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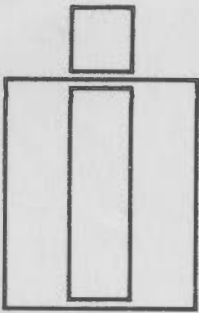
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A SIMULATION APPROACH TO THE CONSTRUCTION
OF INVESTMENT COEFFICIENTS

by

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SIXTH INTERNATIONAL CONFERENCE
ON input-output techniques



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1962-63

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I. CONSTRUCTION OF INVESTMENT COEFFICIENTS

It is supposed in theoretical works especially when studying the dynamic models that in connection with models of structural analysis the matrix of investment coefficients exists. Besides the investment coefficients are considered as time-constant. From the practical point of view both assumptions are rather optimistic.

The existence of investment coefficients matrix does assume to performe a special investigation and/or, for the permanent aplication of input-output analysis for the planning it assumes the construction of a system which would secure the informations for the investment coefficients matrix. From practical experiences of socialist states one can draw the conclusion that the information system concerning the past investment processes does not allways give the right informations, there are rather often differences towards reality. It is possible to reduce these shortcommings by improving the quality of informations but it is very difficult to take such a danger definitely off.

The second assumption about the matrix of investment coefficients being constant is also wrong. Namely the new investment processes differ from the previons ones and owing to this change with the time, the investment coefficient of the whole national economy is also changed as well as the investment coefficients of majority of branches. The investment coefficients are constant only in few branches of national economy. Of course at the same time one should not forget that as a rule the rank of investment coefficients matrix is lower than that of the technological coefficients. Therefore when considering the dynamic systems we have to respect this property. Few unfinished computation procedures exist which take into consideration the property mentiond above /1-6/.

The process of construction of investment coefficients matrix may be divided into two basic group, if considering the transformation processes of the investment modelling :

a/ transformations

$$I_t \longrightarrow dF_{t+\tau} \longrightarrow dX_{t+\tau}$$

where I_t are the investments during time t , $dF_{t+\tau}$ is increment of the capital stock during time $t+\tau$ where τ is the time lag and, $dX_{t+\tau}$ is the output increment during time $t+\tau$.

b/ transformations

$$dX_{t+\tau} \longrightarrow dF_{t+\tau} \longrightarrow I_t.$$

The transformations ad a/ require usually a special investigation of the investment coefficients. The investment coefficients are calculated by using the formula :

$$\varepsilon_{ij} = \frac{I_{ijt}}{dX_{jt+\tau}} \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array}$$

where indexes i belong to the input brauches, and the indexes j belong to the output brauches.

In the more analytic models the investment coefficients are constructed on the ground of plan informations. From the point of view of time as for instance for the middle-term models the appropriate coefficients may be divided as follows :

- the coefficients belonging to the investments which will be finished during the planned term and,
- the coefficient belonging to the investments which will be finished after the planned term.

Such principles for building his model has used G.L.Šagalov /7/.

In the connection with that we wish to note that in the socialist economies there is a real possibility of creating the investment coefficients matrices for the need in the middle-term plans on the basis of plan informations. The reason why is, that there are informations about the investment intentions which from the point of view of a middle-term planning are of obligatory character. Of course, the deviations from the appropriate intentions arise and, the character and the direction of those deviations is carefully watched. The deviations mentioned are mainly of one direction and they mainly mean that the planned expenses of constructions are overdo.

In Czechoslovakia such a motion has been investigated for 1154 constructions in the year 1971 and, 685 constructions in the year 1972. In both time-terms we have noted the planned expenses increase /20,5 % in the year 1971 and, 24,1 % in 1972/. As a main reason for overdo the cost is the improvement of the original parameters /37,9% and 36,9%/, the undervaluation of the origin budget /20,2 % and 18,1 %/, of the original budget /20,2 % and 18,1 %/, the price influences /25,0 % and 17,0 %/, and others influences not improving the technical and technological parameters /16,9 % and 28,04%/. There is not any essential change at the present time. Even more detailed research concerning the individual stages of constructions of hydro-electric power plants has been performed in the mentioned area in USSR /8/. The appropriate deviations were minimal during the first years of construction being of the value of about 3 %. At the end of construction /7 to 8 years/ they were 19,1%, both being overdone. In the hungarian economy the deviation was even as large as 22,2 % /9/. In spite of the rather easy estimated deviations, when constructing the investment coefficients, than there is a trend in the socialist economies towards exploitation the plan informations for long-term models also.

The transformations ad b/ are in most cases connected with the simulation approaches. There is a close coherence between them and appropriate production and investment functions.

The basis idea of such a treatment is to deduce the investment coefficients by means of production and investment function when at the same time a series of factors may enter the function which are simulated. The model may be constructed in such a way that the not allowed simulations may be seen as infeasible solutions.

