A 2D discrete model for crowd-structure interaction

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Summary (maximum 15 lines)

An emerging problem for civil engineering in the field of dynamics is the modelling of dynamic effects due to crowd-structure interaction. We are particularly concerned with structures such as footbridges, which oscillate due to the crossing of a group of pedestrians. The objective of this study is double: to model the movement of pedestrians with consideration of pedestrian-pedestrian and pedestrian-obstacle interactions and to include a pedestrian-structure coupling in the proposed model. This 2D discrete model would be able to study the problem of synchronization between pedestrians and structure on footbridges. Our idea is to extend the modelling of particles movements to study the movements of a crowd. The non-smooth discrete model of Frémond initially proposed to simulate a granular assembly, is chosen. It applies a rigorous thermodynamic framework in which the local interactions between particles are managed by the use of dissipation pseudo-potentials. Social forces as well as a desired direction/ velocity are introduced in order to simulate the behaviour of pedestrians. Concerning the pedestrian-structure coupling, a differential equation of Kuramoto allows one to manage the evolution of the pedestrians' phase. Two cases are studied depending on the sensitivity of pedestrians to the footbridge's oscillations. Numerical simulations on the Millenium Bridge are performed and discussed.

Keywords: granular assembly, contact, crowd movement, crowd-structure interaction, synchronization, vibrations, resonance.

1. Introduction

Several footbridges, built recently in accordance with the architectural criteria of lightness and elegance, such as the Millennium Footbridge in London and the Solferino footbridge in Paris have proved sensitive to the excitation induced by the crossing of pedestrians. Several experimental measurement campaigns have allowed a better understanding of this phenomenon: the crowd walking on a bridge imposes a lateral excitation on the structure which has a frequency of about 1Hz. When a lateral mode of vibration of the bridge, often the first, has a frequency close to this value, a resonance phenomenon is activated. It follows that the amplitude of oscillations of the bridge increases. If the number of pedestrians is not high, the amplitude of oscillations stays small and pedestrians continue to walk as they would on a rigid floor. If instead the number is high enough (beyond a certain critical number), the oscillations' amplitude becomes larger, enough so that pedestrians are encouraged to change their way of walking, in particular their frequency of walking, until there is a phase synchronization between pedestrians and structure. The behaviour of a pedestrian is also influenced by the crowd around him: if the density of the crowd is very low, the walking is "free", i.e. each individual walks as if there was nobody close to him. Conversely, if the crowd is dense, the pedestrian is forced to walk "at the same speed" as the others.

In this communication, the first part deals with the crowd movement microscopic model. To describe the acceleration of each pedestrian along his trajectory, we use the non-smooth discrete model of Frémond [9, 10], proposed to simulate a granular assembly, in order to manage the local pedestrian-pedestrian and pedestrian-obstacle interactions. The adaptation of the Frémond's model to the crowd gives a desired trajectory to each pedestrian. To take into account the pedestrian's oscillations around his trajectory due to his walking and his action on the bridge, an alternating (sinusoidal) sideways force is used. This force allows one to define the acceleration of each pedestrian's oscillations around his trajectory. In the second part, we give the crowd-footbridge interaction model in the case of lateral oscillations of a bridge. The synchronization of the walking frequency of each pedestrian with the lateral oscillations frequency of the bridge can be managed thanks to a differential equation of Kuramoto [3, 21], which allows one to govern the evolution of the total phase of the walking force generated by each pedestrian on the bridge. In the last part, the whole model is applied to the case of the North span of the Millennium Bridge and results are presented and discussed.

2. Representation of the crowd

We need to model the dynamic loading of a single pedestrian on a vibrating structure like footbridges in order to study the phenomenon of crowd-structure synchronization. The human body is a very complex mechanical system made of several parts in mutual interaction. It can be modeled according to the desired degree of refinement and objectives to be achieved. In [11], each part of the human body's skeleton is represented by a rigid body connected to the others by means of springs, dampers, and torques which impose a relative movement between rigid bodies in order to reproduce the walking. In [18, 22], the human body is represented with a set of masses, springs and dampers, in order to reproduce the vertical force of foot's impact on the ground during a footstep.

The human body can also be represented by a simple mass interacting with the ground. The expression of the restoring force between the pedestrian and the ground allows one to distinguish between models. Erlicher and al. [8] suggest a self-sustained single degree of freedom oscillator, able to accurately predict the lateral walking force. In [1, 21], the action of a pedestrian on a bridge is represented by a sinusoidal lateral force with amplitude of 30N and the evolution of its total phase is managed by a differential Kuramoto equation. Bodgi [2, 3] was inspired by this equation to achieve an original macroscopic model of pedestrian-structure coupling.

The knowledge of the trajectory of each pedestrian and of his action on the ground is sufficient for us to conduct our study on the synchronization. Hence, we represent a pedestrian i by a mass m_i, supposed lumped at the gravity centre of the pedestrian, and subjected to the inertial, gravitational and restoring forces. In the Galilean reference frame $(0,\mathbf{e_1},\mathbf{e_2},\mathbf{e_3})$, the fundamental principle of dynamics applied to each pedestrian i gives:

$$m_i\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{abs}}(\mathbf{q_i},t)+\mathbf{F_i}(\mathbf{q_i},t)=m_i\mathbf{g} \tag{1}$$

where the dot indicates the derivative function of time; $\mathbf{g}$ is the vector of the gravity's acceleration; $\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{abs}}$ is the absolute acceleration of m_i; $\mathbf{q_i}$ is the position of the pedestrian i in the Galilean reference frame; and $\mathbf{F_i}(\mathbf{q_i},t)$ is the exchanged force between the pedestrian i and the ground.

