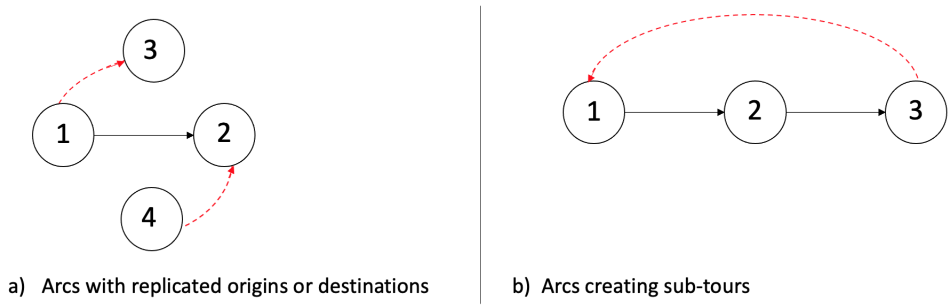


Figure 4.5: Examples of vehicle routing infeasibility

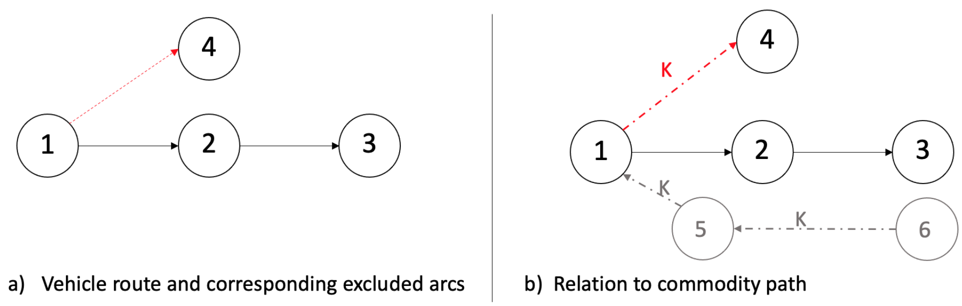


Figure 4.6: Example of commodity path infeasibility

that nodes 1 and 2 will be visited more than once. In the example shown in Figure 4.5b) arcs (1,2) and (2,3) are assigned to v^* and arc (3,1) is in A^f. Assigning arc (3,1) to v^* would result in sub-tour in the vehicle route. In both examples to prevent the discussed arcs from being assigned to v^*, the set ν_{ij} is updated by removing v^*.

Feasible Commodity Set Updates

The MCARP heuristic can also result in an infeasibility related to commodity transfer at non-transshipment node. This is best illustrated using the example in Figure 4.6. Consider a vehicle v^* travelling route 1-2-3 seen in Figure 4.6a). Furthermore, consider the additional unassigned arcs $(i, j) \in A^f$: (6,5) , (5,1) and (1,4) (seen in Figure 4.6b)). If route feasibility only, is being considered, then v^* is removed from set of allowed vehicles ν_{14}. Accordingly, arcs (6,5) and (5,1) can be iteratively added to v^* because they do not compromise the feasibility of the route.

Now consider an unassigned commodity k path 6-5-1-4 seen in Figure 4.6b). The path does not contain any transshipment nodes. Hence, all arcs in the path must be assigned to the same vehicle. If arcs (6,5) and (5,1) are added to v^* as discussed in the previous paragraph then the path of commodity k will be split at node 1. Meaning that the arcs(6,5) and (5,1) will be assigned to v^*, while the remaining arc (1,4) will be assigned to a different vehicle.

To significantly reduce the chance of this infeasibility we update the feasible commodity sets by removing any unassigned $k \in K_{ij}$ from $K_{ij}^{v^*}$ if $v^* \notin \nu_{ij}$. If k is removed from $K_{ij}^{v^*}$ then k is removed from subsequent feasible commodity sets $K_{i_1 j_1}^{v^*}$, if all the following conditions apply:

- k's path includes arc (i_1, j_1)
- k is unassigned (i.e. $k \in K_{i_1 j_1}$)
- (i_1, j_1) is connected to arc (i, j) so if $(i_1 = j \vee j_1 = i)$
- nodes i_1 and j_1 are non-transshipment nodes ($i_1, j_1 \notin H^*$)

Finally if k is removed from the feasible commodity set $K_{i_1 j_1}^{v^*}$, then the steps are repeated to update sets $K_{i_2 j_2}^{v^*} : i_2 = j_1 \vee j_2 = i_1$. This iterative procedure is repeated till all relevant feasible commodity sets are updated.

This procedure and an additional step (discussed in Section 4.4.4) reduces the chance of a commodity changing vehicle at non-transshipment center but does not eliminate the infeasibility completely. Accordingly, we allow the solution of some iterations of the matheuristic to be infeasible.

4.4.4 Assignment costs

Initially the cost ($\bar{c}_{ij}^{v}$) of assigning any arc $(i, j) \in A^f$ to any vehicle $v \in \nu_{ij}$ is approximated as $c^v + c_{ij}^v$. After the selection of (i^*, j^*) and v^* pair, and the selection of a subset of commodities $K^* \subseteq K_{ij}$ to assign to the pair; the costs ($\bar{c}_{ij}^{v^*}$) of all remaining arcs $(i, j) \in A^f$ are updated as follows:

- $\overline{c}^{v^*}_{ij} = c^{v*}_{ij} \, \forall (i,j) \in A^f$: To encourage adding arcs to v^*, the cost of assigning any arc to v^* is reduced to routing cost

- $\overline{c}^{v^*}_{ji^*} = r \, c^{v*}_{ji^*} : i^* \notin H^*, (j, i^*) \in A^f$: If any arc precedes (i^*, j^*) and i^* is not an open transshipment facility, the cost of assigning the arc to v^* is reduced further using reduction factor r. This encourages addition of connected arcs (in a commodity path) to the vehicle reducing the chance of the infeasibility discussed in Section 4.4.3.

- $\overline{c}^{v^*}_{j^*i} = r \, c^{v*}_{j^*i} : j^* \notin H^*, (j, i^*) \in A^f$: If any arc succeeds (i^*, j^*) and j^* is not an open transshipment facility, the cost of assigning the arc to v^* is reduced further using reduction factor r. This encourages addition of connected arcs (in a commodity path) to the vehicle reducing the chance of the infeasibility discussed in Section 4.4.3.

4.4.5 Vehicle Size Change

As explained in Section 4.4.1, the arc-routing heuristic initially only considers the largest vehicle capacity. Therefore, it is possible that a smaller vehicle can be used for the the exact same route.

Let v^* be one of the maximum capacity vehicles selected by the matheuristic and R^* be the route of the vehicle such that $R^* \subseteq A^f : x^{v^*}_{ij} = 1$. If there exists a vehicle $v_1 \in V$ such that $\Gamma^{v_1} \leq \Gamma^{v^*}$ and the capacity of the vehicle is respected for every $(i, j) \in R^*$, allowing v_1 to replace v^* is feasible. If using a smaller vehicle is feasible, the cost of using v_1 and v^* is calculated using equation (89). Finally, if the cost of using v_1 is less than the cost of v^*, v^* is replaced with v_1 to transverse the route R^*.

$$c^v + \sum_{(i,j) \in R^*} x^v_{ij} c^v_{ij} \tag{89}$$

4.4.6 Feasible Route IP

The aim of this step in the MCARP heuristic is to construct a feasible route for every vehicle chosen by the heuristic. A feasible vehicle route is one that starts and ends at node 0, is fully connected and does not contain sub-tours. To construct feasible routes additional arcs may be needed to ensure connectivity.

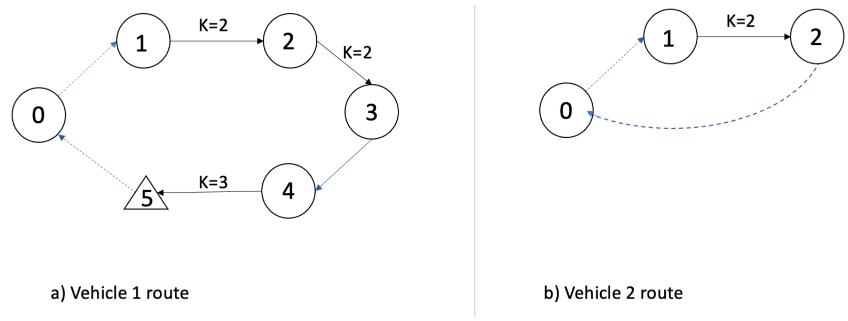


Figure 4.7: Examples of feasible vehicle routes

Figure 4.7 shows two examples of feasible vehicle routes. In the first example (Figure4.7a)), vehicle 1 is assigned arcs (1,2) , (2,3) and (4,5) during the heuristic step. Additional arc (3,4) is required to ensure connectivity. Arcs (0,1) and (5,0) are required so that the vehicle 1 starts and ends at node 0. In the second example (Figure4.7b)), vehicle 2 has only one arc assigned (1,2). Accordingly, only arcs (0,1) and (2,0) are required for the route to be feasible.

Let V' be the set of chosen vehicles such that $V' \subseteq V$. We define $N^v \subseteq N$ be the nodes visited by vehicle v and A^v be the complete graph between nodes $i \in N^v$. $A^{cv} \subseteq A^v$ are the arcs $(i,j) \in A^f$ assigned to vehicle v using the VAP heuristic. If all arcs assigned to vehicle v are connected (i.e $|A^{cv}| = |N^v| - 1$) then an arc from origin to starting node and from end node to origin are added to the vehicle.

However if the arcs are disconnected, similar to the example in Figure4.7a), then the Feasible Route IP is solved to determine the cheapest arcs to add to ensure connectivity of route. In the example in Figure4.7a) it is possible to add arc (4,1) instead of (3,4) to ensure connectivity.

