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Abstract

Births postponement is a key demographic trend of the last decades. To examine its social desirability, we study how utilitarian criteria rank histories equal on all dimensions except the age at which individuals give birth to their children. We develop a T-period dynamic overlapping generations economy with a fixed living space, where individual welfare is increasing in the available space per head, and where agents have children in one out of two fertility periods. When comparing finite histories with an equal total number of life-periods, classical, average and critical-level utilitarian criteria select the *same* fertility timing, i.e. the one leading to the most smoothed population path. When comparing infinite histories with stationary population sizes, utilitarian criteria may select different birth timings, depending on individual utility functions. Those results are compared with the ones obtained when agents value coexistence time with their descendants. Finally, we identify conditions under which a shift from an early births regime to a late births regime is socially desirable.

Keywords: population ethics, utilitarianism, fertility, birth timing.

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1 Introduction

In *Reasons and Persons*, Parfit (1984) emphasized some major shortcomings faced by utilitarian social welfare criteria in the context of different numbers choices. On the one hand, the classical utilitarian social welfare function suffers from the Repugnant Conclusion: for any large population, there exists a much larger population with a much lower welfare per head, but which yields a larger aggregate welfare than the former population, and which is thus regarded as strictly better under classical utilitarianism. On the other hand, average utilitarianism suffers from the Mere Addition Paradox: the addition, to some population, of a given number of persons with a welfare slightly lower than the initial average welfare is regarded as socially undesirable under average utilitarianism, even if the welfare of the initial population is left unaffected.

Following Parfit's pioneer work, utilitarian population ethics has been developed in various directions. The search for alternative social welfare criteria that can escape from the two counterintuitive results mentioned above has received a strong attention. That research program has produced various new social welfare criteria, such as critical-level utilitarianism (Blackorby and Donaldson 1984), number-dampened utilitarianism (Hurka 1983, Ng 1986), and critical-band utilitarianism (Blackorby *et al* 2005).[1] Besides new utilitarian social welfare criteria, new paradoxes and impossibility results have also been produced (Arrhenius and Bykvist 1995, Bykvist 2007, Arrhenius 2013). Moreover, utilitarian population ethics has been applied to some complex ethical dilemmas, such as the treatment of future generations (Broome 1992, Arrhenius and Bykvist 1995), or the trade-off between the addition of new persons and the extension of the life of existing persons (Broome 2004, Arrhenius 2008).

Most of the population ethics literature relies on an abstract representation of human societies, which is often reduced to two dimensions: (1) a population size; (2) a level of utility for each person. More recently, economists revisited utilitarian population ethics within economic environments, where other dimensions of life are modeled. Renstrom and Spataro (2011) applied the critical-level utilitarian criterion to the question of the optimal population growth in a dynamic overlapping generations (OLG) economy with physical capital accumulation, which was first formulated by Samuelson (1975) under average utilitarianism. Jouvet and Ponthiere (2011) revisited the trade-off between adding new persons and extending the life of existing persons in a dynamic OLG economy with a fixed living space. Boucekkine and Fabbri (2013) reconsidered Parfit's Repugnant Conclusion within a dynamic economic model of endogenous growth. More recently, Boucekkine et al (2014) examined the extent to which equalizing the consumption of all individuals in all generations can be the optimal solution to a utilitarian social planning problem consisting in choosing the optimal age structure dynamics under the constraint of a given initial age structure.

Besides theoretical developments, the population ethics literature also evolves through the new challenges it faces. New phenomena can raise new ethical

[1] For other utilitarian criteria under a variable population, see Blackorby *et al* (2005).

dilemmas. One of those new phenomena consists of the recent postponement of births. Since the 1970s, demographers have observed a significant postponement of parenthood. That postponement is illustrated on Figure 1.[2]

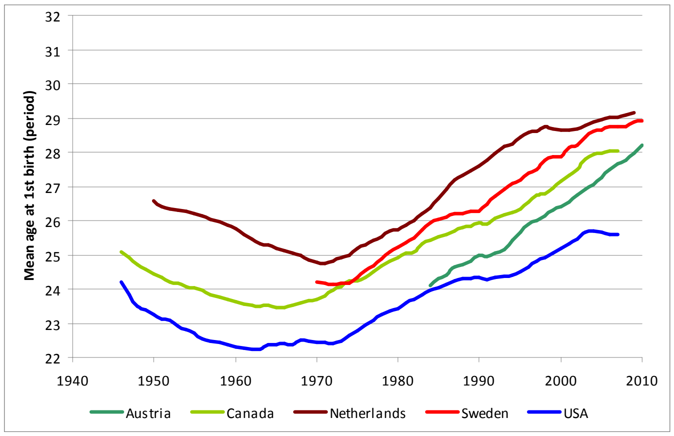

Figure 1: Average age of women at first birth (period)

In the case of Sweden, for instance, the average age of women at first birth has increased from age 24 in 1970 to age 29 in 2010. The postponement of births is a fundamental evolution of the last decades, which, as such, raises several questions, about its causes, its consequences, and about its social desirability. Is the postponement of births socially desirable?

The goal of this paper is to study the social desirability of births postponement in the long-run. For that purpose, we study, within a dynamic framework, whether the postponement of births is socially desirable or not from the perspective of utilitarian population ethics. As this is well-known, utilitarian ethics answers questions about social desirability of actions or policies by means of the calculus of social welfare under those different actions or policies (without emphasis on individual choices). In the present context, we propose to examine how utilitarian population ethics criteria would rank histories that are equal on all dimensions except one: the *timing of births*. The problem can be presented as follows. Suppose that a life includes not one, but several fertility periods, during which one can give birth to children. The question that we ask is the following: which fertility profile is optimal from a utilitarian perspective?

To illustrate the problem, Figure 2 shows, for a lifetime equal to four periods and including two fertility periods (periods 2 and 3), two possible alternatives, coined "early births" and "late births" respectively. One can also refer to those two societies as two distinct islands, i.e. *Early Island* and *Late Island*. The fertility rates n and m denote, respectively, the number of children born during the first and the second reproduction periods. As shown on Figure 2, the fertility

[2] Source: The Human Fertility Database (2012).

timing implies two major differences. A first difference concerns the coexistence time between the parent and the child. At each point in time, four generations coexist when births are early, whereas only two cohorts coexist when births are late. Moreover, whereas children can coexist with their grand-parents on Early Island, this is not the case on Late Island, where grandparents are dead when the grandchildren are born. Another difference lies in how the current cohort size is related to the past cohort size. In Early Island, the size of a newborn cohort is directly related to the size of the cohort that is born at the previous period. However, on Late Island, the size of a newborn cohort is related to the size of the cohort born two periods before, which generates a different population dynamics.

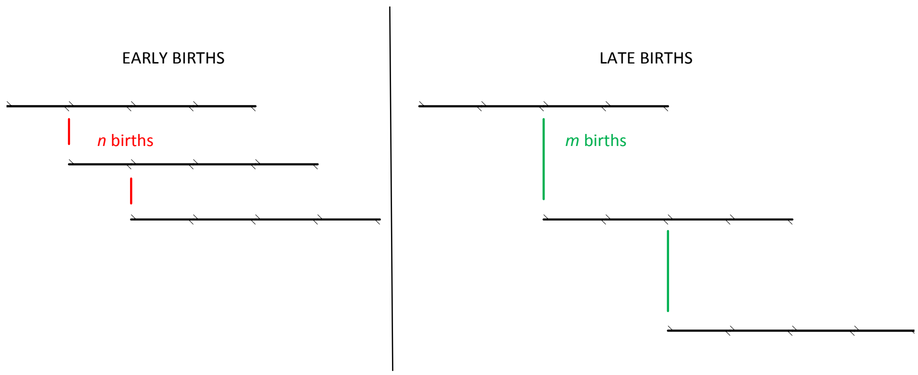

Figure 2: early births *versus* late births

In order to study the social desirability of birth postponement, we will focus here on an economy with a *fixed* living space.[3] That living space is a common resource, which is used, at a given point in time, by *all* individuals who are alive at that time. One can regard that economy with fixed living space as a "spaceship economy", in line with Boulding's metaphor of the Earth as a spaceship (Boulding 1966). On that spaceship, individual welfare is assumed to depend positively on the available space per head, space being here equally distributed among the living individuals.[4] The positive effect of space per head on individual welfare can be interpreted in several ways. One may consider that space consumption matters for its own sake, so that a too large population can lead to welfare-reducing congestion (Marshall 1890).[5] Another interpretation consists of considering that space is used in production, so that a too large population can also reduce welfare through output reduction (Hardin 1968).

[3] See Pestieau and Ponthiere (2014) for the study of the optimal fertility profile in an economy with physical capital accumulation, in line with Samuelson (1975).

[4] See Cramer *et al* (2004) on the negative effects of congestion on individual welfare.

[5] Indeed, as noticed by Marshall (1890), the existence of phases of increasing returns to scale for non-natural production factors, if combined with a sufficiently large amount of natural resources, could make the "population problem" disappear, except if a too large population density reduces individual welfare through congestion.

The timing of births being an inherently dynamic issue, we will study the social desirability of birth postponement in a dynamic framework where distinct generations overlap with each others. The advantage of that framework is that it includes, in addition to information about population size and utility, information about the age-structure of the population, and on the duration of overlap between generations, which is most relevant for the issue at stake. We develop a T-period dynamic overlapping generations economy where individual temporal welfare depends positively on the available space per head, and where agents have children in one out of two reproduction periods.

Then, we compare, by means of various utilitarian population ethics criteria, such as classical utilitarianism (CU), average utilitarianism (AU) and critical-level utilitarianism (CLU), two economies that have exactly the same initial conditions, the same longevity, but differ regarding the fertility timing: in one economy, all births take place early in life (i.e. Early Island), whereas in the other economy, all births take place later on in life (i.e. Late Island).

The welfare comparisons are carried out in four stages. In a first stage, the comparison focuses on *finite* histories, i.e. histories where an equal finite number of life-periods is lived by humans. Then, in a second stage, we compare *infinite* histories, i.e. histories where an infinite number of life-periods is lived by humans, by focusing on histories with a constant long-run population size. In a third stage, the robustness of those comparisons is examined by introducing an additional determinant of human well-being: the coexistence time with descendants, which varies across the different births timing (see Figure 2). Finally, we examine the social desirability of a sudden *transition* from an early births regime to a late births regime, which amounts to compare histories differing in the timing of births and in their starting points.

Anticipating on our results, we first show, when comparing finite histories with an equal number of life-periods but different fertility timing, that CU, AU and CLU criteria select the *same* fertility timing, which is the one leading to the most smoothed population pattern over time. That optimal birth timing depends on the initial age-structure within the reproductive age-group. When the comparison focuses on infinite histories with constant long-run population sizes, the fertility timing recommended by CU, AU and CLU depends on the form of the temporal utility function, as well as on the initial age-structure. Those results are qualitatively robust to the introduction of coexistence concerns across generations, except insofar as those coexistence concerns push towards earlier births, in such a way as to allow for longer periods of coexistence between humans. Hence the socially optimal birth timing depends, at the end of the day, on how congestion and coexistence concerns are weighted against each other. Regarding the social desirability of a transition from an early births regime to a late births regime, we show that such a transition is, for an equal total number of life-periods, socially desirable when individuals have no coexistence concerns, but may be undesirable under strong coexistence concerns.

In sum, the major contribution of this paper is to provide, by means of a general dynamic overlapping generations model, a formal analysis of the social desirability, from a utilitarian perspective, of the postponement of births. Our key findings are: (1) the social desirability of birth postponement depends, in general, on the postulated population ethics criterion; (2) the social desirability of birth postponement depends strongly on the initial age structure of the population; (3) the social desirability of births postponement varies also depending on the importance of spacial congestion and coexistence concerns within individual's preferences; (4) the conclusions that can be drawn are sensitive to the precise framing of the model, in the sense that a study of the desirability of different simultaneous histories is not formally similar to a study of the desirability of a sudden transition from one births regime to another regime.

By its findings, the present paper contributes to the recent renewal of the optimal fertility literature. In a canonical paper, Golosov et al (2007) revisited the optimal fertility by extending Pareto efficiency to situations of varying population sizes. Their extension gave birth to two concepts: A-efficiency (focusing on the well-being of each individual alive in all situations under comparison) and P-efficiency (which treats alive and potential individuals in symmetric ways). More recently, Conde-Ruiz et al (2010) proposed to revisit optimal fertility, by means of the concept of Millian efficiency (constrained A-efficiency). In comparison with the present study - which is based on utilitarian criteria -, those two recent papers rely on more general normative foundations. However, both Golosov et al (2007) and Conde-Ruiz et al (2010) rely on a framework with a unique fertility period, in the sense that individuals have all their children at the *same* point in their life. The present study, on the contrary, relies on a framework that allows also for the study of the optimal timing for births.

The rest of the paper is organized as follows. Section 2 presents the framework. Section 3 compares histories with different birth timings, in the case of finite histories with an equal total number of life-periods. Section 4 compares histories with different birth timing in the case of infinite histories with a constant asymptotic population sizes. Section 5 considers an extended model where individuals value coexistence with their descendants. Section 6 examines the social desirability of a transition, at a given point in time, from an early births regime to a late births regime. Section 7 concludes.

2 The framework

We consider an economy where time is discrete, and goes from $t = -T$ to $t = \infty$. That economy takes the form of a T-period dynamic overlapping generations economy (OLG): at any point in time, T distinct generations coexist. The specificity of that economy is that it involves a fixed living space. That fixed living space can be regarded as the Earth or any planet, or even a "spaceship", where living conditions allow humans to reproduce themselves.

2.1 Assumptions

Here are the assumptions that we will make throughout this paper.

Assumption A1 A place of finite surface Q is available for the life of a population.

Assumption A2 Only living persons occupy a part of the space Q.

Assumption A3 Each person alive at time t enjoys an equal share of the total available space: $q_t = \frac{Q}{L_t}$, where L_t denotes the population size at time t, whereas q_t denotes the space available per person.

Assumption A4 Each person lives exactly T periods of life for sure.

Assumption A5 At each period, the temporal welfare of all persons who are not alive (either non-existing or dead) is normalized to zero.

Assumption A6 At each period, the temporal welfare of a person who is alive depends on the space available per person, according to the function:

$$u_s = u(q_s) = (q_s)^{\sigma} + \alpha \;\; \forall s < t + T$$

where $0 \leq \sigma \leq 1$ and $\alpha \gtreqless 0$ is the intercept of the temporal utility function.

Assumption A7 Lifetime welfare is the sum of temporal utilities: $w = \sum_{s=1}^{T} u_s$.

Assumption A8 Each person gives birth *either* to $n > 0$ children in the second period of his life, *or* to $m > 0$ children in the third period of his life.

Assumption A9 The population age-structure at $t = -1$ is $\left\{N_{-T}, N_{-(T-1)}, ..., N_{-1}\right\}$ where $N_{-T}, N_{-(T-1)}, ..., N_{-1} > 0$.

