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Control Design of Variable-Geometry Suspension Considering the Construction System

Balázs Németh and Péter Gáspár

Abstract

The paper proposes a method in which the construction of a variable-geometry suspension and the design of a robust suspension control are performed simultaneously in order to enhance vehicle stability. The control system guarantees various crucial performances which are related to the chassis roll angle and half-track change. Moreover, by changing the camber angles of the front wheels the yaw rate of the vehicle is modified, which can be used to reduce the tracking error from the reference yaw rate. The design of a suspension control system is based on robust LPV methods, which meet the performance specifications and guarantee robustness against model uncertainties. Since there is an interaction between the construction design and the control design a balance must be achieved between them.

I. Introduction

Since the requirements in terms of technology, economy and consumer demand for vehicle systems have increased significantly automotive companies must respond to them in different innovation strategies. In research and development one of the new technologies is based on the variable-geometry suspension. Its advantages are the simple structure, low energy consumption and low cost compared to other mechatronic solutions, see [1].

The height of the roll center has an important role in the roll dynamics of the vehicle. A possible way to minimize the chassis roll angle is the minimization of the height of the roll center. The roll center

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depends on the camber angle of the front wheels, which can be modified by the variable-geometry suspension system. The lateral movement of the contact point of the variable-geometry system is also relevant from the aspect of tire wear, when the suspension moves up and down while the vehicle moves forward, see [2]. Moreover, by changing the camber angles of the front wheels the yaw rate of the vehicle is modified, which can be used to reduce the tracking error from the reference yaw rate. With the control of the camber angles, the variable-geometry system is able to focus on trajectory tracking . Thus, in an emergency such as a sharp bending the variable-geometry system focuses on trajectory tracking instead of the conventional performances.

Several papers for various kinematic models of suspension systems have been published. A review of the first variable-geometry suspensions was presented by [3]. The selection of the suspension components and the influence of the distribution of vertical forces were analyzed in [4]. A nonlinear model of the McPherson strut suspension system was published by [5], [6]. By using this model the kinematic parameters such as camber, caster and king-pin angles were examined. The kinematic design of a double-wishbone suspension system was examined by [7]. The vehicle-handling characteristics based on a variable roll center suspension was proposed by [8]. A rear-suspension active toe control for the enhancement of driving stability was proposed by [9]. Another field of variable-geometry suspension is the steering of narrow vehicles. These vehicles require the design of an innovative active wheel tilt and steer control strategies in order to perform steering as if for a car on straight roads and leaning in the bends as a motorcycle, see [10]. The active tilt control system, which assists the driver in balancing the vehicle and performs tilting in the bend, is an essential part of a narrow vehicle system, see [11]. As it can be seen, several papers concern the effects of suspension construction on the performances of the controlled system. In an important paper the coupling between the model and the controller optimization is analyzed. Different approaches are used such as sequential, iterative or nested approaches. The combined system optimum is not guaranteed, see [12].

The paper proposes a method in which the construction of the variable-geometry suspension and the design of robust suspension control are performed simultaneously in order to meet performance specifications. Since there is a trade-off between the control design and the construction of the variable-geometry suspension, an optimization criterion will be formulated. In the paper robust LPV (Linear Parameter Varying) methods are proposed for control design, see [13].

II. Lateral dynamics of vehicle model

Using the bicycle model the lateral dynamics of the vehicle is formulated. Although the Magic Formula gives a highly accurate description of the lateral tire force, see e.g. [14], a simplified form is constructed for numerical reasons.

A control-oriented lateral tire model in the direction of the wheel ground contact velocity is approximated $F_{yf} = C_1\alpha_f + C_\gamma \gamma$ and $F_{yr} = C_2\alpha_r$, where γ is the wheel camber angle, α_f, α_r are the tire side slip angles at the front and rear, C_1, C_2 are cornering stiffnesses and C_γ is a coefficient which represents the degree of offset, see [6].

