On Excessive Mathematatization, Symptoms, Diagnosis and Philosophical bases for Real World Knowledge

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ON EXCESSIVE MATHEMATIZATION,
SYMPTOMS, DIAGNOSIS,
AND PHILOSOPHICAL BASES FOR REAL WORLD KNOWLEDGE

Several articles on the misuse of mathematics in economics have already appeared in this journal. They all denounce this excess and list numerous weaknesses of liberal economics and theoretical economics that are due to, or at least related to, too much math.

This subject is worthy of further comment because it seems to me that these articles have mostly described symptoms, albeit a great many symptoms, but have barely begun to diagnose the causes and have given no hint of the kind of knowledge that would enable us to escape this no-man's-land of using a little math but not too much.

The most recent contribution, by Michael Hudson (RWER No. 54), focuses on the important issues that escape mathematical models, such as the structural and historical evolution of societies, prevention of crises, psychological phenomena, long-term thinking. It emphasizes the normative nature of marginal analysis and equilibrium models, and denounces rough quantifications such as GNP and the staggering increase in debt. He acknowledges Marx's openness to the big issues in society that are currently excluded from political debate by an economic philosophy that tries to impress its opponents with sophisticated mathematics. These questions are analyzed thoroughly. On several occasions, however, one feels that the criticism is that the math is being misused and should be developed in some other direction (e.g. a statistical analysis of the financial tendencies that polarize wealth and income, or a study of the positive feedback mechanisms, etc.). This leaves a certain dissatisfaction — on a philosophical level — a feeling that the problem of excess math has not been addressed in all its aspects.

My thesis is that economics adds its own particular difficulties to these issues (because of its status as "conseiller du prince", and because through teaching it gives useful professional skills, etc.) and that things become clearer when we step back and frame the question in terms of knowledge in general. As the reader will see, this enables us to trace, with great epistemological force, the direction of a different type of knowledge. This allows us to escape from the addiction of mathematization while building a better quality knowledge.

We will take in a number of examples in economics and finance, but the fact remains that economics has many distinctive characteristics, as several authors have noted, which tend to prevent a reasoned consideration of its social function. Consequently there remain several points that will need to be developed further.

A. The contribution of mathematics to knowledge: some history and preliminary remarks.

Since the beginnings of civilization mathematics has been associated with most forms of knowledge. Early examples are Archimedes's work in engineering and, from the same era, *The Nine Chapters* about land measures and economy in China[1]. Few areas have not been influenced in some way by mathematics. From this long and multi-faceted history we extract some key features.

I. The Baconian program served by mathematics. It is in *Il Saggiatore* (The Assayer) in 1623 that Galileo posits that the universe is written in the language of mathematics. This *αποφτεγμα* as it is called, became the foundation of all Western science. This clarifies Francis Bacon's program, which asserts that man has a Promethean perspective, because he is subject to God choosing to share his power. He can conquer, dominate and transform nature. Galileo tells us how he can know

[1] SHEN Kangshen *The Nine Chapters on the Mathematical Art*, Oxford 1999; K. Chemla et G. Shuchun, *Les Neuf Chapitres,* Dunod, 2004.

and understand it. In fact later in his work — as Alexander Koyre has clearly shown[2] — Galileo proceeds essentially by thought experiments following mathematical reasoning, not by experiments providing data for subsequent modeling.

He believed that mathematics was a sufficient sign of the essence of God in nature that nature would reveal its secrets purely by geometric and algebraic deductions. Over a century later, Kant built his philosophy around the explicit idea that mathematics, although not based in sensory experience (*a priori* judgments), nevertheless teaches about the world (synthetic judgments). Subsequently mathematics has gradually yielded the philosophical throne of *synthetic a priori judgments*, but without ever losing the prestige of a natural fertility. In the early 19th century there was a separation with mathematics on one side, taking a modern and rigorous turn in the writings of Gauss, Cauchy and Bolzano, and philosophy on the other side, which, with Hegel's *Logic*, had no mathematical element. But then the emergence of non-Euclidean geometries and crises in the foundations of mathematics gave rise to a plurality of views about mathematics and its role in the development of scientific knowledge. At the end of the 19th and 20th centuries, with the development of physics that became the focus of epistemology, mathematics is, with variations depending on the authors, mainly considered as a servant of the natural sciences; we refer to this as its ancillary role.

II. The appearance of mathematics in economics. Sociology, as introduced by Auguste Comte, takes a non-mathematical road, except through the use of statistics, particularly by Durkheim. Subsequently it acquired its own methodological bases with Max Weber in the early 20th century. Economics, on the other hand, was mathematized as early as the mid 19th century with Jules Dupuit and Augustin Cournot, without really using statistics. Prior to this, economics presented itself as a kind of philosophy of accounting operations. After Dupuit and Cournot economics was full of talk of derivatives, equations and integrals. How did math come to be accepted into the very heart of this social science?

To answer this we follow the path of Jules Dupuit (1804-1865). A civil engineer, he realized that one can do better than simply fixing a single price for the tolls on a bridge since, whatever the price, some users will find it too expensive, while others would happily pay an even higher toll. He is the inventor of what today is called market segmentation. Having a good mathematical training he had the intuition that with a single price one cannot recover all of the integral of the curve that quantifies the willingness to pay; one can only recover that of a truncated curve. This idea of an integral is quite clear in his articles.