The absolute acceleration is expressed as:

$$\begin{aligned}\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{abs}}(\mathbf{q_i},t)&=\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{ent}}(\mathbf{q_i},t)+\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{rel}}(t)+\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{cor}}(t)\\&=\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{ent}}(\mathbf{q_i},t)+\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{tr}}(t)+\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{osc}}(t)+\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{cor}}(t)\end{aligned} \tag{2}$$

where $\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{ent}}$ is the ground's acceleration; $\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{rel}}$ and $\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{cor}}$ are the relative and the Coriolis accelerations of m_i respectively; $\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{tr}}$ is the acceleration of the pedestrian i along is trajectory on the ground; and $\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{osc}}$ is the acceleration of the pedestrian's oscillations around his trajectory.

The Coriolis acceleration for each pedestrian i is neglected: $\ddot{\mathbf{u}}_{\mathbf{i}}^{\mathbf{cor}}=\mathbf{0}$. In the following, expressions of $\ddot{\mathbf{u}}_{\mathrm{i}}^{\mathbf{tr}}$ and $\ddot{\mathbf{u}}_{\mathrm{i}}^{\mathbf{osc}}$ which are the components of the relative acceleration of the pedestrian i are given. In this purpose, the ground is considered rigid and the reference frame becomes $(0,\mathbf{e_1},\mathbf{e_2})$.

2.1 Expression of $\ddot{\mathbf{u}}_{\mathrm{i}}^{\mathbf{tr}}(t)$

In order to determine the acceleration $\ddot{\mathbf{u}}_{\mathrm{i}}^{\mathbf{tr}}$ of each pedestrian i along his mean trajectory on the ground, we control the movements of the N pedestrians in the plane $(0,\mathbf{e_1},\mathbf{e_2})$, taking into account local pedestrian-pedestrian and pedestrian-obstacle interactions. The microscopic model of crowd presented in [19, 20] is used to obtain $\ddot{\mathbf{u}}_{\mathrm{i}}^{\mathbf{tr}}(t)$. This model has been first proposed by Frémond [9, 10] to model the rigid particles' movement, and is then adapted to the crowd movement.

The model of Frémond, presented in the first part of this section, is based on the theory of rigid bodies' collisions and follows the lead of the works of Moreau [16]. The numerical aspects were later developed in [6, 7]. In this model, the set of N particles is considered as a deformable system composed of rigid solids. Each particle i is rigid, circular, of radius r_i and of mass m_i. We introduce : $\mathbf{M}$, the $2N\times 2N$ mass matrix of the particles; $\dot{\mathbf{u}}^{\mathbf{tr}}(t)$, the velocity vector of dimension $2N$, ${}^{\mathbf{t}}\dot{\mathbf{u}}^{\mathbf{tr}}(t)=({}^{\mathbf{t}}\dot{\mathbf{u}}_{\mathbf{1}}^{\mathbf{tr}}(t),{}^{\mathbf{t}}\dot{\mathbf{u}}_{\mathbf{2}}^{\mathbf{tr}}(t),\dots,{}^{\mathbf{t}}\dot{\mathbf{u}}_{\mathbf{N}}^{\mathbf{tr}}(t))$; the indices - and + referring to values before and after collisions, respectively; $\mathbf{f}^{\mathbf{ext}}$ (resp. $\mathbf{f}^{\mathbf{int}}$), the exterior forces vector (resp. interior forces vector) of dimension $2N$, applied to the system; and $\mathbf{\Delta_{ij}}(\dot{\mathbf{u}}^{\mathbf{tr}}(t))$, the relative deformation velocity between the particles i and j, $\mathbf{\Delta_{ij}}(\dot{\mathbf{u}}^{\mathbf{tr}}(t)) = \dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{i}}(t) - \dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{j}}(t)$. The motion equations of the system are:

$$\mathbf{M}\ddot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{i}}(t) = -\mathbf{f}^{\mathbf{int}}(t) + \mathbf{f}^{\mathbf{ext}}(t) \qquad \text{almost everywhere} \tag{3}$$

$$\mathbf{M}(\dot{\mathbf{u}}^{\mathbf{tr},+}_{\mathbf{i}}(t) - \dot{\mathbf{u}}^{\mathbf{tr},-}_{\mathbf{i}}(t)) = -\mathbf{p}^{\mathbf{int}}(t) + \mathbf{p}^{\mathbf{ext}}(t) \qquad \text{everywhere} \tag{4}$$

