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**Abstract:** This paper discusses the robust model-based fault detection filter design problem for faults in the air path of diesel engines. Two failure modes in the air path, namely, the Exhaust Gas Recirculation (EGR-Actuator) and Variable Geometry Turbocharger (VGT-Actuator) bias faults were considered. The objective of the design is the detection of failures, which is robust against model uncertainties and external disturbances. By using aH-infinity optimization approach the filter robustness is ensured by the application of a design trade-off that is made between the worst-case disturbance and the  $L_2$  norm of the filter error. Beyond the classical state estimation, the method requires the solution of a linear-quadratic optimization problem that leads to the solution of the Modified Filter Algebraic Riccati Equation.

**Keywords:** Diesel engine diagnostics, fault detection, H-infinity optimization, robust estimation, actuator fault model.

## **1. Introduction**

With the increasing complexity of combustion engines in current automotive vehicles, the early detection of failures for engine diagnostics plays an increasingly important role. From view of operational reliability of combustion engines, the assurance of error-free operation of the air path is an ultimately important design task. Possible faults are due to actuator, sensor and component failures, which can lead to engine malfunctions or even damages in the worst case.

There are many different types of Fault Detection and Isolation (FDI) methods available in the literature. Data-driven approaches are based on the

comparison of healthy and faulty data collected from the system. It can be quite expensive to collect, store and process data in real-time, especially in case of processes with fast dynamics. Model-based fault diagnosis [17, 18, 19] does not need stored data for analysis but, instead, it uses the mathematical model of the system, which is driven by temporary measurements acquired from the system. The response (output) of this model is compared with the output of the real physical process thus creating the residual. The residual characterizes the behaviour of the system. Perfect model matching creates zero residual meaning a fault-free operation, while a residual significantly different from zero (i.e., in engineering terms, when the residual value exceeds a certain threshold) indicates deviances from normal systems operation that may be caused by a fault. Because of the advantages, the model-based detection approaches became popular in the past decades, see e.g. [7, 15, 16].

Combustion engines can typically be represented by highly nonlinear processes that may have very fast dynamics. This property poses additional requirements for the design, modelling and filter implementation. Complexity of modelling and simplicity of implementation are design properties mutually opposed. From the one hand, the filter should be capable of running recursively, real-time, in few milliseconds cycles by taking the constrained computational capability of on-board microcontrollers into account. On the other hand, the computational complexity of the model might need processing power reasonably not available for the specific application.

The creation of an accurate mathematical model of the system is not an easy engineering task. Combustion engines, for instance, are complex, nonlinear physical systems that are subject to noise and other disturbances originated from many sources. These physical processes can best be approximated by using nonlinear system models. By making reasonable simplifications and conditionally neglecting nonlinearities the complexity of the models can be effectively reduced on the price that modelling will necessarily contain uncertainties. One of the major concerns of model-based detection filter design is, therefore, to ensure robustness of the detection process, meaning that the filter maintains its detection sensitivity to faults in the presence of disturbances and modelling uncertainties.

Model-based diagnosis of automotive engines has been considered earlier in [4], where a Nonlinear Unknown Input Observer (NUIO) for detection of actuator faults in diesel engines is presented. In [3] an adaptive Lyapunov-based fault estimation for leakage in the air path of diesel engines is used. To comply with the severe requirements on recursion speed of the filter by using nonlinear models a fuzzy filtering approach was proposed for sensor fault detection and isolation in diesel air path in [6]. For similar reasons, the study in [5] proposes the usage of neural network for filter implementation.

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Advanced methods of robust detection filter design are based on geometric approaches and optimal filtering. Earlier results, see e.g. [7] have shown that the  $H_{\infty}$  fault detection filtering is more flexible and more robust than other approximate detection filter design techniques for example the unknown input observers. In this sense the  $H_{\infty}$  fault detection filter is to be implemented in our paper, whether it would be a suitable method for fault detection of actuator faults in the air path of diesel engines at the chosen operating point. For further investigation, according to our concept, for extending a filtering procedure for the entire engine operation range, a set of local Linear Time Invariant (LTI) models is to be created for different operating points and from these a corresponding filter gains can be calculated (Gain scheduling). In spite of the disturbance and model nonlinearities the  $H_{\infty}$  fault detection filtering approach may contribute to finding a useful solution.