Assumption A1 is standard: the fixity of land is a major ingredient of discussions on the "population problem" in early political economy (see Botero 1588, Cantillon, 1755, Malthus 1798). Assumption A2 is made for analytical simplicity. Assumption A3 amounts to suppose that, at a particular point in time t, all living persons, whatever their age is, occupy an equal amount of space, defined as the total space divided by the number of persons alive at that time. That assumption allows us to abstract from problems of intragenerational distribution of space. Assumption A4 states that all individuals, whatever the point in time at which they are born, will all enjoy the same longevity, equal to T periods. Assumption A5 is a standard normalization. Assumption A6 reflects the fundamental trade-off between the quantity and the quality of life.[6] It states that individuals derive some disutility from congestion, in line with Cramer et al (2004). Adding one person tends, *ceteris paribus*, to reduce the welfare of all

[6] Indeed, adding a person to the population would, under a fixed temporal utility $u_s = \alpha$, necessarily increase the sum of individual utilities. However, once $u(q_s)$ is decreasing in the population size, adding a person to the population does not necessarily increase the sum of individual utilities, leading to a trade-off between the quantity of life (i.e. the population size) and the quality of life (i.e. $u(q_s)$).

other persons, leading, at the aggregate level, to a quantity/quality trade-off.[7] Assumption A7 is standard in the literature. Assumption A8 states that agents can have children either in the second period of their life, or in the third period of their life. The existence of a double reproduction period is the specificity of this framework. Assumption A9 provides conditions on the initial age structure of the population, at time $t = -1$.

2.2 Problem setting

The comparison of economies with distinct fertility timings is a special case of more general comparisons, which focus on what we can call *histories*. A history includes, conditionally on an initial age structure, all demographic information: period-specific fertility rates n and m, and longevity T.

Definition 1 *Given an initial age structure* $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$ *prevailing at* $t = -1$*, a history is a triplet* $\{T, n, m\}$ *where* T *is the duration of each life, and* n *and* m *are period-specific fertility rates.*

A history includes all relevant piece of information in order to fully characterize population dynamics, conditionally on the initial age structure of the population.

There exist lots of possible histories. A particular family of histories consists of histories that involve exactly the same total number of life-periods. These particular histories are called *lifetime-equal* histories.

Definition 2 *Given an initial age structure* $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$ *prevailing at* $t = -1$*, two histories* $\{T, n, m\}$ *and* $\left\{\tilde{T}, \tilde{n}, \tilde{m}\right\}$ *are lifetime-equal if and only if these exhibit an equal total number of life-periods for people born at* $t \geq 0$*.*

The reason why we will focus here on lifetime-equal histories is merely that we would like to examine the social desirability of birth postponement *ceteris paribus*, that is, for a given total number of births. Put it differently, our focus on lifetime-equal histories allows us to compare histories that differ only regarding the temporal locations of births, without differing on the total number of births.

The demography of our economy can be described as follows. The total number of life-periods for all cohorts born at $t \geq 0$, denoted by $P_{0\rightarrow}$, is:

$$P_{0\rightarrow} = T\left[N_0 + N_1 + N_2 + N_3 + ... + N_\infty\right]$$

[7] Note that Q can also be interpreted as an amount of fully renewable natural resource, in the sense that, at the beginning of each period, an amount Q of the resource is available for all living persons. However, we prefer, throughout this paper, to keep the spatial interpretation of Q, since its constancy over time would amount, under the alternative interpretation, to assume a constant consumption technology for the fully renewable resource, which is a strong assumption.

where N_t denotes the number of individuals born at time t. The size of cohort t, i.e. the number of persons born at a period t, can be written as:

$$N_t = nN_{t-1} + mN_{t-2}$$

where n and m are, respectively, the early and late fertility rates. The total fertility rate (TFR) equals $n + m$. Multiplying and dividing P by N_{-1} yields:

$$P_{0\rightarrow} = TN_{-1}\left[\frac{N_0 + N_1 + N_2 + N_3 + ... + N_\infty}{N_{-1}}\right]$$

Let us now define the cohort growth factor as $g_t \equiv \frac{N_t}{N_{t-1}}$. Using that definition, the total number of life-periods $P_{0\rightarrow}$ can be written as:

$$P_{0\rightarrow} = TN_{-1}\left[g_0 + g_0g_1 + g_0g_1g_2 + ... + g_0g_1g_2...g_\infty\right]$$

Hence, given that $g_t \equiv \frac{N_t}{N_{t-1}} = \frac{nN_{t-1}+mN_{t-2}}{N_{t-1}} = n + \frac{m}{g_{t-1}}$ for $t \geq 0$, $P_{0\rightarrow}$ can be rewritten as:

$$\begin{aligned}
P_{0\rightarrow} &= TN_{-1}\left[\begin{array}{l} g_0 + g_0\left(n+\frac{m}{g_0}\right) + g_0\left(n+\frac{m}{g_0}\right)\left(n+\frac{m}{g_1}\right) \\ +... + g_0\left(n+\frac{m}{g_0}\right)\left(n+\frac{m}{g_1}\right)...\left(n+\frac{m}{g_{\infty-1}}\right) \end{array}\right] \\
&= TN_{-1}g_0\left[\begin{array}{l} 1 + \left(n+\frac{m}{g_0}\right) + \left(n+\frac{m}{g_0}\right)\left(n+\frac{m}{g_1}\right) \\ +... + \left(n+\frac{m}{g_0}\right)\left(n+\frac{m}{g_1}\right)...\left(n+\frac{m}{g_{\infty-1}}\right) \end{array}\right] \\
&= TN_{-1}g_0 + TN_{-1}g_0\left(\sum_{t=1}^{\infty}\prod_{s=1}^{t}\left(n+\frac{m}{g_{s-1}}\right)\right)
\end{aligned}$$

Note that the second term is a sum of an *infinite* number of terms. An infinity of life-periods is somewhat problematic for social welfare comparisons. Various solutions can be used to escape from that problem. One could, when aggregating the welfare associated to all those life-periods, assign a weight that is decreasing with time. However, as stressed by Ramsey (1928), such a way to proceed is completely arbitrary. Therefore we will, in the rest of this paper, focus on two kinds of histories: first, *finite histories*, where only a finite total number of periods are lived by humans (Section 3); second, *infinite histories*, where we focus only on the well-being prevailing when the population size is stabilized at a strictly positive level (Section 4).

3 Birth timing and welfare: finite histories

This section examines how a utilitarian social planner ranks, conditionally on a given initial age structure $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$ prevailing at $t = -1$, histories that are equal on all dimensions except regarding the timing of births. For that purpose, we will assume, throughout this section, that only a *finite* total number of life-periods P is lived.[8]

Assumption A10 Only a *finite* number of life-periods will be lived by people born at $t \geq 0$:

$$P_{0\rightarrow} = TN_{-1}g_0 + TN_{-1}g_0\left(\sum_{t=1}^{\infty}\prod_{s=1}^{t}\left(n+\frac{m}{g_{s-1}}\right)\right) < \infty$$

Assumption A10 imposes some restrictions on the levels of the two fertility rates n and m. To see this, let us consider two histories $\{T, n, 0\}$ and $\{T, 0, m\}$ equal on all dimensions except fertility timing. The only difference is that, in one history (early births case), we have $n > 0$ and $m = 0$, whereas, on the other, we have $n = 0$ and $m > 0$ (late births case).

If $n > 0, m = 0$, the total number of life-periods $P_{0\rightarrow}$ is equal to:

$$\begin{aligned} P_{0\rightarrow} &= TN_{-1}\left[g_0 + g_0g_1 + g_0g_1g_2 + ... + g_0g_1g_2...g_{\infty}\right] \\ &= TN_{-1}n\left[1 + n + n^2 + ... + n^{\infty}\right] \end{aligned}$$

Hence, a finite total number of life-periods requires $n < 1$. Then, the total number of life-periods beyond $t \geq 0$ in the early births case is:

$$P_{0\rightarrow} = \frac{TN_{-1}n}{1-n}$$

If $n = 0, m > 0$, the total number of life-periods $P_{0\rightarrow}$ is equal to:

$$\begin{aligned} P_{0\rightarrow} &= TN_{-1}\left[g_0 + g_0g_1 + g_0g_1g_2 + ... + g_0g_1g_2...g_{\infty}\right] \\ &= TN_{-1}\left[\frac{m}{g_{-1}} + \frac{m}{g_{-1}}\frac{m}{g_0} + \frac{m}{g_{-1}}\frac{m}{g_0}\frac{m}{g_1} + ...\right] \\ &= TN_{-2}m + TN_{-2}m\left[\frac{m}{g_0} + \frac{m^2}{g_0g_1} + \frac{m^3}{g_0g_1g_2} + ...\right] \\ &= TN_{-2}m + TN_{-2}m\left[\frac{m}{g_0} + m + \frac{m^2}{g_0} + m^2 + ...\right] \\ &= TN_{-2}m + TN_{-2}m\left(1+\frac{1}{g_0}\right)\left[m + m^2 + ... + m^{\infty}\right] \end{aligned}$$

as $g_t = \frac{m}{g_{t-1}}$, which implies $g_tg_{t-1} = m$.

[8] By "finite histories", we mean histories with a finite total number of periods *lived* by (some) individuals. Note that it remains true that, in our OLG economy, time goes from $-T$ to ∞, despite of the fact that only a finite number of those periods are lived by some persons.

Hence, a finite total number of life-periods requires $m < 1$. Then, the total number of life-periods beyond $t \geq 0$ in the late births case is:

$$\begin{aligned} P_{0\rightarrow} &= TN_{-2}m + \frac{TN_{-2}m^2\left(1+\frac{1}{g_0}\right)}{1-m} \\ &= \frac{TN_{-2}m\left(1+\frac{m}{g_0}\right)}{1-m} \\ &= \frac{TN_{-2}m\left(1+\frac{N_{-1}}{N_{-2}}\right)}{1-m} \end{aligned}$$

The following lemma states the condition, on the fertility rates n and m, such that two histories $\{T, n, 0\}$ and $\{T, 0, m\}$ are lifetime-equal.

Lemma 1 *Suppose an initial age structure* $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$ *prevailing at* $t = -1$. *Take two histories* $\{T, n, 0\}$ *and* $\{T, 0, m\}$ *with a finite number of life-periods for people born at* $t \geq 0$. *Those histories are lifetime-equal histories if and only if:*

$$m = n\frac{N_{-1}}{N_{-1} + N_{-2}(1-n)} \implies m < n$$

Proof. Equalizing $P_{0\rightarrow}$ in $\{T, n, 0\}$ and $\{T, 0, m\}$ yields:

$$\begin{aligned} \frac{TN_{-1}n}{1-n} &= \frac{TN_{-2}m\left(1+\frac{N_{-1}}{N_{-2}}\right)}{1-m} \\ &\iff \frac{\frac{n}{1-n}}{\frac{m}{1-m}} = \frac{N_{-2}+N_{-1}}{N_{-1}} \end{aligned}$$

Given that $\frac{N_{-2}+N_{-1}}{N_{-1}} > 1$, we have $m < n$. Let us now write the level of m such that the two histories are lifetime-equal. We need $\frac{\frac{n}{1-n}}{\frac{m}{1-m}} = \frac{N_{-2}+N_{-1}}{N_{-1}}$. Hence:

$$\frac{n}{1-n}N_{-1} = m\left[(N_{-2}+N_{-1}) + \frac{nN_{-1}}{1-n}\right]$$

Hence the equality of $P_{0\rightarrow}$ in $\{T, n, 0\}$ and $\{T, 0, m\}$ requires

$$\begin{aligned} m &= \frac{n}{(1-n)\left[(N_{-2}+N_{-1}) + \frac{nN_{-1}}{1-n}\right]}N_{-1} \\ m &= n\frac{N_{-1}}{N_{-1}+N_{-2}(1-n)} \end{aligned}$$

■

The intuition behind Lemma 1 goes as follows. As reproduction takes place after two life-periods in history $\{T, 0, m\}$, instead of one life-period in history

$\{T, n, 0\}$, there is a larger dependency on the size of past cohorts in the former. Given that the fertility rates are below the replacement level, so that the size of cohorts tends to shrink over time, it is clear that, under the same fertility rates $n = m$, there would be, after a given time period, *more* total life-periods lived in $\{T, 0, m\}$ than in $\{T, n, 0\}$. Hence the equality of the total number of life periods for people born at $t \geq 0$ in $\{T, n, 0\}$ and $\{T, 0, m\}$ requires a lower fertility in $\{T, 0, m\}$. Given that longevity T is equal across histories, Lemma 1 states that equal births can only be achieved with unequal fertility rates.

In order to carry out our welfare comparisons, we will use the three following utilitarian criteria, which are known as classical utilitarianism (CU), average utilitarianism (AU) and critical-level utilitarianism (CLU).

Definition 3

- *Classical utilitarianism (CU):*

$$\{T, n, m\} \succeq_{CU} \left\{\tilde{T}, \tilde{n}, \tilde{m}\right\} \iff \sum_{t=0}^{\infty} N_t \sum_{s=t}^{t+T-1} u(q_s) \geq \sum_{t=0}^{\infty} \tilde{N}_t \sum_{s=t}^{t+T-1} u(\tilde{q}_s)$$

- *Average utilitarianism (AU):*

$$\{T, n, m\} \succeq_{AU} \left\{\tilde{T}, \tilde{n}, \tilde{m}\right\} \iff \frac{\sum_{t=0}^{\infty} N_t \sum_{s=t}^{t+T-1} u(q_s)}{\sum_{t=0}^{\infty} N_t} \geq \frac{\sum_{t=0}^{\infty} \tilde{N}_t \sum_{s=t}^{t+T-1} u(\tilde{q}_s)}{\sum_{t=0}^{\infty} \tilde{N}_t}$$

- *Critical-level utilitarianism (CLU):*

$$\{T, n, m\} \succeq_{CLU} \left\{\tilde{T}, \tilde{n}, \tilde{m}\right\} \iff \sum_{t=0}^{\infty} N_t \sum_{s=t}^{t+T-1} [u(q_s) - \hat{u}] \geq \sum_{t=0}^{\infty} \tilde{N}_t \sum_{s=t}^{t+T-1} [u(\tilde{q}_s) - \hat{u}]$$

where $\hat{u}$ is the critical utility level for continuing existence, making a life-period neutral, while $\succeq_i$ denotes the standard preference relation (binary, reflexive and transitive) for the type-i planner. As usual, $\succ$ and $\sim$ denote strict preference and indifference.

The CU social welfare function ranks histories depending on which one leads to the largest sum of lifetime well-being for all individuals born at $t \geq 0$. On the contrary, AU ranks histories according to the average lifetime well-being enjoyed among individuals born at $t \geq 0$. Finally, CLU ranks histories according to the sum of lifetime well-being net of a critical level $\hat{u}$ making a life-period neutral from a social perspective. Note that CLU vanishes to CU when the critical utility level for continuing existence $\hat{u}$ is equal to 0.[9]

[9] Note that the nature of the intercept of the temporal utility function α is quite different from the nature of the critical utility level for continuing existence $\hat{u}$. Indeed, whereas the former is a purely descriptive parameter, capturing how individuals value longevity with respect to congestion, the latter is an ethical parameter, which reflects the preferences of the social planner.

