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Identity, Continuity and Consciousness

by

Mark R Whittington

**A Thesis submitted in partial fulfilment of the requirements for the degree
of Doctor of Philosophy in Philosophy**

**University of Warwick, Department of Philosophy
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Declaration

I declare that all the work presented in this thesis was undertaken by myself (unless otherwise acknowledged in the text) and that none of the work has been previously submitted for any other academic degree. All sources of quoted information have been acknowledged by means of references.

M. R. Whittam

12/11/2004

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Abstract.

It is my intention in this thesis to demonstrate that there exists a clear and explicit formal relationship between the seemingly exclusive descriptions of spatio-temporal and purely temporal continuity, and further, that this relationship manifests itself within our most fundamental understanding of *the physical world* itself, namely; within our understanding of the identity, diversity and re-identification of material bodies (Book 1). It may therefore be claimed that behind that cultural understanding which leads us to imagine that *the physical world* is located in both space and time, whereas our thoughts and feelings are located in time alone, there lies a formal logical framework, or an explicit formal description of how being in space and time relates to being in time alone – leading us to wonder, perhaps, whether these two things are really as distinct as we might at first imagine.

That I should then go on (albeit without a formal methodology) to apply to this analysis a philosophical interpretation of Bergson's conception of the relationship between the intuition and the intellect (Book 2) is of lesser importance – indicating as it does little more than my own philosophical inclinations. However, something will be gained, I hope, from this further exercise. Along the way it will allow me to clarify a number of technical points of which the general philosopher may be unaware; for example the unobservable nature of numerical identity and re-identification, the importance of the principle of special relativity to the topic of mind and the technical difficulties of claiming that mental events are 'in time' at all.

Notwithstanding these latter points, however, the intentions of this work are predominantly analytical and are adequately described as an attempt to consolidate spatio-temporal and purely temporal description under a unified logical framework.

1 Introduction.

1.1 A Question Which Has about It at Least the Form of Epistemological Query?

If S is an individual who knows, or believes, or thinks, or who finds it convenient in certain circumstances to accept, that the world is one of material bodies moving about in space and time, then we may be tempted to ask how S may come to ‘see’ the world in this way, or to ask what is necessary and sufficient (either for S or the world) in order that S should arrive at this view:

Q1. How does S know (or believe, or think) that the world is one of material bodies moving about in space and time?

That this question should be of genuine interest to the philosopher or the psychologist, or that it should form the basis of wide ranging academic investigations (or indeed that it should be of interest to anyone other than S) relies upon the rarely emphasised assumption that S is not alone in holding this view. We assume, in posing Q1, that S could in fact be any one of us, or more formally perhaps, we say that S is a variable within a range or set of individuals (including me and you)¹. But if this ‘range of individuals’ is determined upon the basis of their ‘knowing’ something (or believing something, or thinking something – in this case; believing or thinking the world to be one of material bodies moving about in space and time), and if we both believe (or ‘know’) this range to exist and believe (or ‘know’) ourselves to be a member of it, then the very possibility of this ‘range of individuals’ (or the possibility of our

¹ $S \in \{G \mid G \text{ knows (or believes or thinks) that the world is one of material bodies moving about in space and time}\}$.

knowledge of it and membership within it) presupposes the more characteristically epistemological question:

Q2. How does S know that T knows that the world is one of distinct material bodies moving about in space and time?²

In other words, how do I know that you, or indeed anyone else, also knows the world to be of this nature?

The question Q2 therefore necessarily accompanies Q1 if it is to be assumed that Q1 is a question of interest to anyone other than S, or if it is to be assumed that S is in some sense a variable amongst a range of individuals³. In claiming that Q2 necessarily accompanies Q1 I mean to suggest, not only that Q2 should be answerable in order to justify the variable status of S (and thus make Q1 of interest to more than one individual) but that our ability to answer Q2, or the extent to which we could possibly answer Q2, determines the nature of Q1 itself. In other words, if we were to ask what exactly is it that S knows in knowing the world to be one of material bodies moving around in space and time, we should need to divide our answer into two categories. Firstly, that element of S's knowing the world to be one of material bodies in space and time which is private to S and unknowable to T (if indeed there exists such an 'element'), and secondly, that element of S's knowing the world to be one of material bodies in space and time which can equally be known by T (again, if indeed there should exist such an 'element'). Only in as much as Q1 may

² Q2 is a question which we might equally ask if our concerns were to lie with establishing an understanding of the methods of an empirical science based upon S's view.

³ In Q2 both S and T are variables within the same range of individuals. If we define the set of ordered pairs $B = \{(H,I) \mid H \text{ knows that } I \text{ knows (or believes or thinks) that the world is one of material bodies moving around in space and time}\}$, then $(S,T) \in B$.

address the second of these categories may S genuinely be a variable within a range of individuals, and S may only be genuinely variable within this range of individuals to the extent that this second category defines⁴. Thus Q1 is in fact itself restricted (in its answer) purely to aspects of S's understanding falling within this second category. In as much as S may also have aspects of understanding which fall within the first of these categories, S is no longer a variable within a range or set of individuals – and thus these 'elements' of S's knowing the world to be one of material bodies in space and time cannot be the subject of Q1, nor any answer which we might propose in response to Q1. Any deferral or avoidance in addressing Q2, or any attempt to suggest that Q2 is somehow secondary to Q1 (or in some sense follows from, or is 'begged by' Q1) must unavoidably leave Q1 somewhat ambiguous (a fuller exposition of this epistemological position – a position which is central to the methods of this current thesis – is outlined in Appendix II).

It is, of course, tempting to ignore this argument and claim that regardless of whether or not S is to be treated as a variable it is still perfectly sensible, for any given individual S, to ask how S knows P. My argument is firstly, that this is

⁴ The formal nature of this relationship between Q1 and Q2 can be captured in the following terms:

a/ S knows P.

b/ If S is a variable: $A = \{G \mid G \text{ knows } P\}$, $S \in A$.

c/ S1 knows that S knows P.

d/ If S1 and S are variables: $B = \{(H,I) \mid H \text{ knows } I \text{ knows } P\}$, $(S1, S) \in B$.

Terminate the regress:

e/ If the 1st projection of B, i.e. $\{J \mid (J, K) \in B\}$ is the set A and the 2nd projection of B, i.e. $\{M \mid (L, M) \in B\}$ is also the set A (as is the case in asking Q1 and Q2)

then B itself may be defined as the Cross Product of the set A, i.e. $B = \{(H,I) \mid H \in A, I \in A\} = A \times A$. Finally:

f/ If S is a variable then A exists. If A exists then $A \times A$ exists. Thus A is defined such that: $S \in A \wedge (S1, S) \in A \times A$ and thus A is defined such that both a/ and c/ are consistently accommodated. a/ is therefore inseparable for c/ if b/ and d/ apply.

disingenuous; since we are only interested in such questions in the first place because we believe that S is a variable (or could in principle be anyone of us), and secondly, that by employing any general term or terms in our answer to this question, or in citing any general terms in the nature of S's knowing P, we must once again reinforce the variable status of S. Neither Q2 nor the variable status of S may be ignored in addressing ourselves to Q1.

Unfortunately, an answer to Q2 (in the sense that it is informally and somewhat ambiguously stated here) is likely to be extremely difficult to formulate. In our everyday lives, of course, we more or less accept a solution by way of its being the simplest explanation. If we see other people acting as though and talking as though they perceive the world in a way similar to ourselves, then the simplest solution is to assume that they do indeed perceive the world in a qualitatively similar fashion. Such a process is, however, unlikely to stand up to the philosophical scrutiny required to establish an epistemological *criterion* of S's knowing that T knows that the world is one of distinct material bodies moving about in space and time. For example, although S may observe T *acting as though* T knows the world to be one of material bodies moving around in space and time, if S 'knows' the world to be of this nature, then could S observe T acting in any other way (particularly if, as may or may not be the case, part of S's 'knowing' this arises from observing T *acting as though* T 'knows' this)? Thus while Q1 may seem to be a perfectly sensible question to ask, it is in fact a question which sits uncomfortably, although necessarily, with a question (Q2) for which we have no immediate epistemological criterion for its solution. We therefore need, I would suggest, to ask Q1 in a slightly different way, or to

replace Q1 with a different question of S knowing something which sits more comfortably with the question of S knowing that T knows something.

The method of arriving at this 'slightly different' version of Q1 is, of course, to firstly find a relevant question referring to S knowing that T knows something (a relevant re-working of Q2 perhaps) for which we feel that a strict epistemological criterion for its solution is possible. I intend to propose the following:

Q3. If S knows that there are n (rather than $n+1$ or $n-1$) material bodies moving about within a given region space over some given interval of time, then how does S know that T knows that there are n (rather than $n+1$ or $n-1$) material bodies moving about within this same region of space at this same time?⁵

So why should we believe that an epistemological criterion for the solution of Q3 is any more likely to present itself than an epistemological criterion for the solution of Q2? The answer to this is that we may formulate it with respect to something which, in theory at least, is independent of what either S or T perceive the world to be like. In addressing Q3 we may refer to the properties of a consistent integer arithmetic – an arithmetic whose properties may, in theory at least, be enumerated quite independently of what S and T know about the world (or independently of S and T's 'knowing' the world to be one of material bodies moving about in space and time). For example, we may

⁵ It could of course be argued that if we had to hand a *criterion of S knowing that T knows there to be n (rather than $n+1$ or $n-1$) material bodies moving about within a given region of space over some given interval of time*, then it would not be unreasonable to assume that we had also developed a criterion for answering Q2 in certain circumstances and to a certain quantifiable degree – since if S knows that T knows there to be n material bodies moving about within a given region of space over some given interval of time (rather than $n+1$ or $n-1$), then surely S must also know that T knows (to a certain quantifiable degree) the world to be one of material bodies moving about in space and time?

develop the system of ordinal numbers from an axiomatic set theory and derive their arithmetic properties directly. Moreover, we may consider the principles employed in the application of this integer arithmetic to what S and T ‘know’ the world to be. For example, in my own understanding of the material bodies which I perceive to be around me I generally think that two such objects cannot be at the same place at the same time, or that one such object cannot be at two places at the same time – and it is principles such as these that allow me to count objects (and add to and subtract from their number) in accordance with an abstract integer arithmetic. These additional ‘applying principles’ are not themselves part of the integer arithmetic of S (or T) but are principles (about S or T’s understanding that the world is one of material bodies moving about in space and time) via which the properties of this arithmetic are applied to the world and revealed.

Consider, for example, that S claims that there are three tea-cups upon a particular table at a particular time and that T agrees with S. From this corroboration alone nothing can be deduced about S’s knowledge of what T knows. If however S removes one of the tea-cups and claims that there are now only two, and if T agrees, or if S adds another tea-cup and claims that there are now four, and if T agrees, then the combination of these corroborations indicate, although perhaps only partially in this limited case, the common application of both a consistent integer arithmetic (for we may independently determine what both S and T mean by ‘two’, ‘three’ or ‘four’) and a common set of applying principles (such as *two tea-cups cannot be at the same place at the same time* and *one tea-cup cannot be at two places at the same time*).

Of course, the simple set of corroborations discussed above do not, in themselves, amount to much; they do not, in themselves, justify that a common integer arithmetic and common set of applying principles are at play between S and T. However, we may imagine that over many such corroborations and over many different conditions and circumstances (in the empirical justification of the theories of classical vector mechanics for example), that S may, in effect, test the properties of T's integer arithmetic and the principles by which these are applied to the world and find them to correspond to the properties of S's own. Thus S may establish (to a certain degree limited, perhaps, by the logical nature of the arithmetic itself) that S and T share a common integer arithmetic and that S and T apply the properties of this arithmetic via common principles (for if T, unlike S, did not 'know', for example, that *two objects cannot be at the same place at the same time*, then we should expect to be alerted to this fact via differences in the counting claims of S and T). There is then no certainty in the epistemological solution to Q3; simply an increasing and repeated corroboration which leads us to believe that S may know that T knows that there are n material bodies moving about within a given region space over some given interval of time. In other words, no matter what is going on inside T's head, no matter how different T's understanding of the world is from S's, S may corroborate that in some sense both S and T apply a similar integer arithmetic via a common set of applying principles.

Now perhaps it may be argued that a suitable criterion for Q2 (a criterion of *S's knowing that T knows that the world is one of material bodies moving about in*

space and time) may be formulated by not dissimilar methods; that by an increasing and repeated corroboration of statements between S and T, S may somehow come to know (independently of Q3) that T represents the world in a qualitatively similar fashion (as indeed we actually do in our everyday lives). My argument is simply that it is extremely unlikely that we could rigorously formulate and express the criterion thus developed – and since this doubt exists, and since no such doubt (or at least a lesser doubt) exists in connection with the epistemological criterion of Q3, then it is to Q3 rather than Q2 that any serious investigation of these matters should be directed – at least in the first instance.

Let me then recap. We (philosophers, psychologists, physicists, biologists) may well want to ask Q1 [*How does S know (or believe, or think) that the world is one of material bodies moving about in space and time?*]. There may well be, of course, many different motives behind asking this question, and equally no doubt, many methods of addressing it. All of these ‘motives’ and ‘methods’, however, unavoidably assume (except via unacceptable construction) that S is not alone in this view, or that S is a variable amongst a range of individuals (including you and me) – otherwise philosophers, psychologists, physicists and biologists would not be interested in asking it (and universities would certainly not be interested in funding research for it). It is possible to argue, of course, that as a question about S (an individual) it stands in its own right. But this is not really how the question is asked (and to pursue this particular line of argument is disingenuous). It is asked specifically in the sense that S is a variable – and if it is asked in this sense then Q1 must be necessarily associated (even if we choose to ignore it) with the epistemological question Q2. My

argument is that Q2 is simply too difficult to answer in a strict epistemological sense; it is unlikely (although I could of course be wrong) to have an explicit criterion. A much better question, in the sense of the availability of an epistemological criterion, is Q3 [*If S knows that there are n (rather than n+1 or n-1) material bodies moving about within a given region space over some given interval of time, then how does S know that T knows that there are n (rather than n+1 or n-1) material bodies moving about within this same region of space at this same time?*]. So if we can actually arrive at an answer to this question (as an epistemological exercise), but cannot arrive at an answer to Q2 in equally rigorous terms, or if we can only answer Q2 (in some quantifiable sense) by recourse to Q3, then let us ask Q3 and (for the moment at least) forget about Q2.

We arrive then, via this somewhat torturous route, at that question which bears to Q3 the relationship that Q1 bears to Q2. or that question which (now that Q2 is to be rejected) must replace Q1:

Q4. How does S know that there are n (rather than n+1 or n-1) material bodies moving about within a given region of space over some given interval of time?⁶

This is essentially the question which (with some slight modification discussed below) will concern me in this current thesis. It is a question which we may at first address purely via analysis – for we may transform the claim that S ‘knows’ there to be n material bodies within a given region of space over a given interval of time into the claim that S ‘claims’ there to be n such objects (as

⁶ This is a question which is not the concern of establishing an epistemological understanding of the methods of empirical science (since this is already contained in the answer to Q3) but a question about S and T themselves, or at the very least perhaps, a question about the types of theories that S and T might put forward for empirical testing.

part of an intersubjective corroboration perhaps), and having expressed Q4 in terms of a statement formulated by S we may ask what is it exactly that is being claimed. Thus I shall be concerned initially with the analysis of statements (or that statement formulated by S) pertaining to there being n material bodies moving about within a given region of space over some given interval of time (Book 1).

However, it should be emphasised that, given the route by which we have arrived at the question Q4 (the need for a strict criterion for Q3), it is not in any sense a trick question. There is nothing hidden within it. It simply asks how S knows that there are n (rather than $n+1$ or $n-1$) material bodies moving within a given region of space over some given interval of time; how S knows that there are three rather than four, or four rather than five – for it is only this number, and its relation to a common arithmetic, which may be intersubjectively corroborated. It does not ask what material bodies are, nor what space and time are, nor what it is to be moving in space and time. In other words, we may express Q4 as: given that S knows the world to be one of material bodies moving around in space and time (regardless of whether or not T knows this also), how does S know that there are n (rather than $n+1$ or $n-1$) material bodies moving about within a given region of space over some given interval of time (because we can know that T also knows this – to some quantifiable degree)?

Now, of course, anyone, be they philosopher, psychologist, physicist, biologist, or whatever, may ask whatever question they wish – and if they can answer it to their own satisfaction, then all well and good (and if they can get funding to

answer it, then even better). My opinion, however (and it is, of course, only an opinion), is that the question Q1 [*How does S know (or believe, or think) that the world is one of material bodies moving about in space and time?*] is not a question which we should, at the outset at least, be asking; or at least not if S is intended to be a variable amongst a range of individuals (it may be of interest to S but it is not of interest to T). The question Q4 is as close as we may come to addressing Q1. Simply because 'S' in Q4 may be a legitimate variable amongst a range of individuals (because we can have an epistemological criterion for Q3) whereas 'S' cannot be a legitimate variable amongst a range of particulars in Q1 (because we have no epistemological criterion for Q2 except that which we have for Q3).

1.2 Re-identification, Continuity and the Infinitesimal Interval.

Although I have therefore presented at least some justification for the question which is to concern me in this theses, I am not yet in a position to outline the analytical claims which I shall make regarding this question (Book 1), nor the philosophically speculative interpretations of these 'claims' which I shall consider later (Book 2). For whereas until now I have been content to simply pass over the expression "*moving about in space and time*" (in the formulation of Q1, Q2, Q3 and Q4) I shall not be able to precede further (in a discussion of this analysis) until I have presented to the reader a more accurate description of what this expression means. Thus while I hesitate to burden the reader so soon with a description of the infinitesimal interval and the derivative, so important is this topic to this current work (and to an understanding of this introduction) that its inclusion at this stage is unavoidable. Let me then, as

briefly as possible, introduce the important relationship between our understanding of continuity (or more specifically ‘continuous motion’) and the formal nature of infinitesimal terms – for it is upon the basis of an understanding of this relationship, as much as anything perhaps, that the analytical claims of this thesis depend.

Unless it can be argued that Kinematic concepts are either reducible to, or derived from, Dynamic concepts⁷, then Kinematics (the analysis of mechanical systems in terms of the concepts of position, time, velocity and acceleration) must capture a description of the continuous re-identification of material bodies (the numerical re-identification of material bodies over continuous spatial and temporal intervals). For we cannot conceive of velocity, in any classical sense, except in terms of the velocity of a single entity, nor can we conceive of acceleration except in terms of the acceleration of that which is accelerated and which remains the same throughout the acceleration. The concepts of velocity and acceleration are therefore inseparable from our understanding of the continuous numerical re-identification of material bodies. If then our description of spatial and temporal continuity, or at least that description which is free of the seeming absurdities and infinite regresses of *Zeno*, is one which employs the derivative and the definite integral (as described in Appendix I), then this ‘description’ must itself be based upon the description of the re-identification of material bodies over infinitesimal spatial and temporal intervals; for the derivative is nothing more than the finite ratio of infinitesimal terms, and the definite integral is nothing more than the finite sum of the infinite addition of

⁷ This would be Kant’s view for example.

infinitesimal terms. The description of numerical re-identification over infinitesimal intervals is therefore central to the formal description of the Kinematic concepts of velocity and acceleration - and thus to the description of the cardinality (total number) of moving bodies within some designated region of space.

This is not to suggest, of course, that the infinitesimal is in some sense a real characteristic of the world – that infinitesimal intervals in some sense exist independently of our own chosen analysis of movement. They almost certainly do not. The infinitesimal interval is a product of logical analysis alone; it is the recognition that an infinite regress lies at the heart of the analysis of continuity (Zeno) and that this infinite regress must be terminated in order to reach a conclusion or avoid logical absurdity. The infinitesimal exists, in as much as it ‘exists’ at all, in order that we may consolidate our understanding of continuity with our understanding of logical analysis. The infinitesimal interval is simply the point at which we decide to stop regressing. Nor should we believe that we have some understanding of the infinitesimal beyond purely its logical role in the analysis of continuity (its logical role in the construction of the derivative or definite integral). To say that an infinitesimal term is neither finite nor Zero but ‘tends towards Zero’ is to say nothing at all. To say that the infinitesimal is indivisible is equally uninformative (although it does exhibit certain important characteristics of indivisibility). The infinitesimal is a term described solely by its logical operations (operations designed solely to facilitate the termination of the infinitely regressive analysis of continuity); firstly, that the ratio of two infinitesimal terms may yield a finite result (and in this operation may ‘mimic’

the properties of finite magnitudes and numbers), and secondly, that they vanish when taken in product with finite terms (and in this operation may ‘mimic’ the properties of Zero), or as Leibniz put it: “*quantities infinitely small such that when their ratio is sought, they may not be considered zero but which are rejected as often as they occur with quantities incomparably greater*” (Kline 1980. p 137). It is in this latter operation, of course, that the infinitesimal term plays its most important role in the termination of infinitely regressive arguments. For if the infinitesimal exhibits the properties of Zero when taken in product with a finite term (i.e. remains itself the same, as in $0 \times 5 = 0$), then the infinitesimal has no properties of finite division. It may be taken in ratio with another infinitesimal term but may not be taken in ratio with a finite one. You cannot have half an infinitesimal, or a quarter of an infinitesimal – although one infinitesimal may ‘mimic’ the property of being half the magnitude of another, or a quarter of the magnitude of another. Thus if the infinitesimal interval is ‘indivisible’ in this somewhat technical sense (not submitting to finite division), then you cannot carry on an infinitely regressive argument across it. If the same arrow is claimed to be re-identified over an infinitesimal interval of space and time, then, in terms of analysis at least, there is no sense in asking if it was also somewhere ‘in-between’ these locations; for there is no ‘in-between’ these locations (the infinitesimal does not submit to finite division) and thus any infinite regression is terminated.

That we should look upon the infinitesimal as some kind of ‘sleight of hand’ on the part of the logician and the mathematician is perhaps understandable. Equally, the accusation that the infinitesimal is simply a response to, rather than

a solution of, the paradoxes of Zeno may be legitimately made (no doubt). Yet it must be admitted that these paradoxes themselves arise only because there are things which we “know darn well to be the case” (that a body may move from A to B for example) but which logic will allow us to approach only via an infinite number of steps. It is then perhaps to the genius of Newton and Leibniz that we are indebted for placing what we ‘*know darn well to be the case*’ above the sterile necessities of logic, or more likely perhaps, for having drawn our attention to the absurdity of explaining physical continuity in terms of the properties of real numbers⁸. It is not that the infinitesimal is real, or corresponds in some way to a characteristic of the world, but that our own concept of the continuous re-identification of material bodies (what we call ‘motion’) is real – or real to us. The formal properties of the infinitesimal term (most notably its resistance to finite division) is therefore to be treated within this thesis as no more than a formal representation of our intuitive and familiar ideas about continuity – or the sense in which these ‘intuitive and familiar ideas’ are consolidated with our (no doubt equally intuitive and familiar) ideas about the logical consistency of arguments.

My task then, as I see it here, is to address the question Q4 [*How does S (or T) know that there are n (rather than n+1 or n-1) material bodies moving about within a given region of space over some given interval of time?*] in terms of

⁸ The application of the arithmetic of real numbers to physics depends solely upon the emergence of finite terms (whose properties are like those of real numbers) in the derivative and the definite integral, and so long as the principles of physics may yield these finite terms the physicist has little need to scrutinise the infinitesimal itself. And yet surely it cannot be the case that physics is really secure in relying upon the derivative and the definite integral (the finite ratio of infinitesimal terms and the finite sum of the infinite addition of infinitesimal terms respectively) unless it has, within its conceptual armoury as it were, some understanding of what the infinitesimal actually is; and where else should we look for this ‘understanding’ than in the concept of motion itself – that concept which relies primarily upon an understanding of what it is to re-identify a material body over infinitesimal intervals of space and time?

this understanding of the relationship between things “moving about” and the infinitesimal term (or between things “moving about” and our intuitive and familiar ideas about continuity). As such, the actual question which I shall address is not Q4 (as stated in the previous question) but a question from which Q4 may itself be constructed via the methods of integration:

Q4a. How does S (or T) know that there are n (rather than $n+1$ or $n-1$) material bodies moving about within a given region of space over some given infinitesimal interval of time?

If we should be capable of finding a solution to this question, then we should have equally answered Q4; since Q4 may be re-captured from Q4a simply by the integration of infinitesimal terms over finite regions – as we do all the time, for example, in the application of the principles of classical mechanics (which are invariably expressed in terms of first and second order differential expressions) to real world (finite) situations.

1.3 A Summary of the Analysis of Lockean Cardinality (A Summary of Book 1).

We have arrived then, finally, at that question which we may submit to the methods of analysis. For we may analyse the statement (formulated by S): “*There are n (rather than $n+1$ or $n-1$) material bodies moving about within a given region of space over some given infinitesimal interval of time*” and may reveal, in formal terms, what it is exactly that is being claimed within this statement.

Of course, in the analysis of any statement we may find nothing essentially new; merely a logical clarification of what is already held within it. In this

case, however, I feel that what we may reveal about S's statement via analysis is of such importance to an understanding of Q4a, that not only shall I dedicate three sections of this thesis to its analysis alone (Sections 2, 3 and 4), but I shall take the risk of presenting a summary of this analysis at the outset. And while it may appear premature to summarise such an analysis before its full derivation has been presented to the reader, in this case I feel that a far greater understanding of my concerns will result from its early presentation and summary.

In addressing Q4a via the methods of analysis (Book 1 of this thesis) I shall in fact do little more than address those identity and diversity relationships which together both indicate and justify the value of n . When we come to consider material bodies, however, we are confronted with various identity and diversity conditions which must somehow be consistently accommodated in order to arrive at our common understanding of this number. Firstly, we have (for historical reasons and perhaps due to a slight informality in our consideration of material objects) what we might call the "identity and diversity properties of material bodies at a given time". These are those principles to which I have already referred, namely; that *two objects cannot be at the same place at the same time* and that *one object cannot be at two places at the same time* (two principles which I shall later refer to as "Locke's Principles" and whose nature will be explored more fully in Section 2). Secondly, we have what we might more commonly refer to as "re-identifications" – the all too common understanding that the same material body may be re-identified over finite intervals of space and time (as when, for example, I claim that the tea-cup which

is currently upon my desk is the same tea-cup that was on the draining board in the kitchen this morning). Finally, we have that peculiar relationship between re-identification and spatial and temporal continuity which we call motion (roughly speaking, we feel that the tea-cup that was on the draining board in the kitchen this morning can only be re-identified as the tea-cup that is currently upon my desk if it has somehow ‘moved’ between these two locations along some sort of continuous spatial and temporal path). Put simply then, in order to break down a cardinality claim concerning material bodies we need to take account of the various senses in which we say that one is the same as another or different from another - ‘identity and diversity’ relationships which are an accommodation of the *‘identity and diversity properties of material bodies at a given time’* with *‘re-identification over space and time’* via our concept of *‘motion’* (this topic, together with its philosophical implications, is covered fully in Section 3).

1.3.1 Identity and Diversity ‘at a given time’ and Re-identification Over Space and Time

In addressing the topic of the identity and diversity of material bodies, whether ‘at a given time’ or over finite regions of space and time, we may do little more (in terms of analysis) than define a class of names appropriate to our understanding of these relationships.

We may capture the re-identification of material bodies over finite intervals of space and time in the three part conjunction of terms⁹:

$$P(a) \neq P(b) \wedge T(a) \neq T(b) \wedge a = b \quad \dots (i)$$

where ‘a’ and ‘b’ are temporary names applied in accordance with two principles [which ensure the transitivity of the identity relationship cited in (i)]; the first being a principle of identity¹⁰ and the second being a principle of diversity¹¹ :

$$P(a) = P(b) \wedge T(a) = T(b) \rightarrow a = b \quad \dots \text{LP.1a}$$

$$P(a) \neq P(b) \wedge T(a) = T(b) \rightarrow a \neq b \quad \dots \text{LP.2a}$$

I shall later refer to these principles as “*Locke’s principles of Identity and diversity*” (see section 2).

1.3.2 Continuity and re-identification.

The relationship between re-identification (over space and time) and continuous motion may be captured by firstly formulating (i) for the small but finite spatial and temporal intervals $\delta P_{a,b}$ and $\delta T_{a,b}$:

$$P(a) = P(b) + \delta P_{a,b} \wedge T(a) = T(b) + \delta T_{a,b} \wedge a = b \quad \dots (ii)$$

⁹ The object which is temporarily named ‘a’ is at a different position [$P(a) \neq P(b)$] and a different time [$T(a) \neq T(b)$] from that object which is temporarily named ‘b’, and ‘a’ and ‘b’ are two names for one and the same object [$a = b$].

¹⁰ If that object which is temporarily named ‘a’ is at the same position and time of that objects which is temporarily named ‘b’, then ‘a’ and ‘b’ are two names of one and the same object (a claim which we might usually express as: *two objects cannot be at the same place at the same time*).

¹¹ If that object which is temporarily named ‘a’ is at a different position from that object which is temporarily named ‘b’ at the same time, then ‘a’ and ‘b’ are not names for one and the same object (a claim which we might usually express as: *one object cannot be at two places at the same time*).

and then allowing the third conjunction (\wedge) of this expression to become an inference (in the sense that the symbol “ \rightarrow ” occurs in the definitions LP.1a and LP.2a) in the limit as $\delta T_{a,b}$ tends towards zero, or as $\delta P_{a,b} / \delta T_{a,b}$ becomes the instantaneous velocity of a material body in the limit as $\delta T_{a,b}$ tends towards Zero:

$$P(a)=P(b)+dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \rightarrow a=b \quad \dots \text{LP.1b}$$

where $dP_{a,b} / dT_{a,b} = \lim_{\delta T_{a,b} \rightarrow 0} \delta P_{a,b} / \delta T_{a,b}$.

Here then we see the role of the infinitesimal term in the formation of logical arguments (we have employed it in the process of transitioning from a mere statement to a principle or definition), and we note that as $dP_{a,b}$ and $dT_{a,b}$ vanish in LP.1b we arrive at the expression LP.1a.

The equivalent continuous form of LP.2a (or an expression which becomes LP.2a as its infinitesimal terms vanish) suffers from the fact that it requires an isolated infinitesimal term, and at the outset we must note that there exists nothing within our understanding of the motion of material bodies which might lead us to believe that such ‘isolated’ (or non-quotiented¹²) infinitesimal terms are possible or meaningful. We may overcome this problem, however, by formulating it in relation to two instances of LP.1b in which an infinitesimal term may be defined in terms of the first order derivative of position with respect to time of a material body:

$$\begin{aligned} P(a)=P(a')+dP_{a,a'} \wedge T(a)=T(a')+dT_{a,a'} &\rightarrow a=a' \\ P(b)=P(b')+dP_{b,b'} \wedge T(b)=T(b')+dT_{b,b'} &\rightarrow b=b' \end{aligned}$$

¹² An infinitesimal term which is defined otherwise than via a derivative, or otherwise than in terms of the ration of two infinitesimal terms.

and

$$P(a) \neq P(b') \wedge T(a)=T(b')+dT_{a,b'} \rightarrow a \neq b' \quad \dots \text{LP.2b}$$

and where $dT_{a,b'} = dT_{a,a'} = dT_{b,b'}$.

Again we note that as $dT_{a,b'}$ becomes zero in LP.2b we obtain LP.2a, i.e. that

LP.2a and LP.2b are logically continuous.

1.3.3 Cardinality.

If the cardinality (total number) of a collection of entities is n , then we must account for n instances of the reflective, symmetric and transitive relationship of identity ($=$), and $\frac{1}{2}(n^2-n)$ instances of symmetric but non-transitive relationship of 'difference' (\neq)¹³. Having formulated the identity and diversity principles of material bodies over a vanishing temporal interval (LP.1b and LP.2b) we may then say that the claim that there are n material bodies within a given region of space over such a given temporal interval requires n instances of LP.1b and $\frac{1}{2}(n^2-n)$ instances of LP.2b.

For the set of objects $a_1, a_2, a_3, \dots, a_n$, we may express this as:

a/	$P(a_1)=P(a_1')+dP_{a_1,a_1'} \wedge T(a_1)=T(a_1')+dT_{a_1,a_1'} \rightarrow a_1=a_1'$	}	n instances of LP.1b
b/	$P(a_2)=P(a_2')+dP_{a_2,a_2'} \wedge T(a_2)=T(a_2')+dT_{a_2,a_2'} \rightarrow a_2=a_2'$		
c/	$P(a_n)=P(a_n')+dP_{a_n,a_n'} \wedge T(a_n)=T(a_n')+dT_{a_n,a_n'} \rightarrow a_n=a_n'$		
d/	$P(a_1) \neq P(a_2') \wedge T(a_1)=T(a_2')+dT_{a_1,a_2'} \rightarrow a_1 \neq a_2'$	}	$\frac{1}{2}(n^2-n)$ instances of LP.2b
e/	$P(a_1) \neq P(a_3') \wedge T(a_1)=T(a_3')+dT_{a_1,a_3'} \rightarrow a_1 \neq a_3'$		
f/	$P(a_{n-1}) \neq P(a_n') \wedge T(a_{n-1})=T(a_n')+dT_{a_{n-1},a_n'} \rightarrow a_{n-1} \neq a_n'$		
..	..		

¹³ For n objects we may consider n^2 relationships of identity and distinction between them. Of these n^2 relationships, n will refer to the identity of each object with itself; thus leaving n^2-n relationships of distinction. However, since the relationship of 'difference' is symmetric (but not transitive) half of these relationships are redundant, and thus the total number of distinction relationships will be $\frac{1}{2}(n^2-n)$.

We note that due to the definition of LP.2b (its necessary relation to at least two instances of LP.1b) and the transitivity of the diversity relationship (or that for each pair of distinct objects within the system there must exist at least one symmetric diversity statement of the form LP.2b) all infinitesimal temporal terms in a/-f/ are equal.

Given the equality of infinitesimal temporal terms in a/-f/ the following condition is true of a/-f/:

$$\text{S: } \quad \exists_{m,m'} P(m)=P(m')+dP_{m,m'} \wedge T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$$

$$T(ax)=T(m) \wedge T(ax')=T(m') \text{ for all } x \text{ in } 1,2,3, \dots, n.$$

where m may be any of the objects a1, a2, a2, . . ., an or any other object whose continuity principle is captured by LP.1b. Substituting this term in a/=f/ then gives:

$$\begin{aligned} 0/ \quad & \exists_{m,m'} P(m)=P(m')+dP_{m,m'} \wedge T(m)=T(m')+dT_{m,m'} \rightarrow m=m' \\ a1/ \quad & P(a1)=P(a1')+dP_{a1,a1'} \wedge [T(a1)=T(m) \wedge T(a1')=T(m')] \rightarrow a1=a1' \\ b1/ \quad & P(a2)=P(a2')+dP_{a2,a2'} \wedge [T(a2)=T(m) \wedge T(a2')=T(m')] \rightarrow a2=a2' \\ \dots & \dots \\ c1/ \quad & P(an)=P(an')+dP_{an,an'} \wedge [T(an)=T(m) \wedge T(an')=T(m')] \rightarrow an=an' \\ d1/ \quad & P(a1) \neq P(a2') \wedge [T(a1)=T(m) \wedge T(a2')=T(m')] \rightarrow a1 \neq a2' \\ e1/ \quad & P(a1) \neq P(a3') \wedge [T(a1)=T(m) \wedge T(a3')=T(m')] \rightarrow a1 \neq a3' \\ \dots & \dots \\ f1/ \quad & P(an-1) \neq P(an') \wedge [T(an-1)=T(m) \wedge T(an')=T(m')] \rightarrow an-1 \neq an' \end{aligned}$$

This gives an expression of those relationships required to claim that there are n material bodies (rather than n+1 or n-1) within a given region of space over an infinitesimal interval of time which is itself defined from the first order derivative of position with respect to time of the object m (this expression is derived fully in Section 4).

We now make the important step of recognising that $0/-f1/$ is not a necessary and sufficient form (or that it contains terms and expressions which contribute nothing to the determination of a cardinality statements). The term $P(m)=P(m')+dP_{m,m'}$ in $0/$ plays no active role in the actual expression of the cardinality statement. Firstly, we note that: (a) The spatial terms in $a/-c/$ are vanishing (infinitesimal) and the meaning of $a/-c/$ is dependent upon these terms being vanishing. (b) That the spatial terms in $d/-f/$ are finite and the meaning of $d/-f/$ is dependent upon these terms being finite. (c) That we cannot substitute a finite, or non-vanishing, spatial term $[P(m) \neq P(m)]$ for a vanishing one $[P(m)=P(m')+dP_{m,m'}]$, nor a vanishing spatial term for a finite one, without losing the meaning of either $a/-c/$ or $d/-f/$. Thus while we may make a common temporal substitution S in $a/-f/$ (as in $0/-f1/$) we cannot make a common spatial substitution within $a/-f/$. The condition $P(m)=P(m')+dP_{m,m'}$ (in $0/$) therefore plays no role in the construction of $a1/-f1/$ (the cardinality statement itself). It is not substituted within $a1/-f1/$, and could not be alternatively employed as a substitute within $a/-f/$ (because $a/-f/$ will not submit to a common spatial substitution).

In other words, all that the term $P(m)=P(m')+dP_{m,m'}$ does is (in conjunction with $T(m)=T(m')+dT_{m,m'}$) ensure that the common temporal infinitesimal term $dT_{m,m'}$ is well defined (in this case via the first order derivative of position with respect to time of the object m) and that in being 'well defined' as an infinitesimal is not therefore subject to finite division and is thus suitable for the inferential term (\rightarrow) employed in $0/$ to $f1/$. The actual determination of those relationships

required to claim that there are n material bodies within a given region of time over a given infinitesimal interval only require that a valid infinitesimal interval be supplied - how it is defined is irrelevant to these relationships themselves. In other words, those relationships required to claim that there exists n material bodies within a given region of space over a given infinitesimal interval are formally independent of the spatial properties of the reference object m .

As such we may equally formulate 0/-f1/ with the omission of the condition $P(m)=P(m')+dT_{m,m'}$, providing we replace the inference in 0/ with a conjunction, i.e.

$$01/ \quad \exists_{m,m'} T(m)=T(m')+dT_{m,m'} \wedge m=m'$$

$$a1/ \quad P(a1)=P(a1')+dP_{a1,a1'} \wedge [T(a1)=T(m) \wedge T(a1')=T(m')] \rightarrow a1=a1'$$

$$b1/ \quad P(a2)=P(a2')+dP_{a2,a2'} \wedge [T(a2)=T(m) \wedge T(a2')=T(m')] \rightarrow a2=a2'$$

...

$$c1/ \quad P(a_n)=P(a_n')+dP_{a_n,a_n'} \wedge [T(a_n)=T(m) \wedge T(a_n')=T(m')] \rightarrow a_n=a_n'$$

$$d1/ \quad P(a1) \neq P(a2') \wedge [T(a1)=T(m) \wedge T(a2')=T(m')] \rightarrow a1 \neq a2'$$

$$e1/ \quad P(a1) \neq P(a3') \wedge [T(a1)=T(m) \wedge T(a3')=T(m')] \rightarrow a1 \neq a3'$$

...

$$f1/ \quad P(a_{n-1}) \neq P(a_n') \wedge [T(a_{n-1})=T(m) \wedge T(a_n')=T(m')] \rightarrow a_{n-1} \neq a_n'$$

The sufficiency of these expressions (to capture those relationships required to claim that there are n material bodies within a given region of space over a given infinitesimal interval) remains unchanged – providing that $dT_{m,m'}$ is still a valid infinitesimal and exhibits the property of resistance to finite division. The necessity of them, however, arises in eliminating the possibility of an interpretation of 01/ in terms of a single instance of LP.2b – i.e. an interpretation which would involve an undefined isolated temporal infinitesimal. We might say that 0/ is a classical interpretation of the necessary and sufficient form 01/, or that 0/ is an interpretation of 01/ in which the infinitesimal temporal reference

$(dT_{m,m})$ is defined in the first order derivative of position with respect to time of a classical material body.