In the bicycle model the first equation considers forces for the lateral dynamics ($F_{yf} + F_{yr}$), while the second one considers the torque balance equation for yaw moments ($F_{yf}l_1 - F_{yr}l_2$).

$$mv(\dot{\psi} + \dot{\beta}) = C_1\alpha_f + C_2\alpha_r + C_{1,\gamma}\gamma, \tag{1}$$

$$J\ddot{\psi} = C_1 l_1\alpha_f - C_2 l_2\alpha_r + C_{1,\gamma} l_1\gamma, \tag{2}$$

where J is the yaw inertia of the vehicle, l_1 and l_2 are geometric parameters, ψ is the yaw of the vehicle, β is the side-slip angle of the vehicle and $\alpha_f = -\beta + \delta - l_1 \cdot \dot{\psi}/v$, $\alpha_r = -\beta + l_2 \cdot \dot{\psi}/v$ also depend on longitudinal velocity v.

In the design of trajectory tracking control it is necessary to guarantee that the lateral position of the vehicle tracks the course of the road. The required lateral motion is controlled by the difference between the actual yaw-rate of the vehicle and the yaw-rate desired by the driver. The desired yaw-rate, which is the reference signal of the system, can be computed by using the following first order reference system [15], which is represented by a transfer function from steering angle δ to reference yaw-rate signal $\dot{\psi}_{ref}$:

$$G_{ref}(s) = \frac{v}{l_1 + l_2 + \frac{\eta}{g}v^2}\frac{1}{\tau s + 1} \tag{3}$$

where η is an understeer gradient [14], and τ can be computed as: $\tau = \frac{Jv}{l_1 C_1 (l_1+l_2)(1+\eta v^2/g/(l_1+l_2))}$. The side-slip angles of the tires influence the actual yaw rate, which differs from the reference yaw rate. Thus, the cornering maneuver may lead to understeering or oversteering.

III. Modeling of a variable-geometry suspension system

The kinematic model of the variable-geometry suspension based on the double wishbone suspension system is presented in Figure 1. This model contains the geometry of the actuator and shows the suspension displacements. Since in this model the masses, inertias and elasticity of the construction elements are ignored, the arms of the suspension are modeled as bar elements. The suspension is analyzed in a coordinate system which is fixed to the chassis. Consequently the rolling of the chassis and the road irregularities have the same effect in terms of the moving of the wheel as the chassis.

