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Sabine Lemoyne de Forges, Ruben Bibas, Hallegatte Stéphane. A dynamic model of extreme risk coverage : Resilience and e fficiency in the global reinsurance market. 2001. <halshs-00800460>

HAL Id: halshs-00800460
https://halshs.archives-ouvertes.fr/halshs-00800460

Submitted on 13 Mar 2013

POLICY RESEARCH WORKING PAPER   5807

A Dynamic Model of Extreme Risk Coverage

Resilience and Efficiency in the Global Reinsurance Market

Sabine Lemoyne de Forges
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Abstract

This paper presents a dynamic model of the reinsurance market for catastrophe risks. The model is based on the classical capacity-constraint assumption. Reinsurers choose every year the quantity of risk they cover and the level of external capital they raise to cover these risks. The model exhibits time dependency and reproduces a market dynamics that shares many features with the real market. In particular, market price increases and reinsurance coverage decreases after large shocks, and a series of smaller losses may have a deeper impact than one larger loss. There is a significant oligopoly effect reducing reinsurance supply, and the market is segregated into strategic large actors that influence market prices and price-taker smaller firms. A regulation trade-off between market efficiency and resilience is identified and quantified: improving the ability of the market to cope with exceptional events increases the cost of reinsurance. This model provides an interesting basis to analyze further capacity needs for the insurance industry in view of growing worldwide exposure to catastrophic risks and climate change.

A dynamic model of extreme risk coverage: Resilience and efficiency in the global reinsurance market

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1 Introduction

Over the last 50 years, insured losses caused by natural disasters have followed an increasing trend.[1] Population increase and economic growth on the one hand, the development of insurance markets on the other hand, have expanded the global insurance industry. In the future, continued growth and development in at-risk zones and possible changes in climate are likely to maintain or amplify this trend. This evolution would represent a significant challenge for the insurance industry (Herweijer et al., 2009). Reinsurance firms are particularly exposed to these issues as they supply disaster coverage to insurance companies for hurricanes, windstorms or earthquakes, among others.

The reinsurance market resilience is a concern for the insurance sector. In particular, insurers need to assess the ability of the reinsurance sector to absorb large shocks. More specifically, they need to know how important losses impact the reinsurance market capacity, reinsurance prices and the time needed for the sector to recover.[2] From a public policy perspective, one may ask if a modified solvency regulation increases the system resilience, and how it impacts the market. Resilience is a key issue in the finance industry in general. It has been raised again following the financial crisis, as the ability of the financial markets to withstand shocks has been questioned. The economic literature has examined the question of regulations for financial industries and especially insurance industry (see Plantin and Rochet (2007) for a review). Our study focuses on the specific case of the robustness of the reinsurance market to large shocks but its conclusions are of interest for the financial sector in general.

In this paper, we analyze the impact of important catastrophic losses on the reinsurance market, using a dynamic model of the reinsurance sector based on a solvency constraint for reinsurance firms. We thus enrich Winter (1994) first dynamic model of the competitive insurance market. This solvency constraint is represented by a bounded probability of default, and the model is related to the classical capacity constraint hypothesis. Furthermore, we take into account the strategic effects of firms individual behavior on the market equilibrium, completing the traditional competitive view of the insurance industry (Hardelin and Lemoyne de Forges, 2009).

Model results show a market dynamics that reproduces many real-world observations, suggesting that the capacity-constraint hypothesis is able to explain the observed variability in reinsurance prices and quantities, provided that strategic behavior is taken into account. We find path-dependency, i.e. the fact that the consequence of a one shock depends on previous years losses and reinsurers response. We characterize and quantify the "resilience" of the sector through the time necessary to restore reinsurance capacity after large disasters. We provide a sensitivity analysis of the key parameters including higher loss levels and series of losses. We show that improving the ability of the reinsurance market to cope with large losses and reducing reinsurer default probability — that is consolidating reinsurance firms with regard to risk — has a cost, due to its negative impact on market prices and capacity. Thus we exhibit a regulation trade-off between market resilience and efficiency.

The paper is organized as follows: Section 2 briefly describes the main characteristics of the reinsurance market for natural catastrophe risks. Section 3 presents the underlying economic

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[1]See Munich Re's long-term statistics: http://www.munichre.com/en/ts/geo_risks/natcatservice/long-term_statistics_since_1950/default.aspx

[2]Capacity refers to the term used by Cummins et al. (2002) and defined as follows: *"For any losses for which the reinsurance companies are liable, the capacity of the reinsurance market is the proportion of the liabilities that is deliverable to their customers, given the financial resources of the companies, and all their risk management arrangements (retrocession, catastrophe bonds)."*

model and its main assumptions and limits. Section 4 describes the modeling approach and details the data used for the calibration. Section 5 proposes a reference scenario and compares it with the stylized facts described in the literature. Section 6 provides a sensitivity analysis on model parameters. Section 7 discusses the regulation trade-off beween efficiency and resilience. Section 8 concludes.

2   The reinsurance market for natural catastrophes - An overview

The reinsurance market is a relatively small market compared to the insurance industry. Its capitalization, all lines included, was estimated at the end of 2008 at $309 bn (Aon-Benfield, 2009). As underlined by Plantin (2006), the demand for reinsurance can be explained from two different points of view. Reinsurance can be used by insurance companies as a *risk management tool* (Borch, 1962; Blazenko, 1986; Lewis and Murdock, 1996; Froot, 2001; Froot and O'Connell, 2008) or from a *capital structure perspective* (Doherty and Tinic, 1981; Mayers and Smith, 1990; Garven and Tennant, 2003). In the specific case of natural disasters insurance, the role of reinsurance is actually threefold: (i) reinsurers provide insurance companies with additional capacity, thereby making it possible for them to increase insurance supply; (ii) reinsurance allows for a worldwide mutualization of losses, thus reducing the cost of risks; and (iii) reinsurers provide an expertise on the risks supported by insurers. This third aspect is particularly important for insurance companies of medium and small sizes, which do not always have access to sufficient knowledge on disasters risks.

The reinsurance market for natural catastrophe risks is a peculiar market as described in Froot (2001) and Cummins and Trainar (2009). Contracts, referred to as *treaties*, are passed between a ceding insurer, called the *cedant*, and the reinsurance company. They determine the covered portfolio, the underlying risk and the level of coverage. For natural catastrophes coverage, the most common contracts are excess-of-loss treaties based on non-proportional reinsurance.[3] The cedants transfer a "layer" of the risk from a defined portfolio to the reinsurer at a contractually defined price. A *layer* is defined by a deductible (the risk remaining at the charge of the insurance companies) and a limit (that is the maximum indemnity that can be paid by the reinsurance companies). For example, an insurer gets coverage for a property portfolio exposed to hurricanes with a treaty of $20 million in excess of $5 million. In this case, if an event (defined in the contract) occurs causing a loss of $12 million, $7 million will be paid by the reinsurer. If an event causes losses amounting to more than $25 million, the reinsurer only pays $20 million to the cedant. Typically, a cedant builds a reinsurance program, dividing coverage among different reinsurers even for the same layer of the risk, thus diversifying its exposure to each reinsurer, and taking into account its quality (for instance its default risk, its claims management system, etc.). Note that there is no standardized market for such risks, and data on these contracts are difficult to obtain.

To model the ability of the reinsurance industry to withstand large disasters, a good under-standing of its specificities is necessary. Numerous contributions see a limited systemic risk from disasters in the sector, as mentioned by the reports of The Group of Thirty (2005) and The Geneva Association Systemic Risk Working Group (2010). But even if capital depletion in the reinsurance industry does not increase the risk of default significantly, it has however an important impact

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[3]Other types of reinsurance contracts are described in Cummins and Trainar (2009).

on reinsurance prices (Guy Carpenter, 2009) and thus on the capacity of insurance companies to maintain the same level of coverage. As a consequence, it impacts significantly insurance prices and the wider economy.

Several market characteristics have to be taken into account to understand better this mechanism which is complicated by the entangled aspect of the reinsurance market and its opacity. First, the market has a small number of participants who share a lot of business risks (exposition to large disasters, asset risks). Second, the reinsurers may themselves use retrocession, i.e. they transfer part of their risk to other companies.[4] Third, the reinsurance market is characterized by several market imperfections that explain the limited use of reinsurance. Indeed, the market volume of the global reinsurance market is quite low compared to the insurance one, and the price of reinsurance has been found quite high, up to several times the actuarial price (the expected loss of the underlying risk's distribution). Market imperfections, as listed by Froot (2001), include important frictional costs linked to the illiquidity of reinsurance treaties; moral hazard (Bohn and Hall, 1999; Doherty and Smetters, 2005); adverse selection (Cutler and Zeckhauser, 1999; Jean-Baptiste and Santomero, 2000); interventions on the reinsurance market by third-parties, from disaster reliefs of all forms to state guaranty funds (Bohn and Hall, 1999); and agency issues. However, K. Froot underlines two aspects of the market that are particularly important: (i) capital market imperfections that lead to capacity shortage; and (ii) reinsurance market power. Our modeling exercise is built on these two mechanisms.

