Late Again with Defiers

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LATE again, with defiers

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Abstract

We show that the Wald statistic still identifies a causal effect if instrument monotonicity is replaced by a weaker condition, which states that the potential propensities to be treated with or without the instrument should have the same distribution, conditional on potential outcomes. This holds for instance if the slippages between these potential propensities and the average propensity are independent of potential outcomes. In this framework, the Wald statistic identifies a LATE on a population which comprises both compliers and always takers.

1 Introduction

Since the seminal work of Imbens & Angrist (1994), the use of instruments for identifying causal effects has been thought of as depending on two crucial assumptions: random assignment and monotonicity. Random assignment states that the instrument is assigned to individuals independently of their potential outcomes and treatments. Monotonicity means that the effect of the instrument on the treatment should go in the same direction for all observations in the sample.

This latter condition may be problematic in some applications. Barua & Lang (2010) argue that quarter of birth, used as an instrument for school entry age, violates this monotonicity condition, because of heterogenous strategic behavior of parents when choosing the entry date at school of their child. Another potential example is the use of sibling-sex composition as an IV when studying the effect of childbearing on labor supply (see Angrist & Evans, 1998). In this paper, it appears that the share of parents who have a third child is 7

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percentage points higher when their first two children have the same sex than when they have different sex, which implies that some parents have a preference for diversity. But the authors also find that the share of parents who have a third child is approximately 1.5 percentage points higher when their first two children are girls than when they are boys, which implies that some parents have a preference for boys. This suggests that monotonicity might be violated. Parents with a preference for diversity will be more likely to have a third child if their first two children are males, whereas parents with a preference for males might decide not to have a third child in such circumstances.

The aim of this note is to show that actually, the Wald statistic still identifies a local average treatment effect (LATE) under a weaker condition than instrument monotonicity. As shown by Vytlacil (2002), monotonicity is equivalent to assuming a single index model for selection into treatment. In this threshold model, the unobserved term does not depend on the instrument. This means that the "taste" for treatment of an individual should be the same whether he receives the instrument or not. This is equivalent to a rank invariance assumption: the rank of an individual in the distribution of taste for treatment should not change with the instrument. What we show is that we can actually make the taste for treatment depend on the instrument, which implies that there may be some defiers. However, the unobserved terms with or without the instrument should have the same marginal distribution, conditional on potential outcomes. In other words, we show that we can replace the rank invariance condition embedded in Vytlacil's model by a rank similarity condition (see Chernozhukov & Hansen, 2005).

This rank similarity condition is substantially weaker than monotonicity. Indeed, it will be verified if for each value of the instrument, individuals' departure from their mean taste for treatment is independent of their potential outcomes, conditional on their mean taste for treatment. In Angrist & Evans (1998)'s context, this basically means that conditional on the average propensity to have a third child, the specific preference of parents for boys should be independent of mother's participation to the labor market.

The issue of whether monotonicity is necessary for identifying local average treatment effects has received attention recently. Small & Tan (2007) replace monotonicity by the assumption that basically, there are more compliers than defiers. In such a case, the standard Wald parameter does not identify a causal effect anymore, but satisfies the no sign reversal property (namely, its sign would be positive if the treatment effect is positive with probability one). Klein (2010) considers "local" violation of monotonicity, and shows that the bias of the Wald parameter can be well approximated if such violations are small. Finally, Huber & Mellace (2012) show that it is possible to identify average treatment effects on compliers, defiers or both if monotonicity only holds conditional on potential outcomes. In contrast with these papers, we show here that it is possible to relax monotonicity while keeping a causal interpretation of the Wald parameter.

This note is organized as follows. In the second section, we introduce our assumption and discuss its link with monotonicity. In the third section, we prove the main result that the Wald parameter still identifies a LATE in our framework. Then, we give in the fourth section a necessary and sufficient condition for our rank similarity assumption to be rejected in the data.

2 Rank similarity versus monotonicity

Our framework is the same as the one of Imbens & Angrist (1994). Let $Y(0)$ and $Y(1)$ denote the potential outcomes with and without treatment. Let Z be a binary instrument which affects the treatment, and let $D(z)$ denote the potential treatment when $Z = z$. We only observe $D = D(Z)$ and $Y = Y(D)$. The first assumption in Imbens & Angrist (1994) is that the instrument is exogenous.