Yet we must note that this "willingness to pay" is a poorly defined concept. It depends on many factors, the weather, time of day, seasons, and a thousand social and economic causes. It seems impossible to measure. A collection of experiments measuring traffic against toll level would not provide a curve but *a cloud of points*. It also depends on the tolls levied on other crossings, and on whether users collude and sell their rights of crossing etc.

In the early 19th century, this concept was debated under the name "*utility"*. Dupuit pursued the belief that the mathematical phenomenon that he had discovered *would help to clarify the concept*. He postulated the existence of this quantity as a property of the commodity being exchanged and its price, which is shared according to the benefits of the seller/manufacturer and the consumer. "*Political economics*," he wrote [as opposed to social economics], "*should measure the utility of an object by the sacrifice that each consumer is prepared to make in order to acquire it*" and he took the still famous example of a bridge: "[the utility of a toll bridge] *can be separated into two main parts: 1) the lost utility, which corresponds to those crossings that would have occurred if the toll were abolished but which do not take place with the current charge, and 2) the utility produced, which corresponds to the crossings which do take place. This latter splits into two*

[2] Cf. A. Koyré *Etudes d'histoire de la pensée scientifique* Gallimard 1973, and Galilée *Dialogues et lettres choisies* Hermann 1966.

further parts: a) utility for the producer, i.e., the money raised by the toll, and b) utility for the consumer, i.e., the excess value of the service over the price it costs." [3]

Dupuit explains: "[In a shop we see] *the fine, the very fine, the super fine, the extra fine, which, though from the same barrel and showing no difference other than the superlative of the label, are sold at very different prices*"[4] and this changes the optimization of public taxes: "*So when the bridge is built and the State establishes a tariff, it stops caring about production costs. It charges less for a heavy cart which wears out the bridge more, than for a carriage with good suspension. Why two different prices for the same service? Because the poor do not value the crossing as highly as the rich, and raising the tariff would only prevent them from using the bridge*." He explains: "*The goal is always the same: to charge for the service rendered, not what it costs, but what the buyer thinks its value to be*." [5]

Dupuit fully realizes that, being defined by thought experiments, this notion of utility is difficult to measure. He acknowledges that it is abstract. "*It may be objected that the calculation for which we have given the formula is based on data that no statistics can provide, thus we will never be able to express precisely the utility provided by a machine, by a road, by any work ...*" But he advances the famous argument, which has been repeated endlessly by neo-classicists ever since, that economic science is only an approximation. It is this argument that led to all the ambiguities in the passage from descriptive to normative and to the performativity of discourse, and which opened economics up to all the mathematical refinements imaginable.

Dupuit starts from a mathematical property and uses it to account for the psychological, and it is interesting to compare his approach with that of Condorcet, who, at the end of the preceding century, proposed a different kind of mathematization of the social.

Condorcet, a great mathematician, aimed to use the calculus of probabilities to understand the propagation and sharing of a "reason to believe", a concept somewhat similar to that of utility but based on the truth or fallacy of judgments[6]. He pursued this program at length, making, along the way, the great discovery of the "paradox of the vote of an assembly". But he did not think that it would be possible to go so far as to calculate peoples' behavior.

"*On the use of language of geometry, the amount of universal commodity, that of a particular commodity, these can be approximated by numbers, but the urge to buy and sell cannot be calculated. Yet the changes in price depend on this moral quantity which, in turn, depends on opinions and passions. It's a beautiful idea to try to calculate everything, but look at the greatest mathematicians of Europe, the likes of d'Alembert and Lagrange. They seek to understand the motion of three attracting bodies: they assume that these bodies are point masses, or are very nearly spherical, and yet this issue, despite being limited by a hundred conditions that make calculation easier, has occupied them for twenty years without an answer. The effect of the forces acting on the head of the dullest shopkeeper is much more difficult to calculate.*" [7]

Condorcet's approach starts from the psychological, the reason to believe, and attempts a mathematization of sociality by the calculus of probabilities. His epistemology is an extension of that of Laplace: we cannot determine everything — principles, laws of forces and their way of acting — only the calculation of probability is relevant. It is an approach with an *a priori* limitation of science. Condorcet had to spell out all his assumptions — independence or correlation of opinions etc. — before doing calculations.

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[3] J. Dupuit *Annales des Ponts et Chaussées* 1849.
[4] *Annales des Ponts et Chaussées* 1944.
[5] *Ibid*. Note that today's large online shops can charge "good" customers more than new customers, thanks to the information they receive from cookies. Good customers are those users attracted to this way of buying and can therefore be charged more for the service. The screen presented to the customer is not a public price tag, it depends on the user's IP number.
[6] The reason to believe is what we call today the degree of certainty. Condorcet studied how it accumulates when we collect uncertain information or when members of an assembly vote.
[7] Letter to P. Verri 1773.

Dupuit, on the other hand, can immediately perform calculations, and does so in his articles, he constructs concepts which interpret price curves (assumed to be obtained). His concepts require very strong assumptions of independence, but he leaves the details of these hypotheses to be spelt out and improved later.

