

An Observer-Based *PID* Regulator

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Abstract: An equivalent transfer function representation (*TFR*) is used to study the state-feedback/observer (*SFO*) topologies of control systems. This approach is applied to combine this methodology with YOULA-parametrization (*YP*) introducing new classes of regulators. Then this method is used to introduce observer based *PID* regulators.

Keywords: *Observer, state-feedback, model error, YOULA-parametrization, PID regulator*

1. Introduction

In our previous paper [6] it was shown that in the classical state-feedback/observer (*SFO*) scheme the model error decreases by the sensitivity function of the observer feedback loop. An equivalent transfer function representation (*TFR*) was used to demonstrate this special feature of these regulators. It was also shown that this principle can be used to generalize for the YOULA-parametrized regulators, too. In this paper the demonstrated new approach is used to introduce further new class of regulators.

2. The Observer Based Youla-Regulator

For open-loop stable processes the all realizable stabilizing (*ARS*) model based regulator \hat{C} is the YOULA-parametrized one:

$$\hat{C}(\hat{p}) = \frac{Q}{1-Q\hat{P}} \Big|_{\hat{p} \rightarrow P} = \frac{Q}{1-QP} = C(P), \quad (1)$$

where the "parameter" Q ranges over all proper ($Q(\omega = \infty)$ is finite), stable transfer functions [5], [7], see Fig. 1a.

It is important to know that the *Y-parametrized* closed-loop with the *ARS* regulator is equivalent to the well-known form of the so-called *Internal Model Control (IMC)*

principle [7] based structure shown in Fig. 1b.

Q is anyway the transfer function from r to u and the closed-loop transfer function (i.e., CSF) for $\hat{P} = P$, when $\ell \rightarrow 0$

$$\hat{T}_{ry} = \frac{\hat{C}P}{1 + \hat{C}P} = QP \frac{1 + \ell}{1 + (1 - QP)\ell} \Big|_{\ell \rightarrow 0} = QP = T_{ry} \quad (2)$$

is linear (and hence convex) in Q .

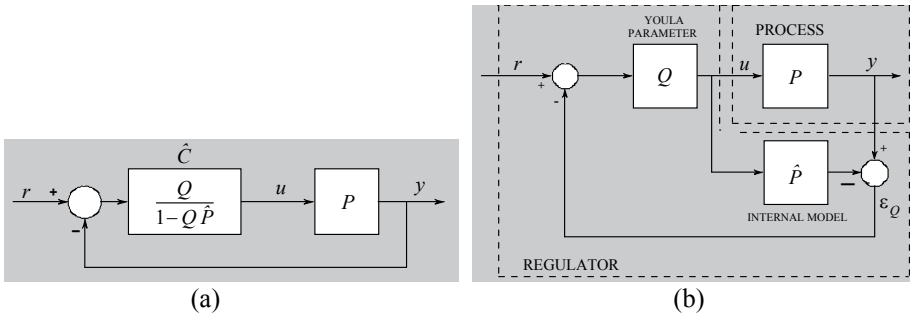


Figure 1. The equivalent IMC structure of an ARS regulator

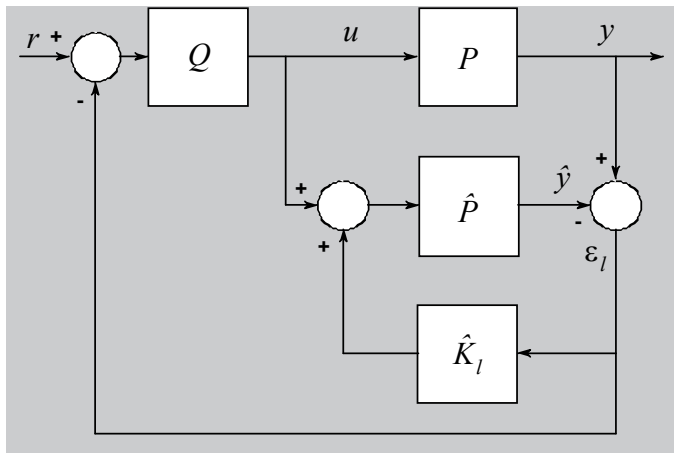


Figure 2. The observer-based IMC structure

It is interesting to compute the relative error ℓ_T of \hat{T}_{ry}

$$\ell_T = \frac{T_{ry} - \hat{T}_{ry}}{\hat{T}_{ry}} = \frac{T_{ry}}{\hat{T}_{ry}} - 1 = \frac{QP}{1 - Q(P - \hat{P})} - 1 = Q(P - \hat{P}) = QP \frac{\ell}{1 + \ell} = T_{ry} \frac{\ell}{1 + \ell}.$$

(3)

The equivalent IMC structure performs the feedback from the model error ε_Q . Similarly to the SFO scheme it is possible to construct an internal closed-loop, which virtually reduces the model error to

$$\varepsilon_l = \frac{1}{1 + \hat{K}_l \hat{P}} (y - \hat{P}u) = \frac{1}{1 + \hat{K}_l \hat{P}} \varepsilon_Q = \frac{1}{1 + \hat{L}_l} \varepsilon_Q = \hat{H} \varepsilon_Q; \quad \hat{L}_l = \hat{K}_l \hat{P} \quad (4)$$

and performs the feedback from ε_l (see Fig. 2), where \hat{L}_l is the internal loop transfer function. In this case the resulting closed-loop will change to the scheme shown in Fig. 3.