#### 2. Modelling the air path of diesel engines

#### 2.1 Linear Time Invariant (LTI) model investigation, plant properties

The robust fault detection filter design methodologies used in this paper require using of Linear Time Invariant (LTI) model. For this end, we use a third order nonlinear mean-value model parametrized for low and medium speed load points, which covers the New European Drive Cycle (NEDC), it was proposed by Jung [1] for robust control purposes.



Figure 1: The scheme of the modelled turbocharged diesel engine [1]

The basic structure of the system is shown in Fig. 1 and presented in the following. The turbocharger dominates the air path, it consists of the turbine and compressor. The turbine driven by the exhaust gas flow has a Variable Geometry Turbocharger (VGT), the position of which corresponds to closing or opening the guide vanes, that increases or decreases the turbine speed respectively. When the turbine speed is increased, the air in the inlet charge is more compressed, i.e. increased Intake Manifold Pressure.

The second path from the exhaust gas to the Intake Manifold is the exhaustgas recirculation, which is needed for  $NO_x$  reduction. The burned gas fraction is re-circulated into the Intake Manifold and it displaces fresh air in it. This is lowering the flame temperature and hence decreasing of  $NO_x$  in the exhaust gas. The basic structure of the system is shown in *Fig. 1*.

All the model parameters proposed by Jung [1] are summarised in *Table 1*. According to study in [1] the differential equations of the nonlinear model are

$$\dot{p}_{i} = \frac{RT_{i}}{V_{i}} \left( \frac{\eta_{c}}{c_{p}T_{a}} \frac{P_{c}}{\left(\frac{p_{i}}{p_{a}}\right)^{\mu} - 1} + \frac{A_{egr}\left(x_{egr}\right)p_{x}}{\sqrt{RT_{x}}} \sqrt{\frac{2p_{i}}{p_{x}}\left(1 - \frac{p_{i}}{p_{x}}\right)} - \eta_{v} \frac{p_{i}NV_{d}}{120T_{i}R} \right)$$

$$\dot{p}_{x} = \frac{RT_{x}}{V_{x}} \left( \eta_{v} \frac{p_{i}NV_{d}}{120T_{i}R} + \frac{A_{egr}\left(x_{egr}\right)p_{x}}{\sqrt{RT_{x}}} \sqrt{\frac{2p_{i}}{p_{x}}\left(1 - \frac{p_{i}}{p_{x}}\right)} - \left(1 - \frac{P_{i}}{P_{x}}\right) - \left(1 - \frac{P_{i}}{P_{x}}\right) + O\left(c\left(\frac{p_{x}}{p_{a}} - 1\right) + d\right)\frac{p_{x}}{P_{ref}} \sqrt{\frac{2p_{a}}{T_{x}}} \sqrt{\frac{2p_{a}}{p_{x}}\left(1 - \frac{p_{a}}{p_{x}}\right)} + W_{f} \right)$$

$$(1)$$

$$\dot{P}_{c} = \frac{1}{\tau} \left( -P_{c} + \eta_{m} \left( ax_{vgt} + b \right) \left( c \left( \frac{p_{x}}{p_{a}} - 1 \right) + d \right) \frac{p_{x}}{p_{ref}} \sqrt{\frac{T_{ref}}{T_{x}}} \sqrt{\frac{2p_{a}}{p_{x}}} \left( 1 - \frac{p_{a}}{p_{x}} \right)^{c} p_{x} T_{x} \eta_{t} \left( 1 - \left( \frac{p_{a}}{p_{x}} \right)^{\mu} \right) \right).$$

Table 1: Nonlinear model parameters

Parameter	Name	Value	Units
τ	turbocharger time constant	0,11	S
$\eta_{_m}$	turbocharger mechanical efficiency	0,98	-
$V_i$	volume of the Intake Manifold	0,006	$m^3$
$V_x$	volume of the Exhaust Manifold	0,001	$m^3$
$\eta_c$	compressor efficiency	0,61	-
$T_a$	ambient temperature	298	K
$c_p$	specific heat at constant pressure	1014,4	J/kgK
C <sub>v</sub>	specific heat at constant volume	727,4	J/kgK