Of course, the expressions $01/-f1/$ will always stand in need of interpretation to ensure the validity of the infinitesimal reference term $dT_{m,m}$ (that the this term exhibits the properties of an infinitesimal interval – most importantly; a resistance to finite division in the sense discussed in section 1.2) but does not insist that this infinitesimal reference term be defined via the first order derivative of position with respect to time of a material body. If there should exist other equally valid ways of defining such a term, then these ‘other equally valid ways’ would do as well for the formal definition of cardinality statements.

In summary then, the analysis of the question Q4a reveals to us the wholly unremarkable fact that if you say that there are n material bodies within a given region of space over a given infinitesimal interval of time, then you have to say what ‘given infinitesimal time’ you are talking about. What is important, however (and indeed so important, in my opinion, that I shall require to dedicate three whole sections of this thesis to demonstrating it rigorously) is that there is nothing in the formal structure of those relationships required to claim that there are n such material bodies which in any way demands that this ‘infinitesimal temporal interval’ is defined from the first order derivative of position with respect to time of a material body. As long as dT_{mm} is ‘supplied’, and ‘supplied’ as a legitimate infinitesimal term possessing the properties (of resistance to finite division) that an infinitesimal term must possess, then these relationships can be fully realised and n can be claimed.

It must therefore be admitted that this analysis leads us directly to two questions more immediately suited to a doctoral thesis in philosophy than the analysis of statements itself. Firstly, it begs the purely logical question of whether an infinitesimal temporal interval can be legitimately defined (i.e. maintaining the properties of an infinitesimal term) other than in terms of the first order derivative of a continuous function of position and time, or other than in terms of the motion of material bodies. Secondly (and this is where the philosophy comes in, or where the analytical concerns of Book 1 of this thesis must give way to the philosophical concerns of Book 2) it begs the question of whether there exists, within any established, accepted, or even merely muted philosophy, an understanding of time within which an infinitesimal temporal interval may be defined otherwise than in terms of the motion of material bodies.

The second of these questions must, of course, stand in need of a positive response to the first; since if an infinitesimal temporal interval defined otherwise than in terms of the first order derivative of position with respect to time of a material body is simply logically impossible (or if Δt has one and only one legitimate solution - that solution held in Δt), then no legitimate philosophy of time may posit such terms. It is essential then, at the outset, that I should argue that isolated (or non-quotiented) infinitesimal intervals are at least logically possible, or that it is possible to define an infinitesimal temporal term otherwise than with respect to the first order derivative of position with respect to time of a material body. As I shall demonstrate in the next sub-section of this introduction, however, it is perfectly logically feasible to define an isolated

infinitesimal temporal term, and further, that we already know how to do it from my previous discussions on motion.

1.4 The Logic of Isolated (non-quotiented) Infinitesimal Temporal Terms.

The analysis that I shall outline and summarise here is more naturally, perhaps, the topic of Book 2 of this thesis – since it concerns more than simply the analysis of question Q4a itself. However, it not only justifies those purely philosophical questions that I shall address in Book 2 (in attempting to address the “How” part of Q4a), but points the way to these questions themselves – it does not therefore sit uneasily between my concerns of analysis (Book1) and philosophical speculation (Book 2) but occupies instead a central role in the method by which I shall pass between these concerns. I should re-emphasise, however, that in addressing the question of isolated (or non-quotiented) infinitesimal temporal terms I am not necessarily addressing a question about the world or ourselves (I shall argue later that it does indeed refer to something about ourselves but, at this stage, this need not concern us). At this stage, or for the purpose of this introduction, I am simply interested in demonstrating that such terms are logically possible and, if indeed they are possible, in considering how this may guide us in the philosophical concerns of Book 2 of this thesis.

Let us firstly then remind ourselves about the importance of the infinitesimal term (and its property of resistance to finite division) in the description of motion.

I have previously argued that the three-part conjunction of terms involved in the claim of the re-identification of a material body:

$$P(a) = P(b) + \delta P_{a,b} \wedge T(a) = T(b) + \delta T_{a,b} \wedge a = b \quad \dots (ii)$$

is continuous with that ‘principle’ which determines the transitivity of identity relationships expressed in terms of what I have called ‘temporary names’ (Locke’s first principle LP.1a: $P(a)=P(b) \wedge T(a)=T(b) \rightarrow a=b$) under the condition:

$$P(a)=P(b)+dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \rightarrow a=b \quad \dots LP.1b$$

where $dP_{a,b} / dT_{a,b} = \lim_{\delta T_{a,b} \rightarrow 0} \delta P_{a,b} / \delta T_{a,b}$.

In this case then, it is the logical properties of the infinitesimal (its resistance to finite division) which allows us to move from the third conjunction (\wedge) of (ii) to the inference (\rightarrow) of LP.1b.

As a logical exercise only we may therefore equally define an infinitesimal temporal term from the description of purely temporal re-identification statements, i.e. statements which claim the re-identification of an entity over time without any reference whatsoever to spatial terms (and whether or not such ‘re-identification statements’ correspond to anything in reality is irrelevant to the concerns of this sub-section).

Suppose, for example, that we were to claim that the object temporarily named m (and I make no assumption as to what object, or even what type of object, m may be) is re-identified as the object which is temporally named n and that the time of m is not the time of n:

$$T(m) \neq T(n) \wedge m=n \quad \dots (iii)$$

The validity of this statement is then dependent upon a principle (referring to the application of such temporary names) which ensures the transitivity of the identity relationship ($m=n$) cited in (iii). By analogy to LP.1a, we might (as a logical exercise only) posit the principle:

$$T(m) = T(n) \rightarrow m = n \quad \dots (iv)$$

We may then (by analogy to the formulation of LP.1b) introduce a continuity term firstly by expressing (iii) in terms of the small but finite temporal interval $\delta T_{m,n}$:

$$T(m)=T(n)+\delta T_{m,n} \wedge m = n$$

and then defining a principle (which is logically continuous with the principle (iv)] in the limit as $\delta T_{m,n}$ tends towards Zero:

$$T(m)=T(n)+dT_{m,n} \rightarrow m = n \quad \dots (v)$$

Where $dT_{m,n}$ is $\lim_{\delta T_{m,n} \rightarrow 0} \delta T_{m,n}$ ¹⁴.

In this case then the infinitesimal term $dT_{m,n}$ is defined at the point of transition from (iv) to (v) – the transition from a conjunction of terms to the inference of one term from another (just as it occurs and is defined in the description of the motion of material bodies) in the temporal continuity of m and n ¹⁵.

¹⁴ This expression of the limit as $\delta T_{m,n}$ ‘tends towards Zero’ is not ideal. Normally we would simply interpret it as Zero. In this case, however, I use it merely as a convenient notation to indicate that (v) should really be expressed:

$$\lim T(m)=T(n)+\delta T_{m,n} \rightarrow m = n$$

¹⁵ It is obvious that the expression $T(m)=T(m')+dT_{m,m} \rightarrow m=m'$ can apply neither to the case where m and m' are material bodies, nor to the case where $T(m)$ and $T(m')$ are times in the sense of the measured times of the physicist. The properties of this expression are also rather abstract (and the implications of applying it as a solution to 01/ are somewhat complex) and thus I shall leave any further discussion of it until Book 2 of this thesis.

We have therefore defined a temporal infinitesimal term ($dT_{m,n}$ – a term which exhibits the logical properties of an infinitesimal, i.e. a resistance to finite division) other than in terms of the first order derivative of position with respect to time of a material body. Thus we may claim, in purely logical terms at least, that the necessary and sufficient expression 01/ (arising in the analysis of Lockean Cardinality statements) is not restricted purely to the classical interpretation 0/ - and thus, with some relief perhaps, my analysis of Lockean Cardinality statements is not trivial.

Really this is all I need to say to summarise the topics of Book 1 of this thesis (whose concerns lie solely with the analysis of statements - those statements formulated by S). However, it will undoubtedly leave the reader somewhat unclear about my intentions if I do not, even at this early stage, give some indication of the solution to Q4a which I hope to develop in Book 2. Let me then return briefly to that question which is to concern me throughout this work.

Given the question Q4a:

Q4a. How does S (or T) know that there are n (rather than n+1 or n-1) material bodies moving about within a given region of space over some given infinitesimal interval of time?

then we may claim (via the methods of analysis alone) the following to be a perfectly logically acceptable, although only partial, answer (which is no doubt simply one logically acceptable answer among several):

S (or T) knows that there are n (rather than n+1 or n-1) material bodies moving about within a given region of space over some given infinitesimal interval of time because S (or T) has reference to, or equally 'knows' about, purely temporal re-identification

statements of the form $T(m)=T(n)+\delta T_{m,n} \wedge m = n$ which are themselves dependent upon a continuity condition $T(m)=T(n)+dT_{m,n} \rightarrow m = n$ where $dT_{m,n}$ is $\delta T_{m,n}$ in the limit as $\delta T_{m,n}$ 'tends towards Zero'.

Or, if I were to be brave enough to make an as yet unjustified leap (but one which I shall attempt to justify in Book 2 of this thesis):

S (or T) knows that there are n (rather than n+1 or n-1) material bodies moving about within a given region of space over some given infinitesimal interval of time because S (or T) is, in part, or in some aspect of S associated with S's knowing things, itself a thing persisting through time alone with no spatial properties (whether actually or only seemingly so to S).

It should be evident to the reader by now that it is my intention to argue that our ability to formulate cardinality statements (or our ability to 'see' the world in terms of material bodies moving about in space and time) is dependent upon our recognition of ourselves as temporally persistent entities – that we can 'feel time passing' in some peculiar way.

In moving from the analytical claims of Book 1 to the philosophically speculative arguments of Book 2, I am therefore interested in philosophies in which S (a thing that can 'know things' about the world) is itself, in part at least, "a thing persisting through time alone with no spatial properties (whether actually or only seemingly so to S)". And there is no point in my pretending to the reader that I do not intend to claim that this is consciousness, or that the condition $T(m)=T(n)+dT_{m,n} \rightarrow m = n$ is not a property of measured time, as the physicist might measure it, but a property of phenomenological time: *time as experienced by consciousness.*

But this need not concern us in Book 1 of this thesis and, as a claim, is best laid to one side for the moment.

Book 1 - An Analysis of Identity, Diversity and Re-Identification Statements.

The Formal Properties of the Identity, Diversity and Re-Identification of Material Bodies over Infinitesimal Spatial and Temporal Displacements.

2. The Lockean Principles of Identity and Diversity.

2.1 Identity, Diversity and Locke

Before we can address ourselves to the topic of re-identification across the infinitesimal interval (Chapter 3) we must firstly acquaint ourselves with the properties of the identity and diversity of material bodies ‘at a given time’; for while neither experience nor measurement may reveal to us the non-extended instant and the non-extended point, these ideas nonetheless play an important role (at least historically) in our understanding of identity.

Two seemingly intuitive principles, both found in Locke’s treatment of “*Identity and Diversity*”, underlie our counting of material objects. The first is the principle that two objects ‘of the same kind’ cannot be at the same place at the same time: “*For we never finding, nor conceiving it possible, that two things of the same kind should exist in the same place at the same time, we rightly conclude that whatever exists anywhere at any time, excludes all of the same kind, and is there itself alone*” [Locke (1690) XXVII, Pringle-Patterson (1934)]. In the “*Identity of Substances*” (XXVII 2) Locke informs us that we have “*but three sorts of substance: 1 God. 2. Finite intelligences. 3. Bodies*” and that “*though these three sorts of substances, as we term them, do not exclude one another out of the same place, yet we cannot conceive but that they necessarily each of them exclude any of the same kind out of the same place*”. Here then, Locke seems to be using the expression “*of the same kind*” to mean either of the type “*God*”, “*Finite intelligences*”, or “*Bodies*”¹⁶. It is in this sense, or in the

¹⁶ He does not therefore use the term “*kind*” in the sense of the contemporary expression “*sortal predicate*” (Wiggins 1980 ch 3), or as a concept by which we may count the number Fs (e.g. donkeys, cats, chairs, or tables) within a given region of space at a given time; since if he did, then there is nothing within the first principle which excludes a table and a chair being at the same place at the same time. Equally, in claiming

sense in which “*Bodies*” constitute a “*kind*”, that Locke’s principle is most clearly applicable to the concerns of, say, classical mechanics – for within this science we do not distinguish the kinematical or dynamical properties of “*Bodies*” upon the basis of their sortal predicate or the peculiarities of their intrinsic properties. There is not, for example, a mechanics of chairs and a separate mechanics of tables; simply a mechanics of material objects, or “*Bodies*”, in general.

At one level we may consider Locke’s first principle to be a practical descriptive principle – one applying, in this case, to the simple fact that we may identify objects by different methods. For example, when Strawson asks “*When shall we say that a hearer knows what particular is being referred to by a speaker?*” (Strawson 1959. p17) he suggests that that we may employ linguistic means to isolate a particular within a given range of particulars which are themselves isolated by a “*demonstrative identification*” – that the hearer may “*pick out by sight or hearing or touch, or can otherwise sensibly discriminate, the particulars being referred to*” (Strawson 1959. p18). It is therefore conceivable that two different speakers may make a hearer know that they each refer to the same particular by different demonstrative identifications and by different linguistic means (or by the use of different words to isolate the particular to which they refer within the range of particulars which they have demonstrably identified). In this sense then we may think of Locke’s first principle as defining a rule regarding different identifications which relate to the

that material bodies constitute a ‘kind’, and in relating this kind to his principles of identity and diversity, we may consider Locke’s principles to constitute a ‘*criterion of identity*’ for material objects in general (see entry under ‘*Identity, criterion of*’ in the Oxford Companion to Philosophy (Ed Honderich T, 1995).

same place and time, i.e. if we identify x by one method and y by another, and if we should then learn that x and y are at the same place at the same time, then Locke's first principle informs us that we must claim that ' x is y '.

Yet the first principle is not without a metaphysical heritage (for we may legitimately ask about the origin of this 'rule' within our understanding and it is not unreasonable that this 'origin' may lead us into metaphysical matters), nor is it treated purely descriptively by Locke. Within the Cartesian tradition, for example, it arises from the argument, or 'Law', of contradiction (Smith 1963, p 409). If the essence of matter is spatial extension (if the essence of a material thing is to be extended in space), as Descartes had claimed it to be, then it is seemingly contradictory to assume that two material bodies may occupy the same place at the same time – since they would then be of the same essence and thus no longer be distinct. This was, however, unacceptable to both Kant and Leibniz; both of whom espoused the view that the characteristics of matter cannot be deduced from extension alone and must instead entail a dynamic element (or an element which cannot be reduced purely to the description of places, times, velocities and accelerations). Most importantly, in order to maintain his empirical theory of knowledge, Kant was forced to reject Cartesian mathematical extension as the 'essence' of matter (i.e. a mathematical property which can be directly grasped by the mind without recourse to the contribution of the senses) and with it, of course, he was forced to reject the application of the 'Law of Contradiction' in the formulation of what I refer to here as "Locke's First Principle". Realising perhaps that the description of motion is itself

impossible without this ‘first principle’¹⁷ Kant then proposes that we re-institute this principle, not from the Law of Contradiction, but from a dynamic metaphysics of matter. For Kant, two material bodies cannot be at the same place at the same time since matter possesses a dynamic ‘force’ or ‘power’ to resist penetration (Kant 1786) – thus establishing (together with Leibniz perhaps) a philosophical tradition within which the topics of identity, diversity and re-identification are forever intimately linked with dynamic (as opposed to purely kinematic) issues¹⁸.

Locke is not then alone in supposing a metaphysical intention for his first principle rather than a merely descriptive one, or in presenting an intention which goes beyond the simple practical interpretation discussed earlier – *“Another occasion the mind often takes of comparing, is the very being of things, when, considering anything as existing at any determined time and place, we compare it with itself existing at another time, and thereon form the ideas of identity and diversity”* [XXVII.1]. In effect, Locke is keen to define for us what it is to be a material body, or more accurately perhaps, what it is to be a single material body (to be counted only once in any act of counting). This is more clearly seen, however, with respect to his second principle.

Locke’s second principle is that one object cannot be at two places at the same time: *“When we see anything to be in any place in any instant of time, we are*

¹⁷ Kant treats the ‘essence of matter’ to be movement. Matter is that which moves or can be moved. Only via movement, argued Kant, may matter effect the senses and thus be known by us as appearances.

¹⁸ The overriding kinematic nature of the analysis presented in Book 1 of this thesis therefore stands in need of justification with respect to this Kantian position.

sure (be it what it will) that it is that very thing, and not another, which at that same time exists in another place, how like and indistinguishable soever it may be in all other respects” (XXVII 1). It is in the formulation of this second principle that Locke is making a more obviously metaphysical claim – since if objects at different places at the same time must be different regardless of whether they are otherwise indistinguishable, then particularity is not to be determined upon the basis of intrinsic properties alone (or that an object, or a substance, is to be considered as something more than simply its properties). Traditionally, however, this metaphysical position is challenged by Leibniz in the principle of the “Identity of Indiscernibles”. For Leibniz, diversity goes beyond mere spatial and temporal properties and must constitute instead an internal principle of distinction [*“it is not true that two substances may be exactly alike and differ only numerically, solo numero”* - Discourse on Metaphysics (Hollis 1973 p284)] - and thus while Leibniz does not deny that Locke’s second principle may be a practical aid to deciding that two objects are ‘different’ (or may help us to *“distinguish things which are not easily distinguished in themselves”*), he argues that the diversity of such objects actually entails something more than simply simultaneous spatial separation. Likewise, Zimmerman describes the idea (which he attributes to Locke) of *“a mysterious substratum, an unreachable kernel that bears properties but is not itself a property”* as *“metaphysics at its most gratuitous and pernicious”* (Zimmerman 1998).

Whether, in addressing Locke’s principles, we should really consider ourselves to be addressing a metaphysical problem is, of course, a difficult question to

answer. Certainly the idea that spatial and temporal position are alone sufficient to characterise the diversity of material bodies goes back as far as St Aquinas. However, when treated as a metaphysical problem, Russell has argued that this position may be reduced either to '*Identity of Indiscernibles*' of Leibniz, or to the belief (which Russell assumes to be the view of most modern empiricists "*if they took the trouble to have a definite view*") that numerical diversity is ultimate and indefinable (Russell 1948). The topic is therefore perhaps more naturally epistemological. For example, when Popper asks for "*something like a sufficient condition, i.e., a criterion of difference or non-identity of material bodies, or bits of matter*" (Popper 1957), he resorts in the end to the claim (equivalent, at least in form, to Locke's 2nd principle) that "*Two qualitatively undistinguishable material bodies or bits of matter differ if they occupy at the same time different regions of space*". Yet even here we are led to propose (as does Bobik) that question which most naturally arises in connection with Locke's principle (and Popper's epistemological formulation of it); "*why are different regions of space different? Are different regions of space to be distinguished by different individuals; or are different individuals to be distinguished by different regions of space?*" (Bobik 1963) – a question which is most naturally pertinent, perhaps, to Kant's treatment of identity and diversity in the *Analytic of Principles*.

When Kant addresses himself to the question of identity and diversity in the *Analytic of Principles* he informs us that "*When an object is presented to us several times but always with the same internal determinations (qualitas et quantitas), it, if an object of pure understanding, is always the same, not several*

things, but only one thing (numerica identitas); but if it is an appearance, it is not a matter of comparing concepts, and although everything may be the same as far as concepts are concerned, the difference of place of appearance at the same time is a sufficient ground for asserting the numerical difference of the object (of sense)” (Politis 1997 pp 117-8). But in what sense does Kant claim that “*difference of place of appearance at the same time*” constitutes a “*sufficient ground for asserting the numerical difference of the object*”, or in what sense, or upon what basis, does Kant claim that one object may not be at two places at the same time? The answer it would seem, or so Kant would have us accept, lies in the inherent diversity of places in space: “*For one part of space, although it may be perfectly similar and equal to another, is still outside it, and for this reason alone is different from the latter, which is added to it to make up a greater space. It follows that this must hold good of all things that are in the different parts of space at the same time, however similar and equal one may be to another*” (Politis 1997 pp 117). In other words, Kant asks us to accept that the origin of numerical diversity of objects (of which we may only know via “*appearances*”) lies in the numerical diversity of the places which they occupy at the same time. Thus our understanding of the diversity of places (at the same time) must in some sense precede, or be more fundamental than, our understanding of the diversity of objects themselves – and thus the pertinence of Bobik’s question of “*why are different regions of space different?*”.

I shall not, however, treat Locke’s principles as metaphysical definitions of the identity and diversity of material objects. Nor do I wish to engage in

metaphysical speculation as to the question of what it is for two particulars to be distinct (and in this much my adoption of the term “Locke’s Principles” is merely in line with common convention and does not imply my equal adoption of Locke’s conception of material bodies). My own position with regards to these principles is (admittedly) somewhat contradictory. For I shall treat them in both a relatively pragmatic sense; in claiming that they refer primarily to our own psychological inclinations to individuate experience, and a more rigorous formal sense; in claiming that they are either the logical pre-requisite for, or the logical consequence of (but in either case necessarily associated with), the ability to formulate numerical re-identification statements for material bodies.

Why then should I adopt two so seemingly different positions with respect to these principles?

One need not venture far into the common discussion of Locke’s principles to be confronted with those questions (or type of questions) which throw doubt, not necessarily upon these principles themselves, but upon our ability to apply them clearly and unproblematically to all objects and object types. For example, can two clouds be at the same place at the same time, or can two waves be at the same place at the same time? Similarly, one need not venture too far into the technical philosophical literature to discover that the rigorous application of these principles may itself seemingly lead to contradiction and absurdity – for example in the classical problem of the ‘Ship of Theseus’ (see Section 6.5). Finally, one need not delve too deeply into the theories of modern physics to discover that these principles themselves start to fail, or become un-helpful in

the description of physical systems, as we move away from the typical conditions and physical scales of our everyday experience (as discussed in section 2.3 below).

In short, we need to consistently address Locke's principles in two ways. Firstly, in a relatively pragmatic sense, or a sense in which the question of their violation is not critical (or where the violation of these principles, as in considering it possible for two clouds to be at the same place at the same time, is not necessarily detrimental to our understanding of the identity, diversity and re-identification of certain objects). Secondly, we require a formal perspective upon these principles; a consideration of the rigorous application of these principles to a class of objects and problems where their violation would lead us to radically re-think our opinions on the identity, diversity and re-identification of these objects.

These two ways of considering Locke's Principles (the 'pragmatic' and the 'formal') correspond to the cases where we are respectively uncertain and certain as to whether we can unproblematically re-identify objects over space and time. For example, the claim that *the cloud which is currently above my head is the same cloud that was just above the Eastern horizon at 10 o'clock this morning* is likely to be subject to a number of irritating questions which may, in extreme cases, lead us to doubt the validity of the claim itself. For example, when is a cloud the same cloud despite its change of shape and mass? When does a cloud become fog or fog become a cloud? Where does the cloud go when it is burnt off by the Sun? We might suspect then that things like

clouds will not only have a complicated criterion of re-identification but may well, in some circumstances, or under some arguments, violate Locke's first principle.

Equally, however, when I claim that *the tea-cup which is currently upon my desk is the same as the tea-cup that was on the draining board in the kitchen this morning* I feel there to be no ambiguity in what is meant by this claim (even if it should turn out to be false). I mean that the same tea-cup has moved continuously from the draining board in the kitchen to my desk. In this case, not only does a criterion of re-identification clearly present itself, but the claim that two tea-cups could actually be at the same place at the same time seems highly contradictory to my understanding of the identity, diversity and re-identification of such objects.

We therefore require a 'pragmatic' approach to Locke's principles for things like clouds and waves (for sometimes we may like to claim that two clouds or two waves may be at the same place at the same time) and a 'formal' approach for things like tea-cups (for we may never wish to admit that two tea-cups may be at the same place at the same time).

The first of these (and unavoidably a somewhat weak philosophical position) is to reformulate Locke's principles in a somewhat protected form (or in a form protected from those occasional questions which at once seem intuitively clear but which nonetheless challenge our ability to apply the first principle). For example:

If there is sufficient reason, or inclination upon our part, to individuate those qualities which we observe at the position p_1 at time t_1 , or to assign to these qualities a single name 'a', and if there is sufficient reason, or inclination upon our part, to individuate those qualities which we observe at the position p_2 at time t_2 , or to assign to these qualities the single name 'b', then if t_1 is (or seems to us to be) numerically identical to t_2 and p_1 is (or seems to us to be) numerically identical to p_2 , then we will often, although not necessarily universally, be inclined to say that 'a is numerically identical to b' (Locke's first principle). If, on the other hand, t_1 is (or seems to us to be) numerically identical to t_2 but p_1 is not (or seems to us not to be) numerically identical to p_2 , then we will often, although not necessarily universally, be inclined to say that 'a is not numerically identical to b' (Locke's second principle).

These principles arise then (or at least are treated as such above), not as the result of a metaphysical principle of identity and diversity, but as a result of those 'sufficient reasons', or 'inclinations upon our part', to individuate those qualities at a place and time and assign to them a single proper name (that there may be reasons well enough for us to have such 'inclinations', and that these 'inclinations' may themselves be described in scientific, philosophical and evolutionary terms will be discussed later).

With respect to this somewhat convoluted definition the reader might well object that it is qualified too strongly, or that its provisos may eliminate from my discussion all violating situations. It does not insist that Locke's principles apply to all situations (even those of our most common experience and understanding) nor even that there be any more to spatial and temporal simultaneity than our own belief in such situations. In short the reader may feel that I have been too timid in my definition, or that I have defended it so strongly from attack that, in effect, it says nothing of interest. To some extent this is indeed the case. For Locke's principles are interesting simply because they

describe for us a 'norm' or a typical situation. The idea that two objects cannot be at the same place at the same time, or that one object cannot be at two places at the same time, is a kind of useful rule of thumb by which we make sense of the world and can successfully interact with it. Of course it is possible to question these rules. Can two clouds be at the same place at the same time? Can two waves be at the same place at the same time? Equally, of course, it is possible to be lead towards absurdity by their strict application – as, for example, in the problem of The Ship of Theseus (which I shall consider in section 6). But to concentrate upon these exceptions at the expense of the 'norm' itself, or to develop a philosophy of individuation which insists upon accommodating these exceptions with the 'norm', is to mistake the methods of philosophy with theory of empirical science. Certainly the need for consistency requires us to accommodate exceptions when they contradict the 'norm' itself - just as we must reject an accepted scientific theory (the 'norm' in this case) when contrary empirical evidence arises (the 'exception'). But this is only if we accept the 'norm' to be incompatible with its exceptions. My somewhat pragmatic formulation of Locke's principles above is intended simply to express them as a 'norm', or to express them simply as a typical response to more or less typical situations. Thus formulated they do not deny the possibility of their own violation and thus are not incompatible with their own exceptions. The philosophy of individuation should (in my opinion) start, not with the metaphysical interpretation of Locke's principles and ultimately their rejection upon the basis of exceptions and logical absurdity, but with the recognition that these principles, first and foremost, serve a purpose; and that purpose is to allow us to make sense of the world by separating one thing from another in order that

we may interact successfully with it. Locke's principles are not, in this sense, immutable truths about the world (truths which must be rejected when found to stand in contradiction to certain know facts – 'exceptions') but are instead a more or less useful guide to our own inclinations (inclinations which have their origin in our evolution); a way of allowing us to arrive at a judgement and act in accordance with it. Locke's principles are neither true nor false. They are either useful or un-useful depending upon the situation in which they are applied (I shall discuss later those situations in which Locke's principles become un-useful). Locke's principles are applied "*often, but not necessarily universally*", and it is to the fact of their being applied "*often*" rather than "*not necessarily universally*" that our attention should be drawn. Likewise any sort of metaphysics of identity and diversity (when applied to material bodies) need only concern me if it is demonstrably the case that these 'sufficient reasons', or 'inclinations upon our part', require an explanation in terms of an ontology of material objects - and any such ontology of material objects, or any such attempt to define their identity independently of our own 'inclinations' to individuate them, is to be strongly opposed in this work.

In addition to this 'pragmatic' and somewhat psychological approach I also adopt a more rigorous formal position with respect to Locke's principles. Effectively, I would suggest that whenever it is claimed that '*a is b*', where *a* is at a different position and time from *b*, it is necessary, in order that this statement be meaningfully formulated, that both *a* and *b* are the type of objects to which Locke's principles rigorously apply. This is not, of course, in anyway contradictory to the pragmatic treatment of these principles outlined above; for

it is a position with respect to the formal properties of re-identification statements, and is thus pertinent only when those '*inclinations on our part*' are such as to lead us to formulate such re-identification statements. The exact nature of this formal position with respect to Locke's principles will be covered fully in section 2.4 when we come to consider the formal symbolic representation of these principles themselves. However, for the sake of completeness, I shall briefly outline this position here.

In claiming, say, that the tea-cup which is currently upon my desk is the same (numerically the same) as the tea-cup that was on the draining board in the kitchen this morning, the important point is the use of the word '*same*'. In this case a numerical identity is implied to exist between *the tea-cup which is currently upon my desk* and *the tea-cup that was on the draining board in the kitchen this morning*. However, numerical identity is not without its own identifiable properties, most importantly; its reflectivity (*a is a* and *b is b*), its symmetry (if *a is b* then *b is a*) and its transitivity (if *a is b* and *b is c* then *a is c*). Transitivity, in this case, is dependent upon a property of the relationship between objects and names. Put simply, if either *a*, *b* or *c* could be the names of more than one numerically distinct object, then the transitivity rule would not apply. In other words, the name '*a*' may refer to one and only one numerically distinct object, the name '*b*' may refer to one and only one numerically distinct object, and the name '*c*' may refer to one and only one numerically distinct object – otherwise it would be possible that *a is b* and *b is c* but *a is not c*. So when I claim that *the tea-cup which is currently upon my desk is the same as the tea-cup that was on the draining board in the kitchen this morning*, and

when I intend, by the formulation of this statement to use the word ‘*same*’ to imply a numerical identity, I must equally imply that the names ‘*the tea-cup which is currently upon my desk*’ and ‘*the tea-cup that was on the draining board in the kitchen this morning*’ are each names which can be the name of only one (numerically distinct) object. In other words, I must imply that there may only be one tea-cup where *the tea-cup which is currently upon my desk* is, and only one tea-cup where *the tea-cup that was on the draining board in the kitchen this morning* was – and thus in simply formulating the statement in the first place (and regardless of whether the statement is true or false) I must implicitly suggest that tea-cups, at least, are the types of things which rigorously adhere to Locke’s first principle (that two objects, of the same kind, cannot be at the same place at the same time). This then is the sense in which I suggest that Locke’s principles (or more accurately the first principle) is a logical prerequisite for, or a logical consequence of (but in either case necessarily associated with), the ability to formulate a numerical re-identification claim for material bodies.

I shall not therefore be concerned with the metaphysics of material objects; with the metaphysics of their particularity or diversity, their relationships to qualities (whether these qualities be universals or not), nor with whether such objects are more than their qualities and relations or nothing more than their qualities and relations. In fact I shall admit no individuation to material objects except that which we ourselves impose in our ‘inclinations’ to individuate them – and if, like Locke, we equate identity with existence [as is equally a position within Logical Metaphysics (Benardete 1989)], then I shall admit no existence to these

objects except in relation to our own ‘inclinations’ to individuate them. I do not, however, adopt this as a philosophical position, nor as one arising from an analysis of traditional and contemporary metaphysics. For regardless of the ontology of reality, or regardless of whether material objects actually exist independently of our concept of them, we must still address the question of how we come to represent the world in the way that we do (or how we come to individuate and characterise it in the way that we do) – and as we shall see in the following section, this latter question has aspects which are quite independent of metaphysical or ontological considerations.

2.2 The Epistemological Status of Locke’s Principles

If justification be sought for my current approach to Locke’s principles, or if justification be required for separating an ontology of identity and diversity from the study of our own ‘inclinations to individuate’ and their formal properties (as though we could treat the subject of identity and diversity, at least for material objects, not as a topic of metaphysics but as a topic of our own psychology), then we might do little better than to inquire as to the empirical, or synthetic, nature of Locke’s first principle (Popper 1959 p39).

2.2.1 The Empirical (or otherwise) Nature of Locke’s First Principle.

If we take Locke’s first principle to be a principle concerning what it is to be a material body in the first place, or what it is for such a material body to be possessed of a singular identity (to be counted only once in any act of counting), then we may certainly question if this principle is synthetic. Whatever the numerical identity, or individuality, of a material object may be, or whatever we

may imply by the term (metaphysically), we are entitled to ask if this ‘individuality’ is itself an observable property. In other words, can experience reveal to us the individuality of a material body?

At one level it seems relatively trivial to claim that we cannot observe, or measure, or detect, the numerical individuality of a material thing, nor can we observe that one material thing has been numerically re-identified as another, nor that one material thing is numerically continuous over time. Neither pure experience nor pure sensation would seem to reveal to us the identity of external material objects¹⁹. For example, Zimmerman (1997) claims that “*All we observe or detect are the properties of things, and a particular substance is nothing more than a bundle of properties*”. Personally, I take this to mean that we may observe and detect properties and qualities at places and times, but that our inability to observe a “*mysterious substratum, an unreachable kernel that bears properties but is not itself a property*” is synonymous with our inability to observe or detect the numerical individuality of a particular. However, we must treat such a claim with considerable caution. If individuality were to be understood in terms of an ‘bundle of properties’, and if the diversity of individuals is then guaranteed by the Principle of the Identity of Indiscernibles, then it may be claimed that this ability to distinguish one individual from another (upon the basis of experience) must itself infer the individuality of that

¹⁹ My labouing of the term ‘material’ is intended to avoid possible philosophical problems with claiming that all types of identities are actually non-observable. My claim here is merely that the identity of a persistent material object is unobservable. As to whether the identities of such things as properties and relations, for example, are observable, I shall not inquire. Nor shall I inquire into what we mean by the term ‘observation’ (for example, in the question of whether a genuine observation must entail the conscious direction of the mind upon a subject within experience – and thus, in effect, an individuation). My claim is simply that in its most embryonic form, or in the form of pure sensation, experience cannot contain or immediately reveal to us the identity of a material object.

which is distinguished from something else²⁰. My own opinion, however, is that we cannot anyway assume that a ‘bundle of properties’ is itself observable. Certainly the properties themselves may be observable (may be revealed to us via pure sensation), but their collection into a ‘bundle’ is not. Of course, experience may lead to our attention being drawn to a collection of properties (for example if they all seem to occupy an isolated region of space and time) and this process of ‘being led’ to a collection of properties is no doubt important to the way in which we represent experience to ourselves. But again this is a process of representation. Experience itself does not reveal that these properties are in fact a ‘bundle’.

Similarly, in Leibniz’ principle of the “*Identity of indiscernibles*” we are presented with the claim that if every intrinsic non-relational property of A is also every intrinsic non-relational property of B, then A is the same as B (or $A=B$). In other words, in enumerating every possible observable property of an object we exhaust all observable means of determining identity or diversity – since identity itself (pure particularity) is not itself observable²¹. Surely, if

²⁰ However, the Principle of the Identity of Indiscernibles (which, it might be claimed, guarantees the diversity of different ‘bundles of properties’) is not itself an observable property. We may observe different ‘bundles of properties’ perhaps (although I would deny even this) but we may not observe that they are therefore distinct individuals. We may go on to represent these ‘bundles’ as distinct individuals if we also, as part of this representation, employ the Principle of the Identity of Indiscernibles, but this is representation. It is perception, not experience. Equally, if it were the case that the Principle of the identity of Indiscernibles were a metaphysical principle, then surely the same is true. If we represent to ourselves two ‘bundles of properties’ as being distinct, then this cannot be upon the basis of experience (pure sensation) alone. It must employ a representation which itself employs a principle (not necessarily the Principle of the Identity of Indiscernibles).

²¹ I do not mean to directly support Leibniz’ principle, nor Zimmerman’s ‘Bundle Theory’, merely that the possibility of their formulation is itself sufficient to demonstrate that identity is not an observable. Most importantly, Leibniz’ ‘identity of indiscernibles’, or the very possibility of this theory as a legitimate philosophical position (one that is not to be immediately abandoned upon the grounds of absurdity) must itself be proof that identity is not a directly observable property.

numerical diversity were observable then there would be no need for philosophical debate about the respective merits of Locke's second principle and Leibniz' 'Identity of Indiscernibles' – since the matter could be settled on purely empirical grounds. This, of course, is a point which could equally be made about a great deal of contemporary philosophy of identity and individuation. For example, could we really accept as genuine the respective philosophical positions of Sortal Dependency and Sortal Relativity (Wiggins 1980), and the philosophical debate between the proponents of each, if it were nothing more than a matter of mere observation which could settle this debate? Numerical identity is simply not an observable property.

Even if we were to admit for the moment that qualitative identity, diversity and re-identification were observable properties of material bodies (and I would deny even this), still we should find it difficult to justify in any strict philosophical sense that numerical identity, diversity and re-identification are likewise observable. Suppose it were possible that A and B were alike in all their observable properties (i.e. they are qualitatively identical) but that at any given time A was at an observably different place from B. Of course, given these observations, our instinct is to claim that A and B are numerically distinct – since one object cannot be at two places at the same time (or so we are inclined to think). But what is it that we have observed which corresponds to this numerical diversity? We have observed two qualitatively identical bodies and we have observed that they are at different places at the same time. We have not observed that *one object cannot be at two places at the same time* since this is merely a principle; not something which is itself at a place and a time and

which can affect our senses (and I shall shortly argue that it is not an empirical principle). If then this *principle* is required in addition to, or to be applied to the interpretation of, these observable properties of A and B in order to arrive at the judgement of their numerical diversity (over and above their qualitative identity), then in what sense may we claim their numerical diversity to be observable?

Similarly, in the observation of the continuous motion of a material body, can this observation reveal anything to us other than the a continuous qualitative re-identification? And surely we must admit that the observation of continuous qualitative re-identification can reveal nothing more to us about numerical re-identification than can the observation of qualitative re-identification over periods of non-continuous observation. For example, Strawson would have it (in reaction to Hume's claim that all re-identifications over periods of non-continuous observation must be treated as essentially qualitative) that a condition for our having a conceptual spatio-temporal scheme with respect to which numerical re-identifications can be described is "*the unquestioning acceptance of particular-identity in at least some cases of non-continuous observation*". In other words, even within the anti-revisionary scheme of Strawson numerical re-identification (what Strawson refers to here as "*particular-identity*") must reference a non-observable (or non-directly-observable) element.

From a somewhat less philosophical perspective we are perhaps similarly drawn to the unobservable nature of individuality by consideration of observable properties within empirical science. The physicist, for example, has long since

subsumed his or her understanding of ‘observables’ under the science of quantum mechanics and must have long since accepted that there is no *eigenvalue* of identity itself (Cassels 1970, p8); there is no operation (no *Hermitian Operator*²²) which we may perform upon the wave equation to yield the measurable result that ‘a=b’. Numerical identity, diversity and re-identification fall outside of the range of measurable and observable things of the physicist²³. There is nothing whatsoever within traditional quantum mechanics, no operation upon the wave equation or superposition of multiple wave equations, which corresponds in anyway whatsoever to the intuitive and familiar claim that *the tea-cup which is currently upon my desk is the same tea-cup that was on the draining board in the kitchen this morning.*

Finally, we must consider that point which stands against the philosophical arguments presented in this section, namely; that individuality should be considered as a primitive notion – one requiring no ‘principle’ or further analysis²⁴. In this case, it may be argued that we can observe that something is an individual because to observe an object is to observe that object-as-an-individual. While this is by far the most philosophically complex position to address, I cannot (personally) see how it follows from the assumption that the notion of numerical diversity is “*ultimate and undefinable*” (Russell 1948) that individuality is itself observable in the sense that I mean here. We certainly

²² Only a certain kind of linear operator (upon the wave function) is suitable for representing an observable within traditional quantum theory. These are known as ‘Hermitian Operators’ (Cassels 1970, p9).

²³ The absence of identity and diversity statements from the expressions of empirical science is not necessarily surprising, nor does it stand in immediate contradiction with philosophical positions other than the one proposed in this thesis.