Another characteristic of the industry is the time-variability of prices and capacities, which is often referred to as a "cycle" (Meier, 2006). A large literature exists on the reinsurance cycle, on which a review has been conducted by Weiss (2007). It provides a wider theoretical analysis of the dynamics of insurance and reinsurance markets. Thus, reinsurance prices are high during "hard" market and low during "soft" markets. Even if this denomination of cycle is debated (Kessler, 2005), the special role of disasters is noted as they may induce an important depletion of capacity (Weiss, 2007). This hypothesis is reinforced by the observation of emerging transitory means for reinsurance companies to gain access to additional capital after large shocks, like catastrophe bonds or newer vehicles (Lane, 2007a; Cummins, 2008), or the emergence of new reinsurance companies as the Bermudians (Lane, 2007b).[5] Figure 1 represents the evolution of natural catastrophe reinsurance price index (given by the rate-on-line) as well as new capital flows into the natural catastrophe reinsurance industry.[6] A distinction is made between equity and IPO (Initial Public Offering) — in dark grey on the graph — and Insurance Linked Securities (ILS) as catastrophe bonds and sidecars — in light grey on the graph. The impact of Andrew (1992), the World-Trade Center's attack (2001) and Katrina (2005) can be observed. Following these losses, the price of natural catastrophe reinsurance increases. There was also an increase of the flow in external capital to the market: $10 bn in 1992/93, $16 bn in 2001/02, and more than $35 bn in 2005/06 for the whole industry (Guy Carpenter, 2009). The development of the ILS market can be seen on the graph

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[4] Retrocession qualifies the cession of risk by reinsurance companies to retrocession companies (i.e. other companies devoted to retrocession or other reinsurance companies). The Group of Thirty (2005) estimate its volume at 10 to 15% of the reinsurance risk.

[5] Risk-linked securities enable insurance or reinsurance companies to transfer a share of their risks to the capital market. Catastrophe bonds are defined by Cummins (2008) as "a fully collateralized instrument that pays off on the occurrence of a defined catastrophic event", side cars are.

[6] The rate-on-line corresponds to the reinsurance limit (that is the maximum indemnity that can be paid by the reinsurance company) of a contract divided by the premium paid. It is an imperfect proxy for reinsurance prices, but it is widely used in the industry.
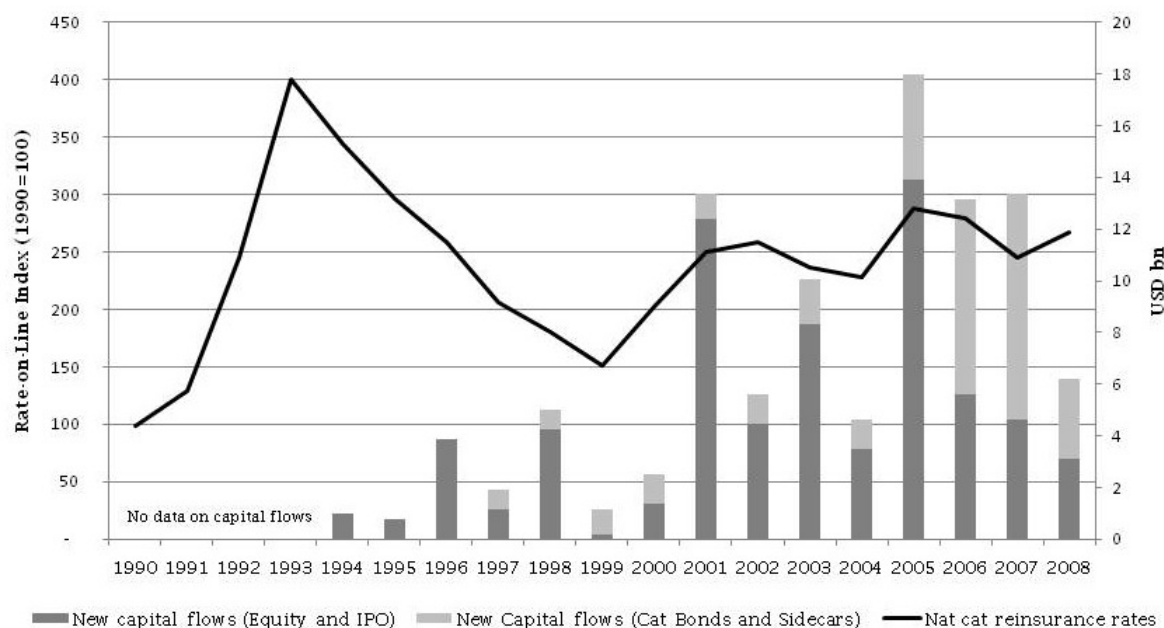
(see Cummins and Weiss (2009) for a review).

Figure 1: New capital flows in the natural catastrophe reinsurance industry and reinsurance rates for natural catastrophes.
Source: The Geneva Association Systemic Risk Working Group (2010)

Two types of models are used in the literature to analyze this "cycle." The first kind focuses on the impact of *capacity constraints* on market prices (Gron, 1994). As Weiss and Chung (2004) mentions, these models are particularly relevant for the study of extreme events impact on the reinsurance market. The second kind is based on a *risky debt hypothesis* in which customers are concerned about the liability of their insurance companies (Cummins and Danzon, 1997; Zanjani, 2002).

Our model builds on these analyzes to address the issue of natural catastrophe reinsurance capacity but proposes a different approach and four main contributions. First, the model is dynamic and can be used to estimate market resiliency to series of disasters. Existing companies resistance to catastrophe losses is modeled. The time needed for the market to replenish itself is simulated. Secondly, it allows for a better understanding of the impact of market imperfections on the reinsurance supply. It takes into account the small size of the reinsurance market and particularly the leading role of the few reinsurers with large market shares. The consequences of financial market imperfections are also scrutinized. Third, the validation of the model behavior compared to what is currently observed allows for a discussion on the ability of the sector to adapt to increasing exposure and losses, due to socio-economic drivers and climate change. Four, the model makes it possible to investigate the costs and benefits of regulation that aims at reducing reinsurers default risk and increasing systemic resilience.

3   The model

Our approach is similar to that of Dynamic Stochastic General Equilibrium (DSGE) models. It is based on a succession of Nash equilibriums on the reinsurance market in which market conditions depend on past losses and reinsurers' actions. Our approach shares common features with Winter (1994) model of the "dynamics of competitive insurance markets", especially the model recursive form. But we extend this approach considering a solvency constraint (and no limited liability), an oligopolistic market, and by providing long-term simulation of the market variables.

3.1   Reinsurance demand

Modeling the process of risk sharing between several insurers and reinsurers is difficult. We propose here a simple approach: we only consider one aggregate demand for reinsurance. This demand consists of one catastrophe layer to be covered by the whole market. Each reinsurance company provides supply for the coverage of a fraction of this layer. These assumptions, even if they are restrictive, provide a tractable way to understand the reinsurance market. This is a proxy since it does not distinguish between different disasters and affected regions, and between risk layers.

We suppose that market demand is characterized by $d(p)$, the share of the insured underlying risk covered by the reinsurance market, in other words the fraction of the insured losses that are transferred to reinsurers. This coinsurance factor only depends on the market reinsurance price p:

$$d(p) = d_0 \exp(-\delta p) \tag{1}$$

For notational simplicity, the subscript t, which refers to time, is omitted in all equations. The parameter δ characterizes the elasticity of demand to market price. The parameter d_0 is such that for an actuary fair premium p^a, $d(p^a) = 1$ and all insured losses are reinsured. The function $d(p)$ is decreasing and convex in p.

3.2   Reinsurance supply

We consider a market with N_R reinsurance firms, indexed by $r = 1, .., N_R$, that produce equivalent goods O_r. The variable O_r is the amount of reinsurance coverage (in terms of coinsurance factor) that is supplied to a unique global insurance market, i.e. to a continuum of insurance companies. For example, in presence of insured losses $\tilde{L}$, reinsurer r will have to pay $O_r \tilde{L}$ to its customer insurers when losses are distributed homogeneously among reinsurers (see Section 3.4 for details).

At the beginning of the period, each reinsurer has an initial wealth W_r that arises from the capital issued in the past and accumulated surplus. This capital is costly to hold, even if it corresponds to the accumulation of retained earnings.

At each period, the reinsurance company chooses O_r. The reinsurer can also acquire some external capital $E_r \geq 0$, if this additional wealth allows it to supply more coverage and to increase the premiums it receives in such a way that it increases its profits in spite of higher capital costs. This new external capital is costly to raise on short notice (Froot and Stein, 1998). Also, the reinsurer can reduce its wealth by $E_r < 0$, through purchases of its own shares to shareholders. This is the case if the cost of capital is larger than benefits that can be derived from it in the

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reinsurance business.

Our model relies on a strong assumption of perfect information. Each reinsurer knows the loss distribution perfectly. Then, when it decides the risk amount it covers, the external capital acquisition and given an initial wealth, it is able to compute a final wealth for all possible losses and quantify its bankruptcy risk.

The probability of default π_D of a given reinsurer is a key variable in our dynamic model. In a nutshell, it represents the quantified risk that this reinsurer goes bankrupt, i.e. the risk that the realized loss at this year added to all costs outweighs the sum of the capital and earned premiums. We then make the following assumption: (A1) Reinsurers limit their default probability under an exogenous below a limit π_{lim} that corresponds to an exogenous solvency constraint, linked to regulatory requirements.

Reinsurer's program

We model the reinsurance companies from a managerial perspective (Shleifer and Vishny, 1997). We assume, therefore, that all reinsurers aim at maximizing the firm total profits rather than maximizing the return on equity. Profits depend on the quantity of risk subscribed by the firm and its price, the level of capital raised or bought back, and the initial wealth of the firm. Raising additional capital allows for a decrease of the reinsurer default risk, and thus larger reinsurance supply. The following equation defines expected profits for one period of time (subscript t is omitted).

$$\mathbb{E}\tilde{\Pi}_r(O_r, E_r) = pO_r - \mathbb{E}\tilde{L}_r + \alpha(E_r + W_r + pO_r) - S_r(E_r + W_r) - c_r(E_r, W_r), \tag{2}$$

where pO_r is the reinsurer revenue from premium, and $\mathbb{E}\tilde{L}_r$ is the expected loss reinsured by reinsurer r (see Section 3.4 for details). Earned premium and capital (new external and initial wealth) are invested at the risk-free rate α. Furthermore, there is a dead weight cost of raising additional capital ($E_r > 0$):

$$c(E_r, W_r) = c_r \left(\frac{E_r}{W_r} \right)^2. \tag{3}$$

We assume that there is no cost for reducing the amount of capital through share purchase programs and special dividend. The term $S_r(E_r + W_r)$ corresponds to the cost of carrying capital, where S_r is the remuneration of capital asked by reinsurer shareholders. The difference $(S_r - \alpha)$ is the corresponding risk premium. The model does not distinguish between the long-term cost of internal and external capital.