As an example of possible approach we introduce a model in the 3-rd chapter. This model allows the calculation of investment coefficients and, at the same time it takes into consideration the limiting conditions of the economic system /the labor forces, investment resources, etc./. Before formulating the model we introduce the factors which are included in the model.

We derive the investment coefficients in accordance with the series of transformations of Leontjef's modified inverse production function. We suppose that in this function the production increments are determined in accordance with the system resources. The inversibility of the function is understood in such a sence that the capital stocks increments are derived from the production increments.

$$dF_{jt} = f_1/dx_{jt}, t, dS_{jt}/$$

where dx_{jt} are the production increments and, dF_{jt} are the capital stocks increments of the j^{th} branch during time t / $j = 1, \dots, 16$ /,

$$I_{jt} = f_2/dF_{jt}, \delta_{jt}F_{jt-1}, \tau_{jt}/$$

where I_{jt} are investments of the j^{th} branch during time t , $\delta_{jt}F_{jt-1}$ are the depreciated capital stocks of the j^{th} branch during the time t , τ_{jt} is the time lag between the investment process and the introduction the capital stocks into operation.

Analytically, in respect to the behaviour of the individual components, we are able to formulate the investment coefficients by the following way /we take into consideration the input branches/ :

$$g_{ijt} = \left[\frac{a_{ij} + b_{ij}/t + \tau_{jt}}{S_{jt}} \right] \frac{S_{jt-1}}{S_{jt}} / 1 + \tau_{jt} r_{jt} / + \quad /a/$$

$$+ \frac{F_{ijt-1} \delta_{ijt} + \tau_{jt} / 1 + \tau_{jt} r_{jt} /}{dx_{jt}} + \quad /b/$$

$$+ \frac{b_{ij} X_{ijt-1} / 1 + \tau_{jt} r_{jt} / \frac{S_{jt-1}}{S_{jt}}}{dx_{jt}} \quad /c/$$

for $i = 1, \dots, m$; $j = 1, \dots, n$

where

a_{ij}, b_{ij} are the capital/output ratio parameters,

S_{jt-1}/S_{jt} is the change of the capital stock time exploitation between time $t-1$ and t ,

$/1 + \tau_{jt} r_{jt} /$ is the linear approximation of speed of growth of the economic system, where r_{jt} is the system growth rate and,

F_{ijt-1} are the capital stocks supplied by the i^{th} branch in the j^{th} branch in time $t-1$.

It gives a real view when constructing the coefficients of the capital/output ratio according to two branches $/i=1,2/$ and in the connection with the building and machinery capital stocks. If input from other branches of national economy are

more important, it is necessary to schedule the coefficients statistically and/or to estimate them.

The most important component of the function mentioned above is the component /a/ which is time dependent according to coefficient b_{ijt} . This component of course does not itself change in respect to the production increments which, on the other hand, respect the economic system resources. The time exploitation of equipment as well as the production increment rate are also contents of the mentioned function.

