Replicating agent-based simulation models of herding in financial markets

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Abstract

Agent-based simulation of herding in financial markets varies in the herding and market mechanism. Replication studies are a cornerstone of the scientific method although it is not applied very often. The research aims to obtain a greater understanding of herding and the related stylised facts, assess the herding models' reproducibility, and find ways to improve reproducibility by replicating two herding models.

The first herding mechanism factor (Tedeschi et al., 2012) controls the extent of the neighbour's influence on the expected returns and hence on the trading decisions. The market mechanism is an artificial market where agents submit either ask or bid orders into the order book, and they trade between themselves. The second replicating herding mechanism (Lux and Marchesi, 2000) is based on transition probabilities to decide whether agents are fundamentalists, optimistic or pessimistic chartists. The market mechanism is demand and supply.

The first replicating study fails to produce the original results, whereas the second does have similar findings to the original paper. The second model's description is done by following the recent STRESS guidelines for specifying models. The guidelines help to cover everything needed for describing the second model. Then features from the STRESS and ODD guidelines are combined to give a slightly revised guideline with a defined structure. This is considered to give an improvement.

Both models have fat tails, and only the second model has volatility clustering. The behaviour of the second model that gives the volatility clustering is called on-off intermittency. This is analysed in detail to understand how the model enters and leaves periods of high volatility. The conclusions are that randomness causes the model to move into high volatility, which happens when the percentage of noise traders is high, and the price effect in the model soon returns the model back to low volatility.

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Declaration

I declare that this thesis is my own work and has not been submitted in substantially the same form for the award of a higher degree elsewhere.

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Chapter 1

Introduction

Agent-based simulation, as a bottom-up method to capture phenomena with a detailed description and flexibility, is a widely used technique to model many systems. Bonabeau (2002) divides the applications into four categories: flows (e.g., evacuations from threats, traffic); markets (e.g., stock or options markets); organisations (e.g., organisational designs) and diffusion (e.g., diffusion of innovation where people are influenced by social connections). It can be a more natural and flexible description to build a model based on the behaviour of agents instead of a simple equilibrium type model. This also makes adding in agents or cutting out agents easy. Additionally, agent-based simulation can represent systems with complex interactions among the agents and with emergent behaviour. Macal and North (2010) divide the applications into three categories: epidemics, markets and socio-technical systems. Macal and North (2014:6) explain the extent of applications as ‘ranging from modelling agent behaviour in supply chains and the stock market, to predicting the success of marketing campaigns and the spread of epidemics, to projecting the future needs of the healthcare system’. The four main aspects of an agent-based simulation are as follows: the agents define the set of agents attributes; the relationships define the set of interactions between each other; the environment defines the reaction to the environment; the system defines the boundary of inputs and outputs (Taylor, 2014). The particular interest in this thesis is the application of agent-based simulation in modelling financial markets.

Agent-based simulation is a simulation technique based on the agent. There is no agreed definition of an agent. One of the possible definitions is an entity that models a cognitive process, such as an individual's intention and belief (Edmonds and Möhring, 2005). Onggo (2016) defines the agent as an independent decision making entity to achieve its objectives. Agents can be human or not. Bandini et al. (2020) define an agent as an autonomous entity, having the ability to decide the action within the environment and interactions among agents, according to its perceptions and internal state. Taking the whole economy into account, consumers, companies, regulators and governments could be agents. Agent-based simulation as a bottom-up method is widely used in financial markets. Compared to traditional equilibrium models, it is a more natural and flexible way to model agents' behaviour and the complex interactions among agents (Bonabeau, 2002).

Financial markets are markets where funds are exchanged. Several common features are shared by different financial instruments and time scales, which are called stylised facts and are based on empirical studies of financial data (Cont, 2001). There are four main stylised facts in financial markets. Firstly, prices follow a random walk or martingale. Secondly, returns do not follow a Gaussian distribution: returns have a distribution that is sharper and narrower in the middle with fatter tails compared with the Gaussian distribution; this distribution feature is known as fat tails. Thirdly, returns are not correlated in each period. Fourthly, squared returns or absolute returns are correlated, which is called volatility clustering. This means, if we use the squared returns or absolute returns instead of real returns, big returns tend to be followed by big returns, and small returns tend to be followed by small returns.

Herding describes behaviour where groups of people keep making similar decisions. This may be due to some types of interactions between them or just because they are using similar rules to make decisions. Herding is an important behaviour identified in financial markets. It may be an explanation for some common features and stylised facts in financial markets. Unfortunately, there are no agreed explanations for these financial statistical features. Using agent-based simulation to study the herding behaviour in financial markets is a useful way to enhance the understudying of herding behaviour and financial markets.

1.1 Research problem

Models are built in financial markets for several purposes. The early models before 1998 mainly aimed to investigate the out of equilibrium phenomena, like historical bubbles and crashes (Samanidou et al., 2007). The Kim-Markowitz (1989) model is an early example of agent-based modelling to reproduce the 1987 crashes from historical observation. The Levy- Levy- Solomon (1994) model is a simple homogenous model to show historical bubbles and crashes. Later models concentrate more on the statistical features of financial time series. The Lux (1995, 1998) models and the LeBaron et al. (1999) model are examples of using a model to explain the universal empirical statistical features, namely the stylised facts. Some models criticise the original model or the fundamental concepts and then modify the model. Chen and Yeh (2001) build a model to illustrate a school concept to fulfil the gap of strategic learning. Schools here can be mass media, national library, information suppliers to give a method to imitate the strategies while we think others' strategy is hard to observe. Only studies about stylised facts are considered in this research. Herding becomes one of the explanations for stylised facts such as fat tails and volatility clustering.

The herding mechanism and the market mechanism are two essential components for herding models of financial markets. Unfortunately, there are neither general market mechanisms nor herding mechanisms. There are different market mechanisms such as excess demand (Lux and Marchesi, 1999; Alfarano et al., 2005) and order-driven markets (LeBaron and Yamamoto, 2007; Tedeschi et al., 2012). For the herding mechanism, some papers (Alfarano et al., 2005; Alfarano and Milaković, 2009; Carro et al., 2015) are based on Kirman's idea of ant behaviour (1993). When facing two identical foods, an ant will tend to choose the option chosen previously by more ants. The herding factor in these models is based on a discrete choice using a probability factor which usually affects the transition probability of two groups, such as pessimistic and optimistic traders. Some papers (Lux, 1995; Lux, 1998; Lux and Marchesi, 1999) describe a herding factor with a continuous time discrete model which is also similar to Kirman's idea but with different probability formulae. Unlike Kirman's switching type model, there are several models mainly based on social imitation. The type of the agents in these models does not change, and the herding effect is modelled through impacting the agent's decisions. The imitation rules in these models vary: the agents in spin models (Chowdhury and Stauffer, 1999; Bornholdt, 2001; Kaizoji et al., 2002) imitate their nearest neighbours; the agents in other models (Markose et al., 2004; Mauri and Tettamanzi, 2012; Tedeschi et al., 2012; Yang et al., 2012; Kaizoji et al., 2015) are influenced by other agents such as the opinions of majorities or successful agents ("gurus"); some autonomous models (Chen and Yeh, 1999; LeBaron and Yamamoto, 2007; Yamamoto, 2011) set the herding mechanism through a genetic algorithm or genetic programming learning; other models divide the agents into different groups with different sizes to replicate the herding by clustering (Chen et al., 2013; Manahov and Hudson, 2013; Lee and Lee, 2015). The literature on this topic is discussed further in Chapter 2.

1.2 Research questions and overview

In order to study the herding behaviour in financial markets, replication is chosen as the research method where an attempt is made to replicate (or reproduce) a model from a previous journal paper. In this context, replicating and reproducing a model are considered to mean the same thing and are used inter-changeable. Replicating involves repeating the research as described in an existing paper. If replicating yields the same or similar results, the replicability or reproducibility of the original paper is proved. Replicating, as a cornerstone of the scientific method, has arisen as an issue in many disciplines, including business and social science. Replication studies in business journals are as low as 10% or less of that research, and most of them do not have similar results as the original ones (Hubbard and Vetter, 1996). For example, one famous paper in economics written by Rogoff and Reinhart (2010) has several errors in data analysis which can be considered to lead to an inappropriate conclusion (Herndon et al., 2014). The result concluded by Rogoff and Reinhart (2010) influenced government policy at that period to some extent. A paper by the Open Science Collaboration (2015) finds that only 39% of 100 experiments in psychology are reproducible. Also, in biomedical science, a low rate of reproducible studies is found and some of them have secondary studies that are based on the original unrepeatable results (Begley and Ellis, 2012). In addition, there are replicating issues in forecasting and social science.

The detailed reason for choosing replication as the method will be discussed in Chapter 3. Inspired by the different agent-based simulation models of herding in financial markets and replication issues in many areas, this research focuses on understanding the herding on the one hand and the replication process on the other hand. The general aims are to investigate whether the chosen models can be replicated and whether the herding structure of the models gives similar types of behaviour to that observed in financial markets. This leads to my two main research questions:

i. How and to what extent can herding produce the stylised facts observed in financial markets?

ii. To what extent can previous modelling results be reproduced, and how can reproducibility in simulation modelling be improved?

The remainder of the thesis structure is as follows. Chapter 2 reviews the literature in three aspects: agent-based simulation, financial markets and herding. Chapter 3 demonstrates the replicating issues in each area, problems of replicating and the way to improve the replicating, guidelines for simulation and why replicating is chosen as my research method. Chapter 4 analyses the first replicating model based on Tedeschi et al. (2012), along with further experiments. Chapter 5 analyses the second replicating model based on Lux and Marchesi (2000), especially understanding the model's on-off intermittency behaviour. The STRESS (Monks et al., 2019) model specification guidelines (discussed in Chapter 3) are applied in the description of the model in Chapter 5. Chapter 6 discusses the evaluation of the STRESS guidelines and suggestions for improvements, the replicating issues found in my replicating experience, and the similarities and differences between the two replicating model results and the implications for understanding herding in financial markets. Chapter 7 summarises all the contributions related to the research questions and suggests future work. All the symbols are in italics.

In terms of the research questions, the question on replicating is addressed by describing the models (Sections 4.1 and 5.1 - 5.3) and by the results of experiments to replicate the results in the original papers (Sections 4.2 and 5.4). The implications and suggestions for improving replicability are discussed in Sections 6.1 and 6.2.

For the question on herding the model descriptions are also relevant to understand the structure of the models. The results from reproducing the original studies also help with understanding the model behaviour. However, the main work on this is the developments of the models and the additional experiments in Sections 4.3 and 5.5. The discussion on this topic is in Section 6.3.

Chapter 2

Literature Review

This chapter introduces the literature for agent-based simulation of financial markets. The different models are explained with the main types being N-type models and autonomous models (Section 2.1). Then, stylised facts in financial markets are introduced and analysed from evidence in empirical data and theoretical studies (Section 2.2). Finally, herding models and results are compared in terms of the types of agents, assets, herding mechanism, price mechanism, and the nature of the results (Section 2.3).

2.1 Agent-based simulation of financial markets

The agent is an essential concept in the agent-based simulation field. The agent is a cognitive process involving an entity (Edmonds and Möhring, 2005). For the financial markets, most agent-based models define the agent by the belief of investors. Fundamentalists and chartists are two main agents used in financial market modelling. Fundamentalists have the perspective that 'the price of an asset is determined solely by its efficient market hypothesis (EMH) fundamental value, as given by the present discounted value of the stream of future dividends and chartists believe the price is 'predicted by simple technical trading rules, extrapolation of trends and other patterns observed in past prices' (Brock and Hommes, 1998:1235). The fundamentalist plays a stabilising force while the chartist plays a destabilising role in the financial markets

(Chen et al., 2012). The fundamentalist thinks the misprice will soon correct to its fundamental value, but the chartist thinks the misprice trend will not disappear in the short run. Therefore, different strategies will be adopted by fundamentalists and chartists. In the market, fundamentalists sell the asset when the price is higher than the fundamental value as they believe the price will decrease to its fundamental value and vice versa. The chartist will buy the asset as they think the price will increase continuously even when the price is higher than the fundamental value and vice versa.

There are different approaches in the modelling of agent-based financial markets, from the fundamentalist-chartist model (Kirman 1991, 1993), which mainly focuses on switching mechanisms between the two types of agent, to the Santa Fe artificial market (SFI market) (Arthur et al., 1996) which uses the genetic algorithms to model learning and adaption. In models with several different types of agents ("N-type models"), agents can only choose the pre-decided types and rules but can change between the different types. The SFI market is an example of autonomous agents (Chen et al., 2012). The agent in this kind of modelling type has more autonomy and flexibility. For autonomous models, agents use the trading belief that is not pre-decided from past experience and can change while the trading takes place (Chen et al., 2012). The main difference between N-type models and autonomous models is the learning process. Learning in N-type models is the same in one group. Different behaviours for homogenous agents are usually caused by randomness. While learning in autonomous models is heterogeneous. Individuals have their self-learning mainly through a genetic algorithm or genetic programming. The following Section 2.1.1 and 2.1.2 include more information on the N-type models and autonomous models. Section 2.2 looks at stylised facts and then Section 2.3 covers more details on herding mechanisms.

2.1.1 N-type model

The N-type model is a series type of model which defines the agents with their belief. N in the N-type model stands for the number of types of agents. For the N-type models, they differ from only one type of agent to numerous types of agents who have heterogeneous beliefs. The simplest one is with just one type of agent who follows the same rules, for example, Yang et al. (2012)'s model.

Gode and Sunder (1993) build a model with two types of agents who are human traders and zero intelligence machine traders. They find human traders converge to the equilibrium, but the computer traders are volatile. The Lux (1995) model focuses on using a fixed number of traders who are optimistic or pessimistic speculators. It is a model to reveal herding behaviour and contagion between the two types of agents and the paper finds that this herding model can explain both excess volatility and mean reversion.

Then Lux (1998) builds a three-type agent model developed from the model built in 1995. In this model, the objective is focused on the social and economic interactions among fundamentalists, optimistic chartists, and pessimistic chartists in the foreign exchange or the securities market. Bubbles and crashes, fat tails, and aggregation gaussianity are found in this particular modelling. Kaizoji (2004) adds noise traders with fundamentalists and chartists, and Sansone and Garofalo (2007) add contrarian traders. Kaizoji (2004) is not in agreement with Brock and Hommes (1998), who categorise the noise trader as the same as the chartist mentioned before. The noise traders differ from chartists as the noise traders will buy when they believe the noise news will be good, while chartists will buy when the anticipated price of the stock is positive. The noise trader is focused on the noisy information, but the chartist is based on their future expectation of the asset price. In the Sansone and Garofalo (2007) model, the contrarian trader performs opposite to the chartist. The chartist will buy when they believe that the price will increase while contrarian traders will sell the stock. Therefore, the chartist performs like the trend chaser while the contrarian performs likes the opposite one.

Brock and Hommes (1998) build an adaptive belief systems model. This model involves heterogeneous beliefs that are more complex than the two and three type model. The most important part of the Brock and Hommes model is the idea of all beliefs. It generates all beliefs in one deterministic function $F_{ht} = g_h * x_{t-1} + b_h$ where g_h and b_h represent the trend and bias respectively, while $x_{t-1} = p_t - p_t^\star$ where p_t means price and $p_t^\star$ means the fundamental value at time *t*. When $b_h = 0$, $g > 0$ the F_{ht} is the belief of the pure trend chaser (strong trend chaser when $g > R$ gross return) and when $b_h = 0$, $g < 0$ the F_{ht} is the contrarian trader (strong contrarian when $g < -R$). The trend chaser believes that the price will increase when the positive trend is observed, while the contrarian believes that the price will decrease when there is a positive trend. If $g = 0$, the model is purely biased with no trend, $b_h > 0$ is upward noise and $b_h < 0$ is downward noise and $b_h = 0$ is the fundamentalist. The fundamentalist has a belief in the fundamental value and believes that F_{ht} equals zero, which is the intrinsic value. As a result, the price is always equal to its intrinsic value. Pure bias investors have a belief in the increase in price when the bias is positive. This model finds that the deviation from the fundamental price will be persistent when the intensity of choice to switch prediction becomes high.

2.1.2 Autonomous model

Lettau (1997) uses a genetic algorithm (GA) to explain portfolio changes to help decide how much of a risky asset to purchase. A single asset is taken into consideration in this model. Investors are trying to maximise their utility function with genetic algorithm learning. The genetic algorithm involves new rule generation and testing for learning evolution (LeBaron, 2001a). It is a biologically inspired learning method developed by Holland (1975) (LeBaron, 2000). The genetic algorithm treats the learning of the investors by replacing the worst with the best and adding new rules through mutation and crossover (LeBaron, 2000). For replacing the bad rule, mutation, new weight and crossover are chosen equally. The new weight is 'Choose one rule from the parent set and choose one weight at random and replace it with a new value chosen uniformly' and the mutation is 'Choose one rule from the parent set and add a uniform random variable to one of the network weights' (LeBaron, 2001a:233). The crossover takes two parents with good rules and chops off a branch of rules from each other and then switches (LeBaron, 2001a).

Arifovic (1996) studies the economy in Kareken-Wallace (1981) with two currencies. Heterogeneous agents learn by genetic algorithm. Arifovic (1996) finds that the exchange rate fluctuates a lot without equilibrium, but the first period of consumption come to a stable equilibrium.

In many models mentioned later, a utility function is used. The general background of the utility function and its risk attitudes are introduced here. The utility theory is a study of people's preferences with respect to the risk attitude. Risk attitude is usually divided into three categories: risk aversion, risk neutral and risk lover. These three attitudes describe different preferences when comparing certainty and uncertainty. Suppose people are faced with the same two expected value payoffs. For example, one will get five pounds in certain, and the other will get ten pounds or nothing with equal probability. If people prefer the certain payoff with five pounds, their risk attitude is risk averse; if people think the two payoffs are the same, their risk attitude is risk neutral; if people prefer the uncertain payoff with ten pounds or zero, their risk attitude is risk lover. The utility represents people's risk attitude through the utility function. For risk aversion preference, the utility function will be concave; for risk neutral preference, the utility function will be linear; for risk lover preference, the utility function will be convex. The concavity of the utility function is used to measure risk aversion. The utility function is divided into absolute risk aversion (CARA) and relative risk aversion (CRRA). The difference between the CARA and CRRA is the variability for the concavity of the utility. For absolute risk aversion, the concavity of the utility stays unchanged no matter how consumption varies. While for relative risk aversion, the concavity of the utility will change when consumption varies. The preference may even change from risk lover to risk averse as consumption varies.

Arthur et al. (1996) build an important model that introduces the SFI model as mentioned before. This model uses the constant absolute risk aversion utility function (CARA), which produces smooth consumption over time. The CARA preference makes the risky assets demand of investors not influenced by the income change or, in other words, the wealth change (Chen et al., 2012). The model suggests the rational equilibrium exists when the rational expectation regime is applied, while the crash and bubbles exist when a complex regime is studied. The regime is the different market behaviour. LeBaron et al. (1999) use the CARA risk preference for investors. The absence of autocorrelations, fat tails, long memory of volume, volatility clustering, and volatility volume correlation are reproduced in this time series model. LeBaron (2001a) builds a model that instead of using the CARA as before, uses the CRRA risk preference. Agents have a different time horizon. The time horizon defines how far back in the past to estimate the future performance of a certain asset. The CRRA is the constant relative risk aversion which describes the demand for a certain investor in a positive linear relationship with the wealth the investor gets (Chen et al., 2012).

Chen and Yeh's model (2001) build a model with a school to produce the fat-tail stylised fact. The concept of school was aiming to solve the criticism of copying the action or strategy from others. The action copying is not so important as the strategy coping because investors usually learn strategy. Hence, it makes the strategy of coping essential. Unfortunately, the financial strategy of others is not easy to imitate. Therefore, school, which can be the national library, information suppliers makes the strategy imitation possible. Then the asset price modelling with utility maximisation is built in heterogeneous agents. Although some people claim the artificial market is efficient, it is interesting that traders behaviourally reject the efficient market hypothesis as the strategies are working in this model.

2.2 Stylised facts

The financial markets are modelled by classifying the pattern of traders with a particular strategy. Millions of trades take place in one day in certain financial markets like the London Stock Exchange. It is not possible to identify the patterns of each trader, and it is not even possible to track the behaviour of a trader. This makes it difficult to build a model to reproduce the activity in financial markets. Therefore, some researchers focus on the psychology of investors and categorise investors into different groups. The uncertainty of model inputs makes output checking vital. As the price of the financial markets is unpredictable, the specific output comparison is not appropriate. The study of stylised facts helps us to identify the statistical features of the data from financial markets, thus making the output comparison work. As a result, understanding stylised facts will help the model building and output checking. Therefore, this section looks at stylised facts empirically and theoretically.

There are several stylised facts divided into the high and low frequency data. High frequency data are intraday data from the financial markets, while low frequency data are daily, monthly, quarterly data, and so on. Based on the stylised facts reviewed in Chen et al. (2012), the stylised facts in the agent-based modelling are usually assumed by day. Thus, the stylised facts found in low frequency data are mainly those under consideration. The three well-known stylised facts are non-Gaussianity of the return distribution; the return of today and the subsequent return correlation is almost zero; the transformations of returns like an absolute return or squared returns are positively correlated in each period (Taylor, 2005). In the following sections, firstly, the general issues for stylised facts are discussed. Secondly, empirical evidence for stylised facts is established. Thirdly, theoretical evidence providing possible explanations for certain stylised facts on returns is revealed in detail.

2.2.1 General background

In the financial markets, prices are usually treated as a random walk. The efficient market hypothesis points out that available information is incorporated in price instantly, not only the historical price information but also the public and private information (Fama, 1991). Therefore, only the new information can influence the price, and the price is not predictable as the new information is not known. Stylised facts of the time series describe the statistical features of the financial markets. On the surface, stylised facts are against the unpredictable financial markets argument as they reveal some statistical regularity of the price returns or the trading volumes. The regularity of returns and trading volumes, to some extent, can help to predict the future price. In fact, the stylised facts are the features of the financial time series, in the long run, sometimes more than half a century. Stylised facts are defined as 'the seemingly random variations of asset prices do share some quite nontrivial statistical properties. Such properties, common across a wide range of instruments, markets and time periods are called stylised empirical facts.' (Cont, 2001:224). So, it is not against the existing financial theory. The methods used in the empirical statistical facts are usually the non-parametric method and semi-parametric (Cont, 2001). The non-parametric method, a model free method, focuses on the patterns and features of the data itself. As the non-parametric method is qualitative for describing the statistic, the semi-parametric method adds the quantitative value to describe the property precisely, for example, the scale of the property.

2.2.2 Empirical studies

Only the main stylised facts are discussed in this section. The stylised facts discussed below have lots of empirical and theoretical studies.

2.2.2.1 Returns

2.2.2.1.1 Absence of autocorrelation

Cont (2001) finds after the testing of linear autocorrelations of asset logarithm returns for each period, the autocorrelations are insignificant. The small intraday time scale for less than 20 minutes is not included. The return from time t to T is defined as (Cont, 2001):

(2.1) $r\ (t,T) = X\ (t+T) - X(t)$

(2.2) $\mathrm{X(t)} = \ln S(t)$

where $S(t)$ is the price of a financial asset at time t.

The autocorrelation function is based on the log return on stock price (equation (2.1) and (2.2)):

(2.3) $C(\tau)\ =\ corr\ (r(t,T), r(t+\tau,T))$

In equation (2.3), the Greek letter τ is the time lag. The letter τ is the multiple of the T typically, $corr$ stands for correlation. Malkiel and Fama (1970) examine the 30 stocks in Dow Jones Industries Average from 1957 to 1962 for every one, four, nine and sixteen days. The log of stock price returns from the market data does not show any evidence for linear autocorrelations. The absence of linear autocorrelations for log returns indicates a further study scope in the non-linear autocorrelations for log returns. However, the absence of linear autocorrelations does not hold for transformation returns of an asset like the absolute return or square return of the stock price, which are defined (Cont, 2001):

(2.4) Absolute return $C_0(\tau)\ =\ corr\ (\ |r(t+\tau,T)|, |r(t,T)|)$

(2.5) Square return $C_2(\tau)\ =\ corr\ (\ |r(t+\tau,T)|^2, |r(t,T)|^2\ )$

The stylised facts related to the absolute or square return will be illustrated in Section 2.2.2.2.

2.2.2.1.2 Bubbles and Crashes

An operational definition of the bubble 'is based on whether the future realised returns of the asset justifies the original price over a time period long enough so that the present value of cash flows received by investors during this periods constitutes at least one-half of that price' (Siegel, 2003: 13-14). Rosser (1997) states the existence of bubbles that fundamentally reflect irrational behaviour. Bubbles can be negative or positive as the return is below or above the fundamental value. Crashes often happen with the positive bubble and the price crashes to its fundamental value suddenly, and investors receive a diametrical loss in returns. When crashes happen, the existence of bubbles can be proved because crashes follow bubbles. Bubbles and crashes happen across time series and countries. The historical examples of crashes are the 1929 Roaring Twenties, 1634 Dutch tulip mania, 1719 Mississippi Bubble, 1720 South Sea Bubble, March 2000 internet share prices (Brunnermeier, 2008). The Roaring Twenties 1929 bubble was also called the Wall Street bubble in America. The roaring twenties describe life before the crash. People were interested in the stock market and bought stock with their borrowed money (Rosenberg, 2012). Thus bubbles emerged in the market and it crashed in 1929. This great crash caused economic depression, impacting not only America but also other countries. The 1634 - 1637 Dutch tulip mania, 1719 - 1720 Mississippi Bubble, and the 1720 South Sea Bubble are three important bubble events in Europe. Dutch tulip mania is the earliest famous bubble event caused by the Dutch tulip. In the sixteenth century, the tulip entered Western Europe and was priced above all flowers. Then the price of tulip reduced rapidly and affected the Dutch economy (Van Der Veen, 2012). The Mississippi Bubble was caused by a French company undertook by John Law. It was a company involved in some financial activity like floating shares. Then the investors lost confidence in the company, and the firm's value was reduced dramatically (Velde, 2009). The South Sea bubble came from the South Sea company, which was granted a monopoly of trade with South American (Carswell, 1960). Then the share price reduced heavily, and the bubble crashed. The 2000 internet example was about speculative activity in the stock market related to the internet, then the price of such companies became much higher than expected (Demers and Lev, 2001). In 2000, the internet companies shares crashed.

2.2.2.1.3 Equity Puzzle

The average returns of stocks and treasury bills are remarkably different over a century in the US markets (Kocherlakota, 1996). The difference between the return of stocks and treasury bills, which is the equity premium, is as high as 6.9 %, while the return of treasure bills is only 1% for over 110 years from 1889 to 2000 (Mehra, 2003). However, the equity premium is much less if the duration of years is expanded to 200 years from 1802 to 1998 with 4.1% (Siegel and Coxe, 2002). This phenomenon is not only in the United States but also in countries like the United Kingdom, France, and Japan (Mehra, 2003). Since the Second World War, the premium of equity is 4.6% over the 1.1% risk free rate in the UK stock market. Treasury bills are usually treated as the risk free rate as the default risk for such bills is very small. The equity puzzle for these statistical features is on two sides: the low risk free rate and the high risk premium. The low risk free rate can be explained by several models like the generalised expected utility model, while the high equity premium is still a puzzle (Kocherlakota, 1996).

2.2.2.1.4 Fat tails

The unconditional return usually has a positive excess kurtosis distribution (Cont, 2007). Ane and Geman (1999) model the returns without any conditions as:

(2.6) $F_T(\mu) = P(r\ (t,T) \leq \mu)$

P in equation (2.6) represents the probability function. Therefore, $F_T(\mu)$ is a cumulative probability when *r (t, T)* is less or equal to μ. The kurtosis represents the deviation from the defined distribution F_T to the normal distribution. The kurtosis κ is defined as κ (Arnoedo et al., 1998):

(2.7) $\kappa = \frac{(r\ (t,T) - r\ (t,T))^4}{\sigma(T)^4} - 3$

In the two equations above, $\sigma(T)$ is the standard deviation of the log returns $r\ (t,T)$ in equation (2.1) where $X(t) = ln\, S(t)$ which is the logarithm of a certain stock price. For kurtosis, if $\kappa = 0$, a normal distribution will be observed. When κ is a positive value, a distribution with fat tails will be produced. In the asset markets, positive values for assets are observed in many financial markets. The stock prices in various markets are non – Gaussian distributed where the kurtosis value is not zero: for US dollars / Deutsche Mark exchange rate futures, US dollars / Swiss Franc exchange rate futures, Standard & Poor's 500 index futures, the kurtosis values are about 74, 60, 16 respectively when the time interval is equal to five minutes (Campbell et al., 1997; Cont et al., 1997; Pagan, 1996). This empirical data provides a persuadable fat tail for intraday data, but low frequency data still holds this property (Cont, 2001). Müller et al. (1998) show the existent of fat tails in the daily foreign exchange markets daily. In recent academic literature the interpretation of kurtosis is disputed. Westfall (2014) argues that kurtosis only reflects the nature of the tails of the distribution and not the peak, but Crack (2019) disagrees with this. As the value of kurtosis varies from market to market, the exact distribution which can reproduce the observed property is difficult to decide. Literature tries to propose lots of models with several parameters to match the property for the statistical findings of fat tail returns, for example, exponentially truncated stable distributions (Barndorff-Nielsen, 1997). Four parameters are needed to make the fat tails 'a location parameter, a scale (volatility) parameter, a parameter describing the decay of the tails, and eventually, an asymmetry parameter allowing the left and right tails to have different behaviours' (Cont, 2001: 226). The residual returns after volatility clustering correction may through generalised autoregressive conditional heteroskedasticity (GARCH) models, still have fat tails. This is treated as a conditional fat tail stylised fact where the tails are not the same heavy as the unconditional one (Cont, 2001). Volatility clustering will be introduced in Section 2.2.2.2.2.

2.2.2.2 Transformation of returns

2.2.2.2.1 Power law of returns

Power law behaviour is observed in returns. With the help of equation (2.1) and (2.2), the probability for an absolute return greater than a certain number of x is (Gabaix et al., 2003):

(2.8) $P(|r\ (t,T)| > x)$

There is evidence showing that the data for the probability in (2.8) follows the power law distribution with an exponent of about 3 (the function of power law distribution: x^{-3}). Therefore, this is also known as the inverse cubic law. This phenomenon is observed in a variety of markets and even consistent with the 1929 to 1987 market crashes data (Gabaix et al., 2003). This demonstrates the universal statistical property for the power law behaviour of returns. Thirty German stocks in the Deutscher Aktien Index for a six-year period from 1988 to 1994 are analysed, and the probability of absolute return data follows the inverse cubic law (Lux, 1996). Gopikrishnan et al. (1999) take an observation in Standard & Poor's 500 indexes. Power law behaviour of returns lasts for 8686 daily data from 1962 to 1996 and 852 monthly data from 1926 to1996 separately.

2.2.2.2.2 Volatility clustering (power law of volatility)

Taylor (2005) reviews one year data for the Standard & Poor's 500 index and finds the speed of price movements varying in one year. When there is less information, the prices move slower than when there are more trading and information. The speed in some months is faster than in other months. The rate of price change is defined as volatility, typically the standard deviation of stock price returns.

The autocorrelation function of absolute returns is positive over several weeks and reaches zero slowly in the SLM stock (Cont, 2007). This indicates that the absolute return of a certain asset is dependent and usually follows a pattern where the big returns tend to be followed by big returns and small returns tend to be followed by small returns. The dependency of the absolute return is questionable on whether it is a long range dependency or a short range dependency. Long range dependency describes the decrease of the autocorrelation function as a power law of τ which is the time lag. While short range dependency describes a geometric decay of the autocorrelation function (Cont, 2007). Several papers study autocorrelation and provide evidence for power law behaviour of returns. For example, Cont (2001) demonstrates that the autocorrelation function of absolute returns decreases slowly as a power law distribution with exponent approximately ranging from 0.2 to 0.4.

2.2.3 Theoretical Studies

The theoretical support for those stylised facts discussed in Section 2.2.2 is explained. The explanation for the equity puzzle is excluded because it is not so relevant to agent-based simulation modelling. Theoretical evidence related to stylised facts is mainly focused on non-herding models. The herding models provide another explanation of how the facts can emerge and are discussed in Section 2.3.

2.2.3.1 Absence of autocorrelation

There is not much literature that discusses the absence of autocorrelations. This may be because it is easy to understand and fulfils the basics of financial markets. On the one hand, Fama's (1991) efficient hypothesis thinks the market incorporates the new information instantaneously. Any irrational behaviour can be detected by arbitragers who eliminate the under or over the value of the price. As the information is unpredictable, therefore, the market price is fluctuating randomly. On the other hand, if the market has a strong correlation of price movement, the pattern of price and expected return can be predicted. Therefore, it will be the right strategy to follow the autocorrelations to earn a positive profit (Cont, 2001). Then the autocorrelation will be reduced as these strategies are used. Then patterns or excess profits pull the price back to random. Thus the price patterns and the autocorrelation of price are eliminated. In a very short time, this inefficiency may emerge, but a simple strategy that follows the pattern can earn a profit so that the pattern disappears through trading. Typically, the correlation time is short for futures markets. The foreign exchange market has an even shorter time (Cont, 2001).

2.2.3.2 Bubbles and Crashes

Bubbles usually occur by four categories of reasons: rational or near rational bubbles, where bubbles are built on the speculators' rational expectation for future asset return or behavioural factors like overconfidence; intrinsic bubbles where bubbles are built on overreaction to news; fads where bubbles are built on social interactions like media amplification and social networks; informational bubbles where bubbles are built on information cascades which the investors are faced with (Montier, 2003). The information bubble states a situation where people have information without aggregation. Thus, there is a trade-off between their own judgment on the basis of their own signal and following the other traders. These four factors all make the misprice emerge.

On the other hand, the persistence of bubbles is caused by limits to arbitrage. In an efficient market, mispricing will be eliminated by arbitragers. The persistence of mispricing is due to arbitrage limitations. An arbitrager is different from the speculators who take a guess of future expectations and make gains through hopefully buying low and selling high. The arbitrager examines the price in different markets to seek the mispricing and believes the mispricing will converge to its fundamental value. Gromb and Vayanos (2010) identify the fundamental risk which arbitragers may bear in the financial markets. As the arbitragers make a profit by finding the mispricing in the market, the existence of substitutes can make this possible. The perfect substitutes are like a benchmark for a particular asset that makes the justification on price from arbitrage realistic. In other words, if there is no substitute, the arbitragers will bear the risk of making mistakes on the price justification and may take the wrong position, like becoming a seller rather than a buyer. Also, the short selling cost may limit arbitrage (Gromb and Vayanos, 2010). In some countries, there is a limit for selling a financial asset in the market to protect the big jump of a certain asset price. When arbitragers need to sell a certain asset suddenly to generate profit, the short selling cost makes this strategy not possible. Abreu and Brunnermeier (2002) introduce a synchronisation risk that arbitragers may be faced with. When arbitragers plan to beat the misprice, they should all do this together at the same time so that the over or under price can be sent back to its fundamental price. The question is, as an arbitrager individually, you may not be able to identify other strategies or actions. Thus, this factor makes arbitrage difficult and limits the profit for an arbitrage opportunity.

DeLong et al. (1990) demonstrate that the fear of loss by investors produced by the noise traders will limit the arbitrage. It is a model to prove the existence of noise traders. It has two types of traders: a noise trader and an informed trader. Two assets are trading: a safe asset with a certain dividend at 1; a risky asset with a certain dividend but in perfectly inelastic supply. With the help of the expected utility based on the negative exponential CARA utility, the utility function for an informed trader and noise trader can be found. Then the maximum point for the utility of each trader's demand can be different. The proportion of noise traders and informed traders multiply their demand separately. Then the expected price can be expressed in terms of the true value, discounted noise trader mispricing and the present value of misevaluation with its proportion and risk adjustment. The price equation demonstrates the relationship between the expected price and the noise trader risk. Also, the demand of the informed trader and the noise trader with the different returns between these two types can produce an expected difference formula, which proves the existence of a noise trader as the expected difference can be positive. The influence caused by the noise traders on the expected price will limit the arbitrage position. Noise traders' misevaluation is unpredictable and also persistent. The informed trader may not be able to go against the misprice heavily as the price may not go back to its fundamentals in the short run. But this model just takes into account the systematic risk, not the idiosyncratic risk and the constant utility function, instead of the relative utility function.

Furthermore, the hedge fund managers may find the predictable price through the investors' sentiment may make trading on the mispricing less attractive than free riding the bubbles (Brunnermeier and Nagel, 2004). This paper observed the hedge fund performance for 71 hedge fund manager's records for a two-year period from 1998 to 2000. The analysis suggests managers ride bubbles instead of taking an arbitrage strategy. This can be explained on two sides. There is an opportunity cost of the arbitrage strategy, which is the amount forgone for this certain activity. Also, the factors discussed above related to limits to arbitrage constrain the arbitrage action further. The predictable sentiment can make the riding bubble strategy more profitable compared to the arbitrage strategy. Riding the bubble can earn more money than doing arbitrage as they can predict the future price through the investor sentiment and sell or buy the financial asset before goes back to the fundamental value.

2.2.3.3 Fat tails

Evolution and heterogeneous arrival rates of information are two factors generating the fat tails that will be introduced in 2.2.3.5. Also, herding models in Section 2.3, for example, the Kaizoji et al. (2002) model, generate fat tails.

Thurner et al. (2012) establish a model with leverage that causes fat tails and the volatility clustering effect. In this model, the leveraged asset with a margin call is considered. The margin call is a security for the creditors who lend money to firms to reduce the default risk. As we all know, if the firm is not in good condition, the money they borrowed from creditors may not be returned in full value. When the firm faces a bankruptcy risk, its assets are depreciated. Thus, they do not have enough money to pay back to the creditors. If the creditors cannot collect enough money from the firms who borrowed money for them, the creditors may fall as well. A margin call is a way to protect the creditors in a certain way. As the leverage contract is settled, the leverage ratio will be decided. The leverage ratio represents the level of loan in a certain project, investment, or total firm value. The more the loan's value, the higher the leverage ratio, and thus, the more cash down payment. 'A buyer at his threshold of x times leveraged loses $x\%$ of his investment for every 1% drops in the asset price, and on top of that will have to come up with \$$(x - 1)/ x$ of new cash for every \$1 drops in the price of the asset' (Thurner et al., 2012:02). The fall in asset price to margin call transfers the buyer to the seller as a non-linear dynamic. In this model, investors have a trade-off of a single asset and cash. The noise trader trades randomly to make the price mean-revert to its fundamental value V. The funds will enter the market when the price is undervalued compared to V. The funds borrow money from creditors at zero fixed rate and treats the asset as collateral. All traders in the market are so small that they cannot influence the market price. There are two demanders: one is a noise trader and the other is the fund. The demand equation for the noise trader and the fund is defined. The wealth dynamic is based on the relationship between the price and demand dynamic. The result increases the leverage ratio x with consideration of margin call from 1 to 10, when $x = 1$, there is no leverage. The negative return generates fat tails and the majority of returns are in the centre. Thus, the leverage effect explains fat tails to some extent.

2.2.3.4 Power law of returns

LeBaron (2001c) introduces a stochastic volatility model and as a result, it produces power law behaviour.

Lux (2006) demonstrated a model with a speculative bubble. The speculative bubble is based on rational expectations. Therefore, the price today *(t)* is defined as the expectation of the dividend tomorrow *(t + 1)* on the condition of today's information *(t)* plus the price for tomorrow *(t + 1)* divided by the appropriate discount factor. The equation for a price today is based on the price tomorrow and can be extended to time *t+2*, *t+3*. Thus the fundamental value of the asset is the expected value of all future dividends on the condition of today's information with an appropriate discount factor. This price relationship is based on the traditional view of efficient markets. This model takes the speculative activity into account, so the price will be no longer at its fundamental value but the fundamental value plus the bubble component. The bubble component is defined as

$B_t = a_t * B_{t-1} + \varepsilon_t$ (9)

The ε_t is a noise term as the bubble prediction from the last period may have some error. 'The equation, with a_t belongs to $A = \{a_1, a_2, \ldots, a_n\}$ occurring with probabilities π_1 $\pi_2, \ldots \pi_n$ and ε_t IID (identically and independently distributed) with the mean of zero. The only additional restriction which the a_i have to meet the condition that $E[a_t]$ is the sum total from *i* to *t*, $\pi_i * a_i = 1 / \delta$, where δ is the discounted factor' (Lux, 2006: 10). As a result, for certain parameter settings for a_t, the speculative model with rational expectations can generate power law behaviour in a certain way.

Also, there are some multi agent models that found the existence of the power law behaviour of return in their model, for example, the LeBaron et al. (1999) model.

2.2.3.5 Volatility clustering (power law of volatility)

As volatility clustering describes the correlation between the volatility, it is not easy to provide explanations that can cause the clustering in a straightforward way. Thus, the researchers try to build a model and add the explanation component to reproduce the volatility. Cont et al. (2004) establish a simple model to find the explanation for volatility clustering. This model has a market with a single asset traded by N agents, which is typically large (N = 1000, 1500) at time $t = 0, 1, 2, \ldots$ The time can be defined as minutes, hours, and days. In this case, the time is days. Each day, the agent can either trade through buying (demand = 1) or selling (demand = -1) or not trade on the basis of the public information, which modelled as a Gaussian random variable. The sign of the information value represents buying or selling strategies, where positive represents buying strategies. The significance of the information value based on the threshold decides the trading activity. Each agent has its own decision threshold on the basis of the agent's subjective view. If the absolute value of information is greater than the threshold value, the information value is significant enough to activate the trade; otherwise, the agent will not trade at this period. The total demand can be calculated through the agent's decision when buying equals to 1 and selling equals to -1. Then the change in price can be reproduced. The thresholds decision may be updated on the basis of the moving average of the absolute return by a certain agent at a certain period with probability q followed by the $[0, 1]$ uniform. This model is simple in several aspects. A single asset is traded and the agent is not categorised in different characteristics like fundamentalist and chartist. The information received by all agents is the same. The different trading rules for each agent are caused by their own judgment through the threshold value. Also, there is no interaction between the agents.

The simulation results are considered in two extreme cases: one is not updating and the distribution of threshold remains identical at the origin where $q = 0$; the other is updating the threshold at the same time on the basis of the last time absolute returns where $q = 1$. At the same time, updating eliminates the heterogeneity. The first case has a non-volatility clustering pattern; the second has a volatility clustering pattern but the distribution of returns on condition of the last period absolute return is unrealistic (Cont et al., 2004). The normal setting of results with agent 1000, $q = 0.01$ and agent 1500, $q = 0.1$ shows the positive correlation with absolute returns. When $q = 0.01$, the threshold is updating every 100 days. When $q = 0.1$, the threshold is updating every ten days. Therefore, the three cases comparison shows heterogeneity and updating from the last period feedback are two important components generating the volatility clustering phenomenon.

Then there are the evolution models, which contain the genetic algorithm first introduced by Holland (1975). LeBaron et al. (1999), as a Santa Fe artificial stock market, reproduced the volatility clustering. Settings in LeBaron et al. (1999) are more complex than the model demonstrated before. There are two tradable stocks instead of a single asset before. The two stocks are risk-free, with constant dividends and infinite supply, while risky assets have stochastic dividends. Investors follow the constant absolute risk aversion (CARA) (LeBaron et al., 1999). Both of the models reproduced the volatility clustering and fat tails successfully. Thus the evolution and heterogeneous arrival rates of information can be two factors influencing both the volatility clustering and fat tails stylised facts.

Also, herding is regarded as one of the explanations for clustering volatility. Some of the models in Section 2.3 generate volatility clustering. For example, Bornholdt (2001)'s model and LeBaron (2001a)'s model has a heterogeneous investment horizon. In LeBaron (2001a), the risk preference of agents is CRRA.

2.3 Herding models and results

Herding usually arises from two factors. One is the direct copying of the others, and the other is the convergence of views based on the information of the public (Markose et al., 2004). The herding behaviour modelled in the literature concentrates more on the imitation from the others through copying. There are mainly 6 aspects to be considered when building an agent-based modelling in financial markets. The agents, trading, securities, evolution, benchmarks or calibration, and time (LeBaron, 2001b). As there are at least 6 factors to be considered, the agent-based simulation will have numerous ways of building the model and studies will vary from each other on the basis of the theory they believe and the area or reality they want to explain. Then detailed analysis will be demonstrated below for twenty-five agent-based models with behavioural herding. As the benchmarks or calibration and time are the factors mainly ignored by most of the models, the remaining 4 factors will be illustrated below. Results are important to be included because this research is interested in stylised facts. Identifying these five factors in agent-based simulation of herding in 25 papers will give us a basic understanding of how to build a herding model and how to study the herding. Also, understanding the similarities and differences of 25 papers from these five factors will help to have a deeper and more detailed understanding of herding and its related results. Hence, it will help to decide which models to choose to be replicated.

2.3.1 Agents

As discussed in Section 2.1, agent-based simulation models are mainly divided into N-type models and autonomous models. Agents in N-type models have N types of beliefs. *N* can be an integer from *1* to *N*, while in the autonomous model, agents have different rules. The papers in the review are ordered by categories and the time of publication.

2.3.1.1 N-type model

Raberto et al. (2001) and Yang et al. (2012) build a one type model. Markose et al. (2004) also build a one type model. Agents decide to buy or sell for one unit of a certain asset. The difference of agents in this model is the length of previous memory they have. Tedeschi et al. (2012) is also a one type model in which agents decide to trade or not on the basis of their expectations. For some experiments in this model, they add fundamentalists and chartists. Chen et al. (2013) introduce a one type model. For each agent, they choose one of three states: buying, not trade or selling. In one-type models, some of them (Raberto et al., 2001; Yang et al., 2012; Chen et al., 2013) divide the agents into different clusters. Those models are classified as one type model because all agents share the same rules. Two remaining ones (Markose et al., 2004; Tedeschi et al., 2012) use the network to connect agents. Also, agents in these two models share the same rules, so they are categorised as a one type model.

Lux (1995) and Carro et al. (2015) have two-type models with optimistic and pessimistic traders. Agents are either fundamentalists or noise traders in Bak et al. (1997) and Alfarano et al. (2005). Chowdhury and Stauffer (1999) build a two-type model that has a similar state function to Chen et al.'s (2013). Agents are either fundamentalists or noise traders, too. Bornholdt (2001) introduces a two-type model with fundamentalists and chartists. Kaizoji et al. (2002) have a two-type model with fundamentalists and interacting traders. Alfarano and Milaković (2009) introduce a two-type model that divides agents into two groups. They can be fundamentalists and chartists like Bak et al. (1997) or optimistic and pessimistic traders like Lux (1995).

Lux (1998), Lux and Marchesi (1999), and Lux and Marchesi (2000) have three-type models with fundamentalists, optimistic noise traders, and pessimistic noise traders. Kaizoji et al. (2015) is a three-type model with rational agents and noise agents who only invest in risky or risk free assets. The noise agents investing in risky or risk-free is similar to optimistic noise traders or pessimistic noise traders.

Lee and Lee (2015) have an N-type model with N types of beliefs. They assume all agents know both fundamentalists and chartists' ways to predict the expected price. Then the weight for these two rules is different among agents by their own belief. That's why the agents in this model are considered to be N-type. N stands for the number of agents.

2.3.1.2 Autonomous model

Chen and Yeh (1999) build an autonomous model based on genetic programming. Two genetic programmings are used in this model. Chen and Yeh (2001) also build an autonomous model based on genetic programming. Mauri and Tettamanzi (2012) introduce an autonomous model based on genetic programming in which the genome for each agent does not change. After a certain period, the fitness for each agent is compared. Then 30% of agents are eliminated. Wealth is redistributed and new agents will enter the market based on a genetic algorithm. LeBaron (2001a), LeBaron and Yamamoto (2007), Yamamoto (2011), and Manahov and Hudson (2013) propose autonomous models based on genetic algorithms. The rules for agents are updated through genetic algorithms.

For agent-based modelling of herding in financial markets, autonomous models differ from N-type models. In N-type models, agents divide into one type, two types, three types and N types. There are 18 models that are N-type models and 7 models that are autonomous models, which indicates that the N-type models are the main focus of the herding ones. Based on choosing a model that can be both relatively simple and have a detailed description, then N-type models are selected to be replicated.

2.3.2 Assets

In most of the models, a single asset is built in to make the model simpler. Four authors (Bak et al., 1997; LeBaron, 2001a; Yamamoto, 2011; Kaizoji et al., 2015) propose models with risky and risk-free assets. For example, in Kaizoji et al.'s (2015) model rational agents maximise their expected utility through risky and risk-free assets while noise agents buy either risky assets or risk-free assets. The number of noise traders invests in the risky asset and the other noise traders invest in the risk free asset. Agents in the two models of Chen and Yeh (2001) and Manahov and Hudson (2013) invest in two assets: a risk asset and cash. As most of the models consider one single asset, for replicating, a one single asset model is considered to be both simple and appropriate.

2.3.3 Herding mechanism

There are four types of rules of herding in 24 of the 25 models. They are group learning through transition probability, networks, and clusters, and individual learning such as genetic algorithms. Group learning is used in the N-type model, and individual learning is used in autonomous models. For group learning, the herding mechanism is ordered by the importance which is the number of models using a certain mechanism.

Only one model focuses on herding in a different way as convergence views based on the information of the public. This means this model does not have special herding rules but is interested in how people react in the same market as doing the same thing on the basis of the same information.

2.3.3.1 Transition probability

The herding mechanism in this part is inspired by Kirman's ants theory. Kirman finds that when ants are faced with two kinds of identical food, groups are not equally divided. That means when people are faced with two similar restaurants A and B, people will prefer to choose restaurant A instead of restaurant B if restaurant A has more customers. The switching between optimists and pessimists depends on the index which is the difference between optimists and pessimists in Lux (1995). Alfarano et al. (2005) also use transition probability as the herding mechanism. Switching between fundamentalists and noise traders is based on the number of fundamentalists and noise traders. Carro et al. (2015) have a similar herding mechanism to Alfarano et al. (2005) with external information. The transition probability only applies to noise traders in

Kaizoji et al. (2015). The opinion index, which is the difference between the noise traders who invest in risk-free assets and risky assets are divided by the total number of noise traders, and price movement decides the transition probability.

Different types of agents in these models have different herding mechanisms. In Lux (1998), Lux and Marchesi (1999), and Lux and Marchesi (2000), the change between noise traders depends on the price trend and the difference between optimistic and pessimistic, while the change between fundamentalists and noise traders depends on the profits earned for each group. The herding mechanism of Chowdhury and Stauffer's (1999) model is based on the transition probability. Agents switch their states (buying, selling, not to trade) through their herding mechanism on the basis of the transition probability. For noise traders, the transition probability from the current state to another state depends on the total opinion of traders. While for fundamentalists, the transition probability is dependent on not only the total opinion of traders but also their individual bias. The transition probability in Bornholdt (2001) is based on the opinions of all agents to buy or sell and the type of agents. If the agents are fundamentalists, they tend to join the minority, and if the agents are chartists, they tend to join the majority. The transition probability in Kaizoji et al. (2002) is based on agents' opinions. Herding in this model only influences the interacting traders. If agents are in the minority group, they have a greater possibility to switch to the majority group for gaining some capital. While if agents are in the majority group, they have a greater possibility to switch to the minority group for avoiding the risk.

In total, ten models are based on the transition probability. Four out of six two-type models are based on the same rules for the transition probability. The other two two-type models and four three-type models are based on different rules for the transition probability.

2.3.3.2 Networks

After the trading, noise traders change from buyers to sellers or sellers to buyers and randomly choose one seller or buyer agent to imitate in Bak et al. (1997). In Markose et al.'s (2004) model, the herding mechanism is based on the networks. Agents take recommendations from connected neighbours to buy or sell. Zero memory agents give random recommendations to buy or sell, while agents with memory give advice on the basis of their past experience. The herding in this model is measured by the number of buyers divided by the number of total agents. Herding in Tedeschi et al. (2012) propose a model with a network where agents' expectation of returns is influenced by their neighbours' expected returns.

Alfarano and Milaković (2009) combine the transition probability with network influences. Unlike the transition probability models introduced before, the transition probability not only depends on the number of agents in each group like Alfarano et al. (2005) but also the social network for each agent.

All in all, four models depend on network herding. Two of them are one-type models, and the other two are two-type models. Tedeschi et al. (2012) propose a model with three-type models in a short discussion, but network herding is only considered in one group.

2.3.3.3 Clusters

Raberto et al. (2001) connect agents into different clusters. The clusters can either be active or inactive. Agents in the same active cluster submit the same orders. Then the cluster is destroyed. The herding mechanism in Chen et al. (2013) divides agents into different clusters. In each cluster, agents make the same decision to buy or sell or hold with the same probability. Chen et al. (2013)'s herding factor is decided by the difference of weighted average opinions of all agents from previous periods and the degree of asymmetry. The degree of asymmetry makes the herding differ from a bull market to a bear market. The bull market is a market where the weighted average opinions are positive and the bear market is one where the weighted average opinions are negative. Then the number of clusters is based on the total number of agents divided by the herding factor.

Yang et al. (2012) have a herding mechanism that combines the network and clusters. Yang et al. (2012) divide agents into different clusters through the network. Agents connected together in the network are in the same cluster and make the same decision.

Three models use clusters as herding mechanisms in which agents divide into different clusters. In each group, agents make the same decision. For models using clusters as a herding mechanism they are categorised as a one-type model.

2.3.3.4 Genetic Algorithm

Genetic algorithms are individual learning with an autonomous type model. In Chen and Yeh (1999), LeBaron (2001a) and Manahov and Hudson (2013), each agent uses their own rules to make independent decisions through the genetic algorithm. Herding emerges when agents change their rules by crossover. Agents in Chen and Yeh (2001) use a function called the school, which is a place allowing direct interaction among agents to imitate strategies. LeBaron and Yamamoto (2007) use the genetic algorithm to update the learning and adaption process. The fitness of 1000 parameters is selected. The strategy with bigger fitness is more likely to be imitated. Yamamoto (2011) use genetic algorithms to update their trading strategy, make agents more likely to imitate the agent who has a better performance. Herding in Mauri and Tettamanzi (2012) is through a genealogy network. Individual’s belief is influenced by their relatives. In total, seven models use the genetic algorithm, and they are all autonomous models.

2.3.3.5 Independent rules

No herding mechanism is used in the Lee and Lee (2015) model. Agents have a chance to change their opinions from buying to selling when prices reach the tipping point.

In total, ten models are based on transition probabilities. Four models are based on networks and three models are based on clustering. Seven models are based on genetic algorithms. One model has no herding mechanism. For different types of herding mechanisms, they are different in the herding formation process as well as the detailed information copying. For transition probability and clusters, agents are usually treated as full information copying. For network connection, some of the models are imitating part of information while some of them are imitating full information. For the genetic algorithm, only part of the information is learned. Transition probability and network are two main mechanisms in N-type models: one imitates full information of others, one copies part of the information.

2.3.4 Price mechanism

2.3.4.1 Demand and supply

Markets clear the price to bring excess demand of noise traders plus the excess demand of expected fundamentalists to zero in Lux (1995). Excess demand also is used in Lux (1998), Lux and Marchesi (1999), Lux and Marchesi (2000) and LeBaron (2001a). A market maker is assumed in the Kaizoji et al. (2002) model to balance the demand and supply from both fundamentalists and interacting traders. Alfarano et al. (2005) clear the price to bring the excess demand of noise traders plus the excess demand of fundamentalists to zero. Kaizoji et al. (2015) have a similar process as Alfarano et al. (2005) to bring excess demand of noise trader plus the rational traders to zero. The price is cleared in Yamamoto (2011). In this model, all agents submit their demand for only one market maker to clear the price. The demand and supply decide the price in Yang et al. (2012).

Unlike the other models formulating the price, prices change in Chowdhury and Stauffer's (1999) model is defined by the difference between the agents of buying or selling. Price change equals the log of excess demand at time *t* minus log of excess demand at time *t-1* in Bornholdt (2001). Chen et al. (2013) introduce a similar method to decide the price change, which is the return. Returns are decided by excess demand in Lee and Lee (2015).

2.3.4.2 Artificial stock market

Unlike the market based on demand and supply, where all buyer and seller orders are traded, only market orders in an artificial stock market are traded. In artificial stock markets, each agent submits either market or limit orders to the market. Orders which get executed are market orders. Orders are matched, where the price of the highest buyer’s order is higher than the price of the lowest seller’s order. Untraded orders left in the market are limit orders. Prices are usually decided by the highest of buyer’s order and the lowest of seller’s order of limit order left in the market. Bak et al. (1997), Chen and Yeh (1999), Chen and Yeh (2001), LeBaron and Yamamoto (2007), Mauri and Tettamanzi (2012), Tedeschi et al. (2012) and Manahov and Hudson (2013) use the artificial stock market. Bak et al. (1997), Chen and Yeh (1999), Chen and Yeh (2001), Raberto et al. (2001), LeBaron and Yamamoto (2007), Mauri and Tettamanzi (2012), Tedeschi et al. (2012) and Manahov and Hudson (2013) use the artificial stock market.

