

The Continuing Distance Education Program M@tmídias: Contributions to the Teaching of Trigonometry

Fábio Henrique Patriarca^{a,b}
Nielce Meneguelo Lobo da Costa^b
Samira Favez Kfourir da Silva^c

^a Governo do Estado de São Paulo, Secretaria da Educação, Escola Estadual Adib Miguel Haddad.

^b Universidade Anhanguera de São Paulo (UNIAN-SP), Programa de Pós-Graduação em Educação Matemática, São Paulo, SP, Brasil.

^c Universidade Norte do Paraná (UNOPAR), Programa de Pós-Graduação em Metodologias para Ensino de Linguagens e suas Tecnologias, Londrina, PR, Brasil.

Received for publication on 20 Dec. 2018. Accepted, after revision, on 27 Mar. 2019.

Assigned editor: Claudia Lisete Oliveira Groenwald.

ABSTRACT

This article presents a clipping of research on a Continuing Distance Learning Program for 600 High School Math Teachers working in public schools of São Paulo state, Brazil. The formative Program entitled M@tmídias aimed to discuss the use of Learning Objects in the Mathematics teaching process. The research objective was to identify contributions of the use of technology for teaching Trigonometry. The theoretical base was developed by Imbernón's ideas on continuing teachers' education, as well as the ones of Mishra and Koehler regarding pedagogical technological content knowledge (TPACK). The research was documentary, and the methodological procedures were: 1) Collection of curricula documents and historical data of the investigated Program; 2) Selection and organisation of materials related to the Trigonometry content, stored in the Program VLE; 3) Treatment and data analysis. The analysis was interpretative and of content, according to Bardin. As a result, in the M@tmídias Program's historical documents, it was verified that all the Mathematics contents from the High School curriculum of São Paulo State were discussed and, in the first module of the course M@tmídias 2, Trigonometry was studied. Evidence of participants' construction was verified of mathematical content knowledge, pedagogical knowledge and technological content knowledge, showing possibilities to the construction of technological pedagogical content knowledge (TPACK) of the participants.

Keywords: Continuing Teacher Education; Educational technology; Learning Objects; Trigonometry.

Corresponding author: Fábio Henrique Patriarca. E-mail: patriark@uol.com.br

O Programa de Formação Continuada a Distância M@tmídias: Contribuições para o Ensino de Trigonometria

RESUMO

Este artigo apresenta um recorte de uma pesquisa sobre um Programa de formação continuada a distância para 600 professores de Matemática do Ensino Médio, atuantes em escolas públicas Estaduais de São Paulo, Brasil. O Programa formativo, denominado M@tmídias, teve por foco discutir o uso de objetos de aprendizagem nos processos de ensino de Matemática. O objetivo da pesquisa foi o de identificar as contribuições do uso de tecnologia para ensino de Trigonometria. A fundamentação teórica foi construída pelas ideias de Imbernón em relação à formação continuada, assim como as de Mishra e Koehler quanto ao conhecimento tecnológico pedagógico do conteúdo (TPACK). A pesquisa foi documental e os procedimentos metodológicos foram: 1) Coleta de documentos curriculares e dados históricos do Programa, 2) Seleção e organização dos materiais estocados no AVA do Programa, relacionados ao conteúdo de Trigonometria, 3) Tratamento e análise dos dados. A análise foi interpretativa e de conteúdo, segundo Bardin. Como resultados, nos documentos históricos do Programa M@tmídias, constatou-se que todos os conteúdos de Matemática do Currículo Oficial Paulista para o Ensino Médio foram enfocados e, no primeiro módulo do curso M@tmídias 2, foi estudada a Trigonometria. Foram constatadas evidências de construção de conhecimento do conteúdo matemático, do conhecimento pedagógico do conteúdo e do conhecimento tecnológico do conteúdo, sinalizando possibilidades para construção do conhecimento pedagógico tecnológico do conteúdo (TPACK) dos participantes.

Palavras-chave: Formação Continuada de Professores. Tecnologia Educacional. Objetos de Aprendizagem. Trigonometria. TPACK.

INTRODUCTION

The research that subsidises this study was developed in the context of a continuing training program entitled “M@tmídias”. This program was developed and implemented by the School of Training and Enhancement of Teachers in the state of São Paulo (EFAP/SP) and had the purpose of subsidising the use, in the classroom, of technological resources, such as video, audio, software and experiments. All activities proposed in the program were allied to the Learning situations appearing on the teachers’ and students’ books, printed materials available in the state public schools of São Paulo.

The research comprised the investigation of the second edition of the course of Continuing Education of Mathematics teachers of Secondary School, called: M@tmídias 2 – Multimedia Learning Objects for the Teaching of Mathematics. This course included, among others, the teaching of trigonometry and had as its target audience teachers working in the second year of Secondary Education. This course was composed of five modules, being the first four for the study of learning objects, which evaluative activities were towed, namely: a discussion forum, a dissertative question and an objective question. The fifth module proposed a didactic experience lived in the classroom, in which the attendants should apply to their students a learning object, – always associated with the curricular content of the respective series. They should also document the application, producing a report to be attached in a virtual learning environment of the course (AVA – EFAP).