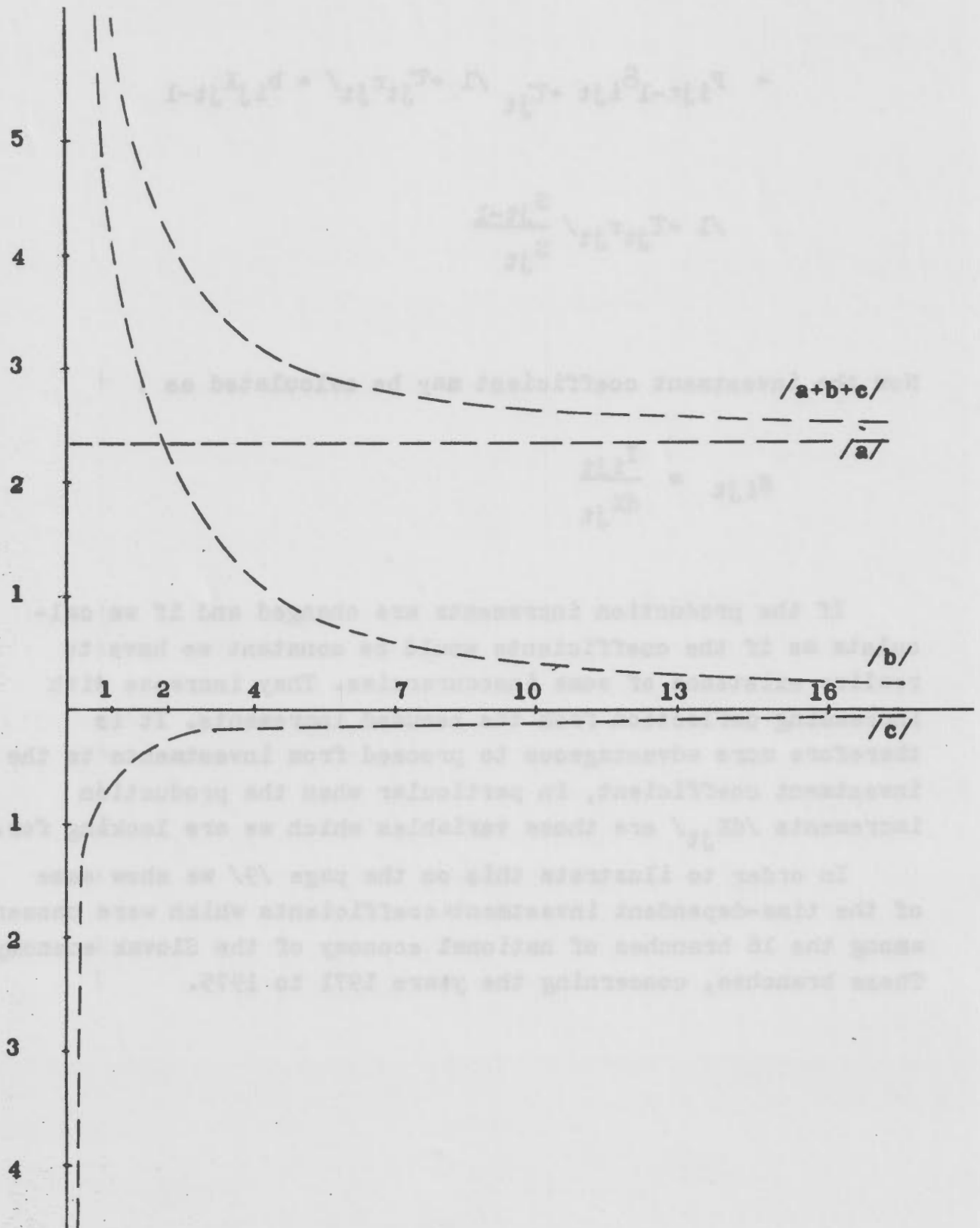
The component /b/ is in coherence with the depreciation of capital stocks. The mean value of this component in Slovak economy is 12%. The depreciation rate in investment coefficient is simulated δ_{ijt} .

The last component /c/ is in the connection with construction of capital/output ratio norm as an average norm of the total output /the production volume/. In individual branches it may gain positive or negative values which depends on the increase or decrease of the capital/output ratio. In accordance to this norm change, the need of investments also changed which is included in the investment coefficient. This component is also changed up to the production increment volume.

The dependence of the investment coefficients g_{ij} on the production increments may be shown by the diagram on the page /7/.

One can see that as far as the investment coefficient is concerned there is a tendency to decline with the production increment. Because of a number of parameters determining the value of the investment coefficient it seems useful to construct it by means of investments in the partial branches F_{ijt} . The considerations will be simplified when multiplying the appropriate a equation increment dX_{jt} .

Figure I: The dependence of the investment coefficients g_{ij} on the production increments



$$I_{ijt} = \left[\frac{1}{a_{ijt}} + b_{ijt} \frac{1}{1 + \tau_{jt}} \frac{S_{jt-1}}{S_{jt}} \right] \frac{1}{1 + \tau_{jt} r_{jt}} \frac{dX_{jt}}{dx_{jt}} +$$

$$+ F_{ijt-1} \delta_{ijt} + \tau_{jt} \frac{1}{1 + \tau_{jt} r_{jt}} + b_{ij} X_{jt-1}$$

$$\frac{1}{1 + \tau_{jt} r_{jt}} \frac{S_{jt-1}}{S_{jt}}$$

Now the investment coefficient may be calculated as :

$$g_{ijt} = \frac{I_{ijt}}{dX_{jt}}$$

If the production increments are changed and if we calculate as if the coefficients would be constant we have to realize existence of some inaccuracies. They increase with increasing deflection from the assumed increments. It is therefore more advantageous to proceed from investments to the investment coefficient, in particular when the production increments dX_{jt} are those variables which we are looking for.

In order to illustrate this on the page /9/ we show some of the time-dependent investment coefficients which were chosen among the 16 branches of national economy of the Slovak economy. These branches, concerning the years 1971 to 1975.

