1991

Quality economics in total quality control.

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Order Number 9203876

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West Virginia University, 1991

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Quality Economics
in
Total Quality Control

DISSERTATION

Submitted to the College of Engineering
of
West Virginia University
In Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy

by
Yang Fu Qiang
Morgantown
West Virginia
1991

ACKNOWLEDGMENTS

The author wishes to express his appreciation to the McNamara Program, World Bank, for their invaluable assistance in making this study a possibility in the United States of America.

The author is grateful to his graduate advisor, Dr. M. Jaraiedi for his guidance for this research, and to Doctors Wafik Iskander, Howard Wall, Thomas F. Torries, Jack Byrd, Jr. and Walter C. Labys who provided constructive criticism on the models and the results.

Most of, the author's heart-felt appreciation is extended to his wife, DanLing Guo, for her encouragement and typing, and to his daughter, Mimi Yang, for understanding why the author spent so little time with her during the study.

The author also expresses his deep thanks to Mrs. Julia Friend. Without her help, the author could not have finished the research.

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Abstract

The purpose of this research is to develop a general approach to quality economics in Total Quality Control. Based on microeconomics theory and the development of prevailing quality control approaches, the research develops a set of economic quality concepts which provide a solid theoretical base for economic design and evaluation in quality control. A number of quality economics modeling procedures are established in the phases of both the consumer's and the producer's decision makings on product quality as well as their interactions.

In the context of a total quality control approach, the research first analyzes consumer behavior under quality discrimination and derives expected quality value, consumer quality loss function, consumer quality surplus, and the consumer quality decision model. Results will provide a useful means for measurement of quality loss and its effect on consumer welfare as well as information on consumer assessment of product quality.

This research also analyzes the producer's behavior under quality risk in an increasingly competitive market and provides both general and approximate forms of the production loss function. Conventional approaches to the analysis of the producer's optimal behavior may not be adequate to illustrate product quality setting and producer behavior in quality competition. A consumer-based approach is established and employed to implement multi-competitive advantages in higher quality setting, lower cost and better consumer satisfaction. The effects of producers' production/market quality strategies, with which the producer affects consumers' quality decisions, is studied and modeled.

This research provides theoretical guidance for quality activity in an environment of increased competition. It also offers a set of simplified and computable functions to be applied in quality control practices.

Chapter 1 Introduction and Objectives

Quality in recent years has become an attractive topic and an important competitive tool in industry and in the marketplace. Consumers and producers have paid more attention to product and service quality which affects both the consumer's decision making and a firm's competitive position. High quality is regarded as one of the key points in explaining Japanese success in economic development and competition in the world markets after the 1960's.

Production management, process control and firms' marketing strategies in the traditional forms have been challenged by the wave of quality improvement in the world. Bhote [1988] points out that published data from more than 3,000 businesses indicate that quality is closely related to productivity, market shares, profits on sales and returns on investments, and that the total costs of poor quality are estimated to "reach an astronomical 50% of the sales dollar." In an increasingly competitive economy, a firm cannot survive and succeed without continuous quality improvement and cost reduction. Quality improvement must be one of the most crucial objectives of any firm.

The whole society has benefited a great deal from quality improvement which is usually accompanied with social cost reduction, resource saving and efficiency improvement in capital utilization. Without a doubt, in this decade quality improvement will dominate policy decision making in production management, process control and market strategy, and economic competition in quality will be more intense than ever before.

A large number of papers have contributed extensively to the fields of quality design, quality control and quality management in theoretical development and practical applications. However, only a few papers have been devoted to the investigation of the economic motivation behind quality activities and the role economic criteria play in decision making for quality design and quality control. This research is mostly aimed at establishing a set of economic quality concepts which provide a solid theoretical base and applied criteria for the economic design of quality activities.

1.1 Quality Control in Transition

The definition of quality and its contents in quality activities change with economic development and technological progress. Evolution of quality may be classified into three stages based on time periods and their applications. They are traditional approaches, modern approaches and current approaches, i.e., total quality control.

1.1.1 Definition of Quality and Quality Control

Quantity and quality are two major properties of any commodity. A competitive environment and consumer satisfaction are the main sources of motivation for quality activities (design, improvement, control and management). When a society needs more commodities supplied to meet its requirement of demand and development, quality plays a supporting role. On the other hand, when demand can be satisfied with a sufficient supply, quality plays a major role in meeting the requirement of social development. The change in quality definition is not a word puzzle, but a reflection and summary of people's insight for quality property and its effects on society.

Garvin [1984] summarizes different perspectives of product quality from the points of view of philosophy, economics, marketing, and operations management. He lists five major approaches to the definition of quality which are described as transcendent, product-based, user-based, manufacturing-based and value-based. Each approach emphasizes some basic elements of quality features and properties and poses a conflict. Following is a set of representative quality definitions.

Leffler [1982] writes, "Quality refers to the amounts of the unpriced attributes contained in each unit of the priced attribute."

Juran [1974] provides a user-based definition of quality: "Quality is fitness for use."

Based on SPC management approach, Crosby [1979] defines quality as "conformance to requirement."

Stephens [1981] gives a definition of quality as "conformance to a given requirement or specification on a product or service." Another related definition is, "The totality of features and characteristics of a product or service that bear on its ability to satisfy a given need." Obviously this definition of quality is ideally related to the properties of product quality which satisfy consumer needs.

Taguchi and Wu [1985] define quality as "the loss imparted to the society from the time a product is shipped." This definition reveals the social effect (economic, environmental and consumer effects) of product quality.

Diversification of quality definitions also is a result of multi-dimensions of quality, as Garvin [1987] points out, which include performance, features, reliability, conformance, durability, serviceability, aesthetics and perceived quality. Progress in quality improvement is closely related to the following factors:

(1) Level of economic development of society. Change in quality level corresponds to societal progress. This is why quality issues have been so competitive in the past 20 years in most developed countries, and why developing countries adopt their quality objectives, in contrast to developed countries, in accordance with their economic status.

(2) Degree of availability of technology. It includes technical knowledge, labor skill, available machinery and the transition period of new technology from the laboratory stage to the production process. Technological progress results in quality development and improvement at the levels that were impossible before and greatly reduces the cost of production.

(3) Change in consumer behavior for quality. Income, commodity price, tastes, hobbies and satisfaction vary with time and determine consumers' attitude toward quality variance.

(4) Market structure and organization pattern. A competitive environment stimulates a firm to improve product quality so as to be able to stay in business.

(5) Degree of harmful side effects. Higher quality will significantly reduce side effects on the environment, human health and safety, and other external factors. For example, quality requirements must be strictly met at a nuclear power station to ensure that its harmful effects on residents around it and on the environment are minimized.

(6) Economic motivation in producer behavior. A firm's market strategy, cost reduction, profitability and competitive advantage should involve quality activities.

Therefore, it would not be meaningful to define quality without consideration of current conditions including economic, technological and social constraints.

Because different people have different views on quality, we define quality as **the functional performance of a product or service to satisfy consumer needs.** This is a consumer-based definition.

Consumer requirements and the effects on society for product quality should be the main objectives of product design. Quality control in a broad range is aimed at ensuring implementation of quality objectives, which includes managerial, design, process and market quality activities. Managerial quality activity includes an effective organization for quality development, quality improvement and quality maintenance in various groups to enhance efficiency and reduce errors in strategy and production process. The human-factor is the largest single cause of poor quality. The difference in product quality levels between American and Japanese manufacturing industries is believed to be mainly due to managerial methods rather than the degree of application of quality control procedures.

Quality design activity optimizes the product design and manufacturing process design under available technology and machinery so that the functional variation of the product is minimized in a wide range of conditions. Process quality control is meant to diagnose and discover unusual states in each stage of the manufacturing process as quickly as possible, and to adjust the process and return it to its normal state. During and at the end of a manufacturing process, inspections are performed in order to avoid the shipment of nonconforming items to consumers. Market quality activity includes surveying the market size and competitor's strategy; measuring the consumer quality requirement, complaints and preference; and feeding information back to the firm's quality development stage. Thus, it reduces design cycle time and white-collar errors.

1.1.2 Traditional Approaches of Quality Control

The traditional approaches of quality control (TAQC) emphasize the use of statistical principles and techniques to diagnose, analyze and adjust process performance at all stages of production, maintenance and service, directed toward the economic satisfaction of demand. Inspection for nonconforming items is also mentioned in these approaches. The statistical Process Control (SPC) approach is an important component of TAQC.

Major sources that cause product quality to vary from the predetermined specification are: (1) inconsistency in the quality of materials and components purchased, (2) operator mistakes and errors, (3) inherent problems in the wear and tear of tools and machines, (4) unstable manufacturing processes or procedures, and (5) environmental disturbances.

Since 1924 when Walter A. Shewhart of the Bell Telephone Laboratories first introduced statistical procedures for the control of manufacturing processes, SPC has been widely used in American industry. In the 1950's and 1960's other quality assurance approaches such as quality, cost, reliability engineering and quality management were developed. The zero-defect approach (ZD) was aimed at stimulating workers to increase product quality and reliability. After World War II, SPC approaches found widespread use in the Japanese industry. The

Japanese also creatively developed Quality Control Circles (QCC) based on their cultural situation. QCC have been extremely successful in quality improvement.

Conventional statistical process control techniques for quality efforts mainly include control charts, flow charts, run charts, cause-effect diagrams, Pareto diagrams, histograms and scatter diagrams. These TAQC techniques are used to manufacture products with the quality specified through identification and control of systemic causes of defects and variations in each process. If the process is under control and capable, the product is considered consistent and defect-free. Any product outside the specification should not be shipped to consumers. An effective SPC program is instrumental in reducing the cost of waste and work and increasing productivity and efficiency.

Although TAQC have made a significant historic contribution to improvements in products and services, with time, some of its inherent disadvantages have become evident. They are: (1) The effect of product quality improvement is assessed by the consumer rather than by the firm. Consumer quality behavior and satisfaction should be first carried out as the requirement of quality control rather than the firm's subjective quality settlement. (2) Quality control is only executed in the manufacturing process and inspection stages. No emphasis is placed on the early stages, quality development and quality design. (3) Those approaches are usually "cost-up" and slow in their ability to solve chronic quality problems. (4) Any product quality characteristic inside the process specification is regarded as a conforming item for which no quality loss is accounted. This kind of product cannot compete with a product that has less quality variation. (5) No consideration is made for the variance of product functions at a wide range of operating conditions.

1.1.3 Modern Approaches of Quality Control

In recent years, many concepts and new methods of quality assurance have been devised to improve product quality. Taguchi's methods are the most outstanding of these modern approaches. Combined with other effective methods in quality control, Taguchi's methods are referred to as the "secret weapon" giving Japanese products the reputation of quality leaders in the world.

These methods were developed by Genichi Taguchi, a well-known Japanese engineering-statistics specialist. Taguchi's methods creatively combine engineering and statistical methods to improve quality and reduce cost by optimizing product design and manufacturing processes. Taguchi's approach is based on the theme that "quality is the loss imparted to the society from the time a product is shipped." Continuous quality improvement and cost reduction is very important for a firm to survive and succeed in business in a competitive environment. More specifically, Taguchi's approach includes quality loss function, parameter design, tolerance design, system design, on-line quality control, design of experiments using orthogonal arrays for product quality and measuring systems.

A product performance variation leads to consumer dissatisfaction and must be considered at the product design stage in order to minimize the deviation of the performance characteristic from its target value at a wide range of environmental conditions. From quality development to the quality manufacturing process, parameter design distinguishes those low-cost factors that possess significant effects on the target value but small (or no) effects on the quality deviation. The "cost-up" is emphasized on the tolerance design where higher-cost factors are used to obtain the gain from variance reduction. These methods optimize product design and manufacturing processes to achieve quality optimization and cost reduction simultaneously. This approach is definitely cost-effective because it eliminates compromises at early stages. A loss-function performing as a cost saving term is used in Taguchi's on-line quality control procedures to obtain optimum diagnosis and adjustment intervals. There are other significant advantages in the approach, such as reducing performance variance by exploiting the non-linear effects of the parameters on the product performance.

With more attention paid and more practice applied to quality activities, some problems associated with the modern approaches have surfaced. They are: (1) Quality is closely related to the progress of technology and the economy. Quality should be redefined. (2) Consumer satisfaction and requirements under quality discrimination have not been fully considered. Since consumer assessment of quality discrimination is subjective, the producer should obtain such consumer information and then feed it back to the quality development stage. (3) Quality loss function is derived from the consumer behavior under quality risk, not from mathematical deduction. (4) Consumer and producer quality behaviors and attitudes interact with each other. (5) These approaches lack a sound economic basis. Quality improvement is an economic activity. Economic motivation is the engine, and quality control methods are the tools to achieve the firm's objectives. Development of quality economic criteria is urgent. (6) These approaches do nothing to reduce the design and manufacturing process cycles.

1.1.4 Total Quality Control Approach

A new approach, the Total Quality Control Approach (TQCA), which seeks the commitment to quality activity in all stages of an entire cycle from consumer to producer and in the way quality information is transferred between those connections, is proposed to cope with the problems or disadvantages mentioned in the previous sections. The idea of the Total Quality Control Approach has been described in various forms by Feigenbaum [1983], Sinha and Willborn [1985], Shores [1989] and many other authors.

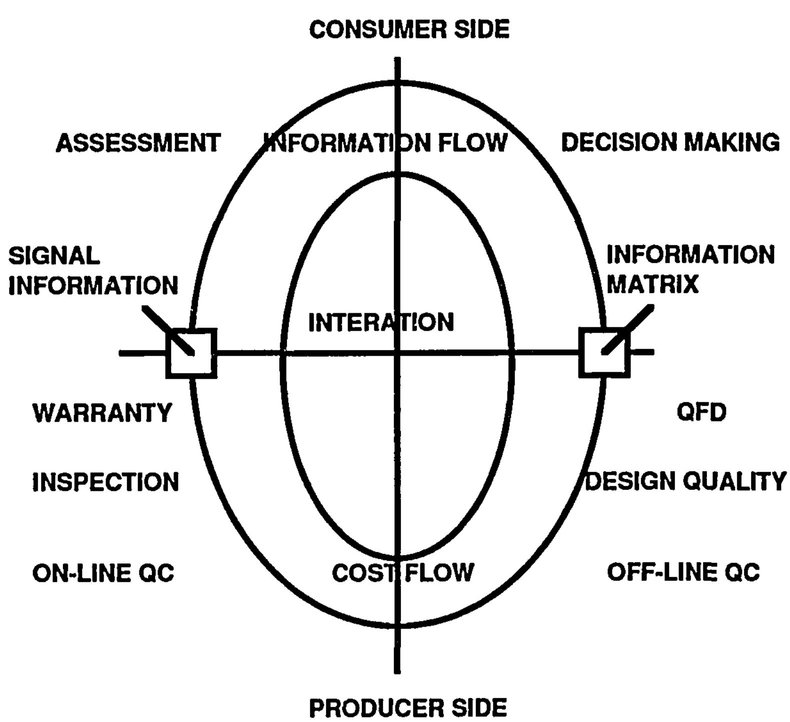

Figure 1. TQC Approach

A schematic of a modified TQCA is illustrated in Figure 1. It consists of two sides, consumer and producer behaviors under quality risk; two links, a transformed consumer information matrix on decision making and a producer information signal on market strategy; and two flows, information flow in the consumer side and cost flow in the producer side. The cycle from consumer to producer is for continuous quality improvement, which should be viewed as a spiral curve in three-dimensional space rather than a circle in two-dimensions. The management of TQCA is aimed at improving product quality in a more efficient and economical way and accelerating the quality improvement cycle by running quality activities parallel rather than in series, i.e. concurrent quality engineering.

Consumer attitude toward quality performance variance depends on the quality deviation pattern, product price, income level, taste and the consumer's subjective judgment of quality. A product with higher quality may not be competitive in the market if it does not meet the consumer quality requirement and consumer preference weight distribution for attributes of quality in comparison with the competitor's product. Quality design and quality control are not concentrated exclusively on meeting firm or technical requirements, but upon consumer quality expectation. An operating mechanism, consumer information matrix, is developed to transform consumer quality expectation into a standard form for quality development. Furthermore, the main and side effects of quality on society can be measured by consumer surplus and social welfare.

A consumer-based firm's attitude toward product quality is determined by the firm's market strategy, market size, demand pattern, and market structure as well as by competitor's decision making. In this phase, the quality of design not only involves statistics and engineering control methods, but also economic criteria and constraints. A trade-off is necessary if there is contradiction among economic and engineering conditions. Moreover, a firm will execute effective management of TQCA in the specific operation environment in design, planning and control of all activities related to quality assurance. The new management approach is a challenge for conventional quality management methods in which quality improvements are accelerated in parallel phases from design to inspection stages. The total cycle of quality improvement can be shortened significantly to achieve a strong competitive advantage. A firm will assure its success if it provides the product faster than other competitors to meet the higher level of consumer satisfaction and needs.

The consumer quality expectation and subjective assessment are reflected in an information matrix and fed back to the quality development stage while the firm's quality information is accompanied with prices or advertisements (or other promises) to influence consumer decisions regarding the purchase of the product. In summary, TQCA is not an approach containing a cradle-to-grave system; rather, it focuses the spot-light on the decision-maker's behavior, consumer-producer decision interaction and competition in quality improvement.

1.2 Thesis Statements

The definition of quality given in Section 1.1.1 is quoted here again.

"Quality is the functional performance of a product or service to satisfy consumer need."

TQCA (Total Quality Control Approach) is the most promising approach to meet this quality definition and easy implementation in practical application. The motivation behind quality assurance is the economic rationale where the consumer spends money to purchase the product with better quality for satisfying his needs, and the producer improves the quality of the product for profitability and competitive advantage.

Since the 1920's statistical and engineering methods of quality control have been developed extensively. A great number of applications have been extremely successful. However, the development of economic concepts of quality control is far behind the progress in other related fields.

Conventional microeconomics theory has mainly concentrated on the product quantity. But with increased quality competition, it is required to develop a theme of economic quality concepts to provide a sound theoretical base and to guide quality activities in the correct direction. Moreover, a number of economic criteria should be established in decision making at all stages of quality activities. Otherwise, present quality approaches cannot cope with the challenge of quality assurance in the future.

Therefore, this research is concentrated on the development of quality economics in TQCA in light of microeconomics theory and conventional quality control approaches. Specifically, the following goals will be pursued.

(1) Development of a theoretical base will explain the decision maker attitude toward quality risk. This is a field purely dominated by economics research.

(2) Quality loss function, transformed consumer information matrix and social assessment for quality improvement will be derived from consumer behavior under quality risk.

(3) Establishment of the product quality function. This function will provide economic criteria for product quality design and manufacturing

process design. Quality cost function will be employed to indicate where corrective action is profitable, illustrate how to accelerate the cycle time of quality improvement, and how to execute the concept of "optimal quality setting for consumer satisfaction."

(4) A consumer-based approach will be developed to describe the producer's product quality setting under various market conditions.

(5) Models will be set up describing firms' quality production strategies under demand uncertainty and firms' information signals on market strategy to influence consumer decision making.

1.3 Research Objective and Organization

The research objective is aimed at implementing the goals mentioned in the thesis statements. It provides comprehensive coverage from basic economic principles to applicable economic criteria in decision making for quality activities. The research will give a sound understanding of quality economics and deduce a number of relationships to carry out calculations for numerical problems. The research has a strong economic orientation and requires an adequate statistical background.

This research consists of seven parts from Chapter 2 to Chapter 8.

Chapter 2 deals with the recently published literature on basic economic concepts, consumer and producer behaviors under risk, measurement of quality assurance on social effects, firms' quality strategies in competitive environments, and product quality function all of which are related to quality assurance, productivity and profitability. This chapter also includes an overview of papers on the economic implication in quality engineering and management.

Chapter 3 presents a theoretical development for consumer attitudes toward quality risk. A general quality value expectation is derived, and a consumer quality loss function is computed on the basis of consumer quality discrimination. The loss function proposed by Taguchi is compared with the consumer quality loss function developed in this research.

Chapter 4 develops a comprehensive model of consumer quality evaluation. A consumer decision model is set up on the basis of the quality utility function reflecting the consumer preference distribution for a multiple-attribute quality product. A transformed information matrix containing consumer subjective judgment and weight distribution for a multiple-attribute quality is extremely useful for quality design. A fuzzy set model with empirical data is employed to illustrate consumer decision making in product quality selection.

Chapter 5 presents the effect of quality variation on social welfare. Two concepts, equivalent quality price and consumer expected price, are developed to evaluate a quality activity. Neither consumer nor producer quality loss functions are capable of assessing the quality's effect on society. Two consumer surplus approaches, the demand shift-based and the expected price-based, are discussed and compared with each other. The total producer quality loss and social welfare are analyzed under the partial equilibrium conditions.

Chapter 6 establishes product quality function and quality cost function based on the experiences and development in quality control practice in past decades. The relationships among price, quantity and quality are discussed. The product quality function and quality cost function are employed to set up the theoretical criteria for product quality design and manufacturing process design. Quality cost function is used to indicate where quality improvement is most profitable and how to accelerate the cycle period of quality design and control in combination with quality management and quality engineering methods. The long-run and short-run effects of quality cost function patterns, which correspond to technological progress and managerial development, on the continuous quality improvement is presented.

Chapter 7 describes producer behavior under quality risk and market uncertainty. The producer's quality loss results from a consumer switch to the competitor's product due to inferior product quality. The conventional approaches to the study of producer's optimal behavior are not adequate to describe product quality setting and producer behavior in quality competition. A consumer-based approach, which contains both consumer and producer behaviors under quality uncertainty as well as the interaction in both sides' decision making, is developed to determine quality settings under various market conditions. A number of production/market strategies, with which the producer attempts to influence consumer decision making and gain the advantage in competition, such as quality leadership, rebate policy and the optimal investment, is studied.

Chapter 8 summarizes the conclusions derived from the research and provides directions for further development.

Chapter 2 Background and Literature Review

Despite the important roles of managerial, engineering and statistical functions in quality assurance, the orientation of this research is focused more on quality economics. The following background and literature review reflect heavily recent developments in these related fields.

2.1 Basic Quality Economic Concepts

Although only a few studies have contributed extensively and directly to quality economics, development in microeconomics theory and other disciplines, such as marketing and management science, have provided a sound base for multifaceted quality economics.

In the early literature on microeconomics, product quality were viewed as differencial products and resulted in shifts in a product's demand curve [Chamberlin, 1953; Dorfman and Steiner, 1954; Akerlof, 1970; White, 1972 and Spence, 1976].

Lancaster [1966] proposes an approach to dealing with the problems of consumer choice of a product with multiple-attributes as well as market penetration of a new product. In spite of being a good illustration for durable quality differences, the model ignores the evaluation weights that individuals usually assign to a set of quality attributes and the difficulty of deriving an effective statistical method to aggregate consumer demand in accordance to widely varying preferences [Garvin, 1984].

How price and quality are correlated is an interesting topic in quality economics. Riesz [1978, 1979] points out that when a consumer is well informed and higher quality can only be produced at a higher cost, price and quality will move together positively. If price information is not completely available, the consumer will use comparative prices to make a purchase decision. Price would not reflect the real quality. Therefore, the availability of information determines the price and quality correlation. Some researchers have indicated a positive correlation between quality and price, especially for durable goods [Gabor and Granger, 1966; McConnell, 1968; Westbrook et al., 1978]. Curry [1985] uses the Law of Comparative Judgment (LCJ) to measure price and quality competition. He concludes that price and quality are positively associated and price is a fairly accurate indicator of quality for a group of manufactured products, whereas some empirical studies reveal negative price/quality correlations [Friedman, 1967; Sproles, 1977]. Other empirical studies on the relationship between price and quality [Morris, 1971; Geisfeld, 1982] concluded that quality/price relation is product-specific and weak in general. Gersner [1985] used real data (1980-1982) to explain the variance in the quality/price relations across products, and he claims that higher price appears to be a poor sign of higher quality. Curry and Riesz [1988] carry out a longitudinal analysis and confirm that price decreases in real terms over time periods. Correspondence between price and quality level diminishes over time, which implies that as pricing flexibility declines, competition would be focused on increased expenditures rather than on relative quality improvement. Jacobsan and Aaker [1987] propose a model to determine the role of product quality in market competition and find that compared with other strategic variables, the product quality is more attractive and facilitates an increase of profitability in both a focus and a market share context.

Mussa and Rosen [1978], Phillips [1983], Cooper [1984], Maskin and Riley [1984] separately develop a similar model of quality distortion under discriminating monopoly, which parameterizes quality discrimination based on the assumption of a positive relationship between total and marginal quality utility across consumer groups. These studies suggest that the discriminating monopolist will not provide distortion at the highest quality level, but degrade quality at lower levels. Srinagesh and Bradburd [1989] provide an alternative model based on the different assumption that total quality utility is negatively associated with marginal quality utility, and they derive the result that the monopolist may enhance quality rather than degrading quality to maintain profitable market segmentation.

Quality improvement is viewed as a strong method to provide higher productivity, lower costs and a better competitive position [Deming, 1982; Gitlow and Hertz, 1983; Day, 1988]. Several advantages, including increased quality, productivity and capacity along with lower cost per unit can be obtained by process quality control. Kraft II [1983] proposes a method of quality improvement in Concurrent Engineering Management. Quality activities are run parallel, not in a series, to reduce the cycle time for product design and manufacturing procedures as well as entry time to market. Quality assurance requires intensive hard technical data on performance, manufacturing yield, testing and inspection in all of the stages in order to take corrective action on time. Simulation methods with sufficient data supplied are strongly recommended. More specifically, observation is emphasized on the data flow rather than on the product flow.

In the service quality field, Behrman and Birdsall [1983] argue that quantity alone rather than the incorporation of quality and quantity will lead to biases in estimated returns to schooling. Their model shows that an estimated social rate of return to quality substantially exceeds the social return to quantity of schooling, and that inclusion of quality may lead to better policy making under the constraints of scarce resources. Research suggests that consideration of quality should be incorporated into analysis of public investment decisions in conventional quantitative methods.

Deaton [1988] points out that the change in price not only affects choice in quantity but also in quality. The quality difference is transformed into a unit measurement value in some agricultural goods, such as beef, meat, fish, cereal and starches. Thus the analysis of price elasticity would be carried out more easily.

2.2 Decision Making Behavior under Quality Risk

Only a few papers deal directly with quality discrimination. However, a large number of papers have been published to deal with the concept of attitude toward risk which is an important concept in the theory of both prescriptive and behavioral decision making. Both consumer and producer behavior in the decision making of purchasing and producing a product with certain quality specifications fall in the scope of decision theory.

Von Neumann and Morgenstern [1953] developed the expected utility theory to measure and rank consumer preference. Pratt [1964] and Arrow [1971] state that a person is risk neutral relative to all lotteries if the utility of the expected value of the lotteries equals the expected utility of the lotteries. A person is risk averse relative to all lotteries if the utility of the expected value is greater than the expected value of its utility. Since Von Neumann and Morgenstern's utility function contains both attitude toward risk and the intensity of preference for riskless outcomes, Dyer and Sarin [1979, 1982] provide a risk utility function and a riskless measurable value function to separately represent the risk attitude and intensity of preference for difference between outcomes. For a region of riskless intensive preference, a person with an attitude toward risk is relatively risk averse, risk neutral or risk seeking if his risk utility function is strictly concave, linear or strictly convex, respectively.

Based on the outcomes of a pair of lotteries involving extreme combinations of attribute values [E] or involving moderate combinations of attribute values [M], Richard [1975] defines a person as multiattribute risk prone, multiattribute risk averse, or multiattribute risk neutral if he prefers E to M, M to E, or is indifferent between M and E.

Either the additive or multiplicative multiattribute utility function is widely employed to capture the behavior of the decision maker in risky multiple objective decisions. The additive utility function implies that the decision maker is consistently multiattribute risk neutral, whereas the multiplicative utility function suggests that the decision maker can be either consistently multiattribute risk averse, or consistent multiattribute risk prone [Richard, 1975]. Some studies [Tversky, 1967; Fischer 1976, 1977; Currim and Sarin, 1984] illustrate that either the additive or multiplicative multiattribute utility function can well approximate the behavior of the decision maker. Many studies [Fishburn and Kochenberger, 1977; Laughhunn, Payne and Crum, 1980; Payne et al., 1981] have proposed that people are risk averse for gains but risk prone for losses. But Keller [1985] and Fowicz [1983] shows that relative risk-preference for gains and losses is not clearly different by employing Dyer and Sarin's relative risk preference approach.

Fischer et al. [1986] also argue that Payne et al.'s finding is ambiguous because of the unsuitable system referred. Fischer et al. develop a reference risk-value [RRV] model to capture a multiattribute reference effect in comparison with Payne et al.'s findings. They conclude that any direct generalization of the findings to a multiattribute context may be questioned.

Wiggins and Lane [1983] assume that consumer risk results not only from quality variables, but also from variation in quality across products as well as consumer inability to evaluate the particular product quality before purchase. Roberts and Urban [1988] suggest that the consumer can only approximately evaluate a new product with an uncertainty which includes the inherent product variability and information uncertainty. Consumers use media, retail salesmen and friends as information sources to resolve the uncertainty of imperfect information. Consumer risk could be reduced through information obtained either by buying advertised products that a firm uses as an implicit signal to affect consumer decisions, or by a direct search (Hey and Mckenna [1981]). Wiggins and Lane construct a full partial-equilibrium model to examine the behavior of both consumers and producers and conclude that the consumers who are risk averse tend to purchase the lower variation in product quality in the advertised array if the information is not perfect and the search cost is high. Furthermore, Chadler [1988] argues that the producer's attitude toward quality improvement should be beyond consumer satisfaction, to achieve the goal of consumer delight to catch up with more serious competition in the future.

2.3 Measurement of Social Benefit in Quality Improvement

In microeconomics social benefits in any social cost-benefit calculation are usually measured in terms of sum of consumer surplus and producer surplus. This is because the market price, even in a perfectly competitive economy, is not a standard measurement to value the social benefit from the analysis [Willing, 1976; Mishan, 1984]. Taguchi and Wu [1985] define quality as the value which can be measured by society loss resulting from imperfect quality. The quality improvement is inversely related to external effects (harmful side effects); that is, higher quality is related to lower society loss. However, this approach actually is cost saving due to quality improvement rather than the social loss approach. Taguchi et al. [1989] employ a similar loss function to estimate society loss at the present time. They also use a time series return method to compute the loss due to quality deterioration over a time period.

Spence and Sheshinski [1976] studied the effect of quality choice by a monopolist under perfect information on social welfare. Shapiro [1982] demonstrated that imperfect information in a monopolistic market tended to reduce the product quality, which might lead to either a gain in social welfare if the quality was overestimated, or a loss if the quality was underestimated. Shapiro [1983] also analyzed the welfare effect of the minimum quality standards in a perfect competition market. Besanko et al. [1987] examined the policies of a price ceiling and minimum quality standards on social welfare improvement. The price regulation showed a sufficiently positive effect. However, it was ambiguous for the minimum quality standards. Specifically, Breshahan [1986] and Trajtenberg [1989] carried out the welfare analysis of technical advances and product innovation for mainframe computers and tomography scanners respectively.

2.4. Producer Quality Objectives and Market Strategies

2.4.1 Producer Quality Objectives

A firm's objectives and its market strategies are usually summarized as maximizing revenue (market share), minimizing cost, maximizing profit and maximizing the firm's utility in traditional microeconomics.

Quality is viewed as a great potential strategy to achieve a strong competitive advantage [Juran, 1981; Deming, 1982; Crosby, 1979, 1984; Taguchi, 1985]. Research has shown a strong positive relationship between quality and market shares. Buzzell and Wiersema [1981] point out that during the 1970's businesses with improved quality increased their market share five to six times faster than those with declined quality, and three times faster than those with unchanged quality. Other researchers have also confirmed the positive relationship between quality and market shares [Gale and Branch, 1982; Phillips et al., 1983].

Recent studies show that the effort devoted to the improvement of quality may reduce the unit costs of products [Daetz, 1987; Gunter, 1987; Sullivan, 1987]. Gavin [1983] indicates that Japanese products possess both higher quality and lower cost than American products. The perception that higher quality is accompanied with higher cost dominates a wide variety of American industries. The objective of minimizing total cost not only includes the quality cost as commonly understood, such as reworking and scrap cost, but, more importantly, involves the cost derived from the loss function [Taguchi and Wu, 1985]. Quality improvement may exert a significant effect on a firm's profitability change with (1) lessening elastic demand and higher prices or an increase in sales and market shares and (2) reducing cost by means of quality control activities [Chamberlin, 1953; Dorfman and Steiner, 1954; Darvin, 1984]. It is confirmed by some empirical studies that quality and profitability have a strong positive relation. Schoeffler et al. [1974] have found that high quality produces a higher return on investment. Businesses with poor quality experienced about one fourth of those with higher quality in return on an investments [ROI]. Bhote [1988] points out that the enormous costs of poor quality, including scrap, rework, warranty, inspection and tests, would range from 10%-20% of the sales. In broader meaning, quality cost including equipment down time, supplier delinquencies, long manufacturing and design cycle time, poor quality management and loss of consumer due to shift to competitors' product, would be estimated as much as 50% of the sales dollar.

Since it is difficult to evaluate the relationship between quality and other variables such as price and direct cost in accordance with current economic theory, more theoretical research and empirical studies are needed. Producer decision making can be represented through a multiattribute utility approach. For the problem of maximization of market shares, Albers and Brockhoff [1977] proposed a solution for equal weight attributes by using a heuristic algorithm, while Zufryden [1979] employed a mixed integer program to get the solution with multiattributes differently weighted. Shocker and Strinivasan [1974] and Green et al. [1981] suggested an approach to profit maximization for a firm's objective. Hauser and Simmie [1981] employ Lancaster's model in a framework for identifying the profit maximizing position. All of the above studies are based on consumer-based methodology to identify a new product. Traditionally, quality is viewed as a new product introduced into the market of a class of products. Gavish et al. [1983] propose an approach to the analysis of requirements on the demand side of the consumer and the cost of producing a new alternative on the supply side as well as competitive equilibrium. They give algorithms for the problem of market share maximization and a heuristic method for the problem of profit maximization.

Garvin [1984] identifies eight dimensions that can be used as a framework to describe properties of product quality. They are performance, features, reliability, conformance, durability, serviceability, aesthetics, and perceived quality. A firm's quality strategy must improve one or more aspects of the dimensions to gain a competitive advantage. Dorfman and Steiner [1954], Schmalensee [1978], and Wiggins and Lane [1983] have indicated that the firm may use advertisements as an implicit signal, other than price, to provide consumers with some information on product quality and to affect consumer purchase decisions. Nelson [1970, 1974] developed a theoretical argument for a positive association between quality and price and introduced two basic properties of goods distinguished with "search" and "experience." A consumer could determine the attribute of the former prior to the purchase while he or she could learn the attribute of the latter after using the purchase. Nelson deducted theoretically that higher advertised products were of better quality. The evidence on this claim is conflicting. Surveys and some studies [Cole et al., 1955; Rotfeld and Rotzoll, 1976; Farris and Buzzell, 1979; Barksdale et al., 1982] indicate that the association is negative and the consumer is no more likely to respond to the advertised product than that without advertising. Nevertheless the positive association is supported by other research [Archibald et al., 1983; Lambin, 1976; Marguardt and McGann, 1975]. Tellis and Fornell [1988] argue that empirical evidence is scarce and that theoretical work is controversial. They employ a product life cycle model to carry out the study and suggest that it is more likely for a highly advertised product to be of better quality when the quality cost is lower and when consumers do not solely rely on advertisements for their information.

2.4.2 Quality Cost Function

In both theoretical concepts and empirical evidence on the quality activity and its cost, an obvious controversy and an inconsistency exist. Lundvall and Juran [1974] claim that cost trade-off analysis should be considered to find the optimal quality level in their economic conformance model, whereas Deming [1982] and Crosby [1979] assert that zero defects should be the objective of the optimal quality level. Taguchi [1985] argues that any quality deviation from the target value is the firm's loss. The zero defects quality concept does not completely capture the cost feature of quality; the quadratic loss function does.

Kackar [1986] argues that if firms want to stay in business, they should adopt a strategy to improve quality and reduce cost on a continuous basis. Crosby [1984] and Taguchi [1985] point out that quality improvement need not increase product cost, and the correspondence could be negative through product quality design and other effective quality activities, such as defect prevention rather than defect detection. The core of off-line and on-line quality control is to provide higher quality at a lower cost. Trade-off made in the early stages are common in U.S. industries, and they preclude the utilization of robust functions for product quality design [Sullivan, 1987]. The "inherent idea" of the positive relation between quality and cost would be the reason why the American product is far behind the Japanese product in a competitive market [Gunter, 1987].

The relationship between quality and cost is ambiguous. One reason is that quality costs are defined differently, based on the assessment, background and understanding of quality control activities in different research and practical areas. Campanella and Corcoran [1983] consider quality cost as any expenditure on manufacturing or service in excess of those that are normally occured before the product design is exactly right the first time. Quality cost usually includes prevention, appraisal, failure, and internal and external failure costs [Batson, 1988]. Compared with Taguchi's methods and concurrent quality engineering, the cost at the stages of quality development and design as well as the cost of cycle time in schedule management are not accounted for. Taguchi et al. [1989] introduce the quality loss function as an important part in total quality cost, in which quality is negatively associated with cost. This approach gives the insight of continuous quality improvement to stimulate a firm to improve quality at a minimized total cost.

Gilmore [1983] suggests that prevention cost is the key ring in quality cost chains. Greater expenditure on prevention would effectively improve quality performance and reduce the total costs of quality in all subsequent stages of quality control activities. Although some empirical studies [Gale and Branch, 1982; Phillips et al., 1983; Bader, 1983] have revealed the form of association between quality and cost in special industries, a general quality cost function should be formed to verify the relationship between quality and cost for a wide range and variety of businesses.

Forrester [1956] first proposed the method of industrial dynamics in which interacting flows of material, staff, money and information were collected and fed back loops so that proper organization policy could be determined for achieving goals. These concepts have seen many applications in social, biological and economic systems [Robert, 1963; 1981]. Knight et al. [1987] have applied these ideas to the quality of manufacturing products by simulating quality circles. Batson [1988] uses a system dynamics flow model as a communication device to construct a quality system in which the quality cost system is prevention oriented to feed information back to the early stages of the product.

Cooper and Kaplan [1988] propose an activity-based cost model to guide corporate strategy selling multiple products. This model, different from conventional cost accounting, is based on the fact that many important cost categories vary not with changes in output in the short term, but with the changes over a period of years in the design, product mix, and other factors. With more accurate cost information, management can take corrective action in the most profitable products and processes as well as in decisions about product design, marketing and pricing.

Hauser and Clausing [1988], Rose [1988], Sullivan [1988] and Delatore et al. [1989] contend that Quality Function Deployment (QFD) plays an important role in quality assurance, which links and transforms the requirement and voice of the consumer into the development of product quality and specifications for the producer. Products should be designed to reflect consumer satisfaction and tastes and management should organize a group, including marketing staffs, design engineering and manufacturing staffs, to carry out the mission through QFD. This approach is more crucial in international market competitions [Slerra, 1988].

2.4.3 Producer Behavior in Quality Competition

Taguchi's approach is widely applied in American industries and has been extremely successful. Taguchi and Wu [1985] claim that off-line quality control, executed at the early stage of quality development and quality design, seems more significant and efficient to achieve both quality improvement and cost reduction than that in on-line quality design for production process control. Off-line quality control consists of two-step procedures: product design and manufacturing process design. The objective of product quality design, which consists of three stages, system design, parameter design and allowance design, is to minimize the effect of noise sources at a lower cost. Parameter design is the core of the first step, because it determines the optimum level of individual parameters of the system within a wide range of performance conditions, but at the lowest price. Through investigation at this stage, a low priced and wide varying element which is resistant to a variation of performance conditions will be selected from a substitutable group while a higher priced and less varying element in the allowance design will be used to narrow the variance of quality performance. It is necessary to decide on a trade-off between cost and quality at this stage.

Manufacturing process design also consists of three stages to meet specifications, i.e. system design, parameter design, and tolerance design. Again, parameter design is an effective "cost-down" stage while tolerance design and system design are "cost-up" stages. Therefore, parameter design is the most important step in quality design and may be used to explain the difference in market shares between Japanese and American industries. The latter puts emphasis on the allowance design and the system design which demand increased expenditure on quality control.

While a large number of studies have concentrated on the engineering and statistical applications of Taguchi's methods, few papers have adequately dealt with the economic design in off-line quality control. No economic criteria have been established for selection of input factors. The production function in microeconomics describes the relationship between output quantity and input quantity. This concept could be applied to off-line quality design to form a product quality function for describing the relationship between output quality and input quality.

Bhote [1988] proposes an approach to identify crucial variables in product and process design to reduce the performance variance of product quality, and to open up the tolerances on the unimportant variables to reduce cost substantially. Actually, this approach is consistent with some aspects of Taguchi's approach.

Lambert [1980], in his study, indicates that some other cues, such as brand name, store image, and country of manufacture would project quality stronger than price. Kiechel [1981] and Porter [1980] suggest that higher quality possesses an advantage for product discrimination to create customer reputation, to lower price elasticity, and to present barriers to competition. There is a significant advertising and quality interaction that makes price elasticity lower than that of quality improvements alone [Farris and Reibstein, 1979]. Rational buyer behavior under imperfect information or product quality discrimination has an attitude toward pioneering brands with reputation in quality [Schmalensee, 1982].

Consumer quality reputation is regarded as the motivation for producers to improve product quality under situations in which there is asymmetric information about product quality. Miller [1988] and Kreps and Wilson [1982] developed a game-theoretic approach to analyze a monopolist behavior for successive generations of new and improved products, and how consumer reputation for high-quality products was established in the sequential equilibria of the product's life cycle.

Following Friedman's [1971] development of trigger strategy equilibria, Shapiro [1982] proposed a decision-theoretic approach for reputation building. The reputation as a consumer's expectation of product quality, was a force to prevent quality distortion in a monopolistic market with imperfect information. The producer took the consumer reputation into quality setting for once-choice and time variation improvement. Dybving and Spatt [1980] and Shapiro [1982] explained how to maintain a reputation.

Klein and Lettler [1981] suggested that consumers are assumed to have knowledge in acting as the firm to induce the product quality at the market price. In a competitive environment, the firm acquired a bad reputation and was excluded from the market for selling the low-quality product at a high price. The firm with a higher quality product would make positive profits through nonprice competition.

In contrast with Klein and Lettler's equilibrium model where price was equal to margin cost, Allen [1984] proposed a model to consider the role of reputation for unobservable product quality. With the aid of a moral hazard curve, the firm produced quantity at the market equilibria where price was equal to average cost but greater than that marginal cost. The consumers were more sophisticated and perceived the firm's behavior to avoid buying low-quality products.

Feenstra [1988] investigated the quality change in Japanese car and truck imports during 1979-1985 and demonstrated that a quota restraint resulted in a significant quality improvement and price increase for Japanese car imports. Rose [1990] studies potential linkages between financial variables and the firm's product safety (product quality) choices with empirical data in airline safety performance. The results of this study strongly support the linkage and reveal the advantage of a high reputation firm in safety competition.

Chapter 3 Consumer Behavior and Quality Loss under Quality Variation

3.1 Theoretical Background and Some Basic Assumptions

Classic microeconomics theory and decision behavior theory are employed to establish a theoretical base for quality economics. However, to some extent, some important concepts in quality economics are not consistent with, and may even violate some of the basic assumptions in the classic microeconomics.

Products actually are different in quality, which definitely affect both the consumer satisfaction and the producer profit. Consumer satisfaction and product quality competitiveness are the forces that promote quality improvement. The term "consumer" used in this paper does not refer to only a single individual but also a unit (household or agent) which makes decisions collectively by delegation of responsibility on a variety of matters affecting the well-being of the unit and its members. Consumers behave rationally as described in classic economic theory. In perfect competitive conditions, all agents behave as price takers. Perfect information is available to both consumers and producers. However, product quality variation is closely associated with information uncertainty and technological effects. External technological effects and market failure connected with uncertainty violate the basic conditions for the perfect competition market. All of the above conditions for information availability and technology effects need to be revised in quality economics.

Neumann-Morgenstern expected utility functions are employed in explanation of consumer behavior and decision making under risk [Von

Neumann and Morgenstern, 1953; Pratt, 1964; Arrow, 1971]. Product quality variation can be treated as a type of risk. Taguchi and Wu [1985] provide a meaningful quality loss function to illustrate social and producers' loss due to quality variation. Since this loss function is derived mathematically, based only on the producer's cost function, development of a comprehensive loss function on a consumer-based utility function is necessary.

The assumptions in decision theory, ordering of outcomes and transitivity, reduction of compound uncertain events, continuity and substitutability, are satisfied in quality economics. Although it has been shown in a number of papers that the assumptions used in this approach have limited its application to some extent, its advantages are still attractive in explaining of a decision maker's behavior under quality risk.

Lancaster [1966] proposed an approach to deal with the problem of consumer choice of a product with multiple-attributes. In spite of good results for quality difference in durable products, the model ignores the evaluation weight that consumers usually assign to quality attributes, and the variation of product quality.

With increased competition, consumer satisfaction and the assessment of product quality are very important for a firm in order to improve product quality. There is a need to develop a comprehensive method to convey consumers' quality information and preference to producers.

3.2 Consumer Behavior under Quality Risk

3.2.1 Quality Discrimination, Quality Value Functions and Quality Information Uncertainty

Quality discrimination for consumers is based on the fact that consumers pay the same price for a certain product but may get different product quality. Compared to the value of the highest quality in the product, consumers experience a loss under quality discrimination. The larger the product quality variation, the more serious the quality discrimination. Quality discrimination is a result of both quality deviation from its product quality mean value, and difference between the mean value and quality target value in evaluation of a brand of products.

Quality activities are not meaningful if the assessment of product quality is separated from the product market value which is the price in a perfect competitive market. If the product price is determined by its quality, no quality discrimination (or quality loss) will occur. However, it is very difficult to establish a price based on a product quality before it is used. The performance of product quality varies with the environment and usage conditions, built-in variability and maintained reliability. For brands of products with the same functions of performance and utilization, if the differences in their prices do not match the differences in their qualities in light of consumer utility function, either quality discrimination, or price discrimination, or both will occur. Consumers want to purchase the product with the smallest quality and price discriminations in a set of products subject to their budgetary constraints.

Phadke [1989] gave four types of quality characteristics in manufacturing processes, which can also be used to illustrate product quality variation. The product quality characteristics are symmetric on either side of the finite target value; such quality characteristics are called the nominal-the-best type. Deviation of the quality characteristics in one-tail may be different from that in the other direction. This is called the asymmetric type of quality characteristics. The further the product quality deviates positively from the target value which is set to zero or small value, the worse the performance of product function. Such quality characteristics are called the smaller-the-better type. On the other hand, if an increase in the product quality characteristics improves the performance of the product, the characteristics are called the larger-the-better type. These definitions are only associated with product quality distribution variances; no quality discrimination resulting from the difference in quality target values is considered.

The desired product quality corresponding to the price paid, in the view of consumers, is the highest quality value of the product. This is also called the consumer quality target value. Figure 3-1 shows the quality values corresponding to the prices paid for the above four types of quality characteristics with normal distributions (except for the asymmetric case). T, μ, and σ in Figure 3-1 represent the consumer quality target value, the mean value, and the standard deviation of product quality distribution, respectively. Product quality deviation is in the range between $\mu + 3\sigma$ and $\mu - 3\sigma$. The quality that the consumer attaches to the price is the mean value in the cases of the nominal-the-best type and the asymmetric type of quality characteristics, while it is the largest value in the case of the larger-the-better type and the smallest value in the case of the smaller-the-better type of quality characteristics. Consumer quality discrimination is the difference between the value provided by the quality of the product purchased and the highest value provided by the best quality product purchased by other consumers.

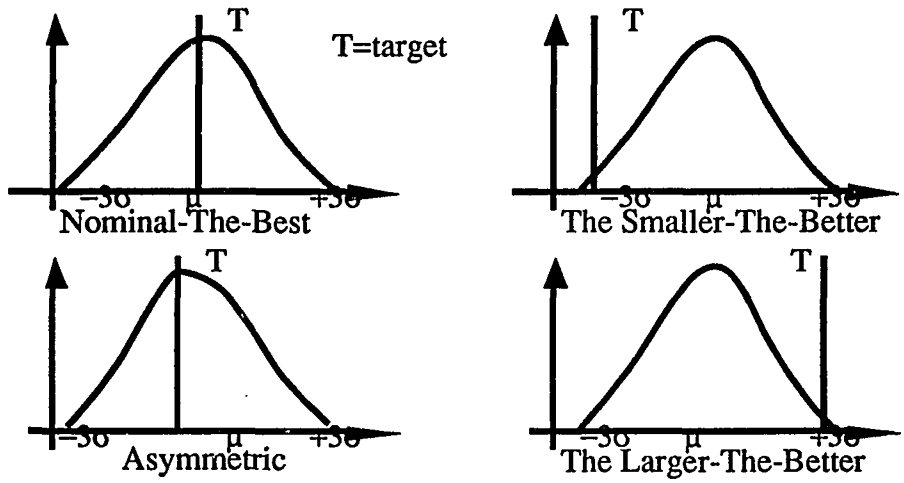

Figure 3-1 Consumer Quality Discrimination

Traditionally, the product quality desired by consumers with respect to the price paid is assumed to be the mean value of product quality distribution. This approach does not account for consumer quality discrimination and fails to describe consumer quality loss and decision making under quality variation. For example, if the consumer desired quality is the mean value in the case of the larger-the-better type of quality characteristics, the consumer compares his product quality value with the mean value of quality distribution, and then this is possibly augmented with other consumer quality surplus (or shortage) provided by the quality better (or worse) than the mean value of the product quality. Actually, quality comparison is not made between the product quality value that the consumer has and the mean value of product quality distribution, but between the quality purchased and the best quality purchased by other consumers. Even if the product quality has a normal distribution for a durable product (in the case of the larger-the-better), the consumer quality target value is not located at the mean value but at $T = \mu + 3\sigma$. Therefore, product quality variation will definitely result in quality discrimination and the consumers experience a quality loss if other conditions remain constant.

The effect of quality discrimination is relative with time change. Since it is impossible to determine the eventual target value of product quality for any time period to make quality assessment, the only thing the consumer can do to determine quality target values is to find the best values of a brand of similar products currently available. With time, quality target values will change, and the consumer's satisfaction met by previous quality value will result in a new consumer quality discrimination and a quality loss. The time-relativity of quality discrimination requires continuous quality improvement to meet the change in consumer satisfaction and to compete with rivals. The consumer's requirement and quality competition is the power to push product quality activities forward.

We define consumer quality loss, CQL, as the difference between the highest quality value, w_H, and the actual quality value, w, realized by the consumer for a workable product.

$$CQL = w_H - w \tag{3.2.1-1}$$

Poor quality results in consumer dissatisfaction, complaints and waste of resources. Poor quality also results in producer loss due to warranty cost, reducing market shares, bad reputation, consumer boycotts and, eventually, going out of business.

The quality value function for a product is derived from consumer assessment for deterministic quality. The consumer is willing to pay more for higher product quality. No uncertainty and risk are involved in the determination of quality value function. Consumer quality value function can be rationally assumed to be a monotonic increasing function with continuous and higher differentiable characteristics over a range of product quality. For instance, if quality level 1 possesses quality value 5 which corresponds to the market price of the product, how much will the consumer be willing to pay for quality level 3? Figure 3-2 shows consumer quality value function over a range of quality values. The vertical axis represents the consumer quality value in terms of monetary units and the horizontal axis indicates quality levels.

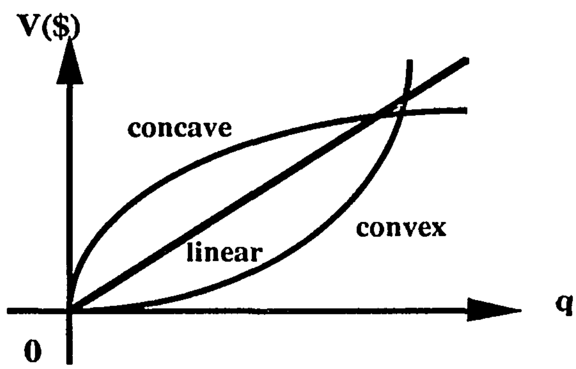

Figure 3-2 Relationship Between Consumer Quality Value and Quality

The shapes of the quality value function for different product characteristics depend on consumer preference strength for deterministic quality, product performance features and product price. Consumer marginal quality value (first derivative of the value function with respect to quality variable) can be decreasing, constant, or increasing, which corresponds to concave, linear and convex curves over a range of product quality. Increasing, constant or decreasing marginal quality values suggest that consumer quality value corresponding to one unit increase in quality level is increasing, constant or decreasing. There may be other quality value functions which are a combination of the above quality value functions over a range of quality levels. The quality value function for which marginal value is constant is a logical candidate and the simplest form to use when one tries to describe the behavior of the consumer under quality certainty. It will be more clear in later sections to illustrate why linear, or transformed linear, quality value function is more convenient to describe the effects of product quality variation and price difference on consumer utility under relative risk aversion. However, our interest is in the examination of the effect of quality value function pattern on expected quality value and consumer quality loss. The procedures for the establishment of the quality value function depend on a specific product quality performance and other conditions, which will not be discussed at this point.

Product quality actually varies over a range of probability distribution. Figure 3-3 illustrates the pattern of the nominal-the-best type of quality characteristics. In this figure, Δ is the specification for a uniform quality distribution, $\Delta=(b-a)/2$ (b and a are the upper and lower bounds for quality variation, respectively). σ_1, σ_2, and σ_3 are the standard deviations corresponding to the three normal distributions shown and $\sigma_3 > \sigma_2 > \sigma_1$.

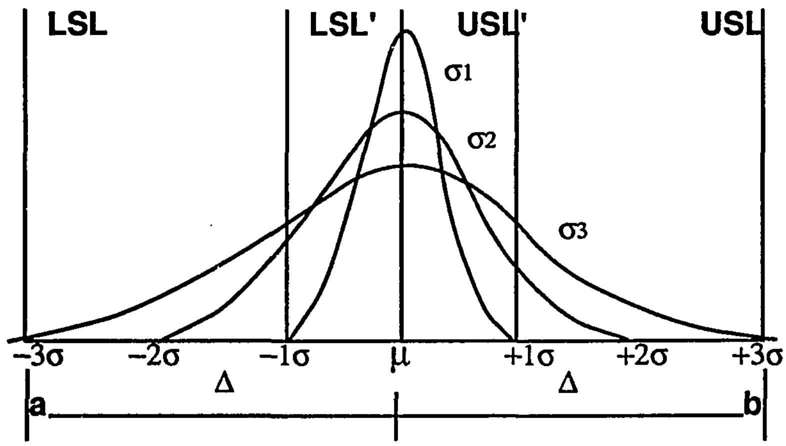

Figure 3-3 Product Quality Variation

Similar product quality distributions are also used in the study of the process in terms of the process capability ratio (PCR). The quality characteristic possesses both upper and lower specification limits (USL and LSL, respectively). The diagram shows the $6\sigma_3$ spread of the product quality distribution. μ is the population mean or the consumer quality target value in the nominal-the-best type of quality characteristics. Items that perform in the region between LSL and USL are considered as conforming. No quality loss for either consumers or producers would be accounted for by these conforming parts.

This traditional method of product quality specification has been challenged by the Japanese industry since the 70's. The Japanese suggest that the emphasis of quality control activities should not be limited to the manufacturing process, but on the product design to minimize deviation of product quality from the target value. In other words, quality loss is a continuous function in the range of quality specification. The more the deviation from the target value, the larger the loss will be. As a result of this procedure, if the quality deviation is only $3\sigma_1$, not $3\sigma_3$, the parts inside the limits shown by LSL' and USL' would result in a significant reduction of the consumer's quality dissatisfaction. This new definition of product quality variation has improved the quality of products and has strengthened the ability of Japanese products to penetrate international markets and compete strongly.

According to consumer discrimination properties, the cumulated distribution function is more useful in capturing the consumer expected quality value under a variety of quality characteristics than the direct use of quality distribution pattern, especially in the asymmetric type of quality characteristics. The product quality distributions in Figure 3-3 are transformed into the cumulated probability distributions, shown in Figure 3-4.