It is worth mentioning that the teaching of trigonometry has been regarded as a great challenge for teachers from the public São Paulo State network, because, as Amaral (2002) states, “it is the content considered as one of the most difficult understanding of students, we believe that this difficulty is due to its degree of abstraction and the expository-transmissive way that it is taught” (p.11). Thus, we assume that a course of continuing formation that discusses with teachers methodologies with use of technology can impact the teaching practice and help to break the expository purely as the content of trigonometry has been approached in the classroom.

To boost the pedagogical practice and assist the students’ learning, it is essential to integrate technology into teaching. However, is it wondered whether the school integrates new technologies into the classroom? In our opinion, the use of these resources should assist in the rapprochement between students and teachers, it must be adapted to the educational project of the school, and its faculty must be prepared for this use.

According to Moran (2013, p; 89),

The technologies arrived at the school, but they always have focused more on the control, the modernisation of infrastructure and management of the change. The administrative management programs are more developed than those focused on learning are. There are advances in virtualisation of learning, but they can only scratch superficially the massive structure in which the various levels of education are structured.

The ideas of Moran (2013) go hand in hand with what we live in our schools today, we have computers, but many times there is only investment for the administrative part, and the pedagogical section does not always receive attention. The teacher who tries to use the resources made available by the school, in general, must do it alone, the pedagogical support that he or she receives is usually minimal or non-existent.

Bittar, Guimaraes, & Vasconcelos (2008, p.86) are in line with the ideas of Moran and complement:

[...] to us what has been done in most schools, is the insertion of the technology, teachers use, but without causing a different kind of learning than what was done before, and what is more, the computer is being a strange instrument to pedagogical practice, used in situations will not be evaluated.

Some schools have computers available for the use of teachers; however, for many of them, the technologies are tools outside the practice. When they insert them, many times, they are dismembered from the Curriculum and the day by day of the class, which means that there is no integration of that technology, only its insertion.

Not only Moran (2013) but also Bittar, Guimaraes, & Vasconcelos (2008) emphasise that generally there is an investment in educational institutions for the insertion of technologies, but for the most part, it tends to be for the administrative sector. However, not always there are investments in the development of innovative pedagogical strategies with the use of technology; and when there is this investment, it tends to be punctual. Consequently, the teacher uses resources sporadically in a way that technology is not part of their daily lessons. In addition, he or she often faces other problems, such as the infrastructure of schools, with the insufficient number of computers, the internet is not always of quality, these problems that may interfere with integrating technology into the teaching practice.

In this article, we favour the discussion of a clipping of the research about the course M@tmidias 2, in which activities were undertaken for the teaching of trigonometry with technologies, seeking to subsidise the teaching practice. Among the activities, are a forum for discussion, a dissertative question and an objective question that will be described and analysed in this text.

THEORETICAL FRAMEWORK

The clipping of the research presented herein, had theoretical support for continuing formation in studies of Imbernón (2000; 2010) and the Teacher Professional Knowledge in studies of Mishra and Koehler (2005, 2006), which we discuss below.

Regarding the teachers' continuing education, we believe that it should help them to rebuild the practice and to build new knowledge, in order to improve each time more the quality of teaching that is offered to students. In this respect, we support ourselves in Imbernón (2000; p.49), for which the continuing formation, should "foster the development of the personal, professional and institutional framework of the teachers, by leveraging a collaborative work to change the practice."

For Imbernón (2000; p.49), continued training should focus on five principles:

1. The practical-theoretical reflection about the practice itself, through an analysis of the educational and social reality of its country, its understanding, interpretation and intervention on the same reality. The teachers' ability to generate pedagogical knowledge through the analysis of the educational practice.
2. The exchange of experiences, school, life, etc. and reflection among equal individuals to allow the update in all fields of educational intervention and increase communication among teachers.
3. The union of formation to project work, and not the other way around (first perform the training and after a project).
4. Training as a critical weapon against labour practices as the hierarchy, sexism, the proletarianization, individualism, etc., and against social practices, such as the exclusion and intolerance.

5. The professional development of educational institution upon the collaborative work, recognising that the school is constituted by all, and we agree with the intention of transforming this practice. Allow the passage of the experience of isolated and cellular innovation for institutional innovation.

With this, in the teaching profession, the teacher needs to mobilise different knowledge in order to plan, develop and evaluate his or her pedagogical actions; it is a context of action. In the research, we look to these principles, related by Imbernón, seeking to identify if they are present in the program of continuing formation investigated.

In relation to Teacher Professional Knowledge, which integrates the theoretical research, a program of continuing formation should consider that teachers need a knowledge base to serve as a foundation for decision-making in his or her teaching work. For the construction of knowledge, Mishra and Koehler (2006) based on the ideas of Shulman, proposed a model explaining their positions on the knowledge needed to professor: the specific, the pedagogical and technological ones. The Model TPACK, Technological Pedagogical Content Knowledge, developed by Mishra and Koehler (2006), can help us in understanding the knowledge necessary to the teacher to teach with the use of Information and Communication Technology.

The TPACK model was constructed from Schulman's ideas of the Knowledge Base (1987) specifically those relating to the Pedagogical Knowledge of content, which is located at the intersection of the Pedagogical Knowledge with the Content Knowledge. Mishra and Khoeler integrated technological knowledge to the knowledge base.

This knowledge, in fact, is more than just the junction between the content knowledge, the pedagogical knowledge and technological knowledge. It is complex and involves the teaching of curricular contents through pedagogical techniques, methods or teaching strategies that use technology appropriately to teach differently and in accordance with the needs of the students.