A complete treatment of the reinsurers program at time t would involve the definition of an expected actualized value function integrating the expected actualized profits in following periods.[7] Here, we propose a simple approach where at each period t, the reinsurance companies maximize their expected profits over one year, taking into account their solvency constraints. Moreover, a limit is introduced on share repurchase and special dividend distribution.

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[7]Indeed, the program could be written in a form recalling Bellman's equations. The problem could then be broken apart following Bellman's principle of optimality, as the choice for (O_r^t, E_r^t) at period t can be separated from all choices for further periods. Such a dynamic model is very difficult to solve due to the recursive term of the equation. Most of the time, it is resolved through linearization or at the stationary mode. However, our situation can not be satisfied with such practices as (1) the equations of the valuation of the firm are quite complicated and cannot be easily linearized, and (2) our interest resides in the analysis of the non stationary (non linear) dynamics after great shocks that can deplete the firms from a large share of their wealth.

Indeed, two situations can be encountered at time t:

- The reinsurance company is under-capitalized and thus is in a situation where the solvency constraint is saturated. In such case, we consider that short term considerations overcome the inter-temporal issues, and the optimization is done over one year only.

- The reinsurance company is overcapitalized at the beginning of the period and thus its quantity choice is not restrained by the solvency constraint. If the reinsurance company only considered short-term effects at the beginning of period t, it would redistribute to shareholders as much capital as it could, until the solvency constraint is saturated. However, the inter-temporal part of the program gives an incentive to limit the redistribution to shareholders, taking into account the value of keeping more wealth in case of losses in the years to come or of better business opportunities in case prices increase. To represent these inter-temporal effects in a simple way, we assume that the quantity of capital that the firm can redistribute over a year is limited.

This simplification leads to the following assumption: (A2) Reducing the amount of capital through share purchase and special dividend is restricted to a share κ of the reinsurer's wealth at the beginning of the period.

From Assumptions (A1) and (A2), we consider that managers choose the quantity O_r they supply to the market and their level of external capital E_r, in order to maximize their expected profit under their solvency constraint and the limit on the amount of capital they can redistribute:

$$\begin{cases} \max_{O_r, E_r} \mathbb{E}\tilde{\Pi}_r(O_r, E_r) \\ \pi_r^d \leq \pi_{lim} \\ E_r \geq -\kappa W_r \end{cases} \tag{4}$$

$\mathbb{E}\tilde{\Pi}_r(O_r, E_r)$ is the expected profit of the firm detailed in Eq.(2).

3.3 Market equilibrium

At each period, we consider that reinsurance companies, under perfect information, maximize their expected actualized profit by choosing their reinsurance supply and their capital level, either by raising new capital or buying capital back. We look for a Nash equilibrium where no reinsurer has an interest to deviate from its choice of quantity and capital. We suppose here that (A3): reinsurers compete in quantity. The equilibrium price is such that market demand equals reinsurance supply.

The demand is given by the inverse demand function $d(p)$, where, at equilibrium, the aggregate output of the industry is $d(p) = \sum_{r=1}^{R} O_r(p)$. By definition, as O_r correspond to the share of the market risk layer covered by the reinsurance companies, we have $d(p) \leq 1$. The equilibrium price is such that market demand equals reinsurance supply.

The equilibrium is defined by the following system at each period (subscript t omitted):

$$\begin{cases} d(p) = \sum_{r=1}^{R} O_r(p) \\ \forall r, \quad \begin{cases} \max_{O_r, E_r} \mathbb{E}\tilde{\Pi}_r(O_r, E_r) \\ \pi_r^d \leq \pi_{lim} \\ E_r \geq -\kappa W_r \end{cases} \end{cases} \tag{5}$$

We solve this system by computing the first order conditions. Details of the methods can be found in the Appendix.

3.4 Model dynamics

At each period t, the model follows a series of steps. At the beginning of the period, each reinsurance company is endowed with an initial level of internal capital W_r^t, derived from the previous time step. Our model is dynamic and can be written in the following recursive form, defining the level of internal capital for all reinsurers at time t, $W^t = [W_1^t, .., W_R^t]$:

$$W^{t+1} = f\left(W^t, L^t\right) \tag{6}$$

At time step $t + 1$, reinsurers' initial wealth depends on the wealth they inherited from the preceding period t, and the loss L^t they incurred at the end of time step t.

Stage 1 Each reinsurer chooses simultaneously its market share and its level of capital to maximize its profit, under the constraint of market equilibrium. The market prices p^t, reinsurer supplies O_r^t, capital choices E_r^t, and default probabilities $\pi_r^{d,t}$ are determined.

Stage 2 Losses are realized following the loss distribution function. The final wealth of all reinsurers is computed as:

$$W_r^{t+1} = W_r^t + p^t O_r^t - L_r^t + \alpha(E_r^t + W_r^t + p^t O_r^t) - S_r(E_r^t + W_r^t) - c_r(E_r^t, W_r^t), \tag{7}$$

where L_r^t is the amount of losses reinsured by the reinsurer r. We proceed in the following manner: when losses are drawn at the end of each period, they are distributed randomly among reinsurers. Each of them supports a loss:

$$L_r^t = L^t O_r^t (1 + \tilde{\epsilon}), \tag{8}$$

where $\tilde{\epsilon}$ is drawn in a normal distribution $\mathcal{N}(0, 0.5)$. When computing their default probability and choosing their capital and quantities, reinsurers anticipate this random distribution of losses. We define a bankruptcy event when a reinsurer wealth goes below a lower limit threshold wealth at the end of the period. This is anticipated in the estimation of reinsurers' probability of default.

Figure 2 summarizes the main steps of model computation.

Table 1 summarizes the variables and parameters of the model.

4 Materials & methods

4.1 Algorithm

For the resolution of the market equilibrium each year, we decided to use the trust-region-reflective algorithm. This algorithm is a subspace trust-region method and is based on the interior-reflective Newton method described in Coleman and Li (1994) and Coleman and Li (1996). Each
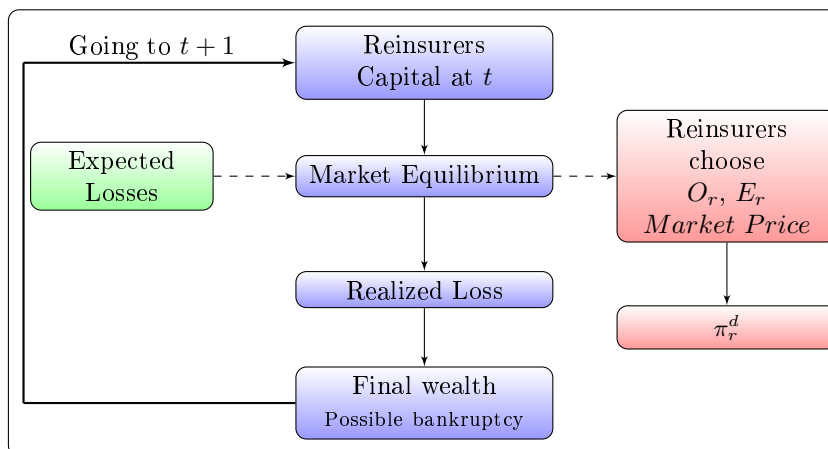
Figure 2: Model timing schematics

Table 1: Variables and Parameters

Market variables	
$\tilde{L}$	Distribution of market loss to be reinsured
p_a	Actuarial price of losses
p^t	Market price of reinsurance at time t
Market parameters	
d_0	Demand parameter
δ	Price demand elasticity
α	Rate of return of risk-free investments
π_{lim}	Limit probability of default
κ	Limit percentage of capital that reinsurers can buy back
c_r	Cost of external capital
S_r	Shareholder cost
Reinsurers' specific variables	
r	Reinsurer index
N_R	Number of reinsurance firms
W_r^0	Initial wealth of the firm
W_r^t	Wealth of the firm at the beginning of time t
E_r^t	External capital acquired by the firm at time t
O_r^t	Share of the market risk covered by reinsurer r at time t
$\pi_r^{d,t}$	Probability of default of the firm at time t

iteration involves the approximate solution of a large linear system using the method of Preconditioned Conjugate Gradients (PCG).

4.2 Scenario

Reinsurance loss coverage

To build the insurance market demand for reinsurance, we proceed in the following manner. We use model-based aggregate insured loss data from Risk Management Solutions Inc. for the four main reinsurance disaster markets: Japan Earthquake, European Windstorm, US Hurricane and US Earthquake, including four types of risk covered (agricultural, commercial, residential and

business). We suppose that there are no correlations between losses in these markets.