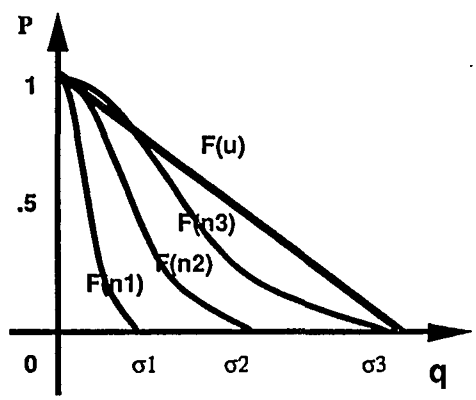

Figure 3-4 Cumulated Probability Distributions

There are several notations used in Figure 3-4. (1) The vertical and the horizontal axes represent the cumulated probability and the quality deviation from the target value, respectively. (2) The cumulated uniform probability distribution F(u) is a straight line corresponding to probabilities between 0 and 1. (3) The absolute first order central moment, $\mathring{a} = |z - \mu|$, is used to measure the deviation from the consumer quality target value in the case of the nominal-the-best type of quality characteristics. (4) $F(n_1)$, $F(n_2)$ and $F(n_3)$ are the cumulated distribution curves corresponding to the normal distributions, $f(n_1) \sim N(\mu, \sigma_1^2)$, $f(n_2) \sim N(\mu, \sigma_2^2)$ and $f(n_3) \sim N(\mu, \sigma_3^2)$. (5) In the nominal-the-best type of quality characteristics, the deviations with the same distance on both sides of the quality target value have the same effect on consumer dissatisfaction or consumer quality loss. The cumulated probability is the sum of two-tailed probability with the same deviation from the target value. (6) The cumulated diagram drawn for the nominal-the-best type of quality characteristics can be expanded to the other three types of quality characteristics, the larger-the-better, the smaller-the-better and asymmetric types.

In the following, an example is used to illustrate the effect of reduction of product quality deviation from the target value on quality competition. Assume that $\sigma_3 = 3\sigma_1$ and $\sigma_2 = 2\sigma_1$, as shown in Figure 3-4. If the quality deviation from the mean is σ_1 on the horizontal axis, it crosses the cumulated probability distribution line of specification 1 ($F(n_1)$) at the point (σ_1, 0.317). It means that 68.3% of product quality is located in the region between σ_1 and $-\sigma_1$, and 31.7% of product quality is outside the region. In the same way, it can be found that 38.3% of product quality in specification 2 ($F(n_2)$) is set in the same region, 25.9% for specification 3 ($F(n_3)$) and 11.1% for uniform distribution, respectively. When the deviation is $3\sigma_1$, 0.25% of product quality in specification 1 is outside the acceptable region between $+3\sigma_1$ and $-3\sigma_1$, 13.4% for specification 2, 31.7% for specification 3, and 66.7% for uniform distribution, respectively. The uniform distribution is the upper bound for outside the acceptable region, or the lower bound for inside the acceptable region. Keep in mind that when specification limit for uniform distribution is set to $6\sigma_3$, the cumulated probability for outside the specification between $+3\sigma_3$ and $-3\sigma_3$ is 0, but about 0.25% for the normal distribution. Obviously, the consumer will select product 1 which provides the smallest quality deviation.

Generally, the uniform distribution can be considered as a baseline system for the specification of product quality distribution. Shown in Figure 3-5, the uniform density function is

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0\ , & \text{otherwise} \end{cases}$$

where f(x) _ uniform density function;

a, b - the lower and the upper bounds for uniform distribution, respectively.

The quality specification, Δ, for uniformly distributed quality is

$$\Delta = (b-a)/2$$

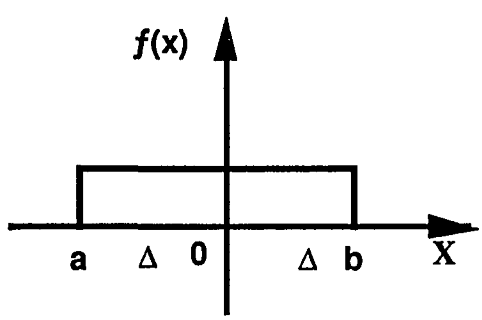

Figure 3-5 Uniform Density Function

It is possible to find a small range, Δ_0, such that the consumer is indifferent for the quality variation within this range Δ_0. In other words, the consumer randomly picks up a product within the range Δ_0 without any change in his utility. The consumer will suffer a quality loss if the specification limit is bigger than Δ_0. In general, the actual interval of the product quality variation is

$$\Delta = \Omega\Delta_0 \qquad (3.2.1\text{-}2)$$

where Δ _ the actual interval of quality variation that would result in consumer quality discrimination;

Δ_0 _ quality indifference interval;

Ω _ coefficient, $\Omega \geq 1$.

If current technology development and manufacturing processes make the indifference quality interval Δ_0 possible, the coefficient $\Omega = 1$ and the consumer would have no loss for the quality interval Δ_0. If the producer provides products within this quality specification, he does not suffer any loss. Any attempt to tighten up Δ_0 will not be beneficial. However, in most cases, $\Omega >> 1$ and the quality interval Δ is much larger than the ideal interval Δ_0.

The probability that a product works is denoted by ρ. $(1-\rho)$ is the probability of the product failure to work. Assuming ρ is a no increasing concave function of absolute value of Δ.

$$\rho = f(\Delta) \qquad (3.2.1\text{-}3)$$

$$\Delta \rightarrow \Delta_0, \ \rho \rightarrow 1$$

In the range $[|\Delta|, \Delta_0]$,

$$\partial f(\Delta)/\partial(-|\Delta|) = f'(\Delta) > 0, f''(\Delta) < 0 \qquad (3.2.1\text{-}3')$$

Equation (3.2.1-3') means that the probability of conforming rate, ρ, is closer to 1 when the product quality specification, $|\Delta|$, becomes smaller. The traditional on-line quality control is focused on equations (3.2.1-3) and (3.2.1-3') for product quality improvement.

In the following, we discuss the consumer quality loss for a workable product. The effect of a nonconforming rate on the consumer quality utility will be described in Chapter 6. We define T as the consumer quality target value, and $\mathring{a}$ as quality deviation from T, i.e.,

$$\mathring{a} = z - T \quad \text{in the smaller-the-better type} \qquad (3.2.1\text{-}4)$$

$$\mathring{a} = T - z \quad \text{in the larger-the-better type} \qquad (3.2.1\text{-}4')$$

$$\mathring{a} = |z - T| \quad \text{in nominal-the-best type} \qquad (3.2.1\text{-}4'')$$

where z - actual variation of quality characteristics, unit.

In the nominal-the-best type of quality characteristics, z takes the value either smaller or larger than the T value. Since the quality deviations which are symmetric on either side of the quality target value have the same effects on consumer quality loss, an absolute value $|z-\mu|$ (i.e. $|z-T|$) is used to express the symmetric effects. However, if the mean value of the product quality distribution is not consistent with the target value, it can be described in the case of the larger-the-better type or the smaller-the-better type of quality characteristics.

In the following, we will discuss three typical quality value functions with constant, decreasing and increasing quality marginal values in the nominal-the-best type of quality characteristics.

Linear quality value function is determined by the following formula:

$$w = w_H - k_1 å \qquad (3.2.1\text{-}5)$$

$$= w_H - k_1 |z - \mu| \qquad (3.2.1\text{-}5')$$

where w_H _ the value corresponding to the highest quality, $;

w _ the value corresponding to the actual quality realized, $;

k_1_ constant quality loss coefficient, $/unit;

μ _ mean value of quality distribution, unit.

The two formulas, (3.2.1-5) and (3.2.1-5'), are equivalent. The use of these two equations depends on the specific situation. However, quality variable å might be more convenient to explain the quality deviation from the quality target value. Figure 3-6 shows the relationship between w and å as well as w and z.

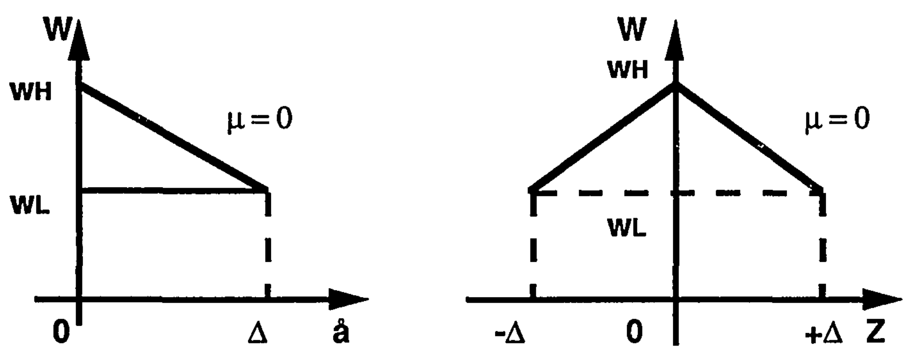

Figure 3-6 Relationships Between w and å, w and z

Two quadratic quality value functions, one with decreasing and the other with increasing marginal quality values, are established in the nominal-the-best type of quality characteristics. The quadratic quality value function with decreasing marginal value is

$$w = w_H - k_2 å^2 \qquad (3.2.1\text{-}6)$$

$$= w_H - k_2(z - \mu)^2 \qquad (3.2.1\text{-}6')$$

and the quadratic quality value function with increasing marginal quality value has the following form

$$w = w_L + k_3(å - \Delta)^2 \qquad å \leq \Delta \qquad (3.2.1\text{-}7)$$

$$= w_L + k_3(|z - \mu| - \Delta)^2 \qquad z \leq \Delta \qquad (3.2.1\text{-}7')$$

where w_L - the quality value corresponding to the lowest quality, \$;

k_2, k_3 - quality loss coefficients, \$/unit^2;

Δ - quality variation range, unit.

The quality loss coefficients k_1, k_2 and k_3 in the above equations can be determined by setting $å = \Delta$ in equations (3.2.1-5) and (3.2.1-6), and $å = 0$ in equation (3.2.1-7), as shown below:

$$k_1 = (w_H - w_L)/\Delta \qquad (3.2.1\text{-}8)$$

$$k_2, k_3 = (w_H - w_L)/\Delta^2 \qquad (3.2.1\text{-}9)$$

Similarly, the quality value function for the asymmetric type of quality characteristics is

$$w = w_H - k_{11}|z_1 - \mu| \qquad z < \mu \qquad (3.2.1\text{-}10)$$

$$w = w_H - k_{12}|z_2 - \mu| \qquad z > \mu \qquad (3.2.1\text{-}10')$$

where k_{11}, k_{12} - quality loss coefficients for the two sides, \$/unit;

z_1, z_2 - actual deviation of quality characteristics in two sides, unit.

Expected quality value (EQV) depends on the shape of the consumer's quality value function and the quality distribution for a given type of product. In mathematical form, it is

$$EQV = E[f(w)] \qquad (3.2.1\text{-}11)$$

In the discrete case:

$$EQV = \Sigma f(w_i)P_i$$

and in the continuous case:

$$EQV = \int f(w)\phi(w)dw$$

where $f(w)$ _ consumer quality value function, \$;

$f(w_i)$ _ value of quality event i, \$;

P_i _ probability associated with quality event i;

$\phi(w)$ _ probability density function of quality characteristic distribution.

We assume that there is no substantial difference in quality preferences and beliefs among groups where consumers are relatively homogeneous.

Average consumer quality loss (ACQL) is defined as the average of consumer quality loss for a given product (or expectation of consumer quality loss under the quality distribution for a given product),

$$ACQL = E(CQL) = w_H - EQV \qquad (3.2.1\text{-}12)$$

Quality variation observed by the consumer is closely related to the degree of availability of quality information. Quality information is available for all agents, consumers and producers, and quality variation only involves inherent product quality variance in the perfect competitive conditions. In the real world, the market is not perfect, and complete information is not available. The consumer needs to gather information to reduce or avoid quality loss. As Roberts and Urban [1988] suggest, consumers can only approximately evaluate a new product with an uncertainty which includes the inherent product variability and information uncertainty.

The degree of information availability determines the pattern of consumer product selection. If no information, including any previously related information, is available, consumers would choose a product randomly from a set of totally substitutable products. If some imperfect but unbiased information is available, consumers use the information to assess the product quality (see Robert and Urban [1988]).

$$x = x_p + \varepsilon_i \quad \varepsilon_i \sim N(0, \sigma^2_{\varepsilon i}) \tag{3.2.1-13}$$

$$E(x) = E(x_p) + E(\varepsilon_i) = \mu_p \tag{3.2.1-14}$$

$$\sigma(x) = \sigma(x_p) + \sigma(\varepsilon_i) \tag{3.2.1-15}$$

$$\sigma^2 = \sigma^2_p + \sigma^2_{\varepsilon i} \tag{3.2.1-16}$$

where x - consumer assessment about product quality;

x_p - inherent product quality;

ε_i - information about product quality. x_p and ε_i are independent.

If some of the information sources are somewhat biased, they could generate the information risk. Consumers could not correctly estimate the product quality and would have a greater quality loss resulting from the information sources.

$$x = x_p \pm \varepsilon_i \qquad \varepsilon_i \sim N(\mu_{\varepsilon i}, \sigma^2_{\varepsilon i}) \tag{3.2.1-17}$$

$$E(x) = E(x_p) \pm E(\varepsilon_i) = \mu_p \pm \mu_{\varepsilon i} \tag{3.2.1-18}$$

$$\sigma(x) = \sigma(x_p) + \sigma(\varepsilon_i) \tag{3.2.1-19}$$

$$\sigma^2 = \sigma^2{}_p + \sigma^2{}_{\varepsilon i} \tag{3.2.1-20}$$

Total quality variance, σ^2, is the sum of two independent components as follows:

$$\sigma^2 = \sigma_i^2 + \sigma_p^2 \tag{3.2.1-21}$$

where σ_i^2 _ quality information variance;

σ_p^2 _ inherent product quality variance.

Under conditions of perfect information, quality information uncertainty equals zero, $\sigma^2 = \sigma_p^2$. In other situations, consumers use magazines, advertisements, retail salesmen and friends as information sources to reduce uncertainty of imperfect quality information. Consumer risk also could be reduced through the information obtained either by buying the advertised product that a firm uses as an implicit signal to affect consumer decision making, or by direct search. Urban et al. [1990] point out that word-of-mouth may have positive or negative effects on a consumer's product assessment. The effect of information on total quality variation should be carefully examined.

Furthermore, the total quality uncertainty is also associated with the stage of market entrance of the product as well as the period in a product's life cycle. Product reputation also affects the shape of consumer total quality information. For instance, even though another company's products have the same inherent quality level as a highly reputed product has, consumers prefer the latter because its quality reputation reduces the risk of consumer decision making under quality information uncertainty. The company can greatly benefit from consumer preference, especially for new products. However, the consumer evaluates product quality from the available information which may be biased and imperfect. For tractability and without significant loss of generality, we will discuss the product quality in the case of unbiased information. If the condition is changed, we will mention it.

Since the quality of a certain product follows a pattern of probability distribution, the consumer faces an uncertainty of product quality in buying a desired product. Product quality variation can be treated as a special risk. Consumer behavior and decision making under quality risk can be described by von Neumann-Morgenstern utility function.

We assume that the consumer obeys the von Neumann-Morgenstern utility axioms. Consumer behavior under quality risk is shown in Figure 3-7. The vertical axis in this figure represents consumer utility while the horizontal axis represents certainty equivalent in terms of money value. The patterns of consumer utility functions for product quality variation can be linear, concave, convex and other specific forms with respect to consumer behavior under risk neutral, risk aversion, risk taking and risk portfolio, respectively.

Specifically, we assume in Figure 3-7 that: (1) The consumer utility corresponds to the specification of product quality value variation, located in the region $\mu \pm \Delta$ ($\Delta = 3\sigma$). (2) The certainty equivalents of the highest and the lowest product quality are w_H and w_L, respectively, whose initial values are determined by consumer quality value function.

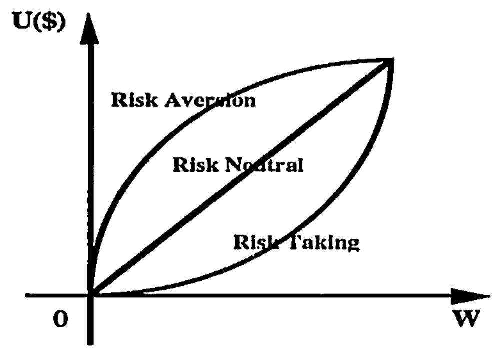

Figure 3-7 Consumer Behavior under Quality Risk

3.2.2 Consumer Behavior Under Quality Risk Neutral

We employ the concepts of the conventional consumer behavior theory to explain consumer quality loss. The consumer utility function suggests that (1) utility is measured in monetary units, (2) utility function is strictly increasing, and (3) utility function is continuous with continuous first and second order derivatives.

A person is risk neutral relative to a quality variation if the utility of the expected value of the quality equals the expected utility of the quality, i.e.

$$U[E(w_i)] = E[U(w_i)] \qquad (\ 3.2.2\text{-}1\ )$$

Obviously, a risk neutral person relates only to the expected quality values and is not sensitive to the quality risk. The expected quality value can be calculated by using certainty equivalents without using the preference (utility) curve.

We define a utility value transfer operator, V, as follows:

$$V[U(cf(w_i))] = cf(w_i) \qquad (3.2.2\text{-}2)$$

$$V[U(c_1 f(w_i)) + U(c_2 f(w_j))] = c_1 f(w_i) + c_2 f(w_j) \qquad (3.2.2\text{-}2')$$

If U_i and U_j are independent

$$V[U_i(c_1 f(w_i))U_j(c_2 f(w_j))] = c_1 f(w_i) c_2 f(w_j) \qquad (3.2.2\text{-}2'')$$

where c, c_1 and c_2 are coefficients.

Applying the above rules on equation (3.2.2-1), then

$$V\{E[U(w_i)]\} = V\{U[E(w_i)]\} = E(w_i) \qquad (3.2.2\text{-}3)$$

The expected utility value under quality neutral equals the expected quality value. The reason for using utility value transfer operator V is to avoid tedious discussion of the features for a specific consumer utility function, and concentrate on the results derived from the general utility function properties.

Equation (3.2.2-1) implies that the consumer has a linear utility function

$$U = b_L + b_1(w - w_L)$$

$$= b_L + (b_H - b_L)(w - w_L)/(w_H - w_L)$$

From equation (3.2.1-8), $(w_H - w_L) = k_1\Delta$

$$U = b_L + (b_H - b_L)(w - w_L)/(k_1\Delta) \qquad (3.2.2\text{-}4)$$

where w _ certainty equivalent value, $;

b_L _ intercept factor of utility function, corresponding to the certainty equivalent w_L, $V[b_L] = w_L$;

b_H _ the utility corresponding to the certainty equivalent w_H,

$V[b_H] = w_H$;

b_1 _ the constant slope of utility function, $b_1 = (b_H - b_L)/(w_H - w_L)$.

For linear quality value function,

$$w = w_H - k_1 å$$

$$dw/d(-å) = k_1 > 0 \qquad (3.2.2\text{-}4')$$

We use the negative deviation value (-å) to indicate that quality deviation from the target value is smaller when (-å) becomes larger.

Taking partial derivative of (-å) for equation (3.2.2-4) and using equation (3.2.2-4'), we have,

$$\begin{aligned} \partial U/\partial(-å) &= (\partial U/\partial w)(\partial w/\partial å) \\ &= k_1(b_H - b_L)/(k_1\Delta) \\ &= (b_H - b_L)/\Delta \end{aligned}$$

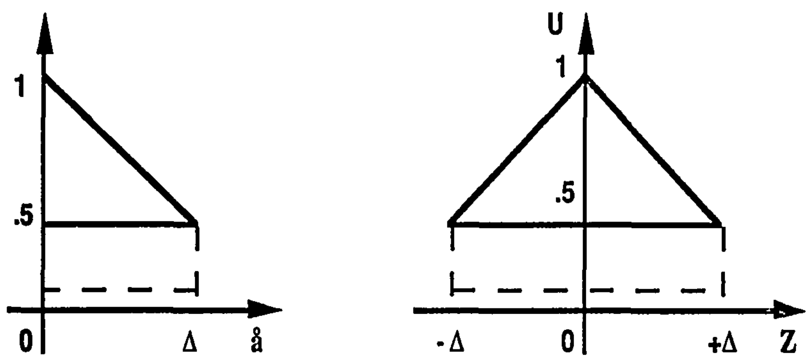

Figure 3-8 Relationships Between U and å, U and z in Nominal-The-Best Type of Quality Characteristics

The above equation implies that under quality neutral conditions, the consumer's marginal utility change is fixed with respect to one unit change in quality deviation when consumer quality value is linear. The positive rate means that when the deviation is closer to the target value, the utility of the consumer increases proportionally. The estimation of the expected utility value can be made through the estimation of expected quality value under quality neutral.

Similarly, for concave and convex quadratic quality value functions, we have

$$\partial U/\partial(-å) = 2å(b_H - b_L)/\Delta^2 > 0,\ U_å'' < 0 \qquad (3.2.2\text{-}5)$$

$$\partial U/\partial(-å) = 2(\Delta - å)(b_H - b_L)/\Delta^2 > 0,\ U_å'' > 0 \qquad (3.2.2\text{-}5')$$

i.e., consumer marginal quality utility is decreasing and increasing with reduction of quality deviation, respectively. Therefore, consumer utility under quality neutral is closely related to the properties of the quality value function.

There are two ways to compute expected quality value, one of which employs the probability distribution of product quality and the other uses the cumulated distribution of product quality. Whichever method is used, quality deviation, å, is adopted to capture the quality deviation from the quality target value. Since the certainty equivalent value is the function of quality deviation, we will examine the relationship between the expected quality value (i.e. the expected utility value) and the quality variation.

Normal and uniform distributions for product quality will be employed to derive their expected quality values in the case of the nominal-the-best type of quality characteristics. For a normally distributed quality characteristic, the expected quality value, EQV_n, is [1]

$$EQV_n = w_H - 0.8k_1\sigma_n \qquad (3.2.2\text{-}6)$$

For the uniform distribution of product quality, the expected quality value, EQV_u, is[2]

$$EQV_u = w_H - k_1(b - a)/4$$
$$= w_H - 0.866k_1\sigma_u \qquad (3.2.2\text{-}7)$$

---

[1] $EQV_n = E(w_H - k_1|z - \mu|) = w_H - k_1E(|z - \mu|)$

$$E(|z - \mu|) = 2\int_0^{\infty}(z - \mu)\frac{1}{\sqrt{2\pi}\sigma}\,EXP\frac{1}{2}\left(\frac{(z - \mu)}{\sigma}\right)^2 dz$$

$$\frac{z - \mu}{\sigma} = t$$

$$E(|z - \mu|) = 2\int_0^{\infty}\frac{t\sigma}{\sqrt{2\pi}}\,EXP(-\frac{t^2}{2})dt$$

$$= \frac{2\sigma}{\sqrt{2\pi}}\int_0^{\infty} t\,EXP(-\frac{t^2}{2})dt$$

$$= -\sqrt{\frac{2}{\pi}}\sigma(e^{\frac{-t^2}{2}})\Big|_0^{\infty}$$

$$= \sqrt{\frac{2}{\pi}}\sigma \quad = 0.8\sigma$$

$EQV_n = E(w_H - k_1|z - \mu|) = w_H - 0.8k_1\sigma$

[2] $EQV_u = w_H - k_1E(|z - \mu|)$

$$E(|z - \mu|) = 2\int_{(a+b)/2}^{b}(z - \frac{a + b}{2})\frac{1}{b - a}dz$$

$$= \frac{z^2 - (a + b)z}{b - a}\Bigg|_{(a+b)/2}^{b} = \frac{b - a}{4}$$

$EQV_u = w_H - k_1E(|z - \mu|) = w_H - k_1\frac{b - a}{4}$

If $(b-a)/2 = \Delta = 3\sigma_n$, where σ_n is the standard deviation corresponding to the normal distribution, then

$$EQV_u = w_H - 1.5k_1\sigma_n \qquad (3.2.2\text{-}8)$$

In comparison of equation (3.2.2-6) and equation (3.2.2-7), the product quality with a uniform distribution results in a larger consumer loss. The expected quality value for a uniform distribution can be considered as a lower quality bound for the expected quality value of any shape of product quality distribution, while the highest quality value, w_H, is the upper bound with the exception of the larger-the-better and the smaller-the-better types of quality characteristics with convex quality value function.

$$E(w_u) \leq E(w_i) \leq w_H \qquad (3.2.2\text{-}9)$$

where $E(w_u)$ _ expected quality value of product quality with a uniform distribution, \$;

$E(w_i)$ _ expected quality value of product quality with the distribution pattern i, \$.

It is found that the expected quality value, or the expected utility value under risk neutral, is located somewhere from the quality target value in the nominal-the-best type of quality characteristics.

We define the absolute ratio of average consumer quality loss, r, for any probability distribution i to the corresponding uniform distribution for a given product as:

$$r = (w_H - EQV_i)/(w_H - EQV_u) \qquad (3.2.2\text{-}10)$$

For linear quality value function,

$$EQV_u = (w_H+w_L)/2$$

then $$EQV_{linear} = w_H - r(w_H - w_L)/2$$
$$= (1 - r/2)w_H + (r/2)w_L$$

Let $$r/2 = p$$
$$EQV_{linear} = (1 - p)w_H + pw_L \qquad (3.2.2\text{-}11)$$

Equation (3.2.2-11) is not related to parameters of any product quality distribution and is very convenient to compute the expected quality value for normal quality distribution if r can be derived mathematically or computed approximately for linear quality value function. For example, the absolute ratio of average consumer quality loss and the expected quality value for a normal distribution are, respectively,

$$r = 0.8/1.5 = 0.5333$$
$$EQV_n = 0.733w_H + 0.267w_L \qquad (3.2.2\text{-}12)$$

For concave and convex quadratic quality value functions in nominal-the-best type of quality characteristics with normal distribution, the simplified computational equations are derived in the following, respectively

$$EQV_u \text{ (concave)} = w_H - 3k_2\sigma^2$$

$$= w_H - 3(w_H - w_L)\sigma^2/\Delta^2$$

$$= w_H - 3(w_H - w_L)\sigma^2/9\sigma^2$$

$$= w_H - (w_H - w_L)/3$$

$$EQV_u \text{ (convex)} = w_H - 6k_3\sigma^2$$

$$= w_H - 6(w_H - w_L)\sigma^2/\Delta^2$$

$$= w_H - 2(w_H - w_L)/3$$

$$EQV_{concave} = (1 - r/3)w_H + (r/3)w_L \qquad (3.2.2\text{-}13)$$

$$EQV_{convex} = (1 - 2r/3)w_H + (2r/3)w_L \qquad (3.2.2\text{-}14)$$

While in the larger-the-better and the smaller-the-better types of quality characteristics with normal distribution, the simplified computational equations are, respectively

$$EQV_{concave} = (1 - r/3)w_H + (r/3)w_L \qquad (3.2.2\text{-}15)$$

$$EQV_{convex} = (1 - 2r/3)w_H + (2r/3)w_L \qquad (3.2.2\text{-}16)$$

We define the relative ratio of average consumer quality losses for any two products with similar quality distribution patterns as

$$r_{nm} = (w_H - EQV_n)/(w_H - EQV_m), \quad \sigma_n^2 \geq \sigma_m^2 \qquad (3.2.2\text{-}17)$$

where r_{nm} - the relative ratio of average consumer quality losses;

w_H - the highest quality value for both products n and m, \$;

EQV_n, EQV_m - expected quality values for products n and m, respectively, \$;

σ_n^2, σ_m^2 - quality distribution variances for products n and m, respectively.

The difference between average consumer quality losses for the two products, d_{nm}, is

$$d_{nm} = ACQL_n - ACQL_m = - d_{mn} \qquad (3.2.2\text{-}18)$$

where $ACQL_n$ and $ACQL_m$ correspond to average consumer quality losses for products n and m, respectively.

Now we use a cumulated quality distribution to compute the above indeces. Assume that $\Delta' = \sigma_1$, $\sigma_2 = 2\sigma_1$, $\sigma_3 = 3\sigma_1$. σ_1, σ_2 and σ_3 are the standard deviations for the normal quality distributions $f(n_1)$, $f(n_2)$ and $f(n_3)$, respectively. Δ is the specification, $\Delta = (b-a)/2$, for the uniform distribution. There are a total of $9\Delta'$ equal intervals between w_H and w_L. For each small interval Δ', the middle quality value is used to compute the loss for the interval. For example, the middle quality value in the third interval ($3\Delta'$) is

$$[(w_H - 2k\sigma_1) + (w_H - 3k\sigma_1)]/2 = (w_H - 5k\sigma_1/2)$$

Thus, the middel quality value for interval i is

$$m_i = [(w_H - (i-1)k\sigma_1) + (w_H - ik\sigma_1)]/2 = (w_H - (2i-1)k\sigma_1/2)$$

The expected quality values in the nominal-the-best type of quality characteristics are computed with cumulated distribution as follows.

$$EQV = \sum_{i=1}^{n} \delta_i m_i$$

where δ_i- the cumulated probability for interval i;

m_i - the middle quality value for interval i.

Therefore, the expected quality values for the above three quality specifications specifically are:

$$E(w_{n1}) \approx 0.6826*(w'-(k\sigma_1)/2) \ +0.2718*(w'-(3k\sigma_1)/2)$$
$$+0.0456*(w'-(5k\sigma_1)/2)$$
$$\approx w_H - 0.863k\sigma_1$$
$$E(w_{u1}) = w_H - 1.5k\,\sigma_1$$
$$r_1 = 0.575$$
$$E(w_{n2}) \approx 0.383*(w'-(k\sigma_1)/2) \ +0.2998*(w'-(3k\sigma_1)/2) + \ldots$$
$$+0.0124*(w'-(11k\sigma_1)/2)$$
$$\approx w_H - 1.6263k\sigma_1$$
$$E(w_{u2}) = w_H - 3k\sigma_1$$
$$r_2 = 0.5421$$
$$E(w_{n3}) \approx 0.2686*(w'-(k\sigma_1)/2) \ +0.2386*(w'-(3k\sigma_1)/2) + \ldots$$
$$+0.0076*(w'-(17k\sigma_1)/2)$$
$$\approx w_H - 2.4132k\sigma_1$$
$$E(w_{u3}) = w_H - 4.5k\sigma_1$$
$$r_3 = 0.5362$$

Compared to equation (3.2.2-12), the two methods, theoretical deduction and approximate computation, are consistent. 0.533 is assigned to parameter r to calculate the expected quality value for the product quality with a linear quality value function.

The relative ratio of consumer quality loss for the nominal-the-best type is

$$r_{21} = 6k_1\sigma_1/3k_1\sigma_1 = 2$$
$$r_{31} = 9k_1\sigma_1/3k_1\sigma_1 = 3$$

$$r_{11} = 1$$

$$r_{11} : r_{21} : r_{31} = \sigma_1 : \sigma_2 : \sigma_3 = 1 : 2 : 3$$

The difference of consumer quality losses for the nominal-the-best type is

$$d_{21} = d_2 - d_1 = 0.8k_1(\sigma_2 - \sigma_1) = 1.6k_1\sigma_1$$

$$d_{31} = d_3 - d_1 = 0.8k_1(\sigma_3 - \sigma_1) = 2.4k_1\sigma_1$$

The quality value functions (QVF), expected quality value (EQV) and average consumer quality discrimination (ACQL) for normal distribution for three types of quality characteristics are computed and shown in Table 3-1. Assuming that the quality specification range, $\Delta(\Delta=3\sigma$ in the nominal-the-best type of quality characteristics and $\Delta=6\sigma$ in the larger-the-better and the smaller-the-better types of quality characteristics), and the quality value difference between the highest quality value and the lowest quality value, $w_H - w_L$, are the same for linear, concave and convex quality value functions, it can be shown that

$$EQV_{convex} < EQV_{linear} < EQV_{concave} \qquad (3.2.2\text{-}19)$$

$$ACQL_{convex} > ACQL_{linear} > ACQL_{concave} \qquad (3.2.2\text{-}20)$$

As shown in Table 3-1, the form of consumer quality value function affects the values of EQV and ACQL significantly.

Table3-1: QVF, EQV and ACQL for Normally Distributed Quality Characteristics

Type of quality characteristics		linear	concave	convex
	QVF	$w_H - k_1\lvert z - \mu\rvert$	$w_H - k_2(z - \mu)^2$	$w_L + k_3(\lvert z - \mu\rvert - \Delta)^2$
The nominal	EQV	$w_H - 0.8k_1\sigma$	$w_H - k_2\sigma^2$	$w_L + k_3(\sigma^2 - 1.6\Delta\sigma + \Delta^2)$
-the-best	ACQL	$0.8k_1\sigma$	$k_2\sigma^2$	$w_H - w_L - k_3(\sigma^2 - 1.6\Delta\sigma + \Delta^2)$
	QVF	$w_H - k_1(T-z)$	$w_H - k_2(T-z)^2$	$w_L + k_3(T-z-\Delta)^2$
The larger	EQV	$w_H - k_1(T-\mu)$	$w_H - k_2(\sigma^2+(T-\mu)^2)$	$w_L + k_3(\sigma^2+(T-\mu-\Delta)^2)$
-the-better	ACQL	$k_1(T-\mu)$	$k_2(\sigma^2 + (T-\mu)^2)$	$w_H - w_L - k_3(\sigma^2+(T-\mu-\Delta)^2)$
	QVF	$w_H - k_1(z-T)$	$w_H - k_2(z-T)^2$	$w_L + k_3(z - T - \Delta)^2$
The smaller	EQV	$w_H - k_1(\mu-T)$	$w_H - k_2(\sigma^2+(\mu-T)^2)$	$w_L + k_3(\sigma^2+(\mu-T-\Delta)^2)$
-the-better	ACQL	$k_1(\mu-T)$	$k_2(\sigma^2 + (\mu-T)^2)$	$w_H - w_L - k_3(\sigma^2+(\mu-T-\Delta)^2)$

It is clear that quality characteristics in the larger-the-better type have similar qualitative and quantitative properties as those in the smaller-the-better type of quality characteristics. In the larger-the-better type of quality characteristics EQV and ACQL computed with linear quality value function depend only on the mean value of quality distribution and are not affected by the quality variance. However, EQV and ACQL computed with nonlinear quality value functions depend not only on the mean value of the quality distribution but also on the variance of quality distribution. Consumers with concave quality value function prefer products with small quality variance since the larger the variance, the larger the consumer quality loss when the mean of quality distributions stays the same. Consumers with convex quality value function will select the product with the larger variance because it provides some products with higher quality if these quality distributions have the same distribution mean. The larger the variance, the smaller the quality loss to the consumer with convex quality value function in the larger-the-better and the smaller-the-better types of quality characteristics. But in the real world, such a consumer behavior is not common.

Table 3-2 lists EQV and ACQL values for uniformly distributed quality characteristics. The indexes of absolute ratio of consumer quality loss, r, relative ratio of average consumer quality loss, r_{nm}, and quality loss difference, d_{nm}, are shown in Table 3-3 assuming $\mu_n=\mu_m$. It can be shown that

$$r_{concave} < r_{linear} < r_{convex} \qquad (\ 3.2.2\text{-}21\ )$$

The ratio r can be substituted into equations (3.2.2-11), (3.2.2-15) or (3.2.2-16) to obtain a simple computation of EQV_i. The relative ratio, r_{nm}, and quality loss difference, d_{nm}, indicate that EQV and ACQL with linear quality value function are only related to the mean of the quality characteristics distributions in the larger-the-better and the smaller-the-better types of quality characteristics. The EQV and ACQL values for concave quality value function vary with quality variance; the larger the variance, the higher the quality discrimination. This conclusion is always consistent without consideration of the type of quality characteristic. However, this conclusion is not held for convex quality value function in the cases of the larger-the-better and the smaller-the-better types of quality characteristics.

Table 3-2: EQV and ACQL for Uniformly Distributed Quality Characteristics

Type of quality characteristics		linear	concave	convex
Nominal the best	EQV	$w_H - 1.5k_1\sigma$	$w_H - 3k_2\sigma^2$	$w_H - 6k_3\sigma^2$
	ACQL	$1.5k_1\sigma$	$3k_2\sigma^2$	$6k_3\sigma^2$
The larger the better	EQV	$w_H - k_1(T-\mu)$	$w_H - k_2(3\sigma^2 +(T-\mu)^2)$	$w_L + k_3(3\sigma^2 + (T-\mu-\Delta)^2)$
	ACQL	$k_1(T-\mu)$	$k_2(3\sigma^2 +(T-\mu)^2)$	$w_H - w_L - k_3(3\sigma^2 + (T-\mu-\Delta)^2)$
The smaller the better	EQV	$w_H - k_1(\mu-T)$	$w_H - k_2(3\sigma^2 +(\mu-T)^2)$	$w_L + k_3(3\sigma^2 + (\mu-T-\Delta)^2)$
	ACQL	$k_1(\mu-T)$	$k_2(3\sigma^2 + (\mu-T)^2)$	$w_H - w_L - k_3(3\sigma^2 + (\mu-T-\Delta)^2)$

Note: σ^2 is the variance corresponding to the normal quality distribution.

Table 3-3: Indexes for Three Types of Quality Characteristics
$(\sigma^2_n \geq \sigma^2_m,\ \mu_n = \mu_m)$

Type of quality Characteristics		linear	concave	convex
	r	0.533	0.333	0.633
The nominal	r_{nm}	≥ 1	≥ 1	≥ 1
-the-best	d_{nm}	$0.8k_1(\sigma_n - \sigma_m)$	$k_2(\sigma^2_n - \sigma^2_m)$	$3.8k_3(\sigma^2_n - \sigma^2_m)$
	r	1	0.833	1.083
The larger	r_{nm}	1	≥ 1	≤ 1
-the-better	d_{nm}	0	$k_2(\sigma^2_n - \sigma^2_m)$	$-k_3(\sigma^2_n - \sigma^2_m)$
	r	1	0.833	1.083
The smaller	r_{nm}	1	≥ 1	≤ 1
-the-better	d_{nm}	0	$k_2(\sigma^2_n - \sigma^2_m)$	$-k_3(\sigma^2_n - \sigma^2_m)$

Based on the above analysis and the assumption of quality distributions with the same mean value of quality characteristics, the following conclusions can be made under quality risk neutral conditions.

1. Consumer quality value function for deterministic quality should be carefully evaluated. EQV and ACQL resulting from quality variation will

be overestimated or underestimated if quality value function is not correctly determined.

2. Consumer quality discrimination always exists if there is variation in product quality. Average consumer quality loss, ACQL, is the difference between the highest quality value and the expected quality value.

3. In the nominal-the-best and asymmetric types of quality characteristics, consumers always favor the product with small variance regardless of which pattern of the quality value function is employed.

4. In the larger-the-better and the smaller-the-better types of quality characteristics, consumers with linear quality function are only concerned with the mean value of product quality distribution. Concave quality value function possesses the tendency for the consumer to make a decision in favor of the product with smaller quality variance. In contrast, consumers with convex quality value function will choose the product with higher quality value and ignore its larger variance.

3.2.3 Consumer Behavior Under Quality Risk Aversion

The wider the quality distribution spreads, the more risk the consumer faces in decision making on a choice of products from a number of available products. In the real world, consumers are usually observed to be risk averse in most cases. Consumer quality risk is caused by (1) quality variation in a certain type of product, (2) quality variation across products, and (3) imperfect quality information. A person is a quality risk averter if the utility of the expected value of the quality is greater than the expected value of its utility, i.e.

$$U[E(W_i)] > E[U(W_i)] \qquad (3.2.3\text{-}1)$$

Assuming equations (3.2.3-1) hold within the domain of the utility function, the consumer utility function is strictly concave if the following condition is satisfied

$$dU/dw > 0, \; d^2U/dw^2 < o \qquad (3.2.3\text{-}2)$$

The consumer utility function under quality aversion is a concave curve in comparison with the straight line utility under quality neutral. The consumer utility function depends on (1) the degrees of quality aversion, (2) the probability distribution pattern of the product quality, and (3) the form of the quality value function. Clearly, a specific utility function for a given product can be drawn on the basis of consumer behavior.

First of all, we examine the consumer quality utility value changes corresponding to the change in quality deviation. We must keep in mind that this analysis only gives us a partial picture of the effect of quality activity on consumer decision making under the assumption of other conditions being constant.

Based on the consumer behavior under quality aversion, the utility function is strictly concave over the domain of product quality variation. The expected quality value would not be linearly related to the quality deviation from the target value.

The relationship between the consumer utility and the quality deviation under the quality aversion in nominal-the-best type of quality characteristics is shown in Figure 3-9. (This relationship can be derived for other types of quality characteristics.) The utility value derived from quality deviation å is less than the value under the quality neutral conditions. Curve U_a is located below U_n. Mathematically, the relation could be written as follows in accordance with the properties of the utility and certainty equivalent functions given before. For linear quality value function:

$$U_a = f(w) \quad d^2U_a/dw^2 < 0 \tag{3.2.3-3}$$

From equation (3.2.2-4')

$$w = f(å) \quad \partial w_a/\partial(-å) = k_1$$

$$U_a = f(w(å)) \tag{3.2.3.-4}$$

$$\partial U_a/\partial(-å) = (\partial U_a/\partial w)(\partial w/\partial(-å))$$

$$= k_1 \partial U_a/\partial w \tag{3.2.3-5}$$

$$d^2U_a/d(-å)^2 = k_1 \partial^2 U_a/(\partial w \partial(-å))$$

$$= k_1 \partial[(\partial U_a/\partial w)(\partial w/\partial(-å))]/\partial w$$

$$= k_1^2 \partial^2 U_a/\partial w^2 < 0 \tag{3.2.3-6}$$

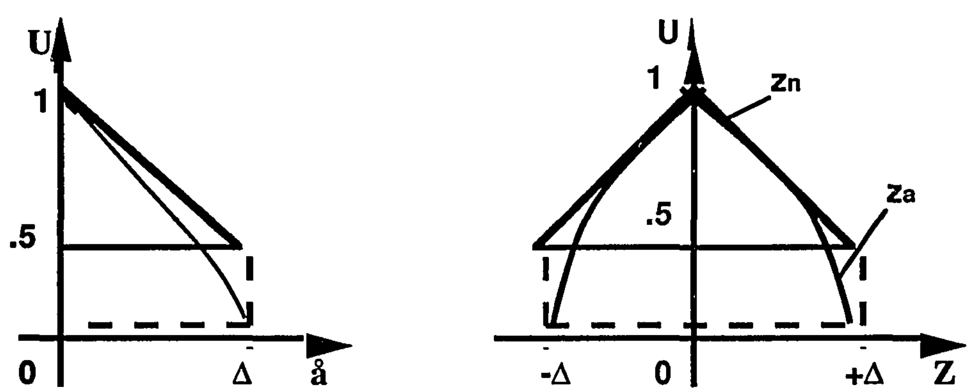

Figure 3-9 Relationships Between U and å, U and z under Quality Aversion in Nominal-The-Best Type of Quality Characteristics

Based on the conditions given before, equations (3.2.3-5) and (3.2.3-6) illustrate that: (1) The slope means the closer the quality deviation from the quality target value, the higher the consumer utility. (2) The marginal quality utility decreases when quality deviation reduces by the same amount of units. This implies that consumers are less and less sensitive to quality improvement. (3) When quality deviation tends to 0, the certainty equivalent value will be the target value and consumer utility will reach the highest point, that is when $dU_a/dw = 0$, then $dU_a/d(-å) = 0$ and $U(å=0) = U_H$. (4) The marginal quality utility is lower than the fixed marginal quality utility under quality neutral conditions.

Equation (3.2.3-6) proves that: (1) Utility function is strictly concave and has a maximum point at $å = 0$ against quality deviation under quality aversion. (2) Utility function is continuous with, at least, first and second-order derivatives with respect to quality deviation (-å).

For concave and convex quadratic quality value functions, respectively, we have

$$U_a = f(w) \quad d^2U_a/dw^2 < 0$$

$$w = w_H - k_2 å^2 \quad \partial w/\partial(-å) = 2k_2 å$$

$$\partial U_a/\partial(-å) = (\partial U_a/\partial w)(\partial w/\partial(-å))$$

$$= 2k_2 å \partial U_a/\partial w > 0 \tag{3.2.3-7}$$

$$-2k_2 \partial U_a/\partial w < 0, \; 4k_2^2 å^2 \partial^2 U_a/\partial w^2 < 0$$

$$d^2U_a/d(-å)^2 = -2k_2 \partial U_a/\partial w + 4k_2^2 å^2 \partial^2 U_a/\partial w^2 < 0 \tag{3.2.3-8}$$

$$w = w_L - k_3(å-\Delta)^2 \quad \partial w_a/\partial(-å) = 2k_3(\Delta - å)$$

$$\partial U_a/\partial(-å) = (\partial U_a/\partial w)(\partial w/\partial(-å))$$

$$= 2k_3(\Delta - å)\partial U_a/\partial w > 0 \tag{3.2.3-9}$$

$$2k_3\partial U_a/\partial w > 0,\ (2k_3(\Delta - å))^2\partial^2 U_a/\partial w^2 < 0$$

$$d^2U_a/d(-å)^2$$

$$= 2k_3\partial U_a/\partial w + (2k_3(\Delta - å))^2\partial^2 U_a/\partial w^2$$

$$\text{If } 2k_3\partial U_a/\partial w + (2k_3(\Delta - å))^2\partial^2 U_a/\partial w^2 > 0,\ d^2U_a/d(-å)^2 > 0 \qquad (3.2.3\text{-}10)$$

$$\text{If } 2k_3\partial U_a/\partial w + (2k_3(\Delta - å))^2\partial^2 U_a/\partial w^2 < 0,\ d^2U_a/d(-å)^2 < 0 \qquad (3.2.3\text{-}10')$$

Compared to the patterns in Figure 3-8, the relationship between U and å as well as the relationship between U and z are nonlinear and may be expressed as a curve with higher order derivatives in quality aversion when the shape of quality value function is concave. For the convex quality value function, the consumer has the higher degree of quality aversion so that his utility shows the properties of risk aversion, otherwise, the consumer is actually a quality risk taker. Curve z_a in Figure 3-9 tells us that one unit change in quality deviation results in a more rapid change in consumer utility than the utility change under quality neutral. In other words, if the product quality decreases, the consumer will face more quality loss than that estimated under quality neutral. It should be pointed out that the certainty equivalent of expected utility under quality aversion is less than that in quality neutral.

However, we cannot draw the final conclusion from the above analysis. The consumer utility obtained from product quality performance should be put into the total consumer utility function which associates consumer assets, budget, product price and information uncertainty.

The expected utility value (EUV) under quality neutral equals the expected quality value (EQV) which is related to the pattern of quality value function and the quality probability distribution. The consumer utility function under quality aversion is associated with the consumer's assets. It is logical to assume that many decision makers would feel they ought to pay less for quality premium against a given risk when they have greater assets. But in quality risk analysis we do not make such an assumption, i.e., we do not assume the utility is decreasing, constant or increasing risk aversion to limit our discussion in this section.

The consumer usually employs the same utility function to evaluate a number of products. Different income groups possess different utility functions. The higher the product price, the more sensitive the consumer attitude toward quality risk.

The consumer's budget for purchasing a product is a function of his assets. For the sake of simplicity, it is rational to assume that the consumer's budget is positively related to his assets. That is, the greater the assets the larger the budget. The consumer is able to search for a desired product in a brand of products subject to his budgetary constraint. The budget is an increasing function of his assets

$$B(x) = f(x) \qquad (3.2.3\text{-}11)$$

$$\partial B(x)/\partial x = f'(x) > 0 \qquad (3.2.3\text{-}11')$$

where B(x) - budget for purchasing a product, \$;

x - consumer's assets, \$.

A total quality risk premium for inherent product quality variation and information uncertainty is defined as the difference between the expected quality value and the certainty equivalent under quality aversion.

$$R = E(w) - C(w) \qquad (3.2.3\text{-}12)$$

where R - quality premium, \$;

E(w) - expected quality value, \$;

C(w) - certainty equivalent under quality aversion, \$.

It should be noted that the total quality risk premium which contains a different meaning from the traditional insurance premium is the utility adjustment under quality reversion. The consumer adjusts his preference to account for the cost of future product repairs, time waste, information availability, the cost of serving warranties, consumer moral hazard, etc.. Quality premium is another type of consumer quality loss under quality risk aversion.

$$C(w) = E(w) - R \qquad (3.2.3\text{-}13)$$

The inherent meaning contained in the certainty equivalent C(w) is the implicit expected utility value for the product under risk aversion.

Quality risk premium depends on the consumer's assets and the pattern of product quality value.

$$R = R(x, w) \qquad R << E(w) \qquad (3.2.3\text{-}14)$$

Assume that the consumer with assets x, budget B(x) and utility function U(x) is indifferent between facing a risk to buy a product with quality variation and receiving a value at the non-random amount E(w) - R. Formula (3.2.3-1) can be expanded as the following comprehensive equation under risk aversion

$$U[x - B(x) + E(w) - R(x,w)] = E[U(x - B(x) + w)] \qquad (3.2.3\text{-}15)$$

$$E(w) = w_H - ACQL$$

$$= w_H - \beta \qquad (3.2.3\text{-}16)$$

where β - average consumer quality loss under risk neutral, \$.

Let $\quad \alpha = w - E(w) \qquad (3.2.3\text{-}17)$

$$E(\alpha) = 0$$

$$\alpha^2 = (w - E(w))^2 \qquad (3.2.3\text{-}18)$$

$$E(\alpha^2) = \sigma^2_\omega \qquad (3.2.3\text{-}19)$$

Let $\quad y = x - B(x) + E(w)$

$$U[x - B(x) + E(w) - R(x + E(w), x - E(w))]$$

$$= E\{U[x - B(x) + E(w) + w - E(w)]\}$$

$$U(y - R) = E[U(y + \alpha)] \qquad (3.2.3\text{-}20)$$

Expanding U around y on both sides of equation (3.2.3-20). (see Pratt [1964].)

$$U(y - R) = U(y) - RU'(y) + f(R^2)$$

$$E[U(y + \alpha)] = E[U(y) + \alpha U'(y) + 0.5\alpha^2 U''(y) + f(\alpha^3)]$$

$$= U(y) + 0.5\sigma^2_\omega U''(y) + E[f(\alpha^3)]$$

Assuming that the third order of α is of smaller than σ^2_ω and the $f(R^2)$ is ignored

$$R = 0.5\sigma^2_\omega r(y)$$

$$= 0.5\sigma^2_\omega r[x - B(x) + E(w)] \qquad (3.2.3\text{-}21)$$

Since - B(x) + E(w) is small compared with x, then

$$R = 0.5\sigma^2_\omega r(x) \qquad (3.2.3\text{-}22)$$

where r(x) - a measure for consumer attitude toward risk in consideration of consumer assets effect. A measure of absolute risk aversion, r(x), is defined as:

$$r(x) = -u''(x)/u'(x) \qquad (3.2.3\text{-}23)$$

If $r(x) = 0$, risk neutral

$r(x) > 0$, risk aversion

$r(x) < 0$, risk taking

where u(x) - a utility function for money value.

Quality risk premium has a form similar to that in traditional consumer theory under risk aversion, which is approximately r(x) times half the variance of product quality value, σ^2_ω. The value of σ^2_ω can be computed by equation (3.2.3-19). The consumer quality loss under risk aversion is simply the sum of the quality loss under risk neutral and the quality risk premium.

We are more interested in the expected utility value rather than the specific utility function form. By using the value operator V, equation (3.2.3-15) becomes

$$U[x - B(x) + E(w) - 0.5\sigma^2_\omega r(x)] = E\{U[x - B(x) + w]\}$$
$$x - B(x) + E(w) - 0.5\sigma^2_\omega r(x) = V\{E[U(x - B(x) + w)]\} \qquad (3.2.3\text{-}24)$$

When $r(x) = 0$, equation (3.2.3-24) becomes the expected utility value under risk neutral, see equation (3.2.2-3).

Table 3-4 shows the relationship between the variance of quality value distribution, σ^2_ω, and the variance of product quality characteristics distribution, σ^2. It indicates that the quality premium is only related to the variance of quality characteristic distribution for linear quality value function. However, the quality premium is associated with squared variance and the squared mean value of quality characteristics distribution, as well as the product of the variance and the mean in the cases of the larger-the-better and the smaller-the-better types for concave and convex quality value functions, which means that the contribution of variance and mean value of product quality distribution on the premium cannot be separated.

The actual variance of product quality distribution is usually so small that the effects of the terms more than the second order of σ on the risk premium can be ignored. The simplified relationship between σ^2 and σ^2_ω is shown in Table 3-5.

Table 3-4: Relationship Between σ^2_ω and σ^2 for Normally Distributed Quality Characteristics

Type of quality characteristics	linear	concave	convex
The nominal -the-best	$0.36k^2{}_1\sigma^2$	$2k^2{}_2\sigma^4$	$k^2{}_3(2\sigma^4+1.44\Delta^2\sigma^2)$
The larger -the-better	$k^2{}_1\sigma^2$	$2k^2{}_2\sigma^2(\sigma^2+2(T-\mu)^2-8\sigma(T-\mu)/\sqrt{2\pi})$ $\approx 2k^2{}_2\sigma^2(\sigma-\sqrt{2}(T-\mu))^2$	$2k^2{}_3\sigma^2(\sigma^2+2(T-\mu-\Delta)^2-8\sigma(T-\mu-\Delta)/\sqrt{2\pi})$ $\approx 2k^2{}_3\sigma^2(\sigma-\sqrt{2}(T-\mu-\Delta))^2$
The smaller -the-better	$k^2{}_1\sigma^2$	$2k^2{}_2\sigma^2(\sigma^2+2(\mu-T)^2+8\sigma(T-\mu)/\sqrt{2\pi})$ $\approx 2k^2{}_2\sigma^2(\sigma+\sqrt{2}(\mu-T))^2$	$2k^2{}_3\sigma^2(\sigma^2+2(\mu-T-\Delta)^2+8\sigma(\mu-T-\Delta)/\sqrt{2\pi})$ $\approx 2k^2{}_3\sigma^2(\sigma+\sqrt{2}(\mu-T-\Delta))^2$

Table 3-5: Relationship Between σ^2_ω and σ^2 for Normally Distributed Quality Characteristics in a Simplified Form

Type of quality characteristics	linear	concave	convex
The nominal -the-best	$0.36k^2{}_1\sigma^2$		$1.44\ k^2{}_3\Delta^2\sigma^2$
The larger -the-better	$k^2{}_1\sigma^2$	$4k^2{}_2\sigma^2(T-\mu)^2$	$4k^2{}_3\sigma^2((T-\mu-\Delta)^2$
The smaller -the-better	$k^2{}_1\sigma^2$	$4k^2{}_2\sigma^2(\mu-T)^2$	$4k^2{}_3\sigma^2(\mu-T-\Delta)^2$

The expected utility value under quality aversion is lower than that under quality neutral. It means that the consumer is actually more sensitive to product quality variation. The expected quality value for the product quality with uniform distribution is the lower bound for the expected quality value of the product with any other distribution under quality aversion. The upper bound changes to the expected quality value of that distribution under the quality neutral.

$$C(w_u)_a \le C(w_i)_a \le E(w_i)_n \qquad (3.2.3\text{-}25)$$

where $C(w_u)_a$ _ expected quality value of uniform quality distribution under quality aversion, $;

$C(w_i)_a$ _ expected quality value of quality distribution pattern i under quality aversion, $;

$E(w_i)_n$ _ expected quality value of quality distribution pattern i under quality neutral, $;

R_u _ the risk premium for uniform quality distribution, $.

The absolute ratio of certainty equivalences for a quality distribution to uniform distribution under the quality aversion is written as follows

$$r_a = (w_H - C(w_i)_a)/(w_H - C(w_u)_a) \qquad (3.2.3\text{-}26)$$

Based on the following rules

$$\min(a/c,\ b/d) \le (a + b)/(c + d) \le \max(a/c,\ b/d) \qquad (3.2.3\text{-}27)$$

$$1 \ge a/c,\ b/d > 0; \quad d \ge b; \quad c \ge a$$

we can find that

$$r_a = (w_H - C(w_i)_a)/(w_H - C(w_u)_a) = (w_H - E(w_i)_n + R_i)/(w_H - E(w_u)_n + R_u)$$

$$r_a \le \max((w_H - E(w_i)_n)/(w_H - E(w_u)_n),\ R_i/R_u)$$

$$R_i/R_u = \sigma^2{}_i/\sigma^2{}_u = 1/3, \quad 1/3 \le r_n$$

$$r_a \le \max(r_n,\ 1/3)$$

$$r_a \le r_n \qquad (3.2.3\text{-}28)$$

where r_a _ absolute ratio under quality aversion;

r_n _ absolute ratio under quality neutral;

$E(w_u)_n$ _ expected quality value of uniform distribution for a product under quality neutral; $;

R_i _ the expected equivalent value of the risk premium for other distribution patterns, $.

Equation (3.2.3-28) is valid regardless of the forms of quality characteristics distribution.