3.1 Classical utilitarianism

According to classical utilitarianism (Bentham 1789), actions should be chosen in such a way as to produce the "greatest happiness of the greatest number", in conformity with the Principle of Utility. Obviously, the selection of a particular timing for births should also satisfy the Principle of Utility. Proposition 1 presents the preferences of a CU social planner in terms of fertility timing.[10]

Proposition 1 *Assume A1-A10 and* $0 \leq \sigma \leq 1$. *Suppose an initial age structure* $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$. *Consider two lifetime-equal histories* $\{T, n, 0\}$ *and* $\{T, 0, m\}$. *We have:*

$$\{T, n, 0\} \succeq_{CU} (\preceq_{CU}) \{T, 0, m\}$$

if and only if:

$$\sum_{t=1}^{\infty} n^t N_{-1} \sum_{s=t}^{t+T-1} (\bar{q}_s)^{\sigma} \geq (\leq) \left[\begin{array}{c} \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-2} \sum_{s=t}^{t+T-1} (\hat{q}_s)^{\sigma} \\ + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-1} \sum_{s=t}^{t+T-1} (\check{q}_s)^{\sigma} \end{array} \right]$$

where

$$\bar{q}_s \equiv \frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r}$$

$$\hat{q}_s \equiv \frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + \sum_{r=1,3,...}^{s} N_{-2} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{(r+1)/2} + \sum_{z=2,4,...}^{s} N_{-1} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{z/2}}$$

$$\check{q}_s \equiv \frac{Q}{\left[\begin{array}{c} \sum_{z=0}^{T-s-1} N_{-(T-s-z+1)} + \frac{nN_{-1}^2}{N_{-1}+N_{-2}(1-n)} + \sum_{r=1,3,...}^{s} N_{-2} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{(r+1)/2} \\ + \sum_{z=2,4,...}^{s} N_{-1} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{z/2} \end{array} \right]}$$

Proof. See the Appendix. ■

In general, CU is known to have a populationist bias, and to lead to excessive population sizes (e.g. in the Repugnant Conclusion). Here, however, despite the fact that the total fertility rate ($TFR = n + m$) is strictly larger under history $\{T, n, 0\}$ that under history $\{T, 0, m\}$ (because of Lemma 1), CU may, under some initial age-structure conditions, regard late births as better than early births. That result is surprising. Note, however, that, given that

[10] Note that the formulas are here shown for the case where T is an even natural number. Obviously, similar formulas can be derived for the case where T is an odd number.

histories $\{T, n, 0\}$ and $\{T, 0, m\}$ are lifetime-equal, the constancy of longevity for all individuals implies that those two histories involve also the same *number* of births, despite the fact that the total fertility rate is larger under history $\{T, n, 0\}$ than under history $\{T, 0, m\}$. Thus the CU ranking on births timing has nothing to do with the number of births. It has to do with the *temporal location* of those births. The CU planner selects the birth timing that leads to the most smoothed population pattern, in such a way as to minimize the social disutility from congestion.

Obviously, in the special case where space does not matter for well-being, that is, $\sigma = 0$, then CU considers any two lifetime-equal histories as equally good, since in that case space congestion does not matter, so that the timing of births does not matter either.

Corollary 1 *Assume A1-A10. Suppose an initial age structure* $\{N_{-T}, ..., N_{-1}\}$. *Consider two lifetime-equal histories* $\{T, n, 0\}$ *and* $\{T, 0, m\}$. *Under* $\sigma = 0$, *we have:* $\{T, n, 0\} \sim_{CU} \{T, 0, m\}$.

Proof. See the Appendix. ■

Another important observation from Proposition 1 is that the CU ranking of lifetime-equal histories does not depend at all on the preference parameter α, that is, the intercept of the temporal utility function. The intuition behind that result lies in the fact that, given that the histories under comparison are lifetime-equal, the number of life periods in each history under comparison is the same, which neutralizes the influence of the parameter α.

Whether history $\{T, n, 0\}$ is, in general, regarded as better or worse than a lifetime-equal history $\{T, 0, m\}$ from a classical utilitarian perspective depends crucially on the initial age structure of the population, i.e. $\{N_{-T}, ..., N_{-1}\}$. To illustrate this, let us focus on a simple 3-period example (i.e. $T = 3$). Let us compare two lifetime-equal histories, with either $n = 0.5$ and $m = 0$, i.e. history $\{3, 0.5, 0\}$, or $n = 0$ and $m = n\frac{N_{-1}}{N_{-1}+N_{-2}(1-n)}$. Numerical simulations show that, under initial conditions $\{N_{-3} = 100, N_{-2} = 100, N_{-1} = 30\}$ leading to $m = 0.1875$, as well as $Q = 1$, $\alpha = 0$ and $\sigma = 0.5$, we obtain that the lifetime-equal history with early births is, under CU, ranked above the lifetime-equal history with late births:[11]

$$\{3, 0.500, 0\} \succ_{CU} \{3, 0, 0.1875\}$$

This result is intuitive, since the TFR is larger on Early Island ($0.500 > 0.1875$). However, under alternative initial conditions $\{N_{-3} = 100, N_{-2} = 100, N_{-1} = 100\}$ leading to $m = 0.3333$, we obtain the opposite ranking, that is, the lifetime-equal history with later births is, under CU, ranked *above* the lifetime-equal history with early births, despite it involves a lower TFR ($0.3333 < 0.500$):

$$\{3, 0.500, 0\} \prec_{CU} \{3, 0, 0.3333\}$$

[11] Simulations cover the first 1000 cohorts, but cumulated social welfare is stabilized far before the end of that time interval.

Figures 3 and 4 show, for each comparison of lifetime-equal histories, the cumulated social welfare across cohorts, in the case of Early Island and Late Island.

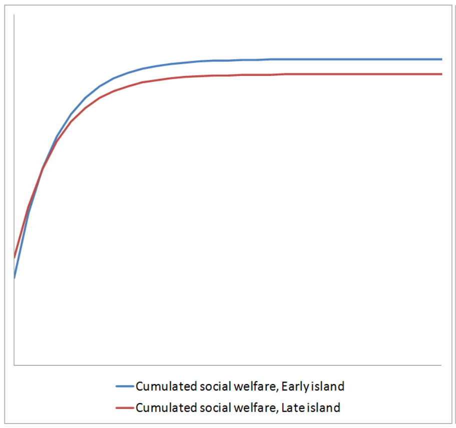

Figure 3: Cumulated social welfare across cohorts under $\{N_{-3} = 100, N_{-2} = 100, N_{-1} = 30\}$. Early Island involves $n = 0.500, m = 0$. Late Island involves $n = 0, m = 0.1875$.

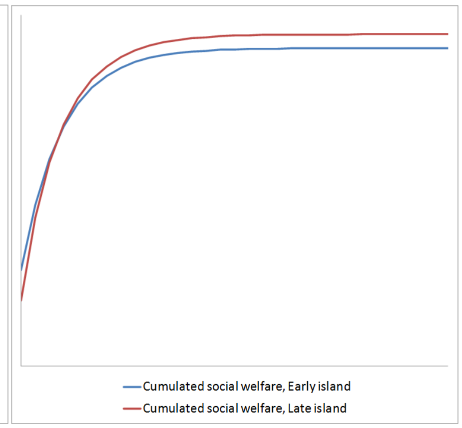

Figure 4: Cumulated social welfare across cohorts under $\{N_{-3} = 100, N_{-2} = 100, N_{-1} = 100\}$. Early Island involves $n = 0.500, m = 0$. Late Island involves $n = 0, m = 0.333$.

Those few numerical simulations suffice to show that CU may, in some cases, opt for histories involving a lower TFR, on the grounds that, under some initial age structures, having fewer births along one's life - but postponed births - may lead to less space congestion, and, hence, to some welfare improvement. However, this result may not hold under alternative initial age structures, as shown by our numerical example. At the end of the day, whether birth postponement is preferred or not under CU depends crucially on the initial age structure that prevails in the population. The underlying intuition is that the initial age structure determines the population dynamics induced by either early or late fertility, and, hence, the pattern of spatial congestion. Given that an equal number of persons are born in lifetime-equal histories, whether a history is more socially desirable than another depends on which history yields the smoothest population pattern, since this minimizes welfare losses due to congestion.

3.2 Average and critical-level utilitarianism

Let us now compare the above findings with what would prevail under other standard utilitarian population ethics criteria, such as average utilitarianism and critical-level utilitarianism.[12]

[12] On the origins of average utilitarianism, Gottlieb (1945) refers to Mill (1859), who was in favour of birth control in the name of social welfare maximization. Note, however, that the

Proposition 2 states that the ranking on histories under AU and CLU are equivalent to the ranking on histories under CU.

Proposition 2 *Assume A1-A10 and* $0 \leq \sigma \leq 1$*. Suppose an initial age structure* $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$*. Consider two lifetime-equal histories* $\{T, n, 0\}$ *and* $\{T, 0, m\}$*. We have:*

$$\{T, n, 0\} \succeq_{AU} \{T, 0, m\} \iff \{T, n, 0\} \succeq_{CU} \{, T, 0, m\}$$

and

$$\{T, n, 0\} \succeq_{CLU} \{T, 0, m\} \iff \{T, n, 0\} \succeq_{CU} \{T, 0, m\}$$

Proof. See the Appendix. ■

The ranking of histories under AU and CLU is exactly the same as the one under CU. That result may seem at first glance surprising: one may expect, for instance, that AU recommends the history where births take place later on in the lifecycle, i.e. $\{T, 0, m\}$, in the spirit of Malthus (1798), who defended birth postponement as a solution to the population problem.

But this is not necessarily the case: it depends on the initial age structure $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$, exactly as under CU. The reason why CU, AU and CLU yield the same ranking is that the two histories under comparison are lifetime-equal, and have thus the same total number of births. Only the temporal location of births differs across the two histories. Here again, AU recommends - exactly as CU - the fertility timing that yields the most smoothed population path, so as to minimize welfare losses due to congestion. Whether this is achieved under an early fertility profile or a late fertility profile depends on the initial age-structure within the reproductive group.

The dependency of the optimal birth timing on the initial age structure is in line with the existing literature focusing on the selection, by a utilitarian social planner, of the optimal age-structure dynamics. By means of a continuous time dynamic model with linear production in labour, Boucekkine et al (2014) showed that the optimal age structure dynamics in the Benthamite case is dependent on the initial age structure that prevails in the economy. The reason for that dependency on initial age-structure conditions in Boucekkine et al (2014) is close to the one prevailing in our discrete time framework. The welfare dynamics induced by a particular birth timing is dependent on the initial age structure of the population on which age-specific fertility rates apply. Hence the optimal birth timing depends also on the initial age structure.

4 Birth timing and welfare: infinite histories

Throughout this section, we depart from the finiteness assumption A10, and consider instead a *stationary* economy, with a finite constant long-run population size. Given that the population size is here stabilized at a strictly positive

distinction between total and average welfare dates back to Sidgwick (1874). Critical-level utilitarianism was introduced by Blackorby and Donaldson (1984).

level in the long-run, the total number of periods lived is now infinite, unlike in the histories considered in Section 3.

The constancy of the population at periods t and $t+1$ requires:

$$\sum_{s=t-T+1}^{t} N_s = \sum_{s=t-T+2}^{t+1} N_s \iff N_{t-T+1} = N_{t+1}$$

that is, the equality of births (RHS) and deaths (LHS). That condition can be rewritten as:

$$N_{t-T+1} = N_{t-T+1} \prod_{s=t-T+2}^{t+1} g_s \iff \prod_{s=t-T+2}^{t+1} g_s = 1$$

The corollaries of that condition for the two kinds of histories compared are straightforward. When all births are early, $g_s = n \; \forall s$, so that the condition collapses to $n^T = 1$ implying $n = 1$. When all births are late, we have $g_s = \frac{m}{g_{s-1}}$, so that the condition collapses to $m = 1$ (assuming that T is an even natural number).[13] These are the assumptions we will make throughout this section.

Assumption A11 The population converges asymptotically towards a positive constant. Hence, when all births take place early in life, we have $n = 1$, whereas, when all births take place later on in life, we have $m = 1$.

Once A10 is replaced by A11, the total number of life-periods becomes infinite. The non-finiteness of total lifetimes is problematic for aggregated ethical doctrines such as utilitarianism. Therefore, to overcome that difficulty, we will, in this section, limit social welfare comparisons to what prevails at the *stationary equilibrium*, that is, at the constant long-run population level.[14]

The following lemma compares the levels of the long-run populations of histories $\{T, 1, 0\}$ and $\{T, 0, 1\}$.

Lemma 2 *Suppose an initial age structure* $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$. *Let us compare the long-run population sizes under the infinite histories* $\{T, 1, 0\}$ *and* $\{T, 0, 1\}$ *where* T *is assumed to be an even natural number. History* $\{T, 1, 0\}$ *has a larger (resp. equal, resp. smaller) long-run population than history* $\{T, 0, 1\}$ *if and only if* $N_{-1} \gtreqless N_{-2}$.

Proof. See the Appendix. ■

It is only in the special case where the initial age-structure satisfies $N_{-1} = N_{-2}$ (i.e. uniform initial age-structure among reproductive age-groups) that

[13] That assumption is indeed necessary to have a constant long-run population when $n = 0$ and $m = 1$. Otherwise, if T is an odd natural number, then the population exhibits a two-period cycle in the long-run.

[14] That exclusive focus on the stationary equilibrium involves some simplifications: it amounts to extract the population problem from the time dimension. But it allows us to compare non-finite histories, since that stationary equilibrium (stationary population and space per head) will, by definition, reproduce itself forever.

the two histories yield the same long-run population size. In general, those two histories, despite the same longevity and the same initial conditions, as well as the same total fertility rates $TFR = n + m = 1$, do not have the same long-run population sizes.

In order to carry out our welfare comparisons, we will concentrate here on the stationary equilibrium of the economy, i.e. the state achieved when the population is stabilized and reproduces itself perpetually in the future. In the context of welfare comparisons at the stationary equilibrium, the definition of the utilitarian criteria needs to be adapted.[15]

Definition 4 •

- *Long-run classical utilitarianism (LRCU):*

$$\{T, n, m\} \succeq_{LRCU} \left\{\tilde{T}, \tilde{n}, \tilde{m}\right\} \iff LTu(q) \geq \tilde{L}\tilde{T}u(\tilde{q})$$

- *Long-run average utilitarianism (LRAU):*

$$\{T, n, m\} \succeq_{LRAU} \left\{\tilde{T}, \tilde{n}, \tilde{m}\right\} \iff Tu(q) \geq \tilde{T}u(\tilde{q})$$

- *Long-run critical-level utilitarianism (LRCLU):*

$$\{T, n, m\} \succeq_{LRCLU} \left\{\tilde{T}, \tilde{n}, \tilde{m}\right\} \iff LT\left[u(q) - \hat{u}\right] \geq \tilde{L}\tilde{T}\left[u(\tilde{q}) - \hat{u}\right]$$

where $\hat{u}$ is the critical utility level for continuing existence, making a life-period neutral, while $\succeq_i$ denotes the standard preference relation (binary, reflexive and transitive) for the type-i planner. As usual, $\succ$ and $\sim$ denote strict preference and indifference.

4.1 Classical utilitarianism

Let us now examine how a classical utilitarian planner ranks infinite histories $\{T, 1, 0\}$ and $\{T, 0, 1\}$ with constant long-run population sizes but different fertility patterns. The following proposition summarizes our results regarding the comparison, from a classical utilitarian perspective, of two histories differing only regarding the timing of births.