This means that the introduction of the observer feedback changes the YOULA-parametrized regulator to

$$\hat{C}'(\hat{P}') = \frac{Q}{1 - Q \frac{\hat{P}}{1 + \hat{K}_l \hat{P}}} = \frac{Q(1 + \hat{K}_l \hat{P})}{1 + \hat{K}_l \hat{P} - Q\hat{P}}. \quad (5)$$

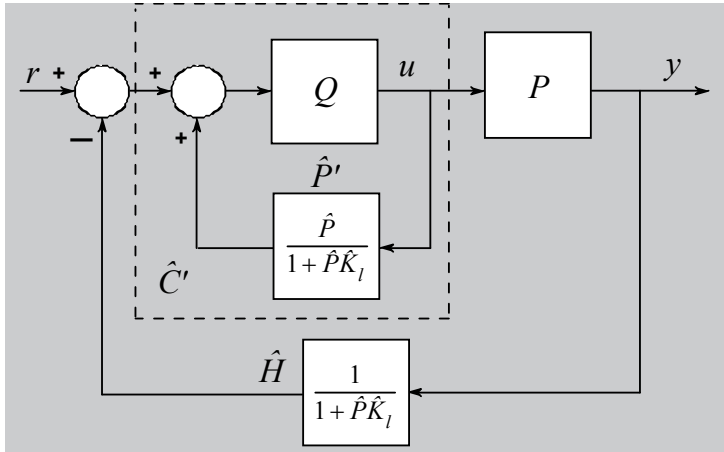


Figure 3. Equivalent closed-loop for the observer-based IMC structure

The form of \hat{C}' shows that the regulator virtually controls a fictitious plant \hat{P}' , which is also demonstrated in Fig. 3. Here the fictitious plant is

$$\hat{P}' = \frac{\hat{P}}{1 + \hat{K}_l \hat{P}} = \frac{\hat{P}}{1 + \hat{L}_l}. \quad (6)$$

The closed-loop transfer function is now

$$T'_{ry} = \frac{\hat{C}'P}{1 + \hat{C}'P} = \frac{QP(1 + \hat{K}_I\hat{P})}{1 + \hat{K}_I\hat{P} - Q\hat{P} + QP} = QP \frac{1}{1 + QP \frac{1}{1 + \hat{K}_I\hat{P}} \frac{\ell}{1 + \ell}} \Bigg|_{\ell \rightarrow 0} = QP = T_{ry}. \quad (7)$$

The relative error ℓ'_T of \hat{T}'_{ry} becomes

$$\ell'_T = \frac{T_{ry} - \hat{T}'_{ry}}{\hat{T}'_{ry}} = \frac{T_{ry}}{\hat{T}'_{ry}} - 1 = \frac{QP}{QP(1 + \hat{K}_I\hat{P})} - 1 = QP \frac{\ell}{1 + \ell} \frac{1}{1 + \hat{K}_I\hat{P}} = \ell_T \frac{1}{1 + \hat{L}_I} \quad (8)$$

$$\frac{1}{1 + Q(P - \hat{P}) + \hat{K}_I\hat{P}}$$

which is smaller than ℓ_T . The reduction is by $\hat{H} = 1/(1 + \hat{L}_I)$.

3. An Observer Based PID-Regulator

The ideal form of a YOULA-regulator based on reference model design [5] is

$$C_{id} = \frac{(R_n P^{-1})}{1 - (R_n P^{-1})P} = \frac{Q}{1 - QP} = \frac{R_n}{1 - R_n} P^{-1}, \quad (9)$$

when the inverse of the process is realizable and stable. Here the operation of R_n can be considered a reference model (desired system dynamics). It is generally required that the reference model has to be strictly proper with unit static gain, i.e., $R_n(\omega = 0) = 1$.

