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HAL Id: hal-00733633

https://hal-upec-upem.archives-ouvertes.fr/hal-00733633

Submitted on 19 Sep 2012

AN EM APPROACH FOR POISSON-GAUSSIAN NOISE MODELING

Anna Jezierska, Caroline Chaux, Jean-Christophe Pesquet, and Hugues Talbot

Université Paris-Est, Lab. Informatique Gaspard Monge, UMR CNRS 8049,
Champs-sur-Marne, 77454 Marne-la-Vallée, France
{first.last}@univ-paris-est.fr

ABSTRACT

This paper deals with noise parameter estimation. We assume observations corrupted by noise modelled as a sum of two random processes: one Poisson and the other a (nonzero mean) Gaussian. Such problems arise in various applications, e.g. in astronomy and confocal microscopy imaging. To estimate noise parameters, we propose an iterative algorithm based on an Expectation-Maximization approach. This allows us to jointly estimate the scale parameter of the Poisson component and the mean and variance of the Gaussian one. Moreover, an adequate initialization based on cumulants is provided. Numerical difficulties arising from the procedure are also addressed. To validate the proposed method in terms of accuracy and robustness, tests are performed on synthetic data. The good performance of the method is also demonstrated in a denoising experiment on real data.

1. INTRODUCTION

In many real world problems, data are corrupted by random noise. Although the physical properties of acquisition systems often lead us to consider a specific probabilistic model for the noise, its parameters are usually unknown. For many simple probability distributions (e.g. Gaussian, Poisson,...), standard estimators are available, such as those provided by the Maximum Likelihood (ML), moment estimates or order statistics. When the noise is Poisson distributed, the Anscombe transform [1] can be also applied so that the noise can be considered as approximately following a Gaussian distribution. However, at low count, the Anscombe transform introduces a significant bias. There exist stabilization methods for more complex forms of noise, but noise parameters must still be known.

When the noise takes a more complicated form such as a combination of Gaussian and Poisson distributions, and the signal-to-noise ratio is low, specific algorithms need to be designed. Such scenarios occur for example in, astronomy [2], medical imaging [10], microscopy imaging [12] but also in MACROscopy imaging.

In the literature, the noise estimation problem has been investigated either from a single or from several signal realizations. The estimation from a single realization is an underdetermined problem. Therefore some prior knowledge concerning the signal of interest must be included into the model. Parametric estimation problems for a Poisson plus zero-mean Gaussian noise from a single image were addressed, for example in [6, 8]. Since in [6] the authors focus on CCD camera applications, the expected signal-to-noise ratio is relatively high and the Anscombe transform can be success-

fully applied. Similarly, the method derived in [8] was developed for CCD camera images where the assumption of a zero-mean Gaussian component is well founded. Estimators based on several realizations [7] are more reliable and they do not necessarily require prior information about the target signal. Moreover, in many practical applications such as microscopy, several signal acquisitions are feasible (possibly through a calibration process).

The literature on parameter estimation for a Poisson plus Gaussian noise with nonzero mean, especially when a low level signal is expected, is very limited. Among the few contributions dealing with this problem, the author in [15] proposes a cumulant-based approach, whereas in [3, 4], the authors make use of the Anscombe transform and a regression based approach. These algorithms can compute noise parameters for denoising procedures, in which they are normally assumed to be known [2, 9].

In this work, we propose a multivariate parameter estimation method when the noise is assumed to be a combination of Poisson and Gaussian components. More precisely, we aim at performing an accurate estimation of the scale parameter α of the Poisson component , the mean c and the variance σ^2 of the Gaussian one. We first point out the limitations of an ML approach. This leads us to develop an Expectation-Maximization (EM) algorithm, the numerical implementation of which is discussed. Although the EM algorithm is a popular solution in statistical signal processing (see [13] for Gaussian mixtures or [5] for Poisson noise), its use in the present context appears to be new. One of the crucial step in the proposed approach is the initialization of the EM iterative procedure. This initialization is realized by an accurate cumulant-based method.

The remaining of the paper is organised as follows: In Section 2 we present the considered model and introduce the notation used in this work. In Section 3, we briefly discuss ML estimation difficulties. Our algorithm is then described in Section 4. Finally, simulations are made in Section 5 showing the good performance and the robustness of the proposed approach.

