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Putting the Isopleth in Its Place

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GEOGRAPHY

Putting the Isopleth in Its Place

The isopleth has long been recognized as one of the geographer's most powerful descriptive tools. As a device for describing geographic relationships, such as the ratio of quantity to area and the ratio of quantities to one another, the isopleth is unexcelled.

ISOPLETH CHARACTERISTICS AND SOURCES OF ERROR

Following the discussions of isopleths by Huntington, Jones, Wright and others in the late 1920's, isopleth maps began to appear in the geographic periodical literature in increasing numbers, reaching a high point in the years immediately prior to World War II.[1] Most of the isopleth maps appeared in agricultural and population studies. In the war years and the post-war period up to 1952, relatively few isopleth maps appeared in the American geographical periodicals. Since that time, however, there has been an apparent revival of interest in the isopleth and an increased recognition of its ability to describe geographic associations. In view of this interest, perhaps a fresh look at isopleths is in order.

While the general utility of the isopleth map is readily conceded, the thoughtful geographer is aware of serious limitations inherent in

[1]Ellsworth Huntington, 1927. The Quantitative Phases of Human Geography, *Scientific Monthly* 25: 289-305; Wellington D. Jones, 1930. Ratios and Isopleth Maps in Regional Investigation of Agricultural Land Occupance, *Annals, Association of American Geographers* 20: 177-195; J. K. Wright, 1930. Isopleth as a Generic Term, *Geographical Review*, 20: 341. A rough count of isopleth maps appearing in the *Annals* of the Association of American Geographers, *Economic Geography*, and the *Geographical Review* from 1900 to 1957 showed that about 440 true isopleth maps have been printed. Of these, 20 percent were published in the peak three year period, 1938-1940, while only 11 percent were published in the period 1944-1952. Since 1952 over 110 (over 25 percent) have appeared.

isopleth construction, limitations which call into question the truthfulness of the patterns he sees. For a symbolic device in such common use as the isopleth, it is important that its characteristics and the sorts of error that enter into its construction be understood. Four variables influence the placement of isopleths on maps: 1. class intervals, 2. shapes and sizes of statistical units, 3. location of central points of statistical units, and 4. methods of interpolation used. A discussion of these variables and of some of the ideas for dealing with them will form a useful background upon which to judge a new method of constructing isopleth maps.

Definition: Before going any further, however, it will be appropriate to define what is meant by the term isopleth. It is a line along which a constant relationship between two sets of information is maintained. It may be a measure of commodity against area (e.g., 20 people per square mile) or it may be a measure of one commodity against another (e.g., income from the sale of livestock as a percentage of total income). The isopleth differs from the isometric line in a fundamental way. Isometric lines express values which can exist at points on the earth's surface (e.g., 40° F., or 1200 ft. above sea level); they are, in short, mensurative of continuous distributions. By way of contrast, isopleths are derived from points which express average values for an arbitrary area, commonly a political unit such as a county. While one knows the average, one does not know the extent of deviation from that average (unevenness) within the statistical unit. Isopleths are, in other words, areal measures of enumerations of discrete phenomena, discontinuously distributed. They describe an areal concept rather than measure actual point values (Wright, 1944; Mackay, 1951).

Class Intervals: The importance of class intervals is generally understood and need not be discussed here. Mackay (1951) illustrated the differences of pattern that emerge when different class intervals are selected. There is need for considerable experimentation with intervals based 1. on natural breaks in a frequency distribution, truly *inter vallum*; 2. on standard deviations; on deciles, quintiles, etc. based on 3. the number of statistical units, 4. the total amount of the commodity being mapped, and 5. the area covered by the statistical units; and 6. on geometric progressions, as well as the more conventional method of selecting isopleths on a rhythmic interval basis (5, 10, 15, 20), that is, isarithmic. This is an interesting line of inquiry, but the three other problems mentioned above are the main concern of this paper.

