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Comparison Tests of the Column Coefficient and the Gravity Coefficient Models

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COMPARISON TESTS OF THE COLUMN COEFFICIENT AND THE GRAVITY COEFFICIENT MODELS

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by

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16. Abstract

This study presents a comparison of the 1947, 1958, 1963, 1970, and 1980 regional outputs estimated using the column coefficient and gravity coefficient models. An evaluation is made of the accuracy of the two models in estimating 1947, 1958, and 1963 outputs. A detailed review of the industrial and regional estimation errors is provided by the authors, supplemented by appendix tables showing the errors for each of 10 industries in each of 9 regions.

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Preface

This report is one of a series being written for the University Research Program, U.S. Department of Transportation, to present analyses of the results obtained using the multiregional input-output (MRIO) model for the United States. An original set of 21 reports prepared for the Economic Development Administration, U.S. Department of Commerce, contained explanations of the methodology used for assembling the MRIO data and of the procedures employed to implement the model. Most of those reports have now been rewritten for publication by Lexington Books, D.C. Heath and Company, Lexington, Massachusetts, in a set of six volumes entitled Multiregional Input-Output Analysis. Five of the six volumes are now available.

The MRIO data have been assembled in a general form, so they can be used with either the column coefficient, gravity coefficient, or other multiregional models. In the present report, Zdenek Fencl and Nathaniel Ng have compared the 1947, 1958, 1963, 1970, and 1980 regional outputs estimated using the two fixed coefficient models and have evaluated the accuracy of the two models in estimating the 1947, 1958, and 1963 outputs. This is the first time that results have been published on the accuracy of the two models in backcasting to 1947 and 1958 using the American data. The results show that the outputs estimated for these two years using the two models have about the same degree of accuracy. In the paper, a detailed review of the industrial and regional estimation errors is provided by the authors, supplemented by appendix tables showing the errors for each of 10 industries in each of 9 regions. The 9 census regions were chosen for this testing because many of the MRIO data were originally disaggregated from data provided in the census publications for the 9 regions, and because testing at a more detailed level of industrial and regional classification would have been too expensive. (In most cases, data from the 9 census regions were used as control totals for the MRIO data assembly.)

This study has indicated a large number of other tests that should be done in order to better determine the accuracy and validity of the two models, as well as the stability of the trade and technical coefficients. Constructive criticism of the material presented in the report would be appreciated.

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April 1974

COMPARISON TESTS OF THE COLUMN COEFFICIENT AND THE GRAVITY COEFFICIENT MODELS

The purpose of this report is to compare the accuracy of the outputs calculated using the column coefficient model and the gravity coefficient model. In the first part of the report, a brief review is given of the two multiregional input-output models, and the data required for implementing the models are explained. In the second part of the report, comparisons of the two models are made in terms of the accuracy of the outputs estimated for the base-year, 1963, and the backcasts of outputs that were made for 1947 and 1958. In addition, the rates of growth of outputs from 1947 to 1958, 1963, 1970, and 1980 are compared for the two models. Most of the comparisons are made at the 10 industry, 9 region, rather than at the 79 industry, 51 region, level of aggregation.[1] The industrial and regional classification schemes are given in the appendix, Tables A-1 and A-2, respectively.

MULTIREGIONAL INPUT-OUTPUT MODELS

Multiregional input-output models are essentially conventional input-output models modified to incorporate interregional trade.[2] Presently, three such models are being tested for use in the United

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[1]The 51 regions are the 50 states plus the District of Columbia.

[2]The reader who is not familiar with multiregional input-output models is advised to refer to Yan [12] for a detailed analysis of national input-output models and to Miernyk [5] for an introduction to regional input-output models. More advanced material on the models can be found in Polenske [6;7;8].

States, namely, the column coefficient, row coefficient, and gravity coefficient. All of these are formulated from one basic economic principle: the total output of an industry is equal to the sum of intermediate demands by various industries (including the industry itself) and demands by final users for that industry's products. Mathematically, this relationship can be expressed as:

$$x = t + y \tag{1}$$

where

x = total output of the industry,

t = intermediate demands for the industry's products,

y = final demands for the industry's products.

Equation (1), however, represents only one industry in one region, and therefore cannot be directly applied to the input-output models. With the other industries in the same region being taken into consideration, and assuming that there is no trade among the regions, an input-output model for m industries and n regions can be represented by the following equation:

$$x_i^{og} = \sum_{j=1}^{m} a_{ij}^{g} x_j^{go} + y_i^{g} \tag{2}$$

where

a_{ij}^{g} = technical coefficient--the amount of input of commodity i required by industry j located in region g to produce one unit of output of commodity j.

x_i^{og} = total consumption--the total amount of commodity i supplied by region g.

x_j^{go} = total production--the total amount of commodity j produced in region g.

y_i^g = final demand--the total amount of commodity i demanded by final users in region g.

i, j = 1, 2, ... , m.

g = 1, 2, ... , n.

For equation (2) to be used to describe a multiregional model, it must be further modified to take into account the amounts of commodities traded among the various regions. Conventionally, the amount of a commodity produced by an industry in one region but consumed in another is considered as part of the output of the industry in the producing region. Because each of the multiregional models has a different accounting scheme for interregional trade, the methods used to develop the column coefficient and the gravity coefficient models will be described separately in the following two sections.[3]

Column Coefficient Model

The column coefficient model uses the following relationship for interregional trade:

$$x_i^{gh} = c_i^{gh} x_i^{oh} \qquad (3)$$

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[3]The reader who desires a more detailed description of the accounting frameworks should refer to Polenske [7]. Testing of the row coefficient model has not been completed yet and will be discussed in a later report.

where

x_i^{gh} = amount of commodity i produced in region g that is shipped to region h.

x_i^{oh} = total amount of commodity i that is consumed in region h.

c_i^{gh} = a trade parameter, indicating the fraction of total consumption of commodity i in region h that is shipped from region g.

Equations (2) and (3) can be combined and transformed to obtain:

$$C^{-1}X = \hat{A}X + Y$$

or

$$X = C(\hat{A}X + Y) \qquad (4)$$

where

X = vector of total outputs, nm·1.

C = matrix of regional trade coefficients, nm·nm, with each of the diagonals of the n·n block containing the coefficients for m traded commodities and all off-diagonal elements set to zero.

$\hat{A}$ = matrix of regional technical coefficients, nm·nm, with each of the n blocks along the main diagonal containing the m·m coefficient matrix derived from each of the n regional input-output tables. The elements in all blocks off the main diagonal are set to zero.

Y = vector of final demands, nm·1, with each element representing the amount of the product of industry m (including that portion that is produced in and shipped from the other regions) demanded by final users in region n.

It is obvious from equation (4) that a number of tests can be made to verify the accuracy of the model, provided that the required data are available. If all of the matrices, X, $\hat{A}$, C, and Y are available or can somehow be estimated from actual statistics, any three of them can be substituted in equation (4) to calculate the remaining one. Then the result obtained, when compared with the actual data, will give an indication of the validity of the model. Unfortunately, the data necessary to assemble the technology and trade coefficient matrices are available only for the year 1963, although both the final demand and total output figures are available for the years 1947, 1958, and 1963. In the present study, the total outputs for each year were therefore calculated from the final demands, using the 1963 technology and trade matrices, and were compared with the actual output data. Equation (4) can be rearranged as:

$$X = C\hat{A}X + CY$$

$$X - C\hat{A}X = CY$$

$$(I - C\hat{A})X = CY$$

$$X = (I - C\hat{A})^{-1} CY \qquad (5)$$

In order to calculate the regional outputs, X, from equation (5), the matrices $\hat{A}$ and C and the vector Y must first be obtained. The procedures used to assemble them will be explained in detail later in this report. This straightforward procedure for calculating X analytically, however, can only be used when there is a small number of industries and regions, because large-scale matrix inversion and multiplication are very costly even when done on the fastest computer available. At the full-scale industrial and regional classification--

79 industries and 51 regions--both the $\hat{A}$ and C matrices will be 4029 by 4029, and inverting or multiplying such large matrices is no easy task. Therefore, testing the model at this level, instead of using the analytical procedure, utilizes a numerical method based upon the following approximation derived from equation (5):

Since, for any diagonally dominant matrix (I - B),

$$(I - B)^{-1} = I + B + B^2 + B^3 + \ldots$$

$$\therefore (I - C\hat{A})^{-1} = I + C\hat{A} + (C\hat{A})^2 + (C\hat{A})^3 + \ldots$$

$$X = [I + C\hat{A} + (C\hat{A})^2 + (C\hat{A})^3 + \ldots]CY \qquad (6)$$

(The number of terms to be enclosed in the brackets is determined by the precision of the inverse required.)

Although equation (6) is already simpler than equation (5), it is still difficult to multiply a matrix dimensioned 4029x4029 by a vector of 4029x1. To solve this problem, Y is converted back into its original form, which is a matrix of m·n, and the A matrix is no longer nm·nm but simply m·m, with each element being a technical coefficient. Similarly, the C matrix is now n·n with each element being a trade coefficient. Since these matrix dimensions do not conform to the rule of matrix multiplication, equation (6) can no longer be used, but, instead, the appropriate matrix elements must be multiplied together one by one and then summed.

A computer program written in CDC Fortran IV has been set up to calculate the outputs, X (which is now an m·n matrix), at the full-scale level (that is, 79x51). (See Appendix G.) The iterative steps performed by the program to calculate X can be summarized as follows:

Step 1. The elements in the first row of Y (79x51) are multiplied by the corresponding elements in the first row of C (51x51) and the products summed to obtain the row 1, column 1, element of X (79x51). Then the same Y elements are multiplied by the corresponding elements in the second row of C and the products summed to obtain the row 1, column 2, element of X, and so on for all the 51 elements in the first row of X. Altogether, there are 64 C matrices for the first 64 traded commodities,[4] and the second row of X is obtained by multiplying the elements in the second row of Y by the corresponding elements in the second C matrix in the same way as described above; the third row of X is obtained by multiplying the elements in the third row of Y by the corresponding elements in the third C matrix; and so on, until the first 64 rows of X are filled. Then the rest of the elements in X are simply set equal to the corresponding elements in Y, since commodities 65 through 79 are not traded.

Step 2. The elements in the first column of the resulting matrix (X) from Step 1 (or Step 3 after the first iteration) are multiplied term by term by the elements in the first row of A (79x79) and the products summed to obtain the row 1, column 1, element of a new matrix, called X' (79x51). Then the same elements of X are multiplied by the elements in the second row of A and the products summed to obtain the row 2, column 1, elements of X'; the same elements of X are multiplied

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[4] Only 61 of the first 64 commodities are traded. All elements in the trade matrices for the other 3 commodities appear on the diagonal of the trade matrix.

by the elements in the third row of A and the products summed to obtain the row 3, column 1, element of X'; and so on, for all the 79 elements in the first column of X'. Altogether, there are 51 A matrices for all the 51 regions, and the second column of X' is obtained by similarly multiplying the second column of X by the second A matrix; the third column of X' is obtained from multiplying the third column of X by the third A matrix; and so on, for all the 51 columns of X'.

Step 3. The resulting matrix from Step 2 (X') is multiplied in exactly the same way by the C matrices, as in Step 1, to obtain a new matrix (79x51).

Step 4. The resulting matrix of Step 1 (or the previous Step 4 after the first iteration) is added to that of Step 3.

Step 5. The result of Step 4 is then compared with that of Step 1 (or the previous Step 4 after the first iteration). If the maximum relative change in any of the industry totals in the output matrix is less than a predetermined maximum allowance (which is presently set at 0.0005), the resulting output matrix is considered to be sufficiently accurate, and the procedure is stopped. Otherwise, the iteration continues with Step 2 for a maximum of 15 times.

Gravity Coefficient Model

The gravity coefficient model is a multiregional input-output model, originated by Leontief and Strout, to reflect the production and consumption relationships between commodities produced and consumed in different regions [3]. The complexity of the gravity model comes from the fact that, generally, no exhaustive data are available for all trade-flow combinations between commodities and regions that would lead to a simple calculation of the total multiregional production by means of the conventional input-output model. In other words, although the conventional single-region input-output model is provided with trade flows (or technical coefficients) from industry to industry, complete trade data (such as the trade flow from industry 3 of region 2 to industry 4 of region 1) do not exist. The problem of nonexistence of exhaustive data is, however, common to all of the three multiregional models.

Introducing the interregional trade flows into the model through the trade coefficients or parameters, the gravity coefficient model defines the following:

$$x_i^{gh} = \frac{x_i^{go} \, x_i^{oh}}{x_i^{oo}} \, q_i^{gh} \qquad (7)$$

where

x_i^{gh} = interregional trade flow--the amount of commodity i produced in region g that is shipped to region h.

x_i^{oo} = the total amount of commodity i produced (consumed) in all regions.

q_i^{gh} = a trade parameter, which is a function of the cost of transferring commodity i from region g to region h (where transfer costs reflect various factors, including transportation costs, that determine interregional trade).

The interregional trade-flow, x_i^{gh}, is a function of the production in region g, consumption in region h, national aggregate production (or consumption) of the commodity, and the trade parameter, q_i^{gh}. From the definition of the production, consumption, and national aggregate, the following equations hold:

$$x_i^{go} = \sum_{h=1}^{n} x_i^{gh} \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ g = 1, 2, \ldots, n \end{array} \tag{8}$$

$$x_i^{oh} = \sum_{g=1}^{n} x_i^{gh} \qquad h = 1, 2, \ldots, n \tag{9}$$

$$\sum_{g=1}^{n} \sum_{h=1}^{n} x_i^{gh} = \sum_{g=1}^{n} x_i^{go} = \sum_{h=1}^{n} x_i^{oh} = x_i^{oo} \tag{10}$$

From equation (2), on page 2, and equations (7) through (10) the following can be derived:

$$x_i^{go} \sum_{r=1}^{n} x_i^{or}(1 - q_i^{gr}) = x_i^{og} \sum_{r=1}^{n} x_i^{ro}(1 - q_i^{rg})$$

where

$$i = 1, 2, \ldots, m$$

$$g = 1, 2, \ldots, n$$

$$q_i^{gg} = 0$$

This is a set of nonlinear equations that can be linearized by a first-order approximation. (See Leontief and Strout [3] and Polenske [6] for details.) Using a reduced form, the basic gravity multiregional system of linear equations is:

$$T'X = S(\hat{A}X + Y) \tag{11}$$

where

X = vector of total outputs, nm·1.

Y = vector of final demands, nm·1.

$\hat{A}$ = technical coefficient matrix nm·nm, with the regional technical coefficients on its diagonal.

S, T' = the parameter matrices with the parameters (non-zero elements) on the diagonal of each regional matrix.

The elements of the matrices S and T are calculated as follows:

$$s_i^{gh} = x_i^{go}(1 - q_i^{gh})$$

$$t_i^{gh} = x_i^{oh}(1 - q_i^{gh})$$

where the trade coefficients q_i's are calculated from equation (7).[5] The forms suitable for computations are as follows:

$$(T' - S\hat{A})X = SY$$

$$X = (T' - S\hat{A})^{-1}\ SY$$

$$X = (S^{-1}T - \hat{A})^{-1}\ Y$$

or $$X = (G - \hat{A})^{-1}\ Y \qquad (12)$$

where

$$G = S^{-1}T'$$

For the multiregional model comparisons, equation (12) was used.