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$\mu = \frac{c_p - c_v}{c_p}$	specific heat ratio	0,286	-
$p_a$	ambient pressure	101325	Ра
$T_x$	exhaust gas temperature	509	K
R	gas constant	287	J/kgK
$\eta_{_{v}}$	engine volumetric efficiency	0,87	-
$T_i$	gas temperature in the Intake Manifold	313	K
$V_d$	total engine displacement volume	0,002	$m^3$
$p_{\mathit{ref}}$	reference pressure	101325	Ра
$T_{ref}$	reference temperature	298	K
$\eta_t$	turbine efficiency	0,76	-
n	number of cylinders	4	-

We linearize this model around the specified equilibrium point (Herceg, 2006), with set values and model variables summarized in *Table 2*.

The operating point has been chosen in the medium speed region at 1900 rpm according to the requirements of NEDC see in study [1].

Variable	Notation	Function	Valuein the equilibrium point
EGR-Actuator effective area	$\Delta A_{egr}$	Input	$4 \cdot 10^{-5} \text{ m}^2$
VGT-Actuator position	$\Delta x_{vgt}$	Input	70 %
Engine speed	$\Delta N$	Disturbance	1900 1/min
Fuelling	$\Delta W_{f}$	Input	0.032 kg/s
Intake manifold pressure (p <sub>i</sub> )	$\Delta x_1$ $\Delta y_1$	State variable 1. Output 1.	124200 Pa
Exhaust manifold pressure (p <sub>x</sub> )	$\begin{array}{c} \Delta x_2 \\ \Delta y_2 \end{array}$	State variable 2. Output 2.	131000 Pa
Turbine power (P <sub>c</sub> )	$\Delta x_3$	State variable 3.	930 W
Air mass	$\begin{array}{ c c }\hline \Delta W_a \\ \Delta y_3 \end{array}$	Output 3.	-

Table 2: States and Input/Output variables of the system

According to study in [1] the control inputs of the LTI system are the position of the Exhaust Gas Recirculation Valve (EGR-Actuator) and Variable Geometry Turbocharger (VGT-Actuator), the outputs are the Sensor for Air Mass Flow into the Intake Manifold (MAF), the Sensor for Exhaust Gas Pressure (EMAP) and the Sensor for Intake Manifold Pressure (MAP).

Noting that in the original model [1] two outputs have been used, namely MAF and MAP, but for the reason of better observability of the system we have extended this to one additional output namely EMAP, which may be nowadays realized in the real application without any problem. Using tree outputs additionally means that the Exhaust Manifold pressure is measured too. This may result in a better performance of the fault detection filtering.

For the sake of simplification, we considered fuelling as a constant input of the air path and not as disturbance. The disturbance was modelled as the fluctuating change of the engine speed.

By linearization we get the LTI model. It can be written in the state space as

$$\begin{bmatrix} \Delta \dot{x}_{1} \\ \Delta \dot{x}_{2} \\ \Delta \dot{x}_{3} \end{bmatrix} = \begin{bmatrix} -5.2643 & 4.7316 & 28.5021 \\ 50.7697 & -156.9827 & 0 \\ 0 & 0.4287 & -9.0909 \end{bmatrix} \begin{bmatrix} \Delta x_{1} \\ \Delta x_{2} \\ \Delta x_{3} \end{bmatrix} + \\ + \begin{bmatrix} 1.6111 \cdot 10^{9} & 0 & 0 \\ -1.5720 \cdot 10^{10} & 8.3514 \cdot 10^{4} 1.46083 \cdot 10^{8} \\ 0 & -141.6484 & 0 \end{bmatrix} \begin{bmatrix} \Delta A_{egr} \\ \Delta x_{vgt} \\ \Delta W_{f} \end{bmatrix} + \\ + \begin{bmatrix} -47.7946 \\ 466.3408 \\ 0 \end{bmatrix} \Delta N, \\ (2)$$

$$\Delta y = \begin{bmatrix} \Delta y_{1} \\ \Delta y_{2} \\ \Delta y_{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3.924 * 10^{-5} \end{bmatrix} \begin{bmatrix} \Delta x_{1} \\ \Delta x_{2} \\ \Delta x_{3} \end{bmatrix}.$$