2. PROBLEM

We consider data $(u_s)_{1 \leq s \leq S}$ where s corresponds to a location index (e.g. locating pixel (x, y) in 2D or (x, y, z) in 3D), which are corrupted by a Poisson-Gaussian noise, and for which we observe T realizations. Each realization will be indexed by $t \in \{1, \ldots, T\}$, which can be a time index.

Such a framework leads us to the following model: $(\forall s \in \{1, \ldots, S\})(\forall t \in \{1, \ldots, T\})$

$$R_{s,t} = \alpha Q_{s,t} + N_{s,t} \tag{1}$$

where $Q_{s,t} \sim \mathscr{P}(u_s)$, $N_{s,t} \sim \mathscr{N}(c, \sigma^2)$, $\alpha \in \mathbb{R}$ is a scaling

parameter, $(u_s)_{1\leq s\leq S}$ is the "clean" signal, and $c \in \mathbb{R}$ (resp. $\sigma > 0$) is the mean (resp. standard deviation) of the Gaussian noise.

The problem is then to estimate $u = (u_s)_{1\leq s\leq S}$, α, c and σ from the available observation field $r = (r_{s,t})_{1\leq s\leq S,1\leq t\leq T}$, which is a realization of a random field $R = (R_{s,t})_{1\leq s\leq S,1\leq t\leq T}$. We have thus $S+3$ parameters to estimate.

In the following, it is assumed that u is deterministic and that $Q = (Q_{s,t})_{1\leq s\leq S,1\leq t\leq T}$ and $N = (N_{s,t})_{1\leq s\leq S,1\leq t\leq T}$ are mutually independent random fields. In addition, the components of N (resp. Q) are assumed to be independent.

3. MAXIMUM LIKELIHOOD ESTIMATOR

The ML estimate of the parameters is defined as

$$(\widehat{u},\widehat{\alpha},\widehat{c},\widehat{\sigma}) = \underset{(u,\alpha,c,\sigma)}{\operatorname{argmax}} \, f_R(r \mid u,\alpha,c,\sigma). \tag{2}$$

where $f_R(\cdot \mid u,\alpha,c,\sigma)$ is the probability density function (pdf) of R. The ML estimator is known to usually have better statistical performance than moment estimates [14].

Let $p_{R,Q}(\cdot,\cdot \mid u,\alpha,c,\sigma)$ denote the mixed continuous-discrete probability distribution of (R,Q). By using Bayes rule, we get

$$p_{R,Q}(r,q \mid u,\alpha,c,\sigma) = f_{R\mid Q=q}(r \mid u,\alpha,c,\sigma)\mathsf{P}(Q=q \mid u)$$
$$= f_N(r-\alpha q \mid c,\sigma)\mathsf{P}(Q=q \mid u) \tag{3}$$

where $f_{R\mid Q=q}(\cdot \mid u,\alpha,c,\sigma)$ is the conditional pdf of R knowing that $Q = q$ and $f_N(\cdot \mid c,\sigma)$ is the pdf of N.

The desired likelihood (using the independence assumption for the components of N (resp. Q)) thus takes the following form:

$$f_R(r \mid u,\alpha,c,\sigma) = \sum_{q\in\mathbb{N}^{ST}} p_{R,Q}(r,q \mid u,\alpha,c,\sigma)$$
$$= \frac{1}{(2\pi)^{ST/2}\sigma^{ST}} \prod_{s=1}^{S} \exp(-Tu_s)$$
$$\prod_{t=1}^{T} \sum_{q_{s,t}=1}^{+\infty} \exp\left(-\frac{(r_{s,t}-\alpha q_{s,t}-c)^2}{2\sigma^2}\right) \frac{u_s^{q_{s,t}}}{q_{s,t}!}.$$

The likelihood takes a rather intricate form, which makes the computation of the ML estimator quite difficult.

4. PROPOSED ALGORITHM

4.1 Expectation-Maximization approach

A possible way of circumventing the aforementioned difficulty consists of resorting to an EM algorithm. In this case, R is viewed as an incomplete random vector that must be completed by another vector. We propose here to consider that the completed vector is (R,Q). For conciseness, let us define $\theta = (u,\alpha,c,\sigma)$. The EM algorithm is given by the following iteration:

$$(\forall n \in \mathbb{N}) \qquad \theta^{(n+1)} = \underset{\theta}{\operatorname{argmax}} \, J(\theta \mid \theta^{(n)}), \tag{4}$$

where $J(\theta \mid \theta^{(n)}) = \mathsf{E}_{Q\mid R=r,\theta^{(n)}}[\ln p_{R,Q}(R,Q \mid \theta)]$. According to (3), we have

$$-\ln p_{R,Q}(R,Q \mid \theta) = \frac{1}{2\sigma^2} \sum_{s=1}^{S}\sum_{t=1}^{T}(R_{s,t}-\alpha Q_{s,t}-c)^2 \tag{5}$$

$$+\frac{ST}{2}\ln(2\pi\sigma^2) + T\sum_{s=1}^{S} u_s - \sum_{s=1}^{S}\ln u_s \sum_{t=1}^{T} Q_{s,t} + \sum_{s=1}^{S}\sum_{t=1}^{T}\ln(Q_{s,t}!).$$