These features — the independence of agents presented as approximation, the progression from prices and quantities to concepts and then, during the 19th century, production function, and problem-solving by local differentiation — these will be the backbone of the neo-classical theory with Stanley Jevons, Carl Menger, Léon Walras (general equilibrium), von Böhm-Bawerk, Vilfredo Pareto (theory of optimum), Irving Fisher, etc. creating an evocative and highly flexible language that is still in use today.

III. Advanced mathematization of finance. This is a very recent and well-known phenomenon, whose history I have recounted elsewhere[8]. I will simply explain how an apparently very clever mathematization of risk, helped lead financiers away from safe practices and facilitated the emergence of the subprime crisis[9].

The crisis has occured in an era when finance is thoroughly mathematized, as a result of the "Black-Scholes revolution". A rediscovery of the work of Bachelier and the use of Brownian motion in modeling, and developments of stochastic calculus after the Second World War, particularly the work of K. Itô (1915-2008), provided a mathematical language (that of semi-martingales) in which the non-arbitrage principle could be expressed under broad assumptions that were suitable for operational cases. Methods for pricing and hedging options were thus provided by partial differential equations. The simplest case is when volatility is constant, but it is clear to everyone that these methods are largely *perfectible*, a point which is epistemologically essential.

This led to three historical phenomena: the development of derivatives markets in the U.S. first, then Japan and Europe, a transformation of professional profiles in banks and a call for new mathematical skills, and an enhanced political role for finance which was felt during the construction of the European Union and then in the globalization movement.

From the hedging of (European or American) options on stocks and currencies, the mathematical formalization then spread to more delicate issues: rate models. In particular, the bond market and the term structure of interest rates. The Cox-Ingersoll-Ross and Heath-Jarrow-Morton models allow the non-arbitrage principle to be applied here. Furthermore the theory can make use of infinite-dimensional models that must be simplified and calibrated to the current data. These model the behavior of agents over five, ten or twenty years and are therefore highly uncertain, this uncertainty being expressed in the language of probability theory.

But the most ambitious level of mathematization goes even further and deals with securitization of debts and risk assessments. Putting risks on the market is *a priori* a good idea, in the sense that it is better not to put all your eggs in one basket. But this assumes that the players (banks, insurance companies) can assess the risks.

This gave rise to a mathematical innovation worth mentioning here. It was noted that to estimate the risk of a portfolio of contingent claims, the classical method known as "value at risk," based on a criterion of the form (level of losses, probability of this level), entailed some logical difficulties. It has been shown that any criterion satisfying the desired consistency was of a particular mathematical form called a "coherent risk measure"[10]. We emphasize that these tools allow calculations for complex portfolios assuming known probability of rare events, i.e., the tails of probability distributions which have great influence on the results. These methods, in other words, yield a quantification based on unknowns.

[8] *Financial Markets and Martingales, Observations on Science and Speculation,* Springer 1998.

[9] For more détails cf. N. Bouleau "Finance et opinion" *Esprit* nov. 1998 and "Malaise dans la finance, malaise dans la mathématisation" *Esprit* fév. 2009, p37-50.

[10] For details see N. Bouleau *Mathématiques et risques financiers* Odile Jacob 2010.

In the credit-risk market financial institutions have mathematical tools to estimate risks on reassembled portfolios for the purpose of exchanging them and improving the situation of each individual with respect to their own utility function and their aversion to risk. It has often been stressed in the commentaries on the crisis that the new tools of these markets especially CDO and CDS (credit default swaps) did not encourage operators to exercise caution. That is correct. The changes in the way agents dealt with risk when protected by insurance, termed “moral hazard” by the Anglo-Saxons, surely had a role in making the “soufflé” of the crisis rise. But equally important is the fact that it was wrong to think that the risk was “in the portfolio”. The risk is interpretative in nature and just as “the beauty of the Parthenon is not found in the dust of the Parthenon”, so these mathematical tools do not see the global economic interpretations related to the decline in U.S. household savings etc.

IV. The quantification of uncertainty is a removal of meaning. From an epistemological point of view this fundamental fact needs to be stressed. It is *the significance* of the event that creates the risk. The probabilistic representation of risk is classically a pair of mathematical quantities: 1) a probability law that governs the states that can arise, 2) a random variable, i.e., a function that maps each state to the damage, that is to say the cost (counted algebraically if there are also benefits). This representation by a pair of quantities is a mathematical model both too simple and too ideal for thinking about risk. It is too ideal because we are almost never in a situation where this model is well informed. We do not know the tails of probability distributions because they concern rare events for which there is insufficient data. We do not know what correlations occur to assess the damage and we do not have a full description of what can happen. Moreover the model is too simplistic because it removes the reasons that make us interested in the events as if their translation into costs could be done automatically and objectively.

The true purpose of risk analysis is to move forward with a little foresight in organizing facts and social practices. It may be the risk that a child be knocked down while crossing the street, the risk that the air of Paris be toxic, that the failure of one business will cause that of others, etc. The intellectual operation of probabilizing a situation is fundamentally one of removing meaning. It is largely problematic for all matters concerning human behavior. Risk analysis necessarily involves *understanding* interpretations.