Table 1. The development of the investment coefficients in the national economy branches during the years 1971 to 1975

Inv. coeff.	years	Chemical industry	Machinery	Food industry	Building industry	Agriculture	Transport and communication
$I_{jt} / \Delta X_{jt}$	1971	1,165	0,820	1,164	0,469	3,729	10,764
	1972	1,104	0,792	1,190	0,486	3,826	10,103
	1973	1,042	0,765	1,215	0,482	3,922	9,684
	1974	0,981	0,738	1,241	0,478	4,019	9,263
	1975	0,918	0,710	1,267	0,474	3,961	9,141
$I_{jt}^1 / \Delta X_{jt}$	1971	0,310	0,366	0,510	0,148	2,375	4,672
	1972	0,294	0,353	0,521	0,147	2,437	4,385
	1973	0,277	0,341	0,532	0,146	2,498	4,203
	1974	0,261	0,329	0,544	0,144	2,560	4,020
	1975	0,224	0,317	0,555	0,143	2,523	3,967
$I_{jt}^2 / \Delta X_{jt}$	1971	0,855	0,454	0,654	0,321	1,354	6,092
	1972	0,810	0,439	0,669	0,339	1,389	5,718
	1973	0,765	0,424	0,683	0,336	1,424	5,481
	1974	0,720	0,409	0,697	0,334	1,459	5,243
	1975	0,674	0,393	0,712	0,331	1,438	5,174

I_{jt} - total investment

I_{jt}^1 - building and construction investment

I_{jt}^2 - machinery investment

II. SIMULATION OF THE INVESTMENT COEFFICIENTS

In the present analysis we have turned our attention to the composition of the investment coefficients and towards factors which create the investment coefficients. In the following part of this paper we are going to analyse the simulation parameters :

- a/ the time exploitation of equipment in the industry branches $\frac{S_0}{S_t}$;
- b/ the time lag between the beginning of the investment process and the introduction of the capital stocks into the operation ;
- c/ the depreciation of the capital stocks.

a/ The time exploitation

Better time exploitation of the capital stocks influences the investment process in such a sence, that the production volume may be increased with the simultaneously lower investment expenses. In our economy, the time exploitation of the capital stocks is measured by means of exploitation coefficient. The exploitation coefficient is a ratio of total number of days during which the labor workers have worked in all three working shifts to the number of working days with the most occupied shift /in most cases it is the first working shift of the day/.

In the individual investigations, the method of measurment of the equipment time exploitation is considered more in detail and, the measurment of exploitation of machine working places is being used.

As we have already mentioned the investment coefficient expresses the change of time utilization of capital stocks during time t in respect to the time t_0 , S_{t_0}/S_t . The development of this coefficient in the economy of our whole industry was irregular. Since 1955 till 1962 there was an upward trend /1,348 in 1955 and 1,485 in 1962/. After 1962 there was downward trend /1,479 in 1963 and, 1443 in 1972/.

$$\frac{S_{1963}}{S_{1972}} = \frac{1.479}{1.443} = 1,025$$

According to the estimation of experts the mentioned drop will stop and the coefficient will be improved. The time exploitation influences the investment coefficient relatively very little when passing from one year to the following one. However for middle-term or long-term planning respectively, the investment coefficient will be influenced by the time utilization on a much wider scale. The influence of the time utilization on the substitution production function will be shown by slowing down the substitution of capital by labour.

In the enclosed table we consider the influence of the time utilization on investment coefficients for pessimistic variant /0,95/ and an optimistic variant /1,05/. These estimations were performed after consulting the experts.

b/ The time-lag τ_{jt}

In respect to the rather high number of branches and the whole structure of the model, the time-lag may be expressed as a mean value. To calculate it, we use the method of statistical reports from which

$$\tau_{jt} = \frac{\text{unfinished investments at the end of an year}}{\text{volume of finished investment}}$$

The appropriate data for economy of Slovakia as a whole were calculated as $\tau_{1973} = 1,2$. It means that between the beginning and the end of an average object building there is a time-lag of 1.2 year. In the majority of branches of Slovak economy the average time-lag is from the interval 1 to 1,5 year. The extremal values of the time-lag are 2,5 year /energetics/ and, 0,5 year /building trade/ respectively. When simulating the mentioned values in optimistic variant one may assume that the building time is shorter by $0,25\tau_t$. It is assumed that the building time prolongation is of the same value.

c/ The depreciation of the capital stocks

For determination of the capital stocks depreciation it is necessary to consider the renovation processes which were performed in the past. Up to the present time the renovation process intensity of the Slovak economy is a rather low one. In the individual branches it is from the interval of 0,5% to 3,0%. An average value for Slovak economy is 1,14%. It is assumed that this intensity will be higher in future according to the exhausted labour force resources and therefore it will be necessary to go to the faster substitution of the fixed capital by the labour forces.

When expressing the depreciation influence on the investment coefficient we assume a pessimistic variant for which the lower renovation limit will not drop under the value 1,0% and, an optimistic variant with the value of renovation as high as 2,0%. The appropriate hypothesis is different for individual branches. The accepted assumptions of course influence the labour/output ratio and, in respect to the hypothesis of full employment which is inherent in socialistic economy, they will influence the change of the production structure also.