2.3.4.3 Reward

Markose et al. (2004) have a reward system. Agents are rewarded randomly or follow the minority game: if there are more buyers than sellers, sellers win.

2.3.4.4 Others

Alfarano and Milaković (2009) do not focus on the price and returns, but only on the variance. In this model, no detailed price mechanism is explained. Carro et al. (2015) do not have a detailed description of price settings. This paper focuses on the effect of external information.

In total, fourteen models are based on demand and supply, eight models are based on an artificial market, one model is based on reward, and two models do not have the price mechanism. For the transition probability mechanism, nine models are based on demand and supply, and one does not have the price mechanism. For the network mechanism, two models are based on artificial stock markets, one is based on the reward system, and one does not have the price mechanism. For the cluster ones, two models are based on demand and supply, and one is based on an artificial stock market. For genetic ones, two models are based on demand and supply. Five models are based on artificial stock markets. The one without the herding mechanism is based on demand and supply. Two main market mechanisms are demand and supply and artificial markets.

2.3.5 Results from herding models

2.3.5.1 Fat tails

Fat tails, bubbles and crashes are found in Lux (1995). Fat tails are found in Lux (1998) through the distribution of price change and also kurtosis. Fat tails are found in a figure of price change distribution in Bak et al. (1997). Fat tails are found in a figure of price change distribution; also bubbles and crashes are generated in Chowdhury and Stauffer (1999). Fat tails are found in Chen and Yeh (2001) with kurtosis. Also, this model rejects the unit root to prove the random walk and the absence of autocorrelation.

2.3.5.2 Volatility clustering

LeBaron and Yamamoto (2007) find volatility clustering through autocorrelation. Also, volatility clustering is obvious in the economy with herding compared to the economy without herding. Only volatility clustering is found in Yamamoto (2011) through the autocorrelation graph and GARCH parameter. They find herding is essential for volatility clustering by comparing models with and without herding. Also, they find if there is less information, the volatility clustering is less obvious.

2.3.5.3 Both fat tails and volatility clustering

Both fat tails (kurtosis) and volatility clustering (GARCH parameters) are found in Chen and Yeh (1999), who build a model with genetic programming. But only one of the two genetic programming designs proves to reject the unit root, which means the random walk of price and absence of autocorrelation. Both fat tails (cumulative distribution of returns) and volatility clustering (scaling of fluctuation function) are found in Lux and Marchesi (1999). Both fat tails through the cumulative distribution of absolute returns and volatility clustering based on autocorrelation are found in Bornholdt (2001), Raberto et al. (2001), Kaizoji et al. (2002), Yang et al. (2012), Chen et al. (2013) and Kaizoji et al. (2015). Kaizoji et al. (2002) also find a positive correlation between volatility and trading volume. Both bubbles and crashes emerge in Kaizoji et al. (2015). In Kaizoji et al. (2015), bubbles and crashes still emerge when there are a small number of noise traders through changing the wealth ratio of rational and noise traders. Also, leverage (negative return-volatility correlation) and anti-leverage (positive return-volatility correlation) effects are produced in Chen et al. (2013). Both fat tails (kurtosis) and volatility clustering (autocorrelation) are found in Lux and Marchesi (2000) and LeBaron (2001a). Also, Lux and Marchesi (2000) reject the unit root to prove the random walk and the absence of autocorrelation. There are three comparable markets LeBaron (2001a): benchmark where the equilibrium price is assumed, long horizon one where the agent's horizon is from 75 to 100, and all horizon one where the agent's horizon is from 5 to 100. All of the markets have fat tails, especially for all horizon ones. Volatility clustering is more obvious for all horizon ones. Both fat tails through the distribution of returns and volatility clustering based on autocorrelation are found in Alfarano et al. (2005). Both fat tails through the distribution of returns and volatility clustering through the ARCH effect are found in Manahov and

Hudson (2013).

2.3.5.4 Others

Markose et al. (2004) find that memory and herding links do not have any impact on emerging from gurus. Herding as a result is calculated by the number of buyers divided by the number of total agents. Herding can be generated either through the memory based on previous information (with and without memory) or network interactions (change of interaction parameter). Alfarano and Milaković (2009) use the network to solve the N-dependence problem in models depending on the transition probability. The N-dependence problem is that certain stylised facts such as fat tails disappear when N gets bigger. Mauri and Tettamanzi (2012) find that herding (sociality) is responsible for the bubbles and crashes through data with and without evolution. Tedeschi et al. (2012) focus on the herding behaviour of gurus, imitators and noise traders. Agents become richer when the herding is higher, by comparing the wealth in three situations: agents are based on their neighbours' opinion; agents are based on half of their own opinion and half of their neighbours; agents are based on their own opinion. Carro et al. (2015) find that the amplification of small external information can cause instability in the market. Lee and Lee (2015) conclude that there are more herding effects (returns change frequently) when more agents share the same opinion on the tipping point.

In total, five models find only fat tails: three are the transition probability model, one is the network model, and one is the genetic model; two genetic models find only volatility clustering. Twelve models find both fat tails and volatility clustering: six are the transition probability models, three are clusters models, and three are genetic models. One transition model, three network models, and one genetic model are focused on other results. For full information copying: three cluster models all find both fat tails and volatility clustering; more than half (six out of ten) of transition probability models find both fat tails and volatility clustering. For part information imitating: none of the network models finds both fat tails and volatility clustering, and three out of seven genetic models find both fat tails and volatility clustering. This indicates the way of information copying influences the generating of fat tails and volatility clustering.

Finally, Table 2.1 below contains the 25 models with the agents, herding, market and result. The models are ordered by herding mechanism and time of publication.

No.	Agent	Herding	Market	Result
1. Lux (1995)	Optimistic and pessimistic traders	Transition probability	D&S	FT, BC
2. Lux (1998)	Fundamentalists and noise traders (optimistic and pessimistic)	Transition probability	D&S	FT
3. Chowdhury and Stauffer (1999)	Fundamentalists and noise traders	Transition probability	Price change (D&S)	FT, BC
4. Lux and Marchesi (1999)	Fundamentalists and noise traders (optimistic and pessimistic)	Transition probability	D&S	FT, VC
5. Lux and Marchesi (2000)	Fundamentalists and noise traders (optimistic and pessimistic)	Transition probability	D&S	FT, VC, AA
6. Bornholdt (2001)	Fundamentalists and chartists	Transition probability	Return (D&S)	FT, VC
7. Kaizoji et al. (2002)	Fundamentalists and interacting traders	Transition probability	D&S	FT, VC
8. Alfarano et al. (2005)	Fundamentalists and noise traders	Transition probability	D&S	FT, VC

9. Kaizoji et al. (2015)	Rational Investors and noise traders (risky and risk-free)	Transition probability (only noise)	D&S	FT, VC, BC
10. Carro et al. (2015)	Optimistic and pessimistic	Transition probability	No	Amplification of external information can cause instability
11. Bak et al. (1997)	Fundamentalists and noise traders	Network (noise)	Artificial market	FT
12. Markose et al. (2004)	One-type (different horizon)	Network	Reward	Memory and interaction
13. Alfarano and Milaković (2009)	Two groups	Combine transition probability and network	No	N-dependence problem
14. Tedeschi et al. (2012)	One-type (add fundamentalists and chartists then)	Network	Artificial market	Herding makes noise trader gain higher profit
15. Raberto et al. (2001)	One-type	Clusters	Artificial market	FT, VC
16. Yang et al. (2012)	One-type	Clusters	D&S	FT, VC
17. Chen et al. (2013)	One-type	Clusters	Price change (D&S)	FT, VC
18. Chen and Yeh (1999)	Autonomous (genetic programming)	GA: crossover	Artificial market	FT, VC, AA
19. Chen and Yeh (2001)	Autonomous (genetic programming)	GA: a function called school	Artificial market	FT, AA
20. LeBaron (2001a)	Autonomous (genetic algorithm)	GA: crossover	D&S	FT, VC
21. LeBaron and Yamamoto	Autonomous (genetic algorithm)	GA: selection (imitate agent with better	Artificial market	VC

(2007)		performance)		
22. Yamamoto (2011)	Autonomous (genetic algorithm)	GA: selection (imitate agent with better performance)	D&S	VC
23. Mauri and Tettamanzi (2012)	Autonomous (genetic programming)	GA: genealogy	Artificial market	BC
24. Manahov and Hudson (2013)	Autonomous (genetic algorithm)	GA: crossover	Artificial market	FT, VC
25. Lee and Lee (2015)	Fundamental value and historical prices component	no herding mechanism	D&S	Herding effect when agents share the same opinions on the tipping point

(D&S: demand & supply; FT: fat tails; VC: volatility clustering; BC: bubbles and crashes; AA: absence of autocorrelation)

Table 2. 1 Twenty-five herding models

From the literature, there are more N type models than autonomous models. And also, from the papers' descriptions, the autonomous model involves many parameter settings and rules that they are not always described in detail. For N type model, the rules are the same in one group which makes the description of an N type model much simpler. On this basis, the replicating will focus on the N-type model. In order to understand the herding from different herding and market mechanisms, two models were chosen that have different mechanisms. In addition, the models need to be described in good detail in the paper in order for them to be replicated. Using these criteria, the Tedeschi et al.

(2012) and Lux and Marchesi (2000) models were selected. Tedeschi et al.'s (2012) model has a network mechanism and artificial markets without stylised facts. The paper says, "the model generates heterogeneity, as indicated by the fat tail distribution of agents' wealth and stock (Tedeschi et al., 2012:91)". The stylised fact of fat tails refers to the distribution of returns, and the paper does not comment on this. Hence fat tails are not included for the paper in table 2.1. However, my analysis of returns from the replicated model in Chapter 6 shows that the returns do have fat tails to an extent with kurtosis values greater than 0. And Lux and Marchesi's (2000) model has the transition probability and demand and supply with both fat tails and volatility clustering.

In summary, this chapter reviews the literature of my research area in three aspects: agent-based simulation of financial markets, stylised facts and herding. Agent-based simulation of financial markets mainly has two types of model: N-type model and autonomous model. The main difference between these two types is the learning process. For N-type model, the learning is the same in one group and agents use pre-decided types and rules. For the autonomous model, the learning is different for each agent and can change during the trading. For stylised facts in financial markets, although there are lots of models to study them, there are no agreed explanations for them except the absence of autocorrelation. Herding is one of the possible explanations for stylised facts, although the herding models differ in a number of ways including the herding mechanism and price mechanism. In the next chapter, the methodology and methods are discussed.

Chapter 3

Methodology and Methods

Chapter 3 discusses the methodology and methods. Four aspects are included in this chapter. One is to introduce the methodology from a philosophy perspective. The beliefs guide the choice of the methods for this research. The second is to introduce replication in science and modelling. Through discussing the importance of replication and identifying the issues in science and modelling, the nature of replication in both science and modelling is demonstrated. The third is to discuss problems in the replicating process and ways to improve reproducibility. After issues are identified in science and modelling, the problems in replicating are identified and also ways to improve the situation are discussed. Replicating guidelines are one of the ways to improve reproducibility, and so these are considered. The fourth is to explain replicating as a method in my research. From the discussion of the previous three aspects, the reasons for choosing replicating as my research method are explained.

Section 3.1 introduces my research approach. Section 3.2 and Section 3.3 discuss replication in science and modelling. Section 3.4 demonstrate the problems in replicating and ways to improve the reproducibility. Section 3.5 is about replicating guidelines for simulation. Section 3.6 discusses replicating as my research method.

3.1 Introduction to the research approach

In the philosophical worldview, ‘a basic set of beliefs guide action’ (Guba, 1990:17). This means that the beliefs guide our choice of research methodology and methods. There are two main aspects of philosophy that influence the choices of research methodology and methods: ontology and epistemology. My research is related to agent-based modelling, and modelling is a representation of part of the real world (Pidd, 1997). Thus in my research, the ontological assumption is based on the positivist perspective that reality does exist externally and objectively. Based on the positivist perspective, the epistemological assumption is that researchers are independent of what is going to be studied. These assumptions lead to the methodology of my research being deductive. From the positivist perspective, the results were tested for validity and reliability. The validation test is to ensure that the model is right. The reliability test is to ensure that the results are reproducible.

The specific approach taken here is one of replicating work from past journal papers. This chapter explains the reason for taking this approach. In particular, why replication is important in science and why it is recognised as a current problem.

3.2 Replication in science

3.2.1 Importance of replication in science

Replicating is a cornerstone of science experiments. The famous philosopher of science Karl Popper says the following:

> “We do not take even our own observations seriously, or accept them as scientific observations, until we have repeated and tested them. Only by such repetitions can we convince ourselves that we are not dealing with a mere isolated ‘coincidence’, but with events which, on account of their regularity and reproducibility, are in principle inter-subjectively testable. Every experimental physicist knows those surprising and inexplicable apparent ‘effects’ which in his laboratory can perhaps even be reproduced for some time, but which finally disappear without trace” (Popper, 2002:23).

This indicates the fundamental importance of repeating experiments in scientific studies. The results based on a single experiment may be caused by errors or chance; hence, replicating them is necessary to have confidence in scientific results.

Replicating involves repeating the research as described in an existing paper. If replicating yields the same or similar results, the replicability or reproducibility of the original paper is proved. Although the standards proposed by Axtell et al. (1996) of reproducibility are focus on the modelling area, they can be used in natural science’s experiment too. Therefore the standards will be applied in both the science and modelling part in sections 3.2 and 3.3. The standards of modelling can be illustrated in detail. Axtell et al. (1996) have proposed replication standards to judge the success of a model replication. There are three standards based on different levels of similarities: numerical identity, distributional equivalence, and rational alignment. Numerical identity describes an exact match between the replication results and the original results. However, this is sometimes difficult to achieve. Distributional equivalence means that the replication results and the original results are statistically similar. Rational alignment means the results show the same relationship.

However, although reproducibility in science is important to ensure scientific results, the reproducibility of published papers in various scientific research areas still has some issues. The current publication system involves peer review to check the paper's work. It is a process to make sure the validation and significance of the work by researchers in the same field. After the peer review, whether the paper should be accepted is decided. But, peer review of journal papers may not guarantee the quality of journal results to some extent, while the process depends on trust rather than checking of evidence (Smith, 2015). Peer reviewers can not do the work again, and their checking depends on their experience, understanding of the field and the description of the paper. Unless there is a significant conflict in the paper description, reviewers trust the paper description with criticised perspective. Ioannidis (2014) argues that the current publication system should value more on the replication studies to ensure the published results. Supporting Ioannidis, Woolston (2014) argues that the current system focuses on amazing results at a statistically significant level but ignores the carefulness of research designs. This makes researchers chase unexpected results at a statistically significant level under time pressure but ignore the reproducibility of their studies. Also, all the above arguments indicate that the current system does not value replicating much, although the importance of replicating is identified. The current system leads researchers to favour building something new rather than replicating existing ones of previously published papers.

3.2.2 Issues in science in general

Although replicating is an important aspect of research in science, replicating issues are still found in science. Baker (2016) produced an online Nature's survey from 1576 researchers to be aware if there is a reproducibility crisis. The survey is a questionnaire about reproducibility and found that 70% of researchers failed to reproduce other's work, and 50% of researchers failed to reproduce their own work. In chemistry, biology, physics and engineering, medicine, earth and environment, and others, more than 60% of researchers in each subject fail to replicate other's work and more than 40% of researchers in each fail to replicate their own work. More than half of people (52%) believe that there is a remarkable replicating crisis. Selective reporting, the pressure on the public and low statistics power and poor analysis are considered to be three main reasons that cause replicating crises.

3.2.2.1 Issues in social science

In social science research generally, there are also some replicating issues. A paper involving 100 experiments will be discussed first and then a paper of one single replication. The Open Science Collaboration (2015) replicated 100 experiments in psychology and found that whereas 97% of original results were statistically significant, only 36% of replicating results were significant. They do this study to be aware of reproducibility in psychological science. In other words, the replicating studies find much weaker results than the original studies. They did this study to be aware of the reproducibility in phycology. The requirements for replication success are based on five indicators such as significant level and size effect with correlation, so distributional equivalence is the replication standard for the Collaboration's work. Only 39% of the replicating studies have similar results to those of the original studies. This finding may lead to doubt of the original findings.

Schooler (2014), together with psychologists in 31 different labs, replicated the verbal overshadowing effect which was himself initially found. This effect is that it is harder for witnesses to identify the person they have seen when there is a physical criminal appearance description. Several researchers questioned the effect, so they decided to repeat the research. They are doing this replication to claim meta-science can help to ensure reproducibility. Rational alignment is the requirement for replication success in Schooler (2014). The replication supports the original finding at the rational alignment level.

3.2.2.2 Issues in biomedical

There are replicating issues in biomedical science in general. Distributional equivalence is the replication standard in Begley and Ellis (2012). This paper aims to outline the ways to improve reproducibility in this area through a discussion about the current issues. In therapeutic research, clinical oncology has the highest failure reproducibility rate (Begley and Ellis, 2012). In preclinical cancer research, the rate of applying cancer research to actual clinical treatment is very low and this is influenced by the related published papers. Only 25% of published papers could be validated and the projects could be continued while the non-reproducible published papers may have follow-up publications based on the original results (Begley and Ellis, 2012).

3.3 Replication in modelling

3.3.1 Importance of replication in modelling

Generally, simulation is a modelling method that aims to find a representative of reality for a certain problem. It has a similar replicating process compared with the process in science. In Science, the findings from experiment results should be general with certain conditions. Then the experiment needs to be tested several times to ensure reproducibility. This makes reproducibility critical in science. The testing process in science is a replicating process. In Science, replication ensures the findings are validated in certain conditions. It is a checking process. Similar to the experiment in science, modelling is an experiment as well. In order to make sure the model is representative of the real world and also the results are not just for one run, reproducibility is crucial in the modelling. To check if the model is built right, it is a verification check. It is a process that ensures the computer programme actually builds the expected model. To check if the right model is built, it is a validation check. This is a process to ensure the confidence that the model is sufficiently realistic for its use. The replication process can help to ensure both verification and validation check.

In modelling, the replicating method has many benefits (Wilensky and Rand, 2007). It proves that the scientific findings in papers are repeatable and not just exceptional cases; hence, it helps the understanding of the research agenda, and it clears the verification and validation of the model. Also, it helps researchers to study beyond the original paper by conducting new experiments. Additionally, Hubbard and Vetter (1996) think that using replicating as a method in business research can help to prove the conclusions statistically by identifying the significance level to avoid type I errors (false positive) and to generalise the results in order to avoid isolated and fragile studies which have type I errors. Hence, replicating will develop business research in both theory and practice.

Diethelm (2012) argues that sometimes the lack of reproducibility is not caused by any other reasons but the computer process based on mathematics algorithm calculation in high-performance computing. This may make two results not the same when you run the model again. In my research, as my objective is to find the general structure which produces certain financial market stylized facts, the standard of replicating is rational alignment at least. If it is possible, the distributional equivalence will also be considered to some extent.

3.3.2 Issues in modelling

Replicating issues in business and forecasting, network and communication, social science, biomedical and ecological science, agent-based simulation will be discussed in order. The order is according to the scale of the related findings and also to the relevance to my own research. Simulation as a bottom-up tool is a popular tool that can be applied in many areas like business and forecasting, network and communication, social science, biomedical and ecological science. For computer modelling, usually applying in economics, forecasting, network and communication, the numerical identity can be achievable when the same condition applies. But, for subjects like social science, biomedical and ecological science, the experiment is mainly used as a method that numerical identity is not achievable. The samples selected for the experiment cannot be the same in the original paper and the replication one. Therefore, usually for those subjects which using experiments, only distributional equivalence is required for replication results. Uhrmacher et al. (2016) found that the replication situation in discrete event simulation is not positive after the practice in healthcare, logistics and so on. In these areas, the replication requirement is different depends on the author's opinions.

Stodden et al. (2018) reviewed 204 computational papers from 2011 to 2012 to evaluate the policy change for data and code access in Science articles as a movement to open science. The main features of open science are process transparency, and data and code sharing (Powers and Hampton, 2019). Out of the 204 articles, 24 have sufficient information without contacting the authors. For the remaining 180 articles, after contacting the authors, still 7% of them refuse to provide information and 2% refuse with a reason. Then they randomly chose 22 from 56 articles that seemed to have enough information to replicate, but even then 4 out of the 22 papers could not be replicated due to insufficient information or intensive hardware and conditions like computation requirements.

As my research focuses on agent-based simulation, in each area the simulation issues will be discussed first. Then the remaining discussion in each area is ordered according to the scale of the related findings and then to the year of publication. In the end, simulation issues in agent-based simulation will be discussed.

3.3.2.1 Issues in business research and forecasting

For simulation in economics, Kleiber and Zeileis (2013) reviewed a total of 40 papers in volume 23 (2008) and 15 papers in volume 153 (2009) focusing on empirical studies of the Journal of Applied Econometrics (JAE) and the Journal of Econometrics (JOE) respectively. They want to be aware of the reproducibility in econometrics. There are 33 out of 40 papers in JAE and 14 out of 15 papers in JOE using simulation in econometrics research. Neither of the journals requires the code of the model. From the review of these published papers, they found lots of essential information for replication is not available. 12 out of 15, which is 80% of the papers in JOE have not provided any data information and the remaining 3 have not used any data. That means none of the paper in JOE provides detailed data information in the paper. Compared to the results in JOE, the JAE is much better, as all of the papers provide information related to the data, except three of them have not used any data. 69.7% and 92.9% of papers in JOE and JAE have not provided files related to replication. And only 15.2% and 7.1 % of papers in JOE and JAE provide the information related to the random seed.

Except for papers that discuss issues in simulation specifically, replicating issues in business research in general will be introduced. Business usually refers to accounting, economics, finance, management, and marketing. Firstly, Hubbard and Vetter's (1996) paper includes all five areas of business research. Hubbard and Vetter did a statistical analysis about published replication and extension studies involving empirical work in 18 leading business journals from 1970 to 1991, a period of 22 years. They aim to identify how common the published replication and extension research work in these areas. Also through finding out the reproducibility in the areas to prove if the research results are non-repeatable fragile work. And hence to conclude if the research works in these areas can give sufficient knowledge to help to build and develop in both practical and theory in this area. In the accounting, economics and finance areas, the replication and extension research in published empirical papers represent less than 10% of all empirical work. In marketing and management areas, the percentage of replication and extension studies in published empirical papers is as low as 5%. Moreover, the replicated results are usually not similar to those of the original studies. Hubbard and Vetter's paper tried to include all empirical replication and extension research results. The standard for the success of replication is not clear in this paper. They reviewed various replication and extension work from each other and different empirical work involved. The exact match is still hard to achieve. Therefore I think most of the works involved in H&V's (1996) paper treated distributional equivalence as the success of replication or rational alignment at least.

Two further papers addressed the replication issues generally in economics research and marketing research. Chang and Li (2015) replicated 67 papers focusing on gross domestic product (GDP) in 13 well-known journals. They tried lots of replication in economics to find if the papers in economics have reproducibility or not. The standard for replication success in C&L's work is demonstrated clearly: rational alignment is required. Their results show that the reproducibility in such economic papers is negative. This means more than half of the papers' results cannot be replicated. In total, only 29 out of 67 (43%) are replicated successfully. Six out of the sixty-seven papers do not have publicly accessible data and cannot be replicated. In the 29 successfully replicated papers, 22 papers out of 67 (33%) are reproducible without contacting the original authors during the replicating process. This indicates the process descriptions in the papers are not detailed enough to replicate and 7 papers cannot be replicated successfully without the help of the original authors. Although the result in Chang and Li's (2015) paper is negative, the percentage of successful replicating rate in Chang and Li's (2015) paper is much higher than that of previous work which just focuses on a single journal: Dewald et al. (1986) found a successful replicating rate of 13% (7 of 54) with the help of the authors in the American Economic Review, while McCullough (2006) found a successful replicating rate of 8% (14 of 186) without contacting the authors in the Journal of Money, Credit and Banking.

Evanschitzky et al. (2007) tried to identify any changes in the replicating rate in marketing from three leading US journals after the emphasis on replication studies in editorial policies. Hubbard and Armstrong (1994) had similar research previously before the emphasis on replication studies in editorial policies. This study tried to find the change after the emphasis on reproducibility by comparing this study and the previous one. The average rate of replicating with extensions in three leading journals from 1974-1989 is 2.4% in Hubbard and Armstrong (1994). This rate reduces to 1.2% from 1990-2004 in Evanschitzky et al. (2007) after H&A's (1994) publication. This indicates that after the emphasis on replication studies was introduced, and the replication rate is still very low and even lower than in previous years. The standard for the replication success varies from each other's in Evanschitzky et al.'s (2007) paper as it generates results based on several papers with replications. The exact match is hard to achieve, then I think the standard of distributional equivalence or rational alignment at least is required. The reproducibility increases in the extension studies from 1974-1989 compared to the studies from 1990-2004: the percentage of original results' confirmation increases from 15% (1974-1989) to 44% (1990-2004), and only 22%

(1990-2004) of extension studies disagree with the original results, a decrease of 60% (1974-1989).

Turning to research in economics, a paper of one single replication is discussed. Herndon et al. (2014) replicated a famous paper in economics written by Rogoff and Reinhart (2010). Rogoff and Reinhart (2010) claimed that GDP growth would be reduced when the debt/GDP ratio exceeds 90%. This finding supports the austerity policy in the US and Europe to reduce the high debt level. However, Herndon et al. (2014) refute this finding by replicating. Herndon et al. found a number of errors in Rogoff and Reinhart's (2010) paper. First, there are some problems with the dataset. The dataset is not adequate; some debt/GDP ratio is missing for some particular countries in certain time periods. Some data are excluded without appropriate reasons and these data are essential to influence the final findings. Second, there is a coding error in the spreadsheet calculation which excludes the entrance of five countries, which affects the summarizing process and final results. Third, the weighting of each category is questionable. In high debt ratio categories, the UK data lasting for 19 years weighs the same as the New Zealand data for just 1 year. This replication failed even at the rational alignment level.

After replicating issues discussed with respect to business, the discussion will focus on the issues in forecasting. Boylan et al. (2015) replicated a famous paper by Miller and Williams (2003) based on a time series model that is using shrinking estimators in seasonal factors. They chose Miller and Williams's paper (2003) is because it won an outstanding paper award. And they choose to replicate it because they want to be aware of the reproducibility in forecasting. Two teams tried to replicate the paper. One of the teams used MATLAB and the other one used visual basic in Excel, as the original paper. Team A contacted the authors twice to produce the first and second sets of results. Team B produced their first set of results based on their own understanding of the original paper. Team A's second set of results disagreed with Team B's first set of results. After the communication between these two teams, Team B obtained their second set of results with additional information provided by Team A, and Team A obtained their third set of results following Team B's steps. However, the results across the teams still did not match. On investigation, the teams found that the result in Excel stops in the local solutions and thus Team B used a different smoothing factor. Then Team B updated their results again and finally they had got similar results in their third set of results. However, the final agreed results of the two teams are still not similar to the original paper. This replication failed even at the rational alignment level.

3.3.2.2 Issues in network and communication

In network and communication, Pawlikowski et al. (2012) reviewed the 2246 papers related to stochastic simulation model on telecommunication network in the proceedings of the IEEE INFOCOM (II) between 1992 and 1998, the IEEE transactions on communications (ITC) between 1996 and 1998, the IEEE/ACM transactions on networking between 1996 and 1998 (ITN), and the performance evaluation journal (PEJ) between 1996 and 1998. They mainly focus on two aspects to ensure reproducibility that is a random generator and also output analysis. The review shows that the majority of papers do not provide the results related information, for example, the time horizon information on a certain result or not properly statistically analysed the potential statistical errors in results such as using a confidence interval. The percentage of papers' results not includes such information or not properly analysed is 76.6% in II, 79.05% in

ITC, 71.6% in ITN and 68.6% in PEJ, separately all over 68% and averagely 76.45%. More than 52% of papers do not provide information about simulation type about whether they are terminating or steady-state. Joerer et al. (2012) reviewed conferences between 2009 and 2011 in intervehicle communication. They selected 116 papers of simulation from more than 1000 papers related to intervehicle communication with short-range communication and found out the missing information in parameter settings. In the simulation of vehicular networks, it is essential to clarify the medium access control (MAC) protocol to indicate the protocol used in order to match the physical layer technology. Still, about 15%, 30%, 20% of papers in 2009, 2010 and 2011 do not clarify the MAC protocol they used. Road traffic simulator is a factor to model different granularity like vehicles' average speed and so on, but nearly 40%, 47%, 60% of papers in 2009, 2010 and 2011 do not mention this. Scenarios of vehicle network are mainly categorised in highway and city with a detailed description. In the city, about 7% of papers, and on the highway, about 27% of papers do not introduce the scenarios in detail.

Kurkowski et al. (2005) reviewed the papers from 2000 to 2005 related to mobile ad hoc networks (MANET). They did this study to be aware of the state of MANET simulation. They mainly focus on the missing information that is mattered in the studied but is ignored in the published papers. Totally, 114 out of the 151 papers in MANET, which is 75.5%, use simulation as a tool. In these 114 papers related to simulation, some replicating issues are found. 66 papers, which are 57.9%, do not even mention the simulation type of their research, whether they are terminating or steady-state. Besides, authors generate terminating results from the steady-state simulation or the other way around. In the review, only eight of the simulation papers, which are only 7.0%, mention the bias in the initialization process. And all eight papers are deleting data based on unreliable arbitrage. In simulation related to MANET, the simulator is one of the essential parameters that will influence the output. Still, 34 papers, which are 29.8% of the overall, do not value the simulator clearly in their description. Even worse, none of the papers discussed the pseudo-random number generator, also known as PRNG. For these simulation papers, the numbers of simulation runs are important factors to be mentioned. But, in MANET protocol simulation papers (109/114), only 39 of the 109 which is 35.8% mention that. Also, for the graphic result (112/114), 12 out of 112 papers draw a graph without legends or labels. Besides, 28 out of 112, which is 25.0%, draws a graph without clear units. These missing information found in these papers will directly influence not only the accuracy of replicating the process and also the result comparison.

3.3.2.3 Issues in social science and ecology

For the simulation in social science research, Rahmandad and Sterman (2012) surveyed the published articles from 2010 to 2011 in the system dynamics review. They mainly focus on whether the essential parts like equations and parameter settings are included in the published papers or not. All in all, not all of the papers include the simulation results. Only 27 out of 34 include simulation results. And for these 27 papers, some of them do not include information such as equations and parameter settings. Only 34 % of them do have all equations and 7% of them have partial equations; for units in the equations, 67% of papers contain no complete equation units, while 11% of papers have partial information of units for the equations; 70% of papers, which is 19 out of 27, provide the parameter value; 30% of papers, which is 8 out of 27, do not introduce the scenarios and parameter settings for results and 19%, which is 5 out of 27, do not introduce them clearly. The results agreed at the distributional equivalence level.

Hales et al. (2003) conclude the Model to Model workshop and found two replication ones in social beliefs related to money as a medium exchange and social science: Juliette Rouchier's one and Edmonds and Hales' (2003) one. Juliette Rouchier tried to replicate Duffy and Ochs's (1999) agent-based model, which originally proposed by Kiyotaki and Wright (1989). After the email contacts with Duffy, she fails to replicate the model. There is no detailed description for this replication. Therefore the replication standard applied for this one is hard to judge. Nevertheless, it should be at least at the rational alignment level. Edmonds and Hales' (2003) one is a successful agent-based simulation replication which will be introduced later in 3.3.3.

For the simulation in ecology, Lauzon-Guay and Lyons (2011) tried to replicate Munguia et al. (2010) model. It is a simulation model in ecology. They built a model with four species within the benthic community to investigate how the disturbance, dispersal, and competition will influence the distribution and abundance. As a result, they cannot replicate the model findings successfully. In the original model, they think the abundance of species is mainly influenced by disturbance and dispersal. But in the replication model, they find, interspecific and intraspecific competitions are also important. Therefore, in this replication, some findings are not agreed with each other even at the rational alignment level. They argue that it is impossible to repeat others' work without plenty of guesses and assumptions.

3.3.2.4 Issues in biomedical and ecological science

A proposed framework in medical research to calculate the reproducibility, including the level of study power and bias, the number of other studies, the probability of no relationship of the relationship study, shows it is hard for the calculation of reproducibility to be more than 50% (Ioannidis, 2005). Hence, Ioannidis concludes that most published papers maybe not valid. This makes replicating processes essential to confirm the results in this area.

3.3.3 Issues in agent-based simulation

In the simulation, the checking process is emphasised as well. Replication is a type of checking process. This is similar to other non-simulation ones: the errors in the data analysis can lead to misleading results, for example, the replicating issues found by Herndon et al. (2014) which explained in detail in 3.3.2.1. Also, the assumptions in the paper can influence the results as some data are excluded from the original data set (Herndon et al., 2014). This can be the same issue as in the simulation replicating process. The issues in science replicating are similar, so the problems and solutions in replicating are similar too. The issues found in other subjects in science also applies to simulation.

The issues for agent-based simulation generally are mentioned by Janssen (2017). Janssen (2017) reviewed agent-based model papers between 1990 and 2014. He concentrates on code sharing of models' issues in a total of 2367 papers. Furthermore, he focuses on how these models are described in three aspects: how the description is ordered; how the relationship is visualised; how the algorithms are listed. Only 236 papers, which are 10% of the total, have public access to the model code. Especially in two journals Physic A and Environment and Planning B, there are 103 papers in Physic A and 30 papers in Environment and Planning B in total, but 0% of the model code is provided for both journals. For the software implementation information, only 52% of journal papers provide such information. For the model description part, 93.3% of the models are described in order. 6.7% of the papers follow the ODD (Overview, Design concepts, and Details) protocol proposed by Grimm et al. (2006). The ODD protocol is a detailed guideline for model description which will be discussed in detail in Section 3.5. 34.2% of models use the flowchart to describe a relationship. 3.2% of models use the unified modelling language diagram. 53.5% of models using mathematic description, 10% using source code, and 9.7% using pseudo code.

Will and Hegselmann (2008) replicated a model for building trust among distant people. In the original paper (Macy and Sato, 2002), the main finding is that US social mobility may cause a breakdown in public trust. In the prisoners' trust game, if they do not trust each other they will choose to exit the game. Will and Hegselmann replicated this agent-based model twice using different software, each has Netlogo and Fortran 95, by using the same set of random numbers. They tried to replicate this model because they are interested in mobility. However, they cannot replicate the result even at the rational alignment level. They think this is because there is some information missing for the transaction cost. There are some successful examples in agent-based simulation as well.

Legendi and Gulyas (2012) replicated a paper about agent-based modelling in macroeconomics that describes labour consumption and credit market in relation to firms, banks and households. They used replication to validate the original results by trying different parameter settings. And also, they tried the model in alternative implementation by using both Matlab and Java. This gives them a chance to validate the results in a different environment. And also, they can compare the different running times in a different environment with different runs to identify the efficiency of Metlab and Java. Then they can suggest how to speed up the running time. The original and replication results are shown in the same graph for production, rate of unemployment, interest rate and so on. The replication was successful with the same graphical patterns (distributional equivalence) with some adjustments.

Wilensky and Rand (2007) replicated a similar agent-based model proposed by Axelrod-Hammond to study ethnocentrism. As they think replication is an important scientific method, there are not lots of replication work have done so far. Then they attempt to replicate a simulation model as a case study. The agents have three traits in this model. The first trait indicates the agent's membership in one of four colours. The second trait indicates the agents' strategy when they meet people of the same colour. The third trait indicates agents' strategy when they meet people of a different colour. The local area has limited space and has four rules: immigration for new people, interaction for helping each other, birth for new reproduction if there is space, death for the disappearing of each agent. The results are measured in three ways in last 100 time: the percentage of agents choose to cooperate instead of defecting; the percentage of agents have chosen to cooperate with agents of own colours and defence with other colours; the percentage of agents chose to cooperate with agents of own colours and defence with other colours all the time. Their replication seems to confirm the original results, although the results are still not an exact match even after several discussions with the authors.

Bruce Edmonds and David Hales tried to replicate an agent-based model by Riolo et al. (2002) in social simulation and successfully replicated the model, though there are shortcomings in results' validation found in the original one. Edmonds and Hales (2003) replicated Riolo et al.'s (2002) model by applying two different implementations in JAVA and SDML. The original one is implemented in JAVA. They (Riolo et al., 2001) tried to find how tags which are observable cues or markings, can make cooperation happened in evolving agents. Patterns from the replicated one and the original one are similar, but the rate for a sudden increase in the donation rate decreased from 3 to 2. After they investigated the model more, they found the average donation rate and tolerance rate used in the model reduces to zero if they change the rule a little bit. That means the high donation rate in the model needs a very specific rule, that is the shortcomings in the model. Then the simulation standard for this one is at the distributional equivalence level.

3.4 Problems in the replicating process and ways to improve reproducibility

Replicating issues are widely spread in many areas and the replication rate can hardly exceed 50% in many areas. However, there are still some successful replicating examples in these areas. It is useful to identify problems found in these replicating issues and draw upon the successful experience to guide my research.

In these replicating processes, there are three kinds of problems are found:

1. Papers without data and code are not easy to replicate. There are 23 out of 39 (59%) reproducible papers with the information provided, while 6 out of 28 (21%) reproducible papers without code and data provided (Chang and Li, 2015).

2. Besides replicating problems caused by data and code, Boylan et al. (2015) argue that the methods description, measurement accuracy, and software used may also mislead in the replicating process. For example, in Will and Hegselmann's (2008) replication, they cannot find a clear description of opportunity and transaction costs. They think these may be factors that cause the research to fail. Wilensky and Rand (2007) misunderstand the description of agents' interactions and fail to produce the exact statistical results. In Lauzon-Guay and Lyons (2011) replication, the inadequate description of equations and methods may be the main factors that cause failure. Culina et al. (2018) argue the importance of data interpretation through open data. For ecological data, the interpretation only takes place under a certain context. Hill (2015) thinks the real number in the binary presentation may cause a problem too. Dalle (2012) argues that the limited space in the published papers and hidden parameters may cause an inadequate description of the model.

3. Some authors did not respond to requests for additional information about their papers (Chang and Li, 2015). In the replicating process, Evanschitzky et al. contacted 52 authors, but 31 of them did not respond to the email for additional information (Evanschitzky et al., 2007). Stodden et al. (2018) contacted 180 authors and 46 of them did not respond to emails. Herndon et al. (2014) tried to use publically accessible data provided by Rogoff and Reinhart's website but were unable to identify the data series, years and methods used in the paper. They contacted the authors and obtained the spreadsheet they used in the paper and consequently, similar results are produced. This spreadsheet helps Herndon et al. to identify the errors in Rogoff and Reinhart's (2010) paper discussed above.

Based on the problems found in the replicating process, ways to improve the replicating issues can be suggested:

1. The replicating process is needed but vague in some cases. Schooler (2014) suggests research should be replicated before it is published to guarantee new results. Wilensky and Rand (2007) emphasize the importance of detailed model description, although it is hard to check by authors themselves if the description is detailed enough or not. Thus, this may need the help of replicators to go through the actual replicating process to confirm the adequacy of a detailed description. Evanschitzky et al. (2007) suggest that editors should select the important papers to be replicated.

2. The requirements for published papers should be improved. Vines et al. (2014) found the data availability was related to the publication year of papers. The older the paper is, the less chance the data provided. It is important to have a policy that requires data sharing. Yilmaz (2011) and Nature (2014) encourage code sharing in research. European Commission promoted new policies such as the probability of access to data to make open science real (Burgelman et al., 2019). In addition, Powers and Hampton (2019) found that policy change has arisen rapidly to increase the accessibility of code and data. Chang and Li (2015) suggest the following: the data and code files should be a requirement for a published paper; the software operating system and running time should be indicated; the seeds used and random generator should be demonstrated if the paper uses random numbers; the file order used to run the program, and raw data should be provided. Stodden (2010) gives six suggestions mainly in coding areas in detail about how to improve the reproducibility: includes code data simulation and statistical results; name each version of code a unique ID and keep it updated; include computer environment and software version; use public accessible software for code; provide

code and data in a readable format; provide public accessible contact. Francois Gygi (2013) emphasises the importance of the software's openly access to improve reproducibility. Boylan et al. (2015) discuss flowchart presentation and code. They think that although the code can guarantee an exact replication, it may conceal the errors and stop thinking about developing new codes. They suggest that a flow chart presentation of the process description is preferable to the code. Liu et al. (2014) argue that besides providing the source code, input data, and input parameters, providing the compiler, round off errors, and computer version will be helpful. Evanschitzky et al. (2007) agree with the previous suggestions to require the data and methods available to the public. Also, sensitive analysis from the original paper may be helpful (Wilensky and Rand, 2007). Pawlikowski et al. (2012) think that the four elements, which are random generators, the type of simulation, method of results analysis and statistical errors in results analysis, are important to clearly state. Dalle (2012) suggests that decreasing human intervention, using an independent platform solution, providing sources code and additional material, improving the review process and avoiding floating point can improve the reproducibility of simulation. Ioannidis (2005) thinks that before we do a test, it is better to consider if there is really a relationship between these factors which we do a test on. The survey (Baker, 2016) shows that three main ways to improve reproducibility are: more robust experimental design, better statistics, and better mentorship.

3. The general guidelines, standards, and even culture may need to be changed. McNutt (2014) suggests an improved guideline to provide a checklist to help researchers to maintain reproducibility. Begley and Ellis (2012) argue that raising standards related to particular research areas to ensure reproducibility is needed. Ioannidis (2005) thinks

interventions are needed in order to ensure reproducibility and agrees with Begley and Ellis (2012) that stronger standards are needed. Large-scale collaborative projects are encouraged. The shareholders should be understood to find the cause of bias and conflicts (Ioannidis, 2014). All in all, such interventions try to reduce conflicts and bias and increase transparency and collaboration. Begley and Ellis (2012) propose an idea of developing a culture that values replicating, and they also suggest a change in a system that is just chasing 'a perfect story'. The current system may change to value reproducibility more than publications to assure the quality of research findings (Ioannidis, 2014).

3.5 Replicating guidelines for simulation

As discussed before, the guidelines will be helpful to improve reproducibility. Then the replication guidelines for simulation will be discussed in the timeline. And it will be helpful for my replication research on how to write my replicating models.

Grimm et al. (2006) propose a model description guideline for agent-based models in ecology called ODD (Overview, Design concepts, and Details). They proposed this guideline because before, there is no such detailed protocol to follow and equations for the model are sometimes just described verbally. They believe that the way researchers describe the information matters. For example, in a sentence, readers usually expect that the opinions are demonstrated at the end followed by the context at the start. Thus, writers better state their opinions at the end rather than at the beginning. Grimm et al. (2006), 28 experienced authors from seven different countries, who totally write over 200 papers, proposed a detailed guideline that includes overviews, design concepts, and details. This is also known as the ODD protocol. In the overviews which stand for O, purpose, state variables and scales, and process overview and scheduling are included. Purpose comes first to help readers understand what this model is about and why this model is built. State variables and scales come to the second to help readers understand the model structure by listing the variables and scales used in the model. Process overview and scheduling help readers to understand the model process, and the order of the process with state variables' updates. Design concepts that stand for D include the general concepts for the model design, like the questions about the emergence and interactions among the agents. For the details which stand for D, initializations, input and submodels are included. After the basic background demonstrated in the overview (O) part and the understanding of the design concepts (D), the details (D) will be described. In the detail part, the initializations which stated how the model gets started for a simulation run come to the first. Input which is what you used in the input file to run the simulation comes to the second. And then, submodels related to the process overview and scheduling are listed and explained in detail to give a whole picture of the model process. Overall, this guideline focuses mainly on the model description part and how to describe the model in an order with a sample application in the ecology area. The output analysis is ignored in this guideline. They are trying to keep updating the ODD guideline, and the second update was published in 2020 (Grimm et al., 2020).

Rahmandad and Sterman (2012) suggest four aspects of guidelines that mainly focus on system dynamic modelling. Although the guidelines have developed from years to years, there is no related one to system dynamic modelling in social science. After reviewing all articles published in System Dynamic between 2010 and 2011, they proposed a guideline for system dynamics in social science in four aspects. They are general visualization guidelines, model reporting requirements, simulation experiment reporting, and optimization experiment reporting. For general visualization guidelines, the graph demonstrating the dynamic system process is required. For model reporting, clear data and model descriptions are required. For experiment reporting, the steps for each process, related scenarios, and sensitivity analysis need to be clearly illustrated. For optimization reporting, the optimal objective and search algorithm and the result should be mentioned. Rahmandad and Sterman (2012) suggestions are specifically based on system dynamics, as certain opinions in the visualization guidelines and optimisation reporting may not be applied in the agent-based models.

Kamon et al. (2012) proposed a guideline in discrete event simulation of the health care area. They proposed a guideline with good examples in five stages, which is discussed and improved by members of the Task Force. The stages are structure and design, parameter estimation, model implementation, analysis, representing and reporting. In the structure and design stage, the representative system and the event which is related to the system are explained. In the parameter estimation stage, different types of variables are considered in the disease course, decision algorithms, resource and health condition costs, and weight of quality life. In the model implementation stage, it is the process to transfer the first stage (structure and design) into the computer process. In the model analysis stage, mean and distribution value and also optimization analysis can be analysed if needed to give estimations or meaningful suggestions answering why you design the system. In representing and reporting stage, check the representation of the model to check if there is some unrealistic result that may influence the validation of the model. Diagrams should be included in reporting the structure and function. This guideline focuses on discrete event simulation in the health care settings which are too detailed to be applied in the agent-based simulation to some extent.

Kleiber and Zeileis (2013) suggest that authors should pay attention to five aspects of the econometric simulation. They developed a new guideline because there is no widely accepted guideline in this area. They reviewed the articles in two leading journals in the econometric area recently and proposed a new guideline. The five aspects are model description, technical information, code, replication files and results generating from the simulation. The model description requires a detailed description of the model itself. But in this paper, what elements should be included in the description are not in detail. Technical information requires computation information such as software versions and environments. The code requires that the code should be provided. Replication files require functions in the computer process to get all the tables and figures. Results generating from the simulation require that the simulation result should be provided in a file. Suggestions from Kleiber and Zeileis (2013) do not include enough detailed information in each rule. And the information is not comprehensive that makes the guidelines hard to follow.

Monks et al. (2019) think guidelines will help to improve reproducibility. Although until then, there are lots of useful proposed guidelines in simulation, there is no one especially for the operational research area. Then they examined guidelines and proposed an initial version. And then, the initial version was discussed and edited with four experts in this area. Detailed checklist guidelines are proposed by them in simulation: discrete event simulation, system dynamics and also agent-based simulation. For agent-based simulation, the guidelines are divided into six parts: objectives (1.1-

1.3), logic (2.1-2.5), data (3.1-3.4), experimentation (4.1-4.3), implementation (5.1-5.4) and code access (6.1). The number in the bracket indicates how many elements are included in each part and it will be listed in Table 6.1 in Chapter 6. Things that need to be done in each part are detailed described to check if the paper provides enough information for the objectives, logic and so on. Taylor et al. (2019) apply the STRESS guideline to a simulation model to demonstrate how to use the guideline.

Grimm et al.'s ODD protocol is used as a guideline for 6.7% of replication papers which are found in Janssen (2017). It is a detailed and comprehensive guideline for the modelling part. Monks et al.'s (2019) STRESS guideline is specifically for simulation and for each type of simulation, it has a specific checklist. Therefore, the proposed replication guideline for my replication models combines the ODD and STRESS guideline with some modifications, which will discuss in Chapter 6.

3.6 Replicating as a method in my research

Despite the fact that the importance of replicating method is discussed a lot to enhance reproducibility, the current system is not sufficient to ensure it. Replicating issues are common in many subjects. In business, replicating papers are not very common and the success rates of replicating models are rarely higher than 50%. And the problem is not an exceptional case. Similar replicating issues are also found in forecasting, social science, network and communication, biomedical and ecological science. The success rate of replicating in these areas is low and there are some secondary papers based on non-reproducible results from published papers. Also, there are some problems that may be found in the replicating process: no data or code needed, no detailed description of methods or measurements, the authors do not respond to emails. This makes replicating even harder. Papers need to increase reproducibility in order to help the replicating process. Guidelines to maintain reproducibility should be provided, and also interventions to reduce bias like large-scale collaborative projects. The stronger standards like requiring the code and data and also the culture in which value reproducibility more is needed. The problems and solutions discussed in Section 3 will guide my research in the replicating process. In this section, replicating as a method in my research will be described in detail.

There are four steps in my research:

1. Selecting: two herding papers will be selected based on the level of model description and the model type.
2. Checking: the models will be built based on the papers to check if the results can be reproduced or not.
3. Extending: an extended model will be produced to obtain a more detailed understanding of the models through new experiments and analysis.
4. Comparing: after reproducing the papers, the results will be compared to see under what conditions similar results are obtained.

Choosing replicating as a method has many benefits in my research particularly:

1. Building a model is actually helpful to understand the internal interventions and parameter settings. Hence, it is useful to reach the research question.
2. It includes a check process to ensure that the general rules are based on reliable conclusions. Some papers may not reproducible, and then our conclusions may be based on the wrong findings.

3. The model is needed to do some extension and sensitive analysis experiments. Thus, building a replicating model is essential.

In summary, this research is based on a positivist perspective and uses replicating as a method to understand the herding and replication issues. Many replicating problems are found in both science and modelling, which makes replicating vital. Through discussing the problems and ways to improve reproducibility, replicating guidelines are one of the useful potential ways to improve the situation.

Other research approaches could be taken. For example, replicating could be examined by a review of a large number of papers to assess what details are provided. Herding and stylised facts could be investigated by analysis of empirical data or by developing new models. The method used here was chosen to enable a detailed study of the ability to replicate the chosen papers. The limitation is that reproducing simulation models is a difficult and time consuming task and so this limits the number of models that can be studied. However, the benefit is that it gives a good understanding of how the models work and so this leads to the further analysis of the herding mechanisms in the models.

The next two chapters use replicating to study two herding models with different herding and price mechanisms. The model in Chapter 4 has a herding mechanism of the agents tending to copy the most successful trader (the "guru"). Agents trade with each other. The model in Chapter 5 has three different types of agents (fundamentalists, optimistic noise traders, pessimistic noise traders) and uses a price structure based on excess demand. The first stages in each case were to try and reproduce the model and the results from the original paper. For the description of the model in Chapter 5, the

STRESS guidelines were applied. Then additional analysis was done to try and improve the understanding of the particular herding mechanism and how it produces the stylised facts. This was done by running some extra experiments and making some changes to the models. Examining the mechanism is a good way to study herding, although because of the complexity of the inner connection among agents and mechanisms, it can be hard to understand the reasons behind certain stylised facts.

Chapter 4

First replicating model

The first replicating model is based on Tedeschi et al. (2012)'s original model in their paper called 'Herding effects in order driven markets: The rise and fall of gurus'. The argument behind their model is that traders tend to imitate the expectations of the most successful trader (the guru) and this creates a herding effect. The model is based on the behaviour of "zero intelligent agents" (Gode and Sunder, 1993), where traders are trading according to random behaviour. In addition, there is also the effect of the guru who is the most successful trader and has the most imitators. The imitators simply copy or follow the guru's future expectations.

The market structure of this model is based on the fact that agents trade with each other. Each trader can view the current bid (buy) and ask (sell) price and submit a market order or a limit order. A market order is an order which can be traded immediately with another agent either fully or partially. A limit order is an order which cannot be traded immediately because no other agent is willing to trade. Instead, it is added to the order book.

Three main aspects are investigated with this model. One is to replicate the model and the results from the original paper. As discussed in Chapter 3, model replication is very important in modelling to give confidence in the results from scientific studies.

The second aspect is the model description. There is a contrast here with Chapter 5. The STRESS guidelines set out in Chapter 3 are applied in Chapter 5 but not in this chapter. This shows how the description works without guidelines. The experience of comparing the description work without (Chapter 4) and with (Chapter 5) the guidelines will be discussed and evaluated in Chapter 6.

The third aspect is to extend the understanding of the model. An altered model is established to try and understand the differences between the replication results and the original results. The aim is to get a better understanding of the reasons for the initial price dropping behaviour in the model.

The model is described in Section 4.1 and this is divided into four aspects: initialisation, the network, the expectation formation mechanism, and the market. All the parameter settings follow the original paper and the details in Section 4.1 are obtained from Tedeschi et al. (2012). Any aspects that are not clear in the paper are mentioned in this section.

In Section 4.2, the results based on the replicated model are analysed and compared with the original results. Unfortunately, I was unable to get the same answers as the original paper. The main problem in my results is the pattern of the price dropping quickly once the simulation starts. In the original paper, the price does drop quickly from 1000 to about 500, but it then stabilizes with fluctuations around 500. The price in my model just keeps dropping to nearly 0. One possible cause of the problem may be a lack of information on some parameter values in the original paper. Even after contacting the paper's authors, there was still not enough information to replicate the original results exactly. Problems in replicating this model will be discussed further in Chapter 6.

One important aspect of the original model is the utility function which affects the trader's order decision. This is not particularly realistic and was probably the cause of the price dropping behaviour. I therefore, produced an altered model with another function that replaces the utility function in the original model. The altered model is described in Section 4.3.1 and 4.3.2 and the results for the altered model are analysed in Section 4.3.3.

4.1 The model description

4.1.1 The initialisation and model overview

The initialisation settings are stated here. The total number of agents is 150, and they have £100,000 in cash and 100 stocks at an initial price of £1000 each. In the original article, the initial value stated for cash is £100. This is not plausible as not even 1 stock can be bought unless the price drops down to less than £100. This appears to be an error in the article. There are two reasons for setting cash to £100,000: one is because in this case traders own half cash and half stock which is consistent with the settings in the Chiarella et al. (2009) paper which guides the market mechanism in Tedeschi et al. (2012), and the other is the graphs for wealth analysis (Tedeschi et al., 2012:90) which show the starting point as £200,000 for wealth. The simulation runs for 1000 time periods. The flow chart (Fig.4.1) below illustrates the steps the model follows.

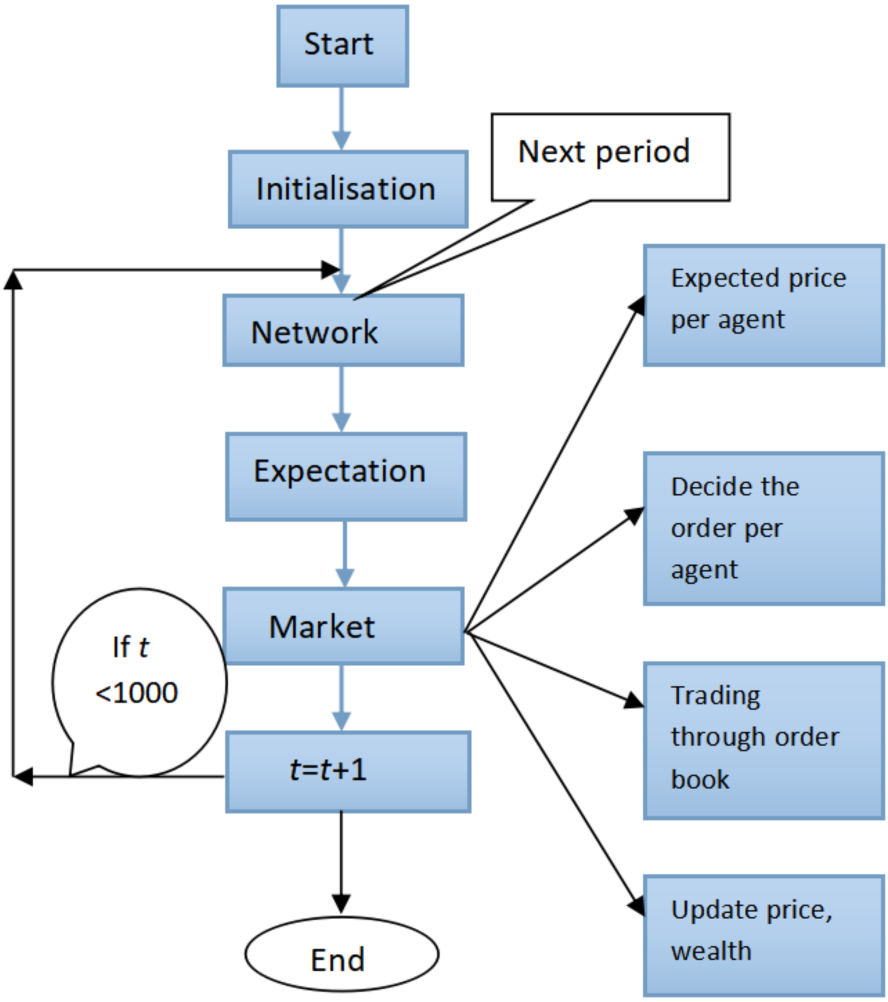


Figure 4. 1 Flow chart of the model

As seen in Figure 4.1, the flowchart of the model starts with initialisation. All agents are initialised with the same cash and with the stock level at the price of £1000. Network connection (in Section 4.1.2) is initialised randomly at the start. Then the network is built following the instructions in Section 4.1.2. In the expectation step, the expected return is built following the instructions in Section 4.1.3 based on the network. Finally, in the market step, based on the expected return, agents decide the expected price and then the number of stock they want to hold as in Section 4.1.4. The order amounts they want to buy or sell are agreed and the order prices they want to submit are formed by the expected price and the cash they hold. They then submit their order into the order book to trade. After the trade, the wealth of each agent is updated as is the price. If the time t is less than 1000, the time is updated to $t+1$, and the network, expectation, and market are rebuilt correspondingly in the next periods with the same rules introduced in Section 4.1.2, 4.1.3, 4.1.4. If the time t is equal to 1000, then this simulation is ended. In all equations, symbols with the superscript i indicate the values are different for each agent.