It is the meaning of the event that creates the risk. As an example, suppose a particular type of cancer is found in a certain proportion of the Swiss population. This proportion is then used to estimate the risk. If it is subsequently found that most of the people with this cancer had consumed cannabis twenty years ago, say, then all cannabis users become potential patients. The risk is much higher; the meaning of the event has changed. Reducing risk to a probability distribution of sums of money amounts to trusting mathematization as an approximation, as if it were describing a physical reality, whereas it is actually a question of meaning whose subjectivity permeates every interaction between the agents. This epistemological point is extremely important. They are interpretations, and hence meanings, that are replaced by numbers.

Recently there have been significant improvements in financial analysis, especially with the so-called coherent risk measures. *All these methods for making decisions in the face of uncertainty have the innate defect of assuming the interpretative process to be closed.* Yet, on the contrary, new interpretations are constantly emerging. Once a new reading is made, new risks are created, but perceived only by those who understand it. If in 2006, nobody had seen the growth of house prices and the decline of household savings in the United States as a phenomenon open to several interpretations, the corresponding risk would not have been perceived. Mathematization of risk conceals these difficulties behind assumptions about the tails of probability distributions. It is not enough to say that those are poorly known. They are by nature provisional and changeable according to the interpretative knowledge that agents bring from their understanding of economic phenomena.

V. In liberal economics, every quantification opens a possible extension to the market. There are numerous examples. The most recent is the quantification of research work. Up until the end of the last century, the quality of researchers was seen in terms of idiosyncratic talents that could only be truly appreciated by researchers themselves experienced in the same type of activities. Putting in place all the machinery of publication indices and journal citations has profoundly disrupted the working relations in the profession. I will say no more. The result has been the emergence of an international market for students, teachers and researchers, with Universities being faced with a new logic where their financial budgets determine what league of intellectual athletes they can afford.

Another example, one which is more serious in its long-term consequences, is biodiversity. Mathematization here is based on separating species into two categories. On the one hand are the *"remarkable"* species, those officially considered as threatened. For these species we calculate the cost of conservation much as for historical monuments. On the other hand for the *"ordinary"* species we calculate the *ecological service* they provide, from prokaryotes (bacteria) to eukaryotes (higher species) by standard methods of cost-benefit analysis. One can then buy and sell any part of nature or exchange it against goods or services already quantified by the economy.

B. When and how is there excessive mathematization?

We now examine the particular type of inefficiency and problem that suggests a diagnosis of excessive mathematization.

VI. We only realize after the fact. The recent financial crisis is quite illustrative in this regard. While the crisis had not yet occurred — except in the eyes of some non-orthodox observers as there always are — every agent and every financial institution believed that they should estimate the risk of their portfolios (comprised of complex products such as credit derivatives) by the methods best suited to the very mathematical nature of these products. Coherent risk measures make assumptions on the tails of laws but enable one to handle multiple scenarios. The weak point is that they omit scenarios based on global interpretations where the value of each portfolio cannot be calculated by considering the others as *ne varietur*.

Once the crisis had started, and after the resultant upheavals, what happened was the result of political forces: on one hand a strong current of opinion emerged urging the adoption of regulatory measures in order to avoid future crises or at least limit their damage, on the other hand most financial workers felt that all that was needed was to take into account the interpretation that had been neglected, to improve, in other words, the global readings of risky situations by strengthening the role of rating agencies in particular. The latter have now been warned, and have learnt to keep in mind the previously neglected facts (resistance to "stress" of the various institutions, etc.). For public opinion we are back where we started, with the same tools with the same defects. [11]

VII. Calculations conceal ignorance. This is obvious for financial risks. Because we do not know how to quantify counterparty risks, or those related to market liquidity, and much less those which are due to human error or to changes in the law, very precise calculations are mixed with crude estimates hoping that they will have no appreciable impact on the outcome. Applying sophisticated calculations, such as coherent risk measures, to complex portfolios supposes that the risks are expressed perfectly in the ontology of the objects considered at the outset. In other words it adds a second level: one ignores one's ignorance. This affects the market (organized or OTC) in credits and their derivatives. By the market, portfolios acquire a value where everyone trusts everybody else's calculations though they are no better. This leads to an instability that may be called "methodological moral hazard" which is the belief that mathematics is able to capture new

[11] It is impossible to predict the next crises, but we can guess that they will revolve around the failure to take into account limits. Bounds, finiteness of the world, resources, raw materials, agricultural land, etc. are all ignored by economics. Anticipation of increasing scarcities in an uncertain environment may provide unpredictable instabilities.

interpretations if the calculations are done by everyone. This kind of instability is worse than in conventional markets in assets and their options because the timescales are much longer (tens of years instead of tens of months) and the punishment of economic reality comes much more slowly.

VIII. The ancillary role of mathematics as servant is confused with that of the subjects being served. The previous idea can be generalized to all situations of mathematized knowledge. Let us take the case of physics. It is obviously helpful to physics when the mathematics used by physicists is improved. There is a real fertility there which has been particularly emphasized by Gaston Bachelard. But it works with the same interpretations as the served science. We are in the syntactic part of normal science in Kuhn's sense. Although Bachelard, with his usual talent, shows that mathematics can suggest questions for physicists, it is impossible to get genuinely new interpretations of phenomena occuring in the domain of the master discipline in this way. Mathematization is an essential component in the phenomenon of scientific crisis as described by Thomas Kuhn.