Proposition 3 *Assume A1-A9 and A11. Suppose an initial age structure $\{N_{-T}, ..., N_{-1}\}$. Consider two histories $\{T, 1, 0\}$ and $\{T, 0, 1\}$:*

- *If $\alpha \geq 0$, then*

[15] Note that we focus here only on the stationary equilibrium, and, hence, we neglect deliberately the transition towards that stationary equilibrium. Taking the transition into account would require to select a pure discount rate. Given that our focus is not on intergenerational justice - but only on the social desirability of births postponement -, we prefer to focus only on the stationary equilibrium without considering the sensitivity of our results to the choice of a pure discount rate.

- *If* $N_{-1} < N_{-2}$, $\{T,1,0\} \prec_{LRCU} \{T,0,1\}$.
- *If* $N_{-1} = N_{-2}$, $\{T,1,0\} \sim_{LRCU} \{T,0,1\}$.
- *If* $N_{-1} > N_{-2}$, $\{T,1,0\} \succ_{LRCU} \{T,0,1\}$.

- *If* $\alpha < 0$, *then* $\{T,1,0\} \succeq_{LRCU} (\preceq_{LRCU}) \{T,0,1\}$ *iff*
$$\alpha\left(\frac{N_{-1}-N_{-2}}{2}\right) \geq (\leq) \left(\frac{Q}{T}\right)^{\sigma}\left[\left(\frac{(N_{-2}+N_{-1})}{2}\right)^{1-\sigma} - (N_{-1})^{1-\sigma}\right].$$

Proof. See the Appendix ■

Proposition 3 states that, when an additional period of life is always worth being lived at the individual level even under a strong congestion (i.e. $\alpha \geq 0$), then the LRCU social planner ranks the two histories $\{T,1,0\}$ and $\{T,0,1\}$ depending on which one leads to the largest long-run population size.

Such a size-based ranking does not prevail once adding a life-period is not necessarily worthy (i.e. $\alpha < 0$). In that case, the ranking between $\{T,1,0\}$ and $\{T,0,1\}$ is ambiguous, except when the two histories have the same long-run population size (i.e. $N_{-1} = N_{-2}$), in which case we have $\{T,1,0\} \sim_{LRCU} \{T,0,1\}$. The reason is that, when $\alpha < 0$, the LRCU ranking over histories depends not only on the number of individuals living at the stationary equilibrium, but, also, of the congestion dimension. Hence the best birth timing depends on how quantities of life and qualities of life are weighted against each other under early and late births.

4.2 Average utilitarianism

Let us now compare the LRCU ranking over $\{T,1,0\}$ and $\{T,0,1\}$ with what prevails under average utilitarianism. Proposition 4 summarizes our results regarding the comparison, from an average utilitarian perspective, of two histories differing on the timing of births.

Proposition 4 *Assume A1-A9 and A11. Suppose an initial age structure* $\{N_{-T},...,N_{-1}\}$. *Consider two histories* $\{T,1,0\}$ *and* $\{T,0,1\}$*:*

- *If* $N_{-1} < N_{-2}$, $\{T,1,0\} \succ_{LRAU} \{T,0,1\}$.
- *If* $N_{-1} = N_{-2}$, $\{T,1,0\} \sim_{LRAU} \{T,0,1\}$.
- *If* $N_{-1} > N_{-2}$, $\{T,1,0\} \prec_{LRAU} \{T,0,1\}$.

Proof. See the Appendix ■

The ranking in Proposition 4 is the exact opposite of the ranking in Proposition 3 when $\alpha \geq 0$. The reason is that, while CU recommends the history with the largest long-run population size under $\alpha \geq 0$, AU does the exact opposite, and recommends the history with the lowest long-run population size. Note, however, that the associated birth timing depends on the initial age structure of the economy.

4.3 Critical-level utilitarianism

The following proposition summarizes our results regarding the comparison of two histories differing on the timing of births, from the point of view of critical-level utilitarianism.

Proposition 5 *Assume A1-A9 and A11. Suppose an initial age structure* $\{N_{-T}, ..., N_{-1}\}$. *Consider two histories* $\{T, 1, 0\}$ *and* $\{T, 0, 1\}$.

- *If* $\alpha - \hat{u} \geq 0$, *then*
  - *If* $N_{-1} < N_{-2}$, $\{T, 1, 0\} \prec_{LRCLU} \{T, 0, 1\}$.
  - *If* $N_{-1} = N_{-2}$, $\{T, 1, 0\} \sim_{LRCLU} \{T, 0, 1\}$.
  - *If* $N_{-1} > N_{-2}$, $\{T, 1, 0\} \succ_{LRCLU} \{T, 0, 1\}$.
- *If* $\alpha - \hat{u} < 0$, *then* $\{T, 1, 0\} \succeq_{LRCLU} (\preceq_{LRCLU}) \{T, 0, 1\}$ *iff*

  $(\alpha - \hat{u}) \left[\frac{N_{-1}-N_{-2}}{2}\right] \geq (\leq) \left(\frac{Q}{T}\right)^{\sigma} \left[\left(\frac{N_{-2}+N_{-1}}{2}\right)^{1-\sigma} - (N_{-1})^{1-\sigma}\right]$.

Proof. See the Appendix ■

When the net contribution of a life-period is always non-negative, that is, when $\alpha - \hat{u} \geq 0$, critical-level utilitarianism recommends the history with the largest asymptotic population size. Here again, this depends on the initial age-structure $N_{-1} \gtreqless N_{-2}$. When the net contribution of a life-period may be negative, the ranking between histories $\{T, 1, 0\}$ and $\{T, 0, 1\}$ becomes more ambiguous, as there is a trade-off between adding new beings and reducing the available space for existing ones.

In sum, when comparing infinite histories with constant long-run population sizes, the social planner's preference over different fertility timings varies depending on the treatment of population size in the social objective. Another crucial determinant of the social ranking depends on the level of the intercept α. Moreover, the initial age-structure still plays an important role. But that role is quite different from the one it played in the finite history case. Indeed, in the comparison of finite histories, the best fertility timing is the one that leads to the most regular population size dynamics *given* the postulated initial conditions. On the contrary, when infinite histories with constant long-run population sizes are compared, the initial age-structure determines which fertility timing leads to the largest or the lowest long-run population size.

5 Valuing coexistence with dynasties

A major difference between societies with different birth timing lies in how long the different cohorts coexist, that is, in the duration of the overlap between generations. Differences in intergenerational overlap can lead to various outcomes in terms of congestion, as this was discussed above. However, different durations of coexistence periods may also be valued for their own sake. For instance, an adult may be happy to see not only his children, but, also, his grand-children and grand-grand children. Such a valuation of coexistence with the dynasty has not been discussed so far, because assumption A6 made temporal welfare independent from coexistence with the descendants. As a consequence, lifetime welfare was only dependent on space (assumption A7).

Our study of the social desirability of births postponement has so far neglected the occurrence of coexistence concerns within the dynasty. However, from a normative perspective, the analysis of the social desirability of birth timing should rely on individual's actual preferences, and not only on some aspect of these. This motivates the generalization of the previous analyses to the case where individual well-being depends not only on space congestion, but, also, on coexistence with descendants. Hence, for the sake of generality, it makes sense to examine to what extent our previous results are robust to introducing coexistence concerns. This constitutes the task of this section.

A first thing to stress is that coexistence time differs strongly across histories with different birth timings. To see this, note first that the total period of coexistence with descendants for an individual on Early Island is equal to:

$$(T-1)\,(n)+(T-2)(n^2)+(T-3)(n^3)+...+(T-T+1)(n^{T-1})$$

The first term consists of the duration of coexistence of the individual with his n early children, equal to $T-1$, multiplied by the number of early children. The second term consists of the duration of coexistence with his n^2 grand-children, equal to $T-2$, multiplied by the number of grand-children, etc.

On Late Island, the total period of coexistence with descendants is equal to:

$$(T-2)(m)+(T-4)(m^2)+(T-6)(m^3)+...+(T-T+2)(m^{T-2})$$

The first term consists of the duration of coexistence of the individual with his m late children, equal to $T-2$, multiplied by the number of late children. The second term consists of the duration of coexistence with his m^2 grand-children, equal to $T-2$, multiplied by the number of grand-children, etc.

Note that the duration of coexistence with descendants is, for each generation, strictly shorter on Late Island, because of the postponement of births.

In order to discuss the social desirability of different fertility timing while taking into account the value of coexistence with dynasties, let us replace assumptions A6 and A7 by the following, which keeps the time-additivity but accounts for valuing coexistence with descendants.[16]

Assumption A12 Individual lifetime welfare takes the following form:

$$w=\sum_{r=1}^{T}\left((q_r)^{\sigma}+\alpha\right)+\sum_{s=1}^{T}\left((T-s)\,(n^s)\right)^{\delta}+\sum_{s=1}^{T}\left((T-2s)\,(m^s)\right)^{\delta}$$

where $0\leq\delta<1$ captures the taste for coexistence with descendants.

[16] Assumption A12 amounts to assume that agents care about coexistence with their descendants, and not with their ancestors. That assumption is made for analytical simplicity. Note that adding a concern for coexistence with both descendants and ancestors would only reinforce our results towards making early births more socially desirable than late births.

Note that, when individuals exhibit no concern for coexistence, then the parameter δ is equal to zero, and we are back to the baseline framework studied in the previous sections.

Let us now examine whether introducing a taste for coexistence with the dynasties affects or not the utilitarian ranking on histories with different birth timing. For that purpose, we will proceed as above, and distinguish between finite histories and infinite histories.

5.1 Dynasties under finite histories

As above, we make assumption A10, which amounts to assume that fertility is below the replacement level. By Lemma 1, we know that lifetime-equal histories $\{T, n, 0\}$ and $\{T, 0, m\}$ must, under that condition, satisfy the condition: $m < n$. The following proposition presents the preferences of a classical utilitarian social planner in terms of fertility timing.

Proposition 6 *Assume A1-A5, A8-A10 and A12 and* $0 \leq \sigma \leq 1$, $0 \leq \delta < 1$. *Suppose an initial age structure* $\left\{N_{-T}, N_{-(T-1)}, N_{-(T-2)}, ..., N_{-1}\right\}$. *Consider two lifetime-equal histories* $\{T, n, 0\}$ *and* $\{T, 0, m\}$.

$$\{T, n, 0\} \succeq_{CU} (\preceq_{CU}) \{T, 0, m\}$$

iff:

$$\begin{aligned}
& \sum_{t=1}^{\infty} n^t N_{-1} \sum_{s=t}^{t+T-1} (\bar{q}_s)^{\sigma} + \frac{nN_{-1}}{1-n} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left(1n^{T-1}\right)^{\delta}\right) \\
\geq \quad & (\leq) \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)}\right)^t N_{-2} \sum_{s=t}^{t+T-1} \hat{q}_s^{\sigma} + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)}\right)^t N_{-1} \sum_{s=t}^{t+T-1} \check{q}_s^{\sigma} \\
& + \frac{nN_{-1}}{1-n} \left[\left(\frac{(T-2)nN_{-1}}{N_{-1} + N_{-2}(1-n)}\right)^{\delta} + ... + \left(2\left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right]
\end{aligned}$$

Proof. See the Appendix. ■

In comparison with the absence of concerns for coexistence (case where $\delta = 0$), CU now recommends early fertility for a larger number cases. Indeed, the above condition differs from the condition stated in Proposition 1 in only two terms. Given that coexistence is unambiguously larger under early births, and given $\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} < n$, the second term of the LHS of the condition exceeds the second term of the RHS, implying that introducing coexistence concerns pushes, *ceteris paribus*, towards earlier births. The following corollary examines the special case where σ equals 0.

Corollary 2 *Assume A1-A5, A8-A10 and A12 and* $0 \leq \delta < 1$. *Suppose an initial age structure* $\{N_{-T}, ..., N_{-1}\}$. *Consider two lifetime-equal histories* $\{T, n, 0\}$ *and* $\{T, 0, m\}$. *When* $\sigma = 0$, *we have* $\{T, n, 0\} \succ_{CU} \{T, 0, m\}$.

Proof. See the Appendix. ■

In that special case, the early fertility timing is regarded as socially desirable. The intuition is that, under σ equals 0, there is no need to smooth the population pattern, unlike under $0 < \sigma$. As a consequence, only the valuation of coexistence with the dynasty matters. From that perspective, early birth timing does better.

Finally, note that the results of Proposition 6 also hold under other criteria, such as AU or CLU, as stated in the following corollary.

Corollary 3 *Assume A1-A5, A8-A10 and A12 and* $0 \leq \delta < 1$. *Suppose an initial age structure* $\{N_{-T}, ..., N_{-1}\}$. *Consider two lifetime-equal histories* $\{T, n, 0\}$ *and* $\{T, 0, m\}$. *AU and CLU lead the same ranking on* $\{T, n, 0\}$ *and* $\{T, 0, m\}$ *as CU.*

Proof. See the Appendix. ■

Hence, the introduction of a value for coexistence with the dynasty affects the social desirability of births timing in a manner that is invariant to the type of utilitarian social criterion considered, since in all histories compared, only the temporal location of births differs, and not the total number of births. That effect makes early births more desirable. Note, however, that the smoothing motive for birth postponement still remains (under $\sigma > 0$), so that introducing a concern for coexistence with the dynasty does not suffice, on its own, to generate a preference for early births.

5.2 Dynasties under infinite histories

In order to check the robustness of our results to the level of total fertility, we now relax assumption A10, and replace it by assumption A11. As above, the stationarity of the population size in the long-run implies that the two fertility rates take their replacement levels, i.e. $n = 1$ and $m = 1$. Hence the two histories under comparison are now: $\{T, 1, 0\}$ and $\{T, 0, 1\}$.

The following proposition summarizes how a CU social planner would rank those histories in the presence of a concern for dynastic coexistence.

Proposition 7 *Assume A1-A5, A8-A9 and A11-A12. Suppose an initial age structure* $\{N_{-T}, ..., N_{-1}\}$. *Consider two histories* $\{T, 1, 0\}$ *and* $\{T, 0, 1\}$.

- *If* $\alpha \geq 0$ *and* $N_{-1} \geq N_{-2}$, $\{T, 1, 0\} \succ_{LRCU} \{T, 0, 1\}$.

- *Otherwise,* $\{T,1,0\} \succeq_{LRCU} (\preceq_{LRCU}) \{T,0,1\}$ *iff:*

$$
\begin{aligned}
& \alpha T^2 \left(\frac{N_{-1} - N_{-2}}{2} \right) \\
\geq \quad & (\leq) TQ^{\sigma} \left[\left(\frac{T(N_{-2}+N_{-1})}{2} \right)^{1-\sigma} - (TN_{-1})^{1-\sigma} \right] \\
& + \frac{T(N_{-2}+N_{-1})}{2} \left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{\delta} + ... + \left(2 \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right) \\
& - TN_{-1} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + ((T-T+1)n^{T-1})^{\delta} \right)
\end{aligned}
$$

Proof. See the Appendix. ■

In comparison to the situation without coexistence concerns, the number of cases where the CU planner prefers early births is unambiguously larger. Indeed, early births are now preferred even when the initial age-structure is uniform across reproductive groups (i.e. $N_{-1} = N_{-2}$). The reason has to do with the larger coexistence possibilities allowed by the early fertility timing, which make early births valuable even when this does not lead to the largest population size in the long-run. However, under $\alpha < 0$ or $N_{-1} < N_{-2}$, it may still be the case, despite the longer coexistence with the dynasty under early births, that late fertility is regarded as socially more desirable than early fertility, on the ground of a higher long-run population size. Hence the CU ranking involves here an arbitrage between total number of persons and how long those persons coexist.