Assumption 2.1 *(Instrument exogeneity) We have* $(Y(0), Y(1), D(z)) \perp\!\!\!\perp Z$ *for* $z \in \{0, 1\}$.

Imbens & Angrist (1994) also suppose that the instrument has a monotonous effect on Z.

Assumption 2.2 *(Instrument monotonicity) Almost surely,* $D(1) \geq D(0)$ *or* $D(0) \geq D(1)$.

As shown by Vytlacil (2002), 2.1 and 2.2 are equivalent to the following threshold model:

Assumption 2.3 *(Threshold model with rank invariance) For* $z \in \{0, 1\}$*, there exists* $v(z) \in \mathbb{R}$ *and* V *a random variable such that* $D(z) = \mathbb{1}\{v(z) \leq V\}$ *almost surely and* $(Y(0), Y(1), V) \perp\!\!\!\perp Z$.

Instead of this threshold model with rank invariance, we consider here a threshold model where the propensity to be treated satisfies a rank similarity condition. We use hereafter the symbol $\sim$ to denote equality in distributions.

Assumption 2.4 *(Threshold model with rank similarity) For* $z \in \{0, 1\}$*, there exists* $v(z) \in \mathbb{R}$ *and* $V(z)$ *random variables such that* $D(z) = \mathbb{1}\{v(z) \leq V(z)\}$ *almost surely,* $(Y(0), Y(1), V(z)) \perp\!\!\!\perp Z$ *and* $V(0)|Y(0), Y(1) \sim V(1)|Y(0), Y(1)$.

Finally, we also consider the following assumption.

Assumption 2.5 *(Threshold model) For $z \in \{0,1\}$, there exists $v(z) \in \mathbb{R}$ and $V(z)$ random variables such that $D(z) = \mathbb{1}\{v(z) \leq V(z)\}$ almost surely, $(Y(0), Y(1), V(z)) \perp\!\!\!\perp Z$ and $(V(0), V(1))$ are exchangeable.*[1]

We prove the following representation result, which generalizes the one of Vytlacil (2002) and clarifies the relationship between all those Assumptions.

Proposition 2.1 *(i) Assumptions 2.1 and 2.5 are equivalent.*

(ii) Assumptions 2.1 and 2.2 together are equivalent to Assumption 2.3.