²⁴ This is a position which becomes important, for example, in consideration of how indistinguishable points in space are to be considered numerically distinct.

perceive material bodies (as the result of a process of representation), and it may well be the case that our notion of the individuality of these ‘perceived’ particulars is “*ultimate and indefinable*”, but it does not then follow that these particulars are presented directly to us via experience (pure sensation) – and it is only the with the denial of this direct presentation of particulars via experience that my current arguments are concerned.

Whatever one’s opinion on the arguments of this current section, or whatever one’s own philosophical position regarding that nature of identity and diversity, it seems a relatively unproblematic claim that pure sensation alone cannot reveal to us the individuality of material bodies. For I may as easily argue that pure sensation may no more reveal to us the identity and diversity of material bodies than may the coloured dots on a photograph capture the identity of the objects which we ourselves recognise within them (or which are realised through them).

The individuality of a material body is not then ‘given’ in experience, and thus Locke’s first principle is not synthetic; not learnt from experience alone.

2.2.2 The Empirical (or otherwise) Nature of Locke’s Second Principle.

We may discover an equally non-synthetic character in the second of Locke’s principles (that *one object cannot be at two places at the same time*). In this case, however, we are immediately drawn, not to its metaphysical implications (of which, as discussed earlier, there are significant aspects), but to the question of its empirical justification. For if this were indeed an empirical principle, or

one learnt from experience, then we might expect that experience was itself sufficient to justify it. Formally, however, this is not the case.

Firstly, since it is patently obvious that we cannot, under any circumstances, observe all places at a given time, then Locke's second principle cannot be strictly justified upon the basis of empirical test. In other words, if the claim that: "*one object cannot be at two places at the same time*" is based upon observation, then it must involve an unjustified induction, or an induction from the observation that this principle applies to a finite range of places to the claim that it applies to all possible places.

Now although the problem of induction [that there is no logical basis by which we may proceed from any number of particular statements to a general statement – Popper 1959 pp27-9)] may indeed be a legitimate philosophical problem, we rarely find it difficult to construct such inductions in practice. As such, this particular argument against the empirical nature of Locke's second principle is not particularly convincing. Of far greater significance, however, is that this principle is not, in practice, a synthetic principle, or would never, in practice, be falsified by comparison to experience. If we admit within our description of the physical world the possibility of the relationship of *qualitative identity* (Strawson 1959 p34, Baillie 1993 p5), or the relationship of two 'different' (not-same) material objects which are at different places at the same time but which are otherwise indistinguishable, then any falsifying event of Locke's second principle (one object actually being at two places at the same time) could always be explained away by citing this relationship of *qualitative*

identity. In other words, the condition of one object actually being at two places at the same time must be empirically indistinguishable from an instance of the relationship of *qualitative identity* (again justifying the non-observable nature of identity), and we may well ask ourselves under what circumstances we would be willing to interpret a given experiment in terms of the former rather than the latter? Locke's second principle therefore can be neither strictly justified nor falsified by comparison to experience, and is thus not strictly an empirical (or synthetic) principle.

We conclude then that neither Locke's first nor second principle is, in any strict sense, an empirical principle, and thus are left with the idea that these principles arise in those processes by which we ourselves represent our experiences, i.e. they are principles pertaining to our own 'inclinations to individuate'.

2.3 The Scientific Limitations of Locke's Principles.

Further support for my approach to these principles, or further support for treating them, not as metaphysical principles, but as principles pertaining to our own 'inclinations to individuate', is to be found in the observation that they may be limited in their application - or that there may be situations (or certain interpretations of situations) in which these principles do not seemingly apply. In the field of quantum theory, for example, we are presented with numerous examples where both the classical conception of the particle and its Lockean identity and diversity characteristics may be brought into question. Experiments such as the experimental realization of the Bose-Einstein Condensate (Anderson

et al 1995 Cornell, Wieman 1998) present us with situations where it is theoretically impossible to associate a unique number with each instance of an object of a kind - or with "*indistinguishable things*" (Simons 1997) - and experiments such as dual slit electron diffraction (Feynman 1983 p79) present us with situations where it is seemingly possible for a particle to pass through more than one place at one time. In quantum theory then, we are presented with many situations in which the identity and diversity properties of particles may be seen (under certain interpretations) to deviate considerably from the Lockean characteristics of classical bodies. However, it would be inaccurate to claim that the violation of Locke's principles in quantum systems is either well understood or universally accepted. Ever since Bohm's illustration of how nonrelativistic Schrödinger theory can be made compatible with the existence of point particles (Bohm 1952) various variants on the 'Real Particle' interpretation of quantum mechanics have been proposed. Within these interpretations something like the conception of the classical particle survives (albeit often with some compromise). Equally, a number of contemporary philosophers [for example French (1989, 1998), Van Fraassen (1985) and Huggett (1997)] have argued that the 'indistinguishable' particles of quantum theory can be treated as individuals to which standard identity conditions apply. The important point is that while the application of Locke's principles of identity and diversity are certainly open to question within many areas of quantum theory, the issue is generally not straightforward.

When we come to look at the transitions between classical mechanics and relativity theory we again find problems with sustaining Locke's principles (but

in this case in a slightly less obvious fashion). In claiming that *one object cannot be at two places at the same time* Locke imposes a principle upon the condition of a finite spatial separation in conjunction with a single (instantaneous) time. These are the conditions associated with the classical concept of temporal simultaneity, and it is the relative nature of this concept, with respect to the state of motion of the observer (or the “*relativity of temporal simultaneity*” - Einstein 1920 p25), which forms the basis of the theory of special relativity. Although relativity theory deals with the spatial and temporal relationships of events (and while events exhibit quite different identity and diversity characteristics from material objects²⁵) we may translate the conclusions of the special theory in the following terms: What one observer sees as two objects at different places at the same time (temporal simultaneity), another observer (in a state of relative motion with respect to the first) may see as two objects at different places at different times. This does not itself imply a violation of Locke’s principles (since we may assume that both of these observers continue to apply them independently) but does raise the question of how it is to be decided that these two objects are distinct. For in the case of the first observer we may apply Locke’s second principle to determine their diversity, but in the case of the second observer we cannot. Thus while we cannot claim that the second principle is actually directly violated in such cases, we may consider that its application as a descriptive principle upon which to base a mechanics must become increasingly problematic – since two different observers can no longer apply it to the same situation.

²⁵ For example, Russell argues that events cannot re-occur and thus cannot be re-identified (as particulars) with each other over time (Russell 1948).

While it is far from straightforward to claim that Locke's principles are directly violated in quantum and relativistic systems, there is certainly evidence to suggest that the application of these principles must become increasingly problematic (and in need of considerable compromise) within certain situations.

2.4 The Symbolic Formalisation of Locke's Principles.

Having established, albeit imperfectly, a philosophical position regarding the application of Locke's principles to material bodies, or having argued that there is sufficient epistemological and scientific evidence for questioning their metaphysical status and attributing them instead to the processes of our own 'inclinations to individuate', I shall now turn to the question of how these principles may be expressed symbolically.

In presenting a symbolic formalisation of Locke's principles I do not claim to be able to fully capture these principles in all their philosophical glory. Nor do I claim that Locke's principles (as originally formulated and as commonly conceived) may be symbolically expressed in a truly non-circular fashion. In fact, as we shall see, there are good reasons to assume that these principles must forever elude a truly consistent non-circular symbolic formulation. All I shall attempt to do here is to develop a symbolic form which is commensurate with, or derived from, at least the intentions of these principles, and then claim that any consequence arising from an analysis of these symbolic expressions is equally a consequence of the adoption of Locke's principles within our 'inclinations to individuate' material objects (justification for this latter claim will be presented in the following section). The symbolic formulations

presented below are therefore largely methodological. The analysis of the subsequent sections, and thus the arguments upon which much of this thesis is based, could equally be formulated with respect to the description of these principles as originally presented by Locke; only the reader would soon tire of the convoluted arguments and the simplicity of the analysis would be lost. Thus while I am readily aware of the limitations of the symbolic approach in this case, the advantages of clarity which result from this formalisation must outweigh any potential logical objections. Let us turn firstly then to the problems which a formalisation of Locke's principles must unavoidably face.

We note firstly that Locke's principles apply to the identity and diversity of particulars (particular material objects), and thus we must firstly consider what constitutes a valid identity statement concerning such 'particulars'. An identity assertion of the form $a=b$, when applied to particulars, is essentially the claim that the particular object whose name is 'a' is also that particular object whose name is 'b'; or that 'a' and 'b' are two names for one and the same particular object²⁶. The proviso here is that the type of names used, or the types of

²⁶ That this simple definition may itself be inadequate is made clear when we apply it to the reflective form '*a is a*' (the *principle of identity*); since here our definition becomes tautologous. As Wittgenstein put it ". . . to say of one thing that it is identical with itself is to say nothing at all." (*Tractatus Logico-Philosophicus* 5,5303) [Wittgenstein's actual objection is that we should not treat identity as a relationship "Roughly speaking, to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing at all", or that anything useful which can be said using the words 'is the same' can equally be said by a sentence using a repeated expression.]. With respect to our original definition we may be tempted to agree with Wittgenstein – as though in recognising some meaning in the claim '*a is b*' (although Wittgenstein may deny even this) we then go on to recognise that we may equally say '*a is a*' without realising that these words no longer have meaning. But even this may not satisfy us completely. For in Logic at least the locution '*a is a*' has some considerable power – as is evidenced, for example, in the use of the existential statement $\exists a(a=a)$ in logical metaphysics (Benardete 1989) and in the implementation of the *principle of identity* within the *axiom of extension* in axiomatic set theory [the identity of two sets, or the claim that one set is the same as another, is determined by these sets having exactly the same membership. While this 'criterion of identity' for sets has many useful applications it clearly suffers from the same paradox of identity which Wittgenstein claims applies to the simple locution '*a is a*']. Thus

relationships that these names may bear to particular objects, must be such as to ensure the reflectivity $[(\forall a) a=a]$, the symmetry $[(\forall a,b) a=b \rightarrow b=a]$ and the transitivity $[(\forall a,b,c) a=b \wedge b=c \rightarrow a=c]$ of the identity relationship²⁷. Not all name types are therefore suitable for formulating valid identity statements for material particulars. Basically, the required condition is satisfied for any class of names where one particular object may have many names but where one name may be the name of one and only one particular object. If this condition is not satisfied for a given class of names, then such names are not suitable for formulating identity statements for particulars²⁸.

Now if Locke's first principle is true (and two material objects of the same kind cannot be at the same place at the same time), then this principle itself defines a class of names suitable for formulating valid identity statements for particulars. Any name type which is formulated with respect to a given position and time (what we might call a 'temporary name') will be a valid name for employment within identity assertions concerning particular material objects. For example, if the material object q moves along the continuous spatial and temporal path

while the locution ' a is a ' may have no meaning and yet have significant ramifications within Logic, in those case where we can see clearly that we have named the same thing twice, or where there is good reason on our part for having named that same thing twice, then the claim ' a is b ' is seemingly straight forward.

²⁷ These being the first three of what are commonly referred to as 'The Axioms of Identity'. It is not uncommon, however, to add a fourth axiom concerning the complete community of properties, or the "*congruence of sameness, affirmed by a principle usually known as Leibniz's Law*" (Wiggins 1980 p19).

²⁸ For example, the name 'man' may be the name of more than one particular object. As such the identities $a=\text{man}$ and $b=\text{man}$ do not imply that $a=b$; or the application of this name type to identity statements does not entail the transitivity of the identity relationship and is thus not suitable for formulating identity assertions about material particulars. If however, we were to talk about the identity of classes, and insist that 'man' is the name of one and only one class, then $a=\text{man}$ and $b=\text{man}$ would imply $a=b$. In other words, names like 'man' can be used in identity assertions about classes (that one class is the same as another) but not about particular material objects.

$p=f(t)$, and if (p_1,t_1) and (p_2,t_2) are two solutions of $p=f(t)$, then how are we to indicate the continuity of q between the locations (p_1,t_1) and (p_2,t_2) ? Simply claiming that $q=q$ tells us no more about q at (p_1,t_1) and (p_2,t_2) than it tells us about q at (p_1,t_1) or (p_2,t_2) . One solution then is to say that q is temporarily named 'a' when it is at (p_1,t_1) and 'b' when it is at (p_2,t_2) . We may then express the continuity of q in the identity $a=b$ (or that 'a' and 'b' are two names of the same particular object). The claim $a=b$ is a valid identity assertion, of course, only if it is reflective, symmetric and transitive, but as long as Locke's first principle is true, then there cannot be more than one particular object at (p_1,t_1) or more than one particular object at (p_2,t_2) , i.e. 'a' can be the name of one and only one particular object, and 'b' can be the name of one and only one particular object.

We might then begin to see the nature of the logical problem facing us in attempting to symbolically formulate Locke's first principle, namely; that this principle (that *two objects cannot be at the same place at the same time*) is likely to posit some kind of identity assertion about particular objects (namely, that what is at one place at one time is one and only one object – or is possessed of a singular identity), and yet we already know that we assume Locke's first principle in formulating such valid identity assertions about material objects.

However, this observation need not restrict us from symbolically formalising this principle. All we need to remember is that any such formalisation will, in effect, constitute a rule, or definition, for applying certain types of names where the nature of these names themselves assume Locke's first principle to be true.

Let us then extend the definition of 'temporary names' in the following fashion:

If q is named 'a' at (p_1, t_1) , then $P(a)=p_1$ and $T(a)=t_1$

and

If q is named 'b' at (p_2, t_2) , then $P(b)=p_2$ and $T(b)=t_2$.

We may then define Locke's first principle (that *two objects of the same kind cannot be at the same place at the same time*) as the inferential form:

$$P(a)=P(b) \wedge T(a)=T(b) \rightarrow a=b$$

... LP.1a

In words, we would say that *if that particular object which is temporarily named 'a' is at the same position and time as that particular object which is temporarily named 'b', then 'a' and 'b' must be two names of the same particular object.* In this case then the inference (\rightarrow) is taken from the prescriptive, rather than descriptive, nature of Locke's principles themselves. In other words, Locke's principles do not simply claim that it is the case that no two objects ever have or never will be found at the same place at the same time (descriptive), but that no two objects ever can be found at the same place at the same time (prescriptive). It is only in this sense that the inference used in LP.1a should be interpreted. It is not therefore an immediate inference as commonly understood (Joseph 1914, pp 232-48), but simply a symbol which captures the prescriptive nature of Locke's claims²⁹.

²⁹ I should add, perhaps, that I have not qualified this expression (neither existentially nor universally). Mainly because I do not need to qualify it in order to carry out the analysis I intend. Any universal qualification would perhaps be largely circular; since this would require the specification of a set of particulars [for example the set P in terms of which the universal qualification $\forall_{a,b \in P}$ could be made] and any such specification would be likely to imply Locke's first principle itself (and thus the circularity). The expression is perhaps more naturally existentially qualified (as in $\exists_{a,b}$) but since LP.1a is little more than a rule for applying certain types of names (what I have termed 'temporary names') such qualification seems somewhat unnecessary. If the reader prefers such statements to be qualified, then I would suggest that they be existentially rather than universally qualified, but (as stated above) this makes little difference to the following analysis.

Having already recognised that the use of such temporary names (in the formulation of the identity statements of material particulars) itself assumes Locke's first principle, then I shall claim that LP.1a is simply a condition defining how such names are to be applied to such objects. This is why I do not claim to have fully captured Locke's first principle (in all its philosophical glory) within a symbolic expression, nor that this symbolic expression is necessarily non-circular, but simply to have developed an expression which is commensurate with, or derived from, this principle. That I shall go on to claim that the consequences arising from the analysis of this symbolic expression are equally consequences of our adoption of Locke's principle will be addressed in the following section.

Turning now to the second of Locke's principles (that *one object cannot be at two places at the same time*), we may equally express this second principle as the inferential form:

$P(a) \neq P(b) \wedge T(a) = T(b) \rightarrow a \neq b$... LP.2a
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In words, we would say: *If that particular object which is named 'a' is at a different place but at the same time as that object which is named 'b', then 'a' and 'b' are not two names of the same particular object.*

2.5 Locke's Principles and Cardinality (a justification of the inferences employed in LP.1a and LP.2a)

Given the logical complexities and circularities involved in the derivation of LP.1a and LP.2a (which I shall address further when I come to consider their continuous forms) it may well be asked upon what basis I may maintain that

these symbolic expressions are commensurate with Locke's principles, or upon what basis I might claim that in analysing these symbolic expressions we might learn something about these principles themselves? The answer to this concerns our treatment of the identity and diversity of material objects themselves, and more specifically, our treatment of the identity and diversity of material objects in determinations of their cardinality – the total number of material objects within a given region of space at a given time. For I shall argue that LP.1a and LP.2a apply to the determination of such a cardinality in exactly that same way that Locke's principles apply to such a determination. Thus while LP.1a and LP.2a may well be little more than definitions of how temporary names are to be applied to particular material objects, they play, in certain circumstances, exactly the same role as do Locke's principles (and as long as the subsequent concerns of this thesis refer solely to these 'certain circumstances' then I am justified in addressing the intuitive principles of Locke in terms of the symbolic forms LP.1a and LP.2a).

Firstly, however, we should understand that when we claim that there are n material objects within a given region of space at a given time, we are making a somewhat specific claim about the identity and diversity relationships which exist between these objects. In effect we are claiming that there are n distinct objects where no one object is being counted twice. Formally, this requires n instances of the identity relationship (one for each object) and $\frac{1}{2}(n^2-n)$ instances of the symmetric but non-transitive relationship of 'difference' (see Section 1). To claim that there are three ($n=3$) such objects, A, B and C, therefore requires

three instance of identity: $A=A$, $B=B$ and $C=C$, and three $[\frac{1}{2}(n^2-n)=\frac{1}{2}(3^2-3)=3]$ instances of distinction: $A\neq B$, $A\neq C$ and $B\neq C$.

Intuitively (for material objects) we arrive at these relationships by the application of Locke's principles. We know, for example, that A and B are at different places at the same time and must therefore be different objects (Locke's second principle), that A and C are at different places at the same time and must therefore be different objects, and that B and C are at different places at the same time and must therefore be different objects. So far then we seem to have decided that there are three objects. However, we only know that there are three objects because we also know that *two objects cannot be at the same place at the same time* (Locke's first principle). In other words, only A can be where A is (there is not another object there as well – adding to our total), only B can be where B is, and only C can be where C is.

My justification for LP.1a and LP.2a (that they are commensurate with Locke's principles and that their analysis will tell us something about these principles) is based upon the fact that they apply to a determination of cardinality in an identical fashion. For example, in claiming that there are three tea cups A, B and C, upon a particular table at given time, we may formulate three ($n=3$) instances of LP.1a:

- (I) $P(A)=P(A) \wedge T(A)=T(A) \rightarrow A=A$
- (II) $P(B)=P(B) \wedge T(B)=T(B) \rightarrow B=B$
- (III) $P(C)=P(C) \wedge T(C)=T(C) \rightarrow C=C$

and three $[\frac{1}{2}(n^2-n)=\frac{1}{2}(3^2-3)=3]$ instances of LP.2a:

- (IV) $P(A)\neq P(B) \wedge T(A)=T(B) \rightarrow A\neq B$

- (V) $P(A) \neq P(C) \wedge T(A) = T(C) \rightarrow A \neq C$
 (VI) $P(B) \neq P(C) \wedge T(B) = T(C) \rightarrow B \neq C$

We note that neither (I), (II) and (III) in isolation, nor (IV), (V) and (VI) in isolation, are sufficient in themselves to claim that the cardinality of this set is 3. For example, given $A=A$ and $B=B$ we do not know that $A \neq B$ unless we stipulate this condition via (IV). Equally, given $A \neq B$ we do not know whether there may be many tea cups at both $P(A)$ and $P(B)$ unless we restrict this possibility by (I) and (II). Thus while in practice we might intuitively feel that we need refer simply to LP.2a to determine the cardinality of a collection of objects at a given time, this is simply due to our familiarity with LP.1a, or our “*never finding, nor conceiving it possible, that two things of the same kind should exist in the same place at the same time*”. Formally, however, we require both n instances of LP.1a and $\frac{1}{2}(n^2-n)$ instances of LP.2a to claim that there are n objects of a kind within a given region of space at a given time.

So despite the logical complexities and circularities of LP.1a and LP.2a, they seem to apply in an identical fashion to the description of cardinality as do Locke’s principles (as we intuitively understand and apply them), and thus are at least commensurate with them. And as long as I restrict myself purely to considerations of cardinality (as I shall in section 4), I may therefore continue to apply LP.1a and LP.2a in the place of Locke’s first and second principles.

We may note one further important point. Lockean Cardinality, as described above in (I) to (VI), is concerned with those relationships of identity and diversity required to claim that there are n material objects within a given region of space at a given time; and this may seem somewhat arbitrary. What is so

special about counting objects within a given region of space at a given time? Why not, for example, count the number of objects within a given region of time at a given place? The answer to this, however, is relatively straight forward; for we simply do not possess the principles necessary to determine such a number.

Suppose, for example, we wanted to know how many objects there were at the place p_1 over the temporal period t_1 to t_2 . We could, of course, still apply Locke's first principle to this problem. For example, if the object x were at the place p_1 at time t' (where $t_1 < t' < t_2$) we could claim that no other objects may be at p_1 at t' and thus would count only one object. The problem arises, however, when we come to distinguish objects. For if we restrict our consideration to one place over an extended period of time then we can no longer apply Locke's second principle. For example, suppose the object at p_1 at t' was a red tea cup and that an indistinguishable red tea cup was at this same place (p_1) at t'' (where $t'' \neq t'$). Could we say whether this was numerically the same tea cup (and thus count both instances as only one object) or qualitatively the same tea cup (and thus count each instance as distinct objects)? The simple answer is that without the application of Locke's second principle we have no way of distinguishing between numerical and qualitative identity, and thus have no means of counting the number of distinct objects at a given place over a given interval of time.

2.6. Summary

We have seen then, that far from being necessarily metaphysical principles, Locke's principles may (depending on one's interpretation of physical theory) exhibit transient applicability across physical scales. Equally, they have about their nature nothing which must force us to assume that they are essentially empirical, or that they are learnt from experience. These are not, then, or not necessarily, metaphysical truths about the world; not immutable principles applying to a class of ontologically real bodies, but principles which we are at least entitled to suggest are imposed by ourselves in our own 'inclinations to individuate'.

Having said this, however, we have also equally seen something of the fundamental nature of these principles in our attempts to symbolically represent them – or the problem that such a representation is not free from the claims of circularity and must be considered as essentially a definition.

The Lockean Identity and Diversity of Material Bodies

Locke's Principles lead to a naming convention (based upon position and time) which ensures the transitivity of identity relationships for particular material bodies, i.e. $P(a)$, $T(a)$, where $P(a)$ is the position of that object which is temporarily named 'a' at the time $T(a)$, and where Locke's principles themselves become:

$$P(a)=P(b) \wedge T(a)=T(b) \rightarrow a=b \quad \dots \text{LP.1a}$$

$$P(a)\neq P(b) \wedge T(a)=T(b) \rightarrow a\neq b \quad \dots \text{LP.2a}$$

Nonetheless, these symbolic representations (although far from perfect perhaps) have proved sufficient at least to describe the concept of Lockean cardinality – and as long as I restrict my analysis of these principles to the description of Lockean cardinality I may continue to assume that this analysis may reveal to us something about the nature of Locke's principles themselves.

3. Identity, Re-identification and the Infinitesimal

3.1 *Introduction.*

Having considered something of the nature of Locke's principles of identity and diversity, and having started to lay upon these principles the rudiments of a symbolic expression, I now wish to turn to the important technical question of the relationship between these principles and our understanding of what it is to re-identify a material body over space and time. More specifically, I intend to ask how these two aspects of our understanding (our understanding of the identity and diversity of material bodies 'at a given time' and our understanding that material bodies may be numerically re-identified over finite regions of space and time) are related. May we, for example, treat Locke's principles as some kind of limiting case of our understanding of re-identification, or are these conceptually distinct understandings of identity and diversity?

Intuitively, we might suppose that some kind of continuity must exist. Suppose, for example, I were to observe two tea-cups upon my desk. In this case I may apply Locke's first principle (that *two objects, of the same kind, cannot be at the same place at the same time*) to conclude that wherever one of these tea-cups may be there may be only one tea-cup at this place. Equally, I may apply Locke's second principle (that *one object cannot be at two places at the same time*) to conclude that these tea-cups (in being at two places at the same time) must be numerically distinct. Thus, upon the basis of these principles I may understand something of the identity and diversity of tea-cups at a given time. However, if I am reliably informed that one of the tea-cups which is currently upon my desk is the same tea-cup that was on the draining board in the kitchen

this morning, then I am no longer dealing with the identity and diversity of tea-cups 'at a given time', but with the identity and diversity of tea-cups at two different times ('now' and 'this morning'). Intuitively, we might assume that as these 'different times' become closer together (as when, for example, I might claim that the tea-cup which is currently upon my desk is the same tea-cup that was at this same position upon my desk one second ago) and as these 'times' ultimately become the same time, our ideas of re-identification must somehow converge with Locke's principles, i.e. that our understanding of re-identification over space and time must somehow become our understanding that two objects cannot be at the same place at the same time.

Thus we may feel that some continuity is inevitable, but still we must understand what it is we mean by 'continuity' in the first place. More specifically, I wish to consider the relationship between re-identification and Locke's principles in relation to two important but significantly distinct ideas about the nature of motion itself: firstly, in relation to the idea that movement can be reduced to a description in terms of distinct places and times (an idea seemingly adopted, for example, by Strawson in his treatment of re-identification), and secondly, in relation to Bergson's denial of such a reducibility, i.e. his "*metaphysical individuality of every movement*" (Mullarkey 1999, p15)³⁰.

³⁰ Bergson makes a number of claims as to the "*metaphysical individuality of every movement*" which I shall neither adopt nor attempt to support here. I merely adopt Bergson's stance (itself fully commensurate, I believe, with at least the formalism of classical mechanics if not its implied metaphysics) that movement (as a concept) cannot be reduced to a description in terms of distinct points in space and instants in time.

Let me firstly, however, address the nature of this continuity (between re-identification and Locke's first principle) in straight forward analytical terms.

3.2 Re-Identification and Locke's First Principle

In the previous section I have claimed that we may express Locke's first principle in the form:

$$P(a)=P(b) \wedge T(a)=T(b) \rightarrow a=b \quad \dots \text{LP.1a}$$

where the inference (\rightarrow) captures the prescriptive, as opposed to descriptive, nature of Locke's first principle (or in the sense that LP.1a is a definition – in this case, a definition of how the temporary names 'a' and 'b' are to be applied). In other words, if we replace this inference with a conjunction (\wedge) then we can no longer claim to have captured Locke's principle - even though, in any given instance, this conjunction might be true. This observation is important because in claiming that the material body 'a' is re-identified as the material body 'b' over the small but finite spatial and temporal intervals $\delta P_{a,b}$ and $\delta T_{a,b}$ we effectively form the conjunction of three terms³¹:

$$P(a)=P(b)+\delta P_{a,b} \wedge T(a)=T(b)+\delta T_{a,b} \wedge a=b \quad \dots \text{RI}$$

At first sight then, our ideas of re-identification and our appreciation of Locke's principles seem to refer to two different aspects of our understanding of material bodies. Most importantly, if we substitute $\delta P_{a,b} = 0$ and $\delta T_{a,b} = 0$ in RI we obtain an expression which is not Locke's first principle (in as much as it does not capture its prescriptive nature):

³¹ This being a slightly modified form of the general claim of a re-identification over finite regions of space and time: $P(a)\neq P(b) \wedge T(a)\neq T(b) \wedge a=b$.

$$P(a)=P(b) \wedge T(a)=T(b) \wedge a=b$$

Now obviously we cannot simply take RI and replace its second conjunction with an inference (since this would mean that any two material bodies which are spatially and temporally separated would be the same). However, intuition would seem to suggest that as long as $\delta P_{a,b}$ and $\delta T_{a,b}$ are very small, then it is often reasonable to directly assume that $a=b$. For example, if there were a red ball on a particular table at three o'clock and an indistinguishable red ball upon this same table at half past three, then we cannot determine (upon the basis of this information alone) whether that ball has simply remained (implying and identity) or has been replaced by an indistinguishable but different red ball (denying an identity). If, however, we were to make our second observation at one second past three o'clock, and were to discover that there was still a red ball upon the table, then we might feel more confident that the ball has simply remained (or would feel there has probably not been sufficient time for someone to exchange the ball) – a confidence which approaches certainty, or so we might assume, as the time between these observations becomes infinitesimal (or as our observations, in effect, become continuous³²). Thus while we cannot simply replace the second conjunction in RI with an inference, common sense might lead us to believe that if we replace $\delta P_{a,b}$ and $\delta T_{a,b}$ with $dP_{a,b}$ and $dT_{a,b}$ (where $dP_{a,b}/dT_{a,b}$ is the ratio $\delta P_{a,b}/\delta T_{a,b}$ in the limit as $\delta T_{a,b}$ 'tends towards zero' – see Appendix I) then we may indeed replace this second conjunction with an inference to obtain:

$$P(a)=P(b)+dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \rightarrow a=b \quad \dots \text{LP.1b}$$

³² This link between our observations becoming continuous and the infinitesimal is important since, as discussed in section 1, the infinitesimal is essentially a formal construct which allows us to maintain our concept of continuity in the face of unremitting regression in logical analysis.

This being an expression which is directly continuous with Locke's first principle (or an expression which immediately becomes Locke's first principle as its infinitesimal terms become Zero).

Here then, we employ the derivative $dP_{a,b} / dT_{a,b}$ to capture our intuitive understanding of the connection between numerical re-identification and spatial and temporal continuity (i.e. motion), and we can easily see that this is fully in line with common sense. Suppose, for example, that having claimed that the tea-cup which is currently upon my desk is the same tea-cup that was on the draining board in the kitchen this morning [a claim of the form $P(a) \neq P(b) \wedge T(a) \neq T(b) \wedge a=b$] I subsequently learn that this is not in fact the case. I do not, of course, need to greatly reconstruct my view of the world in order to accommodate this news. A fact which I assume to be true has simply turned out to be false. It is simply the case (I assume) that the tea-cup which is currently upon my desk is qualitatively identical to, but numerically distinct from, the tea-cup which was on the draining board in the kitchen this morning. I have simply made a mistake. My original claim had been nothing more than a three part conjunction of different conditions; one of which ($a=b$) has turned out to be false. In claiming that the tea-cup which is currently upon my desk is the same tea-cup that was on the draining board in the kitchen this morning I am not claiming some principle of the world whose violation would astound me or would force me to radically re-think my understanding of the identity and diversity of material bodies. However, had I continually observed the tea-cup, from its being on the draining board in the kitchen this morning, to its moving continuously from the kitchen to my study, to its finally arriving upon my desk

and not moving again till now, then I might well feel there to be some greater degree of certainty in my original claim, or some greater degree of certainty than can be captured merely by the claimed conjunction of these conditions. Indeed, were I to be reliably informed in this case that the tea-cup which is upon my desk is not in fact the same tea-cup that was in the kitchen this morning, then I may well feel that I have somehow misunderstood the nature of material bodies and may well feel that some more definite principle of this understanding has been violated. This then is our understanding of the intimate relationship between spatial and temporal continuity and numerical re-identification (our concept of motion) – that while we may well accept that we are wrong in reidentifying a body which we have not seen move from one place to another, we would vigorously defend any reidentification where such a motion was observed, and this is why LP.1b contains an inference (\rightarrow) and not a second conjunction. For if LP.1b is true of any part of the continuous observation of the passage of the tea-cup from the kitchen to my desk, then its inference will survive the integration of its infinitesimal terms over finite spatial and temporal regions and we shall arrive at a conviction whose violation is equally a violation of a principle of our understanding (a principle of our understanding of material bodies).

Thus LP.1b is not only a valid analytical solution to our question of the relationship between re-identification and Locke's first principle, but it is one fully in tune with the intuition.

Now this expression (as thus derived) is simply an intuitive extension of Locke's first principle (if indeed we can accept that there is any such thing as an intuitive interpretation of the derivative $dP_{a,b}/dT_{a,b}$) and thus suffers from the same limitations as Locke's first principle itself, namely (as we shall see); that it does not apply to all of those classes of bodies which we may wish to refer to as 'physical', nor to all philosophical theories and interpretations of re-identification (see Chapter 6). What is important about this expression (LP.1b) is its formal relationship, firstly to temporality (as described in Chapter 4), and secondly, to our conscious movements and actions (as described in Chapter 6). For now, however, it is the intention of the remainder of this section to derive and consider this expression (LP.1b) in more detail and to consider its relation to Bergson's claim of the irreducibility of movement to a description in terms of points in space and time.

3.3 Re-identification and Movement.

That the topic of re-identification should be closely related to the idea of movement is perhaps obvious. For example, Strawson claims that "*. . . for many kinds of thing, it counts against saying that a thing, x, at one place at one time is the same as the thing, y, at another place at another time, if we think there is not some continuous set of places between these two places such that x was at each successive member of this set of places at successive times between these two times and y was at the same place member of the set of places at the same time*" (Strawson 1959 p37). Equally, when Wiggins asks: "*Is a, the man sitting on the left at the back of the restaurant, the same person as b, the boy who won the drawing prize at the school I was a pupil at in 1951?*" he admits that; "*. . . what*

organizes our actual method is the idea of a particular kind of continuous path in space and time which the man would have had to have followed in order to end up here in the restaurant . . .” (Wiggins 1980, p49). Thus while both Strawson and Wiggins go on to address this topic of re-identification in their own respective terms, they both agree that, in principle at least, what we mean (or what we imply) when we claim that a material body is re-identified over space and time is that a single (numerically identical) body has moved continuously from one place and time to another³³.

While Wiggins’ seemingly remains non-committal on the subject of the reducibility of movement to places and times (at least in this single quotation) Strawson gives a far clearer indication of his belief in such a reducibility, specifically; in his *“some continuous set of places between these two places”* but does not perhaps define his use of the term *“continuous”* so explicitly that we may be sure of his beliefs from this single quotation alone. For Bergson, however, movement is irreducible, or more accurately, movement cannot be reduced to locations in space and time; *“Bergson’s solution [to Zeno’s paradox of the arrow] is that the arrow is only at a point if it stops there; any other point that we might pick along its course will only represent a possible co-ordinate rather than a real resting place. Like the overtaking steps of Achilles [in Zeno’s paradox of Achilles and the tortoise], the course of the arrow is a single unique*

³³ We may, of course, make an objection to this claim and argue that motion must itself entail re-identification (thus reducing a description of re-identification in terms of motion to a circularity). However, we might then simply argue that all Wiggins and Strawson are proposing above is that there is, deep within our understanding of the world of material bodies, an inseparable intimacy between numerical continuity (re-identification) on the one hand, and spatial and temporal continuity on the other; and that this ‘inseparable intimacy’ is what we call movement.

bound” (Mullarkey 1999, p15). Thus if, according to Bergson, movement is irreducible (if the course of each movement is “*a single unique bound*”), and if, according to both Strawson and Wiggins, our understanding of re-identification is based upon our understanding that a “*solid thing*” has moved, then there is some element of our understanding of re-identification which is likewise irreducible. Effectively then, Bergsonian philosophy insists upon the existence of essentially irreducible re-identifications (and these ‘essentially irreducible re-identifications’ will play a significant role in the more formal aspects of the analysis which I shall present in this thesis)³⁴.

Put simply, or so I shall shortly demonstrate, if movement can be reduced to distinct places and times, then we cannot claim a continuity between Locke’s principles and our understanding of re-identification. If, on the other hand, movement cannot be reduced to distinct places and times (if there exist irreducible re-identifications as Bergsonian philosophy implies) then we find a continuity between these principles and our ideas of re-identification which is both intuitive and logically compelling. I therefore intend to argue that given our intuitive inclination towards such a continuity (as discussed earlier) we must equally be intuitively inclined towards a view of movement which cannot be reduced to a description in terms of ‘points’ in space and ‘instants’ in time.

Firstly, however, we must consider the important role of transitivity within the formulation of our everyday re-identification statements.

³⁴ Bergson presents a number of arguments for the metaphysical irreducibility of every movement which I personally find unsatisfactory and which are not directly supported here. In this work I adopt a Bergsonian view only in as much as it is claimed that movement cannot be reduced to a description in terms of non-extended ‘points’ in space and ‘instants’ in time.

3.4 Re-identification and the Infinitesimal Interval of a Path.

When I claim that the tea-cup which is currently upon my desk is the same tea-cup that was on the draining board in the kitchen this morning, I make a re-identification claim, i.e. a claim that a material body that was at one place and time is numerically identical with (is 'the same as') a body that is at a different place and time. Suppose, however, that I was learn later that at three o'clock this afternoon there was a tea-cup on the table in the breakfast room and that my wife claims that this was the very same tea-cup that was on the draining board in the kitchen this morning.

There seems to be no conceptual difficulty in accommodating this new fact. The tea-cup has simply moved from the draining board in the kitchen to the table in the breakfast room and, at some stage, moved again from the table in the breakfast room to my desk – and has not moved since. If, as Strawson and Wiggins agree, each re-identification is effectively the belief that the same tea-cup has moved, then I can recover my original claim by exploiting the transitivity of the identity relationship.

Firstly, the tea-cup on the draining board in the kitchen this morning (A) is the same as the tea-cup on the table in the breakfast room this afternoon (B) because, at some stage, the tea-cup on the draining board moved to the table in the breakfast room (A=B).

The tea-cup on the table in the breakfast room this afternoon (B) is the same as the tea-cup which is currently upon my desk (C) because, at some stage, it moved from the table in the breakfast room to my desk and has not moved since (B=C).

Finally then, from the transitivity of the identity relationship, if A=B and B=C then A=C, i.e. the tea-cup which is currently upon

my desk is the same tea-cup that was on the draining board in the kitchen this morning.

These transitive arguments apply then to the case where the tea-cup is known to have moved in a series of distinct movements (firstly in the movement from the draining board to the table in the breakfast room and then from the table in the breakfast room to my desk). Each of these ‘distinct movements’ are separated by an instance of stopping (or of ceasing to move) and thus the transitive arguments above are equally applicable to Bergson’s claim that these individual movements are “*distinct bounds*”. However, even if its movement had not been punctuated by these instances of stopping, we still find little difficulty in applying some kind of reduction to its movement. For example, if I were to observe the motion of the tea-cup in moving from the draining board, first to the table in the breakfast room and then on to the desk in my study, I would certainly be able to observe different phases and formulate different elements of description for this movement. For example, I would see it being lifted off the draining board and carried through the doorway to the breakfast room. Equally, I would see it moving towards the table in the breakfast room and then arriving at the table and being put down. I would then, at some stage, see it being picked up from the table in the breakfast room and carried to the door of my study, and finally, I would see it approaching the desk in my study and being put down there. Indeed, the transitivity employed in the argument above could still be employed in relation to these ‘phases’ and ‘elements of description’ and could therefore account fully for my original claim that the tea-cup on my desk is the same as the one that was in the kitchen this morning (i.e. the use of a transitive argument in the formulation of a re-identification statement does not require that the movement of the object concerned must have at some stage stopped).

Further, these 'phases' or 'elements of description' are themselves distinguishable, or are distinguishable descriptions of the various stages of the movement of the tea-cup, so how can Bergson still maintain that movement is irreducible; for it is patently obvious to everyone that it is reducible to these identifiable and distinguishable 'phases' and 'elements of description'?

Now Bergson does not, of course, deny that we may make this kind of reduction to a movement in terms of such 'phases' or 'stages' or 'elements of description'. What he denies (or what is ultimately of significance within his arguments) is that we can reduce movement to a description of points in space associated with instants in time; and in terms of re-identification we can see why this may be so. There are an infinite number of points on the path of the tea-cup in moving from the draining board to my desk. Thus if its motion were reducible to some idea of its 'being at' each of these points at certain times (as Strawson seems to imply) then I should need to construct an infinite number of transitive arguments to explain what I mean in claiming that the tea-cup which is currently upon my desk is the same tea-cup that was on the draining board in the kitchen this morning. In fact, we could never reach a justification of the claim that the tea-cup is re-identified over any finite interval, no matter how small, upon the basis of transitive arguments alone.

