Viscosity of multimodal suspensions predicted from solid fraction model for mixtures

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Objectives:

The model is able to predict the viscosity of a suspension of non colloidal rigid spherical particles in a Newtonian fluid. The theory is developed to highlight a new relation between relative viscosity and the solid volume fraction. A new version of the Compressible Packing Model (CPM), the 4-parameter CPM, is introduced to predict the solid fraction of maximally dense disordered packings of spherical particles. It is apt to account for the geometrical interactions between particles.

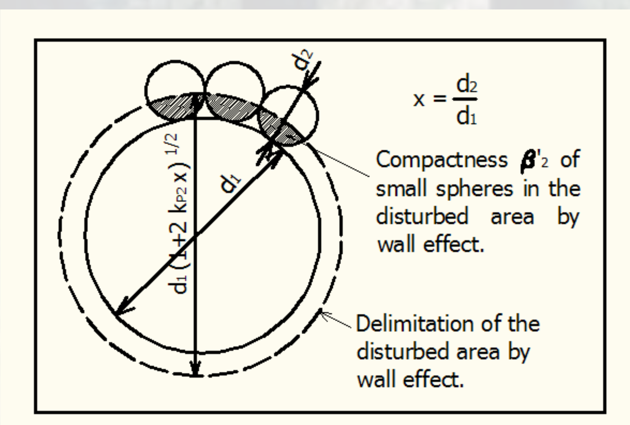
4-parameter Compressible Packing Model (CPM):

The 4-parameter CPM is apt to account for the loosening effect on big particles from interstitial small ones and for the wall effect within assemblies of small particles near a big one. The new theory is based on a specific treatment of configurations of one secondary class particle surrounded by dominant class neighbours. Four parameters are: wall effect and loosening effect coefficients, critical cavity size ratio and compaction index.

$$\gamma_i = \frac{\beta}{1 - \sum_{j=1}^{i-1}\left(1-\beta\right)\left(1-b_{ij}\right)y_j - \sum_{j=i+1}^{n}\left(1-a_{ij}\right)y_j}$$

β: compactness value, supposed to be constant, of each elementary class ;
y_j: volume proportion of each elementary class, by reference of the total solid volume;
b_{ij}: wall effect coefficient;
a_{ij}: loosening effect coefficient;
γ_i: compactness value of the mixture when the class i is dominant.

Wall effect:



$$x = \frac{d_2}{d_1}$$

Delimitation of the spherical disturbed area by the wall effect:

$$d_{hyp} = d_1\sqrt{1+2k_{P2}x}$$

Compactness of small spheres in the disturbed area by wall effect:

$$\beta'_2(x) = \frac{\pi(1+x)}{4x\left(\left(1+2k_{P2}x\right)^{\frac{3}{2}}-1\right)\arcsin\left(\frac{x}{1+x}\right)} \cdot \left(2\left(1+2k_{P2}x\right)^{\frac{3}{2}} - 3\left(1+2k_{P2}x\right)\left(1+\frac{k_{P2}x}{1+x}\right) + \left(1+\frac{k_{P2}x}{1+x}\right)^3 - x^3 + \frac{3k_{P2}x^3}{1+x} + x^3\left(1-\frac{k_{P2}}{1+x}\right)^3\right)$$

Wall effect coefficient for a binary mixture:

$$b_{21}(x) = \frac{\beta-\beta'_2(x)}{1-\beta}\left(\left(1+2k_{P2}x\right)^{\frac{3}{2}}-1\right)$$

k_{p2} is determined by respecting the boundary condition: $b_{21}(1)=1$

Viscosity of a suspension:

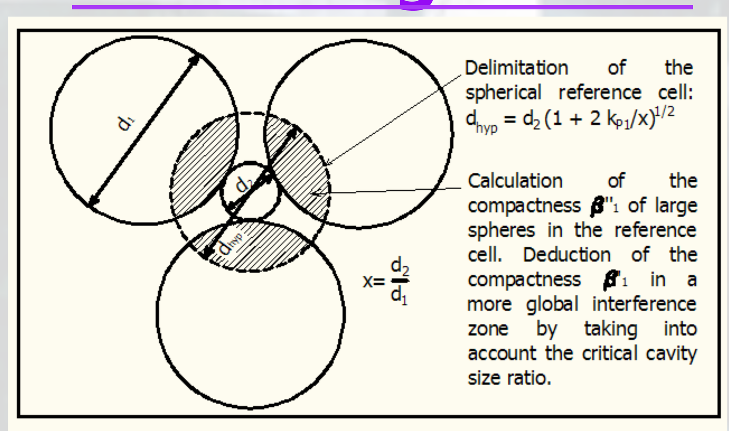
$$\eta = \eta_0 \prod_{i=1}^{n}\left(1-\frac{\psi_i}{\psi_i^{MAX}}\right)^{-C_E\beta} \qquad C_E = 2{,}5$$

$$\psi_i = \frac{y_i}{\frac{1}{\phi} - \sum_{j=1}^{i-1} y_j} \qquad \psi_i^{MAX} = \frac{\beta\left(\frac{1}{\phi}-\frac{1}{\gamma_i}\right) + y_i\left(1-\beta\right)}{\frac{1}{\phi} - \frac{\beta}{\gamma_i} - \left(1-\beta\right)\sum_{j=1}^{i-1} y_j}$$