By dropping the terms that are independent of θ, we see that the EM algorithm reduces to

$$(\forall n \in \mathbb{N}) \qquad \theta^{(n+1)} = \underset{\theta}{\operatorname{argmin}} \, \widetilde{J}(\theta \mid \theta^{(n)}), \tag{6}$$

where

$$\widetilde{J}(\theta \mid \theta^{(n)}) = \frac{1}{2\sigma^2}\sum_{s=1}^{S}\sum_{t=1}^{T}\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[(r_{s,t}-\alpha Q_{s,t}-c)^2]$$

$$+\frac{ST}{2}\ln(\sigma^2) + T\sum_{s=1}^{S} u_s - \sum_{s=1}^{S}\ln u_s \sum_{t=1}^{T}\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}]. \tag{7}$$

This leads us to the following iterative solution: for every n,

$$(\forall s \in \{1,\ldots,S\}) \quad u_s^{(n+1)} = \frac{1}{T}\sum_{t=1}^{T}\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}] \tag{8}$$

$$\begin{bmatrix} ST & \sum_{s,t}\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}] \\ \sum_{s,t}\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}] & \sum_{s,t}\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}^2] \end{bmatrix}\begin{bmatrix} c^{(n+1)} \\ \alpha^{(n+1)} \end{bmatrix}$$
$$= \begin{bmatrix} \sum_{s,t} r_{s,t} \\ \sum_{s,t} r_{s,t}\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}] \end{bmatrix} \tag{9}$$

$$(\sigma^2)^{(n+1)} = \tag{10}$$
$$\frac{1}{ST}\sum_{s=1}^{S}\sum_{t=1}^{T} r_{s,t}\left(r_{s,t}-\alpha^{(n+1)}\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}] - c^{(n+1)}\right).$$

So, provided that we are able to compute $\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}]$ and $\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}^2]$, the implementation of the EM algorithm is quite simple.

Let us now turn our attention to the computation of the required conditional mean values. For every $(t,s) \in \{1,\ldots,T\}\times\{1,\ldots,S\}$, we have

$$\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}] = \sum_{q_{s,t}=1}^{+\infty} q_{s,t}\mathsf{P}(Q_{s,t}=q_{s,t} \mid R=r,\theta^{(n)}). \tag{11}$$

In addition,

$$\mathsf{P}(Q_{s,t}=q_{s,t} \mid R=r,\theta^{(n)}) = \frac{p_{R_{s,t},Q_{s,t}}(r_{s,t},q_{s,t} \mid \theta^{(n)})}{f_{R_{s,t}}(r_{s,t} \mid \theta^{(n)})}. \tag{12}$$

Using again (3), this implies

$$\mathsf{E}_{Q\mid R=r,\theta^{(n)}}[Q_{s,t}] = \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}, \tag{13}$$

where

$$\zeta_{s,t}^{(n)} = \sum_{q_{s,t}=1}^{+\infty} \exp\left(-\frac{(r_{s,t} - \alpha^{(n)} q_{s,t} - c^{(n)})^2}{2(\sigma^2)^{(n)}}\right) \frac{(u_s^{(n)})^{q_{s,t}}}{(q_{s,t}-1)!} \tag{14}$$

$$\eta_{s,t}^{(n)} = \sum_{q_{s,t}=0}^{+\infty} \exp\left(-\frac{(r_{s,t} - \alpha^{(n)} q_{s,t} - c^{(n)})^2}{2(\sigma^2)^{(n)}}\right) \frac{(u_s^{(n)})^{q_{s,t}}}{q_{s,t}!}. \tag{15}$$