The matrices

$$A = \begin{bmatrix} -5.2643 & 4.7316 & 28.5021 \\ 50.7697 & -156.9827 & 0 \\ 0 & 0.4287 & -9.0909 \end{bmatrix},$$

$$B = \begin{bmatrix} 1.6111 \cdot 10^9 & 0 & 0 \\ -1.5720 \cdot 10^{10} & 8.3514 \cdot 10^4 & 1.46083 \cdot 10^8 \\ 0 & -141.6484 & 0 \end{bmatrix},$$
(3)  
$$B_{\omega} = \begin{bmatrix} -47.7946 \\ 466.3408 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3.924 \cdot 10^{-5} \end{bmatrix}.$$

are appropriate constant matrices,  $B_{\omega}$  is the matrix for disturbance acting on the system.

#### 2.2 Modelling of faults in the air path of diesel engines

The model for the faults considered in this paper is described below. First we introduce the concept of fault modelling.

Considering the LTI system model according to study in [7] subjected to disturbance and unknown faults, which can be represented in state space form

$$\dot{x}(t) = Ax(t) + Bu(t) + B_{\omega}\omega(t) + \sum_{i=1}^{k} L_i v_i(t),$$

$$y(t) = Cx(t).$$
(4)

with  $x \in \mathbb{R}^n$ ,  $y \in \mathbb{R}^p$ ,  $u \in \mathbb{R}^m . \omega \in \mathbb{R}^p$  are the process disturbances in  $L_2[0, T]$ . A, B, C and  $B_{\omega}$  are appropriate constant matrices. Assume, that (A, C) is an observable pair.

The cumulative effect of k number of faults appearing in known directions  $L_i$  of the state space is modelled by additive linear term  $\sum L_i v_i(t) \cdot L_i \in \mathbb{R}^{nxs}$  and  $v_i(t)$  are the fault signatures and failure modes respectively.  $v_i(t)$  are arbitrary unknown time functions for  $t \ge t_{ji}$ ,  $0 \le t \le T$ , where  $t_{ji}$  is the time instant when the *i*-th faults appears and  $v_i = 0$ , if  $t < t_{ji}$ . If  $v_i(t) = 0$ , for all *i*, then the plant is assumed to be fault free. Assume, however, that only one fault appears in the system at a time.

The LTI model including the engine speed disturbance is extended to include two actuator faults: an EGR-Actuator fault and a VGT-Actuator fault, denoted by  $f_{egr}(t)$  and  $f_{vgt}(t)$  respectively. The disturbance was modelled as a changing engine speed caused by variable load (wind, ramp, break, etc.) that corresponds the real driver situation.

According to equation (4) the EGR-Actuator fault and a VGT-Actuator fault can be modelled as additive terms in the state equations as

$$\dot{x}(t) = Ax(t) + Bu(t) + B_{\omega}\omega(t) + Bf_{a}(t),$$
  

$$y(t) = Cx(t).$$
(5)

As the actuator faults enter the system in the same direction as the input does, the fault directions matrices can be signed as the input matrices.

The faults can be represented as a vector with two arbitrary unknown time functions as



Figure 2: Additive fault model

As mentioned above, the state space realisation of the LTI model of the faulty system is described as