4.1.2 The network

The network is the structure of the model to enable communication among the agents. All agents are nodes in the network and the edges are the communication links. For each node, there is just one out-going link to keep it simple. This constrains the system so that one agent can just get advice from one other agent.

The directions of the links define the imitating relationship between the two agents. Supposing there are two agents called Agent *1* and Agent *2*, and the link between them goes from Agent *1* to Agent *2*. The link represents Agent *1* imitating Agent *2*, who is called the neighbour of Agent *1*. Information transfers the opposite way in that Agent *1* uses information from Agent *2* to make trading decisions. The diagram (Fig.4.2) below illustrates the interaction between Agent *1* and *2*.

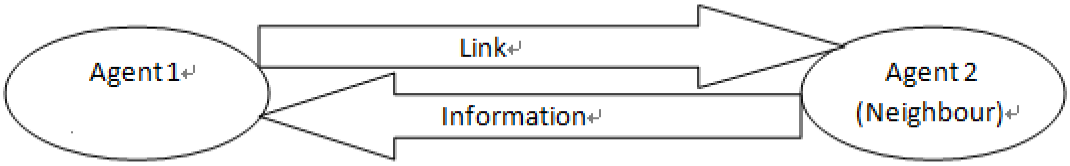


Figure 4. 2 The interaction between Agents *1* and *2*

The network is randomly chosen for initialisation at the start. Changes to the network are based on a simple wealth fitness function. At the start, all agents are in the same position with the same cash holding and stock holdings as given in Section 4.1.1. In all the formulae here, the superscript i is the particular agent i from 0 to 149 (150 agents), and the subscript t is the time t from 0 to 1000.

The wealth is equal to the current value of the stock plus the cash holding in the following equation (4.1):

(4.1) $$W_t^i = S_t^i p_t + C_t^i$$

where W, S, C and p with subscript and superscript are the symbols of wealth, stock, cash and price. Thus W_t^i is the wealth for agent i at time t. p does not have a superscript because all agents are faced with the same price at a certain time.

The fitness function simply measures the level of wealth for each agent relative to the wealthiest agent, as shown in equation (4.2):

(4.2) $$f_t^i = \frac{W_t^i}{W_t^{\max}}$$

In equation (4.2), f represents the fitness value, $W_t^{\max}$ is the wealth of the agent who has the most wealth of all agents at time t. Hence, for agent i, the more wealth agent i has, the larger the fitness.

A probability function based on the fitness function changes the whole communication network. Each agent has an existing assigned neighbour from the previous period. At the beginning of each period, a potential new neighbour is chosen for each agent randomly. Each agent is faced with a choice of keeping the existing neighbour or changing to the new neighbour. As this choice is made randomly, the probability function in equation (4.3) gives the probability of a switch to the new link. The P_r^i with a superscript in equation (4.3) is the symbol of agent i's probability of switching to the newly formed link. The probability of keeping the existing link is 1 - P_r^i.

$$(4.3) \quad p_r^i = \frac{1}{1+e^{-\beta^i(f_t^j - f_t^k)}}$$

In equation (4.3), β^i is a random number sampled from the uniform distribution with a minimum value of 5 and a maximum value of 45 for each agent i. This random number protects against locking if an agent always imitates the same guru. The superscript k is the existing neighbour of agent i, and j is the potential new neighbour of agent i. The higher the fitness value for the new agent, the more likely he would be imitated, and hence the agent has more chances of becoming the new guru.

The probability function makes the sum of two probabilities based on two fitness differences which are minus x and plus x equals to 1. This is when one fitness difference is -0.2 and one fitness difference is 0.2, the total probabilities of these two fitness differences are 1. This keeps the new and existing neighbour agents consistent as if the absolute differences of the fitness and beta are the same and the probabilities of linking to the agent with greater fitness are the same. A more detailed explanation follows after Table 4.1 below.

Beta/Difference	-0.3	-0.2	-0.01	0.01	0.1	0.2	0.5	0.8
5	0.1824	0.2689	0.4875	0.5125	0.6225	0.7311	0.9241	0.9820
10	0.0474	0.1192	0.4750	0.5250	0.7311	0.8808	0.9933	0.9997
15	0.0110	0.0474	0.4626	0.5374	0.8176	0.9526	0.9994	1.0000
20	0.0025	0.0180	0.4502	0.5498	0.8808	0.9820	1.0000	1.0000
25	0.0006	0.0067	0.4378	0.5622	0.9241	0.9933	1.0000	1.0000
30	0.0001	0.0025	0.4256	0.5744	0.9526	0.9975	1.0000	1.0000
35	0.0000	0.0009	0.4134	0.5866	0.9707	0.9991	1.0000	1.0000
40	0.0000	0.0003	0.4013	0.5987	0.9820	0.9997	1.0000	1.0000
45	0.0000	0.0001	0.3894	0.6106	0.9890	0.9999	1.0000	1.0000

Table 4. 1 Probability of choosing the new link

From Table 4.1, the row is the fitness difference and the column is beta. For agent i, if the existing link is agent k and the new neighbour is agent j, the difference fitness for agent k and j is -0.2, the beta is 5, the probability for changing to a new neighbour j will be 0.2689. The probability for keeping the existing link is 1-0.2689 = 0.7311. For another agent i called agent *2*, if the existing link is agent j and the new neighbour is agent k, the difference fitness for agent j and k can be known as 0.2, if the beta is 5 as well, then the probability for changing to a new neighbour k is 0.7311. This is the same as what has been calculated and means that if the absolute differences of the fitness and beta are the same, the probabilities of linking to the agent with greater fitness are the same. Also, from Table 4.1, when beta gets bigger, if the fitness difference is big, the probability is close to 0 or 1.

4.1.3 The expectation

The agent's expected return is based on the agent's idiosyncratic expected return and his neighbour's expected return. Expected returns in this model are the expected values of returns in profit for trading the stock.

The returns in these formulae are spot returns with the time interval from time t to time $t + \tau$. The τ which is the time horizon for traders to make their expected price is fixed at 200.

The agent's idiosyncratic (individual) expectation is based partly on a volatility factor:

(4.4) $\sigma_t^i = \sigma_0^i(1 + l_{i,t}^{\%}(1 - w))$

The σ_t^i is the return's volatility for agent i at time t, w is a herding factor, and $l_{i,t}^{\%}$ is the percentage of incoming links for agent i at time t. σ_0^i is a uniformly distributed value for agent i from 0 to σ_0. Although there is no detailed description of the value of σ_0, after several attempts to make the replication results closer to the original results, the value was decided to be 0.01 and σ_0 will be further discussed in Section 6.2.1. $l_{i,t}^{\%}$ is the total incoming links for agent i divided by the total incoming links for all agents, which is 150 in this case. The herding factor is an experiment scenario setting chosen by the modeller to vary the amount of herding.

The idiosyncratic expectation is based on the result obtained from formula (4.4) multiplied by a normal noise factor:

(4.5) $\hat{r}^i_{t,t+\tau} = \sigma^i_t \in^i_t$

The $\hat{r}$ is the symbol of an agent's idiosyncratic return and $\in$ is a noise term with normal distribution *N (0,1)* with mean 0 and standard deviation 1. The return's volatility is a positive factor to decide how large the volatility of the expected returns can be. The noise term decides if the agents are pessimistic or optimistic about the expected returns. According to equation (4.4), with the herding factor *w* smaller than 1, the agents with more incoming links can have a higher chance to have a higher return's volatility. Thus, according to equation (4.5), traders who have more imitators have more chances to be either too optimistic or pessimistic.

The overall expected return of each agent is calculated by a combination of their own idiosyncratic return and their neighbour's return, as follows:

(4.6) $r^i_{t,t+\tau} = w\hat{r}^i_{t,t+\tau} + (1-w)\hat{r}^j_{t,t+\tau}$.

In this formula, *r* is the symbol of return and $\hat{r}^j_{t,t+\tau}$ is the neighbour *j*'s idiosyncratic return when agent *j* is the neighbour of agent *i*.

In equations (4.4) and (4.6), the smaller the herding term w is, the bigger the herding influence is. This follows the terminology in the original paper, although it can be confusing in the sense that the higher w means less herding. When w gets smaller, the return is affected more by the neighbour's return rather than by the agent's own idiosyncratic return.

When all the agents have their idiosyncratic expectation returns formed as in equation (4.5), the return in equation (4.6) is revised in a random order one by one for each agent. Once the revised return in equation (4.6) is updated for a particular agent i following the random order, the agent's idiosyncratic return is updated to the revised return, which means after the agents have updated their revised returns, their idiosyncratic returns are the same as the revised returns.

4.1.4 The market

The general approach taken in the model is to calculate the ideal stock holding of the agent. Comparing this with the actual current stock holding determines how much the agent will buy or sell. A utility function is used to find the ideal stock holding. This depends on the expected price and on the agent's estimate of the variance of returns. This section explains the detailed mechanisms used and how they are derived.

In the model, the market is an order driven market where agents trade with each other. All traders at a certain time period submit an order with a price, the amount of stock and a buy or sell position. Traders submit orders in a random sequence to the order book.

The best bid price is the highest buying price and the best ask price is the lowest selling price in the order book. Trading takes place when the price of buying is higher than the price of selling in the order book. When trading takes place, the best bid and best ask will be updated as the orders in the order book are changed. The executed orders are called market orders while the orders remaining in the order book are called limit orders. At the start, all orders are limit orders until trading takes place.

In the model, each trader has only one active order at time *t*. In the original paper, all orders are kept for 200 time periods and have just one active order for each trader at time *t* (Tedeschi et al., 2012). This may not be that reliable because the price observed by all traders at any one time is updated according to the trading taking place in the order book based on the updated price, agents' expected returns and the order they want to submit in the order book is changed. The expectation and the order submitted are revised each time.

After the expectation is formed by each agent at a particular time *t*, as described in Section 4.1.3, the market can work based on the expectation. The expected future price at time $t + \tau$ based on the expected return is defined as:

(4.7) $\hat{p}^{i}_{t,t+\tau} = p_t \, e^{r^{i}_{t,t+\tau}\sqrt{\tau}}$

The symbol p_t is the market price at the current period *t* which is known by all the traders. It starts at the price p_0 equal to 1000. The traders make a forecast for period τ after time *t*.

When the interest rate is continuously compounded, the basic finance equation is $p_{t,t+\tau} = p_t e^{r^i_{t,t+\tau}\tau}$. In the order driven market, each agent submits his order based on his own expectations on returns. Therefore, the value of the future price is not exactly the same as in this equation. The difference is that equation 4.7 uses the square root of time τ. This square root describes a geometric random walk that occurs when traders are purely noise traders (Tedeschi et al., 2012). This is consistent with financial theories that the price is unpredictable and follows a random walk.

The preferred stock holdings for each agent are deduced using a utility function. In this model, the CARA (constant absolute risk aversion) exponential utility function is used to get the optimal number of stock holding.

Another well known utility function is CRRA which is the "constant relative risk aversion". The difference between these two utility functions is the type of risk aversion. The CARA utility function assumes that risk aversion will not change with wealth or consumption, whereas the CRRA utility function assumes that risk aversion does change with different levels of wealth or consumption. The exponential function is the basic structure for CARA. By contrast, CRRA is an isolated utility function defined as $u(c) = \frac{C^{1-\rho}-1}{1-\rho}$ for the consumption utility when ρ is positive and not equal to 1. When it equals 1, $u(c) = log\ (c)$. C is consumption and ρ is the risk aversion parameter.

In this model, the CARA utility function is defined as:

$$(4.8)\quad U(W_t^i, \alpha) = -e^{-\alpha W_t^i}$$

In this equation, W is the wealth and α is the risk aversion parameter that controls the investor's risk preference (Tedeschi et al., 2012). Risk aversion ranges from where α = 0, and risk is neutral; to $\alpha > 0$ which indicates risk averse; and $\alpha < 0$ shows risk-seeking. In this model, α is fixed at 0.01, which means agents are risk averse.

When people are risk averse, they prefer a return that is guaranteed rather than an uncertain return when these two options have the same expected return or even when the uncertain one has a slightly bigger expected return. Risk neutral indicates that people treat the two options the same when they have the same expected return. The risk seeking investors are prepared to gamble on the uncertain return in the hope of a higher payoff even though the expected value is lower. The optimal solution for balancing the expected return and risk is decided by the individual's risk perceived variance. The risk perceived variance is determined from the unconditional variance of returns.

As the time horizon τ is 200, the unconditional variance is calculated from the 200 previous returns. The equation for mean returns is:

$$(4.9)\quad \bar{r}_t = \frac{1}{\tau}\sum_{j=1}^{\tau} r_{t-j} = \frac{1}{\tau}\sum_{j=1}^{\tau} \ln \frac{p_{t-j}}{p_{t-j-1}}$$

$\bar{r}$ is the mean spot return based on the previous price. The return from period *t-j-1* to *t-j* is the relative change in price (e.g., a price moving from 500 to 505 gives a return of 5/500 or 1%). The equation uses the logarithm of the price at period *t-j* divided by the price one period before that. This is because the logarithm value is nearly equivalent to

$p_{t,t+\tau} / p_t - 1$ (Tedeschi et al., 2012). The total value of all previous 200-period returns is then summed up and the average number of the spot return is obtained through dividing the total by 200, the number of periods.

The paper does not state the initial position for the return. At time period 0, there is no return as no previous price is available. Also there is not much information to calculate the average spot return for the time period less than 200. At time period 1, there is just one return value but in equation (4.9), 200 returns are needed.

The unconditional variance then can be calculated by the usual variance formula. The variance is based on the returns for the previous 200 periods:

$$(4.10)\quad V_t = \frac{1}{\tau} \sum_{j=1}^{\tau} [r_{t-j} - \overline{r_t}]^2 .$$

V_t is the unconditional variance based on return r_{t-j} and average spot return $\overline{r_t}$.

At the start, in period *0* there is no return to calculate the variance and in period *1* if just one period is taken into account, the average spot rate of return is the return itself which makes the unconditional variance zero. This is not acceptable as equation (4.12) illustrates later as the variance, the denominator, cannot be zero. Therefore, the variance is set at 0.001 at the start of initialisation.

The variance for the individual agent taking the network into account is:

(4.11) $V_t^i = V_t \ (1-(1-w)l_{i,t}^{\%})$

The individual's variance depends on the herding parameter *w* and the percentage of incoming links for that agent. The smaller the w is, the smaller the individual's variance is. This indicates a herding effect which causes a decrease in the individual's unconditional variance. Hence, herding decreases the risk assessment. If the herding phenomena in the market increases, the agents are copying each other and so the prediction may be less risky because their beliefs are similar. They are confident with the information from the other agents.

In addition, the larger the percentage of incoming links, the smaller the variance V_t^i is. The percentage of incoming links indicates the level of success of the agent. If the agent has more links their prediction will be less risky because others will copy that agent making it more likely that the prediction becomes true.

The portfolio which the agent needs to hold to be an optimal solution according to the exponential CARA utility function based on the wealth is:

(4.12) $\pi^i(p) = \dfrac{\ln(\hat{p}_{t,t+\tau}^i / p)}{\alpha V_t^i p}$.

This equation is derived from the utility function (4.8), with the derivation provided in Tedeschi et al. (2012). $\pi^i(p)$ is the preferred number of stock to hold for agent *i* at certain price *p*. The optimal solution for stock holdings given the price *p* is related to the risk aversion parameter alpha which is equal to 0.01 in this model, the future expected price $\hat{p}^i_{t,t+\tau}$ from equation (4.7), the individual's variance V^i_t from equation (4.11) and the price level itself.

In this model, the 150 traders trade with each other and there is no short selling and borrowing. No short selling means that traders are constrained and can only sell the stock available at hand; no borrowings constrains traders borrowing money from others to buy stock. Hence at each time period, the stock position and cash position for each trader are crucial. Equation (4.12) establishes the relationship between the price and optimal portfolio amount of stock, and from this the price related to the stock and cash position for each trader can be found.

The cash position limits how much a trader can buy. The price at which a trader is willing to buy, multiplied by the optimal stock amount at this buying price, cannot exceed the total cash. The p^i_m, controlled by the total amount of cash is as follows in this equation:

(4.13) $p^i_m(\pi^i(p^i_m) - S^i_t) = C^i_t$

In this equation, p^i_m is the price limit due to the cash position which needs to be found and $\pi^i(p^i_m)$ is the optimal stock amount at p^i_m. The existing stock amount is S^i_t. The $\pi^i(p^i_m)$ minus the S^i_t is the number of stock the trader wishes to have minus the number the trader now has and is the maximum amount to buy at this cash position. The smaller the price is, the larger the amount the trader can buy with the given available cash. p^i_m is the minimum price for the trader to submit in the order driven market in order to get the optimal buying amount as decided by equation (4.13).

The biggest highest price a trader can submit is decided by equation (4.7) by the symbol $\hat{p}^i_{t,t+\tau}$. The reason is that the stock amount for holdings must be positive in this model. If the actual price p is greater than this, the stock holding in equation (4.12) will be negative. Therefore, the maximum value for order price p is $\hat{p}^i_{t,t+\tau}$. In the model, the order price p for each trader to submit is chosen randomly from a uniform distribution using the interval (p^i_m , $\hat{p}^i_{t,t+\tau}$). Once the trading price is selected, then the amount they wish to hold related to this price can be calculated using equation (4.12).

The price p^* relates to the existing stock amount and determines whether the trader's order is a buy order or a sell order. The price p^* is the price at which the trader would be happy with their current stock holding according to the utility function. The value of p^* can be determined by solving equation (4.14) where the utility quantity from equation (4.12) equals the current stock:

$$(4.14)\quad \pi^i(p^*) = \frac{\ln(\hat{p}^i_{t,t+\tau} / p^*)}{\alpha V^i_t p^*} = S^i_t$$

In this equation, the S^i_t and the denominator are all positive numbers which means p^* cannot be larger than $\hat{p}^i_{t,t+\tau}$. The relationship between the price and stock amount to buy or sell is always a negative relationship. The difference of S^i_t and $\pi^i(p\ )$ is the amount to trade. If the submit order price is more than p^*, that means the trader i values the stock at more than p^*. This will lead to a smaller amount to hold where there is a negative relationship between price and demand than S_t^i which is the demand at p^*. This makes the trader a seller with an amount to sell at $S_t^i - \pi^i(p)$. If the submit order price is less than p^*, the trader values the stock at less than p^*. This will lead to a larger amount being held compared to S_t^i and will make trader *i* a buyer with an order amount of $\pi^i(p) - S_t^i$. In fact, we do not need to calculate p^*. We can simply compare S_t^i and $\pi^i(p)$. The market mechanisms for order price and amount to trade are summarised in Table 4.2:

	Position	Type of order	Volume
$p_m^i < p < a_t^q$	Buy	Limit order	$s_i = \pi^i(p) - S_t^i$
$a_t^q \leq p < p^*$	Buy	Market order	$s_i = \pi^i(a_t^q) - S_t^i$
$p = p^*$	No order placement		
$p^* < p \leq b_t^q$	Sell	Market order	$s_i = S_t^i - \pi^i(b_t^q)$
$b_t^q < p \leq p_M^i$	Sell	Limit order	$s_i = S_t^i - \pi^i(p)$

Table 4. 2 Trading mechanisms (Tedeschi et al., 2012:87)

In Table 4.2, the a_t^q and b_t^q are the best ask and best bid price in the market. The relationship between the different prices p_m^i, p^*, $\hat{p}_{t,t+\tau}^i$ and their related stock level are shown in the graph (Fig.4.3) below:

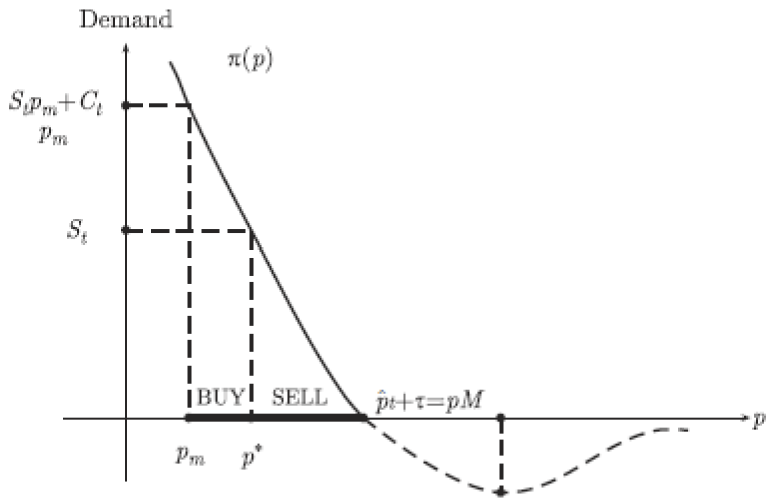


Figure 4. 3 The relationship between price (p_m^i , p^* , $\hat{p}_{t,t+\tau}^i$) and demand (Chiarella et al., 2009:528)

The first demand related to p_m is $\frac{S_t p_m + C_t}{p_m}$, the budget constraint. This graph shows the negative relationship between price and demand. The relationship between price p, p_m^i , p^* , $\hat{p}_{t,t+\tau}^i$ with the related demand amounts, and the buy and sell conditions are also illustrated.

When all the above parameters have been decided, the market can start to trade. The orders enter the market one at a time in random order. The best bid and best ask is the highest buying price and lowest selling price and changes with orders entering and with trading happening. When there is no buying or selling order in the order book, the best bid or best ask is non-existent. Trading happens when one condition is met: a buyer price in the order book is higher than a seller price. The amount to trade is the smaller amount of the buyer amount and the seller amount. At each time, one order comes into the order book, and then the orders are separated into buy orders and sell orders. For buy orders the prices are sorted from highest to lowest, while for sell orders the prices are sorted from lowest to highest. When an order comes in, the trading condition is checked. When the first line in the order book meets the trading condition as mentioned, trading happens. The amount for the buyer or seller for the first line changes to zero and the order is deleted from the order book. The cash and stock positions are changed for the trading buyer and seller at their submitted price with the amount traded. The next one in the order book then becomes the first. The process continues until the trading condition is not matched and a new trader enters. When no more traders come into the order book, the trading for time t is completed.

All stock and cash positions for the trading agent are updated according to the amount traded. The observed price for the next period is the average of the highest bid and lowest ask price. If these are not available, the price for the next period is the previous price. The wealth is updated according to the change in cash, stock and price. Then the time t becomes $t+1$ until the end of the simulation at time 1000. The code description for this model is in appendix A.

4.2 The results

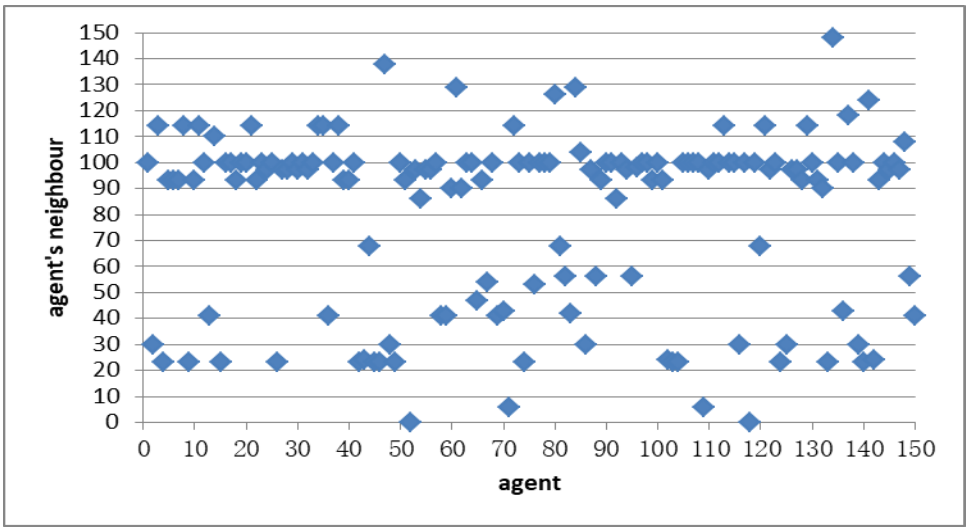
The replicating results in Section 4.2.1 to 4.2.2 are based on a single run with run length 1000. The time unit for the run length is not specified in the original paper. The total agents are 150. The herding factor w in this model is 0.1, 0.5 and 1, respectively. Section 4.2.3 has a herding coefficient based on a single run and average returns for ten runs with run length 1000.

4.2.1 The results from network analysis

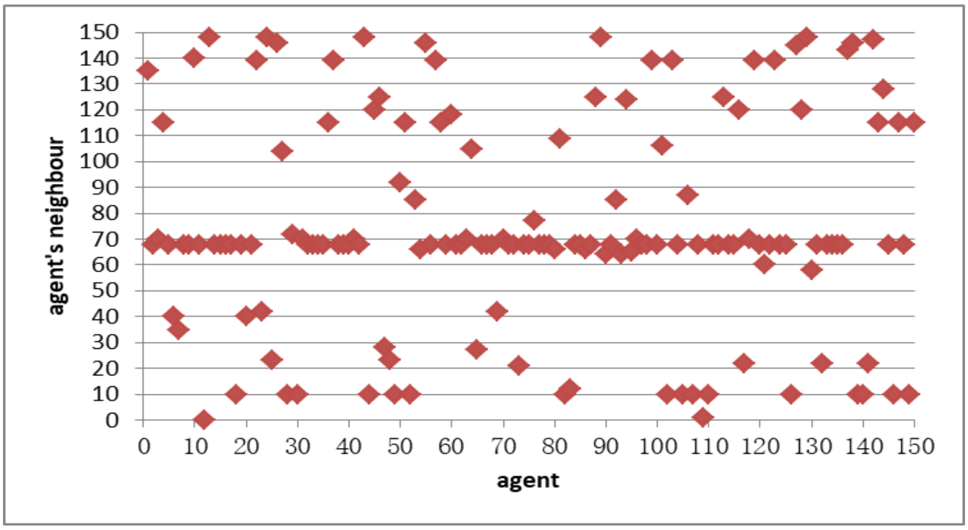
The network analysis is based on the network connection introduced earlier. The index of the guru, the incoming links with the guru and the fitness of the current guru, are discussed. The connections between them for the final period are as shown in Figure 4.4 below for one typical run: the first one with a blue point is the connection network with *w*=0.1; the second one with a red point is the connection network with *w*=0.5; the third one with a grey point is the connection network with *w*=1. The x axis is the agent himself and the y axis is the neighbour of that agent.

At the final period, the agent connections in the change of herding are similar when *w* is 0.1 and 0.5. They all have a guru who has many imitators and the number of agents who are imitated by others is not large. But at the communication network when *w* is 1, there is a guru who has many imitators and an agent who has many imitators as well, though fewer than the guru. Therefore herding exists to some extent which limits the number of agents being imitated and increases the incoming links of the guru.

w = 0.1:



w = 0.5:



w = 1:

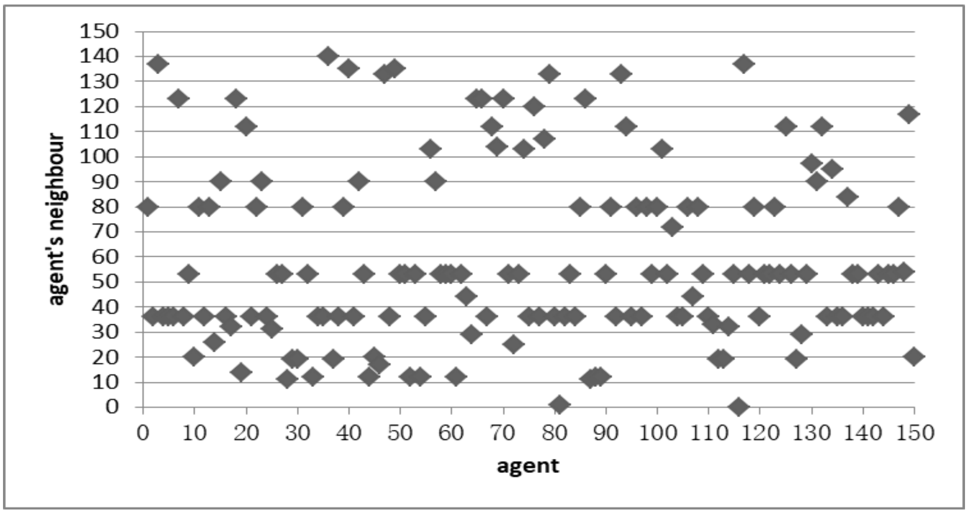


Figure 4. 4 The network when *w*=0.1, 0.5, 1 at the last period in the replication model

Figure 4. 5 The network when w=0.1, 0.5, 1 from left to right in the original model (Tedeschi et al., 2012:88)

agent	w=0.1	w=0.5	w=1
1	4	3	2
2	4	11	3
3	1	1	1
4	1	1	3
5	1	1	1
6	16	1	1
7	1	2	3
8	1	2	1
9	4	17	1
10	4	1	8
11	4	1	1
12	1	1	2
13	1	2	1
14	1	1	1
15	1	8	1
16	61	58	1
17	1	1	42
18	4	14	26
19	39	3	1
20		1	1
21		2	2
22		18	2
23			1
24			2
25			1
26			11
27			30
total	150	150	150

Table 4. 3 Agents connections in the original model

As seen in Figure 4.5 and Table 4.3, when *w* increases, there is less herding in the original model; the network is less centralised which means the guru has fewer followers. The pattern from the replication model is different. Figure 4.4 shows *w* is 0.1 and 0.5, the guru has a similar amount of followers and the network is similarly centralised when *w* is 0.1 and 0.5. By counting, the guru in the replication model is 45, 57, 35 when *w* is 0.1, 0.5 and 1. The amount of agents' connections is reducing from *w* = 0.5, 1 to 0.1.

The index of current guru, the percentage of incoming links of the guru and the fitness of guru are shown in the three figures below where *w* is equal to 0.1, 0.5, and 1 respectively. The index of the current guru is shown in which the agent is the guru. The value used is the number of the guru agent divided by the total number of agents. For example, if the guru is agent 75 the index value will be 75/150 = 0.5. The guru is the trader who has the most incoming links thus indicating that he has the most followers.

In the Figures 4.6-4.8 below the x–axis represents the time from 0 to 1000. The black line is the index of the current guru from 0 to 1 and shows how often the guru changes over time. The green line is the fitness - it is almost one because fitness in the equation (4.2) is $f_t^{\,i} = \frac{W_t^{\,i}}{W_t^{\max}}$. A fitness of 1 means the guru is the wealthiest agent. The red line is the percentage of incoming links to the guru.

Figure 4.6 shows when *w* = 0.1, the percentage of incoming links increases at the start to about 0.3 on average. Fitness is one most of the time. The guru remains the same and lasts more than 500 periods.

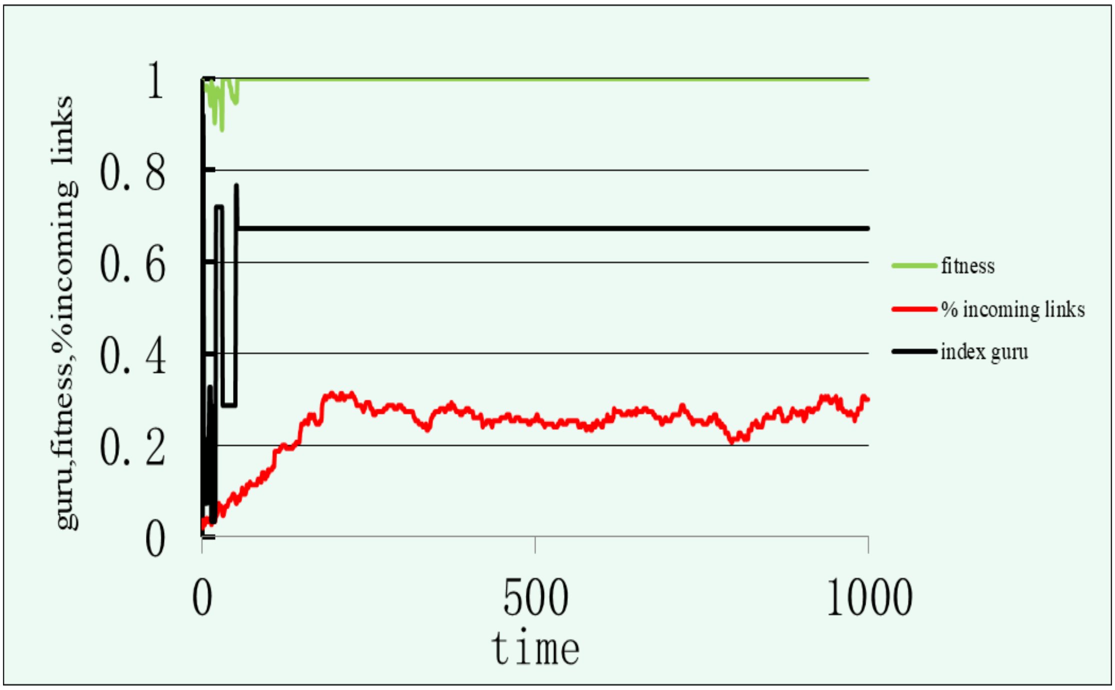


Figure 4. 6 Guru, incoming links and fitness when w=0.1 in the replication model

Figure 4.7 shows that the incoming links are growing at first with an average of 0.3 when w = 0.5. If the herding w works as expected from the original paper, when w increases, the herding effect is reduced and the percentage of incoming links should be lower than Figure 4.6. Also, the guru remains the same and lasts longer than in Figure 4.6. Therefore, it would seem that the herding effect is not reduced by the herding parameter from 0.1 to 0.5. Fitness is 1 most of the time.

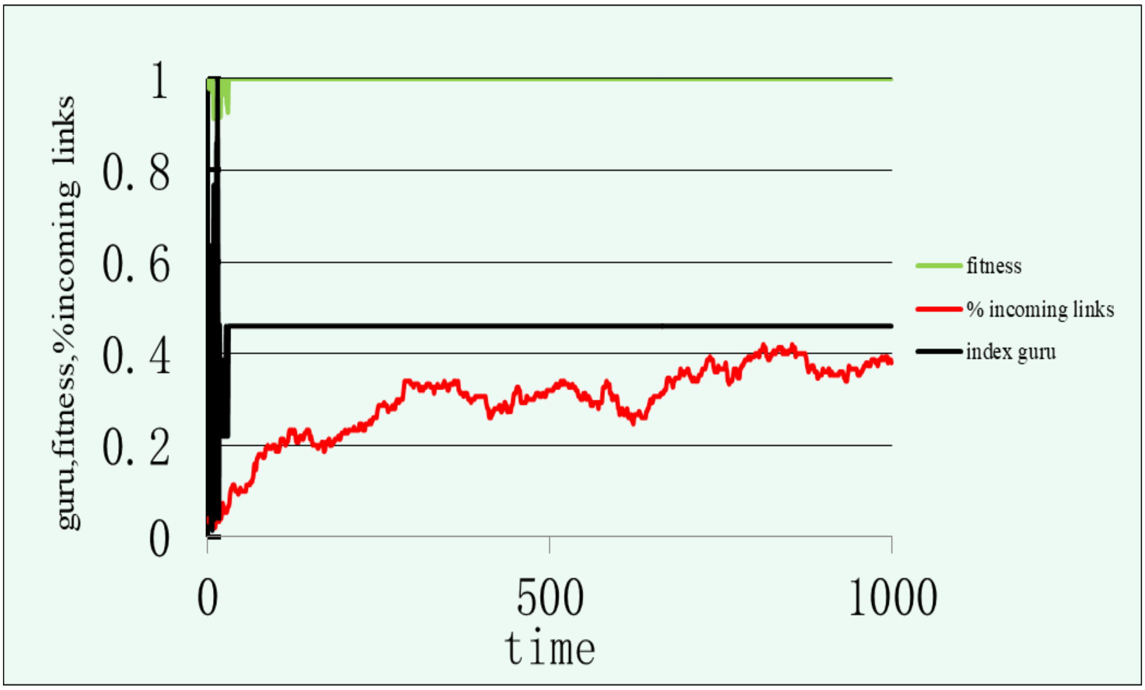


Figure 4. 7 Guru, incoming links and fitness when *w*=0.5 in the replication model

The fitness for the guru at each time is nearly one and the guru remains unchanged for more than 500 periods until the end when *w* is 0.1 or 0.5 in Figure 4.6 and Figure 4.7.

Figure 4.8 shows the guru, incoming links and fitness changes in the time period from 0 to 1000 when *w*=1.

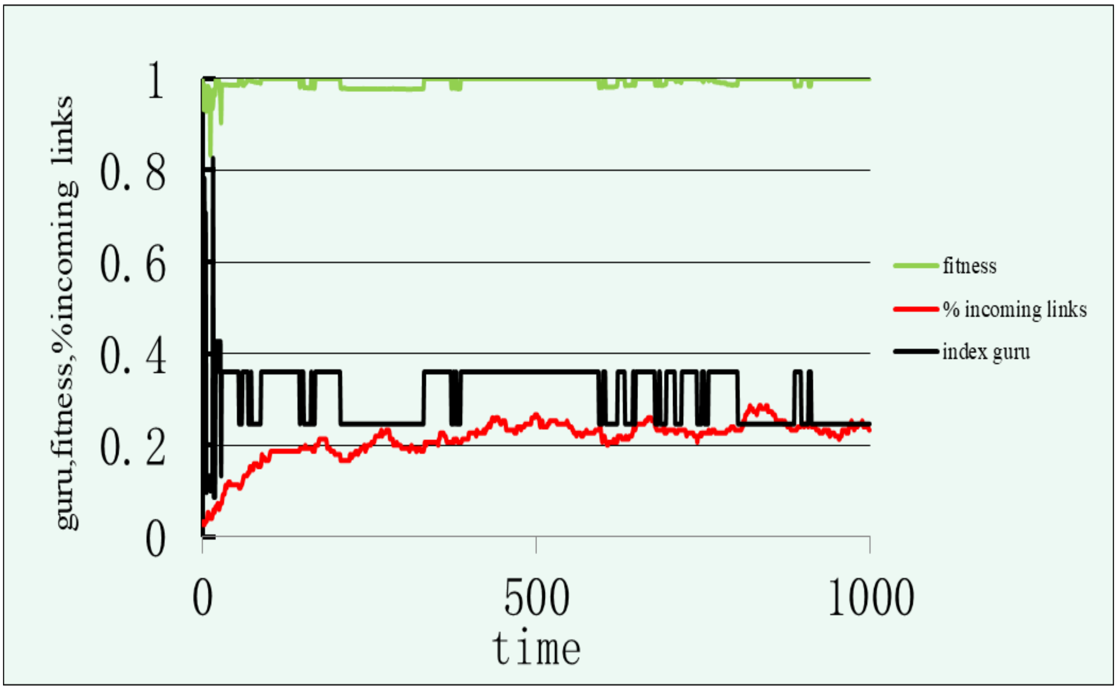


Figure 4. 8 Guru, incoming links and fitness when w=1 in the replication model

In this case, the red line with a proportion of incoming links is lower than the previous two graphs and it seems that the herding is reduced from $w = 0.1$ to 1 and also from 0.5 to 1. The fitness for the guru at each time is not exactly one as in the other two graphs indicating that the traders here do not imitate the most successful one which has the maximum wealth at each time to give him a fitness of 1. The guru keeps changing and does not last very long compared to Figures 4.6 and 4.7 when w is 0.1 and 0.5.

All in all the herding effect is not that obvious from 0.1 to 0.5, but it can be observed when comparing the graphs from $w = 0.1$ to $w = 1$ and from $w = 0.5$ to $w =1$.

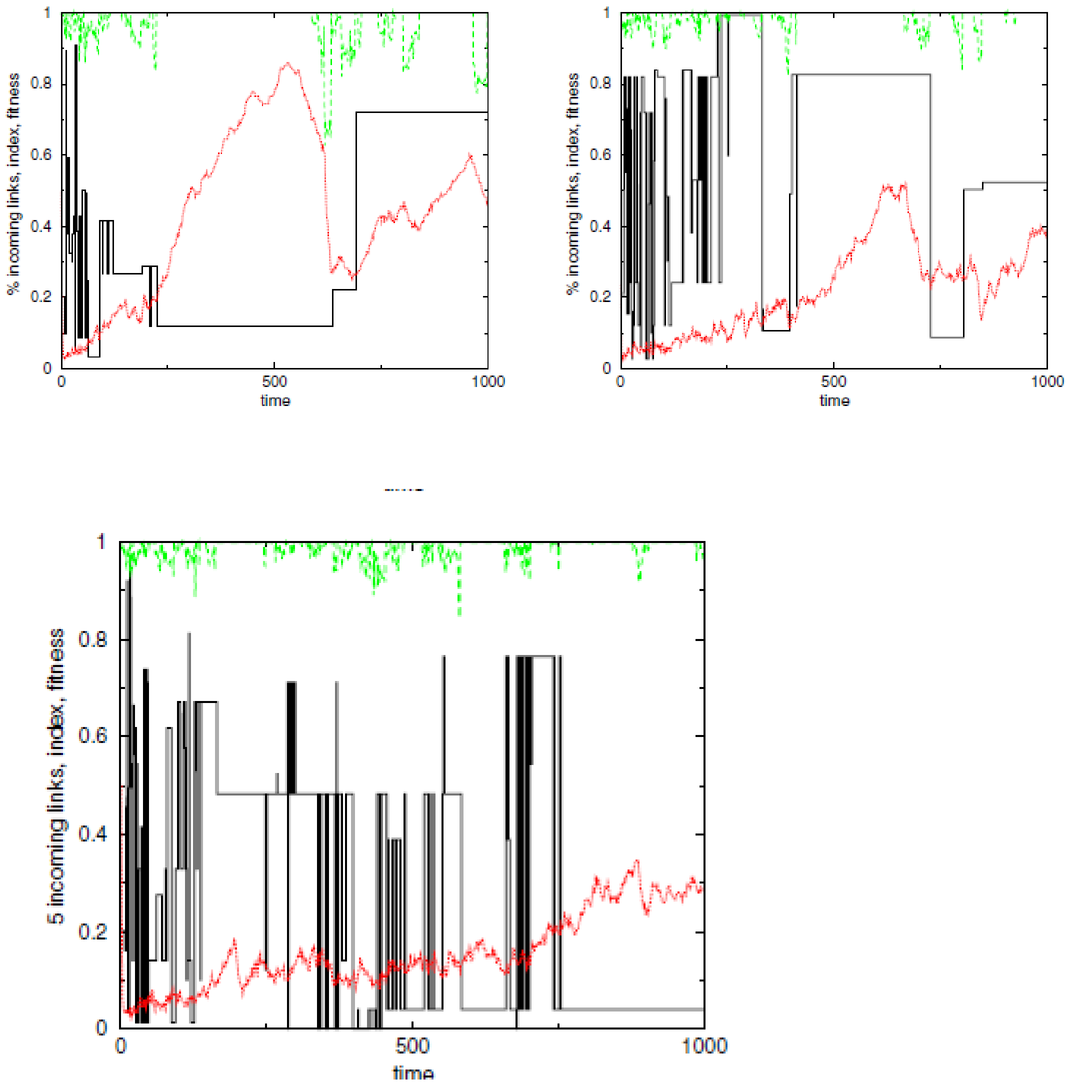


Figure 4. 9 Guru, incoming links and fitness when w=0.1 (top left) 0.5 (top right) 1 (bottom) in the original model (Tedeschi et al., 2012:89)

In the original model, the incoming links (red line) are much higher compared to the replication model with almost all under 0.4. The guru (black line) changes little when w=0.1 and 0.5. The fitness (green line) is not always one compared to the replication model.

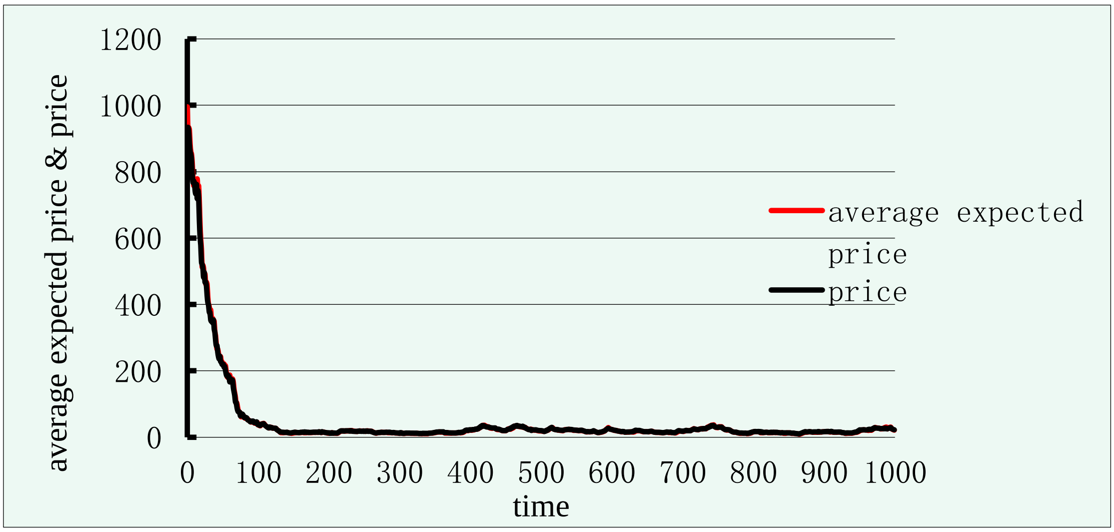
The results of the replication model and the original one are different but when comparing the patterns generated when w changes from 0.1 to 1, the trend is similar. Incoming links increase and the guru lasts longer from w=1 to w=0.5 and w=0.1. The fitness is almost 1 when w=0.5 and 0.1but it is not when w=1 which is different from the original model.

4.2.2 The results from the price analysis

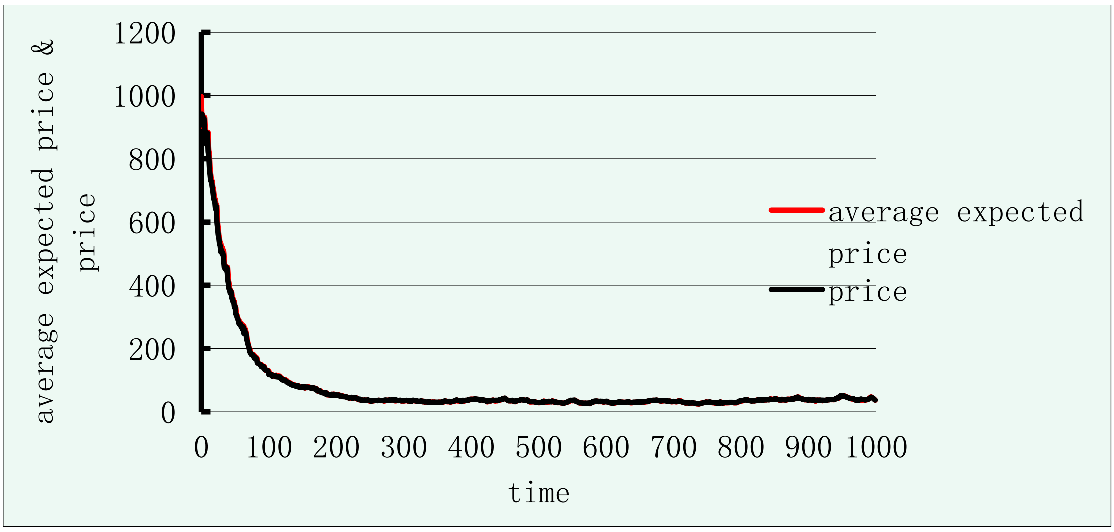
The price analysis focuses on the market price p and the average expected price which is the $\hat{p}$ in equation (4.7). The expected price is the maximum price a trader i can submit in the order book at that time t. The three graphs below show the price change at w = 0.1, w = 0.5 and w = 1 respectively.

Figure 4.10 shows more or less the same pattern: the black line for the price drops sharply at first and then the price fluctuates around a very small price. The green line for the average expected price in these graphs is hard to observe. This shows that the average expected price and the final price are almost the same at any one time.

$w = 0.1$:


average expected price & price
1200
1000
800
600
400
200
0
0
100
200
300
400
500
600
700
800
900
1000
time
average expected price
price

$w = 0.5$:


average expected price & price
1200
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200
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time
average expected price
price

w= 1:

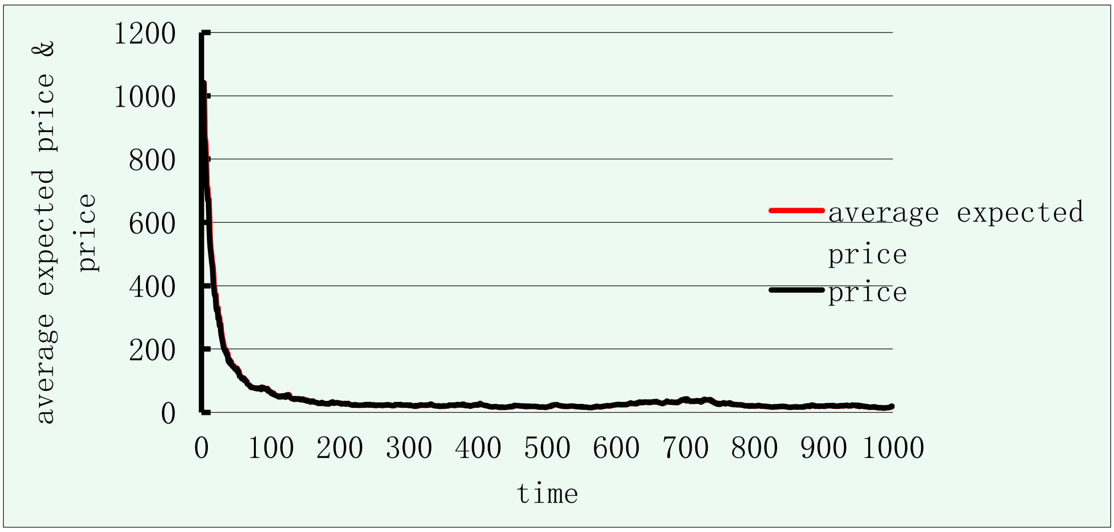


Figure 4. 10 Prices and average expected prices when *w*=0.1, 0.5, 1 in the replication model

In order to evaluate the variation of price under the three different conditions of *w,* the relatively stable periods are picked up to study.

For all the three values of *w* the stable period starts close to period 200. The price variation for *w* = 0.1 after period 200 is as below:

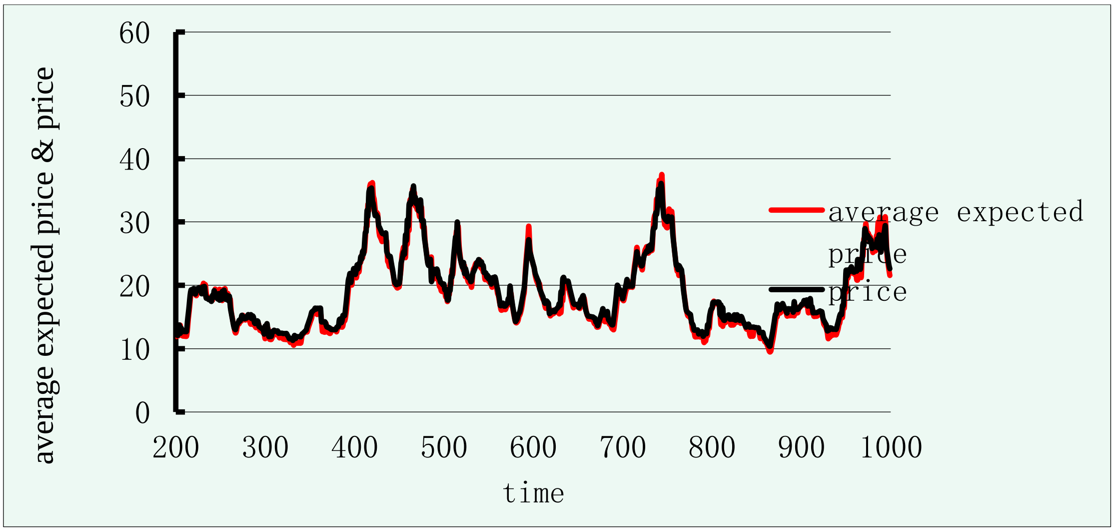


Figure 4. 11 Prices and average expected prices when w=0.1 after period 200

The price movements for w = 0.5 are shown next.

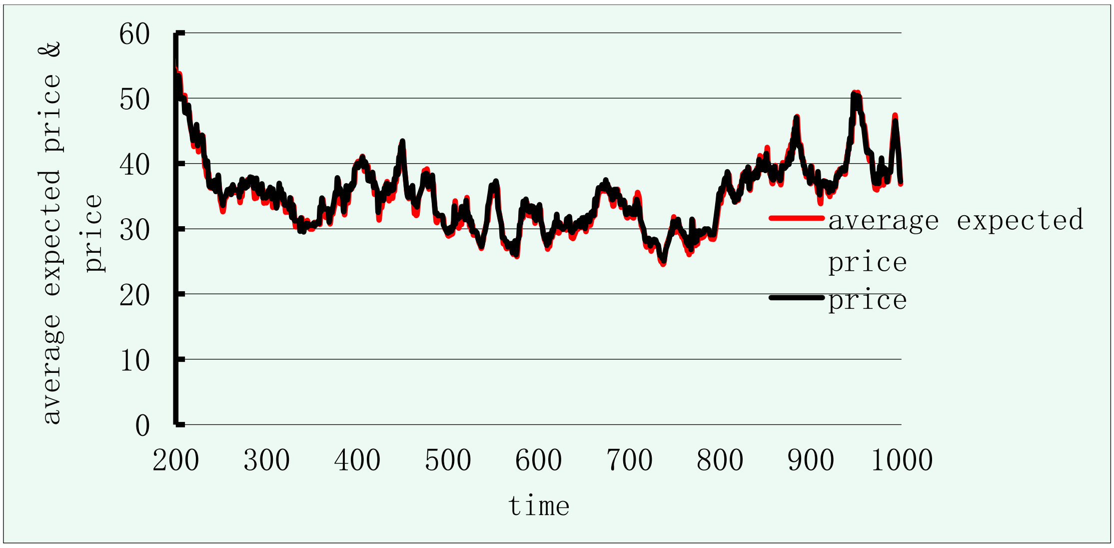


Figure 4. 12 Prices and average expected prices when w=0.5 after period 200

The average expected price and price behave in a much more stable way in Figure 4.12 compared to the same period in Figure 4.11.

The price trend for *w* = 1 is shown below from period 200.