Thus while the scientist may translate the common reduction of a movement to 'phases' and 'elements of description' into the measurements of position and

time³⁵, this process must stop somewhere. As we divide the path of a movement into ever smaller intervals we require ever more steps in the sequence of transitive arguments by which we may claim that what started moving is also what stopped moving. Finally, either our measuring instruments or our patience will fail us and we will have to accept that there are very small intervals within the path of a movement over which re-identification is just accepted (or else descend into Zeno-like paradoxes); and it is the sense in which these re-identifications are ‘just accepted’ which distinguishes a Bergsonian conception of re-identification from that which I have ascribed (hopefully not incorrectly) to Strawson.

Formally, the physicist terminates the infinitely regressive sequence of transitive arguments employed in a re-identification claim at the point of ‘the infinitesimal’ (see Appendix 1). For example, in defining the instantaneous velocity of a body as the ratio $dx/dt = \delta x/\delta t$ in the limit as δt ‘tends towards zero’, dx and dt are infinitesimal intervals of distance and time, i.e. intervals which are neither finite nor zero but which ‘tend towards zero’, or “*quantities infinitely small such that when their ratio is sought, they may not be considered zero but which are rejected as often as they occur with quantities incomparably greater*” (Kline 1980. p 137). For the scientist, the infinitesimal is largely a practical device (albeit a remarkably fruitful one). It is an admission that a problem involves an infinite regress and that to get an answer you will have to

³⁵ Physicists do not actually perform such a reduction, or do not conceptually reduce movement to places and time. Their use of ‘functions’ of position and time is not to be mistaken for a conceptual description of movement itself but as a formal definition of certain forms of co-ordinate systems - as for example in the definition of the inertial reference system as a system of co-ordinates with respect to which the spatial positions of a “freely moving” particle are a simple linear function of time.

stop somewhere. The infinitesimal interval is where scientists from the Seventeenth Century to today have decided that infinite regress will stop; for they have so contrived the infinitesimal that when one is divided by another a finite number results, but when one is divided by a finite number it exhibits the properties of zero and is equal once again to itself. In other words, you cannot chop up the isolated infinitesimal into bits and so you cannot carry on a regressive transitive argument over it. The re-identification of a body over an infinitesimal interval of its path is an irreducible claim.

It is relatively easy to see why this should be so. An infinitesimal interval is one which is neither zero nor finite but which ‘tends towards zero’, or may be ‘as small as we like’. Suppose then that it is claimed that a is b but that a and b are separated only by an infinitesimal interval. To apply the transitivity of identity to explain $a=b$ would, in this case, require us to posit the existence of an object, y say, which lays somewhere in the interval between a and b , i.e. somewhere which is closer to a than is b and closer to b than is a . But how could we possibly describe the location of this object y ? If the interval between a and b already ‘tends towards zero’ then we would need to locate y at an interval which tends more closely towards zero than an interval which already ‘tends towards zero’. Even more ridiculous perhaps, if the interval between a and b is already ‘as small as we like’, then y would have to be located at an interval which is smaller than an interval which is already ‘as small as we like’. In other words, transitive arguments cannot sensibly apply across an infinitesimal interval (this property of the infinitesimal is described more fully in Appendix I).

We can see this property of the infinitesimal clearly from a simple example of the derivative of the continuous and differentiable function $y=f(x)$ at the value of the free variable $x=a$:

$$f'(a) = \lim_{h \rightarrow 0} (F(a+h) - F(a))/h$$

For example, if our function were $y=x^2$, then we may define the derivative at the value of the free variable $x=a$ in the following fashion.

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} ((a+h)^2 - a^2) / h && \dots (a) \\ &= \lim_{h \rightarrow 0} (a^2 + 2ah + h^2 - a^2) / h && \dots (b) \\ &= \lim_{h \rightarrow 0} 2a + h && \dots (c) \\ &= 2a && \dots (d) \end{aligned}$$

In this simple example we may clearly see Leibniz' methodological claims in action. Step (a) to (b) is simply the expansion of $(a+h)^2$. In moving between step (b) and (c), however, we not only remove a^2-a^2 to leave $(2ah + h^2)/h$, but we divide $2ah + h^2$ by h to leave $2a + h$. However, this process requires that we divide h^2 by h to obtain h and divide h by h to obtain 1. Here then we encounter our "*quantities infinitely small such that when their ratio is sought, they may not be considered zero*", which may more accurately be expressed:

$$\lim_{h \rightarrow 0} h^2/h = \lim_{h \rightarrow 0} h$$

$$\lim_{h \rightarrow 0} h/h = 1$$

In the step between (c) and (d) however, h has somehow vanished – it has been replaced by zero, i.e. it has become one of those values which can be "*rejected as often as they occur with quantities incomparably greater*". More accurately we may express the step (c) to (d) as

$$2a + \lim_{h \rightarrow 0} h = 2a$$

In other words, h (as it tends towards zero) exhibits the properties of ratio ($h^2/h=h$ and $h/h=1$) but not the properties of magnitude with respect to finite values ($2a+h = 2a$).

We may use this latter property of the infinitesimal to demonstrate its indivisibility with respect to finite terms. Consider the two limits:

and
$$\lim_{h \rightarrow 0} (2a+h) = 2a$$

$$\lim_{h \rightarrow 0} (4a+h)/2 = \lim_{h \rightarrow 0} (2a+h/2) = 2a$$

In the first case we have the expansion

$$\lim_{h \rightarrow 0} (2a+h) = 2a + \lim_{h \rightarrow 0} h = 2a \quad \dots (e)$$

In the second we have

$$\lim_{h \rightarrow 0} (4a+h)/2 = 2a + \lim_{h \rightarrow 0} h/2 = 2a + \frac{1}{2} \lim_{h \rightarrow 0} h = 2a \quad \dots (f)$$

Thus from (e) and (f)

$$\lim_{h \rightarrow 0} h = \frac{1}{2} \lim_{h \rightarrow 0} h$$

In other words, the infinitesimal:

$$\lim_{h \rightarrow 0} h$$

cannot be divided by a finite term; or in as much as we try to divide it it exhibits the properties of zero - and thus, in all interpretations to which we are sensible, is not divisible at all. As such we cannot distinguish (for a single differential coefficient) the case where ΔT 'tends towards zero' from the case where $\frac{1}{2} \Delta T$ 'tends towards zero'. We cannot distinguish between a single value 'tending towards zero' and half of this single value 'tending towards zero' – and thus the

role of the infinitesimal in terminating infinitely regressive arguments (such as those of Zeno) is evident.

3.5 Re-identification and its Continuity with Locke's Principles.

The derivative is therefore employed in the termination of infinitely regressive arguments. However, the two 'ideas' of motion discussed above (the first being that motion is reducible to a description in terms of points in space and time and the second denying this) will result in our placing different interpretations upon the significance of the derivative itself. Consider, for example the claim that an object is re-identified over small but finite spatial and temporal intervals $\delta P_{a,b}$ and $\delta T_{a,b}$. In terms of my earlier terminology of 'temporary names' we may easily express this claim as the three part conjunction (as discussed above):

$$P(a)=P(b)+ \delta P_{a,b} \wedge T(a)=T(b)+\delta T_{a,b} \wedge a=b$$

If we now allow these 'small but finite' terms to become infinitesimal, or define the ratio $dP_{a,b}/dT_{a,b}$ as the ratio of $\delta P_{a,b}/\delta T_{a,b}$ in the limit as $\delta T_{a,b}$ 'tends towards zero', then we may express this as:

$$P(a)=P(b)+ dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \wedge a=b$$

If we believe that the motion $dP_{a,b}/dT_{a,b}$ can ultimately be reduced to a description in terms of points in space and time, then we must treat the employment of the derivative simply as an admission of our own limitation (either of our patience or our measuring devices) and can make no further reduction to the description of this re-identification. The re-identification $a=b$ must for ever remain simply conjoined with the conditions $P(a)=P(b)+ dP_{a,b} \wedge$

$T(a)=T(b)+dT_{a,b}$. If, however, we believe that that motion cannot ultimately be reduced to a description in terms of points in space and time, then the derivative $dP_{a,b}/dT_{a,b}$ becomes intimately related with the re-identification $a=b$ itself. Put simply, we may claim that the re-identification $a=b$ is, in this case, an irreducible re-identification. The nature of this intimate relationship can, however, be simply derived (as described earlier) from our knowledge of the prescriptive, as opposed to descriptive, nature of Locke's first principle (which I have previously formulated as the expression LP.1a).

If we allow the infinitesimal terms $dP_{a,b}$ and $dT_{a,b}$ in the above description of re-identification to actually become zero, then we arrive at the expression:

$$P(a)=P(b) \wedge T(a)=T(b) \wedge a=b$$

But we already know that Locke's first principle insists that:

$$P(a)=P(b) \wedge T(a)=T(b) \rightarrow a=b \quad \dots \text{LP.1a}$$

Where (as described in section 2) the inference ' \rightarrow ' captures the prescriptive, as opposed to the descriptive, nature of Locke's principles. We may therefore express our description of re-identification in a form which we know is logically continuous with Locke's first principle (or which becomes Locke's first principle as its infinitesimal terms actually become Zero):

$$P(a)=P(b)+ dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \rightarrow a=b \quad \dots \text{LP.1b}$$

My argument, of course, is that this expression (LP.1b) cannot be derived if it is believed that motion can be reduced to points in space and time – since, as discussed above, under this interpretation of motion the derivative is merely an admission of our own limitation in patience and measurement. The claim that

motion can be reduced to points in space and time must force us to no more than conjunct a re-identification $a=b$ with conditions of spatial and temporal separation. We may therefore conclude that this interpretation of motion (that it may be reduced to points in space and time) must lead us to an understanding of re-identification which is forever separated from Locke's principles, or where Locke's principles cannot be a limiting case of our conception of motion – for in simply allowing infinitesimal terms to become zero we may never miraculously transcend from a conjunction (\wedge) to an inference (\rightarrow). If our ideas of re-identification and Locke's principles are to be logically continuous, as intuition might suggest, then our description of re-identification must contain an inference (like that employed in Locke's first principle) even before we allow infinitesimal terms to vanish. But in what sense can we assume this inference to exist within our understanding of re-identification unless we assume that such re-identification is intrinsically irreducible across the infinitesimal interval; where the derivative $dP_{a,b}/dT_{a,b}$ is intimately related to the re-identification $a=b$ itself? The answer, of course, is in the sense that movement cannot be reduced to a description in terms of points in space and time (as Bergson claims).

Bergson's claim as to the irreducibility of movement to points in space and time is therefore of special significance to our understanding of re-identification. Re-identification is not simply conjoined with the infinitesimal interval of a movement's path – it is not simply that we have an infinitesimal interval AND a re-identification, but that the interval and the re-identification are intimately wound up in each other – or that this is our understanding of movement; the inseparable combination of numerical continuity with spatial and temporal

continuity. The derivative is not a limitation of either our patience or our measurements. It is the irreducible association between movement and re-identification – movement which cannot ultimately be reduced to a description in terms of points in space and time.

Of course it is somewhat abstract to attach such significance to the use of an inference rather than a conjunction (since both LP.1b and the conjunctive form discussed above are equally commensurate with Locke’s principles). However, we must none the less differentiate between the claim that two objects cannot be at the same place at the same time (Locke’s principle), and the far weaker claim that no two objects (anywhere within the physical universe over all time) happen to be at the same place at the same time. The former is a prescriptive principle whereas the latter is merely a descriptive statement (possibly of fact). My argument is therefore that a consolidation of our understanding of re-identification with Locke’s principles, or the claim that the latter is a limiting case of the former, requires a principle of continuity as expressed in LP.1b.

3.6 The Symbolic Expression of the Continuous Form of Locke’s Principles

I have therefore argued that, with respect to the Bergsonian claim of the irreducibility of movement, our citing of continuous motion as an ‘explanation’ for the re-identification statement:

$$P(a)=P(b)+\delta P_{a,b} \wedge T(a)=T(b)+\delta T_{a,b} \wedge a=b$$

is based upon the principle:

$$P(a)=P(b)+dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \rightarrow a=b \quad \dots \text{LP.1b}$$

where $\text{Lim}_{\delta T_{a,b} \rightarrow 0} \delta P_{a,b} / \delta T_{a,b} = dP_{a,b} / dT_{a,b}$

and which is logically continuous with Locke's first principle:

$$P(a)=P(b) \wedge T(a)=T(b) \rightarrow x=y \quad \dots \text{LP.1a}$$

This continuity between LP.1b and LP.1a is, however of a specific nature. If we allow both $dP_{a,b}$ and $dT_{a,b}$ to become zero in LP.1b then we simply obtain LP.1a.

Likewise, if we allow just $dP_{a,b}$ to become zero, then we obtain:

$$P(a)=P(b) \wedge T(a)=T(b)+ dT_{a,b} \rightarrow a=b$$

which, on the face of it, seems to claim little more than the continuous identity of an object through continuous displacements in time with no associated spatial displacement (the persistent identity of a 'stationary object'). However, if we start with LP.1b and allow $dT_{a,b}$ to become zero, then we obtain:

$$P(a)=P(b)+ dP_{a,b} \wedge T(a)=T(b) \rightarrow a=b$$

which corresponds to nothing in our experience and which is seemingly in contradiction to the inference in LP.2a [or at least would force us to assume some radical alteration – some logical discontinuity - in the persistent qualities of an object as its simultaneous displacements pass from infinitesimal to finite values]. We cannot therefore allow that $dT_{a,b}$ may become zero unless $dP_{a,b}$ also becomes zero.

That we should find such a restriction, and that this restriction should apply to the relationship between the first and second of Locke's principles, is to be expected. For if $dT_{a,b}$ may become zero while $dP_{a,b}$ remains vanishing, then the instantaneous velocity of the object would tend towards infinity. In other words, we would 'tend towards' the case where one object can be at two places at the same time (a violation of LP.2a).

We must, however, be careful of this intuitive amendment to LP.1a. For we may readily conceive of situations in which LP.1b can be violated. I refer here to the case of vanishing or ‘point-like’ particles – of the type often presented in the simplified explanation of classical mechanical descriptions (for example, when we describe a mechanical system in terms of the motion of a single ‘point-like’ particle at its centre of gravity). In being themselves non-extended, it is evident that two such imaginary objects may interact over vanishing spatial and temporal intervals and thus: $P(a)=P(b)+dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \wedge a \neq b$. It is therefore implicit in the following work that I am concerned with the identity, diversity and cardinality of extended objects, or that the principles which I shall address are concerned, not with spatial and temporal objects in general, but with a particular sub-class of such objects. In fact, the application of LP.1b will require us to be somewhat more selective even than this (as described in section 6), but in this section I am concerned simply with giving some justification (regardless of how philosophically tenuous) for my adoption of it.

Having addressed the continuous expression of LP.1b with respect to the properties of vanishing terms, we may now perform a similar operation upon LP.2a (the principle that *one object cannot be at two places at the same time*). However, in this case we must be somewhat more careful. Suppose, for example, we were to formulate (in the terminology of ‘temporary names’) the statement that a and b are finitely spatially separated over a small interval of time and that a is not the same as b, i.e.

$$P(a) \neq P(b) \wedge T(a) = T(b) + \delta T_{a,b} \wedge a \neq b$$

and that that this statement becomes the principle (continuous with LP.2):

$$P(a) \neq P(b) \wedge T(a) = T(b) + dT_{a,b} \rightarrow a \neq b \quad \dots \text{LP.2a}$$

where $\lim_{\delta T_{a,b} \rightarrow 0} \delta T_{a,b} = dT_{a,b}$

This method does indeed seem to be equivalent to our derivation LP.1b and provides us with a principle (LP.2b) which is obviously continuous with LP.2a. In other words, if we allow $dT_{a,b}$ to become zero in LP.2b, then we obtain LP.2a. The difference, however, is that whereas the limit we employed to derive LP.1b actually means something (the instantaneous velocity of a material body), the limit employed to obtain LP.2b does not seem to refer to anything tangible. In the form derived above it does not, for example, appear within a finite ratio of two infinitesimals. In other words, it is simply a mathematical operation upon the small but finite temporal interval $\delta T_{a,b}$ (some abstract process of ‘tending towards zero’) which may or may not mean anything in reality.

Fortunately, however, this particular point need not concern us greatly; for what is important is the way in which LP.2b is applied rather than the way that it is derived (and, as we shall see, this makes any doubt over the validity of the infinitesimal used in LP.2b redundant). When we move from the application of Locke’s principles ‘at a given time’ to their application over a vanishing temporal interval, it is only with some degree of construction that we may apply a single principle of diversity in the first place. More specifically, this ‘single principle of diversity’ now applies to the diversity of continuants themselves, i.e. to the diversity of objects which are themselves continuous in time. Consider, for example, two instances of the continuous form of Locke’s first

principle (LP.1b) applied to the re-identification of two continuants $a=a'$ and $b=b'$:

$$P(a)=P(a')+dP_{a,a'} \wedge T(a)=T(a')+dT_{a,a'} \rightarrow a=a' \quad \dots (i)$$

$$P(b)=P(b')+dP_{b,b'} \wedge T(b)=T(b')+dT_{b,b'} \rightarrow b=b' \quad \dots (ii)$$

The claim that these are indeed two different, or 'distinct', continuants seems to require, not one, but four statements of diversity, i.e.

- (1) $a \neq b$
- (2) $a \neq b'$,
- (3) $a' \neq b$
- (4) $a' \neq b'$,

and the violation of any of these statements would be sufficient to disrupt our intuitive comprehension of the diversity of moving objects – and thus all four are necessary for this 'intuitive comprehension'.

However, if we assume that we already know that $a=a'$ and $b=b'$ (i.e. if we assume that we already know the two instances of LP.1b above), then given any one of these four statements we may derive the remaining three, i.e. we only need to argue that one of these is true in order to claim that all are true. For example:

Given	$a \neq b'$	(2)
From (2) and (i)	$a' \neq b'$	(4)
From (4) and (ii)	$a' \neq b$	(3)
From (3) and (i)	$a \neq b$	(1)

In other words, all we need is one principle which will tell us that $a \neq b'$, and this principle is LP.2b as defined above, i.e.

$$P(a) \neq P(b') \wedge T(a)=T(b')+dT_{a,b'} \rightarrow a \neq b' \quad \dots \text{LP.2a}$$

However, since this single application of LP.2b only works if we already have access to two instances of LP.1b, then we no longer need to rely upon an abstract definition of the infinitesimal term $dT_{a,b}$. In this case we can merely substitute the meaningful temporal infinitesimal from the instances of LP.1b, i.e. $dT_{a,b} = dT_{a,a} = dT_{b,b}$. Put simply, in applying a single instance of LP.2b to determine the diversity of two continuants, the infinitesimal term employed within this instance of LP.2a is not a primitive temporal interval but a temporal interval 'borrowed' from the infinitesimal temporal intervals employed in the instances of LP.1b to which it relates.

In summary then, we have a continuous expression of Locke's principles contained within the expressions LP.1b and LP.2b, but where LP.2b can only be applied (as a single principle) in conjunction with at least two instances of LP.1b, or else its infinitesimal term is not defined (i.e. not defined as an infinitesimal, or not defined as a term which is resistant to finite division). This is not to suggest that Locke's second principle is in some sense dependent upon the first. It is simply that we have chosen to express the diversity of continuants in terms of a single principle.

3.7 Summary.

We have seen then that Locke's principles apply not simply to the identity and diversity properties of material bodies at a given time, but also determine the nature of (or our understanding of) the re-identification of material bodies over space and time. We may summarise this conclusion as follows:

The Lockean Identity and Diversity Material Bodies

Locke's Principles lead to a naming convention (based upon position and time) which ensures the transitivity of identity relationships for particular material bodies, i.e. $P(a)$, $T(a)$, where $P(a)$ is the position of that objects which is temporarily named 'a' at the time $T(a)$, and where Locke's principles themselves become:

$$P(a)=P(b) \wedge T(a)=T(b) \rightarrow a=b \quad \dots \text{LP.1a}$$

$$P(a)\neq P(b) \wedge T(a)=T(b) \rightarrow a\neq b \quad \dots \text{LP.2a}$$

The Lockean Continuity of Material Bodies

With respect to this terminology, the re-identification claim over a small but finite spatial and temporal interval:

$$P(a)=P(b)+\delta P_{a,b} \wedge T(a)=T(b)+\delta T_{a,b} \wedge a=b$$

is based upon the principle:

$$P(a)=P(b)+dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \rightarrow a=b \quad \dots \text{LP.1b}$$

where $dP_{a,b} / dT_{a,b}$ is $\delta P_{a,b} / \delta T_{a,b}$ as $\delta T_{a,b}$ 'tends towards zero', and where LP.1b is continuous with LP.1a. With respect to two instances of LP.1b we may construct the diversity of $a=a'$ and $b=b'$ by a single instance of the principle:

$$P(a)\neq P(b') \wedge T(a)=T(b')+dT_{a,b'} \rightarrow a\neq b \quad \dots \text{LP.2b}$$

Where $dT_{a,b'}$ is a substitution from the related instances of LP.1b, and where LP.2b is continuous with LP.2a.

4. Temporality and Lockean Cardinality

4.1 Introduction.

In concluding the topic of the previous section we now have two important expressions at our disposal: firstly, an expression referring to the identity of material bodies over infinitesimal temporal intervals (LP.1b), and secondly, an expression referring to the diversity of material bodies over infinitesimal temporal intervals (LP.2b). As such we have to hand those expressions required for describing cardinality statements over infinitesimal intervals of time - for a 'Cardinality Statement' is composed of nothing more than n statements of identity and $\frac{1}{2}(n^2-n)$ statements of diversity (as already discussed in section 2.5).

In this section I therefore intend to examine the logical nature of Locke's principles as they are applied to the problem of cardinality, i.e. the claim that there are n distinct objects within a given region of space over an infinitesimal interval of time. As such it is the intention of this section to finally submit to analysis that statement formulated by S (see Section 1) which I take within this work to be indicative of S's ability to 'see' the world in terms of distinct material bodies moving about in space and time.

In presenting more fully that analysis which has already been outlined in the introduction to this work, my intention is merely to convince the reader that isolated (non-quotiented) infinitesimal temporal terms arise naturally within the analysis of cardinality statements, and thus arise naturally within our common understanding of the world of material bodies.

Of course, analysis may often be a somewhat arbitrary process. One may choose to follow one route rather than another, or to highlight one logical property at the expense of others. The only advantage being that, once presented, the steps in any such analysis are open to the scrutiny of the reader and they may decide for themselves whether the methods employed have lead us naturally to the conclusion, or whether the conclusion has itself been artificially coaxed out of the analysis by some selective process.

4.2 An Analysis of Lockean Cardinality (a necessary and sufficient formulation).

In section 2 I discussed those relationships necessary in order to claim that there exists three material objects within a given region of space at a given time - a description in terms of three ($n=3$) instances of Locke's first principle (LP.1a) and three [$\frac{1}{2}(n^2-n)=3$] instances of Locke's second principle (LP.2a):

- (I) $P(A)=P(A) \wedge T(A)=T(A) \rightarrow A=A$
- (II) $P(B)=P(B) \wedge T(B)=T(B) \rightarrow B=B$
- (III) $P(C)=P(C) \wedge T(C)=T(C) \rightarrow C=C$

- (IV) $P(A)\neq P(B) \wedge T(A)=T(B) \rightarrow A\neq B$
- (V) $P(A)\neq P(C) \wedge T(A)=T(C) \rightarrow A\neq C$
- (VI) $P(B)\neq P(C) \wedge T(B)=T(C) \rightarrow B\neq C$

By extension therefore, those relationships necessary in order to claim that there are n continuant objects ($a_1=a_1'$, $a_2=a_2'$, . . . , $a_n=a_n'$) within a given region of space over a given infinitesimal temporal interval may be constructed by n instances of Locke's first principle as captured in LP.1b and $\frac{1}{2}(n^2-n)$ instances of Locke's second principle as captured in LP.2b – where LP.1b and LP.2b (as described in section 3) are the continuous forms of Locke's principles:

$$P(a)=P(b)+dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \rightarrow a=b \quad \dots \text{LP.1b}$$

$$P(a) \neq P(b) \wedge T(a)=T(b)+dT_{a,b} \rightarrow a \neq b \quad \dots \text{LP.2b}$$

and where the isolated infinitesimal term in LP.2b requires that LP.2b be formulated in relation to at least two instances of LP.1b.

i.e.

$$\begin{array}{l} \text{a/} \quad P(a_1)=P(a_1')+dP_{a_1,a_1'} \wedge T(a_1)=T(a_1')+dT_{a_1,a_1'} \rightarrow a_1=a_1' \\ \text{b/} \quad P(a_2)=P(a_2')+dP_{a_2,a_2'} \wedge T(a_2)=T(a_2')+dT_{a_2,a_2'} \rightarrow a_2=a_2' \\ \dots \\ \text{c/} \quad P(a_n)=P(a_n')+dP_{a_n,a_n'} \wedge T(a_n)=T(a_n')+dT_{a_n,a_n'} \rightarrow a_n=a_n' \end{array} \left. \vphantom{\begin{array}{l} \text{a/} \\ \text{b/} \\ \dots \\ \text{c/} \end{array}} \right\} \begin{array}{l} n \\ \text{instances} \\ \text{of LP.1b} \end{array}$$

$$\begin{array}{l} \text{d/} \quad P(a_1) \neq P(a_2') \wedge T(a_1)=T(a_2')+dT_{a_1,a_2'} \rightarrow a_1 \neq a_2' \\ \text{e/} \quad P(a_1) \neq P(a_3') \wedge T(a_1)=T(a_3')+dT_{a_1,a_3'} \rightarrow a_1 \neq a_3' \\ \dots \\ \text{f/} \quad P(a_{n-1}) \neq P(a_n') \wedge T(a_{n-1})=T(a_n')+dT_{a_{n-1},a_n'} \rightarrow a_{n-1} \neq a_n' \end{array} \left. \vphantom{\begin{array}{l} \text{d/} \\ \text{e/} \\ \dots \\ \text{f/} \end{array}} \right\} \begin{array}{l} \frac{1}{2}(n^2-n) \\ \text{instances} \\ \text{of LP.2b} \end{array}$$

The first thing we notice about this formulation of a cardinality statement is that all temporal terms must be equal. This is because of the way in which I have defined LP.2b (or the decision on my part to express the diversity of two material bodies in terms of a single diversity statement). For any two continuants, $ax = ax'$ and $ay = ay'$, their diversity is contained within the expression

$$P(ax) \neq P(ay') \wedge T(ax)=T(ay')+dT_{ax,ay'} \rightarrow ax \neq ay'$$

but this expression requires:

$$P(ax)=P(ax')+dP_{ax,ax'} \wedge T(ax)=T(ax')+dT_{ax,ax'} \rightarrow ax=ax'$$

$$P(ay)=P(ay')+dP_{ay,ay'} \wedge T(ay)=T(ay')+dT_{ay,ay'} \rightarrow ay=ay'$$

where $dT_{ax,ax'} = dT_{ay,ay'} = dT_{ax,ay'}$.

Therefore, because there must exist at least one diversity statement for each pair of distinct continuants within the set of counted continuants, all infinitesimal terms in a/ to f/ must be equal.

Given the equality of infinitesimal temporal terms in a/-f/ the following condition is true of a/-f/:

$$\text{S: } \quad \exists_{m,m'} P(m)=P(m')+dP_{m,m'} \wedge T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$$

$$\text{and } T(ax)=T(m) \wedge T(ax')=T(m') \quad \text{for all } x \text{ in } 1,2,3, \dots, n.$$

Which claims that the objects m and m' exist and are re-identified over infinitesimal intervals of space and time, and that the temporal locations of $a_1, a_2, a_3, \dots, a_n$ are equal to the temporal location of m , and that the temporal locations of $a_1', a_2', a_3', \dots, a_n'$ are equal to the temporal location of m' . This condition (which may equally be interpreted as having the form of a substitution) contains the case where the pair (m,m') may be any of the pairs $(a_1, a_1'), (a_2, a_2'), \dots, (a_n, a_n')$, i.e. that we may take the temporal interval of any of the re-identification statements in a/-f/ and substitute it within the temporal term of all other re-identification statements in a/-f/. For example, if $x=1$, i.e. $m=a_1$ and $m'=a_1'$, then we may construct S as:

$$\exists_{a_1,a_1'} P(a_1)=P(a_1')+dP_{a_1,a_1'} \wedge T(a_1)=T(a_1')+dT_{a_1,a_1'} \rightarrow a_1=a_1'$$

Substitute $T(ax)=T(a_1)$ and $T(ax')=T(a_1')$
 For all x in $1, 2, 3, \dots, n$

The substitution S is not, however, restricted to this interpretation alone and may apply an interpretation of m and m' in terms of any material body (contained within the counted set or otherwise) whose re-identification is subject to LP.1b

Thus with respect to the substitution S: we may formulate a more generalized form of a Lockean cardinality claim over an infinitesimal temporal interval

which is itself defined by the first order derivative of position with respect to time of the material body m at the time $T(m)$:

- 0/ $P(m)=P(m')+dT_{m,m'} \wedge T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$
- a1/ $P(a1)=P(a1')+dP_{a1,a1'} \wedge [T(a1)=T(m) \wedge T(a1')=T(m')]\rightarrow a1=a1'$
- b1/ $P(a2)=P(a2')+dP_{a2,a2'} \wedge [T(a2)=T(m) \wedge T(a2')=T(m')]\rightarrow a2=a2'$
- ...
- c1/ $P(an)=P(an')+dP_{an,an'} \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an=an'$
- d1/ $P(a1)\neq P(a2') \wedge [T(a1)=T(m) \wedge T(a2')=T(m')]\rightarrow a1\neq a2'$
- e1/ $P(a1)\neq P(a3') \wedge [T(a1)=T(m) \wedge T(a3')=T(m')]\rightarrow a1\neq a3'$
- ...
- f1/ $P(an-1)\neq P(an') \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an-1\neq an'$

Having therefore constructed this expression of those relationships (of identity and diversity) required in order to claim that there are n material bodies within a given region of space over the infinitesimal interval $dT_{m,m'}$ (where $dT_{m,m'}$ is itself defined in terms of the first order derivative of position with respect to time of the material body m) we may now 'stand back', as it were, and address ourselves purely to its logical structure – forgetting (or ignoring) for the moment that these symbols are intended to mean anything in particular.

We note firstly that the n statements of identity and $\frac{1}{2}(n^2-n)$ statements of diversity necessary to justify a cardinality claim are held in a1/ to f1/ (which are merely the expressions a/ to f/ under the substitution S). The additional expression 0/ is included merely to provide an infinitesimal temporal interval ($dT_{m,m'}$) which is substituted within a/ to f/. In this case the expression 0/ merely guarantees that $dT_{m,m'}$ is a valid infinitesimal term (i.e. cannot be subjected to finite division) and is thus suitable for forming the infinitesimal terms in a1/ to f1/.

However, upon examination, it is obvious that the spatial term in $0/$ ($P(m)=P(m')+dP_{m,m'}$) plays no role in the definition of a cardinality statement other than (in combination with the temporal term in $0/$) guaranteeing this property of $dT_{m,m'}$. For example:

- (i) The spatial terms in $a/-c/$ are vanishing (infinitesimal) and the meaning of $a/-c/$ is dependent upon these terms being vanishing.
- (ii) The spatial terms in $d/-f/$ are finite and the meaning of $d/-f/$ is dependent upon these terms being finite.

Thus while we may make a common temporal substitution in $a/-f/$ (as in $0/-f1/$) we cannot make a common spatial substitution within $a/-f/$ since:

- (iii) We cannot substitute a finite, or non-vanishing, spatial term [$P(m)\neq P(m')$] for a vanishing one [$P(m)=P(m')+dP_{m,m'}$], nor a vanishing spatial term for a finite one, without losing the meaning of either $a/-c/$ or $d/-f/$ ³⁶.

The condition $P(m)=P(m')+dP_{m,m'}$ (in $0/$) therefore plays no role in the construction of $a1/-f1/$. It is not substituted within $a1/-f1/$, and could not be alternatively employed as a substitute within $a/-f/$ (because $a/-f/$ will not submit to a common spatial substitution). In other words, the expression of cardinality captured in $a1/-f1/$ is totally independent of the spatial nature of that reference continuant ($m=m'$) with respect to which the common temporal interval is determined³⁷. The only role played by the spatial condition $P(m)=P(m')+dP_{m,m'}$

³⁶ Here we see the significance of formulating the “continuous form of Locke’s principles” in terms of formally vanishing, of ‘infinitesimal’, terms. Since if we had not made this move, or if we have formulated these ‘continuous forms’ in terms of small but finite spatial and temporal intervals, then the inability to substitute the spatial terms of $a/-c/$ for the spatial terms of $d/-f/$ (or visa versa) would not be a formal condition.

³⁷ This point is of some importance. Although the spatial term $P(m)=P(m')+dP_{m,m'}$ is not substituted in the formulation of $0/-f1/$, if it were the case that an alternative substitution of this spatial term were possible for $a/-f/$, and if this alternative substitution were in some determinable way related to $0/-f1/$, then we would have to claim that the spatial term in $0/$, whilst not substituted within $01/-f1/$, is still necessary in the formulation of $0/-f1/$.

(in 0/) is in contributing, together with the condition $T(m)=T(m')+dT_{m,m'}$, to the identity of m and m' in a manner commensurate with Locke's first principle.

What consequences will arise then if we omit the spatial term of 0/ from my formulation of the cardinality statement 01/ to f1/?

The most obvious result is that if we omit the condition $P(m)=P(m')+dP_{m,m'}$ from 0/ we can no longer maintain the inference (\rightarrow) to the identity of m and m' – since we may no longer employ Locke's first principle to arrive at this identity. We therefore have a choice (one of those arbitrary choices of analysis discussed earlier perhaps?); either we replace the inference (\rightarrow) in 0/ with a conjunction (\wedge), thus claiming that the temporal term $T(m)=T(m')+dT_{m,m'}$ is merely conjoined with the identity $m=m'$, or else we drop this identity altogether (along with the spatial term $P(m)=P(m')+dP_{m,m'}$).

In other words, as long as we continue to assume that $dT_{m,m'}$ is in some way well defined as an infinitesimal (or exhibits resistance to finite division) we may replace 0/ with either

- 01/ $T(m)=T(m')+dT_{m,m'} \wedge m=m'$
or
02/ $T(m)=T(m')+dT_{m,m'}$

As it turns out, however, and as I shall demonstrate below, there is in fact no arbitrary choice to be made between these options. The logic of our system will insist that we choose 01/ and thus formulate our cardinality statement as:

- 01/ $T(m)=T(m')+dT_{m,m'} \wedge m=m'$
- a1/ $P(a1)=P(a1')+dP_{a1,a1'} \wedge [T(a1)=T(m) \wedge T(a1')=T(m')]\rightarrow a1=a1'$
- b1/ $P(a2)=P(a2')+dP_{a2,a2'} \wedge [T(a2)=T(m) \wedge T(a2')=T(m')]\rightarrow a2=a2'$
- ...
- c1/ $P(an)=P(an')+dP_{an,an'} \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an=an'$
- d1/ $P(a1)\neq P(a2') \wedge [T(a1)=T(m) \wedge T(a2')=T(m')]\rightarrow a1\neq a2'$
- e1/ $P(a1)\neq P(a3') \wedge [T(a1)=T(m) \wedge T(a3')=T(m')]\rightarrow a1\neq a3'$
- ...
- f1/ $P(an-1)\neq P(an') \wedge [T(an-1)=T(m) \wedge T(an')=T(m')]\rightarrow an-1\neq an'$

Here then we have no change in the expressions a1/-f1/ (since they are totally independent of the spatial properties of the reference continuant) but have replaced the instance of Locke's first principle in 0/ with a conjunction between a temporal interval $[T(m)=T(m')+dT_{m,m'}]$ and an identity $[m=m']$. This, I intend to claim, is:

The *necessary and sufficient formulation* of those relationships needed to claim that there are n distinct continuants within a given region of space over a vanishing temporal interval $dT_{m,m'}$.

But in what sense may it be claimed that this expression is both necessary and sufficient? It is obviously sufficient in as much as it fully captures an expression of those relationships needed to claim that there are n continuants within a given region of space over a vanishing temporal interval. But why is it necessary? Why, specifically, should it involve the conjunction $T(m)=T(m')+dT_{m,m'} \wedge m=m'$ (01/) and not just the simple temporal condition $T(m)=T(m')+dT_{m,m'}$ (02/)?

If the necessary and sufficient formulation of Lockean cardinality over a vanishing temporal interval were formulated simply with respect to the condition $T(m)=T(m')+dT_{m,m'}$ (i.e. without being conjoined with $m=m'$ in 0/),

then it would submit to an interpretation in terms of an instance of Locke's second principle:

$$03/ \quad P(m) \neq P(m') \wedge T(m) = T(m') + dT_{m,m'} \rightarrow m \neq m'$$

However, we know that this principle (LP.2b) contains an infinitesimal temporal term which is not primitive – since LP.2b cannot be applied (as a single principle) independently of at least two instances of LP.1b with respect to which its infinitesimal temporal interval is defined. As such we cannot use an instance of LP.2b to supply the temporal term $T(m) = T(m') + dT_{m,m'}$ for substitution within a/-f/ (since this would be the substitution of a term which itself requires definition in terms of other expressions) i.e. we cannot use an instance of LP.2b in place of an instance of LP.1b in expression 0/. Therefore, in requiring us to rule out any such a possibility, the conjunction $T(m) = T(m') + dT_{m,m'} \wedge m = m'$ in 0/ is required (i.e. is necessary). Alternatively, we might claim that in attempting to apply 03/ to the definition of a cardinality statement as the source of the substituted infinitesimal temporal term $dT_{m,m'}$ we would be forced to revert to two additional instances of LP.1b which might equally, and more suitably, be used to define this temporal interval and thus we would be forced to revert to 01/.

So finally then, we see that 01/-f1 is a necessary and sufficient form in which the conjunction between $T(m) = T(m') + dT_{m,m'}$ and $m = m'$ must be maintained in order to obviate interpretations (in terms of material bodies) which would be invalid.

It is to be admitted, perhaps, that this analysis has revealed to us little more than the not altogether surprising fact that in order to claim that there are n material bodies within a given region of space over a given infinitesimal interval of time you have to say what this 'given infinitesimal interval of time' is. The proviso being, however, that this 'given infinitesimal interval of time' must be defined as a valid infinitesimal term (one resistant to finite division) and thus we must resist its definition via a single instance of LP.2b (which we do by the conjunction of the temporal term $T(m)=T(m')+dT_{m,m'}$ with the identity $m=m'$).

Now, of course, the simple statement $T(m)=T(m')+dT_{m,m'} \wedge m \neq m'$ (the statement 01/) does not itself guarantee that $dT_{m,m'}$ is well defined as an infinitesimal. It merely rules out an interpretation in terms of that one principle currently within our system which is unacceptable for the definition of $dT_{m,m'}$ (i.e. LP.2b). The statement 01/ will always stand in need of interpretation to ensure the resistance to finite division of its infinitesimal temporal term $dT_{m,m'}$. The aim of this analysis has been to demonstrate that there is nothing in the formal analysis of a material cardinality statement which insists that this infinitesimal term has to be defined in terms of the first order derivative of position with respect to time of a material body. Any alternative definition of the isolated infinitesimal $dT_{m,m'}$ (such as that presented in the introduction to this work) may equally be employed (even if only as a valid logical exercise) in the formulation of a material cardinality statement.

4.3 Summary of the Analysis.

Lockean Identity and Diversity of Continuants.