TPACK is supported in teaching action, whose construction is in the pedagogical practice. Mishra and Koehler (2006) define it as the knowledge necessary for the teacher of how to use the technology to the teaching quality of content, using their databases in an integrated way and observing their complex relations:

Technological knowledge and pedagogical content is an emerging form of knowledge that goes beyond all three components (content, pedagogy and technology) [...]. The integration of productive technology in education needs to consider all three issues, not isolated but within the complex relationships in the system defined by three key elements. (Mishra & Koehler, 2006, pp.1028-1029)

Thus, the Model TPACK identifies the basis for effective teaching and highly qualified, which encompasses the integration of technologies and pedagogy that teachers use when they teach curricular contents. Thus, according to this model, the teacher lacks to dominate the three fields of knowledge and their relations, to develop the teaching.

THE RESEARCH CONTEXT: STUDY METHODOLOGY

The methodology of documentary research was inserted into the group of qualitative research. Following Gil (2008), this qualitative research is classified, for purposes as exploratory and descriptive. We consider as being exploratory research because it seeks to identify the possibilities of a distance continued training to assist the teacher (1) in integrating technology into teaching and (2) to construct technological pedagogical knowledge of the content. The research was descriptive because it considered and described the perceptions of teachers participating in the course. Regarding the means, the research should be inserted in the category of bibliographic and documental, once bibliographical sources were used for the understanding of the context of the research and documents, such as regulations, guides to the attendants and tutors, assessment questionnaires, etc., in order to answer the research questions.

The use of documents in a survey should be valued and appreciated because according to Cellard (2008), we can remove and redeem a richness of information, enlarge the understanding of objects whose understanding requires historical and sociocultural contextualization. In his words:

[...] the written document constitutes an extremely valuable source for every researcher in the social sciences. It is, evidently irreplaceable in any reconstitution for a relatively distant past, because it is not uncommon that it represents almost all the traces of human activity at certain times. In addition, very often, it remains as the only testimony of private activities that have occurred in the recent past. (Cellard, 2008, p.295)

We agree with the author when he says that a document can be the only testimony of an event, frequent or in the past. We emphasise that, in the case of this study, documents and records are stored in AVA of *online* achievement of the course M@timidias 2, which bear the facts that occurred.

According to Gil (2008), in the documental research materials are used that have not received an analytical treatment yet or may be re-elaborated in accordance with the research objectives. In addition to analysing the documents of “first hand,” there are also those already processed, but which may receive other interpretations. Thus, this research is going in that direction, because the analysed materials are “first hand” documents receiving the first treatment in this study.

The study was developed in three steps:

- Step 1: Collection of curricular documents and historical data of the Program M@tmidias relating to the design and structure.
- Step 2: Selection and organisation of materials stored in the virtual learning environment of the examined Course (AVA-EFAP) related to the content of trigonometry,
- Step 3: Treatment, interpretative data analysis, and the establishment of the conclusions.

The data analysis was interpretative, by the method of content analysis and documentary analysis according to (Bardin, 2011).

We see interpretative analysis as the one in which, as explained by Severino (2007, p.94), the researcher “[...] adopts a position in respect of ideas laid down, overcome the strict message text, it has to do with reading between the lines, forcing the author to a dialogue, it is exploring the fruitfulness of the ideas exposed, it is to confront them with others, it is a dialogue with the author...”. Which means that, from the collected data, the researcher takes a position, interpreting them from his or her vision and theoretical referential support.

We chose to undertake the analysis by the method of content analysis, and also by documentary analysis. We emphasise that such methods are similar in some procedures. However, there are critical differences between them. For Bardin (2011), the documentary analysis works with documents; unlike the content analysis, which examines messages (communication). For the author, the documental analysis is accomplished mainly by classification and indexing, being such a categorical analysis topic one of the standard techniques between it and the analysis of content, however,

“the goal of the documentary analysis is the condensed representation of information, consultation and storage [while] the analysis of content is the manipulation of messages (content and expression of this content) to highlight the indicators that allow us to infer a different reality than that of the message” Bardin (2011)

In this research, the records relating to the dissertative question, to the investigated Discussion Forum were scanned, identified the similarities and differences by means of tables compiled in Excel, and then grouped the records to show categories of analysis. From the establishment of these categories, the interpretative analysis was using content analysis and documentary analysis.

The Program M@Tmídias

The program aimed to 1) provide training to mathematics teachers in Secondary School of the state network, discussing methodologies for the use of learning objects in different media, such as “complementary” material for the implementation of the curriculum, in a manner consistent with the policy’s pedagogical SEESP; 2) encourage reflection and socialization among the mathematics teachers of secondary education about the use of learning objects in the teaching and learning process of mathematics, by means of interaction in the Virtual Learning Environment – AVA-EFAP and 3) create the conditions to ensure that the teacher use materials developed by other institutions, which are freely available on the internet and that can enrich the development of the curriculum of mathematics.

Thus, we believe that the main focus of this program was to promote continuing education for Mathematics teachers of the network geared to the development of practice with the technological resources tied to the curriculum of Secondary Education, as a support in the teaching and learning processes. In addition, the Program aimed to allow discussion and reflection on practice, in a virtual learning environment, the AVA-EFAP. The program also proposes to assist the teacher in using digital resources free and available on the internet.