The IP formulation below is solved for $\forall v \in V'$ using CPLEX.

$$\min \sum_{(i,j)\in A^v/A^{vc}} c^v_{ij} x^v_{ij} \tag{90}$$

$$x^v_{ij} = 1 \quad \forall (i,j) \in A^{vc} \tag{91}$$

$$\sum_{(i,j)\in A^v} x^v_{ij} = \sum_{(j,i)\in A^v} x^v_{ji} \quad i \in N^v \tag{92}$$

$$\sum_{(0,j)\in A^v} x_{0j}^v = 1 \quad i \in N^v \tag{93}$$

$$\sum_{(i,0)\in A^v} x_{i0}^v = 1 \quad i \in N^v \tag{94}$$

$$\sum_{(i,j)\in A^v} x_{ij}^v = 1 \quad \forall i \in N^v \tag{95}$$

$$\sum_{(i,j)\in A^v} x_{ji}^v = 1 \quad \forall j \in N^v \tag{96}$$

$$\sum_{i'\in S}\sum_{j'\in N^v/S} x_{i'j'}^v + \sum_{i'\in S}\sum_{j'\in N^v/S} x_{j'i'}^v \geq 1 \quad \forall i \in S, S \subset N^v, |S| \geq 2 \tag{97}$$

$$x_{ij}^v \in \{0,1\} \quad \forall (i,j) \in A^0, v \in V \tag{98}$$

The objective function (90) minimizes the costs of additional arcs to open to create a feasible route. Equation (91) ensures that any arc assigned to vehicle in the VAP heuristic step is included in the vehicle's route. Equations (93) - (95) are the same as (12) - (14) but defined only over the subset of nodes visited by the vehicle. Equations (95) - (97) are the same as (16)- (18).

4.5 Updating Cost Coefficient

At the end of each iteration for every $k \in K$ the approximate **per commodity unit** routing and fixed costs for arcs $(i,j) \in A$ where $f_{kij}^v = 1$ are estimated using equations (100) and (99) respectively.

$$\overline{F}_{ij}^{k(t)} = \frac{\dfrac{c^v}{\sum_{(i,j)\in A} x_{ij}^v}}{\sum_{k\in K} f_{kij}^v} \tag{99}$$

$$\overline{c}_{ij}^{k(t)} = \frac{c_{ij}^v}{\sum_{k\in K} f_{kij}^v} \tag{100}$$

4.6 Randomizing the search

For some iterations of the proposed matheuristic, we randomly select if any of the following randomization procedures should be applied.

4.6.1 Removing a vehicle from the solution

Given the subset of used vehicles $V' \subseteq V$, a ratio R^v is computed for every $v \in V'$. The vehicle with the largest non-zero ratio is removed from the solution and the arcs are re-assigned using the steps 3 to 10 presented in algorithm 2 .

$$R^v = \frac{Total\, arcs\, that\, can\, be\, reassigned\, to\, a\, smaller\, vehicle}{Total\, arcs\, that\, require\, full\, vehicle\, capacity} \tag{101}$$

4.6.2 Iterative MCFP

The MCFP explained in Section 4.2 is solved for every $k \in K$. The commodity path selection is guided by the cost coefficients $\overline{F}_{ij}^{k(t)}$ and $\overline{c}_{ij}^{k(t)}$ calculated at the end of each iteration of the matheuristic.

A more random approach is described below:

(1) Randomly selecting $k \in K$ to solve the MCFP for.

(2) Solving MCFP for k with the coefficients calcaulated $\overline{F}_{ij}^{k(t)}$ and $\overline{c}_{ij}^{k(t)}$ to determine path of the commodity

(3) For each arc chosen in the path of commodity k (i.e. $(i, j) \in A$: $f_{ij}^k = 1$). The cost coefficients: $\overline{F}_{ij}^{k'(t)}$ and $\overline{c}_{ij}^{k'(t)}$ for any commodity k' that can be bundled with k ($b_{kk'} = 1$) are assigned a small value

(4) The steps 1-3 are repeated till the MCFP is solved for all commodities

4.6.3 Cost Coefficients Update

At the end of each iteration $\overline{F}_{ij}^{k(t)}$ and $\overline{c}_{ij}^{k(t)}$ are updated using the equations (99) and (100). If this random modification step is selected, a subset of arcs $A^s \in A^f$ are randomly selected and the costs are reset to initial values (equations (43) and (42)).

4.7 New Solution Space

The matheuristic steps described in Sections 4.2 - 4.6 are solved with a predetermined selection of the subset $K^c \subseteq K$ of commodities that must be delivered using couriers. We refer to this assignment as a **K-G solution space**. The K-G solution space can be modified by adding or removing commodities selected for courier delivery.

First we define P^k ($\forall k \in K$) to be the per unit cost of the path of commodity k. P^k is calculated using equation (102). If $k \notin K^c$ (the commodity is not preassigned to a courier) the second term is equal to 0.

$$P^k = \sum_{(i,j)\in A^f} (\overline{c}_{ij}^{k(t)} + \overline{F}_{ij}^{k(t)}) f_{ij}^k + \sum_{i \in C} g_{ik} c_{cour} \tag{102}$$

Through the iterative procedure of the matheuristic we retain the path costs associated with the best solution found (P^k_{best}) and the costs associated with the cheapest path found for each commodity ($P^k_{cheapest}$).

4.7.1 Adding Commodity-Courier Assignment

We consider three strategies for selecting a commodity to assign to courier deliveries. The first is selecting the commodity $k \notin K^c$ with the maximum P^k_{best}. The second strategy is selecting the commodity k with the maximum $P^k_{cheapest}$. Finally, a commodity $k \notin K^c$ can be randomly selected regardless of its relative path cost.

4.7.2 Removing Commodity-Courier Delivery

Given the best solution costs $P^k_{best}(\forall k \in K^c)$, three strategies can be used when selecting a commodity to remove from the G-K solution space. The first strategy is to select the commodity assigned to courier delivery with the most expensive P^k_{best} to remove. The second strategy is to use a random weighted probability selection where commodity k with the most expensive P^k_{best} is most likely to be chosen and commodity k with the least expensive P^k_{best} is the least likely to be chosen. Finally, a commodity k can be randomly selected from the set K^c.

Chapter 5

Instance Generation

The 3E-LRP-MC-PD is a new variant of the location routing problem and accordingly there are no benchmark instances available. To conduct our computational experiments we develop an instance generator, coded in C, to randomly generate several classes of instances. In this chapter, we describe the procedure we follow to generate random instances that include all the characteristics of the 3E-LRP-MC-PD.

5.1 Network

All instances are built on a base graph in the Euclidean x,y plane where the x and y co-ordinates range from [0,1000]. To ensure that the nodes are adequately distributed, the graph is divided into clusters (areas) seen in Figure 5.1.

Given the set of nodes N, C is the subset of courier nodes and S is the remaining subset of non-courier nodes.

5.1.1 Non-Courier Nodes

The generation of the nodes in the subset S are based on the following parameter values:

- Cardinality of subset $|S|$
- Number of clusters on the graph and their corresponding x and y limits.

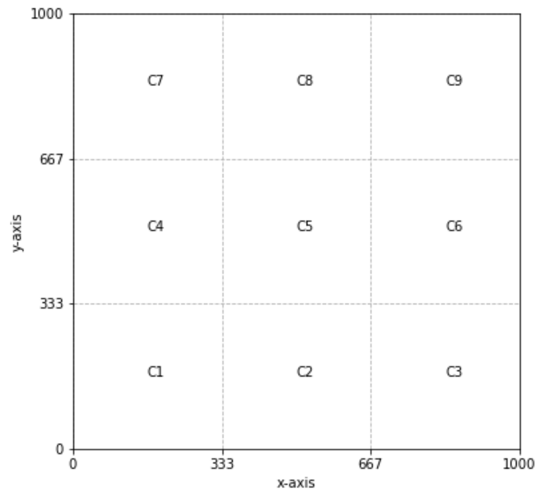


Figure 5.1: Cluster representation

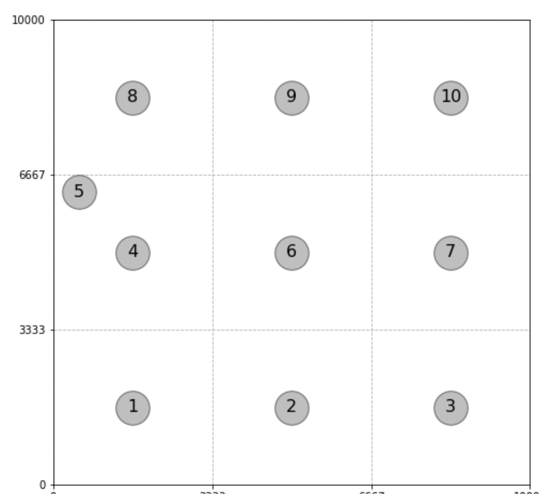


Figure 5.2: Example Nodes Generated

To generate locations of nodes $i \in S$:

- An approximate of nodes per cluster is determined using the equation below

$$\frac{|S|}{Total\ Clusters} \tag{103}$$

- For nodes in a given cluster the x and y co-ordinates are randomly generated within the x and y limits of the cluster.

- Additional nodes are randomly generated by choosing a cluster at random and accordingly randomly selecting the x and y co-ordinates of the node within the cluster limits

An example of nodes generated for $|S| = 10$ can be seen in Figure (5.2)

5.1.2 Courier Nodes

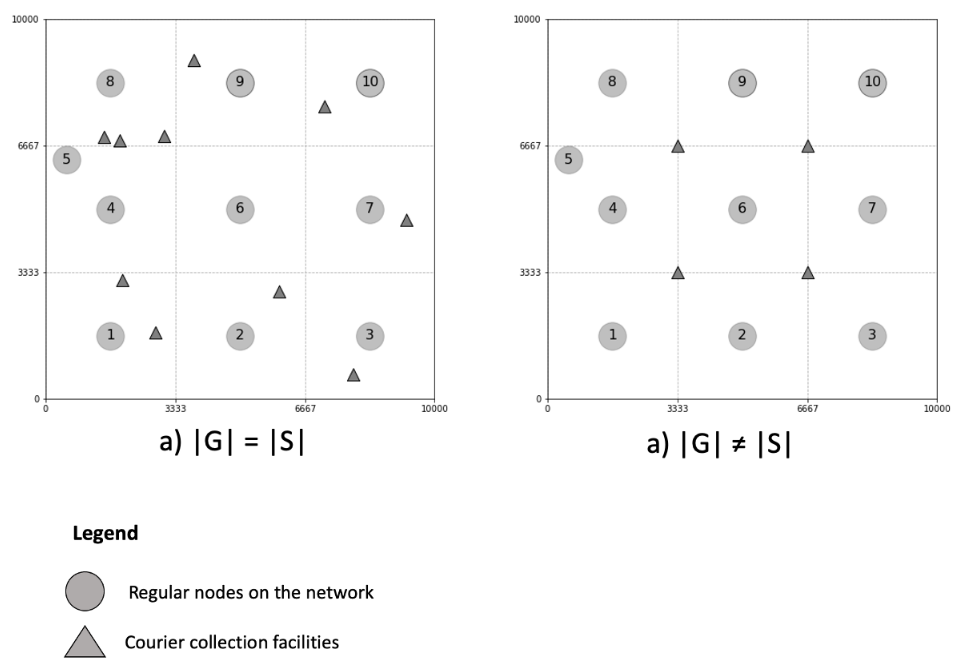


Figure 5.3: Example Courier Nodes Generated

Courier deliveries provide a key advantage of conveniently located collection points. We consider this feature when generating instances using two strategies. The first strategy assumes that there is a close courier node to every non-courier node in the network. Accordingly, the generation of the nodes in C are based on the following:

- $|C| = |S|$
- C_{radius}.