For each line $l = 1..4$, we consider the industry-wide demand for coverage. We note $\tilde{L}^l_I$ the loss to which the insurance industry is exposed. Let us consider $\tilde{L}^l$ the demand for reinsured loss in the lth line of business as a function of $\tilde{L}^l_I$. We suppose, at first, that the insurance industry chooses to cover losses for each line of business between a fixed limit and a fixed exhaustion point defined by their return periods, respectively T^l_L and T^l_E that define two level of losses L^l_{min} and L^l_{max}.

$$
L_l = \begin{cases} 0 & \text{for } L^I_l \leq L^l_{min}, & (9) \\ L^l_I - L^l_{min} & \text{for } L^l_{min} < L^I_l < L^l_{max}, & (10) \\ L^l_{max} - L^l_{min} & \text{for } L^l_{max} \leq L^I_l. & (11) \end{cases}
$$

This could be interpreted as if there was one layer of the risk that could be covered for each line of risk. In practice, each reinsurer would define its own layers (limits and retentions) for each risks. This allows the generation of a cumulative distribution function of potential reinsured losses for the whole disaster market. In this version of the model, we assume that $T^l_L = 10$ years and $T^l_E = 300$ years. We obtain the potential market for reinsured losses by looking at the aggregate potentially reinsured loss distribution. We fit this distribution with a log-normal distribution. Figure 3 represents the obtained loss distribution, in billion of dollars. The stairs arise from the limits and exhaustion points calculation. Expected loss is of \$22 bn, standard deviation is \$26 bn, and maximum market loss is \$568 bn.
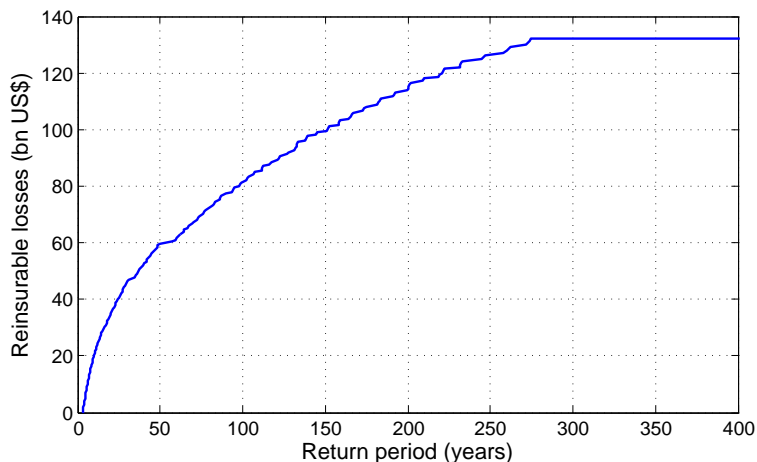

Figure 3: Distribution of market loss considered for reinsurance coverage.

This model of reinsurance demand is quite limited as reinsurance companies do not all have an equal exposition to catastrophic risks (e.g. earthquake, wind etc...). However, we account here for the main problem reinsurers have to address that is the covariance of the risks and the systematic component of catastrophe risk. One limitation of our model is that we do not take into account diversification aspects as only one line of risk is considered.

5 Reference simulation: The oligopoly effect

The reference simulation is a starting point to understand the main effects reproduced by the model as well as its limitations. It is based on an fictive market in which all reinsurers are identical. We consider a 50-year simulation with a scenario of annual market losses, assuming

that the form of reinsurance demand does not evolve over time (it only depends on the price). Each year, losses are randomly drawn from the distribution described above. For the purpose of illustration, we introduced a 100-year loss at year 30 and use the same loss scenario for the whole paper. No stochastic distribution of losses among reinsurers is carried out in this first simulation, and reinsurers incur losses proportional to their market shares (i.e. $\epsilon = 0$ in Equation 8).

The aim is here to understand the dynamics of the present market and check that the model is able to reproduce a realistic dynamics. For these simulations, a bankruptcy event is defined when reinsurer wealth falls below 0.

Table 2 presents the values of the parameters used in this reference case. There are little data available on most on these values in the literature. We base our reference parameters on the available studies.[8] The value of S_r is consistent with the magnitude obtained in Zanjani (2002).[9] The value of π_{lim} corresponds to a return period of 200 years, which is used to calculate the level of required capital in Solvency II.[10] Concerning c_r, we calibrated this parameter such that the recourse to external capital following a high loss would be of a magnitude comparable with the one shown on Figure 1, but without considering new insurers. Finally, available information concerning share repurchase program are from Aon Benfield (2010). Share repurchase are estimated at 2%, peaking at 11% of shareholders fund for three of their reinsurance companies in the first semester of 2010. This cannot be used directly to calibrate κ, since most of these reinsurers are multiline, but it provides orders of magnitude for this process. To assess the robustness of our results, a sensitivity analysis is conducted on each of these parameters.

Table 2: Parameters

Parameters	Reference simulation
elasticity[a]	0.2
α	4%
S_r	12%
c_r	$4.5 \cdot 10^4$
π_{lim}	0.5%
κ	1%

[a]The elasticity parameter is defined as $\delta = \frac{elasticity}{\mathbb{E}L}$.

We consider 5 reinsurers that each have an initial wealth W_0 equal to \$20 bn. Our reference market can appear as highly capitalized but as reinsurers are monolines, there is no diversification effects that can play here.

5.1 Simulation results

Results are presented on Figure 5 and commented below. All the graphs in this paper will follow the same pattern. The different output graphs are disposed following the order presented on Figure 4. The black curves represent market variables, and the gray curves individual reinsurance companies variables.

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[8]Moreover most empirical studies are based on data that do not include years following Katrina.

[9]A more detailed analysis by Cummins and Phillips (2005) is focused on the insurance market.

[10]See http://www.gccapitalideas.com/2010/02/17/higher-pressure-on-cat-risk-under-solvency-ii-part-i-standard-formula-approach/ for a CEIOPS discussion on Solvency II cat risk approach.

The model reproduces a market showing many characteristics similar to those observed empirically. In particular, prices are higher than actuarial prices, market capacity decreases after high loss shocks, as the market price increases, and additional capital is raised by reinsurers after large shocks. Since losses are distributed proportionally to market shares, the reinsurers remain identical during the entire simulation (this assumption is relaxed in Section 6.2).
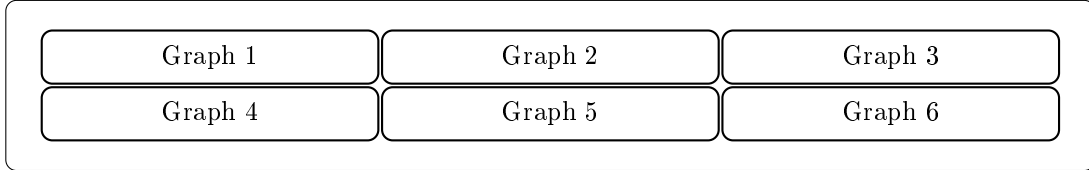
| Graph 1 | Graph 2 | Graph 3 |
| Graph 4 | Graph 5 | Graph 6 |

Figure 4: Output graphs layout

Graph 1: random reinsurance market losses scenario. The $90bn loss at year 30 corresponds to a return period of about 100 years. From year 38 to 45, there is a cluster of several medium losses.

Graph 2: market price of reinsurance as the ratio of price on expected loss. A price equal to 1 corresponds to the expected loss. Market price always exceeds twice the actuarial price. The price spikes after high loss events. Depending on the size of the shock, the time needed to return to the lower prices is different.

Graph 3: reinsurers' probability of default. The probability of default is capped by π_{Lim} as our solvency constraint requires. After loss events, the solvency constraint is saturated until market capacity has replenished itself.

Graph 4: market wealth at the end of the year. The total market capitalization (black curve) is the sum of reinsurer capitalizations (gray curve). Each important loss is followed by capital depletion. After the loss, the market rebuilds itself as capitalization rises. Time needed to recapitalize depends on the intensity of the shock.

Graph 5: reinsurance supply. The gray curve corresponds to the risk covered by one reinsurer, and the black curve to the risk covered by all the market. The total market supply always remains below a ceiling value. This corresponds to an oligopoly effect: serving a higher part of the market would decrease the market price, and not be as beneficiary to the reinsurers. As a consequence, reinsurers voluntarily ration the market to drive prices upward.

Graph 6: market external capital acquisition. The sum of all external capital movements (black curve) corresponds to the sum of the identical individual reinsurers' external capital movements (gray curve). At the beginning of the simulation, it is negative corresponding to a purchase of reinsurer's own shares, as it is the case each time the solvency constraint is not saturated. In such a situation, the market is overcapitalized. Most of the time, external capital is positive and small, but after high losses it becomes much higher (year 15 and 31), as well as in the middle of the series of medium losses (year 41).
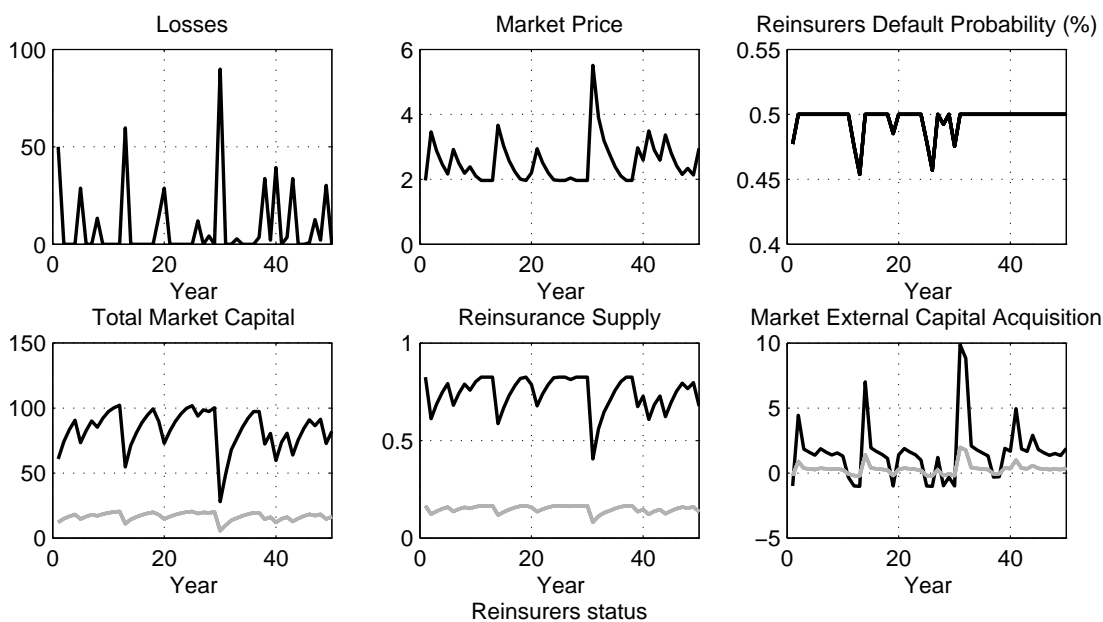
Figure 5: Reference simulation's outputs.