η: suspension viscosity;
η_0: viscosity of the newtonian suspending fluid;
ϕ: volume fraction of the suspended spheres in a total volume unity;
ψ_i: volume fraction of the class i considering the presence of finer class;
ψ_i^{MAX}: maximal volume fraction of the class i considering the presence of finer class;

In agreement with a previous work of [BOUR05], the iterative approach advocated by Farris and a power-law relation (Krieger-Dougherty type) are used for the relative viscosity. The theory is developed to highlight a new relation between relative viscosity and the solid volume fraction, compatible with the Einstein relation. When the solid volume fraction reaches its critical value, the suspension is jammed and the mixture reaches the packing density of the solid skeleton.

Loosening effect:



$$x = \frac{d_2}{d_1}$$

Delimitation of the spherical reference cell: $d_{hyp} = d_2\sqrt{1+\frac{2k_{P1}}{x}}$

Compactness of large spheres in the reference cell:

$$\beta''_1(x) = \frac{\pi(1+x)}{4\left(1+\frac{2k_{P1}}{x}\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{1+x}\right)} \cdot \left(2\left(1+\frac{2k_{P1}}{x}\right)^{\frac{3}{2}} - 3\left(1+\frac{2k_{P1}}{x}\right)\left(1+\frac{k_{P1}}{1+x}\right) + \left(1+\frac{k_{P1}}{1+x}\right)^3 - \frac{1}{x^3} + \frac{3k_{P1}}{x^2(1+x)} + \left(\frac{1}{x}-\frac{k_{P1}}{1+x}\right)^3\right)$$

Compactness of large spheres in a more global interference zone by taking into account the critical cavity size ratio x_0:

$$\beta'_1(x) = \frac{\beta''_1(x)}{\beta''_1(x_0)\left(1+\frac{(x-x_0)}{(1-x_0)}\left(\sqrt[3]{\frac{2\beta''_1(1)}{\beta''_1(x_0)}}-1\right)\right)^3}\beta_1 \qquad x_0 \approx 0{,}2$$

Volume proportion of small particles at the « eutectic » point:

$$\phi_2^*(x) = \beta + \left(\left(1-\beta\right)\left(1-b_{21}(x)\right)-1\right)\beta'_1(x)$$

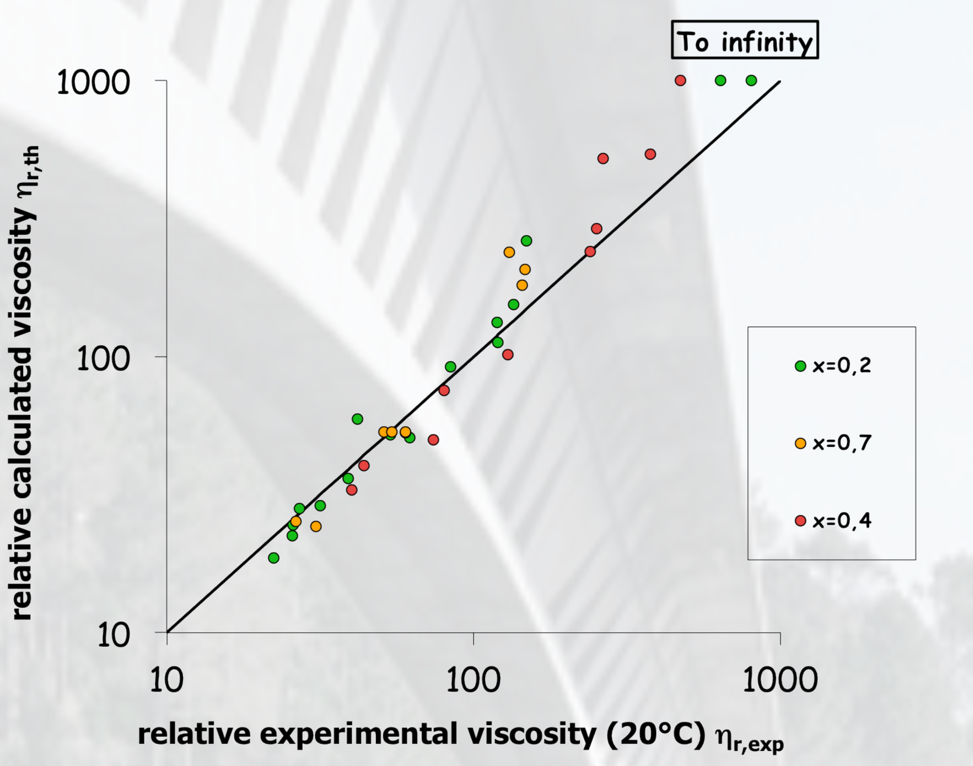
Loosening effect coefficient for a binary mixture:

$$a_{12}(x) = \frac{\beta-\beta'_1(x)}{\phi_2^*(x)} \quad \text{pour } x \geq x_0$$