Similarly, we have

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2] = \frac{\xi_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}, \tag{16}$$

where

$$\xi_{s,t}^{(n)} = \sum_{q_{s,t}=1}^{+\infty} q_{s,t} \exp\left(-\frac{(r_{s,t} - \alpha^{(n)} q_{s,t} - c^{(n)})^2}{2(\sigma^2)^{(n)}}\right) \frac{(u_s^{(n)})^{q_{s,t}}}{(q_{s,t}-1)!}. \tag{17}$$

In these formulas, $q_{s,t}$ acts as a summation index. As we can only perform finite summations, one can stop the summing at convergence. Alternatively, for improved efficiency we have derived lower and upper bounds for $q_{s,t}$ present in (14), (15) and (17). The bounds are functions of $r_{s,t}$, $\alpha^{(n)}$, $c^{(n)}$, $(\sigma^2)^{(n)}$ and $u_s^{(n)}$. The lower bound is denoted by $\underline{q}_{s,t}^{(n)}$ and upper bound by $\overline{q}_{s,t}^{(n)}$. Due to the lack of space, this point will be discussed in an expanded version of this paper.

4.2 Initialization

In Section 4.1, an EM algorithm for Poisson-Gaussian noise parameter identification was derived. As with many implementations of the EM algorithm, an important consideration is how to initialize θ. The problem is now to find appropriate initial values of $\alpha^{(1)}$, $c^{(1)}$, $(\sigma^2)^{(1)}$ and $u^{(1)}$. We propose a moment based approach to set these values. Due to the mutual independence assumption,

$$\kappa_n[R_{s,t}] = \alpha^n \kappa_n[Q_{s,t}] + \kappa_n[N_{s,t}] \tag{18}$$

where $\kappa_n[A]$ designates the cumulant of order n of some random variable A. Using classical results about cumulants, we obtain:

- mean value: $\quad \kappa_1[R_{s,t}] = \mathsf{E}[R_{s,t}] = \alpha u_s + c \quad$ (19)
- variance: $\quad \kappa_2[R_{s,t}] = \mathsf{Var}[R_{s,t}] = \alpha^2 u_s + \sigma^2 \quad$ (20)
- higher-order cumulants: $\quad n \geq 3,\ \kappa_n[R_{s,t}] = \alpha^n u_s. \quad$ (21)

Let $\widehat{\mathsf{E}}[r_{s,t}] = \frac{1}{T}\sum_{t'=1}^{T} r_{s,t'}$ and let similar sample estimates $\widehat{\mathsf{Var}}[r_{s,t}]$ and $\widehat{\kappa_3}[r_{s,t}]$ be used for the other cumulants. Different procedures may be derived from (19), (20), and (21) in order to estimate θ, but they are not equally reliable. For example according to our observations, $\frac{\widehat{\kappa_4}[r_{s,t}]}{\widehat{\kappa_3}[r_{s,t}]}$ does not provide a very good estimate of α. Instead, we propose to use:

$$\alpha^{(1)} = \frac{S\sum_{s=1}^{S} \widehat{\mathsf{E}}[r_{s,t}]\widehat{\mathsf{Var}}[r_{s,t}] - \sum_{s=1}^{S}\widehat{\mathsf{E}}[r_{s,t}]\sum_{s=1}^{S}\widehat{\mathsf{Var}}[r_{s,t}]}{S\sum_{s=1}^{S}(\widehat{\mathsf{E}}[r_{s,t}])^2 - \left(\sum_{s=1}^{S}\widehat{\mathsf{E}}[r_{s,t}]\right)^2}, \tag{22}$$

which is quite accurate, as only first and second order statistics are used. However, $(\sigma^2)^{(1)}$ cannot be computed in a similar manner and third order cumulants need to be considered. One of the possibilities is to compute $(\sigma^2)^{(1)}$ as median $\left\{\widehat{\mathsf{Var}}[r_{s,t}] - (\alpha^{(1)})^{-1}\widehat{\kappa_3}[r_{s,t}]\right\}$, but it can be shown that the cumulant estimate becomes sensitive when T is small or when u_s takes large values. To account for this latter problem, we propose the following weighted least squares estimate of σ^2:

$$(\sigma^2)^{(1)} = \frac{\sum_{s\in\mathbb{I}}\widehat{\mathsf{Var}}[r_{s,t}]^{-6}\left(\widehat{\mathsf{Var}}[r_{s,t}] - (\alpha^{(1)})^{-1}\widehat{\kappa_3}[r_{s,t}]\right)}{\sum_{s\in\mathbb{I}}\widehat{\mathsf{Var}}[r_{s,t}]^{-6}} \tag{23}$$