For a simple, but robust *PID* regulator design method assume that the process can be well approximated by its two major time constants, i.e.,

$$P \cong \frac{A}{A_2}, \quad (10)$$

where

$$A_2 = (1 + sT_1)(1 + sT_2). \quad (11)$$

According to (9) the ideal YOULA-regulator is

$$C_{id} = \frac{R_n P^{-1}}{1 - R_n} = \frac{R_n (1 + sT_1)(1 + sT_2)}{A(1 - R_n)}; \quad T_1 > T_2. \quad (12)$$

Let the reference model R_n be of first order

$$R_n = \frac{1}{1 + sT_n}, \quad (13)$$

which means that the first term of the regulator is an integrator

$$\frac{R_n}{1 - R_n} = \frac{\frac{1}{1 + sT_n}}{1 - \frac{1}{1 + sT_n}} = \frac{1}{1 + sT_n - 1} = \frac{1}{sT_n}, \quad (14)$$

whose integrating time is equal to the time constant of the reference model. Thus the resulting regulator corresponds to the design principle, i.e., it is an ideal *PID* regulator

$$C_{PID} = A_{PID} \frac{(1 + sT_I)(1 + sT_D)}{sT_I} = A_{PID} \frac{(1 + sT_1)(1 + sT_2)}{sT_1} \quad (15)$$

with

$$A_{PID} = \frac{T_1}{AT_n}; \quad T_I = T_1; \quad T_D = T_2. \quad (16)$$

The YOULA-parameter Q in the ideal regulator is

$$Q = R_n P^{-1} = \frac{1}{A} \frac{(1 + sT_1)(1 + sT_2)}{1 + sT_n}. \quad (17)$$

It is not necessary, but desirable to ensure the realizability, i.e., it is reasonable to use

$$Q = R_n P^{-1} = \frac{1}{A} \frac{(1 + sT_1)(1 + sT_2)}{(1 + sT_n)(1 + sT)}, \quad (18)$$

where T can be considered as the time constant of the derivative action ($0.1T_D \leq T \leq 0.5T_D$). The regulator \hat{C} and the feedback term \hat{H} must be always realizable. In the practice the *PID* regulator and the YOULA-parameter is always model-based, so

$$\hat{C}_{PID}(\hat{P}) = \hat{A}_{PID} \frac{(1 + s\hat{T}_1)(1 + s\hat{T}_2)}{s\hat{T}_1}; \quad \hat{A}_{PID} = \frac{\hat{T}_1}{\hat{A}\hat{T}_n}, \quad (19)$$

$$\hat{Q} = R_n \hat{P}^{-1} = \frac{1}{\hat{A}} \frac{(1 + s\hat{T}_1)(1 + s\hat{T}_2)}{1 + s\hat{T}_n}. \quad (20)$$

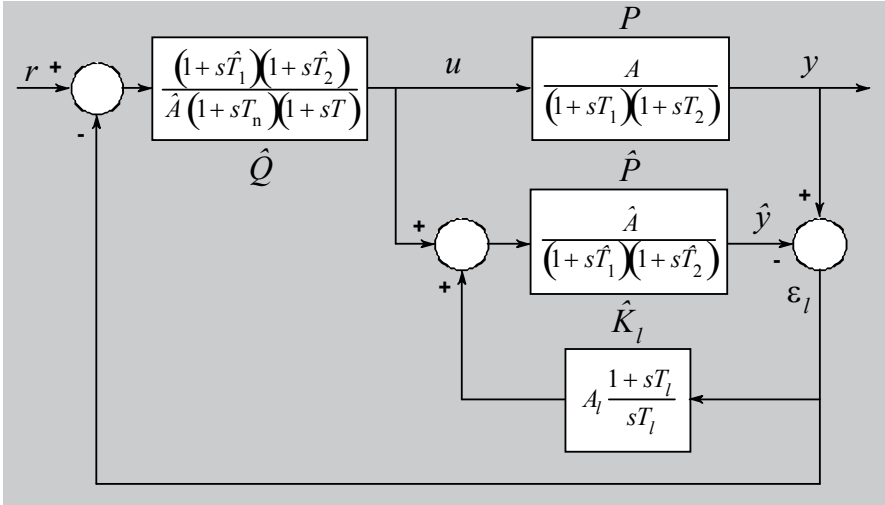


Figure 4. An observer based PID regulator

The scheme of the observer based *PID* regulator is shown in Fig. 4, where a simple *PI* regulator

$$\hat{K}_l = A_l \frac{1 + sT_l}{sT_l} \quad (21)$$

is applied in the observer-loop. Here T_l must be in the range of T , i.e., considerably smaller than T_1 and T_2 .

Note that the frequency characteristic of \hat{H} cannot be easily designed to reach a proper error suppression. For example, it is almost impossible to design a good realizable high cut filter in this architecture. The high frequency domain is always more interesting to speed up a control loop, so the target of the future research is how to select \hat{K}_l for the desired shape of \hat{H} .

4. Simulation Experiments

The simulation experiments were performed in using the observer based *PID* scheme shown in Fig. 4.