For every $i \in S$ a node $j \in C$ the x and y co-ordinates of j are randomly generated such that the maximum x or y distance node j is from node i is equal to C_{radius}. For all the instances tested we assume C_{radius} to be 300. An example of courier nodes generated based on the first strategy can be seen in Figure 5.3a).

Assuming that $|C| = |S|$ can increase the complexity of the instances solved, especially because the x^v_{ij}, f^v_{kij} decision variables exponentially increase with the increase of the number of nodes on the network. Accordingly, a less restrictive assumption is needed for courier locations. For our second strategy, we consider instances where the number of courier collection points is less than the number of non-courier nodes. We locate the courier collection points at central locations as seen in figure 5.3b).

5.1.3 Transshipment Nodes

Given a number of transshipment potential locations $|H|$ such that $H \subset S$, nodes $i \in H$ are selected by:

- Selecting a cluster at random
- Choosing the center node of the cluster as the transshipment facility
- If $|H|$ is more than the number of clusters then additional transshipment facilities are selected at random from $i \in S$

5.1.4 Travel time

The Euclidean distances c_{ij} among nodes are computed. A parameter for the t_{max} between the furthest nodes is predetermined, for the instances generated we assume t_{max} to be 8 hours. The travel times t_{ij} between the remaining nodes are scaled accordingly.

5.2 Commodities

K is the set of commodities being transported through the network. We define additional subsets $DD, CD, RD \subseteq K$ to ensure that for each instance all relevant commodity attributes are represented. DD is a subset of commodities that we initially assume will use direct delivery and their attributes are generated accordingly. Similarly, CD is a subset of commodities that we initially assume will use courier delivery. Finally, RD is a subset of commodities for which, all the attributes are randomly generated. The following parameters are initially set before generating commodity attributes:

- $|K|$ the total number of commodities
- $|DD|$, $|CD| and |RD|$ the number of (assumed) direct, courier and random commodities.
- Maximum and minimum quantity (units) for commodity Q_{min}, Q_{max}

For the instances generated, we assume the ratio $DD : CD : RD$ to be $1 : 1 : 1$. Additionally, we assume Q_{max} to be 20 units and Q_{min} to be 10% of Q_{max}, to represent a wide range of commodity sizes.

Direct delivery is assumed to be more favorable for commodities that fill vehicle capacities and where the origin and destination are relatively close. Accordingly, given $k \in DD$:

- Origin o_k is assigned by selecting a node at random from $i \in S$
- Destination d_k of commodity flow is determined by finding the j such that $c_{o_k j}$ is minimum
- q_k is randomly generated in range [$0.7Q_{max}$,Q_{max}]

Courier delivery is assumed to be more favorable for small commodities where the origin and destination are far apart. Accordingly, given $k \in CD$:

- Origin o_k is assigned by selecting a node at random from $i \in S$
- Destination d_k of commodity flow is determined by finding the j such that $c_{o_k j}$ is maximum
- q_k is randomly generated in range [Q_{min}, $0.4Q_{max}$]

Random commodities are added to ensure that the instances are generated without bias. Accordingly, given $k \in RD$:

- Origin o_k is assigned by selecting a node at random from $i \in S$
- Destination d_k is determined by selecting a node at random from $i \in S$ such that $o_k \neq d_k$
- q_k is randomly generated in range $[Q_{min}, Q_{max}]$

For all $k \in K$ the remaining commodity attributes are randomly generated as follows:

- Transit time restrictions $tmax_k$ are randomly selected from range $[1.5t_{o_k d_k}, 5t_{o_k d_k}]$
- Bundling restrictions between pair of commodities $b_{kk'}$ are randomly generated from range [0,1]

5.3 Vehicles

This section aims at generating vehicles of different capacities where the fixed costs of vehicles are relative to capacities and the routing costs of arcs are subject to economies of scale.

The following parameters are used:

- Maximum fixed cost per vehicle c_{vmax}
- Approximate number of vehicles in solution $V_{predict}$
- Number of vehicle sizes V_{type} in heterogeneous vehicle fleet
- Economies of scale factor e_{factor}

We assume that c_{vmax} is 100 times the maximum possible distance on the graph and e_{factor} to be 0.6. We also assume $V_{predict}$ to be $|K|/5$.

Minimum and maximum vehicle capacities τ_v are determined using:

- The minimum vehicle capacity τ_v^{min} is set to be Q_{max} allowing direct delivery commodities to fully utilize one type of vehicles

- The maximum vehicle capacity τ_v^{max} is calculated using

$$\frac{\sum_{k \in K} q_k}{V_{predict}} \tag{104}$$

The remaining vehicle attributes $v \in V_{type}$ are generated as follows:

- Routing cost $c_{ij}^{vmax} = 10\, c_{ij}$
- τ_v is randomly generated in range [τ_v^{min},τ_v^{max}]
- Fixed cost of acquiring vehicle is calculated using:

$$c^v = c^{vmax} \frac{\tau_v}{\tau_v^{max}} \tag{105}$$

- To allow for economies of scale:

$$c_{ij}^{vmax} = e_{factor} * c_{ij}^{vmin} \tag{106}$$

  and the remaining routing costs for all vehicles are scaled accordingly

The 3E-LRP-MC-PD assumes an unlimited vehicle fleet, we model this assumption in the instances generated by assuming that there are $|K|$ vehicles from each vehicle type. However, this assumption exponentially increases the size of the problem. Hence, we also consider instances where there are $0.25|K|$ vehicles from each vehicle type. We verify that the instances considering $0.25|K|$ vehicles per type do not violate the unlimited vehicle fleet assumption during testing.

5.4 Courier costs

The costs of couriers per unit are calculated using either average routing per unit or the maximum routing per unit cost. The parameter r^c is determined such that it r^c is 1 the courier per unit cost is equal to the per unit routing cost below.

- The average per unit routing cost is calculated using:

$$r^c \sum_{(i,j)\in A} \sum_{(v)\in V_{types}} \frac{c_{ij}^{v}}{\tau^{v}} \tag{107}$$

- The maximum per unit routing cost is:

$$r^c \sum_{(i,j)\in A} \sum_{(v)\in V_{types}} \frac{c_{ij}^{vmin}}{\tau^{vmin}} \tag{108}$$

Chapter 6

Computational Experiments

In this chapter, we present our computational results. First we describe the CPLEX configuration that provides the best performance when solving the 3E-LRP-MC-PD. Then we present four computational sessions. The aims of the sessions are:

(1) Evaluating the performance of the four formulations summarized in table 3.1.

(2) Assessing the performance of the decomposition matheuristic

(3) Examining the impact of the 3E-LRP-MC-PD attributes on its complexity

(4) Establishing the importance of considering these attributes when optimizing distribution logistics.

In this research, all solution procedures and algorithms are coded in C and we use CPLEX 20.1 as the MIP solver. All computational experiments are conducted on an Intel(R) Xeon(R) CPU E5-2687W v3 processor @ 3.10 GHz with 750 GB of RAM running on Linux environment. The solver can use up to seven threads for solving any instance of the 3E-LRP-MC-PD and a total time limit of 4h of CPU time (14400s) is imposed.

6.1 CPLEX Parameters

We initially test CPLEX at default settings to solve the 3E-LRP-MC-PD and for most medium sized instances finding an initial feasible solution is difficult. We test several non-default parameters to improve CPLEX's performance. Our tests show that solving the 3E-LRP-MC-PD using most of CPLEX's parameters at default settings and the following modified parameters provide the best results:

- Emphasising feasibility over optimality
- Selecting branching variable at a node based on pseudo reduced costs
- Using relaxation induced neighborhood search (RINS) heuristic at a high frequency

For formulations 1 and 2, the seperation and addition of the sub-tour elimination constraints (18) are implemented using CPLEX callback functions (*User Cuts* and *Lazy Constraints*) to reduce the time required to solve the LP relaxation. At an integer or a fractional solution found by CPLEX, we solve a series of minimum s-t cut problems for the vehicles used ($v \in V : a^v > 0$). We use the Concorde callable library by Applegate, Bixby, Chvátal, and Cook (2011) to solve the minimum s-t cut problems and identify the violated constraints to be added to the 3E-LRP-MC-PD formulation.

Additionally, we find that adding the following parameter works best with the *Lazy Constraints*:

- Performing primal reductions only during preprocessing

These parameters are used for all the computational experiments using CPLEX to solve medium sized instances of the 3E-LRP-MC-PD.

6.2 Categorizing Instances

The size of any instance of the 3E-LRP-MC-PD is defined by the following dimensions:

- $|S|$: the total number of non-courier nodes
- $|H|$: the total number of potential transshipment locations
- $|C|$: the number of courier nodes

- $|K|$: the number of commodities being transported in the network
- V_{type}: the number of vehicle types
- V_{rep}: the available number of vehicles from each type

Given that there are six dimensional features, it is difficult to define a fixed criteria to categorize the size of the instances as small, medium or large.

For simplicity we mainly focus on varying the number non-courier nodes ($|S|$) and the total number of commodities ($|K|$) when defining different sized instances. For our computational experiments, we classify small instances as instances with $|S| \leq 10$ and $|K| \leq 10$. Additionally, we define medium sized instances as instances with $|S| \leq 10$, but with total commodities ranging from 20 to 40 ($20 \leq |K| \leq 40$).

Initially, we solve instances of varying $|S|$ and $|K|$ dimensions using CPLEX. We use the average computational time required to find an initial feasible solution and the quality of the final solution to confirm that this categorization is sensible.

6.3 Session 1: Comparing Formulations

The aim of the first computational session is to assess the performance of the four formulations summarized in Table 3.1. Due to the difficulty in finding any feasible solutions to the 3E-LRP-MC-PD, we use the time taken to find an initial solution as a performance indicator. We also consider the common indicators: the quality of CPLEX's final solution (optimality gap); and if solved to optimality, the time take to solve the instance.