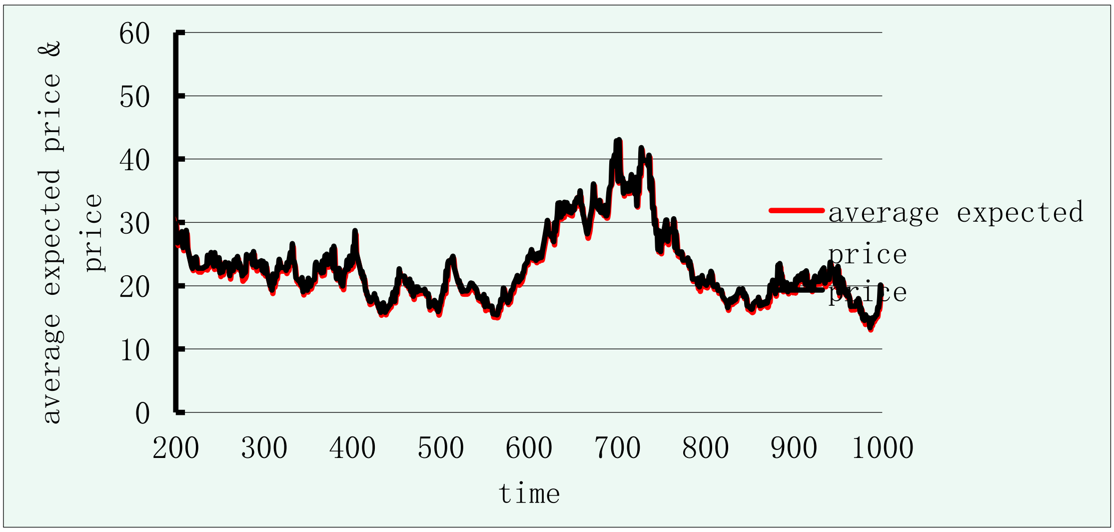


Figure 4. 13 Prices and average expected prices when *w*=1 after period 200

The expected price and price is not very stable as might be expected especially around the 500 period. The variation in price and the average expected price in Figure 4.13 and Figures 4.11 and 4.12 are hard to compare.

The price trend in the original model is shown in Figure 4.14.

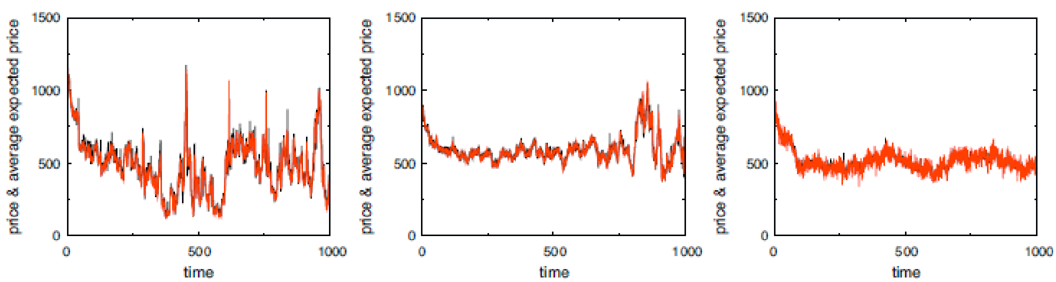


Figure 4. 14 Prices and average expected prices when *w*=0.1 0.5 1 (left to right) in the original model (Tedeschi et al., 2012:91)

To conclude, Figure 4.14 shows herding causes price fluctuation to some extent as in previous studies such as that of LeBaron and Yamamoto (2007). However, a pattern is not clear in the replication model except when w=1 and prices become stable more quickly than w=0.1 and 0.5. As both sets of results are based only on one run, the pattern may not be that obvious in multiple runs.

4.2.3 Price dropping analysis

The herding effect can be assessed in this model by herding coefficient H = total buyers / total traders (Tedeschi et al., 2012). If H is equal to 0.5, that means half of the total are buyers and half are sellers in the markets. When H is nearly 1 this would indicate that there is a lot of herding and the numbers of buyers and sellers are not equal. The average herding coefficient for one run with 1000 time are 0.9500, 0.9333, 0.9413 respectively for w=0.1, w=0.5 and w=1. The minimum value, maximum value and average value for the herding coefficient with time series from 1-1000 are summarised in Table 4.4 below:

w	0.10000	0.50000	1.00000
Min	0.43000	0.46000	0.43333
Max	0.95055	0.93330	0.94135
Average	0.90361	0.87423	0.89357

Table 4. 4 The minimum, average, maximum values for herding coefficient

As seen in Table 4.4, the range of herding coefficients decreases from w = 0.1 to w = 0.5, but increases from w = 0.5 to w = 1. This indicates a slight herding effect that is produced by parameter w where the range of the herding coefficient is shrinking when w gets bigger. The average herding coefficient is not as in the original paper which was equal to 0.5. Also, the average is greater than 0.85, which means the market has many buyers under the three conditions for different *w*. This may indicate that in most periods the markets do not have a lot of trading which might influence the final price.

The average and variance of returns for ten runs are shown in Table 4.5:

	average	variance
w=0.1	-0.00471	0.00253
w=0.5	-0.00405	0.00115
w=1	-0.00432	0.00256

Table 4. 5 Average returns and variance

In Table 4.5, the average returns are all negative when *w* is 0.1, 0.5 and 1. To some extent, this indicates the price dropping as well. And there is not much difference for average returns when *w* is 0.1, 0.5 and 1, which means in this case, the herding *w* does not have much influence for returns. And the variance is reduced when *w* is 0.5. More discussion about replication results based on ten runs are in Chapter 6.3 for herding and related stylised facts.

4.3 The altered model and results

As the original results and the replication results were different and the prices dropped a lot in the replication model, some modifications were made. The model is modified to some extent to stop the price from dropping. There are three reasons for changing the utility function in the market mechanism. Firstly, the utility function changes very little when the wealth is large. Secondly, the utility function in the model makes an unrealistic assumption that the noise traders balance returns and risks to make optimal decisions. Thirdly, the price function based on the utility function constrains the maximum submitted price to the expected price, which is not realistic in the real world. Traders may submit a price to sell above expectations in order to make more profit.

4.3.1 Altered initialisation

The traders in the model are in the same situation at the start. They all have 100 stocks at a price of £1000 each and £100,000 cash. Hence, in the beginning, the values of stock and cash are exactly the same for each trader. For initialization, the trading rules for the first period are not the same as for the remaining periods. It is a warm-up period with no price change after each trading during the period and no change of traders' stock and cash positions. Therefore, it is more or less dummy trading in the beginning period. The closing market price then changes according to the best-bid price, as illustrated in Section 4.1.4 (the market mechanism description). The remaining un-traded orders are the active orders in the next period and are traded on a rolling basis (Section 4.1). There are 150 traders in this model for 1000 periods from starting point period 0 to period 1000 with 100 runs. The following section will discuss the altered market. The network and the expectation are unchanged.

4.3.2 Altered market

After the expectation for each agent is formed, the future price for each agent is also formed based on their expectations. In order to have more variety in expected returns, the σ_0 is set to 0.07. The expected price at time $t + \tau$ when the agents are at time t is:

(4.15) $\hat{p}^{i}_{t,t+\tau} = p_t \, r^{i}_{t,t+\tau}$

The $\hat{p}$ here in the equation (4.15) is the future price. After agents get information like the current price and the expected price, they make decisions about their prices and amounts submitted to the order driven market, which are mainly based on the work of Chiarella et al. (2009).

At any given time, each agent always has just one order in the market. The discount factor in this model is ignored. Then instead of considering the utility function, the noise traders in this model submit orders based on a very simple rule. They compare the current price with the expected price from equation (4.15) to submit a buy or sell order. Obviously, if the current price is greater than the expected price, traders sell now and hope to buy in the future to make gains; otherwise, the traders buy now at a lower current price and hope to sell at the higher future price to make a profit. After the traders have made their decisions to submit their buy or sell orders through the comparison between the current price and expected price, the prices and amounts for the orders are generated. When the difference between the current price and expected price is greater than or equal to 50, the buy order price for a certain agent is the current price plus 50 and the sell order price is the current price minus 50 to ensure some gains. Otherwise, traders submit limit orders at buy order prices or sell order prices which are expected price minus 50 or expected price plus 50 to ensure at least 50 profit if the expectation is right. This reflects the two attitudes of traders. When traders think there is a satisfactory gain in the market, they make a concession and make orders to be traded to make some gain. When no satisfactory gain in the market is expected, they make an unrealistic stricter rule to ensure their profit, but their order does not easily trade.

The amount of the order for a certain agent to submit is based on their cash, the probable gain they make and also the risk attitude. The probable gain is equal to:

$$(4.16) \quad g_t^i = \frac{|Po_t^i - \hat{p}_{t,t+\tau}^i|}{Po_t^i}$$

The g is the probable gain for the trading and Po is the price submitted into the order driven market. The equation for the amount the agent to submit is:

$$(4.17) \quad s_t^i = \frac{C_t^i g_t^i}{Po_t^i Max} \quad (\text{when } g_t^i \leq Max)$$

$$s_t^i = \frac{C_t^i}{Po_t^i} \quad (\text{when } g_t^i > Max)$$

The s and Max stand for the stock amount to submit in the market and the acceptable percentage of gain for using the full cash. C is cash. There is an assumption that the relationship between the cash in hand and the stock amount to submit is a straight line. The amount of cash to use depends on the probable gain and the gain which the agent is expecting. Max is a uniform distribution from 0.5 to 1 which means the agent is willing to invest all of the cash into the market when the profit is at least from 50% to 100%. Once the probable gain exceeds the Max, the agent invests all of his cash into the market. When the level factor of cash to invest $\frac{g_t^i}{Max}$ has been decided, the stock amount to submit into the market is the cash to be invested divided by the price expected to trade which is Po.

The market mechanism in the order driven market is now discussed. The agent's order is submitted on a rolling basis in a random sequence. The rolling basis means that when a certain agent enters the market, the last period order for that particular agent is cancelled if it is still in the order book. During time period t, when agents are entering the market, the current price is equal to the average of the best bid and best ask. The best bid is the highest buy order price in the order book and the best ask is the lowest sell order price in the order book. If there is no best ask the current price is the best bid. If there is no best bid the current price is the best ask. When all the agents have submitted their orders to trade, the current price at the end of the period is the average of the best bid and best ask. If there is no best bid or best ask or both, the current price is the previous end market price. When the new order comes into the market, the price of the new buy or sell order is compared to the best ask or best bid price. If the price of the buy order is greater than the best ask or the price of the sell order is smaller than the best bid, there is a match. The new order is the market order and trades at best ask or bid. The same rules apply to the newly updated best bid or ask price when the previous best bid or ask has the amount of 0 after the trade unless the amount of new order is fully satisfied or there is no match. If this is the case and the new order still has some untraded amount of order, the remaining amount of new order goes into the order book and the best bid or best ask is updated accordingly. The new order comes into the order book directly if there is no match and the best bid or best ask is updated accordingly.

4.3.3 Results based on the altered model

4.3.3.1 Illustrative results from a single run

Results in this part are all from a single run which was chosen from several single runs for its typical results. These results give an understanding of the patterns of this model. They follow the format in the original paper. However, these results are illustrative only and we need to be careful when drawing conclusions from just one run. Some results from 100 runs are given in Section 4.3.3.2.

4.3.3.1.1 The network

The three graphs below show information about the guru for one particular run based on the same random seed when *w* is 0.1, 0.5 and 1. The black line is the current guru index using the guru's agent number divided by the total number of agents (150). The red line is the percentage of incoming links for the guru which are the total number of imitators divided by the total number of agents (150). The green line is the fitness for the guru which is the wealth of the guru divided by the richest agent's wealth. At the start, there may be more than one agent who has the most incoming links. The guru is the agent with the smaller agent number for limiting the current guru to obtain 1 for each period.

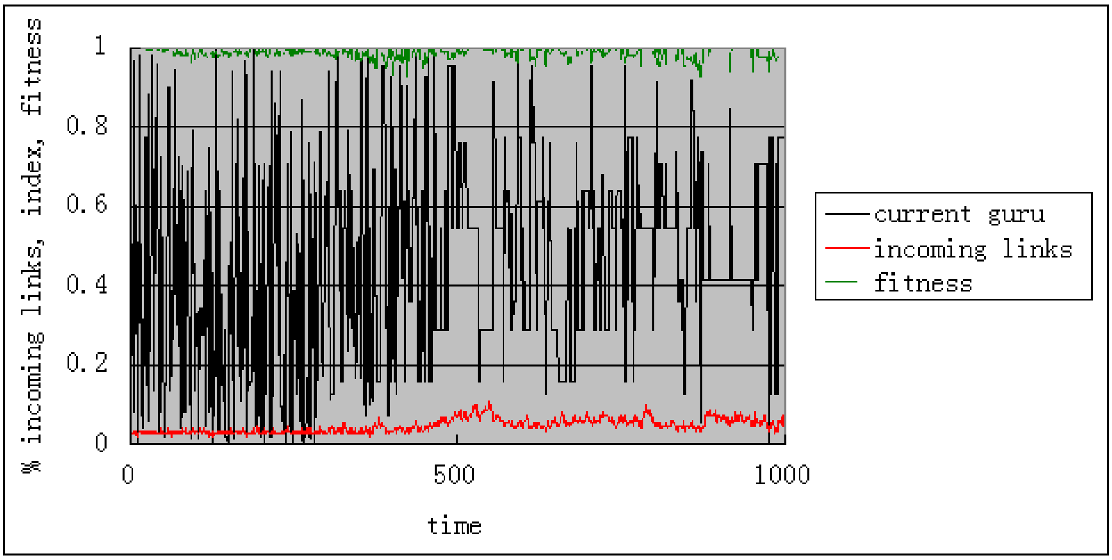


Figure 4. 15 Guru, incoming links and fitness when *w*=0.1

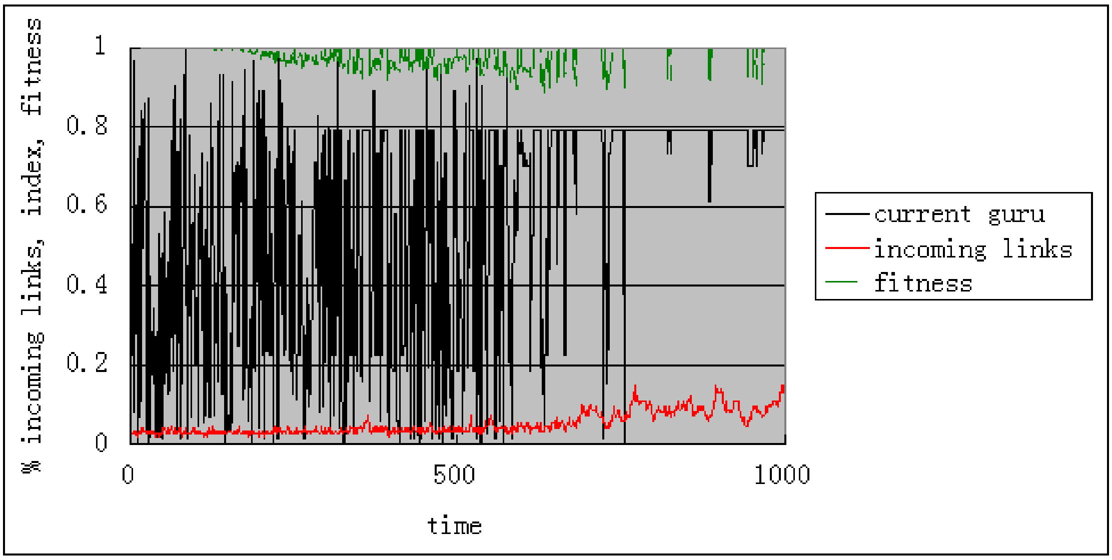


Figure 4. 16 Guru, incoming links and fitness when *w*=0.5

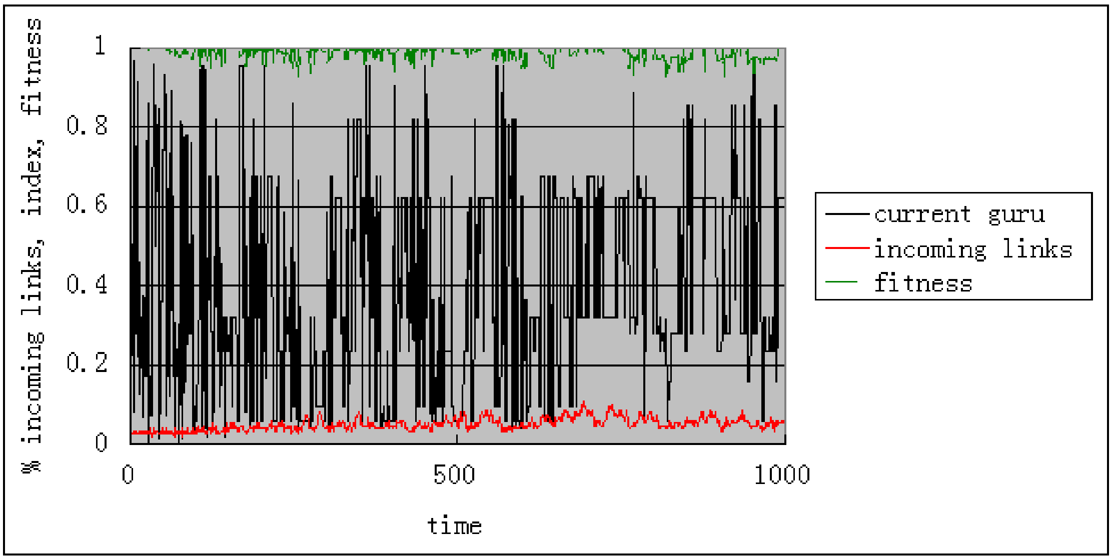


Figure 4. 17 Guru, incoming links and fitness when *w*=1

From the above Figures 4.15, 4.16, 4.17, the fitness for the guru in each situation is more or less the same: they are all close to 1. This shows the wealth of the guru is larger than that of the majority of agents. The guru performance is better than other agents in this market. Comparing the three graphs for incoming links, there are no obvious differences between them. The incoming links for each figure are all smaller than 0.2 with an average below 0.1. As for the black lines, an unchanged horizontal segment indicates the guru lasting longer than one period. Unfortunately, these three graphs do not show any big variations for the guru's life among the different herding factors. This is not the situation in the original paper as seen in Figure 4.9: when *w* gets smaller, the red line for incoming links gets larger and the black line for the current guru has fewer fluctuations indicating that the guru's life is increasing when *w* is decreasing.

4.3.3.1.2 The wealth

The three graphs below show the information about the wealth for the guru, guru imitators and noises based on one particular run with the same random seed when *w* is 0.1, 0.5 and 1. The black line is the guru's wealth at each period (there is only one guru for each period). The red line is the wealth for the imitators who follow the guru's decisions. The green line is the wealth of the noises.

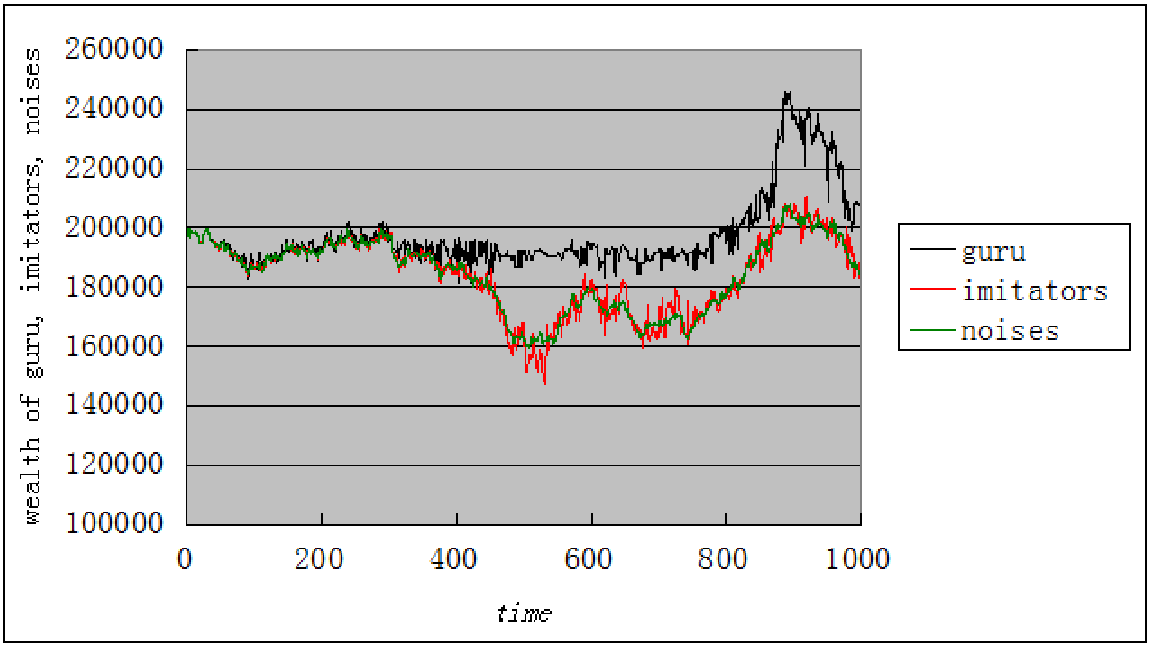


Figure 4. 18 Wealth of the guru, imitators and noises when w=0.1

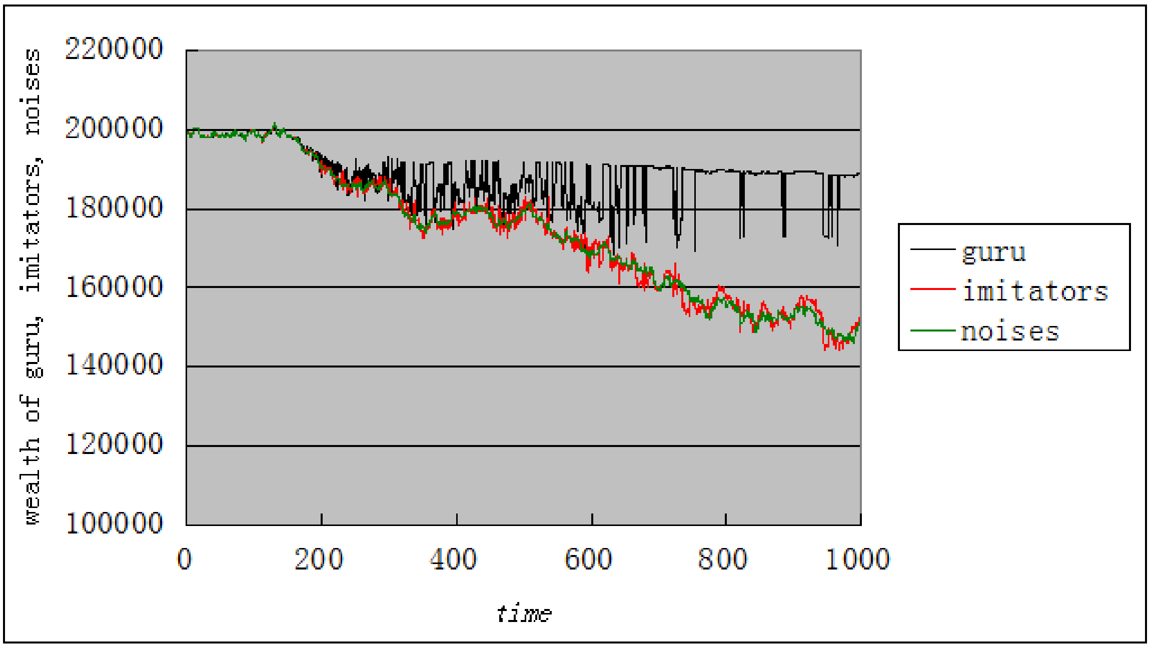


Figure 4. 19 Wealth of the guru, imitators and noises when *w*=0.5

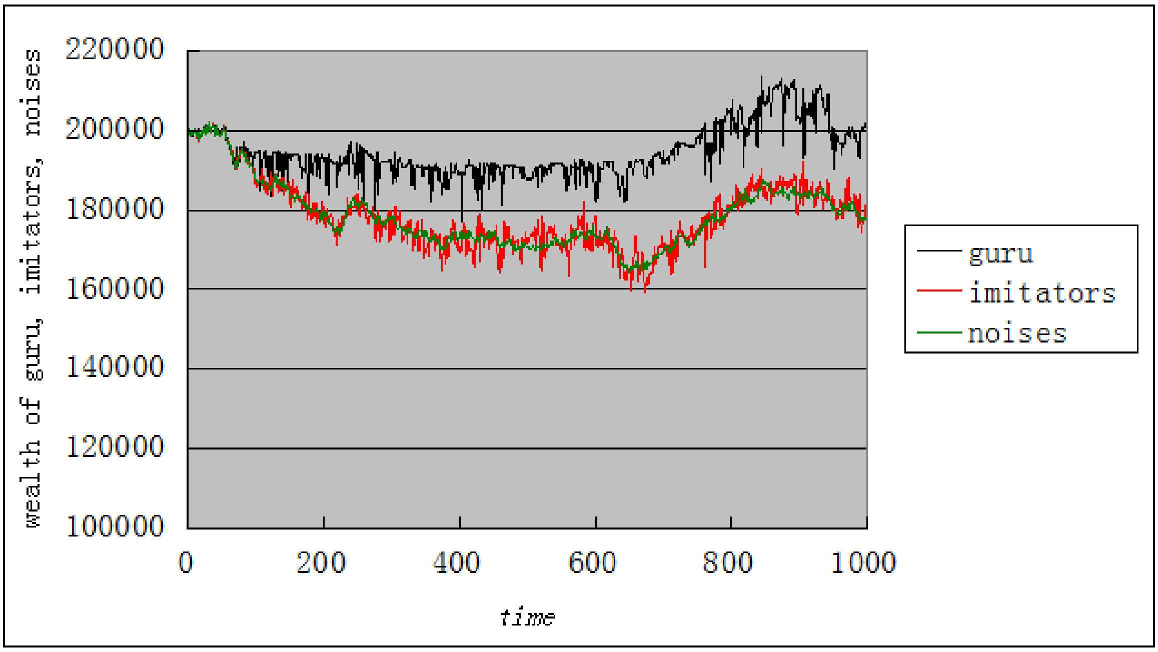


Figure 4. 20 Wealth of the guru, imitators and noises when *w*=1

The above figures, 4.18, 4.19 and 4.20, show that the guru always performed better than the imitators and noises while there is no difference between the wealth of imitators and noises.

In the original paper in Figure 4.21, when *w* gets smaller, the average wealth of the guru, imitators and noises, and the wealth gap between the guru, imitators and noises, become larger and wider which suggests the guru may perform (in wealth) much better than imitators and that the imitators may perform much better than noises. This pattern is confirmed by the 100 run graph for wealth in the original paper.

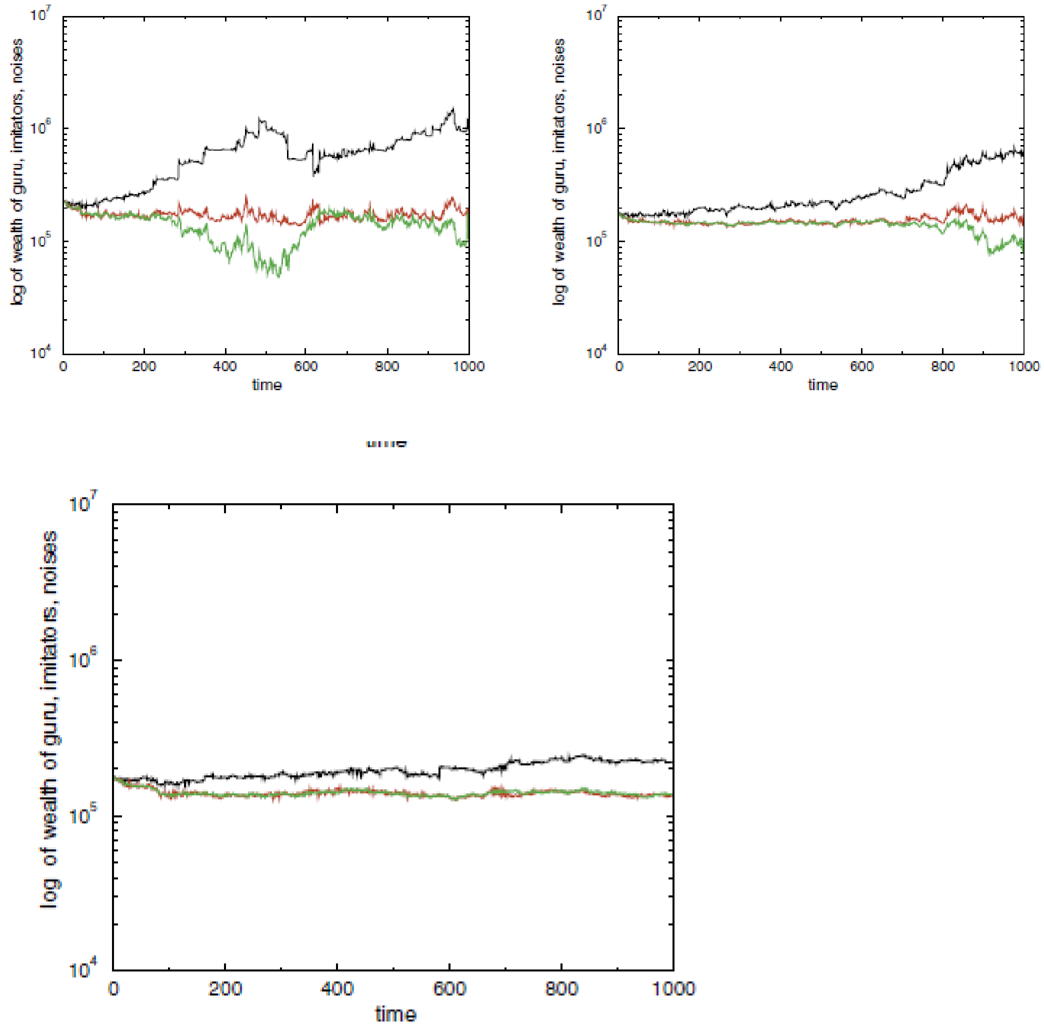


Figure 4. 21 Wealth of the guru, imitators and noises when w=1 in the original model (Tedeschi et al., 2012:90)

4.3.3.1.3 The price

The three graphs below show prices and expected prices for one particular run based on the same random seed when *w* is 0.1, 0.5 and 1. The black line traces the prices based on the time series and the red line is the average expected prices based on the total expected prices for all agents divided by the number of total agents.

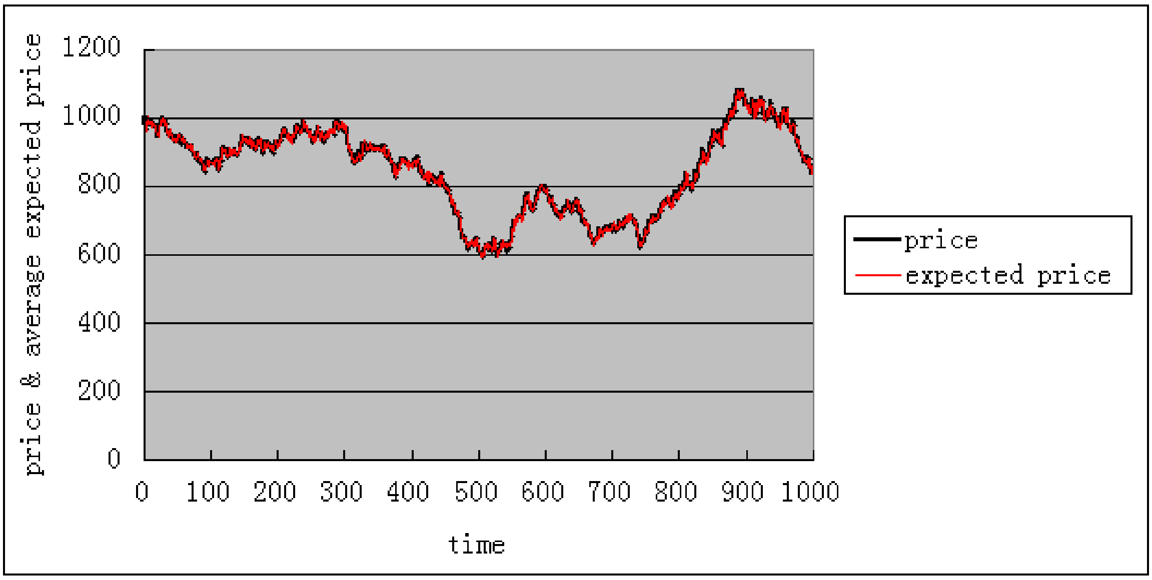


Figure 4. 22 Prices and average expected prices when *w*=0.1

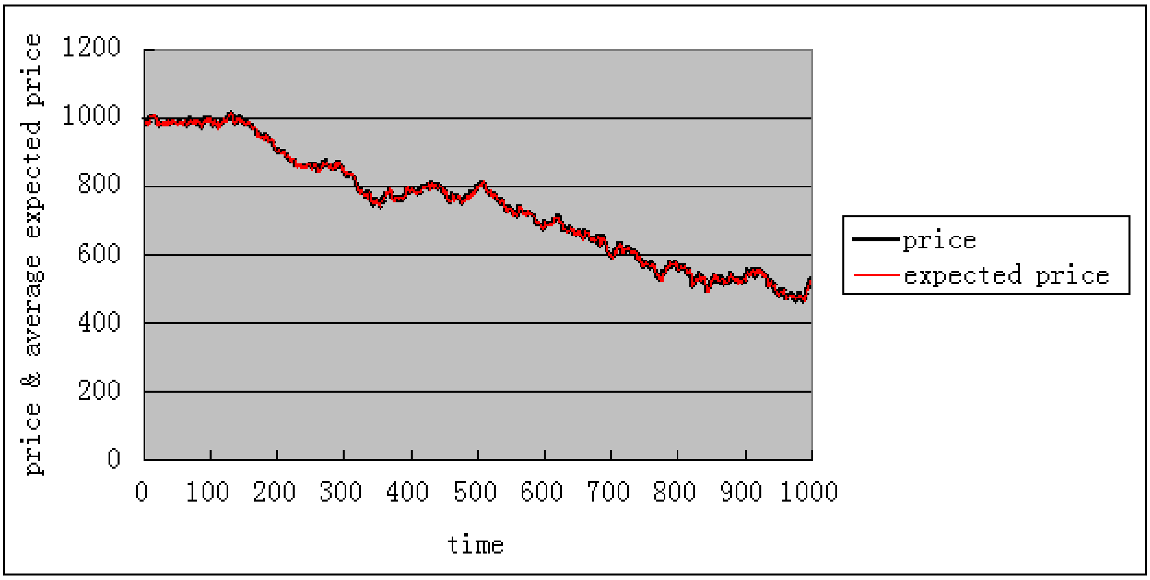


Figure 4. 23 Prices and average expected prices when *w*=0.5

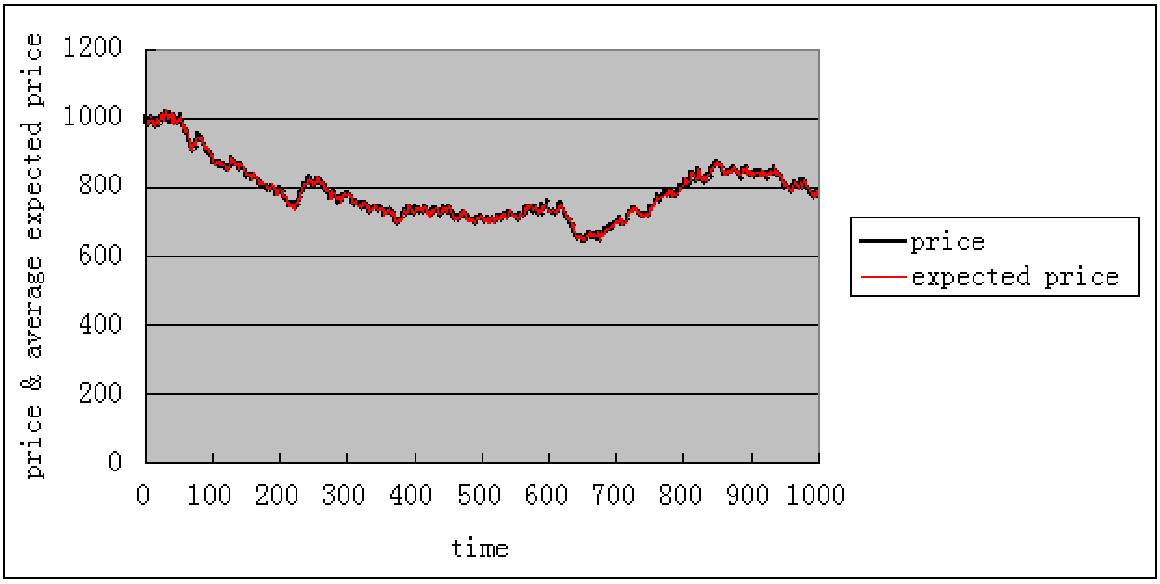


Figure 4. 24 Prices and average expected prices when *w*=1

From the three graphs above, it can be seen that all the prices fluctuate on the time series with the average expected price. Comparing the three graphs, the fluctuations in prices are minimal for *w*=0.5 and *w*=1. The prices for *w*=0.1, however, fluctuate a lot more Figure 4.14 in the original paper shows that the price fluctuations increase as the herding factor gets smaller.

4.3.3.2 Results from 100 runs

The five graphs below are the results generated from the average values from 100 runs. These results follow the format in the original paper. It is the value for each run divided by 100 for the first four graphs. The fifth graph is the mean and standard deviation of returns from 100*1000 data. For each graph, the left side line is the information for herding factor *w*=0.1, the middle line is the information for herding factor *w*=0.5 and the right side line for herding factor *w*=1. The middle point is the average value and the two bars represent ±1 standard deviation.

In Figure 4.25, the wealth of the guru, imitators and noises match the information in Figure 4.18, 4.19 and 4.20 in that the guru's wealth is bigger than those of the imitators and noises which are similar. There is also not much difference in the results when w=0.1 and w=1.

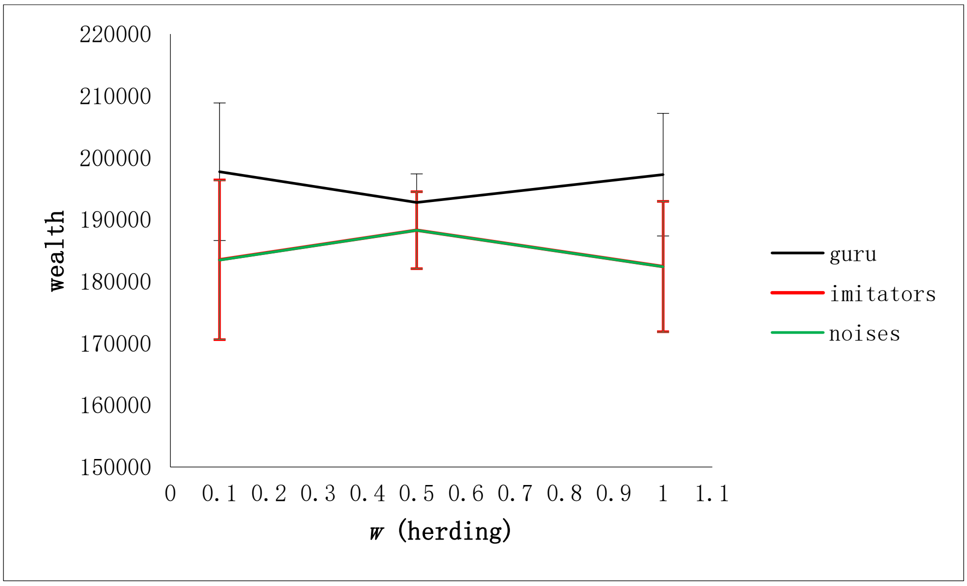


Figure 4. 25 Average wealth of guru, imitators and noises when w=0.1, w=0.5, w=1

Figure 4.26 from the original paper shows that the guru and imitators gain higher profits than noises because of the herding effect.

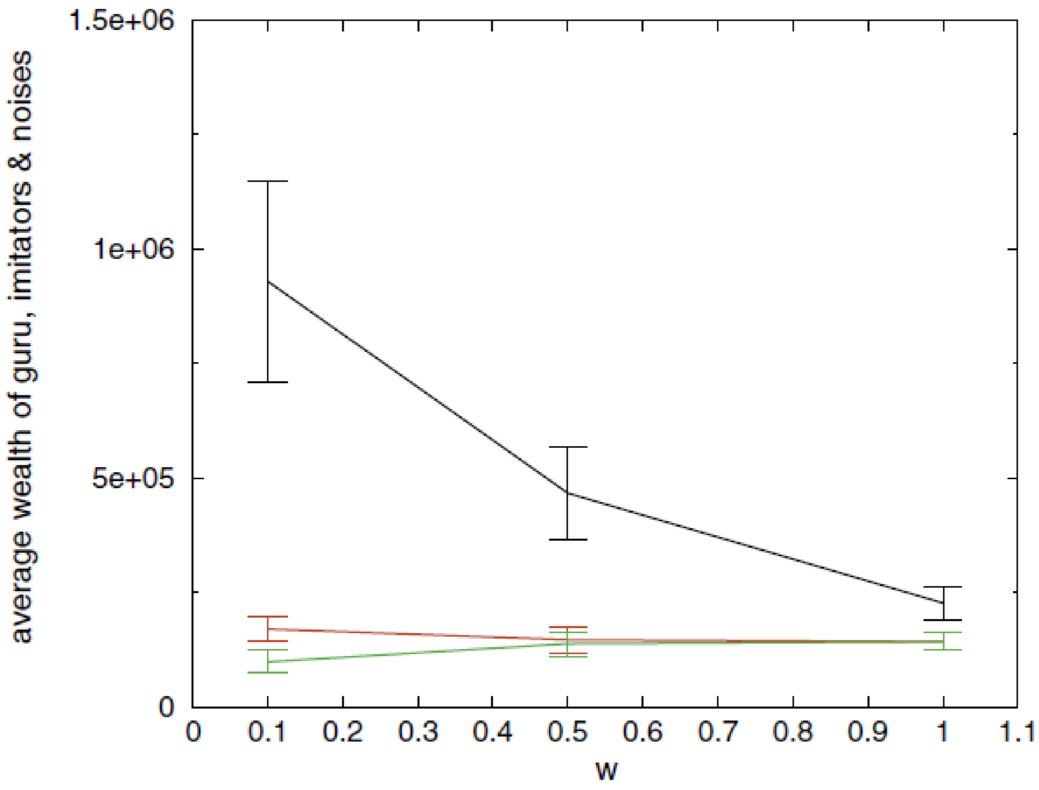


Figure 4. 26 Average wealth of guru, imitators and noises when *w*=0.1, *w*=0.5, *w*=1 in the original model (Tedeschi et al., 2012:90)

Comparing Figures 4.25 and 4.26, the herding effect is not clearly generated as the gaps between the guru and imitators and also imitators and noises are expected to increase when *w* gets smaller (as in the original paper).

In Figure 4.27, the guru's average life is the total period of 1000 divided by the number of gurus. When *w*=0.5 the average guru life is shortest while *w*=1 has the longest average guru life.

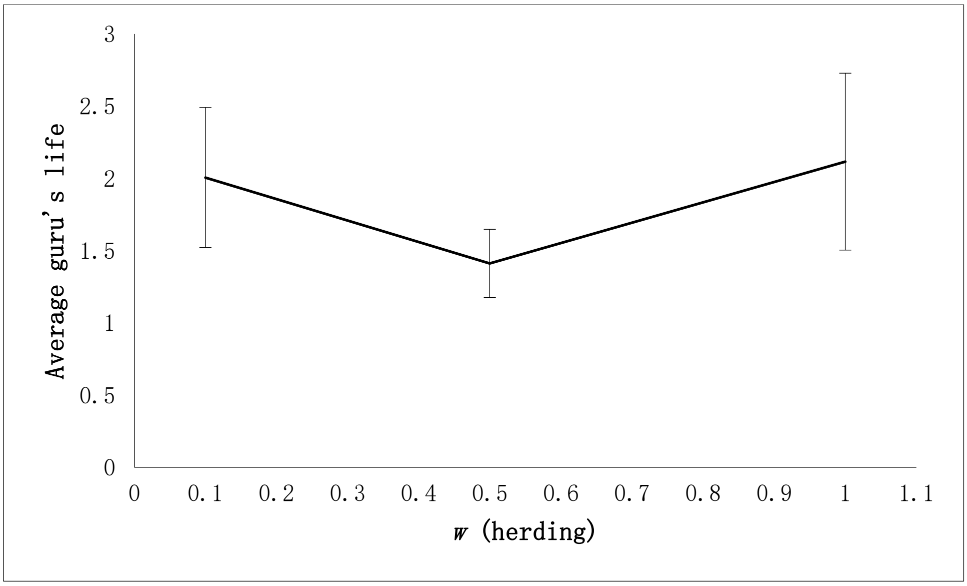


Figure 4. 27 Guru's average life when *w*=0.1, *w*=0.5, *w*=1

From the results in the original paper in Figure 4.28, the average guru life increases when *w* gets smaller. Herding generates more social imitation which helps the guru to be successful and so to last longer. Comparing the results of Figure 4.27 and Figure 4.28, the guru's average life is very different from 100 (original) to only 2 (replication).

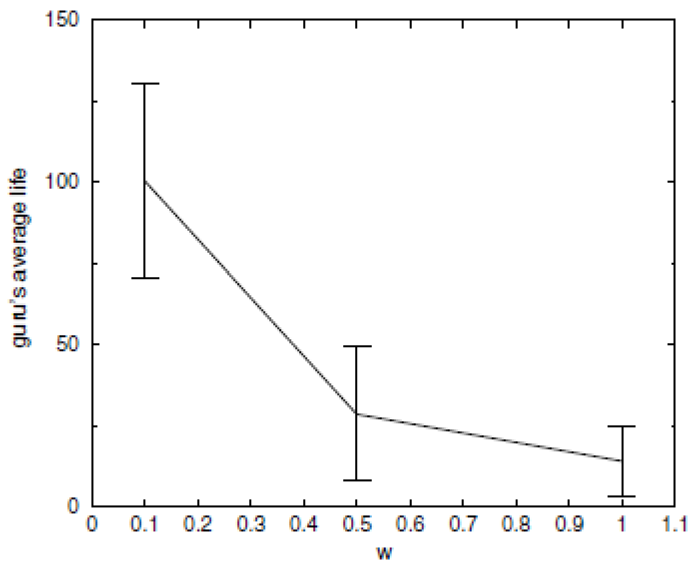


Figure 4. 28 Guru's average life when *w*=0.1, *w*=0.5, *w*=1 in the original model (Tedeschi et al., 2012:89)

The average percentage of marketable sell orders are similar to marketable buy orders. They are the volume of marketable sell orders divided by the total volume of orders. In Figure 4.29, the marketable orders are only a small percentage of the total volume of orders which means that many orders are limit orders. And when *w* gets smaller, the marketable sell orders first decrease and then increase.

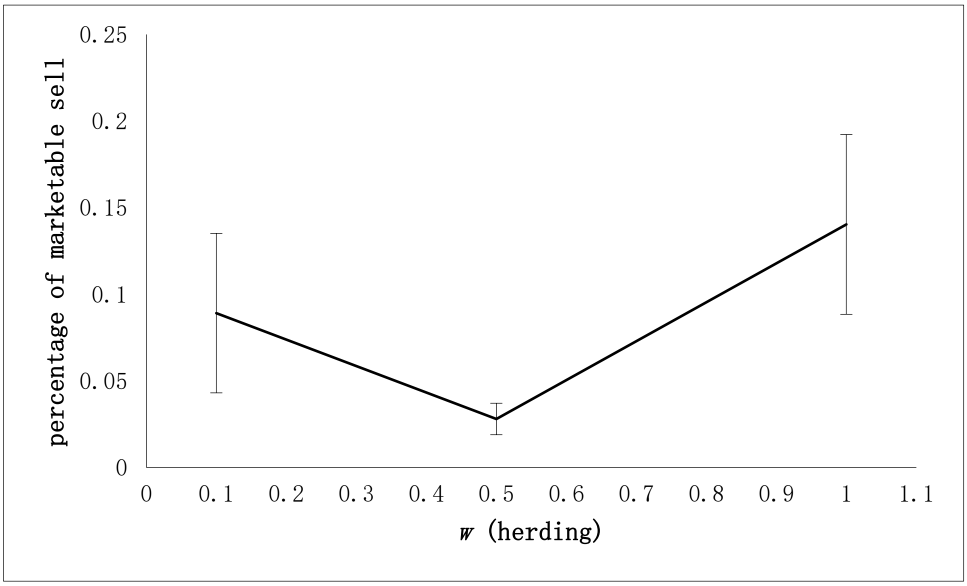


Figure 4. 29 Average percentage of marketable sell orders when *w*=0.1, *w*=0.5, *w*=1

In the original paper in Figure 4.30, herding generates more marketable orders.

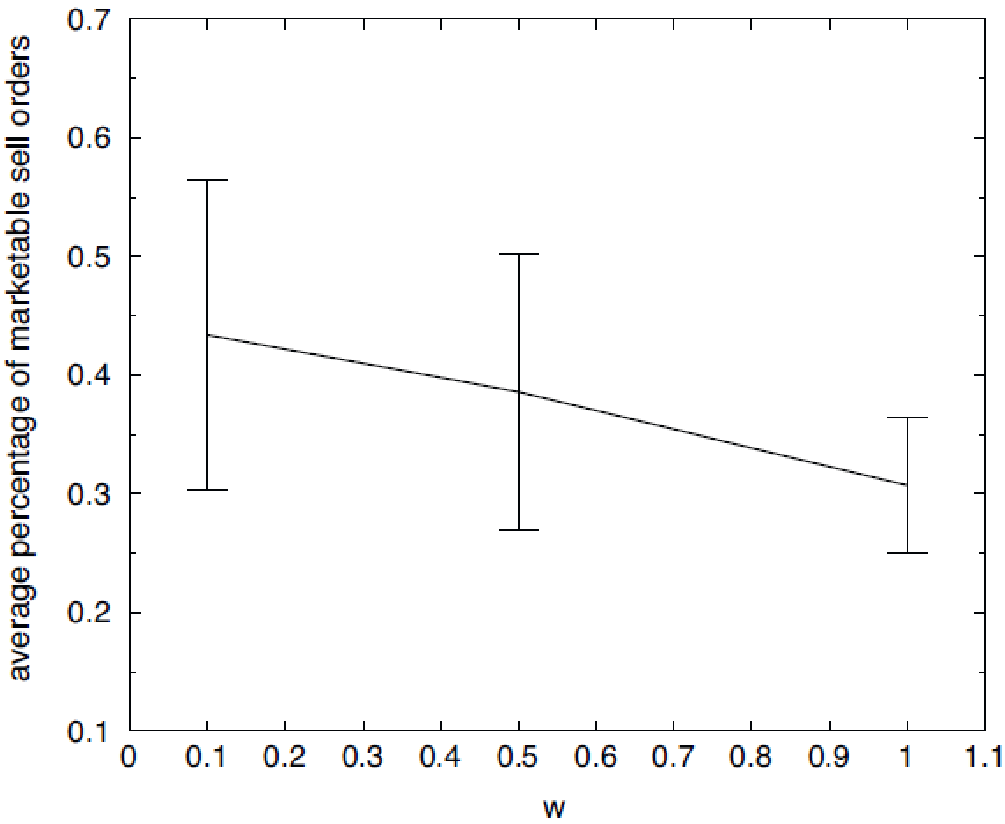


Figure 4. 30 Average percentage of marketable sell orders when *w*=0.1, *w*=0.5, *w*=1 in the original model (Tedeschi et al., 2012:92)

The herding coefficient is the number of agents to buy divided by the total number of agents. In Figure 4.31, although the standard deviation increases when *w* gets smaller the average value is almost the same. This indicates that although there is a herding factor in the model, it did not cause the different clustering for buyers and sellers but just influenced the volatility of the number of buyers and sellers.

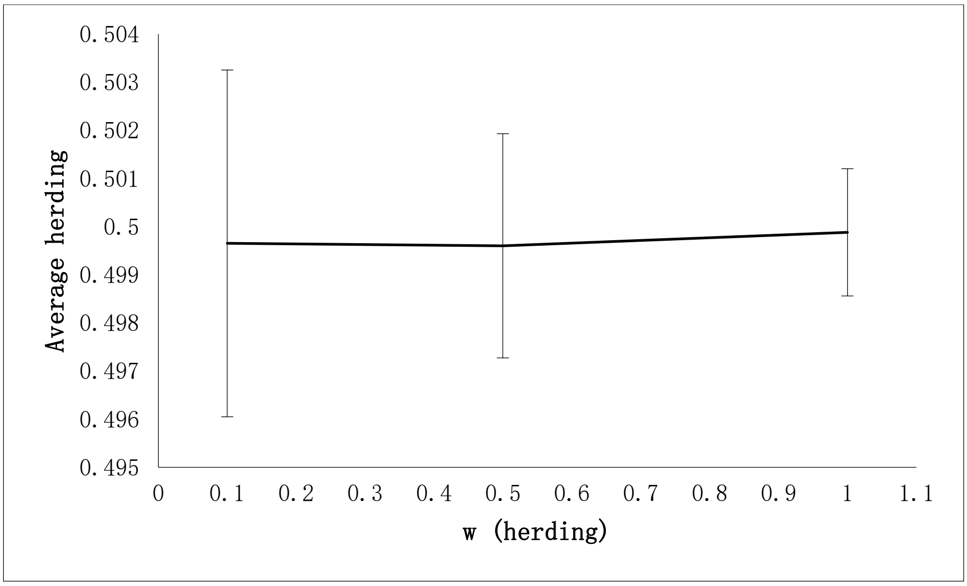


Figure 4. 31 Average herding coefficient when w=0.1, w=0.5, w=1

In the original paper in Figure 4.32, the herding coefficients of w= 0.5 and 1 are similar, but bigger when w=0.1.

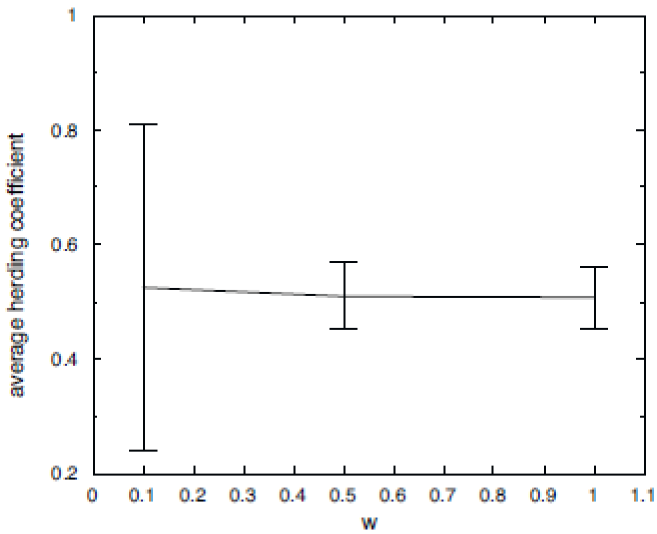


Figure 4. 32 Average herding coefficient when w=0.1, w=0.5, w=1 in the original model (Tedeschi et al., 2012:92)

The returns are the current closing price minus the closing price one period before divided by the closing price. It is a percentage change for one period of price change. In Figure 4.33, the mean of returns does not change when herding changes and is nearly 0. The volatility of returns increases with more herding indicating that herding may be one of the factors for price fluctuations.

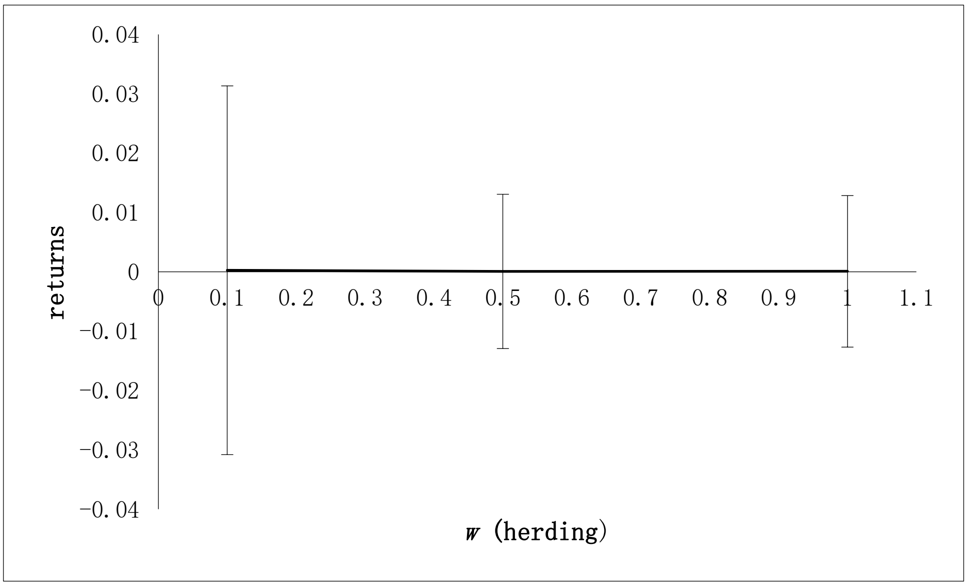


Figure 4. 33 Mean and standard deviation of returns when w=0.1, w=0.5, w=1

In the original paper in 4.34, the pattern is similar to Figure 4.33.