To achieve the proposed objectives, the basic design of the program indicated the proposition of courses and the methodology for the development and implementation of those courses. There were three online courses: M@tmídias 1 – Learning Objects with multimedia intended mainly for teachers of 1st year of high school, M@tmídias 2 – Learning Objects with multimedia – offered to teachers of 2nd year of high school and M@tmídias 3 – Learning Objects with multimedia – preferably proposed to teachers of 3rd year of high school, with the aim of preparing them for use in the classroom technology resources tied to learning situations that are in the specification of the Teacher’s and the Student’s Book, materials that make up the official curriculum of the state of São Paulo, in such a way that they can give to their students the construction of mathematical knowledge allied to technologies. The Basic Design of the program indicates that each course is set in an online platform with monitoring of the attendants through teachers, tutors and coordinators. Thus, we believe that the teacher tutor would have the technological pedagogical content knowledge (TPACK) as defined by Mishra and Koehler. He or she needs to “have the knowledge on the subject to be taught or learned, “must “have knowledge originated from different fields, such as the Pedagogy, Curriculum and Didactics, that are applied to learning” and needs to have “the knowledge on the standard technologies, such as books, chalk, and chalkboard, and more advanced technologies, such as the Internet and digital video”. We interpreted that, according to advocated by Mishra and Koehler, with such knowledge the teacher tutor will perform their work, intervening in the forums, arguing to fix the dissertative issues, and knowledge of the technology to guide their class. The tutors and coordinators must have knowledge of the content, in this case mathematical knowledge (MK), the pedagogical knowledge (PK), the technological knowledge (TK) and their intersections, pedagogical knowledge of content (PCK), the

technological pedagogical knowledge (TPK) and the technological knowledge of the contents (TCK), since they must draw up focusing on the actions of mentoring.

Each course in the Program M@tmídias was composed of 5 modules, offered entirely at a distance, by the Virtual Learning Environment (AVA-EFAP), with a duration of 12h each module. Figure 1 presents the schema that illustrates this structure.

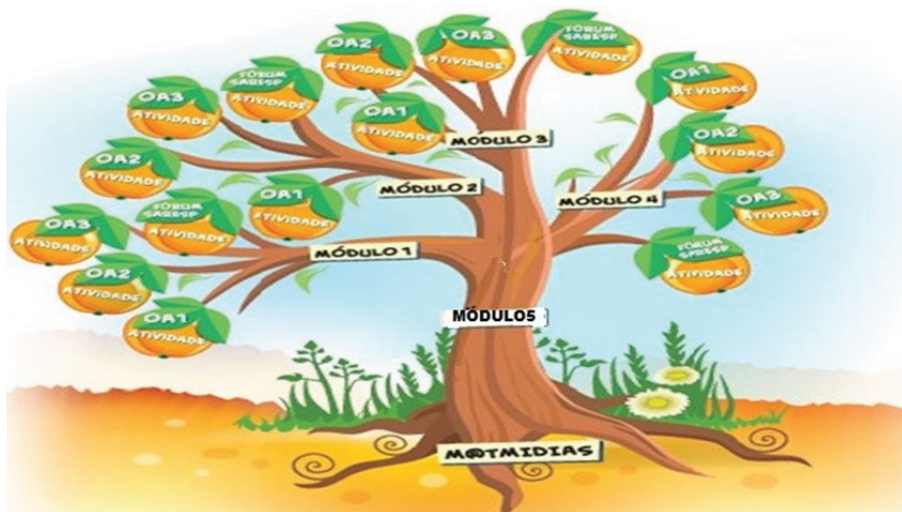


Figure 1. Structure of each course in the Program M@tmídias 2 (Patriarca, 2016, p.98).

In each of the first four modules, three learning objects are studied, with an evaluative activity associated with each one – namely, a discussion forum or a dissertative question and an objective question. From one module to another, the presentation order of activities may be different but following this pattern. Module five of each course proposes an activity of experience, in which the attendants should apply in practice a learning object, associated to the curriculum of mathematics, documenting the implementation and produce a report to be attached at AVA-EFAP.

The modules of the courses cover all the curriculum of mathematics in Secondary School, in this text will discuss the activities related to the Module 1 of the course M@tmídias 2, for the second series of average education, regarding trigonometry.

Activities of Trigonometry: Teaching Contributions

In the module examined, three activities were developed: a dissertative question, a discussion forum and an objective question. Each activity is related to a learning object, removed from the repository called M3 of the State University of Campinas (Unicamp), available at www.m3.ime.unicamp.br.

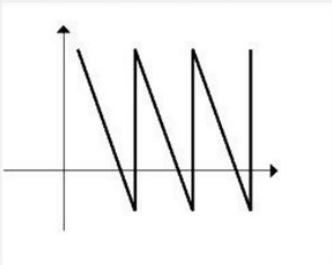
The dissertative question was about the first learning object studied, in this case, the video “Dance of the Sun” that had as its focus the study of the periodicity and the East-West movement of the sun.

The dissertative question presented to the applicants in the study of this learning object was the following (Figure 2):

1. Uma atividade da Situação de Aprendizagem 1 do Caderno do Professor, volume 1, da 2.ª série do Ensino Médio (versão 2009), propõe que o professor solicite ao aluno que registre o comprimento da sombra de uma estaca vertical em função do horário, durante alguns dias. Após o registro, a SA propõe a seguinte atividade:

“Representem em um gráfico cartesiano a evolução do comprimento da sombra da estaca durante a passagem de, por exemplo, 3 dias.”