Equation (3) is applied almost everywhere except at the instant of the collision where it is replaced by Equation (4). When contact is detected, the velocities of colliding particles are discontinuous and we introduce in Equation (4) the percussions $\mathbf{p}^{\mathbf{int}}$ and $\mathbf{p}^{\mathbf{ext}}$, interior and exterior to the system respectively. By definition, percussions have the dimension of a force multiplied by a time. The $\mathbf{p}^{\mathbf{int}}$ percussions are unknown; they take into account the dissipative interactions between the colliding particles (dissipative percussions), and the reaction forces in order to avoid overlapping among particles (reactive percussions). Frémond [9, 10] showed that these interior percussions are defined in duality with the velocity of deformation at the moment of impact $\left(\mathbf{\Delta}(\dot{\mathbf{u}}^{\mathbf{tr},+}) + \mathbf{\Delta}(\dot{\mathbf{u}}^{\mathbf{tr},-})\right)/2$, in the sense of the work of internal forces. $\mathbf{p}^{\mathbf{int}}$ depends on $\left(\mathbf{\Delta}(\dot{\mathbf{u}}^{\mathbf{tr},+}) + \mathbf{\Delta}(\dot{\mathbf{u}}^{\mathbf{tr},-})\right)/2$, and Frémond used a pseudopotential of dissipation Φ (a convex function [15]) defined as: $\Phi = \Phi^{diss} + \Phi^{reac}$, where Φ^{diss} and Φ^{reac} are two pseudopotentials which define the dissipative and reactive interior percussions respectively, to express these quantities:

$$\mathbf{p}^{\mathbf{int}} \in \partial\Phi\left(\frac{\mathbf{\Delta}(\dot{\mathbf{u}}^{\mathbf{tr},+}) + \mathbf{\Delta}(\dot{\mathbf{u}}^{\mathbf{tr},-})}{2}\right) \tag{5}$$

where the operator ∂ is the subdifferential which generalizes the derivative for convex functions [10]. The pseudopotential Φ^{diss} is chosen to be quadratic: $\Phi^{diss} = (K/2)\left(\left(\mathbf{\Delta}(\dot{\mathbf{u}}^{\mathbf{tr},+}) + \mathbf{\Delta}(\dot{\mathbf{u}}^{\mathbf{tr},-})\right)/2\right)^2$ where K is a coefficient of dissipation. This choice allows one to find the classical results of the coefficient of restitution. Others choices of Φ^{diss} allow one to obtain a large variety of behaviours after impact [4, 10].

In Equation (4), the problem is to find the velocity after collision of grains $\dot{\mathbf{u}}^{\mathbf{tr},+}$. To determine $\dot{\mathbf{u}}^{\mathbf{tr},+}$, we have to solve the following constrained minimization problem:

$$\dot{\mathbf{v}} = \underset{\dot{\mathbf{w}} \in \Re^{2N}}{\arg\min} \left[{}^{\mathbf{t}}\dot{\mathbf{w}}\mathbf{M}\dot{\mathbf{w}} + \Phi(\mathbf{\Delta}(\dot{\mathbf{w}})) - {}^{\mathbf{t}}\left(2\dot{\mathbf{u}}^{\mathbf{tr},-} + \mathbf{M}^{-1}\mathbf{p}^{\mathbf{ext}}\right)\mathbf{M}\dot{\mathbf{w}}\right] \tag{6}$$

where the solution is $\dot{\mathbf{v}} = \left(\dot{\mathbf{u}}^{\mathbf{tr},+} + \dot{\mathbf{u}}^{\mathbf{tr},-}\right)/2$.

The existence and uniqueness of the solution are proved in [7, 9, 10] and the minimization problem (6) is numerically solved using a time-stepping scheme and the classical Uzawa algorithm [7].

So, when there is no contact, Equation (3) gives directly the real value of $\ddot{\mathbf{u}}^{\mathbf{tr}}(t)$. When a contact is detected, Equation (4) allows to get an approximated value of $\ddot{\mathbf{u}}^{\mathbf{tr}}(t)$ after the determination of $\dot{\mathbf{u}}^{\mathbf{tr},+}$:

$$\ddot{\mathbf{u}}^{\mathbf{tr}}(t) \approx \frac{\dot{\mathbf{u}}^{\mathbf{tr},+} - \dot{\mathbf{u}}^{\mathbf{tr},-}}{h} \tag{7}$$

where h is the chosen time step of the numerical scheme.

In the second part of this section, we show how the model of Frémond initially applied to rigid grains' movement can be adapted to the crowd movement. The first step is to give a desired trajectory to each particle. The strategy of the shortest path to get from one point to another [14] is implemented through a Fast Marching algorithm and is used to obtain the desired direction, $\mathbf{e}^{\mathbf{tr}}_{\mathbf{d,i}}$, of an individual i. This direction depends on the evolution space (obstacles, etc.), the time and also the characteristics of the individual (gender, age, hurried steps or not, etc.). It is defined by: $\mathbf{e}^{\mathbf{tr}}_{\mathbf{d,i}}(t) = \dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{d,i}}(t) / \left\|\dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{d,i}}\right\|$, where $\dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{d,i}}$ is the desired velocity of the pedestrian i.