$$\begin{bmatrix} \Delta \dot{x}_{1} \\ \Delta \dot{x}_{2} \\ \Delta \dot{x}_{3} \end{bmatrix} = \begin{bmatrix} -5.2643 & 4.7316 & 28.5021 \\ 50.7697 & -156.9827 & 0 \\ 0 & 0.4287 & -9.0909 \end{bmatrix} \begin{bmatrix} \Delta x_{1} \\ \Delta x_{2} \\ \Delta x_{3} \end{bmatrix} + \begin{bmatrix} 1.6111 \cdot 10^{9} & 0 & 0 \\ -1.5720 \cdot 10^{10} & 8.3514 \cdot 10^{4} 1.46083 \cdot 10^{8} \\ 0 & -141.6484 & 0 \end{bmatrix} \begin{bmatrix} \Delta A_{egr} \\ \Delta w_{rgt} \\ \Delta W_{f} \end{bmatrix} + \begin{bmatrix} -47.7946 \\ 466.3408 \\ 0 \end{bmatrix} \Delta N + \begin{bmatrix} 1.6111 \cdot 10^{9} & 0 & 0 \\ -1.5720 \cdot 10^{10} & 8.3514 \cdot 10^{4} 1.46083 \cdot 10^{8} \\ 0 & -141.6484 & 0 \end{bmatrix} \begin{bmatrix} \Delta f_{egr} \\ \Delta f_{vgt} \\ 0 \end{bmatrix}, \quad (7)$$

$$\Delta y = \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \\ \Delta y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3.924 \cdot 10^{-5} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix}.$$

### **3.** Robust $H_{\infty}$ detection filter

#### 3.1 The optimal $H_{\infty}$ detection filtering problem

Our objective is the detection of failure modes of two actuator faults in the presence of the modelling uncertainties and disturbances.

The goal of  $H_{\infty}$  filtering is minimization of the magnitude of perturbations' effects on the filter output through the appropriate choice of filter gain, maximizing the magnitude of the transfer function from failure modes to the filter error.

We can represent this estimation problem as a mixed  $H_2 / H_{\infty}$  filtering problem (Edelmayer, 2012).

The filter gain matrix has to take under consideration that the magnitude of transfer function computed from unknown input to the output error of the filter must be always less than a pre-specified level  $\gamma > 0$  (Grimble, 1987).

To propose the design of  $H_{\infty}$  filter we must first consider the LTI system model (see equation (4)) extended to an observer. The state estimate can be obtained as

$$\widehat{\mathbf{x}}(t) = A\widehat{\mathbf{x}}(t) + K(C(\mathbf{x}(t) - \widehat{\mathbf{x}}(t))) + Bu(t),$$

$$\widehat{\mathbf{y}}(t) = C\widehat{\mathbf{x}}(t),$$

$$\widehat{\mathbf{z}}(t) = \mathbf{C}_{z} \,\,\widehat{\mathbf{x}}(t).$$
(8)

with the observer state  $\hat{x} \in \mathbb{R}^n$ , output estimation  $\hat{y} \in \mathbb{R}^p$  and weighted output estimation  $\hat{z} \in \mathbb{R}^p$ . *K* is the observer gain matrix and  $C_z$  is the constant estimation weighting.

The filter error system can be derived as

$$\dot{\tilde{\mathbf{x}}}(t) = (A - KC)\,\tilde{\mathbf{x}}(t) + B_w w(t) + \sum_{i=1}^{\kappa} L_i v_i(t),$$

$$\varepsilon(t) = \mathbf{C}_z \tilde{\mathbf{x}}(t).$$
(9)

where  $\tilde{x}(t)$  and  $\varepsilon(t)$  are the state error and weighted output error, respectively. They are defined as

$$\widetilde{\mathbf{x}}(t) = \mathbf{x}(t) - \widehat{\mathbf{x}}(t), \tag{10}$$
$$\varepsilon(t) = \mathbf{z}(t) - \widehat{\mathbf{z}}(t).$$

In the presence of faults, the estimation error does not converge asymptotically to zero, but converges asymptotically to the subspace which is different from zero.

In the following step of the design procedure we have to choose the filter gain, by minimizing the magnitude of the effects of perturbations on the filter output subject to maximize the magnitude of the transfer function from failure modes to the filter error. According to the representation in [7] we consider the auxiliary system written in the form

$$\dot{x}(t) = Ax(t) + Bu(t) + B_{\kappa}\kappa(t) + \sum_{i=1}^{k} L_{i}v_{i}(t),$$

$$y(t) = Cx(t).$$
(11)

This does not includes parametric uncertainty.

 $B_{\kappa} = [B_{w}, L_{\Delta}]$  is the worst-case input direction and  $\kappa(t) \in L_2[0, T]$  is the input function for all  $t \in \mathbb{R}_+$  representing the worst-case effects of modelling uncertainties and external disturbances.