When simulating the investment coefficient we proceed in such a way that we change the simulation parameters or, when

choosing the variant we consider the average parameters of the simulation.

To express the influence of the simulation parameters on the investment norm an appendix is enclosed where we show the optimistic as well as the pessimistic variant of the investment coefficients.

The model consists of 16 productive and 1 nonproductive branches and it is designed for forecasting purposes and/or middle planning accomplished in practice by the State Planning Commission. The submodel is based on the exogenously given production or other productivity production increments according to branches determined by individual forecasts which are transformed by means of behavioral functions into the demands for resources which represent in the model the volume of investment or better the volume of national income reserved for investment and labour force.

The submodel of production and investment

The demands for resources (labour force and investments) are computed with the total disposable resources while the output increments are given exogenously and the replacement of equipment, depreciated capital stocks and the like are simulated. Apparently, the forecasted growth of output (volume of production) will not be constant with the total capacities of corresponding subsystems. Therefore a detailed process which shows the way of correcting the input parameters of the subsystem must be applied.

III. SIMULATION MODEL OF ECONOMY WITH USING OF INVESTMENT COEFFICIENT MATRIX

In this chapter the simulation model is presented and its partial result also is the investment coefficient matrix.

The model consists of 16 productive and 1 nonproductive branches and it is designed for forecasting purposes and/or middle planning accomplished in praxis by the Slovak Planning Commission.

The submodel is based on the exogenously given production or more precisely production increments according to branches determined by individual forecasts which are transformed by means of behavioural functions into the demands for resources which represent in the model the volume of investment or better the volume of national income reserved for investment and labour force.

The subsystem of production and investment

The demands for resources /labour force and investments/ are compared with the real, disposable resources while the output increments are given exogenously and the time exploitation of equipment, depreciated capital stocks and time lags are simulated. Apparently, the forecasted growth of output /volume of production/ will not be consistent with the real capacities of corresponding subsystems. Therefore a decision process which shows the way of correcting the input parameters of the subsystem must be applied.

The following input parameters are defined in the time period $t-1$:

1/ F_{ijt-1}^0 - capital stocks /machines and buildings/ in both productive and nonproductive areas according to branches
 $i = 1, 2 \quad j = 1, \dots, 17 \quad t = 1$

2/ I_{ijt-1}^0 - the finished works and deliveries not included into the capital stocks /unfinished investments/
 $i = 1, 2 \quad j = 1, \dots, 17 \quad t = 1$

3/ L_{jt-1}^0 - labour force in both productive and non-productive areas according to branches
 $j = 1, \dots, 17 \quad t = 1$

4/ X_{jt-1} - volume of output /gross output/
 $j = 1, \dots, 17 \quad t = 1$

The following parameters are either estimated or resulting from the econometric analysis. In some cases both procedures are used to define a parameter.

5/ δ_{ijt} - depreciation charges of capital stocks
 $i = 1, 2 \quad j = 1, \dots, 17 \quad t = 1, \dots, 20$

6/ $\frac{S_{jt_0}}{S_{jt}}$ - the given ration of time exploitation of equipment in the time period t and t_0
 $j = 1, \dots, 17 \quad t = 1, \dots, 20$

7/ $FN_{ijt} = a_{ij} + b_{ij}t$ - analytically given capital/output ratio
 $i = 1, 2 \quad j = 1, \dots, 16 \quad t = 1, \dots, 20$

8/ $LN_{jt} = c_j + d_j t$ - analytically given labour/output ratio
 $j = 1, \dots, 16 \quad t = 1, \dots, 20$

9/ τ_{jt} - given time lags between investing and starting the usage of capital stocks
 $j = 1, \dots, 16 \quad t = 1, \dots, 20$

10/ dL_t - given increment of labour force
 $t = 1, \dots, 20$

11/ dL_{17t} - given increment of labour force in nonproductive area
 $t = 1, \dots, 20$

12/ $FV_{i17t} = a_{i17} + b_{i17}t$ given capital/labour ratio in non-productive area
 $t = 1, \dots, 20$

13/ ω_t - given share of national income in social product
 $t = 1, \dots, 20$

14/ τ_{it} - given share of investments in national income for machines and buildings

- 15/ r_{jt} - percentual increment of output in the
j-th branch and the time t
j = 1, ..., 16 t = 1, ..., 20
- 16/ f_{ijt} - input-output coefficient
- 17/ K_v, K_N - given coefficients transforming the
volume of capital stocks into the
volume of investments/not including
the value of projects/