The amplitude $\left\|\dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{d,i}}\right\|$ of the desired velocity represents the speed at which the pedestrian i wants to move on the considered structure, it can be influenced by the nervousness of the pedestrian. It depends on the desired step length $L_{step,0,i}$ of the pedestrian i and on the lateral angular frequency ω_i of his free desired walking [3]:

$$\left\|\dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{d,i}}\right\| = \frac{L_{step,0,i}}{\pi}\omega_i \tag{8}$$

In a second step, we introduce the desired velocity of each pedestrian i in the model of grains' movement to obtain the one of crowd movement. Let $\mathbf{f}^{\mathbf{ext}}(t) = \mathbf{f}^{\mathbf{ext}}_{\mathbf{a}}(t)$, where the external acceleration force $\mathbf{f}^{\mathbf{ext}}_{\mathbf{a}}(t)$ [12] allows one to give a desired direction and amplitude of the velocity to each pedestrian. Each component $\mathbf{f}^{\mathbf{ext}}_{\mathbf{a,i}}(t)$ of the force vector of dimension $2N$: ${}^{\mathbf{t}}\mathbf{f}^{\mathbf{ext}}_{\mathbf{a}}(t) = \left({}^{\mathbf{t}}\mathbf{f}^{\mathbf{ext}}_{\mathbf{a,1}}(t), {}^{\mathbf{t}}\mathbf{f}^{\mathbf{ext}}_{\mathbf{a,2}}(t) \ldots, {}^{\mathbf{t}}\mathbf{f}^{\mathbf{ext}}_{\mathbf{a,N}}(t)\right)$, is associated with pedestrian i and can be expressed as:

$$\mathbf{f}^{\mathbf{ext}}_{\mathbf{a,i}}(t) = m_i \frac{\left\|\dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{d,i}}\right\| \mathbf{e}^{\mathbf{tr}}_{\mathbf{d,i}}(t) - \dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{i}}(t)}{\tau_i} \tag{9}$$

where τ_i is a relaxation time, allowing the pedestrian to recover the desired velocity after a contact. Smaller values of τ_i let the pedestrians walk more aggressively [13, 20]. The pedestrians' behavior can be enriched with the adding of other external social forces [13, 17] in $\mathbf{f}^{\mathbf{ext}}(t)$, in order to become more realistic (socio-psychological force, attractive force, group force, etc.).

2.2 Expression of $\ddot{\mathbf{u}}^{\mathbf{osc}}_{\mathbf{i}}(t)$

When a pedestrian is walking, its center of gravity oscillates around its mean trajectory. Thus, to define the acceleration of the pedestrian's oscillations around his trajectory $\ddot{\mathbf{u}}^{\mathbf{osc}}_{\mathbf{i}}$, we consider the case where the pedestrian is walking in place. We assume that the forces $\mathbf{F}^{\mathbf{osc}}_{\mathbf{i}}$ applied by the two legs of the pedestrian i on the floor are identical. In the frame $(0, \mathbf{e_1}, \mathbf{e_2})$, Equation (1) becomes:

$$m_i \ddot{\mathbf{u}}^{\mathbf{osc}}_{\mathbf{i}}(t) + \mathbf{F}^{\mathbf{osc}}_{\mathbf{i}}(t) = \mathbf{0} \tag{10}$$

As each pedestrian is walking in a plane, the force applied by the pedestrian i on the floor has a normal $\mathbf{F}^{\mathbf{osc}}_{\mathbf{N,i}}$ and a tangential $\mathbf{F}^{\mathbf{osc}}_{\mathbf{T,i}}$ component to the pedestrian's motion. We assume that each pedestrian imparts an alternating (sinusoidal) sideways force to the bridge [21]. As the frequency of $\mathbf{F}^{\mathbf{osc}}_{\mathbf{T,i}}$ is usually taken to be twice the frequency of $\mathbf{F}^{\mathbf{osc}}_{\mathbf{N,i}}$ [3], the expression of $\mathbf{F}^{\mathbf{osc}}_{\mathbf{i}}$ can be written as follows:

$$\begin{aligned} \mathbf{F}^{\mathbf{osc}}_{\mathbf{i}}(t) &= F^{osc}_{T,i}(t)\mathbf{e}^{\mathbf{tr}}_{\mathbf{i}}(t) + F^{osc}_{N,i}(t)\mathbf{e}^{\mathbf{tr},\perp}_{\mathbf{i}}(t) \\ &= T_i \sin(2\phi_i(t))\mathbf{e}^{\mathbf{tr}}_{\mathbf{i}}(t) + N_i \sin(\phi_i(t))\mathbf{e}^{\mathbf{tr},\perp}_{\mathbf{i}}(t) \end{aligned} \tag{11}$$

where T_i (resp. N_i) is the maximum amplitude of $F^{osc}_{T,i}$ (resp. $F^{osc}_{N,i}$); $\mathbf{e}^{\mathbf{tr}}_{\mathbf{i}}$ (resp. $\mathbf{e}^{\mathbf{tr},\perp}_{\mathbf{i}}$) is the unit direction vector of the pedestrian's motion (resp. normal to the pedestrian's motion), $\mathbf{e}^{\mathbf{tr}}_{\mathbf{i}}(t) = \dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{i}}(t) / \left\|\dot{\mathbf{u}}^{\mathbf{tr}}_{\mathbf{i}}(t)\right\|$; and $\phi_i(t)$ is the total phase of the walking force generated by the pedestrian i on the bridge, such that $\dot{\phi}_i(t) = \omega_i$ when the pedestrian is not influenced by external excitations such as the bridge's oscillations.