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[5] It should be noted that since the elements s_i^{gh} and t_i^{gh} are in the range of $(1, 10^8)$ or $(-10^4, -10^8)$, the gravity model calculations at any level of aggregation should be done in double-precision on the IBM 370/165.

The total production, X, can be computed either by the product of the inverse of the matrix $(G - \hat{A})$ and the vector of final demands, Y, or by the following simple iterative method:

$$X = Y + (I - F)Y + (I - F)^2 Y + \ldots$$

where

$$F = (G - \hat{A})$$

However, since the sufficient condition for convergence of this method (that is, the matrix F must be diagonally dominant) cannot generally be fulfilled for a gravity model, and the calculation of the necessary condition for convergence is as difficult as the solution of the model itself, the direct method--the inversion of the matrix $(G - \hat{A})$, as shown in equation (12), was used.

In the past, several attempts have been made to implement an iterative method for the solution of the gravity model, because an iterative method can be ten to one hundred times faster than the direct method. These time- and cost-differences mean that the iterative method, though not an exact method, has a strong advantage over the direct method despite the fact that the inverse matrix $(G - \hat{A})^{-1}$ in the direct method needs to be computed only once, and different solutions can be provided by multiplying the matrix $(G - \hat{A})^{-1}$ by different vectors of final demands. Even the fact that the matrix multiplier $(G - \hat{A})^{-1}$ can be used for unit production and consumption analysis may not be a strong argument for the direct method, since the matrix multiplier has to be recalculated whenever technology changes. Moreover, the interpretation of the matrix multiplier $(G - \hat{A})^{-1}$ for the gravity model is different from that of the conventional input-output matrix multiplier $(I - A)^{-1}$.

These considerations, together with the fact that inversion of the matrix $(G - \hat{A})$ of the model with 79 industries and 51 regions, which is 4029x4029, would take over 100 hours on the IBM 370/165,[6] have led to thorough investigations of iterative methods and the computational characteristics of the gravity model. A historical survey of these investigations and the reports on the latest research are contained in a report by Fencl [2]. So far, no iterative method has been found that would converge under the given properties of the matrix $(G - \hat{A})$, nor has any method been found that would modify or restructure the matrix $(G - \hat{A})$ to fulfill the conditions of convergence. Even though some gravity model calculations at an aggregated level were done by an iterative method (the Japanese data for 9 regions and 10 industries and for 9 regions and 24 industries), the problem of the solution of very large gravity models will continue to exist unless an iterative or fast method that takes advantage of the special structure of the gravity model can be found.

During the computation of the inverse matrix S^{-1} for the present study, an overflow situation occurred several times. This was apparently caused by the large range of the elements of the matrix S. When the matrix S was scaled down before and after the inversion by multi-

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[6] A detailed study of the hours and cost is contained in a report by Luft [4]. That study was extended in another report by Cohen, Solenberger, and Tucker [1].

plying it by a scaling factor, 0.0001, for example, underflows were encountered, but after further adjustments of the scaling factor there was only one underflow. No further experiments with scaling the S matrix were undertaken. Judging from the results shown in Table D-3 in the appendix, which gives one set of results using a scaling factor, the underflows apparently did not affect the accuracy of the computation, especially since the results obtained using different scaling factors were identical.

Data Assembly

As was mentioned earlier, because the input-output tables, actual trade flows, secondary transfers-out (STRO), and service industries residual (SIR) data were available only for the year 1963, they were used, together with total final demands for each year, to calculate the the industry outputs for 1947, 1958, 1963, 1970, and 1980 in all of the comparison tests. Two sets of data were used in the model-comparison tests--79 industries and 51 regions and 10 industries and 9 regions. Most of the tests and results, however, were based upon the second set, the aggregated version, because any detailed analysis of a large data base, such as 79x51, is extremely expensive and difficult.

Technology and Trade Coefficients

The technical coefficient matrix, $\hat{A}$, and trade coefficient matrix, C, were assembled from the 1963 input-output tables and adjusted trade flows, respectively, in order to calculate the regional outputs, X. Matrix $\hat{A}$ was obtained by dividing each column of each regional input-output table by the column total and placing these matrices of technical coefficients as blocks along the main diagonal of one large matrix, as shown in Figure 1. For the 10x9 set of calculations, the input-output

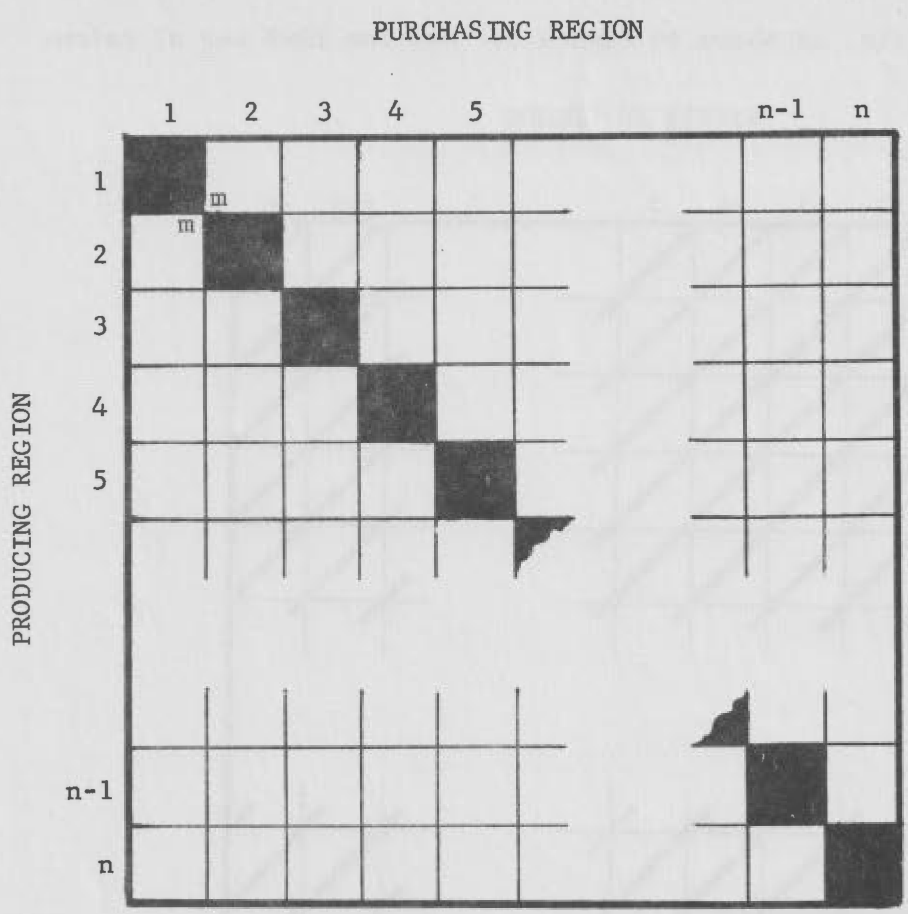

Figure 1. Interindustry Technical Coefficient Matrix

tables were aggregated from 51 matrices (79x79) to 9 matrices (10x10) before the technology coefficients were calculated. Because the input-output tables contain a double-counting of the secondary products, steps must be taken during the aggregation process to eliminate unnecessary double-counting.[7]

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[7] This procedure is explained in the volume by Polenske and others entitled State Estimates of Technology, 1963 [10, pp. 19-21]

Matrix C was obtained by dividing each column (hence the name, "column coefficient model") of each commodity trade-flow matrix by the column total and placing the coefficients along the diagonals of a large square matrix, as shown in Figure 2. For the 10x9 set of calcu-

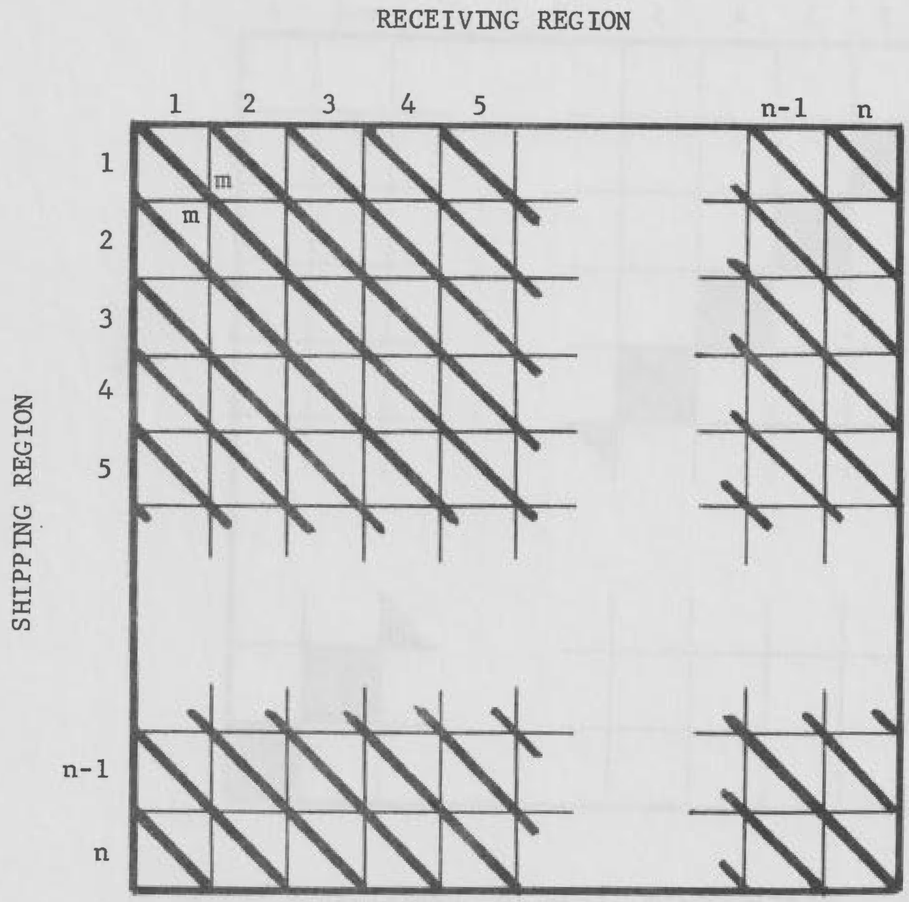

Figure 2. Multiregional Input-Output Trade Matrix

lations, the trade-flow matrices were aggregated from 61 matrices (51x51) to 8 matrices (9x9) before the trade coefficients were calculated. The aggregated commodity trade-flow matrix used to calculate the C matrix had to be adjusted so as to be consistent with the regional input-output tables. The adjustment procedure is described in detail in the MRIO guide [9, pp. 31-37].

Total Final Demands

To obtain appropriate final demands for the model comparisons, four adjustments were made to the six final demand components for the years 1947 and 1958.

(1) No net foreign exports (by port of exit) figures were available for years other than 1963. The necessary 1947 and 1958 export figures were therefore estimated from the 1963 exports (by port of exit) by dividing the figures in each row (industry) in that matrix by the row sum (industry total) and then multiplying these coefficients by the 1947 and 1958 net foreign exports by state of production. This estimation procedure is based on the assumption that the regional exports by port of exit remained in the same proportions to the industry totals in the period 1947-1963.

(2) No final demand components had been assembled for Alaska and Hawaii for years other than 1963. They were therefore estimated from 1963 figures by assuming that the percentage of final demand in each of the two states with respect to the rest of the United States remained constant throughout the period under study. Thus, the row coefficients (obtained by dividing the 1963 row totals minus the figures for Alaska and Hawaii by the entries for Alaska and Hawaii for each 1963 final demand component) were used to multiply the row totals of 1947 and 1958 final demand components (without Alaska and Hawaii) to obtain the 1947 and 1958 entries for Alaska and Hawaii (columns 50 and 51).

(3) In order to make the state final demands consistent with the state output data, two industries, IO-74, Research & development, and

IO-81, Business, travel, entertainment, & gifts, were eliminated from the final demand components by distributing the entries in rows 74 and 81 to the other industries. To distribute the data, columns 74 and 81 were taken from the national input-output transfer matrix [11], and the figures in each column were divided by the column total to obtain two vectors of coefficients. These were used as proportions to distribute each element in rows 74 and 81 to the elements in each respective column. The values in the original rows 74 and 81 were then set to zero.

(4) The adjusted component figures were deflated to 1963 dollars by first deflating the 1947 figures to 1958 dollars, then from 1958 dollars to 1963 dollars. For 1947, deflators were available only for the total final demand, while for 1958, they were available for each of the six components [9 , pp. 126-128]. This deflation was necessary in order that the 1963 technology, trade, STRO, and SIR matrices (all in 1963 dollars) could be used to calculate the outputs.

After these adjustments had been made to the 1947 and 1958 data, the six components were summed for each of the five years (1947, 1958, 1963, 1970, and 1980), and the 1963 STRO and SIR data were added to obtain the total final demand, Y, for each respective year.

Total Outputs

Three adjustments had to be made to the output figures for 1947 and 1958 before they could be used for comparison. The first two, similar to the second and fourth adjustments described for the total final demands, were as follows:

(1) Exactly the same adjustment as in (2) of total final demands was made to obtain the 1947 and 1958 output estimates for Alaska and Hawaii, except that establishment output data [10, Appendix D] were used instead of final demand data.

(2) The 1947 and 1958 data in current values were deflated to 1963 dollars using the appropriate output deflators [9, pp. 149-151].

(3) After these adjustments had been completed, a final adjustment was made to convert the 1947 and 1958 data, which are establishment outputs and therefore do not include secondary transfers-in and imports, into production outputs. The ratio of 1963 production output to 1963 establishment output was multiplied by the 1947 and 1958 establishment outputs [10, Appendix D] to obtain estimates of 1947 and 1958 production outputs, respectively.

All of the output and final demand matrices contain 87 industries and 52 regions (50 states and the District of Columbia, plus other U.S. posessions), but in calculating the outputs, only the first 79 industries and the first 51 regions were used.

METHODS OF MODEL COMPARISON

The validity and accuracy of the column coefficient and gravity coefficient models, which are very useful tools for projecting and backcasting regional and industrial outputs, were tested by comparing the output figures obtained from each of the models with the actual figures at the aggregated level (10 industries and 9 regions). The 1970 and 1980 estimated outputs were used only for comparing changes in the industrial rates of growth, because no actual output figures were available to use in comparing the accuracy of the estimated outputs for the two years. The column coefficient model was also further studied and the results analyzed through the use of the full-scale (79 industries and 51 regions) data, but due to the difficulty caused by the nonconvergence of the S matrix at the disaggregated level, only the aggregated data were used to study the gravity coefficient model. Nevertheless, some simple comparison tests were applied to the full-scale data, and the methods used for both sets of data are described in the following two sections.