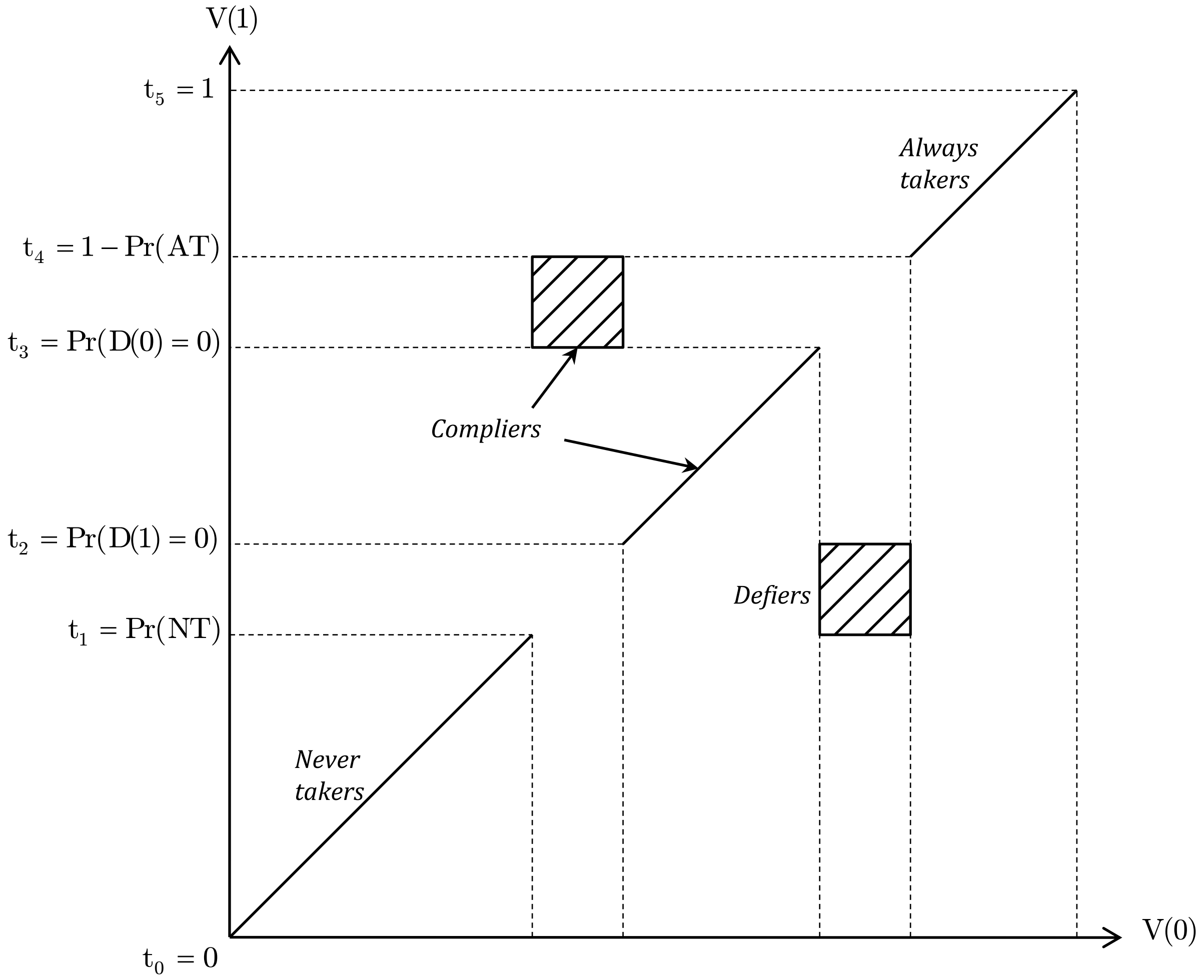
(iii) Assumption 2.3 $\Rightarrow$ Assumption 2.4 $\Rightarrow$ Assumption 2.5.

Proof: Let us show i). That Assumption 2.5 implies Assumption 2.1 is trivial. Now, assume that Assumption 2.1 holds. We have to build $(V(0), V(1))$ exchangeable and satisfying $D(z) = \mathbb{1}\{v(z) \leq V(z)\}$. Let $v(z) = P(D(z) = 0)$. We suppose without loss of generality that $P(D(0) = 0) \geq P(D(1) = 0)$. Otherwise what follows would still hold, simply by reverting $P(D(0) = 0)$ and $P(D(1) = 0)$. Since the shares of compliers and defiers are respectively equal to $P(D(0) = 0) - P(D(0) = 0, D(1) = 0)$ and $P(D(1) = 0) - P(D(0) = 0, D(1) = 0)$, this implies that there are more compliers than defiers. Before building formally the joint distribution of $V(1)$ and $V(0)$, let us consider Figure 1 which represents its support graphically. Defiers must verify $V(0) \geq P(D(0) = 0)$ and $V(1) < P(D(1) = 0)$ which means that part of the support of $(V(0), V(1))$ should be in $[t_3, 1] \times [0, t_2]$. Since $(V(0), V(1))$ should be exchangeable, this means that another part of the support of $(V(0), V(1))$ must lie in $[0, t_2] \times [t_3, 1]$. This corresponds to a population of compliers of same size than defiers. Finally, for all other observations (never takers, the remaining part of compliers, and defiers), we set $V(0) = V(1)$. Note that when there is no defier, $t_1 = t_2$ and $t_3 = t_4$, and we get $V(0) = V(1)$, as in Vytlacil (2002).