We first test small instances of $|S| = [5, 8]$, with total commodities $|K| \leq 10$. Set 1 considers one transshipment center ($H = 1$) and two vehicle types ($V_{types} = 2$). The instances are generated based on the initial assumptions (discussed in Sections 5.1.2 and 5.3). We assume that the number of courier nodes are equal to the number of non-courier nodes ($|C| = |S|$) to represent the conveniently located courier collection points. We also assume that the number of available vehicles from each type (V_{rep}) is equal to the total number of commodities ($|K|$), to represent the unlimited vehicle fleet and guarantee that there is at least one intuitive feasible solution.

Thirteen instances are solved using CPLEX. Since the small instances are relatively easy to solve, CPLEX is used at default parameters. The computational results for Set 1 are summarized in Table 6.1. Columns 2-5 summarize the instance parameters. Columns 2 and 3 are the number of non-courier and courier nodes $|S|$ and $|C|$, respectively. The fourth column is the number of commodities $|K|$ and the fifth column V_{rep} is the number of vehicles available (repetitions) from each vehicle type. The next three columns correspond to the time taken to find an initial feasible solution (IFS), the best upper bound and the optimality gap found by CPLEX. The column *%GAP* displays the relative gap between the best integer feasible solution and the lower bound found by CPLEX at termination. A %GAP of zero indicates that CPLEX solves the instance to optimality. Finally, if an instance is solved to optimality, the time taken to find the optimal solution is shown in column 8 (*CPU*). Columns 6-8 are repeated for each formulation. We use **time** in the column *CPU* to indicate that the optimal solution is not found in the given time limit.

Table 6.1: Formulation Comparison for Set 1

Instance					F1			F2			F3			F4		
	$\|S\|$	$\|C\|$	$\|K\|$	Vrep	Time to IFS (s)	%GAP	CPU (s)	Time to IFS (s)	%GAP	CPU (s)	Time to IFS (s)	%GAP	CPU (s)	Time to IFS (s)	%GAP	CPU (s)
1	**5**	**5**	5	5	3.80	0.00	10.27	1.69	0.00	5.27	0.60	0.00	2.06	0.53	0.00	1.95
2			6	6	34.95	0.00	95.41	6.91	0.00	112.05	1.55	0.00	58.51	1.34	15.22	time
3			6	6	14.59	0.00	56.90	2.38	0.00	61.83	2.01	0.00	38.16	0.90	18.33	time
4			8	8	85.74	0.00	250.28	23.69	0.00	2524.89	8.31	0.00	179.18	41.11	14.01	time
5	**8**	**8**	5	5	6.90	0.00	36.37	2.67	0.00	12.72	1.78	0.00	10.83	1.86	0.00	20.09
6			5	5	16.71	0.00	644.35	3.52	0.00	20.60	1.16	0.00	8.19	1.08	6.04	time
7			6	6	25.83	0.00	50.74	9.00	0.00	72.34	3.37	0.00	26.33	233.27	29.80	time
8			6	6	43.19	0.00	94.80	17.09	0.00	33.62	2.15	0.00	24.97	2.11	14.32	time
9			6	6	13.63	0.00	24.18	4.54	0.00	10.67	2.83	0.00	12.75	2.79	6.71	time
10			8	8	271.16	0.00	2433.59	63.47	0.00	399.35	53.98	0.00	200.88	7.41	18.07	time
11			8	8	41.63	0.00	56.55	9.13	0.00	54.39	17.38	0.00	26.14	4.42	8.10	time
12			8	8	249.59	0.00	4336.35	20.76	0.00	722.20	47.15	0.00	320.38	4.50	8.08	time
13			10	10	88.20	0.00	2706.71	87.74	0.00	1176.34	19.56	0.00	67.44	3358.60	18.07	time
Average					**68.9**	**0.0**	**830.5**	**19.4**	**0.0**	**400.5**	**12.4**	**0.0**	**75.1**	**281.5**	**12.1**	**11.0**

The results summarized in Table 6.1 show that for formulations F1, F2 and F3; CPLEX solves all thirteen instances to optimality in reasonable computational time. F4 appears to be the worst formulation as only two of the small instances are solved to optimality. Additionally, the average %GAP for F4 is 12.1 which is a large optimality gap relative to the size of the instances. For most of the data set, F3 outperforms all other formulations in terms of the time taken to find an initial feasible solution and the time taken to solve the instances to optimality. We also observe that relaxing some of equality constraints (F2) (discussed in Section 3.3.2), results in better performance for most instances, as seen through the comparison between F1 and F2.

We also test the performance of the four formulations for larger instances. For Set 2, we consider $H = 2$ and $V_{types} = 2$. The instances are generated based on the initial assumptions for both $|C|$ and V_{rep}. At default settings, CPLEX does not find an initial feasible solution in the 14400s of allowed computational time. Accordingly, CPLEX parameters are set to the values discussed in Section 6.1.

Table 6.2 shows the results of the second data set. We set 0.1 as an acceptable relative tolerance between the upper bound and the lower bound found by CPLEX. Accordingly, instances with 0.1 displayed in the *%GAP* column are the instances solved to optimality. As seen in Table 6.2 only one of the instances (14) is solved to optimality. The time taken by CPLEX to find an optimal solution for F3 is 82% less than the time taken for F2. Based on the solutions for instance 14, F4 is the worst formulation, because no feasible solution can be found for an instance that can be solved to optimality using the other formulations. For instance 15, an initial feasible solution is found only using F1. For the remaining six instances no feasible solution is found regardless of the formulation solved. Most importantly, we observe that the assumptions: $|C| = |S|$ and $V_{rep} = |K|$ can redundantly increase the complexity of the instances and hence can be restrictive when solving larger data sets using CPLEX.

Table 6.2: Formulation Comparison for Set 2

Instance					F1			F2			F3			F4		
	$\|S\|$	$\|C\|$	$\|K\|$	Vrep	Time to IFS (s)	%GAP	CPU (s)	Time to IFS (s)	%GAP	CPU (s)	Time to IFS (s)	%GAP	CPU (s)	Time to IFS (s)	%GAP	CPU (s)
14	5	5	15	15	251.25	1.42	time	129.44	0.10	14038.30	318.31	0.10	2438.76	time	-	time
15	**6**	**6**	20	20	9253.1	17.47	time	time	-	time	time	-	time	time	-	time
16			30	30	time	-	time	time	-	time	time	-	time	time	-	time
17	**7**	**7**	25	25	time	-	time	time	-	time	time	-	time	time	-	time
18			30	30	time	-	time	time	-	time	time	-	time	time	-	time
19	**8**	**8**	20	20	time	-	time	time	-	time	time	-	time	time	-	time
20			25	25	time	-	time	time	-	time	time	-	time	time	-	time
21	10	10	20	20	time	-	time	time	-	time	time	-	time	time	-	time

Table 6.3: Formulation Comparison for Set 3

Instance					F1			F2			F3			F4		
	$\|S\|$	$\|C\|$	$\|K\|$	V rep	Time to IFS (s)	% GAP	CPU (s)	Time to IFS (s)	% GAP	CPU (s)	Time to IFS (s)	% GAP	CPU (s)	Time to IFS (s)	% GAP	CPU (s)
22	**6**	4	20	5	197.43	0.10	546.83	101.20	0.10	2244.00	12.64	0.10	201.48	time	-	time
23		6	20	5	878.25	6.46	time	276.88	0.10	9784.25	674.17	0.10	5816.24	time	-	time
24		4	40	10	time	-	time	time	-	time	3338.80	15.81	time	time	-	time
25	**7**	4	30	7	time	-	time	time	-	time	time	-	time	time	-	time
26		**7**	30	7	time	-	time	time	-	time	time	-	time	time	-	time
27	**8**	**4**	30	7	time	-	time	time	-	time	13805.48	1.69	time	time	-	time
28	**10**	**4**	25	6	7982.43	3.41	time	3658.35	1.27	time	2793.54	0.55	time	time	-	time
29			30	7	time	-	time	time	-	time	2076.67	1.52	time	time	-	time
30			35	8	time	-	time	time	-	time	3089.40	3.51	time	time	-	time

To be able to analyze larger sets, we relax this assumption and consider instances where $V_{rep} \neq |K|$ and for most instances $|C| \neq |S|$. Set 3 considers nine instances, of varying sizes summarized in Table 6.3. All instances consider $H = 2$ and $V_{types} = 2$. Comparing the results for Set 3 (Table 6.3) with the results of Set 2 (Table 6.2) show that relaxing the assumptions related to the $|C|$ and V_{rep} can significantly reduce complexity.

F3 is able to find feasible solutions for seven out of the nine instances. While formulations F1 and F2 find solutions for only three instances. On the other hand F4 is unable to find a feasible solution for any of the instances. The optimal solution is found for instances 22 and 23 using F1,F2 and F3. F3 still outperforms the other formulations in terms of computational time. Overall the performance of F1 and F2 are relatively similar and it is difficult to predict which one will perform better.

The results from the first computational session show that F3 is the best proposed formulation for the 3E-LRP-MC-PD. This contradicts our initial intuition. Mainly because the sub-tour elimination constraints used in F1 and F2 are only added if violated (Section 6.1). We initially expected that using callback functions to add violated constraints will reduce the complexity of the problem and hence make formulations F1 and F2 more likely to outperform other formulations.

6.4 Session 2: Matheuristic Performance

The aim of the second computational session is to assess the performance of the slope scaling decomposition matheuristic (proposed in chapter 4) in solving the 3E-LRP-MC-PD. We use a total of fifty three instances to test the matheuristic and for all instances CPLEX is used to solve formulation F3 within a 14400s time limit.

We divide our instances to two test sets:

(1) Set 1: Small instances solved to optimality using CPLEX

(2) Set 2: Larger instances, for which most of the data set are not solved to optimality and for a subset instances CPLEX does not find any feasible solutions in 14400s of computational time

For the small instances in Set 1, we run the matheuristic once for a maximum of 300 iterations.