<u>Aggregated Level (10 Industries, 9 Regions)</u>

For each of the comparison years, 1947, 1958, and 1963, the full-scale total final demand matrix was aggregated to 10x9 using the industrial and regional classification schemes given in Tables A-1 and A-2 in the appendix. Then the aggregated and appropriately adjusted 10x9 secondary transfers-out and service industries residual matrices for 1963 were added to the aggregated total final demand matrix, which was then transformed into a 90x1 column vector, with the first 10 elements being the 10 total final demands in region 1, the second 10 those in region 2, and so on. The result of premultiplying this total final demand vector, Y, by the 1963 trade coefficient matrix C, 90x90, assembled as described on page 16, was further premultiplied by the inverse of $(I - C\hat{A})$, where $\hat{A}$ is the 90x90 technical coefficient matrix for the year 1963 and I is a 90x90 identity matrix. The final result of all these matrix operations was a 90x1 column vector of estimated outputs, the first 10 elements of which are the 10 industry outputs in region 1, the second 10 those in region 2, and so on. After this column vector had been rearranged into a 10x9 matrix, X', the elements in each row and each column were added to obtain the row and column sums, respectively. This was done so that a comparison could be made not only of the individual industry outputs in each region, but also of the total national outputs by industry, total regional outputs, and the aggregate national output for all industries and regions. Two schemes were used in making the comparisons between the calculated and actual outputs at this aggregated 10x9 level--the percentage difference and the weighted percentage difference.

The first scheme is described in the following equation:

$$p_{ij} = \frac{x'_{ij} - x_{ij}}{x_{ij}} \times 100 \qquad (13)$$

where

p_{ij} = percentage difference between the estimated and actual outputs.

x'_{ij} = the ith row and jth column element of the estimated output matrix, X'.

x_{ij} = the ith row and jth column element of the actual output matrix, X.

$i = 1, 2, \ldots, 10, 11$ (11 is the column sum).

$j = 1, 2, \ldots, 9, 10$ (10 is the row sum).

The results from this comparison are shown in Tables C-1 through C-3 and D-1 through D-3 in the appendix. A detailed analysis is given in the following section.

The second scheme, weighted differences, used the actual outputs as the weighting base so that industries with larger shares of output would show larger percentage differences, while small industries would show relatively smaller percentage differences. This comparison was made because, rather than assigning equal weights to all industries, it is important to find out whether the large percentage errors in some cases are indeed due to the fact that those industries had relatively very small outputs or whether they had experienced rapid growth rates in that period. The weighting procedure is as follows:

<u>Step 1</u>. Put the original 10x9 actual output matrix, X, into a 90x1 column vector.

<u>Step 2</u>. Sum all the elements in X, divide each element by the sum, and multiply by 100.

<u>Step 3</u>. Put the result of Step 2 into a 10x9 matrix and multiply element by element into the percentage-difference mátrix obtained from the first comparison.

The results for the years 1947 and 1958 are shown in Tables C-7, C-8, D-4, and D-5 in the appendix. It is important to point out that these numbers do not represent in any way the percentage differences between the estimated and actual outputs but are merely a measure of relative differences. For instance, in Table C-7, industry 10 in region 7 has a relative difference of -42.8, but its difference from actual outputs is only -13 percent, as shown in Table C-1. This means that the small percentage differences obtained in Table C-7 could be very significant as a whole, and, on the other hand, the large percentage differences could be relatively unimportant.

In addition to the 1947, 1958, and 1963 comparisons described above, 1970 and 1980 outputs were also calculated using the 1963 technology and trade matrices and the projected 1970 and 1980 total final demands. They are represented in Tables E-1 through E-10 in the appendix as percentage increases from the fixed base year (1947) data and in Tables F-1 through F-10 as annual compound growth rates. In each table, the first two columns were calculated from the actual data, the third and fourth columns were based upon figures projected using the column coefficient model, and the last two columns were based upon figures projected using the gravity coefficient model. The following equation was used to construct the first set of tables (Appendix E):

$$\text{percentage increase} = \frac{\text{current outputs - 1947 outputs}}{\text{1947 outputs}} \times 100 \qquad (14)$$

The second set of tables (Appendix F) was constructed according to equation (15) obtained from the following derivation.

$$x = (1 + r)^n x_o$$

$$\log(x/x_o) = n \log(1 + r)$$

$$\log(1 + r) = \frac{\log x - \log x_o}{n}$$

$$r = \left[\text{antilog}\left(\frac{\log x - \log x_o}{n}\right) - 1\right] \times 100 \qquad (15)$$

where

x = output value in the year N,

x_o = output value in the year N_o ,

r = annual compound growth rate,

n = number of years between N and N_o , $N - N_o$.

Full-Scale Level (79 Industries, 51 Regions)

This large-scale comparison of total outputs, X, calculated by the iterative method described on pages 6-8, was necessary to determine which disaggregated components of industries were responsible for the large percentage differences obtained at the aggregated level, and therefore to pinpoint the causes for such large differences. Exactly the same comparison methods as were used in the aggregated version were used to compare the calculated and actual outputs for 1947, 1958, and 1963. The results are shown in Tables C-4, C-5, and C-6 in the appendix. Because of the enormous amounts of data involved, however, no attempt was made to test the gravity model at this level, to estimate the weighted percentage differences, or to construct the percentage-increase and growth-rate tables, all of which were done at the aggregated level.

RESULTS OF MODEL COMPARISON

The results obtained from the comparison tests of the column coefficient and gravity coefficient models are explained in the following two sections.

Column Coefficient Model

The results obtained from the testing of the column coefficient model can be divided into two categories, the first of which, shown in Tables C-1 through C-5 in the appendix, can be used to determine how accurately the model, given a set of actual data as input, replicates another set of actual data. The second category, shown in Tables C-6, C-7, C-8, and E-1 through F-10, can be used to study the relative importance and growth pattern of each industry in the economy, and consequently to investigate the causes of some of the very large percentage errors, such as 515 percent in Table C-1 (row 7, column 8).

The percentage differences between estimated and actual outputs for 1947, 1958, and 1963, for the column coefficient model are shown in Tables C-1, C-2, and C-3, respectively. As was anticipated, the 1947 percentage errors are considerably larger than the 1963 errors. This is not surprising since the output estimates for the three years were computed using 1963 technology, trade, final demand, secondary transfers-out, and service industries residual data. (In fact, the model was first implemented for 1963 to assure that errors for the base year were as close to zero as possible.) Furthermore, several adjustments and approximations had to be made to the 1947 and 1958 figures, partly because of the lack of appropriate data and partly for the sake of consistency. The use of 1963 data as the basis, and the fact that there is a longer span between 1947 and 1963 than between 1958 and 1963, meant that the 1947 outputs naturally could not be estimated as accurately as the 1958 outputs. It is interesting to note, however, that most of the calculated outputs, when overestimated (positive entries) in 1947, are also overestimated in 1958 and, similarly, when underestimated (negative entries) in 1947, are also underestimated in 1958. A change in sign between 1947 and 1958 occurred for only 20 of the 90 estimates. Moreover, there are more overestimated than underestimated entries in all the three percentage-differences tables (Tables C-1, C-2, and C-3). Of the 90 entries (excluding the industrial and regional totals), 34 are underestimated in 1947, 36 in 1958, and 20 in 1963. These observations tend to prove that the column coefficient model is at least stable and consistent.

The validity of the model can best be shown by Table C-3 (the percentage differences between 1963 estimated and actual regional outputs). In the table, all but 5 of the 90 percentages are less than 1 percent, and 59 of the figures are 0.1 percent or less. This strongly indicates that the model is certainly valid, although a more conclusive statement cannot be made until further tests have been performed when more data become available. For the three tables, it can also be observed that the estimates of regional and industrial total output (row 11 and column 10 in each table) are generally much more accurate than the estimates of individual regional or industrial outputs, apparently because the estimation errors tend to average out when the data are in a more aggregated form. For instance, the estimation error in 1947 total outputs of Region 4, West North Central, is 0 percent (Table C-1), while three of the individual industry estimates in the same region (column 4) have errors of over 30 percent. Likewise, the estimation error in 1958 total outputs of Region 9, Pacific, is 0 percent (Table C-2), while five of the individual industry estimates in the same region (column 9) have errors of over 10 percent. In spite of all these large individual percentage differences, however, the largest estimation error in any regional or industrial total for 1947 is 20 percent (Region 8, Mountain, in 1947) and for 1958 is 9 percent (Industry 7, Transportation equipment & ordnance). The 1958 outputs, in general, are accurately estimated, given that estimates for this recession year are based upon data for a boom year, 1963. The estimated total national outputs for both 1947 and 1958 (row 11, column 10) differ from the actual figures by a mere 2 percent.

Some of the individual errors can be readily explained on the basis of the assumption of the fixed technology and trade coefficients. The 1947 underestimate of the New England output of Industry 6, Fabrics & textile products, was expected, given that by 1963 the industry had relocated in the South Atlantic region, where the 1947 output was overestimated. The 1963 trade coefficients were therefore causing 1947 production to be misallocated. The underestimate for New England was reduced from 44 percent in 1947 to 13 percent in 1958, while the overestimate for the South Atlantic region was reduced from 46 percent in 1947 to 11 percent in 1958. This would indicate that an analyst interested in the repercussion of regional shifts in the fabrics and textiles industry could make some selective changes in the technology and trade coefficients to provide more accurate estimates of the results. The individual percentage errors for 1947 and 1958 are summarized in Table 1. Of the 1947 output estimates, over 50 percent are within ±15

Table 1

DISTRIBUTION OF PERCENTAGE OUTPUT DIFFERENCES FOR 1947 AND 1958, COLUMN COEFFICIENT MODEL

	1947		1958	
Range	No. of Elements	Percent of Total	No. of Elements	Percent of Total
±1%	3	3.3	5	5.6
±5%	22	24.4	38	42.2
±10%	33	36.7	60	66.7
±15%	46	51.1	68	75.6
over ±15%	90	100.0	90	100.0

percent error. For 1958, more than 75 percent of the output estimates are within that range.

In order to determine why some 1947 and 1958 errors are extremely large (especially those for row 7--the transportation equipment and ordnance industry--and column 8--the Mountain region), the results shown in the rest of the Appendix C tables must also be examined. First, Tables C-4 and C-5, which are, respectively, the comparisons between estimated and actual 1947 and 1958 outputs for 8 industries at the 79x51 level, show that the large percentage differences in Industry 7, Transportation equipment & ordnance, at the aggregated level are primarily due to large estimation errors in IO-13, Ordnance & accessories, and IO-60, Aircraft & parts. (At the 10x9 level, Industry 7 is composed of the sum of the data for IO-13, Ordnance & accessories; IO-59, Motor vehicles & equipment; IO-60, Aircraft & parts; and IO-61, Other transportation equipment.) Second, the percentage errors in the national totals of IO-1, Livestock & livestock products, and IO-2, Other agricultural products, (which are the same two industries at the 10x9 level) in Tables C-4, C-5, and C-6 are almost the same, even though individual regional estimates are vastly different, thus again proving the stability and consistency of the model. Third, it is apparent from comparing these tables that wherever several industry components are aggregated to form one industry at the 10x9 level of aggregation, the estimation errors in these components are also similarly added. In other words, if two industries, say IO-14, Food & kindred products, and IO-15, Tobacco manufactures, are combined to form one industry, Industry 5, Food & tobacco, at the 10x9 level, the large number of overestimated entries in one industry tend to be cancelled out by the large number of underestimated entries in the other. This can be seen by comparing columns 3 and 4 of Table C-4 or C-5. Although some very large figures occur in Tables C-4 and C-5 (such as the 494,515 percent in IO-13, Ordnance & accessories, in Region 9, Florida), the estimates of the total national outputs at the 79x51 level in all three years are nearly as accurate as the estimates at the 10x9 level, with the 1947 estimation error being 7 percent and the 1958 being 3 percent. These and other observations all lend support to the theory that the more disaggregated computations tend to yield larger errors than the aggregated ones.

The extraordinarily large percentage figures in Tables C-1 through C-5 should not cause undue concern, because the method used to compute them gave large and small industries the same weight. The 1947 output of Industry 5, Fabrics & textile products, in the Mountain region, for example, was overestimated by 78 percent, but the output of this industry represented less than 1 percent of the total output of all products in the region and also less than 1 percent of the total 1947 outputs of the fabrics and textile products industry in the nation. The significance of the percentage errors can best be studied by referring to Tables C-7, C-8, E-1 through E-10, and F-1 through F-10. Tables C-7 and C-8 give the weighted percentage differences between estimated and actual 1947 and 1958 outputs. Tables E-1 through E-10 show the increases in outputs in each of the 9 regions and the national total, and Tables F-1 through F-10 give the annual compound rates of growth of industries. (In comparing the respective elements from Tables C-1 and C-7, or Tables C-2 and C-8, reference should be made to Tables B-1 and B-2, which contain the actual 1947 and 1958 outputs.)

On the whole, the largest errors in Tables C-1 and C-2 are relatively unimportant, because they occurred in industries with fairly small actual outputs; whereas some of the seemingly insignificant errors are important, because they occurred in industries with large actual outputs. For instance, the large 1947 error of 515 percent for Industry 7, Transportation equipment & ordnance, in the Mountain region (column 8 of Table C-1) reduced to a mere 5.5 weighted percentage difference, as shown in Table C-7, and that of 78 percent for Industry 6, Fabrics & textile products, to 0.4. Similarly, the difference in 1958 of 78 percent for Industry 9, Machinery & equipment, in the Mountain region (column 8 of Table C-2) actually corresponds to only a 3.4

weighted percentage difference (Table C-8). These differences in error magnitudes are caused partly by the fact that these industries had relatively very small outputs in the years concerned (see Tables B-1 and B-2), and partly by the fact that the production levels of these industries in the Mountain region had increased substantially from 1947 to 1963. As can be observed from Table E-8, the outputs in the Mountain region of Industry 7 increased by 1301 percent and of Industry 9 by 398 percent from 1947 to 1963, compared with an average increase of 119 percent for other industries in the region in the same period. Their annual compound growth rates were more than 17 percent from 1958 to 1963 (Table F-8). On the other hand, the comparatively small percentage differences obtained in the regional total and national industry total estimates (row 11 and column 10 of Tables C-1 and C-2) become quite large when they are weighted by their respective proportions of total national output (see Tables C-7 and C-8).

The results obtained from using the column coefficient model are on the whole rather satisfactory and reasonable and are comparable with the gravity coefficient model comparison results, which are described in the following section.

Gravity Coefficient Model

All the comparison methods and data used for the gravity model were the same as those used for the column coefficient model, except that they were carried out only at the aggregated level (10 industries, 9 regions).