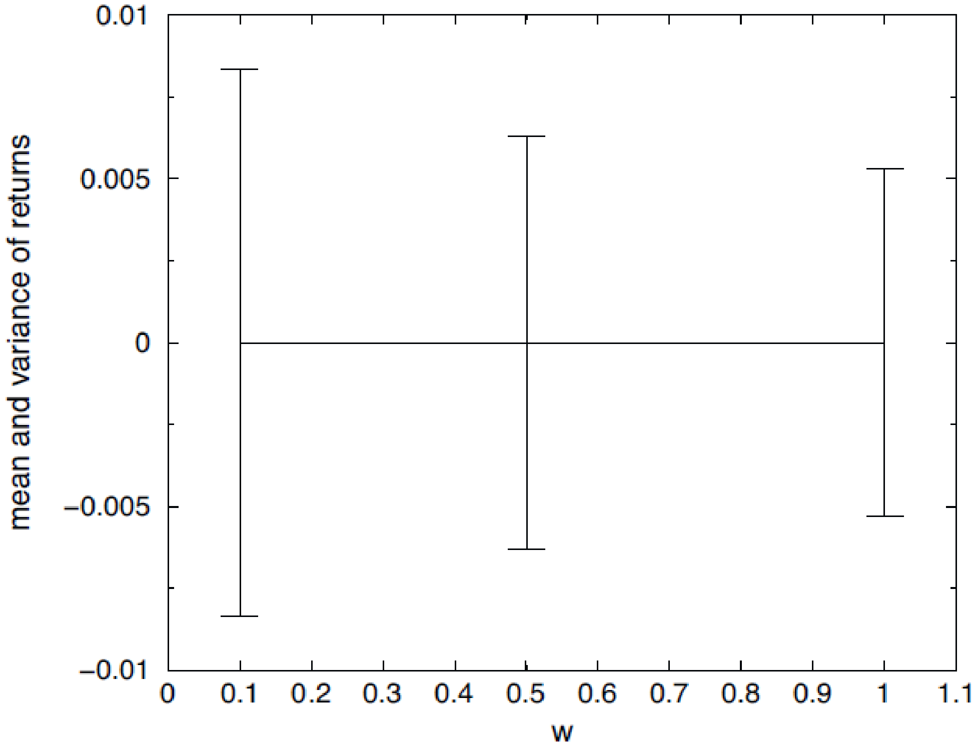


Figure 4. 34 Mean and standard deviation of returns when *w*=0.1, *w*=0.5, *w*=1 in the original model (Tedeschi et al., 2012:93)

As shown in all the graphs above, there is no clear herding effect in this market. Although the standard deviation of the herding coefficient and returns gets larger when *w* gets smaller, it did not increase the stability of gurus (the guru's average life), the gap among the wealth of guru, imitators and noises, and the marketable orders.

All in all, the first replication model is not a successful one. The results are different from the original paper even after the model was modified. There are only partially similar results with the original model. Comparing the results from the replication and altered model, the price dropping issue is stopped. The main differences from the results are the guru's life and herding coefficient. From the replication model, the guru lasts a long period. While, from the altered model, the guru lasts a short time, and it changes a lot during the time period. For the herding coefficient (total buyers / total traders), the average is greater than 0.85 in the replication model, which means there are a lot more buyers than sellers. But, the average herding coefficient is almost 0.5 in the altered model. This may indicate that the price dropping is caused by the bias in the market of an inequal number of buyers and sellers. A second replicating model will be analysed and discussed in Chapter 5.

Chapter 5

Second replicating model

This chapter describes the work on the second replicating model. The model is a financial markets model developed by Lux and Marchesi (2000) where there are different types of trading behaviour and the agents switch between the different types. This is a well known a highly cited paper, with related works including a letter in Nature (Lux and Marchesi, 1999).

Three main aspects have been investigated with this model. One is to try and reproduce the model and the results from the original paper. As discussed in Chapter 3, model replication is very important in science and in modelling to give confidence in the results from scientific studies.

The second aspect is to apply the model description guidelines set out in Chapter 3. The guidelines are used in this chapter to describe the model. This shows how the guidelines work for a real modelling example. The guidelines are used as a checklist which is the recommended method of the STRESS guidelines. The experience of applying the guidelines will be discussed and the guidelines evaluated in Chapter 6.

The third aspect is to extend the understanding of the model. This is done by some detailed analysis of the model behaviour and additional experiments. The aim is to get a better understanding of the reasons for the patterns of behaviour in the model.

Section 5.1 gives an overview of the model. The design concepts are in Section 5.2, and the model details in Section 5.3. Section 5.4 gives the results obtained from the model and compares these with the results from the original paper. Section 5.5 explains the additional work and analysis done on the model. Conclusions are given in Section 5.6.

5.1 Model description overview

5.1.1 Purpose

The overall purpose of the modelling project is to get a better understanding of how stylized facts in financial markets could be produced by particular types of behaviour of the traders. The project takes the approach of replicating a model from a journal paper, which is Lux and Marchesi's (2000) paper entitled: Volatility clustering in financial markets: a microsimulation of interacting agents.

From the literature discussed in Section 2.3.3, there are five herding mechanisms in the agent-based literature. They are transition probability, networks, clusters, genetic algorithm, and independent rules. In my first model described in Chapter 4 of the thesis, the herding mechanism is networks and fat tails are produced, but there is no volatility clustering. Then this second paper is selected to be one with both fat tails and volatility clustering with a different herding mechanism – transition probability. The transition probability used in the model is inspired by Kirman's ants' theory – when ants are faced with two identical foods the groups are not equally divided. This is a similar situation to that when people are faced with two similar restaurants A and B, where people will generally prefer to choose restaurant A instead of restaurant B if restaurant A has more customers. Hence, there is a feedback mechanism that leads to one restaurant being more popular than the other. The detailed herding mechanism will be illustrated in Sections 5.3.3.1 and 5.3.3.2.

Although this paper was written about 20 years ago, it is still a suitable paper with a detailed description of the modelling part and results with an analysis of the main stylised facts. Also, it is a popular paper that is cited more than 800 times in Google Scholar statistics.

After Lux and Marchesi's paper (2000), other models were developed based on similar mechanisms. For example, Alfarano et al. (2005) developed another model with asymmetric transition probabilities. Alfarano and Milaković (2009) investigated alternative network structures that decide the agents' connection in four different ways: regular networks, random networks, small-world networks, and scale-free networks. Also, it considered individual heterogeneity. Other papers tend to concentrate on these models' parameter settings compared with the actual financial markets. From the above discussion, the Lux and Marchesi (2000) model is rarely improved by the authors since the related papers are mostly focused on the parameter settings or changes in herding mechanism settings.

In the model of Lux and Marchesi (2000), there are three types of traders: fundamentalists, optimistic chartists, and pessimistic chartists. Fundamentalists are traders who think the price will go back to its fundamental value while chartists are traders who chase the price trend. Optimistic chartists are chartists who think the price will go up and pessimistic chartists are those who think the price will go down. Agents switch between the different types of traders.

The market mechanism is based on excess demand. Excess demand is decided by the agents' opinion and the number of agents of each type. For example, if the agent is a fundamentalist and the price is lower than the fundamental value, the agent believes the price will go back to its fundamental value. Therefore, the agent thinks the price will increase. The detailed marketing mechanism will be explained later in Section 5.3.3.

My aim of replicating this model is to find out how interactions among these agents produce stylised facts like fat tails and volatility clustering.

5.1.2 Variables and scales

All the parameters used in the model are discussed in this section. Three tables (Table 5.1, 5.2, and 5.3) are produced to draw together the constants and their notation, the alternative parameter sets of values for the constants, and dynamic variables that change during the simulation run.

Symbol	Meaning
N	total number of agents
α_1	importance of individuals place on the majority opinion
α_2	importance of actual price trend
v_1	frequency of revaluation of opinion
α_3	pressure exerted by profit
r	nominal dividends of the asset
v_2	frequency of transition
R	average real returns from other investments
s	discount factor
p_f	fundamental value
t_c	number of units of all chartists either buy or sell
γ	reaction strength
β	speed of the auctioneer
μ	small noise term
Δt	time interval

Table 5. 1 Constant variables

Symbol	Parameter set 1	Parameter set 2	Parameter set 3	Parameter set 4
N	500	500	500	500
α_1	0.6	0. 9	0. 75	0. 8
α_2	0.2	0. 25	0. 25	0. 2
v_1	3	4	0. 5	2
α_3	0.5	1	0. 75	1
r	0.004	0.004	0.004	0.004
v_2	2	1	0. 5	0. 6
R	0.0004	0.0004	0.0004	0.0004
s	0.75	0. 75	0. 8	0. 75
p_f	10	10	10	10
t_c	0.02	0. 015	0. 02	0. 01
γ	0.01	0. 01	0. 02	0. 01
β	6	4	2	4
μ	N ~ (0, sd 0.05)	N ~ (0, sd 0.1)	N ~ (0, sd 0.1)	N ~ (0, sd 0.05)
Δt	0.01 or 0.002	0.01 or 0.003	0.01 or 0.004	0.01 or 0.005

Table 5. 2 Parameter sets of values for the constants

Symbol	Meaning
n_c	number of chartists
n_f	number of fundamentalists
n_+	number of optimistic chartists
n_-	number of pessimistic chartists
x	opinion index
z	fraction of chartists
U_1	decision effect factor
$\dot{p}$	price trend
$\pi_{-\to+}$	probability of changing from pessimistic to optimistic chartists
$\pi_{+\to-}$	probability of changing from optimistic to pessimistic chartists
$U_{2,1}$	decision effect factor
$\pi_{f\to+}$	probability of changing from fundamentalists to optimistic chartists
$\pi_{+\to f}$	probability of changing from optimistic chartists to fundamentalists
$U_{2,2}$	decision effect factor
$\pi_{f\to-}$	probability of changing from fundamentalists to pessimistic chartists
$\pi_{-\to f}$	probability of changing from pessimistic chartists to fundamentalists
ED_c	excess demand of chartists
ED_f	excess demand of fundamentalists
ED	total excess demand
$\pi_{\uparrow p}$	probability of price going up
$\pi_{\downarrow p}$	probability of price going down

Table 5. 3 Other variables

In appendix B, the coding and the variables in the code are explained.

5.1.3 Process overview and scheduling

In this model, there are 500 agents. Each agent has three states representing the type of trader: fundamentalists (f) or optimistic chartists (+) or pessimistic chartists (-).

A flowchart of the process that happens in each time increment is shown in Figure 5.1.

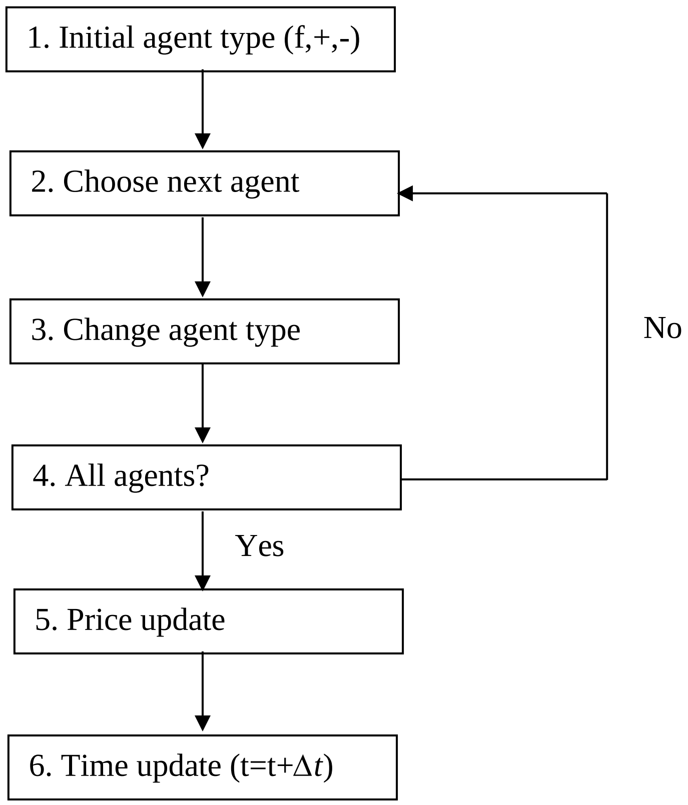


Figure 5. 1 Flowchart of the model process

Further details of some of the steps in the flow chart are:

Step 1. Initial agent type: agents are divided randomly. To maintain the number of fundamentalists at the start, the probabilities of agents to be optimistic chartists or pessimistic chartists are 10% each at the start. As a result, the probability of agents to be fundamentalists is 80% at the start.

Step 2. Choose next agent: the order of choosing agent does not make any differences. The price only updated when all agents are changed.

Step 3. Whether to change agent type: each agent only switches once in each interval time Δt. The agents switch from one to another according to the switching rules given later in Section 5.3.3.1 and 5.3.3.2:

For fundamentalists – they will switch to either optimistic chartists or pessimistic chartists according to the equation (5.9) and (5.12) respectively in Section 5.3.3.2.

For optimistic chartists – they will switch to either pessimistic chartists or fundamentalists according to the equation (5.7) in Section 5.3.3.1 and (5.10) in Section 5.3.3.2 respectively.

For pessimistic chartists – they will switch to either optimistic chartists or fundamentalists according to the equation (5.6) in Section 5.3.3.1 and (5.13) in Section 5.3.3.2 respectively.

Step 5. Price update: the price updates by increasing or decreasing by 0.01 or staying unchanged according to equations (5.17) and (5.18) in Section 5.3.3.3.

Step 6. Time update: the time updates by a time increment. The time increment Δt can either be 0.01 time units or 0.002 time units. This means that in 1 time unit there will either be 100 or 500 time increments. The main results are just recorded at the end of each whole time unit. The paper indicates that the time increment value depends on whether the change in price is greater than the average price change but the exact criteria for changing the time increment is not entirely clear from the paper. This is investigated further in Section 5.5. In my model, if the price changes so frequently that it exceeds 0.405 per time unit, then the time increment changes from 0.01 to 0.002. The value for the average of price fluctuation of 0.405 is calculated from two models built-in *c* and *witness*. It is calculated by the average price change in each model from time unit 0-20000 when the increment is 0.01 with parameter set 1. As we think the switching of chartists happens instantly, the components of the switching formula (for the number of each type of agent and the price) will not update during the time increment Δt but only after each time unit.

5.2 Design Concepts

Environment: agents interact with themselves.

Agents: data access - according to the equations 5.6, 5.7, 5.9, 5.10 explained below, each agent knows the price trend, the price now, the total number of optimistic chartists, pessimistic chartist, fundamentalists, and the total number of agents;

objective - maximise profit;

rules - equations 5.6, 5.7, 5.9, 5.10. If there are more optimistic chartists and a positive price trend, agents prefer to switch from pessimistic to optimistic chartists. If optimistic chartists' excess profit is more than that of the fundamentalists, agents prefer to switch from fundamentalists to optimistic chartists, and vice versa. If pessimistic chartist's excess profit is more than that of the fundamentalist, agents prefer to switch from fundamentalists to pessimistic chartists, and vice versa;

working in group and prediction - not applicable as agents work alone – i.e., they do not collaborate directly with their trading.

Interaction: Three agent groups are agent states. Agent states can be optimistic chartists, pessimistic chartists and fundamentalists. The state changes at each small time increment. There are two switching strategies among these agents: the switching between optimistic chartists and pessimistic chartists, and the switching between fundamentalists and chartists.

Enter/exit: no new agent will enter the market and no agent will leave the market– i.e., the market always has the original 500 agents.

Emergence: the price changes when the number of optimistic chartists, pessimistic chartists, and fundamentalists is changed. The price also has an effect on whether agents switch and so there are complex interactions between the number of each type of agent and the price.

Assumption: It is assumed that people follow Kirman's ant's theory. For the price changes, it is assumed that the auctioneer will adjust the price to the next higher or lower possible value within the next small time increment based on the imbalance between demand and supply. The auctioneer will adjust the price by increasing or decreasing the market price by a fixed amount of 0.01.

5.3 Model Details

5.3.1 Initialisation

At the initialisation stage in Figure 5.1, agents are randomly separated into different types with one requirement: the number of fundamentalists is maintained to be reasonably high for stabilisation purposes. This requirement is vague in the original paper. I set the probabilities of agents to be optimistic chartists, pessimistic chartists or fundamentalists as 10%, 10%, and 80% respectively. The total number of agents is fixed at 500 shown in Table 5.1. At the start, the price is equal to the fundamental price of 10.

5.3.2 Input

Returns in financial markets are usually defined with log returns. That means when the time scale is 1, the return is $\mathrm{ret_t} = \ln(p_t) - \ln(p_{t-1})$. There are two main reasons to use the log return instead of the raw return which is $\mathrm{ret_t} = (p_t - p_{t-1})/p_{t-1}$. First, log returns and raw returns have similar results. Second, log returns have time additivity which makes sure that the return from time 0 to time *t* is $ln(p_t) - ln(p_0)$.

The input file for code access provided in Appendix B is called parameter in a *txt* form and the random number generator used in the model is the one built-in to *c*.

5.3.3 Submodels

Agents in the model try to maximise their gains. The number of chartists is n_c, and the number of fundamentalists is n_f. The total number of agents is equal to the total number of chartists plus the total number of fundamentalists which gives equation (5.1):

(5.1) $n_c + n_f = N$

The number of chartists is divided into the number of optimistic chartists n_+ and the number of pessimistic chartists n_-. From the meaning of these symbols, we can obtain equation (5.2):

(5.2) $n_+ + n_- = n_c$

The changes between fundamentalists, optimistic chartists, and pessimistic chartists are based on the herding mechanism. The herding mechanism in this model is based on the switching between these agents through the transition probabilities. Also, inspired by Kirman's ants' theory, the transition probabilities are mainly influenced by the number of agents. A detailed explanation will be given in Sections 5.3.3.1 and 5.3.3.2.

The reference numbers for the equations for switching between the agent types are as follows:

$n_f \to n_+$ Eq. (5.9)
$n_f \to n_-$ Eq. (5.12)

$n_+ \to n_-$ Eq. (5.7)
$n_+ \to n_f$ Eq. (5.10)

$n_- \to n_+$ Eq. (5.6)
$n_- \to n_f$ Eq. (5.13)

5.3.3.1 Switching between chartists

There are two components of switching between chartists: the decision making influenced by the individuals' opinions called "flows" inspired by Kirman, and also a supplementary component influenced by market "charts". The first one is influenced by the majority decision of agents and the second one is influenced by the price trend. Hence, the equations deciding the probability of changing between chartists are based on these two components. And in the process of switching, if the number of fundamentalists, optimistic chartists, and pessimistic chartists is lower than 4, they will not switch to other types.

The opinion index *x* which captures the individuals' opinion is defined in equation (5.3). This is a simple factor showing the net majority opinion of the chartists which is the difference between the number of optimistic chartists and pessimistic chartists divided by the total number of chartists. If there are more optimistic chartists than pessimistic chartists, the value of *x* is positive; otherwise, the value of *x* is negative.

(5.3) $x = \frac{(n_{+} - n_{-})}{n_c}$

The fraction of chartists out of the total number of agents *z* is:

(5.4) $z = \frac{n_c}{N}$

The price trend $\dot{p}$ is captured by the price changes in continuous time. In this model, the price change during the last increment from *t*-Δt to *t* can only be -0.01, 0, and +0.01. To get more possibilities of price trend, $\dot{p}$, it is calculated by the average price change with a time lag of 0.2. That means when we calculate the price change at time *t*, $\dot{p}$ is the average change in price from *t -0.2* to *t* (i.e., the total change in price divided by 0.2).

The decision effect factor U_1 is used in the equations for the probability of agents changing their type between optimistic and pessimistic chartists. The equations for U_1 is given in equation (5.5). In this equation, α_1 is a factor for the amount of importance which individuals place on the majority opinion, the value of which for parameter set 1 is 0.6; and α_2 is the factor for the amount of importance in the actual price trend which in parameter set 1 is 0.2; v_1 in the model represents the frequency of revaluation of opinions which is 3 in parameter set 1. The original paper has a note that: "α_1 and α_2 need not sum up to 1".

The decision effect factor U_1 is affected by "flows" and "charts" which is discussed before. If x is positive, which means there are more optimistic chartists than pessimist chartists, and $\dot{p}$ is positive and so moves in the same direction as the majority opinion, then U_1 will be greater than zero. Similarly, if x is negative and $\dot{p}$ is negative then U_1 will be less than zero. If $\dot{p}$ moves to the contrary direction with the majority view given by x, the influence of the opinion will be weakened or even reversed.

(5.5) $U_1 = \alpha_1 x + \alpha_2 \frac{\dot{p}}{v_1}$

In equations (5.6) and (5.7), $\pi_{-\to+}$ and $\pi_{+\to-}$ are the probability of changing from pessimistic to optimistic chartists, and the probability of changing from optimistic to pessimistic chartists respectively. From the equations, we can see the transition probabilities in this model are symmetrical. Δt is the time increment in this model. It may change during the running of the model. When prices fluctuate a lot, investors will check and update their strategy more frequently. Therefore, the time increment will be decreased. In this model, the time increment is 0.01 at the start and in the case when prices are relatively stable. The time increment is 0.002 when prices fluctuate a lot. The prices fluctuate a lot when the price changes so frequently that it exceeds the average price change (as discussed in step 6 in Section 5.1.3). In equations (5.5) and (5.6), v_1 is the parameter used to represent the frequency of revaluation of opinions and is one of the input parameters in Table 5.2. Then the probability of changing between optimistic chartists and pessimistic chartists in time increment Δt is:

(5.6) $\pi_{-\to+} = v_1 \left(\frac{n_c}{N}\exp(U_1)\right)\Delta t$

(5.7) $\pi_{+\to-} = v_1 \left(\frac{n_c}{N}\exp(-U_1)\right)\Delta t$

5.3.3.2 Switching between fundamentalists and chartists

The switching probability between fundamentalists and chartists has more complex components compared with the one in Section 5.3.3.1. The optimistic chartists are assumed to buy fixed units. In optimistic chartists' opinion, they think the price will go up. Therefore, they will buy now at a lower price. At the same time, pessimistic chartists are assumed to sell fixed units in the financial market. In pessimistic chartists' opinion, they think the price will go down. Therefore, they will sell now at a higher price.

The probabilities for switches between fundamentalists and optimistic chartists are based on the decision factor $U_{2,1}$ given in equation (5.8). This has the change between fundamentalists and optimistic chartists as the difference between excess profit per unit for optimistic chartists and excess profit per unit for fundamentalists. In equation (5.8), α_3 measures the pressure exerted by the pay-off differential and this is fixed at 0.5 for parameter set 1. r is a nominal dividend of the asset which is 0.004 for all parameter sets. v_2 represents the frequency of this type of transition, which is 2 for parameter set 1. $\dot{p}$ represents the price trend, while p represents the actual price. R is the average real returns from other investments and is set in the model for all parameter sets at 0.0004.

The excess profit for optimistic chartists per unit is $\left(r + \frac{\dot{p}}{v_2}\right)/p - R$. That means chartists avoid losses by comparing the asset portfolio return $\left(r + \frac{\dot{p}}{v_2}\right)/p$ with the return from other investments R. The parameter s means the discount factor which is set at 0.75 for all parameter sets. The profit for chartists is realised immediately whereas the expected gain for fundamentalists will be realised when price goes back to its fundamental value in the future and hence a discount factor is applied. p_f is the fundamental value of the asset which is fixed at 10. The excess profit for fundamentalists is given by: $s\left|\frac{p_f - p}{p}\right|$.

In equation (5.8), agents compare the excess profit between optimistic chartists and fundamentalists times the pressure depending on the price differential and then switch to the more successful one.

(5.8) $U_{2,1} = \alpha_3\left(\left(r + \frac{\dot{p}}{v_2}\right)/p - R - s\left|\frac{p_f - p}{p}\right|\right)$

In equation (5.9) and (5.10), $\pi_{f\to+}$ and $\pi_{+\to f}$ are probabilities of the change from fundamentalists to optimistic chartists and the change from optimistic chartists to fundamentalists in the time interval Δt respectively. v_2 represents the frequency of this type of transition as in equation (8). And the probabilities are:

(5.9) $\pi_{f\to+} = v_2\left(\frac{n_+}{N}\exp\left(U_{2,1}\right)\right)\Delta t$

(5.10) $\pi_{+\to f} = v_2\left(\frac{n_f}{N}\exp\left(-U_{2,1}\right)\right)\Delta t$

The factor $U_{2,2}$ that decides the change between fundamentalists and pessimistic chartists is the difference between excess profit per unit for pessimistic chartists and excess profit per unit for fundamentalists and is specified in equation (5.11). It is a similar equation to equation (5.8). In equation (5.11), the excess profit for pessimistic chartists per unit is $R - \left(r + \frac{\dot{p}}{v_2}\right) / p$. That means that chartists avoid losses by comparing the asset portfolio return $\left(r + \frac{\dot{p}}{v_2}\right) / p$ with the other investment return R. Agents compare the excess profit between pessimistic chartists and fundamentalists times the pressure depending on the price differential and switch to the more successful one. The factor $U_{2,2}$ in equation (5.11) decides the changes between fundamentalists and pessimistic chartists:

(5.11) $U_{2,2} = \alpha_3 \left(R - \left(r + \frac{\dot{p}}{v_2}\right) / p - s \left|\frac{p_f - p}{p}\right|\right)$

In equation (5.12) and (5.13), $\pi_{f \to -}$ is the probability of changing from fundamentalists to pessimistic chartists. It is influenced by the profit factor $U_{2,2}$ and also the number of pessimistic chartists. $\pi_{- \to f}$ is the probability of changing from pessimistic chartists to fundamentalists. Similarly, it is influenced by the profit factor $U_{2,2}$ and also the number of fundamentalists. v_2 represents the frequency of this type of transition as in equations (5.8) - (5.11). The probabilities based on the factor $U_{2,2}$ are:

(5.12) $\pi_{f \to -} = v_2 \left(\frac{n_-}{N} \exp\left(U_{2,2}\right)\right) \Delta t$

(5.13) $\pi_{- \to f} = v_2 \left(\frac{n_f}{N} \exp\left(-U_{2,2}\right)\right) \Delta t$

5.3.3.3 Market mechanism

The market mechanism is based on excess demand. There is an assumption behind it: the auctioneer will adjust the price to the next higher or lower possible value within the next small time increment based on the imbalance between demand and supply. The auctioneer will adjust the price by increasing or decreasing the market price at a fixed amount ± 0.01. In the equation, t_c is the number of units that all chartists either buy or sell which is set at 0.02 for parameter set 1 and γ is the reaction strength set at 0.01 for parameter set 1. The excess demand for chartists ED_c and excess demand for fundamentalists ED_f are:

(5.14) $ED_c = (n_+ - n_-)t_c$

(5.15) $ED_f = n_f\ \gamma\ (\mathrm{p_f} - p)$

The total excess demand is ED:

(5.16) $ED = ED_c + ED_f$

The probability of the price changing is given by equations (5.17) and (5.18). In the equations, β is the speed of the auctioneer fixed at 6 for parameter set 1, and μ is the small noise term following the normal distribution with N ~ (0, σ (standard deviation)) where the standard deviation for parameter set 1 is 0.05. $\pi_{\uparrow p}$ is the probability of price going up and $\pi_{\downarrow p}$ is the probability of price going down.

(5.17) $\pi_{\uparrow p} = \max[0, \beta\ (ED + \mu)]$

(5.18) $\pi_{\downarrow p} = -\min[0, \beta\ (ED + \mu)]$

For the code, It runs in Windows with language c++. The software Code Block 10.05 is used to run the code. The random number generator is the built-in version in c. It runs with fixed time steps. The runtime is about 12 minutes for one long run. It runs on the laptop. The code is accessible on the DOI 10.5281/zenodo.5524993. Or by the email request by x.liu1@lancaster.ac.uk or liuxinbess@126.com.

5.4 Results and Comparisons

This section will analyse results related to the model which are introduced in Section 5.1-5.3. In this model, there is no warm-up period. This follows what was done in the original paper. This is based on designing the initial conditions to have a particular structure and mix of the different type of agents and wanting to see what will happen in the model from those initial conditions.

Following what was done in the original paper, only one long run is used for each parameter set mentioned in Table 5.2 with a run length of 20000 time units. What the time unit in this model represents in real time is not specified in the original paper. This experimentation method is running the model for one long run.

Some of the statistics in the original paper come from calculating values for sub-periods within the run, such as 10 sample values for each 2000 time units for the tail index. This is effectively a batch means approach.

Each unit of time is actually divided up into time increments Δt, in which agents are switched among each other, and then the price is updated. As explained earlier, depending on the simulation conditions Δt is either 0.01 or 0.002 (Step 6 of Section 5.1.3). This means that there are actually either 100 or 500 updates every unit of time. Hence one run of the model has at least 2,000,000 time increments in which the agents can change their type, and the price can change. Hence, this seems reasonable as the long run.

The number of agents in each group and the price are recorded at the end of every unit time interval. That means the result will be recorded in every 100 increments when Δt is 0.01 or every 500 increments when Δt is 0.002.

In the results analysis, returns, the fraction of chartists, unit root, measures of fat tails, and d for volatility clustering are calculated. The returns $\mathrm{ret_t} = \ln(p_t) - \ln(p_{t-1})$ measure the price difference between the price now and the last previous price using the logarithm; in other words, they measure the fluctuation of prices.

The fraction of chartists among traders z is introduced in the equation (5.4) in Section 5.3.3.1. It is the total number of chartists divided by the total number of agents which are 500.

In the original paper, Lux and Marchesi (2000) use unit-roots to test if the data is a random walk or not. The Dickey-Fuller test is used. The 20000 data values are divided into 40 subsamples. Hence, in each subsample, there are 500 values. The null hypothesis and alternative hypothesis for the one-sided test are:

H0: $\rho = 1$ and *H1:* $\rho < 1$ in the regression： $pt = \rho pt\text{-}1 + \epsilon$

If the data rejects the hypothesis, the prices are not dynamic to conceal systematic motion. If the hypothesis is rejected by $\rho > 1$, the unit root is rejected by explosive roots of the dynamics. That means extreme instability occurred temporarily and it could be bubbles. On the other hand, if the hypothesis is rejected by $\rho < 1$, the unit root is rejected by mean-reverting dynamics.

In this model, there are two methods to measure fat tails. The same as the measure of tails in the first replication model, they use kurtosis to represent fat tails. There is another method to measure fat tails as well to get a median of a tail index from samples. 20000 data are divided into ten subsamples. Hence, in each subsample, there are 2000 values. To calculate the tail index from 10 samples, the 2000 values in each sample need to be ordered. The tail index is calculated by using the equation (5.19) where *H* is the percentage of the tail, *k* is the number of observations locating in the tail distribution:

$$\alpha_H = \frac{1}{\frac{1}{k}\sum_{i=1}^{k}[\ln(x_{(n-l+1)}) - \ln(x_{(n-k)})]}$$

(5.19)

The equation 5.19 is copied from the paper, there is a typo in it: *n–l+1* should be *n-i+1*. For kurtosis, the bigger the value indicates more fat tails. For a tail index, the smaller the value indicates more fat tails. From empirical results, the tail index usually ranges from 2 to 5.

To test volatility clustering the 20000 data values are divided into ten subsamples. Hence, in each subsample, there are 2000 values. From 2000 entries in one sample, the autocorrelation function *ρ(k)* is calculated. Then *d* is calculated while *ρ(k) ~ k^(2d-1)*. A value of *d* which exceeds 0 indicates that there is volatility clustering. And if it is greater than 0.5, it indicates a non-stationary aspect of the volatility process. From the empirical data, *d* usually ranges from 0.22 to 0.6 for absolute returns.

The replication results comparing with the original paper will be illustrated. Throughout the comparison, the reproducibility of the original paper will be illustrated.

In Figure 5.2, the returns from time 1000 to 4000 is presented with the parameter set 1.

This follows the original paper which showed charts for this time period. It was not clear how typical this data was in the original paper. In my results, I produced charts for the whole time period and from a visual inspection this data does appear to be similar to the rest of the data.

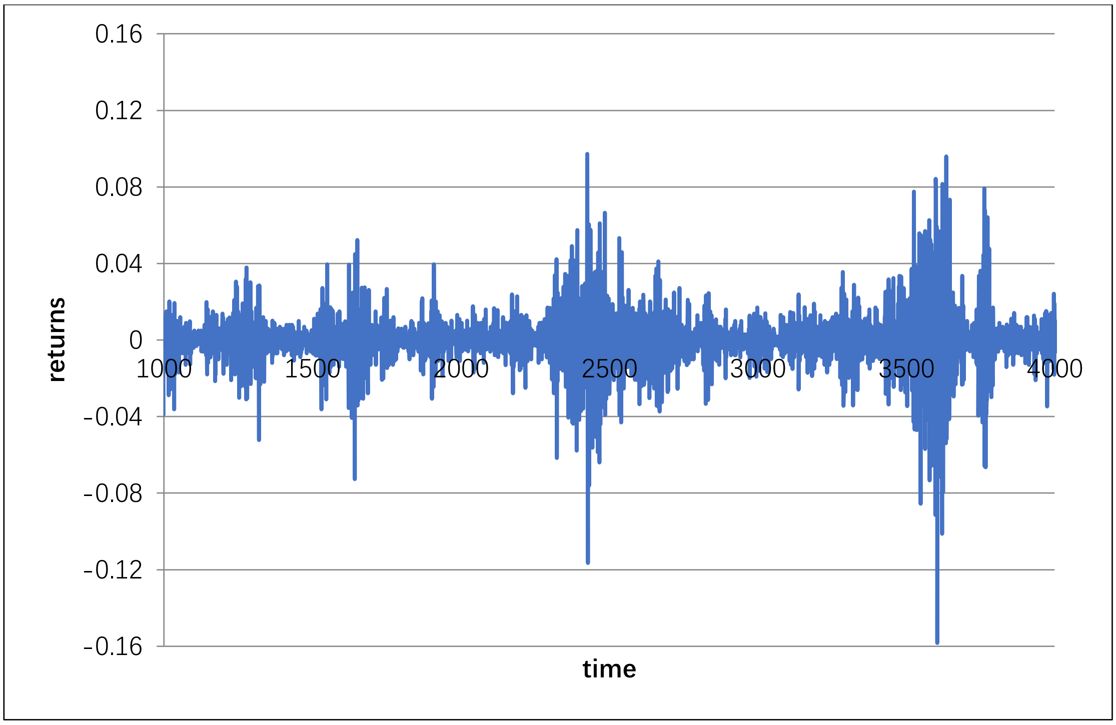


Figure 5. 2 Returns from 1000-4000 in the replication model

In Figure 5.3, the fraction of chartists among traders which is z from time 1000 to 4000 is presented with the parameter set 1.

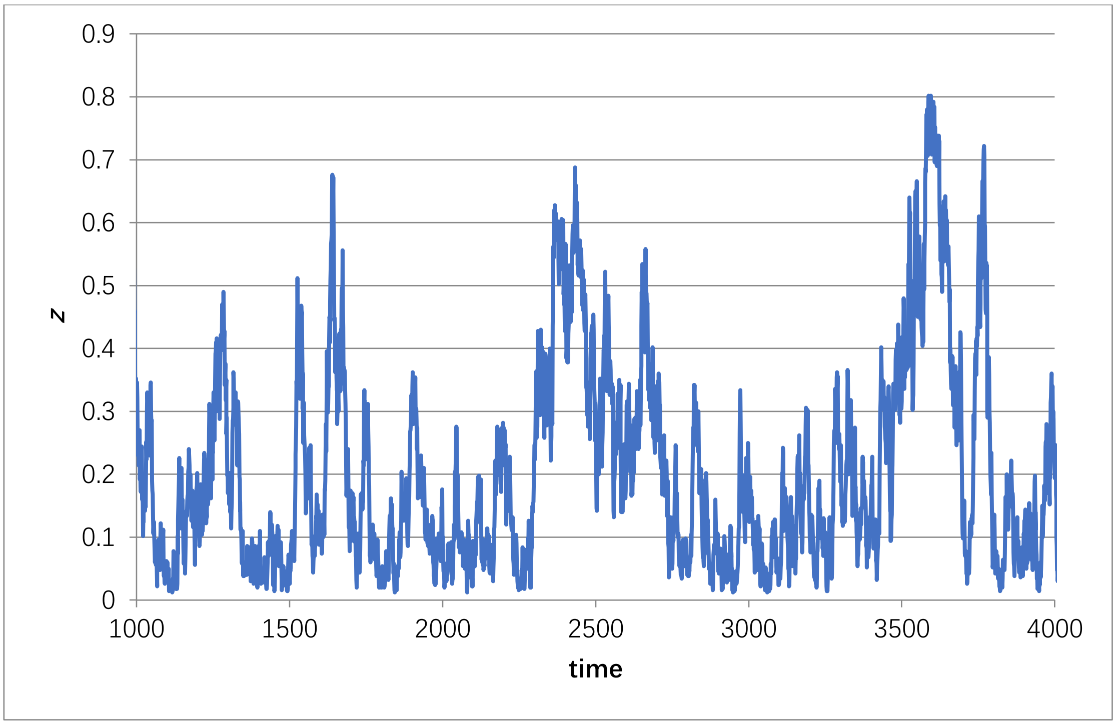


Figure 5. 3 *z* from 1000-4000 in the replication model

In Figure 5.4, returns and *z* are presented in the same graph to compare the patterns of returns and chartists' fraction with the parameter set 1. Chartists' fraction *z* is drawn on the left y-axis while returns are drawn on the right y-axis. From Figure 5.4, it is clear that when *z* gets larger, the magnitude of returns gets bigger too.

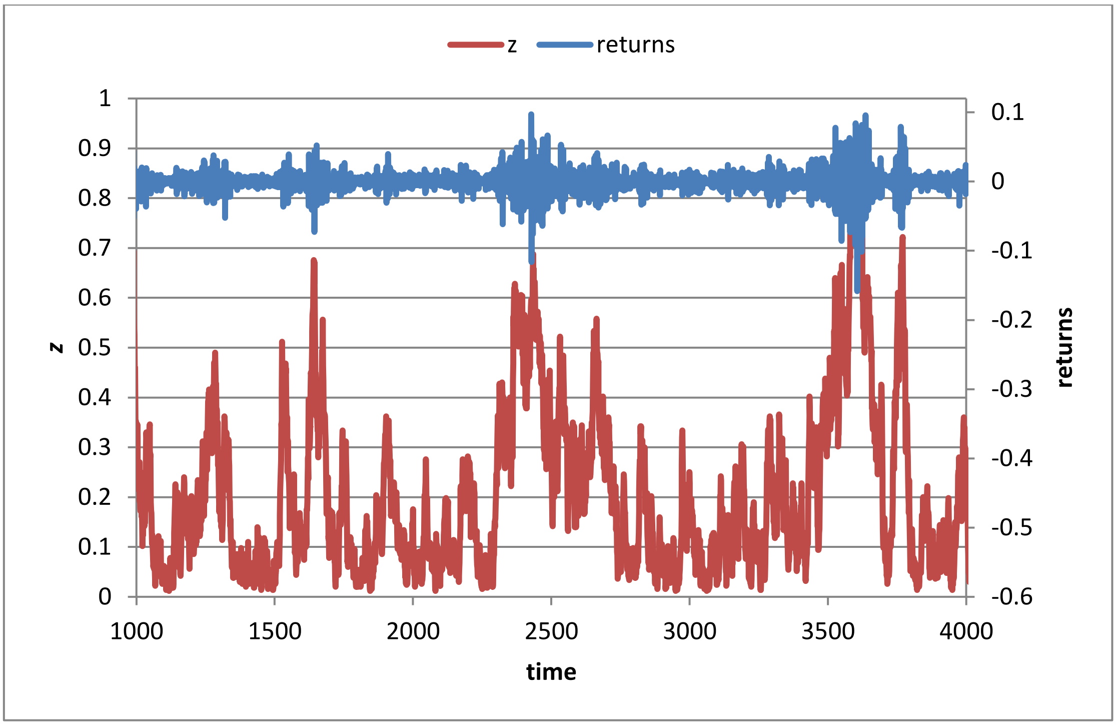
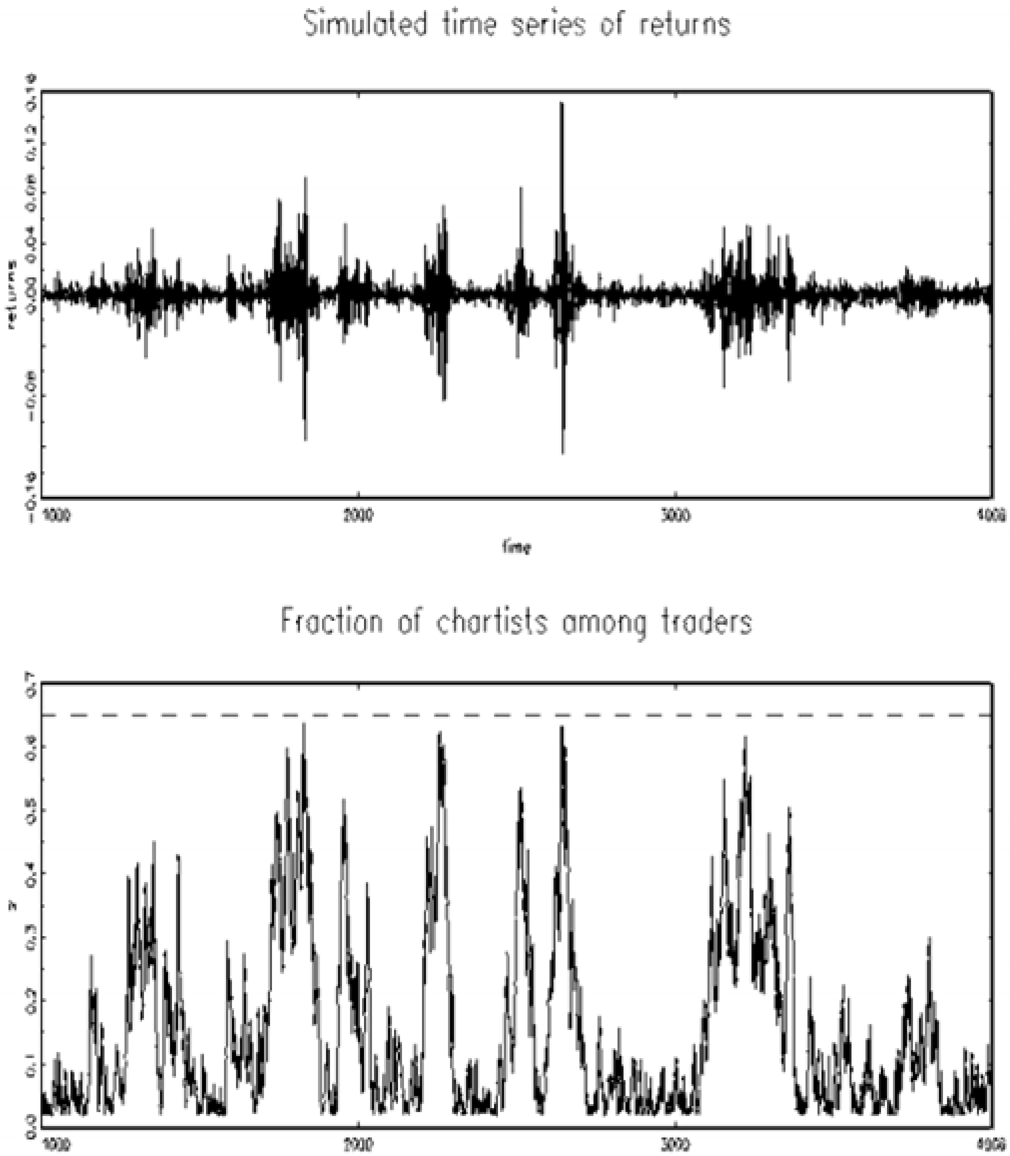


Figure 5. 4 Returns and *z* from 1000-4000 in the replication model

In Figure 5.5, the returns and fraction of chartists in the original paper are presented with the parameter set 1. In the original paper, returns are stable most of the time. When the fraction of chartists among traders get extremely high, returns become big.



Upper part: typical simulated time series of returns. Bottom part: simultaneous develop-... the fraction of chartists, $z = n_c/N$. The broken line indicates the critical value $\bar{z} = 0.65$ where a loss of stability is expected. Parameter values underlying this simulation are those of parameter set I, cf. main text.

<u>Figure 5. 5</u> Returns and z in the original model (Lux and Marchesi, 2000:689)

Comparing the patterns in Figure 5.4 and 5.5, they are visually similar. The returns are mostly stable when z is less than 0.5. When z is more than 0.5, which means there are more chartists than fundamentalists, the variability of returns becomes large. This is called "on-off intermittency" in the original paper.

In Table 5.4, it shows the results of unit-roots from my replication model.

Parameters	range
Parameter set 1	0.999301-0.999996
Parameter set 2	0.999819-1.000021
Parameter set 3	0.999899-1.000061
Parameter set 4	0.999874-1.000031

Table 5. 4 Unit root in the replication model

In Table 5.5, it shows the results of unit-roots from the original paper.

Results of unit-root tests.

Parameters	range of $\hat{\rho}$	No. of rejections for one-sided test at 95% level	No. of rejections for two-sided test at 95% level
Parameter set I	0.999819–1.000022	0	0
Parameter set II	0.999977–1.000021	0	0
Parameter set III	0.999957–1.000030	0	3
Parameter set IV	0.999972–1.000014	0	2

Note: The test statistic of the Dickey–Fuller test is identical to the usual t-statistic of p_{t-1}. The difference is that, in the case of a unit root, this statistic has a non-standard distribution. Critical points can be looked up in, for example, Dickey *et al.* [11]. Parameter sets are given in footnote p.

Table 5. 5 unit root in the original model (Lux and Marchesi, 2000:691)

Comparing the results in Table 5.4 and 5.5, the ranges of ρ in the replication model and the original paper are similar. The replication model has a little wider range of unit root than in the original paper except for parameter set 1. And the range in the replication model for the parameter set 1 does not quite include the value 1. For Table 5.5 and Table 5.9, the number of rejections for unit root and volatility clustering is provided. The number of rejections is not included in the replication model because the way to do the test is not clearly described.

Again following the results in the original paper, in Figure 5.6, the price from period 14950 to 15450 with parameter set 4 based on the replication model is shown.

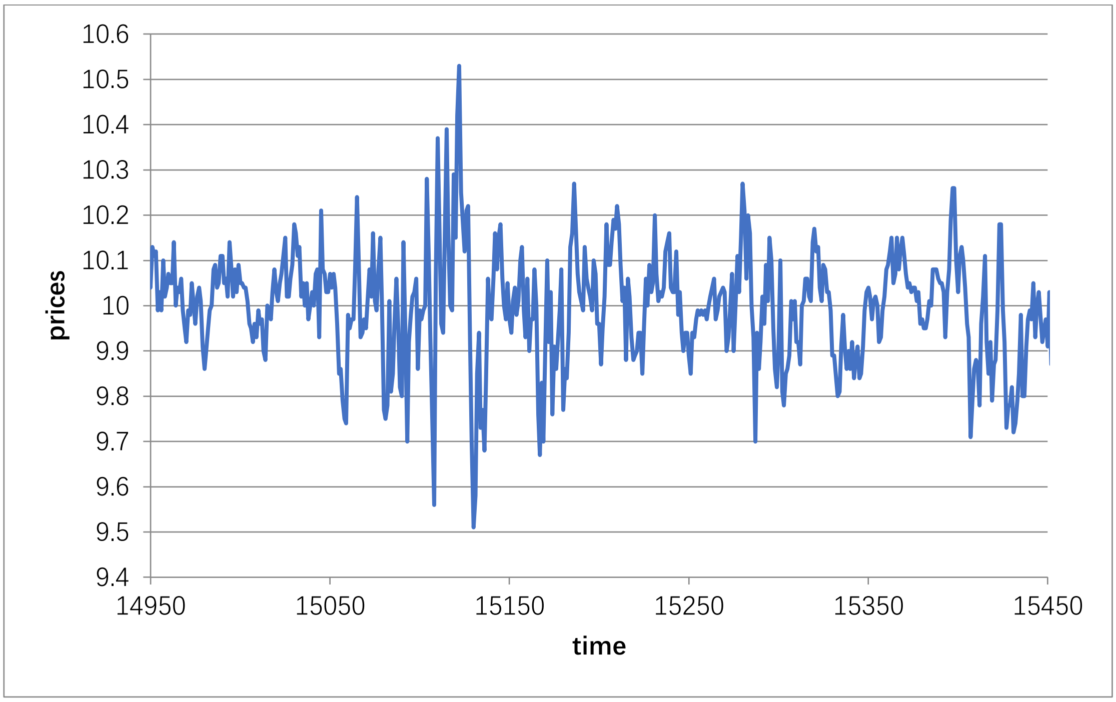


Figure 5. 6 Prices from 14950-15450 in the replication model

In Figure 5.7, the price from period 14950 to 15450 with the parameter set 4 from the original paper is shown.

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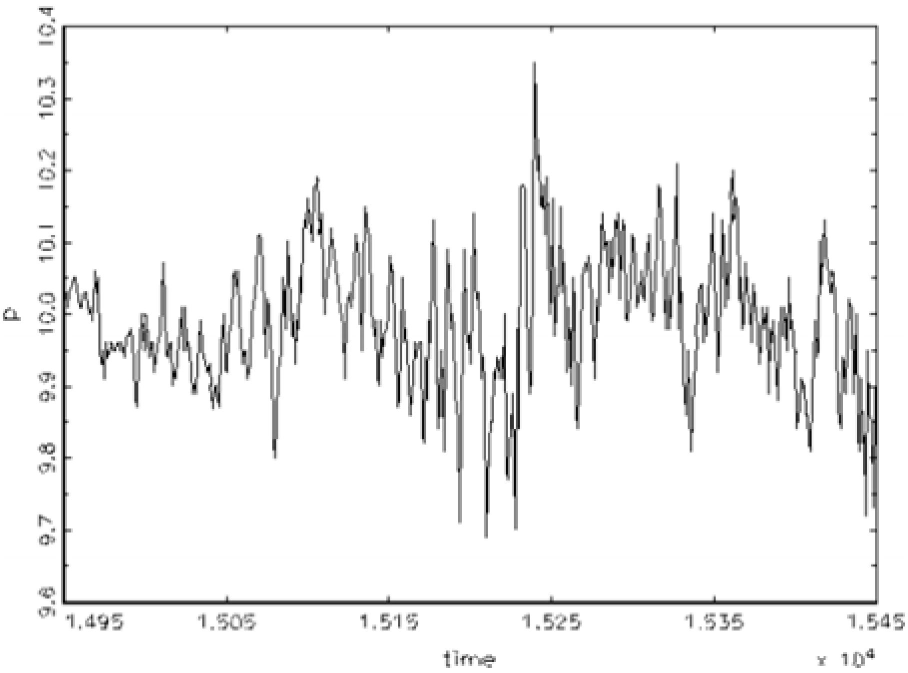

Time series of prices. It is hard to decide by inspection whether the dynamics is stationary or non-stationary. This example is from a simulation with parameter set IV.

Figure 5.7 Prices from 14950-15450 in the original model (Lux and Marchesi, 2000:692)

Comparing the prices in Figures 5.6 and 5.7, the patterns of prices are visually fairly similar. However, obviously the exact pattern is not the same since this will vary with using different random numbers and the precise details of the model. All in all, the prices in this model go up and down with some extreme values like about 10.5 and 9.5 in the replication model and 10.4 and 9.7 in the original model. It is difficult to make a precise comparison, particularly when only a small section of values over 500 time units is provided in the original paper.

Measures of fat tails based on my replication model are shown in Table 5.6. The results of kurtosis and tail index of 2.5% tail, 5% tail, and 10% tail are shown following that in the original paper. The kurtosis is the excess kurtosis statistic where a normal distribution has a value of 0.

The tail index is calculated for the ten samples of 2000 observations across the whole run, and so there are ten values of each tail index. In Table 5.6, the median and the range of the tail index are shown. For example, for the parameter set 1 the median of the 5% tail index is 2.51, ranging from 2.20 to 3.20 across the 10 sample values.

median *αH* from 10 samples of 2,000 observations (in parentheses: range of estimates)				
Parameters	kurtosis	2.5%tail	5%tail	10%tail
Parameter set 1	19.34	3.06 (2.15-3.93)	2.51 (2.20-3.20)	2.21 (1.98-2.81)
Parameter set 2	11.57	3.72 (2.18-4.27)	3.09 (2.37-3.60)	2.63 (2.16-3.20)
Parameter set 3	8.72	4.21 (3.03-5.67)	3.62 (2.51-4.20)	3.05 (2.29-3.39)
Parameter set 4	13.94	3.36 (2.43-4.66)	3.00 (1.99-3.98)	2.60 (1.71-3.28）

Table 5. 6 Fat tails in the replication model

Measures of fat tails based on the original model are shown in Table 5.7.

. Fat tail property of the data: kurtosis and tail index estimates.

Parameters	kurtosis	median α_H from 10 samples of 2,000 observations (in parentheses: range of estimates) 2.5% tail	5% tail	10% tail
Parameter set I	135.73	2.04 (1.61–4.50)	2.11 (1.51–2.64)	1.93 (1.26–2.44)
Parameter set II	16.10	2.82 (2.28–3.73)	2.52 (2.00–3.17)	2.18 (1.55–2.36)
Parameter set II	27.11	4.63 (2.41–6.82)	3.48 (2.33–8.60)	2.86 (1.80–4.84)
Parameter set IV	37.74	3.08 (2.11–4.06)	2.46 (2.13–7.86)	1.97 (1.65–3.18)

Note: Parameter sets are those given in footnote p.

Table 5. 7 Fat tails in the original model (Lux and Marchesi, 2000:693)

Comparing the kurtosis in Table 5.6 and 5.7, in the replication model, it is 19.34 while in the original paper, it is 135.73 when we use the parameter set 1. This is a big numerical difference but might not imply a big difference in model behaviour. The comments about the kurtosis values in the original paper are “As can be seen, we have excessive fourth moments in each case. As compared to empirical data at daily frequency, the results from parameter sets 2 to 4 look very realistic while the first variant may seem to exhibit too high a degree of leptokurtosis [footnote q: However, such high numbers are not uncommon for thinly traded assets at daily frequencies and for more frequently traded ones at intra-daily frequencies.] However, note that kurtosis is a somewhat ambiguous concept and it is not entirely clear how to compare the statistics obtained for various time series. Furthermore, empirical power-law tails with exponents in the range 2 to 4 imply non-convergence of the fourth moment which also makes empirical estimates of the kurtosis statistics unreliable.” Hence, it is difficult to assess the practical significance of the differences in the kurtosis values between the original paper and my results.

Also, even in the fairly recent academic literature the interpretation of kurtosis is disputed. For example, Westfall (2014) argues that kurtosis only reflects the nature of the tails of the distribution and not the peak, but Crack (2019) disagrees with this.

The main kurtosis result is that for both my results and the original paper, the kurtosis values are considerably higher than 0, indicating fat tails.

The tail index is an estimate of the shape of the tail. A value of α means than $Pr(X>x)$ is distributed in proportion to $x^{-\alpha}$ and so follows a power law.

For parameter set 1, the median from 10 samples of 2000 observations of 2.5% tail in my model is 3.06 ranging from 2.15 to 3.93. While the median of 2.5% tail is 2.04 from the original paper, ranging from 1.61 to 4.50.

Looking at all the median values there appears to be a systematic difference. With the exception of the 2.5% tail for the parameter set 3, the median values are higher in my model than in the original paper with the differences being between 0.14 and 1.02. The average difference for all twelve values is 0.42. Given such a consistent difference this probably indicates a difference in the model.

In the original paper, the comment about the results for the tail index values is “The results are, in fact, close to the usual empirical finding of tail indices somewhere between 2 and 5. There is also some indication of an increase of the estimate with decreasing tail size - a pattern which is also often encountered in empirical investigations." Although there is a difference in my values compared to the paper, they still follow this description. The difference corresponds to a relatively small change in shape for the tail. For example, for the 5% tail with parameter set 1, a power law shape with index 2.51 compared to 2.11.

In Figure 5.8, the result of the autocorrelations of raw, squared and absolute returns with the parameter set 4 are shown based on my replication model.