Table 6.4: Matheuristic Performance: Set 1

Instance						CPLEX		Matheuristic	
	$\|S\|$	$\|C\|$	$\|K\|$	Vrep	$\|H\|$	% GAP	CPU (s)	%Opt GAP	CPU (s)
1	**5**	**5**	5	5	1	0.00	2.06	1.71	3.38
2			6	6	1	0.00	58.51	0.20	2.54
3			6	6	1	0.00	38.16	0.16	17.54
4			8	8	1	0.00	179.18	0.24	20.00
5	**8**	**8**	5	5	1	0.00	10.83	0.00	3.40
6			5	5	1	0.00	8.19	3.64	3.45
7			6	6	1	0.00	26.33	4.97	15.78
8			6	6	1	0.00	24.97	5.81	15.86
9			6	6	1	0.00	12.75	5.85	16.02
10			8	8	1	0.00	200.88	4.60	17.13
11			8	8	1	0.00	26.14	6.60	16.54
12			8	8	1	0.00	320.38	7.05	16.03
13			10	10	1	0.00	67.44	5.07	17.01
Average							75.06	3.53	12.67

The results for Set 1 can be seen in Table 6.4. CPLEX solves all thirteen instances in Set 1 to optimality. Hence, the column *% Opt Gap* records the optimality gap between the matheuristic solution and the optimal solution. As seen in Table 6.4, the matheuristic solves five out of the thirteen instances to an optimality gap of less than 2%. The average optimality gap is 3.53% which shows that the matheuristic reaches high quality solutions for small instances. The computational time of the matheuristic is consistently less than CPLEX, however, for the small instances (Set 1) they are both negligible.

Table 6.5: Matheuristic Performance: Set 2

Instance						CPLEX				Matheuristic						
										Best			Average			Avg CPU(s)
	$\lvert S \rvert$	$\lvert C \rvert$	$\lvert K \rvert$	V rep	$\lvert H \rvert$	UB	LB	% GAP	CPU (s)	Best OFV	% GAP UB	% GAP LB	Avg OFV	% GAP UB	% GAP LB	
14	**6**	4	20	5	2	366,334.08	365,969.03	0.10	2,244.00	456,996.10	24.75	24.87	460,740.91	25.77	25.90	255.32
15		6	20	5	2	413,342.73	412,931.33	0.10	9,784.25	479,261.82	15.95	16.06	479,261.82	15.95	16.06	363.52
16		4	25	6	2	477,354.02	395,040.55	17.24	6,054.45	510,066.96	6.85	29.12	518,792.08	8.68	31.33	387.90
17		6	25	6	2	505,520.47	467,619.32	7.50	5,730.90	591,282.04	16.97	26.45	591,282.04	16.97	26.45	661.26
18		4	30	7	2	449,989.14	431,830.20	4.04	6,983.39	567,064.91	26.02	31.32	610,952.24	35.77	41.48	502.47
19		4	35	8	2	537,883.09	487,906.86	9.29	13,199.70	647,315.77	20.35	32.67	681,447.09	26.69	39.67	657.06
20		4	40	10	2	618,471.32	520,675.90	15.81	11,859.05	729,904.84	18.02	40.18	786,098.46	27.10	50.98	816.79
21	**7**	4	20	5	2	427,431.54	396,509.36	7.23	1,492.89	468,594.58	9.63	18.18	475,128.32	11.16	19.83	394.22
22		4	25	6	2	530,526.36	488,880.51	7.85	13,232.63	623,892.65	17.60	27.62	629,182.07	18.60	28.70	414.17
23		4	30	7	2	614,470.53	497,184.23	19.09	13,090.82	687,902.72	11.95	38.36	704,574.84	14.66	41.71	686.80
24		4	40	10	2	2,347,452.06	550,217.02	76.56	11,112.41	907,278.77	-61.35	64.89	934,545.37	-60.19	69.85	917.45
25	**8**	4	20	5	2	391,704.21	391,313.93	0.10	1,518.85	416,888.25	6.43	6.54	432,599.96	10.44	10.55	353.46
26		4	25	6	2	354,165.80	340,844.87	3.76	14,041.00	448,601.50	26.66	31.61	483,843.26	36.61	41.95	548.03
27		4	30	7	2	367,838.45	361,632.00	1.69	14,235.70	533,211.23	44.96	47.45	572,446.07	55.62	58.30	670.90
28	**9**	4	20	5	2	371,533.27	371,163.25	0.10	12,474.66	475,074.61	27.87	28.00	485,036.59	30.55	30.68	365.87
29		6	20	5	2	367,902.63	350,676.51	4.68	8,271.94	395,047.09	7.38	12.65	399,644.36	8.63	13.96	538.10
30		8	20	5	2	367,437.05	349,642.93	4.84	6,809.11	391,508.60	6.55	11.97	397,417.69	8.16	13.66	644.28
31		4	25	6	2	482,660.73	365,398.33	24.29	13,216.99	473,296.53	-1.94	29.53	475,521.94	-1.48	30.14	574.51
32		6	25	6	2	359,418.27	322,273.28	10.33	3,752.53	388,079.07	7.97	20.42	425,803.79	18.47	32.13	775.09
33		8	25	6	2	360,284.48	319,753.32	11.25	14,211.36	382,140.65	6.07	19.51	387,486.37	7.55	21.18	1,018.38
34	**10**	4	20	5	2	327,550.39	327,227.10	0.10	612.31	361,450.32	10.35	10.46	392,091.20	19.70	19.82	452.39
35		6	20	5	2	323,855.05	323,531.20	0.10	2,596.62	358,621.49	10.74	10.85	381,331.34	17.75	17.87	646.78
36		8	20	5	2	324,787.83	324,463.07	0.10	2,442.65	353,644.03	8.88	8.99	365,455.33	12.52	12.63	753.40
37		4	25	6	2	446,592.97	444,150.80	0.55	3,509.17	496,763.55	11.23	11.85	511,339.21	14.50	15.13	597.59
38		6	25	6	2	441,092.67	435,616.87	1.24	10,940.23	504,395.99	14.35	15.79	545,587.66	23.69	25.24	862.50
39		4	30	7	2	497,202.26	489,664.82	1.52	13,141.64	552,226.96	11.07	12.78	557,857.93	12.20	13.93	1,133.78
40		4	35	8	2	509,027.74	491,140.83	3.51	14,307.98	534,422.40	4.99	8.81	576,416.38	13.24	17.36	1,213.85
Average								8.63	8550.64		11.49	23.59		15.90	28.39	637.25

To assess the matheuristic performance for Set 2, we run the matheuristic three independent times for each instance (in Tables 6.5 and 6.6) for 2000 iterations every run.

Table 6.5 summarizes the instances for which CPLEX finds at least one feasible solution. The table shows the best objective function value found (*Best OFV*) and the average objective function value found (*Avg OFV*) of the three runs. We calculate the %GAP between the matheuristic solution and the upper bound found by CPLEX as $100 \times \frac{OFV-UB}{UB}$. We record the GAP between the matheuristic's best and average OFVs and CPLEX's UB in the corresponding *%GAP UB* columns. We also calculate the GAP between the matheuristic solutions and the lower bound found by CPLEX at termination as $100 \times \frac{OFV-LB}{LB}$.

Additionally, the column *CPU* for CPLEX displays the time taken to find the optimal solution when an optimal solution is found (%GAP ≤ 0.10). Otherwise, it displays time take by CPLEX to find best feasible solutions. Finally, the average time take to run the heuristic for 2000 iterations is shown in column *Avg CPU*.

The computational results in Table 6.5 show that the matheuristic finds a solution with less than 10% deviation from CPLEX's upper bound for ten out of the twenty seven instances. For two of these instances the matheuristic finds a solution better than CPLEX's best solution. The matheuristic's worst deviation from CPLEX's upper bound is 44.96% for instance 27. However, the average deviation based on both the best solution is 11.49%. On the other hand the average gap between the algorithms best solution and the LB is 23.59%. The results of the best and average matheuristic performance also show that the quality of the solutions produced by the matheuristic are relatively consistent. Finally the results show a good performance of the matheuristic against CPLEX in terms of speed. The algorithm is able to generate a reasonably good solution within an average of 10 minutes. While CPLEX requires an average of 2.3 hours.

Table 6.6: Matheuristic Performance: Set 2 continued

Instance						CPLEX				Matheuristic: Best OFV		Matheuristic: Avg OFV		Avg CPU(s)
	$\|S\|$	$\|C\|$	$\|K\|$	V rep	$\|H\|$	UB	LB	% GAP	CPU (s)	Best OFV	% GAP LB	Avg OFV	% GAP LB	
41	**7**	4	35	8	2	0.00	527,170.38	-	time	708,395.74	34.38	758,732.72	43.93	726.66
42	**8**	8	25	6	2	0.00	405,423.86	-	time	602,192.38	48.53	634,392.54	56.48	1,013.04
43	**9**	4	30	7	2	0.00	412,217.52	-	time	605,741.26	46.95	614,183.20	48.99	674.05
44		6	30	7	2	0.00	402,971.36	-	time	545,785.44	35.44	621,782.31	54.30	1,059.53
45		8	30	7	2	0.00	399,062.47	-	time	499,021.59	25.05	527,043.98	32.07	1,468.78
46		4	35	8	2	0.00	366,944.73	-	time	625,409.94	70.44	649,120.56	76.90	984.85
47		6	35	8	2	0.00	343,745.83	-	time	590,974.47	71.92	611,257.72	77.82	1,456.15
48		8	35	8	2	0.00	372,105.45	-	time	573,411.06	54.10	627,833.42	68.72	2,085.18
49		4	40	10	2	0.00	432,971.04	-	time	731,826.02	69.02	758,655.93	75.22	1,146.81
50	**10**	8	25	6	2	0.00	437,793.46	-	time	494,101.88	12.86	523,716.59	19.63	1,217.99
51		6	30	7	2	0.00	483,929.29	-	time	546,648.87	12.96	564,856.81	16.72	1,531.57
52		8	30	7	2	0.00	483,103.34	-	time	540,383.43	11.86	553,008.96	14.47	2,271.84
53		4	40	10	2	0.00	577,103.35	-	time	753,808.23	30.62	766,260.68	32.78	1,195.89
Average											40.32		47.54	1,294.80

The results from Table 6.6 show that Set 2 is much harder than Set 1. For thirteen out of the forty instances in Set 2, CPLEX does not find a feasible solution in 4h of computational time. Table 6.6 shows that the matheuristic provides solutions within an average 40.3% GAP from CPLEX's LB at termination in an average of 21 mins.