where $\mathbb{I} = \left\{s \in \{1,\dots,S\} \mid \widehat{\mathsf{Var}}[r_{s,t}] - (\alpha^{(1)})^{-1}\widehat{\kappa_3}[r_{s,t}] \geq 0\right\}$. Finally, the initialization is completed by:

$$c^{(1)} = \frac{1}{S}\sum_{s=1}^{S}\left(\widehat{\mathsf{E}}[r_{s,t}] - (\alpha^{(1)})^{-1}\widehat{\mathsf{Var}}[r_{s,t}]\right) + \frac{(\sigma^2)^{(1)}}{\alpha^{(1)}}, \tag{24}$$

and

$$(\forall s \in \{1,\dots,S\}) \qquad u_s^{(1)} = \frac{1}{\alpha^{(1)}}(\widehat{\mathsf{E}}[r_{s,t}] - c^{(1)}). \tag{25}$$

4.3 Overview of the proposed method

The tools introduced in Sections 4.1 and 4.2 constitute the main two ingredients of our estimation method, the pseudo-code of which is summarized in Algorithm 1.

The iterative process stops when the maximum number of iterations N is reached, or if $\max_{i=\alpha,\sigma^2,c}\left|i^{(n+1)} - i^{(n)}\right| \leq \delta$, where $\delta > 0$ is some tolerance.

5. SIMULATION EXAMPLES

The experimental results presented here aim at providing information about the performance of the proposed algorithm under different working conditions. In particular, in Section 5.1 the influence of the values of parameters c, α, σ^2 and T is studied. The proposed initialization is shown to be accurate for $T > 200$ and values of c, α and σ^2 neither too low nor too high. EM algorithm is investigated for smaller data set, when its benefits in terms of accuracy are more significant. Finally, an application of the proposed algorithm to real data is demonstrated (Section 5.2). The noise parameters are identified based on sequences of images corrupted with Poisson-Gaussian noise.

5.1 Validation of the proposed approach

We evaluate the proposed algorithms using S randomly generated u_s values uniformly distributed over $[0,100[$. This range was chosen in order to show the performance of our algorithm in the conditions when the Anscombe transform is less reliable. Signal $R_{s,t}$ is generated according to (1) for different set of parameter values for θ and T. Poisson and Gaussian noises are simulated using random number generators as proposed in Park *et al.* [11].

Identification accuracy is evaluated in terms of relative absolute error, defined as:

$$\mathrm{err} = \frac{\left|\frac{\sigma^2 - \hat{\sigma}^2}{\sigma^2}\right| + \left|\frac{\alpha - \hat{\alpha}}{\alpha}\right| + \left|\frac{c - \hat{c}}{c}\right|}{3} \tag{26}$$

Algorithm 1 Proposed algorithm.

Initialization:
Compute $\alpha^{(1)}$ using (22)
Find $\mathbb{I}$
Compute $(\sigma^2)^{(1)}$ using (23)
Compute $c^{(1)}$ using (24)
Compute $u^{(1)}$ using (25)
Set $\theta^{(1)} = \left(u^{(1)}, \alpha^{(1)}, c^{(1)}, (\sigma^2)^{(1)} \right)$

EM Algorithm:
For $n = 1 \ldots N$
  Set $\zeta_{s,t}^{(n)} = 0, \xi_{s,t}^{(n)} = 0, \kappa_{s,t}^{(n)} = 1$
  $\eta_{s,t}^{(n)} = \exp \left(-\frac{(r_{s,t} - c^{(n)})^2}{2(\sigma^2)^{(n)}} \right) u_{s,t}^{(n)}$
  Compute $\underline{q}_{s,t}^{(n)}$ and $\overline{q}_{s,t}^{(n)}$
  For $q_{s,t} = 1 \ldots \underline{q}_{s,t}^{(n)} - 1$ (see (14), (15), (17))
    $\kappa_{s,t}^{(n)} \leftarrow \kappa_{s,t}^{(n)} \frac{u_{s,t}^{(n)}}{q_{s,t}}$
  For $q_{s,t} = \underline{q}_{s,t}^{(n)} \ldots \overline{q}_{s,t}^{(n)}$
    $\kappa_{s,t}^{(n)} \leftarrow \kappa_{s,t}^{(n)} \frac{u_{s,t}^{(n)}}{q_{s,t}}$
    $\chi_{s,t}^{(n)} = \kappa_{s,t}^{(n)} \exp \left(-\frac{(r_{s,t} - \alpha^{(n)} q_{s,t} - c^{(n)})^2}{2(\sigma^2)^{(n)}} \right)$
    $\zeta_{s,t}^{(n)} \leftarrow \zeta_{s,t}^{(n)} + \chi_{s,t}^{(n)} q_{s,t}$
    $\xi_{s,t}^{(n)} \leftarrow \xi_{s,t}^{(n)} + \chi_{s,t}^{(n)} q_{s,t}^2$
    $\eta_{s,t}^{(n)} \leftarrow \eta_{s,t}^{(n)} + \chi_{s,t}^{(n)}$
  Update $\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}]$ using (13)
  Update $\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2]$ using (16)
  Update $u_s^{(n+1)}$ using (8)
  Update $\alpha^{(n+1)}$ and $c^{(n+1)}$ using (9)
  Update $(\sigma^2)^{(n+1)}$ using (10)
  $\theta^{(n+1)} = \left(u^{(n+1)}, \alpha^{(n+1)}, c^{(n+1)}, (\sigma^2)^{(n+1)} \right)$