5.2 Market dynamics

This reference simulation allows the analysis of the path dependency on the reinsurance market dynamics.

Market behavior with regular losses: From year 1 to 30, losses are globally low and easily absorbed by the market: the global market capacity is high. The risk served by the market does not exceed 80%. The corresponding market price is low, but still twice the actuarial price.[11] The probability of default of the reinsurers is capped at 0.5%, until the reinsurers have replenished themselves during soft markets. In such cases, external capital is slightly negative, corresponding to share repurchase program. Higher losses, as the one that occurred at year 14, lead to a rise in market price. Simultaneously, there is a large increase on the acquisition of external capital that limits the correction on reinsurance supply, as is observed in the market (The Geneva Association Systemic Risk Working Group, 2010).

Impact of a 100-year loss: The impact on the market of the 100-year loss, that happens here on year 30, has three main characteristics. First, market capacity decreases brutally leading to reduced reinsurance capacity. Consequently, the following year sees a double correction from the reinsurers: they lower their supplies, and raise external capital ($12 bn here over two years, that is one fifth of amount of reinsured losses of the 100-year event). Their main aim is to limit their probability of default that is capped here at 0.5%. Finally, there is a delay of more than 7 years for the reinsurance industry to recover its initial capacity. At the same time, the market price rises (80% rise). Note that we do not consider any change in reinsurance supply or demand linked to a change in risk evaluation from either insurers and reinsurers.[12] Froot (2001) shows that after a large event, reinsurance coverage increases, but with a bigger retention. Smaller events are then less covered. Further model development

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[11] Transactions and administrative costs are excluded.

[12] This issue is explored in depth in Lai et al. (2000)'s model.

will introduce a more sophisticated treatment of reinsurance demand.

Impact of a series of medium losses: From years 38 to 43, several losses occur, which have a return period between 25 and 30 years. The market has no time to recover between them. Therefore, market capacity remains low and reinsurance companies keep acquiring external capital although in a limited amount. Market supply decreases to 50%, showing the importance of considering series of shocks and introducing path dependency and the dynamics of the market. This clearly indicates a path dependency in the market. This result emphasizes the need to take into account possible clustering in extreme events, as suggested by Bunde et al. (2005).

This reference simulation already exhibits some of the characteristics of the reinsurance market dynamics. The main ones are (i) the rise of the price following an extreme event, (ii) the correlation between market capacity and the price of risk, and (iii) the fact that reinsurers raise significant amounts of capital after large shocks. These effects arise from the capacity constraint due to the limit of 0.5% of probability of default. The high rise of the price after year 30 does not take into account the impact of a change in reinsurance demand, nor of a risk reevaluation by reinsurance companies, that Froot (2001) and Guy Carpenter (2009), among others, also mention to explain market reaction.

5.3   Sensitivity analysis

Little data are available on the reinsurance market and there is a large uncertainty in all model parameters. In such a situation, it is useful to carry out a sensitivity analysis to assess the robustness of our results and to better understand important mechanisms. It is based on the reference reinsurance-market scenario detailed above. We carry out sensitivity analysis on the main parameters presented on Table 1. For each of those, we take varying values around the one used in the reference scenario. Exact values and figures are presented in Appendix (Figures 11 to 15). Results are the following:

Demand elasticity (e): The higher the demand elasticity, the lower the share of the market covered after the large loss of year 33. This result is consistent with intuition: the increase in prices after a large shock reduces reinsurance demand for insurers. For the year following a high loss, higher elasticity leads to lower ex-post prices. It is in line with the economic theory literature: Cagle and Harrington (1995) show that in the case of capacity constraint and endogenous solvency risk (with limited liability) the increase in price after a shock with inelastic demand price increases is lower than the amount necessary to offset the shock. The higher the elasticity, the lower the impact of shocks on prices. However, when considering market recovery over several years, a lower elasticity leads to an earlier recovery of the market.

Cost of carrying capital (S_r): As can be expected, the higher the cost of holding capital, the less capitalized the reinsurance companies are, and the less they are able to supply the market. Hence, the equilibrium capital level (i.e. the stabilized level if losses are set to the expected loss) depends on the cost of carrying capital. Following a large shock, external capital acquisition is less important when the share cost is high.

Cost of external capital (c_r): This parameters has a main impact after important losses when the reinsurers acquire external capital. The lower the cost of external capital, the more the

firms relay on it, and the less the market correction on prices. This leads to a higher capacity of the reinsurance market to supply coverage after important losses when cost of external capital is low. This impact is only visible during tight markets.

Impact of initial capital (W_r^0): In the present case with a perfectly symmetric market with no entry nor exit, all reinsurers have an optimal capital target that they all tend to reach. A difference in initial capital disappears after periods of soft market during which the reinsurers can replenish their capital.

Impact of the number of reinsurers (N_r): When we distribute the same total capital among different numbers of reinsurers, the oligopoly effect is quite clear: the fewer the reinsurers, the lower market capacity and the higher the prices during soft markets. There is therefore a clear gain from increased competition in the market. However, impact of important losses lead to the same depletion of capacity.

Impacts of the solvency constraint (π_{lim}): Results show that the lower the solvency constraint, i.e. the higher π_{lim}, the higher the capacity of the reinsurance market in soft market and the higher the time needed for recovery after an important loss. However, as path dependency is important, a refined analysis of the impact of this constraint, particularly on the resistance to high shocks, is conducted in Section 7.

Impacts of the limit share of buy backs (κ): The sensitivity analysis shows that market capitalization stays higher over the scenario when κ is small, as firms can not buy back as many shares as they would for higher level of κ. Consequently, market resilience to large losses is higher.

6 Asymmetric market and reinsurer bankruptcy

6.1 Market segmentation between two sizes of reinsurers

To investigate a more realistic market, we build a fictive reinsurance market from available market data. The reinsurance market is characterized by a certain concentration that has increased in the last decade. The current market involves big reinsurance firms such as Swiss Re and Munich Re, smaller reinsurers, and new reinsurers as the Bermudian. However, as reinsurance companies offer coverage on several markets (life, disasters...), we do not have specific data on the capital linked to disaster risks and on the market shares on the natural-disaster market. We use as a proxy the market shares as obtained from Standard and Poor's (2008) on the reinsurance market for all lines of risk: in 2007, the four biggest reinsurance companies accounted for 48 % of the market. We consider a market with 4 large reinsurers that each have an initial wealth W_0 of \$22.5 bn, and 10 small reinsurers that have an initial wealth of \$3 bn. This market is still fictive, but it is consistent with the type of concentration observed in the industry. The Geneva Association Systemic Risk Working Group (2010) presents a concentration curve for the reinsurance market where the 10 biggest reinsurers share 80% of the market and the five biggest more than 50% in 2008. The market parameters used are the same than in the reference reinsurance-market simulation. Figure 6 presents the results of this simulation. The smaller reinsurance companies are represented in solid gray and the larger ones in dashed gray. The interpretation of the output graphs is essentially the same as in the reference simulation. However, several interesting features appear due to the
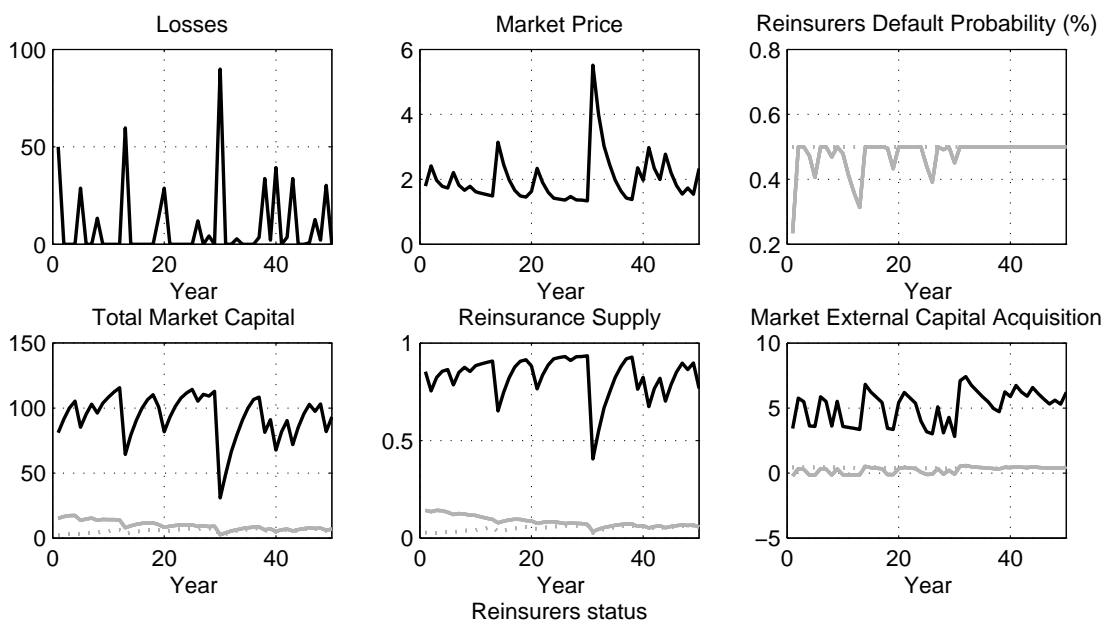
Figure 6: Fictive market outputs.

difference between large and small reinsurance companies. First, a higher number of reinsurance companies are present on the market. Hence, the strategic impact of each of the firm is lower than in the preceding case, and the oligopoly effect is reduced. Consequently, the total share of the market served by reinsurers is higher, up to 93%.