The re-identification claim $P(a)=P(a')+\delta P_{a,a'} \wedge T(a)=T(a')+\delta T_{a,a'} \wedge a=a'$ is based upon the principle:

$$P(a)=P(a')+\delta P_{a,a'} \wedge T(a)=T(a')+\delta T_{a,a'} \rightarrow a=a' \quad \dots \text{LP.1b}$$

Where $dP_{a,a'}/dT_{a,a'} = \delta P_{a,a'}/\delta T_{a,a'}$ in the limit as $\delta T_{a,a'}$ 'tends towards zero'. For two such re-identifications $a=a'$ and $b=b'$, the diversity of these continuants is based upon the principle:

$$P(a)\neq P(b') \wedge T(a)=T(b')+\delta T_{a,b'} \rightarrow a\neq b' \quad \dots \text{LP.2b}$$

where $dT_{a,b'}$ is defined in terms of $dT_{a,a'}$ and $dT_{b,b'}$ and is thus not primitive.

Lockean Cardinality of Continuants.

Given n instances of LP.1b and $\frac{1}{2}(n^2-n)$ instances of LP.2b applied to the continuants $a1=a1'$, $a2=a2'$, . . . , $an=an'$, we may capture an expression of Lockean cardinality over a vanishing temporal interval ($dT_{m,m'}$) by the substitution **S**:

$$T(m)=T(m')+dT_{m,m'}$$

$$\text{and } T(ax)=T(m), T(ax')=T(m) \text{ for all } x \text{ in } 1, 2, 3, \dots, n.$$

and thus arrive at the necessary and sufficient form

(one in which $T(m)=T(m')+dT_{m,m'}$ cannot be obtained from an instance of LP.2b):

- | | |
|-----|---|
| 01/ | $T(m)=T(m')+dT_{m,m'} \wedge m=m'$ |
| a1/ | $P(a1)=P(a1')+\delta P_{a1,a1'} \wedge [T(a1)=T(m) \wedge T(a1')=T(m')] \rightarrow a1=a1'$ |
| b1/ | $P(a2)=P(a2')+\delta P_{a2,a2'} \wedge [T(a2)=T(m) \wedge T(a2')=T(m')] \rightarrow a2=a2'$ |
| . | . |
| c1/ | $P(an)=P(an')+\delta P_{an,an'} \wedge [T(an)=T(m) \wedge T(an')=T(m')] \rightarrow an=an'$ |
| d1/ | $P(a1)\neq P(a2') \wedge [T(a1)=T(m) \wedge T(a2')=T(m')] \rightarrow a1\neq a2'$ |
| e1/ | $P(a1)\neq P(a3') \wedge [T(a1)=T(m) \wedge T(a3')=T(m')] \rightarrow a1\neq a3'$ |
| . | . |
| f1/ | $P(an-1)\neq P(an') \wedge [T(an-1)=T(m) \wedge T(an')=T(m')] \rightarrow an-1\neq an'$ |

5. Summary of Book 1.

In the introduction to this thesis I proposed that question which, while not equivalent to the question: “*How does S know the world to be one of material bodies moving around in space and time?*” (Q1) is more well suited to the formulation of such questions when one adopts a certain position with respect to the epistemological description of variables such as ‘S’ (Section 1.1):

Q4a. How does S (or T) know that there are n (rather than n+1 or n-1) material bodies moving about within a given region of space over some given infinitesimal interval of time?

The first Book of this thesis has therefore been dedicated to an analysis of that statement which we may consider to be formulated by S:

There are n (rather than n+1 or n-1) material bodies moving about within a given region of space over some given infinitesimal interval of time.

I have firstly argued that we must construct this statement from n instances of a continuity condition (what I have called the “continuous form of Locke’s first principle”, LP.1b) and $\frac{1}{2}(n^2-n)$ instances of a diversity condition (what I have called the “continuous form of Locke’s second principle”, LP.2b); both of which arise from a consolidation of Locke’s principles of the identity and diversity of material bodies with our common understanding that such bodies may be numerically re-identified over finite intervals of space and time.

The result of this analysis was to arrive at a necessary and sufficient formulation of this statement which employs reference to an isolated infinitesimal temporal term: $T(m)=T(m')+dT_{m,m'} \wedge m=m'$, or an infinitesimal temporal term ($dT_{m,m'}$) which, while necessarily maintaining the properties of a legitimate infinitesimal

(most importantly a resistance to finite division) does not have its own definition specified within this necessary and sufficient formulation.

The question which naturally arises is: must such an infinitesimal term be invariably defined from the first order derivative of position with respect to time ('instantaneous velocity') of a material body (in which case my analysis is trivial and tells us nothing new about Q4a), or is there perhaps some other sense in which such a term may be defined? As I have already argued in Section 1.4 (and upon which I shall expand in Section 7) it is perfectly possible (as an exercise in logic alone perhaps) to define an isolated, non-quotiented, infinitesimal term from the analysis of purely temporal re-identification statements, or from continuity statements of the form $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$. Thus not only is the analysis of Book 1 non-trivial (or tells us something new about the question Q4a) but we have before us a means of progressing from purely analytical concerns, or concerns arising purely from the analysis of statements, to the philosophical concerns of the possibility of purely temporal re-identification and continuity and the systems within which such re-identification and continuity may be manifested.

But even if the reader is indisposed to progressing from analysis to philosophy in this fashion (and it is an indisposition with which I have some sympathy) something has nonetheless been gained from the arguments of Book 1. For it has been demonstrated that the analysis of spatio-temporal continuity and the analysis of purely temporal continuity coincide at the issue of isolated (or non-quotiented) infinitesimal temporal terms; the first revealing such terms in the

analysis of material cardinality statements, and the second in the direct analysis of purely temporal re-identification statements. And should not our curiosity be aroused by this ‘coincidence’ when we remember that this distinction, between spatio-temporal description on the one hand, and purely temporal description on the other, has strong historical echoes in the philosophy of mind (e.g Descartes). And should we not be forgiven (by everyone except the philosopher perhaps) for then proceeding from these analytical ‘facts’ to the philosophy of mind itself – carrying with us, as it were, at least the certainty of this logical framework within which any future philosophical speculations may be both formulated and bound?

Book 2 - A Philosophical Speculation Upon the Nature and Origin of the Individuation of Material Bodies.

An Interpretation of the Isolated Infinitesimal Term.

6. Locke's Principles and Our Movement and Actions.

6.1 Introduction.

In moving from the analytical concerns of Book 1 of this thesis to the philosophical concerns of Book 2, I have already stated that it is my intention to place an interpretation upon the isolated infinitesimal temporal term $T(m)=T(m')+dT_{m,m}$ which, I have claimed, arises within the necessary and sufficient expression of a Lockean Cardinality statement (that statement formulated by S). However, even before we can reach the stage of constructing such an interpretation we must be drawn unavoidably towards those philosophical issues surrounding the terms and expressions of my previous analysis itself. More specifically, I have until now employed Locke's principles within this analysis without any great regard to those purely philosophical objections which may, and have, been made against them. In this section then, I wish to address at least two existing philosophical objections which might naturally arise in connection with my treatment of Locke's principles. This is not, however, a deviation from the stated aims of Book 2 of this thesis, but is instead a natural way to firstly ease the transition from analytical to philosophical concerns (or to argue that such a transition is itself natural in this context), and secondly, to start to pave the way to that interpretation which I shall finally place upon the isolated infinitesimal temporal term of the analysis of Book 1.

6.2 An Outline of Method.

The Lockean conditions of identity and diversity which have primarily concerned me in this thesis refer to the inferred continuity (or persistent

numerical identity) of objects upon the basis of their spatial and temporal continuity alone, i.e. what I have previously referred to as “the continuous form of Locke’s principles”:

$$P(a)=P(b)+dP_{a,b} \wedge T(a)=T(b)+dT_{a,b} \rightarrow a=b \quad \dots \text{LP.1b}$$

$$P(a) \neq P(b) \wedge T(a)=T(b)+dT_{a,b} \rightarrow a \neq b \quad \dots \text{LP.2b}$$

I have already noted, however, that there are cases in which these principles simply fail to apply, or where they do not apply equally to all object classes that we might wish to refer to as 'physical' or 'material'. For example, I have already considered (Sections 2 and 3) how LP.1b is inapplicable to the interactions of imaginary 'point-like' classical particles, and equally, that the 'objects' of quantum theory (if it is sensible to refer to them as such) may seemingly violate both LP.1b [as in the Bose-Einstein Condensate] and LP.2b [as in electron diffraction experiments]. Not all object classes which we might wish to refer to as 'physical', or 'material', therefore have identity and diversity characteristics which are compatible with LP.1b and LP.2b.

But even for the objects of our common experience (those “*material bodies*” to which we may assume Locke’s principles refer unproblematically - such things as tea cups, tables, chairs, cats, dogs, trees, and mountains), or those objects with whose spatial and temporal continuity we are seemingly familiar, there is considerable philosophical doubt as to whether LP.1b and LP.2b can be universally (and unproblematically) applied. More accurately, there are at least two philosophical problems where LP.1b would seem to be directly violated or where it is seemingly denied that we may infer the continuity of a material object purely upon the basis of spatial and temporal continuity alone. I shall

refer to these problems respectively as: **A.** *The Problem of Lot's wife*, following Wiggins (1980 pp 60-1), and **B.** *The problem of the Ship of Theseus*, following Hobbes [De Corpore (II, 7, 2)] and Hughes (1997a).

In part, my approach to these problems, **A** and **B**, will be critical (in as much as I shall present limited arguments against them) but it is not my intention, nor is it within my limited powers of philosophical argument, to dismiss them. Instead, I shall argue that there is one sense in which they are irrelevant to the topic of the continuity of material bodies, or one sense in which we may discuss the topic of continuity without reference to the terms and concepts around which these problems centre. This section will therefore be largely concerned with the philosophical analysis of these two problems themselves and the sense in which LP.1b and LP.2b (my 'continuous forms' of Locke's principles of identity and diversity) may survive them unscathed – or with isolating that sense in which LP.1b and LP.2b may be applied unproblematically.

That I should restrict my arguments within this section to a purely philosophical analysis requires, perhaps, some justification. At first sight it does not seem unreasonable to assume that the application of LP.1b and LP.2b might be the subject of some form of empirical investigation. For example, the empirical work of Spelke and Van de Walle (Eilan, McCarthy and Brewer 1999 pp132-62) seems to point towards the employment by infants of two kinematic principles (principles linking position, velocity and acceleration): (a) a Continuity principle – "*objects move only in connected paths from one place and time to another*", and (b), a principle of Solidity – "*no parts of distinct*

objects ever coincide in space and time”³⁸. These “*kinematic principles*” would seem then to refer respectively to LP.1b and LP.1a (with which LP.1b is continuous). However, if valid, such claims do little more for the arguments of the current thesis than reiterate the already known fact that in our more mature descriptions we often employ the idea of continuity to ‘explain’ re-identification over finite spatial and temporal intervals [as considered in section 3]. Thus while such experiments may well be employed (albeit somewhat selectively) to support the claims of this thesis, I shall restrict myself in this section to a purely philosophical analysis.

6.3 Problem A: The Problem of Lots Wife.

In section 3 we have noted that when Wiggins asks: “*Is a, the man sitting on the left at the back of the restaurant, the same person as b, the boy who won the drawing prize at the school I was a pupil at in 1951?*” he admits that; “. . . *what organizes our actual method is the idea of a particular kind of continuous path in space and time which the man would have had to have followed in order to end up here in the restaurant . . .*” (Wiggins 1980, p49). Here, of course, Wiggins does not suggest that this “*actual method*” is one of observing this continuous spatial and temporal history – since such a method would be unavoidably restricted to relatively small spatial and temporal intervals [or to the “*discontinuities and limits on observation*” (Strawson 1959 p33)]; merely that it is the assumption of the existence of such a history which constitutes (in part) what it is to be re-identified as the same object over space and time. However,

³⁸ In contrast to this, the idea that “*reasoning about objects*” actually involves the employment of dynamic principles, or principles employing concepts of force and mass, is favoured by Peacock (Eilan, McCarthy and Brewer 1999 p163-76).

there is one important case in which Wiggins would deny that spatial and temporal continuity (or a continuous spatial and temporal history) is sufficient to infer re-identification.

As far as Wiggins is concerned, the tracing of continuants through space and time is a sortally dependent process, or is subject to the *Thesis of Sortal Dependency D* (Wiggins 1980 p48):

D $a=b$ if and only if there exists a sortal concept f such that

- (1) a and b belong to a kind which is the extension of f ;
- (2) to say that x falls under f - or that x is an f - is to say what x is (in the sense that Aristotle isolated)³⁹
- (3) a is the same f as b , or a coincides with b under f , i.e. coincides with b in the manner of coincidence required for members of f , . .

In other words, if a is b , then there must exist a sortal concept (whether known or not) f , such that a is an f , b is an f , and a is the same f as b . It is within the Thesis of Sortal Dependency (**D**) that we first encounter problems in claiming that LP.1b applies universally to material bodies, or that spatial and temporal continuity is itself sufficient to infer continuant identity. For the Thesis of Sortal Dependency is directly incompatible with LP.1b if the following possibility (either real or imagined) can occur: That there may exist two objects with a continuous spatial and temporal history but which can be subsumed under no

³⁹ This phrase "In the sense that Aristotle isolated" [D(2)] refers to the sense in which Aristotle attempted to define the category of second substance. For Aristotle the ultimate subject of predication is the concrete individual (first substance). For example, Adam, Red Rum, or the stone in your engagement ring. But if you ask what this 'concrete individual' essentially is, then you will have to specify some kind of substance (or second substance) (Joseph 1925 p50). For example, Adam is a man, Red Rum is a horse, and the stone in your engagement ring is a diamond. Here then, 'man', 'horse' and 'diamond' are types of f s. The reference to a and b belonging to "a kind which is the extension of f " [(1)] employs the idea of extension (as opposed to intension) as in the relationship between genus and species - "in intension the species includes the genus, in extension is included in it" (Joseph 1925 p135). For example, we might say that man (a species) is a kind of animal (a genus), and thus we include the genus in the definition of a species (intension), or we might say that the genus of animal includes (amongst others) the species man (extension). In modern logic, however, the term extension has become more closely associated with the mathematical definition; as, for example, when we replace the intentional definition of 'multiply an integer by two' with the set of ordered pair (1,2), (2,4), (3,6), . . In D(1) the "extension of f " is the set of all things which are f s

common sortal concept. In this case, if there is no sortal commonality, and if the Thesis of Sortal Dependency is true, then spatial and temporal continuity is not itself sufficient to determine the continuant identity of material bodies - and thus, in this case, LP.1b is violated.

Although such cases do not obviously occur in reality, at least not without some degree of construction (as when we reidentify a butterfly with a caterpillar), they may occur in imaginary and allegorical situations. For example, Wiggins cites the case of Lot's wife who, in Genesis chapter 19, is transformed into a pillar of salt. In this case, or so we might imagine, even though there is a continuous spatial and temporal history between Lot's wife and the pillar of salt there is no sortal concept under which both Lot's wife and the pillar of salt may be subsumed (unless, as is rightly repugnant to Wiggins, we invent some kind of intermediate sortal concept 'woman-pillar'). Wiggins is then forced to argue, in accordance with D, that there is no continuant identity between Lot's wife and the pillar of salt, or that Lot's wife cannot be re-identified as the pillar of salt (Wiggins 1980 pp66-7). Now such examples as Lot's wife are, of course, somewhat artificial. However, they raise the serious question of our fundamental ideas about the continuity and re-identification of physical objects. For even if such events do not actually occur in reality, their analysis should reveal something of our actual ideas about continuant identity - i.e. whether, given such an example, we believe there to be a continuant identity between Lot's wife and the pillar of salt [in which case we are applying LP.1b - since there is nothing here but spatial and temporal continuity], or whether we believe there to be no continuant identity between Lot's wife and the pillar of salt [in

which case we must be denying LP.1b]. My argument is that at the most fundamental level we do in fact believe there to be some form of continuant identity between Lot's wife and the pillar of salt, and that this arises purely because of their continuous spatial and temporal histories.

6.4 How Lot should treat his wife.

Genesis Chapter 19 does not tell us about Lot's reaction to the transformation of his wife and so we may take the liberty of elaborating a number of possibilities. Suppose that having just witnessed this miraculous transformation, Lot (temporarily a strong believer in **D**) now believes his wife to be gone and this pillar of salt to bear no continuity or re-identification with his wife. Let us suppose also, that tired and hungry after his flight from Sodom, he produces a piece of meat from his pocket and, being partial to salt, sets about chipping a lump off the pillar (which, for the sake of argument, we shall assume to be Sodium Chloride) . Now in being a strong believer in **D** Lot feels no sense of shame at the possible impropriety of this action. There is no continuant identity between his wife and this pillar. There is no sortal concept under which they may both be subsumed. The principle by which his wife persisted is not the principle by which this pillar persists. What grief he may feel for the demise of his wife has nothing to do with his obtaining salt for his lunch.

The question, of course, is: How do we feel about these imagined actions of Lot? Do we feel that he is acting correctly? Do we feel that his actions are free of any moral sanction because, after all, he is merely taking salt from a pillar and grinding it up for his food? Or do we feel slightly uneasy about his actions?

Do we feel that his action is in some way callous or disrespectful to the memory of his wife? For if we feel uneasy about Lot's actions, then this can only be because we believe, at some fundamental level which is perhaps unavailable to philosophical analysis, that Lot's wife and this pillar of salt are in some sense the same, or that there is some kind of continuant identity between them (unless, of course, we harbour some kind of affection for that region of space which would have been occupied by Lot's wife but is now occupied by the pillar of salt).

Let us now consider another situation (although in this case we must alter the original story slightly). Suppose that upon turning to look upon the destruction of Sodom, God's wrath is such that he makes Lot's wife instantly vanish (to reappear nowhere and thus to cease to exist). Let also assume that at the very instant that this act is performed a pillar of salt miraculously appears some twenty feet from where Lot's wife was standing. Once again, Lot sets about preparing his lunch and once again turns to the pillar for a supply of salt.

How do we feel about the actions of Lot in this second situation? The true supporter of **D** must feel the same about Lot in both these cases. It must make no difference to them whether or not the pillar of salt is spatially and temporally continuous with Lot's wife. In both cases there is no sortal under which both Lot's wife and the pillar of salt may be subsumed, and thus in both cases there is no continuity of identity between these objects.

Let us go one step even further with this argument. Suppose that at the instant that Lot's wife vanished a pillar of salt miraculously appears some twenty miles away. It may, of course, be many hours before Lot and his daughters reach this pillar, but let us assume that there is some way in which Lot can be reliably informed that this pillar appeared at exactly the same time that his wife disappeared. If Lot now goes about obtaining salt for his lunch, then are we to express any moral indignation about his actions?

Now I doubt that anyone, even the most staunch supporter of **D**, could really say that they were perfectly happy with Lot in the first example. All of us, I suspect, would feel some degree of unease or moral indignation with Lot for hacking off lumps of salt from the pillar which was spatially and temporally continuous with his (now 'ex') wife. Equally, I believe that all of us would feel a lesser degree of uneasiness, or a lesser degree of moral indignation (if indeed we felt these things at all) in the second example - since in this case we are more likely to condemn Lot simply for eating at this terrible time rather than for his actions towards the pillar. Finally, I suspect that none of us (although of course I may be wrong) would condemn Lot in the third case - despite his knowledge that the pillar appeared at exactly the same time that his wife disappeared. It is only the true believer **D** who must feel exactly the same about Lot's actions in each of these three cases, and I suspect (although of course I may be wrong) we would be hard pushed to find such a person.

Now I do not, of course, propose this as a serious analytical justification of my claims. But I would ask the reader to honestly reflect upon their moral or

emotional opinions of these three situations. And even if it should be the case that one is only slightly more uncomfortable or indignant in the first case than the second or third (but that there is a noticeable distinction, or that we do not feel exactly the same about each of these cases), then my point is made. For the only distinction between these cases is that in the first there is a continuous spatial and temporal history between the pillar of salt and Lot's wife, and in the second and third there is no such history. If then we can admit to such a difference in our response to these situations, then we have learnt something, not about the philosophy of individuation and re-identification perhaps, but about how we as human beings actually go about identifying and re-identifying material objects. It is not then philosophical analysis which draws us to condemn Lot (in the first of our examples) but the fundamental nature of our ideas about the continuity of material bodies.

The important point, however, is that this belief manifests itself as a response on our part. Our feelings of indignation towards Lot (our 'response') arise because, in this case, we assign continuant identity upon the basis of spatial and temporal continuity regardless of the lack of sortal commonality. But such a 'response' to the lack of sortal commonality is certainly not universal. For example, had we observed Lot's wife leaving Sodom and later encountered a pillar of salt outside the city, then I suspect that there is nothing which would convince us that there is any form of continuity whatsoever between these two things – and in arguing this position we might well cite the fact that there is no sortal concept under which both this pillar of salt and Lot's wife can be subsumed (as no doubt Lot would in the third of our examples above). In this case then, or in the case

where we do not know of a spatial and temporal continuity, the lack of sortal commonality between two objects might be the most important aspect in determining our 'response' to the claim of such an identity. What is important is that there are certain responses upon our part (pertaining to the identity and re-identification of material bodies) which can be characterised with respect to certain principles of identity and re-identification. In some cases these responses (and thus these principles) may depend upon sortal commonality, and in others they may depend upon spatial and temporal continuity. What I am interested in (in my interpretation of LP.1b) are those cases where we know there to be a spatial and temporal continuity and nothing else matters in terms of our 'response' to the claim of a continuant identity, i.e. when we may infer numerical continuity upon the basis of spatial and temporal continuity alone, or when LP.1b may be applied unproblematically.

I have not then, of course, dispatched Wiggins' thesis in these informal discussions on moral indignation⁴⁰. I am merely arguing that there are certain

⁴⁰ A more serious criticism of Wiggins' thesis would centre upon asking how he would consolidate his earlier claim that in re-identifying an object; "*what organizes our actual method is the idea of a particular kind of continuous path in space and time*" with the denial of some form of continuity between Lot's wife and the pillar of salt. For as I have previously argued (Section 3), the citing of such a path to explain re-identification implies the re-identification of an object over a vanishing spatial and temporal interval as a 'brute fact'. If Wiggins is then prepared to deny this 'brute fact' in the case of Lot's wife, then must he not equally abandon it in explanations of re-identifications citing a continuous path – and are we not then left with recourse to nothing more than an infinite number of transitivity arguments for accounting for this method which supposedly "*organizes our actual method*" ? It is not, however, my intention to attack Wiggins' theory here. For at the most fundamental level I would disagree with Wiggins as to what is at issue in the case of Lot's wife, and thus generally, with what is at issue in the question of re-identification (at least for the case of material objects – the re-identification of 'persons' is no doubt more problematic). In claiming that a is re-identified as b, we should not suppose there is some answer to the question; 'is a the same as b?' which is independent of our belief that 'a is the same as b'. In other words, there is not some truth to the matter which is independent of our knowing, although we may know by other means that what we once thought to be re-identified is not. All there is to a re-identification claim is our belief in a re-identification.; there is not some truth of the matter which can be revealed by philosophical or logical analysis. For the 'objects of our everyday experience' re-identification is not the subject

'responses' which we make to certain situations and which are such that we believe that spatial and temporal continuity is indicative of numerical continuity, regardless of any other condition (i.e LP.1b). And if this is the case, then at a fundamental level there are ideas about the continuity of material objects, or of tracing continuants through space and time, which make no reference to the question of "what type of thing is it that moves?" or "what sortal concept it is subsumed under?", or more accurately perhaps, there are ideas which lead us to formulate responses to situations as though it did not matter to us what sort of thing an object is. Indeed, Wiggins himself readily admits that we do not need to know what it is (or what kind of thing it is) that moves in order to know that it moves. And more importantly, that "*Perhaps the man that makes the claim that something moves does not need to know the answer to this question, .*" (Wiggins 1980 p15). It is here, however, in this latter quotation from Wiggins, that part of our successful interpretation of LP.1b lies. For why should it ever be the case, or in what circumstances might it be the case, that someone "*does not need to know the answer to this question*"? In other words, can we generally define those situations wherein it does not matter to us what sortal concept an object (like a tea cup, table, chair, cat, dog, tree, or mountain) is subsumed under? There are, of course, numerous answers to this question. If something is moving towards us very fast, then we do not need to know what it is, we just get out of its way. If something is blocking our path, then we do not need to know what it is, we just move around it. If something is blocking a drain pipe, then we do not need to know what it is, we just push it out. In other words, in our basic mechanical interactions with (and 'responses'

of metaphysics or ontology; it is the subject of psychology (the psychology of our 'inclinations to individuate').

to) the physical world; picking things up, putting things down, walking around things, jumping over things, we do not necessarily need to know what 'kinds of things' things are. Of course, as a matter of survival, we have learnt that there are some things with respect to which we have to be more careful than others, or some things with respect to which we must (as a matter of survival) modify our fundamental mechanical movements. For example, it is useful to know that the thing before you is a Scorpion before you try to pick it up, or that the thing before you is a Lion before you try to jump over it - and perhaps this is all there is to sortal concepts; a sort of modification to 'normal' mechanical action in order to avoid danger or gain reward, or a way of reacting with objects which goes beyond (or is in addition to) a simple 'response' to the properties of their continuity and diversity.

Underlying our common-sense conception of the world is a fundamental framework of identity, continuity, and diversity; and it is this 'framework' which we exploit when we move and act in such a way that it does not matter to us what type of thing we believe a particular object to be. It is this 'fundamental framework' which we exploit when we move identically with respect to a chair, or a table, or a television set, and it is this 'fundamental framework' which we formalise most explicitly within the descriptive basis of classical Mechanics (within which we do not have a mechanics for chairs and a separate mechanics for tables; simply a mechanics of objects in general). That in addition to this framework we also have concepts which lead us to avoid picking up Scorpions and jumping over Lions, simply means that we have additional capabilities which allow us to modify our actions, or to formulate actions which go beyond

a mere response to the identity, continuity and diversity of material objects. It is these 'additional capabilities' that we must fall back upon when we are presented with the question of re-identification over periods of non-continuous observation (when the spatial and temporal continuity of a material body is unknown to us via observation), and thus it is these 'additional capabilities' which manifest themselves within such ideas as the Thesis of Sortal Dependency.

If I then claim that LP.1b and LP.2b are principles of this 'fundamental framework' of identity, continuity and diversity, then LP.1b and LP.2b are not principles of the identity and diversity of a natural kind or class of objects (or a class of objects for which membership of this class is defined by LP.1b and LP.2b)⁴¹. They are instead principles which account for, or describe, or arise within the analysis of, a certain 'response' upon our part to identity and re-identification questions. This is why I have been careful to avoid an ontological interpretation of Locke's principles, and have referred to them instead as principles pertaining to our own 'inclinations to individuate' (Section 2). LP.1b and LP.2b refer to our response to the identity and diversity of the material objects of our common experience, but only in those senses in which it does not matter to us what type of things objects essentially are. The identity and diversity of material bodies (in as much as this identity and diversity is unproblematically subsumed under LP.1b and LP.2b) is defined, not by virtue of their being members of a natural kind, but by our own 'response' to them, or in terms of our own movement and actions with respect to them.

⁴¹ Since I have argued here that they are the principles of identity and diversity of a class of material which are not subsumed under sortal predicates - and the philosopher is likely to question the existence of such a class of material bodies,

6.5 Problem B: The Problem of the Ship of Theseus.

I am not yet, however, free of contradiction in claiming that such a class is compliant to the principles of identity and distinction held in LP.1b and LP.2b. For there is another case in which it would seem that LP.1b is violated for the 'objects of our common experience' - albeit only indirectly. This concerns the question of the identities of composite objects in relation to the replacement of their parts. For example, both Hughes (1997a) and Simons (1997) find the following two claims (about ships) non-problematic:

(REPL) A ship may survive gradual but total part-replacement.

(REAS) A ship may survive disassembly and subsequent reassembly of its parts.

[Although Hughes goes on to modify these principles (Hughes 1997b), this modification need not concern us here]

Here, (REPL) is essentially a statement of LP.1b, since the gradual replacement of parts is compatible with the continuous spatial and temporal history of the ship. (REAS), on the other hand, would seem to be a new principle, or an additional means of determining re-identification - since the ship (as opposed to its part) is not continuous through space and time but disappears at one place and time and reappears at another.

Now although (REAS) would seem to assign re-identification over an extended spatial and temporal interval (without a continuous spatial and temporal history - or the ship ceases to exist at one place and time and reappears at another), this is not itself in contradiction to LP.1b. All that LP.1b claims is that if you have

spatial and temporal continuity then you have continuant identity. That (REAS) might be true only affects our consideration of LP.1b because when taken in conjunction with (REPL) these two principles lead to a conclusion which is seemingly contradictory with LP.1b (or, more accurately, is in contradiction with LP.1a with which LP.1b is continuous). It is relatively easy to see that, when taken together, (REPL) and (REAS) may seemingly lead to the direct violation of Locke's first principle (LP.1a). In fact, as Simons has pointed out, they may lead both to the conclusion that two objects of the same kind may be at the same place at the same time (a violation of LP.1a), and that one object can be at two places at the same time (a violation of LP.2a).

Let us then firstly consider how these violations of Locke's principles arise. Suppose we have two indistinguishable ships [in the sense of what Baillie refers to as a "*qualitative identity*" (Baillie 1993)], S1 and S2, which are at two different places, P1 and P2, at a given time Ta. If we now systematically replace each part of S1 with its corresponding part from S2, and each part of S2 with its corresponding part from S1, then at time Tb we will once again have two identical ships at P1 and P2. The question, of course, is what ship is where. If (REPL) is true, then S1 is still at P1 and S2 is still at P2. However, if (REAS) is true, then S1 is now at P2 and S2 is now at P1. If, therefore, we accept both (REPL) and (REAS) as equally true, then at Tb we have both S1 and S2 at P1, and both S1 and S2 at P2. In other words, we not only have two objects of the same kind at one place at the same time, but we also have one object at two places at the same time (since either S1 or S2 may be considered to be at two places at the same time).

6.6 How Theseus should treat his ship.

To analyse further the nature of these assumption, lets us modify this description slightly. Suppose that as well as S1 and S2 at P1 and P2 at Ta, there was also a third ship S3 at P3 at Ta. And let us further suppose, that while we may inspect the disassembly and reassembly of S1 and S2, we cannot see what happens to the parts in passing between S1 and S2, nor what happens to S3. For example, we might imagine a screen which allows us to see S1 and S2 but not the space in between them or S3. If them S1 is disassembled and reassembled from parts appearing from behind the screen, and if S2 is disassembled and reassembled from parts appearing from behind the screen, then can we now make any assumptions about where S1 and S2 are at Tb? We note firstly that (REPL) can be applied as before - since this principle does not rely upon the origin of those parts which are used for replacement, only that they are at least qualitatively identical to the parts which they replace. Also, (REPL) is commensurate with LP.1b (i.e. S1 and S2 have a continuous spatial and temporal history throughout the process of part replacement). In this case then, we still have the solution that S1 is at P1 at Ta and Tb, and that S2 is at P2 at Ta and Tb. The problem, of course, arises when we come to apply (REAS). For once the screen is in place we no longer know if the parts with which we are replacing parts of S1 come from S2 or S3 (or indeed S1 itself), and we no longer know if the parts with which we are replacing parts of S2 are coming from S1 or S3 (or indeed S2 itself). In this case then, we are probably not even tempted to apply (REAS) at all, and would probably content ourselves with simply claiming (REPL), or that S1 and S2 have persisted through the gradual

replacement of their parts - Locke's principle, and the principle that one object cannot be at two places at the same time, therefore remains intact.

But what is really different between this second situation (where Locke's principle survives) and the first (where Locke's principle is seemingly violated)? The answer to this, of course, is simply that we no longer know the spatial and temporal histories of the parts. We do not know whether the parts which subsequently reassemble S1 are spatially and temporally continuous with parts from S2 or S3 (or indeed S1 itself).

What we are really asserting when we claim that the identity of a ship may survive its disassembly and subsequent reassembly (REAS), is that if we take the same parts and put them together in the same form, then we have the same ship. But the criterion by which we decide if these parts are indeed 'the same parts' is (in this case) based upon LP.1b - that the parts are known to be the same parts because they have continuous spatial and temporal histories. In other words, as formulated here, or as formulated by Hughes and Simons, or as implied within the classical problem of the Ship of Theseus, both (REAS) and (REPL) are ultimately dependent upon LP.1b. (REPL) is essentially just LP.1b reworded, because the continuity of the ship and the continuity of its parts are dependent upon their continuous spatial and temporal histories (although it is not assumed that the same ships must have the same parts). In (REAS), whilst the re-identification of the ship is not dependent upon a continuous spatial and temporal history (of the ship itself) it is dependent upon the continuous spatial and temporal history of its parts.

Now (REAS) is peculiar in that it deals, not with two different ways of re-identifying objects within a common class (of which both ships and their parts are members), but with two essentially different types of objects - or objects which submit to different principles of re-identification. And this is due, I would argue, simply to a subjective element of analysis - of choosing those objects which will be considered as parts (and will thus be irreducible within the analysis) and those considered as ships (and thus emergent from, and reducible to, the arrangement of their parts). It is always possible, of course, to shift this subjective division. There is no actual fundamental division between, say, ships and planks. No doubt we could break down a plank into its parts (a number of splinters perhaps) and then reassemble them elsewhere to obtain [via a modified version of (REAS)] the same plank. However, we are still left with the case that the re-identification of these parts (this collection of individual splinters) is dependent upon their spatial and temporal continuity - or dependent upon LP.1b. The principle (REAS), in isolation, is regressive. We may break a fleet of ships down into individual ships, and individual ships into individual planks, and individual planks into individual splinters, and we could then reconstruct this same fleet at a different place and time from the same splinters, from the same planks, and from the same ships - all by successive modifications of (REAS). But at the bottom of this process we would require that the splinters are re-identified upon the basis of their spatial and temporal continuity, or their compliance to LP.1b. We require LP.1b to terminate this regress, and it is at the point where LP.1b is applied that the distinction between object types is drawn - or the distinction between parts (subject to no further reduction in the analysis)

and composite objects (or emergent objects which are reducible to the arrangement of their parts).

Now if I am to claim that there is a non-contradictory class of objects which satisfy LP.1b and LP.2b, then I have to get rid of this subjective division - and thus get rid of the combination of (REAS) and (REPL) for this class. Quite simply because we cannot allow that Locke's principle [with which LP.1b is continuous] may be violated. This can be done immediately, of course, simply by not allowing any composite objects (or objects made up of parts) within this class. However, this seems rather drastic since I have already said that this class includes (in a particular sense) such things as tea cups, tables, chairs, cats, dogs, trees and mountains - and we should hardly wish to have to accept that these things are not (in some sense) reducible to parts. In what sense then may we consider such things as tea cups, tables, chairs, cats, dogs, trees and mountains, to be irreducible - and thus in what sense may we claim them to be consistently compliant with LP.1b and LP.2b [and free of the violation of Locke's principle implicit in (REPL) and (REAS)]? The answer to this question, I would suggest (because it is the same answer that I gave above), is in the sense in which we move with respect to them. For example, in walking around a tree in order to get out of a forest I act as though the tree were simply an object to be avoided (since this tree and myself cannot be at the same place at the same time). That I may suspect that the tree may be reduced to parts has no influence upon my physical movement with respect to it (in this case). Likewise, in picking up a radio to take it to another room, I act as though this radio were a simple irreducible object. The fact that I know it to be composed of transistors and resistors and the

such like, makes no difference to my physical interaction with it (in this case). In the most fundamental elements of our own mechanics; picking things up, putting things down, walking around trees, jumping over rivers, etc, our interactions with 'physical objects' are interactions with things which are treated as singular irreducible entities. That we later learn that these things are in fact reducible has no effect upon our most fundamental interactions with them.

There are, of course, specialized actions which we perform which rely upon our understanding of the fact that things often have parts (just as there are specialized actions which rely upon our understanding that things may be subsumed under a sortal concept). If I wanted to repair my radio rather than take it into another room, then I would exploit just such a understanding. But even in the process of this repair my most fundamental movements (picking things up, putting things down, avoiding bumping one thing into another) would be formulated with respect to objects which are subject to no further conscious reduction into parts. My picking up a capacitor and soldering it to a printed circuit board does not require me to act as though the capacitor were reducible to parts. Our most fundamental movement and actions are formulated with respect to objects which are treated as irreducible.

So finally then, I may define that class of objects which I claim are subject to the principles of identity and diversity contained in LP.1b and LP.2b, i.e. that class of objects whose numerical continuity may be directly and unproblematically inferred from their spatial and temporal continuity. But this is a definition, not of the intrinsic properties of objects, but of the relationship

between objects and our movement and interaction with respect to them. These are:

The objects of our immediate experience (such as tea cups, tables, chairs, cats, dogs, trees, and mountains) but only in the sense that: (a) It is with respect to their individuation that our physical actions are conducted, or with respect to their continuity that our successful motion is formulated, and (b) that they are individuated, identified and distinguished as irreducible entities, or that while they may be further reduced to parts our interactions with them are independent of this possible reduction, and (c) that while they may be further subsumed under sortal concepts our interactions with them are independent of these sortal characterizations

(b) avoids the violation of Locke's principle which is seemingly inherent within the combination of (REPL) and (REAS), and (c) avoids the possible violation of LP.1b implicit within the Thesis of Sortal Dependency (**D**). In those cases where our more mature and reflective actions may depend upon either the reduction of an object to its parts, or the characterization of an object under a sortal concept, then those objects with respect to which these actions are formulated are not a member of the class I here define. Likewise, if it is the case that our actions towards an object are in part independent of its possible reduction to parts, or characterization under a sortal concept, and equally, in part dependent upon these reductions and characterizations, then this object is a member of the class I have here defined only with respect to those aspects of our actions which are independent of this reduction to parts and characterization under a sortal.

Now it may well seem that I have laboured the definition of this class somewhat; or that I have realised the limitations of LP.1b and LP.2b and am now desperately trying to save them from philosophical criticism. But this is not the case. Individuation and action are intimately connected in the human being.

For in what sense may we say that we are ‘inclined to individuate’, if not in order to formulate some ‘response’ to those things which we individuate, and in what way may we more readily formulate a ‘response’ than in moving and acting in the physical world?

6.7 Summary

In as much as we may describe our movement and actions with respect to material bodies, and in as much as some of these movements and actions are independent of the material body type (its sortal predicate) and its reduction to parts, we move and act in accordance with their identity, diversity and re-identification characteristics alone. In moving and acting with respect to simply the identity, diversity and re-identification properties of material bodies we move and act in accordance with Locke’s principles (or my ‘continuous form’ of them as captured in LP.1b and LP.2b).

But what are the ramifications of making such a claim about the way we move and act? What does it tell us about our movement and actions themselves? Suppose, for example, that it were the case that we moved and acted as though the world were made up of nothing but perfect cubes (and whether the world was actually made up of perfect cubes is neither here nor there). In this case, when we moved from A to B we would do so in a series of straight lines punctuated by right angle turns (because we ourselves would be cubes of course), and when we tried to put one thing on top of another we could do so only in a manner commensurate with the geometric packing properties of cubes. Equally, the rotational symmetry properties of cubes would be reflected in the

rotational symmetry properties of our actions. In this case then, it is relatively easy to see that what is true of cubes is true also of our (successful) movement and actions⁴². What we could say about cubes, or what properties we might derive from their nature, we could also say, and derive, about our movement and actions.

Now we do not, of course, move and act as though the world were one of perfect cubes. Instead we move and act as though the world were one of material bodies whose identity and diversity characteristics are captured in LP.1b and LP.2b⁴³. Must we not therefore equally assume that what is true of material bodies whose identity and diversity principles are captured in LP.1b and LP.2b is true also of our (successful) movement and action? And if, as demonstrated in section 4, LP.1b and LP.2b exhibit (even if as nothing more than a logical possibility) a clear relationship to purely temporal continuity, then can we not equally say that our movement and action may likewise exhibit

⁴² In this case we are imagining a world which may or may not be made up of perfect cubes but is of such a nature that acting as though it were made up of perfect cubes is a way of successfully moving and acting within it. We then assume that some process of evolution has taken place via which has emerged a species of creatures that move and act as though the world were made up of perfect cubes. While these creatures can move and act in anyway they wish, only when acting as though the world were made up of perfect cubes would their movement and actions be successful or beneficial. So in the current example I should really replace the expression “moving and acting” with “moving and acting successfully”, but this is a detail upon which I do not wish to concentrate in this simple example.