Shapes and Sizes of Statistical Units and Location of Central Points: The shapes and sizes of statistical units with which the geographer is forced to work are a never ending frustration. In drawing isopleths one point must be selected to be expressive of the average value of the commodity contained within the statistical unit. This may be called the centroid, or the point at which the commodity, distributed on a weightless surface, will balance. Needless to say, this center is not always at the geographic center of the statistical unit. The center of corn production in a county whose eastern half is in forest is certainly not at the geographical center of the county.

Even if there is a uniform distribution of a commodity, the shape of the county may be such that the geographical center and the centroid lie outside the county boundaries. What is the geographic center of a doughnut? Where is that center with one bite removed? To draw isopleths accurately one must begin with central points located at the centroids of the commodities they are representing.

The size of statistical units determines the density of the network of central points from which isopleths are interpolated, and thus the degree of precision with which the isopleths can be drawn. An enormous county in a state may give statistical averages and a geographic central point that are virtually worthless. In Minnesota, St. Louis County with Duluth at its southern extremity is an example, particularly as a Bureau of the Census "Standard Metropolitan Area" (Fig. 1). St. Louis County is about 40 times the size of Ramsey County, the smallest in Minnesota. Thus the central point of St. Louis County will give information that is 40 times more generalized than the data for Ramsey County. Clearly such differences in the density of central points are undesirable.

Methods of Interpolation: The most commonly used method of interpolation assumes a uniform gradient between two central points. For example, if the values of two central points are 5 and 15, they are one inch apart on the map, and one wants to find the point through which the 10 isopleth passes, one measures half-way between 5 and 15, that is, one-half inch from either point. It happens, however, that

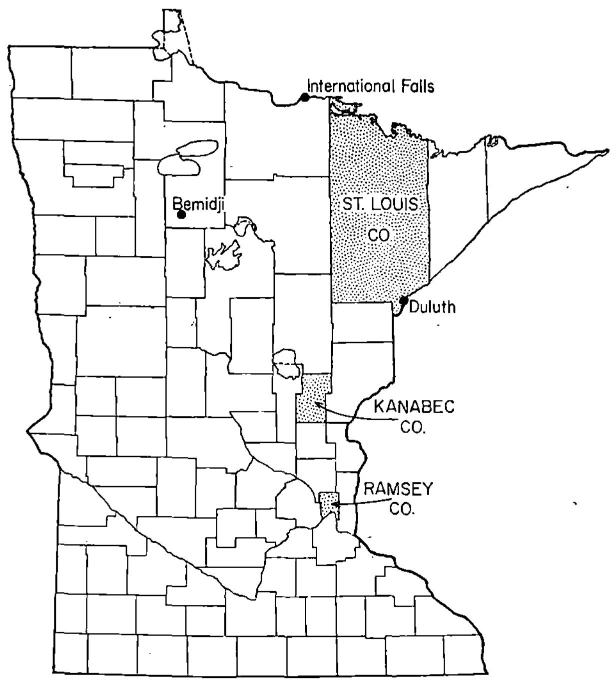


Fig. 1

many of the things geographers map are so distributed that the rates of change are not uniform; rather than being rectilinear (straight line), they are curvilinear. The density of population per square mile passing from city apartment district through single-family dwelling areas to suburban and finally to rural lands has a gradient profile which is curved. The same is true of many other distributions that can be mapped as isopleths.

To look at the sort of placement error that is introduced when account is not taken of differences in the rate of change, there are shown in the lower part of Fig. 2 three statistical squares with average values of 36.3, 8.3, and 1.3. This is a steep though not at all impossible gradient. Assuming that the interest is in drawing the 3 and 15 isopleths, strict linear interpolation will make the isopleths pass through the places shown by dashed lines. Converting this into a graph the same thing is seen: straight lines connect the centers of "terrace levels"and the 3 and 15 isopleth positions can be read off the vertical scale. As was mentioned earlier, geographical phenomena seldom change rates of gradation at angular break points; the gradations are commonly smooth and curved. In fact, the gradient in the example used here is a geometric progression with the differences between spaced points on the base doubling along the curve. The progression is 0, 1, 3, 7, 15, 31 and 63, with interval differences of 1, 2, 4, 8, 16 and 32.