The results of the first category of comparison, the percentage differences between the estimated and the actual outputs, are shown in Tables D-1, D-2, and D-3 in the appendix. For the 1947 and 1958 outputs, the differences between the estimated and the actual national industry totals are about 2 percent (as shown in row 11, column 10, of Tables D-1 and D-2). Some of the errors in the 1947 outputs are probably due to the long (16-year) interval from 1947 to 1963. For the 1947 and 1958 estimates, some industries, such as Industry 4, Construction; Industry 5, Food & tobacco; and Industry 10, Services, show smaller differences than others for most regions. The error for Food & tobacco, for example, is 1 percent in Region 3, East North Central, in both the years, while for the same industry it is 8 percent in 1947 and 4 percent in 1958 in Region 2, Middle Atlantic, as shown in Tables D-1 and D-2. However, the national total error for Food & tobacco is slightly greater than the national total errors of the other industries (10 percent in 1947 and 6 percent in 1958). This is due to the consistently positive errors in all regions for this industry. For other industries, overestimates in some regions are compensated for by underestimates in other regions. As the weighted percentage differences in Tables D-4 and D-5 show, the estimates for Food & tobacco mentioned above are actually rather accurate estimates. The weighted percentage error for Food & tobacco in the East North Central region is 2.7 in 1947 and 2.3 in 1958, which are relatively small errors. The reasons for these better estimates may lie in the accuracy of the data, in the stability of technology and trade coefficients, or in the stability of other factors that are not explicitly quantified within the model.

Most of the results shown in Tables D-1, D-2, and D-3 are comparable with those of the column coefficient model. The gravity model provides very accurate estimates of the 1963 base-year outputs, as shown in Table D-3. Only 5 outputs out of 90 yielded errors greater than 1 percent, and the largest of those 5 was only 2.8 percent (for Industry 3, Mining, in Region 4, West North Central). The 1947 regional totals (Table D-1, row 11) show, on the average, approximately the same percentage differences as the industry totals (column 10). The average absolute value of the regional and industrial total error is about 9 percent. This means that the outputs for all industries within one region were estimated with approximately the same accuracy as were the outputs for one commodity produced in all regions for that year. However, the 1958 regional total differences (Table D-2, row 11) are, on the average, significantly smaller (by 50 percent) than the industry total differences (column 10), which means that the regional outputs were estimated more accurately than the industrial outputs. (This is true, however, only when the weights of outputs for each region are not taken into consideration.) For the base-year 1963 (Table D-3), the regional total differences are, on the average, the same as the industrial total differences.

Table 2 shows the percentage difference distribution for the two observed years. As compared with the same percentage distribution table for the column coefficient model (Table 1), the gravity model (Table 2) shows a slightly larger number of estimation errors less than 15 percent. This difference may support the belief formed from a previous research study [7] that the gravity model is at least as accurate as the column coefficient model; however, no convincing conclusion can be drawn from these small differences in the distribution of percentage errors.

Table 2

DISTRIBUTION OF PERCENTAGE OUTPUT DIFFERENCES FOR 1947 AND 1958, GRAVITY COEFFICIENT MODEL

	1947		1958	
Range	No. of Elements	Percent of Total	No. of Elements	Percent of Total
±1%	4	4.4	11	12.2
±5%	20	22.2	37	41.1
±10%	37	41.1	52	67.7
±15%	51	56.6	70	77.7
over ±15%	90	100.0	90	100.0

Tables D-4 and D-5 show the same percentage differences for the years 1947 and 1958 as in Tables D-1 and D-2, but they are weighted by the industry outputs relative to the total national industry outputs. From Tables D-4 and D-5, the weights or importance of the corresponding errors in Tables D-1 and D-2 can be seen. Most of the large errors of the 1947 and 1958 outputs are unimportant relative to the national total outputs. For example, the estimation error of 639 percent for Industry 7, Transportation equipment & ordnance, in the Mountain region in 1947 is unimportant, since the weighted percentage error is 6.8--a relatively small error. The Industry 9, Machinery & equipment, estimation error of 234 percent in the Mountain region in 1947 (Table D-1) is also unimportant, since its corresponding weighted error is 6.4. On the other hand, the errors for 1958 of 9 percent for Industry 8, Manufacturing products, excluding machinery, and -7 percent for Industry 10, Services, in the

East North Central region (Table D-2) are not as accurate as they appear to be, because their corresponding weighted percentage errors are 41.8 and -63.6, which are actually the two largest errors in Table D-5. For Industry 1, Livestock; Industry 2, Other agriculture; and Industry 5, Food & tobacco, most of the percentage differences in Tables D-1 and D-2 are not greatly altered when they are expressed in terms of weighted percentage errors.

Tables E-1 through E-10 show the percentage changes in outputs relative to the base-year 1947 for different periods of time for the column coefficient and gravity coefficient models. These tables reveal the industrial changes within each region and provide a comparison between the results of the two models. (Only the gravity model results are summarized here.) For most regions, the largest percentage changes in output occurred for Industry 4, Construction; Industry 7, Transportation equipment & ordnance; Industry 9, Machinery & equipment; and Industry 10, Services. The output increases in Region 7, West South Central, for example, of Transportation equipment & ordnance from 1947 to 1958, 1963, 1970, and 1980 are 339, 563, 1246, and 1705 percent, respectively, compared with output increases for all industries in the region of 50, 81, 169, and 327, respectively (Table E-7). Industry 1, Livestock; Industry 2, Other agriculture; and Industry 3, Mining, show the smallest output changes in most of the regions. For example, in Region 2, Middle Atlantic, the output increases of Livestock from 1947 to 1958, 1963, 1970, and 1980 are 18, 21, 31, and 68 percent, respectively (Table E-2).

Tables F-1 through F-10 represent the annual compound rates of growth of output for the same yearly intervals as in the previous tables for the column coefficient and gravity coefficient models. These rates are important for information on the general industrial trends. For example, the most drastic reduction in the rates of growth for the West South Central region (Table F-7) occurred for Industry 7, Transportation equipment & ordnance, where the compound rate of growth drops from 14.4 percent in 1947 to 1958 to a projected 3.0 percent for 1970 to 1980. On the other hand, for Industry 4, Construction, the rates of growth increase in the Mountain region from 1.5 to 6.9 percent in the period 1963 to 1980 (Table F-8). The industries with the highest output growth rates for most regions in the last period, 1970 to 1980, are Industry 4, Construction; Industry 9, Machinery & equipment; and Industry 10, Services. For the same period, the lowest output growth rates for most regions are projected to occur for Industry 1, Livestock; Industry 2, Other agriculture; and Industry 7, Transportation equipment & ordnance.

CONCLUSION

The column coefficient and gravity coefficient models are two basically very similar multiregional input-output models. In addition to the interregional trade-flow data, the total regional consumption and total national production statistics are used in the gravity coefficient model as the normalizing bases to calculate the S and T matrices, while only total regional consumption is used in the column coefficient model to calculate the C matrix. Because of this, the gravity model is generally believed to be more accurate and consistent. The results obtained from tests performed for this report indicate, however, that the two models are comparable in both backcasting the 1947, 1958, and 1963 outputs and projecting the 1970 and 1980 outputs, as can be seen from the tables in Appendices E and F. This is probably due to the nature of the data base used, which is the aggregated 10x9 level, and to the procedures employed to assemble the various input data for the models.

Because all of the multiregional input-output models tend to have more accurate estimates when more aggregated data are used, it is difficult to judge conclusively, from the results obtained, which of the two compared here is a better model. The gravity model, however, cannot be tested on the more disaggregated data base (79x51) until the problem of nonconvergence of the S matrix is solved. Furthermore, as pointed out earlier in this report, the actual 1947 and 1958 outputs were approximated, using the assumption that the outputs of all industries remained in constant ratios to each other throughout the period from 1947 to 1963. The outputs (as well as final demands) could have been estimated more accurately if the actual statistics of imports, inventory depletions, and regional exports by port of exit had been available. In addition, the 1947 and 1958 final demand estimates for Alaska and Hawaii would be better if only the West Coast region (California, Oregon, and Washington), rather than the entire United States, had been used as the proportional base, since economically the Alaska and Hawaii regions resemble the West Coast more closely than they resemble the total U.S. economy.

Judging from the results obtained, the models were very accurate when the 1963 actual data were used, as had been expected. In fact, the models were first tested using only the 1963 data. Then, having obtained an average error of only 0.1 percent using these data, each model was used to calculate the outputs for 1947, 1958, 1970, and 1980 in order to compare the accuracy of the two models.

For 1947 and 1958, outputs were estimated more accurately for some industries than for others. Industry 7, Transportation equipment & ordnance, for example, has the largest estimation errors in both 1947 and 1958 for both models. The regional total outputs are in general more accurately estimated than the industry total outputs. Moreover, it can be observed from the last two sets of tables (Appendices E and F) that Industry 7, Transportation equipment & ordnance, and Industry 9, Machinery & equipment, experienced the largest production increases from 1947 to 1963, while Industry 1, Livestock, and Industry 3, Mining, experienced the smallest increases in that same period. It is important to note that significant decreases in both the actual production levels and the annual rates of growth occurred for some of the industries. For example, there was a 38 percent decrease in Industry 3, Mining, in the Middle Atlantic region from 1947 to 1958, and in the same period there was an annual growth rate of -2.2 percent in Industry 6, Fabrics & textile products, in the New England region. Based upon the 1963 data, the 1970 and 1980 outputs projected using the two models indicate that there will be a steady decline in annual growth rates of Industry 7, Transportation equipment & ordnance, in the nation. They also indicate that the annual growth rate of Industry 4, Construction, will increase from 1.4 percent in the period 1963-1970 to 6.2 percent in the period 1970-1980.

From the results of the comparison tests made, it can be safely concluded that the column coefficient and gravity coefficient models are quite accurate and certainly valid, although more extensive testing is needed to study the dynamic behavior of the models. Some of the tests that can easily be done using the present data base are as follows:

1. The relative estimation errors in regional total and industry total outputs can be compared by weighting them by the national total.

2. In addition to using the national total production as the weighting base, the total regional outputs or industry outputs can be used as the weighting bases to show the comparative importance of each industry in the regions.

3. The annual compound growth rates for the 1947 and 1958 estimated outputs can be calculated and compared with the actual growth rates.

4. The relative error values obtained from the estimation of 1947 and 1958 outputs using the two models can be shown more effectively by calculating the deviations of the errors from the mean of the errors.

5. The gravity model, like the column coefficient model, should be further tested using the disaggregated data (79x51) when the problem of the nonconvergence of the S matrix has been solved.

6. The models can be used to calculate the 1947 and 1958 interregional trade flows so that further consistency tests of the models can be done.

As emphasized throughout this report, the primary difficulty encountered in testing both models is the lack of appropriate data. It is therefore of the utmost importance that new data be assembled from available statistics. Future research efforts to collect new data should be directed to the following areas:

1. Technology and trade data for 1967, so that coefficient stability tests can be performed.
2. Final demand data for 1967, so that the models can be tested on another set of actual data.
3. Values of imports, inventory depletions, and exports by port of exit for 1958, and possibly 1947, so that the outputs for those two years can be more accurately estimated.
4. Tonnage data, so that similar model comparisons can be made not only in terms of dollars as in all the tests done so far, but also in terms of tons.

Finally, it must be emphasized again that the conclusions reached in this report are not final and that further study is needed to determine the relative accuracy and stability of the two models.

APPENDIX A

INDUSTRIAL AND REGIONAL CLASSIFICATIONS

Table A-1

MULTIREGIONAL INPUT-OUTPUT CLASSIFICATION

Industry Number MRIO	Industry Number IO	Industry Title
1	1	Livestock & livestock prdts.
2	2	Other agricultural prdts.
3		Mining
	5	Iron & ferro. ores mining
	6	Nonferrous metal ores mining
	7	Coal mining
	8	Crude petro., natural gas
	9	Stone & clay mining
	10	Chem. & fert. mineral mining
4		Construction
	11	New construction
	12	Maint. & repair construction
5		Food & tobacco
	14	Food & kindred prdts.
	15	Tobacco manufactures
6		Fabrics & textile prdts.
	16	Fabrics
	17	Textile prdts.
	18	Apparel
	19	Misc. textile prdts.
7		Transportation equip. & ordnance
	13	Ordnance & accessories
	59	Motor vehicles, equip.
	60	Aircraft & parts
	61	Other transport. equip.
8		Manufactured prdts., exc. mach.
	20	Lumber & wood prdts.
	21	Wooden containers
	22	Household furniture
	23	Other furniture
	24	Paper & allied prdts.
	25	Paperboard containers
	26	Printing & publishing
	27	Chemicals, selected prdts.
	28	Plastics & synthetics
	29	Drugs & cosmetics
	30	Paint & allied prdts.
	31	Petroleum, related inds.
	32	Rubber, misc. plastics
	33	Leather tanning & prdts.
	34	Footwear, leather prdts.
	35	Glass & glass prdts.
	36	Stone & clay prdts.
	37	Primary iron, steel mfr.
	38	Primary nonferrous mfr.
	39	Metal containers
	40	Fabricated metal prdts.
	41	Screw mach. prdts., etc.
	42	Other fab. metal prdts.
9		Machinery & equipment
	43	Engines & turbines
	44	Farm mach. & equip.
	45	Construction mach. & equip.
	46	Materials hand. mach. & equip.
	47	Metalworking mach. & equip.
	48	Special mach. & equip.
	49	General mach. & equip.
	50	Machine shop prdts.
	51	Office, computing machines
	52	Service industry machines
	53	Elec. transmission equip.
	54	Household appliances
	55	Electric lighting equip.
	56	Radio, TV, etc., equip.
	57	Electronic components
	58	Misc. electrical mach.
	62	Professional, scien. instru.
	63	Medical, photo. equip.
	64	Misc. manufacturing
10		Services
	3	Forestry & fishery prdts.
	4	Ag., for., & fish. services
	65	Transportation & warehousing
	66	Communications, exc. brdcast.
	67	Radio & TV broadcasting
	68	Elec., gas, water, & san. serv.
	69	Wholesale & retail trade
	70	Finance & insurance
	71	Real estate & rental
	72	Hotels; repair serv., exc. auto
	73	Business services
	74	Research & development
	75	Automobile repair & services
	76	Amusements
	77	Med., ed. serv., nonprofit org.
	78	Federal gov't. enterprises
	79	State & local gov't. enterp.