For linear quality value function, the simplified calculation equation for $C(w_{linear})_a$ with nomal distribution is

$$C(w_u)_a = (w_H + w_L)/2 - R_u$$

$$C(w_{linear})_a = (1 - r_a/2)w_H + (r_a/2)(w_L - 2R_u)$$

Let $r_a/2 = P$

$$C(w_{linear})_a = (1-p)w_H + p(w_L - 2R_u) \quad (3.2.3\text{-}29)$$

For concave and convex quadratic quality value functions in nominal-the-best type of quality characteristics with normal distribution, the simplified computeral equations are, respectively

$$C(w_{concave})_a = (1 - r_a/3)w_H + (r_a/3)(w_L - 3R_u) \quad (3.2.3\text{-}30)$$

$$C(w_{convex})_a = (1 - 2r_a/3)w_H + (2r_a/3)(w_L - 3R_u/2) \quad (3.2.3\text{-}31)$$

While in the larger-the-better and the smaller-the-better types of quality characteristics, the simplified equations are, respectively

$$C(w_{concave})_a = (1 - r_a/3)w_H + (r_a/3)(w_L\text{-}3R_u) \qquad (3.2.3\text{-}32)$$

$$C(w_{convex})_a = (1 + 2r_a/3)w_H - (2r_a/3)(w_L\text{-}3R_u/2) \qquad (3.2.3\text{-}33)$$

The relative ratio of certainty equivalences for two same functional products with the same functional characteristics and distribution form under quality aversion becomes

$$\sigma^2{}_n \geq \sigma^2{}_m$$

$$(r_{nm})_a = (w_H - C(w_n)_a)/(w_H - C(w_m)_a) \qquad (3.2.3\text{-}34)$$

$$= (r_n/r_m)_a(w_H - C(w_{un})_a)/(w_H - C(w_{um})_a)$$

$$= (w_H - E(w_{un})_n + R_{un})/(w_H - E(w_{um})_n + R_{um})$$

$$R_{un}/R_{um} = \sigma^2{}_n/\sigma^2{}_m, \quad (r_n/r_m)_a \approx 1$$

From equation (3.2.3-27)

$$(w_H\text{-}E(w_{un})_n+R_n)/(w_H\text{-}E(w_{um})_n+R_m) \geq$$

$$\min((w_H\text{-}E(w_{un})_n)/(w_H\text{-}E(w_{um})_n), R_{un}/R_{um}), \quad R_{un}/R_{um} \geq (r_{nm})_n$$

$$(r_{nm})_a \geq (r_{nm})_n$$

where $(r_{nm})_a$ _ relative ratio under the quality aversion;

$(r_{nm})_n$ _ relative ratio under quality neutral;

$C(w_n)_a$ _ expected quality value for product n under quality aversion, \$;

$C(w_m)_a$ _ expected quality value for product m under quality aversion, \$;

R_{un} _ the risk premium for product n, \$;

R_{um} _ the risk premium for product m, \$.

The relative difference defined in the previous section is revised under quality aversion.

$$\sigma^2_n \geq \sigma^2_m$$

$$(d_{nm})_a = C(w_m)_a - C(w_m)_a \qquad (3.2.3\text{-}35)$$

$$= E(w_m)_n - E(w_n)_n + R_n - R_m$$

$$(d_{nm})_a \geq (d_{nm})_n$$

In comparison of the coefficients and the ratios under quality neutral and quality aversion, the following results can be obtained

$$r_a \leq r_n \qquad (3.2.3\text{-}36)$$

$$(r_{nm})_a \geq (r_{nm})_n \qquad (3.2.3\text{-}37)$$

$$(ACQL)_a \geq (ACQL)_n \qquad (3.2.3\text{-}38)$$

$$(d_{mn})_a \geq (d_{mn})_n \qquad (3.2.3\text{-}39)$$

Since theoretical deduction can be nicely done under quality neutral, the above formula can be employed to set the lower and upper bounds for expected quality value, consumer quality loss, absolute and relative quality loss ratios, and relative quality difference under quality aversion. Generally speaking, the relative systems (relative ratio and relative difference for quality loss) are more useful than the absolute systems (absolute ratio of quality losses). The consumer usually makes his decision on the basis of comparison of relative quality loss in selection from a brand of products.

The expected utility value (EUV), average consumer quality loss (ACQL) and the relative quality loss under risk aversion are shown in Table 3-6 under the assumption that all probability distributions are normal with the same mean value. The EUV under risk aversion is smaller than that under risk neutral, while ACQL is larger in this situation. The consumer is more sensitive to quality variation, such that he will choose the product which provides the highest utility from a set of available products. Even for the convex quality value function, the degree of tendency for favoring larger quality variance under risk neutral is noticeably reduced in risk aversion. The relative quality loss indices in nominal and asymmetric types of quality characteristics are ranked as

$$(d_{nm})_{convex} > (d_{nm})_{linear} > (d_{nm})_{concave} \qquad (\ 3.2.3\text{-}40\ )$$

while in the larger-the-better and the smaller-the-better types of quality characteristics, they are in the order

$$(d_{nm})_{concave} > (d_{nm})_{linear} > (d_{nm})_{concave} \qquad (\ 3.2.3\text{-}41\ )$$

Table 3-6 d_{nm}, EUV and ACQL in Three Types of Quality CharacteristicDistributions

		linear
	d_{nm}	$0.8k_1(\sigma_n-\sigma_m)+0.18k^2{}_1(\sigma^2{}_n-\sigma^2{}_m)r(x)$
Nominal	EUV	$w_H-0.8k_1\sigma-0.18k^2{}_1\sigma^2r(x)$
the best	ACQL	$0.8k_1\sigma+0.18k^2{}_1\sigma^2r(x)$
	d_{nm}	$k_1(\mu_n-\mu_m)+0.5k^2{}_1(\sigma^2{}_n-\sigma^2{}_m)r(x)$
The larger	EUV	$w_H-k_1(T-\mu)-0.5k^2{}_1\sigma^2r(x)$
the better	ACQL	$k_1(T-\mu)+0.5k^2{}_1\sigma^2r(x)$
	d_{nm}	$k_1(\mu_n-\mu_m)+0.5k^2{}_1(\sigma^2{}_n-\sigma^2{}_m)r(x)$
The smaller	EUV	$w_H-k_1(\mu-T)-0.5k^2{}_1\sigma^2r(x)$
the better	ACQL	$k_1(\mu-T)+0.5k^2{}_1\sigma^2r(x)$
		concave
	d_{nm}	$k_2(\sigma^2{}_n-\sigma^2{}_m)+k^2{}_2(\sigma^4{}_n-\sigma^4{}_m)r(x)$
Nominal	EUV	$w_H-k_2\sigma^2-k^2{}_2\sigma^4r(x)$
the best	ACQL	$k_2\sigma^2+k^2{}_2\sigma^4r(x)$
	d_{nm}	$k_2(\sigma^2{}_n-\sigma^2{}_m+(T-\mu_n)^2-(T-\mu_m)^2)+0.5k^2{}_2(\sigma^2{}_{\omega n}-\sigma^2{}_{\omega m})r(x)$
The larger	EUV	$w_H-k_2(\sigma^2+(T-\mu)^2)+0.5k^2{}_2\sigma^2{}_\omega r(x)$
the better	ACQL	$k_2(\sigma^2+(T-\mu)^2)+0.5k^2{}_2\sigma^2{}_\omega r(x)$
	d_{nm}	$k_2(\sigma^2{}_n-\sigma^2{}_m+(\mu_n-T)^2-(\mu_m-T)^2)+0.5k^2{}_2(\sigma^2{}_{\omega n}-\sigma^2{}_{\omega m})r(x)$
The smaller	EUV	$w_H-k_2(\sigma^2+(\mu-T)^2)+0.5k^2{}_2\sigma^2{}_\omega r(x)$
the better	ACQL	$k_2(\sigma^2+(\mu-T)^2)+0.5k^2{}_2\sigma^2{}_\omega r(x)$
		convex
	d_{nm}	$3.8k_3(\sigma^2{}_n-\sigma^2{}_m)+k^2{}_3[(\sigma^4{}_n-\sigma^4{}_m)+0.72\Delta^2(\sigma^2{}_n-\sigma^2{}_m)]r(x)$
Nominal	EUV	$w_H-3.8k_3\sigma^2-k^2{}_3(\sigma^4+0.72\Delta^2\sigma^2)r(x)$
the best	ACQL	$3.8k_3\sigma^2-k^2{}_3(\sigma^4+0.72\Delta^2\sigma^2)r(x)$
	d_{nm}	$-k_3(\sigma^2{}_n-\sigma^2{}_m+(T-\mu_n-\Delta)^2-(T-\mu_m-\Delta)^2)+0.5k^2{}_3(\sigma^2{}_{\omega n}-\sigma^2{}_{\omega m})r(x)$
The larger	EUV	$w_L+k_3(\sigma^2+(T-\mu-\Delta)^2)+0.5k^2{}_3\sigma^2{}_\omega r(x)$
the better	ACQL	$w_H-w_L-k_3(\sigma^2+(T-\mu-\Delta)^2)-0.5k^2{}_3\sigma^2{}_\omega r(x)$
	d_{nm}	$-k_3(\sigma^2{}_n-\sigma^2{}_m+(\mu_n-T-\Delta)^2-(\mu_m-T-\Delta)^2)+0.5k^2{}_3(\sigma^2{}_{\omega n}-\sigma^2{}_{\omega m})r(x)$
The smaller	EUV	$w_L+k_3(\sigma^2+(\mu-T-\Delta)^2)+0.5k^2{}_3\sigma^2{}_\omega r(x)$
the better	ACQL	$w_H-w_L-k_3(\sigma^2+(\mu-T-\Delta)^2)-0.5k^2{}_3\sigma^2{}_\omega r(x)$

3.3 Relative Quality Risk Attitude and Price Effect

3.3.1 Relative Quality Risk Attitude and Potential Quality Tendency

In the previous sections, we have discussed the Pratt-Arrow classic approach on consumer attitude toward quality risk. It is shown that this approach can well describe consumer behavior and decision making under risk in some situations. Compared with the strength of consumer quality value function under quality certainty, this classic concept of consumer attitude toward risk may not be able to explain consumer behavior clearly in some cases.

Dyer and Sarin [1982] pointed out that von Newmann-Morgenstern utility function confounded the consumer's risk attitude with the strength of his preference, and they proposed an approach of relative risk aversion which was used to separate the effects of risk from strength of preference in a choice situation. Although this approach may not be consistent with Pratt and Arrow's definition of classic risk aversion over a range, the interest is focused on the relationship between choices under conditions of risk and certainty. As we have shown the results derived in risk neutral and risk aversion for various types of quality characteristics, the Pratt-Arrow definition of classic risk attitude may not be appropriate to describe the inherent tendency of the consumer's product selection. It should be noted that in nominal-the-best type of quality characteristics, the consumer is only concerned with quality variance regardless of his quality value function. Since the quality value function is symmetric in both sides of the mean value within the domain of actual quality variation, it violates the assumption of monotonical increase for quality value function. Thus, the following discussion does not apply to the cases of nominal-the-best and asymmetric types of quality characteristics.

We have assumed that both the von Neumann-Morgenstern utility function $u(w(å))$ and the quality value function $q(å)$ are monotonically increasing in $(-å)$ and continuously higher order differentiable. By using the coefficient of value satiation defined by Dyer and Sarin, one has

$$dq(å)/d(-å) = q'(å), \quad d^2q(å)/d(-å)^2 = q''(å)$$

$$m(å) = -\, q''(å)/q'(å) \qquad (3.3.1\text{-}1)$$

$m(å) = 0$ indicates constant marginal quality value, and $m(å) > 0$ and $m(å) < 0$ correspond to decreasing and increasing marginal quality values at $-å$, respectively.

Dyer and Sarin state that an individual acts as relatively risk neutral, relatively risk aversion, and relatively risk prone, if

$$m(å) = r(å) \qquad (3.3.1\text{-}2)$$

$$m(å) < r(å) \qquad (3.3.1\text{-}3)$$

$$m(å) > r(å) \qquad (3.3.1\text{-}4)$$

In the above formulas

$$r(å) = -\, u''(å)/u'(å) \qquad (3.3.1\text{-}5)$$

First, we employ the above definition to examine the relationship between the utility and the strength of quality value function in the classic risk neutral, $d^2u/dw^2 = 0$. For linear quality value function

$$w = w_H - k_1 å$$

$$m(å) = r(å) = 0$$

For concave quality value function

$$w = w_H - k_2 å^2$$

$$m(å) = r(å) = 1/å > 0$$

For convex quality value function

$$w = w_L + k_3(å - \Delta)^2 \qquad (\Delta \geq å)$$

$$m(å) = r(å) = -1/(\Delta - å) < 0$$

Therefore, with respect to the strength of quality value function in classic risk neutral, the consumer is relatively risk neutral for the above three types of quality distribution. We further define an index of potential risk tendency for quality value function in the case of relative risk neutral as follows:

$$I = 0.5(v(w+h) + v(w-h)) - v(w) \qquad (3.3.1\text{-}6)$$

where I - index of potential risk tendency;

v(w) - quality value function, \$;

h - small interval of quality value variation around w, \$.

Expanding v around w, one has

$$\begin{aligned} I &= 0.5[v(w) + hV'(w) + h^2v''(w)/2 + 0(h^3) + \\ &\quad v(w) - hV'(w) + h^2v''(w)/2 + 0(h^3)] - v(w) \\ &= h^2v''(w)/2 \end{aligned} \qquad (3.3.1\text{-}7)$$

The quality value function is defined to be absolute risk neutral, potential risk aversion and potential risk taking when the index $I = 0$, $I > 0$ and $I < 0$, respectively, under the condition of relative quality neutral. For the above case, the linear, concave and convex quality value functions are absolute quality risk neutral, potential quality risk aversion and potential quality risk taking. In Table 3-1, the expected quality value (or expected utility value) with linear quality value in the larger-the-better type of quality characteristics (or the smaller-the-better) is related to the quality mean value, and not sensitive to product quality variance. Meanwhile, the expected quality values with both the decreasing marginal quality value function and the increasing marginal quality value function are sensitive to the change in both mean values and variances of quality distributions. In terms of the above definitions the consumer in this situation is classified as absolute risk neutral, potential risk aversion or potential risk taking with respect to his relative risk neutral. These distinctions are not meaningful in the condition of non-relative risk neutral, because the utility function may over- or under-componsate the potential effect of quality value function on the expected utility value.

The above concept can also be used to describe the attitude toward quality variation among consumers. For instance, the concave quality function, loosely speaking, can be employed to illustrate the consumer attitude under quality risk aversion, if the consumer quality value function is linear or convex. It would have been nice if it was possible to have expected quality value depend only on the mean and variance of the quality distribution. However, it is not the function to describe the behavior of the consumer quality aversion, if consumer quality value has the same degree of concavity as the utility function. Therefore, it does not help to argue that Taguchi's concave loss function is the decreasing risk aversion case whether for consumer or producer (see Tang and Tang [1989]).

In the classic risk aversion case, as can be seen from equations (3.2.3-5) to (3.2.3-10') in section 3.2.3, r(å) for three types of quality value functions are as follows

$$r(å) = -\ k_1(\partial^2 U_a/\partial w^2)/(\partial U_a/\partial w) \ > \ m(-å) = 0; \quad \text{for linear}$$

$$r(å) = 1/å + 2k_2 å(\partial^2 U_a/\partial w^2)/(\partial U_a/\partial w) \ > m(-å) = 1/å; \quad \text{for concave}$$

$$r(å) = -1/(\Delta - å) + 2k_3(\Delta - å)(\partial^2 U_a/\partial w^2)/(\partial U_a/\partial w)$$

$$> m(å) = -1/(\Delta - å); \quad \text{for convex}$$

Therefore, any concave utility function over the range of quality values is relative quality aversion for an individual consumer with any of the three types of quality value functions. If a decision maker employs a utility function to determine the choice of product, he has to account for the effect of strength of quality value function on his utility. For example, quality aversion usually would appear as a decreasing quality aversion when quality variation decreases, which implies that the degree of quality aversion and the quality premium decrease as quality improves. The reason for such an attitude toward quality is that, as the quality variation is reduced, the consumer behaviors become less sensitive toward quality risk. Assume that the consumer possesses a decreasing quality aversion such that

$$U = \ln(2\Delta - å) \qquad \Delta/2 > å$$

$$dU/d(-å) = 1/(2\Delta - å)$$

$$d^2U/d(-å)^2 = -\ 1/(2\Delta - å)^2$$

$$r(å) = 1/(2\Delta - å) > 0$$

If $\quad w = w_H - k_1 å$

$$r(\mathring{a}) > m(\mathring{a}) = 0$$

If $$w = w_H - k_2\mathring{a}^2$$

$$r(\mathring{a}) < m(\mathring{a}) = 1/\mathring{a}$$

If $$w = w_H - k_3(\Delta - \mathring{a})^2$$

$$r(\mathring{a}) > m(\mathring{a}) = -1/(\Delta - \mathring{a})$$

Compared with the strength of quality value function, the consumer is relative quality risk neutral, relative quality risk prone, or relative quality risk averse, if his quality value function is linear, concave, or convex, respectively. We can arrive at similar conclusions in classic risk aversion without using any specific von Neumann-Morgenstern utility function. Any concave utility function in classic risk aversion with respect to the strength of linear quality value function illustrates that it is relative risk aversion. Any concave utility function in the classic sense of Pratt's definition with respect to the strength of concave quality value functions is respectively relative risk neutral if the degrees of two concave functions are the same, is relative risk averse if the degree of utility function concavity is higher than quality value function, or is relative risk prone if the degree of utility function concavity is lower than quality value function. The same conclusions can be made for the strength of convex quality value function.

One of the purposes of introducing relative risk attitude is to simplify the complex problems in practical application without any violation of basic assumptions about consumer behavior and decision making under risk. If we consider the attitude of the consumer's quality aversion as the major behavior in quality activities, only those utility functions which possess a higher degree of concavity than the strength of quality value function could be considered in the case of relative risk aversion. With this explanation, Taguchi's loss function may not be adequate to describe the consumer (producer) behavior under risk aversion.

3.3.2 Price Effect, Price/Quality Substitution and Budget/Quality Dilemma

Price reflects the product market value in a perfect competition condition. The quality discrimination resulting from quality variation destroys the implication of unique price for a certain product under perfect competitive conditions. If one could buy a product at the price determined by its quality, no quality discrimination (or quality loss) would appear. No quality activity without consideration of the quality/ price effect is complete. The difference in prices among a number of product brands will definitely affect consumer decision making on product quality choice.

Assume consumer utility is a decreasing and convex function of price such that

$$U'_p(å, p) < 0,\ U''_p(å, p) \leq 0 \qquad (3.3.2\text{-}1)$$

Assume that the consumer employs the same utility function to assess product qualities in a set of products such that a specific utility function discussion can be ignored. The consumer's expense in purchasing a product is actually equal to the product price he paid other than his budget. The higher the budget, the more products the consumer can search in a wider price range. If prices for the searched products are the same, there is no budget saving effect and consumer decision making only depends on the quality advantage and other product features. The effect of price (or the budget saving effect) cannot be ignored if prices are not the same. It is observed that the lower the consumer's assets, the more significant the price effect will be. It is rational to assume that the consumer becomes decreasingly price averse with higher assets. Budget saving is negatively related to the price.

The general expected quality utility value, equation (3.2.3-24), is quoted here again,

$$V\{E[U(x - B(x) + E(w) - R)]\} = x - B(x) + E(w) - 0.5\sigma^2_{\omega}r(x)$$

The following equation is used to establish the indifferent curves of an individual consumer quality utility. Assuming the consumer assets and budget are fixed in the short-term, quality information is imperfect but unbiased, and price and quality deviation are treated as variables, then

$$U[x + (B(x) - f(p)) + (E(w) - 0.5\sigma^2_{\omega}r(x))]$$
$$= U[x + (B(x) - f(p) + (w - \beta - 0.5\sigma^2_{\omega}r(x))]$$
$$= U[x + B(x) - f(p) + f(å)] = U(G) \qquad (3.3.2\text{-}2)$$
$$\partial U/\partial G = U'_G > 0$$
$$\partial U/\partial p = U'_p = - U'_G f'_p < 0 \qquad U''_p \leq 0 \qquad (3.3.2\text{-}3)$$
$$\partial U/\partial(-å) = U'_å = U'_G f'_å > 0 \qquad U''_å \leq 0 \qquad (3.3.2\text{-}4)$$
$$dU = U'_p dp + U'_å\, d(-å)$$

Let $dU = 0$

$$dp/d(-å) = - U'_å/U'_p = f'_å/f'_p$$

By following the definition made by Cooper [1984], the price/quality marginal rate of substitution, MRS, is

$$MRS_{på} = dp/d(-å) = - U'_å/U'_p \qquad (3.3.2\text{-}5)$$

where G - function of price and quality deviation, $G = A - f(p) + f(å)$;

$f(å)$, $f(p)$ - quality and price functions, respectively and the quality premium, \$;

$U'_å$, U'_p - the consumer marginal utility of quality and price, respectively;

$f'_å/f'_p$ - the marginal value of quality and price functions, respectively.

Equation (3.3.2-3) indicates that if quality deviation remains constant, consumer utility decreases with an increase in the product price. The same conclusion can be made for equation (3.3.2-4) if the quality deviation increases. Changing p and product quality deviation (-å) simultaneously to keep U value no change, we can get an indifferent curve u_1. When both price and quality vary simultaneously along with u_1, consumer utility does not change. The consumer is indifferent for two pairs of prices and qualities at the curve u_1. Similarly, we can draw a group of utility indifference curves under relative quality risk aversion, such that $u_1 < u_2 < u_3 < ... < u_n$.

As mentioned in the previous sections, the consumer quality utility is concave over the range of quality values under relative quality risk aversion. That is:

$$d^2U/dw^2 < 0, \quad d^2U/d(-å)^2 < 0$$

For consistency, assume that consumers have the similar patterns for price effect on consumer utility as the quality. We have observed that consumers usually are price averse, which means that consumer utility is a decreasing function over the price range. The higher the price, the more the decrease in consumer utility.

Assume utility functions are quasi-concave against quality, the utility indifference curves which describe the substitution relationship between the price and the quality is concave toward the origin, shown in Figure 3-10. The utility indifference curve can be convex or linear. The patterns of indifference curves depend on the consumer behavior toward quality and price (see Cooper, 1984). The consumer is indifferent as long as utility moves along the same curve. When price is constant, the higher the quality, the better off the consumer, and vice versa.

We now use the following example to explain how a consumer makes a decision to trade off price and quality.

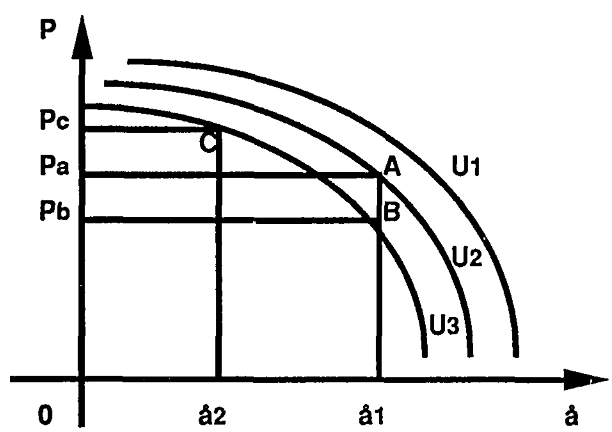

Figure 3-10 Substitution Between Price and Quality

As shown in Figure 3-10, curves u_1, u_2 and u_3 represent an individual consumer's utility in the order of satisfaction, i.e. $u_1 < u_2 < u_3$. Y and X axes represent product price p and quality deviation å, respectively. With respect to an indifference curve, an individual consumer is indifferent to paying a higher price for higher quality, or paying a lower price for lower quality. However, the individual is better off if he buys a higher quality product but pays the same price. The individual is worse off if he buys the same quality product but pays a higher price. For example, an individual wants to buy a TV set. Model A possesses quality deviation $å_1$ and is priced at P_a. Point A on the indifference curve u_2 is the utility provided by model A. Meanwhile, the individual may buy model B at price P_b with quality deviation $å_1$, or model C at price P_c with quality deviation $å_2$. Model A is unable to compete with model B in price and to compete with model C in quality. Obviously, the individual will not buy model A. He is indifferent to buying model B or model C. The final decision will depend on his budgetary constraints. If the consumer budget is less than p_c, the consumer will be better off to buy model B instead of model C.

Therefore, we can derive some strategies of price/quality effects to influence the costumer's decision making: decreasing the price if other competitive products have the same quality level or improving the quality to gain a competitive advantage. When a producer intends to increase price and quality simultaneously, he should make sure that the positive benefit from an increase in product quality would cover or exceed the negative price effect resulting from price increase. Otherwise he would face the risk of losing potential customers and market shares in a competitive environment. A product possesses the strongest competitive advantage if it is in the position with the highest quality and the lowest price among a set of competing products.

The decision to purchase a product is constrained by the consumer's budget. The quality utility, with the property of diminishing marginal utility, implies that consumer satisfaction from product quality must increase as the budget allocation increases. A rational consumer seeks to maximize his utility resulting from product quality subject to his budgetary constraints. In equation (3.3.2-2), consumer assets are no longer fixed, but vary over the longer-term. If the assets increase, consumer indifferent utility curves will be relatively steep compared to his previous standing, shown in Figure 3-11. Curves u'_1, u'_2, u'_3 ...u'_i represent the new individual utility and are ranked in the order of satisfaction $u'_1 < u'_2 < u'_3 \ldots < u'_i$.

There is an interesting budget/quality effect that when quality improvement is accompanied with price increase along the new indifference curve, the consumer is willing to pay a higher price for quality improvement than before. In other words, the consumer now pays several extra unit of price to compensate for the gain of a single unit of product quality. Thus, in this sense, when the consumer's budget increases, product quality is more important to the consumer than before. In comparison with previous indifferent curves, there is a point I, the intersection of two curves, at which consumer utility is independent of budget change. We call this point I "saturation point." The consumer is willing to pay a higher price for the product quality above the saturation point, but would like to pay a lower price for the product quality below the point I. However, these two families of indifferent curves cannot be used to describe a consumer quality utility simultaneously. Mussa and Rosen [1978], Cooper [1984], Srinagesh and Bradburd [1989] demonstrated that monopolistic market segmentation is profitable if the characteristics of heterogeneous consumers utility can be distinguished by different product qualities.

If the consumer's budget decreases to the previous level, the consumer utility indifference curve u^+ lies between the above two indifference curves. In the price range above the saturation point I, the curve u^+ is closer to the curve u corresponding to the lower budget. But in the price range below the saturation point I, the curve u^+ is closer from the curve u' corresponding to the higher budget. It is called budget-price dilemma. The unreversed effect, shown in the dark area in Figure 3-12, is called quality resistance. With the change in the consumer income over time, the preference of quality/price is changed to higher quality with higher price and lower quality with lower price. The producer must improve the product quality continuously to meet the consumer preference change and compete with rivals in quality, or he may adopt the alternative strategy to reduce the product price. The product with no change in quality and price would not gain any advantage in competition and would be forced out of the market when other conditions would change.

The explanation for the budget/price dilemma in practice is that consumer taste and enjoyment from product quality performance change in the income-up period. The consumer is no longer the previous consumer because his taste and expectation for both price and quality are not the same as before. This dilemma is different from the other phenomena that the outcome is reversible when the conditions change back to the original state. When the income decreases to the previous level, the consumer taste and attitude quickly respond to this change in the range above the saturation point I. Below the saturation point, consumer response to the change in budget is very slow. This is called quality resistance. Quality resistance may be used to explain the phenomena that some consumers account for quality effect more than price effect in the lower price range, but opposite in the higher price range when their income is down. Therefore, the producer prefers to adopt the strategy for higher quality with higher price rather than to decrease quality with lower price from the present quality position, because the latter is more risky to him due to consumer quality resistance. Quality improvement is always required whether the economy is booming or is in a recession. In general theoretical development, the quality indifference curves and analysis correspond to a single period of time. Hence, the effect of the quality resistance can be ignored. The above illustration of substitution between price and quality by using the indifference curves is based on the assumptions that other conditions are kept constant.

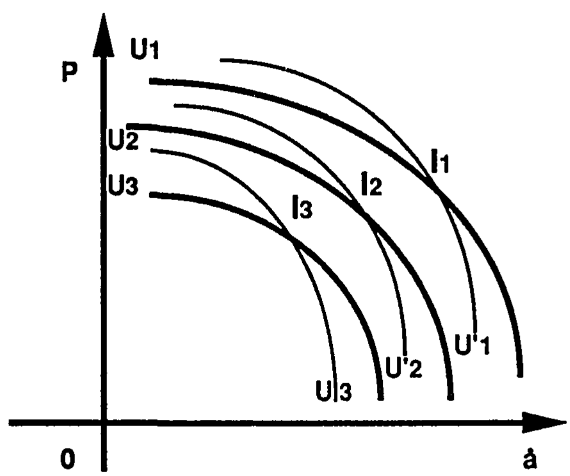

Figure 3-11 Relationships Between Price and Quality

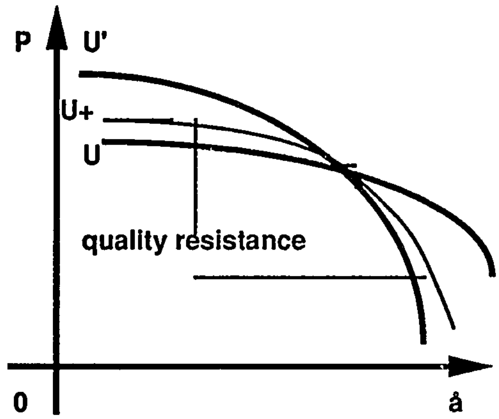

Figure 3-12 Quality Resistance Due to Income Decrease

We define consumers as quality lover, quality neutral and price lover if the price/quality marginal rate of substitution, $MRS_{på}$, is greater, equal, or smaller than 1. In practice, a linear weight function is employed to replace the nonlinear function of price/quality substitution over a smaller quality range, such that

$$MRS_{på} = dp/d(-å) = -U'_{å}/U'_{p} = f'_{å}/f'_{p}$$
$$= (1-\eta)/\eta \qquad (3.3.2\text{-}6)$$

where η - weight for price effect on product quality selection.

We discuss consumer choice on two products which are different in price, quality mean values, quality variances and quality premiums under risk aversion. (It is easy to expand to n products.) Quality loss is divided into two parts, one of which is the loss due to product quality variation and the other due to quality premium. The consumer searches for a product from two types of products, based on the criterion of which product is able to provide higher utility.

$$V_1\{E[U(x-B(x)+E(w_1)-R_1)]\} = x-B(x)+w_H-\beta_1-0.5\sigma_{\omega 1}^2 r(x) \qquad (3.3.2\text{-}7)$$

$$V_2\{E[U(x-B(x)+E(w_2)-R_2)]\} = x-B(x)+w_H-\beta_2-0.5\sigma_{\omega 2}^2 r(x) \qquad (3.3.2\text{-}8)$$

$$\beta_1 = f_1(\delta_1, \sigma_1) \qquad (3.3.2\text{-}9)$$

$$\beta_2 = f_2(\delta_2, \sigma_2) \qquad (3.3.2\text{-}10)$$

where $f(\delta, \sigma)$ indicates quality loss from the quality variation which is related to the difference between the quality target value and the quality mean, δ, and the quality variance, σ^2. This quality loss is independent of its location in linear quality value function. The quality premium cannot be separated into effects of quality mean value and quality variance, with the exception of the linear quality value function in which the quality premium is only related to the variance of quality distribution.

Expense equals the price paid. The higher the price, the lower the consumer utility when other conditions are constant. The budget savings, α, for these two types of products are, respectively

$$\alpha_1 = B(x) - p_1$$

$$\alpha_2 = B(x) - p_2$$

$$B(x) = \max(p_1, p_1) \qquad (3.3.2\text{-}11)$$

where p_1, p_2 - prices for product 1 and product 2, respectively, \$.

We define the product with both higher price and quality as the superior product, whereas the inferior product is characterized with both lower quality and price. Since different income groups may be engaged in product quality search in the same set of products, it is observable that the higher income group (i.e., more wealthy people) puts less weight on price effect than the lower income group does in purchasing a superior product under budgetary constraints. In contrast, the high income group may put higher weight on the price for an inferior product. For example, the upper class people are willing to pay more for a high quality luxury car, but are willing to pay less than the lower income group for a used car. The weight for price effect is a function of the consumer's assets and product quality,

$$\eta_i = \eta(x_i) \qquad 1 \geq \eta_i \geq 0 \tag{3.3.2-12}$$

$$\partial\eta/\partial x = \eta_x < 0,\ \eta_x'' \geq 0 \tag{3.3.2-13}$$

where η_i - weight of price effect for income group i;

x_i - income and other assets for group i.

The weight for quality effect is $(1- \eta_i)$. Thus, the consumer utility function and the difference of the expected utility values, d, for these two types of products are

$$U[(x + \eta(B(x) - p) + (1 - \eta)(E(w) - R)]$$
$$= E\{U[(x + \eta(B(x) - p) + (1 - \eta)w]\} \tag{3.3.2-14}$$

$$d = V_1\{[U(x + \eta(B(x) - p_1) + (1 - \eta)(E(w_1) - R_1)]\} -$$
$$V_2\{[U(x + \eta(B(x) - p) + (1 - \eta)(E(w_2) - R_2)]\} \tag{3.3.2-14'}$$
$$= \eta(p_2 - p_1) + (1- \eta)[\beta_2 - \beta_1 + 0.5r(x)(\sigma_{\omega 2}^2 - \sigma_{\omega 1}^2)]$$
$$= \eta(p_2 - p_1) + (1- \eta)[f_2(\delta_2, \sigma_2) - f_1(\delta_1, \sigma_1) + 0.5r(x)(\sigma_{\omega 2}^2 - \sigma_{\omega 1}^2)]$$

If the effects of quality target value differences and the variance on quality loss can be separated from function $f(\delta, \sigma)$, the above equation becomes

$$d = \eta(p_2 - p_1) + (1-\eta)[f_2(\delta_2, \sigma_2) - f_1(\delta_1, \sigma_1) + 0.5r(x)(\sigma_{\omega 2}^2 - \sigma_{\omega 1}^2)]$$
$$= \eta(p_2-p_1)+(1-\eta)[c_1(f_2(\delta_2)-f_1(\delta_1))+$$
$$c_2(f_2(\sigma_2)-f_1(\sigma_1))+0.5r(x)(\sigma_{\omega 2}^2-\sigma_{\omega 1}^2)] \qquad (3.3.2\text{-}15)$$

where c_i - quality coefficients for quality losses, i=1, 2;

$f_i(\delta_i)$ - the quality loss from difference between the mean value and the target value for product i;

$f_i(\sigma_i)$ - the quality loss resulting from quality variance of product i;

$r(x)$ - measurement of risk attitude toward risk for von Neumann-Morgenstern expected utility function.

Note that the consumer total quality variance contains quality information uncertainty and inherent product quality variance, as seen in equation (3.2.1-21).

$$\sigma^2 = \sigma_i^2 + \sigma_p^2,$$

The final form of the expected utility value difference is

$$d = \eta(p_2 - p_1) + (1-\eta)[c_1(f_2(\delta_2)-f_1(\delta_1))+c_2(f_2(\sigma_{i2}+ \sigma_{p2})- f_1(\sigma_{i1}+ \sigma_{p1}))$$
$$+ 0.5r(x)(\sigma_{i2}^2 - \sigma_{i1}^2 + \sigma_{p2}^2- \sigma_{p1}^2)]$$
$$= \eta\Delta p + (1-\eta)[\Delta f(\delta) - \Delta f(\sigma) - \Delta r(x)\sigma^2_{\omega}/2]$$
$$= \eta\Delta p + (1-\eta)\Delta q \qquad (3.3.2\text{-}16)$$

where Δp - difference between prices, \$;

Δq - difference between quality losses which include quality deviation (quality mean and variance) from the target value and the quality premium, $.

For income group i

$$d_i = \eta_i \Delta p + (1- \eta_i)\Delta q \qquad (3.3.2\text{-}17)$$

This is a very useful equation to capture the consumer's decision making behavior in the choice of a product.

Based on the implication of equation (3.3.2-17), it is found that consumer quality decision making on product selection depends on price, pattern of deterministic quality value function, consumer budget, quality variation, quality premium (quality information uncertainty and inherent product quality variance), and the consumer assets effect. All these factors, to some extent, influence consumer decision making. Generally speaking, these influences on consumer decision making can be classified into two parts, as seen in equation (3.3.2-17), one of which is the price effect and the other is the quality effect. The quality effect contains the quality variation and the quality premium. Equation (3.3.2-17) also suggests that a higher price should be compensated by higher quality to meet consumer's requirements. When $d = 0$, the consumer is indifferent for these two types of products. When $d > 0$, the consumer is better off either from price saving or from quality gain, or from both in the choice of product 1. When $d < 0$, the consumer is definitely worse off in buying product 1.

Consumer assets change with time, which depends on the situation of economic development. When consumer assets vary, the weight for quality/price effects, budgetary limit, consumer quality value function and quality premium will be altered. To better understand the time and consumer assets effects on consumer utility, equation (3.3.2-14) is rewritten as:

$$U = U[(x(t)+\eta(x)(B(x)-p)+(1-\eta(x))(E(w(x, å))-R(x, w))] \quad (3.3.2\text{-}18)$$

Chapter 4 Consumer Quality Decision Making and Information Transformation

4.1 Properties of Consumer Quality Utility Function

In Chapter 3 we developed the consumer behavior under quality variation and the consumer decision making model on product selection. In this chapter we will discuss the properties of consumer multi-attribute quality utility and its application on selection of multi-attribute products.

4.1.1 A Simplified Equation to Calculate Utility Value for Single Attribute Quality

The expected utility value under quality risk aversion derived in previous sections can be employed as a basis for mathematical modeling performed in the following chapters. Since different products provide different utilities for individual consumers, a consumer chooses the product which provides the highest utility among a set of products. However, in practice it is difficult to determine the specific form of the utility function for any given product. Fortunately, we have set up the general form for expected quality value under quality neutral conditions and found it to be the lower bound for establishment of the formula in other instances. If the quality utility function possesses the properties that are continuous and higher order-differentiable, it may be approximated by a quadratic function in the domain of quality variation limitation. It should be noted that this local utility function (i.e. quality utility only) does not account for price effect, consumer budget, quality information uncertainty, difference in quality target value and consumer attitude toward risk. Based on the properties of utility function under quality aversion, utility function can be expanded by the Taylor series as follows:

$$U_a = f(w-å)$$
$$= f(w) + (-å)f'(w) + (-å)^2 f''(w)/2! + ... + (-å)^n f^n(w)/n! + ... \quad (4.1.1\text{-}1)$$

The the highest value of the quality utility, U_H, can be obtained at å = 0. If the terms higher than power 2 are ignored, then the above equation becomes

$$U_a = U_H - (å)f'(w) + å^2 f''(w)/2! \quad (4.1.1\text{-}2)$$
$$E(U_a) = E[U_H] - E[(å)f'(w)] + E[(å^2)f''(w)/2!]$$

Using value operator V,

$$V[E(U_a)] = V(U_H) - E(å)V[f'(w)] + E(å^2)V[f''(w)/2!]$$

Under conditions of quality risk aversion

$$f''(w)/2! < 0$$
$$EUV = w_H - c_1E(å) - c_2E(å^2) \quad (4.1.1\text{-}3)$$

where EUV - consumer expected utility value, \$;

c_1, c_2 - coefficients.

The first term in equation (4.1.1-3) is the highest utility value without quality variation. The second term $c_1E(å)$ and the third term $c_2E(å^2)$ represent the quality deviation from the target value, and the quality premium under quality risk aversion, respectively. For instance, å = |z - E(z)| in nominal-the-best type of quality characteristic distribution with linear quality value function,

$$E(\mathring{a}^2) = E(|z - E(z)|^2)$$
$$= E[(z - E(z))^2]$$
$$= \sigma^2$$
$$E(\mathring{a}) = E(|z - E(z)|)$$
$$= 0.8\sigma$$
$$EUV = w_H - c_1 0.8\sigma - c_2\sigma^2$$

Equation (4.1.1-3) is the general form of approximation of expected quality value for the linear quality value function under quality aversion. Any specific utility function can be approximately expressed by the above quadratic function. The expected utility value under quality aversion is always smaller than the expected utility value under quality neutral. In other words, the quality loss is larger under quality aversion.

If we substitute the concave quadratic quality value function into equation (4.1.1-3), then equation (4.1.1-3) has the following form in nominal-the-best type of quality characteristics

$$EUV = w_H - c_1E(\mathring{a}) - c_2E(\mathring{a}^2) \qquad \mathring{a} = (z - \mu)^2$$
$$= w_H - c_1E[(z - \mu)^2] - c_2E[(z - \mu)^4]$$

Under quality neutral

$$EUV = w_H - c_1E[(z - \mu)^2] = w_H - c_1\sigma^2$$
$$ACQL = w_H - E(w) = c_1\sigma^2 \qquad (4.1.1\text{-}4)$$

Meanwhile, the average quality utility value loss under quality neutral in the larger-the-better and the small-the-better types of quality characteristics is

$$EUV = w_H - c_1 E[(T - z)^2]$$
$$= w_H - c_1[(T - \mu)^2 + \sigma^2]$$
$$ACQL = c_1[(T - \mu)^2 + \sigma^2] \qquad (4.1.1\text{-}5)$$

Taguchi and Taguchi et al. [1984, 1985, 1989] first introduced the concept of quality loss function, which gave an insight of product quality variation. The loss function contains a significant economic meaning to guide quality control activities, which has made a tremendous contribution in many applications. Equation (4.1.1-4) and equation (4.1.1-5) have similar forms as Taguchi's loss functions. Taguchi's loss functions inform the producers of how much benefit can be gotten from product quality improvement. However, there are some significant differences between Taguchi's loss functions and the consumer quality loss functions derived in this chapter:

1. Taguchi's loss function is derived from the producer's behavior; it does not consider consumer behavior completely.

2. Taguchi's loss function is based on concave quadratic quality value function, and producer attitude under quality risk aversion is not considered.

3. Product quality variation results in consumer utility change. No quality discrimination for the consumer means no loss for the producer.

4. Consumer quality loss function is based on the observable consumer behavior for quality variation; that is, the consumer pays the same price but may get different product quality in comparison with the highest quality other consumers bought.

5. The consumer assesses quality variation with consumer quality value regardless of the producer's cost. Quadratic loss function is a

simplified function and is easily employed in practice. However, the pattern of quality value function has a considerable effect on expected utility value and consumer quality loss.

6. Quality discrimination is tightly associated with price. There would be no quality loss if different product qualities are associated with different prices in light of consumer utility function. Quality activities are not meaningful without consideration of the price effect.

7. Total quality risk premium includes quality information uncertainty and inherent product quality variation. Quality information can exert a positive or a negative effect on reduction of quality risk. This can explain why consumers are willing to pay some premiums to collect and search quality information before purchasing a product.

8. Consumer quality loss function will underestimate society quality loss (or gain) because it does not consider the producer's surplus in quality activity (This will be discussed in Chapter 5). Therefore, consumer quality loss function underestimates producer loss due to product quality disadvantage. Taguchi's producer loss function is not an appropriate approach to illustrate society quality loss.

9. Consumer quality loss results from quality deviation (mean and variance) from the target values. Quality premium is also one kind of consumer quality loss that results from consumer utility adjustment under risk. Consumer behavior under quality aversion is a more appropriate approach for most consumers.

10. Consumer attitude toward quality risk is related to product price, consumer's assets, income level, etc. Quality loss function should describe consumer decision making under various situations (quality risk aversion, quality risk taking and quality risk neutral.)

Application of equation (3.3.2-4) on consumer decision in choosing a product needs sufficient information, and the quality loss from both effects of quality mean value and variance differences is confounded in Neumann-Morgestern utility function. It limits equation (3.3.2-4) application in practice. From previous analysis, the linear quality value function is the simplest form with the following advantages:

1. Price value can be incorporated linearly into quality mean value function.

2. The quality loss resulting from quality variation is associated with the mean value of quality distribution, which the consumer can easily perceive. Quality premium is only related to the quality variance that reflects consumer attitude toward quality risk.

3. The effects of quality mean value and variance differences on quality loss can be separately computed. The effect of quality information uncertainty is independent of other factor effects.

4. Any concave utility functions with respect to the strength of quality value function reflect the consumer's relative risk aversion attitude.

5. Linear quality value function is more conservative than the concave quality value function in calculation of consumer quality loss, and in activities of quality improvement within the same quality value domain. If the specific form of quality value function is unknown for a given product, linear quality value function can be adopted easily.

6. Some nonlinear quality value functions can be transferred into linear forms.

We use the linear form of quality value function and assume that the consumer attitude is relative risk aversion. Quality premium is associated with product quality variance. Thus, equation (3.3.2-14') is simplified as:

$$d = \eta(p_2 - p_1) + (1-\eta)[f_2(\delta_2, \sigma_2) - f_1(\delta_1, \sigma_1) + 0.5r(x)(\sigma_{\omega 2}{}^2 - \sigma_{\omega 1}{}^2)]$$
$$=\eta(p_2-p_1)+(1-\eta)[k_1(\mu_1-\mu_2)+0.5r(x)k_2(\sigma_2{}^2-\sigma_1{}^2)] \qquad (4.1.1\text{-}6)$$

where p_j, μ_φ and $\sigma_j{}^2$ - the product price, the quality mean value and the quality variance for product j, respectively; j = 1, 2.

The quality target value, T, can be written as:

$$T = \mu_0 + \delta \qquad (4.1.1\text{-}7)$$
$$\mu_0 = \max(\mu_2, \mu_1) \qquad (4.1.1\text{-}8)$$

Compared with μ_0, δ is very small and can be ignored.

$$T \approx \mu_0 \qquad (4.1.1\text{-}9)$$

0.5r(x) is assumed constant for an individual consumer and can be set to 1 without the loss of generality in two product comparison.

Let $k_1 = p/T \qquad (4.1.1\text{-}9')$

$$p/T = (w_H - w_L)/\Delta$$
$$p/(T\Delta) = (w_H - w_L)/\Delta^2$$
$$k_2 = p/(T\Delta) \qquad (4.1.1\text{-}9'')$$
$$d = \eta(p_2-p_1)+(1-\eta)[(p/\mu_0)(\mu_1-\mu_2)+p/(T\Delta)(\sigma_2{}^2-\sigma_1{}^2)] \qquad (4.1.1\text{-}10)$$

where Δ - the range of quality variation.

Equation (4.1.1-10) can now be easily employed to evaluate consumer decision on product selection from a choice of products with different prices and qualities. If one has the specific quality value function, utility function and other information, he should use equation (3.3.2-14') to get accurate results.

In practice, the consumer usually needs to compare more than two product qualities. The consumer may set up an ideal product which possesses a zero quality loss and a price equal to the highest price in a set of products. Each product is compared with the ideal product, and the product with the highest utility value, the sum of quality effect and price effect, will be chosen. Equation (4.1.1-10) becomes

$$d_i = \eta(p - p_i)+(1 - \eta)[(p/\mu_0)(\mu_i - \mu_0) - p/(T\Delta)\sigma_i^2] \qquad (4.1.1\text{-}11)$$

$$\sigma_0^2 = 0$$

where d_i - product comparison between product i and the ideal product;

p, μ_0, σ_0^2 - price, quality target value and variance of the ideal product;

p_i, μ_i, σ_i^2 - price, quality mean value and variance of the actual product.

4.1.2 Multi-Attributes Quality Utility Function Properties

Consumers measure an overall product quality with subjective quality attribute values, which may or may not be consistent with the product component qualities. In microeconomics, utility functions are assumed to be strictly quasi-concave, differentiable and increasing. Separability and additive assumptions for utility function are usually the basis for model establishment. We also employ these concepts in quality economics. A strong separable quality utility function and a weak separable utility function in their argument are written as, respectively

$$U = F[\Sigma f_i(å_i)] \quad i = 1, 2, ..., n \tag{4.1.2-1}$$

or

$$U = F[f_1(å_1, ..., å_k), f_2(å_{k+1}, ..., å_n)] \tag{4.1.2-1'}$$

The rate of quality substitution (RQS) is

$$RQS = Ff'_i/F'f'_j = f'_i/f'_j \tag{4.1.2-2}$$

A utility function is strongly additive or weak additive if it can be written as

$$U = \Sigma f_i(å) \quad i =1, 2, ..., n \tag{4.1.2-3}$$

or

$$U = f_1(å_1, ..., å_k) + f_2(å_{k+1}, ..., å_n) \tag{4.1.2-3'}$$

The additive utility function has the property that all cross partial derivatives equal zero, i.e.

$$\partial^2 U/(\partial å_i \partial å_j) = 0 \tag{4.1.2-4}$$

where U _ the utility function expressed in terms of monetary unit, \$;

F, f_i _ increasing quality utility function;

$å_i$ _ quality deviation for component i, $å = f(\mu, \sigma^2)$;

$f_i(å)$ - quality value in terms of quality deviation, \$.

Weak separability means that the RQS for pairs of attribute qualities within the same group are unaffected by attribute qualities outside the group, while weak additivity means cross partial derivatives for pairs of attribute qualities in different groups are zero. Additivity of quality utility is a special case of separability. If each of the product quality components is independent, the overall utilities are either separable or additive. In many cases the consumer's overall utility is a linear combination of the qualities of each component of the product. Assume that each product component quality is normally and independently distributed. Then, the overall utility is the sum of individual component qualities.

$$U = s_1 f_1(\mathring{a}_1) + s_2 f_2(\mathring{a}_2) + \ldots + s_n f_n(\mathring{a}_n) \qquad (4.1.2\text{-}5)$$

For instance, in the larger-the-better type of quality characteristics with linear quality value function, equation (4.1.2-5) has the following form

$$U = \Sigma s_i[w_{Hi} - k_{1i}(T_i - \mu_i) - k_{2i}\sigma_i^2] \qquad (4.1.2\text{-}6)$$

where s_i - weight for attribute quality i, $\Sigma s_i = 1$;

w_{Hi} - quality value corresponding to the best attribute quality i, \$;

k_{1i}, k_{2i} - quality loss coefficients, \$/unit and \$/unit^2, respectively;

T_i, μ_i and σ_i^2 - the target value, mean value and variance for quality attribute i distribution, respectively.

Equations (4.1.2-3) to (4.1.2-5) imply that the overall quality can be decomposed into a number of components which are linearly or approximately linearly related to the overall quality. Thus, the overall quality utility in most cases can be expressed in the additive form. The utility derived from a product quality is the sum of the utility derived from the individual attribute qualities. The marginal quality utility for attribute x_i is determined by the quality deviation of x_i alone and independent of the quality of any other attribute. The overall quality of a product consists of a number of independent component qualities; if other quality components remain constant, the marginal quality utility for component x_i increases when quality deviation $\mathring{a}_i$ decreases.

In some cases, the overall utility function for a product may be a nonlinear function of the n quality components.

$$U = F[x_1(\mathring{a}_1), x_2(\mathring{a}_2), \ldots, x_n(\mathring{a}_n)] \qquad (4.1.2\text{-}7)$$

A conventional approach usually uses a linear function of the x_i in its domain to approximate the nonlinear function. Assume $\xi_1, \xi_2, \xi_3, \ldots \xi_n$ are the nominal dimensions associated with the components $x_1, x_2, x_3, \ldots x_n$. By expanding equation (4.1.2-7) in a Taylor series about $\xi_1, \xi_2, \xi_3, \ldots \xi_n$, we obtain the consumer utility over the average

$$U = E\{g(\xi_1, \xi_2, \xi_3, \ldots \xi_n) + \Sigma(x_i - \xi_i)\partial F/\partial x_i \,|_{\xi 1, \xi 2, \xi 3, \ldots \xi n} + C\}$$
$$E[\Sigma(x_i - \xi_i)\partial F/\partial x_i \,|_{\xi 1, \xi 2, \xi 3, \ldots \xi n}] = 0$$

where C represents the higher order terms. Neglecting the terms of higher order, then

$$U = E[g(\xi_1, \xi_2, \xi_3, \ldots \xi_n)] \qquad (4.1.2\text{-}8)$$

For example, if U = E(yz) in the larger-the-better type of quality characteristics with linear quality value function

$$
\begin{aligned}
U &= E[g(\xi_1, \xi_2, \xi_3, \dots \xi_n)] \\
&= (w_{Hy} - ACQL_y)(w_{Hz} - ACQL_z) \\
&= [w_{Hy} - k_{1y}(T_y - \mu_y) - k_{2y}\sigma_y{}^2][w_{Hz} - k_{1z}(T_z - \mu_z) - k_{2z}\sigma_z{}^2]
\end{aligned}
\quad (4.1.2\text{-}9)
$$

If the attribute or component qualities can be substituted with each other, the overall utility is either separable or weakly separable. Some nonlinear separable utility functions can be transformed into linear additive forms without difficulty. Therefore, producers are able to distinguish how much each of the attribute qualities affects the consumer overall quality utility.

For some special quality assessment, the weak separable and weak additive definition for consumer quality utility may be used to expose the relationship between the overall utility and the individual attribute qualities. For instance, the utility resulting from a reliable system quality, or a uniform quality control system, may have the following forms

$$U = F[\min(å_1, \dots, å_k), \max(å_{k+1}, \dots, å_n)] \quad (4.1.2\text{-}10)$$

or

$$U = U1 + U2 = \Sigma s_i f_i(å_i) + \min(å_{k+1}, \dots, å_n) \quad (4.1.2\text{-}11)$$

The system quality or subsystem quality does not depend on the sum of individual component quality, but on the worst component quality (or the best component quality) in the system. In quality utility systems, if component or attribute qualities are not substituted with or complemented by each other, they are called quality independent. The component or attribute qualities which can satisfy the same need of the consumer show up to have the property of quality substitution, while the component or attribute qualities that are used jointly to satisfy some particular needs of the customer are said to have the property of quality complement. Since consumer utility functions are derived from consumer behavior, the above utility-based relationship of attribute qualities may or may not be consistent with the relationships among these component or attribute qualities found in quality engineering. In order to satisfy consumer requirements, quality control activities should combine consumer information on product quality evaluation into engineering and economic quality design.

4.2 Consumer Utility Improvement in Two Attribute Qualities

4.2.1 Relationship between Two Attribute Qualities

A product usually consists of a number of components or attributes. The quality component approach proposes that change in consumer utility due to quality improvement could be equally transferred from the overall product quality to a number of component or attribute qualities. For instance, when an individual wants to buy a TV set, he evaluates the TV's quality based on a number of attribute qualities which can be observed and easily evaluated, such as color fidelity, sound, picture clarity and convenience of operation. An improvement in any one of these attribute qualities may affect consumer decision making.

We will discuss the properties of two attribute qualities in detail to get an insight into how the product quality can be improved in the most efficient and economic way. The two-attribute quality/price decision model can be expanded to more complicated cases. If two attribute qualities are independent, an improvement in any attribute quality will make the consumer better off. The amount of increase in consumer utility is

$$
\begin{aligned}
\Delta U &= \Delta U_2 + \Delta U_1 \\
&= \Delta s_1 f(å_1) + \Delta s_2 f(å_2) \\
&= s_1[f(å_{11}) - f(å_{12})] + s_2[f(å_{21}) - f(å_{22})]
\end{aligned}
\qquad (4.2.1\text{-}1)
$$

where ΔU _ gain in consumer utility due to quality improvement, \$;

ΔU_i _ utility increase from improvement in attribute quality i , \$;

s_i _ weight for attribute quality i, $\sum s_i = 1$;

$å_{i1}$ _ attribute quality i deviation from its target value before quality improvement, unit;

$å_{i2}$ _ attribute quality i deviation from its target value after quality improvement, unit;

In equation (4.2.1-1), the consumer assigns the weights for the two attribute qualities. As mentioned before, different people give different weights for various attribute qualities, based on their preference of attribute qualities. However, the weight distribution for product attribute qualities is assumed to be the same for a homogeneous group. If the producer knows the information on consumer quality preference, he ought to pay more attention to the improvement of the level of the attribute quality that consumers are more concerned with. It should be noted that there is no substitution between the two attribute qualities under quality independence assumptions. Although the utility may be constant when the two attribute qualities change simultaneously, it does not imply that there is a substitution between them.