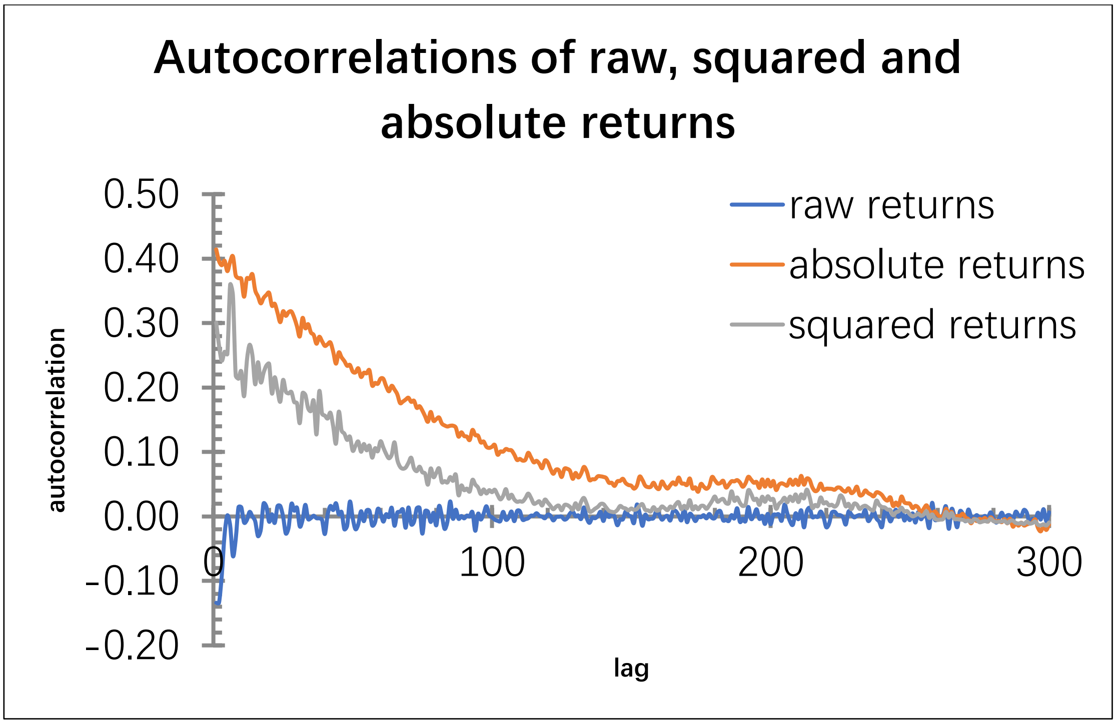

Figure 5. 8 Autocorrelations in the replication model

In Figure 5.9, the autocorrelation based on the original model with the parameter set 4 is shown.

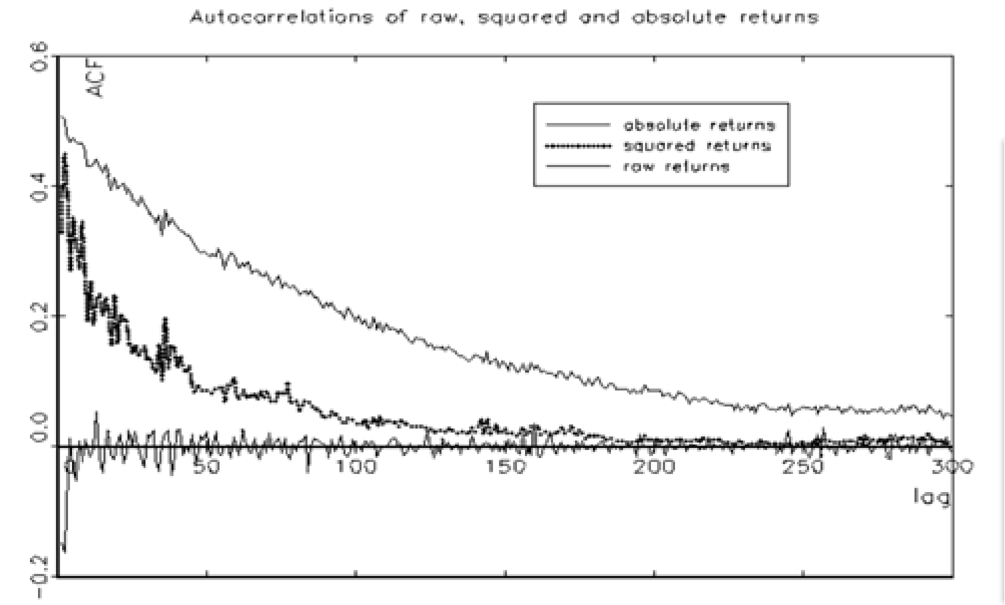

Fig. 3. Typical behaviour of the autocorrelations of raw, squared and absolute returns (bottom to top). The autocorrelation function here is computed from a simulation extending over 20,000 time steps. Underlying parameter values are those of parameter set IV.

Figure 5. 9 Autocorrelations in the original model (Lux and Marchesi, 2000:695)

Comparing the pattern in Figure 5.8 and 5.9, they are similar. Autocorrelation reaches zero when the lag is around 250 in Figure 5.8. Autocorrelation of absolute returns reaches zero when the lag is 300 and autocorrelation of squared returns reaches zero when the lag is 250 in Figure 5.9. The differences between the autocorrelation decrease in absolute returns and squared returns are bigger in the original model than in the replication model.

Table 5.8 shows the value of *d* in my replication model. The median *d* is 0.00 ranging from -0.09 to 0.23 with the parameter set 1 of squared returns based on my replication model. This indicates the volatility clustering is not that obvious with the parameter set 1 based on my replication model.

	squared returns	absolute returns
Parameters	median *d* from 10 samples of 2000 observations (in parentheses:range of estimates)	median *d* from 10 samples of 2000 observations (in parentheses:range of estimates)
Parameter set 1	0.00 (-0.09)-0.23	0.16 0.06-0.30
Parameter set 2	0.10 (-0.06)-0.22	0.20 0.08-0.28
Parameter set 3	0.15 0.03-0.26	0.21 0.18-0.34
Parameter set 4	0.14 0.00-0.28	0.24 0.05-0.36

Table 5. 8 *d* in the replication model

Table 5.9 shows the value of *d* in the original model.

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Table 3. Long-term dependence in squared and absolute returns.

	squared returns		absolute returns	
Parameters	median d from 10 samples of 2,000 observations (in parentheses: range of estimates)	% rejections of $d = 0$ at 95% level	median d from 10 samples of 2,000 observations (in parentheses: range of estimates)	% rejections of $d = 0$ at 95% level
Parameter set I	0.17 (0.06–0.56)	40%	0.38 (0.21–0.64)	80%
Parameter set II	0.54 (0.37–0.86)	100%	0.63 (0.43–0.75)	100%
Parameter set II	0.50 (0.29–0.80)	100%	0.64 (0.26–0.81)	100%
Parameter set IV	0.52 (0.20–0.70)	90%	0.64 (0.17–0.88)	90%

Note: Parameter sets are those given in footnote p.

Table 5. 9 *d* in the original model (Lux and Marchesi, 2000:696)

In my model (Table 5.8), the median *d* is 0.16, ranging from 0.06 to 0.30 of absolute returns from 10 samples of 2000 observations based on the parameter set 1. While *d* is 0.38, ranging from 0.21 to 0.64 in the original model. The values are not the same. This may be caused by different calculations as the description for this part is not so clear and it will be discussed in detail in the replication reflect part in Chapter 6.

Overall, my replication model and the original model have similar patterns, and both have the stylised facts of fat tails and volatility clustering. The statistics on the results are fairly similar but with some systematic differences. Therefore the model has been replicated to the level of rational alignment.

5.5 Model Extension

This section describes work to extend the experiments and analysis on the model to get a better understanding of the model behaviour. One area of uncertainty in the description of the original paper is in how the time increment changes. The effect of this was investigated by keeping the increment fixed and comparing the results. This is described in Section 5.5.1.

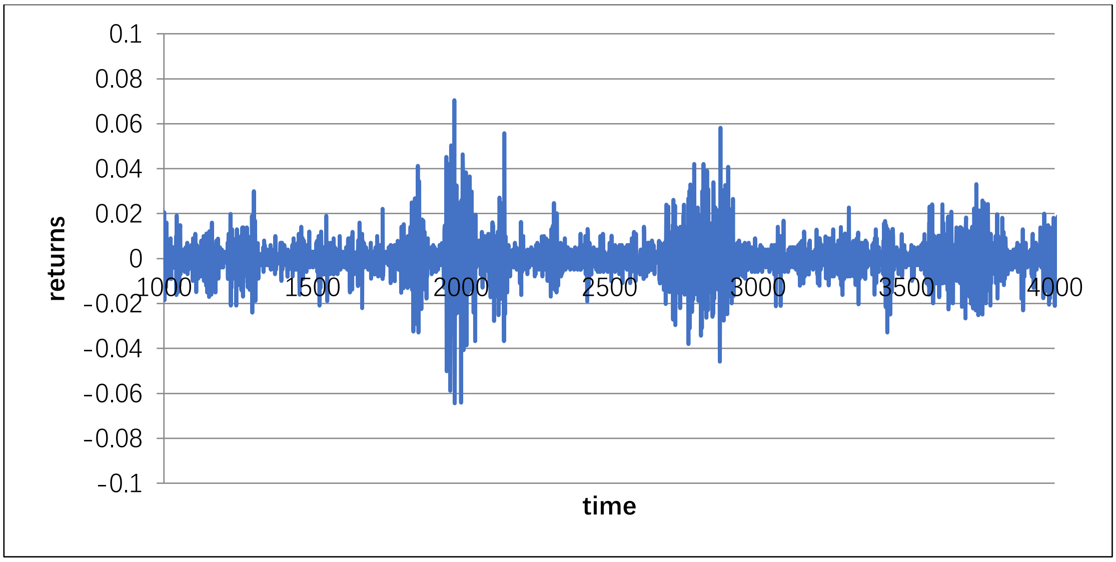
One particular aspect of interest is to try and get a better understanding of the reasons for the behaviour of the model. The model results show the short period of high volatility and then periods in between of more stable behaviour, described as “on-off intermittency” in the original paper. Various analysis was done to investigate the reasons for this behaviour and why the model enters and leaves the high volatility periods. This is described in Sections 5.5.2 – 5.5.6.

5.5.1 Time increment value

The requirement for the change of time interval from 0.01 to 0.002 is vague. Although it is said in the original paper that the time interval will change to 0.002 when the price changes so frequently that it exceeds the average prices change, there is no description of how to calculate the average prices change. Besides, in the original paper, there are four parameter settings, and the average price change may be different each time. If the value is set dynamically for each run, then this will need some data during the run in order to calculate the average and it is not clear how or when to incorporate this into the simulation.

Because the change of time interval may increase the value of extreme returns and the reasoning behind the change of time interval is not particularly persuasive, comparative experiments were run in which the time increment does not change and is always 0.01. The run length and number of runs are the same as the original one, which is 20000 and 1. The parameter set 1 was used and the results are presented in the following paragraphs. The time increment is 0.01 with run length 20000, and parameter set 1 with one long run is used for the remaining sections in Section 5.5.

In Figure 5.10, the pattern of returns and the fraction of chartists among traders are shown.



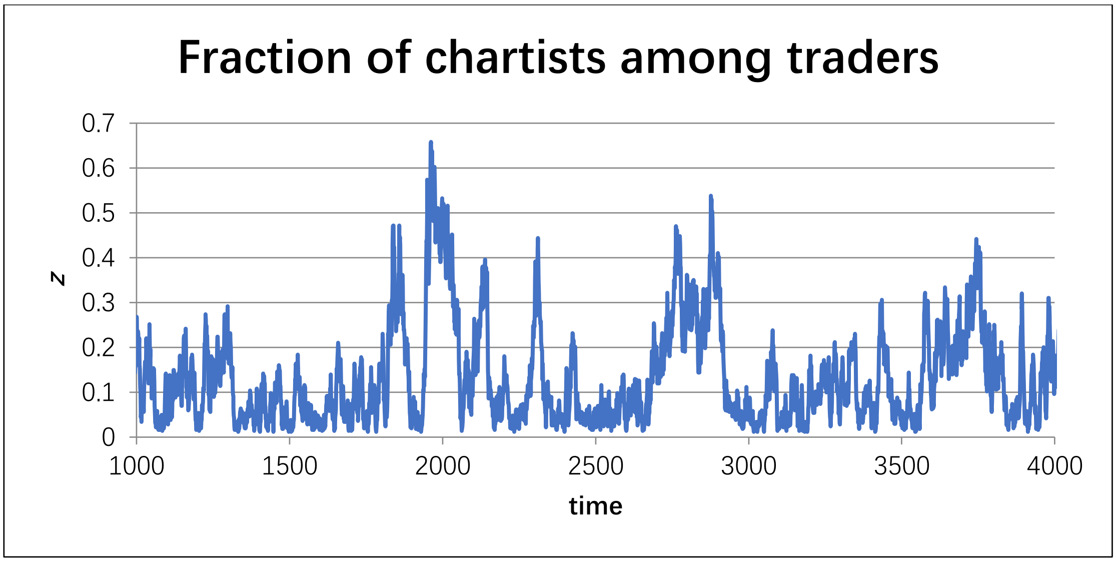


Figure 5. 10 Returns and z from 1000-4000 in the replication model with $\Delta t = 0.01$

From Figure 5.10, when the time interval is not changed, the returns get higher when the fraction of chartists is bigger. This is the same pattern that is found in my replication model with the change of time interval and also in the original model. The difference is that the extreme values of both returns and fraction of chartists are a bit lower here compared to the original replication model which has the change of time increment. These, on the other hand, confirm the effect of the time increment that change of time interval may increase the value of extreme returns.

In Figure 5.11, the pattern of price change from 14950 to 15450 is presented.

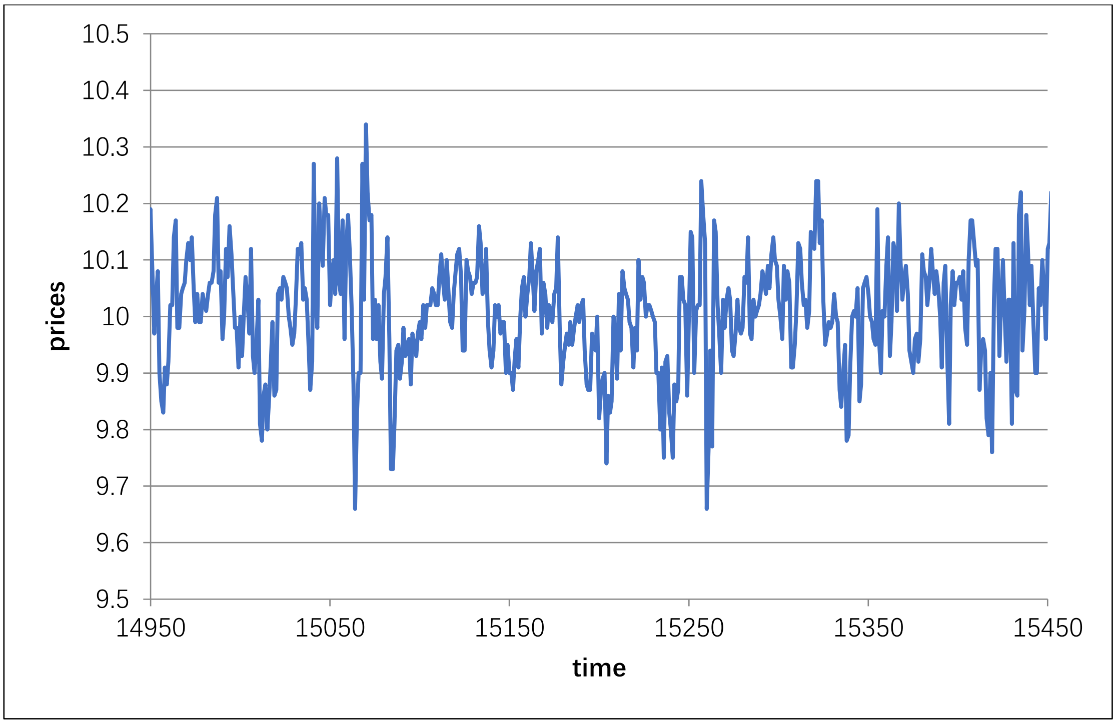


Figure 5. 11 Prices from 14950-15450 in the replication model $\Delta t = 0.01$

The pattern of price change from time 14950-15450 in Figure 5.11 is similar to the original replication model with the change of time increment and also the original model although the parameter sets are different. The result in Figure 5.11 is based on parameter set 1, while in the replication model (Figure 5.6) and original paper (Figure 5.7) are all based on parameter set 4.

In Figure 5.12, the result of autocorrelations of raw, squared and absolute returns with the parameter set 4 are shown.

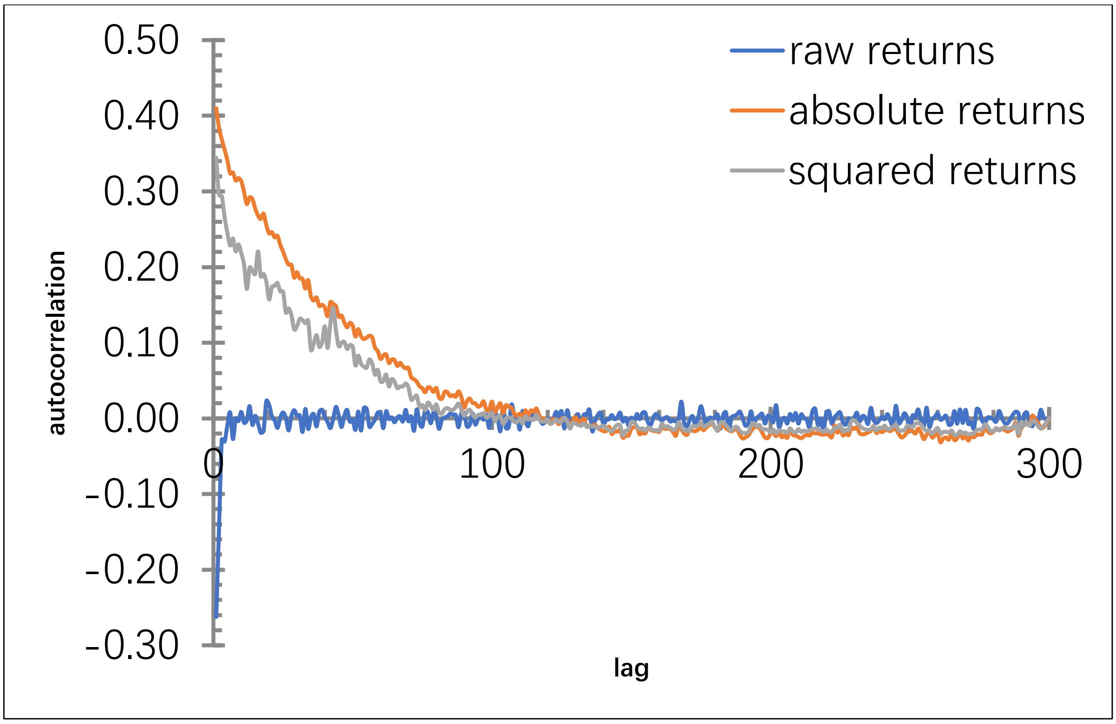


Figure 5. 12 Autocorrelations in the replication model $\Delta t = 0.01$

The patterns of autocorrelations change are generally similar to the results in Figure 5.8 and 5.9 before, although the result in Figure 5.12 is based on parameter set 1 while the result in Figure 5.8 is based on parameter set 4. The autocorrelation in Figure 5.12 decreases to 0 of both absolute returns and squared returns when the lag is 100. The autocorrelation in Figure 5.12 decays much more quickly than in the replication model with the change of time increment and the original model.

In Table 5.10, 5.11 and 5.12, the unit root, fat tails, and *d* in the replication model while $\Delta t = 0.01$ are shown.

Parameters	range
Parameter set 1	0.999796-1.000026

Table 5. 10 Unit root in the replication model $\Delta t = 0.01$

median αH from 10 samples of 2,000 observations				
Parameters	kurtosis	2.5%tail	5%tail	10%tail
Parameter set 1	8.48	3.40 (2.94-3.65)	2.79 (2.26-3.62)	2.29 (2.05-2.87)

Table 5. 11 Fat tails in the replication model $\Delta t = 0.01$

	squared returns	absolute returns
Parameters	median d from 10 samples of 2000 observations (in parentheses:range of estimates)	median d from 10 samples of 2000 observations (in parentheses:range of estimates)
Parameter set 1	0.08 (-0.04)-0.18	0.13 0.04-0.24

Table 5. 12 d in the replication model $\Delta t = 0.01$

In Table 5.10, the values of unit root based on parameter 1 are closer to the values in the original model in Table 5.5 than the values in the replication model with the change of time increment in Table 5.4. In Table 5.11, the kurtosis value is lower than that in the replication model with the change of time increment in Table 5.6 that in the original model in Table 5.7. The tail indices are slightly higher than the previous values.

In Table 5.12, the values of d in both Table 5.12 and in the replication model with the change of time increment in Table 5.8 are much lower than the value in the original model in Table 5.9.

All in all, the result without change of time increment is quite similar to the replication model with the change of time increment and the original model. Whilst the statistics are slightly different, the prices follow a random walk, and returns have fat tails and volatility clustering. Hence, changing the time increment does not appear to have a very large effect and does not seem to be a factor in causing the stylized facts. A fixed time increment was therefore used for the other experiments described in Sections 5.5.2 – 5.5.6.

5.5.2 Relationship analysis

From the results in Section 5.4, the periods of high volatility happen when the fraction of chartists is high. Hence, it is important to understand how the number of each type of agent changes. The model enters a period of high volatility when the number of chartists increases and so a key question is how does this occur?

The relationship between the number of agents and probabilities will be analysed in this section. The experiments are run using parameter set 1 and with the time increment fixed at 0.01.

From the probabilities equation of changing between agents, the probabilities are influenced by the number of agents in each type and by the price. The relationship between each probability and the number of a certain type of agent will be analysed one by one.

The following analysis shows that the probability is mainly based on the number of agents in each type rather than the price. In each case there is essentially a linear relationship. The mathematical reasons why this is the case will also be discussed.

From equation (5.6), the probability of changing from pessimistic chartists to optimistic chartists is influenced by the total number of chartists. Hence, it will be partly due to the number of optimistic chartists. Of course, changing from a pessimistic to optimistic chartist does not change the total number of chartists.

In Figure 5.13, the relationship between the number of optimistic chartists and the probability of changing from pessimistic chartists to optimistic chartists is shown.

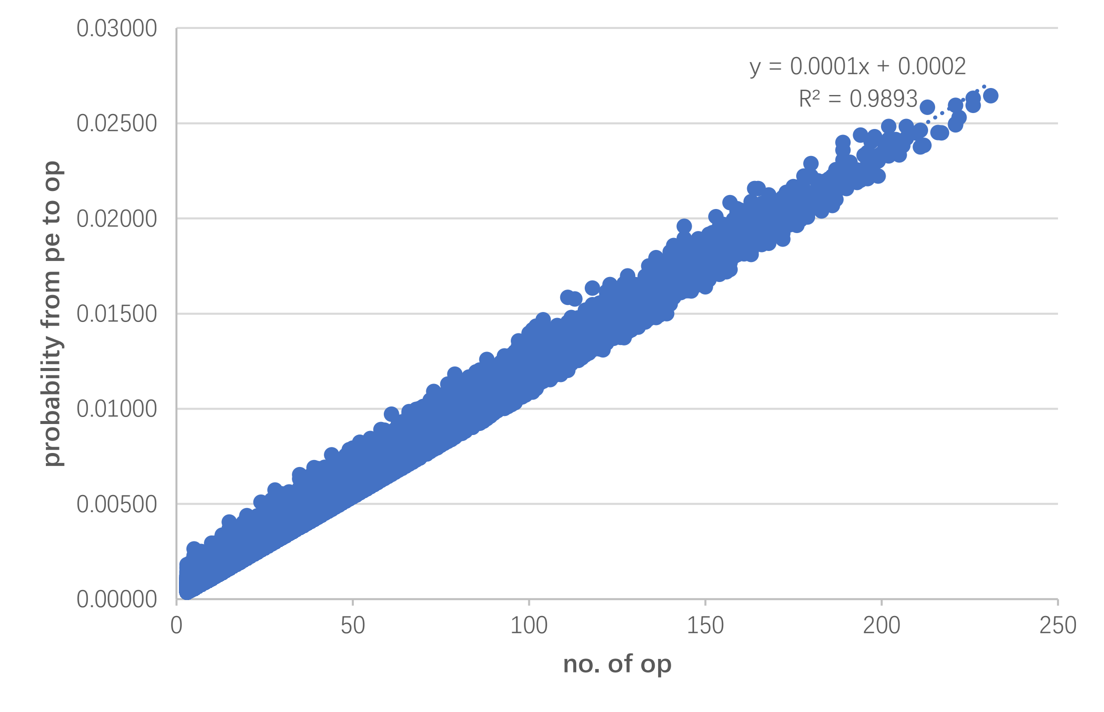


Figure 5. 13 Relationship between the number of optimistic chartists and the probability of changing from pessimistic chartist to optimistic chartists

From Figure 5.13, the relationship between the number of optimistic chartists and the probability of changing from pessimistic chartists to optimistic chartists follows approximately a straight line with a slope of about 0.0001169 and an intercept of 0.0001561. R square is 0.9893, indicating that there is a strong linear relationship, which is also clear from the chart.

From equation (5.7), the probability of changing from optimistic chartists to pessimistic chartists is influenced by the total number of chartists. In Figure 5.14, the relationship between the number of pessimistic chartists and the probability of changing from optimistic chartists to pessimistic chartists is shown.

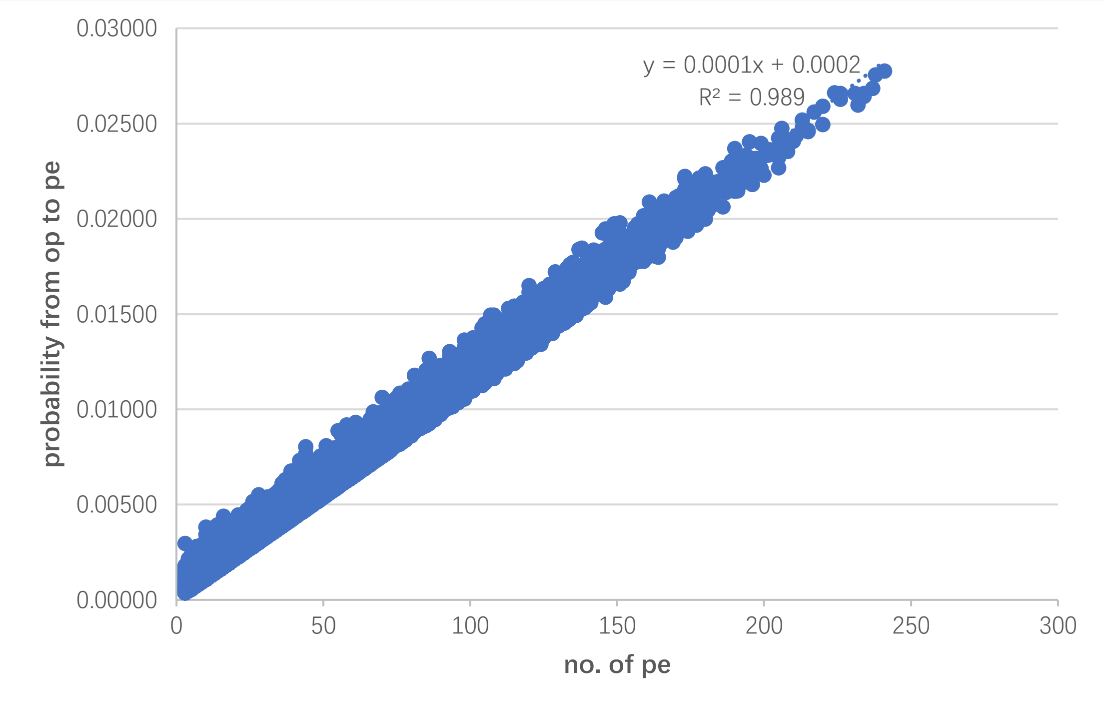


Figure 5. 14 Relationship between the number of pessimistic chartists and the probability of changing from optimistic chartists to pessimistic chartists

From Figure 5.14, the relationship between the number of pessimistic chartists and the probability of changing from optimistic chartists to pessimistic chartists follows a straight line with a slope of about 0.0001163 and an intercept of 0.0002461. R square is 0.989, indicating that there is a strong linear relationship.

Comparing the patterns in Figure 5.13 and 5.14, they are very similar. That indicates the probabilities of changing from pessimistic chartists to optimistic chartists and changing from optimistic chartists to pessimistic chartists are symmetric. If the number of optimistic chartists and pessimistic chartists is the same, the probability of changing between these two types of agents is the same, too.

From equation (5.9), the probability of changing from fundamentalists to optimistic chartists is influenced by the number of optimistic chartists. In Figure 5.15, the relationship between the number of optimistic chartists and the probability of changing from fundamentalists to optimistic chartists is shown.

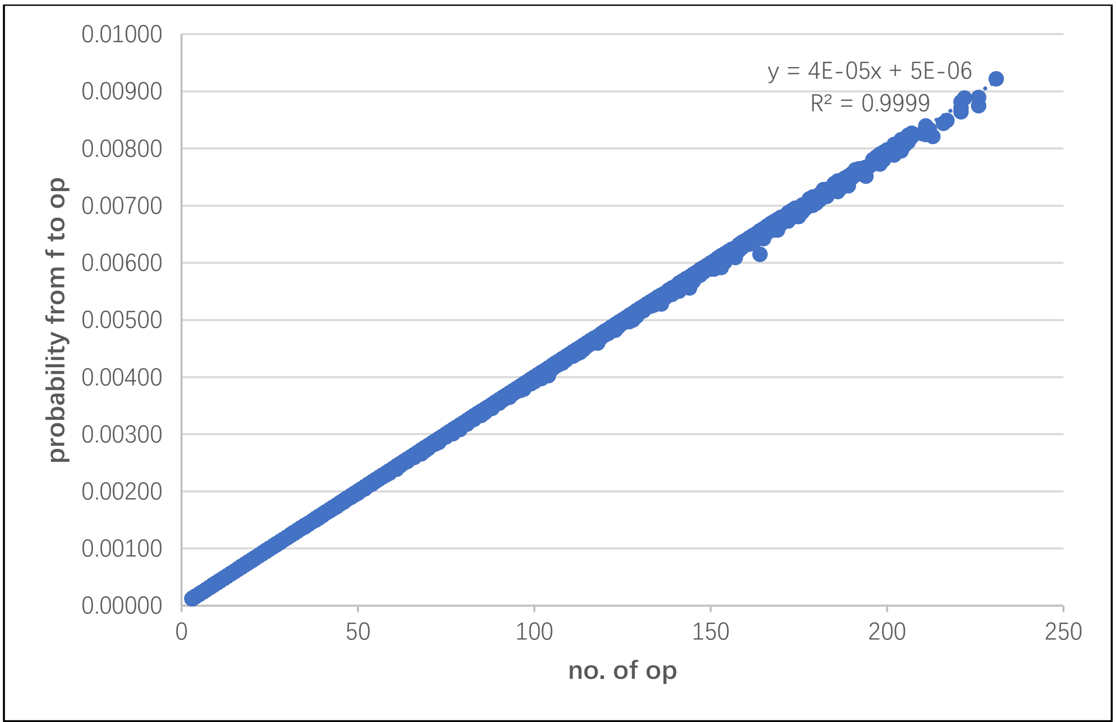


Figure 5. 15 Relationship between the number of optimistic chartists and the probability of changing from fundamentalists to optimistic chartists

From Figure 5.15, the relationship between the number of optimistic chartists and the probability of changing from fundamentalists to optimistic chartists can be modelled by a straight line with a slope of 0.0000397 and an intercept of 0.0000051. R square is 0.9999, indicating that there is a very strong linear relationship.

From equation (5.10), the probability of changing from optimistic chartists to fundamentalists is influenced by the number of fundamentalists. In Figure 5.16, the relationship between the number of fundamentalists and the probability of changing from optimistic chartists to fundamentalists is shown.

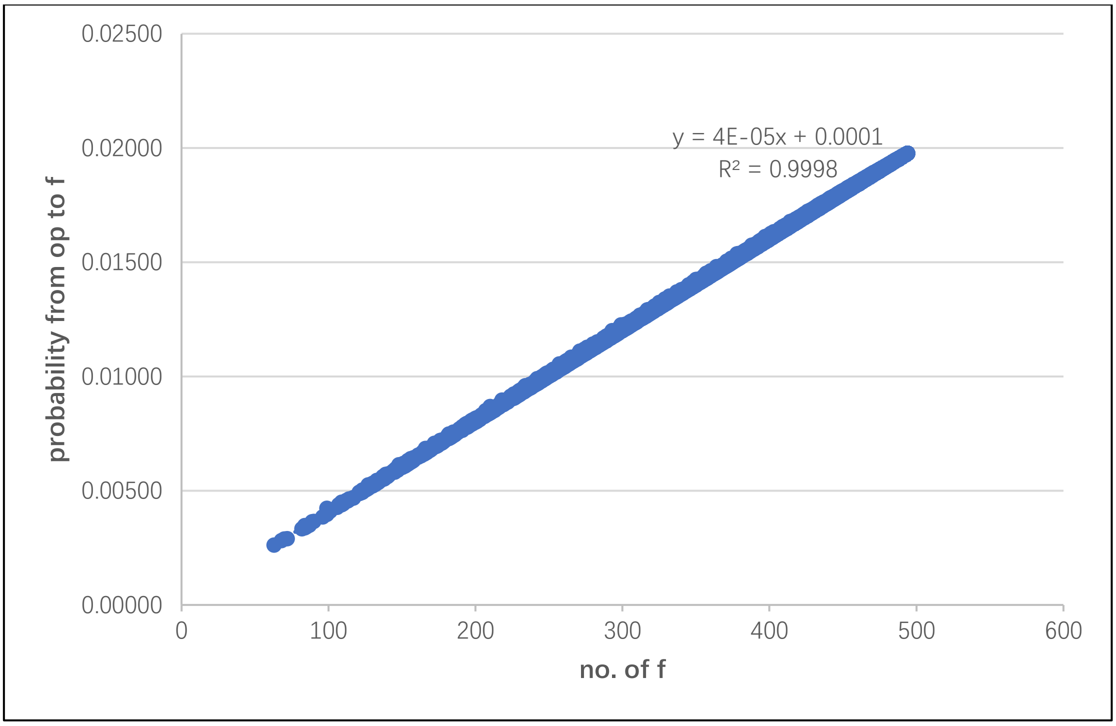


Figure 5. 16 Relationship between the number of fundamentalists and the probability of changing from optimistic chartists to fundamentalists

From Figure 5.16, the relationship between the number of fundamentalists and the probability of changing from optimistic chartists to fundamentalists follows a straight line with a slope of 0.0000398 and an intercept of 0.0001157. *R* square is 0.9998, indicating again that there is a very strong linear relationship.

From equation (5.12), the probability of changing from fundamentalists to pessimistic chartists is influenced by the number of pessimistic chartists. In Figure 5.14, the relationship between the number of pessimistic chartists and the probability of changing from fundamentalists to pessimistic chartists is shown.

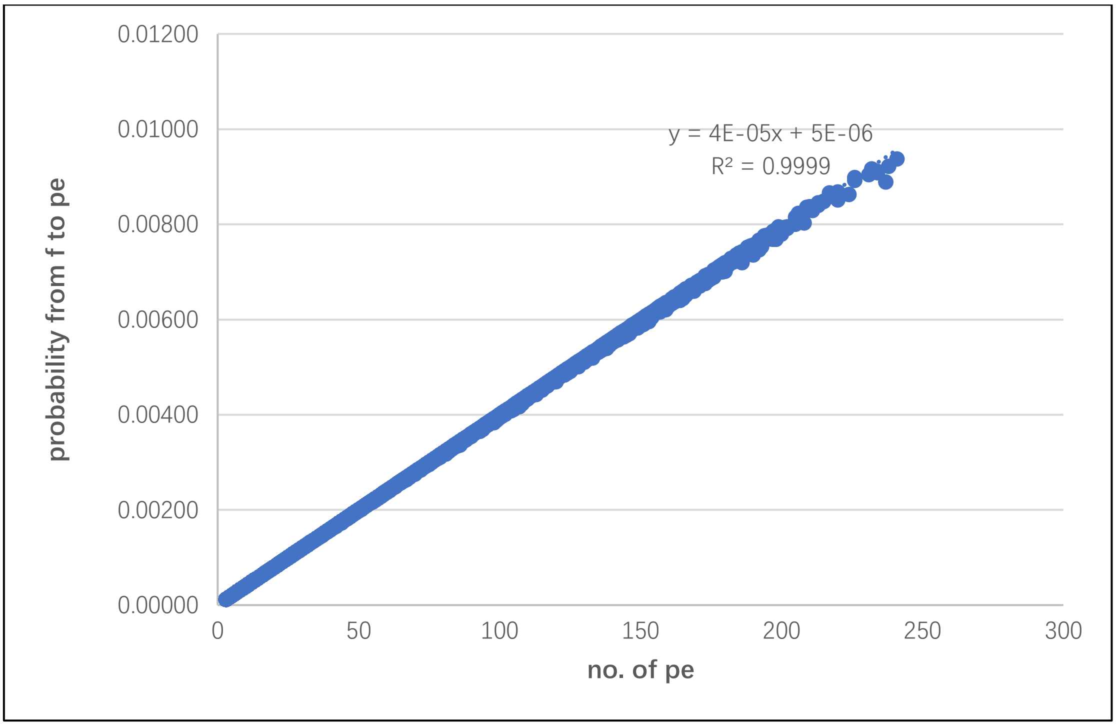


Figure 5. 17 Relationship between no of pessimistic chartists and the probability of changing from fundamentalists to pessimistic chartists

From Figure 5.17, the relationship between the number of pessimistic chartists and the probability of changing from fundamentalists to pessimistic chartists follows a straight line with a slope of 0.0000397 and an intercept of 0.0000050. *R* square is 0.9999, indicating that there is almost exactly a straight-line relationship.

From equation (5.13), the probability of changing from pessimistic chartists to fundamentalists is influenced by the number of fundamentalists. In Figure 5.18, the relationship between the number of fundamentalists and the probability of changing from pessimistic chartists to fundamentalists is shown.

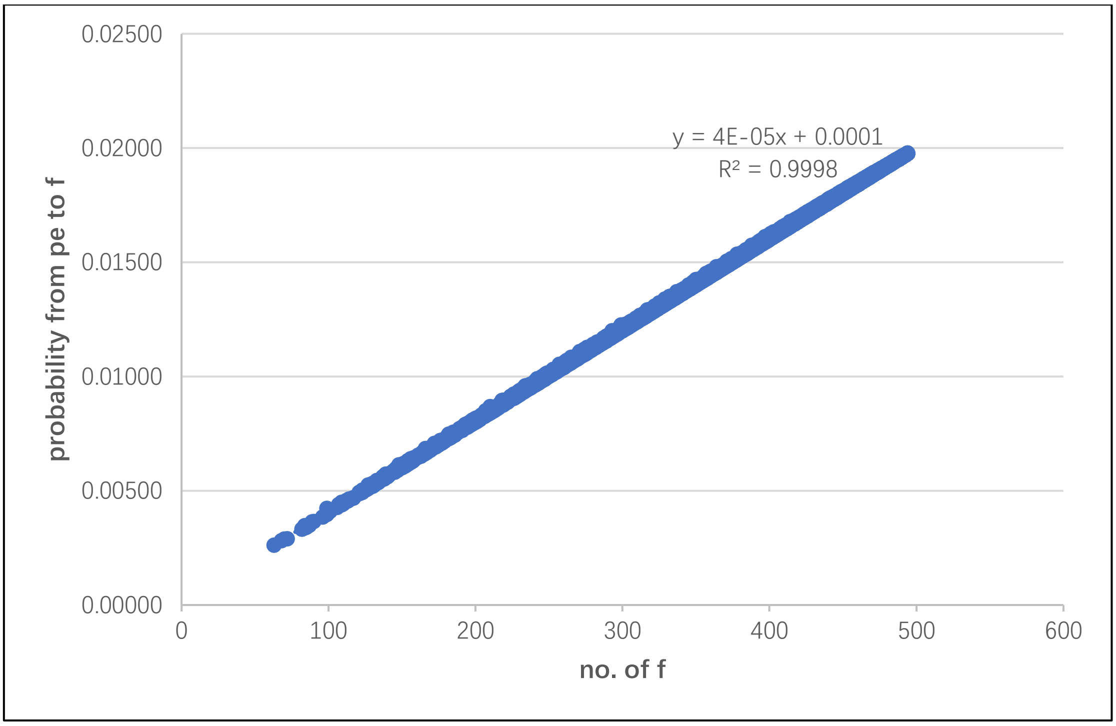


Figure 5. 18 Relationship between no of fundamentalists and the probability of changing from pessimistic chartists to fundamentalists

From Figure 5.18, the relationship between the number of fundamentalists and the probability of changing from pessimistic chartists to fundamentalists follows a straight line with a slope of 0.0000398 and an intercept of 0.0001159. *R* square is 0.9998, indicating that there is almost exactly a straight-line relationship.

From Figure 5.15, 5.16, 5.17, and 5.18, the slope of the straight line is about the same (0.00004), only the intercepts are different. The intercepts in Figure 5.15 and 5.17 are the same (0.000005) and those in Figure 5.16 and 5.18 are also the same (0.0001). The straight-line relationships between the number of agents and probabilities of changing between agents are:

(5.20) $\pi_{f\rightarrow+} = 0.00004 * n_{+} + 0.000005$

(5.21) $\pi_{+\rightarrow f} = 0.00004 * n_{f} + 0.0001$

(5.22) $\pi_{f\rightarrow-} = 0.00004 * n_{-} + 0.000005$

(5.23) $\pi_{-\rightarrow f} = 0.00004 * n_{f} + 0.0001$

The reason for the values for the slopes is considered in the next section. However, what the equations imply is that if the number of a particular type of agent increases, then the probability of swapping to that type also increases. In particular, if the number of chartists increases just through random variability, then the probability of fundamentalists changing to chartists increases and so this can lead to a high proportion of chartists and the start of a period of high volatility. The next question is what factors might cause the period of high volatility to end.

5.5.3 Price effect

The relationship lines in figure (5.13) and (5.14) from the probabilities of changing between optimistic chartists and pessimistic chartists are almost the same. The probabilities which influence the change of the number of fundamentalists depend on the relationships in figure (5.15) (5.16) (5.17) and (5.18).

The on-off intermittency is a pattern that the returns change with the change of chartists' fraction. The fraction of chartists depends on the relative numbers of chartists and fundamentalists. Therefore, the price effect discussion only focuses on the change between fundamentalists and chartists.

For the relationship between fundamentalists and chartists, the slope is the same, only the intercept is different.

Considering the probability equations 5.9, 5.10, 5.12, 5.13, the price effect in these equations is due to the value of $U_{2,1}$ and $U_{2,2}$. These depend on the rate of change of the price and the difference in the price from the fundamental value. In times of stability these will both be small and so $U_{2,1}$ and $U_{2,2}$ will be small.

In the probability equations the terms involving $U_{2,1}$ and $U_{2,2}$ are the exponential of either plus or minus $U_{2,1}$ or $U_{2,2}$. For the exponential function, if x is close to 0 then $exp(x) \approx 1+x$.

And usually, their values are close to 0, which leads to the values of $\exp(U_{2,1})$ and $\exp(U_{2,2})$ being very close to 1. If the price effect is ignored and the probability is only dependent on the number of agents then the slope for equation (5.9) (5.10) (5.12) and (5.13) is equal to $\left(\frac{v_2}{N}\right)\Delta t$ which is 0.00004. This is because for parameter set 1: $v_2 = 2$. We also have $N = 500$ and $\Delta t = 0.01$. Hence: $\left(\frac{v_2}{N}\right)\Delta t = 0.00004$. This is the reason why the relationships in the previous section have a slope of 0.00004.

If the *prie* stands for the price effect, then the equations (5.9) (5.10) (5.12) and (5.13) can be changed to:

(5.9.1) $\pi_{f\to+} = \left(\frac{v_2}{N}\right)\Delta t\ (1 + prie) * n_+$

(5.10.1) $\pi_{+\to f} = \left(\frac{v_2}{N}\right)\Delta t\ (1 + prie) * n_f$

(5.12.1) $\pi_{f\to-} = \left(\frac{v_2}{N}\right)\Delta t\ (1 + prie) * n_-$

(5.13.1) $\pi_{-\to f} = \left(\frac{v_2}{N}\right)\Delta t\ (1 + prie) * n_f$

The simulation model was run and the price effect was calculated at each time unit using the probability equations. Figures 5.19, 5.20, 5.21, and 5.22 show the change of price effect (*prie*) in equations 5.9.1, 5.10.1 5.12.1, and 5.13.1 along with the returns in simulated time from 0-20000.

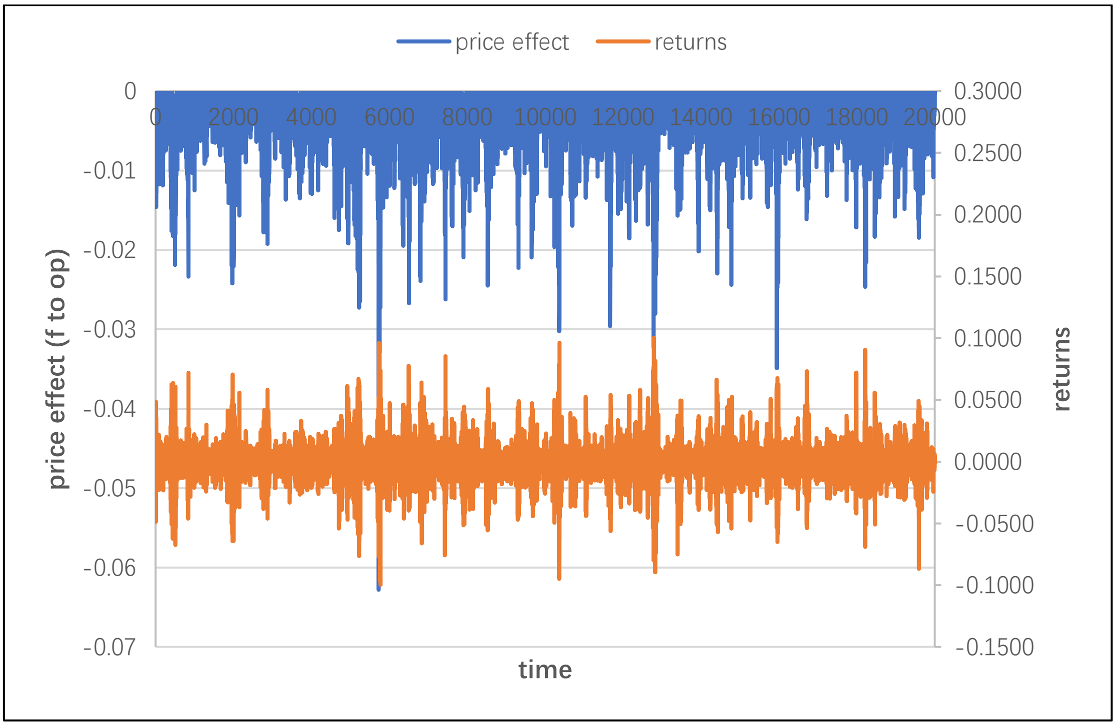


Figure 5. 19 Price effect in Eq. (5.9.1) and returns from 0 – 20000

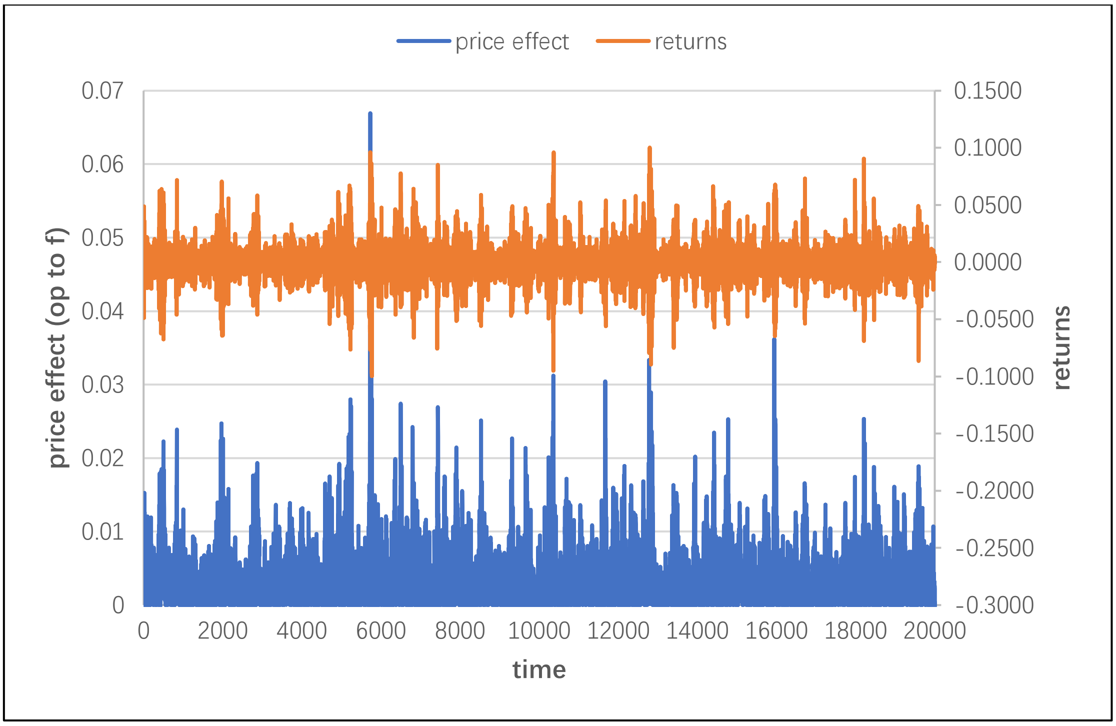


Figure 5. 20 Price effect in Eq. (5.10.1) and returns from 0 – 20000

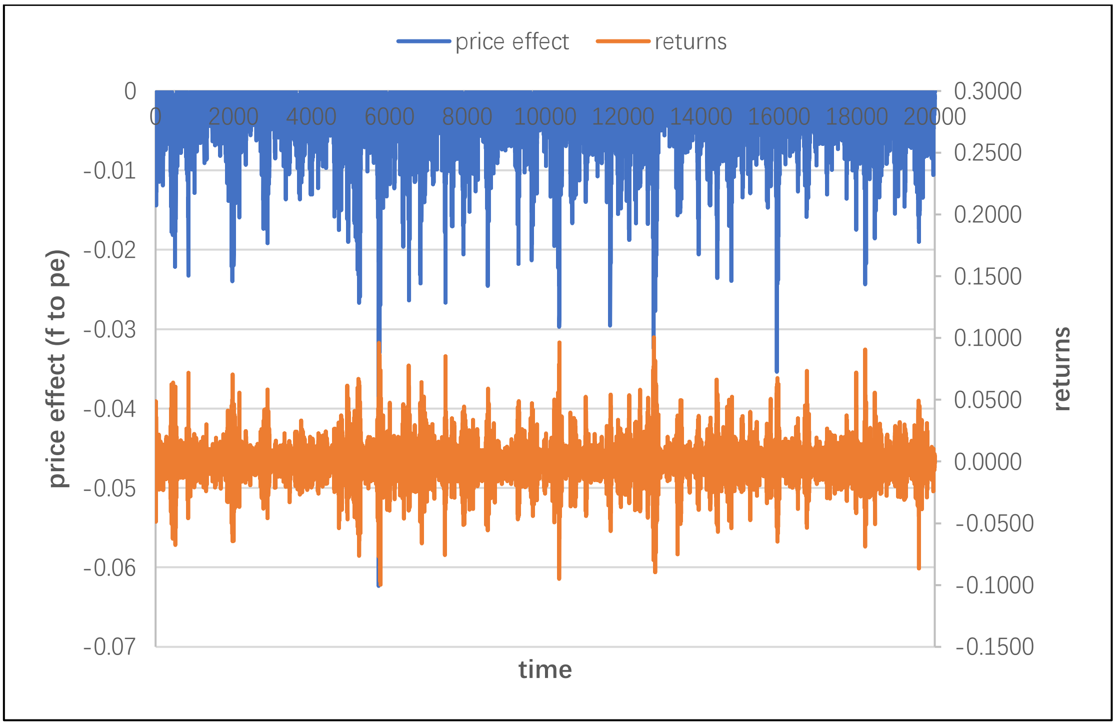


Figure 5. 21 Price effect in Eq. (5.12.1) and returns from 0 – 20000

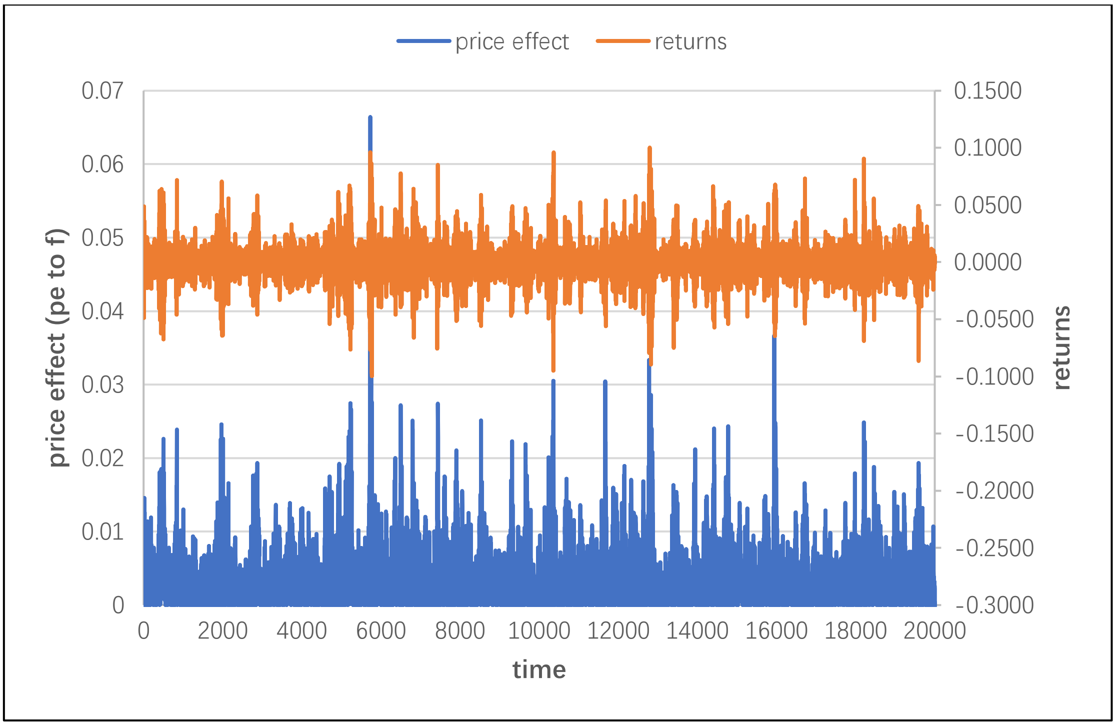


Figure 5. 22 Price effect in Eq. (5.13.1) and returns from 0 – 20000

Figure 5.19 and 5.21 show that the price effect from fundamentalists to chartists is negative. While Figure 5.20 and 5.22 show that the price effect from chartists to fundamentalists is positive. Hence, the price effect is a factor that increases the probability of agents being fundamentalists.

From Figure 5.19, 5.20, 5.21, 5.22, the absolute value of the price effect increases when the magnitude of returns are big. That means the price effect from fundamentalists to chartists reduces the probability more from fundamentalists to chartists when returns are big. In contrast, the price effect from chartists to fundamentalists increases the probability more from chartists to fundamentalists when returns are big. Therefore, when volatility is high the price effect is big which will increase the probability of agents changing to fundamentalists. Hence, this provides a mechanism for ending the period of having a high fraction of chartists and high volatility.

5.5.4 Without price effect

From the discussion before, the price effect seems to be a stabilising force that drives the returns back to their stable stage by changing chartists to fundamentalists. To confirm the discussion of the price effect, the model without the price effect will be analysed below.