Table A-2

MRIO REGIONAL CLASSIFICATION

Regions 9*	States 51	Name
1	6	Connecticut
	18	Maine
	20	Massachusetts
	28	New Hampshire
	38	Rhode Island
	44	Vermont
2	29	New Jersey
	31	New York
	37	Pennsylvania
3	12	Illinois
	13	Indiana
	21	Michigan
	34	Ohio
	48	Wisconsin
4	14	Iowa
	15	Kansas
	22	Minnesota
	24	Missouri
	26	Nebraska
	33	North Dakota
	40	South Dakota
5	7	Delaware
	8	District of Columbia
	9	Florida
	10	Georgia
	19	Maryland
	32	North Carolina
	39	South Carolina
	45	Virginia
	47	West Virginia
6	1	Alabama
	16	Kentucky
	23	Mississippi
	41	Tennessee
7	3	Arkansas
	17	Louisiana
	35	Oklahoma
	42	Texas
8	2	Arizona
	5	Colorado
	11	Idaho
	25	Montana
	27	Nevada
	30	New Mexico
	43	Utah
	49	Wyoming
9	4	California
	36	Oregon
	46	Washington
	50	Alaska
	51	Hawaii

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*The names of the 9 census regions are:

1 New England
2 Middle Atlantic
3 East North Central
4 West North Central
5 South Atlantic
6 East South Central
7 West South Central
8 Mountain
9 Pacific

APPENDIX B

1947, 1958, 1963 OUTPUTS

TABLE B-1

1947 PRODUCTION OUTPUTS
(THOUSANDS OF 1963 DOLLARS)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	495898	1549085	4661726	6471429	1614409	1266748	1921524	1227481	1249759	20458048
2 OTHER AGRICULTURE	298211	884130	3841326	5835820	2003600	1419806	2544875	1294132	2518657	20640560
3 MINING	320238	2956586	1727488	1026089	1636064	1003874	4322476	1149671	1380282	15522770
4 CONSTRUCTION	2290739	7234466	9294958	4441960	5801820	2251962	5905534	2014231	7936594	47172304
5 FOOD, TOBACCO	1673021	9244964	11888569	9429399	7915658	2874651	4306332	1207747	5350280	53890624
6 FABRICS, TEXTILE PRODS.	3913774	9365313	1539141	512812	6235483	1282244	409711	33801	516301	23808576
7 TRANSP. EQUIP., ORDNANCE	901246	4214581	15394428	1284195	888489	334363	384803	64528	2250630	25717264
8 MANUF. PRODS., EXC. MACH.	8455240	31804176	34630720	5187513	8724464	5052037	8031397	1868676	9706384	113460624
9 MACHINERY, EQUIPMENT	4781018	12094371	18031440	2158241	863337	429363	735020	165807	1689357	40947952
10 SERVICES	15441988	64248816	51524656	21696208	24634768	10625590	20208432	7380109	30719088	246479712
11 REGIONAL TOTAL	38571424	143596496	152534480	58043664	60318096	26540640	48770112	16406186	63317344	608098304

TABLE B-2

1958 PRODUCTION OUTPUTS
(THOUSANDS OF 1963 DOLLARS)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	624540	1822282	5427699	7898096	2499705	1786359	2438132	1823237	1851281	26171328
2 OTHER AGRICULTURE	307551	980536	4154750	6667852	2905122	1374003	3967112	1701934	3674564	25733424
3 MINING	380882	1831347	1642010	1178234	1499674	942932	6553152	2275559	1621579	17925360
4 CONSTRUCTION	3921719	12415115	14437851	7021597	9852735	3557451	9226139	4539160	11708623	76680384
5 FOOD,TOBACCO	2422345	12049864	14644927	10347139	10672159	4749777	4691770	1846760	8316266	69741008
6 FABRICS,TEXTILE PRODS.	3077503	10929929	1748199	597630	9846309	1983079	707144	60442	1087138	30037360
7 TRANSP.EQUIP.,ORDNANCE	2343242	5839139	17735696	3607868	2804316	832955	1690427	404367	7365508	42623520
8 MANUF.PRODS.,EXC.MACH.	9173084	35318240	40308048	7539130	13614242	7227469	14576691	3476006	16231620	147464528
9 MACHINERY,EQUIPMENT	5372287	16429108	20897776	3208676	2200559	1514492	1460154	363390	3908252	55354704
10 SERVICES	21041952	88250832	74118416	28384608	40103952	14669947	27843264	12162373	47337648	353912832
11 REGIONAL TOTAL	48665104	185866400	195115408	76450832	95998768	38638464	73154000	28653232	103102480	845644544

TABLE B-3

1963 PRODUCTION OUTPUTS
(THOUSANDS OF CURRENT DOLLARS)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	646868	1867903	5019572	7932120	2426875	1758841	2649951	2036699	2345440	26684256
2 OTHER AGRICULTURE	378892	1353558	4945011	6135930	3584673	2186424	3311047	1755181	3615429	27266144
3 MINING	471034	1935715	1860052	1389041	1676676	1061013	7817033	2695202	1623018	20528832
4 CONSTRUCTION	4516780	12331969	14592089	6369497	11507175	3933318	8937204	4811962	18313328	85313328
5 FOOD, TOBACCO	2787122	13244751	16345917	12165254	12780586	5520470	6230238	2509859	10094592	81678784
6 FABRICS, TEXTILE PRODS.	3192302	12280149	2231345	696244	13147044	2799702	987948	87801	1498503	36921024
7 TRANSP.EQUIP., ORDNANCE	2828724	7500551	29841440	4816593	3963477	1423739	2551923	904116	9904477	63735040
8 MANUF.PRODS., EXC.MACH.	11702098	43975792	52906688	9370589	19346800	9658969	20027456	4521499	20275376	191785312
9 MACHINERY, EQUIPMENT	7659476	20593024	27827968	4601197	3940996	2111608	2656114	826476	7430432	77647296
10 SERVICES	25911344	104662096	84876816	33441056	52451792	18100128	33102400	15759664	60140368	428445696
11 REGIONAL TOTAL	60094688	219745520	240446928	86917520	124826096	48554208	88271328	35908448	135240976	1040005632

APPENDIX C

COLUMN COEFFICIENT MODEL COMPARISONS

TABLE C-1

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1947 OUTPUTS
(COLUMN COEFFICIENT MODEL, 10x9 LEVEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	-1	-7	-20	-12	13	3	0	16	25	-5
2 OTHER AGRICULTURE	2	20	-8	-35	34	5	2	2	1	-6
3 MINING	9	-51	-22	4	-23	-23	29	46	-31	-7
4 CONSTRUCTION	-15	-11	-6	4	-12	-9	7	4	-11	-6
5 FOOD,TOBACCO	17	7	2	-1	15	45	12	36	19	10
6 FABRICS,TEXTILE PRODS.	-44	-8	-2	-3	46	52	67	78	94	7
7 TRANSP.EQUIP.,ORDNANCE	28	-16	-12	69	106	99	194	515	75	10
8 MANUF.PRODS.,EXC.MACH.	-12	-10	-3	19	36	19	63	41	16	6
9 MACHINERY,EQUIPMENT	-2	5	-5	32	170	205	125	196	145	16
10 SERVICES	-10	5	-5	1	7	-11	-13	9	-2	-1
11 REGIONAL TOTAL	-11	-2	-5	0	19	10	13	20	9	2

TABLE C-2

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1958 OUTPUTS
(COLUMN COEFFICIENT MODEL, 10x9 LEVEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	-2	-3	-13	-4	-10	-9	3	5	16	-4
2 OTHER AGRICULTURE	22	37	13	-16	19	40	-28	-3	-14	-3
3 MINING	7	-9	-3	5	-4	-3	2	-4	-22	-3
4 CONSTRUCTION	-2	-3	-4	-1	-2	-2	-4	0	-1	-2
5 FOOD, TOBACCO	5	3	2	9	8	6	20	20	4	6
6 FABRICS, TEXTILE PRODS.	-13	-5	8	1	11	18	19	25	16	3
7 TRANSP. EQUIP., ORDNANCE	-8	-4	20	0	5	26	11	68	-1	9
8 MANUF. PRODS., EXC. MACH.	7	5	10	7	18	11	16	8	-3	8
9 MACHINERY, EQUIPMENT	7	-4	1	11	32	7	40	78	39	6
10 SERVICES	-1	2	-7	3	2	-2	-6	5	-2	-1
11 REGIONAL TOTAL	1	1	1	2	6	5	2	6	0	2

TABLE C-3

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1963 OUTPUTS
(COLUMN COEFFICIENT MODEL, 10x9 LEVEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	-0.0	-0.0	-0.0	0.0	-0.0	0.1	0.0	0.2	0.5	0.1
2 OTHER AGRICULTURE	1.0	0.9	0.7	-0.1	0.9	0.7	0.3	-0.1	0.2	0.4
3 MINING	0.5	0.1	0.9	2.7	0.3	1.4	0.4	-2.0	1.2	0.4
4 CONSTRUCTION	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0	0.0
5 FOOD, TOBACCO	-0.1	-0.1	-0.1	-0.0	-0.2	-0.0	-0.1	0.4	0.7	0.0
6 FABRICS, TEXTILE PRODS.	0.1	0.1	0.0	0.1	-0.0	0.0	-0.1	-0.0	0.0	0.0
7 TRANSP.EQUIP., ORDNANCE	0.5	0.4	0.1	0.2	0.4	0.4	0.3	0.4	-0.0	0.2
8 MANUF.PRODS., EXC.MACH.	0.2	0.1	0.1	0.1	0.1	0.1	0.0	0.3	0.5	0.1
9 MACHINERY, EQUIPMENT	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.2	0.1
10 SERVICES	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.1	0.0
11 REGIONAL TOTAL	0.1	0.1	0.1	0.1	0.1	0.1	0.1	-0.1	0.2	0.1

TABLE C-4

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1947 OUTPUTS
(COLUMN COEFFICIENT MODEL, 79x51 LEVEL)

	1 IO-1	2 IO-2	3 IO-14	4 IO-15	5 IO-13	6 IO-59	7 IO-60	8 IO-61
1 ALABAMA	42	-9	83	150	20766	190	11568	9
2 ARIZONA	100	82	63	*	588832	-76	9075	-8
3 ARKANSAS	13	27	135	0	*	285	582	856
4 CALIFORNIA	52	9	19	-98	60185	46	26	25
5 COLORADO	59	-23	42	0	1376	27	5605	-75
6 CONNECTICUT	-10	8	65	-3	-59	61	17	94
7 DELAWARE	7	52	0	0	*	3038	42	-90
8 DISTRICT OF COLUMBIA	*	*	21	*	0	0	0	*
9 FLORIDA	79	208	119	-28	494515	-59	2486	64
10 GEORGIA	53	7	50	-2	546	464	17200	232
11 IDAHO	1	4	47	*	-94	497	437	586
12 ILLINOIS	-15	-37	-12	82	211	7	40	-45
13 INDIANA	-25	10	16	-36	753	-27	14	35
14 IOWA	-16	-51	21	0	8202	11	-83	83
15 KANSAS	14	-26	-29	*	0	71	203	160
16 KENTUCKY	-16	8	2	97	*	115	509	-7
17 LOUISIANA	-7	40	-19	-45	*	233	6336	82
18 MAINE	63	-4	91	0	*	108	1049	99
19 MARYLAND	12	-4	16	42	551	280	-24	57
20 MASSACHUSETTS	-23	6	5	-94	9	22	-6	-17
21 MICHIGAN	-23	30	30	-45	1594	-14	129	67
22 MINNESOTA	-13	-23	-7	0	146	158	-4	-39
23 MISSISSIPPI	29	19	93	*	*	148	0	-87
24 MISSOURI	-29	46	1	-80	50295	70	250	-6
25 MONTANA	-12	-3	19	*	*	-60	0	339
26 NEBRASKA	4	-44	8	*	323	214	14	845
27 NEVADA	-32	13	-15	*	*	-65	-44	*
28 NEW HAMPSHIRE	-18	-7	35	0	*	-20	0	1038
29 NEW JERSEY	-18	13	32	-82	-79	-35	2	-28
30 NEW MEXICO	34	26	96	*	*	-83	-89	*
31 NEW YORK	-5	27	-3	-78	3	-3	91	-42
32 NORTH CAROLINA	27	39	135	-10	2941	56	6715	340
33 NORTH DAKOTA	-27	-33	-20	*	*	*	0	212
34 OHIO	-29	19	17	-60	3433	66	36	-14
35 OKLAHOMA	-9	-5	-10	*	*	226	4730	304
36 OREGON	-17	-20	23	0	1015	450	3618	114
37 PENNSYLVANIA	-5	22	22	-15	436	-18	436	-22
38 RHODE ISLAND	-19	46	34	-84	-43	144	3406	451
39 SOUTH CAROLINA	-15	2	83	122	*	1	0	72
40 SOUTH DAKOTA	-2	-62	22	*	*	114	0	2607
41 TENNESSEE	-17	5	35	-18	*	-51	2315	167
42 TEXAS	2	-4	23	-82	2965	441	7	206
43 UTAH	-9	-26	12	0	*	174	147121	224
44 VERMONT	11	40	16	*	0	0	15578	-9
45 VIRGINIA	-14	7	57	-16	71	606	1168	-43
46 WASHINGTON	-13	-7	25	0	16718	16	69	154
47 WEST VIRGINIA	-41	44	60	8	*	-21	-96	83
48 WISCONSIN	-11	57	5	-94	1403	89	-74	11
49 WYOMING	-13	-3	66	*	*	*	163	4051
50 ALASKA	8	10	16	*	*	*	*	*
51 HAWAII	-5	5	7	*	*	18	*	0
52 NATIONAL TOTAL	-5	-5	13	-11	324	12	62	2

*Zero output

TABLE C-5

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1958 OUTPUTS
(COLUMN COEFFICIENT MODEL, 79x51 LEVEL)

	1 IO-1	2 IO-2	3 IO-14	4 IO-15	5 IO-13	6 IO-59	7 IO-60	8 IO-61
1 ALABAMA	-2	23	29	56	192	16	319	55
2 ARIZONA	59	-15	16	*	-27	57	43	268
3 ARKANSAS	10	19	67	*	-51	73	1104	129
4 CALIFORNIA	29	-15	3	21	68	12	-5	29
5 COLORADO	38	-18	14	*	-59	7	394	-25
6 CONNECTICUT	-14	18	1	-80	-9	183	7	-62
7 DELAWARE	-2	-16	-3	*	0	34	60	-38
8 DISTRICT OF COLUMBIA	*	*	-18	*	0	0	0	0
9 FLORIDA	-1	34	19	-29	182	17	397	31
10 GEORGIA	-7	16	8	-33	-30	-8	-23	201
11 IDAHO	-6	15	35	*	188	-22	2848	702
12 ILLINOIS	-18	-8	0	-28	-6	-7	-60	4
13 INDIANA	-19	26	7	12	145	-5	-10	17
14 IOWA	-9	-2	17	*	38	-3	-77	111
15 KANSAS	41	-36	17	*	0	-11	-23	248
16 KENTUCKY	-7	47	-8	-1	0	-34	-70	-34
17 LOUISIANA	-21	43	27	-35	393	20	13213	89
18 MAINE	10	11	20	*	-57	-69	-39	12
19 MARYLAND	6	3	3	-40	-81	130	-55	-1
20 MASSACHUSETTS	-8	29	7	-52	-59	39	38	-45
21 MICHIGAN	-3	17	4	-29	-63	16	-10	11
22 MINNESOTA	-9	-5	4	0	-4	4	7	-31
23 MISSISSIPPI	-10	39	22	*	0	-18	0	-95
24 MISSOURI	-15	19	1	55	-35	31	-30	31
25 MONTANA	-23	0	30	*	0	-41	0	69
26 NEBRASKA	8	-34	6	*	-93	128	3	64
27 NEVADA	-33	30	13	*	0	-51	-70	*
28 NEW HAMPSHIRE	-14	20	21	0	0	-46	0	100
29 NEW JERSEY	-21	12	-2	-38	-87	-48	-5	-22
30 NEW MEXICO	11	5	31	*	-1	-9	-90	*
31 NEW YORK	0	46	0	2	-88	1	44	-10
32 NORTH CAROLINA	-11	9	24	3	-80	19	667	119
33 NORTH DAKOTA	-20	-18	7	*	0	0	0	*
34 OHIO	-13	19	3	-7	91	20	-14	12
35 OKLAHOMA	10	-11	9	*	-67	190	469	-7
36 OREGON	-15	-15	-0	*	-40	128	-2	66
37 PENNSYLVANIA	-1	49	12	9	-29	6	162	13
38 RHODE ISLAND	-12	39	17	-79	-99	6	210	77
39 SOUTH CAROLINA	-19	21	43	106	0	15	0	307
40 SOUTH DAKOTA	-10	-24	22	*	0	58	0	-25
41 TENNESSEE	-15	38	22	-62	31	-30	809	26
42 TEXAS	4	-44	11	-46	103	16	-26	23
43 UTAH	-7	27	9	*	266	15	880	133
44 VERMONT	14	132	13	*	0	0	4342	-47
45 VIRGINIA	-13	16	9	1	-95	109	144	-54
46 WASHINGTON	-8	-9	11	*	516	5	-52	89
47 WEST VIRGINIA	-26	86	11	-14	3590	1	-95	200
48 WISCONSIN	-7	102	1	-70	383	31	-78	37
49 WYOMING	-18	46	83	*	0	0	119	-21
50 ALASKA	7	-11	7	*	*	*	*	*
51 HAWAII	-2	-1	5	*	*	16	*	10
52 NATIONAL TOTAL	-4	-2	7	-1	-21	12	-2	7