If an increase in the overall quality utility depends on a simultaneous improvement in both attribute qualities, an increase only in one attribute quality does not improve consumer utility. This is called quality complement. A typical fixed quality complement for two attribute qualities is shown in Figure 4-1. The utility indifferent curves are no longer the smooth quadratic convex curves, but straight lines with a 90^0 angle toward the origin. Simultaneous improvement in both attribute qualities could improve consumer utility, and the most efficient way is along the fixed proportion line OO' whose slope equals the ratio of the two attribute inherent relationship, s_1/s_2. Assume that original attribute qualities 1 and 2 were $å_{11}$ and $å_{21}$, respectively, which provided consumer utility U_1. Attribute quality 1 now changes from $å_{11}$ to $å_{12}$, but attribute quality 2 still stays at $å_{21}$. The consumer utility is still on U_1 at which the consumer feels no better off. If and only if attribute quality 2 changes in corresponding to the improvement in attribute quality 1 from $å_{21}$ to $å_{22}$, the consumer utility will increase, and the increment amount of the utility depends on the lower attribute quality in these two attribute qualities. The increased amount of consumer quality utility is

$$U = \min(f_1(å_1)/s_1, f_2(å_2)/s_2)$$
$$\Delta U = \min(\Delta U_2, \Delta U_1)$$
$$= \min [f_1(å_{11} - å_{12})/s_1, f_2(å_{21} - å_{22})/s_2] \qquad (4.2.1\text{-}2)$$

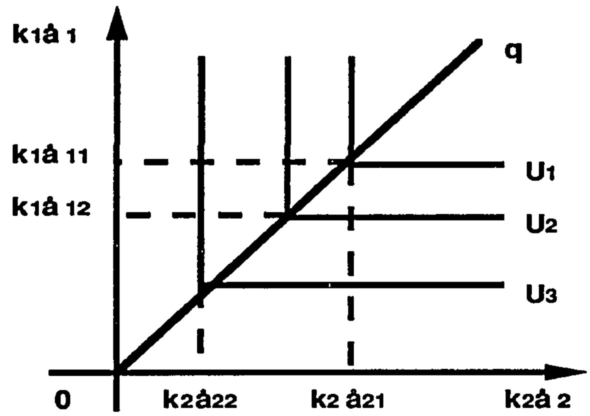

Figure 4-1 Complement Quality Properties

The above relationship between attribute quality and overall quality is quite important in quality control, especially for a reliable system and for the product with symmetrical characters, such as a pair of shoes. For example, the overall quality of a turbine blade is evaluated on the one blade which has the worst quality among the blades. Other blades with higher quality do not contribute anything to the overall quality of the turbine. This situation often requires a uniform quality control over all attributes in the product (or all attributes in a subsystem) to satisfy the consumer. More engineering and economic resources should be pooled into the lowest attribute quality to improve the whole system quality.

If two attribute qualities have quality substitution property based on the diminishing law of marginal utility, any quality increase in one attribute can be realized by the quality decrease in another attribute. One could find an optimal combination of two attribute qualities to maximize consumer utility if other conditions remain unchanged. Figure 4-2 shows that if attribute quality 2 is increased from $å_{22}$ to $å_{21}$ while reducing the quality in attribute 1 from $å_{12}$ to $å_{13}$, the overall product quality does not change. However, the consumer quality utility does not achieve the optimal point if the weights assigned for different quality attributes are not considered. Consumers try to maximize their overall utility by substituting the two attribute qualities, then

$$dU = s_1 df_1(å_1) + s_2 df_2(å_2) = 0$$

$$df_1(å_1)/df_2(å_2) = - s_2/s_1 \qquad (4.2.1\text{-}3)$$

The point ($å_{11}$, $å_{23}$) in the overall quality indifferent curve is optimal to meet the consumer requirement. This is the most efficient way to maximize consumer quality utility for two attributes i and j by adjusting the substitution rate equal to the weight ratio of s_i/s_j.

A special case, in which the substitution indifference curves between the two attribute qualities are no longer quadratic, but straight lines, is shown in Figure 4-3. If the weights are known, the corner solution would be obtained. This quality preference shows that consumer utility largely relies on one attribute quality when the other sacrificed attribute quality is reduced to an acceptable level.

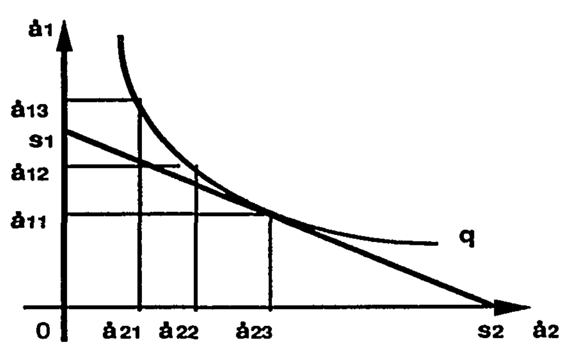

Figure 4-2 Two Attribute Qualities with Substitution

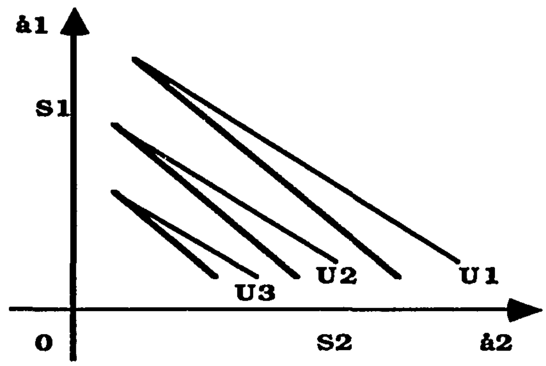

Figure 4-3 Two Attribute Qualities with Fixed Substitution

4.2.2 Comprehensive Decision Model and Price/Quality Decision Table

Since the overall quality utility is usually in the forms of separability and additivity for individual attribute qualities, an evaluation model to illustrate how the consumer makes a decision on product quality selection can be set up. The Lancaster attribute approach cannot be directly applied in the evaluation model for these reasons: (1) The quality deviation from the target value possesses a pattern of probability distribution. (2) The relationship between price and quality is usually nonlinear. (3) The implicit price is not an adequate index to measure the value of individual attribute quality. We develop a comprehensive quality attribute approach (CQAA) which uses consumer expected quality loss to capture the consumer evaluation of the product with multiple attribute qualities.

The consumer decision making in purchasing a TV set is used as an example to illustrate how the CQAA works. There are three TV models, A, B and C. Two TV attribute qualities are evaluated, picture clarity and lifetime. The former is expressed in terms of levels from 1-8 and the latter in hours. As seen in Table 4-1, these three models have the same quality mean values but different quality variances.

Table 4-1 Quality Data for Three TV Models

Model	Price/Unit	Clarity	Lifetime
A	300	8 ± 0.2	16000 ± 300
B	295	8 ± 0.6	16000 ± 600
C	305	8 ± 0.6	16000 ± 300

As mentioned before, the budget should equal the highest price in the set of products under consideration. In this case the consumer budget equals \$305, the price of model C. If the consumer is typically a money saver (η =1), he would choose model B, whereas if he is a perfect quality lover (η =0), he would select model A. But the consumer usually makes his decision in light of his weight distribution for price and quality as well as weights for various attribute qualities.

Generally, a product possesses a feature with three dimensions, quantity, quality and image index. The quantity dimension includes price, component unit and product functions. The quality dimension contains the quality target values and quality variations for various attributes. The image dimension, which is assumed independent of consumer assets, consists of color, shape and fashion which are difficult to measure in terms of physical units. In order to avoid biased estimates for weight assignment for price/quality effect, it is necessary first to evaluate weight distribution for these three dimensions and then evaluate weights in the subsystems for each dimension.

The following matrices are used to demonstrate the weight distribution for a product. The comprehensive weight matrix consists of a price/quality/image weight submatrix and an attribute weight submatrix. Mathematically, it is

$$W_{1x(1+n+m)} = M_{1x3}A_{3x(1+n+m)} \tag{4.2.2-1}$$

where W _ comprehensive weight matrix;

M _ price/quality/image weight matrix;

A _ attribute weight matrix;

n, m - the numbers of quality attributes and image index attributes, respectively.

Matrix M is related to the consumer's assets level which determines his attitude toward the product price, quality risk and image index preference, and has the following form

$$M_{1x3} = (s_p,\ s_q,\ s_i) \tag{4.2.2-2}$$

where s_p, s_q, s_i - weights for price effect, quality effect and image effect, respectively; $s_p + s_q + s_i = 1$.

From equation (3.3.2-6)

$$s_p/(s_p + s_q) = \eta, \quad s_q/(s_p + s_q) = 1 - \eta \tag{4.2.2-3}$$

Matrix A represents the degree of consumer preference for various attributes in quality and image index and is relatively fixed with respect to the weight assignment for product attributes from various groups of people. Matrix A has the following pattern:

$$\begin{pmatrix} 1 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 \\ 0 & s_{q1} & s_{q2} & \ldots & s_{qn} & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & s_{i1} & s_{i2} & s_{im} \end{pmatrix}$$

$$\sum_{j=1}^{1+n+m} s_{1,j} = 1$$

One assumes that there are no differences in product functions and image index among the three models. The consumer is a "quality preferrer" and his weight distribution for price/quality is 0.3/0.7. The weights for attribute quality are 0.4 for clarity and 0.6 for lifetime, respectively. Then the matrix W is :

$$(0.3 \;\; 0.28 \;\; 0.42) = (0.3 \;\; 0.7) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0.6 \end{pmatrix}$$

The comprehensive evaluation model for these three TV models is

$$\begin{aligned} C_{1xk} &= W_{1x(1+n+m)} Q_{(1+n+m)xk} \\ &= M_{1x3} A_{3x(1+n+m)} Q_{(1+n+m)xk} \end{aligned} \qquad (4.2.2\text{-}4)$$

where C _ comprehensive evaluation matrix;

Q _ single factor effect matrix, including price effect, quality effect and image index effect;

k - the number of products under consideration.

The consumer will choose the product which has the highest value of the comprehensive effect. The criterion, D, is defined as :

$$D = \max(C_1, C_2, ..., C_k) \qquad (4.2.2\text{-}5)$$

In matrix Q the first row is the price effect, the next n rows are for quality effects with n attributes and the last m rows are for image effects with m attributes. The number of columns, k, corresponds to the number of products in searching. The pattern of matrix Q is

$$\begin{pmatrix} p_{e1,1} & p_{e1,2} & \cdots & p_{e1,k} \\ ACQL_{2,1} & ACQL_{2,2} & \cdots & ACQL_{2,k} \\ ACQL_{3,1} & ACQL_{3,2} & \cdots & ACQL_{3,k} \\ \cdots & \cdots & \cdots & \cdots \\ ACQL_{(n+1),1} & ACQL_{(n+1),2} & \cdots & ACQL_{(n+1),k} \\ I_{(n+2),1} & I_{(n+2),2} & \cdots & I_{(n+2),k} \\ I_{(n+3),1} & I_{(n+3),2} & \cdots & I_{(n+3),k} \\ \cdots & \cdots & \cdots & \cdots \\ I_{(n+m+1),1} & I_{(n+m+1),2} & \cdots & I_{(n+m+1),k} \end{pmatrix}$$

In the real world there are no products with exactly the same functions, prices and qualities. In quantity dimension, the consumer compares the product functions with a common ideal product. The functions that a product possesses but the ideal product does not should be measured in terms of monetary value and then deducted from the product price. The total extra function cost, f_c, is

$$f_c = \Sigma c_i \qquad (\ 4.2.2.\text{-}6\ )$$

where c_i - cost for extra function i, \$; i = 1, 2, ..., n. f_c is not necessarily equal to the real manufacturing costs.

The actual price effect is

$$p_{ej} = p_h - p_j - f_{cj} \qquad (4.2.2\text{-}7)$$

where p_{ej} - the actual price effect for product j, $;

p_h - the highest price in the set of products under consideration, $;

p_j - the actual price of product j, $;

f_{cj} - the total extra function cost for product j, $.

Similarly, the image index cost can be determined by the following formula

$$I_i = (\sum_{j=1}^{L} C_{ij})/L \qquad j = 1, 2, \ldots, L \qquad (4.2.2\text{-}8)$$

where i_i - the average image cost for attribute i, $;

c_{ji} - the cost for image attribute i assigned by income group j, $.

The equation employed to compute the average consumer quality loss under quality aversion is

$$ACQL_j = -\Sigma[k_{1i}(\mu_{0i} - \mu_{ji}) + k_{2i}\sigma^2{}_{ji}] \qquad (4.2.2\text{-}9)$$

For two quality attributes

$$ACQL_j = - k_{11}(\mu_{01} - \mu_{j1}) - k_{21}\sigma^2{}_{j1} - k_{12}(\mu_{02} - \mu_{j2}) - k_{22}\sigma^2{}_{j2}$$

$$i = 1, 2, \ldots, n; \; j = 1, 2, \ldots, k.$$

where k_{1i} _ quality loss coefficient for the difference between the quality mean value and the target value for quality attribute i, $/unit;

k_{2i} _ quality loss coefficient for the total quality premium for attribute i, $/unit^2;

μ_{0i} _ the highest quality mean value for attribute i, unit;

$\mu_{0i} = \max(\mu_{1i}, \mu_{2i} \dots , \mu_{kn})$.

Generally, the quality loss coefficient for the total quality premium is determined in equation (4.1.1-9")

$$k_{2i} = p_h/(\mu_{0i}\Delta_i) \qquad (4.2.2\text{-}10)$$

where p_h _ the highest price in the set of available products, \$;

Δ_i _ specification for product quality attribute i, deviation unit;

μ_{0i} - the largest mean value for attribute i for three products, unit.

In practice, equation (4.2.2-10) should be modified to fit the specific case. When the attribute quality i must be within the specification, or the product is defective (or nonconforming), the quality loss coefficient is determined by

$$k_{2i} = p_h/\Delta_i^2 \qquad (4.2.2\text{-}11)$$

When the attribute quality i is measured in terms of levels or degrees, the coefficient is

$$k_{2i} = p_h/(L_{0i}\Delta_i^2) \qquad (4.2.2\text{-}12)$$

where L_{0i} - the highest level value for attribute i for these products, unit.

In the above example

$k_{11} = p_h/L_{01} = 305/8 = 38.125$

$k_{12} = p_h/\mu_{02} = 305/16000 = 19.06$

$k_{21} = p_h/(L_{01}\Delta^2{}_1) = 305/(8 \times 36) = 38.125/36 = 1.059$

$k_{22} = p_h/(\mu_{02}\Delta_2) = 305/(16000 \times 600) = 0.000032$

The price and quality effects are shown in Table 4-2.

Table 4-2 Calculation for Quality Loss and Price Saving

Model	P	$ACQL_1$	$ACQL_2$
A	5	-0.471	-0.32
B	10	-1.88	-1.28
C	0	-4.236	-0.32

Substituting the weights the data in Table 4-2 into equation (4.2.2-4), we have

$$C = WQ$$

$$D = \max(1.23 \quad 1.94 \quad -1.32)$$

The highest value in the comprehensive evaluation matrix is 1.94, and model B is chosen from these three TV sets. If one is more interested in the effect of consumer net worth on consumer decision making, it is possible to decompose the comprehensive evaluation matrix C into two submatrices

$$C = MAQ = \begin{pmatrix} 0.3 & 0.7 \end{pmatrix} \begin{pmatrix} 5 & 10 & 0 \\ -0.38 & -1.52 & -1.89 \end{pmatrix}$$

If the weight for the price effect changes gradually from 1 to 0, spaced by 0.1, we can draw a quality/price decision table, shown in Figure 4-4.

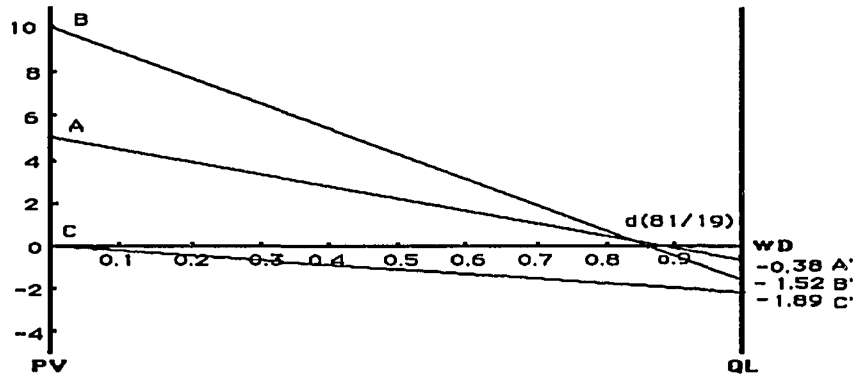

Figure 4-4 Price/Quality Decision Table for Example 1

In Figure 4-4, the horizontal axis, from the left to the right, indicates an change in the weight of the quality effect from 0 to 1. The left vertical axis and the right vertical axis represent the price effect and the quality effect, respectively. They are two extreme cases. The lines that connect the two values in the price effect axis and the quality effect axis, such as AA', BB' or CC', represent the product position in consumer decision making. For example, model B dominates the other two models in the range of weight ratio for price/quality effects from 1:0 to 0.19:0.81. However, if the consumer weight ratio for price/quality effects is less than 0.19:0.81, model A is superior to the other two models and will be chosen. If a product is superior to the other products, its price/quality decision line is above other decision lines. If two lines intersect, it shows which line is superior to another in what range. Line AA' and line BB' intersect at point d which corresponds to the weight ratio 0.19:0.81 for price and quality effects. At this point, a consumer is indifferent to models A and B. Before point d, the consumer prefers model B to model A, and after point d, the consumer prefers model A to model B. The consumer selection depends on the price/quality weight ratio assigned. Model C would not be selected regardless of the consumer's preference for the price/quality effects. Model C cannot compete with either models A or B in price or in quality and would be forced out of the market. It should be noted that the above conclusions derived from the present conditions will be no longer valid if these conditions are changed.

Since the differences in price for the three models are not considerable, it is expected that the weight assignment for price/quality effects for a majority of consumers would be in favor of model A. This could lead to the conclusion that quality improvement will incentive consumers to choose the product with higher quality when the prices of alternatives are almost the same. If the improvement in product quality is very difficult in current levels of technological and economic development, the lower price strategy may be adopted to affect consumer decision making on a product choice.

Now we will discuss the case with significant differences in both product quality effect and price effect and see how the price/quality decision table and the CQAA model work. Suppose that three alternatives of production/market strategies can be adopted. Producer A has a good market share and does not want to change the price and the quality of model A. Producer B adopts the strategy of reducing the price significantly from \$290 to \$250 to attract consumers to his product. Producer C cannot change his higher production cost margin to sell model C at a lower price and hopes to increase product quality to gain the competition's advantage in quality. As a result of this strategy, both the price and the quality for model C increase considerably. The new data are listed in Table 4-3.

Table 4-3 New Data for the Three TV Models

Model	Price/Unit	Clarity	Lifetime	Remote Control
A	300	8 ± 0.2	16000 ± 300	the same as before
B	250	8 ± 0.4	16000 ± 600	the same as before
C	370	10 ± 0.3	18000 ± 300	more functions

The highest effective price is p_e = the highest market price - extra remote control function cost = \$370 - \$20 = \$350. If the Lancaster attribute approach is applied to the consumer decision of selection of three TV sets, obviously model B would be chosen again because its attributes are superior. The CQAA is employed to evaluate these three models again. The quality/price weight assigned is 0.7/0.3 and the weights for attribute qualities are the same. The overall weight distribution matrix, W, is

$$W = (0.3 \quad 0.28 \quad 0.4)$$

$$M = (0.3 \quad 0.7)$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0.6 \end{pmatrix}$$

The quality loss coefficients are determined by the following equations

$$k_{11} = p_{e1}/\mu_{01} = 350/100 = 35$$

$$k_{12} = p_{e1}/\mu_{02} = 350/18000 = 0.01944$$

$$k_{21} = k_{11}/36 = 0.972$$

$$k_{22} = k_{12}/600 = 3.2 \times 10^{-5}$$

The price effect and the quality effect are listed in Table 4-4.

Table 4-4 Data for Quality and Price Effects

Model	PV	$ACQL_1$	$ACQL_2$
A	50	-70.65	-39.20
B	100	-71.30	-40.16
C	0	-0.97	-0.32

The comprehensive evaluation matrix C is

$$D = \max(-21.25 \quad -6.83 \quad -0.41)$$

Model C is selected from these three products. The price/quality decision table is shown in Figure 4-5.

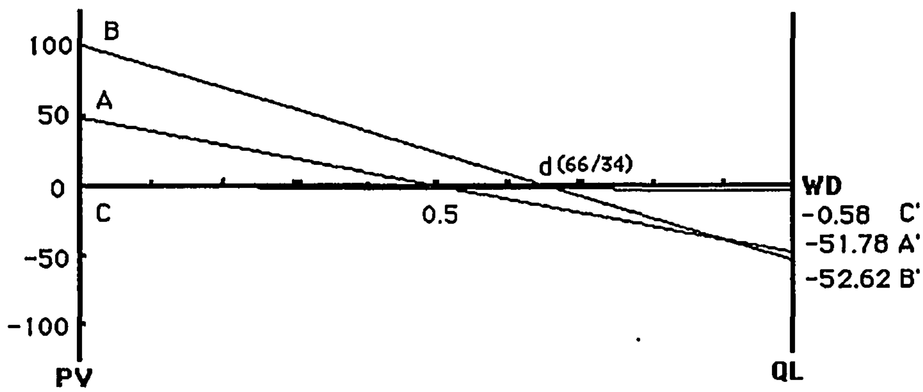

Figure 4-5 Price/Quality Decision Table for Example 2

Line CC' almost overlaps the horizontal axis (quality/price weight distribution) and intersects line BB' at d that corresponds to weight 0.66/0.34 for quality/price effects. Before point d, model B dominates the other two models; after point d, model C is selected. The advantage in lower price or in higher quality are weighed by the consumer on the basis of his preference. Products with neither price advantage nor quality advantage cannot compete with other products, whereas products with both price and quality advantage will offer the strongest competition with other rivals.

In the above example, the consumer budget should reach $350 so that the consumer evaluation for these three models can be carried out. The benefit from the quality/price increment strategy is considerable in the higher price/quality market section. This strategy significantly changes the position of model C, because model C could not compete with both models A and B in the previous production/market situation. Therefore, the advantage in quality improvement is very powerful for producers. Model B in the short-term is still strong in market share competition. However, in the long-term its advantage in lower price would be offset by the lower quality because consumers will look for higher product quality with increases in their incomes. Any improvement in product quality for model B will strengthen its advantage. The strategy of no change in both quality and price has weakened model A's competitive ability.

4.2.3 The Effects of Budget, Weight Distribution and Information Availability on Consumer Decision Making

It should be pointed out that no comparison between the conclusions derived from the first example and the second example in the above section can be made. This is because the budgetary constraint has been changed from $305 to $350. One cannot say that producer C's strategy is superior to producer B's strategy without consideration of constraint conditions.

The budget effect on searching for a product in the separated market sections is observable and considerable. A consumer's budget is a function of his assets (see equation (3.2.3-11)). The total budget distribution of the population has a pattern similar to the national income distribution. Assume that the budget for the above examples of three TV sets follows a normal distribution, i.e., $B(x) \sim N(\mu, \sigma^2)$ with $\mu = 300$ and $\sigma^2 = 1600$.

The differences in the prices in the first example (see Table 4-1) is so small that the consumer is able to adjust his budget to the highest price to search for a desired product. However, the whole market in the second example is separated into three sections corresponding to the significant differences in prices for the three products. The consumer is unable to make a budget adjustment to search for products in higher price sections and only searches for the products whose prices are equal to or less than the budget. Thus, the budget effect on product searching in the separated market sections can be defined in a column matrix as follows

$$B_e = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_L \end{bmatrix} \begin{array}{c} P_j \le B \le P_{j+1} \\ P_{j+1} \le B \le P_{j+2} \\ \vdots \\ P_k \le B \end{array} \qquad (4.2.3\text{-}1)$$

where B_e - a column matrix determined by the budget effect on product searching;

i - number of sections of market separation, $i = 1, 2, \ldots, L$; $L \le k$;

P_j - price of product j, $P_j \ll P_{j+1}$; $j = 1, 2, \ldots k$;

s_i - probability of consumers searching for products in market section i.

In equation (4.2.3-1)

$$\sum_{i=1}^{L} S_i \leq 1$$

$$S_i = 1 - Z\{\frac{P_j - \mu}{\sigma}\} - \sum_{t=i+1}^{L} S_t$$

where Z(x) - cumulative standard normal distribution.

For the first example

$$S_1 = 1 - Z(295\text{-}300/40) = 1 - 0.435 = 0.565$$

$$B_e = (\ 0.565\ )$$

For the second example

$$S_1 = 1 - Z(350\text{-}300/40) = 0.105$$

$$S_2 = 1 - Z(300\text{-}300/40) - S_1 = 0.395$$

$$S_3 = 1 - Z(250\text{-}300/40) - S_1 - S_2 = 0.395$$

$$B_e = \begin{pmatrix} 0.395 \\ 0.395 \\ 0.105 \end{pmatrix}$$

The potential market increases from 0.565 in the first example to 0.895 in the second example, which is the result of the budget effect on product searching in the separated market.

The product searching in the separated market is based on the following important assumptions: (1) the differences in product prices are significant; (2) consumers maximize their total utility subject to their budgetary constraints; and (3) the quality information is totally available.

The consumer weight distribution is determined by consumer attitude toward quality risk and the consumer's assets. The greater the assets, the higher the weight for quality effect. One can estimate the market share of various products under the conditions that the quality information is perfect and completely available. Different income groups give different weights for quality/price effects. The weights possess a certain probability distribution pattern which can reflect the consumer's assets and the price difference effect on the distribution shape. In the first example the price differences are quite small, such that the weight distribution for price/quality effects does not widely spread. However, in the second example, the price differences are significant so that the pattern of weight distribution, both the mean and the variance, is shaped by the price difference. That is

$$S \sim D[\mu = f(d_p), \sigma^2 = g(d_p)] \qquad (4.3.2\text{-}2)$$

where S - weight assignment for price/quality effects;

D[μ,σ] - weight distribution pattern;

d_p - price difference.

Assume that the weight distribution follows a Weibull distribution in the following form

$$f(s) = \frac{\beta}{\delta}\left(\frac{s-\gamma}{\delta}\right)^{\beta-1} \exp\left(-\left(\frac{s-\gamma}{\delta}\right)^{\beta-1}\right) \qquad s \geq \gamma \qquad (4.2.3\text{-}3)$$

where γ - the location parameter, $0 < \gamma < 1$;

δ - the scale parameter, $\delta > 0$;

β - the shape parameter, $\beta > 0$;

s - weight variable.

The mean and the variance of the Weibull distribution are, respectively

$$\mu = \gamma + \delta\Gamma(1 + 1/\beta) \qquad (4.2.3\text{-}4)$$

$$\sigma^2 = \delta^2[\Gamma(1 + 2/\beta) - \Gamma(1 + 1/\beta)^2] \qquad (4.2.3\text{-}5)$$

where $\Gamma(\cdot)$ - gamma function.

The Weibull distribution is very flexible, and by appropriate selection of the parameter γ, δ and β, the distribution can assume a wide variety of shapes. γ represents the bias of weight distribution toward the quality effect. The smaller the value of γ, the higher the weight for quality effect. δ and β are related to the price effect. The larger the differences in product price, the bigger the values of δ and β. Thus, the mean value moves toward the price effect side, and the variance of the Weibull distribution is expanded widely. The cumulative Weibull distribution is

$$F(d) = 1 - \exp[-((d - \gamma)/\delta)^{\beta}] \qquad (4.2.3\text{-}6)$$

where d - intersection point for the two product price/quality decision lines, as seen in Figure 4-4 and Figure 4-5. F(d) is the cumulated probability for the weight distribution for the quality effect.

The market section share percentage matrix F is

$$F = \begin{bmatrix} F_{1,1}(d_1) & F_{1,2}(d_2)-F_{1,1}(d_1) \cdots & F_{1,j+1}(d_{j+1})-F_{1,j}(d_j) & \cdots 1-F_{1,k}(d_k) \\ F_{2,1}(d_1) & F_{2,2}(d_2)-F_{2,1}(d_1) \cdots & F_{2,j+1}(d_{j+1})-F_{2,j}(d_j) & \cdots 1-F_{2,k}(d_k) \\ \vdots & \vdots & \vdots & \vdots \\ F_{i,1}(d_1) & F_{i,2}(d_2)-F_{i,1}(d_1) \cdots & F_{i,j+1}(d_{j+1})-F_{i,j}(d_j) & \cdots 1-F_{i,k}(d_k) \\ \vdots & \vdots & \vdots & \vdots \\ F_{L,1}(d_1) & F_{L,2}(d_2)-F_{L,1}(d_1) \cdots & F_{L,j+1}(d_{j+1})-F_{L,j}(d_j) & \cdots 1-F_{L,k}(d_k) \end{bmatrix}$$

where $F_i(d_j)$ - cumulated probability for intersection point j in market section i, j=1, 2, ..., k; i=1, 2, ..., L.

The total potential market share matrix, T_p, is

$$(T_p)_{1xk} = (B_e)^T{}_{1xL}\ (F)_{Lxk} \qquad (4.2.3\text{-}7)$$

Assuming that $\gamma = 0$, $\beta = 1$ and $\delta = 0.1$ (actually, the Weibull distribution becomes the exponential distribution with parameter $1/\delta$) for Example 1, the percentage of total consumers who prefer quality effect to price effect in product searching is

$$F(d) = 1 - \exp(-0.19/0.1) = 0.85$$

The total potential market share is

$$T_p = 0.565\ (\ 0.85,\ \ 0.15,\ \ 0\ )$$
$$= (\ 0.48,\ \ 0.085,\ \ 0\ )$$

The actual market share, T_a, is

$$T_a = (\ 0.85,\ \ 0.15,\ \ 0\ )$$

Therefore, models A, B, and C share 85%, 15% and 0% of the actual market, respectively. For Example 2, assume $\gamma = 0$, $\beta = 1.5$ and $\delta = 0.25$ for market section 1 (d=0.34/0.66, \$350 $\leq B_1$), and $\gamma = 0$, $\beta = 1$ and $\delta = 0.2$ for market section 2 (d=0.022/0.978, \$300 $\leq B_2 <$ \$350). The matrix F is

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0.1 & 0.9 & 0 \\ 0 & 0.205 & 0.795 \end{bmatrix}$$

The total potential market share and the actual market share are, respectively

$$T_p = (B_e)^T F = (\ 0.04,\ \ 0.772,\ \ 0.083\ )$$

$$T_a = (\ 0.044,\ 0.863,\ 0.093\ )$$

Compared with the Tp and Ta in Example 1, the strategy of price reduction adopted by producer B not only gains the potential market section of lower income groups, but also strengthens its competition against other rivals. The strategy of quality improvement employed by producer C makes model C share 9.3% of total actual market. It expects that model C will show its advantage in quality competition when productivity and people's incomes move to a higher level. The strategy of no change in both price and quality for model A would lead to the loss of a large part of the previous market share. However, the kind of strategies that ought to be used also depends on a firm's long-term goal.

All the above analyses are based on the assumption that the quality information is totally available. The procedures in consumer decision making are deterministic. In an imperfect world, consumers often face information uncertainty in decision making due to imperfect and biased (or unbiased) information sources as well as unobservable product quality.

In equation (3.2.1-21) the total quality variance contains quality information uncertainty which affects consumer estimation of inherent product quality. In order to reduce the total quality risk, the consumer is willing to pay an information premium to assess the information sources.

If quality information is obtained by a consumer's search, the cost of the search is equivalent to the increase in price. Whether the consumer benefits or not from the price increase resulting from the quality information cost depends on the amount of price increase and the information quality. If a signal does not change an individual's state of knowledge, it will not qualify as information. Information only has value if it results in some change in the actions to be taken by a consumer. In imperfect competition, the information available is not perfect, but offers the positive potential to reduce the uncertainty associated with product quality.

The beliefs of a consumer can be changed if the consumer receives a signal containing information. Bayes' theorem provides a formal model for revising probabilities on the basis of the new information. It is usually very difficult for consumers to get perfect information to help them make decisions. We use Bayes' theorem of prior information to study the experience of consumer satisfaction for a product. The derived posterior distribution is used to compute a new set of probabilities of a choice from these products. The information premium for the consumer, in general , is

$$C_i = (1-\eta)R_i$$

Specifically in this case

$$C_i = \omega(\Sigma p'_i w_i - \Sigma p_i w_i) \qquad (4.2.3\text{-}8)$$

where C_i - quality information premium, \$;

ω - degree of consumer attitude toward quality information uncertainty, $\omega = 1$, risk neutral, $\omega > 1$, risk aversion;

p'_i - the posterior probability revised by the imperfect information;

p_i - the probability of prior information.

For the first example of the three TV sets, assume that the consumer makes his decision based on the information provided by the consumer who is satisfied by the product bought. Table 4-5 shows the collected information about consumer satisfaction for these three products.

Table 4-5 Consumer Satisfaction for Three Products

	A	B	C
Completely Satisfied	0.6	0.8	0.3
Fairly Satisfied	0.3	0.2	0.4
Unsatisfied	0.1	0	0.3

The survey in column 1 indicates that for sales of product A, 60% of the consumers said they were satisfied, 30% of the consumers were fairly satisfied, and 10% of the consumers were unsatisfied by the product. If such information were not available, the consumer's decision would depend on the product market share in a general sense (not a specific consumer is mentioned here): 30% of the market shares for product A, 60% for product B and 10% for product C. By using the information based on consumer surveys, the probability of satisfaction from these products is

$$p'_i(\text{product } i/\text{satisfaction}) = p_i(\text{satisfaction/product } i)p_i(\text{product } i) \,/\Sigma p_i(\text{satisfaction/product } i)p_i(\text{product } i)$$

The p'(product i/satisfaction) for product A, B and C is 0.26, 0.696 and 0.043 respectively. The information premium is

$$C_i = \omega(\Sigma p'_i w_i - \Sigma p_i w_i)$$
$$= \omega[1.17(0.26\text{-}0.3) + 2.07(0.696\text{-}0.6) - 1.32(0.043\text{-}0.1)]$$
$$= 0.227\omega$$

If the consumer can get a set of imperfect information sources which could provide the same qualitative and quantitative information, the quality information premium is

$$\max(C_1, C_2, ..., C_i, ..., C_n) \leq \omega(\Sigma p'_i w_i - \Sigma p_i w_i) \qquad (4.2.3\text{-}9)$$

The consumer would select the information source whose cost is

$$C_i = \min(C_1, C_2, ..., C_i, ..., C_n) \qquad (4.2.3\text{-}10)$$

For a new product, the consumer will use his subjective quality preference (i.e., likelihood probability in Bayes' theorem) for the products he bought before adjusting the prior probability. The subjective quality preference reflects the quality credit of a company's products and the consumer quality reputation. This subjective quality assessment would in high probability reduce consumer information uncertainty. Therefore, producers should provide quality information as much as possible for consumers to penetrate the consumer subjective quality barrier, if producers introduce a new product to the market, or significantly improve the quality of the product that already exists in the market. Furthermore, the consumer would use the warranty provided by the producer to reduce the quality uncertainty for a new product. The warranty offers consumers an opportunity to make comparisons for the products newly introduced into a market.

When quality information is provided by a producer through advertisements, word of mouth and other signal channels, the cost to provide quality information is totally, or partially, added to the product cost so that the product price may increase.

Suppose the quality of product A has been improved, as shown in Table 4-6. Producer A provides consumers with very detailed information on product A and wants to have 20% of the total actual market share. The question is how much of the cost of advertising could be added to the price under the assumption of all other conditions being constant.

Table 4-6 Quality Improvement for Product A

	Price	Clarity	Lifetime	Remote Control
Before	300	8 ± 0.2	16000 ± 300	no change
After	310	9 ± 0.2	17000 ± 300	no change

The actual market share of product A in the competitive environment is approximately computed at prices \$310, \$320 and \$330. Product A at a price higher than \$310 is not able to compete with product C in quality and with product B in price in the separated market section 1, even though enough information on product quality is provided to the consumers. However, product A shows a strong advantage in quality improvement in market section 2 (price of product A < B < \$350). The AQ in equation (4.2.2-4) and the intersection point d are, respectively

$$AQ_1\,(\text{price} = \$310) = \begin{pmatrix} 0 & 60 \\ -0.35 & -26.16 \end{pmatrix} \qquad d_1 = {}^{0.7}\!/_{0.3}$$

$$AQ_2\ (\text{price} = \$320) = \begin{pmatrix} 0 & 70 \\ -0.36 & -26.95 \end{pmatrix} \qquad d_2 = {}^{0.725}\!/_{0.275}$$

$$AQ_3\ (\text{price} = \$330) = \begin{pmatrix} 0 & 80 \\ -0.37 & -27.23 \end{pmatrix} \qquad d_3 = {}^{0.749}\!/_{0.251}$$

By using equation (4.2.3-6) at $\gamma = 0$, $\beta = 1$ and $\delta = 0.2$, the share of market section 2 for product A and B are, respectively

$F_1 = (\ 0.776,\ \ 0.224\ )\quad F_2 = (\ 0.748,\ \ 0.252\ )\quad F_3 = (\ 0.712,\ \ 0.288\ )$

$$B_{e1} = \begin{bmatrix} 0.494 \\ 0.296 \\ 0.105 \end{bmatrix} \qquad B_{e2} = \begin{bmatrix} 0.598 \\ 0.204 \\ 0.105 \end{bmatrix} \qquad B_{e3} = \begin{bmatrix} 0.668 \\ 0.122 \\ 0.105 \end{bmatrix}$$

The total potential market share and the actual market share are :

$T_{p1} = (\ 0.230,\ \ 0.582,\ \ 0.083\ )\qquad T_{a1} = (\ 0.257,\ \ 0.650\ \ 0.093\ )$

$T_{p2} = (\ 0.153,\ \ 0.659,\ \ 0.083\ )\qquad T_{a2} = (\ 0.171,\ \ 0.736,\ \ 0.093\ )$

$T_{p3} = (\ 0.087,\ \ 0.725,\ \ 0.083\ )\qquad T_{a3} = (\ 0.097,\ \ 0.810,\ \ 0.093\ )$

An approximately linear relationship between the price and the actual market share for product A is shown in Figure 4-6. With an increase in price, the actual market share decreases proportionately. The product shares 20% of the total market at \$317, which means \$7 out of \$10 in advertisement costs may be added onto the price and the rest should be absorbed by the producer to meet the goal of 20% of the total actual market share. For a more complicated situation in the real world, a computational equilibrium model should be used.

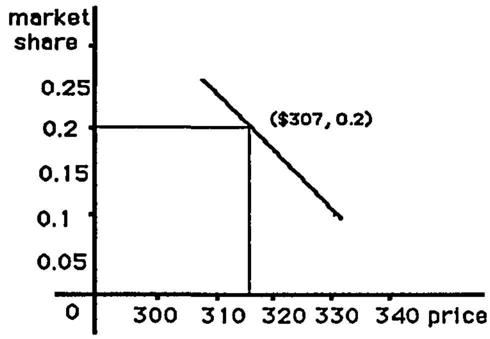

Figure 4-6 Relationship Between Product Price and Market Share

The conclusions derived from the above analysis can be summarized as follows.

(1) Market segments result from heterogeneous consumer behaviors and their budget constraints.

(2) Product market share depends on the budget effect, income distribution, product price and product quality as well as consumer weight distributions for three dimensions.

(3) The degree of information availability determines the patterns of consumer decision making on product selection. If consumers have completely reliable information sources, their behavior on decision making can be exposed by a deterministic choice model, which is developed in this research. If the information is completely unavailable, consumers would choose products randomly. When the information is imperfect, the probabilistic models, such as the Bayes updating model, are suitable to capture the consumer's behavior on decision making.

(4) The product with inferior quality at a lower price could still exist in the market, even though the quality information is available for consumers. The main reason is that consumers search for a desired

product and the weights for price/quality effects possess a distribution pattern which can be modeled by the Weibull distribution.

4.3 Consumer Decision Model in Multiple Attribute Qualities

We will extend the consumer utility and decision making model with the two attribute qualities in the above sections to n multiple attribute qualities mathematically.

If n attribute qualities are quality independent and additive, consumer utility function can be written as

$$U = \Sigma s_i f(\mathring{a}_i) \tag{4.3-1}$$

Any change in one of the attribute qualities will result in consumer utility change

$$\Delta U = \Sigma s_i f(\Delta\mathring{a}_i) \tag{4.3-2}$$

If n attribute qualities are quality complementary, consumer utility gain depends on the specific relationship among the attribute qualities. For the fixed proportional quality complement

$$U = \min[s_i f(\mathring{a}_i)] \tag{4.3-3}$$

$$\Delta U = \min[s_i f(\Delta\mathring{a}_i)] \tag{4.3-4}$$

For the CQAA (comprehensive quality attribute approach) with n attribute qualities, the weight distribution matrix W is

$$W = MA \tag{4.3-5}$$

$$= (s_p \ s_{q1} \ s_{q2} \ \dots \ s_{qn} \ s_{i1} \ s_{i2} \dots s_{im})_{1x(1+n+m)}$$

$$= \left(s_p, s_q, \ s_i\right)_{1x3} \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & a_1 & a_2 & \dots & a_n & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & a_{n+1} & a_{n+2} & \dots & a_{n+m} \end{pmatrix}_{3x(1+n+m)}$$

where $s_{qj} = s_q a_j$, $s_{ij} = s_i a_j$, $s_p + s_q + s_i = 1$, and $\Sigma s_{qj} = s_q$, $\Sigma s_{ij} = s_i$, $s_p/(s_p + s_q) = \eta$, $s_q/(s_p + s_q) = 1 - \eta$.

Based on the specific requirements in quality activities, a number of functional operators can be defined. We define an operator $\oplus$ as the ordinary matrix multiplication operation such that

$$\left(C_1 \ C_2 \ \dots \ C_n\right) \oplus \begin{pmatrix} B_1 \\ : \\ B_n \end{pmatrix} = C_1 B_1 + C_2 B_2 + \dots + C_n B_n \qquad (4.3\text{-}6)$$

and define an operator $\perp$ as the operation of taking the smallest product from a set of products such that

$$\left(C_1 \ C_2 \ \dots \ C_n\right) \perp \begin{pmatrix} B_1 \\ : \\ B_n \end{pmatrix} = \min\left(C_1 B_1, \ C_2 B_2, \ \dots, \ C_n B_n\right) \qquad (4.3\text{-}7)$$

In contrast, an operator $\otimes$ is defined as the operation of taking the largest product from a set of products such that

$$\left(C_1 \ C_2 \ \dots \ C_n\right) \otimes \begin{pmatrix} B_1 \\ : \\ B_n \end{pmatrix} = \max\left(C_1 B_1, \ C_2 B_2, \ \dots, \ C_n B_n\right) \qquad (4.3\text{-}8)$$

Of course, we can define a set of operators according to the particular requirement in a specific quality activity. A combination of a number of these operators can be used in an operation. For instance, if the relationships among attribute qualities are combination of operator $\perp$ and operator $\oplus$, then the equation can be written as

$$W = (S_p \quad S_q)_{1\times2} \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & & 0 \dots 0 \\ (0 & a_1 & a_2 & \dots & a_i)_{\oplus} & a_{i+1} & (1 \dots 1)_{\perp} \end{pmatrix}_{2\times(1+n)}$$

where submatrix $(0 \ a_1 \ a_2 \ \dots \ a_i)\oplus$ is operated by ordinary matrix regulation and submatrix $a_{i+1}(1 \ \dots 1)\perp$ is operated by the operator $\perp$ defined above, $a_1 + a_2 + \dots + a_i + a_{i+1} = 1$.

The single factor effect matrix Q is a (1+n+m)xk matrix. The CQAA final step is

$$\begin{aligned} C &= w_{1x(1+n+m)} Q_{(1+n+m)xk} \\ &= (C_1 \ \ C_2 \dots C_k)_{1xk} \end{aligned} \qquad (4.3\text{-}9)$$

The decision criterion is

$$D = \text{Max}\,(C_1, \ C_2, \dots, C_k) \qquad (4.3\text{-}10)$$

The weight distribution matrix W can be decomposed into two submatrices M and A. Matrix M is related to consumer net worth, consumer attitude toward quality risk, and fashion preference. Different income classes possess different tastes and preferences for quality, price and image index effects. People with higher net worth are less sensitive to price effect but are willing to pay more for the higher quality product.

Matrix A containing the weights that the consumer assigns to various attribute qualities and image index attributes reflects the consumer's preference order for the attributes. Matrix A is relatively fixed for various income groups in comparison with matrix M. The price/quality decision table is a very useful tool to illustrate consumer decision making for a choice of products with multiple-attribute qualities.

In the classical consumer theory, a rational consumer maximizes his utility subject to his budget. This rational assumption is also the basis for consumer behavior in quality economics. The quality utility value function is $V(E(U)) = x+\eta(B-p)+(1-\eta)(f(å))$, and the budget and weight distribution for attribute qualities are, $B \geq p$ and $f(å) = \Sigma s_i f(å_i)$, respectively. The consumer maximizes his utility subject to the budgetary and weight distribution constraints

$$\text{Max } V(E(U)) = x+\eta(B-p)+(1-\eta)(f(å))$$

$$\text{s.t. } B \geq p$$

$$f(å) = \Sigma s_i f(å_i)$$

$$L = V(\cdot) + \lambda_1(B - p) + \lambda_2(f(å) - \Sigma s_i f(å_i))$$

$$\partial L/\partial(-å_i) = (1-\eta)f_{åi} + \lambda_2 s_i = 0$$

$$\lambda_1(B - p) = 0$$

$$\lambda_1 \geq 0$$

$$\partial L/\partial\lambda_2 = f(å) - \Sigma s_i f(å_i) = 0$$

$$f_{åi}/f_{åj} = - s_i/s_j \qquad (4.3\text{-}11)$$

The above conclusion provides an important insight into quality improvement efforts in which quality of components should be improved with a given order of priority. The ratio of the marginal quality utility must be equal to the ratio of weights in the consumer utility maximization. The marginal attribute quality utility divided by its weight is the same for all attribute qualities. This ratio means that consumer satisfaction would increase if an additional dollar were spent on a particular quality attribute at this ratio. If consumer satisfaction could be increased by spending an additional dollar on improvement of quality attribute i rather than attribute quality j, he is not in the position of maximizing his utility. His satisfaction could be increased by selection of lower attribute j when other conditions remain constant. The marginal utility of budget is positive. As the budget increases, consumption of higher quality goods also increases. Increase in budget is equivalent to decrease in price. Consumer utility will decrease along with as increase in product price. The positive marginal budget requires higher quality improvement when incomes increase.

Although the weights of attribute qualities are very important, they cannot be easily captured by the use of conventional methods. One reason is that consumer preference is related to, or heavily depends on, the choices of other consumers. Another reason is that there is a lack of a powerful method to describe unobservable weight distribution which really reflects consumer decision making behavior.

4.4 Consumer Information Transformation Matrix

4.4.1 Black Box of Consumer Decision Making and Fuzzy Set Concepts

Some of the useful models and methods employed in the analysis of decision making under uncertainty usually are based on statistical concepts. A decision model may be more complicated and sophisticated if the uncertainty involves human behavior and multiple-criteria. In quality activities, a producer's market strategy and production planning are executed to meet the long-term goal given the consumer behavior and preference. A lot of information is collected and evaluated on the nontechnical bases, such as a 5 point or 6 point evaluation system of product attribute qualities, such as the Consumer Reports. The conventional model is unable to deal with these types of information.

As shown in Figure 4-7, we can observe the input signals, such as attribute qualities, price and image index, and the consumer characteristics, such as income, age, sex, risk attitude, etc. The final decision results can also be easily obtained. Meanwhile, the noise of correlation among consumers, market structures and subjective judgment errors affects and complicates the consumer decision making procedures. The process of decision making is directly unobservable, which is called the black box. The conventional methods designed to capture the process of consumer decision making are concentrated on the survey and the questionnaires about consumer assessment of the targeted problems. These methods do not consider the environment noise factors, such as consumer interactions. The final output through this process may be somewhat inconsistent with the real final signal output from the consumer's decision. Moreover, these methods are time and resource consuming. Therefore, it is necessary to find a convenient and inexpensive method to figure out the consumer decision making. A fuzzy set model may be able to play such a desired role.

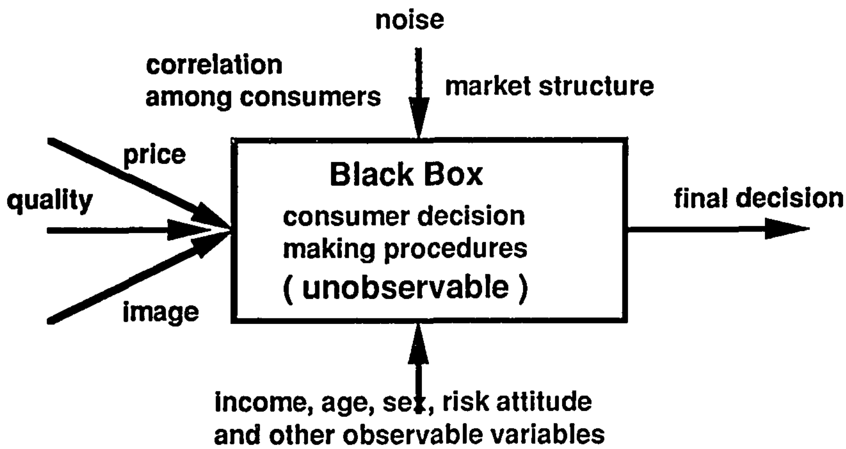

Figure 4-7 Black Box of Consumer Decision Making

In the following, we introduce a number of definitions in Fuzzy Set Theory used in a comprehensive decision making model. These definitions can be found dispersed throughout the published literature on fuzzy sets.

A fuzzy set is defined as a class of objects for which class membership is not clear. We define three fuzzy sets employed in comprehensive decision making model as

1. Factor set $U = \{u_1, u_2, ...u_n\}$;

2. Decision-making set $V = \{v_1, v_2, ... v_n\}$;

3. Single factor decision set. It is a fuzzy map from U to V. Define R as a matrix of fuzzy transformation from U to V.

The comprehensive decision making model can be written as

$$D = f\,(U, V, R) \qquad (4.4.1\text{-}1)$$

A decision associated only with a single factor is easily made. Although the decision procedures are complicated in the case of multiple attribute qualities, the fuzzy set decision model may provide a solution for such a complicated case.

The decision model in general possesses the following properties. Given a set U, define ϖ as a subset of U which could be the weight set for multiple-attribute qualities. The comprehensive decision set is

$$b = \varpi \bullet R \qquad (4.4.1\text{-}2)$$

where b is a fuzzy subset of V.

The assumptions of preference transitive, substitutability among groups, relatively homogeneous in their preferences and beliefs among groups are again adopted in the fuzzy set model.

For example, we will investigate the consumer preference for the TV set selection. The problem is to investigate a number of TV sets involving Sony, GE, Emerson and Samsung for 3 major attribute qualities: picture clarity, color fidelity and sound. Consumers would evaluate each TV set in four levels, excellent, good, fair and unfavorable. In the fuzzy set concept, we have attribute quality set U = (clarity, color, sound) and evaluation set V = (excellent, good, fair, unfavorable). Suppose we ask 100 people to evaluate the quality of an Emerson TV set. For color fidelity attribute quality, no one says it is excellent, 40 people say good, 40 people say fair and 20 people say unfavorable. Therefore, the evaluation for color fidelity attribute quality is

$$U(\text{color}) = (0 \quad 0.4 \quad 0.4 \quad 0.2)$$

The matrix R can be written as

$$R = \begin{array}{c} \begin{array}{cccc} \text{excellent} & \text{good} & \text{fair} & \text{unfavorable} \end{array} \\ \begin{pmatrix} 0.2 & 0.7 & 0.1 & 0 \\ 0 & 0.4 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0.4 & 0.2 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{clarity} \\ \text{color} \\ \text{sound} \end{array}$$

Suppose the consumer weights for attributes be known as

$$\varpi = (0.2 \quad 0.5 \quad 0.3)$$

Using the Zadeh operator, the comprehensive evaluation is

$$b = \varpi \bullet R = (0.2 \quad 0.5 \quad 0.3) \begin{pmatrix} 0.2 & 0.7 & 0.1 & 0 \\ 0 & 0.4 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0.4 & 0.2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.2\hat{}0.2 & 0.7\hat{}0.2 & 0.1\hat{}0.2 & 0\hat{}0.3 \\ 0\hat{}0.5 & 0.4\hat{}0.5 & 0.4\hat{}0.5 & 0.2\hat{}0.3 \\ 0.2\hat{}0.3 & 0.2\hat{}0.3 & 0.4\hat{}0.3 & 0.2\hat{}0.3 \end{pmatrix}$$

$$= (0.2 \quad 0.4 \quad 0.4 \quad 0.2)$$

After unifying the matrix,

$$b = (0.17, \quad 0.33, \quad 0.33, \quad 0.17)$$

The results indicate that 17% of the consumers say Emerson TV set is excellent, 33% say good, 33% say fair and 17% say unfavorable. Of course, we can use other operators, depending on the specific requirement of the problem. Selection of operators should be made and tested in the model developed.

As mentioned before, it is difficult for the consumer to conceptualize this probability. If we ask consumers to give probabilities for various attributes, some of them may be confused and then give some incorrect figures. The weights for various attributes may be easily collected by other methods rather than with the probability concept. We ask consumers to make a simple evaluation, expressing their preference degree (number 1-10) for color quality, excellent, good, fair and unfavorable. Consumers can fill in one level, several levels, or all levels with numbers 1 through 10, which correspond to their real evaluation of the product. The information obtained, therefore, is more reliable. The weight, whether or not it could be evaluated by sophisticated methods, actually exists. To find the weight distribution through the inverse of the evaluation process is more valuable and meaningful because this method takes into consideration for the noise influence on the consumer's decision. If we know b, the final evaluation set, and attribute evaluation matrix R, we may obtain the solution. However, there may be no solution or infinite solutions, depending on the specific question. Since computation details are complicated and tedious, we will not mention them here. The inverse decision solution could tell us the quantitative information to improve the quality design and manufacturing process control to meet consumer requirements.

We can adopt another method, called the principle of closer relationship selection from a number of sets prepared, to compute the consumer weight distribution. But this method is somewhat subjective. Some valuable information may be missed and the results of computation should be carefully explained. Mathematically, there are n fuzzy subsets in the domain U($\varpi_1, \varpi_2, \varpi_3, .., \varpi_n$). If subset ϖ_i satisfies the following requirement based on the relationship defined

$$(B, A_i) = \mathrm{Max}[(B, A_1), (B,A_2), ..., (B, A_n)]$$
$$A_j = \varpi_j R \qquad j = 1, 2, ..., n$$

where ϖ_i - the subject set one looks for;

B - the observable consumer final decision making matrix;

A_i - the computed consumer decision making matrix.

Suppose we have a set J which consists of a number of evaluation weight sets, that is

$$J = (\varpi_1, \varpi_2, \varpi_3) \quad (4.4.1\text{-}3)$$

The close relationship is defined as:

$$\sigma(A, B) = 1 - \frac{1}{n} \left\{ \sum_{k=1}^{n} \left| \mu_A(u_K) - \mu_B(u_K) \right| \right\} \quad (4.4.1\text{-}4)$$

where $\sigma(A, B)$ - distance between the actual and the computed consumer decision making;

$\mu_A(u_k)$- cell k in the computed consumer decision making matrix;

$\mu_B(u_k)$- cell k in the actual consumer decision making matrix;

The selection criterion is

$$D = \max \left[(\underline{A_i}, \underline{B}), \ldots, (\underline{A_n}, \underline{B}) \right] \quad (4.4.1\text{-}5)$$

Specifically,

B = (0.25, 0.45, 0.20, 0.1)

ϖ_1=(0.2, 0.5, 0.3)

ϖ_2=(0.5, 0.3, 0.2)

ϖ_3=(0.2, 0.3, 0.5)

One calculates that

$\varpi_1 \bullet$ R=(0.2, 0.4, 0.5, 0.1)

$\varpi_2 \bullet R=(0.2, 0.5, 0.3, 0.1)$

$\varpi_3 \bullet R=(0.2, 0.3, 0.4, 0.1)$

then by using equation (4.4.1-4)

$(\varpi_1 \bullet R, B)=0.90$

$(\varpi_2 \bullet R, B)=0.95$

$(\varpi_3 \bullet R, B)=0.905$

$D = \max(0.90, 0.95, 0.905) = 0.95$

Thus, ϖ_2 is closer to B in set J. Note that when this method is employed in practical cases, attention should be paid to the specific requirement in the activity, selection of capable operator(s) and an approximate close relationship should be carefully found.

The weight set is obtained for a specific income group. Although the small difference in individual preference arises in a group, it can be treated as a fuzzy boundary. The difference in the tastes of the members of a group is often small and can be ignored. If investigation shows that there are considerable differences in weight distribution for different income groups, it will provide the information that is necessary for diversified product quality design to meet different consumer requirements.

4.4.2. Applications of Fuzzy Decision Making Model on Product Quality Selection

There are few information sources available about consumer assessment of product (service) quality in detail. One reason is that the data collected through a survey is very expensive. The fuzzy decision making model can be employed as a filter which transfers input signals to output signals. Its effectiveness may be evaluated by examining the observable final results. The performance of a mathematical model depends on the designer's knowledge and the input signal quality.

Scales and ratings are currently the most popular methods used to measure consumer evaluation for product or service performance. Consumer Report is one of the most popular information sources on quality assessment. This information provides a direct ranking with five levels for various attributes for different brands of products or services, which consumers easily understand. However, this data does not contain some valuable information about consumer evaluation procedures, such as weight distribution for various attributes. As Curry [1985] has pointed out, some foreign testing agencies publish not only attribute-by-attribute judgments for each brand in a five-point rating, but also explicitly list the weights with which these scales are combined into an overall quality rating.