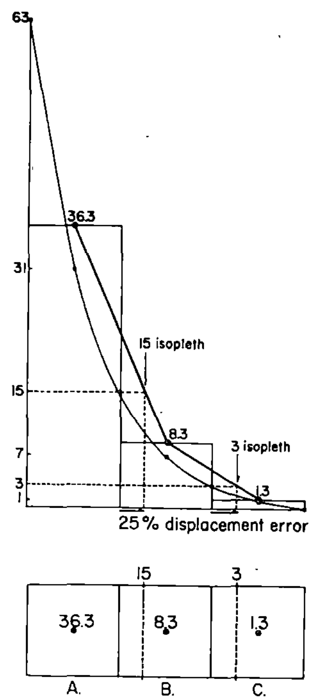


Fig. 2.

Shown as a curve it can readily be seen that the 3 and 15 isopleths coincide with the boundaries between statistical units. By linear interpolation the isopleths have been drawn so that 25 percent of squares B and C are on the wrong side. This is a serious error. This error can become ludicrous if the gradient between statistical units is steep enough, requiring, in an extreme case, a negative quantity in the remaining part of the statistical unit. It should be noted here that the difficulty stems from the fact that the geographical centers of the squares are not the true centroids of the distributions contained within the squares (Alexander and Zahorchak, 1943).

One other problem of minor concern in isopleth interpolation must be mentioned. This problem, which has been discussed by Mackay (1951: 5), involves situations commonly encountered when using square counties (Fig. 3), Assume that an isopleth is drawn whose value is 10. The isopleth passes through points a, b, c and d; but it is not known whether the isopleths should join as shown in Fig. 3b., or as shown in Fig. 3c. Does a ridge of more than 10 pass through point 0, or does a valley of less than 10 cross the same area at right angles to this possible ridge? On the basis of these square statistical units the answer cannot be known.

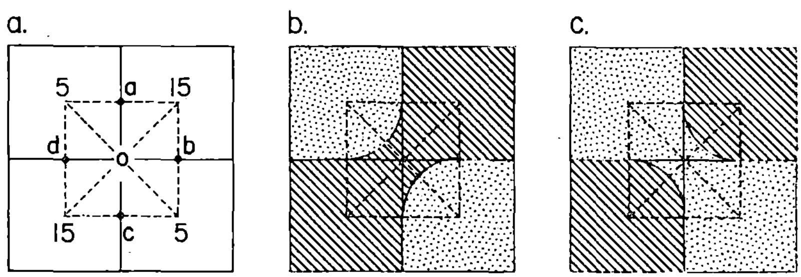


Fig. 3.

A NEW METHOD OF ISOPLETH CONSTRUCTION

The problems which have been discussed above trouble the geographer. All manner of doubts and uncertainties assail him as he studies the published isopleth map whose outward appearance is one of precision and reliability. As the author of the map confidently points to a striking gradient here and an interesting lobate pattern there, the thoughtful reader cannot but be somewhat skeptical. Leaving aside all the errors introduced by human frailty, and those inherent in, say, the Department of Agriculture's 20 percent sampling, the errors of mechanics in drawing isopleths must make each of us somewhat doubtful. As Stewart and Warntz have so clearly demonstrated in a recent paper (1958: 168), "much mischief can be done by the excessive use of the isopleth technique, in which an arbitrary system of areal subdivision is used as the basis for computing density ratios."