*Zero output

TABLE C-6

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1963 OUTPUTS
(COLUMN COEFFICIENT MODEL, 79x51 LEVEL)

	1 IO-1	2 IO-2	3 IO-14	4 IO-15	5 IO-13	6 IO-59	7 IO-60	8 IO-61
1 ALABAMA	-.6	.1	-.5	.4	.6	-.1	.4	.9
2 ARIZONA	-.8	-.6	.3	.8	.6	-.3	.1	.9
3 ARKANSAS	-.7	-.1	-.4	.1	.7	-.2	.3	.9
4 CALIFORNIA	-.3	-.3	.5	.1	.6	-.2	-.9	1.5
5 COLORADO	-.9	-1.2	.1	.1	.6	-.2	.3	1.0
6 CONNECTICUT	-.3	.6	-.1	.2	.7	-.0	.5	.9
7 DELAWARE	-.5	.3	-.3	-.7	.6	.0	.2	.9
8 DISTRICT OF COLUMBIA	*	*	-.2	-.4	.7	-.1	.4	.9
9 FLORIDA	-.5	.6	-.3	.1	.6	.0	.4	.8
10 GEORGIA	-.6	.1	-.6	.1	.7	.0	.3	.8
11 IDAHO	-.7	-.8	.4	.9	.6	-.1	.3	1.0
12 ILLINOIS	-.6	.1	-.2	.4	.6	-.1	.4	.8
13 INDIANA	-.6	-.1	-.3	.4	.6	-.1	.4	.8
14 IOWA	-.9	-1.0	-.4	.5	.6	-.2	.4	1.0
15 KANSAS	-1.1	-1.3	-.3	.1	.7	-.2	.3	1.1
16 KENTUCKY	-.6	.4	-.4	.5	.6	-.1	.2	.8
17 LOUISIANA	-.7	.0	-.4	.2	.6	-.2	.3	.9
18 MAINE	-.4	.6	-.2	.3	.6	-.0	.4	.9
19 MARYLAND	-.5	.5	-.3	-.1	.7	.0	.4	.9
20 MASSACHUSETTS	-.3	.6	-.1	.0	.6	-.0	.5	.9
21 MICHIGAN	-.5	.3	-.2	.2	.6	-.1	.5	.8
22 MINNESOTA	-.8	-1.0	-.3	.2	.6	-.2	.3	1.0
23 MISSISSIPPI	-.7	.0	-.5	.9	.7	-.1	.4	1.2
24 MISSOURI	-.8	-.8	-.3	.5	.6	-.2	.2	1.0
25 MONTANA	-1.0	-1.5	.3	.1	.7	-.2	.1	.8
26 NEBRASKA	-1.0	-1.3	-.3	.1	.6	-.2	.2	1.0
27 NEVADA	-.8	-1.1	.4	.9	.7	-.3	.2	1.1
28 NEW HAMPSHIRE	-.3	.5	-.1	.8	.6	-.0	.6	.9
29 NEW JERSEY	-.4	.6	-.2	.1	.6	-.0	.4	.8
30 NEW MEXICO	-.8	-.9	.1	.9	.6	-.4	.6	.9
31 NEW YORK	-.3	.6	-.1	-.1	.6	-.0	.5	.8
32 NORTH CAROLINA	-.5	.3	-.4	.0	.6	.0	.4	.8
33 NORTH DAKOTA	-1.0	-1.4	-.3	.9	.7	-.1	.4	.6
34 OHIO	-.5	.3	-.2	.3	.7	-.1	.4	.8
35 OKLAHOMA	-.9	-.9	-.3	.1	.7	-.2	.3	1.0
36 OREGON	-.4	-.6	.6	-.2	.7	-.2	.2	1.1
37 PENNSYLVANIA	-.4	.5	-.2	.6	.7	-.0	.4	.9
38 RHODE ISLAND	-.3	.5	-.1	-.3	.6	-.0	.4	.9
39 SOUTH CAROLINA	-.5	-.0	-.4	-.8	.6	.0	.3	.8
40 SOUTH DAKOTA	-1.0	-1.7	-.4	.9	.7	-.2	.1	.9
41 TENNESSEE	-.6	-.0	-.5	.4	.6	-.1	.4	1.0
42 TEXAS	-.8	-.3	-.3	.2	.7	-.2	.3	.9
43 UTAH	-.6	-.8	.4	.9	.6	-.3	.3	1.0
44 VERMONT	-.3	.7	-.2	.7	.9	.1	.5	.6
45 VIRGINIA	-.5	.4	-.4	-.5	.7	.0	.3	.8
46 WASHINGTON	-.3	-.6	.8	-.2	.7	-.2	.0	1.3
47 WEST VIRGINIA	-.4	.4	-.3	.1	1.0	.0	.9	.7
48 WISCONSIN	-.6	-.3	-.2	.4	.6	-.1	.4	.9
49 WYOMING	-1.1	-1.9	.0	.8	.6	-.1	.2	1.1
50 ALASKA	.3	.1	.9	.9	-.1	-.1	.2	-.0
51 HAWAII	1.1	.4	.7	.9	.2	-.2	.1	1.2
52 NATIONAL TOTAL	-.7	-.3	-.1	.1	.6	-.1	.1	.9

*Zero output

TABLE C-7

WEIGHTED PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1947 OUTPUTS
(COLUMN COEFFICIENT MODEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC
1 LIVESTOCK	-0.0	-1.8	-15.3	-12.4	3.5	0.5	0.0	3.3	5.2
2 OTHER AGRICULTURE	0.1	2.5	-4.8	-33.1	11.3	1.1	0.7	0.4	0.6
3 MINING	0.5	-24.8	-6.3	0.7	-6.1	-3.8	20.9	8.7	-6.9
4 CONSTRUCTION	-5.6	-13.2	-8.7	3.2	-11.2	-3.2	6.6	1.3	-14.6
5 FOOD, TOBACCO	4.8	11.4	4.4	-0.9	19.6	21.4	8.3	7.1	16.5
6 FABRICS, TEXTILE PRODS.	-28.1	-12.9	-0.5	-0.2	47.0	11.0	4.5	0.4	8.0
7 TRANSP. EQUIP., ORDNANCE	4.1	-10.8	-30.2	14.5	15.5	5.5	12.3	5.5	27.7
8 MANUF. PRODS., EXC. MACH.	-16.9	-51.8	-14.8	16.0	51.9	15.8	79.0	12.6	26.1
9 MACHINERY, EQUIPMENT	-1.5	9.4	-13.4	11.2	24.1	14.5	15.1	5.4	40.2
10 SERVICES	-26.6	49.6	-44.8	3.9	29.8	-19.0	-42.8	10.5	-9.8

TABLE C-8

WEIGHTED PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1958 OUTPUTS
(COLUMN COEFFICIENT MODEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC
1 LIVESTOCK	-0.2	-0.7	-8.6	-4.0	-2.9	-2.0	0.9	1.0	3.4
2 OTHER AGRICULTURE	0.8	4.3	6.3	-12.4	6.6	6.4	-12.9	-0.6	-6.2
3 MINING	0.3	-1.9	-0.5	0.7	-0.7	-0.4	1.9	-1.1	-4.2
4 CONSTRUCTION	-0.9	-4.7	-6.1	-0.8	-2.2	-0.7	-4.2	-0.0	-1.2
5 FOOD, TOBACCO	1.6	4.0	3.2	10.9	9.6	3.6	11.0	4.3	4.3
6 FABRICS, TEXTILE PRODS.	-4.6	-6.5	1.6	0.0	13.2	4.3	1.6	0.2	2.1
7 TRANSP.EQUIP.,ORDNANCE	-2.3	-2.7	42.1	0.0	1.8	2.5	2.3	3.3	-1.3
8 MANUF.PRODS.,EXC.MACH.	7.1	20.9	46.5	6.6	28.5	9.6	27.2	3.5	-6.2
9 MACHINERY, EQUIPMENT	4.6	-8.7	3.3	4.1	8.4	1.3	6.9	3.4	17.9
10 SERVICES	-3.0	16.6	-61.2	9.7	9.7	-3.4	-18.3	7.1	-10.1

APPENDIX D

GRAVITY COEFFICIENT MODEL COMPARISONS

TABLE D-1

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1947 OUTPUTS
(GRAVITY COEFFICIENT MODEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	-7	-5	-20	-7	10	0	2	17	18	-4
2 CTHER AGRICULTURE	2	19	-6	-36	34	-5	12	9	-5	-6
3 MINING	15	-52	-28	18	-21	-25	36	21	-29	-6
4 CCNSTRUCTICN	-15	-11	-6	5	-12	-9	7	4	-11	-6
5 FOOD,TOBACCO	12	8	1	8	10	47	11	31	15	10
6 FABRICS,TEXTILE PRODS.	-46	-4	-17	4	43	62	63	98	96	8
7 TRANSP.EQUIP.,ORDNANCE	-1	-10	-9	62	104	107	196	639	60	11
8 MANUF.PRODS.,EXC.MACH.	-17	-5	-2	24	25	10	67	33	8	6
9 MACHINERY,EQUIPMENT	-2	5	-1	26	131	215	139	234	114	16
10 SERVICES	-11	5	-5	2	6	-12	-11	8	-3	-1
11 REGIONAL TOTAL	-13	0	-5	3	15	8	16	19	5	2

TABLE D-2

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1958 OUTPUTS
(GRAVITY COEFFICIENT MODEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	-5	0	-14	-1	-11	-17	5	7	14	-4
2 OTHER AGRICULTURE	22	38	13	-14	24	30	-31	4	-18	-3
3 MINING	10	-9	-8	13	-5	-5	5	-6	-24	-2
4 CONSTRUCTION	-2	-3	-4	-1	-2	-2	-4	0	-1	-2
5 FOOD,TOBACCO	3	4	1	13	6	6	19	20	3	6
6 FABRICS,TEXTILE PRODS.	-12	-1	1	5	7	25	15	14	20	3
7 TRANSP.EQUIP.,ORDNANCE	-2	-1	15	15	12	17	4	55	0	9
8 MANUF.PRODS.,EXC.MACH.	4	7	9	15	15	8	18	13	-7	8
9 MACHINERY,EQUIPMENT	4	-2	2	17	19	11	44	93	30	6
10 SERVICES	-1	2	-7	4	2	-2	-5	5	-2	-1
11 REGIONAL TOTAL	0	2	0	5	5	3	2	7	-1	2

TABLE D-3

PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1963 OUTPUTS
(GRAVITY COEFFICIENT MODEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC	10 NATIONAL INDUSTRY TOTAL
1 LIVESTOCK	-0.0	-0.0	-0.0	-0.0	-0.0	0.2	0.0	0.1	0.6	0.1
2 OTHER AGRICULTURE	1.1	0.9	0.6	-0.2	0.8	0.7	0.3	-0.0	0.3	0.4
3 MINING	0.5	0.2	0.9	2.8	0.3	1.5	0.4	-2.2	1.3	0.4
4 CONSTRUCTION	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0	0.0
5 FOOD, TOBACCO	-0.0	-0.1	-0.1	-0.1	-0.2	-0.0	-0.1	0.4	0.7	0.0
6 FABRICS, TEXTILE PRODS.	0.2	0.1	0.1	0.1	-0.0	0.1	-0.1	-0.1	0.1	0.0
7 TRANSP. EQUIP., ORDNANCE	0.5	0.4	0.2	0.1	0.4	0.4	0.3	0.5	-0.0	0.2
8 MANUF. PRODS., EXC. MACH.	0.3	0.1	0.0	0.1	0.1	0.2	-0.0	0.2	0.5	0.1
9 MACHINERY, EQUIPMENT	0.2	0.1	0.1	0.1	0.1	0.2	0.1	-0.1	0.2	0.1
10 SERVICES	0.0	0.0	0.0	0.0	0.0	0.1	0.0	-0.1	0.1	0.0
11 REGIONAL TOTAL	0.2	0.1	0.1	0.1	0.0	0.1	0.1	-0.1	0.2	0.1

TABLE D-4

WEIGHTED PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1947 OUTPUTS
(GRAVITY COEFFICIENT MODEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC
1 LIVESTOCK	-0.5	-1.2	-15.2	-7.0	2.6	-0.1	0.7	3.5	3.7
2 OTHER AGRICULTURE	0.1	2.8	-3.7	-34.5	11.1	-1.2	5.1	1.9	-1.9
3 MINING	0.8	-25.1	-8.1	3.0	-5.5	-4.1	25.9	3.9	-6.7
4 CONSTRUCTION	-5.7	-12.9	-8.6	3.4	-11.4	-3.3	7.0	1.2	-14.9
5 FOOD,TOBACCO	3.2	11.6	2.7	12.0	12.9	22.4	8.0	6.2	13.2
6 FABRICS,TEXTILE PRODS.	-29.4	-6.8	-4.4	0.4	43.9	13.0	4.2	0.5	8.1
7 TRANSP.EQUIP.,ORDNANCE	-0.2	-7.2	-23.4	13.2	15.2	5.9	12.4	6.8	22.3
8 MANUF.PRODS.,EXC.MACH.	-23.5	-27.4	-8.8	20.8	35.9	8.7	88.1	10.0	13.5
9 MACHINERY,EQUIPMENT	-1.7	10.6	-1.7	9.2	18.6	15.2	16.8	6.4	31.7
10 SERVICES	-28.6	55.5	-42.3	7.1	24.8	-20.3	-38.1	9.5	-15.3

TABLE D-5

WEIGHTED PERCENTAGE DIFFERENCES BETWEEN ESTIMATED AND ACTUAL 1958 OUTPUTS
(GRAVITY COEFFICIENT MODEL)