⁴³ In this case we are imagining a world which may or may not be made up of material bodies whose identity and diversity principles are captured in LP.1b and LP.2b but is of such a nature that acting as though it were made up of such bodies is a way of successfully moving and acting within it. We then assume that some process of evolution has taken place via which has emerged a species of creatures that move and act as though the world were made up of material bodies whose identity and diversity principles are captured in LP.1b and LP.2b. While these creatures can move and act in anyway they wish, only when acting as though the world were made up of such material bodies would their movement and actions be successful or beneficial. So once again I should really replace the expression “moving and acting” with “moving and acting successfully”, but this is a detail with which I do not wish to distract the reader.

(even if as nothing more than a logical possibility) just such a relationship to purely temporal continuity?

This question would, of course, be nothing more than a mere abstract enquiry if it were not for the fact that when moving and acting in the way that we do (or when moving and acting as though the world were one of material bodies) we are indeed aware of a relationship to some form of (seemingly) purely temporal continuity. I refer here, of course, to our conscious awareness of our movement and actions and the fact that our mental events seem to be continuously located in time but not in space.

There is nothing in these arguments, of course, which must lead us directly to the conclusion that the temporality of our consciousness and the nature of our movement and actions are intimately related. Just the suspicion that if what is true of LP.1b and LP.2b is true of our movement and actions, then it is perfectly logically consistent to claim that they are. What these arguments do achieve, however, is to provide us with a means of moving forward with this speculative philosophical exercise. For we may now legitimately direct our attention to the possible philosophical relationship between Locke's principles and the temporality of consciousness – its “temporal phenomenology” (or what it feels like to feel time passing).

7. Phenomenological Time – Its Properties and Relation to Locke’s Principles

7.1 Introduction

It has been my stated intention to pursue an interpretation of the analysis of Book 1 (the necessary and sufficient description of that statement formulated by S) in terms of infinitesimal temporal intervals which are defined otherwise than with respect to the ‘instantaneous velocity’ of a material body. Likewise, I have indicated in the introduction to this thesis that I shall be interested in the definition of isolated (non-quotiented) infinitesimal terms which arises within the analysis of purely temporal re-identification statements and the condition of purely temporal continuity which is associated with them [$T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$]. In the previous section I have argued that, not only do the continuous form of Locke’s principles apply unproblematically (or free from the philosophical criticisms discussed in the last section) to a certain description of our movement and actions, but that in being unproblematically related to these principles these ‘certain aspects of our movement and actions’ are equally subject to the analytical conclusions of Book 1. Thus I have arrived at the problem of describing our movement and actions (or the movement and action of S) in relation to our ‘knowing’ (or S’s ‘knowing’) that there are n material bodies moving around within a given region of space over a given interval of time.

There is, however, no immediately satisfying route (other than by the arguments of Section 6.7) by which I can move from this position to the claim that it is the temporality of consciousness which must naturally concern us – or that it is the temporal phenomenology of S which is itself purely temporal (whether actually

or only seemingly so to S) and whose continuity may provide for S that very term ($dT_{m,m'}$) with respect to which S may know n. We may, however, hint at such a claim, or more accurately perhaps, we may propose those arguments which make such a claim less extraordinary than it may at first appear. Firstly, there is the suggested link between the brain and our movement and action itself. For example, as Greenfield puts it; *"So the brain then, in whatever, shape, size and degree of sophistication, is somehow connected in a very basic way to ensuring survival as both a consequence and a cause of movement"* (Greenfield 1997). So if it is to the function of the brain that we must turn for the origin of our moving and acting in the way that we do, and if part of this 'moving and acting' is to *"move and act accordingly and independently of their [material bodies] sortal characterisation and reduction to parts"*, and if this itself requires of S at least reference to a *"system of purely temporal re-identification statements $[T(m) \neq T(m') \wedge m = m']$ and a purely temporal continuity $[T(m) = T(m') + dT_{m,m'} \rightarrow m = m']$* , then where else should we look for this system of statements and continuity than within the function of the brain itself. And whether actually or only seemingly so to ourselves we do indeed seem to find such a pure temporality in those operations of the brain with which we are most intimate and familiar, namely; within consciousness: *"The things around us normally have spatial characteristics, such as size, shape, and location. By contrast, it makes no sense to think of our experiences, desires, thoughts, and feelings as having size and shape, and it is even unclear whether we can assign bodily location to these things"* (Rosenthal 1991) – an idea which finds its most explicit formulation perhaps in what Ryle critically refers to as the *"official doctrine"* of Cartesianism: *"It is a necessary feature of what has physical*

existence that it is in space and time, it is a necessary feature of what has mental existence that it is in time but not in space” (Ryle G, 1949).

Such arguments are, however, largely informal and will not provide for us the rigour which we require in order to at last place a philosophical interpretation upon the isolated infinitesimal term and, ultimately, question Q4a as posed at the beginning of this work. Instead, I wish to address the topic of phenomenological time, or time as experienced by consciousness, directly (admitting for now that it is only a suspicion that leads us in this direction) and enquire as to its characteristics - in as much as it may be said to have such ‘characteristics’. Only if these ‘characteristics’ may themselves be shown to correspond, even if only more or less so, to the formal properties of the purely temporal continuity which we seek $[T(m)=T(m')+dT_{m,m'} \rightarrow m=m']$ may we then feel more confident in the leap from the formal properties of temporal continuity to the temporal phenomenology of consciousness which the above informal arguments would seem to imply⁴⁴.

Let us then turn firstly to the topic of phenomenological time itself and those issues (and problems) surrounding its seemingly purely temporal nature.

⁴⁴ I do not mean to sound overly pessimistic in these comments, nor to be overly apologetic for my methods. The simple truth is that there is no strict methodology for moving from formal arguments (such as those presented in Book 1) to philosophical arguments (such as those which are the concern of Book 2). Analysis may only guides us towards, and limit our stupidity in, the formulation of speculative philosophical arguments. Thus the comments here are merely intended to highlight the fact that in moving from the analysis of purely temporal re-identification statements to the claim that the temporal phenomenology of consciousness exhibits the same properties I am, in fact, still, making a highly speculative leap.

7.2 Phenomenological Time and its properties.

Upon what basis upon do we at first distinguish phenomenological time from the 'measured time' of the physicist, or more accurately, what are the senses in which the temporal nature of mental events both coincide with and differ from the temporal nature of physical events? Given that the temporal nature of physical events are now firmly subsumed under the Theory of Special Relativity (Einstein 1922)⁴⁵, then the question of whether, or to what extent, phenomenological time is either identifiable as, or distinct from, measured time comes down to the following question. Upon what basis do we proceed from the recognition of the temporal nature of mental events to the assumption that their temporal properties are subsumed under this theory? Let me firstly consider a specific and highly relevant instance in which this assumption is seemingly made.

It would seem that there is a suggestion [hinted at by Russell (1927, p 384), but certainly more specifically formulated by Weingard (1977) and Lockwood (1984a, 1984b, 1985)] that despite the traditional denial of the spatial location of mental events (e.g. Descartes – see the quotation from Ryle above), relativity theory can somehow be employed to demonstrate that such 'events' are indeed spatially located. For example, Lockwood proposes the following argument (Lockwood 1989, p 72): "*. . . according to special relativity, any two events which are temporally separated with respect to one frame of reference must be spatially separated with respect to some other frame.*" If mental events are

⁴⁵ Perhaps I should refer here, not to the special theory, but the general theory of relativity. However, my arguments will not require me to move beyond the predictions of this 'special theory' and the nature of the 'General Theory' is so removed from common sense that to cite it in the context of this thesis would seem unnecessarily pedantic.

therefore temporally separated, Lockwood argues, then they must be spatially located ("*somewhere*").

While I do not agree with this particular argument⁴⁶, the real question is why Lockwood, or indeed anyone, should consider that Special Relativity can be directly applied to mental events in the first place? In answer to this, Lockwood proposes that; "*mental events are located in time, in the same sense that physical events are*", or that they "*belong to the very same temporal order as do physical events*" [a position which I believe Gibbins (1985) is justified in suggesting introduces a level of circularity into Lockwood's argument].

Yet Lockwood must, in some sense, be right. For it will lead to absurdities if we do not recognise that in certain circumstances, or under certain arguments, the temporal nature of mental events must submit to some of the prescriptions of relativity theory. Most importantly, there are quite clear cases in which we may arrive at the conclusion that, like physical events, mental events exhibit temporal inertial variance (that the temporal interval between two events may be different in two mutually inertial reference frames). Suppose, for example, that a man were to be observed picking up a sea shell, examining it, and putting it down again. We may assume that whatever mental processes accompany these

⁴⁶ It seems to me to imply that mental events may be spatially located with respect to some reference frames but not others – this itself violating, not only the principle of special relativity (since it implies the existence of privileged reference systems for the description of mental events), but also the co-ordinate transformation rules of Special Relativity itself [since it is only spatial and temporal separations which may vanish under the Lorentz Transformation (Lorentz 1892), not spatial and temporal locations] Thus Lockwood's argument would seem to be incapable of escaping the inevitable circularity, namely; that to be spatially separated with respect to "*some other frame*" requires them to be at least spatially located in all. Finally, the whole of Lockwood's argument would seem to rely upon the acceptance that mental events can, in any case, be unproblematically located with respect to an inertial reference frame – something which, as we shall shortly see, is not altogether obvious.

actions they are in some way closely related to these events themselves. The man is thinking about picking up the shell when he picks up the shell. He is thinking about examining it when he is examining it and he is thinking about putting it down when he puts it down. In other words, we may assume that there is some degree of simultaneity between these events and the mental processes of the man himself. Let us then consider what happens when these actions are observed by two different observers. If the observer A observes the actions of this man from the perspective of the inertial reference frame K^{47} , and the observer B observes the actions of this man from the perspective of the inertial reference frame K' , and if K and K' are in a state of uniform relative motion (they are 'mutually inertial'), then the Special Theory of Relativity predicts that A and B will measure a different temporal interval between the man picking up the sea shell and putting it down. The only sensible solution is then to assume that the man is thinking faster with respect to one of these observers than with respect to the other – and thus we arrive at the opinion (as Lockwood suggests) that mental events "*belong to the very same temporal order as do physical events*". However, seductive as it may be, this argument is (as I shall shortly argue) itself at odds with the Principle of Special Relativity.

Now the argument above looks attractive because it seems to be addressing the right topic. In co-ordinating space and time in relation to the principle of inertia, or in terms of 'inertial reference systems', classical mechanics is dependent upon those properties of physical systems which remain unchanged, or are 'invariant', under the simple linear transformations between one inertial reference system

⁴⁷ An inertial reference system is a system of co-ordinates with respect to which the principle of inertia holds true, or with respect to which the spatial co-ordinates of a 'freely moving' particle are a simple linear function of time.

and another. These 'invariants' are not spatial and temporal locations themselves, but are the magnitudes of the intervals between locations. Hence in Newtonian physics we have a mechanics based upon the invariance of independent spatial and temporal intervals under the Galileian Transformation, and in relativistic physics we have a mechanics based upon the invariance of the spatio-temporal interval under the Lorentz Transformation (Lorentz 1892). In mechanical theory therefore, it is the properties of invariants which are of primary importance, not the properties of variant location terms. The attempted absorption of the temporal properties of mental events under the Special Theory of relativity should not then be based upon attempting to argue that mental events are temporally located in the same sense as physical events (and thus also spatially located), but that their temporal intervals exhibit the same properties under transformation as do the temporal intervals of physical events. The argument above (about the man picking up and putting down sea shells) therefore seems to support the claim that the temporal nature of mental events is inertially variant in the same way as the temporal nature of physical events (which itself seems to lead us back to Lockwood's claim that mental events are spatially located). But this overlooks the most important aspects of mental events themselves.

The temporal intervals of mental events are simply the wrong type of thing to be inertially variant; since they cannot be directly, or objectively, measured by an observer and there is no possible situation in which such intervals may be determined with respect to different states of relative motion. Only the man picking up and putting down sea shells knows what the temporal interval 'feels

like' between his thinking of picking it up and his thinking of putting it down. All that the observers 'A' and 'B' can observe are the physical events themselves. In other words, the arguments above overlooks the first person subjectivity of the temporal nature of mental events.

Let us consider another simple thought experiment which will make this position clear. Suppose that I were at a firework display when I see the flash of a particular firework and shortly after hear the sound of its bang. In this case, I am aware that the flash occurred before the bang and that between these two events there was a single definite temporal duration. Suppose now that during this display there is a neuroscientist (equipped with the appropriate apparatus) who is observing my brain activity in minute detail. Let us assume that as a result of the stimulation of my eyes by the flash this neuroscientist observes an event 'a' within my brain. Similarly, as a result of the stimulation of my ears by the bang, the neuroscientist observes the event 'b' within my brain. Being a physicalist, the neuroscientist then claims that the physical event 'a' corresponds to 'what is going on inside my head' when I perceive the flash, and that the event 'b' corresponds to 'what is going on inside my head' when I perceive the bang. Now the events 'a' and 'b' are, of course, perfectly normal observable physical events and could have been seen by anyone who had taken the trouble to observe my brain. The question that we want to ask however is; what is it, which can be observed and measured by the neuroscientist (and which can thus be considered as part of the physical world), which corresponds to my awareness that the flash occurred before the bang and that between these events there was a single definite temporal duration? More accurately perhaps,

can we say that my awareness of the temporal relationship between the flash and the bang (what this interval ‘feels like’ to me, or how I judge its duration) corresponds to the observed and measured temporal relationship between ‘a’ and ‘b’ within my brain?

The answer to this is absolutely and fundamentally “No”. The neuroscientist can measure nothing that corresponds to my intuition of the temporal duration between the bang and the flash – and we can see this by considering the consequences of the reverse claim. Let us designate my intuition of this duration as ΔT_{ab}^i and the temporal interval between the events ‘a’ and ‘b’, as measured by the neuroscientist, as ΔT_{ab}^1 , and let us claim that $\Delta T_{ab}^1 = \Delta T_{ab}^i$ (or that in measuring the time between the events ‘a’ and ‘b’ the neuroscientist has actually also measured my intuition of the duration between my awareness of the flash and the bang). Let us now posit the existence of another neuroscientist (equipped with perhaps even more remarkable equipment than the first) who is travelling towards both the first neuroscientist and myself in a state of uniform relative motion, and let us assume that this second neuroscientist also measures the temporal interval between the events ‘a’ and ‘b’ within my brain - ΔT_{ab}^2 say. Now the Special Theory of Relativity insists that because of the uniform relative motion between the two neuroscientists $\Delta T_{ab}^2 \neq \Delta T_{ab}^1$, and thus $\Delta T_{ab}^2 \neq \Delta T_{ab}^i$.

We should then immediately see the problem. If the first Neuroscientist’s measurement is a measurement of my awareness of the temporal relationship between the flash and the bang, then the second neuroscientist’s measurement is not. In which case we have a violation of the Principle of Special Relativity

(that all mutually inertial reference systems are equally suitable for the formulation of physical laws, or that there exist no 'privileged' inertial reference systems for the description of physical events). Claiming that any measurement whatsoever of the temporal interval between the events 'a' and 'b' (within my brain) is a measurement of my intuition of the interval between the flash and the bang, or a measurement of my awareness of this interval, is to claim that there must exist privileged inertial reference frames – and thus to negate the Principle of Special Relativity⁴⁸.

The temporal properties of mental events cannot therefore be subsumed under the terms and principles of relativity theory - since paradoxically perhaps, the claim that they may is itself a violation of the principle of special relativity - and thus it is not true that they "*belong to the very same temporal order as do physical events*". But surely we should know this anyway (regardless of these more formal arguments from relativity theory). My awareness of the temporal interval between two events (what this interval 'feels like' to me, or how I judge its duration) cannot anyway be related to the measurement of the interval between two events in 'measured time'. Two students sitting through a second year Mechanics lecture may experience completely different intuitions of the

⁴⁸ It is tempting, perhaps, to suggest that the first of these neuroscientists does indeed occupy a special position with respect to my own temporal phenomenology, namely that, providing that there is no relative motion between this neuroscientist and myself, then we are both located within the same inertial reference system. But this can mean nothing more than that this neuroscientist and myself would 'measure' (using a clock) the same temporal interval between the flash and the bang of the firework (and we should note that this neuroscientist, in 'measuring' the interval between the events 'a' and 'b' within my brain, is, in effect, 'measuring' nothing more than the interval between the flash and the bang within his or her own inertial reference system). We must accept the Principle of Special relativity to be telling us that there is nothing of significance in the claim that x and y 'measure' time with respect to the same inertial reference system. Further, there is nothing within this claim which insists that the phenomenological experience of the duration between the flash and the bang is the same for this neuroscientist and myself.

last ten minutes of the lecture – for one the time may seem to pass quickly (such that upon looking up at the clock she is surprised to find the lecture nearly ended), while the other may find the time to pass slowly (such that no matter how often he looks at the clock the last ten minutes seem interminable). Measuring the temporal intervals between physical events cannot measure the phenomenological duration between those events that we experience, nor can we say that the magnitude of the phenomenological duration between two events as experienced by one individual is the same as, or different from, the magnitude of the phenomenological duration between two events as experienced by another individual – for what is the objective criterion by which we could claim that one such magnitude is the same as another?

These then, or so it would seem, are at least two of the defining characteristics of phenomenological durations (the time between physical events as experienced by consciousness):

- (a) Their first person subjectivity; that no one individual may experience the phenomenological durations of another (these durations are somehow separate – as minds are separate perhaps – and exist in independent temporal domains).
- (b) That we may not identify (either as objects or as magnitudes) the extent of one individual's phenomenological durations with the phenomenological durations of another.

7.3 Are mental events located in time at all.

It seems somewhat paradoxical, perhaps, that having laboured my interest in the temporal nature of mental events I should now ask if such events are located in time at all. What I mean to ask, of course, is are mental events located in time; where by 'time' we mean the measured time of the physicist? The initial

response to this question might be to suggest that I can at least correlate my mental events with physical events (the positions of the hands on a clock face for example) and in this much my mental events must be in time in the same way that these physical events are in time. However, from the discussions of the previous section we now know that physical events themselves (like the positions of the hands on a clock face for example) are not simply 'in time' (or at least their intervals and durations are not simply 'in time') they are located with respect to an inertial reference system – and it makes all the difference when talking about the times and the simultaneity and the intervals and the durations of physical events, to state what inertial reference system these simultaneities, intervals and durations are measured with respect to.

Fortunately, we have to hand a relatively simply definition of what it is to be located with respect to a given inertial reference system. If two physical events 'a' and 'b' are 'located' with respect to the inertial reference system K, then the interval (both spatial and temporal) between them can be equally determined, either by repetitive operations performed upon measuring instruments (i.e. by actually measuring the interval between 'a' and 'b' with a measuring rod and a clock) or by calculation based upon the known locations of 'a' and 'b' (within the reference system) and the known geometry of the inertial reference system itself. Only if these two methods of determining the interval between 'a' and 'b' yield the same answer may we claim that 'a' and 'b' are located with respect to the inertial reference system K – since in what sense may we claim that two events are located with respect to an reference system if their interval (as

actually measured) is not that predicted from the geometry of the reference system itself.

But surely I have already argued that it is itself contrary to the principle of special relativity to claim that the phenomenological interval between two mental events can be identified with the temporal interval between two physical events within any inertial reference system (my arguments about the phenomenological interval between my awareness of the flash and the bang of a firework)? If this is so, then our definition of location within an inertial reference system simply cannot apply to mental events.

Mental events may be correlated with physical events which are themselves legitimately located with respect to an inertia reference system. However mental events cannot be likewise located. And this, of course, should be obvious. The example of the two students in the final ten minutes of a second year Mechanics lecture should be sufficient to convince us that there is no intersubjectively testable geometry of phenomenological time – no geodesic along which phenomenological time may be claimed to characteristically pass. Two ‘clock watching’ students may indeed correlate their own mental events with the same physical events (the clock indicating 9:50 and the clock indicating 10:00 say) and these physical events may well be located with respect to the inertial reference system K , but if one of these students feels time to be passing quickly while the other feels it to drag, then each in their turn would ‘feel’ their own phenomenological temporal intervals as different from the other (although if

questioned each could only claim that ten minutes had passed), and thus there can be no geometry of phenomenological time.

The principle of Special Relativity tells us that there is no privileged inertial reference system for the description of the intervals between physical events (there is no 'ultimate' space and time of Newton). Thus the description of physical events must always imply (even if not stated) the stipulation of the reference system with respect to which they are described. If the phenomenological temporal intervals between mental events fail even the most fundamental definition of location (with respect to an inertial reference system), then what sense is left to us in the claim that mental events are located in 'measured' time?

Thus we arrive at that conclusion which we knew all along. There is no possible way in which the phenomenological intervals of two different individuals may be compared (be said to be the same or different). More worrying perhaps, nor is there any way to say that the phenomenological temporal locations of mental events of two different individuals are actually correlated with the same physical event – i.e. if the individual 'x' correlates the mental event 'm_x' with the physical event 'P1', and the individual 'y' correlates the mental event 'm_y' with the same physical event 'P1' (if indeed it is possible to say that the mental events of different individuals are correlated with the 'same' physical event – and I suspect that there are serious difficulties with claiming this), then there is no sense whatsoever in saying that 'm_x' and 'm_y' are 'at the same time'. In other words, the temporal correlation of mental events with physical events is not

transitive. If 'm_x' is correlated with 'P1' and 'm_y' is correlated with 'P1', then we cannot infer that 'm_x' is correlated with 'm_y'.

Thus to those properties of phenomenological time [(a) and (b)] identified in Section 7.2 we may add an important third:

(c) The temporal correlation of mental events and physical events is not transitive.

7.4 A Derivation of the Properties of Purely Temporal Continuity.

We have concluded then, that phenomenological time (or time as experienced by consciousness) is not to be identified with measured time, and further, that far from being illusory at least some of the properties of phenomenological time are easily characterised and familiar to us all⁴⁹. In this section I wish to argue that these easily characterised and familiar properties of phenomenological time may themselves be derived (or at least implied) from that condition which we have already derived upon purely logical grounds as a legitimate solution to the necessary and sufficient formulation of Lockean cardinality statements (Section 1.4), i.e.

$$T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$$

It should be remembered that this continuity condition is proposed (and its term $dT_{m,m'}$ determined as resistant to finite division) in the logical consolidation of

⁴⁹ Of course this is somewhat contradictory to my previous arguments in the introduction to this thesis. What I seem to be saying is that these properties are familiar to S where S is a variable amongst a range of individuals. In this case, however, we would be hard pressed to come up with a strict criterion of S's 'knowing' that T 'knows' these properties. My only justification for this claim is that in Book 2 of this thesis we are simply placing a philosophical speculation around the more rigorously derived statements of Book 1 – and in this much a degree of informality is unavoidable.

two statements. Firstly, a purely temporal re-identification statement over the small but finite temporal interval $\delta T_{m,m'}$:

$$T(m)=T(m')+\delta T_{m,m'} \wedge m=m'$$

and secondly a definition which ensures the transitivity of the identity relationship in such finite purely temporal re-identification statements, i.e.

$$T(m)=T(m') \rightarrow m=m'$$

It was claimed, in Section 1.4, that in a manner directly analogous to the non-regressive description of continuous classical motion, the second conjunction of the expression $T(m)=T(m')+\delta T_{m,m'} \wedge m=m'$ becomes an inference [and thus continuous with $T(m)=T(m') \rightarrow m=m'$] in the limit as $\delta T_{m,m'}$ 'tends towards zero'.

It is perhaps obvious from initial inspection that this inference cannot apply to the description of re-identifications in 'measured time', or what Searle refers to as "*real time*" (Searle 1994 p127). If we start with two instances of this expression:

$$T(m)=T(m')+\delta T_{m,m'} \rightarrow m=m' \quad \dots \text{(i)}$$

$$T(n)=T(n')+\delta T_{n,n'} \rightarrow n=n' \quad \dots \text{(ii)}$$

and if we allow the vanishing terms $\delta T_{m,m'}$ and $\delta T_{n,n'}$ to actually become Zero (as a logical exercise only perhaps), then we obtain:

$$T(m)=T(m') \rightarrow m=m' \quad \dots \text{(iii)}$$

$$T(n)=T(n') \rightarrow n=n' \quad \dots \text{(iv)}$$

from which we may deduce:

$$T(m)=T(n) \rightarrow m=n \quad \dots \text{(v)}$$

In other words, any two objects (of the type m and n – whatever ‘ n ’ and ‘ m ’ may refer to) which are at the same time [$T(m)=T(n)$] must be the same object [$m=n$]. Thus (i), for example, cannot refer to ordinary material bodies and measured time – or else at any one time there could be no more than one such entity. How then may we account for a diversity of the form $m \neq n$ – assuming, that is, that this diversity is sensible (since we do not at present have any idea what it is that m and m' refer to)?

If the expression $m \neq n$ is valid, then we may claim the following conjunction to be unproblematically true:

$$m \neq n \wedge [T(m)=T(m') \vee T(m) \neq T(m')]$$

However since the negation of $m \neq n$ (i.e. $m=n$) is directly inferred from the condition $T(m)=T(m')$, then we may deduce that the only condition which may be conjoined with $m \neq n$ is $T(m) \neq T(n)$:

$$m \neq n \wedge T(m) \neq T(m') \quad \dots \text{(vi)}$$

In other words, if ever we claim (of entities of the type m and n) that $m \neq n$, then the only condition that we may apply to the temporal locations of m and n is $T(m) \neq T(n)$. So if m and n are different (not numerically identical), then m and n must be at different times⁵⁰. This is, of course, a somewhat peculiar conclusion; for it claims that if $m \neq n$, then m and n cannot be at the same time. If then there exists a class of objects for which the condition $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$

⁵⁰ I am holding back from claiming that the condition $m \neq n$ directly infers the condition $T(m) \neq T(n)$ since, strictly speaking this conclusion requires the additional (independent) assumption that given $m \neq n$ we may directly infer whether $T(m)=T(n)$ or $T(m) \neq T(n)$.

applies, then each of these objects exists within its own unique time, and any time at which one such object exists cannot be a time at which any other such object (numerically distinct from the first) may exist. In other words, such objects exist in independent isolated temporal extensions – they exist, if you like, within their own personal and isolate temporal domain (as perhaps minds exist independently and inaccessibly to each other) – and surely this is at least something ‘like’ the property (a) of phenomenological time addressed above?

Let us now apply the condition (vi) in conjunction with (i) and (ii), i.e.

$$\begin{aligned} m \neq n \wedge T(m) \neq T(n) & \dots \text{(vi)} \\ T(m) = T(m') + dT_{m,m'} \rightarrow m = m' & \dots \text{(i)} \\ T(n) = T(n') + dT_{n,n'} \rightarrow n = n' & \dots \text{(ii)} \end{aligned}$$

If $m \neq n$ and $m' \neq n'$, then we may deduce [from (vi)] that:

$$\begin{aligned} T(m) \neq T(n) & \dots \text{(xi)} \\ T(m') \neq T(n') & \dots \text{(xii)} \end{aligned}$$

and if

$$dT_{m,m'} = T(m) - T(m') \text{ and } dT_{n,n'} = T(n) - T(n')$$

then there is no condition under which we can ever claim that $dT_{m,m'} = dT_{n,n'}$ ⁵¹.

Thus if $m \neq n$, then we cannot claim that any interval of the time over which m and m' are re-identified is the same as the interval of time over which n and n' are re-identified. In other words, not only are objects such as m and n located in independent temporal domains (different times), but we cannot equate any interval (duration) of time in one with any interval of time in the other - and

⁵¹ Strictly speaking this formal argument applies to the claim that $dT_{m,m'}$ cannot be the same interval as $dT_{n,n'}$ (i.e. $dT_{m,m'}$ and $dT_{n,n'}$ cannot be two names for one and the same interval). It does not mean that the magnitude of $dT_{m,m'}$ cannot be the same as the magnitude of $dT_{n,n'}$. However, given that $T(m) \neq T(n)$ and $T(m') \neq T(n')$, any claim that the magnitude of $dT_{m,m'}$ is the same as the magnitude of $dT_{n,n'}$ would require a criterion of the identity of magnitudes which cannot be derived from these expressions alone.

surely this is at least something 'like' the property (b) of phenomenological time addressed above?

We have therefore derived, from the purely temporal continuity $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$, two properties which are at least reminiscent of those properties of phenomenological time derived earlier:

(a) Their first person subjectivity; that no one individual may experience the phenomenological durations of another (these durations are somehow separate – as minds are separate perhaps – and exist in independent temporal domains):

$$T(m)=T(n) \rightarrow m=n, \\ m \neq n \wedge T(m) \neq T(m')$$

(b) That we may not identify (either as objects or as magnitudes) the extent of one individual's phenomenological durations with the phenomenological durations of another.

$$dT_{m,m'} \neq dT_{n,n'}$$

and to these two properties we may add an important third:

(c) The temporal correlation of mental events and physical events is not transitive.

If $m \neq n \wedge T(m) \neq T(n)$ then we cannot correlate $T(m)$ and $T(n)$ via a third term, since if $T(m)=T_1$ and $T(n)=T_1$ then $T_1 \neq T_1$

Now in demonstrating that the formal properties of the continuity condition $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$ are 'like' the familiar properties of phenomenological time, I have not, of course, proved that it is the temporal phenomenology of S which allows S to 'know' that there are n material bodies within a given region of space over a given interval of time (and I am not even sure what would constitute the proof of such a claim). I shall try to take this final step (in full acceptance of the fact that I shall not take it as anything other

than a speculation) upon the basis of consistent philosophical argument in the next Section where I shall concentrate upon the philosophy of Bergson and his description of the relationship between the intuition and the intellect. For now I shall simply complete this section by returning to the topic of Locke's principles themselves (or at least my continuous form of them).

7.5 Locke's Principles and Phenomenological Time.

We arrive then at a remarkable (although not perhaps highly satisfactory) interpretation of my previous analysis of Lockean cardinality; one which embodies not only the characteristics of our common sense understanding of physical systems (in terms of material bodies in space and time), but which equally captures the nature of our own temporal phenomenology (in, of course, some limited degree). This then is an interpretation which captures the nature of physical systems on the one hand, and the nature of the mind on the other. It links our understanding of the world with our appreciation of ourselves – or in Bergsonian terms (which will concern me in the following section) it links our “*comprehension of matter*” with our “*intuition of life*”. It describes the origin of our temporal consciousness in evolution by telling us how the nature of consciousness may “*serve our ends*”⁵².

01/	$T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$	The properties of Phenomenological time?
a1/	$P(a1)=P(a1')+dP_{a1,a1'} \wedge [T(a1)=T(m) \wedge T(a1')=T(m')]\rightarrow a1=a1'$	The Lockean Cardinality of material bodies
b1/	$P(a2)=P(a2')+dP_{a2,a2'} \wedge [T(a2)=T(m) \wedge T(a2')=T(m')]\rightarrow a2=a2'$	
...	.	
c1/	$P(an)=P(an')+dP_{an,an'} \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an=an'$	
d1/	$P(a1)\neq P(a2') \wedge [T(a1)=T(m) \wedge T(a2')=T(m')]\rightarrow a1\neq a2'$	
e1/	$P(a1)\neq P(a3') \wedge [T(a1)=T(m) \wedge T(a3')=T(m')]\rightarrow a1\neq a3'$	
...	.	
f1/	$P(an-1)\neq P(an') \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an-1\neq an'$	

⁵² The expression presented here is based upon the direct substitution of the temporal interval in 01/ ($dT_{m,m'}$) for the temporal intervals within expressions a1/-f1/. If we repeat this operation for the temporal interval defined in the continuity term $T(n)=T(n')+dT_{n,n'} \rightarrow n=n'$ where $n\neq m$, then we shall encounter a problem. For example the two substitutions

$$\begin{aligned} & dT_{n,n'} = dT_{a1,a1'} \\ \text{and} & dT_{m,m'} = dT_{a1,a1'} \end{aligned}$$

become contradictory if we apply the property of purely temporal continuants derived in the previous sub-section, namely $n \neq m \wedge dT_{n,n'} \neq dT_{m,m'}$ since this would imply $dT_{a1,a1'} \neq dT_{a1,a1'}$ – which we may interpret either by saying that $dT_{a1,a1'}$ does not exist (since all things which exist must be identical to themselves), or else that we should not use direct substitution within the formulation of Lockean cardinality statements but should instead use some defined symbolic non-transitive form.

How much faith we can place in this interpretation (as it stands) is, of course, questionable; and I should be pushing the credibility of the reader too far to ask them to accept that there is, within this description, anything approaching a formal proof. All that I have done here is attempt to construct an argument with respect to which the philosophical descriptions of the following section may appear less abstract, or with respect to which these ‘philosophical speculations’ may arouse within us the suspicion that they deal directly with a problem which (after the trials of this thesis) are quite familiar to us.

8. A Philosophical Speculation Based Upon Mitchell and Bergson

8.1 *The Need for a Philosophical Interpretation*

In claiming in this section to present a ‘philosophical speculation’ (or an example of a ‘philosophical speculation’) I mean simply to propose an interpretation of my previous (largely analytical) arguments within the wider context of a philosophical system, or within the wider context of some theory of the world, or of ourselves, or of our knowledge⁵³.

Firstly, however, let me summarise those claims which have until now concerned me.

1. The analysis of Lockean Cardinality statements (or the claim that there exists n material bodies within a given region of space over a given interval of time) reveals a formal relationship to the purely temporal expression $T(m)=T(m')+dT_{m,m'} \wedge m=m'$ ⁵⁴ (Section 4).
2. Such an expression may be directly interpreted in terms of the analysis of purely temporal re-identification statements and their associated expression of purely temporal continuity $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$ (Section 1).
3. The purely temporal continuity expression $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$ pertains to a continuity which exhibits properties notably similar to the properties of the temporality of consciousness (Section 7).

If it is these claims themselves which are to be consolidated within a philosophical speculation (or consolidated within some wider context of our understanding of the world, or of ourselves, or of our knowledge), then I am

⁵³ As I have already stated, it is my belief that there exists no rigorous route for progressing from analysis to philosophy. The ‘philosophical speculation’ which I present in this section is therefore merely an example of the type of philosophical problems which must be accommodated and overcome in order to establish my analytical claims within a wider philosophical context. I make no claims as to this ‘philosophical speculation’ being a self-contained philosophical argument; merely a demonstration of the way in which my analytical claims may be seen to both bound and limit the nature of our speculations themselves.

⁵⁴ It should be remembered that in Section 1 I take Lockean Cardinality statements to be numerically quantifiable derivatives of the more informal claim that the world is one of material bodies moving about in space and time.

interested in identifying at least one philosophical system within which (a) the temporality of consciousness has about it the nature of a persistence [since the expression $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$ itself implies a description of time in terms of the persistence of entities through it], and (b), where there is some clear relationship between this 'persistence', on the one hand, and our understanding that the world is composed of material bodies in space and time on the other.

My choice of the work of Mitchell for this purpose arises (as we shall shortly see) not simply because his work relates a temporally persistent view of consciousness to our understanding of physical systems, but because his work will also allow us to accommodate another topic discussed within this current work, namely;

4. That numerical identity, diversity and re-identification are not observable properties.

In other words, Mitchell will present us with an example of a philosophical system (a wider philosophical context) within which we shall be able to accommodate each of the points 1 to 4 above.

Given my objectives for this section (that of demonstrating how my analytical claims may be accommodated within a wider philosophical context) I have no need to progress beyond the claims of Mitchell himself – for while there may be limitations to, and inconsistencies within, Mitchell's claims, it is not the intention of this section to propose a definitive philosophical argument (that is best left to the philosophers themselves). However, as we shall discover, Mitchell treats persistence as a primitive property of the temporality of

consciousness (or as just one of many properties of consciousness which lay beyond further analysis – ‘consciousness’ itself begin some undefined thing in itself which can possess properties such as temporal persistence). As it turns out, this view presents a number of technical problems which, in the end will lead us away from Mitchell’s arguments upon the basis of contradiction.

Thus even though it is not my aim here to present a purely philosophical argument (but merely to demonstrate how my analytical claims may be accommodated within a wider philosophical context) I would have failed in my objective if the philosophical context within which my analytical claims are accommodated is itself inconsistent.

My solution to this problem is to move from Mitchell (maintaining what is useful in his claims) to Bergson – whose philosophy will more consistently accommodate the claims 1 to 4 above. However, in moving to the philosophical ideas of Bergson we depart radically from the philosophical context of the work of Mitchell. Most importantly, Bergson does not treat the temporality of consciousness as merely a property of consciousness (‘consciousness’ itself being a thing which may possess properties like temporality), instead, Bergson treats consciousness as identical with temporality. For Bergson, consciousness is time (in a rather specific sense). Not only does Bergson differ from Mitchell in this respect, but he also differs on the nature of time itself. While Bergsonian philosophy does indeed allow of a description of time in terms of persistence [and thus in terms within which we may accommodate the purely temporal expressions $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$ and $T(m)=T(m')+dT_{m,m'} \wedge m=m'$] he

also introduces the idea of “*real duration*” and claims this to be more primitive. In Bergsonian terms then, not only is consciousness to be identified with temporality, but temporality may have a description in terms other than that of a ‘persistence’.

It might seem then that Bergson is a rather unpromising example to pick for the purposes of this current section. After all, Bergson’s “*real duration*” cannot itself be interpreted in terms of those purely temporal expressions which have concerned me up to now in this thesis (since it is not of the nature of a persistence). However, Bergson introduces us to a rather useful distinction between what he calls “*intuition*” and “*intellect*”. For Bergson, the “*intellect*” is basically our “*comprehension of matter*” (or to use an expression of Russell’s: the power of “*separating one thing from another*”). On the face of it then, Bergson’s definition of the “*intellect*” is the perfect place to look for an interpretation of Locke’s principles (by which one thing is numerically distinguished from another) and their application in Lockean Cardinality statements. The problem is that Bergson’s definition of the “*intuition*” is one of an immediate awareness of our own “*real duration*” (which is not of the nature of a persistence). Thus while Bergson’s definition of the “*intellect*” provides a perfect point for discussing Lockean cardinality statements, his definition of the “*intuition*” will not subject itself to an interpretation in terms of the purely temporal expressions $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$ and $T(m)=T(m')+dT_{m,m'} \wedge m=m'$. My argument, however, will be that if we can formulate Lockean Cardinality Statements via the “*intellect*” (our “*comprehension of matter*”), and if such Lockean Cardinality Statements reveal a relationship to the purely

temporal expressions $T(m)=T(m')+dT_{m,m'} \rightarrow m=m'$ and $T(m)=T(m')+dT_{m,m'} \wedge m=m'$, then there must (within the Bergsonian context) be a description of time via the “*intellect*” which (unlike the “*intuition*”) has about it the nature of a persistence. In other words, if consciousness is time (according to Bergson), then there are equally both two types of time and two types of consciousness – “*real duration*” on the one hand and persistence on the other – or at least two different ways of describing the same thing (one through the “*intuition*” and one through the “*intellect*”) ⁵⁵.

I should re-iterate, however, that whatever the reader’s opinion about the philosophy of Bergson, it is the intention of this section, not necessarily to promote a Bergsonian view, but to demonstrate the types of problems which we may need to overcome in placing my analytical claims within any wider philosophical context.

8.2 Philosophical Context.

This conception [of Bergson’s] of the simultaneous growth of matter and intellect is ingenious, and deserves to be understood. Broadly, I think what is meant is this: Intellect is the power of seeing things as separate one from another, and matter is that which is separated into distinct things. In reality there are no separate solid things, only an endless stream of becoming, in which nothing becomes and there is nothing that this nothing becomes.

Bertrand Russell 1946.

⁵⁵ We might say (or so I, at least, would argue) that Bergson is committed to a kind of second order perception (a perception of persistence via the “*intellect*”) of a passage of a first order (“*real duration*”).