Formally, let t_i be as in Figure 1 and consider $(W_0, ..., W_4, B)$, mutually independent and also independent of $(Y(0), Y(1), D(0), D(1), Z)$. Take W_i uniform on $[t_i, t_{i+1}]$ and B Bernoulli with probability $(t_2 - t_1)/(t_3 - t_1)$. This ratio is simply the proportion of defiers divided by the proportion of compliers. Then let

$$\begin{aligned} V(0) &= (1-D(0))(1-D(1))W_0 + (1-D(0))D(1)(BW_1 + (1-B)W_2) + D(0)(1-D(1))W_3 \\ &\quad + D(0)D(1)W_4, \\ V(1) &= (1-D(0))(1-D(1))W_0 + (1-D(0))D(1)(BW_3 + (1-B)W_2) + D(0)(1-D(1))W_1 \\ &\quad + D(0)D(1)W_4. \end{aligned}$$

[1] $V(0)$ and $V(1)$ are exchangeable if and only if $(V(0), V(1))$ has the same distribution than $(V(1), V(0))$.



Note : the support of (V(0),V(1)) includes the thick lines and the dashed squares.
We let here Pr(NT) = Pr(D(0)=0, D(1)=0) and Pr(AT) = Pr(D(0)=1, D(1)=1).

Figure 1: Construction of $(V(0), V(1)$.

By construction, $D(z) = \mathbb{1}\{v(z) \leq V(z)\}$ almost surely, so it suffices to check that $(V(0), V(1))$ are exchangeable. This amounts to check that their joint density $f_{V(0),V(1)}$ is a symmetric function. For $x < y$, $f_{V(0),V(1)}(x, y) = 0$ except when $(x, y) \in [t_1, t_2] \times [t_3, t_4]$. In this latter case,

$$\begin{aligned} f_{V(0),V(1)}(x, y) &= P(D(0) = 0, D(1) = 1)P(B = 1)f_{W_1}(x)f_{W_3}(y) \\ &= P(D(0) = 1, D(1) = 0)f_{W_3}(y)f_{W_1}(x) \\ &= f_{V(0),V(1)}(y, x). \end{aligned}$$

This proves i).

ii) follows from Vytlacil (2002). Finally, let us prove iii). That Assumption 2.3 implies Assumption 2.4 is trivial. Assumption 2.4 implies Assumption 2.1 which is equivalent to Assumption 2.5. This completes the proof. □

Following Vytlacil (2002), the second part of Proposition 2.1 shows that Assumption 2.4 is weaker than exogeneity and monotonicity, since both together imply a threshold model with rank invariance. The first part of the lemma shows that compared to the exogeneity assumption alone, we only add the fact that $V(0)$ and $V(1)$ should have the same distribution *conditional on potential outcomes*, not only marginally. A sufficient condition for this to hold is that departures from mean taste for treatment induced by the instrument should be independent of potential outcomes, as the following proposition shows.

Proposition 2.2 *Suppose that Assumption 2.5 holds. Let* $\overline{V} = (V(0) + V(1))/2$, $\varepsilon(z) = V(z) - \overline{V}$, *and assume that for every* $z \in \{0; 1\}$,

$$\varepsilon(z) \perp\!\!\!\perp (Y(0), Y(1))|\overline{V}. \tag{2.1}$$

Then Assumption 2.4 holds.

Proof: for any random variables S and T, let $f_{S|T}$ denote the density of S conditional on T with respect to an appropriate measure. Let $P^{\overline{V}|Y(0),Y(1)}$ denote the probability measure of $\overline{V}$ conditional on $Y(0)$ and $Y(1)$. We have

$$\begin{aligned} f_{V(0)|Y(0),Y(1)}(v|y_0,y_1) &= \int f_{V(0)|Y(0),Y(1),\overline{V}}(v|y_0,y_1,\overline{v})dP^{\overline{V}|Y(0),Y(1)}(\overline{v}|y_0,y_1) \\ &= \int f_{V(0)|\overline{V}}(v|\overline{v})dP^{\overline{V}|Y(0),Y(1)}(\overline{v}|y_0,y_1) \\ &= \int f_{V(1)|\overline{V}}(v|\overline{v})dP^{\overline{V}|Y(0),Y(1)}(\overline{v}|y_0,y_1) \\ &= \int f_{V(1)|Y(0),Y(1),\overline{V}}(v|y_0,y_1,\overline{v})dP^{\overline{V}|Y(0),Y(1)}(\overline{v}|y_0,y_1) \\ &= f_{V(1)|Y(0),Y(1)}(v|y_0,y_1). \end{aligned}$$