$$a_{12}(x) = 0 \quad \text{pour } x \leq x_0$$

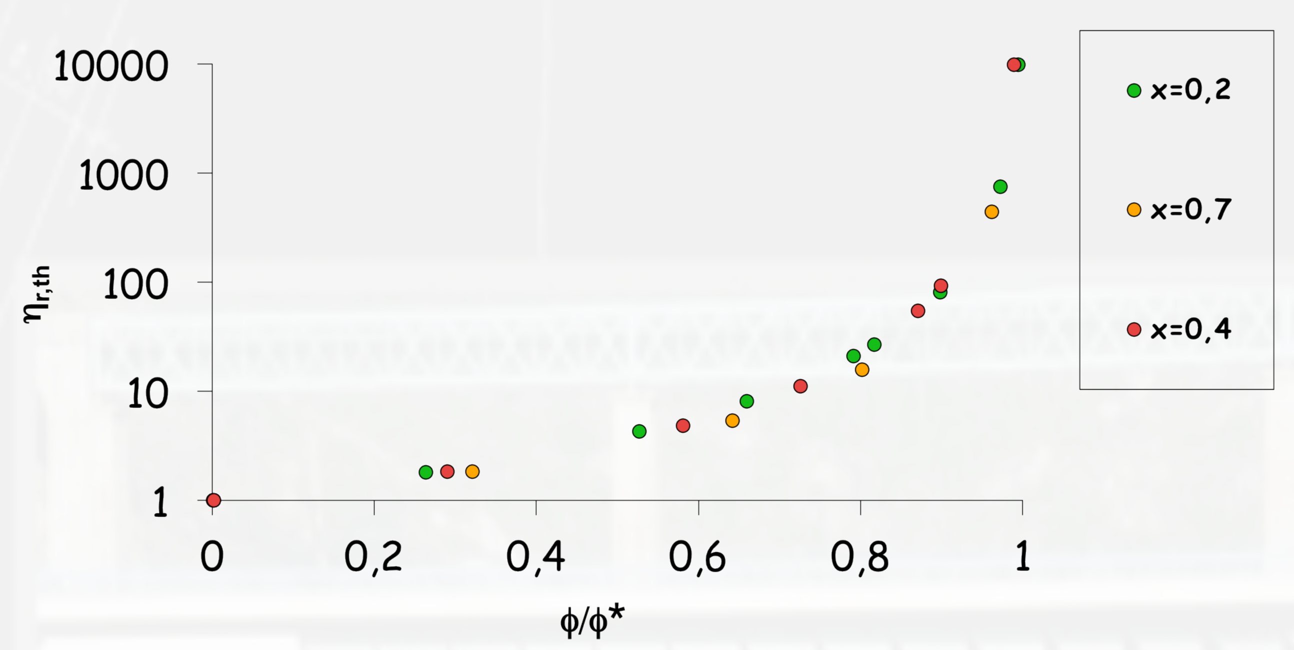
k_{p1} is determined by respecting the boundary condition: $a_{12}(1)=1$

Theoretical relative viscosity versus experimental one:



Bidisperse suspensions: comparison between relative predicted viscosities and relative experimental values from Stovall, Buil and Such [STOV87], with 3 different size ratios (x=0,2, x=0,4, x=0,7), for glass beads in silicone oil.

Theoretical relative viscosity versus normalized total solid volume fraction:



Bidisperse suspensions relative predicted viscosities as a function of the normalized total solid volume fraction for 3 different size ratios (x=0,2, x=0,4, x=0,7), for a volume proportion of the fine class y_2=0,30 and for β=0,60.

Conclusion:

The viscosity of a multimodal suspension is predicted by taking into account geometrical interactions and hydrodynamic interactions between particles. The model initially proposed by Farris is modified to include interactions used in the 4-parameter compressible packing model (CPM) to predict the compactness of a dry granular mixture. The wall effect and the loosening effect are the subject of a new theory. A spherical reference cell, concentric with a foreign sphere lost in a monodimensional mixture, can be used for both types of geometrical interactions. Its diameter is adjustable, depending on fine/coarse diameter ratio and on the specific packing density of the elementary dominant class.

References:

[BOUR05] BOURNONVILLE B., COUSSOT P., CHATEAU X., «Modification du modèle de Farris pour la prise en compte des interactions géométriques d'un mélange polydisperse de particules», *Rhéologie*, Vol.7, pp 1-8, 2005.
[DELA00] DE LARRARD F., «Structures granulaires et formulation des bétons», *Etudes et recherches des laboratoires des ponts et chaussées*, Ouvrages d'art OA34, 2000.
[FARR68] FARRIS R.J., «Prediction of the viscosity of Multimodal Suspensions from Unimodal Viscosity data», *Trans. Soc. Rheol.*, 12, (2), pp.281-301, 1968.
[PABS04] PABST W., «Fundamental considerations on suspension rheology», *Ceramics - Silikáty* 48 (1), pp. 6-13, 2004.
[STOV87] STOVALL T., BUIL M., SUCH C., «Viscosité des suspensions multimodales», *Journées Physiques – Les milieux granulaires*, Les Arcs, pp. 73-78, 1987.