6.5 Session 3: Complexity of the 3E-LRP-MC-PD

As discussed in Sections 6.1 and 6.3, and seen in Table 6.6, medium sized instances of the 3E-LRP-MC-PD can be relatively hard to solve. The aim of this session is to identify the characteristics that contribute the most to this complexity. Accordingly, we investigate the impact of bundling restrictions, transit time constraints and considering a courier transportation strategy on the complexity of the 3E-LRP-MC-PD.

Similar to the first computational session (Section 6.3) we define complexity based on:

- How difficult it is to find an initial feasible solution
- The quality of the final solution found in the given time limit

6.5.1 Bundling & Transit Time

Unlike the standard LRP, the 3E-LRP-MC-PD considers bundling and transit time restrictions which increases the number of constraints in the model. More importantly bundling restrictions also increase the inter-dependencies between flow decision variables which may contribute to the complexity of the problem.

To investigate how influential are the additional constraints and connectivity, we consider instances of the 3E-LRP-MC-PD for which in the given time limit: (1) no optimal solution is found (Table 6.7) and (2) no feasible solution is found (Table 6.8). We investigate the impact of relaxing the bundling restrictions and the transit time constraints on the time taken to find an initial feasible solution and the quality of the final solution. The relaxed F3 formulations are solved with the same computational limit of 14400s.

The computational results in Table 6.7 show the impact of relaxing the bundling and transit time constraints on nine instances that are not solved to optimality. CPLEX does not find the optimal solution for any of the instances when either sets of constraints are relaxed. However, relaxing the transit time constraints improve the %GAP for five out of the nine instances. It is interesting to observe that relaxing the transit time constraints increases the time needed to find an initial feasible solution for eight out of the nine instances.

The results in Table 6.7 show that relaxing the bundling constraints improves the optimality gap for four out of the nine instances. It also reduces the time taken to find an initial feasible solution for seven instances. However, the results from Table 6.7 show that for five instances, the formulation with the relaxed bundling constraints has a greater optimality gap than the original F3 formulation. For instance 7, the new problems defined by relaxing the (1) bundling constraints and (2) the transit time constraints is more difficult to solve than the original problem.

Table 6.8 summarizes the impact of relaxing the constraints on five instances for which no feasible solution is found. For instance 10 relaxing either constraints results in finding an IFS. Relaxing the transit time constraint has a greater impact on complexity reducing both the time to IFS and optimality gap. For instances 11 and 14 relaxing bundling constraints allows an IFS to be found and provides a final solution with an optimality gap of less than 8%. While relaxing transit time constraints appears to have no impact on the complexity. Finally for two instances, both constraints do not seem to affect the complexity of the instance and no solution is found for the new relaxed problems.

Based on the results from Tables 6.7 and 6.8, we observe that relaxing bundling restrictions can help reduce the time needed to find an initial feasible solution for most instances (71%). We conclude that relaxing the bundling constraints may reduce the complexity related to finding feasible solutions. However, the new problem defined can still be challenging to solve to optimality. The problem defined by relaxing the transit time constraints may be less complex since CPLEX finds a better final solution in the same computational time. However, relaxing the transit time constraints does not usually improve the time taken to find initial feasible solutions. Hence, it may not affect the complexity of instances where no initial solution is found for the original problem. Finally, it is important to note that the correlation between relaxing the transit time constraints and the optimality gap aspect of complexity is only valid for 42% of the instances tested; so it is not as strong as the relationship between the bundling constraints and complexity.

Table 6.7: Relating Bundling and Transit Time Requirements to Complexity

Instance					CPLEX F3 (RESULTS)			F3 with Relaxed Bundling restrictions			F3 with Relaxed Transit Time restrictions		
	$\|S\|$	$\|C\|$	$\|K\|$	V rep	Time to IFS (s)	% GAP	CPU time (s)	Time to IFS (s)	% GAP	CPU time (s)	Time to IFS (s)	% GAP	CPU time (s)
1	**6**	4	25	6	1,798.97	17.24	time	126.58	4.53	time	3,683.32	10.17	time
2		4	30	7	4,182.45	4.04	time	901.48	6.32	time	23.07	3.03	time
3		4	40	10	3,338.80	15.81	time	882.56	4.77	time	time	-	time
4		4	35	8	367.63	9.29	time	6,446.40	16.16	time	time	-	time
5	**7**	4	25	6	2,128.73	7.85	time	427.35	3.63	time	1,408.34	7.70	time
6		4	30	7	13,090.80	19.09	time	3,426.84	19.60	time	14,253.44	25.42	time
7		4	40	10	11,015.28	76.56	time	time	-	time	time	-	time
8	**8**	4	25	6	2,917.44	3.76	time	737.09	4.66	time	4,363.56	3.60	time
9	**9**	4	25	6	4,056.76	24.29	time	1,594.91	6.56	time	11,530.55	10.10	time

Table 6.8: Relating Bundling and Transit Time Requirements to Complexity (2)

Instance					CPLEX F3 (RESULTS)			F3 with Relaxed Bundling restrictions			F3 with Relaxed Transit Time restrictions		
	$\|S\|$	$\|C\|$	$\|K\|$	V rep	Time to IFS (s)	% GAP	CPU time (s)	Time to IFS (s)	%GAP	CPU time (s)	Time to IFS (s)	% GAP	CPU time (s)
10	**7**	4	35	8	time	-	time	6,446.40	16.16	time	2,868.09	14.03	time
11	**8**	8	25	6	time	-	time	1,446.55	7.37	time	time	-	time
12		4	35	8	time	-	time	-	-	time	time	-	time
13		8	40	10	time	-	time	-	-	time	time	-	time
14	**9**	9	25	6	time	-	time	3,338.12	7.10	time	time	-	time

6.5.2 Courier Collection Nodes

Considering a courier collection transportation strategy is a key characteristic of the 3E-LRP-MC-PD. Although courier collection nodes act as conveniently located sink nodes, they add complexity, because on the very least, they increase the number on nodes and arcs on the network and accordingly the related decision variables.

It is therefore intuitive that considering courier nodes will increase complexity. To assess the contribution of courier nodes to the complexity we first compare the CPLEX solutions for instances of the 3E-LRP-MC-PD with solutions to the same instance but without the courier delivery option. Additionally, we observe the impact of increasing the number of couriers being considered on the complexity.

Table 6.9: Relating Courier Transportation Strategy to Complexity

Instance					F3 Courier Delivery			F3 without Cour Delivery		
	$\|S\|$	$\|C\|$	$\|K\|$	V rep	Time to IFS (s)	% GAP	CPU time (s)	Time to IFS (s)	% GAP	CPU time (s)
1	**6**	4	20	5	101.20	0.10	2,244.00	62.52	0.10	1,248.92
2		6	20	5	9,253.13	17.47	time	46.32	0.10	14,267.37
3		4	25	6	1,798.97	17.24	time	126.58	4.53	time
4		6	25	6	5,730.90	7.50	time	392.34	7.63	time
5		4	30	7	4,182.45	4.04	time	901.48	6.32	time
6		6	30	7	time	-	time	1,037.37	14.44	time
7	**7**	4	25	6	2,128.73	7.85	13,232.63	427.35	3.63	time
8		4	30	7	13,090.80	19.09	13,090.82	4,345.68	15.72	time
9	**8**	4	20	5	183.08	0.10	1,518.85	88.99	0.10	6,892.46
10		4	25	6	2,917.44	3.76	time	737.09	4.66	time

We use a data set of ten instances (seen in Table6.9) to evaluate the impact of of considering courier transportation strategy on complexity. The results show that for all instances tested, the time needed to find an initial feasible solution is reduced if the courier strategy is not considered. For five of the seven non-optimal instances, the % GAP is also reduced if couriers are excluded. For one of the five instances CPLEX can not find a feasible solution for 3E-LRP-MC-PD. On the other hand, the optimality gap is 14% when the the same instance is solved without considering couriers. Finally, for the three instances of the 3E-LRP-MC-PD that are solved to optimality; the new problem excluding couriers is solved in less computational time.

We also study the impact of the number of couriers on the complexity. For each instance described in Table 6.10, we solve for $|C| = [4, 6, 8]$. The results show that in general for each instance (described by $|S|$,$|C|$ and $|K|$) as the number of couriers increase, the time to find an initial feasible solution also increases. For instances 5 and 6 increasing the number of couriers leads to instances which CPLEX can not find an initial feasible solution for. It is worth noting that based on the instances summarized in Table 6.10, there is no clear correlation between increasing the number of courier nodes and the quality of the final solutions (% GAP).

The third computational session shows that considering courier delivery has the most direct impact on the complexity of the 3E-LRP-MC-PD. Additionally, there appears to be a positive correlation between considering bundling restrictions and the time required by CPLEX to find a feasible solution. The transit time constraints have the least impact. However, for 42% of the instances tested, the problems resulting from relaxing transit time constraints have better quality of final solutions.

Table 6.10: Relating Number of Courier Nodes $|C|$ to Complexity

Instance					$\lvert C\rvert$ =4			$\lvert C\rvert$ = 6			$\lvert C\rvert$ = 8		
	$\lvert S\rvert$	$\lvert C\rvert$	$\lvert K\rvert$	V rep	Time to IFS (s)	% GAP	CPU time (s)	Time to IFS (s)	% GAP	CPU time (s)	Time to IFS (s)	% GAP	CPU time (s)
1	**9**	4	20	5	29.17	0.10	12,474.66	1,010.48	4.68	8,271.94	4,182.45	4.04	time
2		4	25	6	4,056.76	24.29	time	395.44	10.33	time	1,919.41	11.25	time
3	**10**	4	20	5	348.68	0.10	612.31	439.69	0.10	2,596.62	1,051.14	0.10	2,442.65
4		4	25	6	2,793.54	0.55	time	6,691.73	1.24	time	time	-	time
5		4	30	7	2,076.67	1.52	time	time	-	time	time	-	time
6		4	35	8	3,089.40	3.51	time	time	-	time	time	-	time

6.6 Session 4: Importance of the 3E-LRP-MC-PD

In the third computational session (Section 6.5), we establish that the unique characteristics of the 3E-LRP-MC-PD, contribute (with varying intensities) to making the problem harder to solve. The aim of the fourth computational session is to justify why these attributes are worth considering, even though they are computationally taxing.

6.6.1 Bundling & Transit Time

Today's current distribution environment is characterized by a wide product range. We consider bundling and transit time restrictions to more realistically model the product variety and their unique transportation requirements. If these unique transportation requirements are not considered when planning distribution; then the optimized costs of the distribution logistics can be misleading. We assess the magnitude of not accounting for the unique transportation requirement by: comparing the costs calculated by solving the 3E-LRP-MC-PD, to the distribution costs calculated if the (1) bundling or the (2) transit time constraints are neglected.