A combination of (10) and (11) allows us to determine $\ddot{\mathbf{u}}^{\mathbf{osc}}_{\mathbf{i}}(t)$:

$$\ddot{\mathbf{u}}^{\mathbf{osc}}_{\mathbf{i}}(t) = -\frac{1}{m_i}\left(T_i \sin(2\phi_i(t))\mathbf{e}^{\mathbf{tr}}_{\mathbf{i}}(t) + N_i \sin(\phi_i(t))\mathbf{e}^{\mathbf{tr},\perp}_{\mathbf{i}}(t)\right) \tag{12}$$

3. Crowd-footbridge interaction

We consider N pedestrians walking on a bridge of length L and width l. Some assumptions are done: the floor is flat and horizontal; the main axis (longitudinal) of the bridge is straight, along $\mathbf{e_1}$; the floor's movement is governed by its lateral oscillations, i.e. the floor's oscillations take place in the horizontal plane while the longitudinal oscillations are negligible; the displacements of the floor are supposed constant along $\mathbf{e_2}$, unit direction vector of the lateral footbridge's oscillations; and the lateral displacements of the floor are small.

Thus, we define the following Cartesian coordinate system with x-axis which is parallel to the main axis, the z-axis vertical and the y-axis oriented accordingly; the structure has a geometrically linear behaviour; and the displacement of the footbridge is given by: $\mathbf{U}(\mathbf{q}, t) = U_y(x,t)\mathbf{e_2}$.

The response of a linear structure can be expanded into a sum of modal responses. For the lateral component, limiting to the N_m first modes, we find:

$$U_y(x,t) = \sum_{j=1}^{N_m} \psi_j(x) U_{y,j}(t) \tag{13}$$

where $\psi_j(x)$ is the j^{th} modal shape and $U_{y,j}(t)$ is the j^{th} modal coordinate of the bridge's displacement.

As the lateral excitation of the pedestrian due to his walking has a frequency closed to 0.9-1 Hz, the footbridge's lateral mode having the nearest frequency to the pedestrian's excitation one, is sufficient to model the dynamics of the footbridge:

$$U_y(x,t) \approx \psi_1(x) U_y(t) \tag{14}$$

where $U_y(t) = A_y(t)\sin(\psi_{str}(t))$ is the lateral modal displacement of the footbridge having an instantaneous phase

$\psi_{str}(t)$ and an instantaneous amplitude $A_y(t)$, and $\psi_1(x)$ is the first lateral modal shape of the footbridge, normalized by: $\max_{x\in[0;L]}(\psi_1(x))=1$.

The equation describing the dynamic behaviour of the footbridge projected onto its first mode shape is:

$$M_{str}\ddot{U}_y(t)+C_{str}\dot{U}_y(t)+K_{str}U_y(t)=\sum_{i=1}^{N}\psi_1(x_i)F_{y,i}(x_i,t) \tag{15}$$

where M_{str}, C_{str} and K_{str} are the modal (generalized) mass, damping and stiffness of the lateral first mode of the footbridge, respectively and where $F_{y,i}(x_i,t)$ is the projection of $\mathbf{F_i}(x_i,t)$ along the y-axis.

The expression of the ground's acceleration (in Equation (2)) is:

$$\ddot{\mathbf{u}}^{\mathbf{ent}}(x,t)=\psi_1(x)\ddot{U}_y(t)\mathbf{e_2} \tag{16}$$

Equation (16) states that the bridge moves in the lateral direction only ($\ddot{\mathbf{u}}^{\mathbf{ent}}(x,t)$ is parallel to $\mathbf{e_2}$), with a spatial distribution proportional to the lateral mode shape $\psi_1(x)$. This assumption is acceptable for numerous practical situations. More complexes bridge motions could be considered, but this is beyond the purposes of this paper. Synchronization can be seen as an adaptation of the frequency of the force generated by a pedestrian to the frequency of the "crowd-structure" system. We choose to study two differential Kuramoto equations [3, 21] to govern the evolution of the total phase $\phi_i(x_i,t)$ (see Equation (11)) of the walking force generated by the pedestrian i on the bridge. Both differential equations allow the instantaneous walking angular frequency of the pedestrian to converge to that of the footbridge.

In the first one, pedestrians are sensitive to the amplitude of the footbridge's displacement:

$$\frac{\partial\phi_i^{dis}(x_i,t)}{\partial t}=\omega_i+\varepsilon_i^{dis}A_y(t)\psi_1(x_i)\sin\left(\psi_{str}(t)-\phi_i^{dis}(x_i,t)+\alpha\right) \tag{17}$$

as in [21] but their instantaneous frequency also depends on the footbridge's modal shape. In Equation (17), ε_i^{dis} quantifies pedestrians' sensitivity to the bridge's oscillations, its value was estimated by Strogatz [21] by comparing its results (without modal shape) with the experimental results achieved on the Millennium bridge [5]. Strogatz chose $\alpha=\pi/2$ to have the lateral force exerted by pedestrian i on the ground in phase with the lateral velocity of the bridge when synchronization occurs.

In the second one, pedestrians are sensitive to the amplitude of the footbridge's acceleration [3]:

$$\frac{\partial\phi_i^{acc}(x_i,t)}{\partial t}=\omega_i+\frac{\varepsilon_i^{acc}}{2}A_{y,m}(t)\psi_1(x_i)\left(\dot{\psi}_{str}(t)\right)^2\sin\left(\psi_{str}(t)-\phi_i^{acc}(x_i,t)+\alpha\right) \tag{18}$$

where ε_i^{acc} quantifies pedestrians' sensitivity to the bridge's oscillations, its value was estimated by Bodgi [3] by comparing its results (with modal shape) with the experimental results achieved on the Millennium bridge [5]; $A_{y,m}(t)$ is the maximum value of $A_y(t)$ during the last 5 seconds of simulation; $a^{foo}(x_i,t)=A_{y,m}(t)\psi_1(x_i)\left(\dot{\psi}_{str}(t)\right)^2$ is the footbridge acceleration; and Bodgi has found that $\alpha=\pi/2$ too.