Let us now turn to average utilitarianism.

Proposition 8 *Assume A1-A5, A8-A9 and A11-A12. Suppose an initial age structure* $\{N_{-T}, ..., N_{-1}\}$. *Consider two histories* $\{T,1,0\}$ *and* $\{T,0,1\}$.

- *If* $N_{-1} \leq N_{-2}$, $\{T,1,0\} \succ_{LRAU} \{T,0,1\}$.
- *If* $N_{-1} > N_{-2}$, $\{T,1,0\} \succeq_{LRAU} (\preceq_{LRAU}) \{T,0,1\}$ *iff:*

$$
\begin{aligned}
& (TN_{-1})^{-\sigma} TQ^{\sigma} + T\alpha + (T-1)^{\delta} + (T-2)^{\delta} + ... + (T-T+1)^{\delta} \\
\geq \quad & (\leq) \left(\frac{T(N_{-2}+N_{-1})}{2} \right)^{-\sigma} TQ^{\sigma} + \left[T\alpha + (T-2)^{\delta} + (T-4)^{\delta} + ... + (2)^{\delta} \right]
\end{aligned}
$$

Proof. See the Appendix. ■

A major difference with respect to the benchmark model without coexistence concerns has to do with the fact that even an initial age-structure that involves a high number of potential young mothers, it may still be the case that early fertility is regarded as better. The reason has to do that, even if this leads to a larger long-run population size, the larger congestion that is generated from early births is compensated by the larger period of coexistence with the dynasty, which supports early births. However, if $N_{-1} > N_{-2}$, it may be the case that late births is socially desirable, on the grounds of a lower long-run population, even though this reduces the coexistence between generations.

Finally, let us conclude our study by focusing on critical-level utilitarianism.

Proposition 9 *Assume A1-A5, A8-A9 and A11-A12. Suppose an initial age structure* $\{N_{-T}, ..., N_{-1}\}$. *Consider two histories* $\{T, 1, 0\}$ *and* $\{T, 0, 1\}$.

- *If* $\alpha - \hat{u} \geq 0$ *and* $N_{-1} \geq N_{-2}$, $\{T, 1, 0\} \succ_{LRCLU} \{T, 0, 1\}$.
- *Otherwise,* $\{T, 1, 0\} \succeq_{LRCLU} (\preceq_{LRCLU}) \{T, 0, 1\}$ *iff:*

$$\begin{aligned} & (\alpha - \hat{u})T^2 \left(\frac{N_{-1} - N_{-2}}{2} \right) \\ \geq \quad & (\leq)\, TQ^{\sigma} \left[\left(\frac{T(N_{-2} + N_{-1})}{2} \right)^{1-\sigma} - (TN_{-1})^{1-\sigma} \right] \\ & + \frac{T(N_{-2} + N_{-1})}{2} \left(\left(\frac{(T-2)nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{\delta} + ... + \left(2 \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right) \\ & - TN_{-1} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + ((T-T+1)n^{T-1})^{\delta} \right) \end{aligned}$$

Proof. See the Appendix. ■

Coexistence concerns tend, here again, to support early births, thanks to the larger coexistence time allowed by that fertility timing. It may still be the case that late births are regarded as better than early births, but that case is less plausible than in the absence of coexistence concerns.

Introducing a taste for coexistence with the descendants can affect which birth timing is the most desirable from a social perspective. However, even though coexistence concerns push towards earlier births, the arguments that were developed in the previous sections are still at work. In finite histories, the time at which agents make children is still a major instrument for smoothing the population pattern, and minimize congestion problems. Moreover, in infinite histories, the timing of births is still crucial to affect the long-run population size. Therefore, the initial age-structure of the economy remains an essential determinant of which birth timing is the most appealing.

6 The welfare consequences of a transition

Up to now, our analysis focused on the comparison of two hypothetical societies differing in terms of birth timing. Whereas our analyses cast some light on the social desirability of each alternative society from a utilitarian perspective, one may also be interested in a slightly different kind of comparison, consisting in comparing social welfare *without* a childbearing age transition and social welfare *with* a childbearing age transition. In other words, instead of comparing two hypothetical societies differing in terms of birth timing, we would like now to contrast an economy with early births with an economy in which cohorts suddenly decide to postpone births.

The study of the desirability of a transition does not consist in comparing histories $\{T, n, 0\}$ and $\{T, 0, m\}$ starting at the same time, but it consists in comparing, from the perspective of the well-being of cohorts born at $t \geq 0$, a history $\{T, n, 0\}$ starting at $t = 0$ with a history $\{T, 0, m\}$ starting at $t = 1$, the term "starting" referring to the first period of births. Hence, when denoting the situations without and with the transition, we will use the following notations, to emphasize the difference in the starting points of those histories:

$$\begin{array}{rl} \text{without transition:} & \{T, n, 0\} \text{ starting at } t = 0 \equiv \{T, n, 0\}|_{0\rightarrow} \\ \text{with transition:} & \{T, 0, m\} \text{ starting at } t = 1 \equiv \{T, 0, m\}|_{1\rightarrow} \end{array}$$

The difference between the comparison of histories studied above and the study of that transition can be seen by computing the number of births at each period under the two situations. Without the childbearing age transition, that is, under $\{T, n, 0\}|_{0\rightarrow}$, we have:

$$N_0 = nN_{-1}, N_1 = nN_0, N_2 = nN_1, N_3 = nN_2, ..., N_t = nN_{t-1}$$

Let us now consider the case of a childbearing age transition, that is, $\{T, 0, m\}|_{1\rightarrow}$. When the transition towards late births take place at time $t = 0$, the cohort born at $t = -1$ suddenly decides not to have children at $t = 0$, but to adopt instead a new fertility profile, $(0, m)$, and will thus only have children at $t = 1$, which is the point at which history starts in the transition case. Note that, in that case, no one is born at $t = 0$, and the number of births follows the pattern:

$$N_0 = 0, N_1 = mN_{-1}, N_2 = 0, N_3 = mN_1, N_4 = 0, ..., N_t = mN_{t-2}$$

A major difference with respect to the previous analysis lies in the fact that, once the transition has taken place, there are lots of periods (i.e. one out of two periods) *without* any births. On the contrary, the analysis carried out in the previous sections of the paper took place in a setting where there were always, at each period, some new persons being added to the population. The analysis of the transition requires to introduce periods without births, which makes this situation really hypothetical. However, we will pursue that analysis, since it is relevant for the study of the social desirability of birth postponement.

Let us now examine whether this transition from a fertility profile $(n, 0)$ to another fertility profile $(0, m)$ is welfare-improving or not. This comparison amounts to examine whether social welfare is larger under $\{T, 0, m\}|_{1\rightarrow}$ than under $\{T, n, 0\}|_{0\rightarrow}$. Here again, we will focus on the comparison of histories where an equal total number of life periods are lived, i.e. lifetime-equal histories. The total number of periods lived is, without the transition (i.e. under the history $\{T, n, 0\}|_{0\rightarrow}$), equal to:

$$\frac{TN_{-1}n}{1-n}$$

Under the transition (i.e. under the history $\{T, 0, m\}|_{1\rightarrow}$), the total number of life-periods is equal to:

$$mTN_{-1} + m^2TN_{-1} + m^3TN_{-1} + ... + m^{\infty}TN_{-1} = \frac{TN_{-1}m}{1-m}$$

Hence, the equality in the total number of life periods lived for individuals born beyond $t \geq 0$ in histories $\{T, n, 0\}|_{0\rightarrow}$ and $\{T, 0, m\}|_{1\rightarrow}$ requires:

$$n = m < 1$$

Thus, the constraint that the histories under comparison are lifetime-equal requires here an equality of age-specific fertility rates, unlike in the previous comparisons, where we were comparing histories starting at the same point in time (i.e. with newborn persons at $t = 0$ in each of these). Having clarified this point, we can now study under which case a transition in terms of childbearing age is welfare improving. Proposition 10 summarizes our results.

Proposition 10 *Assume A1-A10. Suppose an initial age structure $\{N_{-T}, ..., N_{-1}\}$. Consider the comparison of a history $\{T, n, 0\}|_{0\rightarrow}$ with a history $\{T, 0, m\}|_{1\rightarrow}$. Suppose that $\{T, n, 0\}|_{0\rightarrow}$ and $\{T, 0, m\}|_{1\rightarrow}$ are lifetime-neutral. Under $\sigma > 0$, we have:*

$$\begin{aligned} \{T, n, 0\}|_{0\rightarrow} &\prec_{CU} \{T, 0, m\}|_{1\rightarrow} \\ \{T, n, 0\}|_{0\rightarrow} &\prec_{AU} \{T, 0, m\}|_{1\rightarrow} \\ \{T, n, 0\}|_{0\rightarrow} &\prec_{CLU} \{T, 0, m\}|_{1\rightarrow} \end{aligned}$$

Proof. See the Appendix. ■

Proposition 10 states that, from a utilitarian perspective, and whatever the precise population ethics criterion is (CU, AU or CLU), the childbearing age transition is welfare improving when space congestion matters. The intuition behind that result goes as follows. By postponing births, the cohort born at $t = -1$ contributes to lower the population size at any posterior period of time. As a consequence, that birth postponement tends to reduce space congestion, and, hence, to increase welfare in all posterior periods. Moreover, given that the two histories $\{T, n, 0\}|_{0\rightarrow}$ and $\{T, 0, m\}|_{1\rightarrow}$ are lifetime-equal, this birth postponement does not prevent the existence of any person. Therefore, the childbearing age transition leads to a Pareto improvement, since this makes all individuals better off, without making any person worst off. In the light of this, it follows mechanically that, whatever the utilitarian criterion we use, the childbearing age transition is socially desirable.

That strong result relies on a particular restriction on individual preferences: the absence of coexistence concerns. As shown by Proposition 11, the introduction of coexistence concerns makes things less clear. It may be the case, when coexistence concerns are sufficiently strong, that the childbearing age transition is not socially desirable.

Proposition 11 *Assume A1-A5, A8-A9 and A11-A12. Suppose an initial age structure* $\{N_{-T},...,N_{-1}\}$. *Consider the comparison of a history* $\{T,n,0\}|_{0\rightarrow}$ *with a history* $\{T,0,m\}|_{1\rightarrow}$. *Suppose that* $\{T,n,0\}|_{0\rightarrow}$ *and* $\{T,0,m\}|_{1\rightarrow}$ *are lifetime-neutral. Suppose* $\sigma > 0$.

- *We have:* $\{T,n,0\}|_{0\rightarrow} \succeq_{CU} (\preceq_{CU}) \{T,0,m\}|_{1\rightarrow}$ *iff*

$$\sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right]$$
$$+\frac{TN_{-1}n}{1-n} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left((T-T+1)n^{T-1}\right)^{\delta}\right)$$
$$\geq \quad (\leq) \sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1,3,5,...}^{s} n^{(r+1)/2}} \right)^{\sigma} \right]$$
$$+\frac{TN_{-1}n}{1-n} \left(((T-2)n)^{\delta} + ... + ((T-T+2)n^{T-2})^{\delta}\right)$$

- *We have also:*

$$\{T,n,0\}|_{0\rightarrow} \succeq_{AU} \{T,0,m\}|_{1\rightarrow} \iff \{T,n,0\}|_{0\rightarrow} \succeq_{CU} \{T,0,m\}|_{1\rightarrow}$$
$$\{T,n,0\}|_{0\rightarrow} \succeq_{CLU} \{T,0,m\}|_{1\rightarrow} \iff \{T,n,0\}|_{0\rightarrow} \succeq_{CU} \{T,0,m\}|_{1\rightarrow}$$

Proof. See the Appendix. ■

Hence, whether the transition towards a postponement of births is socially desirable or not depends on the relative strengths of congestion concerns and coexistence concerns. When coexistence concerns are weak, we are back to Proposition 10, and the transition is socially desirable, since this reduces spacial congestion for each individual. If, on the contrary, coexistence concerns are strong, this may make the childbearing age transition not socially desirable.

7 Conclusions

Our societies are witnessing, since the early 1970s, a postponement of births. Our goal is here to determine whether births postponement is socially desirable or not. For that purpose, we developed a simple dynamic overlapping generations framework where individual welfare depends on the available space per head, and compared, in the light of standard utilitarian criteria, and conditionally on the same initial age structure, histories equal on all dimensions except the fertility timing: either early births only or late births.

What kind of fertility timing is socially desirable, and why? Our answer depends on the assumption that we make regarding the future survival of Mankind. If the human species will, like many other species in the past, become extinct in the long-run, then birth timing affects social welfare through its influence on the intertemporal population pattern. When comparing finite histories with the same total number of life-periods, both CU, AU and CLU select the birth timing that leads to the most smoothed population path. That socially desirable birth timing can involve either early or late fertility, depending on the initial age-structure that prevails within the reproductive age-group. However, if one supposes that there will be a stabilization of the human population in the future, then the birth timing has another function: it can lead to a higher or a lower asymptotic population size, which can be socially desirable or not, depending on the postulated individual and social welfare functions.

Those results were shown to be qualitatively robust to the introduction, among the determinants of human welfare, of a concern for coexisting with the descendants. That coexistence concern clearly supports early births rather than late births, on the grounds of the larger duration of coexistence between generations under early births. Late births have indeed an unambiguous tendency to shorten the period during which successive generations can coexist, and that tendency may be regarded as a major shortcoming. However, the optimal birth timing still depends, within that extended framework, on the fertility level (below the replacement level or not), and, also, on the prevailing age structure.

We also examined the social desirability of a transition from a regime with early births to a regime with late births. We showed that shifting from an early births regime to a late births regime is welfare-improving when individuals exhibit no coexistence concerns, but may not be socially desirable when coexistence concerns are sufficiently strong.

All in all, whether birth postponement is, from a utilitarian perspective, socially desirable or not depends on whether welfare gains related to lower congestion at the social level can dominate (or not) welfare losses from coexisting less time with one's *own* descendants (grandchildren, etc.). If concerns for coexistence with the dynasty are strong, then birth postponement can hardly be maximizing the greatest happiness of the greatest number, whatever the precise way in which we specify the social welfare agregator.

Finally, it should be stressed that this paper exhibits some limitations. First of all, we assumed here that the duration of life is certain and equal to a constant T. This constitutes a significant simplification, in particular when considering coexistence issues. Having stressed this, it remains nonetheless true that introducing varying lifespan would make the comparison of demographic regimes even more complex. We thus leave this for future research. Second, our analyses focused on the comparison of histories in which all individuals have the same fertility pattern (which, of course, varies across histories). It would be quite interesting to consider the more general comparison of histories in which some children are born early, and some other children are born late, that is, "mixed" histories. This task is also left on the research agenda.