In being concerned with phenomenological time, or time as experienced consciousness, we are presented at first with a choice regarding the nature of the relationship between consciousness and its temporality. Is temporality a property of consciousness or is temporality consciousness itself? Is the 'feeling' of time passing something which consciousness facilitates or is this 'feeling of time passing' consciousness itself? In adopting here a Bergsonian philosophical context it is the latter of these options to which we must subscribe. For while an intimate association between time and mind has a heritage going back as far as Augustine, who describes time as "*affections of the mind*" (Confessions, 11.27.36), and Kant, who argued that time is a [form of] "*internal sense*" (Kant 1781 – Politis 1997 p56), it is not until Bergson (1859-1941) that we find a more explicit identification between time and consciousness itself; "*When we consider a living being, however, we find that time is the very essence of its life, the whole meaning of its reality.*" (Wildon-Carr 1911 p 17). Thus while both Augustine and Kant would have us remove time from its traditional role as a property of the world and place it instead clearly within the properties of the mind – as though the temporality of consciousness were, as Lockwood puts it, "*a kind of paradigm of temporal relatedness, which we then extend to the world at large*" (Lockwood 1989) – Bergson would have it that time exists within the world as manifested in life; or that life is time (or "*real duration*"; the *durée*) and that the awareness or intuition of life (consciousness) is the awareness or intuition of this real duration: "*The principle then of this philosophy is that reality is time, that it can only be expressed in terms of time, that there is no stuff more resistant nor more substantial than time, that it is the very stuff of which life and consciousness are made.*" (Wildon-Carr 1911 p76). Thus within

this context I may unashamedly identify consciousness with “*real duration*”⁵⁶, or to put it in more contemporary terms; phenomenological time is not a property of consciousness but is consciousness itself.

It is Bergson then, more than any other philosopher perhaps, who takes ‘real duration’ seriously; to the extent that the idea of time as an objective quantifiable phenomenon, such as that envisaged by Newton (or time in which measurements may be made by setting temporal states side by side in juxtaposition so that they may be counted) is rejected upon the grounds that it “*surreptitiously brings in the idea of space*” (Bergson 1910 p85) and thus “*fails to capture time’s true essence*”. (Bergson 1922, Ed. Durie R, 1999 vi). Phenomenological time, or time as experienced by consciousness, Bergson argues, is best characterised by what he refers to as a “*multiplicity of interpenetration*”, or as Durie puts it; “*Pure duration . . . is encountered when consciousness refrains from separating its current state from previous states, from trying to set psychic states alongside one another*” (Durie R, 1999 vi).

The context of Bergson’s extraordinary claims as to the distinctions between time and space lies in his attempts to validate the reality of human free will from an analysis of our immediate experience of time (Bergson 1910). More specifically, in order to validate the reality of human free will, Bergson subscribes to a dualism between this “inner experience of time” (“*real duration*”, the *durée*) and space outside (Bergson 1910). Most importantly, Bergson

⁵⁶ More specifically, Bergson would claim that “*true duration is known to us by direct inner perceiving, an intuition . . . And the true duration which we know when we have this intuition is life*” (Wildon-Carr 1911 p 21).

argues that real duration is qualitative and heterogeneous with “no hint of predictability or linear determinism” (Mullarky 1999, p9), whereas space is quantitative, homogeneous and static. It is therefore in the scientific description of time (where the similarities between time and space are most importantly highlighted) that Bergson would argue that the concepts of space are surreptitiously brought into the concept of time (or the scientific description thereof). It is the description of ‘space-like’ scientific time that involves the “elimination of real change” (Robinet 1972).

After Einstein, of course, some such distinction between ‘measured time’ and ‘real duration’ is necessary if we are to maintain the special status of the temporality of consciousness, or to single it out (as I intend to do) as being of special relevance to the description of the mind. If time is forever left to refer exclusively to the ‘measured time’ of the physicist then the Theory of Special Relativity makes a nonsense of Ryle’s characterisation (and derision) of the doctrine of Cartesianism and thus a nonsense of applying any particular significance to the temporality of consciousness in the description of the mind.

More commonly, however, many contemporary philosophers simply choose to avoid the subject of temporality altogether; as, for example, in Searle’s characteristically honest claim that “*Two subjects are crucial to consciousness, but I will have little to say about them because I do not yet understand them well enough. The first is temporality . . .*”, and later that; “*Notoriously, phenomenological time does not exactly match real time, but I do not know how to account for the systematic character of these disparities*” (Searle 1994 p127).

Yet Searle is nonetheless right to claim that temporality is “*crucial to consciousness*”, and equally right to mirror Bergson in distinguishing

phenomenological time from what he calls “*real time*”; for if Bergson is correct, then the temporality is not merely a feature of consciousness (to be lumped together with other features such as its subjectivity, intentionality and physical efficacy); it is consciousness itself⁵⁷.

While Bergson will therefore largely define the philosophical context of this work, or account (in part at least) for my insistence upon addressing the topic of consciousness via its temporal phenomenology (its ‘real duration’), I shall equally exploit other aspects of Bergsonian philosophy within my methods. Most importantly, as outlined above, I shall exploit that aspect of Bergson’s distinction between “*intuition*” and “*intellect*” which may be captured in the claim “. . . *true duration is known to us by direct inner perceiving, an intuition, and not by an intellectual act such as that by which we perceive the objects around us and the laws of their successive states*” (Wildon-Carr 1911 p 21). While this is, of course, partial support for the special status of “*real duration*” in the description of consciousness, it is also, and more importantly, a distinction between our knowledge of that which we essentially are (“*real duration*”) and that which we consider the physical world to be (“*the objects around us and the laws of their successive states*”) – this being, as much as

⁵⁷ This much may itself be evident from the central role that we might expect temporality to play in an understanding of the phenomenological nature of consciousness. For example, when Nagel suggests that we cannot know “*what it is like to be a bat*” (Nagel 1975), he is no doubt largely correct - or correct in the proposition that one conscious individual may not directly know the phenomenology, or the “*what-its-like-ness*”, of another. However, if bats are indeed conscious (and I have no reason to assume that they are not) then they have at least this in common with ourselves; their intuition of life is equally Bergson’s ‘real duration’. To perceive by the emission and detection of high frequency sound (as Nagel puts it: “*Their brains are designed to correlate the outgoing impulses with the subsequent [my emphasis] echoes, . . .*”) is itself indicative of a process of perception which is dependent upon ‘real duration’ – unless, that is, we are willing to admit that for the bat, unlike ourselves, what is “*subsequent*” is subsequent in ‘measured time’ and thus abandon any hope of unifying bat consciousness and human consciousness under any common understanding.

anything perhaps, the true concern of this current section. More accurately, I shall pursue the suggestion that what Bergson refers to here as “*an intellectual act . . . by which we perceive the objects around us and the laws of their successive states*” is in fact an act of personification (for want of a better word), or is an application of a Bergsonian intuition (a “*direct inner perceiving*”, or the direct inner perceiving of our own “*true duration*”) to the representation of our experiences.

This step requires, however, that we should firstly turn from Bergson to his contemporary Mitchell for a description of the individuation of those objects which we perceive around us. For Mitchell (whose book “*The Structure and Growth of the Mind*” was published in the same year, 1907, as Bergson’s “*Creative Evolution*”) informs us that the individuality of a material body (indeed any entity) “*is borrowed from our own*” (Mitchell 1907, pp154-5). Mitchell therefore provides for us a mechanism of representation via personification; a familiarity with the world or a “*fellow feeling*” towards it (Mitchell 1907 pp 146-163). The individuation of the world around us, its division into those “*objects around us and the laws of their successive states*” - our knowledge of which Bergson separates from the “*intuition*” of our own “*real duration*” - therefore results from the imposition of our own individuality upon it or the recognition of our own individuality within it.

It is with the consolidation of Bergson’s identification of the significance of ‘real duration’ (in the description of life and our awareness of it - consciousness) and Mitchell’s identification of personification, or “*fellow*

feeling”, in the processes of individuation that the concerns of this section lie; for I shall demonstrate that Mitchell’s claim is itself intimately related to, and indeed dependent upon, both Bergson’s interpretation of time and his distinction between the intuition and the “*intellect*”.

8.3 Mitchell’s Personifying Claims.

In attempting to account for our formulation and application of Locke’s principles (within our ‘inclinations to individuate’), and in attempting to account for those formal properties of Lockean cardinality revealed in section 4, and in attempting to accommodate within this account my claim that identity and diversity are not observable properties (section 2), I adopt at first the personifying claims of Mitchell, namely; that “*Our thoughts of an object must consist entirely of what we have experienced, and merely for that reason we may be said to read nothing into these things except ourselves, meaning by ourselves our experience present and past. In this sense we read ourselves into the ultimate properties of matter; into those, namely, by which we account for the change and permanence of things*” (Mitchell 1907, p152) . That this claim extends even to the identity or individuality of objects themselves is evident in the later quotation: - “*A tune, a shape, a movement, a thing, a group of things, may seem to need no constructing for our apprehending them. But they do; they need construction as mere sensations, and further constructing as objects of thought. Both constructions imply our individuality, and the individuality that we ascribe to the objects as real is borrowed from our own*” (Mitchell 1907, p155).

The attraction of this claim, of course, lies in its ability to accommodate the fact that the numerical identity, or individuality, of a material body is not itself observable (or that mere sensation may not reveal to us the identities of external things – Sections 2). In Mitchell's claim the individuality of an object is "*borrowed from our own*", or has an "*individuality*" like our own imposed upon it. Mitchell's claim does not require that the identity of a thing must be made available to us via experience and thus provides for us an explanation of the very origin of individuation itself. Individuation is, if you like, simply the projection of our own "*individuality*" (or, more accurately, the properties of this "*individuality*") upon the world in our attempts to represent it. In Mitchell's claim the individuality of a material body presupposes our own.

Yet we cannot consider Mitchell's claim fully, nor can we even begin to accept it as a theory of individuation, until we have understood something of what he intends by the term "*individuality*"; for while he avoids any explicit definition of this term he clearly identifies this "*individuality*" with our own predominantly temporal character: "*Our sensations occupy time, and there is usually, if not always, some sense of their duration and the order of their coming; and to feel continuity or an order of sensations we must be the same throughout the change*" (Mitchell 1907 p 155). It would therefore seem that Mitchell at first locates our own "*individuality*" within the nature of our own temporal persistence. If to be individual, or to be possessed of an individuality or identity, can mean little more (perhaps) than the ability to enter as a subject into relationships of identity and diversity, then to remain unchanged, or the "*same throughout the change*", implies the necessary re-identification of that which remains permanent. It would seem then that the very notion of persistence (or

permanence with respect to change) must itself imply some notion of continuant identity (the ability to re-identify that which persists with itself) and thus, in the loosest sense, some form of “*individuality*”⁵⁸. If then individuality, or the sense in which something is the same, is (for Mitchell) to be “*the same throughout the change*”, then Mitchell characterises our own “*individuality*” in terms of our own temporal persistence – we are, if you like, that unavoidable sense of permanence which must seemingly accompany any perception of change. We are that which persists, or that which persists unchanged throughout change⁵⁹.

While this notion of our ‘persistence’ (and its relation to our “*individuality*”) stands patently in contradiction to the overriding Bergsonian tones of this current section (as discussed in Section 8.1), we should nonetheless be clear on the indispensability of this idea in any attempt to consolidate Mitchell’s claim with the formal properties of Lockean cardinality outlined in the previous sections.

That Mitchell’s claim (when thus formulated) is itself consistent with the formulation and application of Locke’s principles, and indeed goes some way to explaining them, can be justified by noting that this idea of our persistence (and thus our “*individuality*”) lends itself to a description in terms of temporal re-identification. For example, persistence between the times T1 and T2 (corresponding to some awareness of change perhaps) would then seem to

⁵⁸ The question of whether either of these notions, ‘persistence’ or ‘individuality’, may be truly said to presuppose the other, or whether they must equally presuppose each other and therefore in some sense are two descriptions of the same thing, shall not be pursued here.

⁵⁹ I shall discuss this characterisation of change further in the following sub-section.

imply that there is something, 'a' say, which is associated with T1, and something, 'b' say, which is associated with T2, and that $a=b$ (although of course in this case, or in the case of our own perceived persistence, we may find it extremely difficult to say what it is that 'a' and 'b' refer to). In terms of my previous terminology of temporary names we can express this idea of persistence as the conjunction of a temporal and an identity condition $T(a) \neq T(b) \wedge a=b$, i.e. the conjunction of a temporal interval and an identity with no necessary reference to spatial terms. It is then a small step to argue that for the persistence $T(a) = T(b) + \delta T_{a,b} \wedge a=b$ over a small but finite temporal interval $\delta T_{a,b}$ we require recourse to an infinitesimal to avoid an infinite regress (as discussed in section 3 and Appendix 1) and thus arrive at the description of persistence captured in a conjunction of the form: $T(m) = T(m') + dT_{m,m'} \wedge m=m'$. If then the Necessary and Sufficient formulation of Lockean Cardinality statements demands only that we reference a conjunction of the form: $T(m) = T(m') + dT_{m,m'} \wedge m=m'$ (Section 4), then Mitchell's claim is fully consistent with the formal properties of the identity and diversity of material bodies⁶⁰. Put simply, any theory of individuation which is consistent with the formal properties of Lockean cardinality outlined in the previous section must, as a minimum requirement, account for the occurrence of conjunctions of the form $T(m) = T(m') + dT_{m,m'} \wedge m=m'$. Mitchell's claim accounts for this

⁶⁰ We might note, however, that while Mitchell obviously intends to suggest that our persistence is temporal, there is nothing in this justification of Mitchell's claim which insists that this should be so. If my claim of consistency is based upon the identification, within Mitchell's claim, of conjunctions of the form $T(m) = T(m') + dT_{m,m'} \wedge m=m'$, then we may equally satisfy this condition with the claim that we are ourselves material bodies which "*persist*" through time (i.e. that we are bodies whose continuity condition is captured in: $P(m) = P(m') + dP_{m,m'} \wedge T(m) = T(m') + dT_{m,m'} \wedge m=m'$). An exclusively temporal solution to Mitchell's claim, and one which will place his claim clearly within the topic of consciousness, will require an interpretation in relation to Bergson's concept of duration (as will be discussed shortly).

conjunction, not in the claim that the identities of material bodies are “*borrowed from our own*” (which merely accommodates the non-observable nature of material individuality), but in his characterisation of our own “*individuality*” in terms of temporal persistence – the characterisation of ourselves as entities which persist unchanged through those changes of which we are aware. Thus Mitchell’s characterisation of our “*individuality*” in terms of our “*persistence*” would seem to be a necessary requirement for the validation of his claim with respect to the formal properties of Lockean Cardinality identified in the previous section.

In Mitchell’s claim then, the familiar formulation of the continuous form of Locke’s principles (LP.1b and LP.2b) become:

$$\begin{aligned}
 & T(m)=T(m')+ dT_{m,m'} \wedge m=m' \\
 & \wedge \\
 & P(a)=P(b)+ dP_{a,b} \wedge T(a)=T(m) \wedge T(b)=T(m') \rightarrow a=b
 \end{aligned}$$

and

$$\begin{aligned}
 & T(m)=T(m')+ dT_{m,m'} \wedge m=m' \\
 & \wedge \\
 & P(a)\neq P(b)+ dP_{a,b} \wedge T(a)=T(m) \wedge T(b)=T(m') \rightarrow a\neq b
 \end{aligned}$$

where the continuity expressed in the conjunction $T(m)=T(m')+ dT_{m,m'} \wedge m=m'$ is in some way related to our own perceived temporal continuity.

8.4 Mitchell and Bergson.

Mitchell’s claim has much to recommend it. Not only may it accommodate the fact that the identity of a material body is unobservable (or that mere sensation

may not reveal to us the identities of external things) but it also presents us with a theory of individuation which is demonstrably consistent with the formal analysis of Lockean cardinality statements – insomuch as Mitchell locates our own “*individuality*” within the nature of our own temporal persistence. However, as soon we start to examine Mitchell’s claim more critically we discover that its structure is somewhat more complicated and indicative of a subtle assumption which will, ultimately, lead us back to Bergson.

To claim, as Mitchell does, that the “*individuality*” of a external object is “*borrowed from our own*”, must require that we may become aware of some element of experience (or feel some “*fellow feeling*” towards it) which, once an “*individuality*” like our own is imposed upon it, has the character of persistence (like our own) – i.e. the character of permanence with respect to change. For if our own “*individuality*” lies in the nature of our persistence, then what other than the persistence of the permanent may result from the imposition of this “*individuality*” upon experience? If it were not persistence of the permanent which resulted from this process, then how could Mitchell claim that individuality is “*borrowed from our own*”. Thus if we are to support Mitchell’s claim, then we must assume that experience may no more reveal to us the persistence of external things than it may reveal to us their individualities - for if persistence could be revealed to us via experience then so also could that which persists (and there would be no need for Mitchell’s claim in the first place).

Mitchell’s claim therefore actually requires that there is something within experience which, for want of a better phrase, we might call ‘persistence

without the persistent permanent', or 'endurance without that which endures' onto which our own "*individuality*" may be imposed – or something which, when our own "*individuality*" (the nature of our own persistence) is imposed upon it, becomes to us a persisting thing – as a chair, or a tea cup, or a tree, may become a persisting thing in our representation of it.⁶¹ Nonetheless, to adopt Mitchell's claim we require also that there is something about this mysterious *persistence without the persistent permanent, or endurance without that which endures* with respect to which we can experience a "*fellow feeling*" – some aspect which is like ourselves, or sufficiently familiar to draw our attention to it. And while we may arbitrarily invent words for this element of experience, and while we may well speculate upon its origin, we need in fact look no further than Bergson for its description – for it is already what Bergson describes as "*real duration*". Bergson claims that we (or our conscious selves) are not things which persist unchanged through time, we are time itself (as I shall consider in more detail in the following section). Thus even in this simple reading of Mitchell we are drawn inexorably towards Bergson's conception of change, or more accurately, are drawn to contrast and consolidate these respective and seemingly contradictory notions of change in a single understanding of the processes of individuation.

⁶¹ The expressions "persistence without the persistent permanent" and "endurance without that which endures" are treated here as largely informal expressions which arise from the recognition that Mitchell's claim suffers from an unavoidable denial that experience may reveal persistence to us. That I should then go on to interpret these informal expressions in terms of Bergson's real duration (the *durée*) is a methodological step on my part by which I shall shift my concerns from Mitchell to Bergson. These informal expressions are not Bergsonian terms, and indeed are expressions to which Bergsonian philosophers may well take exception. Nonetheless, and regardless of whether these expressions have any real meaning, they capture the consequences of the analysis of Mitchell's claim (the denial that experience may present persistence to us), and having captured these consequences they lead us to ask if such a description of change has any heritage within philosophy - and having asked this question we arrive, almost inevitably I would suggest, at Bergson.

8.5 Bergson and Change.

Disputes over the nature of change are, of course as old as Philosophy itself. Heraclitus (c.500 BC), for example, argued that *“everything is in a state of flux; there is no real permanence in things”* (Thilly 1914, p15) – a position denied by The Eleatics⁶² who considered change to be inconceivable; *“a thing cannot become something other than itself; whatever is, must remain what it is; permanence, not change, is the significant characteristic of reality”* (Thilly 1914, p15). Resolution of these opposing ideas comes at first from Empedocles (c.495-435 BC) who agrees with the Eleatics that absolute change is impossible but holds with Heraclitus that change does occur, namely; in the form of the rearrangement of *“permanent unchanging elements”*. It is then in Empedocles and the Atomists that the idea of the persistence of the permanent through change at first appears within Western Philosophy; and although not universally adopted, has become a dominant position. For example Kant claims that *“In all changes of appearance, substance is permanent and the quantum thereof in Nature is neither increased nor diminished”* (Politis 1997 p 168), or *“only the permanent can change”* (Joseph 1970, p13).

There is then something fundamental in this idea, which Mitchell adopts, that change involves the persistence of the permanent. Indeed Joseph claims that this concept of change is a consequence of the more fundamental law of Identity

⁶² The ‘Eleatics’ take their name from the town of Elea in southern Italy and include within their number Xenophanes (c.560-470 BC), Parmenides (c.480 BC) and of course Zeno (c.470 BC) whose paradoxes of motion we have already considered.

(one of the “*Laws of Thought*”⁶³); “*It is because what is must be determinately what it is . . . This is why we find a difficulty in admitting the reality of ultimate change, change where nothing remains the same; for then we cannot say what it is which changes*” (Joseph 1970, p13). I do not therefore wish to dismiss this idea of change; for in dismissing it I would abandon, not only the demonstrable consistency between Mitchell’s claim and my previous analysis of Lockean cardinality, but also an element of our understanding of ourselves which seems, if not altogether undeniable, then at least extremely familiar.

Even in adopting this pragmatic position, however, we may still discern problems with consolidating Mitchell’s major claim (that the individuality of an external thing is “*borrowed from our own*”) with his adopted position on change. If the identity of an object is “*borrowed from our own*”, then the very concept of diversity (as the not-sameness of two objects – or a relationship between two things whose identities are each known to us) cannot enter into our comprehension of the world until we have imposed our own “*individuality*” (or the nature of our own “*individuality*”) upon those objects which we ourselves have individuated; for how might we comprehend the idea of diversity if the only entity which we have perceived is that embraced by our own “*individuality*”. Wherein then lies our understanding of that diversity within change with respect to which we are supposed to persist? Surely the idea that we are ‘things’ which persist through change implies that change may itself be characterised, and how else might we characterise change (as distinct from our

⁶³ Normally taken to be general principles exemplified in all thinking and comprising of *The Law of Identity* (‘what ever is, is’), *The Law of Contradiction* (‘a thing cannot be and not be so’) and *The Law of Excluded Middle* (‘a thing either is or is not so’).

own permanence) than via terms of diversity? Mitchell's claim might therefore appear to become somewhat contradictory if we allow both that the individualities of objects are "*borrowed from our own*" and that our own "*individuality*" is a 'thing' which persists through change.

Now this may, of course, be too strong a criticism of Mitchell. It may perhaps be the case that we can characterise change without recourse to a fully fledged notion of diversity⁶⁴. However, the idea of permanence with respect to change would still seem to be dependent upon some notion of diversity embedded, as it were, within our very perception of persistence itself. If nothing else, that which persists unchanged must seemingly be diverse (different from) change itself. But these are perhaps problems which I have little hope of solving here.

In Bergson, however, we find a radically different description of change. For Bergson claims that reality is first and foremost time (a *constant becoming*) and denies a reality of timeless things persisting through change (and thus echoes, perhaps, the earlier opinions of Heraclitus). We are not then, according to Bergson, permanent 'things' which persist unchanged through change, we are change itself. Furthermore, surely this Bergsonian view is less paradoxical than Mitchell's adopted idea of change; for we no longer need to argue that the ideas of identity and diversity precede it. Surely the whole point of Bergson's "*multiplicity of interpenetration*" is that, while employing the language of diversity, it denies the role of individuation and diversity within the intuition of our duration? As Russell puts it (in his description of Bergson's concept of 'real

⁶⁴ One in which diversity is a relationship between entities whose identity are known to us, or supposed by us.

duration’): “*It forms the past and present into one organic whole, where there is mutual penetration, succession without distinction*” (Russell 1946 p759) In our appreciation of a musical tune, for example, we do not individuate its notes in time but allow them to coincide without diversity; to form, as it were, a single intuition of duration. Perhaps I am guilty of personalising my argument, but it seems to me that at any given time (or any instant in the measured time of the physicist) that I am enjoying a tune, it is not memory which relates the past to the present (in the sense that I may remember my first day at school, or my first kiss) but simply the feeling of the past still bearing upon the present – as though the immediate past (the preceding notes of the tune) were still tangible to me; still within the grasp of my immediate perception. In appreciating music we (in effect) de-individuated events in the immediate past and immediate future and refuse to distinguish them in time (refused to perceive their diversity in time). Our “*real duration*” is one in which the immediate past, the present and the immediate future collapse into the enduring instant free of the individuations of reflective analysis - and thus by our “*duration*” we must mean something other than our “*persistences*”; for whereas our “*persistence*” entails identity and diversity, our “*duration*” does not. Try to listen to a tune and individuate its notes, or assign each to a time and recognise the diversity between these individuated times. It can be done, of course, but in so doing all pleasure and feeling in the tune will be lost. The very act of individuating its notes and attending to their diversity in time is enough to distract us from an appreciation of its musical character – the “*intuition of life*” (which we feel each time we lay back and let the tune flow over us) is fragmented by individuation and diversity and dies with them.

But if, as Bergson argues, we do not persist through time (as permanent things) but are time itself, and if (as I argue) “*real duration*” does not presuppose the relationships of identity and diversity, then why do we seem to have both the intuition of our own duration (as when we may appreciate a musical tune) and the idea of our own individuality (as something persisting unchanged through change)? Why is the image of ourselves as something persisting unchanged through time so attractive when we may so readily de-individuate our perceptions of time itself (and let the past, the present, and the future melt, un-individuated as it were, into the single enduring instant)? Why do we have both the perception of our “*real duration*” and the perception of our persistence when each is the antithesis of the other – for we cannot persist through that which is not itself diverse.

The answer to this question, I would suggest, is that our “*individuality*” and our “*real duration*” are merely two terms for one and the same thing – “*duration*” being that name which we give to an intuition (the irreducible intuition of our own “*real duration*”) and “*individuality*” being that name which we give to this intuition when viewed via the intellect, i.e. when it stands in relation to other distinguishable durations (or other distinguishable durations within our comprehension of matter) – and just as a tune may be appreciated without any effort but becomes something else when we try to individuate and distinguish its notes, so the nature of our ‘real duration’ (our “*intuition of life*”) becomes something else when we construct about it relationships of identity and diversity – or when we notice its diversity from other durations within our

comprehension of matter. It is our comprehension of matter (the “*intellect*”), that crude awakening from the innocent state of our ‘real duration’, which accounts for our “*individuality*” and our persistence.

Joseph may himself be a guide to us in this matter. The Laws of Thought, which he argues are the basis for our inability to admit to ultimate change, are themselves an aspect of Bergson’s conception of the intellect (whose role it is to “*comprehend matter*”); “. . . if we think about anything, then (1) we must think that it is what it is; (2) we cannot think that it at once has a character and has it not; (3) we must think that it either has it or has not it not” (Joseph 1970, p13). Surely these “*anythings*” to which the Laws of Thought apply are first and foremost objects of the “*intellect*” (our “*comprehension of matter*”) and only as an afterthought perhaps, and only then when fully developed, projected back upon the qualities of ourselves as made apparent in the intuition (the intuition of our ‘real duration’).

Thus Mitchell’s adopted notion on persistence (that it is the persistence of the permanent through change) survives within this Bergsonian interpretation – it has simply moved from the “*intuition*” to the “*intellect*”; the view of ourselves as things which persist through time is a view which exists only in relation to our comprehension of matter (before we comprehend matter we are nothing but “*real duration*”). And surely we have gained something beneficial to Mitchell’s claim from this shift; for we need no longer assume that our appreciation of “*individuality*” and diversity spring, fully formed as it were, from the perception of our own persistence. Our own individuality is as much a product

of the “*intellect*” (our “*comprehension of matter*”) as are the individualities of those objects which we perceive around us. Equally, we have moved from a mechanistic description of individuation (one which assumes the *a priori* appreciation of identity and diversity) to a more evolutionary or developmental description. For in having the intuition of our own “*real duration*” we may be drawn to similar durations within experience (we experience a “*fellow feeling*” towards them) and thus must unavoidably recognise our distinction from it - and thus make available to ourselves both the idea of our own individuality and its diversity from other persistence. In short, our ideas of identity and diversity arise, not from an *a priori* intuition of our own persistence but from that process by which we impose the nature of our own duration upon experience, or recognise our own nature within it. Of course this is not a simple mechanism. As the intellect slowly forms (as we grow to comprehend matter) the feeling of our own individuality will grow also (fuelled by the increasing appreciation of its diversity from other durations) and will then be echoed back ever more rigorously upon those durations which we recognise within experience until we find ourselves unavoidably an individual within a world of individuals (and our duration has become our persistence). Thus Mitchell is right in claiming that the identity of a material body is “*borrowed from our own*” but is wrong, in my opinion, in assuming that our own “*individuality*” springs fully formed from the prior perception of our own persistence. Our persistence (as unchanging things through change), and those relationships of identity and diversity which characterise this ‘persistence’, arise only within our comprehension of matter. When we forget this ‘comprehension’ (as when we refuse to individuate the notes of a musical tune in time) we revert to our un-individuated selves; we

cease to persist through time but become time itself. We are in part persistence; for we may perceive ourselves to persist unchanged in relation to those objects which we ourselves have individuated in our comprehension of matter, and in part ‘real duration’; for we may forget our comprehension of matter and neglect to individuate events in time (as when we appreciate a musical tune). Thus we do not need to argue that there is some ‘self’, or some entity, which must persist as the same thing throughout our life or any part of it. Our persistence is not continuous but is punctuated by our ‘real durations’. We exist, if you like, in two states (the transitions between which are natural and unproblematic). The first is a state of persistence which we perceive via the “*intellect*” and which exists in relation to our comprehension of matter. The second is a state of “*real duration*” which we perceive directly via the “*intuition*” (or Bergson’s conception there of) when we refuse or neglect to individuate events in time. Neither is more characteristic of ourselves than the other.

Thus we arrive at a philosophical context (that of Bergson’s) within which the four points (1 to 4) with which I began this section may be consolidated. A notion of consciousness as persistence $[T(m)=T(m')+dT_{m,m'} \rightarrow m=m']$ survives within this Bergsonian scheme, and survives within the “*intellect*”. It is within the “*intellect*” that we also find our comprehension of matter and thus the formulation of Lockean cardinality statements whose description may be captured in the necessary and sufficient form:

$$01/ \quad T(m)=T(m')+dT_{m,m'} \wedge m=m'$$

$$a1/ \quad P(a1)=P(a1')+dP_{a1,a1'} \wedge [T(a1)=T(m) \wedge T(a1')=T(m')] \rightarrow a1=a1'$$

$$b1/ \quad P(a2)=P(a2')+dP_{a2,a2'} \wedge [T(a2)=T(m) \wedge T(a2')=T(m')] \rightarrow a2=a2'$$

..

- c1/ $P(an)=P(an')+dP_{an,an'} \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an=an'$
- d1/ $P(a1)\neq P(a2') \wedge [T(a1)=T(m) \wedge T(a2')=T(m')]\rightarrow a1\neq a2'$
- e1/ $P(a1)\neq P(a3') \wedge [T(a1)=T(m) \wedge T(a3')=T(m')]\rightarrow a1\neq a3'$
- ...
- f1/ $P(an-1)\neq P(an') \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an-1\neq an'$

Thus it is persistence, or our own persistence $[T(m)=T(m')+dT_{m,m'} \rightarrow m=m']$, which serves here for 0/ and which enables us to formulate such statements – or which enables us to ‘see’ the world in terms of material bodies in space and time and act (and act successfully) accordingly. And yet the numerical identity of material bodies need not be observable within this scheme, nor need our own individuality be presupposed from theirs. Both our own individuality and the identities of material bodies which are “*borrowed*” from it arise simultaneously within the formation of the “*intellect*”. It is in that very process by which we come to “*comprehend matter*” (the process by which we come to separate one thing from another) that we come to ‘see’ ourselves as individuated and to persist (as the same thing) through time.

It is not, however, my intention to defend this view as a self-contained philosophical argument. It has simply served to highlight the types of issues which we shall need to address in attempting to accommodate the analytical aspects of this work with any wider philosophical context (and no doubt there may be many such *contexts* with respect to which these analytical aspects may be accommodated). Thus while there is no clear route from the analytical aspects of this thesis to purely philosophical issues, we can clearly see that these *analytical aspects* lead us directly to consideration of certain distinctively philosophical topics – most importantly perhaps the nature of temporal change itself.

9. Summary of Book 2

I started Book 2 armed only with the analysis of that statements formulated by S (that there are n material bodies within a given region of space over a given infinitesimal interval) expressed in the necessary and sufficient form:

$$01/ \quad T(m)=T(m')+dT_{m,m'} \wedge m=m'$$

$$a1/ \quad P(a1)=P(a1')+dP_{a1,a1'} \wedge [T(a1)=T(m) \wedge T(a1')=T(m')]\rightarrow a1=a1'$$

$$b1/ \quad P(a2)=P(a2')+dP_{a2,a2'} \wedge [T(a2)=T(m) \wedge T(a2')=T(m')]\rightarrow a2=a2'$$

...

$$c1/ \quad P(an)=P(an')+dP_{an,an'} \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an=an'$$

$$d1/ \quad P(a1)\neq P(a2') \wedge [T(a1)=T(m) \wedge T(a2')=T(m')]\rightarrow a1\neq a2'$$

$$e1/ \quad P(a1)\neq P(a3') \wedge [T(a1)=T(m) \wedge T(a3')=T(m')]\rightarrow a1\neq a3'$$

...

$$f1/ \quad P(an-1)\neq P(an') \wedge [T(an)=T(m) \wedge T(an')=T(m')]\rightarrow an-1\neq an'$$

and the logical claim that we may account for the infinitesimal term $dT_{m,m'}$ (otherwise than in terms of the first order derivative of position with respect to time of a material body) from an analysis of purely temporal re-identification claims of the form $T(m)\neq T(m') \wedge m=m'$, i.e. the argument that the conjunction in $T(m)=T(m')+dT_{m,m'} \wedge m=m'$ becomes an inference [and thus continuous with $T(m)=T(m') \rightarrow m=m'$] in the limit as $\delta T_{m,m'}$ 'tends towards zero'.

I have therefore approached the philosophical concerns of Book 2 with little more than a demonstration that the analysis of spatio-temporal continuity converges with the analysis of purely temporal continuity at the point of interpreting isolated infinitesimal terms in the necessary and sufficient analysis of Lockean Cardinality statements.

In section 7, however, I have presented arguments which may lead us to believe that the unproblematic philosophical interpretation of those expressions employed within the analysis of Book 1 refers, not to an interpretation of object types themselves, but to an interpretation of object types only in as much as we ourselves may move and act with respect to them. In other words, the arguments of section 7 have brought the analytical claims of Book 1 squarely into the realm of talking about ourselves, our movement, and our actions. It is then a small step from recognising the significance of my earlier analysis to our movement and actions (an analysis which demonstrates the convergence of the properties of spatio-temporal continuity with purely temporal continuity) to suspecting that it is S's own temporal phenomenology (time as experience by consciousness for S) which is to concern us in the interpretation of that statement formulated by S.

I have attempted to strengthen this suspicion in section 8 where I have argued that the familiar properties of phenomenological time are 'like' the formal properties of the purely temporal continuity $T(m)=T(m')+dT_{m,m} \rightarrow m=m'$. It is not until Section 8, however, that (bounded by the formal and philosophical claims of the previous sections) the real philosophically speculative element of Book 2 begins. For it is here that I have 'pinned my colours to the mast', as it were, and have opted to interpret my analysis in terms of Mitchell and Bergson rather than in terms of any number of other philosophers who may have equally served my purpose. And thus, via this not altogether satisfactory route, I arrive at an answer to that question which has concerned me throughout this thesis:

Q4a. *How does S (or T) know that there are n (rather than n+1 or n-1) material bodies moving about within a given region of space over some given infinitesimal interval of time?*

The answer (or that answer which I have presented here) is that S is conscious (possesses an “*an inner perception*” of S’s Bergsonian ‘real duration’ - the “*intuition*” of which is consciousness) and “*comprehends matter*” via the intellect. In as much as S is conscious in this sense, S is not a thing which persists through time, but is time itself [a unique time that is $S - m \neq n \wedge T(m) \neq T(n)$]. It is within the intellect that S perceives himself or herself to persist – in the sense to which Mitchell refers – and thus it is within the intellect (S’s “*compression of matter*”) that S has access to purely temporal re-identification statements of the form $T(m) \neq T(m') \wedge m = m'$ which are themselves based upon the purely temporal continuity condition $T(m) = T(m') + dT_{m,m} \rightarrow m = m'$. These are properties, not of S’s consciousness (S’s ‘real duration’), but of S’s intellect (S’s “*comprehension of matter*”). It is in the very process by which S “*comprehends matter*”, or by which S “*sees things as separate one from another*” that S himself or herself becomes a persisting thing, and thus it is the properties of S’s own persistence which (via Mitchell’s claim) become recognised by S in S’s personification of experience (S’s “*comprehension of matter*”). In reality, of course: “*there are no separate solid things, only an endless stream of becoming, in which nothing becomes and there is nothing that this nothing becomes.*”⁶⁵

⁶⁵ Not unlike the collapse of the wave function perhaps?

Appendix I: Infinitesimal Terms and Their Role in the Termination of Infinitely regressive Arguments.

The problem of infinite regress (any infinite regress) would not be a problem unless we did not believe, in some peculiar sense which we are unwilling to abandon, that a definite answer exists to a problem which reason will allow us to approach only by an infinite number of steps. Where such a regress involves the infinite division of a continuous mathematical function (as in the determination of the derivative or the definite integral) its termination is achieved by the introduction of the infinitesimal term.

The origins of the infinitesimal start, perhaps, with Zeno of Elea (c. 470 BC). In the *paradoxes of motion* (reported by Aristotle) Zeno argued that change, and particularly those changes which we refer to by the motion of material bodies, is impossible. For example, in the paradox of the ‘race course’ (also referred to as the ‘stadium’ or the ‘dichotomy’) a “*runner has to run a given length. Before running the whole length he must run half of it. Then, before running the second half, he must run half of that half. And so on. Since the division again never terminates, the whole stretch is composed of infinitely many successive pieces, each of some length. But the runner cannot finish the task of traversing infinitely many substretches in succession.*” (The Oxford Companion to Philosophy, 1995, pp 922-3). Now we know, in some peculiar sense which we are unwilling to abandon, that a runner can run a given length. We are therefore faced with a conflict between intuition and reason – and it is reason, in this case, which must give way (and no doubt rightly so). But we should not suppose that by the introduction of the infinitesimal by the mathematicians and philosophers of the 17th Century that a new method of reasoning was introduced by which

such infinitely regressive arguments were eliminated. Quite the opposite. The introduction of the infinitesimal, in whatever terms one may wish to dress it, is simply the admission that if intuition is to be maintained then infinite regression must be terminated somewhere. The infinitesimal is simply the place where the thinkers of the 17th century decided that regression would stop.

It is not surprising then that Cavalieri chose to describe the infinitesimal as an ‘indivisible’ (Kline 1980 pp 132-3), nor that this interpretation was, initially at least, supported by Newton. For if there are such indivisibles (in the description of motion, for example), then the infinitely regressive arguments of Zeno are terminated. It is to Leibniz, however, that we owe our more contemporary view of the infinitesimal (a view at which Newton equally, albeit eventually, arrived), namely; that the infinitesimal is neither zero nor finite but ‘tends towards zero’, or may be ‘as small as we please’, or “*quantities infinitely small such that when their ratio is sought, they may not be considered zero but which are rejected as often as they occur with quantities incomparably greater*” (Kline 1980. p 137). But it is clear from these terms that the properties of the infinitesimal must arise when a ratio is “*sought*”, or when a number is “*considered*”, or “*rejected*”, or “*compared*” – and it is we ourselves, not points and lines and motion, who ‘seek’ or ‘consider’ or ‘reject’ or ‘compare’. The infinitesimal is a solution to infinite regress, or more accurately it is the admission that infinite regress must be terminated somewhere. The properties of the infinitesimal are the consequences of this decision, not a consequence of the fact that in the world there are really such things.

AI.1 The Derivative and the Differential Coefficient.