The performance can be formulated as min-max problem by worst-case disturbance  $\kappa$ , which is minimizing the H-infinity norm of the transfer function of the worst-case disturbance denoted by  $H_{\mathcal{E}_k}$  to the filter output. The worst-case performance measure is given by

$$J(\mathbf{K},\kappa) = \sup \frac{\left\| z - \widehat{z} \right\|_{2}}{\left\| \kappa \right\|_{2}} = \left\| H_{\varepsilon\kappa}(s) \right\|_{\infty}.$$
 (12)

The filter gain K can be obtained by solving a linear-quadratic optimization problem, which's procedure is interpreted in the following.

With substitution of the decision variable  $Q \in R^{nxn}$  which is a positive definite matrix, the observer equation can be obtained as

$$\dot{\widehat{\mathbf{x}}}(t) = (A - QC^T C)\,\widehat{\mathbf{x}}(t) + Bu(t) + QC^T \,y(t),$$
  
$$\widehat{\mathbf{z}}(t) = C_z\,\widehat{\mathbf{x}}(t).$$
(13)

The goal of the linear-quadratic optimization is to obtain the smallest  $L_2$  – gain of the disturbance input of the system that is guaranteed to be less than a pre-specified positive constant  $\gamma_{min}$ , and, at the same time to increase filtering sensitivity as much as possible (Edelmayer, 2012). The algorithm, which is used iteratively reduces  $\gamma$  until Q no longer has a positive definite solution. The $\gamma_{min}$ , which is reached, is within a given arbitrary small tolerance $\varepsilon > 0$ .

The procedure is based on the solution of Modified Filter Algebraic Riccati Equation (MFARE). From the bounded-real lemma, we have  $||H_{sx}||_{\infty} < \gamma$  if and only if there exists  $Q \ge 0$  such as

$$AQ + QA^{T} - Q(C^{T}C - \gamma^{-2}C_{z}^{T}C_{z})Q + B_{\kappa}B_{\kappa}^{T} = 0.$$
 (14)

After solving (equation (14)) and getting Q, the filter gain matrix can be obtained as

$$K = QC^T.$$
(15)

With the use of  $\gamma_{min}$  the detection threshold of the filter can be given as

(16) 
$$\tau(C_z) = \gamma_{\min} \left\| \kappa \right\|_2.$$

It is important to note, that the failure modes, which's magnitude is smaller than that of the detection threshold, cannot be detected by the filter.

### 3.2 $H_{\infty}$ detection filter for detection of actuator faults in the air path of diesel engines

A  $H_{\infty}$  detection filter has been implemented to the fault simulations performed in Matlab software. The filter was designed for a fixed operating point, but it must work properly in a certain region around the operating point even in spite oft he disturbance.

The disturbance was modelled as a fluctuating change of the engine speed caused by variable load (wind, ramp, break, etc.) that corresponds to the real driver situation. Both actuator faults were bias fault, in which the amplitude of control input signal as fault signature was increased up to 30% in the presents of the disturbance.

It is important to note that in our examination it was assumed, that only one fault appears in the system at a time.

The proposed robust fault detection filter scheme (Edelmayer, 1994), that we have implemented for the air path of diesel engines, is shown in *Fig.3*.



Figure 3:  $H_{\infty}$  detection filter scheme with different weightings C<sub>i</sub>

The simulation of the filter responses was performed by solving the filter error system (equation (9)). The filter acts by estimating the states of the LTI model around the specified equilibrium point.