IX. That a theoretical representation be perfectible does not mean it is the only way to deal with reality and does not guarantee that it is capable of taking into account every aspect of the situation in question. By theoretical representation I mean a semi-artificial language using mathematics, as in physics or modeling. The fundamental point is that perfectibility gives the illusion of completeness. Ptolemy's geocentric planetary system provides a good example: the excess of mathematization lies in cycles and hypocycles that can be added at will. The original system was improved by Tycho Brahe and is infinitely perfectible, and the excess only became apparent after the new interpretation given by Copernicus. The only flaw in Ptolemy’s system is that it has no place for this new interpretation. Yet the new interpretation was much less precise, at least initially, when Copernicus was proposing heliocentric circles. But this is astronomy not planar geometry, and the new reading acquires legitimacy from the fact that it too could be a starting point for improvements; it also has room for possible enhancements. Galileo cannot depart from this new interpretation because he recognized in Jupiter and its satellites a Copernican system. Nevertheless, having, at that pre-Newtonian time, only a *kinematic* description of phenomena, he has no compelling argument against the geocentric system. He was accused during his trial of basing his position on "beliefs" that are not in the sacred texts. It is a case of one interpretation against another, a situation cleverly analysed by Augustin Cournot[12]. The position of Cardinal Robert Bellarmine is that faith has a monopoly of beliefs and that science must remain a means of describing what is allowed in God’s creation.

X. There is confusion between creativity of the representation and creativity of the world. Within a system of thought, especially one that is perfectible, one cannot see a reason to escape the system. This is related to Quine's remarks on ontological commitment and on the near impossibility of talking about things we either don’t know about or deny the existence of. Quine emphasizes our strong tendency to “talk and think about objects”[13] both in ordinary language and in physical or economic theories where agents and objects are subject to certain relationships. “It is hard to say how else there is to talk, not because our objectifying pattern is an invariable trait of human nature, but because we are bound to adapt any alien pattern to our own in the very process of understanding or translating the alien sentences.”[14] Quine also takes into account the ontological conflicts in order to clarify them. The novelty of the famous article “On What There Is”[15] is the proposal of a definition of ontological commitment which in principle applies quite generally. In fact these fine arguments inspired by mathematical logic are based on the use of logical quantifiers and are quite

[12] cf. N. Bouleau *Risk and Meaning* Springer 2011, chap. II. Cournot's "philosophical probabilities".
[13] "Speaking of objects" in *Ontological Relativity and Other Essays,* Columbia University Press, 1969
[14] *Ibid*
[15] in *From a Logical Point of View* (1953), Harpers & Row 1963.

abstract, and they do not focus on the emergence of new objects.

A more concrete historical example is very illuminating: the abandonment of the natural scale in music. The octave, fifth and other basic musical intervals correspond initially to the division of a vibrating string into simple fractions, one-half for an octave, two-thirds for the fifth, three-fourths for the fourth, etc. This is a strict mathematization of the harmony that is actually perceived by the ear through sound frequencies. If we move from fifth to fifth by iterating the operation of taking two-thirds of the length, then we find that twelve fifths are approximately seven octaves. Hence, translating these divisions back onto the original octave yields the twelve intervals of the so-called Pythagorean scale. It is approximate since 12 fifths are not exactly 7 octaves, but it is very close to the mathematics of vibrating strings, which is the natural (and scientific) basis of sound. It took more than twenty centuries before the natural scale and its improvements were abandoned and the so-called "even-tempered" scale, which gives exactly the same role to all intervals, was adopted. The instruments built on the even-tempered scale do not give preference to a particular key, but they do not respect fully the laws of vibrating strings. The creativity of the musicians has won over that of mathematics in music. The victory is in fact not total, because of some harmonics that are heard as dissonance, etc. But the point to emphasize here is that the idealized world of mathematics has been put to one side in favour of a world based on practice.

C. Why normal science and jolts of revolutions? Why orthodox economics and crises?

Things seem to move like tectonic plates, in jolts. Why is this? How can we implement a production of knowledge that goes beyond the Kuhnian epistemology?

XI. As Kuhn thought, normal science is very close to the Popperian vision. Only the modalities of its functioning are seen with a more social emphasis on paradigms as shared understandings of scientific communities. The real difference with Popper is that the disorder that precedes a crisis is more complex than simply encountering a decisive experiment that could refute the theory: there are also attempts to negotiate with the forms of interpretations. Usually the plasticity of the paradigms allows the acceptance of new facts or events in the theory. Kuhn takes the example of a child learning to distinguish ducks, swans and geese in a zoo, with his father playing the role of experimental verdict. He stresses the importance of slightly fuzzy categories whose vagueness is not mathematically quantified[16]. But in certain historical situations, the various ways of arranging things lead to choices that are too artificial (properties of the ether, for example), which gives rise to the search for and the legitimization of more radical interpretative changes.