The second equality holds because of (2.1). The third follows from the fact that $V(0)$ and $V(1)$ are exchangeable and $\overline{V}$ is symmetric in $V(0)$ and $V(1)$ □

Proposition 2.2 shows that it is sufficient to assume that conditional on the mean taste for treatment ($\overline{V}$), departures from mean taste for treatment induced by the instrument ($V(z) - \overline{V}$) are independent of potential outcomes, and that Assumptions 2.1 holds, to obtain our rank similarity condition. Note that Condition 2.1 trivially holds under monotonicity, because in this case we can choose $V(0) = V(1)$, yielding $V(1) - \overline{V} = 0$.

To illustrate condition (2.1), consider the example of Angrist & Evans (1998). Treatment is having a third child or not, while the outcome is participation to the labor market. To simplify the discussion, consider only couples whose first two children are boys, and those whose first two children have different sex. Therefore, the instrument Z is equal to 1 if the first two children are boys and to 0 if they have different sex. Taste for a third child might depend on the instrument because some parents may have preferences for boys or girls. In this context, $\varepsilon(z)$ can be interpreted as the departure from the average propensity to have a third child induced by preference for boys. For instance, couples who have a strong preference for boys will be less prone to have a third child when $Z = 1$, so that their $\varepsilon(1)$ will be negative. Assumption 2.4 holds if this departure from the average propensity to have a third child is independent of potential participations to the labor market, conditional on the average propensity to have a third child. This is still a strong assumption, but, in this particular context, it is more credible than the standard monotonicity assumption.

3 Identification of treatment effects under similarity

We show now that under Assumption 2.4, the Wald parameter still identifies a LATE within the subpopulation satisfying $V(1) \in [v(1), v(0))$. It comprises some compliers but also some always takers, because under Assumption 2.4 we can have simultaneously $V(1) \in [v(1), v(0))$ and $V(0) \geq v(0))$. Of course, it reduces to compliers under the rank invariance condition $V(1) = V(0)$. We also obtain a result on the distribution functions of potential outcomes within this subpopulation which generalizes the result obtained in Imbens & Rubin (1997) under monotonicity. We suppose hereafter that $P(D = 1|Z = 1) > P(D = 1|Z = 0)$ but the result also holds when $P(D = 1|Z = 1) < P(D = 1|Z = 0)$, by simply reverting $v(0)$ and $v(1)$ in (3.1) and (3.2).

Theorem 3.1 *Suppose that Assumption 2.4 holds and* $P(D = 1|Z = 1) > P(D = 1|Z = 0)$. *Then*

$$E\left[Y(1) - Y(0)|V(1) \in [v(1), v(0))\right] = \frac{E(Y|Z = 1) - E(Y|Z = 0)}{E(D|Z = 1) - E(D|Z = 0)} \quad (3.1)$$

$$F_{Y(d)|V_1 \in [v(1), v(0))}(y) = \frac{P(Y \leq y, D = d|Z = 1) - P(Y \leq y, D = d|Z = 0)}{P(D = d|Z = 1) - P(D = d|Z = 0)} \quad (3.2)$$

Proof: first, remark that for $z \in \{0,1\}$,

$$P(D=1|Z=z) = P(D(z)=1|Z=z) = P(V(z) \geq v(z)) = P(V(1) \geq v(z)),$$

where the second equality stems from the threshold model and $V(z) \perp\!\!\!\perp Z$, and the third follows by rank similarity. Thus, $P(D=1|Z=1) > P(D=1|Z=0)$ implies that $v(1) < v(0)$. Moreover,

$$\begin{aligned} E(D|Z=1) - E(D|Z=0) &= P(D=1|Z=1) - P(D=1|Z=0) \\ &= P(V(1) \in [v(1), v(0))). \end{aligned} \quad (3.3)$$