Suponha que um aluno tenha esboçado o seguinte gráfico para representar a evolução do comprimento da sombra da estaca vertical:



a) Aponte quais questionamentos podem ser feitos ao aluno, a partir do gráfico produzido, para que ele reconheça que alguns fenômenos periódicos podem ser representados por funções trigonométricas.

b) Na coleção **Matemática Multimídia**, há objetos que podem ser utilizados para a execução do item a. Pesquise e aponte o(s) *link(s)* referentes aos objetos encontrados, justificando a(s) sua(s) escolha(s).

Figure 2. Dissertative Question – Wave Length (Patriarca, 2016, p.109).

This activity covers a hypothetical situation in which the student represented the outline of a graphic containing the evolution of shadow in a vertical stake. The attendants must answer two questions related to the type of intervention that the teacher can make in his or her classroom. This issue is an integral part of the assessment.

We understand that the proposal of this activity may lead the teacher to reflect on mathematics teaching and to articulate the knowledge built on the course with his or her pedagogical knowledge and content (Schulman, 1987). This is evidenced in the responses of the attendants extracted from AVA, for the question: “Point which questions can be made to the student, from the graph produced, so that he or she recognises that some periodic phenomena can be represented by trigonometric functions.”

As an example, the excerpts below:

Professor QD1 We can ask the student:

What does “periodic phenomenon” mean?

Does the Sun set in a radical way? For example, at 18:00 “does it turn night”. It that what happens?

Or Does the Sun “hide” slowly?

In the graph that the student built, how does the sun “hide”? Quickly or slowly?

Let’s imagine that it crosses the abscissa axis vertically at an angle of 90° at 06:00 p.m. What would happen at 05:59 p.m.?

The teacher QD1 fully met the request of the question, presents the questions for the students, and demonstrated that there is a mobilisation of pedagogical knowledge of content (PCK), because he or she creates daily situations of students so that they can understand the questions they proposed.

Professor QD2 could ask if the variation that he realised of the shadow was always constant (at the same time), always decreasing or increasing in the same proportion, which characterises the representation through straight lines. Why, in his chart, does the shadow increase sharply in one moment and then decrease in a uniform way? Ask why in the graphic measurements appear in the 4th quadrant, whether we are representing the length of the stake shadow.

Teacher QD2 answered when asked the question, relating the situation with the studied contents, shows evidence of mathematical knowledge (MK) when he or she used the language of mathematics, and also evidence of pedagogical knowledge (PK) when related questions with the day-to-day reality of the students, therefore pedagogical knowledge of content (PCK).

Professor Ch3 The activity itself already brings interesting questions and in addition to these we can do other things, such as the period in which the phenomena are repeated; if there is symmetry in the graph trace and why it occurs; if this phenomenon ends at the point represented in his or her chart; what function this phenomenon would represent, etc.

Teacher QD3 showed evidence of mathematical knowledge (MK) when in his or her questioning suggested a reflection on symmetry, also attended at the request of the question. In the same way as in the previous examples, there was evidence of mobilisation of pedagogical knowledge of content (PCK).

The question of letter (b), with the following wording: “In the Collection Mathematics Multimedia, there are objects that can be used for the execution of the item a. Research and point the *link(s)* to the objects found, justifying your choice(s)”.

As examples of resolutions of the attendants, we present the excerpts below:

Professor QD1 <http://m3.ime.unicamp.br/recursos/1080>. The dance of the Sun, same video used in this course, where there is the observation of the path that the Sun travels and relates to the same with the construction of a house or shed;

<http://m3.ime.unicamp.br/recursos/1284>. Displays the trigonometry as an instrument in the history of humankind, as an instrument that is and was very important in major human constructions.

We can observe that teacher QD1 proposed links, as requested in the dissertative question, we noticed that the attendant researched to relate the two items of the question. This leads us to conclude that there was evidence of integration of technology to education (PIE), on a proposal from the teacher, because it has introduced technology to teach trigonometry, proposing the use of learning objects suitable for the study.

Professor QD2 I have found some objects that could be used to represent the function in the collection M^3 , at the following link: “<http://m3.ime.unicamp.br/recursos/search:fun%C3%A7%C3%B5es+trigonom%C3%A9tricas>”, which follows below:

The curves of Lissajous (software), Solar Storms (audio), Sine Waves (audio), Trigonometric Waves (software), which I believe to be the most indicated, because through it we can model the most diverse situations through experimental or observational data, making adjustments to approximate elementary functions and allowing the construction of the graph for the representation of the same.

We can observe that teacher QD2, answered the request of the question; there was evidence of mobilisation of technological pedagogical content knowledge (TPACK) when the attendant examined and concluded that the software “Trigonometric Waves” was the most suitable for modelling various situations that involve periodic phenomena.

Professor QD3 to substantiate the questions as the previous ones it is interesting that are worked on educational objects, leading the student to conceptualise the periodic phenomena. Therefore, I suggest that the objects below be worked:

The video “Dance of the sun” (<http://m3.ime.unicamp.br/recursos/1080>) is very interesting because it is a real situation and that provokes curiosity, making significant learning as the teacher can ask the students to observe, at home, how the sun behaves during a specific period, creating a table for later build the graph.