	1 NEW ENGLAND	2 MIDDLE ATLANTIC	3 EAST NORTH CENTRAL	4 WEST NORTH CENTRAL	5 SOUTH ATLANTIC	6 EAST SOUTH CENTRAL	7 WEST SOUTH CENTRAL	8 MOUNTAIN	9 PACIFIC
1 LIVESTOCK	-0.4	0.0	-9.1	-1.1	-3.3	-3.5	1.3	1.6	3.0
2 OTHER AGRICULTURE	0.3	4.4	6.5	-10.7	8.2	4.9	-14.4	0.7	-8.0
3 MINING	0.4	-2.1	-1.5	1.8	-0.9	-0.6	3.7	-1.5	-4.5
4 CONSTRUCTION	-1.0	-4.5	-6.2	-0.5	-2.2	-0.8	-4.1	0.0	-1.4
5 FOOD, TOBACCO	0.9	5.0	2.3	15.6	7.2	3.5	10.4	4.3	3.1
6 FABRICS, TEXTILE PRODS.	-4.5	-1.7	0.1	0.3	8.0	5.9	1.3	0.1	2.6
7 TRANSP.EQUIP., ORDNANCE	-0.6	-0.6	30.7	6.5	3.9	1.7	0.7	2.6	-0.2
8 MANUF.PRODS., EXC.MACH.	4.5	30.5	41.8	13.1	23.3	6.9	30.5	5.2	-13.8
9 MACHINERY, EQUIPMENT	2.4	-4.1	3.9	6.6	5.0	2.1	7.5	4.0	14.1
10 SERVICES	-3.6	20.1	-63.6	13.8	7.9	-4.3	-17.7	7.8	-12.8

APPENDIX E

PERCENTAGE CHANGES IN OUTPUTS, 1947 TO 1958, 1963, 1970, 1980

TABLE E-1

NEW ENGLAND
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	(58-47)/47	(63-47)/47	(70-47)/47	(80-47)/47	(70-47)/47	(80-47)/47
1 LIVESTOCK	26	30	47	98	49	106
2 OTHER AGRICULTURE	3	27	38	89	42	88
3 MINING	19	47	96	196	81	154
4 CONSTRUCTION	71	97	98	248	97	247
5 FOOD,TOBACCO	45	67	90	162	93	174
6 FABRICS,TEXTILE PRODS.	-21	-18	13	60	4	36
7 TRANSP.EQUIP.,ORDNANCE	160	214	360	469	382	464
8 MANUF.PRODS.,EXC.MACH.	8	38	84	176	74	152
9 MACHINERY,EQUIPMENT	12	60	133	276	107	212
10 SERVICES	36	68	126	252	124	246
11 REGIONAL TOTAL	26	56	107	216	100	198

TABLE E-2

MIDDLE ATLANTIC
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	(58-47)/47	(63-47)/47	(70-47)/47	(80-47)/47	(70-47)/47	(80-47)/47
1 LIVESTOCK	18	21	37	84	31	68
2 OTHER AGRICULTURE	11	53	56	112	47	88
3 MINING	-38	-35	-12	32	-15	27
4 CONSTRUCTION	72	70	111	286	111	286
5 FOOD,TOBACCO	30	43	64	121	62	116
6 FABRICS,TEXTILE PRODS.	17	31	83	161	75	136
7 TRANSP.EQUIP.,ORDNANCE	39	78	161	236	163	215
8 MANUF.PRODS.,EXC.MACH.	11	38	85	177	83	167
9 MACHINERY,EQUIPMENT	36	70	154	312	161	321
10 SERVICES	37	63	110	213	110	211
11 REGIONAL TOTAL	29	53	101	203	101	198

TABLE E-3

EAST NORTH CENTRAL
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 (58-47)/47	2 (63-47)/47	3 (70-47)/47	4 (80-47)/47	5 (70-47)/47	6 (80-47)/47
1 LIVESTOCK	16	8	23	67	24	67
2 OTHER AGRICULTURE	8	29	38	87	37	87
3 MINING	-5	8	46	120	46	120
4 CONSTRUCTION	55	57	92	233	92	233
5 FOOD,TOBACCO	23	37	62	125	61	123
6 FABRICS,TEXTILE PRODS.	14	45	105	194	117	236
7 TRANSP.EQUIP.,ORDNANCE	15	94	177	267	163	254
8 MANUF.PRODS.,EXC.MACH.	16	53	106	208	105	203
9 MACHINERY,EQUIPMENT	16	54	132	275	130	259
10 SERVICES	44	65	128	264	127	262
11 REGIONAL TOTAL	28	58	114	227	112	223

TABLE E-4

WEST NORTH CENTRAL
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 (58-47)/47	2 (63-47)/47	3 (70-47)/47	4 (80-47)/47	5 (70-47)/47	6 (80-47)/47
1 LIVESTOCK	22	23	38	86	31	69
2 OTHER AGRICULTURE	14	5	12	52	5	37
3 MINING	15	35	84	176	78	156
4 CONSTRUCTION	58	43	69	179	68	178
5 FOOD,TOBACCO	10	29	51	108	45	91
6 FABRICS,TEXTILE PRODS.	17	36	90	171	81	150
7 TRANSP.EQUIP.,ORDNANCE	181	275	461	655	512	746
8 MANUF.PRODS.,EXC.MACH.	45	81	140	260	134	252
9 MACHINERY,EQUIPMENT	49	113	222	435	208	459
10 SERVICES	31	54	102	216	100	213
11 REGIONAL TOTAL	32	50	90	185	87	179

TABLE E-5

SOUTH ATLANTIC
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	(58-47)/47	(63-47)/47	(70-47)/47	(80-47)/47	(70-47)/47	(80-47)/47
1 LIVESTOCK	55	50	73	138	78	153
2 OTHER AGRICULTURE	45	79	94	171	95	179
3 MINING	-8	2	37	110	33	108
4 CONSTRUCTION	70	98	124	337	124	338
5 FOOD,TOBACCO	35	61	98	185	103	199
6 FABRICS,TEXTILE PRODS.	58	111	201	339	219	389
7 TRANSP.EQUIP.,ORDNANCE	216	346	589	856	674	1042
8 MANUF.PRODS.,EXC.MACH.	56	122	197	363	195	383
9 MACHINERY,EQUIPMENT	155	356	592	1077	619	1243
10 SERVICES	63	113	207	435	208	440
11 REGIONAL TOTAL	59	107	182	362	186	380

TABLE E-6

EAST SOUTH CENTRAL
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	(58-47)/47	(63-47)/47	(70-47)/47	(80-47)/47	(70-47)/47	(80-47)/47
1 LIVESTOCK	41	39	59	119	66	145
2 OTHER AGRICULTURE	-3	54	59	120	40	105
3 MINING	-6	6	43	121	40	135
4 CONSTRUCTION	58	75	129	341	129	342
5 FOOD,TOBACCO	65	92	137	238	145	257
6 FABRICS,TEXTILE PRODS.	55	118	211	351	189	298
7 TRANSP.EQUIP.,ORDNANCE	149	326	552	791	507	686
8 MANUF.PRODS.,EXC.MACH.	43	91	160	303	162	339
9 MACHINERY,EQUIPMENT	253	392	662	1192	704	1413
10 SERVICES	38	70	146	327	145	331
11 REGIONAL TOTAL	46	83	150	306	149	318

TABLE E-7

WEST SOUTH CENTRAL
PERCENTAGE CHANGES IN OUTPUTS
(1958, 1963, 1970, 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 (58-47)/47	2 (63-47)/47	3 (70-47)/47	4 (80-47)/47	5 (70-47)/47	6 (80-47)/47
1 LIVESTOCK	27	38	59	119	72	145
2 OTHER AGRICULTURE	56	30	79	146	112	198
3 MINING	52	81	145	271	165	295
4 CONSTRUCTION	56	51	79	198	80	200
5 FOOD,TOBACCO	9	45	82	167	87	180
6 FABRICS,TEXTILE PRODS.	73	141	247	410	271	472
7 TRANSP.EQUIP.,ORDNANCE	339	563	1108	1535	1246	1705
8 MANUF.PRODS.,EXC.MACH.	81	149	245	424	274	467
9 MACHINERY,EQUIPMENT	99	261	529	967	667	1224
10 SERVICES	38	64	142	311	148	319
11 REGIONAL TOTAL	50	81	154	304	169	327

TABLE E-8

MOUNTAIN
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	(58-47)/47	(63-47)/47	(70-47)/47	(80-47)/47	(70-47)/47	(80-47)/47
1 LIVESTOCK	49	66	86	156	71	121
2 OTHER AGRICULTURE	32	36	52	109	41	85
3 MINING	98	134	192	354	167	337
4 CONSTRUCTION	125	139	165	418	165	417
5 FOOD,TOBACCO	53	108	145	253	137	234
6 FABRICS,TEXTILE PRODS.	79	160	248	398	261	416
7 TRANSP.EQUIP.,ORDNANCE	527	1301	2027	2738	1795	2441
8 MANUF.PRODS.,EXC.MACH.	86	142	218	402	204	397
9 MACHINERY,EQUIPMENT	119	398	692	1276	700	1210
10 SERVICES	65	114	186	383	183	379
11 REGIONAL TOTAL	75	119	179	358	171	346

TABLE E-9

PACIFIC
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 (58-47)/47	2 (63-47)/47	3 (70-47)/47	4 (80-47)/47	5 (70-47)/47	6 (80-47)/47
1 LIVESTOCK	48	88	116	210	116	217
2 OTHER AGRICULTURE	46	44	69	135	69	140
3 MINING	17	18	50	137	54	143
4 CONSTRUCTION	48	131	87	266	87	266
5 FOOD,TOBACCO	55	89	134	254	135	260
6 FABRICS,TEXTILE PRODS.	111	190	318	519	300	482
7 TRANSP.EQUIP.,ORDNANCE	227	340	543	728	549	724
8 MANUF.PRODS.,EXC.MACH.	67	109	173	328	170	331
9 MACHINERY,EQUIPMENT	131	340	558	1028	538	1034
10 SERVICES	54	96	168	360	168	360
11 REGIONAL TOTAL	63	114	173	350	172	351

TABLE F-10

NATIONAL INDUSTRY TOTAL
PERCENTAGE CHANGES IN OUTPUTS
(1958,1963,1970,1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 (58-47)/47	2 (63-47)/47	3 (70-47)/47	4 (80-47)/47	5 (70-47)/47	6 (80-47)/47
1 LIVESTOCK	28	30	48	103	47	100
2 OTHER AGRICULTURE	25	32	48	103	48	103
3 MINING	15	32	76	168	78	172
4 CONSTRUCTION	63	81	99	264	99	264
5 FOOD,TOBACCO	29	52	81	157	81	157
6 FABRICS,TEXTILE PRODS.	26	55	119	216	119	215
7 TRANSP.EQUIP.,ORDNANCE	66	148	265	381	264	381
8 MANUF.PRODS.,EXC.MACH.	30	69	127	244	126	244
9 MACHINERY,EQUIPMENT	35	90	186	369	186	369
10 SERVICES	44	74	137	285	137	285
11 NATIONAL TOTAL	39	71	128	256	128	256

APPENDIX F

ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS 1947 TO 1980

TABLE F-1

NEW ENGLAND
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	47 TO 58	58 TO 63	63 TO 70	70 TO 80	63 TO 70	70 TO 80
1 LIVESTOCK	2.1	0.7	1.8	3.0	2.0	3.3
2 OTHER AGRICULTURE	0.3	4.3	1.2	3.2	1.5	2.8
3 MINING	1.6	4.3	4.2	4.2	3.0	3.4
4 CONSTRUCTION	5.0	2.9	0.1	5.8	0.0	5.8
5 FOOD,TOBACCO	3.4	2.8	1.9	3.3	2.2	3.5
6 FABRICS,TEXTILE PRODS.	-2.2	0.7	4.8	3.6	3.4	2.7
7 TRANSP.EQUIP.,ORDNANCE	9.1	3.8	5.6	2.2	6.2	1.6
8 MANUF.PRODS.,EXC.MACH.	0.7	5.0	4.2	4.1	3.3	3.8
9 MACHINERY,EQUIPMENT	1.1	7.4	5.5	4.9	3.7	4.2
10 SERVICES	2.9	4.3	4.4	4.5	4.2	4.5
11 REGIONAL TOTAL	2.1	4.3	4.1	4.3	3.6	4.1

TABLE F-2

MIDDLE ATLANTIC
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 47 TO 58	2 58 TO 63	3 63 TO 70	4 70 TO 80	5 63 TO 70	6 70 TO 80
1 LIVESTOCK	1.5	0.5	1.8	3.0	1.2	2.5
2 OTHER AGRICULTURE	0.9	6.7	0.3	3.1	-0.7	2.5
3 MINING	-4.3	1.1	4.2	4.2	3.8	4.1
4 CONSTRUCTION	5.0	-0.1	3.1	6.2	3.1	6.2
5 FOOD,TOBACCO	2.4	1.9	1.9	3.0	1.8	2.9
6 FABRICS,TEXTILE PRODS.	1.4	2.4	4.9	3.6	4.2	3.0
7 TRANSP.EQUIP.,ORDNANCE	3.0	5.1	5.6	2.6	5.7	1.8
8 MANUF.PRODS.,EXC.MACH.	1.0	4.5	4.2	4.1	4.1	3.9
9 MACHINERY,EQUIPMENT	2.8	4.6	5.9	4.9	6.3	4.9
10 SERVICES	2.9	3.5	3.7	4.1	3.7	4.0
11 REGIONAL TOTAL	2.4	3.4	4.0	4.2	3.9	4.0

TABLE E-3

EAST NORTH CENTRAL
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 47 TO 58	2 58 TO 63	3 63 TO 70	4 70 TO 80	5 63 TO 70	6 70 TO 80
1 LIVESTOCK	1.4	-1.6	1.9	3.1	2.0	3.0
2 OTHER AGRICULTURE	0.7	3.5	1.0	3.1	0.8	3.1
3 MINING	-0.5	2.5	4.4	4.2	4.3	4.2
4 CONSTRUCTION	4.1	0.2	2.9	5.7	2.9	5.7
5 FOOD,TOBACCO	1.9	2.2	2.4	3.3	2.3	3.3
6 FABRICS,TEXTILE PRODS.	1.2	5.0	5.0	3.7	6.0	4.5
7 TRANSP.EQUIP.,ORDNANCE	1.3	11.0	5.2	2.9	4.4	3.0
8 MANUF.PRODS.,EXC.MACH.	1.4	5.6	4.3	4.1	4.3	4.0
9 MACHINERY,EQUIPMENT	1.4	5.9	6.0	4.9	5.9	4.6
10 SERVICES	3.4	2.7	4.7	4.8	4.7	4.8
11 REGIONAL TOTAL	2.3	4.3	4.5	4.3	4.3	4.3