Small and large companies: Until year 30, our market is naturally segmented into (i) price-maker big reinsurers, which have a strategic behavior and reduce their market share to drive the price upward; and (ii) "followers", which are price-taker and try to capture the largest market share. Interestingly, small reinsurers have a saturated default probability most of the time, contrary to the larger ones. This difference is due to their lower capitalization and the lower impact of their decisions on the market price. Big reinsurance firms can indeed ration the market by not providing the maximum supply they would be allowed to supply with their capitalization. By reducing the amount of reinsurance they provide, they drive prices upward, increasing their profits. Small reinsurers do not have the same impact on the market price, and they have an interest to capture as much of the market as possible, taking into account their default probability boundary, making small reinsurers more vulnerable than big ones to large disasters.

Convergence of the market With time, the difference between both capitalization levels decreases, as can be seen on Graph 6. This convergence is natural in such cases of repeated Nash games as small reinsurers capture more and more of market shares until all reinsurers converge to their optimal size for a market of 15 symmetric reinsurers.[13]

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[13]Indeed, losses can be interpreted as a capital depletion on average. Simultaneously, we have increasing cost of raising new external capital that can be interpreted as decreasing return to scale. Thus an optimal level of capital for firms is quite classic.

6.2 Reinsurers' bankruptcy

To take into account asymmetries among reinsurers, we include the stochastic distribution of losses among insurers, as described in section 3.4. This allows to include reinsurer bankruptcy in the model. The bankruptcy event is set here when reinsurers' wealth fall below a $0.5bn threshold, that corresponds to bankruptcy costs. Figure 6 presents the results of a simulation with the same parameters as above and the randomization of the losses. The additional graph below the others represents the number of reinsurers on the market.
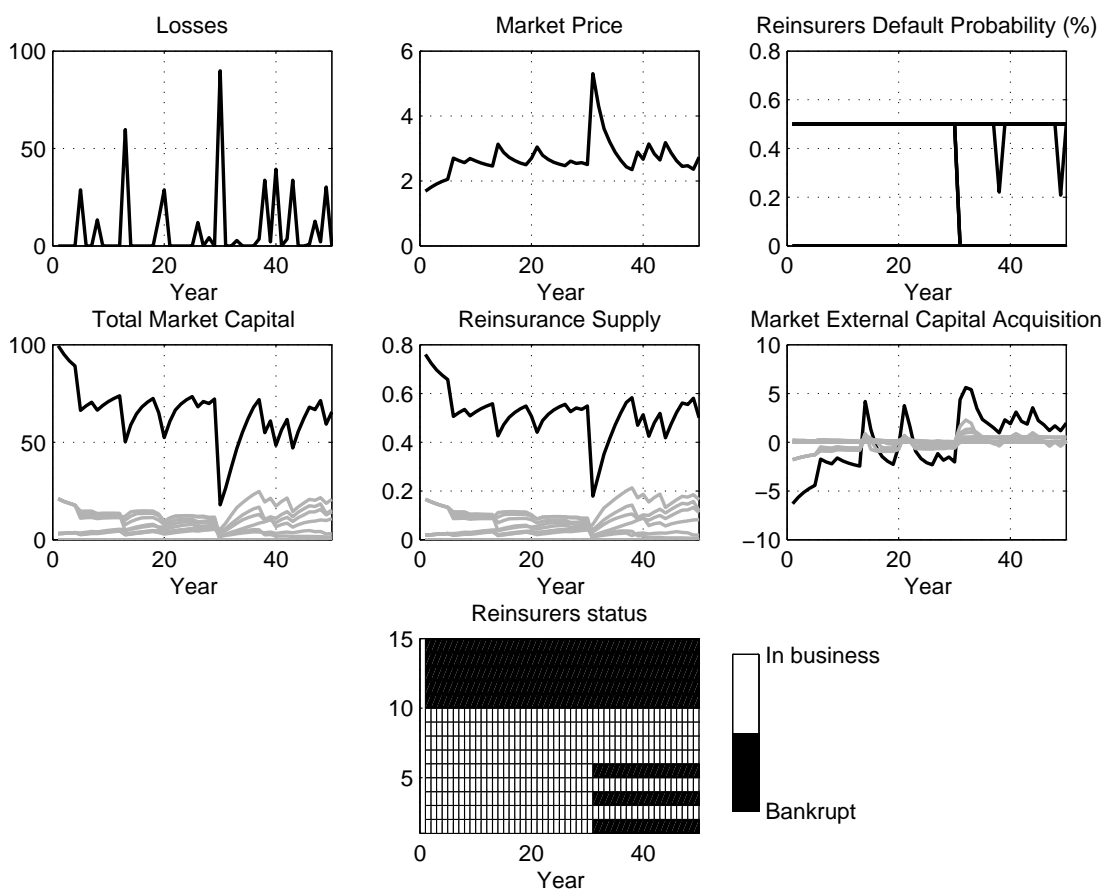
Figure 7: Fictive market outputs with loss randomization.

Analysis of the graph's results are quite straightforward. Randomization of losses impact lessens the convergence among asymmetric reinsurers, suggesting that the existence of multiple lines and imperfect correlation of losses may explain the persistence of reinsurance of various sizes. Moreover, path dependency is clearly seen on the graph. The impact of the huge loss at year 30 is globally the same on the whole market: the price increases, total market supply is contracted, external capital enters the market. However, when looking at all reinsurers, impacts are very different due to the randomization of loss impacts. Three reinsurers go bankrupt. Others, which are less impacted by the loss, benefit from the negative impact on the others, and grow more steadily during the years following the shock, benefiting from the price rise following the shock and from the increased supply they can provide. This corresponds to the intuition discussed by Cagle and Harrington (1995) although they do not address it in their model. Of course, this is still a very

simple representation of the real market, and a deeper study of the impacts of the market structure would be of great interest.

The model allows to get a first insight on the importance of market structure on the price and capacity dynamics. Interesting results are obtained, as the model reproduces a segmented market with both strategic and follower reinsurers that evolve through time depending on losses impact on them. The former are large enough to have a real impact on prices and to reduce the amount of reinsurance they supply to drive the price upward, especially in hard markets. The latter do not have the same influence on the market, and behave in some ways as price-taker agents and benefits from the price driven by the other ones, as shown on the asymmetric idealized market.

One major insight from this article is the modeling of new capital flows into the market. Two different kinds of flows currently exist. First, there is a flow of external capital in surviving reinsurance companies, corresponding to equity emission or development of sidecars and catbonds. Second, one can observe the development of new reinsurance companies when the reinsurance market price is high (Lane, 2007b). The total of these two flows is important. Following hurricanes Katrina, Rita and Wilma, for instance, Guy Carpenter (2009) estimates that more than $35bn were injected in the industry through these various alternative means of capital. This effect has taken more importance during the last decade, and impacts on prices have been less pronounced since hurricanes Andrew (The Geneva Association Systemic Risk Working Group, 2010). Some calibration is still needed to better assess flows of capital in the market. The model presented in this paper does only model the first of these two flows, namely the flow of external capital into existing reinsurers. A forthcoming version of the model shall include new reinsurers entry on the market.

7   A regulation trade-off: Efficiency vs. resilience

Our modeling of reinsurer's risk aversion can be interpreted as a regulatory constraint: reinsurers are not allowed to exceed an exogenous probability of default. This is of course a very simple way to model insurance solvency regulations. Such a constraint has an impact on the market behavior of the firms, as holding more capital to cover the same quantity of risk is costly for the firms. On Figure 8, we represent the sensitivity analysis conducted on the symmetric market for variations of the threshold default probability π_{Lim} between 0.1% and 0.9%. The impact of this parameter on market variables is important. When the solvency constraint is strong (π_{Lim} low), the share of total loss covered by the reinsurance industry is low, prices are high, and the probability of defaults of reinsurers is saturated even in soft markets. As the limit probability of default is lower, reinsurers need a higher capitalization to cover the supply they provide. Market prices evolve correspondingly.

This sensitivity analysis suggests the existence of a classical trade-off between a higher market resistance to large and rare shocks with more stringent regulation and an efficiency criterion in the most frequent situations. Intuitively, a strong solvency constraint increases market resilience. But simultaneously, it decreases market efficiency as holding the same amount of risk requires more capital - and thus is more expensive. To understand better the impact of this *solvency* constraint,
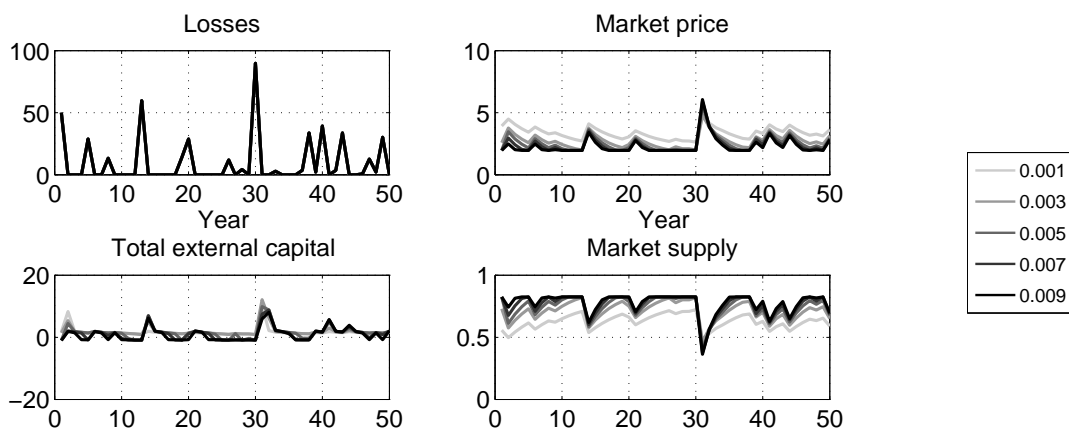
Figure 8: Sensitivity analysis for the solvency constraint

we first studied the impact of a loss on the market price. To do so, we impose a large loss on a stabilized market of 5 symmetric reinsurers, assuming they were exposed to the expected loss level during the 10 previous years (to remove path dependency from the analysis). Figure 9 represents the price in the year following the shock for the limit default probabilities 0.1%, 0.5%, and 1%. This range is centered on a 200-year risk.
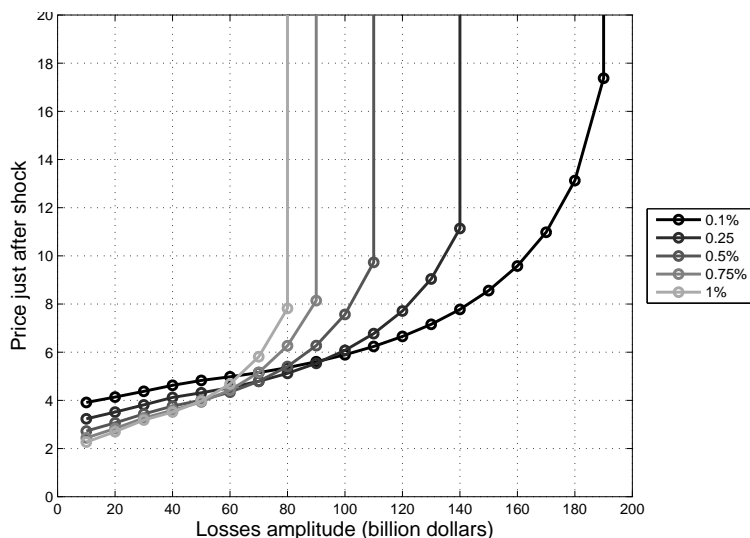