With the experiment “Ferris Wheel” <http://www.m3.ime.unicamp.br/recurso/1033>) will be possible to introduce concepts of oscillatory movements, periods and maximum and minimum points of periodic functions. The activity involves the construction of a Ferris wheel in reduced size made of recyclable material.

In the software “Trigonometric Waves” (<http://m3.ime.unicamp.br/recurso/1240>), the student will be able to study periodic phenomena and learn to model such phenomena by varying the parameters of a sine function of type $f(x)=a*\text{sen}(b*x+c)+d$.

The students will see that this function can be used to model the most diverse situations, such as, for example, the rotation of a Ferris wheel, and the oscillations of the tide or the brightness of a star.

In answer to the question, the attendant teacher QD3 explained the learning objects that would utilise and added the links as it is requested. Therefore, we realised that this attendant related the two items (a and b) to the dissertative question. This leads us to conclude that there was evidence of the presence of class integration of technology to education (PIE) and also the mobilisation of technological pedagogical content knowledge (TPACK) on the proposal of the teacher.

The dissertative question, with its items (a and b), attended the first principle of Imbernón (2010), considered essential in continuing formation, which is the practical-theoretical reflection about the practice itself, thus generating pedagogical knowledge. The question promoted reflections on the mathematics teaching and helped to articulate the knowledge that is being constructed on the course with the pedagogical knowledge of the attendant teachers. The ability of the teacher to relate the theory with the day-to-day routine of students, using everyday situations may have aided in the construction of the pedagogical content knowledge (PCK). Thus, with this proposal, we conclude that the course helped the teacher in the mobilisation of technological pedagogical content knowledge (TPACK) while the attendants analysed what learning object was the most suitable for modelling a particular situation.

The discussion forum was related to the second learning object studied, the experiment “Ferris Wheel”, which proposes a deepening in the study of periodic phenomena, in the construction of graphs of the function in the sine, cosine, through the construction of a miniature of a Ferris wheel. The provocation of the discussion forum, in which all the attendants should participate, is in the figure below.

A Situação de Aprendizagem 2 propõe a interpretação de fenômenos periódicos, que podem ser representados por funções trigonométricas. De acordo com o Caderno do Professor (versão 2009), "um ponto girando em torno de uma circunferência é o modelo ideal para analisar a periodicidade de determinados fenômenos e para expressá-la por intermédio de equações matemáticas".

Como analogia, foi proposta, neste curso, a realização do experimento *A roda-gigante*, em que os estudantes medem as alturas de um banquinho da roda-gigante de acordo com o ângulo correspondente ao giro.

Discuta com seus colegas e tutor como você poderia abordar com os alunos a representação das medidas das alturas do banquinho da roda-gigante, relacionadas com o ângulo, a fim de que compreendam que:

- o fenômeno pode ser expresso por uma função trigonométrica e;
- o formato da onda que se apresenta nesse gráfico é de mesma natureza do que foi esboçado na Situação de Aprendizagem 1.

Figure 3. The screen of the discussion forum (Patriarca, 2016, p.115).

The discussion on the forum revolved around how to discuss with the students the experiment "Ferris Wheel" in order to understand the trigonometric function and the wave format that the graph shows.

We believe that the discussion forum enabled the teacher, from the experience of a learning object, discuss with their peers in the course, which may have helped to build pedagogical knowledge (PK) and content (CK), and may develop, in consequence, the Pedagogical Content Knowledge (PCK).

This is evident in the excerpts taken from the forum on the learning object "Ferris Wheel":

8/3/2014 11:54 A.M...I BELIEVE THAT THE CRUCIAL POINT IN THIS ACTIVITY IS IN THE REPETITIONS THAT OCCUR, ESPECIALLY AFTER THE FERRIS WHEEL COMPLETES A LAP. IN GENERAL, THE STUDENTS ARE ACCUSTOMED TO PAYING ATTENTION TO THE PHENOMENA THAT DO NOT EXHIBIT PERIODICITY. IN THOSE, THE MOVEMENTS ARE ACCOMPANIED BY THE COLLECTION OF INFORMATION AND MEASURES ARISING FROM THE SAME. I FIND THE ACTIVITY PRODUCTIVE AND EDIFYING REGARDING THE CONSTRUCTION OF CONCEPTS.

In the excerpt, we identified the mobilisation of pedagogical knowledge of content (PCK), once the attendant can check in the experiment the periodicity and also the importance of this information for the construction of knowledge on the part of their students.

8/4/2014 12:19 P.M ...I ALSO THINK THAT THE COMPLETION OF THE EXPERIMENT FAVORS THE DEVELOPMENT OF THE CONCEPTS INVOLVED IN AN INVESTIGATIVE AND SIGNIFICANT WAY. [...]

BUT, WHAT I LIKE MOST AT WORK IN CONJUNCTION WITH THE CURRICULUM IS HOW WE GAIN IN THE SYSTEMATIZATION OF INVOLVED TRIGONOMETRIC CONCEPTS, IN A SIGNIFICANT AND CONTEXTUALIZED WAY.

This section highlights the mobilisation of pedagogical knowledge (PK) by the attendant who observes the various systematisations of the content of trigonometry involved in this experiment.