Here is one attempt to make the isopleth travel a more geographic path than is possible on the basis of county data alone. The map subject is "Harvested Cropland as a Percentage of Total Area." In Minnesota there are counties which have on the average nearly 80 percent of their land in crop, whereas in some counties the figure is less than one percent. Cropland is something which shows quite distinctly on air photographs, and is readily differentiated from areas of lake, forest, bog, waste and urban uses. The first task was thus to draw a map in which the areas of cropland were precisely delimited. This was done by the inspection of air photo mosaics for each county of the state. Areas of dense cultivation were differentiated from areas of less dense and scattered cultivation, and all the information was plotted on county maps at a scale of 1:750,000. A visual estimate was then made of the percentage of each county's total cropland lying within the densely cultivated area. This figure was entered next to the map of the county. Such descriptive terms as "very dense, dense, moderate scattered, very scattered, and none" were used with percentage estimates for the various terms as they appeared in any county. Thus there are for Kanabec County (Fig. 4), areas of dense, scattered, and very scattered cropland, with an estimate that 97 percent of all cropped land lies within the area of dense cropland.

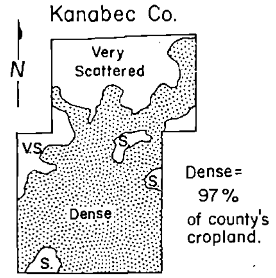


Fig. 4.

If the Department of Agriculture's data on cropland are placed within the cropped area, and isopleths are drawn from statistical units which are sensitive to this distribution, a more geographic description will be obtained. One way to do this is to place dots in the cropped areas, each dot representing so many acres of cropland.[2] The acreage in dots would bear a percentage relationship to the number of acres in any statistical grid unit superimposed over the dots. An example will make this clear. Assume that each dot equals 1,000 acres. The Census of Agriculture reveals that there were 71,000 acres of harvested cropland in Kanabee County in 1954. This means that 71 dots must be distributed throughout the county and that 69 of these dots should be placed in the area of dense cropping. If over this is superimposed a grid of statistical units, each of which covers a map area representative of 100,000 acres (about the size of Ramsey County), then each dot which falls within a grid square will count as one percent in cropland, one percent of the total area of the grid square. Presumably there could be nearly 100 dots under one grid square, in which case the area would be nearly 100 percent in cropland.

This was done. Each county was dotted at the rate of one dot for every thousand acres in cropland divided proportionately among the areas of various cropland densities. Over this dot distribution a uniform grid of 100,000 acre units was superimposed; and counts were made of dots, which gave figures of percent of total area in cropland. From this network of uniformly spaced central points, isopleths were drawn.

Two refinements need further explanation. A grid based on hexagons was used rather than one based on squares. This was done for two reasons. First, the ideal statistical unit is a circle, since the border is a uniform distance from the central point. The hexagon is the closest approximation of a circle which is capable of being arranged so as to cover all areas with statistical units of uniform shape and size. Second, another advantage in using hexagons stems from the fact that central points of adjacent hexagons always form equilateral triangles; hence, there can never be any question of interpolation between matched pairs of central points comparable to the problem encountered in using square statistical units (Fig. 3). In Fig. 5, points b and d are 70 percent farther apart than are points a and c. The second refinement concerns the central points from which the isopleths were drawn. While most dot distributions were even

[2]A useful by-product in constructing such a map is a more precise guide for "dotting the dot map".

within any hexagon, there were cases in which the dots were clustered in one section of the hexagon. In these instances the centroid obviously was not at the geographical center, and the point from which to control isopleths was accordingly moved to the center of the dot cluster. This compensated for errors that otherwise would have displaced the isopleth in the direction of lower values. It copes, therefore, with the problems discussed earlier relative to curvilinear gradients. Now to the maps themselves.

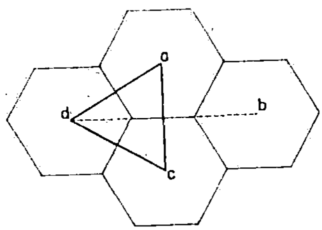


Fig. 5.