In this section, by applying fuzzy set concepts, we use available rating data to illustrate consumer information processing and consumer decision making for product selection with this rating information. More specifically, the fuzzy decision making model will answer the following questions:

(1) Does the consumer assign weights for various attributes of a product (or service)? How can it be illustrated by using the data available in the Consumer Reports?

(2) Is the weight distribution significantly different for different income groups? Can the difference be pointed out?

(3) Does the decision criteria developed in the previous section still work well with such rating information?

The data are obtained from the survey on hotel service quality, which are published in the Consumer Report issued in September 1990. [Note: The consumer assessment for overall quality data is estimated.] The survey asked consumers about their experience and evaluation with the hotels' service quality they had stayed at most recently. The study then sorted these chains of hotels into four price categories: economy, moderately priced, high-priced, and luxury. The information covers the overall satisfaction index to the reservations line. In order to meet the requirements of our study, we chose the following attributes of hotel service quality. Room quality includes cleanliness, size, bed comfort, climate control, noise, and amenities. Staff service is another factor that is a crucial quality attribute which must be accounted for in the model. Although food quality and swimming pools are other factors, especially the food quality, which affects consumer satisfaction. The data for these two factors are not completely available for all chains. Moreover, the data available for some chains are sorted at the same level, which means the effects of food quality and swimming pool on the total assessment can be equally taken out without any influence on the outcome. Therefore, seven attributes of hotel service, cleanliness, size, bed comfort, climate control, noise, amenities, and staff will be used in the evaluation of hotel quality. The input signals of attributes had been processed in five points already. The input signal matrix, for example, for Hampton Inn in the economy category can be written as

1	2	3	4	5	
0	1	0	0	0	cleanliness
0	0	1	0	0	size
0	1	0	0	0	bed comfort
0	0	1	0	0	climate control
0	0	1	0	0	noise
0	0	1	0	0	amenities
0	1	0	0	0	staff

This matrix contains 1 (preference) and 0 (nonpreference). No other information could be provided with the exception of comparison with the other chain's preference in five levels. It is now necessary to transform the data in the above matrix into the form of fuzzy membership matrix, based on the output patterns.

The level of satisfaction output reflects the percentage of respondents who reported one of the three levels for the hotel overall rating, completely satisfied, very/fairly satisfied, and somewhat/very/completely dissatisfied. Three satisfaction levels for each hotel can be roughly figured out. These levels of satisfaction are the final result of the consumer evaluation process, which is also the basis to transfer the input signal matrix in one level to all five levels. Since the difference between chains less than 4 point in overall satisfaction index is not meaningful, the chains in each category are classified into several groups, shown in table 4-7. The input signals in these chains whose attributes are sorted at the same level are impossible to detect consumer weight assignment in a strong white noise, because no operator, data transformation, or weight simulation has an effect on the output signals. These chains are eliminated from the analysis.

Table 4-7 The Classification of Chains and Their Final Signals

Group	Category	Satisfaction Index	Level of Satisfaction high	medium	low
1-1	Economy	85	0.38	0.60	0.02
1-2		74, 73, 71	0.176	0.694	0.13
1-3		66, 63	0.15	0.56	0.29
3-1	High-Price	86	0.45	0.49	0.06
3-2		82, 81, 79	0.29	0.63	0.08
4-1	Luxury	83, 82, 81, 80	0.32	0.60	0.08

Based on the pattern of final signals, the following principles should be obeyed in data transformation of input signals from 0-1 matrix to fuzzy membership matrix.

(1) Even in the case of the most perfect satisfaction, a small number of consumers are still dissatisfied. Thus, zero can not appear in the membership matrix.

(2) The level arising in the rating table (or 1 in the 0-1 matrix) has the highest score.

(3) One and only one peak is allowed for a single attribute. The adjacent levels can have the same score as the highest level.

(4) The further the level from the highest rating level, the lower the score.

The reason for establishment of the fuzzy membership matrix for input signals comes from the fact that not all consumers agree with the rating of attribute at a certain level; instead, the ratings may be distributed at some or all levels. The membership matrix of input signal transformation is

1	2	3	4	5
0.5	0.45	0.3	0.1	0.05
0.1	0.45	0.45	0.35	0.05
0.1	0.3	0.6	0.3	0.1
0.1	0.15	0.4	0.55	0.2
0.05	0.15	0.4	0.4	0.4

This matrix is somewhat subjective. (An objective matrix can be derived from the survey.) A number of similar forms can also be built in light of the level of understanding of the problem. Of course, we can use the original data to establish a reliable membership matrix if data is available. The average input signal matrix in 0-1 form for group 1-1 is transformed as follows

0.0714	0.3214	0.3214	0.25	0.0357
0.0714	0.2143	0.4286	0.2143	0.0714
0.0714	0.3214	0.3214	0.25	0.0357
0.0714	0.2143	0.4286	0.2143	0.0714
0.0714	0.2143	0.4286	0.2143	0.0714
0.0714	0.2143	0.4286	0.2143	0.0714
0.0714	0.3214	0.3214	0.25	0.0357

Corresponding to the three levels of satisfaction in the final signals, the column 1 and 2, column 3 and 4 in the above 7x5 transformed matrix are combined together, respectively, to form a 7x3 matrix. It actually is the single factor matrix R in fuzzy set concepts. That is

$$R_{1\text{-}1} = \begin{bmatrix} 0.3929 & 0.5714 & 0.0375 \\ 0.2857 & 0.6429 & 0.0714 \\ 0.3929 & 0.5714 & 0.0375 \\ 0.2857 & 0.6429 & 0.0714 \\ 0.2857 & 0.6429 & 0.0714 \\ 0.2857 & 0.6429 & 0.0714 \\ 0.3929 & 0.5714 & 0.0375 \end{bmatrix}$$

The comprehensive decision making set, B, is

$$B_{1\text{-}1} = (\ 0.38 \quad 0.60 \quad 0.02\ )$$

By analogy, we can derive single factor matrices for group 1-2, 1-3, 3-1, 3-2, and 4-1. As described before, the weight distribution is just the solution of the inverse problem. From the solution conditions for the defined fuzzy sets, the above inverse problem has many solutions. However, our purpose is to examine consumer weight distributions and their patterns related to different hotel categories (i.e. different income levels). Adopting the selection principle in fuzzy set theory, we prepare a set of weight distributions to carry out consumer decision processing. We predetermine that the weight for no single attribute may exceed 0.3. The prepared weight sets are

$$\varpi_1 = (\ 0.143,\ 0.143,\ 0.143,\ 0.143,\ 0.143,\ 0.143,\ 0.143\ )$$
$$\varpi_2 = (\ 0.25,\ 0.15,\ 0.0875,\ 0.0875,\ 0.0875,\ 0.0875,\ 0.25\ )$$
$$\varpi_3 = (\ 0.2,\ 0.2,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.2\ )$$
$$\varpi_4 = (\ 0.25,\ 0.21,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.25\ )$$
$$\varpi_5 = (\ 0.3,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.2\ )$$
$$\varpi_6 = (\ 0.2,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.3\ )$$

The implications of the above weight sets are briefly described in Table 4-8.

Table 4-8 Implications for Various Weight Sets

Weight Sets	Roles
ϖ_1	Equal weight for each attribute
ϖ_2	Cleanliness, staff attributes, and room size can be distinguished from other attributes
ϖ_3	Equal higher weight for cleanliness, room size, and staff attributes
ϖ_4	Cleanliness and staff are more important
ϖ_5	Cleanliness is weighted more than staff attribute
ϖ_6	Staff is weighted more than cleanliness attribute

By using equation (4.4.1-4), the selection criterion is defined as

$$(B, A_i) = \mathrm{Max}[\Sigma(B, A_1)/n, \Sigma(B, A_2)/n, ..., \Sigma(B, A_n)/n]$$

$$A_i = \varpi_i R$$

where $\Sigma(B, A_1)/n$ - the average close relationship, n = 3 for the economy category and for the high-priced and luxury categories, respectively.

In order to examine the effect of consumer groups with different incomes, the economy chains are separated from high-priced and luxury chains. Two operators are used in the comprehensive decision model, one of which is ⊗, Max. Product, and the other is ordinary matrix operator ⊕, Sum.Product. Table 4-9 and Table 4-10 show the computation results with two operators. The close relationships are ranked in order in the economy chains and in the high-priced and luxury chains, respectively.

Table 4-9 Close Relationship for Various Weight Sets for Economy Chains

Weight Set	Operator	Computation	Rank
$\varpi 1$	Sum.Product	0.966	6
	Max.Product	0.957	6
$\varpi 2$	Sum.Product	0.977	4
	Max.Product	0.972	3
$\varpi 3$	Sum.Product	0.967	5
	Max.Product	0.964	6
$\varpi 4$	Sum.Product	0.973	2
	Max.Product	0.973	2
$\varpi 5$	Sum.Product	0.974	1
	Max.Product	0.988	1
$\varpi 6$	Sum.Product	0.972	3
	Max.Product	0.958	5

The operators Sum.Product, $\oplus$, Max.Product, $\otimes$, were defined in section 4.3 (see equations (4.3-6) and (4.3-8)). In the economy chains, all of the sets with different weights for seven attributes are better than the equal weight for seven attributes. It means that the consumer's different weight distribution for various attributes actually exists. Consumers pay more attention to the attributes of cleanliness and staff in hotel service, which is consistent with the survey in which the study reveals the best predictors of consumer satisfaction turned out to be how clean the room was, whether or not the staff was helpful and efficient, and how well the front desk was run. Weight set 5 is ranked number 1, which may imply that consumers might be more concerned with room quality than with the staff. However, more data is needed to confirm such a claim.

Table 4-10 Close Relationship for Various Weight Sets for High-Priced and Luxury Chains

Weight Set	Operator	Computation	Rank
ϖ_1	Sum.Product	0.973	4
	Max.Product	0.967	5
ϖ_2	Sum.Product	0.977	2
	Max.Product	0.970	3
ϖ_3	Sum.Product	0.980	1
	Max.Product	0.971	2
ϖ_4	Sum.Product	0.973	5
	Max.Product	0.968	4
ϖ_5	Sum.Product	0.972	6
	Max.Product	0.963	6
ϖ_6	Sum.Product	0.974	3
	Max.Product	0.974	1

Compared with Table 4-9, several changes can be seen in Table 4-10. Not all of the weight sets are better than the equal weight set. The higher income group may be more concerned with the staff service than with the cleanliness. The room size attribute seems to become more attractive than other room quality attributes, which are claimed by the hotel service survey. Improvement in all aspects of various attributes will result in gaining as advantage in competition in high-priced and luxury chains groups. However, more information is needed to support the above conclusions. We pointed out that the membership matrix which was somewhat subjective plays a crucial role in this analysis. The quality and reliability of the analysis depend largely on the fidelity of input signals and the fuzzy membership model. Nevertheless, the following findings can be derived from the above analysis.

(1) Consumers assign different weights for various attributes of the hotel service quality.

(2) In the economy chains, the cleanliness and the staff attribute qualities are good predictors of consumer satisfaction.

(3) Consumer's weight distribution for service attributes changes with their income levels and the hotel categories. The higher the price paid, the better the service required.

(4) An improvement in any attributes would gain a competitive advantage in high-priced and luxury chains.

In general, the ratings information roughly provides an evaluation for product attribute qualities on the 5 point scale. Consumers really do not know the exact cost pattern for each attribute of the product, and they are simply the price takers. The fuzzy membership among 5 point ratings has the linear form in the most simple way. The utility in terms of money value provided by the attribute quality is proportional to the rating level. If consumers are concerned more with a specific attribute quality, the membership among the levels in this attribute could be the nonlinear one. In the following example for selection of moderately priced 19- and 20-inch TV sets, we will show that this imperfect rating information can be used to help consumers make decision after the fuzzy membership is determined.

Suppose the consumer predetermines his need of a 19- or 20-inch color TV with remote control. He selects RCA F20700DG, Montgomery Word 12690, Zenith SF2033Y, General Electric 20GT612, and Sylvania RKK191WA as candidates (see Consumer Report, 1988). The attribute qualities, picture clarity, black-level retention and color fidelity determine the picture quality, on which more weights are put. The attributes of cable channels, inputs/outputs, S input and warranty are similar for all sets, or not important. These attributes can be ignored for selection of the TV sets. The lowest rating is taken for couples of attribute qualities, such as airplane flutter and spark reject, fringe reception in VHF and UHF. The information about quality variation from the quality target value for each TV set can not be provided by the ratings of attributes because the test designers do not take a large enough sample for data collection. Consumers, on the other hand, rely on historical repair data as the quality variation information. As mentioned in previous sections, the consumer quality premium includes the consumer loss due to product repair and time waste. The repair rate really reflects the product quality variation from its quality target value. Specifically, consumer repair loss is computed by the following formula

$$C_r = C \times L \qquad (4.4.2\text{-}1)$$

where C_r - consumer repair loss, \$;

C - repair cost at present value, \$;

L - level of historical repair rate from 1985-1989, \$.

Detailed data about various attribute qualities, price, weight distribution and repair cost are shown in Table 4-11.

Table 4-11 Data for 5 Types of Color TV Sets with Remote Control

Weight	Quality Attributes	RCA	MW	ZENITH	GE	SYLVANIA
3	Picture clarity	1	2	2	2	3
3	Black-level retention	1	1	1	1	2
3	Color fidelity	1	1	1	1	1
3	Tone quality	2	2	2	3	4
2	Geometric distortion	2	3	3	3	2
2	Color control	2	3	3	4	3
2	Adjacent channel rejection	3	3	4	3	1
1	Brightness performance	1	2	3	2	2
1	Interlace	1	3	4	2	2
1	Auto color control	1	1	2	1	1
1	Fringe reception	2	2	3	3	2
1	Spark rejection	3	4	4	4	3
1	Resolution	1	1	1	2	2
	Repair cost $30	3	3	4	2	5
	Price $	470	327	405	375	383

The linear form of fuzzy membership function is adopted. The consumer decision making criterion is (see equation (4.3-10))

$$D = \text{Max}\,(C_1, C_2, C_3, C_4, C_5)$$

$$C_j = \eta(P_0 - P_j) + (1 - \eta)(\Sigma P_0 S_i(1 - L_i)/5 + C_{rj})$$

where P_0 - the highest price in the products searched, $P_0 \leq B$;

P_i - the price for product i;

S_i - the weight for quality attribute i;

C_{rj} - the repair cost.

The price effect and quality effect are shown in Table 4-12.

Table 4-12 Price/Quality Effects for Color TV

Model	RCA	MW	ZENITH	GE	SYLVANIA
Price effect	0	143	65	95	87
Quality effect	-66	-108.9	-139.1	-126.2	-141.3

Compared with the score ranking in the study, the quality effect ranks of Zenith and GE sets in Table 4-12 are modified with consideration of the consumer repair loss. If the consumer prefers higher quality and assigns weights 0.1/0.9 for price/quality effects, he would choose the RCA model.

$$D = \text{Max}(-59.4, \ -82.9, \ -118.7, \ -104.1, \ -118.5)$$

From Table 4-12, it can be predicted that the RCA F20700DG would dominate the other 4 models with higher quality at a higher price, and the Montgomery Ward 12690 model would gain the advantage in competition in both price and quality with Zenith, GE and Sylvania.

Chapter 5 Evaluation of Consumer Benefit in Quality Improvement

5.1 Total Consumer Quality Loss and Consumer Expected Price

With developments in technology, science, and the economy, product quality should be continuously improved in a competitive environment. The consumer quality gain from quality improvement (or the consumer quality loss from quality deviation from the target value) is also a continuous process, which can be measured by the consumer surplus.

In Chapter 3, we defined the average consumer quality loss (ACQL) as the difference between the quality target value and the consumer expected quality value for a certain product; see equation (3.2.1-12). Table 3-6 lists the ACQL for four types of quality characteristics under quality aversion. Taguchi and Taguchi et al. [1984, 1985, 1989] introduced a quality loss function to compute quality loss with the following formula

$$L = k[(\mu - T)^2 + \sigma^2]$$

where L- quality loss function, \$;

k - coefficient;

T - the product quality target;

μ, σ^2 – the product quality mean value and the quality variance.

It has the same form as the ACQL in the nominal-the-best type of quality characteristics for concave quality value function under quality neutral. The ACQL under quality aversion is the more appropriate approach to describe consumer behavior for most consumers. The consumer quality loss comes from not only quality deviation (variance and mean value) from the target value, but also quality premium due to consumer utility adjustment under risk. Nevertheless, the ACQL only measures consumer quality loss for the product unit purchased. The quality target value is relatively stable with product market time. It is very difficult to determine the terminal quality target value so that the cosumer quality loss is also a relative concept against time. It is common to use the following formula to approximately compute the consumer quality loss and consumer quality gain from quality improvement.

$$L = ACQL \times Q \qquad (5.1\text{-}1)$$

$$G = (ACQL_1 - ACQL_2)Q \qquad (5.1\text{-}2)$$

$$= L_1 - L_2$$

where L - total consumer quality loss from quality deviation,$;

L_1, L_2 - total consumer quality loss before and after quality improvement, respectively, $;

$ACQL_1$, $ACQL_2$ - average consumer quality loss for an individual before and after quality improvement, $/unit;

Q - total production output, unit;

G - total consumer gain from quality improvement, $.

Equations (5.1-1) and (5.1-2) are conveniently employed in practice to compute the approximate total consumer quality loss or quality gain, respectively. Equation (5.1-2) is a more meaningful measurement of quality activities on consumer welfare. Obviously, these approximate computations are based on the assumption of no change in other conditions except product quality.

However, the above equations would underestimate total consumer quality loss or social quality gain. As a matter of fact, consumer quality loss (gain) not only results from consumer quality loss in utilization of the product purchased, but also from less consumption of the product due to reduction in the purchase of an inferior quality product in the general sense. Furthermore, it is rare to have quality improved without a change in other conditions in quality activities, especially price. The following questions might arise in evaluation of total consumer quality loss (gain): Is it gain or loss for consumers if a quality activity improves product quality accompanied with an increase in price? Does quality activity reduce product quality and product price simultaneously? How can the pure effect of both price and quality be separated on the consumer quality loss (gain)?

We introduce two concepts: consumer expected quality price and quality equivalent price. By using the consumer indifferent utility curves developed in previous chapters, we will illustrate the implications of these two concepts in detail. A consumer is indifferent between two products with different qualities and prices if and only if these two products have the same indifference curve. Consumers will be better off if either price, quality, or both move onto the higher utility indifferent curves. In contrast, consumers will be worse off if either or both price and quality move onto the lower utility indifferent curves. The quality equivalent price on the basis of substitution between price and quality can be derived from equation (3.3.2-5). As shown in Figure 5-1a, the product price remains at p_1, and product quality is improved from $\mathring{a}_1$ to $\mathring{a}_2$, which cause the consumer utility to move from u_1 to u_2. If the utility moves along with u_2 from $\mathring{a}_2$ to $\mathring{a}_1$, the consumer is indifferent. In other words, quality improvement from $å_1$ to $å_2$ is equivalent to the price change from p_1 to p_2. In Figure 5-1b, the consumer utility increases from u_1 to u_2 due to simultaneous reduction in price and improvement in quality. The difference between p_1 and p_2 is the price effect, while the difference between p_2 and p_3 corresponds to the quality effect. Therefore, the price/quality effect can be distinguished from the overall outcomes.

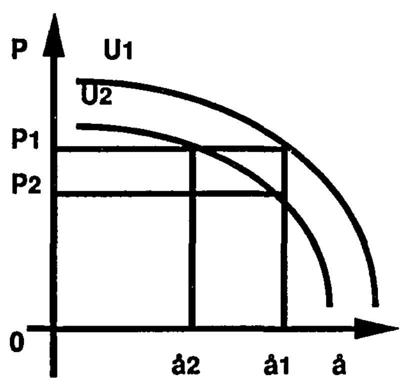

Figure 5-1a Quality Equivalent Price

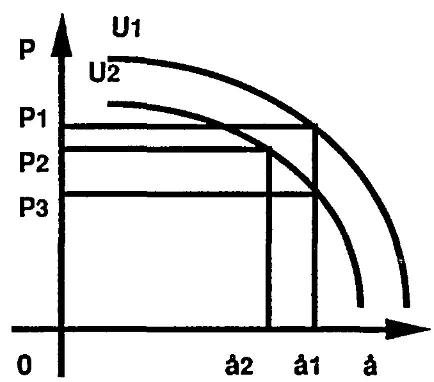

Figure 5-1b Price/Quality Effects

From equation (5.1-1), assume that the consumer utility is additive for both price effect and quality effect, and the relationship between price and quality value (not quality deviation variable) is linear. Then define dp and d(-p) as an increase and a decrease in price, respectively. The same definitions are made for changes in quality value.

Assume that the consumer utility increments are derived from price reduction and quality improvement respectively. From equations (3.3.2-5)

$$dp = f'(å)d(-å)/f'(p) \qquad (5.1\text{-}3)$$

Quality improvement is equivalent to the reduction in price. The aggregated demand is easily executed for the higher quality product in meeting the needs of consumers if consumers and product quality are homogeneous. The individual consumer evaluates product quality by using his quality value function, which is different from the other income groups. Therefore, it is very difficult to induce the aggregated demand curve for nonhomogeneous products due to widely varying quality value functions. For tractability and without significant loss of generality, assume that the relationship between quality and price is linear such that

$$f'(å)/f'(p) = (1-\eta)/\eta$$

$$dp = (1-\eta)d(-å)/\eta \qquad (5.1\text{-}4)$$

The equivalent quality price, ΔP_q, is

$$\Delta P_q = dp = (1-\eta)d(-å)/\eta \qquad (5.1\text{-}5)$$

The weight distribution variable η is assumed to have the pattern of Weibull distribution. The consumer expected price increment is the sum of the difference in the actual prices and the difference in the quality equivalent prices. The consumer expected price is the sum of the actual price before the change in the price and the expected price increment. Mathematically,

$$\Delta P_c = \Delta P_a - \Delta P_q \qquad (5.1\text{-}6)$$

$$P_c = E(P_a + \Delta P_c) = P_a + \Delta P_a - E(\Delta P_q)$$

$$= P_a + \Delta P_a - [\Gamma(1 - 1/\beta)/\delta - 1]d(-\mathring{a}) \qquad (5.1\text{-}7)$$

$$E[(1 - \eta)/\eta] = \Gamma(1-1/\beta)/\delta-1$$[1]

where ΔP_c - consumer expected price increment for the product, \$/unit;

ΔP_a - difference in the actual price for the product, \$/unit;

ΔP_q - difference in quality equivalent price for the product, \$/unit;

P_c - consumer expected price for the product, \$/unit;

P_a - actual market price for the product before change, \$/unit.

The difference between the actual price corresponding to the quality target value and the consumer expected price reflects consumer quality loss in terms of price equivalent.

In previous sections, the quality loss for a unit of a product was measured in terms of monetary value. The ACQL is transformed into the equivalent price to measure total consumer quality loss for the reasons: (1) Total consumer quality loss is not equal to the sum of the individual consumer quality loss. (2) Conventional theoretical concepts of total consumer loss (gain) measurement can be directly employed in quality loss. (3) It is easy to illustrate the effects of quality activities for various

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[1] $f(x) = (\beta/\delta)[(x-\gamma)/\delta]^{\beta-1}\exp(-[(x-\gamma)/\delta]^{\beta}) \quad x \geq \gamma$

Let $\gamma = 0,\ x \geq 0$

$f(x) = (\beta/\delta)(x/\delta)^{\beta-1}\exp(-(x/\delta)^{\beta})$

$E(1/x) = \int 1/x\{(\beta/\delta)(x/\delta)^{\beta-1}\exp(-(x/\delta)^{\beta})\}dx$

$E(1/x) = 1/\delta \int \upsilon^{-1/\beta}\exp(-\upsilon\}d\upsilon$

$\Gamma(1-1/\beta) = \int \upsilon^{-1/\beta}\exp(-\upsilon\}d\upsilon,\ E(1/x - 1) = \Gamma(1-1/\beta)/\delta - 1$

products on consumers by using price elasticity of consumer demand. (5) A partial equilibrium mechanism can explain the results of competition in substitutable products.

The consumer quality price index, I, is defined as the difference in consumer expected price increments between the substitutable products

$$I = \Delta P_{c1} - \Delta P_{c2} \qquad (5.1\text{-}8)$$
$$= \Delta P_{a1} - \Delta P_{a2} + \Delta P_{q1} - \Delta P_{q2}$$

One important condition for the partial equilibrium for these products is I = 0. Assume that positive changes in both actual price and quality equivalent price for product 1 are higher than those for product 2, such that

$$I = \Delta P_{a1} - \Delta P_{a2} + \Delta P_{q1} - \Delta P_{q2} > 0$$

Consumers prefer product 1 to product 2 and will gain from a switch from product 2 to product 1. This switch will not stop until the consumer quality price index equals 0. In the competitive environment, product 2 could survive only by either reducing the actual price or increasing the quality equivalent price or both. On the other hand, with demand increase in product 1, the price would go up due to the upward slope of production cost function. The market reaction resulting from both product 1 and 2 adjustments will force index I to zero under competitive market condition. Equations (5.1-7) and (5.1-8) can be used to measure total consumer quality gain in partial equilibrium.

5.2 Consumer Quality Surplus in Consumer Welfare Evaluation

The welfare function usually comprises both consumer surplus and producer surplus. Since producer surplus in quality activity is well defined and easy measured, the light is shed on the consumer benefit from quality activity in this section. In order to simplify the exposition, this section makes some formal assumptions: (1) Follow Willig's [1976] statement that consumer surplus could be employed to estimate the unobservable compensation and equivalent variations for consumer welfare. (2) The concept of "presentive consumer" is used here to avoid the spillover effect in demand. (3) Consumer surplus does not allow one to assess directly the initial impact of quality improvement (Trajtenberg,1989). (4) Consumer surplus can be treat as the lower bound of social welfare in some special cases.

The consumer surplus concept in quality cost-benefit analysis is not purely an accounting system, but an evaluation method based on the applied welfare theory. Because the prevailing market price, even in a perfect competitive market, is an inadequate index of the real value of product quality in society, consumer surplus is, therefore, a very crucial concept in the measurement of consumer benefit (or loss) in quality improvement.

The basic idea is that the utility of a certain good is at least equal to the price paid for it, so that a person buying a good at a certain price and attaching to a product value greater than its price will derive a net "profit," which is called consumer surplus. This implies that the real value of a product's quality is determined by the buyer's willingness to pay. Therefore, benefits taken into account in quality loss-gain analysis are not the market value of a good that buyers actually pay, but the real value reflected by his willingness to pay.

Several measures of such a consumer's surplus have been proposed. Here we employ the incremental consumer surplus based on the duality theory to illustrate the effect of quality activity.

The following terms are used in this section.

DD' - demand curve for a specific type of product;

P_a - actual price, or the price the consumer is willing to pay for the product under no quality variation, \$/unit;

P_{c1} - consumer expected price, or the price the consumer is willing to pay under quality variation before quality activity implementation, \$/unit;

P_{c2} - consumer expected price after quality activity implementation, \$/unit;

Q_1, Q_2, and Q_3 - total product demand at price P_a, P_{c1}, and P_{c2}, respectively; \$/unit;

G - consumer surplus, \$;

ΔG - consumer surplus gained from quality activity, \$;

H(q) - inverse demand function, \$/unit.

In conventional microeconomics, all products are assumed to be homogeneous in the quality equaling the quality target value. The consumer expected price equals the actual price of the product under no quality variation. The consumer surplus could normally be written as:

$$G = \int_0^{q_1} H(q)\, dq - P_1 Q_1 \qquad (5.2\text{-}1)$$

However, a consumer suffers a loss from utility reduction or adjustment under quality variation. With respect to the price for the highest product quality, the consumer expected price at which the consumer is willing to pay for the product purchased on average is lower than the actual price. No consumer quality loss would arise if the consumer pays for the product at the expected price. As a matter of fact, consumers actually pay the same price for the product without accounting for the quality discrimination. The total consumer quality loss, L, from quality variation is

$$L_{12} = -\Delta G = \int_{q_2}^{q_1} H(q)\, dq - (P_1Q_1 - P_2Q_2) \qquad (5.2\text{-}2)$$

As shown in Figure 5-2, consumer surplus under quality homogeneity is P_aDR_1, which is simply the difference between the amount the consumer is willing to pay for the product, ODR_1Q_1, and the amount he actually pays, $OP_aR_1Q_1$. Now the product quality varies but the selling price is still set at P_a. However, the consumer expected price for the product with quality variation on average is P_{c1}, lower than P_a. Consumer surplus will be taken by $P_aR_1R_2P_{c1}$ which consists of two components, $P_aR_1SP_{c1}$ as the quality loss component and R_1R_2S as the product loss due to the quality loss for consumers. The quality loss rectangle alone could be regarded as a minimum calculation of quality loss under quality variation because it does not account for the product loss. Suppose the product quality be improved, but the actual price be kept at P_a. The consumer expected price is P_{c2}. The total consumer quality loss becomes

$$L_{13} = -\Delta G = \int_{q_3}^{q_1} H(q)\, dq - (P_1Q_1 - P_3Q_3) \qquad (5.2\text{-}3)$$

and the total consumer quality gain from quality improvement is

$$\Delta G = L_{12} - L_{13} = \int_{q_2}^{q_3} H(q)\, dq - (P_3Q_3 - P_2Q_2) \qquad (5.2\text{-}4)$$

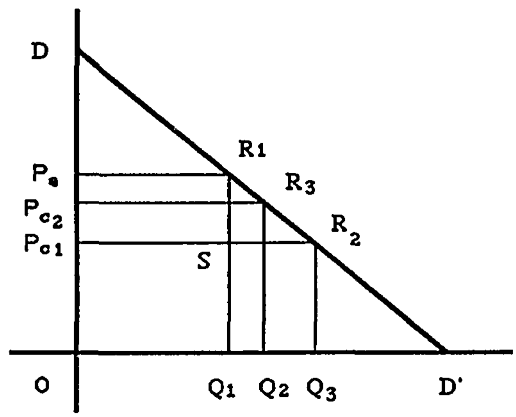

Figure 5-2 Consumer Surplus from Quality Variation

Quality improvement strengthens the competitive ability with close substitute products in price and in the market share. The increment of consumer surplus from quality improvement is regarded as the collective improvement in society that will be calculated as a part of the benefit in quality loss-gain analysis. It should be noted that the increment in consumer surplus arising from quality improvement will be valid to measure an increase in consumer welfare based on the following assumptions: (1) all other prices remain unchanged, (2) all the inputs are combined efficiently, (3) the product is in a perfect market; consumer expected price reflects the product quality variation. However, condition (3) is a weak assumption. In traditional economics, if the product is in a monopoly market, the price prevailing in the market is not the real price; the reduction of price may simply be reduction in monopoly profit. The quality improvement component in consumer surplus does reflect a major part of an increase in the benefit to consumers. Since the price is not changed, no benefit is redistributed from producers to consumers. The consumer is actually benefited from reduction of quality loss, while the producer increases his profit (or revenue) from higher production output.

The consumer surplus for evaluation of consumer quality benefit is still valid in the monopoly market for quality improvement. The maximization of profit under quality variation is the power source to stimulate the producer to improve product quality. However, less competitive pressure will slow down the continuous quality improvement in the monopoly environment.

If a quality activity is accompanied with a change in price, the net value in the consumer expected price increment is used to evaluate whether the activity is taken, see equation (5.1-7). If $P_c > 0$, consumer is better off from such activity; if $P_c = 0$, the consumer is indifferent; if $P_c < 0$, the consumer is worse off from such activity.

Therefore, a quality activity does not necessarily imply a benefit for consumers as some people might think obvious. A quality activity should be carefully evaluated before its implementation in quality policy making.

The consumer surplus approach with the expected consumer price is equivalent to the approach of consumer surplus with a demand curve shift due to product quality improvement. It is easy to illustrate the relationship between these two approaches. As shown in Figure 5-3, p and Q' are the price and the quantity at the market equilibrium. For the former approach, the consumer surplus is Dp_cA. The quality improvement results in the demand curve shift from D to D'; the consumer surplus in the latter approach is D'pB. Obviously, when DPcA = D'pB both consumer surplus and producer surplus are the same in these two approaches.

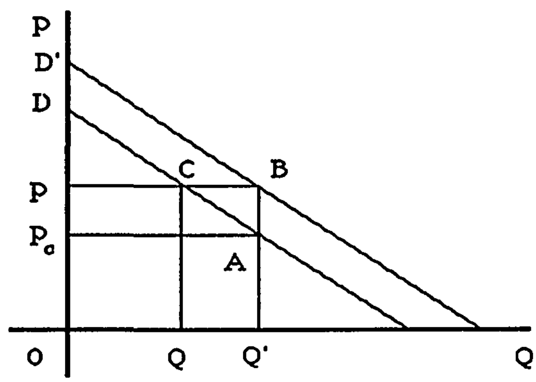

Figure 5-3 Equivalent Relationship Between Two Consumer Surplus Approaches

The latter approach can be induced from the former one, which has been well developed in previous chapters. The effect of change in the consumer expected price from p to p_c can be treated as the demand curve shift from D to D' in most cases. The adoption of the consumer surplus approach in which the quality differences can be transformed into the difference in quantity (or in price) depend on the specific requirement and the property of the problem solved. The consumer demand is the function of product price and the quality. That is:

$$Q = f(p, å) \qquad (5.2\text{-}5)$$

$$dQ = f'_p dp + f'_å d(-å) \qquad (5.2\text{-}6)$$

$$f'_p < 0, f''_p \le 0 \qquad (5.2\text{-}7)$$

$$f'_{å} > 0,\ f''_{å} \leq 0 \qquad (5.2\text{-}8)$$

where Q - product demand;

p - product price;

å - product quality.

The quantity is negatively related to the price increase, but positively related to the quality improvement. The second term in equation (5.2-6) is the amount of up-right shift in the demand curve corresponding to the change in quality. The demand function can be written in another form such that

$$Q = f(p, p(å)) \qquad (5.2\text{-}9)$$

$$dQ = f'_p dp - f'_p dp(å) = f'_p dp - f'_p MRS_{åp} d(-å)$$

In the linear form

$$dQ = f'_p dp - f'_p (1-\eta) d(-å)/\eta$$

$$= f'_p (dp - (1-\eta) d(-å)/\eta) \qquad (5.2\text{-}10)$$

If the product is searched across consumer groups, the average change in product quantity is

$$E[(1-\eta)/\eta] = \Gamma(1-1/\beta)/\delta - 1$$

$$E(dQ) = f'_p (dp - [\Gamma(1-1/\beta)/\delta - 1] d(-å)) \qquad (5.2\text{-}11)$$

The above equation is used in the consumer expected price approach. Any change in product quality is transformed into an equivalent price effect. If we are interested in the effect of quality improvement in a specific product from a set of substitutable products on consumer welfare, a conditional multinominal logic model can be employed for consumer welfare computation (Bresnahan [1986], Trajtenberg [1989]).

5.3. Evaluation of Consumer Quality Welfare at Partial Equilibrium

Evaluation of consumer gain from quality improvement for a set of substitutable products in a competitive environment may be better accomplished by using the concept of consumer surplus at the partial equilibrium. The additional product component, R_1R_2S in Figure 5-2, in consumer surplus is closely related to the pattern of demand for the product and may be estimated under the assumption of approximate linear demand function in the domain of quality variation range, such that

$$\Delta G_1 = (P_{c2} - P_{c1})(Q_1 - Q_2)/2 = \Delta P_c \Delta Q/2$$

If ΔP_c and ΔQ are very small

$$\Delta G_1 = e\Delta P_c{}^2 Q_1/(2p_1) \qquad (5.3\text{-}1)$$

where e - price elasticity of the product demand, defined as PdQ/QdP;

Q_1, Q_2 - demand for the product before and after the quality improvement, respectively;

P_{c1}, P_{c2} - consumer expected prices for the product before and after the quality improvement, respectively.

Equation (5.3-1) reveals that the additional product component in consumer surplus is proportional to the increase in demand and the expected price increment. The consumer surplus from the product with higher price elasticity of demand is more sensitive to the quality improvement.

The quality loss (gain) component is the easy component in the calculation of consumer surplus and it can be regarded as a minimum estimate of consumer gain in quality improvement. The component is

$$\Delta G_2 = (p_{c1} - p_{c2})Q_1 = \Delta P_c Q_1 \qquad (5.3\text{-}2)$$

$$\Delta G = \Delta G_1 + \Delta G_2$$

$$\Delta G = \Delta P_c(Q_1 + \Delta Q/2) \qquad (5.3\text{-}3)$$

The above results contain an important implication for a government policy decision maker to implement quality activity. In some countries, such as Japan, the government stimulates the firm, whether it is private or public, to improve product quality (or big companies stimulate the smaller firm) by taxation, fund distribution, partnership and share of profit. Quality improvement not only promotes the product competition capability, but benefits the consumers when consumer surplus increases. The higher the quality reputation for the firm, the more the consumers would buy the firm's products.

We have to account for the effect of quality activity more carefully in cases in which a change in one product quality has an effect on the other in the competitive environment. We examine the effect of quality improvement on two competitive commodities. Assume that X and Y are close substitutes and their production costs are constant. As shown in Figure 5-4, demand output is Q_{y1} for commodity y at price P_{y1}, while Q_{x1} is the demand output for X at price P_{x1}. These two commodities are relatively stable in the current market before a quality improvement is implemented for product X. Consumers notice a significant quality improvement in product X and evaluate product X at the expected price P_{x2}, such that $P_{x2} < P_{x1}$. The consumer surplus increases by $P_{x1}P_{x2}C_2C_1$. Since an increase in demand for product X would cause the demand curve of product Y to shift from D to D', the total demand for product Y reduces. Obviously, with a quality improvement in product X, consumers are better off and some consumers would switch from buying product Y to product X. The reduction of area $R_1R_3D'D$ under demand curve D for product Y is not to be regarded as a loss of consumer surplus because this reduction in the area is simply the consequence of the consumer bettering himself by switching from product Y to the higher quality product X. Otherwise, double accounting would occur in the evaluation of consumer's gains from quality improvement. The reduction in demand for product Y is definitely the loss for firm Y whereas it is the gain for firm X. These benefit redistribution in quality competition among the producers.

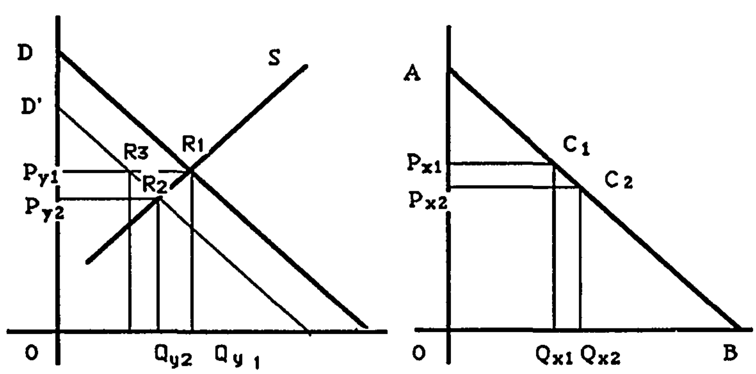

Figure 5-4 Quality Effect on Close Substitute Product

Moreover, in the perfect competitive market, the market mechanism forces the equilibrium price of product Y to reduce to P_{y2} because of the right-upward supply curve S in the short term market equilibrium. The demand output of Y now is Q_{y2} at price P_{y2}. In this dynamic situation, the increment of consumer surplus resulting from decrease in the price of product Y is $P_{y2}P_{y1}R_3R_2$. The total increment of benefit to the consumer would be equal to the initial fall in the real price of product X due to quality improvement plus the further induced fall in the equilibrium price of product Y. If the consumer chooses the price of product X as a base standard of price, the same conclusion derived from this situation can be obtained. Symmetrical reasoning applies also to products that are complements. It should be pointed out that the above discussion of pure quality improvement is based on the assumption of all other conditions being constant. The product with inferior quality could compete with the higher quality product either by increasing its quality or by decreasing its price.

The total loss for firm Y includes not only the part of cost uncovered but also the part of profit previously earned. That is

$$\begin{aligned}\Delta L &= (P_{y1} - C_1)Q_{y1} - (P_{y2} - C_2)Q_{y2} \\ &= (- C_1Q_{y1} + C_2Q_{y2}) + Q_{y1}(P_{y1} - P_{y2}) + P_{y2}(Q_{y1} - Q_{y2})\end{aligned} \tag{5.3-4}$$

where ΔL - total loss for firm Y due to relative poor quality, \$;

Q_{y1}, Q_{y2} - the demand for product Y in market equilibrium before and after a quality improvement in product X, respectively;

P_{y1}, P_{y2} - the price for product Y in market equilibrium before and after a quality improvement in product X, respectively;

C_1, C_2 - the cost for product Y before and after a quality improvement in product X, respectively.

The first part in the right hand side in equation (5.3-4) is the loss due to the uncovered cost (or cost saving), and the second and the third part are the losses due to the profit reduction in lower price and lower output.

As mentioned before, the above evaluation of consumer surplus is a static model only accounted for the effects of quality improvement and price change in quality activities. The production cost, taste of consumers, income, population and other factors are assumed constant. However, quality improvement is a continuous process involving time factors. A dynamic model should be employed to evaluate quality effect over a time period, and changes in other factors over time must be incorporated into the model. The information uncertainty definitely has a negative impact on consumer welfare not only because of larger consumer quality loss on undeterministic product quality selection under risk aversion, but also because of the price increase resulting from advertisement cost and other resource waste.

A change in any product quality does not merely have direct effects on the close substitutes, the complement products, and final (or intermediate) demand pattern. It may have significant second or third order effects on many other sectors in the economy. The quality effect can be fully understood in the context of operation of the whole economic system. A more powerful method, such as input/output model, could be used to evaluate the effect of quality improvement on the whole society.

Chapter 6 Product Quality Function and Quality Cost Function

6.1 Background in Quality Function Development

A firm produces and supplies commodities with varied quality levels to society. In quality economics the firm is regarded as a unit which decides what level of the commodity quality will be produced in order to cover costs and gain profits. Implementation of product quality plans and the operation of technical process are assumed at optimal conditions. Compared with the consumer behavior in previous chapters, the firm consumes inputs which possess the required quality levels (raw material, labor, capital, energy and other products) and produces commodities for consumers. Product quality function is objective and the quality cost function can be determined by quality specification.

In microeconomics theory the product function which states the quantity of output as a function of quantities of variable inputs has fully been developed in guiding firm production policies. Product quality function, however, has not been developed to respond to the phenomenon of increased competition in quality. However, some basic concepts of product quantity function can be directly applied to the development of product quality function.

A firm product quality function is defined as the relationship between the quality of inputs and the quality of output. Mathematically, it is expressed as follows

$$q = T(q)f(\, x_1, x_2, x_3, \ldots, x_i, \ldots, x_n \,) \qquad (6.1\text{-}1)$$

where q - quality of an output product or output attribute, quality unit;

x_i - quality of input factor i, quality unit;

T(q) - system function; it is related to the adopted production system and the manufacturing process.

Input factors in the above function include not only physical items, such as raw materials and intermediate products, but non-physical items, such as labor skill, environmental factors and process conditions. System function, T(q), is related to current technology utilization and prevailing manufacturing processes. The better the utilization of technology, the higher the value of T(q). Economic system design is employed to select an alternative from a number of T(q) values which correspond to a set of available technologies to meet economically the requirement of product quality development. At first one can determine T(q), that is what system should be employed; then one can choose a specific quality function from a number of available input factor functions, $f(x_1, x_2, x_3, ..., x_i, ..., x_n)$. Engineering quality design is aimed at enhancing technical efficiency to optimize the output quality response which may be an intermediate product quality, or final product quality, or an attribute of the final product resulting from reduction of environment noise and manufacturing process influences. The best utilization of any particular input quality combination is a technical problem, but the selection of the best input quality combinations for the particular quality output belongs to the realms of economic analysis. The output quality is related to input quality parameters, manufacturing process ability in the short term and technological innovation and progress in the long term. The output quality response contains two major characteristics: quality target value which is determined from a set of input quality combinations under the current process capability and environment disturbance, and quality deviation from the target value. Taguchi and Phadke [1984], Leon et al. [1987] and Phadke and Dehnad [1988] have developed and discussed a two step procedure for product quality optimization. The first step is to find values of control factors that minimize signal to noise ratio (s/n). It is followed by the second step which adjusts the mean on target. Mathematically, it is

$$\min \ \sigma^2(q)$$

$$\text{subject to} \ \ \mu(q) = \mu_0$$

Phadke [1989] gives the details for such quality optimization.

Assume that quality variation is normally distributed. An output quality and an input quality can be written as, respectively

$$q = q(\mu, \sigma^2) \qquad (6.1\text{-}2)$$

$$x_i = x_i(\mu_i, \sigma_i^2) \qquad (6.1\text{-}3)$$

where μ - the mean of quality distribution;

σ^2 - the variance of quality distribution.

Substitute equation (6.1-2) and equation (6.1-3) into equation (6.1-1) to obtain

$$q(\mu, \sigma^2) = T(q)f[\ x_1(\mu_1, \sigma_1^2), x_2(\mu_2, \sigma_2^2), \ldots, x_n(\mu_n, \sigma_n^2)] \qquad (6.1\text{-}4)$$

In traditional quality control, engineers and managers often concentrate their attention on controlling the deviations of manufacturing processes or promoting the quality of output response by introducing new technology and machinery, i.e. specifying higher T(q) function. These methods are usually costly. The selection of input factor quality level for output quality response -- that is the determination of optimal levels of various input factors -- is most effective in quality control which corresponds to the optimization of quality design and quality engineering.

In recent decades quality competition has required new developments in quality control and management methods. There have been a number of methods developed and employed in engineering product quality design in order to meet the challenge. This progress in quality activity provides a basis for the establishment of product quality function as well as quality economics. Taguchi's method is a good example of these quality improvement methods, which contains an understandable economic meaning. Taguchi methods consist of product quality design and manufacturing process design. Taguchi and Wu [1985] gave the following exposition of this modern approach.

The purposes of product quality design are to minimize the effect of noise sources on product quality performance, to minimize the quality variation from the target value, and to produce products that are insensitive to the component variation. Product quality design consists of three steps:

(1) System Design (or Primary Design)

(2) Parameter design (or Secondary Design)

(3) Allowance design (or Tertiary Design)

System design determines the specification of the system needed to meet the objectives. A new material, new operating method and availability of advanced technology are major factors to change or improve system properties significantly. New system design also includes the role of adding new attributes of performance with setting target values. Parameter design determines the optimum level of individual parameters of the system within a wide range of performance conditions at the lowest cost.

This is the core of product quality design. In spite of the details of engineering methods, the meaningful economic policy is the first to select the low priced and wide varying element (or material) which is resistant to the variation of performance condition from a substituted group, instead of using high priced ones. Allowance design must be performed after parameter design. Allowance design which determines the amount of variation allowed from the target value emphasizes those noise factors imparting a large effect on the quality response. This is a "cost-up" step at which cost should be considered for controlling noise in a narrower range. Elements with higher price and less variation are chosen instead of low priced elements for which noise factors have a large contribution to product quality performance.

Manufacturing process design implements the specification of product quality performance in production processes. It also consists of three steps:

(1) System design

(2) Parameter design

(3) Allowance design

The current manufacturing process may or may not be suitable for the above product quality specification. System design determines a manufacturing process system at current technology availability to produce the product designed in the R&D department. Parameter design is a cost effective step in manufacturing process design which determines the optimum conditions of each process, including materials and components. Thus, the influence of causes on the process capability can be reduced considerably. On the other hand, through parameter design, factors which do not affect quality substantially but make noticeable differences in cost for different levels can be found. In parameter design, the existing process is adjusted to its optimal operating condition and continuous innovations are made on the existing process in the short-term. Similar to allowance design in product quality design, allowance design in manufacturing process design is costly. It is used to determine to what extent a factor causes variation in quality or defective rate, and controls quality by narrowing those factor tolerances which have a large influence on quality variation. It can be used to open the tight tolerance for those factors which have little effect on quality but are costly to operate with narrow tolerances.

In this research, however, we are interested in the relationship between output quality and the input factor qualities in economic analysis. As mentioned in Chapter 3, consumers measure product quality with its performance under a wide range of operation conditions. A product that has been designed and built without consideration of noise disturbance is definitely inferior to a product that has undergone robust quality design under environment variation. By using the quality design method, the product quality function (6.1-4) can be simply expressed by (see Taguchi, 1984; Kackar, 1985)

$$q(\mu, \sigma^2, \zeta /\text{noise}) = T(q)f(Y_1, Y_2, Y_3, Y_4/\text{noise}) \qquad (6.1\text{-}5)$$

$$\partial q/\partial \mu_1 = T(q^0)f_1(\mu) \qquad \partial q/\partial \sigma_1{}^2 = T(q^0)f_1(\sigma^2)$$

$$\partial q/\partial \mu_2 = T(q^0)f_2(\mu) \qquad \partial q/\partial \sigma_2{}^2 = 0$$

$\partial q/\partial \mu_3 = 0$ $\qquad\qquad$ $\partial q/\partial \sigma_3{}^2 = T(q^0)f_3(\sigma^2)$

$\partial q/\partial \mu_4 = 0$ $\qquad\qquad$ $\partial q/\partial \sigma_4{}^2 = 0$

where μ - response quality mean;

σ^2 - response quality variance;

ζ - product performance measure, $\zeta = 10 \log(\mu/\sigma)^2$, called signal to noise ratio (s/n);

Y_1 - input factors influencing ζ only;

Y_2 - input factors influencing μ only;

Y_3 - input factors influencing σ^2 only;

Y_4 - input factors with no detectable influence;

$T(q^0)$ - constant system function.

The noise contains environmental conditions and uncontrollable product variations. Experimental design methods are used to perform quality design under noise disturbance. In the analysis of factors' effects on response quality, the first step identifies factors Y_1 which affect ζ significantly. The maximum performance measure ζ is usually selected from a set of values of ζ which are observed in settings factors Y_1. Taguchi [1985] proposed the optimization of "signal to noise" ratio in product quality design to distinguish input factors Y_1.

The second stage distinguishes factors Y_2 which affect μ only from a set of input factors. The levels of factors Y_2 are chosen in a way to force the response quality mean to equal the target value.

A third stage identifies factors Y_3 that influence σ^2 only. This is a costly procedure and should be carried out carefully.

Factors Y_4 have no detectable influence upon ζ, μ and σ^2. These factors are usually ignored in traditional quality activity but are meaningful in quality economics.

The effect of input factors on output response quality will be discussed in the product quality function, and the system function will be analyzed in quality cost function.

6.2 Basic Properties of Product Quality Function

6.2.1 Quality Mean Function

Product quality is usually characterized by the mean value and the variance. All quality is expressed in terms of one quantity unit.

Following the concepts developed in the context of product quantity function, we will define a set of concepts to describe product quality properties. The mean marginal quality (MMQ) is partially derivative with respect to the quality mean of factor i.

$$\begin{aligned} MMQ &= \partial q / \partial \mu_i \\ &= \partial f[\, x_1(\mu_1, \sigma_1^2), x_2(\mu_2, \sigma_2^2), \ldots, x_n(\mu_n, \sigma_n^2)] / \partial \mu_i \end{aligned} \qquad (6.2.1\text{-}1)$$

The effect of the mean value μ_i on the product quality target is defined as follows

$$q(\mu)|_{xj=0} = f[x_1^0(\mu_1), x_2^0(\mu_2), .., x_i(\mu_i), ..., x_n^0(\mu_n)]|_{xj=0}$$

$$j = 1, 2, \ldots n;\ i \neq j \qquad (6.2.1\text{-}2)$$

where x_j^0 - input factor j that remains constant.

The shape of the relationship between μ_i and μ is shown in Figure 6-1 when the mean values of other input factors remain constant. The quality range between a and b is limited by current available technology and manufacturing process. Changes in other input factor qualities may alter the relationship between μ_i and μ.

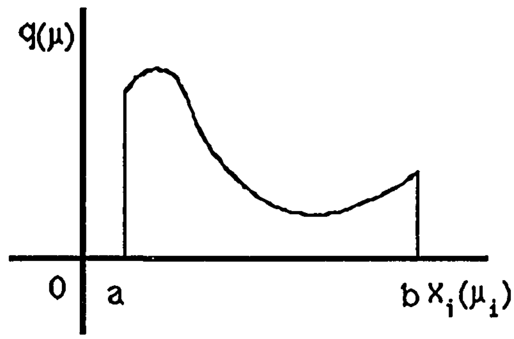

Figure 6-1 Relationship Between Output Quality and Input Quality

In practice, the mean quality curves and their corresponding mean marginal quality (MMQ) have various patterns, which are based on the assumption that the response quality mean is related to input factor i quality while the other factor qualities remain constant. The procedure for finding the optimal mean value for the output quality from a set of input factor quality means is deterministic; no stochastic process is involved. Therefore, it is usually possible to find a combination of specific input factor qualities to maximize the output product quality. It should be noted that the product quality mean function is constructed from left to right. All input factor quality means commonly determine an optimal output quality target, and their relations are fixed. Any increase in the output target must require another specific combination of the quality means of input factors. To optimize the response quality mean, the MMQ for input factor i should be

$$\partial q/\partial \mu_i = f_i = 0 \qquad (6.2.1\text{-}3)$$

where $f_i = \partial f[\, x_1(\mu_1, \sigma_1^2), x_2(\mu_2, \sigma_2^2), ..., x_n(\mu_n, \sigma_n^2)]/\partial \mu_i$.

If we are interested in the optimization of the response quality mean under current conditions of manufacturing process and prevailing technology, equation (6.2.1-3) can be rewritten as:

$$\partial q/\partial \mu_i \big|_{bi \le \mu i \le ai} = f_i \big|_{bi \le \mu i \le ai} = 0 \qquad (6.2.1\text{-}4)$$

where a_i, b_i - upper and lower bounds for the input factor i at current conditions, a_i and b_i vary in the long term. Some factors may satisfy the boundary conditions.

Therefore, we can find a combination of input quality levels under the current manufacturing system to optimize the output response quality target. Assume all input factors are independent. If the relationship between the quality response mean and the quality mean of input factor i is nonlinear, the higher order (more than power 2) can be ignored. That is

$$\begin{aligned} x_i(\mu) &= g(\mu_i) + k(\mu_i^2) + R \\ &= C_1\, \mu_i + C_2\, \mu_i^2 \qquad (6.2.1\text{-}5) \end{aligned}$$

where μ_i _ mean value of input factor i;

C_1, C_2 _ coefficients of first order and second order of μ_i, respectively;

R - higher orders of μ_i, power > 2.

The mean marginal quality, MMQ, for input factor i is

$$\partial q/\partial \mu_i = C_1 + 2C_2\,\mu_i = 0$$

$$\mu_i = - C_1/2C_2 \qquad (\ 6.2.1\text{-}6\ )$$

The linear relationship between input factor quality mean μ_i and the response quality mean μ is

$$q[x_i(\mu)] = C\,\mu_i$$

$$\partial q/\partial \mu_i = C \qquad (\ 6.2.1\text{-}7\ )$$

Based on the boundary conditions, the desired point for input factor i can be determined.

If C = 0, input factor quality mean μ_i does not have any effect (or little effect, C ≈ 0) on the response quality mean.

If C < 0, input factor quality mean μ_i has a negative effect on the response quality mean. The extreme point at the low side boundary corresponding to the optimum value of the response quality mean should be chosen.

If C > 0, input factor quality mean μ_i has a positive contribution to the response quality mean. The extreme point at the upper boundary should be selected to optimize the response quality mean.

If an interaction among factors occurs, it changes the optimum response quality value. The higher order of interaction may generally be ignored. The effect of interaction on the output quality target should be carefully examined and interpreted. If the reduction in the quality target value due to interaction is considerable, an alternative combination of input factors may be needed. The contribution of a single factor quality on the response quality is fixed without accounting for the effects of other input factors when no interaction is present. Otherwise, the levels of the interacting factors must also be taken into consideration.

If two input factors are independent but their qualities can be substituted, the priority order for individual factor quality depends on (1) which one is easier to control, and (2) their cost ratio. Suppose the voltage from a DC circuit is required to be 200 ± 2V. The component current I and resistance R are normally and independently distributed, such that

$$V = IR$$

$$\mu_V = \mu_I \mu_R \qquad \sigma^2_V = \mu^2_R \sigma^2_I + \mu^2_I \sigma^2_R \qquad (6.2.1\text{-}8)$$

where μ_V, μ_I and μ_R - the mean values for voltage, current and resistance, respectively;

σ^2_V, σ^2_I and σ^2_R - the variances for voltage, current and resistance, respectively.

There are many combinations of I and R to meet the requirement of voltage output. The selection criterion means that the combination possesses the lower cost and is more easily controlled. If the combination of I = (2 ± 0.6) and R = (100 ± 4) is easier to control than the combination of I = (1± 0.6) and R = (200 ± 4), the first alternative is accepted.