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9 Appendix

9.1 Proof of Proposition 1

Consider two lifetime-equal histories $\{T, n, 0\}$ and $\{T, 0, m\}$. In history $\{T, n, 0\}$, total welfare for all individuals born at $t \geq 0$ can be written as:

$$
\begin{aligned}
& nN_{-1}T\alpha + nN_{-1}\left[\frac{Q}{N_{-(T-1)} + ... + N_{-1}(1+n)}\right]^{\sigma} \\
& + nN_{-1}\left[\frac{Q}{N_{-(T-2)} + ... + N_{-1}(1+n+n^2)}\right]^{\sigma} \\
& + ... + nN_{-1}\left[\frac{Q}{N_{-1}\left(n+n^2+...+n^{T+1}\right)}\right]^{\sigma} \\
& + n^2N_{-1}T\alpha + n^2N_{-1}\left[\frac{Q}{N_{-(T-2)} + ... + N_{-1}(1+n+n^2)}\right]^{\sigma} \\
& + n^2N_{-1}\left[\frac{Q}{N_{-(T-3)} + ... + N_{-1}\left(1+n+n^2+n^3\right)}\right]^{\sigma} \\
& + ... + n^2N_{-1}\left[\frac{Q}{N_{-1}\left(n^2+n^3+...+n^{T+2}\right)}\right]^{\sigma} \\
& + ...
\end{aligned}
$$

This can be simplified to:

$$\sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-1} N_{-(T-s-z)}} \right)^{\sigma} \right]$$

$$= \sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right]$$

Let us suppose, for the sake of presentation, that T is an even number. Then, in history $\{T, 0, m\}$, total welfare can be rewritten as:

$$mN_{-2}T\alpha + mN_{-2} \left[\frac{Q}{N_{-(T-1)} + ... + mN_{-2}} \right]^{\sigma} + mN_{-2} \left[\frac{Q}{N_{-(T-2)} + ... + mN_{-2} + mN_{-1}} \right]^{\sigma}$$
$$+mN_{-2} \left[\frac{Q}{N_{-(T-3)} + ... + N_{-2}(m+m^2) + mN_{-1}} \right]^{\sigma}$$
$$+... + mN_{-2} \left[\frac{Q}{N_{-2}\left(m+m^2+...+m^{T/2}\right) + N_{-1}\left(m+m^2+...+m^{T/2}\right)} \right]^{\sigma}$$
$$+mN_{-1}T\alpha + mN_{-1} \left[\frac{Q}{N_{-(T-2)} + ... + mN_{-1}} \right]^{\sigma} + mN_{-1} \left[\frac{Q}{N_{-(T-3)} + ... + N_{-2}(m+m^2) + mN_{-1}} \right]^{\sigma}$$
$$+mN_{-1} \left[\frac{Q}{N_{-(T-4)} + ... + N_{-2}(m+m^2) + N_{-1}(m+m^2)} \right]^{\sigma}$$
$$+... + mN_{-1} \left[\frac{Q}{N_{-2}\left(m^2+m^3+...+m^{T/2}\right) + ... + N_{-1}\left(m+m^2+...+m^{(T/2)+1}\right)} \right]^{\sigma}$$
$$+m^2N_{-2}T\alpha + m^2N_{-2} \left[\frac{Q}{N_{-(T-3)} + ... + N_{-2}(m+m^2) + mN_{-1}} \right]^{\sigma}$$
$$+m^2N_{-2} \left[\frac{Q}{N_{-(T-4)} + ... + N_{-2}(m+m^2) + N_{-1}(m+m^2)} \right]^{\sigma}$$
$$+m^2N_{-2} \left[\frac{Q}{N_{-(T-5)} + ... + N_{-2}(m+m^2+m^3) + N_{-1}(m+m^2)} \right]^{\sigma}$$
$$+... + m^2N_{-2} \left[\frac{Q}{N_{-2}\left(m^2+...+m^{(T/2)+1}\right) + N_{-1}\left(m^2+...+m^{(T/2)+1}\right)} \right]^{\sigma}$$
$$+...$$

This expression can be rewritten as:

$$\begin{aligned}
& \sum_{t=1}^{\infty} m^t N_{-2} T\alpha + \sum_{t=1}^{\infty} m^t N_{-1} T\alpha \\
& + \sum_{t=1}^{\infty} m^t N_{-2} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-1} N_{-(T-s-z)}} \right)^{\sigma} \right] + \sum_{t=1}^{\infty} m^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-1} N_{-(T-s-z)}} \right)^{\sigma} \right] \\
= & \sum_{t=1}^{\infty} m^t N_{-2} T\alpha + \sum_{t=1}^{\infty} m^t N_{-1} T\alpha + \sum_{t=1}^{\infty} m^t N_{-2} \left[\sum_{s=t}^{t+T-1} (\hat{q}_s)^{\sigma} \right] + \sum_{t=1}^{\infty} m^t N_{-1} \left[\sum_{s=t}^{t+T-1} (\check{q}_s)^{\sigma} \right]
\end{aligned}$$

where

$$\begin{aligned}
\hat{q}_s & \equiv Q / \left[\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + \sum_{r=1,3,5,\dots}^{s} (N_{-2})\, m^{(r+1)/2} + \sum_{z=2,4,6,\dots}^{s} (N_{-1})\, m^{z/2} \right] \\
\check{q}_s & \equiv Q / \left[\sum_{z=0}^{T-s-1} N_{-(T-s-z+1)} + mN_{-1} + \sum_{r=1,3,5,\dots}^{s} (N_{-2})\, m^{(r+1)/2} + \sum_{z=2,4,6,\dots}^{s} (N_{-1})\, m^{z/2} \right]
\end{aligned}$$

When the two histories are lifetime equal, we have: $m = n \frac{N_{-1}}{N_{-1}+N_{-2}(1-n)}$. Total well-being in the second history can now be written as:

$$\begin{aligned}
= & \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-2} T\alpha + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-1} T\alpha \\
& + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-2} \left[\sum_{s=t}^{t+T-1} \hat{q}_s^{\sigma} \right] + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-1} \left[\sum_{s=t}^{t+T-1} \check{q}_s^{\sigma} \right]
\end{aligned}$$

Hence social welfare is larger in history $\{T, n, 0\}$ than in history $\{T, 0, m\}$ if and only if:

$$\begin{aligned}
& \sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right] \\
\gtrless & \\
& \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-2} T\alpha + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-1} T\alpha \\
& + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-2} \left[\sum_{s=t}^{t+T-1} \hat{q}_s^{\sigma} \right] + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-1} \left[\sum_{s=t}^{t+T-1} \check{q}_s^{\sigma} \right]
\end{aligned}$$

That expression can be simplified as follows. Note that $\sum_{t=1}^{\infty} n^t N_{-1} T\alpha = N_{-1}T\alpha\left(\frac{1}{1-n}-1\right) = \frac{nN_{-1}T\alpha}{1-n}$ and that $\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t T\alpha\left(N_{-2}+N_{-1}\right) = T\alpha\left(N_{-2}+N_{-1}\right)\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t = \frac{T\alpha nN_{-1}}{(1-n)}$. Hence we have:

$$\sum_{t=1}^{\infty} n^t N_{-1} T\alpha = \sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-2}T\alpha + \sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-1}T\alpha$$

and the condition thus becomes:

$$\begin{aligned}
&\sum_{t=1}^{\infty} n^t N_{-1}\left[\sum_{s=t}^{t+T-1}\left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1})\sum_{r=1}^{s} n^r}\right)^{\sigma}\right] \\
\gtrless\; & \\
&\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-2}\left[\sum_{s=t}^{t+T-1}\hat{q}_s^{\sigma}\right] + \sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-1}\left[\sum_{s=t}^{t+T-1}\check{q}_s^{\sigma}\right]
\end{aligned}$$

9.2 Proof of Corollary 1.

Take the special case where space does not matter: $\sigma = 0$. The condition becomes:

$$\begin{aligned}
&\sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1} T \\
\gtrless\; & \\
&\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-2}T\alpha + \sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-1}T\alpha \\
&+\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-2}T + \sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-1}T
\end{aligned}$$

That condition can be simplified as:

$$\begin{aligned}
&\sum_{t=1}^{\infty} n^t N_{-1} \gtrless \left(N_{-2}+N_{-1}\right)\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t \\
\iff\quad & N_{-1}\left(\frac{n}{1-n}\right) \gtrless \left(N_{-2}+N_{-1}\right)\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)-nN_{-1}}\right) \\
\iff\quad & \frac{1}{1-n} \gtrless \frac{N_{-2}+N_{-1}}{\left(N_{-1}+N_{-2}\right)(1-n)}
\end{aligned}$$

That condition is always valid. Hence, independently from initial conditions, if space congestion does not matter, histories $\{T, n, 0\}$ and $\{T, 0, m\}$ bring the same total welfare. This is not surprising, since these two lotteries are, by construction, lifetime-equal, meaning that these yield to exactly the same number of life periods, and, hence, in the absence of concern for congestion, this makes the two lotteries equally good.

9.3 Proof of Proposition 2

Consider first the case of average utilitarianism. Assuming that the two histories are lifetime equal, the total number of individuals born at $t \geq 0$ in history $\{T, n, 0\}$ and in history $\{T, 0, m\}$ is: $\frac{TnN_{-1}}{1-n}$. In the light of this, the average total welfare in history $\{T, n, 0\}$ is:

$$\frac{\sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right]}{\frac{TnN_{-1}}{1-n}}$$

whereas the average total welfare in history $\{T, 0, m\}$ is:

$$\frac{\sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-2} T\alpha + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-1} T\alpha}{\frac{TnN_{-1}}{1-n}}$$

$$+ \sum_{t=1}^{\infty} \frac{\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-2}}{\frac{TnN_{-1}}{1-n}} \left[\sum_{s=t}^{t+T-1} \hat{q}_s^{\sigma} \right] + \sum_{t=1}^{\infty} \frac{\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^t N_{-1}}{\frac{TnN_{-1}}{1-n}} \left[\sum_{s=t}^{t+T-1} \check{q}_s^{\sigma} \right]$$

Given that $\frac{TnN_{-1}}{1-n}$ divides both the LHS and the RHS of that condition, it is straightforward to see that average welfare for individuals born at $t \geq 0$ is larger under history $\{T, n, 0\}$ than in history $\{T, 0, m\}$ if and only if the same condition as under CU is satisfied. The same rationale can be used to show that CLU yields exactly the same ranking as CU as far as the comparison of lifetime equal histories is concerned.

9.4 Proof of Lemma 2

Let us compute the total long-run population size under the two histories $\{T, n, 0\}$ and $\{T, 0, m\}$. In general, the total population follows the dynamics:

$$\begin{aligned}
L_0 &= N_{-(T-1)} + N_{-(T-2)} + ... + N_{-2} + N_{-1} + N_0 \\
L_1 &= N_{-(T-2)} + N_{-(T-3)} + ... + N_{-1} + N_0 + N_1 \\
L_2 &= N_{-(T-3)} + N_{-(T-4)} + ... + N_0 + N_1 + N_2 \\
&\quad ... \\
L_{T-2} &= N_{-1} + N_0 + N_1 + ... + N_{T-2} \\
L_{T-1} &= N_0 + N_1 + N_2 + ... + N_{T-1} \\
&\quad ... \\
L_t &= N_{t-T+1} + ... + N_{t-1} + N_t
\end{aligned}$$

Under the history $\{T, 1, 0\}$, that evolution takes the form:

$$\begin{aligned}
L_0 &= N_{-(T-1)} + N_{-(T-2)} + ... + N_{-2} + N_{-1} + N_{-1} \\
L_1 &= N_{-(T-2)} + N_{-(T-3)} + ... + 3N_{-1} \\
L_2 &= N_{-(T-3)} + N_{-(T-4)} + ... + 4N_{-1} \\
&\quad ... \\
L_{T-2} &= TN_{-1} \\
L_t &= TN_{-1} \text{ for } t \geq T-2
\end{aligned}$$

Under the history $\{T, 0, 1\}$, that evolution takes the form (we assume T is an even number):

$$\begin{aligned}
L_0 &= N_{-(T-1)} + N_{-(T-2)} + ... + N_{-2} + N_{-1} + N_{-2} \\
L_1 &= N_{-(T-2)} + N_{-(T-3)} + ... + N_{-1} + N_{-2} + N_{-1} \\
L_2 &= N_{-(T-3)} + N_{-(T-4)} + ... + N_{-1} + N_{-2} + N_{-1} + N_{-2} \\
&\quad ... \\
L_{T-2} &= N_{-1} + N_{-2} + N_{-1} + ... + N_{-2} \\
L_{T-1} &= N_{-2} + N_{-1} + N_{-2} + ... + N_{-1} \\
&\quad ... \\
L_t &= \frac{T}{2}N_{-2} + \frac{T}{2}N_{-1} = \frac{TN_{-2}(1 + \frac{N_{-1}}{N_{-2}})}{2}
\end{aligned}$$

Hence, the asymptotic population size under $\{T, 1, 0\}$ and $\{T, 0, 1\}$ are ranked according to:

$$TN_{-1} \gtreqless \frac{TN_{-2}(1 + \frac{N_{-1}}{N_{-2}})}{2} \iff N_{-1} \gtreqless N_{-2}$$

Hence, if $N_{-1} > N_{-2}$, the asymptotic population under $\{T, 1, 0\}$ exceeds the one under $\{T, 0, 1\}$. If $N_{-1} = N_{-2}$, the asymptotic population under $\{T, 1, 0\}$ equals the one under $\{T, 0, 1\}$. If , $N_{-1} < N_{-2}$, the asymptotic population under $\{T, 1, 0\}$ is smaller than the one under $\{T, 0, 1\}$.

9.5 Proof of Proposition 3

Social welfare at the stationary equilibrium is equal to (abstracting from time indexes):

$$LTu(q) = LTu\left(\frac{Q}{L}\right) = LT\left(\frac{Q}{L}\right)^{\sigma} + LT\alpha = L^{1-\sigma}TQ^{\sigma} + LT\alpha$$

where L denotes the asymptotic population size, and q the asymptotic space per head.

Under history $\{T, 1, 0\}$, that formula becomes:

$$\begin{aligned} TN_{-1}Tu(q) &= T^2 N_{-1}\left[\alpha + \left(\frac{Q}{TN_{-1}}\right)^{\sigma}\right] \\ &= T^2 N_{-1}\alpha + T^{2-\sigma}\left(N_{-1}\right)^{1-\sigma} Q^{\sigma} \end{aligned}$$

Under the history $\{T, 0, 1\}$, that formula becomes:

$$\begin{aligned} LTu(q) &= \frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2} T\left[\alpha + \left(\frac{Q}{\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}}\right)^{\sigma}\right] \\ &= \frac{T^2 N_{-2}(1+\frac{N_{-1}}{N_{-2}})\alpha}{2} + \left(\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}\right)^{1-\sigma} TQ^{\sigma} \end{aligned}$$

Hence the ranking of the CU planner depends on:

$$T^2 N_{-1}\alpha + T^{2-\sigma}\left(N_{-1}\right)^{1-\sigma} Q^{\sigma} \gtreqless \frac{T^2 N_{-2}(1+\frac{N_{-1}}{N_{-2}})\alpha}{2} + \left(\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}\right)^{1-\sigma} TQ^{\sigma}$$

That expression can be written as:

$$\begin{aligned} T^2\alpha\left(\frac{N_{-1}-N_{-2}}{2}\right) &\gtreqless Q^{\sigma}T^{2-\sigma}\left[\left(\frac{(N_{-2}+N_{-1})}{2}\right)^{1-\sigma} - \left(N_{-1}\right)^{1-\sigma}\right] \\ &\iff \alpha\left(\frac{N_{-1}-N_{-2}}{2}\right) \gtreqless \left(\frac{Q}{T}\right)^{\sigma}\left[\left(\frac{(N_{-2}+N_{-1})}{2}\right)^{1-\sigma} - \left(N_{-1}\right)^{1-\sigma}\right] \end{aligned}$$

If $N_{-1} = N_{-2}$, the LHS and RHS are equal to 0, so that indifference holds. If $N_{-1} < N_{-2}$, and $\alpha > 0$, the LHS is negative, while the RHS is positive (given $\sigma \leq 1$). Hence the history $\{T, 0, 1\}$ is better. If $N_{-1} > N_{-2}$, and $\alpha > 0$, the LHS is positive, while the RHS is negative (given $\sigma \leq 1$). Hence the history $\{T, 1, 0\}$ is better. If $N_{-1} < N_{-2}$, and $\alpha < 0$, the LHS is positive, while the RHS is positive (given $\sigma \leq 1$), so that the ranking depends on $\alpha\left(\frac{N_{-1}-N_{-2}}{2}\right) \gtreqless \left(\frac{Q}{T}\right)^{\sigma}\left[\left(\frac{(N_{-2}+N_{-1})}{2}\right)^{1-\sigma} - \left(N_{-1}\right)^{1-\sigma}\right]$. If $N_{-1} > N_{-2}$, and $\alpha < 0$, the LHS is negative, while the RHS is negative (given $\sigma \leq 1$), the same indeterminacy prevails.