The model without price effect changes the $\dot{p}$ in equation (5.5) to 0. Then the equation (5.5) changes to equation (5.5.1) for the change between chartists as below:

(5.5.1) $U_1 = \alpha_1 x$

The value of $U_{2,1}$ and $U_{2,2}$ changes to one, which makes the price effect (*prie*) in equation 5.9.1, 5.10.1, 5.12.1, and 5.13.1 equal to 0. That reduces the intercept in original straight-line relationships to 0. In other words, the probability of changing between chartists to fundamentalists in the model without the price effect purely depends on the number of agents. Then the straight-line relationships without price effect change the equations (5.20) (5.21) (5.22) (5.23) to:

(5.20.1) $\pi_{f\to+} = 0.00004 * n_+$

(5.21.1) $\pi_{+\to f} = 0.00004 * n_f$

(5.22.1) $\pi_{f\to-} = 0.00004 * n_-$

(5.23.1) $\pi_{-\to f} = 0.00004 * n_f$

The returns and fraction of the chartists of the model without the price effect are presented in Figure 5.23.

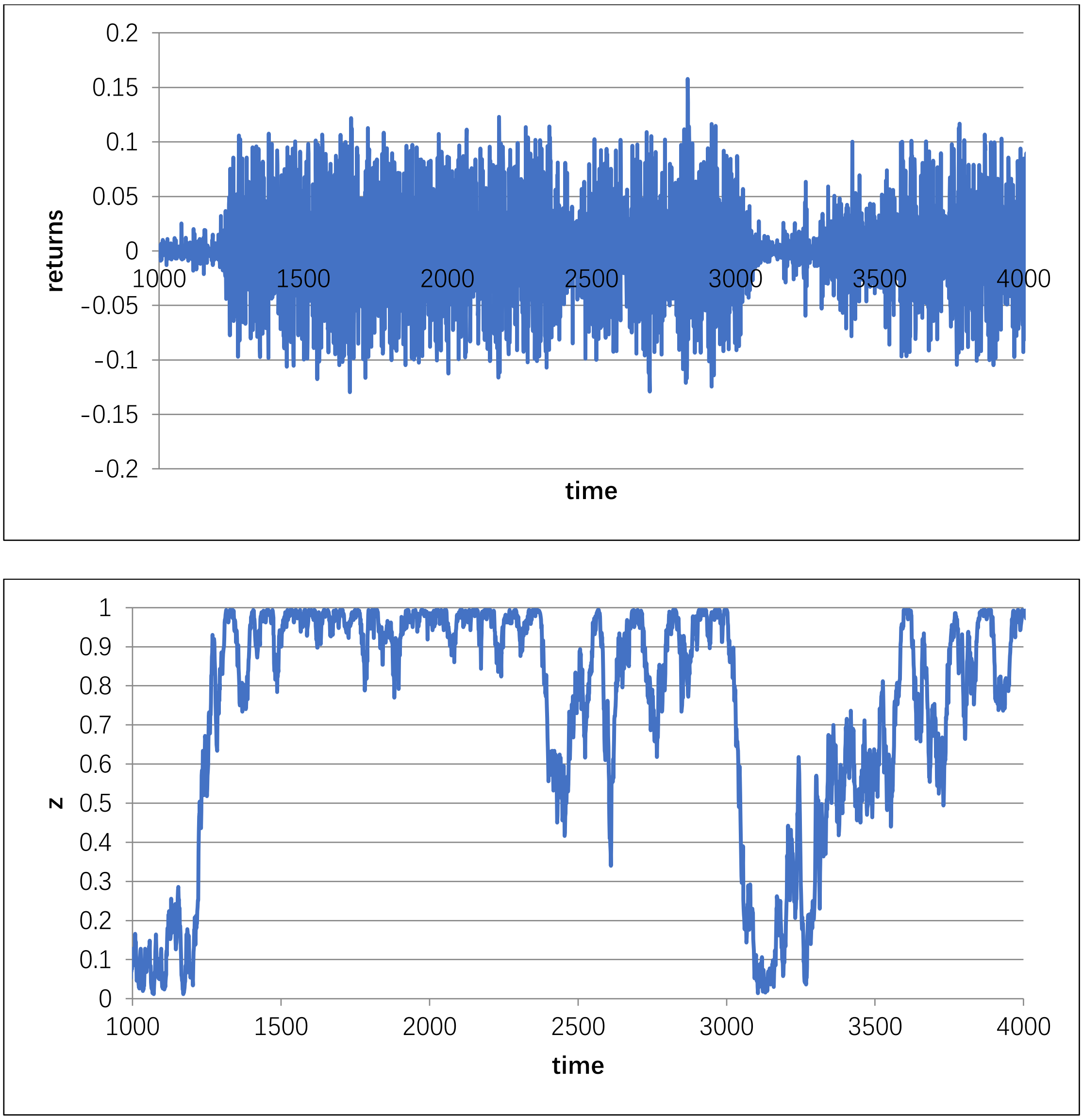


Figure 5. 23 Returns and *z* from 1000-4000 in the replication model without price effect

The patterns of returns and fraction of chartists are very different from the patterns before in Figure 5.10. Although the returns are big when the fraction of chartists is big, the volatile period when the returns are big lasts much longer than in Figure 5.10. And the stable periods when the returns are small are very short. Most of the time the system shows high volatility. These patterns influence the statistical results.

In Table 5.13, 5.14, and 5.15, the unit root, fat tails, and *d* in the model without price effect are shown.

Parameters	range
Parameter set 1	0.997269-1.000104

Table 5. 13 Unit root in the replication model without price effect

median αH from 10 samples of 2,000 observations				
Parameters	kurtosis	2.5%tail	5%tail	10%tail
Parameter set 1	0.25	9.09 (5.64-13.76)	8.33 (4.68-9.63)	6.39 (3.21-7.02)

Table 5. 14 Fat tails in the replication model without price effect

	squared returns	absolute returns
Parameters	median d from 10 samples of 2000 observations (in parentheses:range of estimates)	median d from 10 samples of 2000 observations (in parentheses:range of estimates)
Parameter set 1	0.25 0.01-0.36	0.30 0.05-0.40

Table 5. 15 d in the replication model without price effect

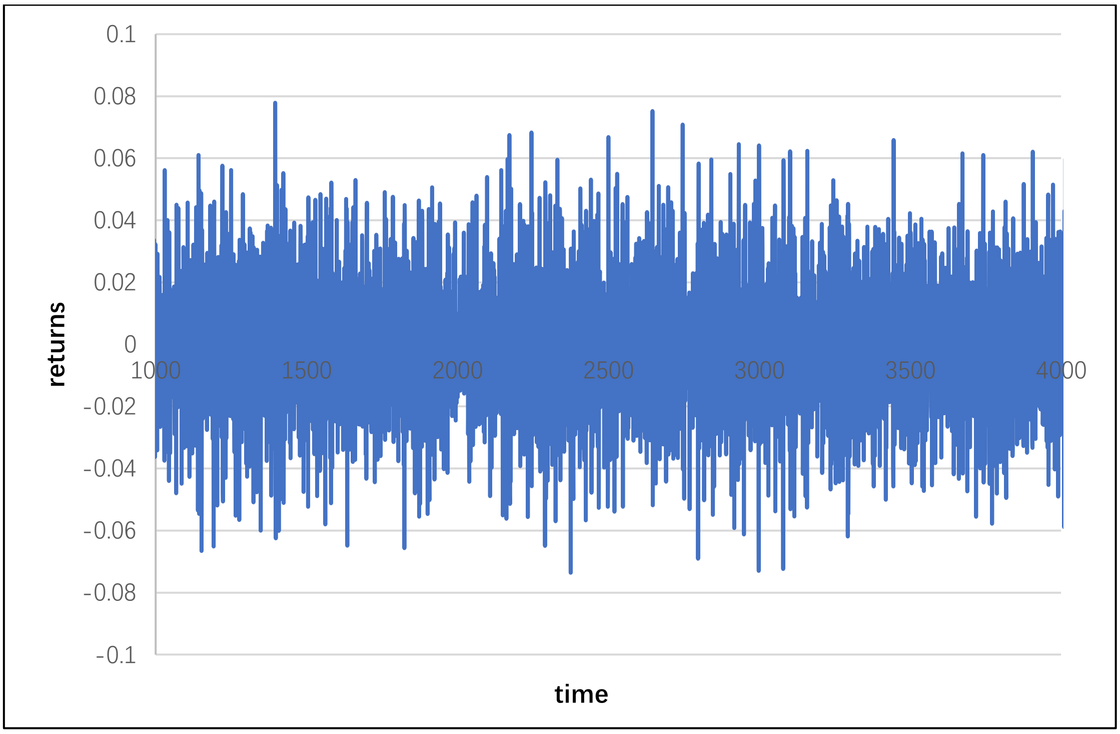
The statistical values in Table 5.13 and 5.15 are relatively similar to the results before in Table 5.10 and 5.12. Comparing the values in Table 5.15 and 5.12, the values for d are higher for this model and in that sense the volatility clustering is much more obvious than for the model with price effect. There is volatility clustering in that there are periods of high and low volatility. The difference is that the periods of high volatility are long and periods of low volatility are short.

The values in Table 5.14 show that there are almost no fat tails in the model without price effect. Compared with the model with price effect and the model without price effect, the model without price effect has on-off intermittency differently and the stable period is really small, which leads to no stylised fact of fat tails.

Overall, the work here indicates that the price effect is one of the main factors that drives the model back to its stable period.

5.5.5 With constant probability

The model with constant probability discusses in this section. The probability changing from fundamentalists and chartists is fixed at 0.005. And the probability of changing between chartists is fixed at 0.015. The scale of the probability setting is based on the generated probability in the replicating model.



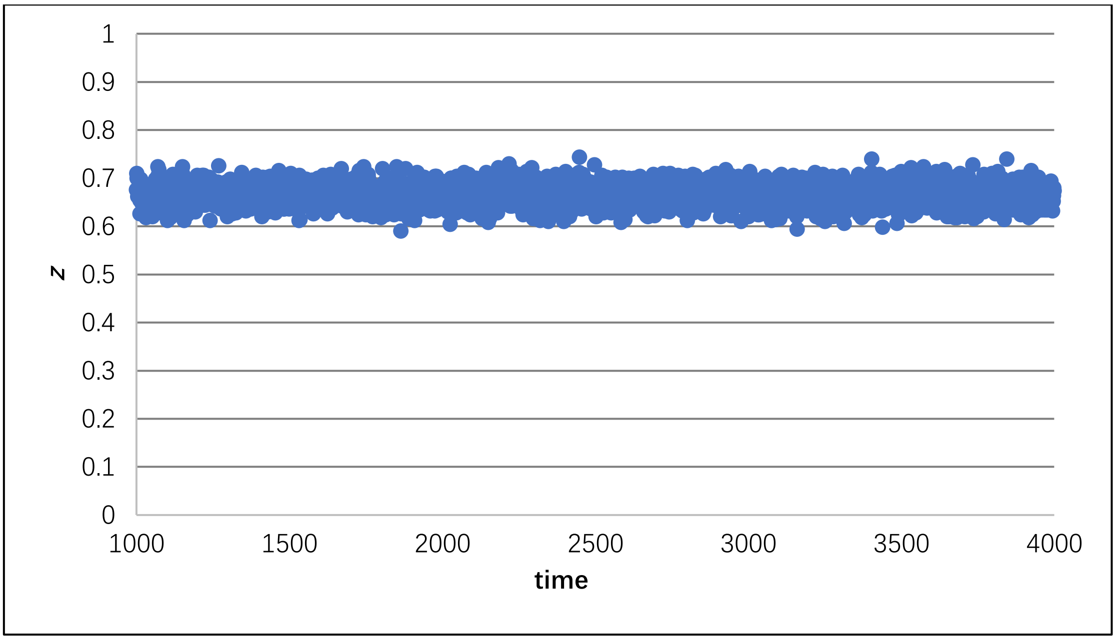


Figure 5. 24 Returns and *z* from 1000-4000 in the constant probability model

The patterns of returns and fraction of chartists are very different from the patterns before in Figure 5.10, 5.23. The fraction of chartists is concentrated between 0.6-0.7 because of constant switching probability. The returns do not have on-off intermittency with the volatile period and the stable. Compared to returns in Figure 5.10, 5.23 and 5.24, returns in Figure 5.23 stay in the volatile period to some extent because of a large fraction of chartists. The settings for the constant probability, in this case, produce a large number of chartists. This confirms that the model does not change the volatile period or stable period without the price effect and randomness.

5.5.6 Understanding the on-off intermittency

If the discussion before is in the right direction, then the analytical model with the probabilities based on the relationships in Figure 5.13, 5.14, 5.15, 5.16, 5.17 and 5.18 will generate a similar result to the model with price effect in Section 5.4.

The probability rules of changing between chartists and fundamentalists are based on equation (5.20) (5.21) (5.22) (5.23) in Section 5.5.2:

(5.20) $\pi_{f\to+} = 0.00004 * n_+ + 0.000005$

(5.21) $\pi_{+\to f} = 0.00004 * n_f + 0.0001$

(5.22) $\pi_{f\to-} = 0.00004 * n_- + 0.000005$

(5.23) $\pi_{-\to f} = 0.00004 * n_f + 0.0001$

The probability rules of changing between chartists are based on equation (5.24) (5.25):

(5.24) $\pi_{-\to+} = 0.0001 * n_+ + 0.0002$

(5.25) $\pi_{+\to-} = 0.0001 * n_- + 0.0002$

The reference numbers for the equations for switching between the agent types are as follows:

$$\begin{tikzcd} & n_+ \text{ Eq. (5.20)} \\ n_f \arrow[ur] \arrow[dr] & \\ & n_- \text{ Eq. (5.22)} \end{tikzcd}$$

$$\begin{tikzcd} & n_- \text{ Eq. (5.25)} \\ n_+ \arrow[ur] \arrow[dr] & \\ & n_f \text{ Eq. (5.21)} \end{tikzcd}$$

$$\begin{tikzcd} & n_+ \text{ Eq. (5.24)} \\ n_- \arrow[ur] \arrow[dr] & \\ & n_f \text{ Eq. (5.23)} \end{tikzcd}$$

For each agent, the switching probability of changing into the other two types are decided. And then, the number of agents in each type is calculated. The price formation process keeps unchanged following Section 5.3.3.3.

Figure 5.25 shows returns and the fraction of the chartists of the analytical model.

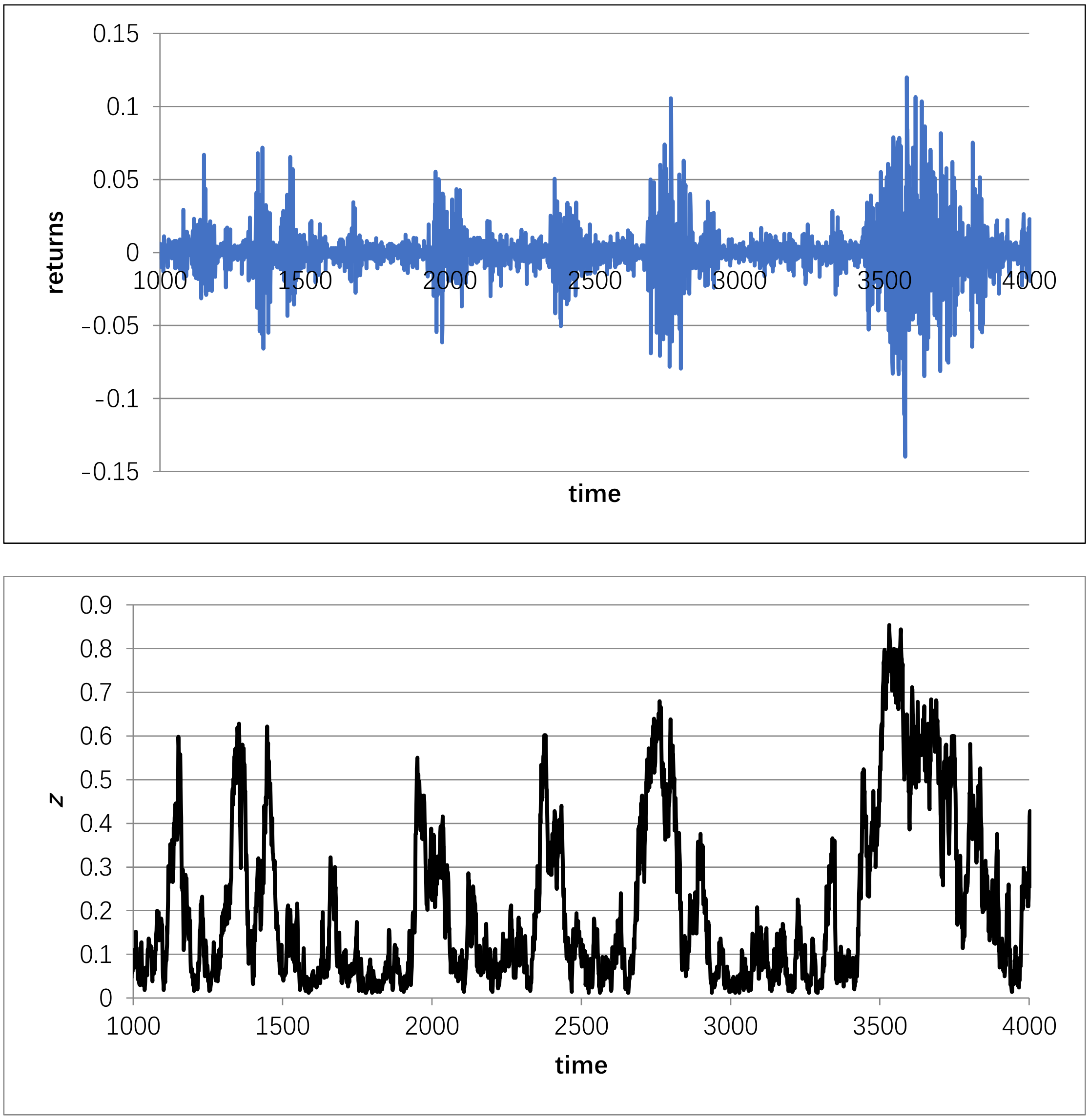


Figure 5. 25 Returns and z from 1000-4000 in the analytical model

The patterns in Figure 5.25 are similar to those in Figure 5.10 to some extent. The autocorrelations of raw returns squared returns and absolute returns are illustrated in Figure 5.26.

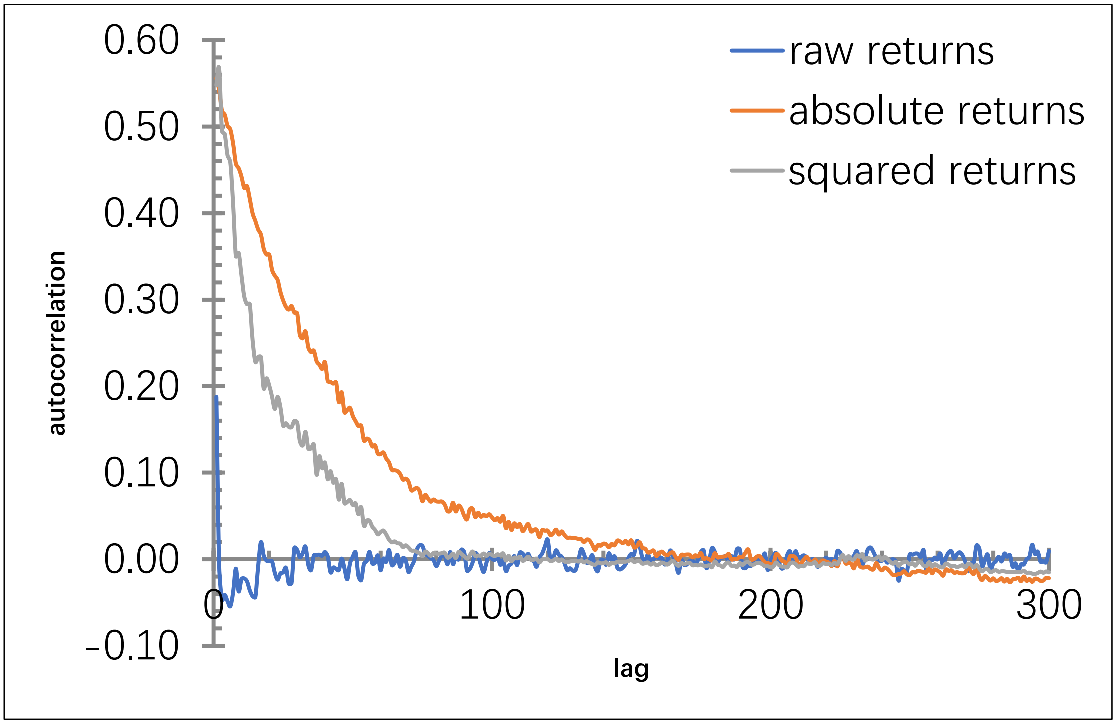


Figure 5. 26 Autocorrelations in the analytical model

The patterns of autocorrelations are similar to the results in Figure 5.12 before. The autocorrelation of absolute returns in Figure 5.26 decreases to 0 when lag is 150 and the autocorrelation of squared returns decreases to 0 when lag is 100. In Figure 5.12, it decreases to 0 with both absolute returns and squared returns when the lag is 100.

In Table 5.16, 5.17 and 5.18, the unit root, fat tails, and *d* in the analytical model are shown.

Parameters	range
Parameter set 1	0.999220-1.000330

Table 5. 16 Unit root in the analytical model

median αH from 10 samples of 2,000 observations				
Parameters	kurtosis	2.5%tail	5%tail	10%tail
Parameter set 1	19.83	3.27 (1.74-5.01)	2.67 (1.69-3.52)	1.99 (1.74-2.63)

Table 5. 17 Fat tails in the analytical model

	squared returns	absolute returns
Parameters	median d from 10 samples of 2000 observations (in parentheses:range of estimates)	median d from 10 samples of 2000 observations (in parentheses:range of estimates)
Parameter set 1	0.07 (-0.01)-0.28	0.17 0.10-0.34

Table 5. 18 d in the analytical model

Compared with the statistical value in Table 5.16, 5.17, 5.18 and 5.10, 5.11, 5.12, they are similar to some extent. That indicates that the model with probabilities just depends on the number of agents and can generate similar results with the replication model and also similar stylised facts like volatility clustering and fat tails. In this model, the price effect is replaced by the intercept part of a model depending only on the number of agents.

The pattern of on-off intermittency is somehow similar to the original one and also the replication model. That gives a clue that the price effect is the key force that drives the returns and fraction of chartists back to its stable stage. The volatile stage starts randomly and the price effect drives the volatile stage quite quickly back to its normal stage.

5.6 Conclusions

In conclusion, my replication model and the original model have the same patterns of prices, returns, and the fraction of chartists. Although the statistical measures of unit root, fat tails, and volatility clustering are varied, most of them are statistically similar. My replication model and the original model generate both fat tails and volatility clustering. The model has been replicated at a rational alignment level.

The probability set by equations is not just related to the number of a certain type of agents but also the price effect. A linear relationship is found between the probabilities and the number of a certain type of agent. The price effect is negatively related to probability from fundamentalists to chartists, and it is positively related to probability from chartists to fundamentalists. The price effect is big when the returns are big then it decreases the number of agents to chartists and increases the number of agents to fundamentalists. This indicates price effect dives the returns back to its stable stage. If the price effect is replaced by a constant intercept found in the linear relationship, the analytical model is built on understanding the herding. The reasons behind the on-off intermittency are identified. Most of the time, returns are stable and the fraction of chartists is relatively low. The volatile period starts mainly because of random variation. From Section 5.5.4, even without the price effect, the probability is only related to the number of certain agents. The volatile period still starts but it lasts longer. From Section 5.5.5, how the returns and number of chartists are changed with constant probability is shown. The analytical model is built in Section 5.5.6. Comparing the differences in Section 5.5.4 and Section 5.5.6, the price effects drive the returns back to their stable stage quicker. The analytical model also confirms that if the model uses the intercept to replace the price effect, similar results can be generated. The effect will drive the model back to its stable stage quicker than the one without price effects.

Chapter 6

Discussion

This chapter discusses the replication issues of the two replicating models in Chapter 4 and 5 related to the replication literature in Chapter 3. Also, this chapter extends the understanding of herding and the related stylised facts by comparing the two models' results.

Three main aspects will be discussed in this chapter. One is to compare the model described in Chapter 4, which is without the help of the Stress Guideline, and 5, which is with the help of the Stress guideline. Besides adding the ODD guideline and other replication guidelines introduced in Chapter 3, this chapter suggests a guideline based on both STRESS guideline and ODD guideline with some other minor suggestions.

The second is to identify all the main and minor replication issues in the replicating models in chapters 4 and 5 separately.

The third is to compare the two models' results to extend the understanding of herding. The first replicating model in Chapter 4 has a network herding mechanism with an artificial market. The second replicating model in Chapter 5 has transition probabilities herding mechanism with a demand and supply market. By discussing the results from the two replication models with different mechanisms, how herding can produce certain stylised facts like fat tails and volatility clustering will be analysed with the herding literature in Chapter 2.

Section 6.1 discusses the replication framework. Section 6.2 discusses the replication issues of the two replicating models found in Chapter 4 and 5 in detail. Then Section 6.3 analyses herding and related stylised facts.

6.1 Replication framework

This section discusses the replication experience of my own in Chapter 4 and 5, also the related guidelines mainly about the STRESS and ODD guidelines. Firstly, it focuses on how the framework can improve the model description by comparing the first model described in Chapter 4, without the STRESS guideline, to the second model in Chapter 5, written with the STRESS guideline. Also, it shows the reflection of applying the guideline in Chapter 5. Secondly, by comparing the similarities and differences of the STRESS guideline and ODD guideline, a new combination of both guidelines is suggested. And also how Chapter 5 will change if it follows the new guideline. Some minor suggestions are added from the guideline literature in Chapter 3 for the new suggested guideline.

6.1.1 STRESS guideline

Although the first model in Chapter 4 tries to cover everything, some parts are still not clearly stated compared to Chapter 5.

In Chapter 4, compared to Chapter 5, the experimentation aims do not clearly describe the investigation objective according to STRESS guideline 1.3. The components for entities, activities, resources, queues, and entry/exit, are not pointed out specifically, according to guideline 2.5. Variables are not listed according to guideline 3.3. The implementation information for software or programming language, random sampling, model execution, system specification is missing according to guideline item 5. For Chapter 4, the implementation and code access are similar to Chapter 5: For the code, It runs in Windows with language c++. The software Code Block 10.05 is used to run the code. The random number generator is the built-in version in c. It runs with fixed time steps. The runtime is about 4 minutes for one run. It runs on the laptop. The code is accessible on the DOI 10.5281/zenodo.5524993. Or by the email request by x.liu1@lancaster.ac.uk or liuxinbess@126.com.

Experimentation aims with investigation objectives help the reader to compare the model results more easily. If several experiments apply for one model, it is better if each experiment's aims are stated.

Each agent's components are clearly outlined in the second model description to help the reader have a better overview and understanding. Components such as environment, interaction and entry or exit of agents, help the reader understand the model easier. Besides, it helps the author conclude the model in a simpler way, including the main and basic components of the model.

It is better to list all the variables and parameter settings in a table at first. It is easier to get an idea of the fixed values in the model and variables that change from time to time. For model replication, it is much easier to look up variables in the variables table than finding the value of variables throughout the whole paper. Besides, building a table for variables gives the author one more time to check the model variables and the scales. That will help the author to include all values of variables appropriately. The replication issues in the original papers had the omission of variables value or errors in the variables from the first and second model, making my replication harder.

For implementation information, including it helps to build the model. Also, it helps to find the reason if the results differ from each other for replication.

The reflection of Chapter 5 applying the STRESS guideline is shown in Table 6.1.

Section/Subsection	Item	Section
1. Objectives		
Purpose of the model	1.1	5.1.1
Model Outputs	1.2	5.4 and 5.5
Experimentation Aims	1.3	5.4 paragraph 2 above Figure 5.2 and 5.5
2. Logic		
Base model overview diagram	2.1	5.1.3 Figure5.1
Base model logic	2.2	5.1.3 and related 5.3.3 (equation 5.6,7,9,10,12,13,17,18)
Scenario logic	2.3	5.4 paragraph 2 above Figure 5.2, 5.5.1, 5.5.4, 5.5.5 and 5.5.6 paragraph 1
Algorithms	2.4	5.3.3
Components	2.5	5.2 first 4 and related 5.3.3
3. Data		
Data sources	3.1	5.1.1
Pre-processing	3.2	not applicable
Input parameters	3.3	5.1.2
Assumptions	3.4	5.2 last 1
4. Experimentation		
Initialisation	4.1	5.3.1
Run length	4.2	5.4 paragraph 1
Estimation approach	4.3	5.4 paragraph 1
5. Implementation		
Software or programming language	5.1	5.3.3.3 last 1
Random sampling	5.2	5.3.3.3 last 1
Model execution	5.3	5.3.3.3 last 1

System Specification	5.4	5.3.3.3 last 1
6. Code Access		
Computer Model Sharing Statement	6.1	Appendix B

Table 6. 1 The checklist applied in the second replication model

From the checklist in Table 6.1, with the STRESS guideline's help, Chapter 5 includes everything in the checklist. Comparing the description in chapters 4 and 5, and the reflection of Chapter 5 shows how the conceptual framework helps to include everything needed clearly. From Table 6.1, the sections reflect the checklist spread out across the table. For this checklist, the writing order does not follow this checklist order, making it more difficult for the reader to find the position of specific information.

6.1.2 Improve the guideline

From my own replication experience and the replication literature, there are some suggestions for the STRESS guideline. It is better to order the framework to help the reader quickly understand and compare the models. Sometimes, when you read a paper, you may not want to understand the entire content but parts of it. The ordered guideline will help you to locate the information you need quickly. If used for many models the reader will learn and remember where to find each item of information. Comparing the descriptions in Chapters 4 and 5, although the titles outline each section's contents, it is still not that easy to locate specific information very quickly to compare as they are written in a different order and have different titles. Especially since these two models have very diverse mechanisms. The framework's order is clearly stated in Grimm et al.'s ODD protocol mentioned in Chapter 3.4 and has been applied in some papers. Also, for the equations presentation, it is better to follow the model flowchart. From my experience, it took lots of time to make sure from the original paper how the model worked in Chapter 4. The equations are not listed by order, and there are many different parameters, making replicating difficult. Also, some equations in the middle are not really used to run the model, which makes the model description even harder for understanding. In the ODD protocol, the design concepts are extra features compared to the STRESS guideline. It focuses on the model's design to include the model design aspects and communication such as emergent behaviour, interaction, stochasticity and observation. That's why the framework suggested below is the combination of the ODD protocol and the STRESS guideline. For the modelling part, the framework is mainly based on the ODD protocol with the order. Because the ODD did not suggest the guidelines for results, for the results part, the framework is ordered by replication experience from Chapter 4 and 5.

Therefore, Table 6.2 below is the modified guideline that ordered the modelling and results combining the STRESS and ODD guideline (Grimm et al., 2006: 116-119; Monks et al., 2019: 20170916_Appendix_STRESS_ABS_R1). The modified guideline applies in agent-based simulation in financial markets particularly, as my experience of modifying the guideline comes from this area. Broadly, it can be used in agent-based simulation as the STRESS guideline applies. But this needs more experience from putting it into practice, as mentioned in Chapter 7, for future work. For the modelling part, the guideline is divided into overview, design concepts and details. For Section 1.1, overview, the purpose of the model needs to start first. Then Section 1.2, variables and scales are discussed. The first two sections demonstrate why the model is built and what variables are used in the model. Hence, in Section 1.3, process overview and scheduling are described to give an overview picture of the model based on the previous two sections. Section 2 is the design concepts, which demonstrate how the model is designed for emergent behaviour interaction and so on. STRESS guideline doesn't have this section in the checklist. Section 3 is the details section. The discussion of this part in the ODD is not as comprehensive as the STRESS guideline. Thus, Section 3 is mainly based on the STRESS guideline. Data sources, pre-processing and input are Section 3.1, 3.2 and 3.3 respectively. These three parts discuss how to get the data, how to sort data out, what data are used in the model in the sequence. In Section 3.4, initialisation and assumptions are described based on the input data. And submodels are described in detail in Section 3.5 with the environment, agents, algorithms, interaction topology and entry/exit. For the results in Section 4: 4.1 states the experiment aims; 4.2 introduces the run length; 4.3 discusses the estimation approach, and 4.4 analyses model scenario and outputs. For Section 4, the type of experiment states first. And then the run length and the number of runs are introduced if it is stochastic. At last, the analysis is discussed based on the information given in Section 4.1 to 4.3. Section 5 discusses the implementation and code access. In Table 6.2, the first column is the section name. The second column is the item number, and the number in the bracket indicates the original item number in the STRESS guideline. The third and fourth column is the recommendation part: the first column is the original description in the STRESS guideline, and the second column is from the ODD guideline. The preferred one is put in bold. For item 2.1, the preferred one is from ODD and some STRESS description from item 3.5.

Section/Subsection	**Item**	**Recommendation**	
1. Overview		STRESS	ODD
Purpose	1.1 (1.1)	Explain the background and rationale for the model	**State why you need to build a complex model, and what, in general and in**

			particular, you are going to do with it.
Variables and scales	1.2 (3.3)	**List all input parameters in the model, providing a description of each parameter and the values used. For stochastic inputs provide details of any continuous, discrete or empirical distributions used along with all associated parameters. Where applicable define the time/spatial dependence of parameters and any correlation structure.** **Clearly state:** • **Base case inputs** • **Inputs used in experimentation, where different from the base case.** • **Where optimisation or design of experiments has been used, state the range of values that parameters can take.** **Where theoretical distributions are used, state how these were selected and prioritised above other candidate distributions.**	Describe the full set of variables and state the scales addressed by the model, i.e. length of time steps and time horizon.
Process overview and scheduling	1.3 (2.1)	Provide one or more of: state chart, process flow or equivalent diagrams to describe the basic logic of the base model to readers. Avoid complicated diagrams in the main text.	**Provide a verbal, conceptual description of each process and its effects; Describe the scheduling of the model processes. This deals with the order of the processes and, in turn, the order in which the state variables are updated.**
2. Design Concepts			
Design concepts	2.1	**Environment** **Describe the environment agents interact within, indicating its structure, and how it is generated. For example, are agents bound within a homogeneous grid, or do they have continuous movement through a detailed landscape incorporating geographic or environmental information?** **Agents** **List all agents and agent groups within the simulation. Include a**	**Provide a common framework for designing and communicating (emergency, interaction, stochasticity, observation, environment, agents, entry/exit)**

		description of their role in the model, their possible states, state transitions, and all their attributes. **Describe all decision-making rules that agents follow in either algorithmic or equation form. Where relevant, authors should report:** • **The data that agents access (i.e. internal attributes or external information from the environment) and how it is used.** • **The objectives agents seek to achieve.** • **The algorithms, optimisations, heuristics and rules that agents use to achieve objectives.** • **How agents work together within a group along with any rules for changes in group membership.** • **Predictions of future events and adaptive action.** **Entry / Exit** **Where relevant, define how agents are created and destroyed in the model.**	
3. Details			
Data sources	3.1 (3.1)	**List and detail all data sources. Sources may include:** • **Interviews with stakeholders** • **samples of routinely collected data,** • **prospectively collected samples for the purpose of the simulation study,** • **public domain data published in either academic or organisational literature. Provide, where possible, the link and DOI to the data or reference to published literature.** **All data source descriptions should include details of the sample size, date ranges and use within the study.**	
Pre-processing	3.2 (3.2)	**Provide details of any data manipulation or filtering that has taken place before its use in the simulation, e.g. interpolation to account for missing data, removal of outliers or filtering of large scale data.**	
Input	3.3 (3.3)	Section 1.2	**All the**

			environmental conditions, which change over space and time, are "input", i.e. imposed dynamics of certain state variables. The model output gives the response of the model to the input. Readers need to know what input data are used, how they were generated or obtained. To really achieve full reproducibility it might be necessary to provide (in online archives) the input files that you used yourself, including even the random number used as seed.
Initialisation and assumptions	3.4 (3.4 &4.1)	**State if a warm-up period has been used, its length and the analysis method used to select it. (State what, if any, initial agent and environmental conditions have been included. For example, the initial agent population size, agent states and attributes, initial agent network structure(s), and resources within the environment. Report whether initialisation of these variables is deterministic or stochastic.** **Where data or knowledge of the real system is unavailable, state and justify the assumptions used to set input parameter values and distributions; agent interactions or behaviour; or model logic.**	Deal with such questions as: How are the environment and the agents created at the start of a simulation run?
Submodels	3.5 (2.2&2.4&2.5)	Give details of the base model logic. This could be text to explain the overview diagram along with extra details including ABS product and process patterns. Include details of all intermediate calculations. 3.5.1 Environment Describe the environment agents interact within, indicating its structure, and how it is generated. For example, are agents bound within a homogeneous grid, or do they have	**Present and explain all submodels representing the processes listed above in 'Process overview and scales' in detail (including ABS product, process patterns and the parameterisation of the model. Give**

	continuous movement through a detailed landscape incorporating geographic or environmental information? 3.5.2 Agents List all agents and agent groups within the simulation. Include a description of their role in the model, their possible states, state transitions, and all their attributes. Describe all decision-making rules that agents follow in either algorithmic or equation form. Where relevant, authors should report: • The data that agents access (i.e. internal attributes or external information from the environment) and how it is used. • The objectives agents seek to achieve. • The algorithms, optimisations, heuristics and rules that agents use to achieve objectives. • How agents work together within a group along with any rules for changes in group membership. • Predictions of future events and adaptive action. 3.5.3 Algorithms Provide further detail on any algorithms in the model that (for example) mimic complex or manual processes in the real world (i.e. scheduling of arrivals/appointments/operations/maintenance, operation of a conveyor system, machine breakdowns, etc.). Sufficient detail should be included (or referred to in other published work) for the algorithms to be reproducible. A Pseudo-code may be used to describe an algorithm. 3.5.4 Interaction Topology Describe how agents and agent groupings are connected with each other in the model [and] define: • with whom agents can interact, • how recipients of interactions are selected • what frequency interaction occurs. • How agents handle and	**details of the base model logic including all intermediate calculations.)**

		assign priorities to concurrent events It is recommended that interactions are described using a combination of equations, pseudo-code and logic diagrams. Report how interactions are affected by agent states and the environment state 3.5.5 Entry / Exit Where relevant, define how agents are created and destroyed in the model.	
4 Result			
Experimentation Aims	4.1 (1.3)	**If the model has been used for experimentation, state the research questions that it was used to answer.** **a.) Theory driven analysis. – Provide details and reference the theories that are tested within the model.** **b.) Scenario based analysis – Provide a name and description for each scenario, including a rationale for the choice of scenarios. (Give details of any difference in the model logic between the base case model and scenarios. This could be incorporated as text or, where differences are substantial)** **c.) Design of experiments – Provide details of the overall design of the experiments with reference to performance measures and their parameters (provide further details in *data* below).** **d.) Simulation Optimisation – (if appropriate). Provide full details of what is to be optimised, the parameters that were included and the algorithm(s) that was be used. Where possible provide a citation of the algorithm(s).**	
Run length	4.2 (4.2)	**Detail the run length of the simulation model and time units.**	
Estimation approach	4.3 (4.3)	**State if the model is deterministic or stochastic. If the model is stochastic, state the number of runs that have**	

		been used. If an alternative estimation method has been used (e.g. batch means), provide full details.
Model Scenario (if any) and Outputs	4.4 (1.2&2.3)	**4.4.1 Model scenario** **Give details of any difference in the model logic between the base case model and scenarios. This could be incorporated as text or, where differences are substantial, could be incorporated in the same manner as 1.3.** **4.4.2 Model outputs** **State the qualitative or quantitative system level outputs that emerge from agent interactions within the ABS.** **Define all quantitative performance measures that are reported, using equations where necessary. Specify how and when they are calculated during the model run along with how any measures of error such as confidence intervals are calculated**
5 Implementation and Code Access		
Software or programming language	5.1 (5.1)	**State the operating system and version and build number.** **State the name, version and build number of commercial or open source ABS software that the model is implemented in.** **State the name and version of general-purpose programming languages used (e.g. Python 3.5.2).** **Where packages, frameworks and libraries have been used provide all details including version numbers.**
Random sampling	5.2 (5.2)	**State the algorithm or package used to generate random samples within the software/programming language used e.g. Mersenne Twister or Java. Random version x.y**
Model execution	5.3 (5.3)	**If the ABS model has a time component, describe how time is modelled (e.g. fixed time steps or discrete-event). State the order of variable updating within the model. In time-stepped execution state how concurrent events are resolved.** **If the model is parallel, distributed and/or uses grid or cloud computing, etc., state and preferably reference**

		the technology used. For parallel and distributed simulations the time management algorithms used. If the HLA is used then state the version of the standard, which run-time infrastructure (and version), and any supporting documents (FOMs, etc.)	
System Specification	5.4 (5.4)	**State the model run time and specification of hardware used. This is particularly important for large scale models that require substantial computing power. For parallel, distributed and/or use grid or cloud computing, etc. state the details of all systems used in the implementation (processors, network, etc.)**	
Computer Model Sharing Statement	5.5 (6.1)	**Describe how someone could obtain the model described in the paper, the simulation software and any other associated software (or hardware) needed to reproduce the results. Provide, where possible, the link and DOIs to these.**	

Table 6. 2 The altered framework

Comparing the guideline from STRESS and ODD, most of the concepts they are using have similar meanings. But for the input, these two guidelines have a different focus. In the STRESS guideline, input means all the input parameters; in the ODD, input emphasises environmental conditions, which change over space and time and input files. The difference between these two guidelines is that the variables and scales are already emphasised in Section 1.2 in the ODD guideline.

For the altered framework in Table 6.2, the preferred recommendation is in bold: for ODD -1.1, 1.3, 2.1 plus STRESS 3.5 for environment, agents, enter/exit, 3.3, 3.5; for STRESS - 1.2, 3.1, 3.2, 3.4, 4.1, 4.2, 4.3, 4.4, 5.1, 5.2, 5.3, 5.4, 5.5.

From this altered framework in Table 6.2, the writing in Chapter 5 would need to be reordered. All the item numbers in Table 6.2 would be modified in the chapter by adding 5 in the front like item 3.1 in Chapter 5 becomes 5.3.1. The changes in Chapter 5 would be as follows:

1. For Section 5.3 submodels, 5.3.1 is the data source section: All data are from the original paper.
2. 5.3.2 is the pre-processing: The model does not involve any pre-processing stages.
3. 5.3.3 is the input moved from the original 5.3.2 and adding: All inputs are discussed in Section 5.1.2 before.
4. 5.3.4 is the initialisation and assumptions changing from the original 5.3.1 and adding assumptions from the original 5.2.1. Delete the assumptions from the original 5.2.1.
5. 5.3.5 is the submodels from the original 5.3.3.
6. For Section 5.4 results and comparisons, 5.4.1 is the experimentation aims section: This model is a scenario based analysis. The scenarios are: replication results in the original Section 5.4 (changes to 5.4.4.1 later at 9); increment fixed model in Section 5.5.1 (changes to 5.4.4.2); model without price effect in Section 5.5.4 (changes to 5.4.4.5); model with constant probability in Section 5.5.5 (changes to 5.4.4.6); analytic model in Section 5.5.6 (changes to 5.4.4.7). The aims are accordingly: Compare replication results with the original paper to confirm the original paper's reproducibility; Uncertainty of time increment changes was investigated by keeping the increment fixed and comparing the results; The model without the price effect is analysed to confirm whether the price effect is a stabilising force; Analytical model with the probabilities based

on the relationships generate a similar result to the model with price effect to confirm the on-off intermittency theory.

7. 5.4.2 is the run length section: This model is for a long period of 20000, and the time unit in this model is not specified.
8. 5.4.3 is the estimation approach - Although it is a stochastic model, only one long run is used for each scenario.
9. 5.4.4 is the model scenario and outputs - the whole output section from 5.4 to 5.5. Combine 5.4 and 5.5 to one Section 5.4.4. All subitem number changes from 5.4.4.1 for each discussion. For example, 5.4 changes to 5.4.4.1 and 5.5.1 changes to 5.4.4.2, and all the headings change accordingly in the original Section 5.5.
10. For Section 5.5 implementation and code access, 5.5.1 is the software or programming language - It runs in Windows, Code Block 10.05, c++.
11. 5.5.2 is the random sampling - The random number generator is the built-in version in c.
12. 5.5.3 is the Model execution - Fixed time steps.
13. 5.5.4 is the System Specification – The runtime is about 12 minutes for one long run. It runs on the laptop.
14. 5.5.5 is the Computer Model Sharing Statement - The code is accessible on the DOI 10.5281/zenodo.5524993. Or by the email request by x.liu1@lancaster.ac.uk or liuxinbess@126.com.
15. After this, Section 5.6 is the conclusions that keep unchanged.

There are some suggestions if the framework has more detailed requirements in these aspects. It will be more helpful to have a clearer coding standard. Stodden (2010) provided a coding standard in six areas mentioned in Section 3.3.

It is better to have meaningful words for the symbols in the equations to help to understand. This can help to avoid my replication issues from my second replication model for wrongly misunderstanding the equations. For simulation models, verification and validation are two very important elements. The STRESS guideline mentioned it and think verification and validation are not that essential to ensure reproducibility. It is often not done well in simulation projects, so including it in the checklist might help with that. It will be helpful to include these in the overview part for verification about how to demonstrate the model's conceptual framework and the result for validation regarding the level of confidence in the model and the validation tests carried out.

There may be too much parameter setting for complex modelling, which makes it impossible to include all of them and also the code. It is reasonable to put some part of the parameter set in the appendix rather than in the main text.

To conclude, by comparing the description between Chapter 4 and 5, Chapter 5 is more straightforward and includes everything needed. That shows how the guideline helps to improve reproducibility. The reflection of Chapter 5 shows how to apply the STRESS guideline in an agent-based simulation example. From my replicating experience, the guideline is improved, and some minor suggestions are stated. It is easier to apply the modified guideline, and it helps the reader to locate information more easily.

6.2 Replication issues

Two models are reproduced separately in Chapter 4 and 5. Although these two reproducing models are different, replication issues are found both in chapters 4 and 5.

The replication issues from the first Tedeschi et al. (2012) model described in Chapter 4 and from the second Lux and Marchesi (2000) model described in Chapter 5 will be discussed in detail.

6.2.1 Replication issues in the first model

In the first replication model in Chapter 4, there are two main issues related to the value of σ_0 and V_t . The discussions about these two main issues and the effects on the results come first. And then the discussion about some minor issues comes later.

There is no clue found in the paper of Tedeschi et al. (2012) to know the value of the parameter σ_0 . σ_0 is illustrated in Chapter 4 Section 4.1.3 to decide the value of σ_0^i . σ_0^i is used in equation (4.4) $\sigma_t^i = \sigma_0^i(1 + l_{i,t}^{\%}(1-w))$. It is introduced in Chapter 4 that in equation (4.4), σ_t^i is the return's volatility for agent *i* at time *t*, *w* is a herding factor, and $l_{i,t}^{\%}$ is the percentage of incoming links for agent *i* at time *t*. σ_0^i is a uniformly distributed value for agent *i* from 0 to σ_0 . The expected return of agent *i* is influenced by the return's volatility, which is σ_t^i in equation 4.4. Finally, the expected future price for agent i is decided by the expected return. After trading, the final market price will be influenced. The market price in this model is one of the essential indexes for two reasons: On one hand, price is an important parameter in the model mechanism itself to decide the agent's reaction for the next period. On the other hand, the market price is one of the essential model output. In this model, the aim is to find the results related to the herding. For price and returns specifically, the price variation caused by herding is proved by this model. The impact of σ_0 on return's volatility in equation (4.4) will be seen. These two tables below show the influence of parameters σ_0 and $l_{i,t}^{\%}$ leading to the change of σ_t^i. According to the equation (4.4), σ_t^i is decided by σ_0^i which is uniform distributed in the interval (0, σ_0), the incoming links $l_{i,t}^{\%}$ and the herding parameter *w*. It is assumed that σ_0^i = σ_0 to look at the influence of σ_0 . In Table 6.3 and Table 6.4, the first row is the value of σ_0 , and the first column is the value of the incoming links $l_{i,t}^{\%}$. The values in the tables are the values of σ_t^i. The first table (Table 6.3) is with *w* = 0.1 and the second table (Table 6.4) is with *w* =0.5:

$l_{i,t}^{\%} / \sigma_0$	0.001	0.010	0.050	0.500	1.000
0.1	0.0011	0.0109	0.0545	0.5450	1.0900
0.2	0.0012	0.0118	0.0590	0.5900	1.1800
0.3	0.0013	0.0127	0.0635	0.6350	1.2700
0.4	0.0014	0.0136	0.0680	0.6800	1.3600
0.5	0.0015	0.0145	0.0725	0.7250	1.4500
0.6	0.0015	0.0154	0.0770	0.7700	1.5400
0.7	0.0016	0.0163	0.0815	0.8150	1.6300
0.8	0.0017	0.0172	0.0860	0.8600	1.7200
0.9	0.0018	0.0181	0.0905	0.9050	1.8100
1.0	0.0019	0.0190	0.0950	0.9500	1.9000

Table 6. 3 σ_t^i with the change in σ_0 and $l_{i,t}^{\%}$ (*w*=0.1)

$l_{i,t}^{\%} / \sigma_0$	0.001	0.010	0.050	0.500	1.000
0.1	0.0011	0.0105	0.0525	0.5250	1.0500
0.2	0.0011	0.0110	0.0550	0.5500	1.1000
0.3	0.0012	0.0115	0.0575	0.5750	1.1500
0.4	0.0012	0.0120	0.0600	0.6000	1.2000
0.5	0.0013	0.0125	0.0625	0.6250	1.2500
0.6	0.0013	0.0130	0.0650	0.6500	1.3000
0.7	0.0014	0.0135	0.0675	0.6750	1.3500
0.8	0.0014	0.0140	0.0700	0.7000	1.4000
0.9	0.0015	0.0145	0.0725	0.7250	1.4500
1.0	0.0015	0.0150	0.0750	0.7500	1.5000

Table 6. 4 σ_t^i with the change in σ_0 and $l_{i,t}^{\%}$ (*w*=0.5)

From Table 6.3 and Table 6.4, σ_t^i increases when σ_0 gets bigger across each row. The influence caused by the percentage of the incoming links is relatively small, which can be observed by the slightly increasing data down each column. It is even smaller when the *w* gets bigger, comparing the difference in the σ_t^i value in each column from the first table to the second one. The data in the same position from these two tables show the influence of *w*. The increase in *w* causes a drop in σ_t^i. All in all, these two tables show all the impacts of these three control variables of σ_t^i, and σ_0 has the most significant impact. The value of σ_t^i is controlled by the parameter σ_0. Hence, it is deduced here that σ_0 has a significant impact on future price and also the price in the market.

Figures 6.1 to 6.3 are from the model testing of σ_0. The model testing runs for 30 agents and a time period of 50 with the same other parameters. For figures 6.1 to 6.3, the initial variance V_t is 0.001. Figure 6.1 is the market price for σ_0 =0.01, 0.05, 0.5 and 1 (blue line for σ_0 =0.01, red line for σ_0 =0.05, green line for σ_0 =0.5, purple line for σ_0 =1).

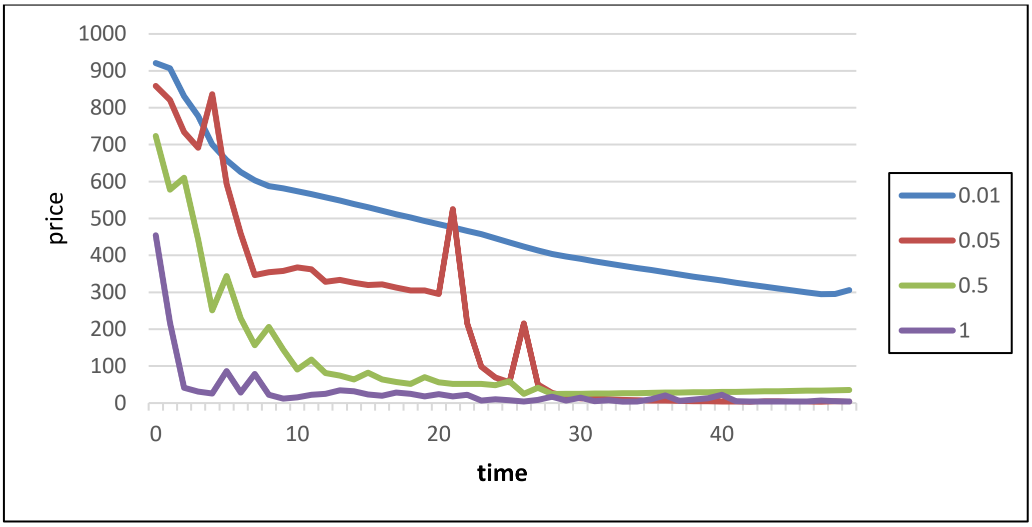
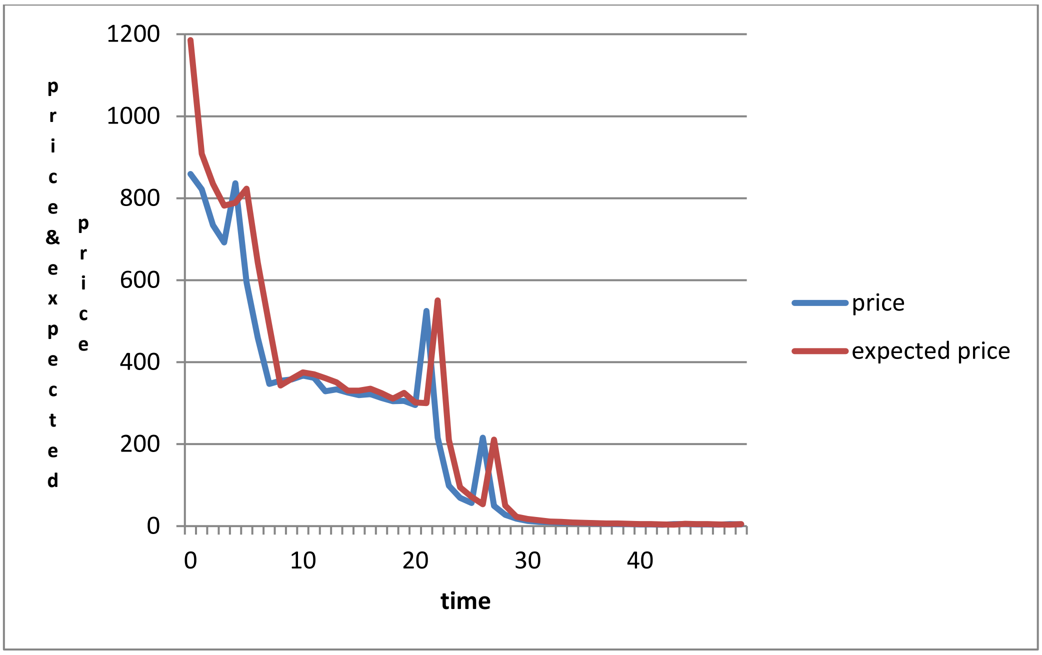


Figure 6. 1 Price with σ_0 =0.01 (blue), σ_0 =0.05(red), σ_0 =0.5 (green) and σ_0 =1 (purple)

Figure 6.1 shows that the market price drops heavily at the first ten periods for σ_0 =0.05, 0.5 and 1. The price behaves so differently compared with the result in the original paper. The price for σ_0 =0.01 doesn't drop that heavily. The price and expected price are shown in Figure 6.2 and Figure 6.3 for σ_0 =0.05 and 0.01.

Figure 6.2 below shows the market price and average expects price change with time series. Figure 6.2 is for σ_0 = 0.05 where the blue line is the market price and the red line is the average expected price.



<u>Figure 6. 2</u> Price & average expected price at $\sigma_0 = 0.05$

Figure 6.2 shows the price drop to almost zero after period 30. This is not the performance in the original paper. The price and expected price are above 500 pounds before period 50 and never get down to zero in the original paper. This indicates σ_0 =0.05 is not an appropriate setting.

Figure 6.3 is for σ_0 =0.01 below (the blue line is the market price, and the red line is the average expected price).