07/25/2014 058:37 pm WITHOUT A DOUBT, BUILDING A FERRIS WHEEL WITH CARDBOARD, WILL BE SOMETHING VERY PLAYFUL; AND YET, MAKING MEASUREMENTS, TABULAR VALUES FOUND, WILL SERVE FOR A BETTER UNDERSTANDING OF GRAPHS OF PERIODIC FUNCTIONS. REGARDING THE INITIAL QUESTION OF THE FORUM, I BELIEVE THAT, WHEN THE STUDENT REPRESENTS IN THE CARTESIAN SYSTEM THE POINTS OBTAINED BY MEASURING THE HEIGHTS RELATED TO ANGLES, IT SHOULD BE NOTED FOR THE PERIOD 2π , WHICH IS THE SAME AS THE FUNCTIONS SINE AND COSINE, AND YET, THE PRESENCE OF THE SAME INTERMEDIATE HEIGHTS, ALSO OCCUR WITH THE SINE AND COSINE VALUES FOR PAIRS OF ANGLES WITH SYMMETRIC MEASURES IN THE TRIGONOMETRIC CIRCLE. THEY ALSO SHOULD NOTE THAT, FOR EACH TURN OF THE STOOL, EVERYTHING IS REPEATED, WHAT HAPPENED WHEN TRYING TO OUTLINE THE SHADOW'S LENGTH OF THE STAKE DURING SOME DAYS OF OBSERVATION (FREQUENCY).

In this contribution of the attendant above, we can see that there is a relationship between the mathematical content (MK) and the experiment "Ferris Wheel" and the practice in the classroom, we conclude that there is a possibility of the course have contributed to the development of the pedagogical content knowledge (PCK).

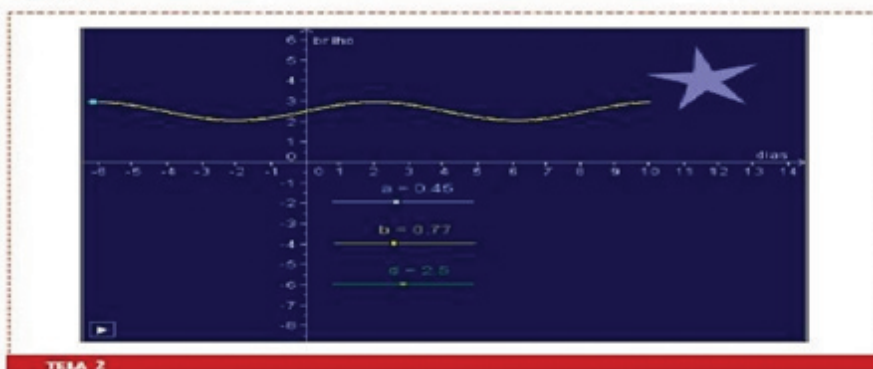
We have identified that the proposal for discussion by the attendants through the forum meets the second principle of (Imbernón, 2010) as being essential in continuing formation, which is to promote the exchange of experiences, school, life, etc. and reflection among equal individuals to allow the update in all fields of educational intervention and increase communication among the teachers.

The third learning object of module 1 of the course is the software "Trigonometric Waves" that initially incorporates the previous object, recovering the concepts of periodic phenomena and the characteristics of the model of a point on a circle. After this study, the third activity of this module is presented, an objective question, as we can see in the figure below.

Na Situação de Aprendizagem 4, do Volume 1 do Caderno do Professor (versão 2009) da 2.ª série do Ensino Médio, um dos problemas propostos envolve a modelagem de fenômenos periódicos. O software *Ondas trigonométricas* utiliza, na atividade 2 – questão 1, o contexto da oscilação do brilho da estrela Delta Cephei (http://m3.ime.unicamp.br/app/webroot/media/software/1240/atividade2_parte1.html).

Quando vemos as estrelas em um céu claro, podemos ter a impressão de que o brilho delas varia um pouco. Geralmente, essa variação se deve a flutuações de transparência da atmosfera. Mas a variação de que trata esta atividade só é observada por sensíveis telescópios profissionais. Os astrônomos descobriram que algumas estrelas brilham com intensidade que varia com frequências muito regulares e permanentes. Essas estrelas foram classificadas como "estrelas variáveis", e são muito importantes, pois servem de referências para medir distâncias intergalácticas.

Esta atividade tem duas partes. Na primeira, os alunos são desafiados a encontrar a função $a \sin (bx + c) + d$ que melhor se ajusta ou se aproxima às observações dadas. Na segunda parte, os alunos verão que a curva de luminosidade mais realista tem alguns aspectos similares à função estudada, $a \sin (bx + c) + d$, mas tem algumas características diferentes também.



Quais são os valores dos parâmetros A, B e D para que a função descreva, de modo aproximado, a oscilação da maré na situação apresentada?

- A = 4, B = 5,4 e D = 1,16
- A = 0,35, B = 5,4 e D = 4
- A = 0,35, B = 1,16 e D = 4

Figure 4. Objective question of Module 1 (Patriarca, 2016, p.121).

This is a multiple-choice question, and the attendants had three alternatives to select the correct one, it was corrected automatically by AVA-EFAP, and thus the attendant noted if he or she hit or not. If the attendant did not hit, he or she could repeat the question again, until he or she did it correctly. Thus, this format does not evaluate if the attendant really knows that content or not. We consider a failure the absence of feedback, when the attendant makes a mistake on this question.