Similarly,

$$\begin{aligned} E(Y|Z=1) - E(Y|Z=0) &= E(Y(\mathbb{1}\{v(1) \leq V(1)\} + \mathbb{1}\{v(1) > V(1)\})|Z=1) \\ &\quad -E(Y(\mathbb{1}\{v(0) \leq V(0)\} + \mathbb{1}\{v(0) > V(0)\})|Z=0) \\ &= E(Y(1)\mathbb{1}\{v(1) \leq V(1)\}) + E(Y(0)\mathbb{1}\{v(1) > V(1)\}) \\ &\quad -E(Y(1)\mathbb{1}\{v(0) \leq V(0)\}) - E(Y(0)\mathbb{1}\{v(0) > V(0)\})) \\ &= E(Y(1)\mathbb{1}\{v(1) \leq V(1)\}) + E(Y(0)\mathbb{1}\{v(1) > V(1)\}) \\ &\quad -E(Y(1)\mathbb{1}\{v(0) \leq V(1)\}) - E(Y(0)\mathbb{1}\{v(0) > V(1)\})) \\ &= E\left[(Y(1) - Y(0))\mathbb{1}\{V(1) \in [v(1), v(0))\}\right], \end{aligned} \quad (3.4)$$

where the second equality follows by independence, the third by rank similarity and the fourth by simply gathering the terms. The first result follows by combining (3.3) and (3.4).

We now turn to (3.2). We prove the result for $d=1$ only, the reasoning being identical for $d=0$. We have

$$\begin{aligned} P(Y \leq y, D=1|Z=1) &= P(Y(1) \leq y, V(1) \geq v(1)|Z=1) \\ &= P(Y(1) \leq y, V(1) \geq v(1)) \\ &= P(Y(1) \leq y, V(1) \geq v(0)) + P(Y(1) \leq y, V(1) \in [v(1), v(0))) \\ &= P(Y(1) \leq y, V(0) \geq v(0)) + P(Y(1) \leq y, V(1) \in [v(1), v(0))) \\ &= P(Y(1) \leq y, V(0) \geq v(0)|Z=0) + P(Y(1) \leq y, V(1) \in [v(1), v(0))) \\ &= P(Y \leq y, D=1|Z=0) + P(Y(1) \leq y, V(1) \in [v(1), v(0))), \end{aligned}$$

where the first equality follows by the threshold model, the second by independence, the fourth by rank similarity and the fifth by independence again. Equation (3.2) follows using $P(Y(1) \leq y, V(1) \in [v(1), v(0))) = F_{Y(d)|V_1 \in [v(1), v(0))}(y) P(V_1 \in [v(1), v(0)))$ and Equation (3.3) $\square$

4 Testability

As shown in Kitagawa (2008), Assumptions 2.1 and 2.2 together are testable. Equation (3.2) shows that similarly, Assumption 2.4 is also testable: the right-hand side of (3.2) should be increasing, as a cumulative distribution function. The next theorem strengthens this idea, by showing that basically this is the only testable implication of Assumption 2.4.

Theorem 4.1 *The three following statements are equivalent:*

(i) Assumption 2.4 can be rationalized by the data;

(ii) Assumptions 2.1 and 2.2 together can be rationalized by the data;

(iii) $y \mapsto P(Y \leq y, D = 1|Z = 1) - P(Y \leq y, D = 1|Z = 0)$ *and* $y \mapsto P(Y \leq y, D = 0|Z = 0) - P(Y \leq y, D = 0|Z = 1)$ *are increasing.*

Proof: because Assumptions 2.1 and 2.2 together imply Assumption 2.4, (ii) $\Rightarrow$ (i). By the previous theorem, (i) $\Rightarrow$ (iii). Thus it suffices to prove that (iii) $\Rightarrow$ (ii). For any Borel set A, let $P_d(A) = P(Y \in A, D = d|Z = 1)$ and $Q_d(A) = P(Y \in A, D = d|Z = 0)$. (iii) implies that $P_1((y', y]) \geq Q_1((y', y])$ for any $y' < y$. Because P_1 and Q_1 are positive measures, this implies that $P_1(A) \geq Q_1(A)$ for any Borel set A. Similarly, $P_0(A) \leq Q_0(A)$ for all A. The result follows then by Proposition 1 of Kitagawa (2008) □

This result shows that DGP rejecting Assumption 2.4 are the same as those rejecting Assumptions 2.1 and 2.2 together: those assumptions are observationally equivalent.

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