Fig. 6 shows isopleths drawn on the basis of county statistical units, Fig. 7 shows isopleths drawn on the dot-hexagon basis just

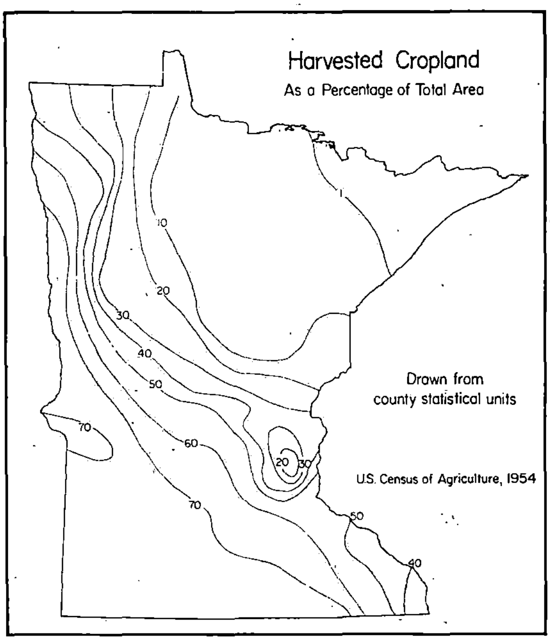
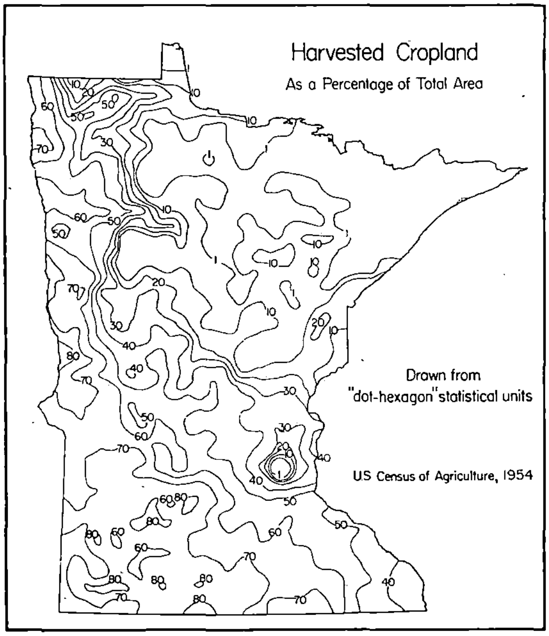
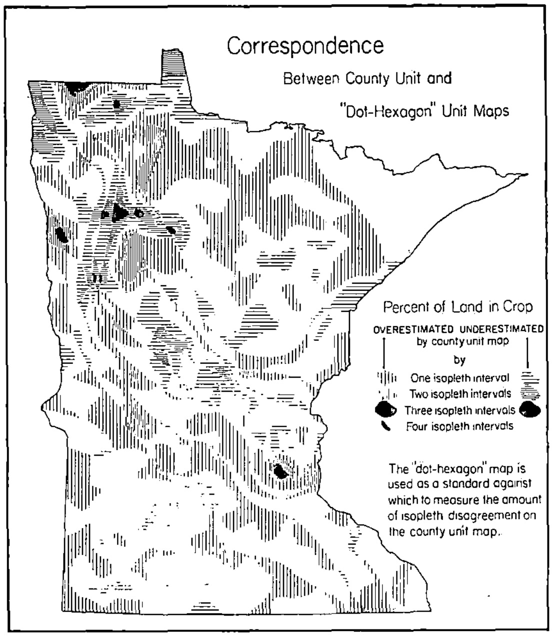


Fig. 6.



described, while Fig. 8 measures the extent of agreement between the two maps. Where the map (Fig. 8) is white there is agreement; where there is a widely spaced line pattern there is a difference of one isopleth between the two maps, i.e. if one map indicates a value of more than 40 percent, the other will indicate a value of less than 40 percent. The areas of more closely spaced lines represent a disagreement of two isopleths; the areas having very closely spaced lines represent a difference of three isopleths; and in the vicinity of Minneapolis there is one area where four isopleths separate the value on one map from the value on the other. The vertical lines indicate an overestimation on the part of the county unit map, at least as measured against the dot-hexagon map, while the horizontal lines represent an underestimation.