FINAL CONSIDERATIONS

Considering the analyses made, we conclude that the methodology used in the course studied with the learning objects associated with learning situations of the support material to the teacher can assist teachers to integrate technology into teaching, because according to Bittar, Guimaraes, and Vasconcelos (2008), for this to occur it is necessary that he or she experiences and participates in the elaboration of didactic sequence that will apply to their students. This format has been provided in the activities that involved the referred to learning objects, i.e. first the teacher experienced the proposal, then watched a video class explaining the possibilities of didactic approach with the students and, finally, activities were applied to their students, adapting them to the peculiarities of their class.

The São Paulo state network teachers pointed out, in a study conducted in all boards of education of the state of São Paulo, to have difficulties teaching Trigonometry in High School, especially by the complexity in justifying the students the importance of learning this topic. In this sense, the Program M@tmédias sought to support an approach to education in which the student could “make sense” to the trigonometric concepts, such as the periodicity, the trigonometric functions, the concept of amplitude, the sine and cosine graph through the use of learning objects can be relevant to the teaching practice. Therefore, a video entitled “Dance of the Sun” was discussed, with a focus on the study of the periodic motion of the sun and the analemma formed on the sky, then, an experiment called “Ferris Wheel”, to discussions about periodic and circular motion of the wheel and on the format of the graph of displacement in function of time, finally, the software “Trigonometric Waves”, whose core was the discussion of graphs of trigonometric functions and thus the entire contents of trigonometry of the official curriculum of the state of São Paulo was approached by making the relationship with the learning objects.

Whereas about the last activity of the module of trigonometry, the objective question, associated with the study of the third learning object, this presented alternatives of answers with only one correct, that could be redone as many times as the attendant wanted until he or she hit, and there was no discussion or feedback about the question. Considering such particularities, we believe that the answers stored at AVA on that question have not provided us with elements to see the possibilities of knowledge construction by the attendants.

In this aspect, for future online courses that use objective questions, we suggest that the number of attempts is limited and that there is feedback to the answers.

Finally, it is worth mentioning that on the activity “Discussion Forum”, connected to the second object of learning studied, it enabled the teacher to discuss with their peers about ways to exploit the learning object in the classroom to help students understand the phenomenon. Thus, we understand that this may have aided the attendants to build pedagogical knowledge (PK) and also specific knowledge about the mathematical content (CK), which indicates the viability of opportunities in the course for the development of pedagogical content knowledge (PCK).

We also conclude that the proposal for discussion in the forum met the second principle of continued formation, according to Imbernón (2010), that is, to promote

the exchange of experiences, school, life, etc. and reflection among equal individuals to allow the update in all fields of educational intervention and boost communication among the teachers.

Thus, in every discussion, the pedagogical knowledge, technological knowledge, and pedagogical content knowledge, as reported by Mishra and Koehler (2006), were being built. In this forum, the tutor played a fundamental role in provoking the attendants for that debate to happen, keeping the focus of discussion and systematising the contents studied.

AUTHORS' CONTRIBUTIONS STATEMENTS

F.H.P. and N.M.L.C. conceived the idea presented in the article, which is a study referring to the doctoral research of F.H.P., guided by N.M.L.C. Suggestions for the treatment of the data presented in the text and theoretical review were made by S.F.K.S. The three authors discussed the results and contributed to the final version of the manuscript.

REFERENCES

- Amaral, F. J. (2002). *Ensino da trigonometria via resolução de problemas mediado por dinâmicas de grupo, analogias e recursos informáticos*. Belo Horizonte: Dissertação de Mestrado.
- Bardin, L. (2011). *Análise de Conteúdo* (1 ed.). (L. A. Reto & A. Pinheiro, Trans.) São Paulo: Edições 70.
- Bittar, M., Guimaraes, S. D., & Vasconcelos, M. (2008). A Integração da Tecnologia na Prática de Professor que Ensina Matemática na Educação Básica: uma proposta de pesquisa-ção. *REVMAT Revista Eletrônica de Educação Matemática*, 3, 84-94.
- Cellard, A. (2008). A Análise Documental. In: J. Poupart, *A Pesquisa Qualitativa: enfoques epistemológicos e metodológicos* (pp.295-316). Petrópolis: Vozes.
- Gil, A. C. (2008). *Como elaborar projetos de pesquisa*. São Paulo: Atlas.
- Imbernón, F. (2000). *Formação docente e profissional: formar-se para a mudança e a incerteza*. São Paulo: Cortez.
- Imbernón, F. (2009). *Formação permanente do professorado – novas tendências*. São Paulo: Cortez.
- Imbernón, F. (2010). *Formação Continuada de Professores*. Porto Alegre: Artmed.
- Mishra, P. & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A new framework for teacher knowledge. *Teachers College Record* 108, 1017-1054.
- Moran, J. M. (2013). *A Educação que Desejamos: novos desafios e como chegar lá* (5ª ed.). Campinas, São Paulo, Brasil: Papirus.
- Patriarca, F. H. (2016). *Contribuições do Programa M@tmídias para a Integração de Tecnologia às Aulas de Trigonometria no Ensino Médio*. Dissertação de Mestrado, Universidade Anhanguera de São Paulo, Programa de Pós-Graduação em Educação Matemática, São Paulo.
- Severino, A. (2007). *Metodologia do Trabalho Científico* (23ª ed.). São Paulo, São Paulo, Brasil: Cortez.
- Shulman, L. S. (1987). Knowledge and teaching: foundations of the new reform. *Harvard Educational Review*, 57, 1-22.