Input balance equations

$$1/ \sum_{j=1}^{17} F_{jt-1}^0 = F_{t-1}^0 \quad t = 1$$

$$2/ \sum_{j=1}^{17} I_{jt-1}^0 = I_{t-1}^0 \quad t = 1$$

$$3/ \sum_{j=1}^{17} L_{jt-1}^0 = L_{t-1}^0 \quad t = 1$$

$$4/ \sum_{j=1}^{16} X_{jt-1} = X_{t-1} \quad t = 1$$

Behavioural equations of the subsystem

$$5/ dx_{jt} = X_{jt-1} r_{jt} \quad t = 1, \dots, 20$$
$$j = 1, \dots, 16$$

This equation represents the rate of output growth in the branches X_{jt} . It is given exogenously for every $t = 1, \dots, 20$ and $j = 1, \dots, 16$.

$$6/ \quad /c_j \pm d_{jt}/dx_{jt} \pm d_j X_{jt-1} = dL_{jt}$$

This equation represents the demands for labour force depending on the increment of output and time.

$$7/ \quad dL_t - dL_{17t} = d\tilde{L}_t$$

$$8/ \quad d\tilde{L}_t \pm \sum_{j=1}^{16} X_{jt-1} = d\tilde{\tilde{L}}_t \quad \begin{array}{l} t = 1, \dots, 20 \\ j = 1, \dots, 16 \end{array}$$

The equation represent the reduction of the real state of labour force to that in the productive area $/d\tilde{L}_t/$ with respect to the change of the labour/output ratio /the equation 8/ between the years t and $t-1$.

$$9/ \quad \sum_{j=1}^{16} /c_j \pm d_{jt}/dx_{jt} = d\bar{L}_t \quad \begin{array}{l} j = 1, \dots, 16 \\ t = 1, \dots, 20 \end{array}$$

This equation represents the demand for labour force in the productive area with respect to the increments of output.

The basic axiom of socialistic economy is the full employment. If $/d\tilde{\tilde{L}}_t - d\bar{L}_t/ > \Delta dL_t$ where Δ is a priori given the increments of output are adapted by means of the coeffi-

coefficient $K_{L_t} = \frac{d\tilde{L}_t}{d\bar{L}_t}$ to establish the full employment of labour force in the model.

New calculated rates of output growth \bar{r}_{jt} derived from the use of coefficient K_L are then used in the equation 5 and the following calculations.

$$10/ \quad K_v \left\{ \left[\tilde{a}_{ij} \pm \tilde{b}_{ij}/t + \tau_{jt}/ \right] / 1 + \tau_{jt} r_{jt} / d\bar{x}_{jt} \pm \right. \\ \left. \pm b_{ij} x_{jt-1} / 1 + \tau_{jt} r_{jt} / + F_{ijt-1} \delta_{ijt} + \tau_{jt} / 1 + \tau_{jt} r_{jt} / \right\} = I_{ijt}$$

$$i = 1, 2 \quad j = 1, \dots, 16 \quad t = 1, \dots, 20$$

The adapted equation

$$K_v \left[\tilde{a}_{ij} \pm \tilde{b}_{ij}/t + \tau_{jt}/ \right] / 1 + \tau_{jt} r_{jt} / d\bar{x}_{jt} = I_{ijt}$$

$$\pm K_v \left\{ / 1 + \tau_{jt} r_{jt} / \left[b_{ij} x_{jt-1} \pm F_{ijt-1} \delta_{ijt} + \tau_{jt} \right] = \bar{I}_{ijt} \right.$$

$$11/ \quad K_N \left\{ \left[a_{i17} \pm b_{i17}/t + \tau_{17t} \right] dL_{17t} + \tau_{17t} \pm b_{i17} L_{17t} + \tau_{17t-1} + \right.$$

$$\left. + F_{i17t-1} \delta_{i17t} + \tau_{17t} / 1 + \tau_{17t} r_{17t} / \right\} = I_{i17t}$$

$$i = 1, 2 \quad j = 1, \dots, 16 \quad t = 1, \dots, 20$$

The equations 10/ and 11/ are mathematically identical. The equation 11/ is constructed for the nonproductive area. It differs from the equation 10/ only by the transformation coefficient /Kv/ which excludes the value of projects from the investment outlays.

$$12/ \quad X_{jt-1} + d\bar{X}_{jt} = X_{jt} \quad j = 1, \dots, 16 \quad t = 1, \dots, 20$$

$$13/ \quad \sum_{j=1}^{16} X_{jt} = X_t \quad t = 1, \dots, 20$$

The equation 12/ and 13/ are used to calculate the output volume in the year t.