In general, equation (6.1.1-6) can be written in the following form

$$q(\mu, \sigma^2, \zeta /noise) = T(q)f[(Y_1, Y_2, Y_3, Y_4)/noise] \qquad (6.2.1\text{-}9)$$

$$f(Y_1/noise) = f_i[x_1(\mu_1, \sigma_1{}^2), x_2(\mu_2, \sigma_2{}^2), \dots x_p(\mu_p, \sigma_p{}^2)]$$

$$f(Y_2/noise) = f_j[x_{p+1}(\mu_{p+1}, \sigma_{p+1}{}^2), x_{p+2}(\mu_{p+2}, \sigma_{p+2}{}^2), \dots x_o(\mu_o, \sigma_o{}^2)]$$

$$f(Y_3/noise) = f_k[x_{o+1}(\mu_{o+1}, \sigma_{o+1}{}^2), x_{o+2}(\mu_{o+2}, \sigma_{o+2}{}^2), \dots x_n(\mu_n, \sigma_n{}^2)]$$

$$f(Y_4/noise) = f_l[x_{n+1}(\mu_{n+1}, \sigma_{n+1}{}^2), x_{n+2}(\mu_{n+2}, \sigma_{n+2}{}^2), \dots x_m(\mu_m, \sigma_m{}^2)]$$

$$i = 1, 2 \dots, p; \quad j = p+1, p+2, \dots, o; \quad k = o+1, o+2, \dots, n; \quad l = n+1, n+2, \dots, m$$

where $f_l[x_{n+1}(\mu_{n+1}, \sigma_{n+1}{}^2), x_{n+2}(\mu_{n+2}, \sigma_{n+2}{}^2), \dots x_m(\mu_m, \sigma_m{}^2)]$ is the part that contains all input factors whose changes in quality levels have little or no effect on the quality response. Although part Y_4 has no effect on engineering design at this stage, it is meaningful in selection from a group of substituted input factors in light of the cost criterion, and should not be dropped from the product quality function. Each factor in part 1 could be selected from the cheap priced group. While the part $f_j[x_{p+1}(\mu_{p+1}, \sigma_{p+1}{}^2), x_{p+2}(\mu_{p+2}, \sigma_{p+2}{}^2), \dots x_o(\mu_o, \sigma_o{}^2)]$ contains the factors whose changes in quality levels affect the quality response significantly, since it contributes nothing to the output quality variance low priced and low grade input factors can be chosen. The above two parts are cost free or cost effective. It needs both engineering and economic attention to determine input factor values in part $f_i[x_1(\mu_1, \sigma_1^2), x_2(\mu_2, \sigma_2^2), \ldots x_p(\mu_p, \sigma_p^2)]$. Hunter [1985] pointed out that even when the response quality function is known, the environment noise produces variability in the response and reduces the quality of product performance. Therefore, the input factor quality values are chosen so that the response quality variance is minimized under disturbance of the environment noise. Or, in order to meet consumer requirements, further reduction of quality variation is needed. A high grade with high priced input factor may be desired in this group. In all, the optimization of the output quality mean is cost-effective. From the product quality function, the input factor means are set in levels such that the output quality target value is uniquely determined. Any optimal combination of input factor means to increase the target value should be evaluated in economic analysis based on the firm's objectives.

6.2.2 Quality Variance Function

In the quality optimization procedure, we search the optimum response quality in conditions in which all input factors are set at their optimal operating quality levels. Nothing is said about the variation from the operating point. Once the response quality target value and all input factor operating levels are determined, we should work out how to specify the allowance derived from the quality deviation from the mean value in real operating environment conditions. Compared with the unique quality target value setting in the above section, the specific quality variance can be obtained by a number of combinations of input quality variances. These steps (mean value determination and the variance setting) are in order and cannot be inversed. We will prove that the quality variation can only be minimized after the determination of the output quality target value. Generally, when the system function is predetermined, in the short term the product quality function is

$$q = f(x_1, x_2, ..., x_n)$$

where q - the output quality;

x_i - the quality of input factor i.

Total differential for the above equation

$$dq = f_1 dx_1 + f_2 dx_2 + ..., + f_n dx_n + ..., + f_{ij} dx_i dx_j \ + R$$

where R are the higher order items and usually can be ignored. The above equation can also be written in another form

$$\Delta q = f_1 \Delta x_1 + f_2 \Delta x_2 + ..., + f_n \Delta x_n + ..., + f_{ij} \Delta x_i \Delta x_j$$

Quality variation is stochastic and may be assumed to be normally distributed. Assume that the value of f_i (MMQ) is constant in the small Δ, and all input factors are independent. For $\Delta q \sim \sigma^2(\mu \neq \mu_0)$ and $\Delta x_i \sim \sigma^2{}_i(\mu_i \neq \mu_{i0})$

$$V(\Delta q) = f^2{}_1 V(\Delta x_1) + f^2{}_2 V(\Delta x_2) + ..., + f^2{}_n V(\Delta x_n) + ..., + f^2{}_{ij} V(\Delta x_i \Delta x_j)$$

$$\sigma^2 = f^2{}_1 \sigma^2{}_1 + f^2{}_2 \sigma^2{}_2 + ..., + f^2{}_n \sigma^2{}_n + ..., + f^2{}_{ij}(\mu^2{}_j \sigma^2{}_i + \mu^2{}_i \sigma^2{}_j)$$

For $\Delta q \sim \sigma^2(\mu = \mu_0)$ and $\Delta x_i \sim \sigma^2{}_i(\mu_i = \mu_{i0})$

$$f^2{}_i(\mu_i = \mu_{i0}) \Rightarrow 0$$

$$f^2{}_i(\mu_i \neq \mu_{i0}) \geq f^2{}_i(\mu_i = \mu_{i0}),\ f^2{}_{ij}\mu^2{}_j(\mu_{j0} \neq \mu_{j0}) \geq f^2{}_{ij}\mu^2{}_j(\mu_{j0} = \mu_{j0})$$

$$\sigma^2(\mu \neq \mu_0) \ \geq \ \sigma^2(\mu = \mu_0) \qquad (6.2.2\text{-}1)$$

These factors, Y_2 and Y_4, which do not affect the response quality variation, are not involved in the allowance quality design procedures such that the quality variance function is written as

$$q(\sigma^2/\mu=\mu_0, \zeta, \text{noise}) = f[(Y_1, Y_3)/\mu=\mu_0, \zeta, \text{noise}] \qquad (6.2.2\text{-}2)$$

$$f(Y_1/\mu=\mu_0, \zeta, \text{noise}) = f_i[\, x_1(\sigma_1{}^2/\mu_1=\mu_{10}), x_2(\sigma_2{}^2/\mu_2=\mu_{20}), \ldots$$

$$x_p(\sigma_p{}^2/\mu_p=\mu_{p0}\,)]$$

$$f(Y_3/\mu=\mu_0, \zeta, \text{noise}) = f_k[\, x_{o+1}(\sigma_{o+1}{}^2), x_{o+2}(\sigma_{o+2}{}^2), \ldots\, x_n(\sigma_n{}^2)]$$

Any deviation from the quality mean values of input factors contributes positively to the response quality deviation from the response target value.

The average variance quality (AVQ) and the marginal variance quality (MVQ) for input factor i are defined as

$$AVQ = q(\sigma^2)/\sigma_i{}^2 \qquad (6.2.2\text{-}3)$$

$$MVQ = \partial q(\sigma^2)/\partial\sigma_i{}^2 \qquad (6.2.2\text{-}4)$$

$\partial\sigma^2/\partial\sigma_i{}^2$ designates how many unit changes in response quality variance due to one unit change in input factor i quality variance, when other factor quality variances remain constant. In other words, it is the weight for one unit change in input factor i quality variance. AVQ is the percentage of contribution for input factor i variance onto the response quality variance. There are many sets of solutions to determine the combination of input factor variances for a desired value of response quality variance. But the problem is to choose a solution in the scope of quality economics analysis. The output quality variance elasticity of σ_i^2, expressed by ω, is defined as the proportionate rate of change of output quality variance $q(\sigma^2)$ with respect to input factor variance σ_i^2:

$$\omega_i = f(\sigma^2_i)\Delta q(\sigma^2)/(q(\sigma^2)\Delta f(\sigma^2_i)) = f(\sigma^2_i)\partial q/q(\sigma^2)\partial f(\sigma^2_i)$$

$$= AVQ/MVQ \qquad (6.2.2\text{-}5)$$

$$\Delta q = \omega_i q \Delta f_i / f_i \qquad (6.2.2\text{-}6)$$

We will examine the relationships between the response quality variation and the input factor i quality variation, shown in Figure 6-2. In part a of Figure 6-2, the response quality variation is simply a linear function of an input factor quality variation. Any change along the location of input factor curve does not alter the output quality variance.

$$q(\sigma^2) = f(\sigma_i^2)$$

$$\sigma^2 = b\sigma_i^2$$

$$\partial\sigma^2/\partial\sigma_i^2 = b \qquad (6.2.2\text{-}7)$$

In part b of Figure 6-2, the response quality variation is a nonlinear function of input factor i quality variation. One can expect that the response quality variation will be minimized when the partial derivative is the minimum along the input factor curve (Taguchi [1985]).

$$q(\sigma^2) = f(\sigma_i^2)$$

$$\partial\sigma^2/\partial\sigma_i^2 = f_i(\sigma_i^2) \qquad (6.2.2\text{-}8)$$

In part c of Figure 6-2, two linear input factor quality variations lead to parallel contour curves for a set of response quality variation while two nonlinear factor quality variations lead to nonparallel contour curves, shown in part d, $\sigma_1^2 < \sigma_2^2 < \sigma_3^2$. This situation can be expanded to n input factor quality variations.

Define the rate of quality technical substitution, $RTS(q_{ij})$, for input factors i and j as:

$$RTS(q_{ij}) = -\,df(\sigma^2{}_i)/df(\sigma^2{}_j) \qquad (6.2.2\text{-}9)$$

In part c of Figure 6-2

$$q(\sigma^2) = f(\sigma_1^2) + f(\sigma_2^2)$$

$$\sigma^2 = b_1\sigma_1^2 + b_2\sigma_2^2$$

$$d\sigma^2 = b_1 d\sigma_1^2 + b_2 d\sigma_2^2$$

$$RTS(q_{12}) = b_2/b_1 \qquad (6.2.2\text{-}10)$$

In part d

$$q(\sigma^2) = f(\sigma_1^2, \sigma_2^2)$$

$$d\sigma^2 = f_1 d\sigma_1^2 + f_2 d\sigma_2^2 + f_{12} d\sigma_1^2 d\sigma_2^2$$

$$d\sigma^2 \approx f_1 d\sigma_1^2 + f_2 d\sigma_2^2$$

$$RTS(q_{12}) = -\,d\sigma_1^2/d\sigma_2^2 = f_2/f_1 \qquad (6.2.2\text{-}11)$$

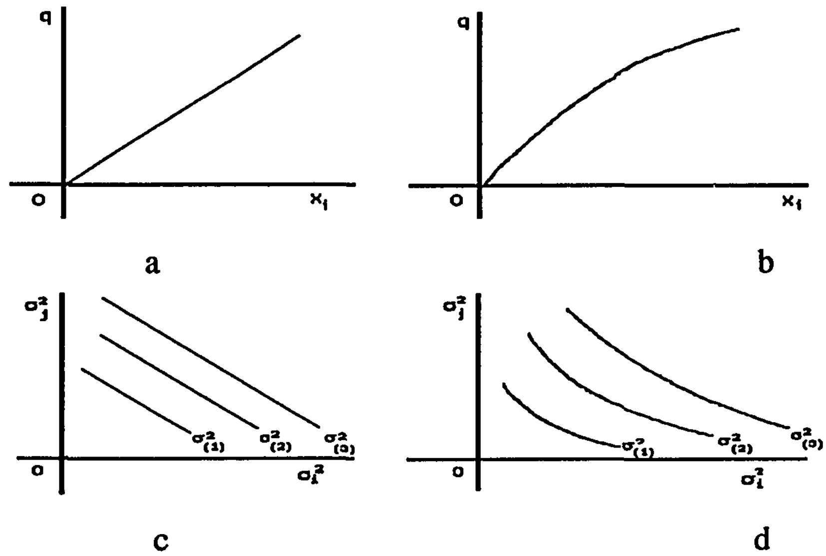

Figure 6-2 Relationship Between Input Factor Variance And Response Quality Variance

Generally, the response quality variance has or can be approximately transformed into an additive function form in which the response quality variance is simply a sum of individual input factor quality variances

$$\sigma^2 = f(\sigma_1{}^2) + f(\sigma_2{}^2) + ..., f(\sigma_i{}^2) + ..., + f(\sigma_n{}^2) \qquad (\ 6.2.2\text{-}12\ )$$

We classify all input factor variances into two groups, one of which contains the input factor variances that can be more efficiently and economically controlled. In the other group, input factor variances are costly or difficult to control.

$$q(\sigma^2/\mu{=}\mu_0,\ \zeta,\ \text{noise}) = g_1[\ x_1(\sigma_1{}^2),\ x_2(\sigma_2{}^2),\ ...\ x_i(\sigma_i{}^2)] +$$

$$g_2[\ x_{i+1}(\sigma_{i+1}{}^2),\ x_{i+2}(\sigma_{i+2}{}^2),\ ...\ x_n(\sigma_n{}^2)] \qquad (\ 6.2.2\text{-}13\ )$$

where g_1 - first group that can be economically controlled.

g_2 - second group that cannot be economically controlled.

There are several possible scenarios to achieve the predetermined response quality specification (variance): We may (1) reduce variances in group 1 or group 2, respectively, (2) reduce variances in group 1 and group 2 simultaneously, and (3) reduce variances in group 1 but increase variances in group 2 when the benefit from group 1 is more than the loss from the group 2. The economic criteria for the response quality variance specification will be discussed in combination with quality cost function in the next section.

The product quality variation usually is specified as a multiple of standard deviation, that is

$$\Delta = \pm 3\sigma$$

$$= \pm 3\{f(\sigma_1^2) + f(\sigma_2^2) + ..., f(\sigma_i^2) + ..., + f(\sigma_n^2)\}^{1/2} \qquad (6.2.2\text{-}14)$$

$$\Delta_i/\Delta_j = \sigma_i/\sigma_j \qquad (6.2.2\text{-}15)$$

where Δ - the product quality specification (or manufacturing specification);

Δ_i - the quality specification for attribute i.

The relationship between Δ and input factor quality variation is no longer linear and additive. One unit reduction in the specification Δ corresponds to more than one unit reduction in the quality variance σ^2. We can derive a number of the same (or similar) formula and conclusions for Δ as for quality variance σ^2.

Quality of production and quality control are implemented through top managers to workers in the production line. It is impossible to produce higher product quality if the human factors are not fully considered in the total quality management. Quality competition also involves management. An effective management approach should be developed in U.S. industry, though it is more difficult to establish than the methods of engineering quality optimization, to meet the increased quality challenge worldwide.

6.3 Quality Cost Function Features

6.3.1 Quality Cost Function in the Short Term

The short term in quality activity is defined as the period in which the quality system function is fixed. However, it is a relatively longer term with respect to other economic parameters, such as quantity output level. The reason is that once the quality level is set, other economic parameters in turn can be determined.

A controversy and an inconsistency in the relationship between quality and cost exist in both theoretical concepts and empirical evidence. There are many different approaches to quality cost modeling to show the role of cost-based quality control and management strategic decisions [Campanella and Corcoran, 1983; Gilmore, 1983; Gale and Brance, 1982; Phillips et al., 1983; Bader, 1983; Batson, 1988]. However, since quality costs are defined differently based on the assessment, background and understanding of quality control activities in different research and practical areas, a general quality cost function should be formed to verify the relationship between quality and cost. Quality cost functions are used to provide an evaluation basis for quality activity and give economic criteria for decision making, allocation of resources, investment, budgeting, etc.

In microeconomics, the term "cost function" is used to denote cost expressed as a function of output and input prices. The term "cost equation" is used to denote cost expressed in terms of input levels and input price. A set of theoretical concepts, such as marginal cost, have been fully developed. A firm's optimal behavior can be illustrated completely on the basis of quality cost function. However there is a need to set up quality cost function and quality cost equation to guide quality planning and quality activities. A number of research and empirical efforts have contributed to the quality cost of production conformance; few papers are related to the quality cost of quality target value settings.

There is a number of cost categories related to quality activities. They are usually classified into four major groups as follows:

(1) Research and Development Cost. Product quality design cost belongs to this group in which the engineering and laboratory resources as well as time taken are the major elements that contribute to quality improvement. The quality design makes the product performance robust to the environment condition variations and keeps the manufacturing and operating costs at a minimum.

(2) Manufacturing Cost. Manufacturing cost includes equipment, machinery, labor, energy, raw materials, reworking, scrapping, and inspection. Through parameter design and allowance design, the lower-grade materials and optimal manufacturing conditions are selected. Processes with less sensitivity to manufacturing disturbances considerably reduce the manufacturing cost expended on the control of operation conditions.

(3) Maintenance Cost. The worker-retraining cost, production

maintenance cost, and inventory of spare parts are in this group. The higher product quality, especially for higher reliability and lower deterioration, significantly reduces the large maintenance and inventory costs.

(4) Investment Cost and Cycle Time Cost. Continuous quality improvement needs capital investment for new technology, advanced process, and new materials. Higher quality provides a higher profit margin and a higher rate of return. Without capital investment, it is impossible to gain the advantage in future quality competition. However, the conventional quality cost classification does not involve the costs of design cycle and manufacturing process cycles. As Stalk Jr. [1989] points out, time is another strong strategic weapon to strengthen a firm's competitive advantage, like money, productivity and quality. Chandler [1989] notes that a company will lose 50% of the potential revenues in a new product if it enters a market nine to twelve months later than its competitors. In the next decade, competition will be focused on quality, price and time. To achieve the advantage of higher quality, low cost and a faster time cycle, a new quality engineering method should be combined with other quality approaches to promote a competitive edge in product quality.

Concurrency is an effective method for schedule compression which challenges the prevailing quality activities. Reduction of cycle time can not be achieved by prevailing quality approaches which run in series from quality development to final inspection. The role of quality assurance in concurrent engineering is to develop hard technical data, quality design information, and quality cost flow data on quality performance, manufacturing processes, controlling and test field in time to determine the corrective action taken. The quality activities are performed in parallel. For two products with comparable quality and price level, success depends upon which one is introduced into the market earlier. Another concept related to time advantage is "do it right the first time." The early stages and crucial processes could be given heavier weights to stimulate the redesign and the corrective action to be taken as soon as possible.

Lundvall and Juran [1974] proposed a traditional model of quality improvement to reveal the relationship between quality and per unit production cost in conforming product. A trade-off between increases in both quality and cost determines the optimal level q*. Beyond the optimal level q*, higher quality results in higher cost. Taguchi's methods in on-line and off-line quality control are designed to provide higher quality at lower prices through the reduced production cost approach. A quality loss function is added into the conventional cost function to achieve higher quality settings (Taguchi et al., 1989; Tang, 1989).

Obviously, these two models are inconsistent. Although Fine [1986] uses a quality-based learning model to explain why quality and low costs need not be inconsistent, the question "What is economic criteria in quality design?" still persists. From a quality economics argument, it is likely that in the short term, the relation between cost and quality at the early stages of product quality design and process design would be in a negative direction, and then, after the implementation of robust product quality design and quality learning, cost and quality would run positively for further quality modification. In the long term, quality cost is associated with the properties of product quality life cycle and the utilization of advanced technology and science.

The ambiguous relationship between quality and cost is more seriously distorted by conventional cost accounting system which uses direct labor hours as an allocation base. As Cooper and Kaplan [1988] point out, effective decisions about pricing, marketing product design, and product mix depend on the degree of accurate knowledge of cost information. They recommended an activity-based cost accounting system to guide corporate strategy for production of multiple products.

One unit of output and input is used to carry out measurement of quality cost. With respect to the output quality target design (see equation (6.2.1-9)), the output quality cost is

$$C(q/\mu)=C_1(Y_1/\mu_1, \sigma^2{}_1) + C_2(Y_2/\mu_2) + C_3(Y_3/ \sigma^2{}_3) + C_4(Y_4) \qquad (6.3.1\text{-}1)$$

$$C_4(Y_4) = \min[C(x_1), C(x_2), ..., C(x_n)]$$

$$C_3(Y_3/\mu_3) = 0$$

$$C_2(Y_2/\mu_2=\mu_{02}) = \min[C(x_1/\mu_1=\mu_{02}), C(x_2/\mu_2=\mu_{02}), ..., C(x_m/\mu_m=\mu_{02})]$$

where C(.) - cost for the quality mean value.

Input factors Y_4 have no detectable influence on the output quality. The input factor with the lowest cost (price) in group Y_4 is chosen, and its cost can be treated as fixed quality cost. Input factors Y_2 affect the output quality mean only; low priced but wide variation factors are selected after the quality design is finished. Group Y_3 has no contribution on the mean value cost function, but its cost will depends on the influence on the quality variance. Input factors Y_1's costs are balanced for both effects in quality mean and in quality variation. It usually requires high-grade materials at higher prices. We always can find a lowest cost alternative corresponding to the specific parameter quality design. For a set of product quality designs, $q_1(\mu_1) \geq q_2(\mu_2) \geq ... \geq q_i(\mu_i) \geq ... \geq q_n(\mu_n)$, it is rationally assumed that the quality mean cost is a nondecreasing function with respect to the change in product quality mean (target), $C_1(q_1/\mu_1) \geq C_2(q_2/\mu_2) \geq ... \geq C_i(q_i/\mu_i) \geq ... \geq C_n(q_n/\mu_n)$. The cost for output quality mean is

$$C_\mu = C_1 + C_2 + C_4 = C(\mu) + b \qquad (6.3.1\text{-}2)$$

$$\partial C_\mu/\partial\mu = C'(\mu) > 0$$

where C_μ - the cost for ouput quality mean value;

C_i - the cost for input quality factor i mean value;

$C(.)$ - the cost function for quality mean value.

Assume the cost without product quality optimization methods is $C_\mu*$, then

$$C_\mu* \geq C_\mu,\ \partial C_\mu*/\partial\mu \geq \partial C_\mu/\partial\mu,\ C*'(\mu) \geq C'(\mu) \qquad (6.3.1\text{-}3)$$

If we define $\mu*$ as the quality target value at current conditions, the difference, å , between $\mu*$ and the product quality mean value μ is

$$å = \mu* - \mu, \qquad -då = d\mu$$

$$dC_\mu/d\mu = dC_\mu/d(-å) = C'(å) > 0 \qquad (6.3.1\text{-}4)$$

The cost function for the quality target is a convex function of quality deviation.

The quality mean value design also provides a smaller quality variance than the quality variance without the optimal methods (see equation (6.2.2-1)). In the output quality variance modification, any further reduction in input factor variances is costly. From the experiments and empirical data, the relationship between input factor variance and cost can be expressed as a decreasing function in the quality variance domain.

$$\partial C_i(\sigma^2)/\partial \sigma^2{}_i = C_i' < 0 \quad \text{or} \quad \partial C_i(\sigma^2)/\partial(-\sigma^2{}_i) = -C_i' > 0 \qquad (6.3.1\text{-}5)$$

The cost of output quality variance is

$$C(\sigma^2) = C_1(\sigma^2{}_1) + C_2(\sigma^2{}_2) + \ldots + C_i(\sigma^2{}_i) + \ldots + C_n(\sigma^2{}_n) \qquad (6.3.1\text{-}6)$$

$$\partial C(\sigma^2)/\partial \sigma^2{}_i = C_i'(\sigma^2{}_i) < 0 \qquad (6.3.1\text{-}7)$$

$$\partial^2 C(\sigma^2)/\partial(\sigma^2{}_i)^2 = C_i''(\sigma^2{}_i) < 0 \qquad (6.3.1\text{-}8)$$

The variance cost function is (1) decreasing and (2) convex over the domain of quality variation. The costs for controllable and noncontrollable input factors in quality variance function have the following obvious patterns:

$$C_{g1}(\sigma^2) \le C_{g2}(\sigma^2), \quad |C'_{g1}| \le |C'_{g2}| \qquad (6.3.1\text{-}9)$$

where g_1 and g_2 are the controllable and the noncontrollable input factors, respectively.

We can adjust the input factor variances to minimize the total quality variance cost such that

$$\text{Min } C(\sigma^2)$$

$$\text{Subject to } \sigma^2 = \sigma^2_0$$

$$L = C(\sigma^2) + \lambda(\sigma^2 - \sigma^2_0)$$

Substitute equation (6.2.2-12) and equation (6.3-6) into the above equation

$$L = [C_1(\sigma^2_1) + C_2(\sigma^2_2) + \dots + C_n(\sigma^2_n)] + \lambda[f(\sigma^2_1) + f(\sigma^2_2) + \dots + f(\sigma^2_n)]$$

$$\partial L/\partial\sigma^2_i = C'_i + \lambda f_i = 0$$

$$\partial L/\partial\sigma^2_j = C'_j + \lambda f_j = 0$$

$$C'_i/C'_j = f_i/f_j = -\text{RTS}(q_{ij}) \qquad (6.3.1\text{-}10)$$

The absolute value of RTS(qij) for every pair of input factor variances i and j, holding the levels of output quality variance and all other input factor variances constant, must equal the ratio of their marginal costs. If the condition of equation (6.3.1-10) is not satisfied, one can adjust the input factor variances to achieve the minimum cost given output quality variance. We can use quality variation specification Δ to derive the same conclusion.

Considering the product quality function and cost equation, the cost is an implicit function of the quality level of output, input costs and the costs of fixed inputs.

$$C(\mu) = \Phi[q(\mu), \Sigma C(\mu_i)] + b \qquad (6.3.1\text{-}11)$$

$$C(\sigma^2) = \Phi[q(\sigma^2), \Sigma C(\sigma_i^2)] \qquad (6.3.1\text{-}11')$$

$$C(å) = \Phi[q(\mu, \sigma^2), \Sigma C(\mu_i, \sigma^2)] + b = \Phi[å, C(å_i)] + b \qquad (6.3.1\text{-}11'')$$

where $C(å)$ - the total quality cost function;

$\Phi[å, C(å_i)]$ - the quality cost function which is related to the input factor quality costs and output quality level;

b - the fixed cost.

Costs in the above equations only include the quality design cost and quality control cost. Lundvall and Juran [1974] presented two distinct classes of conformance-related costs. The concepts in this static economic model can be expanded to all the quality stages from quality design to service quality after product sale. The patterns of various costs are still valid in the total quality function. Quality of design and quality of conformance were regarded as two distinguished quality activities in the conventional quality approach. As proved in the previous section, the optimal quality design also increased the conformance quality and reduced its cost. The conformance quality cannot be separated from the quality design procedures, and its cost is regarded as a part of total quality cost.

There are three distinct classes of quality costs in light of their relationships with quality change in the conventional quality approach. They are quality operation cost, failure cost, and prevention and quality setting cost. Quality operation cost consists of time and resource costs in R and D, equipment maintenance cost, inventory cost, energy consumption cost, quality planning, data acquisition and reporting. Failure cost includes internal failure costs, such as scrap, rework, downtime and yield losses, and the external failure costs, such as warranty, penalty, service, cash return, rebate, recall, uncovered material and labor. Prevention and quality setting cost contains design cost, inspection cost, process control, high-grade materials, test cost and quality planning. The costs of the product parameter and allowance designs (see equations (6.3.1-11) and (6.3.1-11')) are classified into this cost class. The operation, failure, and prevention and quality setting costs are constant, strictly decreasing, and strictly increasing as the quality deviation decreases, shown in Figure 6-3. The total quality cost, therefore, is

$$C(å) = C_o(å) + C_f(å) + C_p(å) \qquad (6.3.1\text{-}12)$$

$$\partial C(å)/\partial(-å) = C'_f + C'_p$$

$$C'_o = 0,\ C'_f < 0,\ C'_p > 0$$

If $C'_f + C'_p = 0$, $\partial C(å)/\partial(-å) = 0$ (6.3.1-13)

Before the minimum cost point

$$C_{fl}(å) > C_{fh}(å) > 0 \qquad (6.3.1\text{-}14)$$

$$C'_{fl} < C'_{fh} < 0, C'' < 0 \qquad (6.3.1\text{-}15)$$

After the minimum cost point

$$C_{fh}(å) > C_{fl}(å) > 0 \qquad (6.3.1\text{-}16)$$

$$C'_{fh} > C'_{fl} > 0, C'' > 0 \qquad (6.3.1\text{-}17)$$

where C_o - operation cost, \$;

C_f - failure cost, \$;

C_p - prevention and quality setting cost, \$;

C_{fl} - cost function for lower product quality setting;

C_{fh} - cost function for higher product quality setting.

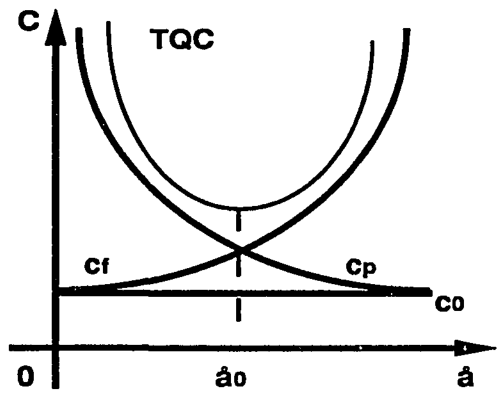

Figure 6-3 Three Types of Quality Costs and Quality Setting

When marginal costs for failure and prevention and quality setting are equal in terms of absolute value, the total quality cost achieves the lowest point. Although constant operation cost has no contribution on the determinant of quality setting, it does have a noticeable effect on the minimum total cost. This trade-off cost approach can be used to illustrate the firm's strategy that under the same quality setting, it minimizes the total cost in order to gain the potential quality cost rent. Any quality activity which reduces quality costs is adopted. For instance, the concurrent engineering method reduces operation cost and time cycle. It also helps product quality redesign through a quality leaning curve. These approaches, which change the shapes of failure cost and prevention cost, can have significant effects on the quality settings. However, traditional quality economics theory assumes that the quality cost function is convex, such that the higher quality products are more costly to be produced. The firm is not stimulated to provide consumers with anything other than the minimum quality product. This assumption ignores the optimal quality engineering approaches, the producer quality learning ability and the effective quality managerial methods which reduce quality cost and increase product quality simultaneously. The quality function with U shape is more rational to describe the quality activity theoretically and practically.

Taguchi [1984] proposed a producer's quality loss function which decreases with quality improvement. Taguchi et al. [1989] added the loss function into the total cost function to determine the quality specification. A noticeable piece of evidence for the loss function addition is that the producer would suffer a serious sell loss (market share reduction) due to consumer switching from inferior product quality to the competitor's product, though all products may be in the conforming limitation. This cost is omitted in conventional quality approaches. Through these methods, equation (6.3.1-12) becomes

$$C(å) = C_o(å) + C_f(å) + C_p(å) + C_L(å) \qquad (6.3.1\text{-}18)$$

$$\partial C(å)/\partial(-å) = C'_f + C'_p + C'_L, \quad C'_L < 0$$

To meet the cost minimization conditions

$$\partial C(å')/\partial(-å) = C'_f + C'_p = 0, \quad \partial C(å^*)/\partial(-å) = C'_f + C'_p + C'_L = 0$$

where $C_L(å)$ - the quality loss function in terms of cost.

Obviously, $q(å^*) > q(å')$. The quality setting is further promoted in the cost minimization approach when the quality loss function is accounted for in the total quality cost. In a competitive environment, both the minimum quality level and the optimal quality setting increase. The strategy for minimizing loss at the same manufacturing unit cost, or minimizing both quality loss and manufacturing cost, can gain both advantages in quality and cost.

As shown in Figure 6-4, in a competitive environment, TQC_1 reflects the total quality cost with an addition of quality loss function due to the consumer switch effect. The cost for the minimum quality standard is C_1 rather than C_0. Both the minimum quality standard and the optimal quality setting are demonstrated higher, i.e. $q(å_1) > q(å_0)$ and $q(å^*_1) > q(å^*_0)$. If no competition is considered, the total quality cost TQC_0 would be employed.

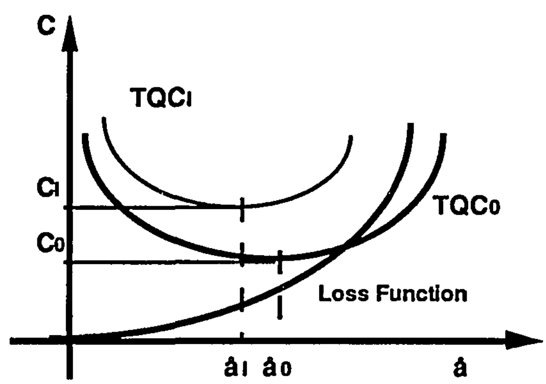

Figure 6-4 Effect of Quality Loss Function on Minimum Quality Standard

The above approaches do not consider the quality reaction and preference on the consumer side (demand side). Some costs, such as warranty, reworking cost, and advertising cost, are directly related to consumer behavior. Without accounting for consumer behavior under quality variation, the total cost is undetermined and, in turn, the quality settings are not fully optimized. The producer's quality loss function should be replaced by the consumer's quality loss function.

The quality cost function is expressed in terms of product unit, item, or attribute, whereas the quantity cost function is related to the output level. The total cost is the quality cost per unit multiplied by quantity output plus the fixed cost

$$C = C(å)Q + b \tag{6.3.1-19}$$

The product quality and quantity settings are processed successively. The firm, based on its long term goal, first sets the product quality level and then determines the output quantity. In quantity production the input factor costs (or price) are predetermined in the quality system evaluation and quality design. The input factors in quantity production, KLME (capital, labor, materials and energy), are also needed in the quality design and quality control. The operation cost, failure cost, and prevention and quality setting costs in quality cost function represent the unit cost for the quantity production. The quality loss function can be treated as either a pure concept of quality loss or the actual waste of KLME, depending on the firm's specific requirement. These quality costs also meet the assumptions in classical microeconomics. It is difficult and unnecessary to distinguish pure quality cost from quantity costs. The conventional cost equation can be rewritten as

$$C = r_1(å)x_1 + r_2(å)x_2 + \dots + r_n(å)x_n + b \tag{6.3.1-20}$$

where input factor cost (price), $r_1(å)$, is not constant, but related to the quality settings. The quantity of input factor x_i is determined by the producer's objective. Combined with the quantity and quality product functions and the path of production expansion, the total cost function is used to denote the output level and the quality cost per unit plus the fixed cost.

$$C = \Phi(å, Q) + b \qquad (6.3.1\text{-}21)$$

where C - total cost;

$\Phi(å, Q)$ - cost function; It is related to the costs of input factor qualities and quantities and the levels of output product quality and quantity.

After the product quality level is determined, the quantity cost function is

$$C = \Phi[Q/(å = å_0)] + b \qquad (6.3.1\text{-}22)$$

If the quality cost per unit is constant (or say, product quality has been determined), the total cost function becomes the quantity cost function, which possesses the general properties in classical microeconomics. The combination of quantity economics (current microeconomics) with development of quality economics will provide a richly interesting research area.

In the following, we will describe the firm's activity of quality improvement in perfectly competitive and monopolistic markets respectively. To maximize profit, for instance, all agents are price takers in perfect competition. Assume that the firm's attitude toward quality variation is neutral and all products are conforming. The firm attempts to maximize its profit π

$$\pi = PQ - \Phi(å, Q) - b \qquad (6.3.1\text{-}23)$$

$$d\pi/d(-\mathring{a}) = \Phi'_{\mathring{a}}(\mathring{a}, Q) = 0 \qquad \Phi'_{\mathring{a}}(\mathring{a}, Q) = 0 \qquad (6.3.1\text{-}24)$$

$$d\pi/dQ = P - \Phi'_{Q}[Q/(\mathring{a} = \mathring{a}_0)] = 0 \qquad P = \Phi'_{Q}[Q/(\mathring{a} = \mathring{a}_0)] \qquad (6.3.1\text{-}25)$$

In classical microeconomics, equation (6.3.1-24) in profit maximization approach can be interpreted as: (1) There is no quality variation for all products, $\mathring{a}=0$. This makes sense in the theoretical concepts. (2) If a homogeneous product is not assumed, the quality is set in the MQS (minimum quality standard), which has the lowest quality cost, or set in the boundary within the quality domain. It seems more rational because quality variation exists. From equation (6.3.1-25), production output is at the point where marginal cost must be equal to the price. The output level is related to the quality cost and varies with a change in quality costs. If heterogeneous consumer behaviors toward product quality result in market segmentation, the price is determined by the product quality and the market mechanism. The producer may set its product quality at the level which corresponds to the largest gap (profit per unit) between the price in the market segment and the firm's quality cost.

The producer's demand curve for its output is the horizontal price line itself if the market is perfectly (or near perfectly) competitive. The producer believes that he can produce as many products as he can and sell them at the prevailing price without influence from other firms' decision making. To cope with the violation of homogeneous product quality, it is rational to assume that the consumer demand curve is the function of quality and price. When the price is constant, the higher the quality, the larger the quantity demanded at the market equilibrium. Quality and quantity have the following relationship:

$$Q = f(p, å), \text{ p is constant}$$

$$dQ/d(-å) = f'(å) > 0, \quad f''(å) \leq 0 \qquad (6.3.1\text{-}26)$$

where Q - demand function.

If all firms in the market are identical, the industry total supply is the sum of an individual firm's production with the same level of quality. The supply equals the demand at the prevailing price.

$$S(Q) = \Sigma Q_i(å_i{=}å_0),\ S(p) - D(p) = 0 \qquad (6.3.1\text{-}27)$$

where S(Q) - product supply function.

However, producers in the industry actually have different manufacturing processes, execute different quality planning and managerial methods, and use diversified resources. Thus, firms provide different product qualities to consumers. Assume that each firm sets its product quality at the point with minimum quality cost. Clearly, the product supply with diversified qualities exceeds the demand at the prevailing price in the assumption of homogeneous consumers.

$$S(Q) = \Sigma Q_i(å_i \leq å_0),\ S(p) - D(p) > 0 \qquad (6.3.1\text{-}28)$$

If the information about each firm's quality is completely available, the effect of consumer selection priority occurs, which means that consumers first select the best quality product, and then the next best quality product until the amount of demand is satisfied. The market mechanism will force the firms with lower quality and higher cost out of business. For the profit maximization approach, if the quality cost takes the form of equation (6.3.1-23), then

$$\pi = PQ - \Phi(å, Q) - b$$

$$d\pi/d(-å) = pdQ/d(-å) - \Phi'_{å}(å, Q) = 0$$

The term pdQ/d(-å) is the revenue resulting from higher quality. The producer providing the higher quality product does not suffer a quality loss at this moment. If other firms make significant quality improvement, a potential quality loss may arise. To avoid quality loss, the producers promote the product quality setting by adding a quality loss function into the total quality cost function. If the loss function is structured such that

$$pdQ/d(-å) = - L'(å)$$

$$dQ/d(-å) > 0, \quad - L'(å) > 0$$

$$L'(å) < 0$$

the profit maximization approach is

$$\text{Max } \pi = PQ - \Phi(å, Q) - b$$

$$d\pi/d(-å) = pdQ/d(-å) - \Phi'_{å}(å, Q) = 0$$

$$L' + C'_p + C'_f = 0$$

$$d\pi/dQ = P - \Phi'_Q[Q/(å = å_L)] = 0 \qquad P = \Phi'_Q[Q/(å = å_L)]$$

$$å_L > å_0$$

where $å_L$, $å_0$ - quality settings with and without the addition of the loss function.

This approach with the quality loss function does not violate the classical profit maximization conditions under a perfect competitive market. The argument is that the producer's quality loss function can truly reflect the loss due to the consumers' reaction to product quality variation if the perfect quality information is available.

Three conclusions may be obtained from the above interpretation: (1) Any quality activity which has the potential ability to reduce cost is profitable and adoptable. (2) Profit is unchanged when a quality activity promotes the product quality, but maintains the lowest cost constant. (3) A quality activity results in profit reduction if the cost increases with quality improvement. This is why in quality control and quality projects, cost minimization is the most popular approach employed by a firm to evaluate the activities. Only those entrepreneurs who have the largest quality rent (the lowest quality cost, or higher quality at the same quality cost) can survive in the quality competition. The reason is that once the quality settings are over, the firms compete with each other with lower price and high-cost service in the short term. Conclusions (2) and (3) are not fully consistent with the analysis in previous chapters in which consumers usually give more weight for product quality and prefer higher quality products. Consumers' reactions and behaviors toward quality are not accounted for. Thus, competition is a necessary condition, but not sufficient to fuel continuous quality improvement if the rational firm maximizes its profit.

In the monopolistic market, the monopolist's total revenue and total cost can be expressed as functions of quality unit cost and output level.

$$R = R(\mathring{a}, Q, p) \qquad C = C(\mathring{a}, Q) + b \qquad (6.3.1\text{-}29)$$

In this situation, an aggregate of the demand curves of individual consumers is the firm's demand curve. Consumer preference and utility for product quality and quantity are reflected in the individual demand curves. The price can be represented as a function of product quantity and quality

$$P = f(å, Q) \qquad (6.3.1\text{-}30)$$

$$R = QP$$

$$dR/d(-å) = pdQ/d(-å) + Qdp/d(-å) \qquad (6.3.1\text{-}31)$$

From equations (3.3.2-6) and (6.3.1-26)

$$dQ/d(-å) > 0, \; dp/d(-å) > 0 \qquad (6.3.1\text{-}32)$$

The better the product quality settings, the higher the revenue. The quality cost function takes the form of equation (6.3.1-12), instead of equation (6.3.1-14), because the consumer switches to a noncompetitor's product and no sale losses occur due to quality problems in the monopolistic situation.

$$dC/d(-å) = C'_f + C'_p$$

If $å = å_0$, such that

$$dC/d[-å/(å=å_0)] = C'_f + C'_p = 0 \qquad (6.3.1\text{-}33)$$

then $$dC/d[-å/(å>å_0)] = C'_f + C'_p < 0 \qquad (6.3.1\text{-}34)$$

$$dC/d[-å/(å<å_0)] = C'_f + C'_p > 0 \qquad (6.3.1\text{-}35)$$

For the monopolist's profit maximization

$$\pi = R(å, Q) - C(å, Q) - b$$

$$d\pi/d(-å) = R'_å(å, Q) - C'_å(å, Q) = 0, \; R'_å(å, Q) = C'_å(å, Q) \qquad (6.3.1\text{-}36)$$

$$d\pi/dQ = R'_Q(Q/å<å_0) - C'_Q(Q/å<å_0) = 0$$

$$R'_Q(Q/å<å_0) = C'_Q(Q/å<å_0) \qquad (6.3.1\text{-}37)$$

Since $R'_{å}(å, Q) > 0$ (see equation (6.3.1-32)), $C'_{å}(å, Q)$ must be greater than 0. Only equation (6.3.1-35) meets the condition for profit maximization. The optimal quality setting is not at a point with the lowest quality cost, but on the higher quality side. The monopolist believes that the higher quality setting puts his product in a position to gain a higher profit with consideration of demand curve patterns (in other words, consumer behavior). Therefore, consideration of the consumer quality behavior can make the product quality setting higher.

One should note that equations (6.3.1-36) and (6.3.1-37) in some cases may not be simultaneously employed to determine the product quality, quantity and price in a firm's profit maximization. Since these three variables are not totally independent of each other, the quality, quantity and price may not have a unique solution set by solving these two equations. This is called undeterministic rule. In two-stage model, one can pre-fix one variable and then find a solution set for the other two variables to meet the objective predetermined. In a perfect competition market, the firm is price taker (even though he can choose one price, but cannot determine the price) and he chooses quality and quantity to maximize its profit. In a monopolistic market, the firm uses either equation (6.3.1-36) to determine quality and price with fixed quantity (or constant quantity cost), or equation (6.3.1-37) to obtain the optimal price setting and quantity output to maximize its profit after the desired quality setting is done.

In the perfect competition environment

$$\pi = pQ - C(å, Q) - b$$

$$d\pi/d(-å) = pdQ/då - C'_{å}(å, Q) = 0, \quad pdQ/då = C'_{å}(å, Q)$$

$$d\pi/dQ = p - C'_{Q}(Q, å) = 0, \ p = C'_{Q}(Q, å)$$

$$dQ/då = C'_{å}(Q, å)/C'_{Q}(Q, å)$$

In the monopoly situation

$$\pi = pQ - C(å, Q) - b$$

$$d\pi/d(-å) = pdQ/då + QdP/då - C'_{å}(å, Q) = 0$$

$$pdQ/då + QdP/då = C'_{å}(å, Q)$$

$$d\pi/dQ = p + QdP/dQ - C'_{Q}(Q, å) = 0$$

$$p + QdP/dQ = C'_{Q}(Q, å)$$

$$dQ/då = C'_{å}(Q, å)/C'_{Q}(Q, å)$$

The same conclusions, the ratio of the marginal quality cost and the marginal quantity cost equals the quantity/quality marginal rate, are obtained from these two market mechanism. The firm can use this ratio to determine which strategy, quality improvement or quantity output, is more profitable.

The following relationships are used to demonstrate the effect of the firm's quality strategy on its profit and consumers' utility. For substitution between quality and price only

$$dQ/dp = q_p \qquad (6.3.1\text{-}38)$$

$$dp/d(-å) = MRS_{åp} \qquad (6.3.1\text{-}39)$$

$$dQ/d(-å) = q_{å} \qquad (6.3.1\text{-}40)$$

$$q_{å} = q_p \, MRS_{åp}$$

For linearity

$$dp/d(-å) = (1-\eta)/\eta$$

$$dQ/d(-å) = q_p(1-\eta)/\eta \qquad (6.3.1\text{-}41)$$

For average

$$E(dp/d(-å)) = \Gamma(1 - 1/\beta)/\delta - 1$$

$$E(dQ/d(-å)) = q_p[\Gamma(1 - 1/\beta)/\delta - 1] \qquad (6.3.1\text{-}42)$$

For elasticity

$$PdQ/(Qdp) = \varepsilon_{qp},\ å dp/(pd(-å)) = \varepsilon_{på},\ å dQ/(Qd(-å)) = \varepsilon_{qå}$$

$$\varepsilon_{qå} = \varepsilon_{qp}\,\varepsilon_{på} \qquad (6.3.1\text{-}43)$$

where q_p, $MRS_{åp}$ and $q_å$ are the demand quantity/price marginal rate, the marginal rate of substitution between price and quality, and the demand quantity/quality marginal rate, respectively. $\varepsilon_{qå}$, ε_{qp} and $\varepsilon_{på}$ are the elasticities for quantity/price, price/quality and quantity/quality on the demand side, respectively.

Similarly, if demand quantity and quality are substitutable for each other, such as food, meat and drugs, they can find the elasticity for demand price/quality. The relationships among these three decision variables are very important for being considered in the economic model, or some confusing conclusions would be derived. For example, Gal-Or [1983] studied the quality and quantity competition in an oligopoly market and demonstrated that the average quality would decrease and the average aggregate output would increase on the base on entry impact. However, the Gal-Or model would break down if the relationship between quality and quantity is accounted for in a realistic model.

The firm, monopolist, will determine how to maximize its profit by increasing the product price or expanding the output derived from the quality improvement. If MRS for price/quality is high and quality cost is low, the firm will adopt the strategy to increase the price with predetermined output level. Traditionally, the firm determines the product price and output level with a fixed quality level. Consumers are indifferent or worse off with the former strategy, if all utility surplus from quality promotion are transferred into the producer surplus through the high price. Consumers are definitely better off if the quality increase is totally transformed into quantity increase at the constant price. Both the consumer and the producer may share the benefit from quality improvement, if the price determined by the traditional profit maximization methods is located between these two prices in the above two approaches.

The above analysis about product quality settings is made in profit maximization under two extreme market conditions. Assume other comparative conditions are the same for these two market situations. The quality setting at higher or lower levels depends on the measurement of quality loss function in the competitive market, whereas it depends on the observation of consumer behavior in the monopolist's market which situation can set higher product quality is undetermined by the use of conventional approaches in a profit maximization. Since the product price is much lower in a perfect competition market, consumers are better off with the activity of quality improvement. This analysis suggests that: (1) Competition and consideration of consumer quality behavior are two sufficient conditions to have the optimal product quality settings. (2) The conventional economic approaches are not enough for the task of evaluating and determining quality planning and quality activities in quality competition. (3) A new quality approach should be developed to describe the producer's behavior under quality risk and market uncertainty. This approach is expected to be more realistic with a better illustration of the optimal quality setting.

6.3.2 Quality Cost Function in the Long Term

In the short term, quality projects, such as process innovation, learning procedures influenced by ongoing redesigns or manufacturing improvement, training production workers, tooling simplification, automation, and design to product life cycle reduce the quality cost and improve quality. In the long-term, system design, adoption of new materials, and employment of advanced technology are the critical factors to push quality improvement forward. The establishment of quality cost function in the long-term is a fully dynamic process.

The current manufacturing system may not be suitable to produce the product that satisfies satisfy both the consumer's requirement and the producer's long-term objectives. If advanced technology is available, producers will quickly adopt it to build new systems and processes such that the total quality cost (operation, prevention, quality design, failure and quality loss) can be significantly cut. Producers can achieve the role of quality leadership for that product through utilization of new science and technology. However, adoption of new systems and processes does not mean the process is automatically set in the optimal conditions to produce a higher quality product. Innovation for current processes may achieve the same product quality as the adoption of new processes, but at a much cheaper cost. It reflects the different attitudes for U.S. and Japanese manufacturing industries toward quality improvement. U.S.

manufacturers prefer to adopt new process system to improve product quality, while the Japanese achieve the quality goal through efficient management approaches and optimal quality design methods. A great potential capability of technology and processes is wasted in the U.S. manufacturing industry. If the U.S. top managers in industry learn and establish a set of better quality management procedures and develop more powerful methods for product quality design supported by advanced technology and science, the products made in the U.S. can catch up and exceed Japanese product quality. Figure 6-6 shows the effect of the system function (technology and process) in the short-term and in the long-term on quality improvement and cost.

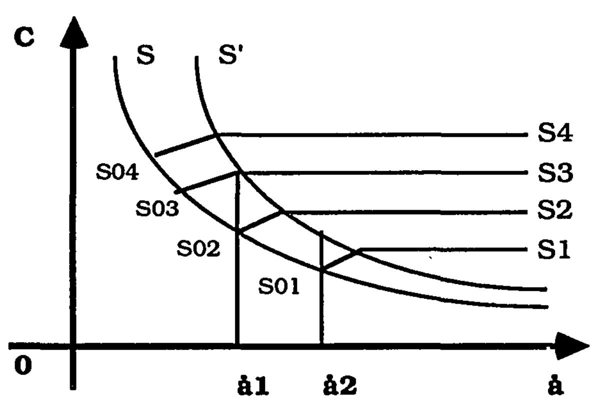

Figure 6-5 Quality System Cost in Long-Term

From the technological point of view, $S_4 > S_3 > S_2 > S_1$. S_{01}, S_{02}, S_{03} and S_{04} are the best utilization for this technology and the processes in the short term through innovation and optimal condition settings. S is the system function in the long-term, which is the focus of the optimal setting points in a number of short term settings. S' is the long-term quality cost function without implementation of the innovation and the optimal quality settings. For a given product quality setting $å_2$, S_2, S_3 and S_4 have enough capability to meet the design requirements. But the innovation and optimal condition setting make the current process S_1 able to produce the product quality needed. For the product quality setting $å_1$, S_2 is the better alternative to implement the producer's quality goal at an acceptable cost. Advanced technology does not mean it can produce higher product quality than disadvantaged technology unless it is set in the optimal conditions. We assume the quality system cost is a function of technological level

$$C_s = C_s(T(q)) \tag{6.3.2-1}$$

$$\partial C_s/\partial q = \partial C_s(T(q))/\partial q = \partial C_s(T(å))/\partial(-å) > 0 \tag{6.3.2-2}$$

$$C'_s \leq C'_{s'} \tag{6.3.2-3}$$

where C_s - system cost function;

$T(q)$ - level of technology system;

$C_{s'}$ - system cost function for an advanced technology system.

On the other hand, the utilization of new technology and advanced processes reduces the manufacturing and design cycle time which is one of the most effective improvement levers to reduce investment, resources, control cost, inventory and manufacturing unit cost (Schein and Berman, 1988), as shown in Figure 6-6. The cycle time cost is a function of technological level, such that

$$C_t = C_t(T(q)) \tag{6.3.2-4}$$

$$\partial C_t/\partial q = \partial C_t(T(q))/\partial q = \partial C_t(T(å))/\partial(-å) < 0 \tag{6.3.2-5}$$

$$C'_t \geq C'_{t'} \tag{6.3.2-6}$$

where C_t - cycle time cost function;

$C_{t'}$ - cycle time cost function for an advanced technology system.

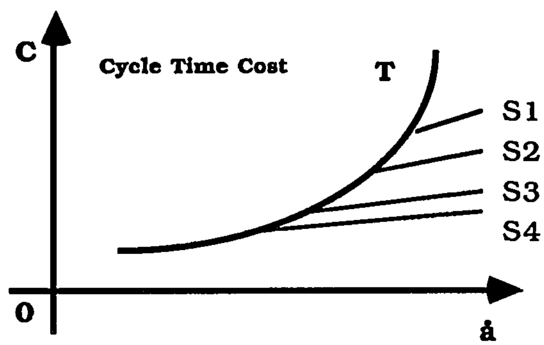

Figure 6-6 System Function and Cycle Time in Long Term

The operation quality cost is variable in the long term and is a trade off between system cost and time cycle.

$$C_o = C_s + C_t \tag{6.3.2-7}$$

$$\partial C_0/\partial(-å) = \partial C_s/\partial(-å) + \partial C_t/\partial(-å) \quad C'_o = C'_s + C'_t \tag{6.3.2-8}$$

$$C'_s + C'_t < 0, \quad C'_o < 0, \quad C'_0 \geq C'_{0'} \tag{6.3.2-9}$$

$$C'_s + C'_t = 0, \quad C'_o = 0, \tag{6.3.2-10}$$

$$C'_s + C'_t > 0, \quad C'_o > 0, \quad C'_0 \leq C'_{0'} \tag{6.3.2-11}$$

where C_0 - operation cost in the long term.

The operation quality cost is strictly convex within the quality domain in the long term.

The long term total cost equation is

$$C(å) = C_0(å) + C_f(å) + C_p(å) + C_L(å) \qquad (6.3.2\text{-}12)$$

$$\partial C(å)/\partial(-å) = C'_o + C'_f + C'_p + C'_L \qquad (6.3.2\text{-}13)$$

Assume the shapes of $C_f(å)$, $C_p(å)$ and $C_L(å)$ are affected by the technology, learning procedures and the progress in quality management, such that

If	$C'_o + C'_f + C'_p + C'_L < 0$	$\partial C(å)/\partial(-å) < 0$
If	$C'_o + C'_f + C'_p + C'_L = 0$	$\partial C(å)/\partial(-å) = 0$
If	$C'_o + C'_f + C'_p + C'_L > 0$	$\partial C(å)/\partial(-å) > 0$

when $\partial C(å)/\partial(-å) = 0$, it is called the long term saturation of the minimum quality standard. The long-term quality cost function is a function of output quality level in the conditions in which product quality is produced in the optimal condition settings. The long term cost function is a strictly convex curve. With adoption and utilization of new technology and process, the total quality cost would decrease until saturation is achieved. After that point, the total quality cost increases. Quality improvement is more and more difficult and costly after some periods. The specific relationship between quality and cost also depends on the market structure and the pattern of product life cycle. Costs are negatively, constant, and positively associated with quality improvement when the product is in the stage of introduction into the market, saturation or deterioration. Based on this approach, products for which the pattern of cycle time cost is not significantly affected by the employment of new science and technology would show a positive relationship between cost and quality improvement in the product life cycle.

Chapter 7 Producer Behavior Under Quality Risk and Consumer-Based Quality Setting Approach

7.1 Conventional Producer Behavior under Quality Risk

Like consumer behavior under quality discrimination, producer decision behavior toward quality risk can be described by decision theory. The term "producer" as used here may have double features. He is a pure producer if he produces the inputs himself that are demanded in production and sells the final products to consumers; while he is a consumer/producer if he purchases inputs from other producers and sells the output to the consumers. In the latter, a firm possesses consumer behavior described in Chapters 3 and 4, and the producer behavior which will be described below.

Producer quality production function is objective, and in most cases product quality can be measured in scientific and technical terms. A single firm may produce more than one product, and its decision behavior under uncertainty can be expressed by the firm's utility function. A firm's action on quality improvement will increase the profit and gain the advantage of competition through strategies such as cost minimization under product quality variation.

The expected utility analysis developed in Chapter 3 concerning consumer behavior and decision making under quality variation can be applied here when the firm is subject to uncertainty. Assumptions made in Chapter 3 are still valid, and the firm should obey the Von Neumann-Morgensterm axioms. Furthermore, it is assumed that the producer's utility function is based on the argument of profit gained from product quality activities and that the product quality has been set by using the optimal quality design methods.