Figure 9: Non linearity between price reaction and loss amplitude

We first observe a non linearity between price and the magnitude of the loss. When the regulation is stricter (lower limit probability of default), the price after small losses is higher, because it requires more capital to supply the same amount of reinsurance capacity. So, the market is most of the time less efficient with a strict regulatory constraint. But after a large loss, the market collapse with a weak constraint (e.g., for losses larger than $80 billion if the limit is at 1%), while the loss can be absorbed with a stricter regulation (however at the expense of a large increase in price). Indeed, the highest loss the market is able to absorb is a market loss of $190 bn for a limit probability of default of 0.1% (1000-year loss), $110 bn for a probability of 0.5% (200-year loss) and $80 bn for a probability of 1% (100-year loss).

A closer look at the impact of the default probability on the market is provided on Figure 10.

On the typical scenario we used in Section 5, we compute the mean price and mean covered risks on the market during the 50th years.[14] We see again that average market price increases with the regulatory constraint whereas correspondingly the capacity of the reinsurance market decreases. Thus tighter regulation leads during a *regular* time period with limited losses to higher prices and less available capacity for the same amount of capital. This is a typical example of an efficiency vs. resilience trade-off as can be found in ecological systems.
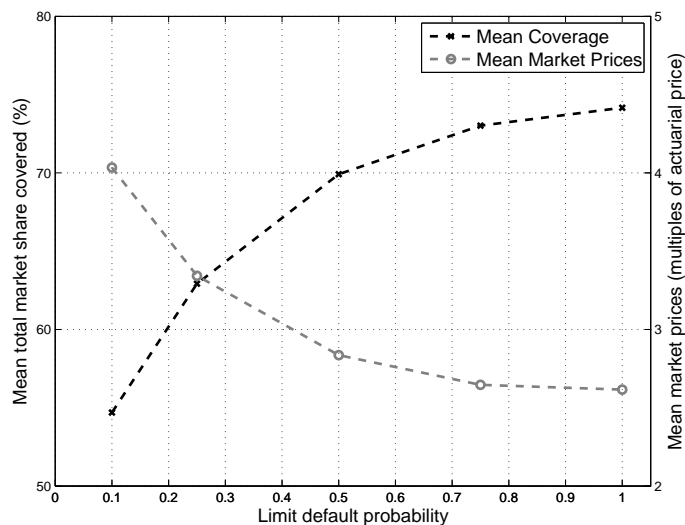
Figure 10: Reinsurance price and market capacity in function of the limit probability of default

This analysis is of course quite limited for several reasons: (i) we only consider one line of business for the reinsurers with only one capital charge; (ii) we only take into account shocks that may arise on the liability side of the reinsurers balance-sheet while asset-side shock play a crucial role; (iii) we disregard any potential correlation between financial market risks and natural disasters. Furthermore, the reinsurance industry reaction to shocks on their asset sides can induce impacts on the financial markets.

The justification and the choice of a proper regulation is a difficult question that Plantin and Rochet (2007) analyze. Agency considerations and moral hazard issues are crucial: policyholders are mostly very diluted and do not exert sufficient monitoring.[15] Furthermore, the complexity of the reinsurance business makes it very difficult to assess the solvency risks linked to each company for technical as well as moral hazard reasons. In this paper, we do not ask the question of the optimal level nor the justification of regulation as we do not consider any welfare perspective and only look at the impact on the reinsurance market. But the efficiency/resilience trade-off in this simple way provides a clear illustration of one of the key regulator issues: what is the acceptable risk for such an industry and at what cost can it be hedged? This question is at the center of regulation questions following the financial crisis, especially following AIG's buy-out (Harrington, 2009). Certainly, the reinsurance industry does not have as much a macroeconomic impact as the insurance industry, but its proper operation is an important condition for the insurance industry to be able to still cover large risks.

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[14]A proper analysis would require running the full model with loss randomization. This will be done in a further version of the model.

[15]In the case of reinsurance companies, it could be argued that insurance companies could have the ability to monitor them efficiently.

8 Conclusion

The model outputs shed a better light on the way classical economic assumptions allow for understanding and reproducing the reinsurance market dynamics. Compared with previous analyses, our approach investigates the issues of reinsurance capacity to withstand important losses in a dynamic framework. Our model relies on a capacity constraint analysis, with a threshold probability of default for reinsurance firms. This factor may enhance the role of regulatory measures on the dynamics and the resilience of the reinsurance market either through the resistance to severe losses or through the time needed for capacity replenishment. Furthermore, we take into account the impacts of the firm choice on the market equilibrium to model potential strategic behaviors of large reinsurers. Our approach is innovative as we compute a dynamics of the market, based on these clear and tractable assumptions.

This analysis is only the first step of a broader research agenda. Incorporating several lines of risk will refine the reinsurance demand model. A better calibration would allow a better understanding of the market features that influence the market resilience. Recent data on reinsurance contracts would be needed to do so, and they are difficult to access. The long-term objectives of this model are to develop insights on the needed market capacity and to understand the insurance industry limits in providing disaster coverage. This question is critical when considering the current trends in insured and reinsured exposure. In addition to the classical economic trend issues (Hallegatte, 2011), the IPCC (2007) suggests a probable rise in the frequency and intensity of some natural disasters (e.g., storm surge and coastal floods). Several studies have proposed projections of future exposure due to these trends (see for instance Hanson et al. (2011) on the exposure in coastal cities) and the consideration of different scenarios will be necessary. How could the reinsurance industry be able to cover increasing risks, and how could specific policies and regulations enhance its ability to do so?

9 Acknowledgements

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A.1 Market Equilibrium Resolution

Intuitively, two situations shall be distinguished for modeling for each reinsurer whether its solvency constraint is saturated or not. First, in the saturated case, the choice of external capital E_r is an implicit function of the choice of supply O_r through the solvency constraint.[16] The equilibrium constraint is then equivalent for the concerned reinsurers to the maximization of their expected profit as a function of reinsurance supply only.

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[16]In practice, due to the form of the capital costs, there may be two solutions E_r to this constraint that lead to the same expected profits. In this case, we take the lower level of capital E_r as it gives the highest yield.

$$(1+\alpha)\left(\frac{\partial p(O_1,...,O_R)}{\partial O_r}O_r + p(O_1,...,O_R)\right) - \mathbb{E}L_r - \left(\frac{\partial c(E_r)}{\partial E_r} + (S_r - \alpha)\right)\frac{\partial E_r}{\partial O_r} \quad = 0 \qquad (12)$$

In the case where the constraint is not saturated, the choices of external capital and reinsurance supply are not linked any more. The first order conditions for the concerned reinsurers are composed of two different elements: (FOC_O) determining the reinsurance supply choice and (FOC_E) determining the external capital level that corresponds to the exogenous limit for reducing the amount of capital through share purchase programs and special dividend.

$$\begin{aligned}(1+\alpha)\left(\frac{\partial p(O_1,...,O_R)}{\partial O_r}O_r + p(O_1,...,O_R)\right) - \mathbb{E}L_r \quad &= 0 \qquad (FOC_O)\\ E_r \quad &= -\kappa W_r \quad (FOC_E)\end{aligned} \qquad (13)$$

A.2 Sensitivity analysis

Sensitivity analyses are conducted on key parameters. Parameters are changed individually from the reference reinsurance-market scenario. The order of the graphs is the same as for the reference simulations. To be able to better read the results, a shorter time period is shown for each analysis, depending on the parameter considered.