9.6 Proof of Proposition 4

Under $\{T, 1, 0\}$, average social welfare is:

$$Tu(q) = T\left(\frac{Q}{TN_{-1}}\right)^{\sigma} + T\alpha$$

Under $\{T, 0, 1\}$, L equals $\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}$, so that average social welfare is:

$$Tu(q) = T\left(\frac{Q}{\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}}\right)^{\sigma} + T\alpha$$

Hence the ranking between $\{T, 1, 0\}$ and $\{T, 0, 1\}$ depends on:

$$T\left(\frac{Q}{TN_{-1}}\right)^{\sigma} + T\alpha \gtrless T\left(\frac{Q}{\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}}\right)^{\sigma} + T\alpha \iff N_{-1} \lesseqgtr N_{-2}$$

Simplifications yield: $N_{-1} \lesseqgtr N_{-2}$. Hence if $N_{-1} > N_{-2}$, $\{T, 0, 1\}$ is preferred over $\{T, 1, 0\}$. If $N_{-1} < N_{-2}$, $\{T, 1, 0\}$ is preferred over $\{T, 0, 1\}$. Indifference holds under $N_{-1} = N_{-2}$.

9.7 Proof of Proposition 5

Under history $\{T, 1, 0\}$, social welfare under CLU at the stationary equilibrium is equal to (abstracting from time indexes):

$$\begin{aligned} LT\left[u(q) - \hat{u}\right] &= TN_{-1}T\left[u(q) - \hat{u}\right] \\ &= TN_{-1}T\left(\frac{Q}{TN_{-1}}\right)^{\sigma} + TN_{-1}T\left(\alpha - \hat{u}\right) \\ &= T^{2-\sigma}\left(N_{-1}\right)^{1-\sigma} Q^{\sigma} + T^{2}N_{-1}\left(\alpha - \hat{u}\right) \end{aligned}$$

Under history $\{T, 0, 1\}$, that formula becomes:

$$LT\left[u(q) - \hat{u}\right] = \left(\frac{N_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}\right)^{1-\sigma} T^{2-\sigma}Q^{\sigma} + \frac{N_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}T^{2}\left(\alpha - \hat{u}\right)$$

Hence the ranking of the CLU planner depends on:

$$T^{2-\sigma}\left(N_{-1}\right)^{1-\sigma} Q^{\sigma} + T^{2}N_{-1}\left(\alpha - \hat{u}\right) \gtrless \left(\frac{N_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}\right)^{1-\sigma} T^{2-\sigma}Q^{\sigma} + \frac{N_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}T^{2}\left(\alpha - \hat{u}\right)$$

After simplifications, that expression becomes:

$$\left(\alpha - \hat{u}\right)\left[\frac{N_{-1} - N_{-2}}{2}\right] \gtrless \left(\frac{Q}{T}\right)^{\sigma}\left[\left(\frac{N_{-2} + N_{-1}}{2}\right)^{1-\sigma} - \left(N_{-1}\right)^{1-\sigma}\right]$$

If $N_{-1} = N_{-2}$, the LHS and RHS are equal to 0, so that indifference holds. If $N_{-1} < N_{-2}$, and $\alpha - \hat{u} > 0$, the LHS is negative, while the RHS is positive (given $\sigma \leq 1$). Hence the history $\{T, 0, 1\}$ is better. If $N_{-1} > N_{-2}$, and $\alpha - \hat{u} > 0$, the LHS is positive, while the RHS is negative (given $\sigma \leq 1$). Hence the history $\{T, 1, 0\}$ is better. If $N_{-1} < N_{-2}$, and $\alpha - \hat{u} < 0$, the LHS is positive, while the RHS is positive (given $\sigma \leq 1$), so that the ranking depends on $(\alpha - \hat{u}) \left[\frac{N_{-1} - N_{-2}}{2}\right] \gtrless \left(\frac{Q}{T}\right)^{\sigma} \left[\left(\frac{N_{-2}+N_{-1}}{2}\right)^{1-\sigma} - (N_{-1})^{1-\sigma}\right]$. If $N_{-1} > N_{-2}$, and $\alpha - \hat{u} < 0$, the LHS is negative, while the RHS is negative (given $\sigma \leq 1$), the same indeterminacy prevails.

9.8 Proof of Proposition 6

Under history $\{T, n, 0\}$, the cumulated social welfare for individuals born at $t \geq 0$ is:

$$
\begin{aligned}
& nN_{-1}T\alpha + nN_{-1}\left[\frac{Q}{N_{-(T-1)} + ... + nN_{-1}}\right]^{\sigma} + nN_{-1}\left[\frac{Q}{N_{-(T-2)} + ... + n^2N_{-1}}\right]^{\sigma} \\
& + ... + nN_{-1}\left[\frac{Q}{nN_{-1} + n^2N_{-1} + ... + n^{T+1}N_{-1}}\right]^{\sigma} + nN_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left(n^{T-1}\right)^{\delta}\right) \\
& + n^2N_{-1}T\alpha + n^2N_{-1}\left[\frac{Q}{N_{-(T-2)} + ... + nN_{-1} + n^2N_{-1}}\right]^{\sigma} + n^2N_{-1}\left[\frac{Q}{N_{-(T-3)} + ... + n^3N_{-1}}\right]^{\sigma} \\
& + ... + n^2N_{-1}\left[\frac{Q}{n^2N_{-1} + n^3N_{-1} + ... + n^{T+2}N_{-1}}\right]^{\sigma} + n^2N_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left(n^{T-1}\right)^{\delta}\right) \\
& + n^3N_{-1}T\alpha + n^3N_{-1}\left[\frac{Q}{N_{-(T-3)} + ... + n^2N_{-1} + n^3N_{-1}}\right]^{\sigma} + n^3N_{-1}\left[\frac{Q}{N_{-(T-4)} + ... + n^4N_{-1}}\right]^{\sigma} \\
& + ... + n^3N_{-1}\left[\frac{Q}{n^3N_{-1} + n^4N_{-1} + ... + n^{T+3}N_{-1}}\right]^{\sigma} + n^3N_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left(n^{T-1}\right)^{\delta}\right) \\
& + ...
\end{aligned}
$$

This can be rewritten as:

$$
\begin{aligned}
& \frac{nN_{-1}T\alpha}{1-n} + \sum_{t=1}^{\infty} n^t N_{-1}\left[\sum_{s=t}^{t+T-1}\left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1})\sum_{r=1}^{s} n^r}\right)^{\sigma}\right] \\
& + \frac{nN_{-1}}{1-n}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left((T-T+1)n^{T-1}\right)^{\delta}\right)
\end{aligned}
$$

Under history $\{T, 0, m\}$, the cumulated social welfare for individuals born at $t \geq 0$ is:

$$
\begin{aligned}
& mN_{-2}T\alpha + mN_{-2}\left[\frac{Q}{N_{-(T-1)} + ... + N_{-1} + mN_{-2}}\right]^{\sigma} \\
& + mN_{-2}\left[\frac{Q}{N_{-(T-2)} + ... + mN_{-2} + mN_{-1}}\right]^{\sigma} \\
& + mN_{-2}\left[\frac{Q}{N_{-(T-3)} + ... + N_{-2}\left(m + m^{2}\right) + mN_{-1}}\right]^{\sigma} \\
& + ... + mN_{-2}\left[\frac{Q}{N_{-2}\left(m + m^{2} + ... + m^{T/2}\right) + N_{-1}\left(m + m^{2} + ... + m^{T/2}\right)}\right]^{\sigma} \\
& + mN_{-2}\left(((T-2)m)^{\delta} + ((T-4)m^{2})^{\delta} + ... + \left((T-T+2)m^{T-2}\right)^{\delta}\right) \\
& + mN_{-1}T\alpha + mN_{-1}\left[\frac{Q}{N_{-(T-2)} + ... + mN_{-2} + mN_{-1}}\right]^{\sigma} \\
& + mN_{-1}\left[\frac{Q}{N_{-(T-3)} + ... + N_{-2}\left(m + m^{2}\right) + mN_{-1}}\right]^{\sigma} \\
& + mN_{-1}\left[\frac{Q}{N_{-(T-4)} + ... + N_{-2}\left(m + m^{2}\right) + N_{-1}\left(m + m^{2}\right)}\right]^{\sigma} \\
& + ... + mN_{-1}\left[\frac{Q}{N_{-2}\left(m^{2} + m^{3} + ... + m^{T/2}\right) + ... + N_{-1}\left(m + m^{2} + ... + m^{(T/2)+1}\right)}\right]^{\sigma} \\
& + mN_{-1}\left(((T-2)m)^{\delta} + ((T-4)m^{2})^{\delta} + ... + \left((T-T+2)m^{T-2}\right)^{\delta}\right) \\
& + m^{2}N_{-2}T\alpha + m^{2}N_{-2}\left[\frac{Q}{N_{-(T-3)} + ... + N_{-2}\left(m + m^{2}\right) + mN_{-1}}\right]^{\sigma} \\
& + m^{2}N_{-2}\left[\frac{Q}{N_{-(T-4)} + ... + N_{-2}\left(m + m^{2}\right) + N_{-1}\left(m + m^{2}\right)}\right]^{\sigma} \\
& + m^{2}N_{-2}\left[\frac{Q}{N_{-(T-5)} + ... + N_{-2}\left(m + m^{2} + m^{3}\right) + N_{-1}\left(m + m^{2}\right)}\right]^{\sigma} \\
& + ... + m^{2}N_{-2}\left[\frac{Q}{N_{-2}\left(m^{2} + m^{3} + ... + m^{(T/2)+1}\right) + N_{-1}\left(m^{2} + m^{3} + ... + m^{(T/2)+1}\right)}\right]^{\sigma} \\
& + m^{2}N_{-2}\left(((T-2)m)^{\delta} + ((T-4)m^{2})^{\delta} + ... + \left((T-T+2)m^{T-2}\right)^{\delta}\right) \\
& + ...
\end{aligned}
$$

Substituting for m in the context of equal lifetime histories, that expression can be rewritten as:

$$\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}N_{-2}T\alpha+\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}N_{-1}T\alpha$$
$$+\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}N_{-2}\left[\sum_{s=t}^{t+T-1}\hat{q}_s^{\sigma}\right]+\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}N_{-1}\left[\sum_{s=t}^{t+T-1}\check{q}_s^{\sigma}\right]$$
$$+\sum_{t=1}^{\infty}N_{-2}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}\left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{\delta}+...+\left(2\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right)$$
$$+\sum_{t=1}^{\infty}N_{-1}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}\left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{\delta}+...+\left(2\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right)$$

That expression can be rewritten as:

$$\frac{T\alpha nN_{-1}}{(1-n)}$$
$$+\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}N_{-2}\left[\sum_{s=t}^{t+T-1}\hat{q}_s^{\sigma}\right]+\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}N_{-1}\left[\sum_{s=t}^{t+T-1}\check{q}_s^{\sigma}\right]$$
$$+\left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{\delta}+...+\left((T-T+2)\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right)\frac{nN_{-1}}{(1-n)}$$

Hence history $\{T, n, 0\}$ has a larger or a lower social welfare than history $\{T, 0, m\}$ if and only if:

$$\frac{nN_{-1}T\alpha}{1-n}+\sum_{t=1}^{\infty}n^{t}N_{-1}\left[\sum_{s=t}^{t+T-1}\left(\frac{Q}{\sum_{z=0}^{T-s-1}N_{-(T-s-z)}+(N_{-1})\sum_{r=1}^{s}n^{r}}\right)^{\sigma}\right]$$
$$+\frac{nN_{-1}}{1-n}\left(((T-1)n)^{\delta}+((T-2)n^{2})^{\delta}+...+((T-T+1)n^{T-1})^{\delta}\right)$$
$$\gtrless\quad\frac{nN_{-1}T\alpha}{(1-n)}+\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}N_{-2}\left[\sum_{s=t}^{t+T-1}\hat{q}_s^{\sigma}\right]+\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{t}N_{-1}\left[\sum_{s=t}^{t+T-1}\check{q}_s^{\sigma}\right]$$
$$\left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{\delta}+...+\left((T-T+2)\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right)\frac{nN_{-1}}{(1-n)}$$

That inequality can be rewritten as:

$$\sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right]$$
$$+ \frac{nN_{-1}}{1-n} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left((T-T+1)n^{T-1}\right)^{\delta} \right)$$
$$\gtrless \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^t N_{-2} \left[\sum_{s=t}^{t+T-1} \hat{q}_s^{\sigma} \right] + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^t N_{-1} \left[\sum_{s=t}^{t+T-1} \check{q}_s^{\sigma} \right]$$
$$+ \left(\left(\frac{(T-2)nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{\delta} + ... + \left((T-T+2) \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right) \frac{nN_{-1}}{1-n}$$

9.9 Proof of Corollary 2

Fixing $\sigma = 0$ in the condition:

$$\sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right]$$
$$+ \frac{nN_{-1} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left((T-T+1)n^{T-1}\right)^{\delta} \right)}{1-n}$$
$$\gtrless \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^t N_{-2} \left[\sum_{s=t}^{t+T-1} \hat{q}_s^{\sigma} \right] + \sum_{t=1}^{\infty} \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^t N_{-1} \left[\sum_{s=t}^{t+T-1} \check{q}_s^{\sigma} \right]$$
$$\left(\left(\frac{(T-2)nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{\delta} + ... + \left((T-T+2) \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right) \frac{nN_{-1}}{1-n}$$

yields:

$$\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left((T-T+1)n^{T-1}\right)^{\delta} \right)$$
$$\gtrless \left(\frac{(T-2)nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{\delta} + ... + \left((T-T+2) \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{T-2} \right)^{\delta}$$

The LHS is unambiguously larger than the RHS, since $m < n$ and coexistence time is necessarily reduced by birth postponement. Hence the history $\{T, n, 0\}$ is preferred over $\{T, 0, m\}$.