According to equation (9), the filter error system of residual generator for the two actuator faults can be written as

$$\tilde{\mathbf{x}}(t) = (A - KC) \,\tilde{\mathbf{x}}(t) + B_w w(t) + B f_a(t),$$
  

$$\varepsilon(t) = C_z \tilde{\mathbf{x}}(t).$$
(17)

The term  $Bf_a(t)$  in the equation (17) can be expressed as

$$Bf_{a}(t) = B_{fegr}f_{egr}(t) + B_{fvgt}f_{vgt}(t),$$
(18)

with the matrices and fault vector

$$B_{fegr} = \begin{bmatrix} 1.6111 \cdot 10^9 \\ -1.5720 \cdot 10^{10} \\ 0 \end{bmatrix}, \quad B_{fvgt} = \begin{bmatrix} 0 \\ 8.3514 \cdot 10^4 \\ -141.6484 \end{bmatrix}, \quad f_a(t) = \begin{bmatrix} f_{egr}(t) \\ f_{vgt}(t) \\ 0 \end{bmatrix}.$$
(19)

 $B_{fegr}$  is a fault direction matrix for the fault  $f_{egr}(t)$ , which is composed from the first column of the *B* matrix corresponding to the first row in the fault vector  $f_a(t)$ .  $B_{fvgt}$  is a fault direction matrix for the fault  $f_{vgt}(t)$ , which is composed

from the second column of the *B* matrix corresponding to the second row in the fault vector  $f_a(t)$ .

As previously mentioned both actuator faults were bias fault modelled, in which the amplitude of control input signal as fault signature was increased up to 30%. Thus the according failure modes are written

$$f_{egr}(t) = 0.3 \cdot u_{egr}(t - t_{jegr}) , \qquad (20)$$
  
$$f_{vet}(t) = 0.3 \cdot u_{vet}(t - t_{ivet}) .$$

where  $u_{egr}(t)$  and  $u_{vgt}(t)$  are the input signal of EGR-Actuator and VGT-Actuator respectively.  $t_{jegr}$  and  $t_{jvgt}$  are the time instants when the fault  $f_{egr}(t)$  and  $f_{vgt}(t)$  appear respectively.  $f_{egr}(t) = 0$ , if  $t < t_{jegr}$  and  $f_{vgt}(t) = 0$ o, if  $t < t_{jvgt}$ . If  $f_{egr}(t) = 0$  and  $f_{vgt}(t) = 0$ , the plant is assumed to be fault free. Assume, however, that only one fault appears in the system at a time.

In order to show the effect of the weighting matrices on the  $H_{\infty}$  bound, we start with  $\gamma_{min} = 3.6 \cdot 10^5$  (that corresponds to a very high value), of the corresponding setting of estimation weighting matrices obtained as

$$C_{Z} = \begin{bmatrix} 10^{4} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
 (21)

To get the optimal value of the filter gain we must perform a  $\gamma$ -iterations on the Riccati equation (equation (14)). The filter gains given as

$$K = \begin{bmatrix} 0.0030 & -0.0005 & 0 \\ -0.0005 & 0.0111 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
 (22)

The plots corresponding to solutions of the filter error system of residual generator (equation (17)) and a comparison between the states of faulty - and faultless system are shown in *Fig.* 6 and *Fig.* 7.

In order to prove the filter performance for disturbance attenuation, the transfer function of the disturbance using the filter gain *K* is described as

$$\mathbf{H}_{\varepsilon\omega}(\mathbf{s}) = C_{Z}(\mathbf{s}\mathbf{I} - \mathbf{A} + \mathbf{K}\mathbf{C})^{-1}\mathbf{B}_{\omega},$$
(23)

the transfer functions calculated from the fault of the EGR-Actuator and VGT-Actuator to the weighted error residual  $\varepsilon(t)$  of the filter, respectively

$$\mathbf{H}_{\varepsilon egr}(\mathbf{s}) = C_Z (\mathbf{s}\mathbf{I} - \mathbf{A} + \mathbf{K}\mathbf{C})^{-1} \mathbf{B}_{fegr},$$
(24)

$$\mathbf{H}_{\varepsilon v g t}(\mathbf{s}) = C_Z (\mathbf{s} \mathbf{I} - \mathbf{A} + \mathbf{K} \mathbf{C})^{-1} \mathbf{B}_{f v g t},$$
(25)

The magnitude of the above mentioned transfer functions can be seen in Fig. 4.