It is impossible for a firm to provide consumers with all products in homogeneous quality. In conventional quality activities, a firm classifies all its products as either "conforming" or "nonconforming." Any product with a characteristic (or attribute) outside the manufacturing specification is regarded as "nonconforming" and results in the firm's profit loss which includes scrapping cost, reworking cost, and waste in material, labor and other resources. This conventional concept, called the on-line approach in quality control, is widely used to express a producer's expected quality value in terms of money unit. That is

$$E(V) = \rho v_1 + (1 - \rho)v_2 \qquad (7.1\text{-}1)$$

where ρ - probability of an item being conforming;

$1-\rho$ - probability of an item being nonconforming;

v_1 - quality value from a conforming item, \$;

v_2 - quality value from a nonconforming item, \$;

$E(V)$ - firm's expected quality value for a product, \$.

The values of v_1 and v_2 are predetermined by the firm's objective goal. For instance, if the firm minimizes the total cost in its product quality activities, then

$$v_1 = C_1 \quad v_2 = C_1 + C_2$$

$$E(V) = \rho C_1 + (1 - \rho)(C_1 + C_2) = C_1 + (1 - \rho)C_2 \qquad (7.1\text{-}2)$$

where C_1 - cost for a conforming items, \$;

C_2 - added cost for a nonconforming item, \$.

When the nonconforming item is reworkable, C_2 contains reworking cost, inspection cost and other costs contributed to the rework process. When the nonconforming item is not reworkable, C_2 represents the cost due to uncovered capital, labor, materials and energy (KLME) resource wastes for the item. It also includes the penalty cost and the consumer credit loss. It is straightforward to minimize the total cost by increasing the conforming rate.

If a firm is quality neutral, then

$$U[\, E(V) \,] = E\{\, \rho U(v_1) + (1-\rho)U(v_2) \,\} \tag{7.1-3}$$

If a firm is quality averse, then

$$U[\, E(V) \,] > E\,\{\, \rho U(v_1) + (1-\rho)U(v_2) \,\} \tag{7.1-4}$$

In the perfect competitive conditions with quality variation, the firm's profit in the short term is computed on the assumption that price is certain and all products are conforming, but output is the decision variable to maximize firm's profit (see Chapter 5.3.1).

$$\pi = PQ - C_1(Q)$$

$$d\pi/dQ = P - C'_1(Q^*) = 0$$

$$C'_1(Q^*) = P \tag{7.1-5}$$

where Q^* - output level for the conforming products.

However, when not all products are conforming, the firm's expected utility of profit under risk neutral is

$$E[U(\pi)] = E(\pi) = \rho[PQ - C_1(Q)] + (1-\rho)[PQ-C_1(Q) -C_2(Q)] \quad (7.1\text{-}6)$$

$$dE(\pi)/dQ = P - C'_1(Q) - (1-\rho)C'_2(Q) = 0$$

$$C'_1(Q') + (1-\rho)C'_2(Q') = P \quad (7.1\text{-}6')$$

where Q' - output level when the conforming rate is less than 1.

Clearly, $Q' < Q^*$ and $\pi' < \pi^*$ when $\rho \neq 1$.

The firm's expected utility of profit under quality aversion is

$$E[U(\pi)] = \rho U_1(\pi) + (1 - \rho)U_2(\pi) \quad (7.1\text{-}7)$$

$$dE[U(\pi)]/dQ = \rho U'_1(\pi)[P - C'_1(Q)] + (1-\rho)U'_2(\pi)[P- C'_1(Q) - C'_2(Q)]$$

$$U'_2(\pi) > U'_1(\pi)\ ,\ U'_2(\pi)/[U'_2(\pi) + \rho(U'_1(\pi) - U'_2(\pi))] \geq 1$$

$$C'_1(Q'') + (1 - \rho)C'_2(Q'')U'_2(\pi)/[U'_2(\pi) + \rho(U'_1(\pi) - U'_2(\pi))] = P \quad (7.1\text{-}7')$$

where Q" - output level under quality risk aversion.

$U'(\pi)$ is a function of price and output. Since marginal quality cost increases with reduction in quality deviation from the minimum cost point, Q" must be lower than Q'. Thus, the firm's profit is also reduced. Insofar, nothing is said about the price at equilibrium in the above cases. If other conditions are constant, the output levels and the profits under conventional quality approaches with the price certain are

$$Q'' < Q' < Q^*, \quad \pi'' < \pi' < \pi^* \tag{7.1-8}$$

Only and only if the rate of conforming, ρ, equals 1, the outputs and the utility of profit would be the same in formula (7.1-8). The lower quality conforming rate and the producer behavior under quality risk aversion lead to lower production output than that which would be produced in the perfect quality conditions. In contrast, the production output increases with improvement in the conforming rate. A relationship between the quantity and quality for a certain product exists and is used to guide the quality control activity in practice.

This is producer-based behavior for the quality control approach. Producers mainly concentrate their attention on the reduction of the nonconforming rate through manufacturing process control and inspection procedures. This approach has contributed to product quality improvement in manufacturing industries in the 1940-1960s. The properties of the quality control approach are mainly: (1) Minimization of total cost is the cornerstone to determine the adoption of quality activity. (2) Quality cost function is convex against the improvement activity in the conforming rate. This means that once the level of product conformance is set by cost criteria, any quality improvement is accompanied with a production cost increase. (3) The price and the quality are positively related.

Although Taguchi's loss function has a form similar to the simplified consumer quality loss function developed in Chapter 3, the meanings contained in their forms are absolutely different. Taguchi's quality loss function is derived from producer behavior and employs production cost to compute the quality loss. This function reveals that producer quality improvement is a continuous activity. Quality control should be concentrated on the activities which promote the quality target value and reduce quality deviation. The nonconforming rate can also be improved through Taguchi's methods.

Taguchi loss function usually is written as

$$L_p = k[(\mu - T)^2 + \sigma^2] \qquad (7.1\text{-}9)$$

where L_p - producer quality loss function, \$;

k - loss coefficient, \$/unit^2;

μ - mean value of product quality distribution, unit;

T - quality target value, unit;

σ^2 – variance of product quality distribution, unit^2.

Substitute equation (7.1-9) into equation (7.1-1), and the producer expected quality value becomes

$$E(V) = \rho(v_1 - L_p) + (1-\rho)(v_2 - L_p) \qquad (7.1\text{-}10)$$

$$= \rho v_1 + (1-\rho)v_2 - L_p \quad \text{for reworkable item}$$

$$E(V) = \rho(v_1 - L_p) \quad \text{for not-reworkable item} \qquad (7.1\text{-}10')$$

Equation (7.1-10) and equation (7.1-10') are much better to reveal the producer's expected quality value which depends on not only the conforming rate but, more importantly, the quality loss due to the deviation from quality target value which causes consumer complaint, dissatisfaction, and switching to another competitor's product. Taguchi methods, the modern quality approach, do not necessarily require a cost increase accompanying quality improvement, and they are cost-effective methods. If product quality variations have the same quality characteristic distribution pattern, reduction in loss function through quality design will also improve the conforming rate ρ. In contrast, increase in ρ does not necessarily mean an improvement in quality design. If the same conditions for equations (7.1-6') to (7.1-7') are applied on equation (7.1-10), it is easily found that the output level would be:

$$Q_I < Q' \quad Q_{II} < Q'' \tag{7.1-11}$$

where Q_I and Q_{II} are the output levels under quality neutral and quality aversion in the modern quality approach. Taking the quality loss into account for production output, the producer would provide fewer products to consumers concerned with quality loss in a competitive environment. In other words, producers would be more likely to improve product quality to strengthen the competitive capability to avoid the quality loss. Compared with the conventional quality control approach, the modern quality approach accounting for the quality loss as a production cost definitely promotes the product quality in production decision making. The producer quality loss may overestimate or underestimate the real loss due to no consideration of consumer quality behavior. The approach of total cost minimization is still the economic tool for quality activity in the modern quality approach. Although there arise questions on the adequacy of loss function and the procedures of optimization (Box, Bisgard and Fung, 1988), the modern approach has been widely employed and has made a tremendous success in quality improvement.

7.2 Consumer-Based Approach in Quality Activity

7.2.1 Consumer-Based Behavior under Quality Variation

The producer-based approaches described in the previous section do not account for consumer attitude toward quality risk. As a matter of fact, consumers are perceptive and sensitive to quality variation. With increased quality competition, the producer's quality loss is now mainly caused by the consumer switching to a competitor's product with better quality. The reasons and the advantages for taking the consumer quality utility value as the producer's basis for quality improvement can be summarized as follows: (1) Product quality is consumed and evaluated by consumers, not producers. Consumer dissatisfaction causes the producer's quality loss. (2) Consumer satisfaction and requirement must be accounted for in the firm's long-term objective goal in the product quality design. (3) Consumer quality loss as opposed to the producer's loss function is more adequate to capture the effects of product quality on consumer decision making. (4) Producer's loss function will underestimate the optimal product quality settings to meet the firm's long-term goal. (5) Consumer quality loss is the criterion to correctly evaluate the effect of the quality activity on the competition advantage and on the consumer quality welfare. (6) The behaviors and decisions of both consumers and producers are related to each other. (7) The market in the real world is operated somewhere between the perfect competition and the pure monopolist's markets. Both producer competition and consumer quality preference can be simultaneously incorporated in a consumer-based approach. The following example gives a better understanding of the changed producer behavior.

One successful example for taking consumer-based behavior is the Xerox Corporation in this transformation. As the 1989 winner of the Malcolm Baldrige National Quality Award, Rickard Jr. [1991], the vice president for Xerox Corporation, points out that "The challenge facing Xerox was to change individual and corporate behavior." Allaire, the Chief Executive Officer for Xerox Corporation, said "Our No.1 priority has not and will not change. It was, and is, and will continue to be customer satisfaction." (See Rickard,1991). As a result of the "meeting consumer requirement," it is found that

- **Parts-reject rates on the assembly line fell from 10,000 parts per million to 300 parts per million, with a goal of 150 per million.**
- **Ninety-five percent of supplied parts no longer needed inspection; in 1989, 30 American suppliers went the entire year defect-free.**
- **Suppliers were cut from 5,000 to fewer than 500.**
- **Cost of purchased parts was reduced by 45 percent.**
- **Three-month inventories were reduced to nearly 20 days.**
- **Six out of seven parts inspectors were reassigned to other jobs.**
- **Despite inflation, manufacturing costs dropped 20 percent.**
- **Product-development time decreased by 60 percent.**
- **Overall product quality improved 93 percent.**
- **Product quality, measured by customer surveys, increased 38 percent between 1985 and 1989.**
- **Unscheduled maintenance calls dropped by 40 percent, reflecting major gains in product reliability.**
- **Xerox became the first American company to, without any tariffs or government-mandated protection, regain market shares in an industry targeted by the Japanese. (Rickard [1991])**

The consumer-based producer expected quality value is

$$E(V) = \rho(v_1 - ACQL) + (1-\rho)(v_2 - ACQL - L_0)$$

$$= \rho v_1 + (1-\rho)(v_2 - L_0) - ACQL \qquad (7.2.1\text{-}1)$$

where ACQL - the average consumer quality loss for a quality deviation;

L_0 - consumer quality value loss due to product failure to work.

Specific forms of ACQL are given in Table 3-1 for quality neutral and in Table 3-6 for quality aversion conditions. L_0 may be expressed in terms of money for inconvenience, time waste or moral hazard.

The difference between equation (7.1-10) and equation (7.2.1-1) is only the term of producer quality loss values replaced by the consumer quality loss value. This change contains a significant meaning. In the view of quality improvement, a quality activity increasing consumers' utility may benefit producers simultaneously. It is easy to prove that the consumer-based approach is better than the producer-based approach for profit maximization and optimal quality settings. From equation (6.3.1-24) and equation (6.3.1-25), assume that the consumer-based loss function is employed and price is fixed in the short term. The quality settings and the quantity output level are

$$\pi = PQ - \Phi(å, Q) - b$$

$$d\pi/d(-å) = \Phi'_{å}(å, Q) = 0 \qquad \Phi'_{å}(å_0, Q) = 0$$

$$d\pi/dQ = P - \Phi'_Q[Q/(å = å_0)] = 0 \qquad P = \Phi'_Q[Q_0/(å = å_0)]$$

When the producer-based loss function is employed, which overestimates the reaction of consumer's quality settings, then the quality setting and the quality output level denoted by subscript 1 are:

$$d\pi/d(-å) = \Phi'_{å}(å, Q) = 0 \qquad \Phi'_{å}(å_1, Q) = 0$$

$$d\pi/dQ = P - \Phi'_Q[Q/(å = å_1)] = 0 \qquad P = \Phi'_Q[Q_1/(å = å_1)]$$

The same procedure is applied to the case with employment of underestimated producer loss function, denoted by subscript 2. We have the results

$$\mathring{a}_2 > \mathring{a}_0 > \mathring{a}_1 \qquad Q_1 < Q_0 < Q_2 \qquad \pi_1 < \pi_0, \pi_2 < \pi_0 \qquad (7.2.1\text{-}2)$$

Assume that there are n identical firms in the competitive market. The demand of products equals the supply at the equilibrium condition.

$$D(p) = S(p) = nQ_0, \qquad D(p) > nQ_1, \qquad D(p) < nQ_2 \qquad (7.2.1\text{-}3)$$

Although consumers prefer the higher quality product resulting from oversetting quality level, the producer can obtain a higher profit by expanding his production level at an appropriate quality level, whereas the lower quality setting arises the problem of supply being greater than the demand. The producer suffers a loss from over production.

The strategy in quality activity adopted by producers can be judged by the consumer-based approach. So far, many top managers and quality experts claim producers must take the consumer's satisfaction as a matter of prime importance in quality improvement to cope with the challenge of the changed world. In fact, successful enterprises emphasize consumer requirements and satisfaction in the implementation of production planning and market strategies. A mathematical model should be developed to reflect the changes in the producer's behavior and the interaction between producer and consumer decision making. The economic parameters, quality setting and price usually go together in quality activity and can be endogenously determined in the consumer-based approach at the market equilibrium. No contract or third party is needed or assumed in this model implementation because any force outside the model would, in some contents, disturb the outcomes derived from the approach.

There are four possible combinations for consumer-based quality utility functions: consumer-neutral producer-neutral, consumer-aversion producer-neutral, consumer-neutral producer-aversion and consumer-aversion producer-aversion. The first, the second and the fourth combinations have more common sense and are observable. The third combination is ignored here. If the producer's expected quality utility value under risk neutral accounts for consumer attitude under quality risk aversion, then

$$E[U(V)]_n = \rho U_{n1} + (1-\rho)(U_{n2} - U(L_0)) - EUV_a \qquad (7.2.1\text{-}4)$$

where $E[U(V)]_n$ - producer's expected quality utility value under quality risk neutral;

U_{n1}, U_{n2}- quality utility for event i, i= 1, 2;

EUV_a - consumer's expected quality utility under quality risk aversion;

$U(L_0)$ - consumer's quality utility loss due to product failure to work.

For the sake of simplicity, the conforming rate ρ and the utility function are expressed in terms of quality deviation variable, å, i.e.

$$E[U(V)]_n = \rho(å)U_{n1}(å) + (1-\rho(å))[U_{n2}(å) - U(L_0(å))] - EUV_a(å)$$

$$= U_{n1}(å)-EUV_a(å)-(1-\rho(å))[U(L_0(å)+U(v(å))] \quad (7.2.1\text{-}5)$$

$$U_{n2}(å) = U_{n1}(å) - U(v(å)) \quad (7.2.1\text{-}6)$$

where $U(v(å))$ - utility loss due to reworking for the failure of the product, \$. $U(v(å))$ is also related to the producer warranty policy and the service after sale, $U_{n1}(å) \geq U(v(å)) \geq 0$. The relationships between $U(L_0(å)$ and $U(v(å))$ for a given quality setting are: $U(v(å))\uparrow$, $U(L_0(å)\downarrow$, $U(v(å))\downarrow$, $U(L_0(å)\uparrow$. When $å \Rightarrow 0$, both $U(L_0(å)$ and $U(v(å)) \Rightarrow 0$, and $\rho(å) \Rightarrow 1$.

In a competitive environment, a producer is faced with a number of uncertainties: (1) Consumer behaviors under quality aversion are actually different from the generalized form. (2) The strategy of quality activity adopted by the competitor is unknown. (3) The product demand curve is related to the consumer's taste and income level, which change from income group to income group and from time to time. The supply curve also changes with variation in input costs (prices). (4) Price is associated with the imperfect market mechanism. Therefore, the producer is more conservative under quality risk. The producer behavior under risk aversion is:

$$E[U(V)]_n > E[U(V)]_a \quad (7.2.1\text{-}7)$$

$$E[U(V)]_a = E[U(V)]_n - U[R(å)] \quad (7.2.1\text{-}8)$$

where $U[R(å)]$ - utility of producer quality premium, \$;

$E[U(V)]_n$, $E[U(V)]_a$ - expected consumer quality value under quality risk neutral and risk aversion, respectively.

U[R(å)] is assumed to take a decreasing function of quality deviation. The higher the product quality, the lower the utility of producer quality premium.

$$dU[R(å)]/d(-å)] < 0 \tag{7.2.1-9}$$

$$d^2U[R(å)]/d[(-å)]^2 < 0 \tag{7.2.1-10}$$

The values for $U_{n1}(å)$ and $\rho(å)$ increase with quality improvement, and the value of U(v(å)) is an implicit function of quality deviation and is related to the producer's warranty policy. The higher the warranty policy compensation for consumer quality loss, the lower the value of U(v(å)).

The producer utility of profit can be directly expressed in the expected profit such that equations (6.2.1-7) through (6.2.1-10) become

$$E(\pi)_n > E(\pi)_a \tag{7.2.1-11}$$

$$E(\pi)_a = E(\pi)_n - R(å) \tag{7.2.1-12}$$

$$dR(å)/d(-å) < 0 \tag{7.2.1-13}$$

$$d^2R(å)/d(-å)^2 < 0 \tag{7.2.1-14}$$

7.2.2 Consumer Quality Value Surplus Model

Sinha and Wilborm [1985] proposed a model called "maximization of value addition" to illustrate the relationship between quality value to consumers and the cost to quality production. Since this model cannot distinguish between producer and consumer quality values corresponding to the quality improvement, we modify this model as a consumer quality value surplus to provide a basis for optimal quality settings in the longer term. In the point of view of the producer, the cost of quality activity is a function of quality deviation. In the short term, no change in production system and manufacturing processes is made, and only some innovations and learning rates are assumed to be carried out. The quality cost is reduced considerably through optimization of product quality design and manufacturing parameter settings as well as effective management. The quality cost function is a convex function in the quality domain, as described in Chapter 6. This pattern of quality cost curve reflects an argument in quality activity. Quality improvement does not necessarily imply an increase in cost over quality range. Producers should first pay attention onto product quality design, processes' parameter setting and innovation to reduce the cost. Taguchi methods and other optimization techniques are powerful and cost effective in quality activity. In contrast, the conventional quality control approaches are focused on the quality specification (conforming rate) and claim that quality improvement is costly, and they ignore the employment of optimization methods and learning effects.

In the long term, the quality cost function will vary with development in technology and science. Previously high-cost quality activities will become the economic ones to realize continuous quality improvement.

The change in quality cost is partially or entirely transformed into the product price. The product price has a negative effect on consumer quality utility. Both price and quality effects in the consumer quality value surplus model simultaneously determine the product quality setting. Product quality determination is regarded as a longer run decision variable with respect to the output variable. In most cases, no price and quantity planning/strategy are changed before the quality setting is made. The quality is usually the first stage decision variable. In some cases, other decision variables will be predetermined. The price decision variable plays two roles in both the producer behavior of profit maximization and the consumer behavior of utility maximization. It links consumer quality utility to the producer profit and finds a point which may satisfy both consumer and producer objectives in the long term.

From equation (3.3.2-14), the consumer quality utility function is

$$U = U[x(t_i) + \eta B(x(t_i)) - \eta f(p) + (1 - \eta)F(w(å))]$$

$$V(U) = x(t_i) + \eta B(x(t_i)) - \eta f(p) + (1 - \eta)F(w(å))$$

For simplicity, assume that a "representive consumer" is characterized, and the consumer's utility is expressed in the form of the expected utility for the consumers. If we are interested in the characteristics of a specific income group, the above equation is directly employed.

$$E[V(U)] = x_k(t_i) + \mu_\omega B_k(x_k(t_i)) - \mu_\omega f(p) + (1-\mu_\omega)F(w(å) \qquad (7.2.2\text{-}1)$$

in the short term with respect to product quality settings

$$E[V(U)] = A - \mu_\omega f(p) + (1 - \mu_\omega)F(w(å)) \qquad (7.2.2\text{-}2)$$

where A - constant, assuming the consumer assets and budget are fixed at time period i, i.e. $A = x(t_i) + \mu_\omega B(x(t_i))$ \$;

μ_ω - mean value of Weibul distribution for quality/price weight;

$x_k(t_i)$, $B_k(x_k(t_i))$ - the average consumer assets and budget for group k at time period i, respectively.

The price variable is exogenous in consumer quality utility function.

For the sake of discussion, assume the price variable is a function of quality deviation, quantity output pattern and profit. However, price is endogenous in the consumer-based producer utility function, and such an assumption is relaxed. Thus, equation (7.2.2-1) is transformed into the following form

$$E[V(U(G))] = A - \mu_{\omega} f(p(\mathring{a}, Q_c, \pi)) + (1 - \mu_{\omega})F(w(\mathring{a})) \qquad (7.2.2\text{-}3)$$

where Q_c - quantity output cost pattern, which affects the quality operation cost;

π - the profit which may or may not be related to the product quality setting.

It was proved in Chapter 3 and Chapter 4 that consumer expected quality value increases with quality improvement when price remains constant. It is no surprise that the consumer achieves the highest utility value when quality deviation is zero. On the other hand, if product quality remains constant, consumer utility will increase with decreased price, but decrease with an increase in price. In the consumer quality value surplus approach, we examine the effects of simultaneous changes in price and expected quality value on consumer utility. The patterns of the expected quality value depend on the consumer assets, taste and preference as discussed in Chapter 3. A rational consumer has a concave function for the expected quality value. For a particular consumer, however, the concavity condition for the utility function may not be necessary.

$$F(w(å)) = f(w(å)) - [1-\rho(å)]L_0(å) \qquad (7.2.2\text{-}4)$$

Substitute equation (7.2.2-4) into the above equation (7.2.2-3), then

$$\text{Max } E(V\{F\}) = A - \mu_\omega f(p) + (1-\mu_\omega)\{f(w(å)) - [1-\rho(å)]L_0(å) \qquad (7.2.2\text{-}5)$$

$$dE(V\{F\})/d(-å) = (1-\mu_\omega)[f'(w)+\rho' L_0(å) - (1-\rho(å))\, L_0'] - \mu_\omega f'(p(å,Q_c,\pi)) = 0$$

$$(1-\mu_\omega)[f'(w) + \rho'\, L_0(å) - (1-\rho(å))\, L_0'] = \mu_\omega f'(p(å,\, Q_c,\, \pi) \qquad (7.2.2\text{-}6)$$

$$df(w)/d(-å) > 0,\;\; f''(w) < 0 \qquad (7.2.2\text{-}7)$$

$$d\rho(å)/d(-å) > 0,\;\; \rho''(å) < 0 \qquad (7.2.2\text{-}8)$$

$$d\, L_0(å)/d(-å) < 0,\;\; L_0''(å) > 0 \qquad (7.2.2\text{-}9)$$

$$df(p)/d(-å) > 0,\;\; f''(p) > 0 \qquad (7.2.2\text{-}10)$$

The optimal consumer quality surplus in the above conditions given is

$$F'(å)/f'(p) = \mu_\omega/(1-\mu_\omega) \qquad (7.2.2\text{-}11)$$

The quantity output cost pattern has an impact on the consumer surplus, but not on the slope of quality cost function. To avoid complexity, we assume a constant return to scale exists. Thus, the price is the sum of the quality cost function and the profit. If the profit is constant, it does not affect quality setting. Otherwise, it does.

The consumer's compensation for quality loss, such as replacement of the product which fails to work, is the producer's cost reflected in the product price. To avoid double counting in the above model, some assumptions should be made. More details will be discussed in the next section. However, the qualitative explanation of the model is meaningful without losing generality. The positive, neutral and negative quality value surplus corresponding to $E(V\{F\}) > 0$, $= 0$, or < 0 are used in the maximization, minimization and equality approaches. All of these three options of optimal approaches are meaningful, and their applications in quality activity depend on the specific requirement of the situation. We mainly discuss the case with positive consumer quality value surplus. The consumer quality utility value reaches the optimum point å* where dE(V{F})/d(-å) = 0, shown in Figure 7-1. Before the maximization point, the gain from one unit increase in the weighed quality function offsets the loss from one unit increase in the weighed price function, while after the maximization point, the weighed quality gain is less than the weighed loss in terms of one unit change in price. The intercept points A and B are called neutral points at which the consumer does not lose or gain from quality activities, and the consumer is indifferent to the high quality product with a high price and the lower quality product with a lower price. The outcome resulting from one neutral point to the other neutral point does not affect the changes in the consumer surplus, demand level, and consumer quality welfare for the uniform consumers assumed.

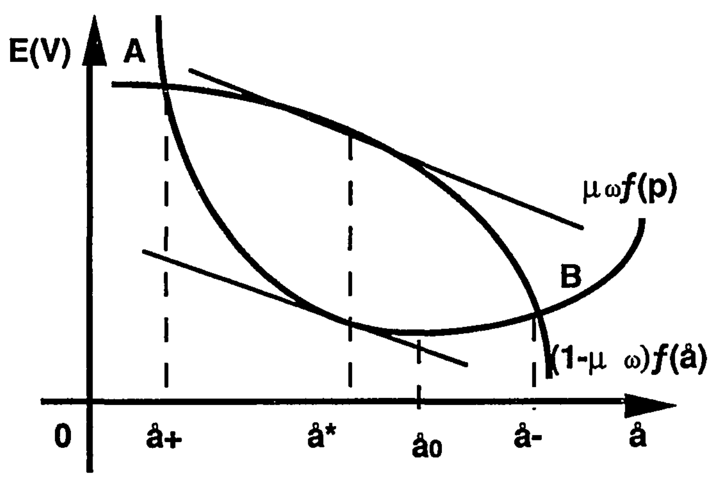

Figure 7-1 Maximization Approach for Consumer Quality Value Surplus

When the consumer quality utility under risk neutral is considered, the optimum point of quality setting corresponding to the maximization of consumer value surplus is lower than the quality deviation under risk aversion. We have

$$\mathring{a}^*_n \geq \mathring{a}^*_a \qquad (7.2.2\text{-}9)$$

where $\mathring{a}^*_n$ and $\mathring{a}^*_a$ represent the product quality settings under consumer risk neutral and consumer risk aversion, respectively.

If consumers give more weight for product quality, the quality setting in the optimal consumer surplus will be higher. In the competitive market, the producer is price taker, and he cannot determine the price. In the monopolistic market, the producer will set the price as high as possible to exploit the consumer surplus in its profit maximization. The consumer surplus in a competitive environment is much higher than in a monopoly market.

The consumer quality value surplus approach provides a criterion for the adoption of quality activities and for comparison among a number of alternatives. A more impressive result from the approach is that the optimum quality setting is not located at the place of MQS (minimum quality standard) where the lowest price is set through cost minimization methods. But the quality setting point is located in the region where higher quality is accompanied with a higher price. The properties of a variety of factors in consumer quality utility, such as assets, budget level, information uncertainty, quality premium, conforming rate, and price developed in Chapters 3 and 4, illustrate consumer quality behavior. The price is the exogenous variable in the model. The producer behavior of product quality settings, and his attitude toward quality risk, have not been incorporated into the model. Competition and consumer satisfaction are sufficient conditions for quality activity and product quality setting. Quality usually is a long-term decision variable with respect to the price and the quantity decision variables. This model should incorporate the competition in it to determine the optimal quality setting in the long-term.

7.2.3 A Simplified Utility Value Function for Implementation of Quality Settings

The complexity of the consumer quality utility under quality risk may reveal a difficulty in applying it to practical problem solving, though equation (7.2.2-5) is very useful in theoretical and qualitative analysis. Generally, the forms of consumer expected utility value (EUV) for the commercial product could be obtained through a survey of consumer quality assessment or analysis of the historical production/market data. A consumer quality value function for implementation of quality setting for coming products in a future market can be established. If a specific form of EUV is not available, a simplified EUV with the linear quality value function developed in previous chapters could be used here for the sake of appropriateness and simplicity. Another reason is that the linear quality value function is more conservative than the concave functions (see Chapter 3 for details). The EUV under quality neutral and quality aversion are summarized in Table 7-1 which is obtained from parts of Table 3-1 and Table 3-6. (The EUV for the smaller-the-better type of quality characteristics has the same patterns as for the larger-the-better type).

Table 7-1. EUV Under Quality Neutral and Aversion with Linear Quality Value Function

	quality neutral	quality aversion
The nominal-the-best	$w_H - 0.8k_1\sigma$	$w_H - 0.8k_1\sigma - 0.18k_1^2\sigma^2$
The larger-the-better	$w_H - k'_1(T-\mu)$	$w_H - k'_1(T-\mu) - (k'_1\sigma)^2$

The quality target value in Table 7-1 is no longer the quality value for the products sold in the current market, but the designed value with the availability of technology and science in the future market. w_H is determined by the price the consumer is willing to pay for the future product quality, while w_L is determined by the price paid for the lowest product quality. Thus, in the nominal-the-best type of quality characteristics

$$w_H = P_H \qquad w_L = P_L \tag{7.2.3-1}$$

$$k_1 = (w_H - w_L)/\Delta \tag{7.2.3-2}$$

where P_H, P_L are the highest and the lowest prices the consumer is willing to pay for the quality designed product, respectively.

The quality target value w_H could be determined by the information available at the present time in the larger-the-better or the smaller-the-better type of quality characteristics:

$$w_H = TP_0/\mu_0 \tag{7.2.3-3}$$

$$k_1' = w'/T = P_0/\mu_0 \tag{7.2.3-4}$$

where T - designed quality target value in the future market, unit;

P_0, μ_0 - price and quality mean value for the current products.

Therefore, a simplified consumer quality utility function can be written as follows. For the nominal-the-best type of quality characteristics under consumer quality aversion

$$E(V\{F\})=A-\mu_\omega f(p)+(1-\mu_\omega)[(w_H-0.8k_1\sigma-0.18k_1^2\sigma^2)-(1-\rho)L_0] \quad (7.2.3\text{-}5)$$

For the larger-the-better type under consumer quality aversion

$$E(V\{F\}) =A-\mu_\omega f(p)+(1-\mu_\omega)[(w_H-k'_1(T-\mu)-(k'_1\sigma)^2)-(1-\rho)L_0] \quad (7.2.3\text{-}6)$$

If the multiple-quality attribute approach is incorporated into the above equations, the multiple utility function value is

$$E(V\{F\})=A-\mu_\omega f(p)+(1-\mu_\omega)\{\Sigma s_i[\, w_{Hi}-(ACQL_i-(1-\rho_i)L_{0i})] \quad (7.2.3\text{-}7)$$

where s_i - weight for attribute quality i;

$ACQL_i$ - average consumer quality loss for attribute i, \$.

If the specific consumer utility function is not available, the above simplified function can be approximately used in the model to determine the quality settings. Another advantage of utilization of this function is that the consumer quality utility becomes computable.

7.3 Producer Optimization Behavior under Quality Uncertainty

Traditionally, the producer strategy on optimization behavior mainly includes constrained cost minimization, output maximization, profit maximization, and producer utility maximization, which are successfully applied on the producer's quantity strategy. Diversity of frameworks on optimization is structured on the specific requirement and conditional limitations of the problem solved. However, the basic framework of the optimal approach to the specific construction of the mathematical model is similar. The traditional optimal strategies on production output quantity are based on the assumption of quality homogeneity.

As mentioned before, quality activity is related to both consumer and producer behaviors. Any change in product quality will cause the variations in cost, price and quantity output level singly or jointly. A degree of uncertainty is introduced into the above optimal models if variation in product quality exists. The conventional production approach under uncertainty, see section 7.1, is employed to explain the producer strategy of the effect of quality variation on the output level. Although this approach needs further modification, it provides meaningful information that the quality output level is reduced, corresponding to the degree of producer aversion toward quality risk. In other words, the product output level will increase with quality and information improvement when other conditions are constant. The optimization models on product quantity should be modified or restructured to account for quality variation. These conventional optimal models are used to implement producer objectives after quality design is implemented. A set of solutions, quality setting, price, demand quantity level, consumer quality value surplus, will be derived from the consumer-based surplus maximization model under a variety of attitude toward quality risk.

7.3.1 Constrained Consumer-Based Quality Value Surplus Maximization in Static State

Assume that the producer behavior and objective about product quality is known. The producer will set a higher product quality standard to cope with the uncertainty he is faced with. It concludes that the firm's attitude toward quality risk and uncertainty depends on both consumer and producer behavior, not only on the firm itself, if the firm wants to benefit from quality activities. It is very important for the firm to transform the consumer quality utility to his own part of utility to avoid resource waste in establishment of a specific firm quality loss function. This approach may find an optimal quality setting point subject to some constraints. These constraints include all requirements, limitation and resources availability for both consumers and producers.

Taguchi's methods and other optimization methods in quality activity are aimed at optimal quality improvement and reduction of quality cost. Whatever effort is made with the conventional quality control approach, the quality value surplus is always smaller than that with employment of optimizing quality engineering methods. However, the cost/quality or price/quality trade-off ideas in the conventional quality control approaches are still alive. Based on the economic point of view, the optimum quality setting through optimal methods in quality engineering may or may not be the desired point which is obtained from the consumer-based quality value surplus. Quality engineering methods lack economic criteria and ignore the consideration for the consumer's utility function shapes and the producer's attitude toward quality risk. In the consumer quality value surplus approach, combined with the optimization methods applied on the product, process and system designs, an economic optimization criterion is used to select the best optimal quality engineering settings.

The consumer-based quality value surplus maximization is employed to determine the quality settings and other decision variables in the long term. If the producer compensates the consumer quality loss by providing some policies, such as recall-repair, advertisement and warranty, the costs for these policies must be added to the cost function. This model has the following form

$$\text{Max} \quad E(V\{F\})=\{x(t_i)+\mu_\omega B(x(t_i))-\mu_\omega f(p)+(1-\mu_\omega)[f(w(å))-[1-\rho(å)]L_0(å)]\}q \tag{7.3.1-1}$$

$$\text{S.t.} \quad \pi = pf(Q) - C(å, Q) - C - R(å, Q) \tag{1}$$

$$S(Q) = D(q) \tag{2}$$

$$B(x) \geq p \tag{3}$$

$$F(w(å)) = \Sigma s_i F(w(å_i)) \tag{4}$$

$$T(q) = T(q_i) \tag{5}$$

For the demand quantity q

$$q = q_0 + eq_0 \Delta p_c / p_0 \tag{7.3.1-2}$$

$$\Delta p_c = (p_0 - p) + (\Gamma(1-1/\beta)/\delta -1) [F(w(å)) - F_0(w(å_0))] \tag{7.3.1-3}$$

where q_0, p_0, F_0 - initial quality demand level, price and quality utility value at stage t_{i-1};

e - demand price elasticity;

Δp_c - consumer expected price increment;

$T(q_i)$ - system function at stage i, i = 1, 2, ... n.

We will discuss the reaction of one side to the change in the other side conditions through this model. To simplify this exploration, the following assumptions are made so that the analysis can be focused on the implications of the model:

(1) Monopolistic competition exists, which contains elements of both monopoly and perfect competition.

(2) Free entry and exit will occur as long as profits are positive. The equilibrium is a zero-profit.

(3) The consumer is rational to maximize his utility subject to the constraints.

(4) Firms act as quality leaders, given consumer utility maximization under the quality attitude taken.

(5) Quality and price are the first-order decision variables determined in the model. The demand and supply quantity levels are the second-order decision variables. The demand quantity is the nominal one and suggests that the firm can produce these volume products at both the price and the quality determined to maximize its utility value without any effect on consumer behavior.

(6) In the maximization of producer utility of profits, each firm chooses quality to maximize profits, if the firm is risk neutral. The firm is willing to pay the quality premium to achieve its long term goal, if it is quality averse.

(7) Consumers are quality risk neutral or averse depending on their attitude toward quality variation and information availability.

(8) Technology for producing an additional unit is constant return of scale.

(9) A representive consumer quality behavior is considered in the approach.

(10) All firms are identical and have the same cost function.

(11) Quality information is completely available.

Assumptions (9) through (11) are weak and will be relaxed in the real market situations. The total industry demand is the sum of individual consumer quality value and is endogenously given such that the market quantity and price are in equilibrium in the long term. The firm decision variables are product quality under various specified conditions and other strategy variables, such as price, consumer value surplus, demand and supply output levels, which are determined in the sequence of the model operation.

The objective function in the model consists of consumer quality utility derived from product quality, product price, inherent quality variation, assets, budget constraints and weight distribution for price/quality effects. It may also include multiple attribute quality. The information premium is excluded from the objective function in this moment because of assumption (11). The consumer will maximize his quality utility under risk aversion or risk neutral. The price variable is endogenous in this model. q is the demand quantity such that the objective of the model is aimed at maximizing the total consumer quality value surplus rather than the single value surplus.

Constraints (1) is from the producer behavior which is centered at profit maximization. In the long term, the firm's expected profit is

$$\pi = pf(Q) - C(å, Q) - C - R(å, Q)$$

The producer's quality premium R(å, Q) equals zero under producer quality risk neutral. The quality cost function can be decomposed into several different parts. For the sake of demonstration, we will use the following quality cost function (see equation(6.3.1-19))

$$C(å, Q) = [C(å)]Q$$

$$C(å) = C_p(å) + C_f(å) + C_L(å) + C_o(å)$$

Free entry drives active an firm's expected profits to zero. Quality cost per unit can be separated from the total cost function, based on the two-step procedure where quantity output level will be determined after the quality setting is implemented.

$$p = C_p(å) + C_f(å) + C_L(å) + C_o(å) + R(å) + C/Q \qquad (7.3.1\text{-}4)$$

R(å) = 0 for producer quality neutral

R(å) > 0 for producer quality aversion

where C/Q - average fixed cost, \$/unit.

When a zero-profit condition is satisfied, the price equals the average cost. The price at the equilibrium is not only related to quantity determination, but also the product quality settings.

Constraint (2) means that the market demand equals the industry supply at the equilibrium conditions. The demand quantity is related to price, quality, weights for price/quality effects and demand price elasticity, shown in equations (7.3.1-2) and (7.3.1-3). To maximize total consumer value surplus, both product quality and quantity must be set under the optimal conditions, respectively.

Constraint (3) may give a boundary solution for the model. The budget constraint may or may not rule out the maximization point. When the budget is greater than the price corresponding to the optimal quality, the consumer arrives at the maximum utility value in the long term. Otherwise, the budget limitation determines the optimum quality level at the boundary conditions. Although the budget constraint may have two solutions for quality settings around the lowest quality cost point, the firm has no reason to operate at the higher cost but lower quality setting condition. If the optimal solution is beyond the scope of quality settings constrained by the budget, the quality setting point at which the budget equals the price will be chosen. The constraint will be meaningful in the spectrum of budgets corresponding to the differentiated income groups and the spectrum of product brands. However, based on the objectives and production conditions, the producer can promote the product into the higher brand category through new quality design and quality control subject to higher budget constraint. When the product quality attribute has an impact on consumer decision making in product selection, constraint (4) must be adapted to maximize the consumer overall quality utility. The last constraint is utilization of advanced technology and science in the production system and processes. System function is related to time, technology and investment. Any variation in consumer assets and producer system function will change all parameters and variables in the model.

Assume that the consumer utility $E\{V(f)\} = V_0$, $p = p_0$, $q = q_0$, and $\mathring{a} = \mathring{a}_o$ at the initial conditions. First of all, we consider the situation under combination of consumer-neutral producer-neutral. To maximize total consumer surplus subject to the constraints, then

$$\partial L/\partial(-å) = \partial E\{V[F]\}/\partial(-å) = 0$$

$$F'(å)/C'(å) = \mu_\omega/(1- \mu_\omega)$$

$$\partial L/\partial q = \partial E\{V[F]\}/\partial q - \lambda_2 = 0$$

$$V = \lambda_2$$

$$\partial L/\partial\lambda_2 = Q - q = 0$$

$$Q = q$$

$$\lambda_3[B(x) - p] = 0$$

$$\lambda_3 \geq 0$$

For multi-attribute quality with additivity form

$$\partial L/\partial(-å_i) = \partial E\{V[F]\}/\partial(-å_i) - \lambda_1 C(å)/\partial(-å_i) = 0$$

$$\partial L/\partial(-å_j) = \partial E\{V[F]\}/\partial(-å_j) - \lambda_1 C(å)/\partial(-å_j) = 0$$

$$s_i F'(å_i)/s_j F'(å_j) = f_{åi} C'(å_i)/f_{åj} C'(å_j)$$

Specifically, the two parts of a quality setting, the quality mean and the quality specification (variance), are determined in the following

$$E[s_i F'(å_i)/s_j F'(å_j)] = E[f_{åi} C'(å_i)/f_{åj} C'(å_j)]$$

$$s_i F'(\mu_i)/s_j F'(\mu_j) = f_{\mu i} C'(\mu_i)/f_{\mu j} C'(\mu_j)$$

$$s_i F'(\sigma^2{}_i)/s_j F'(\sigma^2{}_j) = f_{\sigma 2i} C'(\sigma^2{}_i)/f_{\sigma 2j} C'(\sigma^2{}_j)$$

Shown in Figure 7-1, in the range between $å^-$ and $å^0$, consumer quality value surplus increases with quality improvement and price reduction. In the range between $å^0$ and $å^*$, the consumer quality value surplus increases though the price also increases. For further quality improvement after the optimal quality, between $\mathring{a}^*$ and $\mathring{a}^+$, the increase in consumer quality utility value is less than the utility value decrease in price-up. The process for the optimal quality setting simultaneously determines the optimal quantity level. Any deviation from the optimal quality setting will result in a decrease in total demand in the long-term. The total industry supply equals the total demand at the market equilibrium where price equals the average cost. For instance, if quality setting is at $\mathring{a}_0$, producers have the potential to produce more products. Since the decrease in consumer value surplus results in total demand reduction, the supply must be cut to match the demand at the equilibrium condition. Therefore, producers suffer a loss from a redundant production because of the lower quality setting at $\mathring{a}_0$. Although the zero-profit is realized in monopolistic competition, the consumer quality surplus is still larger in a competitive market with lower average cost than in a monopolistic market. Consumer budget saving is discounted by factor μ_ω. If $\mu_\omega < 0.5$, consumers are willing to pay more for higher product quality. For multiple-attributes quality, the above condition should be met to maximize consumer quality surplus. In short, we can find a solution set for quality, quantity, price and consumer value surplus. Let $\mathring{a}^*$, q^*, p^* and V^* be the solutions for quality, quantity, price and consumer quality value surplus in this exercise under combination of consumer-neutral producer-neutral. Any firm's gain from output level increase is the other firms' loss in quantities at the market equilibrium, if the firms' cost function are not identical.

Any increase in the total cost function induces $p' > p^*$, $V' < V^*$ and $q' < q^*$. If the marginal quality cost remains constant, $C^* = C'$, the quality level does not change at all, $\text{å}' = \text{å}^*$. If $C^* > C'$, $\text{å}' < \text{å}^*$. If $C^* < C'$, $\text{å}' > \text{å}^*$. If consumer surplus increases, the demand quantity increases too, and vice versa. The increase in demand can be regarded as demand curve shifts up-right.

In comparison with the conventional producer-based quality setting approach, assume that the producer considers consumer behavior in the partial equilibrium separately rather than taking the consumer-based utility value into the entire equilibrium model. He determines the product quality at the market situation where both quantity and price are in equilibrium. Free entry and exit drive pure profit to zero:

$$\pi = PQ - C(\text{å}, Q) - C = 0 \qquad (7.3.1\text{-}5)$$

Suppose the cost function can be separated into quality cost and quantity cost function

$$C(\text{å}, Q) = [C(\text{å}) + r(Q)]Q \qquad (7.3.1\text{-}6)$$

$$\text{å} = C^{-1}[P - r(Q) - C/Q] \qquad (7.3.1\text{-}7)$$

where C^{-1} is quality cost function expressed in terms of price and quantity-related cost and fixed cost.

Generally, the optimization related to product quantity and quality can be treated as a two-step model in engineering and manufacturing quality activity. The quantity cost and quality cost function in the model are independently separated. The procedures usually determine quality setting first, and then the quantity level. In contrast, with the procedure in the conventional producer-based model, the quantity, quality and price variables have no decision-order among them, and quality setting could be made after the quantity level is given. From equation (7.3.1-7), any increase in the cost function would reduce the product quality because quantity and price are exogenous variables at market equilibrium. For example,

$$\mathring{a}_1 = C^{-1}[P - r_1(Q) - C_1/Q] \quad \mathring{a}_2 = C^{-1}[P - r_2(Q) - C_2/Q]$$

$$\text{if} \quad r_1(Q) > r_2(Q) \text{ or } C_1 > C_2, \text{ then } \mathring{a}_1 > \mathring{a}_2$$

When the demand exceeds the supply in the market, this model may provide an insight to illustrate a reduction in average product quality, but the market equilibrium condition is broken down. A change in product quality, however, will induce the change in market equilibrium, which also breaks the condition for the market equilibrium. Obviously, this conventional producer-based model lacks the ability to determine the optimal quality setting and ignores the fact of decision order between quality and quantity variables as well as the uncertainty involved in the future market. This model limits its application on quality activity and may lead to some confusing results. With the consumer-based value surplus maximization approach, if the cost function increases or uncertainty exists, the producer attempts to improve product quality and reduce production output, rather than to reduce quality to maintain the same output level. It is rare that the quantity can stay at the previous equilibrium when quality setting is changed. This behavior explained by the consumer-based approach is consistent with conventional theoretical concepts in decision making and also with an observable producer's attitude toward uncertainty in practice.

As described in Chapters 3 and 4, it is more realistic for consumers to behave in a quality averse manner. The consumer is willing to pay the inherent quality premium to avoid risk in buying the product with larger quality variation. Clearly, the consumer utility value function is steeper than the value function under quality neutral. Let $\underline{\mathring{a}}$, $\underline{q}$, $\underline{P}$ and $\underline{V}$ be the solution set for quality, quantity, price and consumer value surplus in the exercise of combining consumer-aversion with producer-neutral. In comparison with the first combination of neutral-neutral, assume that the quality cost function is not changed. Thus, $\underline{\mathring{a}} < \mathring{a}^*$, $\underline{p} > p^*$, $\underline{q} < q^*$. Since no consumer could possess quality neutral and quality-aversion behaviors at the same time for a specific product, no quality value surplus is compared. Both product price and product quality are higher, but production output level is lower under consumer quality aversion.

If a cost having no effect on the shape of marginal quality cost function is added onto the total cost function, the quality level is not affected, but the price increases and the consumer quality value surplus decreases, which results in output level reduction. If the added cost affects the shape of the marginal quality cost function, the price definitely increases, and quality setting change depends on the property of the added cost. In turn, the consumer value surplus and quantity demand will decrease. Note that these conclusions are derived from the assumption of quality information available.

The producer is quality conservative rather than neutral when he faces uncertainties described in previous sections. He intends to maximize the consumer-based value surplus under risk aversion. In this case, the producer's cost function is higher but more flat in the slope than that under quality neutral, because of an addition of the producer's premium which is related to the quality setting strategy, see equation (7.2.1-13) and equation (7.2.1-14). The same procedures are carried out for the quality setting, quantity level, price and utility value surplus represented by $\mathring{a}$, q, P and V in the case of combination of consumer-aversion and producer-aversion. The producer quality premium is a very flexible policy and changes with time, event, market place and interest rate, etc., which the producer uses to cope with uncertainty. Compared with the previous case, $\mathring{a} < \underline{\mathring{a}}$. If the quality premium is totally acted as the real price reduction for consumer compensation, then $V < \underline{V}$, $p > \underline{p}$, $q < \underline{q}$. The basic idea is that the producer prefers the strategy having both higher price and quality, but lower output level, because price and output level could be quickly adjusted to correspond to the change in the uncontrollable market conditions, but the quality variable cannot be reset in the short term. Furthermore, the quality premium compensation may affect consumer behavior. This approach implies that the producer maximizes his utility to gain the advantage in quality under uncertainty and, on the other hand, is willing to pay consumers the partial or total quality premium to secure his quality position in the market. This premium value can be regarded as the value transformation from the producer to the consumer who makes a decision based on his preference of the quality utility value. The producer premium is also a function of degrees of uncertainty, which is more flexible than change in price, quantity and quality alone.

In summary, the following results can be obtained in the static model when other variables are constant.

$$\mathring{a}_{nn} \geq \mathring{a}_{an} \geq \mathring{a}_{aa} \qquad (7.3.1\text{-}8)$$

$$p_{nn} \le p_{an} \le p_{aa} \qquad (7.3.1\text{-}9)$$

$$q_{nn} \ge q_{an} \ge q_{aa} \qquad (7.3.1\text{-}10)$$

where subscripts nn, an and aa represent the combinations for consumer-neutral producer-neutral, consumer-aversion producer-neutral and consumer-aversion producer-aversion, respectively.

The solutions for consumer quality value surplus under a variety of combinations depend on income group, interaction between consumer and producer behaviors, and the impact of producer quality strategy.

If the conventional optimal approaches are employed to achieve the producer's objective, some modification for quality setting must be made. The quality variable is exogenous and predetermined by the producer's objective. For constrained cost minimization,

$$\text{Min } Z = C(\mathring{a}^*, Q^*)$$

$$\text{S.t. } \mathring{a}^* = f(\mathring{a})$$

$$Q^* = Q(x/\mathring{a}^*)$$

$$L = C(\mathring{a}^*, Q^*) + \lambda_1(\mathring{a}^* - f(\mathring{a})) + \lambda_2(Q^* - Q(x/\mathring{a}^*))$$

Considering quality variation

$$E(L) = C([\mu^*, \sigma^2], Q^*) + \lambda_1 f(\sigma^2) + \lambda_2(Q^* - Q(x/[\mu^*, \sigma^2])$$

$\partial E(L)/\partial \mu i = f_{\mu i} = 0$ (To classify the input factors into different groups, then select the factors with lower costs, see Chapter 6.)

$$\partial E(L)/\partial \sigma^2{}_i = r_i - \lambda_1 f'_i = 0$$

$$\partial E(L)/\partial \sigma^2{}_j = r_j - \lambda_1 f'_j = 0$$

$$\partial E(L)/\partial x_i = R_i - \lambda_2 F'_i = 0$$

$$\partial E(L)/\partial x_j = R_j - \lambda_2 F'_j = 0$$

$$RTS(q_{ij}) = f'_i/f'_j = r_i/r_j$$

$$RTS(Q_{ij}) = F'_i/F'_j = R_i/R_j$$

where å* and Q* are the quality and the output quantity level predetermined.

The above conditions must be satisfied simultaneously so that the minimum total cost can be achieved. RTS for quality specification should equal the ratio of component costs. Meanwhile, the RTS for quantity output should equal the input price ratio. The cost for product quality target design has to be minimized. The above conditions are also required in the consumer-based value surplus approach. The optimal quantity level setting in the conventional approach is a local, not global, solution for total cost minimization. Similarly, the constrained output maximization is

$$\text{Max } V = Q(x/å_0)$$

$$\text{S. t. } C^0 = C(å_0, Q)$$

If the quality setting is given, the above model returns to the conventional one. If quality setting varies, the quality level is set at the lowest quality cost such that the maximum output can be obtained if the conditions in the cost minimization approach are also satisfied here.

In quality competition, the following two models are more powerful. One is to maximize the producer's profit subject to the product providing more consumer quality surplus than the rival's. The other maximizes consumer surplus value subject to the profit level predetermined. They are:

$$\text{Max } \pi = R(å, Q) - C(å, Q)$$

$$\text{s.t. } V_1 > V_2$$

and

$$\text{Max } V = f(å, p)$$

$$\text{s.t. } \pi = \pi_0$$

7.3.2 Quality Rent and Quality Information Effects

In the previous section, we assumed that all the firms in the industry were identical and had the same cost function and that the consumers in the market were uniform and indifferent. We discussed the behavior of "average" consumer decision making and the objective of representative firm in quality activity. Under realistic economic and production conditions, the cost functions for all firms are not identical, and the consumer quality preferences are not uniform. Lack of efficient access to quality information sources significantly affects both consumers' and producers' decision making. One reason for the temporary survival of firms which produce inferior product quality shipped to society is that perfect quality information is not available. Assumptions 9 through 11 in the consumer-based value surplus model are now relaxed to match the real world situations.

The higher-income consumers have higher budget distribution, lower quality premium, and higher weight assigned for the product quality effect. Significant budget differences divide the whole market into several market segments. With lower quality cost, through quality optimization methods, a firm can produce higher product quality at a lower average cost than the other firms. In the consumer value surplus model, the firm can determine its product quality setting in one of the market sections. The higher-income consumers or higher taste consumers can search for the desired product across the market segments. A firm may provide different brands of product quality for different income groups. The low-cost and high-quality firm possesses a potential advantage to earn more profit than an ordinary firm. If information is completely available and product quality is observable, consumers will select the product with higher quality at a lower price. If products are the same in quality, the firms compete with each other in price, while firms compete in quality if the prices are the same. The traditional cost rent for differential cost conditions can be regarded as one of quality rent. The cost difference on KLME in some cases actually arises from the quality difference in KLME. Quality rents can be classified into two categories, quality cost rent and quality managerial rent. The former diminishes in the monopolistic competition in the long term, but the latter depends on what quality management method is applied in the production system. Because the quality managerial rent is related to human factors, the top managers must create an effective quality management most suitable for the people in the production system.

The cost function affected by the effective quality managerial method is lower than the cost function without it. We define $f(M.E)$ as an operator for producer learning. The operator contains the advanced managerial methods and engineering quality systems. If the producer exercises this powerful instrument, the cost function will be significantly reduced. Mathematically,

$$C(å, Q)*f(M.E) = C(å, Q/M, E) \qquad (7.3.2\text{-}1)$$

$$C(å, Q) > C(å, Q/M, E) \qquad (7.3.2\text{-}2)$$

$$C'(å > å_0, Q) < C'(å > å_0, Q/M, E) < 0 \quad (7.3.2\text{-}3)$$

$$C'(å < å_0, Q) > C'(å < å_0, Q/M, E) \geq 0 \quad (7.3.2\text{-}4)$$

$$\text{If } å^* > å^{**}, \quad dC(å^*, Q/M, E)/dt < dC(å^{**}, Q/M, E)/dt \quad (7.3.2\text{-}5)$$

where f(M.E) - the operator for producer quality learning;

C(å, Q), C(å, Q/M, E) - quality cost functions without and with impact of advanced managerial and engineering methods;

å*, å** - different quality settings.

Equation (7.3.2-5) implies that the learning rate for the higher quality setting is faster than for the lower one.

There is a substantial body of work examining the impact of information on decision making (Simon, 1981; Schwartz et al., 1982; Kambhu, 1982; Matthews et al., 1985; Milgrom et al., 1986; and Wernerfelt, 1988). For instance, two products with different quality levels are supplied for the same income group with $å_1 < å_2$ and $p_1 > p_2$. If $V_1 > V_2$ and consumers prefer product 1 to product 2, firm 2 should either improve product quality or reduce the price to compete with firm 1. If complete quality information is available, consumers gain from switching from product 2 and purchasing product 1 to avoid quality loss. Firm 2 can improve the product quality or reduce the real price to survive the competition by using information-guide production planning. Both consumers and producers are well-informed about the reaction of quality strategy and quality preference on opposite-side decision making procedures. Social welfare is enhanced by a well-informed economic system, and the quality setting is determined in the long term quality competition.

Due to the shortcomings of the imperfect market mechanism, information is not totally available. It is difficult (or impossible) for the product quality to be observed. Decision making is subject to information uncertainty or the risk generated by imperfect information. For diversified cost functions, price is not a reliable signal of product quality except for the case in which the cost functions for the firms in the industry are identical. Consumers obtain the information about product quality through direct search or advertisements provided by firms in order to avoid incorrect decision making. On the other hand, firms want to obtain information about the consumer preference patterns and the rival's quality strategy in their quality design and production. Positive, correct, and truthful information reduces the degree of consumer aversion toward information risk, whereas negative information will increase the degree of riskiness in the long term. If product quality and other information sources are not totally available, consumers would choose a product randomly or in light of the product price. Consumer quality loss on the average is larger, and the firm producing the inferior quality product could survive in competition. Social and consumer welfare experiences a loss under poor information availability.