For a walking on a rigid floor, both of previous equations are reduced to: $\dot{\phi}_i(t)=\omega_i$.

Hence, the expression of $F_{y,i}(x_i,t)$ becomes:

$$\begin{aligned}F_{y,i}(x_i,t)=&-m_i\psi_1(x_i)\ddot{U}_y(t)-m_i\ddot{u}_{y,i}^{tr}(t)\\&+T_i\sin\left(2\phi_i(x_i,t)\right)\sin\left(\theta_i(t)\right)+N_i\sin\left(\phi_i(x_i,t)\right)\cos\left(\theta_i(t)\right)\end{aligned} \tag{19}$$

where $\theta_i(t)$ is the angle between $\mathbf{e_i^{tr}}(t)$ and the direction $\mathbf{e_1}$ of the longitudinal axis of the bridge.

To take into account the oscillations of the structure in the crowd movement model, the amplitude of the desired velocity of the pedestrian i in Equation (8) is modified and noted $\left\|\dot{\mathbf{u}}_{\mathbf{d,i}}^{\mathbf{tr}}\right\|_{\mathrm{mod}}(x_i,t)$. It depends on $A_y(t)$, $\psi_1(x_i)$ and $\psi_{str}(t)$ through the instantaneous step length $L_{step,i}(x_i,t)$ and the instantaneous angular frequency $\frac{\partial\phi_i}{\partial t}(x_i,t)$ of the pedestrian i:

$$\left\|\dot{\mathbf{u}}_{\mathbf{d,i}}^{\mathbf{tr}}\right\|_{\mathrm{mod}}(x_i,t)=\frac{L_{step,i}(x_i,t)}{\pi}\frac{\partial\phi_i(x_i,t)}{\partial t} \tag{20}$$

Three expressions of the instantaneous step length can be proposed according to the model we chose to apply.

If we don't take into account the deceleration of the pedestrian, then:

$$L_{step,i}(x_i,t) = L_{step,0,i} \tag{21}$$

If we take into account the deceleration and Equation (17), so as in [19]:

$$L^{dis}_{step,i}(x_i,t) = L_{step,0,i}\max\left(0{,}1 - \frac{A_y(t)\psi_1(x_i)}{A_{\max,i}}\right) \tag{22}$$

where $A_{\max,i}$ is the maximum amplitude of the lateral displacement of the footbridge that the pedestrian i can stand. When $A_y(t)\psi_1(x_i)$ reaches $A_{\max,i}$, the amplitude is so large that the pedestrian stops walking.

If we take into account the deceleration and Equation (18), so as in [3]:

$$L^{acc}_{step,i}(x_i,t) = L_{step,0,i}\begin{cases} 1 & if\ a^{foo}(x_i,t) \le a_{\min} \\ \dfrac{a_{\max} - a^{foo}(x_i,t)}{a_{\max} - a_{\min}} & if\ a_{\min} < a^{foo}(x_i,t) < a_{\max} \\ 0 & if\ a_{\max} \le a^{foo}(x_i,t) \end{cases} \tag{23}$$

where $a_{\min}$ (resp. $a_{\max}$) is the amplitude of the lateral footbridge's acceleration from which pedestrians feel the oscillations (resp. stop to walk).

4. Simulations

The experimental study made by Arup [5] on the North span of the Millennium Bridge showed that when the average walking frequency of pedestrians is equal to the modal frequency of the structure (1.03 Hz), the critical number of pedestrians Nc, above which the frequency synchronization between a portion of the crowd and the structure is triggered, is: Nc = 166 pedestrians. We have tried to find this result with our proposed model of interaction.

In this section, we consider the case where pedestrians cross the North span of the Millennium Bridge in London and we present the results obtained. For each simulation, the number of pedestrians on the structure is constant and all the pedestrians move in the same direction. To keep the number of pedestrians walking on the bridge constant, once one pedestrian exits the bridge on one side, one similar pedestrian enters the bridge with a random position along the y-axis on the other side. The parameters used in simulations are listed in Table 1:

Pedestrian's parameters		**Footbridge's parameters**	
symbol	value	symbol	value
r_i	chosen randomly between [0.2 m, 0.25 m]	L	81 m
m_i	$500\pi r_i^2\ kg$	l	4 m
$L_{step,0,i}$	normal distribution: mean 0.71 m standard deviation 0.071 m	M_{str}	113000 kg
N_i	35 N	C_{str}	$11000\,kg.s^{-1}$
T_i	120 N	K_{str}	$4730000\,kg.s^{-2}$
ω_i	normal distribution: mean $2\pi\times1.03 rad.s^{-1}$ standard deviation $2\pi\times0.094 rad.s^{-1}$	$U_{y,0}$	0 m
$A_{\max,i}$	0.2 m	$\dot{U}_{y,0}$	$0\,m.s^{-1}$
$\phi_{0,i}$	chosen randomly between $[-\pi,\pi]$ With mean value 0	$[a_{\min}, a_{\max}]$	$[01\ m.s^{-2}, 1.35\ m.s^{-2}]$
ε_i	1) $\varepsilon_i^{dis} = 16\,m^{-1}.s^{-1}$ (estimation given in [21] without modal shape) 2) $\varepsilon_i^{acc} = 1.1819\,s.m^{-1}$ (estimation given in [3] with modal shape)	$\psi_1(x)$	1) with modal shape : $\sin(\pi x/L)$, (case of a simply supported beam at both ends) 2) without modal shape : 1