Table 6.11: The Impact of Relaxing Bundling Restrictions on Costs

Instance						3E-LRP-MC-PD		No Bundling		**% decrease in Cost**
	$\|S\|$	$\|C\|$	$\|K\|$	V rep	$\|H\|$	UB	% GAP	UB	% GAP	
1	**6**	4	20	5	2	366,334.08	0.10	252,828.88	0.10	**30.92**
2		4	30	7	2	449,989.14	4.04	308,594.11	10.84	**28.54**
3		4	35	8	2	537,883.09	9.29	311,968.45	0.10	**36.06**
4	7	4	25	6	2	530,526.36	7.85	306,113.75	0.10	**37.38**
5	**8**	4	20	5	2	391,704.21	0.10	162,238.72	0.08	**58.54**
6		4	25	6	2	354,165.80	3.76	249,415.46	0.10	**26.82**
7		4	30	7	2	367,838.45	1.69	261,829.28	9.83	**27.60**
8	**10**	4	20	5	2	327,550.39	0.10	256,341.95	32.45	**21.66**
9		4	25	6	2	446,592.97	0.55	262,237.35	27.26	**40.96**
10		4	30	7	2	497,202.26	1.52	309,140.18	32.91	**36.87**
Average										**34.53**

We first choose 10 instances of the 3E-LRP-MC-PD where CPLEX finds a solution with an optimality gap of less than 10%. This allows us to make valid inferences about the distribution costs (objective function value). The data set chosen is summarized in Table 6.11. We then solve the same instances without considering bundling restrictions. The column *UB* for the 3E-LRP-MC-PD shows the distribution cost accounting for the bundling attributes. While the column *UB* for the relaxed bundling problem shows the costs if bundling restrictions are not considered. The results show that failing to account for bundling restrictions can lead to underestimating distribution costs for all instances by an average of 34.53%.

We also consider how relaxing transit time requirements can result in underestimating distribution costs. We study four scenarios with identical parameters and different transit time requirements (seen in Table 6.12). The first scenario considers the most restrictive transit time constraints where the transit time required for a commodity k : $tmax_k$ is strictly equal to the travel time between origin and destination. For the remaining three scenarios we randomly generate the transit time for a commodity k (as explained in Section 5.2) based on the ranges specified in Table 6.12. The fourth scenario has the most relaxed transit time requirements such that the minimum transit time for a commodity is three times the time required for a direct delivery between its origin and destination.

Since the first scenario is the most restricted, it is most likely to have the highest distribution costs. This is because for most commodities only direct delivery can be considered and the cost-saving potential of using courier deliveries or indirect delivery can not be exploited. We use the first scenario as a benchmark and compare the impact of varying levels of relaxation on the reduction in distribution costs. For four instances relaxing transit time restrictions to scenarios 2 and 3 lead to a decrease in costs. For instances 8 to 11 relaxing the distribution costs to scenario 2 is not enough to impact costs. However a more extreme relaxation (scenario 3) leads to more than 10% decrease in costs for all scenarios. The results for the first 3 scenarios show that in general the more aggressive the relaxation of the time constraints the greater the cost reductions. This is indicative of how neglecting transit time requirements can result in underestimating distribution costs.

It is worth noting that relaxing beyond a certain level (between scenarios 2 and 3) does not lead to significant cost reductions. It is also important to note that inaccurately relaxing bundling restrictions has a greater impact on underestimating costs compared to relaxing transit time constraints.

Table 6.12: The Impact of Transit Time Restrictions on Costs

Instance						$tmax_k$ = [1,1] $t_{o_k d_k}$		$tmax_k$ = [1,1.5] $t_{o_k d_k}$			$tmax_k$ = [1.5,5] $t_{o_k d_k}$			$tmax_k$ = [3,5] $t_{o_k d_k}$		
	$\|S\|$	$\|C\|$	$\|K\|$	V rep	$\|H\|$	UB	% GAP	UB	% GAP	% decrease in Cost	UB	% GAP	% decrease in Cost	UB	% GAP	% decrease in Cost
1	**6**	4	20	5	2	417,052.20	0.01	416,501.00	0.09	**0.13**	366,334.08	0.10	**12.16**	366,334.08	0.10	**12.16**
2		4	30	7	2	529,637.46	0.10	537,114.76	3.38	**-1.41**	449,989.14	4.04	**15.04**	447,094.16	3.03	**15.58**
3		4	35	8	2	665,856.48	0.34	582,809.39	1.12	**12.47**	537,883.09	9.29	**19.22**	0.00	time	**-**
4	7	4	25	6	2	530,098.59	7.70	613,324.90	0.10	**-15.70**	530,526.36	7.85	**-0.08**	530,098.59	7.70	**0.00**
5	**8**	4	20	5	2	392,476.19	0.00	393,086.19	0.04	**-0.16**	391,704.21	0.10	**0.20**	391,704.21	0.10	**0.20**
6		4	25	6	2	447,413.13	0.10	365,258.53	0.06	**18.36**	354,165.80	3.76	**20.84**	354,165.80	3.60	**20.84**
7		4	30	7	2	445,749.30	0.10	406,213.56	0.10	**8.87**	367,838.45	1.69	**17.48**	366,884.39	1.61	**17.69**
8	**10**	4	20	5	2	427,522.76	0.10	427,522.76	0.10	**0.00**	327,550.39	0.10	**23.38**	327,550.39	0.10	**23.38**
9		4	25	6	2	539,597.26	0.10	539,597.26	0.10	**0.00**	446,592.97	0.55	**17.24**	446,592.97	0.62	**17.24**
10		4	30	7	2	583,232.36	0.34	583,232.36	0.34	**0.00**	497,202.26	1.52	**14.75**	0.00	time	**-**
11		4	35	8	2	573,894.06	1.00	573,894.06	1.00	**0.00**	509,027.74	3.51	**11.30**	0.00	time	**-**
Average										**2.05**			**13.78**			**13.39**

6.6.2 Courier Collection Nodes

Finally, we analyze the cost saving opportunities provided by considering courier deliveries. We consider eight instances that do not consider courier deliveries and have an optimality gap of less than 10%, to ensure the validity of the analysis. The instances are summarized in Table 6.13.

We solve additional instances with identical parameters but considering courier delivery as an additional transportation strategy. We estimate the courier delivery cost as the average per unit routing costs (equation (107)) and use the procedure described in Section 5.1.2 to determine the location of courier nodes. The results from table 6.13 show that considering courier deliveries reduce distribution costs for all instances. For two instances, considering courier deliveries reduce costs by more than 50% .

Table 6.13: The Cost Saving Potential of Considering Courier Delivery

Instance					F3 without Courier Delivery		3E-LRP-MC-PD F3 Courier Delivery		**% decrease in Cost**
	$\lvert S\rvert$	$\lvert C\rvert$	$\lvert K\rvert$	V rep	UB	% GAP	UB	% GAP	
1	**6**	4	20	5	469,562.27	0.10	366,334.08	0.10	**28.18**
2		6	20	5	469,562.27	0.10	457,556.44	17.47	**2.62**
3		4	25	6	613,286.62	4.53	477,354.02	17.24	**28.48**
4		6	25	6	610,502.93	7.63	505,520.47	7.50	**20.77**
5		4	30	7	685,327.83	6.32	449,989.14	4.04	**52.30**
6	**7**	4	25	6	713,690.32	3.63	530,526.36	7.85	**34.52**
7	**8**	4	20	5	454,040.89	0.10	391,704.21	0.10	**15.91**
8		4	25	6	532,704.72	4.66	354,165.80	3.76	**50.41**
Average									**28.04**

The fourth computational session shows that not accounting for transit time and bundling restrictions can lead to underestimation of transportation costs. The results from this session show that it is more misleading to neglect bundling restrictions than to fail to account for transit time requirements. As seen in Tables 6.11 and 6.12, relaxing bundling restrictions results in a more significant decrease in distribution costs. Finally, Table 6.13 shows that considering courier strategies has a notable cost saving potential.

Chapter 7

Conclusion and Future Research

In this thesis we studied a 3 echelon location routing problem with multiple commodities and allowing pick-up and delivery (3E-LRP-MC-PD). It is an extension of the multi-echelon location-routing problem that considers multiple commodities, three transportation strategies and allows flexible vehicle routing between echelons. Moreover, we consider bundling restrictions and transit time requirements as unique attributes for all commodities. To the best of our knowledge, no problem in the location-routing literature addressed all these features simultaneously. We modeled the problem as an integer program and proposed three additional formulations based on relaxing equality constraints and modifying the sub-tour elimination constraints. We also developed a slope-scaling matheuristic which utilizes a new decomposition scheme.

We presented a computational study of the four formulations. The formulation with relaxed equality constraints and the Miller-Tucker-Zemlin subtour elimination constraints outperformed others when solved using CPLEX. We observed that even with this formulation there are medium sized instances, for which it is difficult to find initial feasible solutions. Furthermore, we studied factors increasing the complexity of the 3E-LRP-MC-PD. We found that considering courier delivery strategy contributes the most to the complexity of the problem, followed by the bundling restrictions. Additionally, we evaluated the impact of neglecting bundling restrictions and transit time on the estimated distribution costs. We concluded that if the bundling restrictions and transit time are unaccounted for, then the distribution costs will be underestimated and hence misleading. We also quantified the cost saving opportunity of using courier deliveries. Finally, we evaluated the performance of the proposed matheuristic in addressing the complexity of the 3E-LRP-MC-PD. Our algorithm obtained near optimal solutions for small instances and an average optimality gap of 11.49% for medium-sized instances.

There are three main future directions for this research. First, developing other heuristic strategies for the arc-routing sub-problem. Secondly, exploring the potential of sub-gradient optimization to improve the updating of coefficients at the end of each iteration of the matheuristic procedure. Finally, using the 3E-LRP-MC-PD to address an industrial case-study.

References

Abbassi, A., Kharraja, S., El Hilali Alaoui, A., Boukachour, J., & Paras, D. (2020). Multi-objective two-echelon location-distribution of non-medical products. *International Journal of Production Research*, 1-17.

Abdulsalam, Y., Gopalakrishnan, M., Maltz, A., & Schneller, E. (2015). The emergence of consolidated service centers in health care. *Journal of Business Logistics*, *36*(4), 321-334.