In the consumer-based value surplus approach, the objective function now contains a consumer quality information premium which is related to product quality and the consumer's willingness to pay for the information access. The producer adds the advertisement cost onto the total cost function. Assuming that the advertisement cost is related to the quantity cost or fixed cost, which means that the cost does not affect quality setting but consumer quality utility, then

$$p = r(Q) + C(å) + R(å) + (C + Cd)/Q \quad \text{for fixed cost} \qquad (7.3.2\text{-}1)$$

$$p = r(Q) + Cd(Q) + C(å) + R(å) + C/Q \quad \text{for variable cost} \qquad (7.3.2\text{-}2)$$

where Cd, Cd(Q) - the fixed advertisement cost and the quantity related advertisement cost, respectively.

The objective function under information uncertainty is (see equation (3.2.1-21))

$$Y = E\{V[F]\} - (1 - \mu_{\omega})f(\sigma^2_I) \qquad (7.3.2\text{-}3)$$

Let $å^*_I$, q^*_I, p^*_I and V^*_I be a set of solutions for quality setting, quantity demand, price and consumer value surplus with no information on the combination of consumer-aversion and producer-aversion. No information cost is added onto the profit constraint. If the information provided by the firm is absolutely reliable to compensate consumer quality loss due to information risk aversion, then

$$Cd(Q) \Rightarrow - (1 - \mu_{\omega})f(\sigma^2_I) \qquad (7.3.2\text{-}4)$$

where $\Rightarrow$ means that an advertisement has an impact on consumer quality utility. The consumer quality utility function becomes that under information neutral. The set of solutions is $å^{**}_I$, q^{**}_I, p^{**}_I and V^{**}_I. Compared with the solutions under perfect information, it can be shown that $å^{**}_I = å^*$, $q^{**}_I < q^*$, $p^{**}_I > p^*$ and $V^{**}_I < V^*$.

Shown in figure 7-2, line AA' and BB', represent $E\{V[F]\}$ and $E\{V[F]\} - (1 - \mu_{\omega})f(\sigma^2_I)$ for consumer quality values without and with an information premium, respectively. CC' and DD' represent the price effects with and without information cost (advertisement). It can be shown that:

$$E\{V[F]/(\mathring{a} = \mathring{a}^{**}_I)\} - \mu_\omega Cd = A'D' - D'C' = A'C' = V^{**}_I \qquad (7.3.2\text{-}5)$$

$$E\{V[F]/(\mathring{a} = \mathring{a}^{**}_I)\} - (1 - \mu_\omega)f(\sigma^2_I) = A'D' - A'B' = B'D' \approx BD = V^*_I$$

$$E\{V[F]/(\mathring{a} = \mathring{a}^{**}_I)\} = A'D' = V^*$$

$$V^* - \mu_\omega Cd = V^{**}_I$$

$$V^* - (1 - \mu_\omega)f(\sigma^2_I) = V^*_I$$

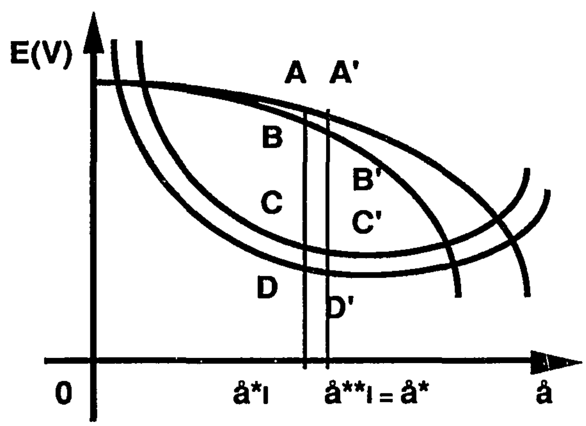

Figure 7-2 Effect of Information Cost on Consumer Value Surplus

We can approximately determine the effect of advertisement by examining that Cd is equal to, greater than or smaller than $(1-\mu_\omega)f(\sigma^2_I)/\mu_\omega$.

If $Cd(Q) > (1 - \mu_\omega)f(\sigma^2_I)/\mu_\omega$, $\mathring{a}^*_I < \mathring{a}^{**}_I$, $q^*_I > q^{**}_I$ and $V^*_I > V^{**}_I$, the consumers are willing to buy the product without an advertisement.

If $Cd(Q) = (1 - \mu_\omega)f(\sigma^2_I)/\mu_\omega$, $\mathring{a}^*_I < \mathring{a}^{**}_I$, $q^*_I = q^{**}_I$ and $V^*_I = V^{**}_I$, the consumers are indifferent between products with and without an advertisement.

If $Cd(Q) < (1 - \mu_\omega)f(\sigma^2_I)/\mu_\omega$, $\mathring{a}^*_I < \mathring{a}^{**}_I$, $q^*_I < q^{**}_I$ and $V^*_I < V^{**}_I$, the consumers prefer the advertised product because the information changes their degree of aversion toward uncertainty and risk.

7.4 Producer Quality Strategies on Production/Market Planning

To avoid the consumer's quality losses and to gain quality advantage, producers implement a number of strategies and quality policies to affect consumer behavior to favor their products in the competitive atmosphere. The producers provide warranty policies, such as money back or product replacement, to make up for the consumer loss due to product failure to work or for inferior quality. Product quality information or imperfect consumer information can be obtained or improved through advertisements, word of mouth, exhibitions or magazines provided by producers or consumer organizations. Repair policies or warranties enhance the total consumer utility over time periods due to product quality deterioration. The differentiated prices, rebate, and cash-back in product sale policy also affect consumer decision making in price competition. The producers play the role of quality leader to build up consumer reputation for their products to cope with the uncertainties in future markets. The competition among producers is reflected not only in product quality and product price, but also in the related strategies which affect consumer quality utility shape. Meanwhile these strategies also change the shapes of producer cost function and profit. Producers strive to determine the most effective methods in the implementation of the strategy.

7.4.1 Producer Behavior in Quality-Leadership Competition

Producers benefit greatly from playing the role of quality-leader in quality competition. Consumers prefer products with higher quality reputations to avoid higher risk and quality loss. Two sides' behaviors react and influence each other, and a particular relationship, called the quality credit between consumers and quality leader producer, can be gradually built up. The higher the quality credit, the lower the consumer quality loss. This relationship also forms a barrier to resist a newcomer to enter, or to keep the market share from new rivals, unless the new comer's or rival's product quality demonstrates a greater consumer utility surplus advantage to break through the barrier. The consumer-based quality value surplus approach is employed to illustrate why and how producers play the role of quality leader.

In the monopolistic quality competition, a firm plays the role of quality leader, which is an important stimulation for it to promote its product quality settings in the market. The fact is that the firm can benefit from such a quality strategy. Not all firms are capable of being recognized as quality leaders by the competitors and consumers. The firms who possess the following conditions have the potential to become quality leaders: (1) substantial market share, (2) lower product quality cost, and (3) effective quality management. For simplicity, some assumptions are made before a detailed discussion of producer behavior in playing the role of quality leader in a dynamic process. All variables in the model vary with time.

(1) The quality leader firm chooses its quality strategy while taking the strategies of other rivals as given.

(2) The quality leader firm possesses the information advantage about

other firms' quality production.

(3) Nash-equilibrium is assumed to exist.

(4) Consumers can observe the firms playing quality leader role over extended time periods.

(5) The cost paid for the role of quality leader in previous periods is ignored.

In this model, the objective is focused on the case of consumer-producer quality aversion which reflects producer behavior and attitude toward quality uncertainty: (1) Quality competition will be more serious in the future market. (2) The rivals will significantly promote their product quality to place themselves in the position of quality leader. (3) The goal for quality leader requires product quality not to be set in meeting consumer satisfaction, but to go beyond as far as possible. (4) Rational producers maximize their utility value of profit over longer time periods.

The producer's quality premium is the value given up to guarantee that his expected utility maximization will be achieved. The producer quality premium is a cost to the producer, but a benefit for consumers through price reduction or another producer's policy. The producer's goal cannot be achieved without consideration of consumer quality attitude.

Recall the model of consumer-based quality value surplus in the case of consumer-aversion and producer-aversion. The best model for a single attribute product is defined as follows:

$$\text{Max} \quad E(V\{F\}) = x + \mu_{\omega}B(x) + \mu_{\omega}f(p) + (1 - \mu_{\omega})F(w(\mathring{a})) \qquad (7.4.1\text{-}1)$$

$$\text{S.t.} \quad \pi = pf(Q) - C(\mathring{a}, Q) - C_q(\mathring{a}) - (C + A)$$

where $C_q(\text{å})$ is producer quality premium. A is the advertisement cost; it may be related to the quantity output.

The second best model takes the budgetary constraint into consideration

$$\text{Max } E(V\{F\}) = x + \mu_\omega B(x) + \mu_\omega f(p) + (1 - \mu_\omega)F(w(\text{å})) \qquad (7.4.1\text{-}2)$$

$$\text{S.T.} \quad \pi = pf(Q) - C(\text{å}, Q) - C_q(\text{å}) - (C + A)$$

$$B \geq p$$

One of the purposes of the firm in playing the quality leader role is aimed at meeting the goal of "beyond the consumer satisfaction" in the serious competitive and uncertainty involvement environment. Let Q*, å*, p* and V* represent the quantity, quality, price and consumer surplus value under consumer-based quality utility maximization. The producer expected profit and its quality leader premium are:

$$\pi = pf(Q) - C(\text{å}, Q) - q(\text{å}) - (C + A) \qquad (7.4.1\text{-}3)$$

$$dq(\text{å})/d(-\text{å}) < 0, \quad q''(\text{å}) < 0 \qquad (7.4.1\text{-}4)$$

where q(å) - producer quality leader premium, \$. q(å) is a special case of producer quality premium $C_q(\text{å})$.

q(å) is a function of product quality; the higher the quality, the lower the q(å). q(å) is used to determine the quality setting for the quality leader. Quality leader premium is a special producer's quality premium, which may or may not be a real cost for the firm. The quality setting for the quality leader's product is determined under the quality leader premium:

$$(1 - \mu_\omega)f'(w(\mathring{a})) = \mu_\omega f'(p) = \mu_\omega(C'(\mathring{a}) + q'(\mathring{a})) \qquad (7.4.1\text{-}5)$$

The quality leader premium is determined in such a way that the solutions for the consumer-based approach under the quality leader's requirement are $\mathring{a}' < \mathring{a}^*$ and $p' > p^*$ (or $p' \leq p^*$), but $V' \geq V^*$ and $q' \geq q^*$.

If product information is not completely available, consumers are not able to observe the product quality directly and producers will have no information on each other's product quality setting. To facilitate exposition, assume that there are two firms with identical demand and cost functions, but differences in quality behavior. Firm 1 takes consumer satisfaction as its goal for quality design, while firm 2 attempts to maximize its profit for quality design and production. It is also assumed that initial price-quality combinations are the same for both firms. In starting year t = 0, both firms designed the quality setting at $\mathring{a}$ in which the quality cost was at the minimum. Consumers are assumed to be uniformly distributed and randomly select a product among the available brands. Clearly, the initial market is equally shared by these two firms. Since firm 1 possesses a changed behavior and takes consumer satisfaction as the objective of quality design at time period 1, t = 1, consumers perceive the change in the quality of product 1 and spread the information in the market. Some consumers switch to product 1 in a time lag, t = 2. The consumer switch will not stop until the market force makes no difference between these two products. It may be the increase in quality of product 2, or a reduction in the price of product 2, or firm 2 going out of business. Assume that firm 2 finds that it suffers from a loss due to inferior product quality and redesigns the product to catch up with the quality competition in time period t = 2. Consumers usually tend to stick to the product they are accustomed to using if other products do not provide more utility surplus. This process will continue for a number of time periods. At the t = n period, although the two firms produce the same product quality, they have different demand curves (number of consumers) and different profits. Firm 1's profit, expected to be greater than firm 2's, depends on the producer and the consumer quality learning rates. The consumers who buy product 1 are better off on the average, because the consumers' quality loss is less than product 2 in the long term.

As a result, either of these two firms recognizes that they greatly benefit from playing the role of the quality leader. Each firm desires to be the quality leader whether or not the other's behavior is governed by its reaction function. It is not efficient to distinguish this quality leadership behavior from the traditional Stackelburg approach in duopoly. It is rational to believe that competition in quality leadership is an incentive to improve quality continuously in monopolistic competition.

It is easy to understand that if a firm possesses a lower quality cost function, advanced quality management, and the behavior of meeting consumer requirements other than the behavior of short-term profit maximization, the firm has the potential to become the quality leader in the industry. The individual firm in monopolistic competition faces his own distinct demand curve of the product which is noticeably different in product quality but with totally substitutable properties for the other products. The product which the firm can sell depends on the consumer decision in product quality selection. The products in the market are different in quality and price. The product quantity demand for firm i is

$$Q_i = f(V_1, V_2, ..., V_n)$$
$$= f[V_1(å_1\text{-}p_1), V_2(å_2\text{-}p_2), ..., V_n(å_n\text{-}p_n)] \quad i = 1, 2, ..., n \quad (7.4.1\text{-}6)$$

where V_i is consumer quality value surplus for product i. With consumer-based utility, the firm production quantity, quality, and profits depend on the consumer's product assessment. If quality information and other information are perfectly available, consumers will first buy the product providing the largest surplus value, and then the product with the next largest value, until the total market demand is satisfied. In the above equation

$$\partial Q_i/\partial V_i > 0, \quad \partial Q_i/\partial V_j < 0 \quad i \neq j \qquad (7.4.1\text{-}7)$$
$$\partial Q_i/\partial V_i + \Sigma \partial Q_i/\partial V_j < 0 \qquad (7.4.1\text{-}8)$$
$$\partial Q_i/\partial V_i + \Sigma \partial Q_i/\partial V_j > 0 \qquad (7.4.1\text{-}9)$$

An increase in quality surplus value for quality improvement of product i with all other product surplus values remaining unchanged results in an increase in the demand for product i. A number of consumers will switch from competitors to firm i, and vice versa. Equations (7.4.1-8) and (7.4.1-9) are the cross consumer surplus effect. Equation (7.4.1-8) assumes that firm i's demand proportion will fall if all firms raise their consumer surplus together. However, firm i demand will increase even though other firms raise consumer surplus in equation (7.4.1-9). In this case, firm i possesses a strong quality leadership.

Although quality decisions made by firm i do not depend on the other firms' decisions, its effects spread among other firms through consumer selection decision. Two specific cases are that when prices are the same, firms compete in quality, and when the qualities are the same firms compete in price.

If $p_i \approx p_j$

$$\partial Q_i/\partial(-å_i) > 0, \quad \partial Q_i/\partial(-å_j) < 0 \tag{7.4.1-10}$$

If $å_i \approx å_j$

$$\partial Q_i/\partial p_i < 0, \quad \partial Q_i/\partial p_j > 0 \tag{7.4.1-11}$$

If consumers are not homogeneous in behavior toward product quality, and budget effect has to be accounted for, equation (7.4.1-7) becomes

$$Q_i = f[V_1(å_1\text{-}p_1), V_2(å_2\text{-}p_2), \ldots, V_k(å_k\text{-}p_k)] \tag{7.4.1-12}$$

$$B \geq p_i, \; i = 1, 2, \ldots, k$$

The consumer will search for the product subject to his budgetary constraint. The quality leader competition is carried out at different market segments.

Under imperfect and biased information, the consumer subjective credit will result in some power or biased product selection. If there are not any special characteristics for consumers to distinguish one product quality from the other, consumers will randomly search and then update quality information to make selection decisions over time periods. As mentioned before, firms compete to play the role of quality leader to gain some advantages, especially under the imperfect information situation. Consumers recognize that the quality leader's products are more reliable and result in less quality loss. This credit reduces the consumer's degree of quality aversion. Since the information premium is one kind of consumer quality loss, the product quantity for the quality leader firm i becomes:

$$\partial Q_i/\partial(\sigma^2_{Ii}) < 0, \qquad \partial Q_i/\partial(\sigma^2_{Ij}) > 0 \qquad (7.4.1\text{-}13)$$

Consumers prefer the quality leader's product to other competitors' products, because it involves less information risk. In most cases, the competition in quality leadership will result in a Stackelburg disequilibrium, which is a benefit for consumers and social welfare.

It is easy to understand that the optimal quality setting is achieved through competition in quality leadership in the quality range between $\mathring{a}^*$ and $\mathring{a}^0$, shown in Figure 7.1. The question is "How do firms compete for quality leadership in a situation in which optimal quality settings have been reached?" In the following cases, firms still compete for quality leadership if a feasible solution can be found. Since some uncertainties exist concerning future markets, and consumer decision procedures and other competitors' actions are unknown, firms are more conservative in product quantity output, but more active in promoting quality settings. Regardless of what happens, one thing is sure: the firm can survive and succeed in the future if it is really a quality leader.

Consumers from time-cumulated experiences and other information sources know that the quality-leader producer usually provides higher-quality products. The degree of consumer risk aversion resulting from quality information and inherent product quality toward the quality-leader's products is reduced or eliminated, which means that the quality loss resulting from the consumer quality premium (information and variation in quality) is much smaller than before. The degree of quality aversion also reflects the consumer quality credit and the consumer quality learning rate under information uncertainty. Quality reputation, therefore, can be expressed in the cumulated quality premium over time periods.

$$R_p = kf(\sigma^2) = k[f(\sigma^2_I) + f(\sigma^2_p)] \qquad (7.4.1\text{-}14)$$

$$\partial f(\sigma^2_p)/\partial x(t) < 0, \;\; \partial f(\sigma^2_I)/\partial t < 0 \qquad (7.4.1\text{-}15)$$

where $f(\sigma^2_I)$ - consumer quality information premium, \$;

$f(\sigma^2_p)$ - consumer premium for variation of product inherent quality, \$;

R_p - consumer reputation for quality leader, \$.

For example, a specific quality reputation may have the following form:

$$R_p = k[(1\text{-}\exp(\text{-}Rt))\sigma^2_i + (1\text{-}\exp(\text{-}rx))\sigma^2_p] \qquad (7.4.1\text{-}16)$$

where R - the consumer quality learning rate;

r - the consumer quality aversion degree.

The higher the learning rate and the asset level, the larger the consumer quality reputation. The consumer quality reputation is related to not only the quality information and the learning rate, but also the inherent product quality produced. Any incorrect quality information and quality cheating can only stand for a very short time and will seriously damage the quality credit for the producer. It will increase the total quality premium and result in a greater consumer quality loss. The quality premium that the consumer would pay under quality reputation is

$$q_p = Q_{po} - R_p \qquad (7.4.1\text{-}17)$$

substitute equation (7.4.1-16) into the above equation

$$q_p = k[\sigma^2{}_I\exp(-Rt) + \sigma^2{}_p\exp(-rx)],\ Q_{po} = k(\sigma^2{}_I + \sigma^2{}_p) \qquad (7.4.1\text{-}18)$$

The product provided by the quality leader's firm may result in higher consumer quality utility than another firm's product designed at the maximization of consumer quality value surplus. As shown in Figure 7-2, the reason is that the higher quality reputation has changed the shape of the consumer's quality preference and the speed of consumer learning under imperfect information. It is easy to determine the quality settings for a one-period product. Assume that product 1 has a historically higher quality reputation, and product 2 has a quality level which is determined by the optimal point of consumer surplus under quality aversion. If the quality leader premium is designed in such a way that

$$(1-\mu_\omega)(f(å') + R_p) - \mu_\omega f(p') \geq (1-\mu_\omega)f(å^*) - \mu_\omega f(p^*) \qquad (7.4.1\text{-}19)$$

then it is easy to derive the results: $å' < å^*$, $p' > p^*$ (or $p' \leq p^*$), $V' \geq V^*$, $q' \geq q^*$. The consumers benefit more from product 1 and buy more of it.

Another situation for producers to play the role of quality leader is in the case of discrete-time product quality settings over multi-periods.

Product quality is a decision variable in the longer term with respect to other variables, such as the quantity output level. For these kinds of products, producers are not able to adjust or improve product quality instantaneously during the product market time. A considerable quality change requires time consuming and capital investment in product design, production system (machinery), manufacturing process, materials and other resources. Assume that the quality change needs T time periods which also corresponds to a noticeable change in consumer behavior and other time-related factors, such as income and interest rate. The interval, in time periods, is determined by the product market time rather than product life, which means a considerable quality change will arise, and could be in monthly, quarterly, or yearly intervals. In the quality-leader approach, the quality level is set up to maximize total consumer quality value surplus over the product market time periods subject to a zero profit condition and both consumer and producer learning constraints. The producer's quality leader premium is chosen so that the maximization of consumer quality surplus value can be implemented in the discrete-time quality choice. For simplicity, assume that all producer surplus in the competition are transformed to the consumer. The consumer quality utility function can be expressed as a function of consumer assets over time periods:

$$F(\mathring{a}/x_t) = (1-\mu_\omega)\mathring{a}(x(t))$$

where $F(\mathring{a}_j/x_t)$ - consumer quality utility function;

$\mathring{a}(x(t))$ - consumer's assets function.

With time change, the consumer utility provided by the product quality designed at t_0 will gradually degrade as the consumer's income and taste changes. This means that the consumer buys the same product but gains less quality utility surplus in comparison with the surplus at the previous period for the same income group. Assume that the consumer assets utility function is concave when the quality setting is fixed over time periods, i.e. $\partial F(å_j/x_t)/\partial x(t) < 0$, $F''(å_j/x_t) > 0$. The product price is discounted in terms of present value. The price function is a convex function over time:

$$F(p) = \mu_{\omega} p[\exp(-it)]$$

where i - discount rate for present value.

Figure 7-3 is used to approximately illustrate the quality leader strategy. Assume that the same price discounted curve is employed for both the quality leader's and the rival's products. The rival firm sets the quality in the position $(å_1, p_1)$ such that the consumer surplus value is maximized at t_0 for discrete-time quality design. The total consumer quality surplus obtained from the rival product over T (the product market time) is the area between these two curves from $å_1$ to $å_1 + \Delta å_1$. The total consumer surplus for the quality leader's product over T time periods is the area from $å_1 - \Delta å$ to $å_1 + \Delta å_1 - \Delta å$. Consumers will have less quality utility surplus during the first couple of periods but will gain more utility surplus in the latter periods than the rival's product. If this situation exists in the market, it can be proved that the quality leader will choose a quality setting $\Delta å$ such that the total consumer surplus over the entire periods can be maximized.

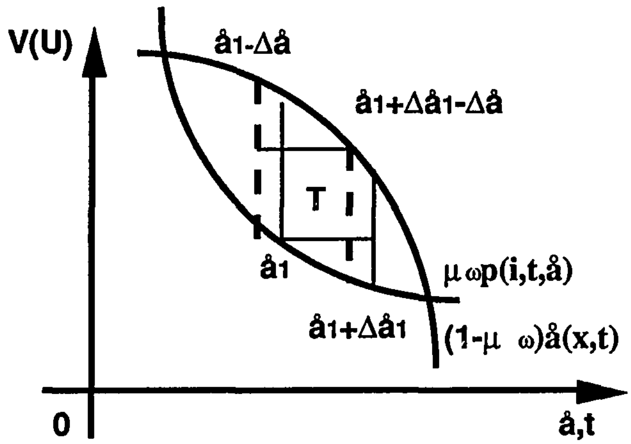

Figure 7-3 Quality Setting Strategy for the Quality Leader Firm

The total consumer surplus over T time periods for the rival's product, S_1, is

$$s_1 = \int_0^T \Delta s_1 dt = \int_{å_1}^{å_1+\Delta å_1} [F(å/x_t) - F(p)]då$$

The function $f(å, p) = F(å/x_t) - F(p)$ is concave. The total consumer surplus for the quality leader's product over the same time periods is

$$s_2 = \int_0^T \Delta s_2 dt = \int_{å_1-\Delta å}^{å_1+\Delta å_1-\Delta å} f(å, p)då$$

$$= s_1 + \int_{å_1-\Delta å}^{å_1} f(å, p)då - \int_{å_1+\Delta å_1-\Delta å}^{å_1+\Delta å_1} f(å, p)då$$

Since $\Delta å_1$ is noticeably larger, and $å_1$ is the optimal quality setting in the static model, then

$$\int_{å_1-\Delta å}^{å_1} f(å, p)då - \int_{å_1+\Delta å_1-\Delta å}^{å_1+\Delta å_1} f(å, p)då \geq 0$$

Thus, the quality leader firm can find a quality setting $\Delta å$ higher than the optimal quality setting in the static model at t_0 by the rival firm to maximize the consumer surplus over the product quality market time. This quality setting, $\Delta å$, depends on the product market time (the longer the market time, the higher the quality setting), and the shapes of the consumer quality utility function and the discounted price function over time periods. The other factors which will be shown in the following model also affect the quality setting. Based on the analysis, the quality leader firm can choose a higher quality setting under the following more advantageous conditions to maximize its long term profit through a consumer-based approach strategy in quality competition.

$$\text{Max} \quad q(å) = \int_0^T [V_1 - V_2]\, dt \qquad (7.4.1\text{-}20)$$

$$= \int_0^T \left[\mu_\omega (p_2 - p_1) \exp(-it) + (1 - \mu_\omega)\left(F\left(å_1/x_t\right) - F\left(å_2/x_t\right)\right)\right] dt$$

s.t. $p_j = [C(å_j) + C(Q) + C/Q]\exp(-m_j t)$

$x_t = x_0 \exp(gt)$

$F(å_j/x_t) = f(å_j/x_t) - q_{pj} = f(å_j/x_t) - k[\sigma^2_{Ij}\exp(-R_j t) + \sigma^2_{pj}\exp(-r_j x)]$

$m_1 > m_2$, $q_{p1} < q_{p2}$, $R_1 > R_2$, $r_1 > r_2$

j =1 for quality leader, j=2 for quality rival where Vi - consumer quality surplus value obtainted from product i;

$F(\mathring{a}_j/x_t)$ - consumer quality value function for product j at year t given the assets x_t;

i - comprehensive discount rate after consideration of interest rate and inflation rate;

T - the product quality market time;

x_0, x_t - consumer assets in time 0 and in time t;

g - income growth rate per year on the average during the product market time;

m - producer quality learning rate.

It should be noted that the model above is different from the product quality surplus value over product life, though the model is somewhat related to the product life cycle. The product market time is much shorter than the product life time. The former depends on the rate of product redesign and innovation as well as the degree of consumer satisfaction. If the product market time is short, $T < \infty$, then the discount rate i in some cases is ignored.

The objective function still contains all values related to consumer quality surplus. All factors in the model vary over time. An increase in consumer income and assets change his or her preference pattern, the quality/price weight distribution μ_ω, the budget level, and the total quality premium. For the sake of comparison, the monetary value which is affected by the inflation rate and interest rate should be discounted into the present value. The quality leader firm introduces the higher quality product at a higher price through an exhibition or advertisements to give consumers the price information as well as images of its quality in order to affect the consumer's decision making. When other conditions change, this product at some later periods would be the best product choice for the consumers. With information available, the consumer's quality premium is eliminated gradually. The producer's strategy for the quality-leader in time periods is to maximize the total consumer-based quality surplus value over time periods of product market time.

Furthermore, in addition to the time needed to achieve the best quality value, the larger quantity output also possesses a time-lag pattern. Considering a product market time-pattern, the quality leader may obtain a better selling record for a higher quality product. This model illustrates the time-characterized quality properties. If the time period required for considerable quality change is longer, the product quality setting is higher in the beginning of quality design. Economic conditions exercise a great influence on time-characterized product quality. The quality competition in economic booming periods pushes producers to set higher quality, while the recession in the economy would force producers to concentrate on price competition and quality improvement, or reduction of quantity output level to cope with future market uncertainty.

The product quality setting heavily depends on the firm's prediction for the economic situation and the income growth rate, g, in the future. i represents the present value discount rate. The term, (g-i), represents the net income growth rate. The bigger the net income growth rate, the higher the quality setting. Since the coefficient of quality/price weight distribution μ_{ω} is determined by the consumer assets level, consumers will put more weight on product quality and prefer the higher quality product when their assets increase. The consumer reputation for the quality leader has less quality value loss in terms of total quality premium. The higher consumer learning rate with a time lag will eliminate the information risk degree sooner than the other competitors' information risk. On the other hand, the producer learning rate is a function of quality setting and management efficiency. Effective quality management methods will enable the quality leader to learn faster in practice to reduce quality cost compared to the others.

The price for the time being is discounted with the rate (i+m) over the periods. The higher the rate m, the larger the consumer value surplus resulting from the lower present value of the product price.

If the second best model with the budgetary constraint is taken into consideration in playing the role of quality leader, and if the rivals set their product qualities at the boundary solution, the quality leader is faced with a decision on quality setting to keep the quality leadership. The firm may still design a higher product quality, which can be distinguished from the competitor's product quality, and sell the product at the same price. The firm may suffer a net loss from such a low price. The firm determines the product quality setting in light of the leader's premium such that the quality leader still benefits from the position. The quality leader premium is determined by the following formula (constraints are the same as the above model except the budget constraint):

$$q(\mathring{a}) = \int_0^T (V_1 - V_2)dt - \int_n^T pe^{-(i-x)t}\,dt \qquad (7.4.1\text{-}21)$$

$$B = p_1 = p_2$$

where V_1 - consumer quality surplus value provided by the quality leader's product;

V_2 - consumer quality surplus value provided by the quality follower's product;

p_1, p_2 - prices for the quality leader's and follower's products, respectively;

x - recovery rate for a loss.

In this case, the optimal quality setting for the quality leader firm may not exist. If $q(\mathring{a}) \geq 0$, the quality leader has the product with the higher quality setting at the price equal to the rival's. The firm loses in the beginning periods for an appropriate price setting but gains more returns in the subsequent periods. The loss is recovered after n periods with a lower discount rate (i-x). Based on the properties of boundary solutions, consumers prefer the higher quality product even when the product price is higher than the rival's in the latter period, which can be compensated by an increase in consumer budget.

If $q(\mathring{a}) < 0$, the firm takes the same strategy as the competitors do. Whichever alternative is adopted, consumers are better off for buying the product provided by the quality leader firm. The product quality provides the same quality performance to consumers when the quality leader produces the product with the same quality as the competitor's. However, from the consumer's point of view, the quality leader's product also provide a moral comfort and reliability, which is another kind of utility that the consumer needs.

The firm plays the role of quality leader in another interesting area, which is to minimize total consumer quality loss over the product life. The higher quality product has a much longer life time. Therefore, the product is designed to be larger than the optimum value gradually wears out during the utilization periods. The firm determines the time lag when the product's quality performance closes at its optimum point of other competitors' products. Promotion in product quality design and its performance also improves other by-product effects, such as reduction in other resource consumption (for example, a car's gas mileage). The negative (positive) by-product effect is external diseconomy (economy). These negative by-product effects put a burden on the consumer and society and are not accounted for in the producer's cost function. The consumer reputation for the quality leader firm results not only from its higher product quality, but also from the responsibility for by-product effects on society in the modern world. This behavior has grown rapidly in the modern world. If the firm does not consider the effect of by-products and social responsibility, it is rarely recognized as the quality leader by consumers.

Minimizing consumer quality utility loss over time periods is equivalent to maximizing the difference between the product quality utilities provided by the quality leader's and the competitor's firms, respectively.

$$\text{Max} \quad q(\mathring{a}) = (1-\mu_\omega)\int[f_1(\mathring{a}_1, s_1, R_1/x_t) - f_2(\mathring{a}_2, s_2, R_2/x_t)]dt +$$

$$(1-\mu_\omega)\int[b_1 - b_2]\exp(-it)dt + \mu_\omega(p_2 - p_1) + \mu_\omega[\varepsilon_1(\mathring{a}_1, \varpi_1) - \varepsilon_2(\mathring{a}_2, \varpi_2)]/(1+i)^T$$

$$\text{s.t.} \quad B \geq \max(p_2, p_1) \tag{7.4.1-22}$$

$$df/ds < 0, \; ds/dt > 0$$

$$df_1/ds_1 > df_2/ds_2, \; ds_1/dt < ds_2/dt$$

$$\mathring{a}_1 < \mathring{a}_2, \; \varpi_1 > \varpi_2; \; \varepsilon_1(\mathring{a}_1, \varpi_1) > \varepsilon_2(\mathring{a}_2, \varpi_2)$$

For positive by-product effect, $b_1 > b_2$

For negative by-product effect, $b_2 > b_1$

where s_j - product quality deterioration rate, j=1,2;

R_j - consumer quality learning rate;

ϖ_j - product value depreciation rate;

ε_j - product residual after T time periods;

b_j - by-product effects in terms of money for product i;

i - consumer utility discount rate.

j = 2 and j = 1 represent the competitor's and the quality leader's products, respectively. The first term is the quality loss due to product quality deterioration over time. S_2 and S_1 are the rates for quality deterioration of the quality-leader's and competitor's product qualities respectively, and $S_2 > S_1$. The total quality premium (inherent quality and information) effect was discussed previously. The consumer utility is discounted as consumer income and assets change with time. The higher the utility discount rate, the higher the quality setting to aviod deteriotation. The second term is the by-product effect, which is usually proportional to the quality setting for the positive effect. The third term is the price effect at the purchase time. The quality-leader firm's cost function determines the product price. The fourth term is related to the residual value of the products after T time periods, which usually is the function of depreciation rate and the quality level remaining in the product.

The higher quality setting has a lower deterioration rate, a higher residual value, and a lower (higher) negatively (positively) by-product effect. Equation (7.4.1-22) is particularly useful for durable goods. In this changing world, the firm can no longer concentrate solely on its profit maximization. Since the consumer-based approach reflects the change in producer behavior in the modern competitive world, the reaction and behavior of consumer attitudes toward the effects arisen by the product quality have a significant impact on the producer profit and utility.

When the quality improvement needs a great deal of investment in the initial research and development stage (R & D), only those producers who have enough capital and intelligence resources can play the role of quality leader. It is called natural quality leadership. Some industries requiring intensive capital and advanced technology possess these characteristics of natural quality leadership, such as computer, automobile, aerospace and military industries. The special quality patents also belong to this case. The natural quality leadership is often associated with the natural monopoly, duopoly and oligopoly markets. The other small sized firms play the role of quality followers to compensate for the shortcoming in R and D investment. They adopt a "wait and see" strategy. Once the quality leader introduces a higher quality product or a new product different from the previous one, the followers can rapidly catch up with the product quality with much lower expenses and then compete with the quality leader in lower price. Therefore, the competition is "advantage in quality but competitive in price" for the quality leader while it is "advantage in price but competitive in quality" for the quality follower. Based on the consumer-based approach, the quality leadership premium is computed as the difference between the quality surplus values resulting from the quality leader's and follower's products respectively:

$$\text{Max } q(å) = \int_0^T V_1[f(å_1, R_1/x_t), f(p_1)]\, dt - \int_0^T V_2[f(å_2, R_2/x_t), f(p_2)]\, dt \qquad (7.4.1\text{-}23)$$

$$\text{s.t. } p_1 = [C(å_1) + C_1(Q_1) + C(I_1)]\exp(-it)$$

$$p_2 = [C(å_2) + C_2(Q_2) + C(I_2)]\exp(-it)$$

$$å_1 \leq å_2, \; I_1 >> I_2$$

$$x_t = x_0 \exp(gt)$$

where I_j - capital investment for quality leader, j =1; for the follower, j=2, $;

R_1 - consumer reputation for quality leader, $;

R_2 - consumer learning rate for follower, $.

To gain the advantage of quality leadership, the firm should examine consumer behavior correctly and devise an effective market strategy, thus returning a significant part of investment in the early years of a product's market time before the follower's product comes onto the market. The quality leader firm should speed up the quality innovation rate to shorten the product market time. Meanwhile, the quality leader firm has to meet the challenge from other large-sized rivals to replace it with the position of quality leadership. The large-sized rivals cannot play the same role of the follower as the small-sized firms do because they would suffer a significant loss for a longer waiting period and market share. They may survive the competition of quality leadership. The difficulty of entering and exiting the natural quality leadership industry also means more gain (or loss) in quality leadership competition.

From the above analysis, the competition in maintaining and replacing a quality leadership position results in benefits for consumers and social welfare in most cases of quality activities. However, in some special cases, the competition in quality leadership may result in an over-utilization, or waste of resources, if either consumer or producer behaviors are not correctly assessed. The military industry is such an example. The consumer, the Pentagon, has a particular interest in product quality, acquiring faster, more accurate, more powerful weapon products. Its cost function (the product price) is not correctly and economically set up for the special consumer, and the weight for quality/price distributions is totally or almost totally put on the quality. Thus, as a requirement of product quality, the Pentagon preference is biased. A part of resources is wasted in the pursuit of high quality, and the whole system quality balance is ignored. The correct way to make a decision for defense expenses is to set up the Pentagon's quality preference and weight for quality/price distribution correctly. In this way, the quality leadership competition in military industry can be effectively carried out.

7.4.2 Quality Related Cash-Back and Rebate Policies under Market Uncertainty

The policies of cash-back and rebate adopted by the producers (or sellers) usually is treated as an effective price reduction to increase the sale under demand and market uncertainty. The higher inventory cost drives the producer's profits down significantly. In the market investigation, the producers providing consumers with higher quality surplus values are less active in implementing the policies above to improve their inventory situations and production stability. These products have lower inventory costs and good selling records. To compete with the other rivals in quality and to cope with demand uncertainty, the producer uses a cash-back or rebate policy to reduce the real price to increase the consumer quality value surplus. The price effect may significantly change consumer demand so that the high-cost inventory and selling loss could be improved.

The information for quality settings of the competitor's products is not totally available. Product quality is predetermined by the producers based on their long-term objectives. Once the products are introduced onto the market, the quality cannot be changed immediately. If the quality is inferior in comparison with other firms' product qualities, some consumers turn to the higher quality products, and loss in sales and high inventory cost will occur and, moreover, an uncertainty or decrease in demand side forces the producers either to cut the quantity output level or reduction of the product price or both. We will illustrate the role of the quality related rebate policy and the effect of quality improvement to cope with the uncertainty.

Rebate is a flexible policy, similar to cash-back, to reduce the real price to cope with demand uncertainty and quality problems. The producer's profit resulting from the rebate policy is

$$\pi = \eta[p-(1-\rho)(b/(1+r)+c)](Q+S) + p(1-\eta)(Q-S)-C_0Q-C_IS \quad (7.4.2\text{-}1)$$

where p - initial price, \$/unit;

ρ - probability of consumer failure to claim rebate;

b - rebate, \$/unit;

r - interest rate during rebate effective periods;

c - rebate service cost, \$/unit;

η - possibility for product demand at rebate;

$1 - \eta$ - possibility for product demand in future market;

C_o - production cost, \$/unit;

C_I - average inventory cost, \$/unit;

S - inventory, unit.

When the demand decreases, the producers first cut the level of production output to reduce inventory cost such that the expected profit becomes

$$\pi_1 = pQ_1 - \eta(1-\rho)(b/(1+r) + c)Q_1 - C_1Q_1 \qquad (7.4.2\text{-}2)$$

$$\pi_1 = [p - \eta(1-\rho)(b/(1+r) + c) - C_1]Q_1$$

$$Q_1 = Q - S, \quad C_1 \geq C_0 \qquad (7.4.2\text{-}3)$$

η reflects the degree of uncertainty in the economy. If η and the rebate service are high, the producer may adopt the strategy of reduction in price. Rebate is a flexible policy to increase selling and decrease inventory, because frequent changes in price will result in the loss of the consumer's credit for the product. If the rebate does not reduce the inventory significantly, the producer may further cut the production level. However, this may create some problems in society. Another effective method is to improve product quality to affect the consumer's behavior. Assume that the producer wants to keep a certain level of the expected profit and to examine the potential effect of the strategy on the future market. In the quality improvement strategy, the price remains unchanged, and the quality is promoted such that

$$\pi_2 = (p - C_2)Q_2 \qquad (7.4.2\text{-}4)$$

If $C_2 = \eta(1-\rho)(b/(1+r) + C) + C_1, \quad Q_1 \approx Q_2,$

$$\mathring{a}_1 < \mathring{a}_2 \quad \pi_2 \geq \pi_1 \tag{7.4.2-5}$$

Recall that due to the effect of quality resistance during the economic uncertainty described in Chapter 4, consumers are more biased in favor of a higher quality product at competitive prices. One of the economic characteristics for a recession period is that the supply exceeds the demand. Consumers are more conservative concerning quality uncertainty and risk. The impact of economic situations on the production output level for higher quality products is much less than that for lower quality products. The production is quite stable and smooth over time, and the policy of lower inventory, or just in time (JIT), can be implemented. The producers with higher quality rent can survive more easily in a changeable environment. This situation is observable in the industries with high inventory costs.

The weight distribution for quality/price, μ_ω, is also changed due to a real income decrease. If the price effect is greater than the quality effect for some products, i.e. $\mu_\omega > 1/2$, the rebate (or cash-back) is more efficient. If $\mu_\omega < 1/2$, the quality promotion is preferred. For the uncertainty in a future market, the reduction in product output is far inferior to the quality improvement.

7.4.3 Quality vs Quantity Under the Constrained Investment

Assume that a firm wants to expand its production capacity and market share through investment to gain higher profit. The firm can adopt the strategy either by increasing output level Q, or by promoting quality level $\mathring{a}$. The firm is also faced with the possibility of employing a mixed strategy to increase both quality setting and quantity level. The firm expects to maximize its profit from the strategy adopted. Several fundamental assumptions about the constrained investment program are described as follows.

(1) The initial product quality setting is located at the point which has the lowest quality cost. Quantity output is oriented at the optimal conditions. Any increase in quantity or quality will result in cost-up.

(2) Production system capability is improved by investment.

(3) No uncertainty is involved in the analysis except when noted.

(4) The firm is monopolist. Consumers are homogeneous.

(5) The explosion, quality and quantity output levels are assumed to be the same in each period over all the periods to return the investment.

A firm's decision variable is the investment I, which determines the quality setting å, quantity output level Q, or both maximize its expected profit. Assume that the investment is a function of quality, quantity and capital cost

$$I_0 = I(å, Q, C_r) \quad (7.4.3\text{-}1)$$

$$dI/d(-å) > 0, \; I''_{å} > 0 \quad (7.4.3\text{-}2)$$

$$dI/dQ > 0, \; I''_Q \geq 0 \quad (7.4.3\text{-}3)$$

$$dI/dC_r < 0 \quad (7.4.3\text{-}4)$$

where I_0 - investment, \$;

C_r - cost of capital, percentage of investment, %.

Equations from (7.4.3-2) through (7.4.3-3) indicate that investment is assumed to be a convex function for quality improvement and quantity increment, respectively. The higher the cost of capital, the lower the investment. For the conventional profit maximization with quantity strategy, as mentioned in the previous chapters, suppose the product quality is set at the minimum cost point. The firm concentrates on quantity output level to maximize its profit. Assume the demand for the product is increasing and the cost of capital is constant. The cost function under investment becomes

$$C_0(Q_0/å=å_0) + C_r I(Q/å=å_0) = C(Q/å=å_0) \qquad (7.4.3\text{-}5)$$

The expected profit is

$$\pi = R(Q/å=å_0) - C(Q/å=å_0)$$

$$= R(Q/å=å_0) - C_0(Q_0/å=å_0) - C_r I(Q/å=å_0) \qquad (7.4.3\text{-}6)$$

$$d\pi/dQ = R'_Q - C'_0 - C_r I'_Q = 0$$

$$R'_Q = C'_0 + C_r I'_Q \qquad (7.4.3\text{-}7)$$

$$Q > Q_0 \qquad (7.4.3\text{-}8)$$

where $C_0(.)$ - cost function before investment;

$C(.)$ - cost function after investment;

$R(.)$ - revenue after investment.

The new production output is set at a level such that the marginal revenue equals marginal cost. The above profit maximization under investment is greater than the previous one, i.e. $\pi > \pi_0$; the producer invests in promotion of the production output level in order to gain more profit. If the output level is greater than before, the consumer and the society are better off, and the consumer surplus is larger.

It has been shown that the consumer quality surplus value increased with quality improvement if the product price was constant or under the following limitation:

$$dp/d(-å) < MRS_{på}$$

specifically in the linear form,

$$dp/d(-å) < (1-\mu_{\omega})/\mu_{\omega} \qquad (7.4.3\text{-}9)$$

The producer invests in the promotion of product quality to maximize his profit under the assumption of constant consumer surplus and constant output level. The unit cost function is

$$C_0(å_0/Q=Q_0) + C_r I(å/Q=Q_0) = C(å/Q=Q_0) \qquad (7.4.3\text{-}10)$$

The expected profit is

$$\pi = R(å/Q=Q_0) - C(å/Q=Q_0)Q_0$$

$$= R(å/Q=Q_0) - [C_0(å_0/Q=Q_0) - C_r I(å/Q=Q_0)]Q_0 \qquad (7.4.3\text{-}11)$$

$$d\pi/d(-å) = p'_{å} - C'_0 - C_r I'_{å} = 0$$

$$p'_{å} = C'_0 + C_r I'_{å} \qquad (7.4.3\text{-}12)$$

$$p = p_0 + (1-\mu_{\omega})d(-å)/\mu_{\omega} \qquad (7.4.3\text{-}13)$$

where p_0 - the initial price before investment;

p - the product price after investment

Since the consumer surplus value is constant, consumers are indifferent between products with and without quality improvement. For heterogeneous consumers, the producer will provide a set of products with different quality utility surplus for consumer self-selection to maximize its total profit.

In the above two investment strategies, the monopolist benefits from quality investment $C_r I'_å$ more than the quantity strategy because the consumer surplus increase from quality improvement is exploited by the firm. However, under a monopolistic competition for these two approaches the consumer will choose the product from the first approach for a larger surplus.

As mentioned in previous chapters, the consumer demand is a function of quality, quantity and price rather than quantity and price only. It is possible for the producer to find an optimal allocation of investment between quality and output levels to maximize its profit. Suppose that all the investment is used up in the belief that the return on investment (ROI) is higher than the alternative of doing nothing.

The model is

$$\text{Max } \pi = R(å, Q) - C(å, Q) \qquad (7.4.3\text{-}14)$$

$$\text{s.t. } I = I(å, Q, C_r)$$

$$L = \pi + \lambda[I - I(å, Q, C_r)]$$

$$\partial L/\partial(-å) = \pi'_å - \lambda C_r I'_å = 0$$

$$\partial L/\partial Q = \pi'_Q - \lambda C_r I'_Q = 0$$

$$\partial L/\partial \lambda = I - I(å, Q, C_r) = 0$$

If C_r is variable,

$$\partial L/\partial C_r = \partial \pi'/\partial C_r - \lambda \partial I/\partial C_r = 0$$

$$\pi'_Q/\pi'_{\mathring{a}} = I'_Q/I'_{\mathring{a}} \qquad (7.4.3\text{-}15)$$

$$R'_{\mathring{a}} = C'_{\mathring{a}} + C_r I'_{\mathring{a}},\ R'_Q = C'_Q + C_r I'_Q$$

$$R'' < 0, C'' > 0, I'' > 0$$

$$\partial I/\partial C_r < 0,\ \partial\pi'/\partial C_r < 0$$

First-order conditions state that the ratio of the marginal quality and the marginal quantity attribution on the total profit must be equal to the ratio of marginal investment in quality improvement and the quantity increment. One can find a point on the quality level which determines the corresponding price and quantity in such a way that the optimal profit can be achieved. If the cost of capital is higher, the profit gained from quality and quantity investment is lower. The second order conditions for profit maximization are satisfied for the model. The quantity production function, quality loss function, price function and cost function as well as investment function are pre-assumed to be strictly concave and convex in the regions of quality and quantity where the first-order conditions are met in order to guarantee the existence of profit maximization.

The producer's profit decreases when the cost of investment, C_r, increases. The results illustrate that the firm can only achieve the profit maximization by satisfying the above investment roles. The conventional investment theory on quantity or quality alone may not be the global optimal. Furthermore, the firm with a low cost of capital will gain more from both quality improvement and quantity increment. The advantage of this approach is to maximize the producer's profit by accounting for consumer behavior regarding quantity, quality and price.

To expose the implication of the above optimal investment in quantity and quality, equation (7.4.3-15) can be explicitly transformed into the following form of functions.

$$\pi'_Q/\pi'_{å} = I'_Q/I'_{å}$$

$$I'_{å} = \pi'_{å} I'_Q/\pi'_Q = I'_Q(R'_{å} - C'_{å})/(R'_Q - C'_Q)$$

$$= (R'_{å} - C'_{å})(I'_Q/R'_Q)/(1 - C'_Q/R'_Q) \qquad (7.4.3\text{-}16)$$

Substitute $I'_{å} = f(\Delta å)$, $I'_Q = f(\Delta Q)$, $R'_{å} = f((1-\mu_\omega)\Delta å/\mu_\omega)$ and $R'_Q = f(\Delta p)$ into equation (7.4.3-12)

$$f(\Delta å) = [f((1-\mu_\omega)\Delta å/\mu_\omega) - C'_{å}](f(\Delta Q)/f(\Delta p)/(1 - C'_Q/f(\Delta p))$$

$$= [f(\mu_\omega, \Delta å) - C'_{å}]f(e)/(1 - C'_Q/f(\Delta p)) \qquad (7.4.3\text{-}17)$$

where e is the demand curve slope. The quality level under the investment is positively related to the marginal quantity cost, the demand curve slope, and the consumer weight distribution for quality/price, but negatively related to the marginal quality cost and the cost of capital. The optimal quality and quantity levels under investment are determined by the first-order conditions. The effective investment decisions should be determined by these factors. For a specific production procedure and process, one should first find the order of effects of each factor on the quality improvement and then invest efficiently in the factor which has the largest effect on quality promotion.

All of the above approaches are producer-based ones rather than consumer-based. However, the uncertainties, such as imperfect information and unknown consumer demand, are in some extent involved in the producer's decision making. The producer-based profit maximization approach may not be realized under uncertainty. This serious problem may be overcome by adoption of the consumer-based surplus approach.

To examine the effect of investment on consumer quality surplus value, assume that free entry and exit derive a zero-profit in quality competition (see section 7.4.1). The initial quality setting is located at the point with the minimum cost, and quantity output is set in the optimal position, which means that any improvement in quality and increment in quantity will result in a cost increase. The cost of capital is assumed to be zero in the discussion of the consumer-based approach in the previous chapters. But the cost of capital is now no longer free for quality activity.

Assume that the optimal quality setting å' and the optimal quantity level Q' can be achieved by investing I. The price is

$$p_{å',Q'} = f\{C_r[I(å', Q')] + C(å^*, Q^*)\} \qquad (7.4.3\text{-}18)$$

If $C_r = 0$, the optimal quality setting in the consumer-based approach is $å^*$. If $C_r > 0$, the optimal quality setting is å'. $å' > å^*$, $V' < V^*$, and $q' < q^*$. The quality setting, the consumer surplus and the demand are negatively related to the cost of capital while the price is positively associated with it. The price is undetermined in comparison with the price at $C_r = 0$. Since uncertainty and competition are involved in the future market and the consumer demand, the consideration for the quality and quantity with profit maximization alone may not be adequate to cope with the situations. The two-stage model (see equation(7.4.3-14)) is employed in combination with the producer's prediction for future market conditions. The first stage determines the optimal quality setting realized by quality investment. The demand curve shift due to quality improvement mainly relies on the change in the consumer quality value surplus plus consideration for other factors affecting the consumer demand. The price and the quantity output are determined in the second stage of investment for profit maximization, which is the same as the conventional monopolist profit maximization. The optimal conditions for the investment described before may not be met, but the consumer surplus is larger when the output level is larger than before, such that the advantage to cope with the uncertainty and competition can be gained. The total investment is

$$I(å^*, Q^*) = [I(å^* = å_0 - \Delta å)] \ I(Q^* = Q_0 + \Delta Q) \qquad (7.4.3\text{-}19)$$

where $å^*$ and Q^* are the optimal quality and quantity settings. The first term in the equation is the unit investment to improve the product quality, and the second term is the amount of investment for output quantity. A dynamic model over time periods will be more adequate to describe the optimal investment procedures.

Chapter 8 Conclusions and Directions for Further Research

In past decades, the practice of quality control and quality design has provided sufficient and necessary conditions for development of quality economics. The increased competition in quality has resulted in the need for theoretical and practical developments to illustrate how a firm can survive and succeed in the future.

This research is focused on the study of both consumer and producer behaviors under quality variation as well as the interactions of both sides in quality related decision making. Producers should take consumer requirements and satisfaction as their objectives in quality activities, whereas consumer quality behavior and attitude are affected by producer market strategies.

This research, for the first time, derives the concept of consumer quality discrimination which reflects the fact that consumers pay the same price but get different product qualities. With respect to the highest product quality other consumers obtained at the same price, the consumer suffers a quality loss due to lower product quality. The average consumer quality loss (ACQL) depends on the consumer quality preference function, product quality performance, consumer assets, budget, attitude toward quality risk, quality information availability and product inherent quality variation. According to the specific research purpose, the comprehensive consumer quality loss function is flexible enough so that it can be easily modified and simplified.

Compared with Taguchi's loss function, the consumer quality loss function is more adequate to illustrate consumer quality evaluation on his choice of a product from a set of available competing brands. The inherent shortcomings in Taguchi's loss function limits its application in consumer quality selection and product quality design to meet consumer requirements.

This research also employs a comprehensive fuzzy set model with empirical data to show that consumers do have their weight distribution for various quality attribute components and price/quality effect. This information should be transformed into the producer's objectives for a quality system design in order to meet consumer satisfaction as much as possible.

The market share pattern among completely substitutable products under information uncertainty relies on the access to the information source and the consumer weight for price/quality which is assumed to follow a Weibull distribution. To measure the effect of quality variation on social welfare, neither Taguchi's loss function nor the consumer quality loss function developed in this research are suitable for such assessment. The consumer quality value surplus, which contains quality loss and price effect, can be used effectively to measure the consumer benefits of quality improvement. Therefore, whether or not a quality activity is adopted depends on its overall effect on consumer quality surplus.

The relationships among quality, quantity and price in consumer demand are established in Chapter 6. An equilibrium in three dimensions may be more rational to reflect the real market mechanism. However, it may be impossible to determine all three variables simultaneously if no one of them is predetermined. This research proposes a two-stage model to cope with the impossibility of simultaneous determination of three variables. That is, one variable is predetermined in the first stage and the other two variables are set in the second stage. From a quality engineering and production procedures point of view, quality in most cases is the variable to be determined in the first stage for the firm's long-term goal.

The producer's production profit is realized through consumer product purchase. The conventional approaches to producer optimal behavior are not adequate to describe quality competition, even though the motivations behind quality improvement are still to make profit and stay in business in the long-term. A consumer-based approach is developed to determine the product quality setting. The model reveals that the higher the market uncertainty, the higher the quality setting. In this model, both consumer and producer behaviors are combined and interact with each other. Besides the factors in consumer quality loss function, the cost of capital, cost function (quality and quantity), producer learning rate and producer quality premium also affect the product quality setting. Quality cost function is the crucial factor for the establishment of the quality model. Some conflicts and incorrect conclusions in previous research have resulted from invalid assumptions about the properties of quality cost function. A new set of criteria is used to confirm whether the quality design or quality activity is economically optimized.

In this research a quality leadership approach is established, for the first time, to illustrate producer behavior in quality competition. The producer is willing to pay the quality premium to play the role of quality leader to reduce the consumer quality loss due to imperfect information and inferior quality. As a result of being in the leadership position, the producer gains consumer good will, the higher learning rate, and the larger market share as well as higher profit. Competition for quality leadership explains why the product quality setting is an issue that must be pursued continuously and why quality setting is higher than that determined in the profit maximization approach, or even in the consumer-based approach. The higher than the optimum quality setting may result in loss of social welfare in some cases.

Two examples, a quality-related market policy and the optimal investment direction, are used to emphasize that conventional approaches excluding quality features is no longer complete and needs modification when quality is involved in the main theme and in the mathematical models.

A set of simplified and computable functions, such as consumer quality loss function, is provided in the research. These functions are not only used in theoretical arguments, but, in a more meaningful way, offer an opportunity for application in quality engineering and quality management practice. This change overcomes the problem that the theoretical concepts, in some contents, are isolated from practical application.

Compared with the development in quality engineering and quality control approaches, the subject of quality economics is relatively underdeveloped and its importance necessarily needs more effort and rapid developments. The quality, quantity and price are three major factors in microeconomics, and their relationships should be fully understood and developed. Therefore, the effect of quality activity which is not separated from the interactions with other variables can be evaluated completely.

Consumer decision making in quality evaluation is complicated, especially under imperfect information. Although there are many models to describe consumer behavior, none of the alternatives is superior to the others. Some important variables may not have been considered in the models. How consumers learn from product quality experience and other imperfect information sources and what factors affect consuming learning patterns require further studies.

How a producer's profit is realized through a consumer-based approach is not clearly answered. What is different for quality settings by using the profit maximization approach and by using the consumer surplus value maximization approach? Which one is more realistic to describe producer quality behavior? Under what conditions is a quality strategy better than other strategies? It is well recognized that the managerial method is one of the most effective ways to achieve the quality goal. How does the managerial effort control variable affect other variables quantitatively? All of these questions need further development in future research.

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APPROVAL OF EXAMINING COMMITTEE

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