Table 1: Parameters used in simulations

We performed simulations of 10 minutes long considering the following numbers of pedestrians: N = 100, 150, 170, 180, 210, 250, 300 and 400. For each N, we study the influence of the modal shape and of the pedestrians' deceleration (see Equations (21-23)) on the triggering of synchronization. In order to study the influence of the modal shape on the results obtained by Equations (17) and (18), we consider the case where the modal shape is not taken into account ($\psi_1(x)=1$) and the case where $\psi_1(x)=\sin(\pi x/L)$.For each studied case, several simulations are achieved with different set of parameters. When the final seconds of simulation correspond to the stationnary state, synchronization has been reached.

Figure 1 shows examples of non-synchronization and synchronization results, obtained in the following case: with Equation (17), without modal shape and without deceleration. Figures 1 a. and b. present a case of non synchronization, obtained with 100 pedestrians crossing the structure. The lateral displacement of the bridge is very low and pedestrians do not synchronize their walking frequency with that of the "pedestrian-bridge" system. Figures 1 c. and d. show a case of synchronization, obtained with 300 pedestrians crossing the structure. The lateral displacement of the bridge reaches 0.15 m, and pedestrians synchronize their walking frequency with that of the "pedestrian-bridge" system.

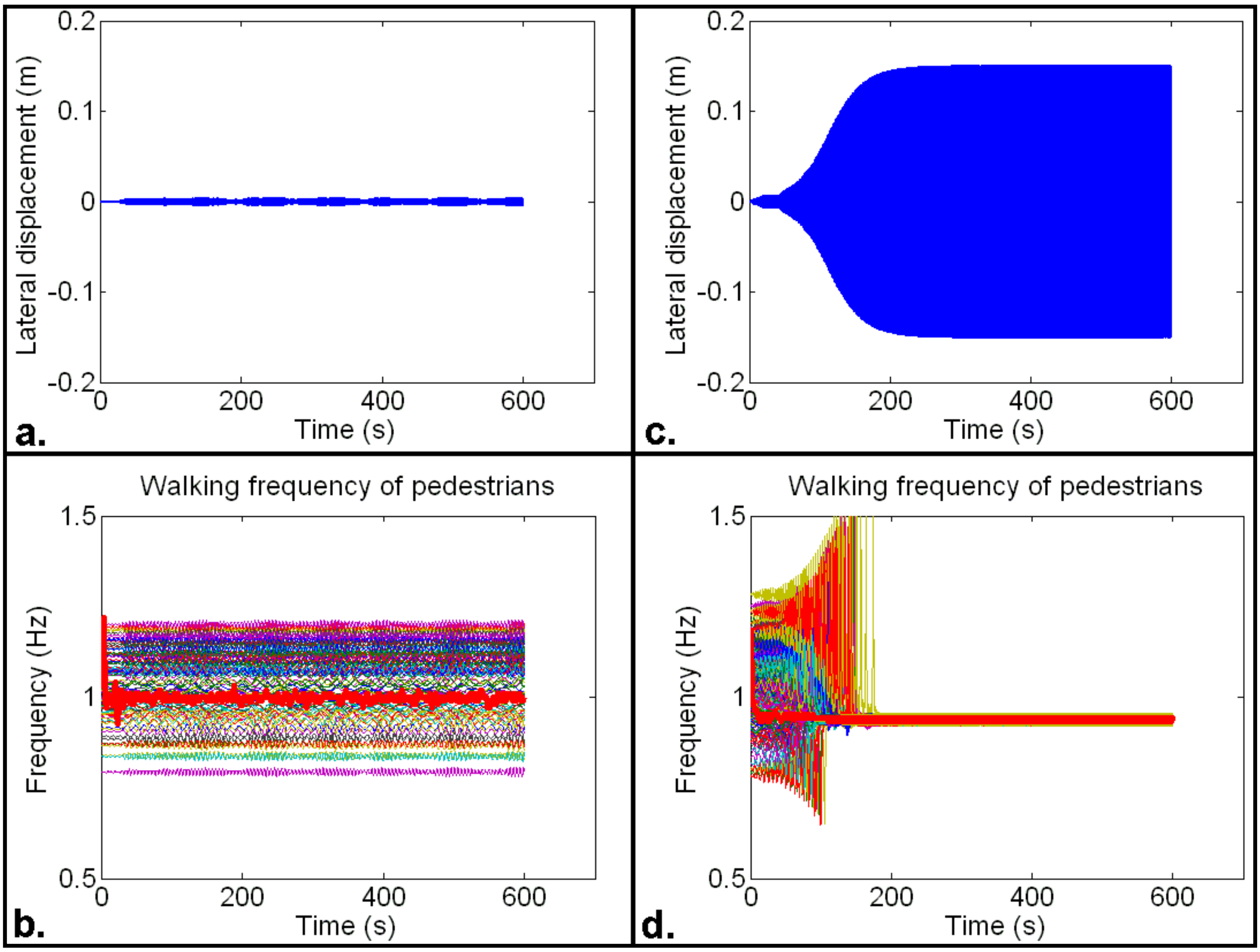

Fig. 1: Phenomenon of synchronization for the model using Equation (17), without modal shape and without deceleration. Figures a. and b. (resp. c and d) show the results obtained with 100 pedestrians (resp. 300 pedestrians) - a. and c. represent the time evolution of the lateral displacement of the bridge; b. and d. represent the time evolution of the walking frequency of pedestrians on the structure. The red curve in bold represents the instantaneous frequency of the "pedestrian-bridge" system.