Param.		Proposed initialization						
S	T	$\hat{\sigma}^2$		$\hat{c}$		$\hat{\alpha}$		$\overline{\text{err}}$
		bias	std	bias	std	bias	std	
1024	50	-14.83	20.29	-1.74	5.07	-0.21	0.14	0.22
	100	-8.58	13.86	-0.54	4.04	-0.09	0.11	0.16
	200	-11.69	10.13	1.18	2.7	-0.04	0.04	0.14
	500	-1.93	6.24	-0.26	1.61	-0.02	0.04	0.06
	1000	-1.16	4.55	-0.24	1.1	-0.01	0.03	0.04
4096	50	-20.86	13.05	-1.62	2.61	-0.19	0.06	0.16
	100	-10.77	6.91	-0.88	1.76	-0.10	0.05	0.09
	500	-2.19	2.80	-0.31	0.80	-0.02	0.02	0.03
	1000	-0.99	2.13	-0.15	0.49	-0.01	0.01	0.02

Table 1: Identified noise parameter versus S and T ($c = 10$, $\alpha = 10$ and $\sigma^2 = 100$).

Param.	Proposed Initialization						
σ^2	$\hat{\sigma}^2$		$\hat{c}$		$\hat{\alpha}$		$\overline{\text{err}}$
	bias	std	bias	std	bias	std	
0.25	-0.14	0.03	-0.11	5.15	-0.19	0.13	0.18
25	-3.59	3.89	- 0.18	4.19	-0.19	1.14	0.19
400	-11.45	84.32	-0.84	10.53	-0.22	0.15	0.34

Table 2: Identified noise parameter versus σ^2($c = 10$, $\alpha = 10$, $S = 1024$ and $T = 50$).

where the estimates are denoted with a hat (e.g. $\hat{\sigma}$).

The bias and standard deviation of estimated parameters computed from 100 different noise realizations are presented in Tables 1, 2, 3 and 4. Moreover the mean error ($\overline{\text{err}}$) over all 100 realizations is given. The following points are highlighted through these results:

- The reliability of cumulant-based approach increases with T (see Table 1);
- the mean of estimated parameters $\hat{\sigma}^2$, $\hat{c}$, and $\hat{\alpha}$ does not strongly depend on S, but the standard deviation of the estimates does (see Table 1);
- the estimates provided by the cumulant-based approach for $T \geq 500$ are very accurate (see Table 1);
- parameter $\hat{\alpha}$ is estimated very accurately with reduced dependence on parameters S and T;
- the initialization estimate is subject to higher errors for very high values of σ^2 (see Table 2);
- this estimate is subject to higher errors for low values of c (see Table 3);
- its accuracy also decreases for very low or very high values of α (see Table 4).

For $T \leq 200$, the results obtained by the cumulant-based

approach need to be further improved by EM. Table 5 provides some numerical results. Here the bias and standard deviation of estimated parameters computed from 20 different noise realizations are presented. One can observe that EM algorithm offers significant improvements in terms of accuracy. Note that, all the presented tests were performed under difficult conditions for cumulant-based method. The following points can be stated:

- Similarly to cumulant-based method, the reliability of EM increases with T (see rows 1, 2, 3 and compare with cumulant-based method results shown in Table 1);
- our EM algorithm performs well even if T is small;
- in contrast with the cumulant-based method, EM does not appear to be sensitive to small values of α and c (see rows 4, 5);
- both methods become less reliable in the presence of high variance of the Gaussian noise (see row 6). However EM still improves results w.r.t. the cumulant-based method (see Table 2). Note that in this case, the initial signal-to-noise ratio is very low.