$$14/ \quad \gamma_{it} / \sum_i X_{it} - \sum_i \sum_j f_{ijt} X_{jt} / + Kv [b_{ij} / 1 + \tau_{jt} r_{jt} / X_{jt-1}] -$$

$$Kv \left[\sum_{j=1}^{16} / 1 + r_{jt} \tau_{jt} / F_{ijt-1} \delta_{ijt} + \tau_{jt} \right] - I_{i17t} = \tilde{I}_{it}$$

$$i = 1, 2 \quad j = 1, \dots, 16$$

The equation 14/ represents the transformation of national income into the real volume of investments / γ_{it} , $t=1,2$ / usable in the productive area/for machines and buildings/ with respect to a change of capital/output ratio decreased by investments serving to replace depreciated capital stocks.

$$15/ \sum_{j=1}^{16} K_v / \tilde{a}_{ij} - \tilde{b}_{ij,t} + \tau_{jt} / (1 + \bar{r}_{jt} \tau_{jt} / d\bar{x}_{jt}) = \bar{I}_{it}$$

The equation 15/ expresses the demand for investment with respect to the output increments $d\bar{x}_{jt}$.

After corresponding calculations the real and calculated volumes of investments $\tilde{I}_{it} / \bar{I}_{it}$ respectively, are compared. If $\tilde{I}_{it} - \bar{I}_{it} > \Delta \tilde{I}_{it}$ in our model Δ given a priori $\Delta = 0.10$ the structure of production is considered to be unfeasible and further computations are not carried on. A new structure of system is established and the model resolved.

If $\tilde{I}_{it} - \bar{I}_{it} \leq \Delta \tilde{I}_{it}$ two cases are considered /regarding a priori defined Δ /, feasible structure $\Delta = 0.05$ and strained structure $0.05 < \Delta \leq 0.10$.

The result has a qualitative nature and characterizes the structure of the system in relation to resources.

If $\tilde{I}_{it} - \bar{I}_{it} \leq \Delta \tilde{I}_{it}$, the technique of goal programming is applied. The purpose of this is to correct the original solution so that it takes into account the given economic resources and possibility to realize the obtained solution in the terms of economic system.

Solving 16/ the new $d\bar{x}_{jt} / j = 1, \dots, 16 \quad t = 1, \dots, 20$ are obtained used further for calculating the increments of the stocks of labour force and other variables of the corresponding subsystem.

$$17/ \left[\frac{1}{a_{ij}} + \frac{b_{ij,t}}{d\bar{x}_{jt}} + b_{ij,t} x_{jt-1} \right] \frac{s_{j,t}}{s_{jt}} * dF_{ijt}$$

$$\tilde{a}_{ij} = \frac{S_{jt-1}}{S_{jt}} a_{ij} \quad \begin{array}{l} i = 1, 2 \\ j = 0, 1, \dots, 17 \\ t = 1, \dots, 20 \end{array}$$

$$\tilde{b}_{ij} = \frac{S_{jt-1}}{S_{jt}} b_{ij} \quad \begin{array}{l} i = 1, 2 \\ j = 0, 1, \dots, 17 \\ t = 1, \dots, 20 \end{array}$$

The equation determines the increment of capital stocks. The increments of labour force /equation 6/, the increments of investments /equations 10,11/ and the volume of output /equation 12/ are calculated similarly.

$$18/ \quad \frac{I_{ijt}}{dx_{jt}} = g_{ijt} \quad \begin{array}{l} i = 1, 2 \\ j = 1, \dots, 16 \\ t = 1, \dots, 20 \end{array}$$

The equation 18/ is used to calculate the investment coefficients for the branches and they are distributed into the building and machinery investment and the time period.

Other equations have a character of final checking balance equations and are not discussed here.

After the first solution of the corresponding subsystem is obtained the simulation technique is applied. First the time exploitation of equipment is simulated/the assumption of decreasing or increasing coefficient of time exploitation of equipment/ then the time lags in investment construction and the replacement policy.

The purpose of simulation is to show the alternations of demands for resources in the subsystem depending on the changes of parameters described above.

16. FORMULATION OF THE GOAL PROGRAMMING PROBLEM

$$\min (\sum U_j - \sum \bar{U}_j)$$

under the conditions

$$d\bar{x}_{jt} + U_j - \bar{U}_j = d\bar{x}_{jt}$$

$$U_j - \bar{U}_j = \sigma dx_{jt}$$

$$i = 1, 2$$

$$j = 1, \dots, 16$$

$$Kv [\tilde{a}_{i1} \pm b_{i1}/t + \tau_{1t}/] / 1 + \tau_{1t}\bar{r}_{1t}/d\bar{x}_{1t} + \dots + Kv [\tilde{a}_{i16} \pm b_{i16}/t + \tau_{16t}/] / 1 + \tau_{16t}\bar{r}_{16t}/d\bar{x}_{16t} = \tilde{I}_{it}$$

$$/c_1 \pm d_{1t}/d\bar{x}_{1t} + \dots \dots + /c_{16} + d_{16t}/d\bar{x}_{16t} = d\tilde{L}_t$$

$$d\bar{x}_{jt}; U_j; \bar{U}_j = 0$$

U_j, \bar{U}_j - are free variables, σ - a constant giving the bounds of deviation/of the free variable//.