*Figure 4*: The magnitude (maximal singular values) of transfer functions:  $T_{\varepsilon_{to}}$  (red line),  $T_{\varepsilon_{fegr}}$  (green line),  $T_{\varepsilon_{fvgt}}$  (cyan line) for estimation weighting  $C_z$ 

Following the same procedure, we choose the estimation weight in special form as

$$C_{Z}^{*} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 25 \end{bmatrix}.$$
 (26)

By performing  $\gamma$ -iterations again in the case of  $C_z^*$ , the optimal value of  $\gamma_{min} = 4.824$  is obtained. The corresponding solution MFARE for the decision matrix and throughout the filter gain is given as

$$K^* = \begin{bmatrix} 158.273 & -46.478 & -0.0001 \\ -46.478 & 692.143 & 0 \\ -2.973 & -0.215 & 0 \end{bmatrix}.$$
 (27)

The magnitude of the transfer functions calculated using the filter gain  $K^*$  is shown in *Fig. 5*.



*Figure 5*: The magnitude (maximal singular values) of transfer functions:  $T_{\varepsilon w}$  (red line),  $T_{\varepsilon fegr}$  (green line),  $T_{\varepsilon fvgl}$  (cyan line) for estimation weighting  $C_z$ 



*Figure 6*: EGR-Actuator bias fault occurring at t= 2s, in the presence of engine speed disturbance

States and residuals:  $X_1$  and  $\varepsilon_1$  (blue line),  $X_2$  and  $\varepsilon_2$  (cyan line),  $X_3$  and  $\varepsilon_3$  (green line) estimation weighting  $C_z^*$ . Dashed lines: States of the faultless LTI system.



*Figure 7*: VGT-Actuator bias fault occurring at t= 2s,in the presence of engine speed disturbance

States and residuals:  $X_1$  and  $\varepsilon_1$  (blue line),  $X_2$  and  $\varepsilon_2$  (cyan line),  $X_3$  and  $\varepsilon_3$  (green line) estimation weighting  $C_z^*$ . Dashed lines: States of the faultless LTI system.

## 4. Conclusion

From the simulation results of the  $H_{\infty}$ -Filter residuals applied for the air path model of diesel engine, it can be concluded that the filter gives the estimates of two actuator faults, namely bias fault of the EGR-Actuator and VGT-Actuator in the presence of disturbance. The disturbance was modelled as a fluctuating change of the engine speed caused by variable load.

It was shown that the estimation weight has an impact to magnitude of transfer function. By appropriate choice of estimation weight the filter sensitivity can be achieved. By selecting estimation weight  $C_z^*$ , as it can be seen in *Fig.5*, the sensitivity is nearby constant in a wide frequency band  $\left(10^5 \frac{rad}{s}\right)$ . The results indicate that the Filter gain  $K^*$ ensures at least min. 50 dB of separation (signal-to-noise-ratio, SNR) between the VGT-Actuator fault and the disturbance effect. From this it can be concluded that this sensitivity is normally enough to detect both the EGR - and VGT-Actuator faults in the presents of the disturbance.

In opposition to them, in the case of the estimation weighting of  $C_z$ , the filter does not provide enough sensitivity, because the transfer function of the VGT-Actuator fault decreases faster at the frequency of  $10^2 \frac{rad}{s}$  and it even reaches the magnitude of the disturbance at the a frequency of  $10^4 \frac{rad}{s}$  as it is shown in *Fig.4*.

If a fault occurs in the EGR-Actuator, the weighted residual  $\varepsilon_1$  is strongly and  $\varepsilon_2$  is slightly changed as it is shown in *Fig. 6*. The residuals correspond to the state  $x_1$  and  $x_2$ , the manifold pressure and exhaust pressure, respectively. It is similar in the case of the VGT-Actuator as it can be seen in *Fig. 7*.

From these it follows that both faults can be detected in spite of worst-case disturbance. As both fault signals do not represent the faults in the residual signal separately, fault isolation with using a single filter could not be guaranteed.

The results confirmed that the  $H_{\infty}$  fault detection filtering is a suitable method for robust fault detection of actuator faults in the air path of diesel engine at the chosen operating point. For further investigation, according to our concept, for extending a filtering procedure for the entire engine operation range, a set of local LTI models is to be created for different operating points and from these a corresponding filter gains can be calculated (Gain scheduling).

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