One can also observe that the EM estimates for $T = 200$ are quite precise as the estimation error is only 5%.

5.2 Unsupervised image denoising

One possible application of our algorithm is the calibration of optical measurement systems, which we simulate in the following experiment. We created a time lapse sequences consisting of 40 images with resolution 100×100, each corrupted with Poisson-Gaussian noise characterized by $\sigma^2 = 416$, $c = 10$ and $\alpha = 50$. This corresponds to an initial SNR value of 0.17 dB. Again, the challenge here stems from the fact that the value of T is low (40). One can observe some remaining noise in the result provided by our cumulant-based method (Fig. 1(c)), which is no longer visible in the EM result (Fig. 1(d)). This is also verified by inspecting SNR values, which are equal to 28.3 dB for the cumulant-based method and 35.6 dB for EM. This example illustrates

No.	Param.					Proposed EM algorithm						Error	
	S	T	σ^2	c	α	$\hat{\sigma}^2$		$\hat{c}$		$\hat{\alpha}$		$\overline{\text{err}}_{\text{cum}}$	$\overline{\text{err}}_{\text{EM}}$
						bias	std	bias	std	bias	std		
1	1024	50	100	10	10	-1.86	14.02	-0.35	1.79	-0.21	0.05	0.22	0.11
2	1024	100	100	10	10	0.54	10.92	0.27	1.38	-0.08	0.02	0.16	0.08
3	1024	200	100	10	10	-0.91	7.89	0.09	0.88	-0.04	0.03	0.14	0.05
4	1024	50	100	5	10	1.30	13.16	0.08	1.61	-0.20	0.05	0.33	0.15
5	1024	50	25	10	1	-0.28	0.50	0.00	0.48	-0.02	0.01	0.64	0.03
6	1024	50	400	10	10	7.51	45.24	0.71	5.13	-0.22	0.07	0.34	0.21

Table 5: Expectation-Maximization algorithm performance under difficult conditions for cumulant-based method.
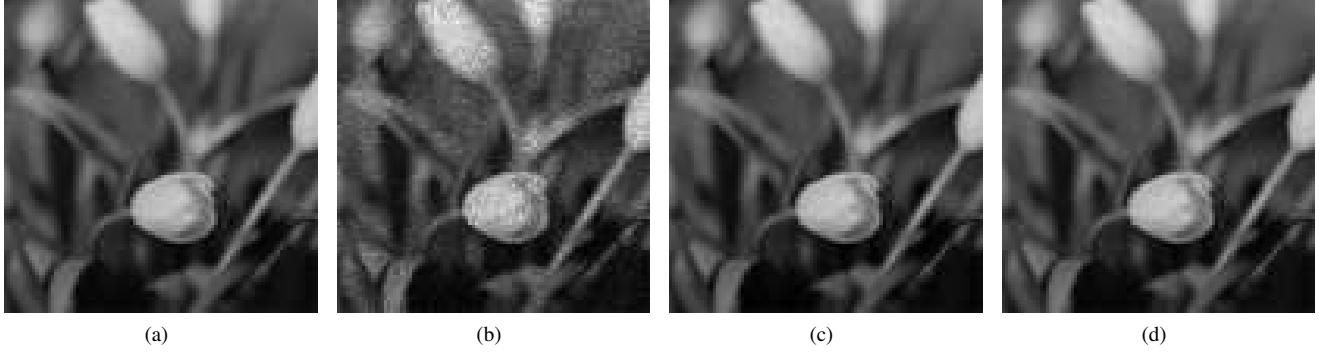

(a)   (b)   (c)   (d)

Figure 1: (a,b,c,d) illustrate original image, its noisy version (scaled with parameter α), cumulant-based method and EM results, respectively.

Param.	Proposed Initialization						
c	$\hat{\sigma}^2$		$\hat{c}$		$\hat{\alpha}$		$\overline{\text{err}}$
	bias	std	bias	std	bias	std	
5	-11.70	19.43	-1.20	5.07	-0.20	0.13	0.33
15	-10.16	26.15	-0.72	5.23	-0.18	0.13	0.17
100	-11.47	24.10	0.21	5.19	-0.17	0.13	0.09

Table 3: Identified noise parameter versus c ($\sigma^2 = 100$, $\alpha = 10$, $S = 1024$ and $T = 50$).