TABLE F-4

WEST NORTH CENTRAL
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

		ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
		1	2	3	4	5	6
		47 TO 58	58 TO 63	63 TO 70	70 TO 80	63 TO 70	70 TO 80
1	LIVESTOCK	1.8	0.1	1.7	3.1	1.0	2.6
2	OTHER AGRICULTURE	1.2	-1.6	0.9	3.1	0.0	2.7
3	MINING	1.3	3.3	4.5	4.1	3.6	3.7
4	CONSTRUCTION	4.3	-1.9	2.4	5.2	2.3	5.1
5	FOOD,TOBACCO	0.8	3.3	2.3	3.3	1.7	2.8
6	FABRICS,TEXTILE PRODS.	1.4	3.1	4.9	3.6	4.1	3.3
7	TRANSP.EQUIP.,ORDNANCE	9.8	5.9	5.9	3.0	7.2	3.3
8	MANUF.PRODS.,EXC.MACH.	3.5	4.4	4.2	4.1	3.8	4.2
9	MACHINERY,EQUIPMENT	3.7	7.5	6.1	5.2	5.4	6.1
10	SERVICES	2.5	3.3	3.9	4.6	3.8	4.6
11	REGIONAL TOTAL	2.5	2.6	3.5	4.1	3.2	4.1

TABLE E-5

SOUTH ATLANTIC
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	47 TO 58	58 TO 63	63 TO 70	70 TO 80	63 TO 70	70 TO 80
1 LIVESTOCK	4.1	-0.6	2.0	3.2	2.5	3.6
2 OTHER AGRICULTURE	3.4	4.3	1.2	3.4	1.1	3.7
3 MINING	-0.8	2.3	4.2	4.4	3.7	4.6
4 CONSTRUCTION	4.9	3.2	1.8	6.9	1.8	6.9
5 FOOD, TOBACCO	2.8	3.7	3.0	3.7	3.4	3.9
6 FABRICS, TEXTILE PRODS.	4.2	6.0	5.2	3.8	6.1	4.4
7 TRANSP.EQUIP.,ORDNANCE	11.0	7.2	6.4	3.3	8.1	4.0
8 MANUF.PRODS.,EXC.MACH.	4.1	7.3	4.3	4.5	4.1	5.1
9 MACHINERY, EQUIPMENT	8.9	12.4	6.1	5.5	6.7	6.5
10 SERVICES	4.5	5.5	5.4	5.7	5.4	5.8
11 REGIONAL TOTAL	4.3	5.4	4.5	5.1	4.7	5.3

TABLE F-6

EAST SOUTH CENTRAL
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 47 TO 58	2 58 TO 63	3 63 TO 70	4 70 TO 80	5 63 TO 70	6 70 TO 80
1 LIVESTOCK	3.2	-0.3	2.0	3.2	2.6	3.9
2 OTHER AGRICULTURE	-0.3	9.7	0.4	3.3	-1.4	3.9
3 MINING	-0.6	2.4	4.4	4.5	3.9	5.3
4 CONSTRUCTION	4.2	2.0	3.9	6.8	3.9	6.8
5 FOOD, TOBACCO	4.7	3.1	3.0	3.6	3.6	3.8
6 FABRICS, TEXTILE PRODS.	4.0	7.1	5.2	3.8	4.1	3.3
7 TRANSP.EQUIP., ORDNANCE	8.7	11.3	6.3	3.2	5.1	2.6
8 MANUF.PRODS., EXC.MACH.	3.3	6.0	4.5	4.5	4.6	5.3
9 MACHINERY, EQUIPMENT	12.1	6.9	6.5	5.4	7.2	6.5
10 SERVICES	3.0	4.3	5.4	5.7	5.3	5.8
11 REGIONAL TOTAL	3.5	4.7	4.6	5.0	4.5	5.3

TABLE 5-7

WEST SOUTH CENTRAL
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	47 TO 58	58 TO 63	63 TO 70	70 TO 80	63 TO 70	70 TO 80
1 LIVESTOCK	2.2	1.7	2.1	3.3	3.2	3.6
2 OTHER AGRICULTURE	4.1	-3.6	4.6	3.3	7.2	3.5
3 MINING	3.9	3.6	4.4	4.2	5.6	4.1
4 CONSTRUCTION	4.1	-0.6	2.4	5.2	2.5	5.2
5 FOOD,TOBACCO	0.8	5.8	3.3	3.9	3.7	4.1
6 FABRICS,TEXTILE PRODS.	5.1	6.9	5.3	3.9	6.3	4.4
7 TRANSP.EQUIP.,ORDNANCE	14.4	8.6	8.9	3.1	10.6	3.0
8 MANUF.PRODS.,EXC.MACH.	5.6	6.6	4.8	4.3	6.0	4.2
9 MACHINERY,EQUIPMENT	6.4	12.7	8.2	5.4	11.3	5.6
10 SERVICES	3.0	3.5	5.7	5.4	6.1	5.4
11 REGIONAL TOTAL	3.8	3.8	5.0	4.7	5.8	4.7

TABLE F-3

MOUNTAIN
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1 47 TO 58	2 58 TO 63	3 63 TO 70	4 70 TO 80	5 63 TO 70	6 70 TO 80
1 LIVESTOCK	3.7	2.2	1.6	3.3	0.4	2.6
2 OTHER AGRICULTURE	2.5	0.6	1.6	3.2	0.5	2.8
3 MINING	6.4	3.4	3.2	4.5	2.2	5.0
4 CONSTRUCTION	7.7	1.2	1.5	6.9	1.5	6.9
5 FOOD,TOBACCO	3.9	6.3	2.4	3.7	1.8	3.5
6 FABRICS,TEXTILE PRODS.	5.4	7.8	4.3	3.6	4.8	3.6
7 TRANSP.EQUIP.,ORDNANCE	18.2	17.5	6.1	2.9	4.3	3.0
8 MANUF.PRODS.,EXC.MACH.	5.8	5.4	4.0	4.6	3.3	5.0
9 MACHINERY,EQUIPMENT	7.4	17.9	6.8	5.7	7.0	5.1
10 SERVICES	4.6	5.3	4.3	5.4	4.1	5.4
11 REGIONAL TOTAL	5.2	4.6	3.5	5.1	3.1	5.1

TABLE F-9

PACIFIC
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	47 TO 58	58 TO 63	63 TO 70	70 TO 80	63 TO 70	70 TO 80
1 LIVESTOCK	3.6	4.8	2.0	3.7	1.9	3.9
2 OTHER AGRICULTURE	3.5	-0.3	2.3	3.4	2.3	3.6
3 MINING	1.5	0.0	3.5	4.7	3.8	4.6
4 CONSTRUCTION	3.6	9.4	-2.9	6.9	-2.9	6.9
5 FOOD, TOBACCO	4.1	4.0	3.1	4.2	3.1	4.3
6 FABRICS, TEXTILE PRODS.	7.0	6.6	5.3	4.0	4.7	3.8
7 TRANSP.EQUIP., ORDNANCE	11.4	6.1	5.6	2.6	5.7	2.4
8 MANUF.PRODS., EXC.MACH.	4.8	4.5	3.9	4.6	3.7	4.8
9 MACHINERY, EQUIPMENT	7.9	13.7	5.9	5.5	5.4	5.9
10 SERVICES	4.0	4.9	4.6	5.5	4.6	5.6
11 REGIONAL TOTAL	4.5	5.6	3.6	5.1	3.5	5.2

TABLE F-10

NATIONAL INDUSTRY TOTAL
ANNUAL COMPOUND RATES OF GROWTH OF OUTPUTS
(1947 TO 1980)

	ACTUAL DATA		COLUMN MODEL		GRAVITY MODEL	
	1	2	3	4	5	6
	47 TO 58	58 TO 63	63 TO 70	70 TO 80	63 TO 70	70 TO 80
1 LIVESTOCK	2.3	0.4	1.9	3.2	1.7	3.1
2 OTHER AGRICULTURE	2.0	1.2	1.6	3.2	1.6	3.2
3 MINING	1.3	2.7	4.2	4.3	4.3	4.3
4 CONSTRUCTION	4.5	2.2	1.4	6.2	1.4	6.2
5 FOOD,TOBACCO	2.4	3.2	2.6	3.5	2.6	3.5
6 FABRICS,TEXTILE PRODS.	2.1	4.2	5.1	3.7	5.0	3.7
7 TRANSP.EQUIP.,ORDNANCE	4.7	8.4	5.7	2.8	5.6	2.8
8 MANUF.PRODS.,EXC.MACH.	2.4	5.4	4.3	4.3	4.2	4.3
9 MACHINERY,EQUIPMENT	2.8	7.0	6.0	5.1	6.0	5.1
10 SERVICES	3.3	3.9	4.6	4.9	4.5	4.9
11 NATIONAL TOTAL	3.0	4.2	4.2	4.6	4.2	4.6

APPENDIX G

COMPUTER PROGRAM TO INVERT A MATRIX BY ITERATION

APPENDIX G

ITER

```
      PROGRAM ITER(INPUT,OUTPUT,TAPE11,TAPE12,TAPE13,TAPE14)
C CDC FORTRAN ITERATION PROGRAM FOR COLUMN COEFFICIENT MODEL
C START, Y ON DS13, A ON DS11, C ON DS12
C    Y=TOTAL FINAL DEMANDS
C    C=COLUMN TRADE COEFFICIENTS
C    A=TECHNICAL COEFFICIENTS
C THE RESULTS(INDUSTRY OUTPUTS) ARE WRITTEN ON DS14, IN STANDARD MOTHER
C FORMAT.
C    X=TOTAL OUTPUTS
C XPRE1 AND XPRE2 ARE DUMMY ARRAYS FOR STORING VALUES OF X DURING
C ITERATION.
       DIMENSION A(79,79),C(51,51),XPREV(79,51),X(79,51),Y(79,51),
     1XPRE1(79,51),XPRE2(79,51),TITLE(20),CREATOR(6),DATSET(6)
      EQUIVALENCE (A(1,1),C(1,1))
      CALL FTNBIN(1,0,DUMMY)
      DATA ACODE,BLK/8H99999998,1H /
C THE NEXT STATEMENET READS IN INFORMATION FOR WRITING MATRIX LABEL
C    CREATOR=NAME OF PERSON MAKING THIS RUN
C    DATSET=NAME OF OUTPUT DATASET 14
C    DATE=DATE OF RUN
C    NPP=NUMBER OF Y MATRICES
      READ 100,CREATOR,DATSET,DATE,NPP
      IP=1
      L=0
      NREG=51
      MIND =79
      DO 250 NP=1,NPP
C THE NEXT STATEMENTS READ IN TITLE OF OUTPUT MATRIX, AND Y MATRIX IN
C STANDARD MOTHER FORMAT(FIRST DUMMY READ IS FOR THE MATRIX LABEL)
      READ 90,TITLE
      READ(13)
      READ(13) ((Y(N,M),M=1,NREG),N=1,MIND)
C ONLY THE FIRST 64 INDUSTRIES NEED TO BE MULTIPLIED BY C BECAUSE THEY
C ARE THE ONLY ONES THAT ARE TRADED
      DO 20 INDUS=1,64
       READ(12)
      READ(12)((C(N,M),M=1,NREG),N=1,NREG)
      DO 10 I=1,NREG
      SUMACC=0
      DO 9 J=1,NREG
    9 SUMACC=SUMACC+C(I,J)*Y(INDUS,J)
      X(INDUS,I)=SUMACC
   10 XPREV(INDUS,I)=SUMACC
   20 CONTINUE
      REWIND 12
      DO 25 INDUS=65,MIND
      DO 25 JREG=1,NREG
      X(INDUS,JREG)=Y(INDUS,JREG)
   25 XPREV(INDUS,JREG)=Y(INDUS,JREG)
C XPREV IS NOW EQUAL TO C TIMES Y
C ITERATION STARTS HERE  WITH A MAXIMUM OF 15 TIMES
```

APPENDIX G (CONT'D)
ITER

```
      DO 150 NITER=1,15
      DO 110 I=1,79
        DO 110 J=1,51
  110   XPRE1(I,J)=X(I,J)
      DO 40 IREG=1,NREG
      READ(11)
      READ(11) ((A(N,M),M=1,MIND),N=1,MIND)
      DO 36 I=1,MIND
      SUMACC=0
      DO 35 J=1,MIND
   35 SUMACC=SUMACC+A(I,J)*XPRE1(J,IREG)
C Y IS NOW USED TO STORE A TIMES THE PREVIOUS X
   36 Y(I,IREG)=SUMACC
   40 CONTINUE
      REWIND 11
      DO 50 INDUS=1,64
       READ(12)
      READ(12)((C(N,M),M=1,NREG),N=1,NREG)
      DO 46 I=1,NREG
      SUMACC=0
      DO 45 J=1,NREG
   45 SUMACC=SUMACC+ C(I,J)*Y(INDUS,J)
   46 XPRE2(INDUS,I)=SUMACC
C XPRE2 IS SET EQUAL TO CA TIMES THE PREVIOUS X
   50 CONTINUE
      REWIND 12
      DO 51 I=65,79
      DO 51 J=1,NREG
   51 XPRE2(I,J)=Y(I,J)
      DO 60 I=1,MIND
      DO 60 J=1,NREG
   60 X(I,J)=XPREV(I,J)+XPRE2(I,J)
C AT THIS POINT, A NEW X HAS BEEN CALCULATED AND MUST BE COMPARED WITH
C THE PREVIOUS ARRAY TO SEE WHETHER THE ITERATION SHOULD BE CONTINUED.
C AS SOON AS THE MAXIMUM RELATIVE CHANGE(RELCHG) IN ANY INDUSTRY TOTALS
C (ROW SUMS) GOES BELOW .0005, THE PROCEDURE IS STOPPED AND THE LAST X
C CALCULATED IS THE OUTPUT MATRIX.
      RELCHG=0.0
      DO 70 I=1,MIND
      SUMACC=0
      SUMAC1=0
      DO 75 J=1,NREG
      SUMACC=SUMACC+X(I,J)**2
   75 SUMAC1=SUMAC1+(X(I,J)-XPRE1(I,J))**2
C THE NEXT TWO STATEMENTS ARE NECESSARY TO AVOID ANY ZERO DIVIDE.
      RELCHG1=0.0
      IF(SUMACC.EQ.0.0) GO TO 70
      PP=SUMAC1/SUMACC
      RELCHG1=SQRT(PP)
   70 IF(RELCHG1.GT.RELCHG) RELCHG=RELCHG1
      IF(NITER.EQ.1) PRINT 80
      PRINT 120,NITER,RELCHG
```

APPENDIX G (CONT'D)
ITER

```
      IF (RELCHG .LT. .0005) GO TO 200
  150 CONTINUE
C THE MATRIX X IS THEN WRITTEN ON DS14 IN STANDARD MOTHER FORMAT
C THE FIRST WRITE STATEMENT IS FOR THE MATRIX LABEL
  200 WRITE(14)NP,ACODE,CREATOR,(BLK,J=1,7),DATE,BLK,MIND,NREG,IP,TITLE,
     1L,L,L,L,DATSET,(BLK,K=1,66)
  250 WRITE(14) ((X(N,M),M=1,NREG),N=1,MIND)
      ENDFILE 14
   80 FORMAT(1H1,10X,*NUMBER OF*,20X,*MAX. REL. CHANGE IN*/10X,
     1*ITERATIONS*,21X,*INDUSTRY OUTPUTS*/10X,10(1H-),20X,19(1H-))
 90   FORMAT(20A4)
 100  FORMAT(6A4,6A4,A10,I4)
  120 FORMAT(1H ,13X,I2,28X,E10.3)
      RETURN
      END
```

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