Example 1

The process parameters are: $T_1 = 20$, $T_2 = 10$ and $A = 1$. The model parameters are: $\hat{T}_1 = 25$, $\hat{T}_2 = 12$ and $\hat{A} = 1.2$. The purpose of the regulation is to speed up the basic step response by 4, i.e., $T_n = 5$ is selected in the first order R_n . In the observer loop a simple proportional regulator $\hat{K}_I = 0.01$ is applied. The ideal form of Q (17) was used. Figure 5 shows some step responses in the operation of the observer based *PID* regulator.

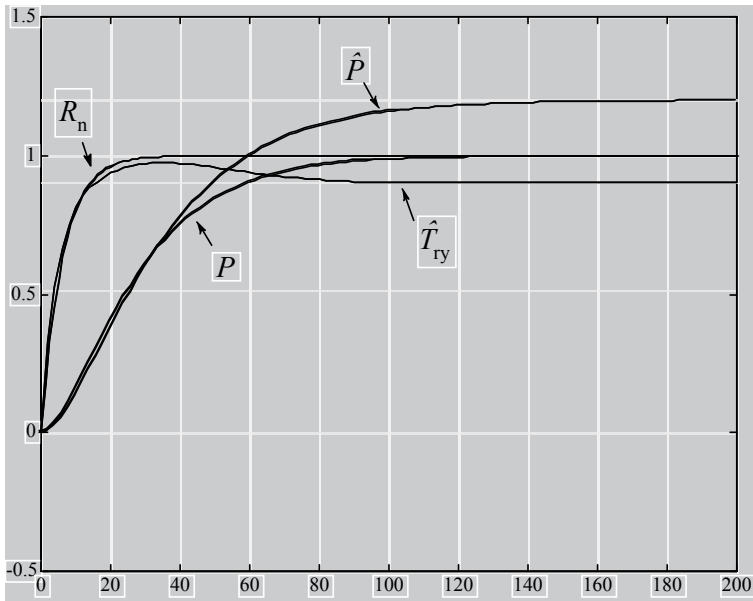


Figure 5. Step responses using the observer based *PID* regulator

It is easy to see that the \hat{T}'_{ty} very well approximates R_n in the high frequencies (for small time values) in spite of the very bad model \hat{P} .

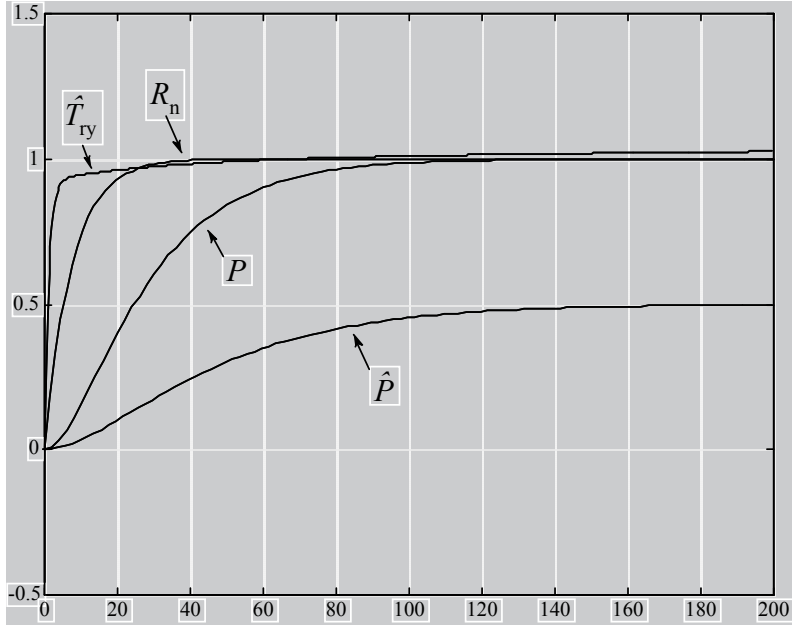


Figure 6. Step responses using the observer based PID regulator

Example 2

The process parameters and the selected first order R_n are the same as in the previous example. The model parameters are: $\hat{T}_1 = 30$, $\hat{T}_2 = 20$ and $\hat{A} = 0.5$. In the observer loop a PI regulator (67) is applied with $A_l = 0.001$ and $T_l = 2$. The ideal form of Q (17) was used. Figure 6 shows some step responses in the operation of the observer based PID regulator.

It is easy to see that the \hat{T}'_{ry} well approximates R_n in the high frequencies (for small time values) in spite of the very bad model \hat{P} .

5. Conclusions

It was shown that the SFO methodology can be applied to the YOULA-parametrized regulators, too. This approach reduces the model error by the sensitivity function of the observer loop similarly to the SFO scheme.

Using this new approach an observer based PID regulator was also introduced. This regulator works well even in case of large model errors as some simulations showed.

Aknowledgement

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