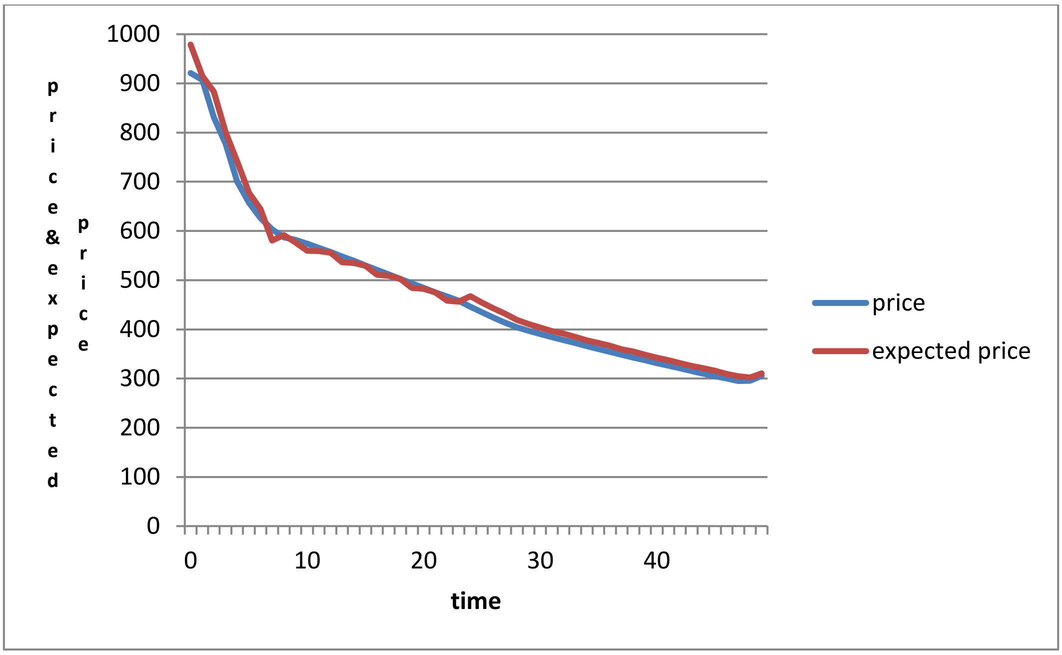


Figure 6. 3 Price & average expected price at $\sigma_0 = 0.01$

The price drops as in all three graphs (Figure 6.1-6.3) related to σ_0 above, but on average, the market price increases as σ_0 reduces. When σ_0 gets bigger, bigger volatility will cause both higher and lower expected price. From Figure 6.1, the bigger difference between agents expected price influences the market price and makes the market price drop. The performance for σ_0 = 0.01 is much better than the other three as the price drop is not that heavy, and in period 50, the price is still 300 pounds. This is a much more reasonable setting for the price. Also, if σ_0 gets even smaller, it may be not that reasonable. As σ_0 gets to 0.01, the idiosyncratic expected return for agents in equation (4.5) based on equation (4.4) becomes in a range of (-0.03, 0.03). If σ_0 gets even smaller than 0.01, the maximum expected return is less than 3%. It is too small to be realistic.

Except for the unclear settings of σ_0 , the description for how to calculate unconditional variance is not that clear as well, which is the V_t . In equation (4.10), $V_t = \frac{1}{\tau}\sum_{j=1}^{\tau} [r_{t-j} - \overline{r_t}]^2$. The unconditional variance depends on the previous returns, but for the start point, there are no previous returns. Therefore, it is hard to decide the value of the unconditional variance at the start. The unconditional variance V_t from equation (4.10) is used in equation (4.11) $V_t^i = V_t (1-(1-w)l_{i,t}^{\%})$ for the variance V_t^i and hence has an impact in (4.12) $\pi^i(p) = \frac{\ln(\hat{p}_{t,t+\tau}^i / p)}{\alpha V_t^i p}$ to decide the number of holdings. Assuming $\hat{p}_{t,t+\tau}^i$ is fixed at 1000 and α is set at 0.01. Table 6.5 below shows the change in the amount to hold $\pi^i(p)$ with the change in individual risk assessment variance V_t^i, which is dependent on the unconditional variance V_t , and the change in price. In each column from the second row to the last row, the amount to hold varies with the value of the price. In each row from the second column to the last column, the amount to hold varies with the value of the variance.

p / V_t^i	0.1000	0.0100	0.0025	0.0001
50	59.91465	599.14645	2396.58582	59914.64547
100	23.02585	230.25851	921.03404	23025.85093
150	12.64747	126.47467	505.89866	12647.46657
200	8.04719	80.47190	321.88758	8047.18956
250	5.54518	55.45177	221.80710	5545.17744
300	4.01324	40.13243	160.52971	4013.24268
350	2.99949	29.99492	119.97967	2999.49178
400	2.29073	22.90727	91.62907	2290.72683
450	1.77446	17.74462	70.97846	1774.46155
500	1.38629	13.86294	55.45177	1386.29436
550	1.08698	10.86976	43.47905	1086.97637
600	0.85138	8.51376	34.05504	851.37604
650	0.66274	6.62743	26.50972	662.74295
700	0.50954	5.09536	20.38143	509.53563
750	0.38358	3.83576	15.34304	383.57610
800	0.27893	2.78929	11.15718	278.92944
850	0.19120	1.91199	7.64795	191.19874
900	0.11707	1.17067	4.68269	117.06724
950	0.05399	0.53993	2.15972	53.99294
1000	0.00000	0.00000	0.00000	0.00000
1050	-0.04647	-0.46467	-1.85867	-46.46682
1100	-0.08665	-0.86646	-3.46582	-86.64562

Table 6. 5 $\pi^i(p)$ with the change in price and V_t^i

Table 6.5 shows that the equation that decides the amount each agent is willing to hold is very sensitive to the price and V_t^i. When the variance is changed, the amount to hold increases hugely with the decrease in variance value. Therefore the initialisation for the unconditional variance matters very much.

Figure 6.4 shows the price change in different variance settings with time series to period 50 where σ_0 is 0.01. The blue line shows the performance of the price when the variance is 0.1. The red line shows the price movements at variance 0.01. The green line shows the price change at the variance of 0.0025. The black line where the variance is 0.0001 and the purple line with variance 0.00001.

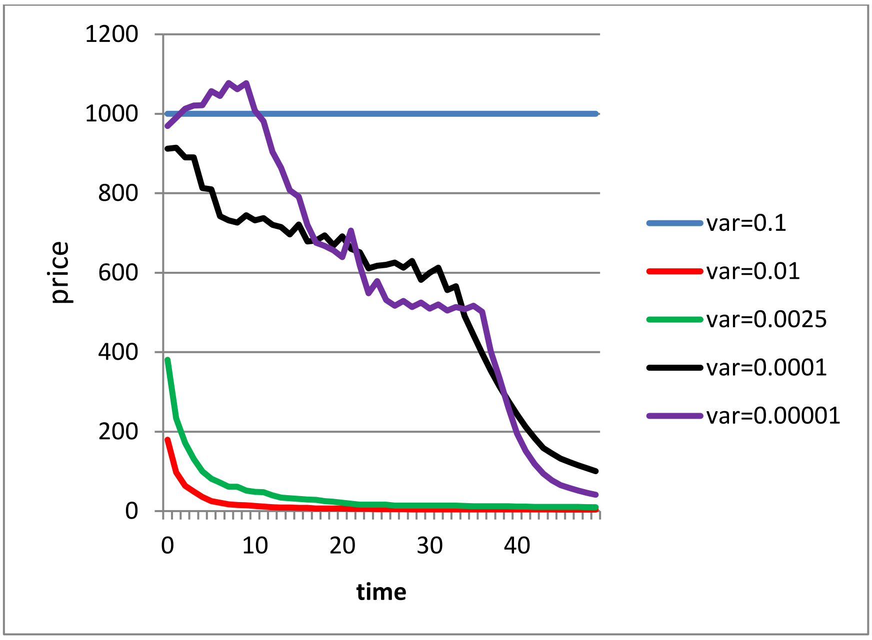


Figure 6. 4 The price change on different level of V_t

From Figure 6.4, when the variance is 0.1 in the initialisation of the blue line, then there are no buyers in the market at that period as the cash position equation (4.13) related to $\pi^i(p\ )$ produces a high price exceeding the p^* got from equation (4.14). Then there is no bid price. The price for the next period is the price before. Therefore the variance is just influenced by the initialisation setting for return at time 0. Then the market for the remaining period is just left with buyers only as the variance is so small to make agents willing to hold more than what they already have. Therefore there is no ask price in the market to change the market price (market price is the average of the highest bid and lowest ask) and also change the variance. As explained before, the price keeps the original level of 1000. The red line shows the price movements at variance 0.01. The price starts at 200 and drops down to nearly zero very quickly which indicates that 0.01 cannot be used when compared to results in Tedeschi et al. (2012). The green line shows the price change at variance equal to 0.0025. The price starts at 400 pounds and drops to nearly zero in just 50 periods. This sudden drop is not a reasonable price performance. The black line where the variance is equal to 0.0001 shows a better performance of the price time series. The purple line with the variance of 0.00001 has performance similar to the black line with some variation. Then the initialisation setting for 0.0001 is used in the model. It still gets down to zero by period 50. Hence, it still does not match the paper.

For the initialisation stage, the cash for each agent to hold is wrongly written in the paper as 100, whereas it should be 100000 according to the graph. For the market part, although the rules are explained in the paper, there is no detailed description of how to update the order book from a limit order to a market order to make the trade happen. For the formula, the time subscript is changing from time t to time $t + \tau$. And some subscript is missing in the original paper. Equations are not introduced in order. That can make readers lose track of how to use the equations easily. We try to contact the authors for all these issues in the model, but they can not precisely remember all these details as there are too many models that were built.

6.2.2 Replication issues in the second model

In the second model replication in Chapter 5, the main issues are how to apply the switching transition probabilities between chartists and fundamentalists, calculate the value of d and apply the measurement tests for both unit root and d. There are also some minor issues about how to set the initial number for fundamentalists and chartists and the symbols in the formulae.

The modelling description part is not detailed enough. In the original paper, there are three elements of the modelling process: (1) switching between chartists, (2) switching between fundamentalists and chartists, and (3) price formation. It is clear that the (3) price formation process is based on the (1) (2) changes among optimistic chartists, pessimistic chartists, and fundamentalists. The problem is how to apply the two switching strategies in the model. The strategies are either applied step by step as first (1) then (2) or first (2) then (1), or applied following in the second switching stage of my flowchart in Figure 5.1. This may not make a big difference as the switching among traders in each time interval is very rare, and the rules for each strategy are the same. The difference is just the order of rules. Hence the number of optimistic chartists, pessimistic chartists and fundamentalists are different when the rules apply.

The measurement d in the results analysis part with returns to measure the dependency for volatility clustering is not clearly described. The d is the exponential index which indicates the data used to calculate d follows the exponential distribution. While we applied the data to calculate the d, some of the data is negative. It is not clear what to do with the negative data when the result of the exponential function is always positive.

The tests used in the original paper in Table 5.5 of the unit root test and 5.9 for *d* test are not clear enough. In Table 5.5, the test applied based on either the Dickey-Fuller test or Augmented Dickey-Fuller test is vague. In Table 5.3, there is no clear description of how to do the test. This makes the comparison between my replication result and the original result not accurate enough.

For the initialisation, the fundamentalists, optimistic chartists, and pessimistic chartists are randomly separated. It is required that the number of fundamentalists is maintained to be reasonably high for stabilisation purposes. It is not explained clearly the definition of reasonably high and how to separate the fundamentalists, optimistic chartists, and pessimistic chartists at the very beginning. For the formula in the second model, it is a little bit misleading that makes me build the wrong model at the start. For the probability change from pessimistic to optimistic, the subscript is +-, which makes me think it was changed from optimistic to pessimistic.

From the two models, there are no apparent clues of how to decide the values, such as the values set, the number of runs, and the run length. And even for the equations, there is no clear clue of the theory about the equations. It may because the financial market is complicated. Therefore the model is built on what people think about how it works.

6.3 Herding

This section tries to understand herding and related stylised facts like fat tails and volatility clustering more by comparing the two replication models. Compared to the two models' results in chapters 4 and 5, the results are not similar. The replication models all have fat tails, but the first model does not have volatility clustering. The results can be easily identified from the returns. When we changed the herding mechanism for the altered first model, the model still did not have volatility clustering.

6.3.1 Herding in the first replication model

For the first replicated model, even when we compared the result with the strongest herding model, which means w=0.1, the pattern of the returns and price are so different from the pattern identified in the second model. The prices are not following the rules: the big price followed by the big price, the small price followed by the small price.

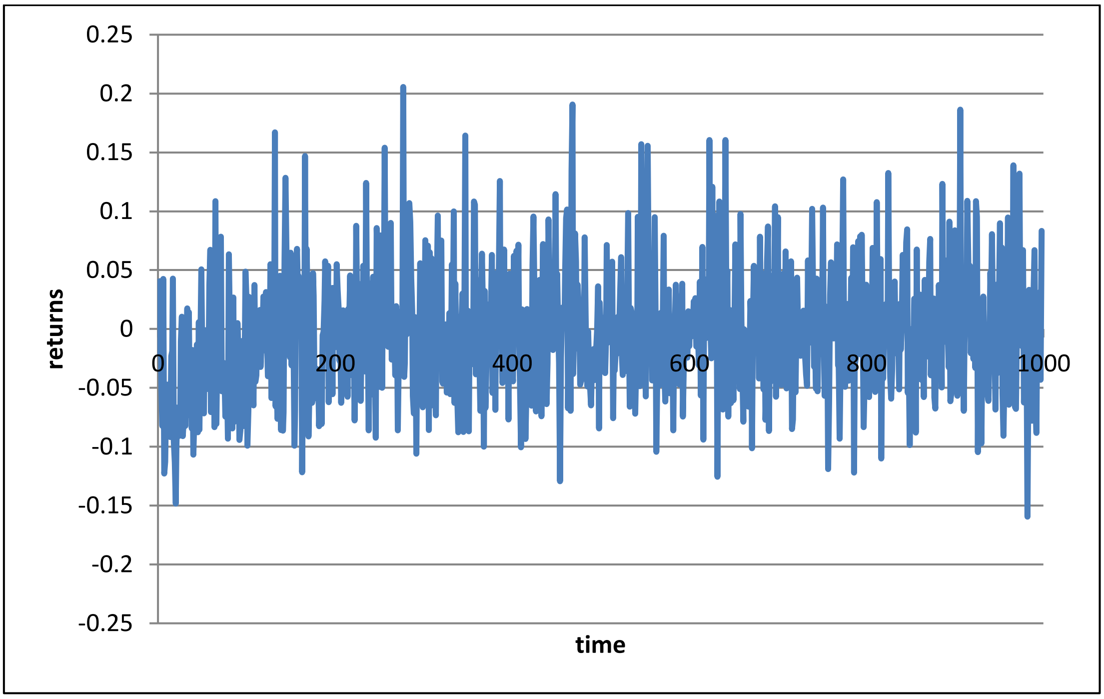


Figure 6. 5 Returns from 0-1000 in the first replication model when w=0.1 (seed 2)

Figure 6.5 shows that returns from the first replication model do have some extreme values and returns fluctuate around 0 with the kurtosis=0.80 when w=0.1. Compared to the second replication model, the kurtosis is much smaller, but it has some fat tails effect when we looked at it in the histogram with returns and the normal distribution.

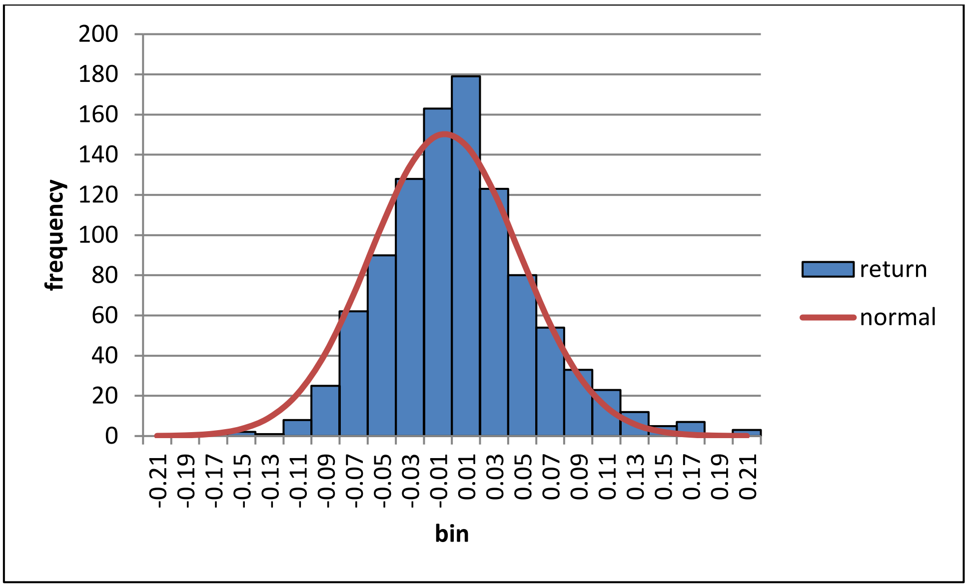


Figure 6. 6 Histogram and normal distribution from 0-1000 in the first replication model when *w*=0.1 (seed 2)

From the features in Figure 6.6, the returns have a slightly sharper distribution in the middle with fatter tails compared to the normal distribution.

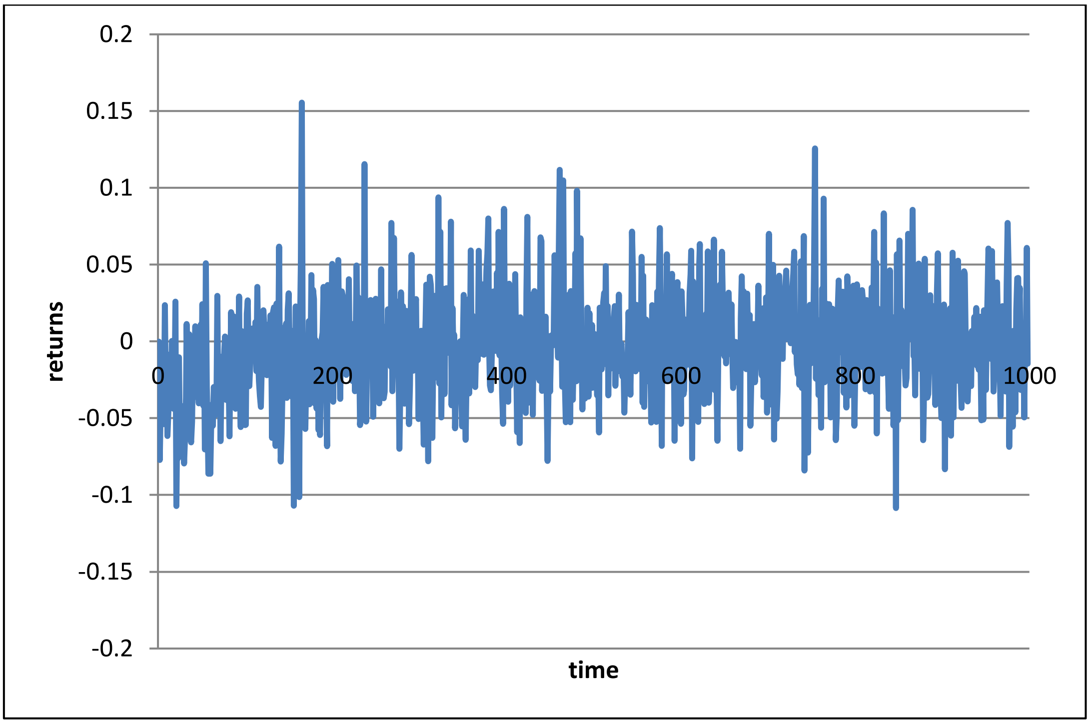


Figure 6. 7 Returns from 0-1000 in the first replication model when *w*=0.5 (seed 2)

From the features in Figure 6.7, the returns from the first replication model do have some extreme values, and returns fluctuate around 0 with the Kurtosis=0.86 when w=0.5. Compared to the first replication model when w=0.1 in Figure 6.6, the kurtosis is similar.

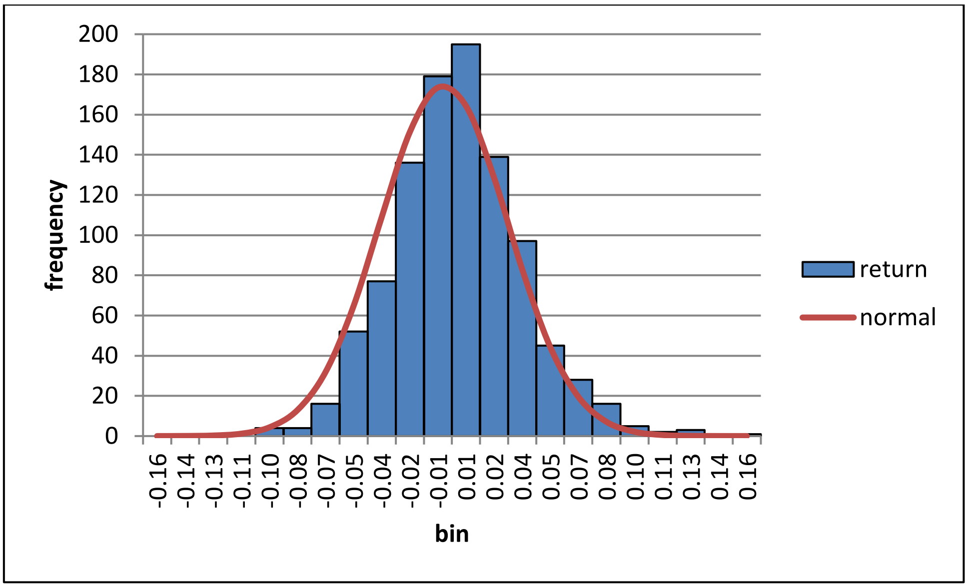


Figure 6. 8 Histogram and normal distribution from 0-1000 in the first replication model when w=0.5 (seed 2)

In Figure 6.8, the returns again have a slightly sharper distribution in the middle with fatter tails compared to the normal distribution. The patterns in Figure 6.8 and 6.6 are similar, while the kurtosis for these two settings is similar as well.

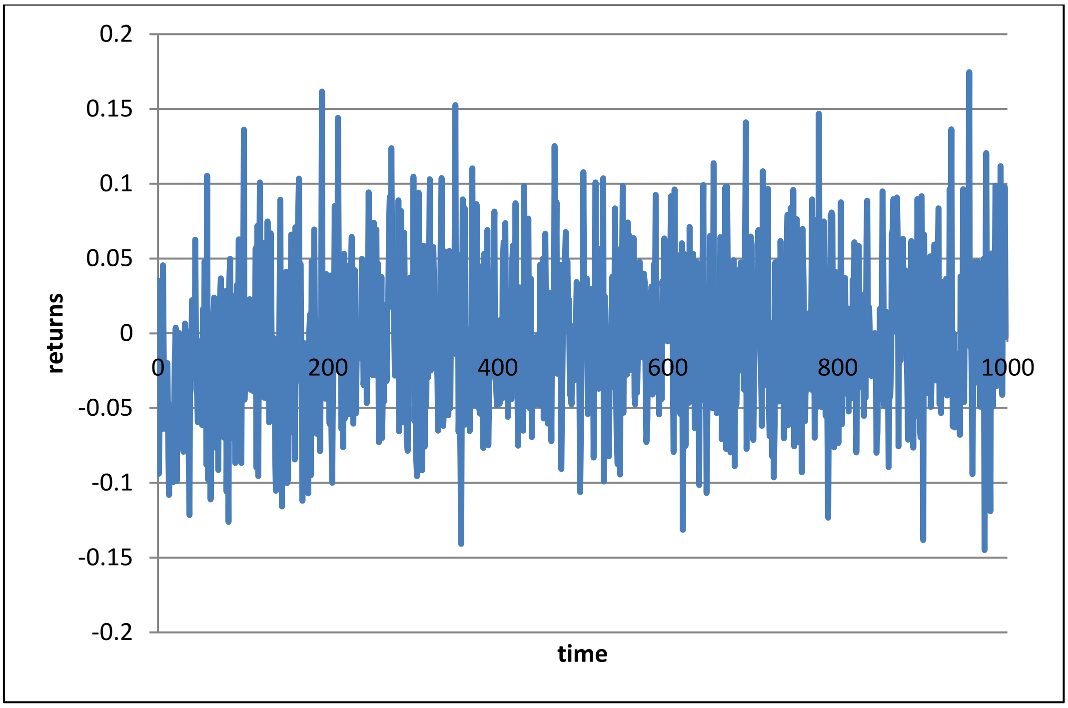


Figure 6. 9 Returns from 0-1000 in the first replication model when w=1 (seed 2)

In Figure 6.9, the returns from the first replication model have some extreme values and returns fluctuate around 0 with the Kurtosis=0.039 when w=1. The kurtosis for w=1 is much smaller compared to the kurtosis for w=0.1 and w=0.5. The kurtosis is keeping reducing while w is increasing, which means the fat tails are less obvious when there is no herding from the first replication model.

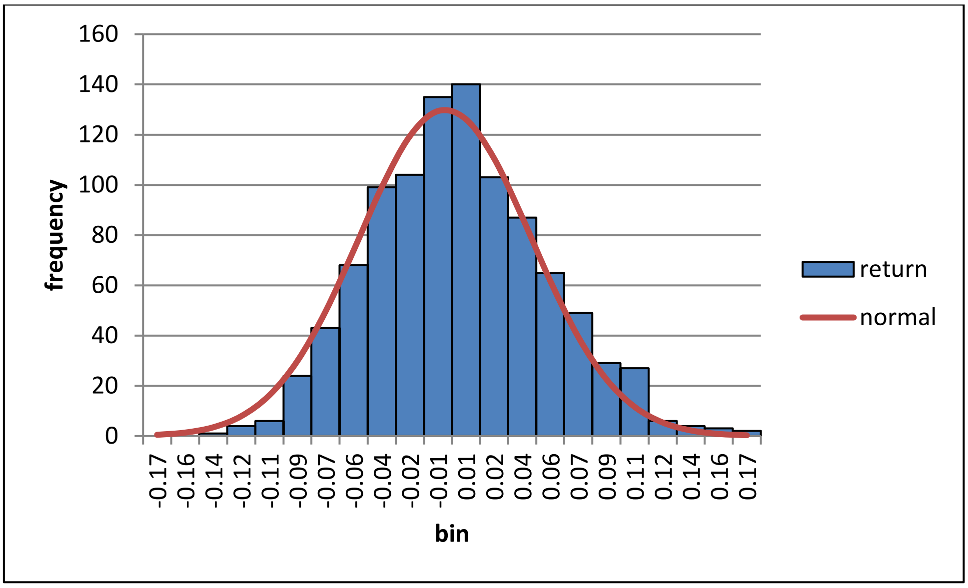


Figure 6. 10 Histogram and normal distribution from 0-1000 in the first replication model when *w*=1 (seed 2)

From the features in Figure 6.10, the returns have a sharper distribution in the middle with fatter tails compared to the normal distribution. Compared to the patterns in Figure 6.8 and 6.6, the returns in the middle are not as sharp and narrow in Figure 6.10.

median αH from 10 samples of 1000 observations (in parentheses: range of estimates)				
Parameters	kurtosis	2.5%tail	5%tail	10%tail
w=0.1	1.24 (0.32-2.74)	5.50 (4.59-10.33)	4.70 (4.24-9.97)	3.87 (3.34-7.10)
w=0.5	0.62 (0.19-1.53)	6.09 (4.39-8.53)	5.53 (4.71-9.33)	4.34 (3.66-7.82)
w=1	0.65 (0.00-0.83)	5.89 (4.93-7.63)	5.08 (4.40-10.34)	4.37 (3.72-9.22)

Table 6. 6 Fat tails in the first replication model (seed 1-10)

The fat tails for *w*=0.1, *w*=0.5 and *w*=1 from 10 runs are not easy to identify. From the tail index, fat tails are more obvious when there is less herding for *w*=0.5 and *w*=1.

Contradictory, from the kurtosis, fat tails are more obvious when there is more herding for *w*=0.1. This is then related to the discussion in Chapter 5 about what are fat tails. The tail index only focuses on the tail, but kurtosis does not. For the first replication model, fat tails are produced. For volatility clustering, most of the *d* is negative, which indicates there is no volatility clustering in the first replication model.

6.3.2 Herding in the first altered replication model

Figure 6.11 shows returns from the altered version of the first model when *w*=0.1, which has the most herding.

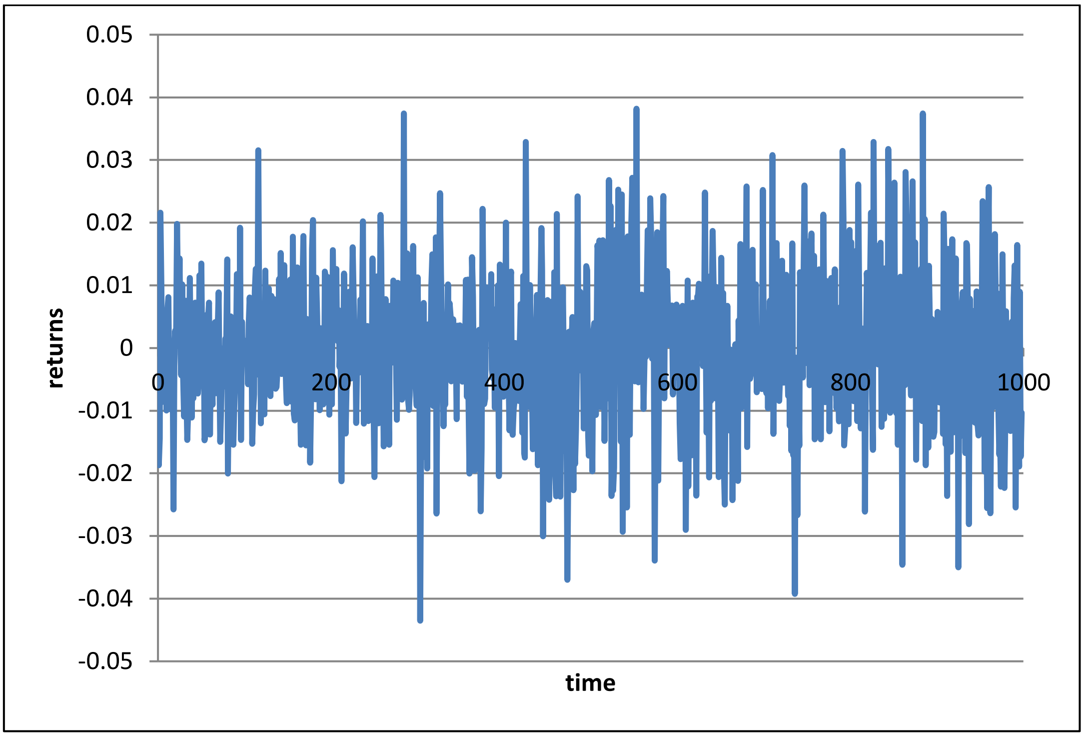


Figure 6. 11 Returns from 1000-4000 in the first altered replication model when *w*=0.1 (seed 2)

From the features in Figure 6.11, the returns from the first altered replication model still have some extreme values and returns fluctuate around 0 with the Kurtosis=0.56 when *w*=0.1. This is smaller than the first replication model when w=0.1 and w=0.5. That means the fat tail in the first altered model is even smaller than the first replication model. This is may because of the change in price settings mechanism which is discussed in Section 4.3.2.

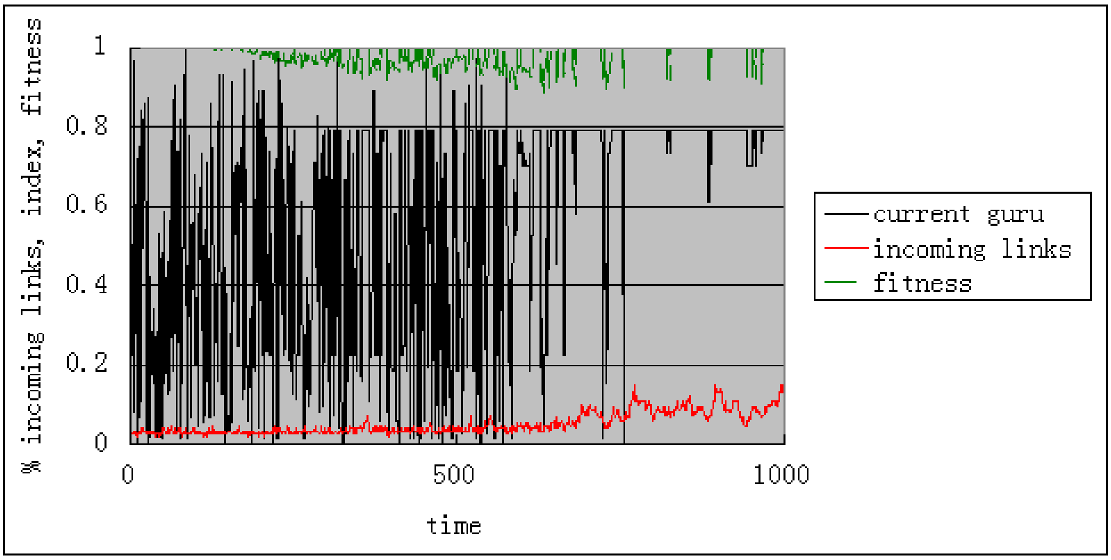


Figure 6. 12 Guru, incoming links and fitness in the first altered replication model when *w*=0.1 (seed 2)

In Figure 6.12, incoming links for the gurus, the percentage of followers is as low as 0.2 when w=0.1, which means lots of herding. This altered mechanism makes incoming links less than the original replication one. That means that the new market mechanism does not make the agents keep imitating each other as much.

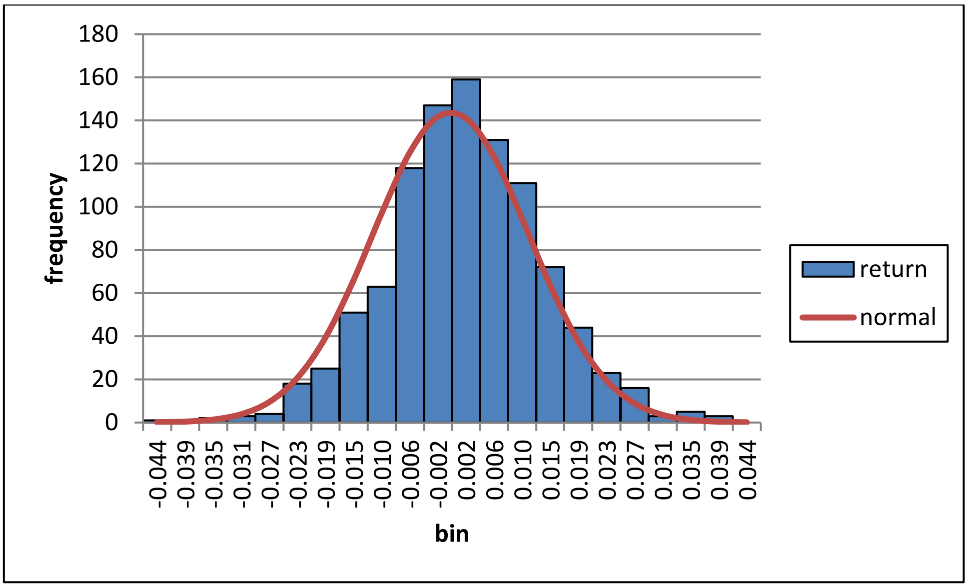


Figure 6. 13 Histogram and normal distribution from 0-1000 in the first altered replication model when w=0.1 (seed 2)

From the features in Figure 6.13, returns again have a slightly sharper distribution in the middle compared to the normal distribution. The fat tails are less noticeable comparing the distribution in Figure 6.13 of the altered model to the distribution in Figure 6.6 of the original replicated model.

median αH from 10 samples of 1000 observations (in parentheses: range of estimates)				
Parameters	kurtosis	2.5%tail	5%tail	10%tail
w=0.1	0.70 (0.29-69.80)	6.08 (0.61-7.09)	4.87 (0.93-10.30)	3.93 (1.37-6.56)
w=0.5	0.90 (0.58-2.46)	6.42 (4.26-10.86)	4.79 (4.00-10.28)	3.70 (2.96-8.73)
w=1	0.24 (0.23-0.54)	6.43 (4.30-8.36)	5.76 (4.68-12.21)	4.66 (3.99-9.12)

Table 6. 7 Fat tails in the first altered replication model (seed 1-10)

Compared to the values in Table 6.7 and Table 6.6, the kurtosis is smaller when *w*=0.1 and *w*=1 in the altered replication model. While, for the tail index in Table 6.7, the data shows not much difference, which indicates the fat tails in the altered replication model are similar when *w*=0.1, *w*=0.5 and *w*=1. Herding does not have much influence in the altered model for fat tails. Also, compared to the tail index in Table 6.6, the data is not that different, which means the fat tails in the altered replication model and replication model are similar. The *d* for the volatility clustering is negative mostly. The first model does not have any volatility clustering.

6.3.3 Herding in the second replication model

In the second model, the returns are in the figure below:

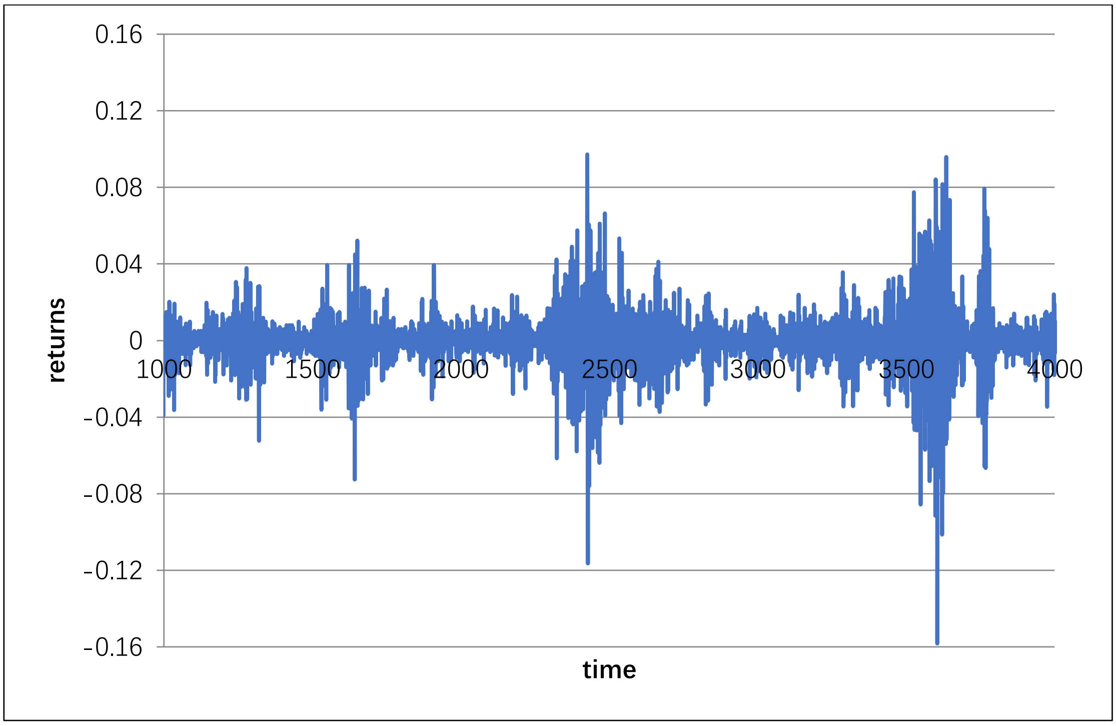


Figure 6. 14 Returns from 1000-4000 in the second replication model

From the features in Figure 6.14, the returns from the second replication model have some extreme values and returns fluctuate around 0. And the returns in Figure 6.14 has different stages like the stable stage and the volatile stage. And it follows the volatility considering the description that: small returns followed by small returns, big returns followed by big returns when we consider the absolute value instead of the real value.

median αH from 10 samples of 2,000 observations (in parentheses: range of estimates)				
Parameters	kurtosis	2.5%tail	5%tail	10%tail
Parameter set 1	19.34	3.06 (2.15-3.93)	2.51 (2.20-3.20)	2.21 (1.98-2.81)
Parameter set 2	11.57	3.72 (2.18-4.27)	3.09 (2.37-3.60)	2.63 (2.16-3.20)
Parameter set 3	8.72	4.21 (3.03-5.67)	3.62 (2.51-4.20)	3.05 (2.29-3.39)
Parameter set 4	13.94	3.36 (2.43-4.66)	3.00 (1.99-3.98)	2.60 (1.71-3.28）

Table 6. 8 Fat tails in the second replication model

From Table 6.8, the kurtosis is 19.34, 11.57, 8.72,13.94 separately with parameter set 1, 2, 3, and 4. The second replication model’s kurtosis is much higher than any of the models in the first replication.

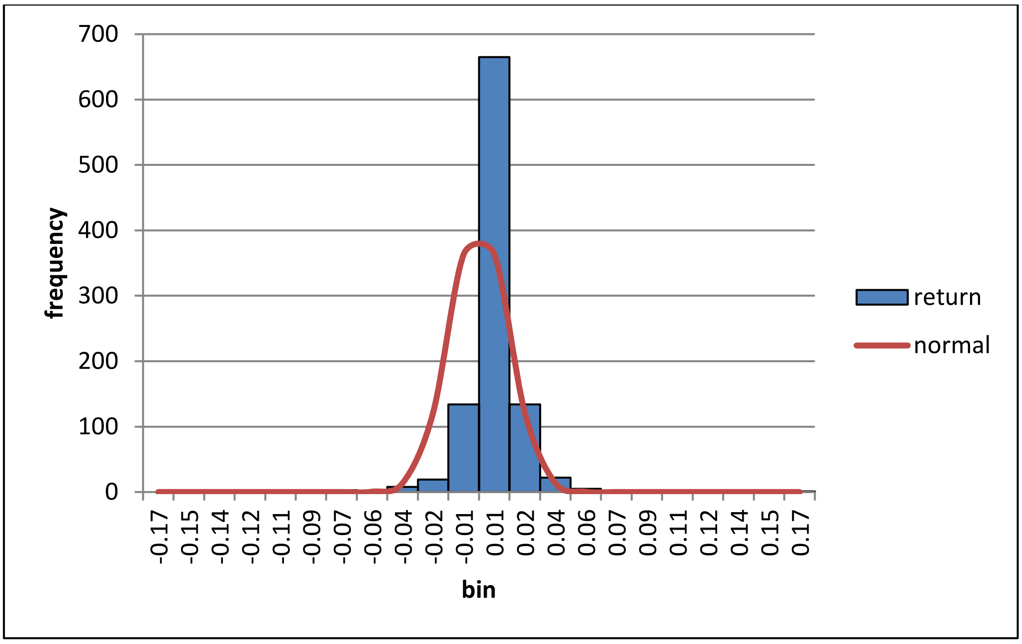


Figure 6. 15 Histogram and normal distribution from 0-999 in the second replication model with the parameter set 1

From the features in Figure 6.15, we can see the returns have a very sharp distribution in the middle with fat tails compared to the normal distribution. The time is from 0 to 999 is to be consistent with the first model, which only has 1000 periods. Returns in Figure 6.14 have some large extreme values to about -0.16 that indicate the distribution has fatter tails compared to the normal. The fat tails should be obvious, but because the frequency differences are so large, the fat tail is not that clear in Figure 6.15.

The second replicating model has volatility clustering, and the on-off intermittency is detailed analysed in Chapter 5.5. In the second replication model, most of the time, the returns are relatively low as the fraction of chartists. The volatile stage starts because of random variation, and then the price effect drives the volatile stage back to the stable stage.

To conclude, the first and second model results differ from each other because of the herding mechanism and the price mechanism. In the first model, agents formed the price expectations through the herding factor w in both the original one and the altered model. Based on agents' expectations, the price and the amount to trade are decided by themselves. In these cases, herding in the first model only influences the agent's opinions but not the final decision directly. Compare this to the herding in the second model, agents are divided into three groups: fundamentalists, optimistic chartists and pessimistic chartists. From the price mechanism in this model, the price will go up or down by 1% depending only on how many agents are in each group. That indicates agents from the same group are making the same decisions. Thus the herding in this model influences agents' decisions rather than just opinions. This may be the reason why the first model is so different from the second one. The herding mechanism is so different: the first one imitates the expectation. Then, they make a different decision whereas the second one imitates the decision and influences the price together. That may lead to the conclusion that volatility clustering more easily happens while agents imitate the result rather than each other. This might be the reason why this second model has both fat tails and volatility clustering, but the first model only has fat tails. From the herding models examined in Chapter 2, 8 out of 12 demand and supply models have volatility clustering and 1 out of 3 artificial market models have volatility clustering. This may also indicate that direct imitation can generate volatility clustering more easily than indirect imitation. For transition probability models, the price effect is one of the stabilising reasons that drive the returns back to the stable stage from understanding the on-off intermittency. The price effect for transition probability models is one of the vital herding components that make both fat tails and volatility clustering.

Chapter 7

Conclusion

This chapter draws conclusions from this whole thesis. Section 7.1 identifies the research contribution in the two aspects of the research questions in Section 1.2: one is the herding, and the other is replicating. Section 7.2 suggests the limitations of the research and future work.

7.1 Contributions

My first research question is how and to what extent can herding produce the stylised facts observed in financial markets? For the literature reviewed in Chapter 2, 68% of models have fat tails, and 60% have volatility clustering. Of the 25 models reviewed, 13 have fat tails and volatility clustering, 4 models have just fat tails, 2 models have just volatility clustering, and 6 models have neither.

For the stylised facts, the first replicating model in Chapter 4 can only generate fat tails. The problem with the first replicating model is the price dropping. Then the first altered model in Chapter 4 is built by mainly changing the utility function to solve the price dropping. The reason behind the price dropping is the utility function. From the analysis of the results, the number of buyers and sellers could be the reasons for price dropping. For the replication model, on average, there are more buyers than sellers and the guru keeps unchanged for a long time. After changing the formation of expectation, for the first altered model, the number of buyers and sellers is equal, and guru keeps changing a lot during periods. Still, the first altered model only produces fat tails. The herding mechanism is through the network for both the first replication model and altered models, and the market mechanism is the order book market. All in all, in Chapter 4, two models (replication and altered) with network herding mechanism and order book market only produce fat tails. The herding in these models only partially influences the agent's expectation and indirectly affects the agent's decision about how much to buy or sell at which price.

The second replicating model, Δt = 0.01 model and analytical model can generate fat tails and volatility clustering. The model without price effect can only generate volatility clustering. These models' herding mechanisms are transition probability, and the market mechanisms are demand and supply. Through analysis (Chapter 5.5.2 and 5.5.3) of the relationship between the transition probability and the number of certain agents and the price effect, the analytical model is built in Chapter 5.5.6 to study the price effect in the transition probability. Understanding on-off intermittency provides a detailed understanding of how herding generates volatility clustering and fat tails to some extent. The model without price effect in Chapter 5.5.4 shows the on-off intermittency but the volatile periods are long and this model only has volatility clustering. Also the model with constant probability in Chapter 5.5.5 does not have on-off intermittency and stylised facts. The analytical model in Chapter 5.5.6 shows on-off intermittency and both fat tails and volatility clustering. The randomness of the transition probability causes the start of the volatile stage of the on-off intermittency to produce volatility clustering. Then the price effect drives the volatile stage back to the stable stage quickly to produce fat tails. The on-off intermittency indicates that the price effect in the transition probability models is one of the vital herding components that make both fat tails and volatility clustering. All in all, four models (replication, Δt = 0.01, without price effect, analytical) in Chapter 5 produce volatility clustering with transition probability and demand and supply market. One model with constant probability with demand and supply market produces neither fat tails nor volatility clustering. This shows how herding in the transition mechanism with price effect makes both volatility clustering and fat tails.

My second research question is to what extent can previous modelling results be reproduced, and how can reproducibility in simulation modelling be improved? The first replicating model in Chapter 4 fails to produce the original results. Some parameter settings and initialisation information are missing in the original paper. That makes the model difficult to be replicated and achieve reproducibility. The first altered model is closer to the original results but still fails to replicate them. The second replicating model in Chapter 5 has the rational alignment level of replicating. Still, for the second model, there is some missing information that makes it hard to replicate exactly. Also the model Δt = 0.01, analytical model are similar to the original model with rational alignment level. In practice, it is challenging to achieve the level of distributional equivalence as different models have different situations and scales and this replicating level will generally require comprehensive information about the model and the experiments. From the two replication models in Chapter 4 and 5, only the second model replicates successfully even with changes in conditions. All in all, from the literature in Chapter 3, replication of studies is essential in science but often does not happen. My experience in replicating agent-based simulation in financial markets gives a taste of reproducibility in this area.

The STRESS guidelines have recently been produced for specifying simulation models. The description of the second replicating model in Chapter 5 was done using the STRESS guidelines. The guidelines help to improve reproducibility by providing a checklist for all the information and parameter settings. The guidelines are in the format of a checklist and the reflection of the guidelines in Chapter 6 shows that it is not easy to locate each item from the checklist. That makes comparison among agent-based simulation harder because they have different mechanisms and structures. It also makes it more difficult to find a specific piece of information. Some papers use the ODD guidelines, which have a suggested order. The new suggested guideline of Table 6.2 is the combination of STRESS and ODD along with structural advice. The new guideline helps include the things needed in the description and makes cross-comparison among papers easier in the field of agent-based simulation in financial markets. The new suggested guideline helps the author to include things needed accurately and in order. Hopefully, the new one is more helpful to some extent to ensure reproducibility. An additional suggestion is to include more verification and validation in the guidelines, as this is very important and can sometimes be overlooked in simulation projects. All in all, the new suggested guideline is one of the ways to improve reproducibility in agent-based simulation in financial markets.

7.2 Limitations and future work

The main limitations of this research have three aspects. It only focuses on the N-type model but ignores the autonomous model. Also, it replicates two models in total, which is not that sufficient. These can lead to future work.

More replicating is needed in autonomous models to understand how herding and the market mechanism works with related stylised facts. Comparing the two already replicated models and the autonomous type of model could identify how the herding and market mechanisms are different from each other. Also, it could help to understand further how certain herding components can produce specific stylised facts. Manahov and Hudson (2013) and LeBaron (2001a) might be good choices. It depends on the extent of the detail of the descriptions in the papers to enable them to be replicated. It is more challenging for the genetic algorithms to be replicated because it needs more detail in the genetic algorithm rules than the N-type model.

More investigation is needed to understand herding and related stylised facts from the understanding of the on-off intermittency. The first replicating model only has fat tails. Suppose the knowledge of the on-off intermittency helps alter the model structure and generates volatility clustering in the first replicating model. In other words, a guru model but with a mechanism that produces high volatility periods but with also a factor that tends to return the model fairly soon to low volatility. In that case, it is sufficient to confirm that the specific herding component can cause certain stylised facts even if the model details are different.

The first replicating model is an indirect imitation from the network, and the second is a direct imitation from demand and supply. From the literature in Chapter 2, 8 out of 12 demand and supply models have volatility clustering, and 1 out of the 3 artificial market models has volatility clustering. Literature may also indicate that direct imitation can generate volatility clustering more easily than indirect imitation. The differences between direct imitation and indirect imitation need more investigation in the future.

Changing the indirect imitation mechanism into the direct imitation in the first replicating model could be a way to develop the model and to show how the direct imitation works. The network herding mechanism in the first model can change to a direct copy of the order price through the network. Or the agents could be grouped by the network connections somehow and change the market mechanism.

More N-type models also need to be replicated to understand different herding and market mechanisms. By comparing the similarities and differences, the understanding of herding and related stylised facts could be enhanced. Besides, it may be possible to find a way to measure the amount of herding mechanism in the future. How the herding mechanism can produce certain stylised facts can then perhaps be generalised.

The improved guideline needs to be applied to evaluate in practice. After that, the reflection of the improved guideline can be produced which may lead to more suggestions. Also, if the guideline can be used in agent-based simulation, it can be tested in practice. It is derived from agent-based simulation guidelines. Hence, there is a large probability that the improved guideline can also apply in a more general area which is agent-based simulation.

Appendix A

Coding for first replicating model

In the code of replication model, there are three matrices with 150 elements to code the network part. Matrix *a* is the existing neighbour for each agent. Matrix *a1* is the newly formed neighbour for each agent. Matrix a2 is the final neighbour through the fitness selection between matrix *a* and *a1*. Then, at the next period, *a* is equal to *a1* for updating the communication network. For each matrix, *a[0]* = 3 means that agent 0 links to agent 3. Hence, from *a[0]* to *a[149]*, the needed neighbour information is collected. For matrix *a*, *a[0]* can not be 0 to rule out copying itself. For matrix *a1*, *a1[0]* can not be 0 or *a[0]* to make sure there are two options for a certain agent to choose and exclude the agent itself. The calculation has followed the formula before calculating the probability of switching from matrix *a* to matrix *a1*. Then a random number is chosen for each agent. If the random number for a certain agent is less than the probability calculated for that agent, *a3* is equal to *a1*. Otherwise, *a3* is equal to *a*.

The expectation coding is reflected mainly in the matrix *ret*. From the network matrix introduced before, the incoming links for each agent are able to get from array *a3*. Hence the percentage of incoming links is known. The array called *volatility* with 150 elements is stored in the calculation results. Then the matrix *ret* is calculated after knowing the value of volatility. Then a random number from 0 to 149 is produced in an array *ran*. At last, the matrix *ret* is updated in the sequence of *ran*.

The market coding part mainly depends on the matrixes *c, s, p2, order_volume, order, new_entrance*, *best_bid* and *best_ask*. The *p2* and *order_volume* are two 150 elements:

order price and order amount for each agent. *New_entrance*, *best_bid* and *best_ask* are the three one row array that has particular information for just one agent which is entering the market, the highest buy price and the lowest sell price. The matrix *order* is recorded the order book information with 300 rows for rolling basis purpose. For *new_entrance* and *order*, there are four aspects of information: the number of the agent, order price, order amount and buy or sell state, with 1 represents a buy and -1 represents a sell. For *best_bid* and *best_ask*, there are five aspects of information: adding in the best bid and best ask agent's row in the order book. Once the random sequence is formed, the *new_entrance* is produced based on the *p2* and *order_volume* for that agent which are calculated according to the equations illustrated in this part. Then the price and amount information in the *new_entrance* are compared with *best_bid* and *best_ask* according to the rules discussed before. Then the *c*, *s* position for a certain agent is updated and also the current price. Then the order book updated, so as the array *best_bid* and *best_ask*.

The code is accessible on the DOI 10.5281/zenodo.5524993. Or by the email request by x.liu1@lancaster.ac.uk or liuxinbess@126.com.

Appendix B

Coding for second replicating model

In the code of replication model, the result will be recorded in every one period. That means the result will be recorded in every 100 periods when Δt is 0.01 or every 500 periods when Δt is 0.002. If the price changes so frequently that it exceeds 0.405, then the time interval changes from 0.01 to 0.002. The value is the average of price fluctuation from two models built in c and witness. It is calculated by the average price change in each model in 20000 time when the interval is 100 by the parameter set 1. There are variables in the code that track: time interval, total small time increments, either 100 or 500, prices before and price after. Those variables change every 100 periods when Δt is 0.01 or every 500 periods when Δt is 0.002. Also, there is a variable that tracks the small time increments, and if this variable equals the total small time increments, then the conditions will be compared to see if there are any changes in the values of time interval settings. As we think the switching of chartists happens instantly, the components of the switching formula will not update during the time interval Δt. The code of this model has three stages in all-the code starts at the initial stage. Then agents switch between each other in the switching stage. And then, the price is changed and the time is changed into the next small time increment from t to t+Δt. Agents start to switch again based on the new information.

1. Initial stage (figure 5.1 step1): agents are divided randomly.

2. Switching stage (figure 5.1 step 2,3,4): Each agent only switches once in each interval time Δt. The agents switch from one to another according to the switching

rules given later in Section 5.3.3.1 and 5.3.3.2. The order for agents' change does not matter.

3. Price formation stage (figure 5.1 step 5): The price updates by increasing or decreasing by 0.01 or staying unchanged according to equations (5.17) and (5.18) in Section 5.3.3.3.

4. Time update stage (figure 5.1 step 6): The time updates by a time increment.

The symbols of code are provided in the table for information to understand the code quicker if required.

Symbol	Code
N	n
α_1	alpha1
α_2	alpha2
v_1	v1
α_3	alpha3
r	r
v_2	v2
R	R
s	s
p_f	pf
t_c	stc(tc/N)
γ	gamma(tf/N)
β	beta
μ	deviation
Δt	t

Symbol	Code
n_c	nc
n_f	nf
n_+	np
n_-	nn
x	x
z	z
U_1	u1
$\dot{p}$	dp
$\pi_{-\to+}$	piepn
$\pi_{+\to-}$	pienp
$U_{2,1}$	u21
$\pi_{f\to+}$	piepf
$\pi_{+\to f}$	piefp
$U_{2,2}$	u22
$\pi_{f\to-}$	pienf
$\pi_{-\to f}$	piefn
ED_c	edc
ED_f	edf
ED	ed
$\pi_{\uparrow p}$	pieup
$\pi_{\downarrow p}$	piedown

The code is accessible on the DOI 10.5281/zenodo.5524993. Or by the email request by x.liu1@lancaster.ac.uk or liuxinbess@126.com.

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