9.10 Proof of Corollary 3

Under AU, the condition for preferring $\{T, n, 0\}$ over $\{T, 0, m\}$ becomes:the average total welfare in history $\{T, n, 0\}$ is:

$$\frac{\left[\begin{array}{l}\sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1}\left[\sum_{s=t}^{t+T-1}\left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1})\sum_{r=1}^{s} n^r}\right)^{\sigma}\right] \\ + \frac{nN_{-1}\left(((T-1)n)^{\delta}+...+((T-T+1)n^{T-1})^{\delta}\right)}{1-n}\end{array}\right]}{\frac{TnN_{-1}}{1-n}}$$

$$\gtrless \frac{\sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-2}T\alpha + \sum_{t=1}^{\infty}\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-1}T\alpha}{\frac{TnN_{-1}}{1-n}}$$

$$+\frac{\left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{\delta} + ... + \left((T-T+2)\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right)\frac{nN_{-1}}{(1-n)}}{\frac{TnN_{-1}}{1-n}}$$

$$+\sum_{t=1}^{\infty}\frac{\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-2}}{\frac{TnN_{-1}}{1-n}}\left[\sum_{s=t}^{t+T-1}\hat{q}_s^{\sigma}\right] + \sum_{t=1}^{\infty}\frac{\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^t N_{-1}}{\frac{TnN_{-1}}{1-n}}\left[\sum_{s=t}^{t+T-1}\check{q}_s^{\sigma}\right]$$

Simplifying by the total number of births leads to the same condition as in Proposition 6. The same kind of argument holds for CLU.

9.11 Proof of Proposition 7

Under history $\{T, 1, 0\}$, social welfare at the stationary equilibrium is equal to (abstracting from time indexes):

$$\begin{aligned} LTu(q) &= TN_{-1}Tu\left(\frac{Q}{L}\right) \\ &= TN_{-1}T\left(\frac{Q}{L}\right)^{\sigma} + TN_{-1}T\alpha + TN_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + (1n^{T-1})^{\delta}\right) \\ &= (TN_{-1})^{1-\sigma} TQ^{\sigma} + TN_{-1}T\alpha + TN_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + (1n^{T-1})^{\delta}\right) \end{aligned}$$

Under the history $\{T, 0, 1\}$, that formula is:

$$\begin{aligned} LTu(q) &= \frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2} T \left[\alpha + \left(\frac{Q}{\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}} \right)^{\sigma} \right] \\ &\quad + \frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2} \left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{\delta} + ... + \left(2 \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right) \\ &= \frac{T^2 N_{-2}(1+\frac{N_{-1}}{N_{-2}})\alpha}{2} + \left(\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2} \right)^{1-\sigma} TQ^{\sigma} \\ &\quad + \frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2} \left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{\delta} + ... + \left(2 \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right) \end{aligned}$$

Hence the ranking of the CU planner depends on:

$$\begin{aligned} & (TN_{-1})^{1-\sigma} TQ^{\sigma} + TN_{-1}T\alpha + TN_{-1} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left((T-T+1)n^{T-1}\right)^{\delta} \right) \\ \gtrless & \frac{T^2 N_{-2}(1+\frac{N_{-1}}{N_{-2}})\alpha}{2} + \left(\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2} \right)^{1-\sigma} TQ^{\sigma} \\ & + \frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2} \left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{\delta} + ... + \left(2 \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right) \end{aligned}$$

That expression can be written as:

$$\begin{aligned} & T^2 \alpha \left(\frac{N_{-1} - N_{-2}}{2} \right) \\ \gtrless & Q^{\sigma} T^{2-\sigma} \left[\left(\frac{(N_{-2}+N_{-1})}{2} \right)^{1-\sigma} - (N_{-1})^{1-\sigma} \right] \\ & + \frac{T(N_{-2}+N_{-1})}{2} \left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{\delta} + ... + \left(2 \left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right) \\ & - TN_{-1} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left((T-T+1)n^{T-1}\right)^{\delta} \right) \end{aligned}$$

If $N_{-2} = N_{-1}$ and $\alpha \geq 0$, the LHS is equal to zero, while the RHS is negative, so that history $\{T, 1, 0\}$ is better. If $N_{-1} > N_{-2}$, the LHS is positive, while the RHS is, under $\alpha \geq 0$, negative, so that history $\{T, 1, 0\}$ is better. If $N_{-1} < N_{-2}$ and $\alpha \geq 0$, the LHS is negative, and the RHS is undetermined. Under $\alpha < 0$, the sign of the RHS is undetermined. Hence whether history $\{T, 1, 0\}$ is preferred depends on whether the above condition holds, which depends on how large α is with respect to T.

9.12 Proof of Proposition 8

Average welfare in the long-run population is, under history $\{T,1,0\}$, to (abstracting from time indexes):

$$\begin{aligned} Tu(q) &= T\left(\frac{Q}{L}\right)^{\sigma} + T\alpha + \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left(1n^{T-1}\right)^{\delta}\right) \\ &= (TN_{-1})^{-\sigma}\, TQ^{\sigma} + T\alpha + (T-1)^{\delta} + (T-2)^{\delta} + ... + (1)^{\delta} \end{aligned}$$

whereas it is equal, under history $\{T,0,1\}$, to:

$$\begin{aligned} Tu(q) &= T\left(\frac{Q}{L}\right)^{\sigma} + T\alpha + \left(((T-2)m)^{\delta} + ((T-4)m^2)^{\delta} + ... + \left(2m^{T-2}\right)^{\delta}\right) \\ &= \left(\frac{T(N_{-2}+N_{-1})}{2}\right)^{-\sigma} TQ^{\sigma} + \left[T\alpha + (T-2)^{\delta} + (T-4)^{\delta} + ... + (2)^{\delta}\right] \end{aligned}$$

Hence the ranking of the AU planner depends on:

$$\begin{aligned} & (TN_{-1})^{-\sigma}\, TQ^{\sigma} + T\alpha + (T-1)^{\delta} + (T-2)^{\delta} + ... + (1)^{\delta} \\ \gtrless\; & \left(\frac{T(N_{-2}+N_{-1})}{2}\right)^{-\sigma} TQ^{\sigma} + \left[T\alpha + (T-2)^{\delta} + (T-4)^{\delta} + ... + (2)^{\delta}\right] \end{aligned}$$

When $N_{-2} = N_{-1}$, the first terms of the LHS and RHS are equal, so that the LHS exceeds the RHS. Hence history $\{T,1,0\}$ is better. If $N_{-2} > N_{-1}$, the LHS is also larger than the RHS, so that history $\{T,1,0\}$ is better. However, when $N_{-2} < N_{-1}$, the first term of the RHS exceeds the first term of the LHS, but the second term of the RHS is smaller than the second term of the LHS, so the ranking is ambiguous.

9.13 Proof of Proposition 9

Under history $\{T,1,0\}$, social welfare at the stationary equilibrium is equal to (abstracting from time indexes):

$$\begin{aligned} LTu(q) &= TN_{-1}T\left[u\left(\frac{Q}{L}\right) - \hat{u}\right] \\ &= TN_{-1}T\left(\frac{Q}{L}\right)^{\sigma} + TN_{-1}T\left(\alpha - \hat{u}\right) + TN_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left(1n^{T-1}\right)^{\delta}\right) \\ &= (TN_{-1})^{1-\sigma}\, TQ^{\sigma} + TN_{-1}T(\alpha-\hat{u}) + TN_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left(1n^{T-1}\right)^{\delta}\right) \end{aligned}$$

Under history $\{T,0,1\}$, that formula becomes:

$$\begin{aligned} & \frac{T^2N_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}(\alpha-\hat{u}) + \left(\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}\right)^{1-\sigma} TQ^{\sigma} \\ & + \frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}\left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{\delta} + ... + \left((T-T+2)\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right) \end{aligned}$$

Hence the ranking of the CLU planner depends on:

$$
\begin{aligned}
& (TN_{-1})^{1-\sigma} TQ^{\sigma} + TN_{-1}T(\alpha - \hat{u}) + TN_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + (1n^{T-1})^{\delta}\right) \\
\gtrless \quad & \frac{T^2 N_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}(\alpha - \hat{u}) + \left(\frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}\right)^{1-\sigma} TQ^{\sigma} \\
& + \frac{TN_{-2}(1+\frac{N_{-1}}{N_{-2}})}{2}\left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{\delta} + ... + \left(2\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right)
\end{aligned}
$$

That expression can be rewritten as:

$$
\begin{aligned}
& (\alpha - \hat{u})T^2\left(\frac{N_{-1}-N_{-2}}{2}\right) \\
\gtrless \quad & TQ^{\sigma}\left[\left(\frac{T(N_{-2}+N_{-1})}{2}\right)^{1-\sigma} - (TN_{-1})^{1-\sigma}\right] \\
& + \frac{T(N_{-2}+N_{-1})}{2}\left(\left(\frac{(T-2)nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{\delta} + ... + \left(2\left(\frac{nN_{-1}}{N_{-1}+N_{-2}(1-n)}\right)^{T-2}\right)^{\delta}\right) \\
& - TN_{-1}\left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + ((T-T+1)n^{T-1})^{\delta}\right)
\end{aligned}
$$

If $N_{-2} = N_{-1}$ and $\alpha - \hat{u} \geq 0$, the LHS is equal to zero, while the RHS is negative, so that history $\{T, 1, 0\}$ is better. If $N_{-1} > N_{-2}$, the LHS is positive, while the RHS is, under $\alpha - \hat{u} \geq 0$, negative, so that history $\{T, 1, 0\}$ is better. If $N_{-1} < N_{-2}$ and $\alpha - \hat{u} \geq 0$, the LHS is negative, and the RHS is undetermined. Under $\alpha - \hat{u} < 0$, the sign of the RHS is undetermined. Hence whether history $\{T, 1, 0\}$ is preferred depends on whether the above condition holds, which depends on how large $\alpha - \hat{u}$ is with respect to T.

9.14 Proof of Proposition 10

Under classical utilitarianism, total welfare for individuals born at $t \geq 0$ is, without the transition:

$$
\sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1}\left[\sum_{s=t}^{t+T-1}\left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1})\sum_{r=1}^{s} n^r}\right)^{\sigma}\right]
$$

whereas, under the transition, it is:

$$
\begin{aligned}
& mN_{-1}\left(\alpha+\left(\frac{Q}{mN_{-1}+N_{-1}+N_{-2}+...+N_{-(T-2)}}\right)^{\sigma}\right) \\
& +mN_{-1}\left(\alpha+\left(\frac{Q}{mN_{-1}+N_{-1}+N_{-2}+...+N_{-(T-3)}}\right)^{\sigma}\right) \\
& +mN_{-1}\left(\alpha+\left(\frac{Q}{m^{2}N_{-1}+mN_{-1}+N_{-1}+N_{-2}+...+N_{-(T-4)}}\right)^{\sigma}\right) \\
& +mN_{-1}\left(\alpha+\left(\frac{Q}{m^{2}N_{-1}+mN_{-1}+N_{-1}+N_{-2}+...+N_{-(T-5)}}\right)^{\sigma}\right) \\
& +...+mN_{-1}\left(\alpha+\left(\frac{Q}{m^{T/2}N_{-1}+...+m^{2}N_{-1}+mN_{-1}}\right)^{\sigma}\right) \\
& +m^{2}N_{-1}\left(\alpha+\left(\frac{Q}{m^{2}N_{-1}+mN_{-1}+N_{-1}+N_{-2}+...+N_{-(T-4)}}\right)^{\sigma}\right) \\
& +m^{2}N_{-1}\left(\alpha+\left(\frac{Q}{m^{2}N_{-1}+mN_{-1}+N_{-1}+N_{-2}+...+N_{-(T-5)}}\right)^{\sigma}\right) \\
& +...+m^{2}N_{-1}\left(\alpha+\left(\frac{Q}{m^{(T/2)+1}N_{-1}+...+m^{2}N_{-1}}\right)^{\sigma}\right) \\
& +...
\end{aligned}
$$

This can be rewritten as:

$$
\sum_{t=1}^{\infty} m^{t}N_{-1}T\alpha+\sum_{t=1}^{\infty} m^{t}N_{-1}\left[\sum_{s=t}^{t+T-1}\left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)}+(N_{-1})\sum_{r=1,3,5,...}^{s} m^{(r+1)/2}}\right)^{\sigma}\right]
$$

Hence the transition is socially desirable iff:

$$
\sum_{t=1}^{\infty} n^{t}N_{-1}T\alpha+\sum_{t=1}^{\infty} n^{t}N_{-1}\left[\sum_{s=t}^{t+T-1}\left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)}+(N_{-1})\sum_{r=1}^{s} n^{r}}\right)^{\sigma}\right]
$$

$$\gtrless$$

$$
\sum_{t=1}^{\infty} m^{t}N_{-1}T\alpha+\sum_{t=1}^{\infty} m^{t}N_{-1}\left[\sum_{s=t}^{t+T-1}\left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)}+(N_{-1})\sum_{r=1,3,5,...}^{s} m^{(r+1)/2}}\right)^{\sigma}\right]
$$

Substituting for $n = m$ and simplifying yields:

$$\sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right]$$

$$\gtrless$$

$$\sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1,3,5,...}^{s} n^{(r+1)/2}} \right)^{\sigma} \right]$$

Given that coexisting generations are always less numerous after the transition, we have that the RHS necessarily exceeds the LHS, leading to a socially desirable transition. The same argument holds when considering AU and CLU.

9.15 Proof of Proposition 11

When coexistence concerns, the condition becomes:

$$\sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right]$$
$$+ \sum_{t=1}^{\infty} n^t N_{-1} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + \left((T-T+1)n^{T-1}\right)^{\delta} \right)$$

$$\gtrless$$

$$\sum_{t=1}^{\infty} n^t N_{-1} T\alpha + \sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1,3,5,...}^{s} n^{(r+1)/2}} \right)^{\sigma} \right]$$
$$+ \sum_{t=1}^{\infty} n^t N_{-1} \left(\left(\frac{(T-2)nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{\delta} + ... + \left((T-T+2) \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right)$$

Simplifications yield:

$$\sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1}^{s} n^r} \right)^{\sigma} \right]$$

$$+ \sum_{t=1}^{\infty} n^t N_{-1} \left(((T-1)n)^{\delta} + ((T-2)n^2)^{\delta} + ... + ((T-T+1)n^{T-1})^{\delta} \right)$$

$$\gtrless$$

$$\sum_{t=1}^{\infty} n^t N_{-1} \left[\sum_{s=t}^{t+T-1} \left(\frac{Q}{\sum_{z=0}^{T-s-1} N_{-(T-s-z)} + (N_{-1}) \sum_{r=1,3,5,...}^{s} n^{(r+1)/2}} \right)^{\sigma} \right]$$

$$+ \sum_{t=1}^{\infty} n^t N_{-1} \left(\left(\frac{(T-2)nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{\delta} + ... + \left((T-T+2) \left(\frac{nN_{-1}}{N_{-1} + N_{-2}(1-n)} \right)^{T-2} \right)^{\delta} \right)$$

The transition towards later births is not necessarily good: it is still true that the first term of the LHS is lower than the first term of the RHS. But the second term of the LHS is larger than the second term of the RHS. Hence the comparison depends on the relative strength of congestion *versus* coexistence concerns. The same condition holds for AU and CLU.