The following results and data are provided for the forecasting purposes of the Planning Commission

- 1a/ Volume of gross output X_{jt} , X_t $t = 1, \dots, 20$ $j=1, \dots, 16$
- b/ National income Y_t
- 2/ Increments of the output volume dX_{jt} , dX_t $t=1, \dots, 20$
 $j=1, \dots, 16$
- 3/ Labour force L_{jt} , L_t $j = 1, \dots, 17$
- 4/ Increments and decrements of labour force dL_{jt} , dL_t ,
 $j = 1, \dots, 17$
- 5/ Capital stocks F_{ijt} , F_{it} , $i = 1, 2$; $j=1, \dots, 17$
- 6/ Increments of capital stocks dF_{ijt} , dF_{it} , $i = 1, 2$;
 $j = 1, \dots, 17$
- 7/ Depreciated capital stocks $\delta_{ijt}F_{ijt-1}$, $\delta_{it}F_{it-1}$, $i=1,2$;
 $j = 1, \dots, 17$
- 8/ Accomplished works and deliveries not included into the capital stocks /unfinished investments/ I_{ijt}^0 , $i=1,2$;
 $j = 1, \dots, 17$
- 9/ Annual investments \bar{I}_{ijt} , I_{i17t} , \bar{I}_{it} , $i = 1,2$;
- 10/ $\frac{I_{ijt}}{dX_{jt}} = g_{ijt}$ $i=1,2$; $j=1, \dots, 16$; $t=1, \dots, 20$;
- 11/ Capital/output ratio
$$\frac{F_{ijt}}{X_{ijt}} = FN_{ijt} \quad \frac{F_{it}}{X_t} = FN_{it} \quad \begin{matrix} i = 1,2 \\ j = 1, \dots, 16 \end{matrix}$$
- 12/ Output/Capital ratio
$$\frac{X_{jt}}{F_{ijt}} = \frac{1}{FN_{ijt}} \quad \frac{X_t}{F_{it}} = \frac{1}{FN_{it}} \quad \begin{matrix} i = 1,2 \\ j = 1, \dots, 16 \end{matrix}$$

Table 2 : Pessimistic and optimistic variant of investment coefficients

Production branch	1971		1972		1973		1974		1975	
	P	O	P	O	P	O	P	O	P	O
Chemical industry	1,111	1,375	1,053	1,337	0,927	1,266	0,936	1,168	0,877	1,098
Machinery	0,739	0,961	0,714	0,931	0,689	0,899	0,664	0,869	0,641	0,840
Food industry	1,071	1,241	1,094	1,252	1,117	1,279	1,140	1,343	1,163	1,371
Building industry	0,447	0,534	0,443	0,529	0,439	0,525	0,435	0,521	0,432	0,516
Agriculture	3,417	4,040	3,503	4,148	3,628	4,262	3,674	4,364	3,623	4,300
Transport and communication	9,736	11,791	9,168	11,038	8,809	10,558	8,453	10,074	8,353	9,928

R E F E R E N C E S

- /1/ Lotoš, Ja.M. :
Issledovanie i interpretacia strukturnych sootnošenij v rešenij odnoj modeli, Ekonomika i matematičeskije metody, Moskva 1970
- /2/ Leontjev, W. :
The Dynamic Inverse, Fourth International Conference on Input-Output Technique, Geneva 1968
- /3/ Clopper, A. :
The American Economy to 1975, Harper Row, New York, Evanston and London 1967
- /4/ Augustinovics, M. :
A Twin Pair of Model for Long-term Planning, Input-Output Technique, ed. by A.Bródy and A.P. Carter, North - Holland Publ. Comp., Amsterdam-London 1972
- /5/ Fisher, H.W. - Chilton, C.H. :
Developing ex ante Input-Output Flow and Capital Coefficients, Input-Output Technique, ed. by A.Bródy and A.P. Carter, North Holland Publishing Comp., Amsterdam-London 1972
- /6/ Szepesi G. - Szekely B. :
Further Examination concerning a Dynamical I/O Model, National Planning Office, Budapest 1972
- /7/ Šagalov, G.L.: Dinamičeskaja model' optimizacii proizvodstva i vnešneekonomičeskich svjazej, Ekonomika i matematičeskije metody, Moskva 1972

/8/ Srednesročnyje programy kapital'nych vloženij, Ekonomika, Moskva 1972

/9/ Kornai, J. :

Matematické programovanie v perspektívnom plánovaní, SVTL, Bratislava 1966.