For the continuous and differentiable function $y=f(x)$ we may define the derivative at the value of the free variable $x=a$ as

$$f'(a) = \lim_{h \rightarrow 0} (F(a+h) - F(a))/h$$

It is conventional, however, to reserve the familiar term dy/dx for the differential coefficient, or the variable value of the derivative over the values of the free variable of a function. For example, if our function were $y=x^2$, then we may define dy/dx (the differential coefficient) in the following fashion.

$$\begin{aligned} dy/dx &= \lim_{h \rightarrow 0} ((x+h)^2 - x^2) / h \\ &= \lim_{h \rightarrow 0} (x^2 + 2xh + h^2 - x^2) / h \\ &= \lim_{h \rightarrow 0} 2x + h \\ &= 2x \end{aligned}$$

In this case $dy/dx = 10$ when $x = 5$, and $dy/dx = 20$ when $x = 10$ - and in this much the familiar dy/dx is merely a symbol, i.e. dy/dx (the differential coefficient) = $f'(a)$ (the derivative) when $x=a$. To discuss the derivative as the ratio of infinitesimal terms, we must invent a new terminology. For example:

$$\begin{aligned} &dy_a/dx_a = f'(a) \\ \text{or} \\ &dy/dx = dy_a/dx_a \text{ when } x=a. \\ \text{or} \\ &f'(a) = \lim_{\Delta x_a \rightarrow 0} (f(a+\Delta x_a) - f(a))/\Delta x_a = \lim_{\Delta x_a \rightarrow 0} \Delta y_a/\Delta x_a = dy_a/dx_a \end{aligned}$$

We may then say (not knowing necessarily whether that we say anything meaningful) that dy_a and dx_a are vanishing or infinitesimal terms, or terms which are neither zero nor finite but which 'tend towards zero', or which can be 'as small as we wish'.

I labour this definition simply to clarify the point that my concerns here lie with the derivative dy_a/dx_a and not the differential coefficient dy/dx , or lie with the value of the differential coefficient of the function $f(a)$ for the single value of the free variable $x=a$.

My concerns with the derivative itself lie only in supporting certain claims as to the indivisibility of the infinitesimal (where ‘indivisibility’ here means nothing more than a condition which may terminate an otherwise infinitely regressive argument) – an indivisibility which is captured, for example, in the theorem:

If dy_a/dx_a is the derivative of $f(x)$ at $x=a$, and if dy_z/dx_z is equally the derivative of $f(x)$ at $x=a$, then $dy_a = dy_z$ and $dx_a = dx_z$.

However, in claiming the infinitesimal to be indivisible (even in this somewhat conditional sense) we must be extremely careful. For one infinitesimal may be divisible by another in the formation and interpretation of ratios (“*quantities infinitely small such that when their ratio is sought, they may not be considered zero*”) but are not divisible by a finite value, and thus, in all interpretations to which we are sensible, are not divisible at all. At the outset, however, we must firstly consider how infinitesimals are divisible by other infinitesimals, or how it is we may apply to such infinitesimals the analogous idea of magnitude – for only in understanding how an infinitesimal is divisible by another may we understand why an infinitesimal is not divisible by a finite value.

AI.2 Apparent Magnitudes of the Infinitesimal Term.

That the infinitesimal term may be treated, in some respect, as a magnitude, or that it may be said in some cases to be greater than or less than something else (some other infinitesimal) arises from its definition in the derivative (or alternatively in its definition in the definite integral – but I shall not consider the definite integral here). For example, if it should be the case that dy_a/dx_a is a finite value greater than 1, then given the definition of this derivative as a ratio (albeit a ratio of terms which are neither finite nor zero, or a ratio of terms which ‘tend towards zero’, or a ratio of terms for which we can find no immediate concept of magnitude) we cannot but help to suppose that dy_a is in some sense greater than dx_a – since this is what we mean by a ratio greater than 1 - or that there must be a sense in which although dy_a and dx_a both ‘tend towards zero’, dx_a somehow tends more closely towards zero than dy_a . It is in this sense then, or in the sense that one infinitesimal may tend more closely toward zero than another, that we feel that we may apply to them the relationships of magnitude (of one value being greater than or less than another). Similarly, if we take the differential coefficient of $f(x)$ at the value of the free variable $x=b$ (where $b \neq a$) to be dy_b/dx_b , and if we were to claim that this finite value is smaller than the finite value of dy_a/dx_a , or that $dy_a/dx_a > dy_b/dx_b$, then we cannot help but conclude that the extent to which dx_a tends more closely towards zero than dy_a is greater than the extent to which dx_b tends more closely towards zero than dy_b . In treating infinitesimals as magnitudes, or in claiming that one such infinitesimal is greater than or less than another, we do not actually claim that these infinitesimal terms have a magnitude in the sense of finite numbers (since they are neither finite nor zero) but that one such

infinitesimal may 'tend more closely towards zero' than another. We might start to see then why an infinitesimal is not divisible by a finite value. For in the ratio of dx_a and 2, say, we cannot claim that dx_a tends more closely towards zero than 2, or that 2 tends more closely towards zero than dx_a — since 2 does not 'tend towards zero' at all (it does not, for example, tend more closely to zero than 3).

More accurately perhaps, in the absence of any clear understanding of what infinitesimals actually are (other than that their use may terminate an infinite regress), we simply deduce their supposed relationships of magnitude from their definition in the derivative (a definition in terms of the ratios of infinitesimals) and the relationships of magnitude that one such derivative may bear to another.

However, if we have reached the conclusion that one such infinitesimal term may be greater than or less than another in the formulation of the derivative, or if one such term may tend more closely to zero than another, then what determines the extent to which any one infinitesimal tends towards zero in any one particular case? In asking this question we must firstly distinguish between free and dependent variables; in the sense that for the function $y=f(x)$ the variable y is dependent upon the value of the free variable x , or in the sense that in the formulation of the differential coefficient it is Δx which we allow to 'tend towards zero'.

AI.3 The 'Tending Towards Zero' of the Dependent Variable.

For the function $y=f(x)$, the extent to which the dependent term (y) 'tends towards zero' in the formulation of the derivative at $x=a$ depends solely on the extent to which the free variable (x) tends towards zero. The terms dy_a and dx_a

are related in such a fashion that their ratio should be commensurate with the function $f(x)$ at the value of the free variable $x=a$. Suppose, for example, that our continuous function were $y=3x$. In this case, when $x=a$ then $y_a = 3a$. If we increase a by a small but finite value Δx_a then we obtain $y_a+\Delta y_a = 3(a+\Delta x_a)$, which may be equally expressed $y_a+\Delta y_a = 3a + 3\Delta x_a$. Subtracting the original identity ($y_a = 3a$) from both sides then gives $\Delta y_a = 3\Delta x_a$. If (as has already been described above) we define the derivative at $x=a$ as

$$\lim_{\Delta x_a \rightarrow 0} \Delta y_a / \Delta x_a = dy_a / dx_a = 3$$

then no matter how closely we allow Δx_a to ‘tend towards zero’ Δy_a will always ‘tend towards zero’ in such a way that $\Delta y_a / \Delta x_a = 3$. The extent to which Δy_a ‘tends towards zero’ in the formulation of the derivative dy_a / dx_a is therefore determined by the extent to which Δx_a ‘tends towards zero’.

Now given such a description it is tempting perhaps to say that a dependent infinitesimal ‘follows’ or ‘precedes’ the free infinitesimal (to which it is related by a function) as this free infinitesimal ‘tends towards zero’. But this is merely our fondness for analogy. When we graphically plot any function which ‘tends towards’ any value (not necessarily zero) using pencil and paper, it is natural to feel that we may follow this curve with the eye as it approaches closer and closer to this value. It is no doubt natural that we may imagine this dynamic process continuing as the function becomes infinitesimally close to its value – and thus apply to our concept of the infinitesimal a dynamic property of ‘tending towards’ which it cannot possibly possess (for there is nothing dynamic about the function $y=x^2$ for example). The absurdity of this analogy is revealed

more clearly when we realise that the finite and the infinitesimal would then be required to be continuous with each other. Dynamic analogies of the idea of 'tending towards zero' will then simply not do. That dx_a 'tends towards zero', or possesses an 'extent towards which it tends towards zero', should not be mistaken for the idea that dx_a is somehow involved in some dynamic process of 'tending towards zero'. 'Tending towards zero' is a static property of an infinitesimal whose quality (which we mistake as its magnitude) is the 'extent to which it tends towards zero'. Indeed it is this very property of the infinitesimal which allows for its employment in the termination of infinitely regressive arguments.

AI.4 The 'Tending Towards Zero' of the Free Variable.

We have seen then that if dy_a/dx_a and dy_b/dx_b are the two derivatives of the function $f(x)$ for the values of the free variable $x=a$ and $x=b$ respectively (where $a \neq b$), then the extent to which values of the dependent terms Δy_a and Δy_b 'tend towards zero' will be dependent upon the extent to which the free terms Δx_a and Δx_b 'tend towards zero'. But what determines the extent to which Δx_a and Δx_b 'tend towards zero' in the determination of dy_a/dx_a and dy_b/dx_b ? Do the free terms Δx_a and Δx_b tend equally towards zero (are they indistinguishable?), or, given that infinitesimals (as we have seen) may be sensibly distinguished in the sense in which they 'tend towards zero' (a distinction which leads us to treat them as magnitudes), is there some sense in which one tends more closely towards zero than another?

To make a start at addressing this question let us first look at finite approximations to the derivative. This is not to suggest that we may extrapolate directly from the properties of such finite approximations to the properties of the derivative, and certainly not that we can extrapolate from the properties of the small finite intervals used in such approximations to the properties of infinitesimals (since infinitesimals are not continuous with finite values). An examination of finite approximations is useful simply because it will reveal to us the nature of the regress that the introduction of the infinitesimal will terminate, and further, that the examination of such 'finite approximations' may guide us to an understanding of what properties the infinitesimal must possess in order to terminate this regress.

Suppose that our continuous function $y=f(x)$ were the function $y=x^2$. We have already seen how the variable differential coefficient for this function (dy/dx) is given by $2x$. In other words, when the value of the free variable x equals 5, then dy/dx equals 10, and when the value of the free variable x equals 10, then dy/dx equals 20. Since these derivatives equal the gradients of the tangent at these values of the free variable (see figure 1) we can obtain an approximation to these gradients in the following fashion. If from the point at $x=5$ we plot the point at $x=5-\Delta x$, then we know that the line passing through these two points ('a' and 'b' in Figure 2) will tend towards the tangent at $x=5$ as Δx 'tends towards zero'. Equally, if from the point at $x=10$ we plot the point at $x=10-\Delta x$, then we know that the line passing through these two points ('c' and 'd' in Figure 2) will tend towards the tangent at $x=10$ as Δx 'tends towards zero'. As an approximation then, we may consider these lines when Δx is small but finite.

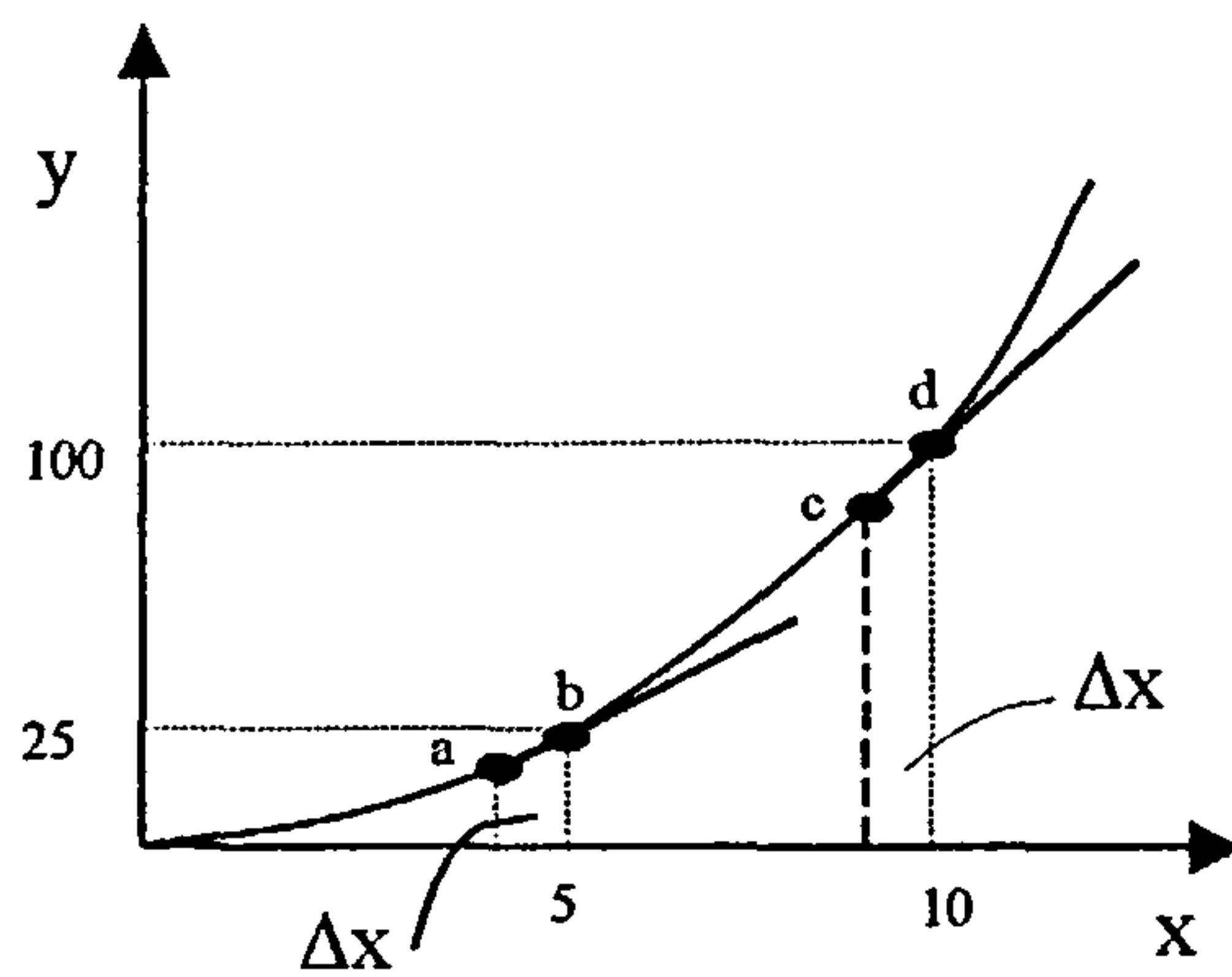


Figure 1 – approximations to the derivative.

Table 1 shows a typical set of results for such an approximation. The first column shows that value of Δx used in each approximation, while the second column (marked ‘Gradient of ab’) shows the calculated value of the gradient of the line passing through the points ‘a’ and ‘b’ (Figure 1) for the given value of Δx . The values in the second column are calculated from the simple formula $[5^2 - (5-\Delta x)^2] / \Delta x$. The third column (marked ‘%’) shows the percentage of the true gradient at $x=5$ captured in the approximation (in the sense that if the calculated gradient is equal to 10 – the known derivative at $x=5$ – then this calculated gradient will be 100% of the true gradient at $x=5$). Columns 4 (marked ‘Gradient of cd’) and 5 are similar to 2 and 3 but apply to approximations to the gradient at $x = 10$, i.e. column 4 shows the value of the approximation at $x= 10$ obtained from the formula $[10^2 - (10-\Delta x)^2] / \Delta x$ and column 5 shows the percentage of the true gradient at $x=10$ captured in the approximation.

With reference to the values captured in Table 1 we may make the following important observations. In the first row, for example, or for the approximations

using a value of $\Delta x=0.5$, we obtain an approximation of the tangent at $x=5$ of 9.5 (or 95% of the true tangent) and an approximation of the tangent at $x=10$ of 19.5 (or 97.5% of the true tangent). In other words, our approximation using $\Delta x=0.5$ seems to give a better result at $x=10$ than at $x=5$. Indeed, this trend is continued through the whole data. In each row the level of the approximation at $x=10$ exceeds that at $x=5$. We might say that as Δx decreases, the approximation at $x=10$ tends more closely to the value of the true tangent at $x=10$ than the approximation at $x=5$ tends towards the value of the true tangent at $x=5$. Alternatively, we might say that in order to obtain the same level of approximation to the true tangent at $x=5$ and $x=10$ (expressed here as a percentage), we would need to use different values of Δx at $x=5$ and $x=10$

Δx	Gradient of ab	%	Gradient of cd	%
0.5	9.5	95	19.5	97.5
0.4	9.6	96	19.6	98
0.3	9.7	97	19.7	98.5
0.2	9.8	98	19.8	99
0.1	9.9	99	19.9	99.5
0.05	9.95	99.5	19.95	99.75
0.01	9.99	99.9	19.99	99.95

Table 1- Calculation of gradients

Do we expect this situation to continue as Δx decreases to ever smaller values, or do we expect there to be a unique value of Δx at which the same level of accuracy, or the same percentage of the true tangent, is reached for both $x=5$ and $x=10$? If there is such a value of Δx , then we may calculate it in the following fashion. The percentage of the true gradient at $x=5$ given for the value of Δx is:

$$\frac{[5^2 - (5-\Delta x)^2]}{10 \cdot \Delta x} \cdot 100\%$$

Similarly, the percentage of the true gradient at $x=10$ given for the value of Δx is:

$$\frac{[10^2 - (10-\Delta x)^2]}{20.\Delta x} \cdot 100\%$$

The value of Δx needed to ensure an identical level of accuracy, or to give the same percentage of the true gradients at $x=5$ and $x=10$ can be resolved from the identity:

$$\frac{[5^2 - (5-\Delta x)^2]}{10.\Delta x} \cdot 100\% = \frac{[10^2 - (10-\Delta x)^2]}{20.\Delta x} \cdot 100\% \quad \dots A$$

which solves at $\Delta x^2 = \frac{1}{2} \Delta x^2$ or $\Delta x = \Delta x/\sqrt{2}$. This condition can only be true for $\Delta x=0$. In other words, as long as Δx is finite and non-zero there is no solution to the identity A.

I now wish to place two interpretations on the infinitesimal term based upon what we have learnt about the nature of finite approximations to the derivative – both of which lead to the same conclusion.

A1.5 First Interpretation.

It would seem then that as long as Δx remains finite, and as long as we use the same value of Δx in our approximations at $x=5$ and $x=10$, then our approximation at $x=10$ will always be closer to the true value of the tangent than our approximation at $x=5$. However, when we apply the methods of the calculus to obtain the variable differential coefficient dy/dx over all values of $x=y^2$ we do not suppose that the accuracy of this value (or its determination of the derivative or the true value of the tangent at a point) is variable with respect to the value of

the free variable x . Somehow, dy/dx is supposed to give us the actual derivative at all values of x .

Now it seems relatively straightforward that if we wanted to determine an approximation to the value of the true tangent at $x=5$ and $x=10$ which (in both cases) was say 99.999% of the true value of the tangent at these points, then we should need to use two different values of Δx in these two approximations. In this case, we would say that the value of Δx_5 needed to give an approximation of 99.999% to the true value of the tangent at $x=5$ is determined by the function $y=x^2$ and the value of the free variable $x=5$. Similarly, the value of Δx_{10} needed to give an approximation of 99.999% of the true value of the tangent at $x=10$ is determined by the function $y=x^2$ and the value of the free variable $x=10$. Can we then believe, that in moving from a finite approximation to the determination of the derivative via the differential calculus, or that by allowing Δx_5 to 'tend towards zero' at the value $x=5$ and by allowing Δx_{10} to 'tend towards zero' at $x=10$, that we refer to the same infinitesimal dx in both these cases, or that dx may be equally expressed by saying that Δx_5 'tends towards zero' or Δx_{10} 'tends towards zero'? Are we not forced to conclude, on the contrary, that Δx must tend more closely to zero at $x=5$ than at $x=10$ in order that the calculus provide us with 100% of the true value of the gradient at these two points (why, for example, should we apply greater significance to a 100% accuracy than a 99.999% accuracy, or why should we assume that some fundamental change in this situation is required in moving between 100% accuracy and 99.999% accuracy)?

Should we not perhaps more accurately say that dy_5/dx_5 is determined when Δx_5 has tended close enough to zero to determine dy_5/dx_5 and that dy_{10}/dx_{10} is determined when Δx_{10} has tended close enough to zero to determine dy_{10}/dx_{10} . And since we know that such infinitesimals are quite sensibly distinguished as magnitudes, or distinguished in the extent to which they 'tend towards zero', then such a claim sits comfortably with both the known properties of infinitesimals and the properties of finite approximations to the derivative.

It is of course dangerous to generalise from such a limited example. But in this case we have arrived at an interpretation of the extent to which Δx_a must 'tend towards zero' in order to arrive at the value of the derivative dy_a/dx_a which is dependent upon the nature of the function $f(x)$ and the value of the free variable $x=a$. If dy_b/dx_b is the derivative of a different function $y=g(x)$ for a value of the free variable $x=b$, then we conclude that while the extent to which Δx_a must 'tend towards zero' in order to arrive at dy_a/dx_a is determined by $f(x)$ and $x=a$, the extent to which Δx_b must 'tend towards zero' in order to arrive at dy_b/dx_b is determined by $g(x)$ and $x=b$. Whether we then go on to claim that these two terms 'tend towards zero' to the same extent ($dx_a=dx_b$) or to different extents ($dx_a \neq dx_b$) depends upon $f(x)$, $g(x)$, $x=a$ and $x=b$.

Finally then, we might suggest: in the determination of the derivative of $f(x)$ at $x=a$, the extent to which the dependent term Δy_a 'tends towards zero' is dependent upon the extent to which the free term Δx_a 'tends towards zero', and the extent to which the free term Δx_a 'tends towards zero' is determined by the function $f(x)$ and the value of the free variable $x=a$.

This interpretation has one significant consequence:

If dy_a/dx_a is the derivative of $y=f(x)$ at the value of the free variable $x=a$ and dy_z/dx_z is the derivative of the same function $y=f(x)$ at the same value of the free variable $x=a$, then $dx_a = dx_z$ and $dy_a = dy_z$.

Now this is a seemingly trivial claim. However, it captures what is perhaps the single most important characteristic of the infinitesimal, namely; that the infinitesimal terms of a single derivative are not divisible except by the infinitesimal terms of another derivative. In other words, given a single derivative there is no meaning whatsoever to dividing its infinitesimal terms.

For the single derivative

$$\lim_{\Delta x_a \rightarrow 0} \Delta y_a / \Delta x_a = dy_a / dx_a$$

there is only a single meaning to the claim that Δx_a 'tends towards zero' [and this 'meaning' is determined by the function $f(x)$ and the value of the free variable $x=a$]. The claim $\frac{1}{2} \Delta x_a$ 'tends towards zero' means exactly that same thing, or refers to exactly the same extent of 'tending towards zero', as the claim that Δx_a 'tends towards zero'.

AI.6 Second interpretation.

In our consideration of finite approximations to the derivatives of $y=x^2$ for the values of the free variable $x=5$ and $x=10$, we concluded that the accuracy of the finite approximation (or its percentage of the true value of the tangent) at $x=5$ could never be equal to the accuracy of the finite approximation at $x=10$ as long as both approximations employed that same value of Δx and as long as Δx remained non-zero and finite (regardless of how small we might allow it to become). This itself was based upon the fact that the condition for the equality

of accuracy required a value of Δx given (in this particular case) by $\Delta x = \Delta x/\sqrt{2}$. Obviously, no finite value can be equal to itself divided by a finite value and thus we concluded that an equality of accuracy at $x=5$ and $x=10$ must remain impossible while Δx remains finite.

In our first interpretation (above) we extended this idea into the realm of infinitesimals and insisted that in the determination of the derivatives at $x=5$ and $x=10$ we actually required that Δx must 'tend towards zero' to different degrees in order to arrive at dy_a/dx_a and dy_b/dx_b . Let us now, however, consider an alternative interpretation. Let us assume that, regardless of the arguments above, it is implicit within the differential calculus that the same value of Δx is to be used in all finite approximations to the tangents of $y=x^2$ and all derivatives for all values of the free variable are to be determined by allowing this single value of Δx to 'tend towards zero. If this is the case, then there will be a value of dx (arising from allowing this single value of Δx to 'tend to zero') applicable to the determination of the derivative at each point on $y=x^2$, or a single value of dx with respect to which the tangent at each point on $y=x^2$ is determined to an accuracy of 100%.

With respect to this argument and the properties of the finite approximations discussed above, we know for the derivatives at $x=5$ and $x=10$ we require of this infinitesimal the property that:

$$dx = dx/\sqrt{2}$$

This being a property which we already know to be true of $\Delta x=0$. In other words, if dx is the extent to which a single value of Δx must tend in order to determine the true (100%) values of the tangents at $x=5$ and $x=10$, then Δx is a term which ‘tends towards zero’ and reflects the properties of zero, i.e. that dx is something which equals itself even when divided by the finite value $\sqrt{2}$. This ‘finite value’ is of course in this case dependent upon our choice of the values of the free variables $x=5$ and $x=10$ of the function $y=x^2$. However, generally we might say that, in the case of the current interpretation, we require of a freely decreasing infinitesimal, which is suitable for the determination of the derivatives of a function, that it be: equal to itself when divided by a finite value.

In other words, that its division by a finite value has no effect upon it, or that it is ‘indivisible’. We note, however, that this is a condition on the infinitesimal expression of the free value of a function and we may thus conclude that: the extent to which the dependent term Δy_a ‘tends towards zero’ is dependent upon the extent to which the free term Δx_a ‘tends towards zero’, and the extent to which the free term Δx_a ‘tends towards zero’ is determined by its ‘tending towards’ an indivisible state. As such if dy_a/dx_a is the derivative of $f(x)$ at $x=a$, then there is no meaning to allowing dy_a and dx_a to tend even more closely towards zero and becoming say $\frac{1}{2}dx_a$ and $\frac{1}{2}dx_a$, since $dx_a = \frac{1}{2}dx_a$. And thus we may conclude, as above, that:

If dy_a/dx_a is the derivative of $y=f(x)$ at the value of the free variable $x=a$ and dy_z/dx_z is the derivative of the same function $y=f(x)$ at the same value of the free variable $x=a$, then $dx_a = dx_z$ and $dy_a = dy_z$.

AI.7 Third Interpretation.

The third interpretation which we may place upon the nature of the infinitesimal term is a pragmatic one, and one which does not refer to the properties of finite approximations to the tangent at a point (as discussed above). This is simply to assume that the infinitesimal has no implicit meaning but must instead be interpreted simply in relation to its emergence and use as a term in the process of determining the derivative. In this sense, infinitesimals are peculiar terms which exhibit the properties of ratio with respect to each other but not magnitude with respect to finite terms

We can see this characteristic of infinitesimals (that they exhibit the properties of ratio but not magnitude with respect to finite terms) in the determination of the differential coefficient of the function $y=x^2$ (as outlined above), ie.

$$\begin{aligned} dy/dx &= \lim_{h \rightarrow 0} ((x+h)^2 - x^2) / h && \dots (a) \\ &= \lim_{h \rightarrow 0} (x^2 + 2xh + h^2 - x^2) / h && \dots (b) \\ &= \lim_{h \rightarrow 0} 2x + h && \dots (c) \\ &= 2x && \dots (d) \end{aligned}$$

Step (a) to (b) simply substitutes $(x+h)^2$ for its expansion. In moving between step (b) and (c) we not only remove x^2-x^2 to leave $(2xh + h^2)/h$, but we divide $2xh + h^2$ by h to leave $2x + h$. However, this process requires that we divide h^2 by h to obtain h and divide h by h to obtain 1, i.e.

$$\lim_{h \rightarrow 0} h^2/h = \lim_{h \rightarrow 0} h$$

$$\lim_{h \rightarrow 0} h/h = 1$$

In other words, although we insist that h is tending towards zero, we still maintain that h exhibits the properties of ratio. In the step between (c) and (d) however, h has somehow vanished – it has been replaced by zero, i.e.

$$2x + \lim_{h \rightarrow 0} h = 2x$$

In this final step then, h has no magnitude with respect to the finite term $2x$ and may thus be removed from any expression in which it is conjoined with $2x$ as an addition. We cannot however simply assume that h is actually zero as it tends ‘towards zero’, since this would mean that $h^2/h = 0^2/0$ and $h/h = 0/0$ in the step from (b) to (c) which is meaningless. All we can assume is that h (as it tends to zero) has no magnitude with respect to the finite term $2x$. In other words, h (as it tends towards zero) exhibits the properties of ratio ($h^2/h=h$ and $h/h=1$) but not the properties of magnitude with respect to finite values ($2x+h = 2x$).

We may use this property of infinitesimals to demonstrate their indivisibility with respect to finite terms. Consider the two limits:

and $\lim_{h \rightarrow 0} (2x+h) = 2x$

$$\lim_{h \rightarrow 0} (4x+h)/2 = \lim_{h \rightarrow 0} (2x+h/2) = 2x$$

In the first case we have the expansion

$$\lim_{h \rightarrow 0} (2x+h) = 2x + \lim_{h \rightarrow 0} h = 2x \quad \dots (e)$$

In the second we have

$$\lim_{h \rightarrow 0} (4x+h)/2 = 2x + \lim_{h \rightarrow 0} h/2 = 2x + \frac{1}{2} \lim_{h \rightarrow 0} h = 2x \quad \dots (f)$$

Thus from (e) and (f)

$\lim_{h \rightarrow 0} h = \frac{1}{2} \lim_{h \rightarrow 0} h$

In other words, the infinitesimal $\lim_{h \rightarrow 0} h$ cannot be divided by a finite term, or is 'indivisible' with respect to finite terms. With respect to the process of division with respect to finite values, the infinitesimal has the same properties as zero.

Appendix II – Numerically Quantifiable Derivatives of Philosophical and Psychological Questions and Propositions as to S knowing P.

AII.1 Variable Subjects within Philosophical and Psychological Questions and Propositions.

I include within this appendix a more detailed exposition of those epistemological arguments outlined in Section 1. These notes are therefore intended to outline the wider philosophical context within which I address numerically quantifiable derivatives of philosophical and psychological questions and therefore outlines the epistemological justification for the chosen route of the analytical aspects of Book 1 of this thesis.

The intersubjective nature of philosophical and psychological questions and propositions, or indeed any question or proposition in which we attribute a property to a variable subject, demands that we justify the intersubjective nature of the property thus attributed, and thus the variable nature of the subject with respect to this property. Most commonly, to claim that *S knows P*, where S is a variable within a range (or set) of individuals (or where S is not the only individual that knows P) presupposes that there exists at least one individual S1 (different from S) and that *S1 knows that S knows P*. Without this condition (which is not itself sufficient to justify the variable status of S), the variable status of S is meaningless – for if there is not at least one S1 that knows that S knows P, then there cannot be an individual that knows that S and some other individual knows P, and thus there can be no meaning whatsoever in attributing to S a variable status with respect to P. However, we might immediately see the beginnings of a significant regress. For if S1 is itself equally a variable (if S1 is not the only individual that knows that S knows P), then there must likewise

exist at least one S2 (different from S1) that knows that S1 knows that S knows P – and may we not then go on to ask as to the variable status of S2, and S3, and S4, and so on *ad infinitum*.

Formally we may express this regress:

a/ *S knows P.*

If S is a variable, then $A = \{G|G \text{ knows } P\}$, $S \in A$

b/ *S1 knows that S knows P.*

If S and S1 are variable, then $B = \{(H,I)| H \text{ knows that } I \text{ knows } P\}$, $(S1, S) \in B$.

c/ *S2 knows that S1 knows that S knows P.*

If S, S1 and S2 are variable, then $C = \{(J,K,L)| J \text{ knows } K \text{ knows } L \text{ knows } P\}$, $(S2, S1, S) \in C$

and so on . . .

The regress may be at first terminated, or so we might expect, in the case where both S_a and S_{a+1} know P and where both S_a and S_{a+1} know each other to know P; since in this case the variable status of both S_a and S_{a+1} can be justified without recourse to a third party (S_{a+2}), and thus without the need to specify that set within which S, S1, S2, . . ., S_{a+2} are variable. However the full termination of the regress actually requires that every individual under consideration (the members of the set A) not only knows P (the condition for membership of A) but also knows that any and every other member of A also knows P.

Under this condition we may terminate the regress by replacing c/ as follows:

c'/ If the 1st projection of $B = \{J|(J,K) \in B\} = A$

And

If the 2nd projection of $B = \{K|(J,K) \in B\} = A$

Then $B = \{(H,I) | H \in A, I \in A\} = A \times A$ – the ‘Cross Product’ of A.

The condition for both the 1st and 2nd projections of B being the set A is, of course, that every member of A knows that every other member of A knows P. This condition is, I would suggest, implicit in every proposition of the form S knows P and where S is a variable within a range of individuals.

Suppose, for example, we were to claim that S knows P and that T knows Q but that S does not know Q and T does not know P. In this case S and T are not equally variables within the same range of individuals defined by their knowledge of either P or Q, i.e.

$$A = \{G | G \text{ knows } P\}, S \in A, T \notin A$$
$$A1 = \{H | H \text{ knows } Q\}, S \notin A1, T \in A1.$$

The variability of S with respect to A (and knowing P) and the variability of T with respect to A1 (and knowing Q) still requires, if regress is to be avoided, that every member of A knows that every other member of A knows P, and that every member of A1 knows that every other member of A1 knows Q.

To try to get around this by claiming that S and T are variables within some range of individuals who can, in principle perhaps, know both P and or Q - whether they actually know P and or Q or not – leaves us with the same condition:

$$A2 = \{I | I \text{ can know } P \text{ and or } Q\}$$

since the members of this range will only avoid the regression above if every member of A2 knows that every other member of A2 can know P and or Q.

Equally, we do not avoid this logical structure by defining a general range of individuals who can 'know things in general' (including P and Q), for this would involve:

$$A_{gen} = \{J | J \text{ can know things including P and Q}\}, S \in A_{gen}, T \in A_{gen}.$$

Once again, the non-regression of this definition requires that every member of A_{gen} knows that every other member of A_{gen} can know things including P and Q.

Any philosophical or psychological question or proposition of the form *How does S know P?* or *S knows P*, where S is a variable within a range of individuals, must therefore be formulated either in the specific form:

$$A = \{G | G \text{ knows P}\}, S \in A.$$

or the generalised form

$$A_{gen} = \{J | J \text{ can know things including P}\}, S \in A_{gen}.$$

The first requires (for the avoidance of regression) that every member of A knows that every other member of A knows P, and the second requires that every member of A_{gen} knows that every other member of A_{gen} can know P. Each of which would seem to pose roughly the same question, namely; how does S know that another individual knows P, or how does S know that another individual can know P? Thus any philosophical or psychological question or proposition referring to S's knowing something must unavoidably be associated with the question of S's knowing that other individuals may also know this 'something'. Formally:

S knows P

If S is a variable, then $A = \{G | G \text{ knows } P\}$, $S \in A$

If A exists then $A \times A$ exists.

$A \times A$ is the range of ordered pairs with respect to which S1 and S are variable and where S1 knows P and S1 knows that S knows P.

And thus we arrive at the conclusion that any philosophical or psychological question or proposition referring to S's knowing P, and where S is a variable within a range of individuals, is only a valid question or proposition if it can be demonstrated that for any two individuals (within this 'range of individuals') each knows P and that each knows that the other knows P. In other words we cannot pose the question *how does S know P?*, nor frame the proposition *S knows P*, unless either we are willing to admit that S is not a variable, or that we have some way of knowing *how S knows that T knows P*, or some way of justifying that *S knows that T knows P*.

More accurately, to claim that *S knows P*, or to ask *how does S know P?* where S is a variable within a range of individuals, is only a valid question if there exists some known or justifiable epistemological *criterion of S's knowing that T knows P*. If there exists no such criterion, or if such a criterion cannot be justified, then such propositions and questions are invalid – or they suffer from an epistemological shortcoming.

The archetypal example in this respect is the *problem of other minds*, i.e. given that, or assuming that, S and T are conscious, how does S know that T is conscious and how does T know that S is conscious? Traditionally, we treat the

problem of other minds as one of the problems of mind – to be lumped together with the problems of its first-person subjectivity, its intentionality and its physical efficacy for example. However, the above epistemological arguments would seem to suggest that to claim that *S is conscious*, where S is a variable within a range of individuals (and the claim is invariably framed in this sense), itself requires, for the epistemological validity of the claim, that we should have before us an answer to the question of how *S knows that T is conscious*. In other words, the *problem of other minds* is not simply one of the problems of mind, it is the central problem of mind – for without its solution we cannot rigorously frame the proposition that *S is conscious* in the first place (not, that is, if by ‘S’ we intend to mean a variable within a range of individuals – including, in this case, or more often than not, you and me). If there exists no epistemological *criterion of S’s knowing that T is conscious*, then the claim that *S is conscious* suffers from the epistemological shortcomings outlined above – and in making this claim I am aware that I condemn the philosophical speculations of Book 2 of this thesis.

Now in presenting these epistemological arguments I mean to imply no criticism of the work of any philosopher or psychologist (present or past). We may know (or at least may feel that we know) that *S may know that T knows P* without being able to write down a strict epistemological *criterion of S’s knowing the T knows P*. In fact most philosophy and psychology works upon this basis and who am I to criticise this approach. In this thesis, however, I have chosen, not the purist approach of insisting that such a criteria must be rigorously formulated and expressed before we can examine the claim that *S*

knows P or the question of *how does S know P?*, but simply the modified approach of claiming that there are certain types of questions and propositions which lend themselves more naturally, or more satisfyingly, to a *criterion of S's knowing that T knows P* than others and that these 'certain types of questions' are both easily identifiable and, in some cases, identifiable as derivatives of more problematic philosophical and psychological questions and propositions. Thus while far from providing a perfect philosophy, addressing these 'certain types of questions and propositions' allows us to both acknowledge the epistemological shortcomings of certain philosophical and psychological questions and propositions and to move one step closer to avoiding them.

AII.2 Numerically Quantifiable Derivatives of Philosophical And psychological Questions and Propositions.

As outlined in the introduction of this thesis, and as evidenced by the content of Book 1, I do not take these comments on the 'epistemological shortcomings' of philosophical and psychological questions and proportions to be insurmountable – merely that we must be extremely careful in the questions and propositions which we frame (for these 'epistemological shortcomings' apply as much to the questions and propositions of empirical science as they do to philosophy and psychology, and yet, to some degree at least, the empirical scientist has overcome them) . More specifically, I suggest that with respect to philosophical and psychological questions and propositions regarding S's knowledge of P, where S is a variable within a range of individuals, there exists a class of questions which are, in certain circumstances, derived from them (or are 'derivatives' of these questions and propositions) which do not suffer from the 'epistemological shortcomings' discussed above – or which at least suffer from

them less. These are what I shall refer to as ‘Numerically Quantifiable Derivatives of Philosophical and Psychological Questions and Propositions’ – and since I shall deal with only one such derivative within this thesis this itself may serve here as an example.

If we consider the question *how does S know the world to be one of material bodies moving around in space and time?*, or the proposition *S knows the world to be one of material bodies moving around in space and time*, then the epistemological arguments outlined above would seem to insist that the validity of this question and proposition relies upon a *criterion of S’s knowing that T knows the world to be one of material bodies moving around in space and time* – and while we may live our lives upon the assumption that we may ourselves know that T does indeed ‘see’ the world in this way, a strict epistemological criterion is, in this case, likely to be extremely difficult to formulated.

A ‘numerically quantifiable derivative’ of this question or proposition is then one which is derived from it (or from them) but whose intersubjective corroboration involves the intersubjective corroboration of a number rather than a concept or an idea. The advantage (or so I have argued in Section 1.1) is that in the intersubjective corroboration of a ‘numerically quantifiable derivative’ of a philosophical or psychological question or propositions we may, in effect, intersubjectively test the properties of an abstract arithmetic and the principles by which this ‘abstract arithmetic’ is applied to the subject of the question or proposition. In this case I therefore claim that the question *how does S know the world to be one of material bodies moving around in space and time?* has the

'numerically quantifiable derivative' question of how does S know that there are n (rather than n+1 or n-1) material bodies within a given region of space at a given time?, and the proposition that S knows the world to be one of material bodies moving around in space and time has the 'numerically quantifiable derivative' proposition: S know that there are n (rather than n+1 or n-1) material bodies within a given region of space at a given time.

Thus in Book 1 of this thesis I am concerned purely with the analysis of 'numerically quantifiable derivative' questions and propositions – for these questions and propositions, while not totally free of the 'epistemological shortcomings' of the philosophical and psychological questions and propositions from which they are derived, are nonetheless somewhat closer to being epistemologically sound - at least to the degree that the questions and propositions of an empirical science are 'epistemologically sound' (and this is, perhaps, as much as we may ask).

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