XII. But most mathematization situations are not Popperian. Economic theories are not likely to be refuted by any observations of facts. The social environment is constantly changing and is never the same twice. Specialized models with predictive aims are probabilistic and cannot be falsified by a single event. More generally, mathematizations useful for studying changes in the environment (pollution, climate change) are always open to several competing models, each based on a different perspective (extrapolation from ice cores or CO2 emmissions), each perfectible as new data become available. The simplest generic example is that of modeling the flow of a river for flood forecasting. Families of models based on Gaussian ARMA factoring in 1) the water depth, 2) the flow rate, 3) the logarithm of the depth and 4) the logarithm of flow, are each infinitely perfectible if new measured data are available yet they do not give the same probabilities of reaching a certain level[17]. This does not mean that these models are useless, far from it. It just shows that it is not because

[16] T. Kuhn "Second thoughts on paradigms" (1974), in *The Essential Tension*, Univ. of Chicago Press 1977.

[17] Auto-Regressive Moving Average processes are the simplest Gaussian models with linear recurrence used in theory of time series. Obviously to the above four families we could add linear alpha-stable models and models with quadratic recurrence (heteroscedasticity) etc. All these examples illustrate the relevance of the remarks of Quine on underdetermination of theories by experience.

reality is plural that it is not scientific. In fact, for one type of phenomenon, the data are always finite in number and a finite number of points can be matched either by polynomials or by combinations of real exponentials or trigonometric functions etc. If you think about the immense range of subjects opened up by modeling, then you quickly become convinced that it is the Popperian cases that are the exception. For a theory to be Popperian it must have a fixed number of parameters, each fixed numerically. It is hard to think of any apart from gravitation and some physical theories. Probabilistic theories never fall into this category because an infinite number of events is needed to determine a probability distribution.

This remark also applies equally well to normal science in the sense of Kuhn. It is an extremely restrictive view of knowledge. Let us be more precise.

XIII. It is the monism required at each step that causes the jolts. Where does the new interpretation that is characteristic of a scientific revolution come from? It can only come from differences in the subject community. In other words, the jolts come from the absolute will that the community accept only one truth. Yet this is one particular vision of knowledge and social organization of science. If we accept instead that "reality" is also, and indeed primarily, people, groups, with their abilities, their habits, their psychology, and their means of interacting with their environment, we see that the only way to capture, or at least to take some account of, the innovation in the world is to make space for the instances where new representations are constructed : users' associations, professional groups, consulting experts, victims of unforeseen circumstances, etc. As Funtowicz and Ravetz have thoroughly analyzed, this route leads to *a better quality* of knowledge, more reliable and in which we can have more confidence[18].

It is a pluralistic knowledge, but that is not to say that it is relativistic. This distinction is crucial. Specifically, as soon as one demands a certain level of rigor and consistency, one is limited to a *small number* of different approaches, just as the major political ideas concern a limited number of parties in multiparty parliamentary systems. To say that departing from the monism of unique truth leads one into relativism is the coarse argument of dominant representations, which the jolts of scientific crises regularly refute.

Nevertheless, if the implementation of such pluralistic knowledge is progressing well in some areas such as climate change or the protection of sensitive areas (despite clashes with political power, which are nothing new), it presents particular difficulties for economics. With globalization, knowledge about economic exchanges has a strong tendency to monism. One would think, however, that the growing environmental problems should lead us to greater tolerance in the implementation of specific economic experiments and their running as a condition of better support for natural equilibriums.

D. Interpretative pluralism is not destructive of knowledge; it is a better type of knowledge.

We now propose to examine more thoroughly the features of that better quality and what role mathematics can play. This will necessitate a step back from science as it is currently most often understood and practiced. Beyond the concept of "confined research" introduced by Michel Callon[19], it appears that what is at stake is the *conquering* character of the Baconian program and the *masculine virtues* connected with them.

For convenience we shall use the term challenge-science to describe the view, held until recently by most scientists, that sees knowledge as *a challenge to nature*. It challenges nature to a duel. The honor in the game is to respect the assumptions that govern the rules for experiments. This includes Popperian science and Kuhn's normal science. In fact it is very old; the induction

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[18] S. O. Funtowicz and J. R. Ravetz "Three Types of Risk Assessment and the Emergence of Post-Normal Science" in *Social Theory of Risk*, Sh. Krimsky and D. Golding eds, Preager 1992.

[19] Cf. M. Callon, P. Lascoumes et Y. Barthe *Agir dans un monde incertain, Essai sur la démocratie technique*, Seuil 2001.

principle advocated by many philosophers and scientists to account for knowledge is similar in nature. Put simply, Popper proposes an induction articulated on a theory. Instead of accepting the thesis that knowledge is essentially philosophical in its ability to spot a pattern and extrapolate it — an idea championed simultaneously (in 1843) by John Stuart Mill and by Augustin Cournot who finely analysed it — thus drawing from a large number of results, or a large number of circumstances, a prospective law that is to be evaluated, Popper strengthens the criterion by requiring that we move from observed facts to a representation with the dress of a theory, that is to say, based on a mathematical syntax like mechanics as formulated by Lagrange or Hamilton. Historically, it is indisputable that during the whole period where industrialization had not yet complexified technology too much, science was practiced with little experimentation and as many challenges were presented to colleagues as to nature. The discoveries at the time of Pascal, Fermat and Father Mersenne were often announced as puzzles, whose answer was known only to the finder, to challenge the wit of contemporaries[20].