Applegate, D. L., Bixby, R. E., Chvátal, V., & Cook, W. J. (2011). *The traveling salesman problem: A computational study*. Princeton University Press.

Archetti, C., & Speranza, M. (2014). A survey on matheuristics for routing problems. *EURO Journal on Computational Optimization*, *2*(4), 223-246.

Armstrong & Associates. (2020). *Global and regional infrastructure, logistics costs, and third-party logistics market trends and analysis* (Tech. Rep.). USA: Armstrong and Associates.

Asgari, N., Rajabi, M., Jamshidi, M., Khatami, M., & Farahani, R. Z. (2017). A memetic algorithm for a multi-objective obnoxious waste location-routing problem: a case study. *Annals of Operations Research*, *250*(2), 279-308.

Azizi, V., & Hu, G. (2020). Multi-product pickup and delivery supply chain design with location-routing and direct shipment. *International Journal of Production Economics*, *226*.

Beaulieu, M., Roy, J., & Landry, S. (2018). Logistics outsourcing in the healthcare sector: Lessons from a canadian experience. *Canadian Journal of Administrative Sciences-revue Canadienne Des Sciences De L Administration*, *35*(4), 635-648.

Boccia, M., Crainic, T., Sforza, A., & Sterle, C. (2018). Multi-commodity location-routing: Flow intercepting formulation and branch-and-cut algorithm. *Computers and Operations Research*, *89*, 94-112.

Boccia, M., Crainic, T. G., Sforza, A., & Sterle, C. (2011). Location-routing models for designing a two-echelon freight distribution system. *Rapport technique, CIRRELT, Université de Montréal*, *91*.

Boudahri, F., Aggoune-Mtalaa, W., Bennekrouf, M., & Sari, Z. (2013). Application of a clustering based location-routing model to a real agri-food supply chain redesign. In N. T. Nguyen, B. Trawiński, R. Katarzyniak, & G.-S. Jo (Eds.), *Advanced methods for computational collective intelligence* (pp. 323–331). Berlin, Heidelberg: Springer Berlin Heidelberg.

Boventer, E. (1961). The relationship between transportation costs and location rent in transportation problems. *Journal of Regional Science*, *3*(2), 27-40.

Brandimarte, P., & Zotteri, G. (2007). Supply chain management. In *Introduction to distribution logistics* (p. 1-52). John Wiley & Sons, Ltd.

Chopra, S. (2019). *Supply chain management : strategy, planning and operation* (Seventh edition ed.). New York, NY: Pearson Education.

Cleophas, C., Cottrill, C., Ehmke, J. F., & Tierney, K. (2019). Collaborative urban transportation: Recent advances in theory and practice. *European Journal of Operational Research*, *273*(3), 801-816.

Crainic, T., Ricciardi, N., & Storchi, G. (2009). Models for evaluating and planning city logistics systems. *Transportation Science*, *43*(4), 432-454.

Cuda, R., Guastaroba, G., & Speranza, M. G. (2015). A survey on two-echelon routing problems. *Computers and Operations Research*, *55*, 185–199.

Dai, Z., Aqlan, F., Gao, K., & Zhou, Y. (2019). A two-phase method for multi-echelon location-routing problems in supply chains. *Expert Systems With Applications*, *115*, 618-634.

Dooley, L. (2009, May). Make logistics the focus of your supply chain plan. *Materials management in health care*, *18*(5), 26—29.

Drexl, M., & Schneider, M. (2015). A survey of variants and extensions of the location-routing problem. *European Journal of Operational Research*, *241*(2), 283–308.

Escabor, J. W., Linfati, R., & Toth, P. (2013). A two-phase hybrid heuristic algorithm for the capacitated location-routing problem. *Computers & Operations Research*, *40*(1), 70-79.

Gianessi, P., Alfandari, L., Létocart, L., & Calvo, R. (2016). The multicommodity-ring location routing problem. *Transportation Science*, *50*(2), 541-558.

Goodarzi, A., & Zegordi, S. (2016). A location-routing problem for cross-docking networks: A biogeography-based optimization algorithm. *Computers & Industrial Engineering*, *102*, 132-146.

Guastaroba, G., Speranza, M. G., & Vigo, D. (2016). Intermediate facilities in freight transportation planning: A survey. *Transportation Science*, *50*(3), 763–789.

Hamidi, M., Farahmand, K., Sajjadi, S., & Nygard, K. (2014). A heuristic algorithm for a multi-product four-layer capacitated location-routing problem. *International Journal of Industrial Engineering Computations*, *5*, 87-100.

Hemmelmayr, V., Doerner, K., Hartl, R., & Vigo, D. (2014). Models and algorithms for the integrated planning of bin allocation and vehicle routing in solid waste management. *Transportation Science*, *48*(1), 103-120.

Hosseini, S., Shirazi, M., & Karimi, B. (2014). Cross-docking and milk run logistics in a consolidation network: A hybrid of harmony search and simulated annealing approach. *Journal of Manufacturing Systems*, *33*(4), 567-577.

Houde, J.-F., Newberry, P., & Seim, K. (2017, April). *Economies of Density in E-Commerce: A Study of Amazon's Fulfillment Center Network* (NBER Working Papers No. 23361). National Bureau of Economic Research, Inc.

Janjevic, M., Winkenbach, M., & Merchán, D. (2019). Integrating collection-and-delivery points in the strategic design of urban last-mile e-commerce distribution networks. *Transportation Research Part E*, *131*, 37-67.

Lam, M., & Mittenthal, J. (2013). Capacitated hierarchical clustering heuristic for multi depot location-routing problems. *International Journal of Logistics Research and Applications*, *16*(5), 433-444.

Landry, S., & Beaulieu, M. (2013). The challenges of hospital supply chain management, from central stores to nursing units. In B. T. Denton (Ed.), *Handbook of healthcare operations management: Methods and applications* (pp. 465–482). New York, NY: Springer New York. doi: 10.1007/978-1-4614-5885-2_18

Landry, S., Beaulieu, M., & Roy, J. (2016). Strategy deployment in healthcare services: A case study approach. *Technological Forecasting and Social Change*, *113*(PB), 429-437.

Li, P., Lan, H., & Saldanha-Da-Gama, F. (2019). A bi-objective capacitated location-routing problem for multiple perishable commodities. *IEEE Access*, *7*.

Mara, S. T. W., Kuo, R., & Asih, A. M. S. (2021). Location-routing problem: a classification of recent research. *International Transactions in Operational Research*, *28*(6), 2941-2983.

Maranzana, F. E. (1964). On the location of supply points to minimize transport costs. *Operational Research Society*, *15*(3), 261-270.

Min, H., Jayaraman, V., & Srivastava, R. (1998). Combined location-routing problems: A synthesis and future research directions. *European Journal of Operational Research*, *108*(1), 1-15.

Mirhedayatian, S., Crainic, T., Guajardo, M., & Wallace, S. (2019). A two-echelon location-routing problem with synchronisation. *Journal of the Operational Research Society*, 1–16.

Nagy, G., & Salhi, S. (2007). Location-routing: Issues, models and methods. *European Journal of Operational Research*, *177*(2), 649-672.

Pichka, K., Bajgiran, A. H., Petering, M. E., Jang, J., & Yue, X. (2018). The two echelon open location routing problem: Mathematical model and hybrid heuristic. *Computers & Industrial Engineering*, *121*, 97-112.

Prodhon, C., & Prins, C. (2014). A survey of recent research on location-routing problems. *European Journal of Operational Research*, *238*(1), 1-17.

Rahmani, Y., Cherif-Khettaf, W. R., & Oulamara, A. (2015). A local search approach for the two-echelon multi-products location-routing problem with pickup and delivery. *IFAC PapersOnLine*, *48*(3), 193-199.

Ranieri, L., Digiesi, S., Silvestri, B., & Roccotelli, M. (2018). A review of last mile logistics innovations in an externalities cost reduction vision. *Sustainability*, *10*(3).

Rieck, J., Ehrenberg, C., & Zimmermann, J. (2014). Many-to-many location-routing with interhub transport and multi-commodity pickup-and-delivery. *European Journal of Operational Research*, *236*(3), 863-878.

Rodrigue, J.-P. (2020). The distribution network of amazon and the footprint of freight digitalization. *Journal of Transport Geography*, *88*, 102825.

Rushton, A., Croucher, P., & Baker, P. (2017). *The handbook of logistics and distribution management: Understanding the supply chain* (Sixth edition ed.). London: Kogan Page Publishers.

Salhi, S., & Rand, G. K. (1989). The effect of ignoring routes when locating depots. *European Journal of Operational Research*, *39*(2), 150-156.

Savelsbergh, M., & Woensel, T. V. (2016). 50th anniversary invited article - city logistics: Challenges and opportunities. *Transp. Sci.*, *50*(2), 579-590.

Schittekat, P., Kinable, J., Sörensen, K., Sevaux, M., Spieksma, F., & Springael, J. (2013). A metaheuristic for the school bus routing problem with bus stop selection. *European Journal of Operational Research*, *229*(2), 518-528.

Schmidt, G., & Wilhelm, W. E. (2000). Strategic, tactical and operational decisions in multinational logistics networks: a review and discussion of modelling issues. *International Journal of Production Research*, *38*(7), 1501–1523.

Schneider, M., & Drexl, M. (2017). A survey of the standard location-routing problem. *Annals of Operations Research*, *259*(1-2), 389-414.

Transport Canada. (2018). *Transportation in canada 2017 overview report* (Tech. Rep.).

Webb, M. H. J. (1968). Cost functions in the location of depots for multiple-delivery journeys. *Operational Research Society*, *19*(3), 311-320.

Winkenbach, M., Kleindorfer, P., & Spinler, S. (2016). Enabling urban logistics services at *La Poste* through multi-echelon location-routing. *Transportation Science*, *50*(2), 520-540.

Wu, X., Nie, L., & Xu, M. (2017). Designing an integrated distribution system for catering services for high-speed railways: A three-echelon location routing model with tight time windows and time deadlines. *Transportation Research Part C*, *74*, 212-244.

Yang, P., & Zeng, L. (2018). Models and methods for two-echelon location routing problem with time constraints in city logistics. *Mathematical Problems in Engineering*, 1-13.

Zhao, Q., Wang, W., & De Souza, R. (2018). A heterogeneous fleet two-echelon capacitated location-routing model for joint delivery arising in city logistics. *International Journal of Production Research*, *56*(15), 5062-5080.