It is of some interest that the areas of greatest disagreement occur in the northwestern part of the state where several counties have a long east-west extent, and where the transition from fertile farmland in



the Red River Valley on the west to unused bog and forest to the east is rather abrupt. The notable finger of disagreement pointing to the west is related to poor land, whereas the two fingers pointing to the east, one toward International Falls and the other toward Bemidji, parallel respectively the tracks of the Great Northern and the Soo Line railways, along which settlement is more dense and the agricultural activity on clay soils is greater. The truthfulness of the dot-hexagon isopleths is startlingly confirmed by a careful look at the population density map of Minnesota (Fig. 9).

Some of the limitations of this dot-hexagon method will readily come to mind. True, it is time consuming, but it effects accuracy. Further, its use is practically limited to data the distribution of which is specifically observable on air photos or maps. Thus, land in farms cannot be shown since farmland includes waste and forested land. Crops are specifically tied to cropped land and are, therefore, adapted to this technique.

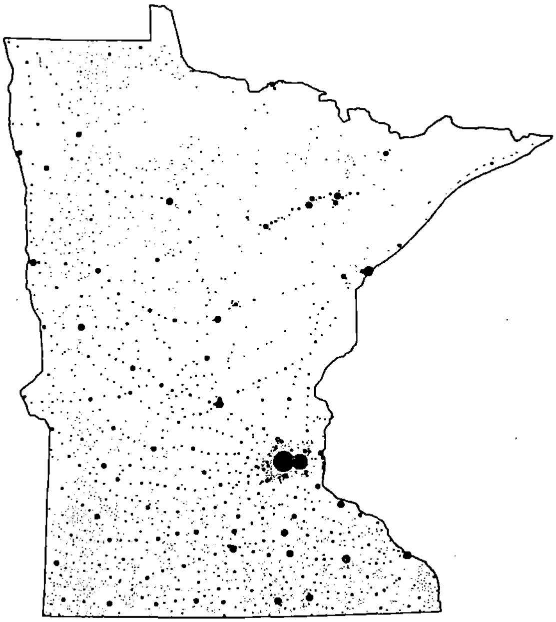


Fig. 9.—Population distribution in Minnesota. Courtesy of the Minnesota State Highway Department.

To summarize the advantages of the technique, it can be claimed that the isopleths are placed much more accurately than on a map drawn from county statistical units. The dot-hexagon method is sensitive to curvilinear gradients and centroids of distributions. What is perhaps equally important is that the same confidence may be placed in the isopleths in all parts of the map. One of the supposed virtues of any isopleth map is that a person can read values from it directly. One can say, "In this general area, 35 percent of the land is in crop." The trouble with maps drawn from county statistical units is that "this general area" means 4,000,000 acres in a St. Louis County, while it means 100,000 acres in a Ramsey County. In the map drawn from dot-hexagon statistical units, *any* point may be selected and one may say "in this general area . . . ", and always mean an area of, in this case, 100,000 acres, roughly a circle whose radius is 7 miles. A "level of confidence" statement with a sample hexagon could even be placed on the map, a sort of isopleth reliability diagram.

The main contention of this paper has been that there is need to be more critical of isopleth maps. One should be aware that there are different confidence levels even within one isopleth map. These levels relate to the density of the network of central points and the methods used in interpolation. If one maps in generalities, one must in all honesty speak in generalities. It is suggested that the dot-hexagon method of drawing isopleths increases greatly the ability to be precise in describing the spatial interrelationships of significance and interest to man.

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