In these early years of the 21st century, a new awareness, unique in the history of man, is happening. Endless continual growth is impossible, and even if the limit is not yet reached, the current pace is so destructive that it must be drastically curbed[21]. It is becoming less and less clear that using challenge-science vis-à-vis the environment, with new technical devices and a progressive mathematization to calculate the economic optimum by cost-benefit analyses in the context of democracy and liberal economy, can overcome the global challenges : arable land, species, climate change, pollution of soil and water, etc.. New options for production and consumption (e.g. use oriented product service systems, etc.) and for democratic structures (new bicameralism[22]) are probably essential. But, more fundamentally, we must also consider the question of what kind of knowledge. The epistemological question of how knowledge is produced also arises.

XIV. What logical status can the new knowledge have? Is there “room” for anything else? What are the characteristics of forms of knowledge that are not falsifiable theories — are there any? They would eventually be forgotten but they are innumerable. Included in this field are all *useful discoveries* that form the logical category complementary to that of refutable hypotheses. The vast majority of knowledge about animal, mineral and vegetable, and a great deal of technical expertise, is of this type.

In this class we find most of the chemistry that has long been viewed as pre-scientific when compared with physics. The great chemist Henry Le Chatelier in the early twentieth century says: “These two sciences have a similar purpose, they both study phenomena that result in transformations of energy, i.e., mechanical, calorific, electrical or chemical power. In teaching physics one refers only to the laws of natural phenomena: the laws of Mariotte, Gay-Lussac, Ohm, Joule, Descartes, Carnot, etc.. [...] In chemistry, on the other hand, there is an endless list of small particular facts [...] the material thus accumulated will be very useful for the subsequent establishment of science but they do not yet constitute it in any way”[23]. Why such a disgrace? Is it justified in terms of services rendered?

This class also contains most medical and environmental knowledge. Long before Popper, Claude Bernard wrote the following about medicine: “in science you can make two kinds of discoveries. Some are predicted by theory; these suppose two conditions: a very advanced science, e.g., physics, and simplicity of the phenomena. The other kind are unexpected: they appear

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[20] See Koyré, *ibid*, on the fact that Galileo never experienced the stone that falls from the mast of a moving ship, and on the conundrum in which he announces to his contemporaries his discovery of the phases of Venus.

[21] Cf. D. Bourg and A. Papaux *Pour une société sobre et désirable* PUF, FNH, 2010.

[22] Cf. B. Latour *Politiques de la nature, Comment faire entrer les sciences en démocratie* La découverte 1999, et D. Bourg et K. Whiteside *Vers une démocratie écologique, Le citoyen, le savant et le politique* Seuil 2010.

[23] H. Le Chatelier, *Leçons sur le carbone, la combustion, les lois chimiques*, preface, Paris, 1908.

unexpectedly in the experiment, not as corollaries of the theory and devoted to confirm it, but always outside of it and therefore contrary to it." [24]

More generally, outside the challenge-science category lies all the knowledge about how the world *is*, what features make it the way we find it, and not another that follows the same laws. This is not inconsistent with general knowledge in Aristotle style, but these innumerable and fortuitous data, that reflect what life and history have made, are essential for nature and the society. Besides, without them challenge-science is nothing. Computers can help us to store them but they do not reduce to dimensions or coordinates. They are interpretative like the new paradigms that Kuhnian revolutions bring. We must therefore accept that some are complementary — plural answers to the same question, differing accounts written in different styles and emphasizing different points.

XV. A knowledge whose social function is not prediction but caution and care. We have to make a place for stories, testimonies, for what makes our current understanding of the world in all its diversity. They are the basis for the uses and values that give meaning to representations, even scientific ones.

With regard to mathematics, there is no reason to exclude it, we need it here too. But symbols may be used more freely than in axiomatized theories. It is perfectly legitimate to reveal a phenomenon, to represent a trend or a natural evolution using existing scientific languages from the established sciences or from engineering which are semi-artificial languages with partial mathematization. For managing natural equilibriums of life and for working on collective decisions of social groups, it is necessary to allow various representations and even different rationalities to coexist. The use of mathematics as thought patterns, for the linguistic value of symbols and combinations thereof, is useful and desirable. They are not reserved for expressing the truths of challenge-science.

XVI. The main tool of a better quality science is critical and contradictory modeling. The models are able first to take into account the distinctive features of situations and to apply proven knowledge to them and secondly to translate, by the ordinary language which forms the internal cement and the external context, an interpretation of the complexity into what we are interested in.

If they are not to be seen as low level or amateur challenge-science, it is essential that models be always viewed as a facet of a plurality. Firstly, they must be validated by data with the same rigor as usually required by scientists. This validation is not a test of truth, but simply a process of eliminating the unlikely. Secondly they must be recognized as a social expression, i.e., a form of communication from an agent (be that a group, association, company, territorial entity, etc.) to an audience in order to contribute to a decision and therefore subject to criticism by other models. Knowledge is no longer formed exclusively by a struggle between theory and nature but by a contest between models. This process obviously requires a specific organizational context, just as challenge-science requires cautious experimental protocols. The "rules" are not currently codified, but the experiments are underway at international level for the IPCC and in the public debates, citizen juries etc., in a kind of applied living epistemology still under development.

To critique a model is difficult. The quantitative arguments are linked together, everything is connected. It is a huge task to draw out all the implicit assumptions of a model. Even though we know that every model is arbitrary in some aspects, we do not see this arbitrariness explicitly. When discussing one model, our thinking remains stuck in a rut. The best way is to build another model from scratch — the options are much clearer then.