Table 2 shows the preliminary results obtained on the phenomenon of synchronization. It gives the number of simulations where the phenomenon of synchronization is detected compared with the number of achieved simulations. Several cases are considered according to N, the type of the differential Kuramoto equation governing the evolution of $\phi_i(x_i,t)$, the expressions of the modal shape and of the instantaneous step length $L_{step,i}(x_i,t)$.

number of pedestrians N	number of simulations: achieved / synchronized							
	pedestrians are sensitive to the amplitude of the footbridge's displacement (Equation (17))				pedestrians are sensitive to the amplitude of the footbridge's acceleration (Equation (18))			
	$\psi_1(x)=1$		$\psi_1(x)=\sin(\pi x/L)$		$\psi_1(x)=1$		$\psi_1(x)=\sin(\pi x/L)$	
	$L_{step,0,i}$ [1,21]	$L^{dis}_{step,i}$	$L_{step,0,i}$	$L^{dis}_{step,i}$	$L_{step,0,i}$	$L^{acc}_{step,i}$	$L_{step,0,i}$	$L^{acc}_{step,i}$ [3]
100	5 / 0	8 / 0	6 / 0	5 / 0	3 / 3	4 / 2	6 / 0	3 / 0
150	5 / 3	8 / 5	6 / 0	5 / 0	3 / 3	4 / 2	6 / 1	3 / 0
170	5 / 3	8 / 5	6 / 0	5 / 0	3 / 3	4 / 3	6 / 2	3 / 2
180	5 / 3	8 / 6	6 / 0	5 / 0	3 / 3	4 / 4	6 / 5	3 / 3
210	5 / 5	8 / 7	6 / 0	5 / 0	3 / 3	4 / 4	6 / 5	3 / 3
250	5 / 5	8 / 8	6 / 2	4 / 1	3 / 3	3 / 3	6 / 6	3 / 3
300	4 / 4	7 / 7	6 / 3	4 / 1	2 / 2	2 / 2	5 / 5	3 / 3
400	1 / 1	2 / 2	1 / 1	2 / 2	1 / 1	2 / 2	2 / 2	1 / 1

Table 2: Results obtained on the phenomenon of synchronization

We find that, when the modal shape is not taken into account ($\psi_1(x)=1$), i.e. the pedestrian close to the bounds of the span is feeling the same oscillations as that at mid-span, synchronization is highlighted. The number of pedestrians being synchronized is higher, and therefore the amplitude of the force generated by pedestrians too.

We consider first the case without modal shape ($\psi_1(x)=1$), and with or without deceleration. The use of Equation (17), where pedestrians are sensitive to the amplitude of the footbridge's displacement, gives the critical number obtained by Abrams [1], around 150 pedestrians. Using Equation (18), where pedestrians are sensitive to the amplitude of the footbridge's acceleration, gives a critical number less than 100 pedestrians. The estimated value of ε_i^{acc} given in [3] with modal shape and with deceleration is not correct for the case without modal shape.

We consider then the case with modal shape ($\psi_1(x)=\sin(\pi x/L)$). With deceleration, the use of Equation (18) gives the critical number obtained by Bodgi [3], around 170 pedestrians. With or without deceleration, using Equation (17) gives a critical number around 250 pedestrians. The estimated value of ε_i^{dis} given in [21] without modal shape and without deceleration is not correct for the case with modal shape.

This application to the interaction model on the North span of the Millennium Bridge shows that the value of the parameter which quantify pedestrians' sensitivity to the bridge's vibrations, has a significant influence on the critical number. The value of this parameter has to be chosen according to the differential Kuramoto equation and to the choice of taking into account the modal shape or not.

5. Conclusion

This paper presents a 2D discrete model for crowd movement on rigid ground obtained from the adaptation of a granular model based on the theory of rigid bodies'collisions developed by Frémond. This crowd movement model is then used in the crowd-structure interaction model. A differential Kuramoto equation allows to manage the evolution of the total phase of the walking force generated by each pedestrian. Depending on the sensitivity of the pedestrians, two differential equations of Kuramoto have been studied.

Numerical simulations on the North span of the Millennium Bridge show that the value of the parameter which quantifies pedestrians' sensitivity to the bridge's oscillations, has to be estimated according to the differential Kuramoto equation and to the modal shape. Other numerical simulations are in progress to fully validate the model from other experimental data. Two studies are planned: first, determining the influence on the phenomenon of synchronization, of the phase shift value between the lateral force exerted by pedestrians on the ground and the lateral velocity of the bridge, and second, comparing the stationary amplitude values of the lateral bridge's displacement when synchronization occurs, with those found in literature.

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