Param.	Proposed Initialization						
α	$\hat{\sigma}^2$		$\hat{c}$		$\hat{\alpha}$		$\overline{\text{err}}$
	bias	std	bias	std	bias	std	
1	12.99	6.17	13.73	6.29	-0.02	0.02	0.64
5	-2.93	5.69	-0.25	2.83	-0.09	0.07	0.15
50	1.07	56.4	-3.55	25.7	-1.10	0.67	0.81

Table 4: Identified noise parameter versus α ($\sigma^2 = 25$, $c = 10$, $S = 1024$ and $T = 50$).

the fact that not only noise parameters α, σ^2 and c are well reconstructed but so are the image intensity values u_s.

6. CONCLUSION

We have proposed a new EM-based approach dealing with Poisson plus Gaussian noise parameters estimation problems. We have shown that the proposed method leads to accurate results. We have also proposed an improved cumulant-based estimation method, which we used to initialize the EM algorithm. The improvement resulting from the EM iterations is especially significant when the number of realizations is small. As a side result, it allows us to obtain a good estimation of the original data when the noise parameters are unknown.

REFERENCES

[1] F. J. Anscombe. The transformation, of Poisson, binomial and negative-binomial data. *Biometrika*, 35(3/4):246–254, Dec. 1948.

[2] F. Benvenuto, A. La Camera, C. Theys, A. Ferrari, H. Lantéri, and M. Bertero. The study of an iterative method for the reconstruction of images corrupted by Poisson and Gaussian noise. *Inverse Problems*, 24(3):20 pp., 2008.

[3] J. Boulanger, J. B. Sibarita, C. Kervrann, and P. Bouthemy. Non-parametric regression for patch-based fluorescence microscopy image sequence denoising. In *5th IEEE International Symposium on Biomedical Imaging: From Nano to Macro (ISBI)*, pages 748–751, Paris, 2008.

[4] S. Delpretti, F. Luisier, S. Ramani, T. Blu, and M. Unser. Multiframe sure-let denoising of timelapse fluorescence microscopy images. In *5th IEEE International Symposium on Biomedical Imaging: From Nano to Macro (ISBI)*, pages 149–152, Paris, 2008.

[5] J. A. Fessler and A. O. Hero. Space-alternating generalized expectation-maximization algorithm. *IEEE Transactions on Signal Processing*, 42(10), 1994.

[6] A. Foi, M. Trimeche, V. Katkovnik, and K. Egiazarian. Practical Poissonian-Gaussian noise modeling and fitting for single-image raw-data. *IEEE Transactions on Image Processing*, 17:1737–1754, 2008.

[7] G. E. Healey and R. Kondepudy. Radiometric CCD camera calibration and noise estimation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 16:267–276, 1994.

[8] C. Liu, W. T. Freeman, R. Szeliski, and S. B. Kang. Noise estimation from a single image. In *Computer Vision and Pattern Recognition, 2006 IEEE Computer Society Conference on*, pages 901–908, 2006.

[9] F. Luisier, T. Blu, and M. Unser. Image denoising in mixed Poisson-Gaussian noise. *IEEE Transactions on Image Processing*, 20:696–708, 2011.

[10] T. E. Nichols, J. Qi, E. Asma, and R. M. Leahy. Spatiotemporal reconstruction of list-mode PET data. *IEEE Transactions on Medical Imaging*, 21(4):396–404, Apr. 2002.

[11] S. Park and K. Miller. Random number generators: Good ones are hard to find. *Communications of the ACM*, 31(10):1–10, October 1988.

[12] P. Paul, H. Duessmann, T. Bernas, H. Huber, and D. Kalamatianos. Automatic noise quantification for confocal fluorescence microscopy images. *Computerized Medical Imaging and Graphics*, 34(6):426–434, Sep. 2010.

[13] D. W. J. Stein. Detection of random signals in Gaussian mixture noise. *IEEE Transactions on Information Theory*, 41:1788–1801, 1994.

[14] H. L. Van Trees. *Detection, Estimation, and Modulation Theory*. Wiley, New York, 2001.

[15] B. Zhang. *Contributions à la microscopie à fluorescence en imagerie biologique : modélisation de la PSF, restauration d'images et détection super-résolutive*. Phd thesis, TELECOM Paris, Nov. 2007.