Figure 11 presents the sensitivity analysis for the elasticity of demand. The elasticity parameter $e = \mathbb{E}L * \delta$ (0.2 corresponds to a low elasticity, and 0.6 to a high demand elasticity with respect to price. Note that for a low elasticity, pale gray, reinsurers buy share back much more frequently since price and quantity can be large together.
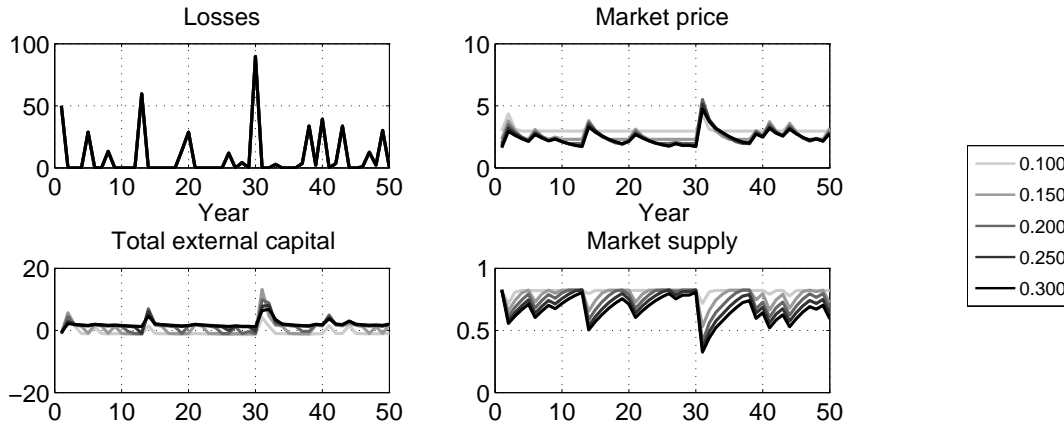

Figure 11: Sensitivity analysis for elasticity

Figure 12 presents the sensitivity analysis for the cost of carrying capital. The parameter S_r, which represents this cost, varies from 10% to 14%.The model is sensitive to this parameter, but qualitative results remain unchanged.
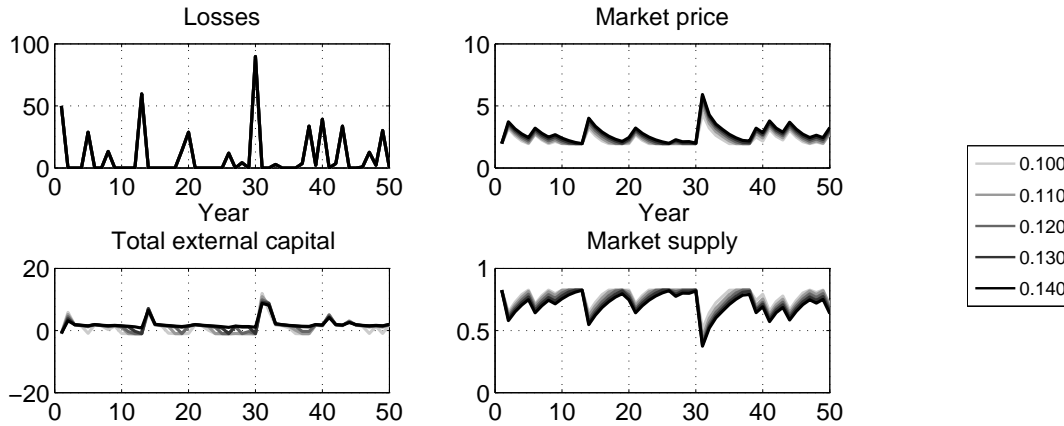
Figure 12: Sensitivity analysis for sharecost

Figure 13 presents the sensitivity analysis on the initial capital of the firms. After 10 years, all simulations converge to the same paths. It illustrates the fact that reinsurers target an optimal level of capital that depends on the number of firms of the market. This convergence is linked to the complete symmetry of the market, and the fact that there is no entry nor exit of reinsurers. The legend indicates the sum of all initial wealth.
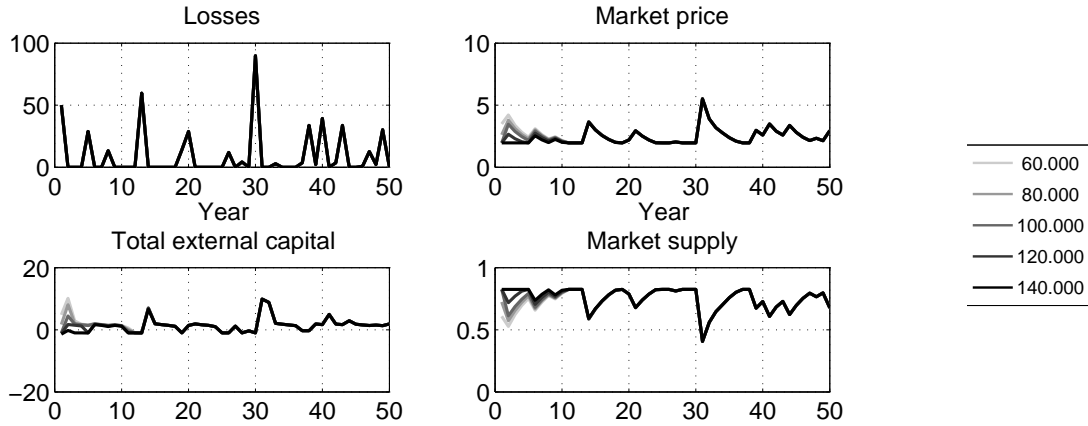

Figure 13: Sensitivity analysis for W_0

Figure 14 presents the sensitivity analysis for the cost of raising new capital. It mainly impacts the quantity of capital raised after large shock and the time necessary for the market to recover. It has only a limited impact on the market other variables.
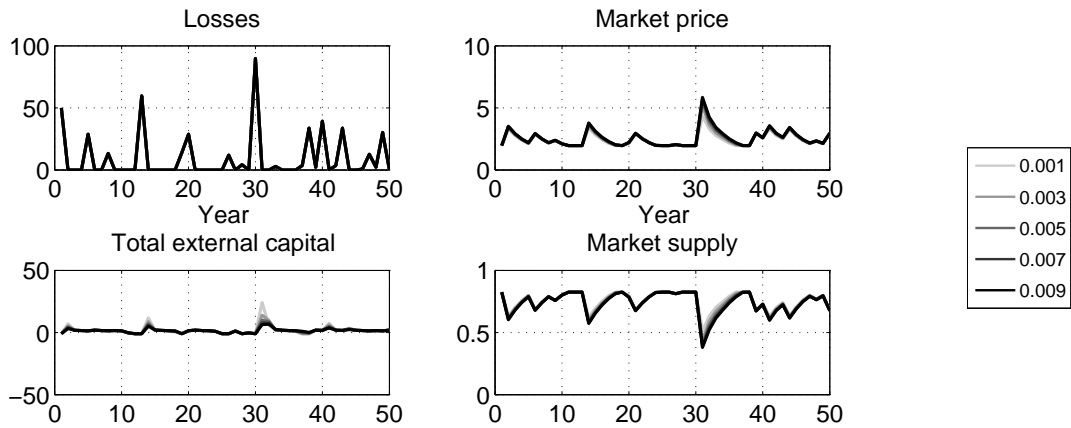

Figure 14: Sensitivity analysis for cost of external capital c

Figure 15 presents the sensitivity analysis for the number of reinsurers sharing the same amount of capital. It illustrates the oligopoly effect: when reinsurers are fewer, they deliver less capacity to the market and prices are higher during soft market, the impact of important losses is however slightly equivalent.
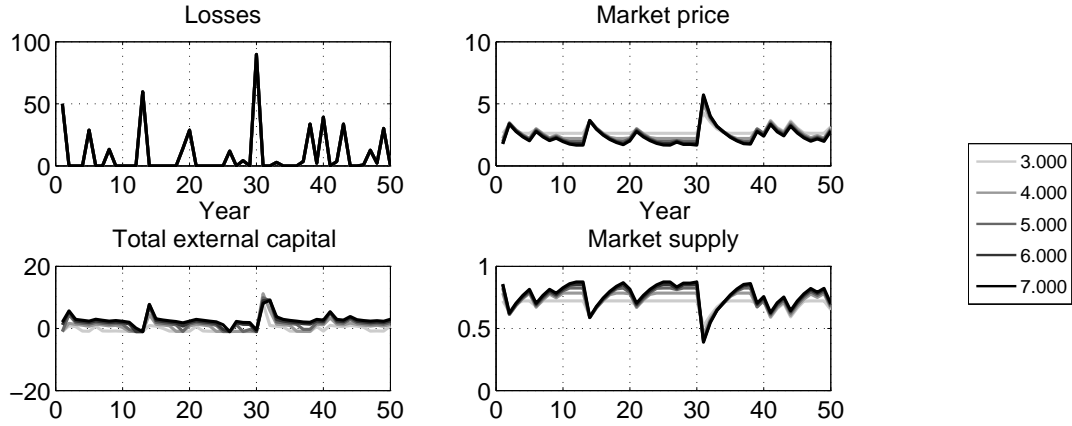

Figure 15: Sensitivity analysis for the number of reinsurers

Figure 16 corresponds to the sensitivity analysis for κ. The lower κ, the more capitalized the industry, and the lower the market prices.
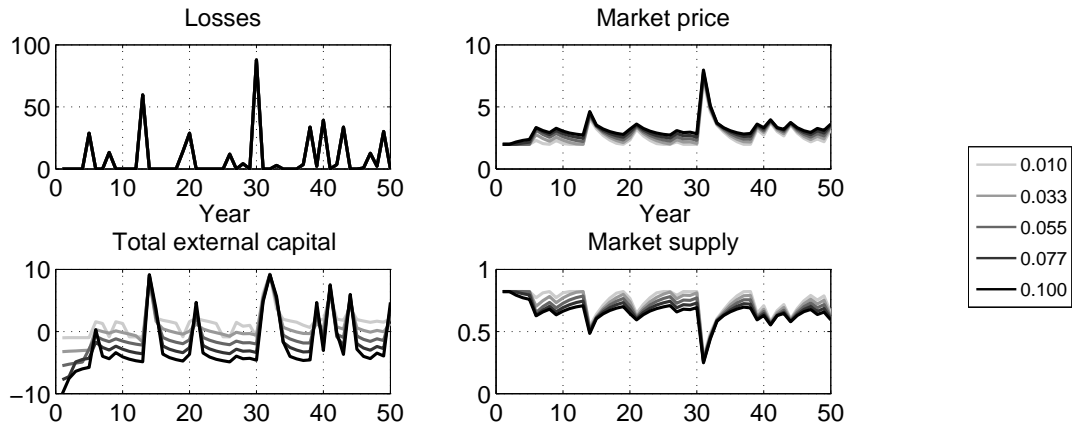

Figure 16: Sensitivity analysis for κ

The Figure corresponding to the sensitivity analysis for π_{lim} can be found in Section 7 (Figure 8) .