To construct another model, the dualities introduced by the philosophy of science are relevant — they facilitate a dialectic setting for the occurence of what may be called co-truths. Let us consider a few examples.

[24] Cl. Bernard *Leçons de physiologie expérimentale appliquée à la médecine faites au collège de France*, Paris, 1885.

Discrete / continuous. Much of the economic theory can be developed without individualizing agents or goods. Some scholars find it illuminating to derive global laws from a micro-economic individual rationality. When studying traffic, depending on the question we may use flow models or we may model each vehicle individually. Sometimes it is thought that discretization, spatial or temporal, simplifies the problems, with the recurrence rules being more elementary than differential equations and finite element algorithms reducing partial differential equations to simple algebra. But often the opposite happens: the discrete probabilities are sometimes intractable and some algorithms (such as Kalman), are best understood in continuous time.

Descriptive / explanatory. In 1970, two American authors, G. E. P. Box and G. M. Jenkins took methods invented by Wiener for signal processing and applied them to economic predictions. Treating annual series without any regard to their economic meaning, they sometimes obtained better predictions. This is the fundamental duality which we began with in this article. In the history of science, it often occurs in successive periods. The purely descriptive approach can be an advance when it frees us from certain loaded interpretations. On the other hand, explanations allow a reading to shed light on situations other than those already considered.

Quantitative / qualitative. The philosophical work of René Thom has brilliantly illustrated that mathematics provides representation tools that go far beyond the quantitative. A huge field of natural phenomena can be addressed qualitatively through a language adapted to the evolution of forms.

Deterministic / random. A huge number of modeling situations involve risks. The instinctive tendency of modelers is to probabilize the uncertainties — we have already discussed this tendency. This provides a very efficient syntax thanks to the stochastic calculus developed in the 20th century. But this, especially in the tails of laws, conceals ignorance. Uncertainty is sometimes better illustrated by some typical or extreme trajectories obtained from different scenarios.

Image / symbol. Let us take the example of dance. Dozens of notation systems have been developed by the choreographers to record ballets, either based on a limited vocabulary of successive steps (Feuillet system 1700) or more elaborate, noting the dancer's energy in each movement (Laban system 1927). The problem is one of modeling, with the usual constraints of relevance for the choreographer and dancers. *But is this not a false problem since film and video can provide us with an almost perfect image of the ballet* ? The image reproduces, it can provide the perfect illusion of reality, but it does not, by itself, allow choreographic creation. The notation systems have the immense superiority of enabling one to record a ballet that has never been danced.

Critiques of models cannot come from recipes or an *a priori* classification, especially since, as we have emphasized, their relevance depends on the social group that proposes them. *The quality* of the plural knowledge thus produced comes particularly from the things that it can draw out of reality but which challenge-science fails to see. Applied in good conditions of open democracy, it is likely to show hidden effects, unnoticed risks, possibly unsuspected solutions. Challenge-science instead, with the successive stages of its rockets, heads only in one direction.

Conclusion: The problem is not that there is too much mathematics, but that it is used exclusively as a framework for theories that claim to univocal truth.

The propensity to mathematize more and more can occur in the development of a classical theoretical line of thought as much as one based on modeling, especially if one assigns a value of absolute truth to the interpretative framework we work in, so that syntactic developments will be seen as revealing reality. This occurs in modeling because the modelers tend to think that their models are reality. But faced with other models they are forced to acknowledge *the scope* of their approach. In contrast, in a Popperian conception, mathematization can be pursued without any restraint, until a crisis occurs. Our analysis of mathematization is an Ariadne's thread that opens up the philosophy of knowledge to a new and immense field of thought. It turns away from the jousts, catapults and knights-in-armor of the conquering knowledge, it takes a step back, whereupon challenge-science starts to look like a very particular way of understanding the world.

It is ultimately a choice between what is important and what is not. A river basin for example, may remain for centuries. But we are faced here with contradictory logics, politicians who want to develop jobs, farmers who want to irrigate, associations that want to respect the landscape, companies that want to build dams for electricity, etc. Often neither the economic interest nor the democratic vote, can overcome the basic dominance of selfishness. Maintaining the scenes of natural life involves intermediate languages between native speech and falsifiable science, languages which oppose but do not destroy each other, which, by their plurality, are open to the *interpretation of data* and the *imagination of eventualities*.

About mathematics itself, there is no need to worry. Real mathematicians know what drives them: the pleasure of an intellectual game[25]. Maths does not need to be the framework for a grand and unique building of knowledge. On the contrary, freedom from applications and doctrines has always been maintained : non-Euclidean geometries, non-standard analysis, etc. Explorations off the beaten track are rewarded with the surprise of the treasures discovered there.

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[25] Cf. N. Bouleau, *Dialogues autour de la création mathématique*, in coll. with Laurent Schwartz, Gustave Choquet, Paul Malliavin, Paul André Meyer, David Nualart, Nicole El Karoui, Richard Gundy, Masatoshi Fukushima, Denis Feyel, Gabriel Mokobodzki, 1997, on line : http://www.enpc.fr/HomePages/bouleau/DialoguesInterferences.html