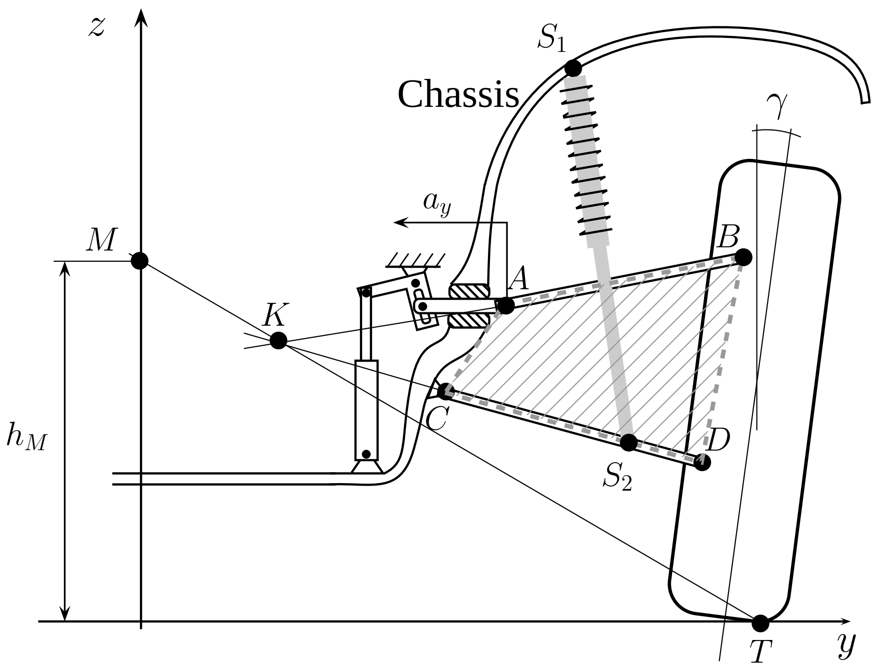

Fig. 1. Applied model of the suspension system

The deduction of the formulation of the wheel camber angle depends on the geometric position of the suspension points, road irregularity and the input of the mechanism. The vertical forces of the suspension are considered as an indirect way in the modeling of the suspension movements. The effects of the movement of the chassis are similar to those of road irregularity. The transformation of double-wishbone suspension parameters to the parameters of a quarter-car model is presented by [16].

The suspension system is analyzed in a local coordinate system, whose center point is C. Point A in the variable-geometry suspension is able to move only in a horizontal direction. The change of point A in the y direction is the real input of the mechanism, which is denoted by a_y. Two further points B and D are marked on the tire, which move in both y and z directions. T is the road-wheel contact point, which moves as a function of the road irregularities, t_y, t_z. The relationship between input a_y, the wheel camber output γ can be expressed, [17].

In Figure 1 the realization of the variable-geometry suspension is also illustrated. The lateral displacement of point A is realized by an electro-hydraulic actuator in the mechanical construction. The actuator is positioned vertically, therefore it requires relatively small space. The necessary stroke of the actuator is influenced by the length of the arms of the rotating element.

In the following the effects of input a_y on different vehicle components are analyzed. The relationship between a_y and γ as a function of t_z is approximated by a linear function:

$$\gamma = \xi_1 t_z + \varepsilon_1 a_y. \tag{4}$$

The construction of the suspension determines the height of the roll center of the chassis (h_M). The roll center is determined by A,B,C,D and T points. The intersection of the arms (A, B) and (C, D) is marked by K. The intersection of the line (T, K) and the vertical centerline of the chassis is the roll center itself. The height of the roll center can be divided into static and dynamic components as follows: $h_M = h_{M,st} + \Delta h_M$. Component $h_{M,st}$ represents the height of the roll center of a stationary vehicle, while Δh_M represents the change of the height during traveling. The dynamic component is expressed by a_y and t_z i in the following form:

$$\Delta h_M = \xi_2 t_z + \varepsilon_2 a_y. \tag{5}$$

During traveling the half-track change (ΔB) is also an important economy dynamic parameter of the suspension system, since it is related to tire wear. Although the relationship between a_y and ΔB can be approximated linearly at low values of (a_y, t_z), the nonlinear effects of the half-track change increase at their high values. Thus the linear and nonlinear parts of the half-track change are separated in the analysis. In the low domain suspension parameter ΔB is approximated in the following form:

$$\Delta B = \xi_3 t_z + \varepsilon_3 a_y. \tag{6}$$

The control input of the system is the lateral movement a_y of the suspension point A. Note that in the real implementation the a_y movement can be realized by using a hydraulic actuator [18] or an electric motor [1].

IV. Performances in the control-oriented LPV model

In this section the performance specifications concerning both the construction of the variable-geometry suspension and the design of the control are formulated.

1./ Trajectory tracking: In the trajectory tracking control the vehicle must follow the reference yaw-rate, which is approximated by (3). The difference between the yaw-rate of the vehicle and the reference yaw-rate must be minimized:

$$z_1 = \dot{\psi}_{ref} - \dot{\psi}; \qquad |z_1| \to min \tag{7}$$

2./ Minimization of the chassis roll angle: It has also been shown that the roll center depends on the controller actuation a_y and t_z. The height of the roll center has an important role in the roll dynamics of the vehicle. In a general case, the vehicle has a generalized planar motion, therefore its acceleration is described as: $\bar{a} = \bar{u}_R \left(\ddot{R}(t) - \frac{v^2(t)}{R(t)} \right) + \bar{u}_\perp \left[\frac{2}{R(t)} \dot{R}(t) v(t) + R(t) \frac{d}{dt} \left(\frac{v(t)}{R(t)} \right) \right]$ where $\bar{u}_R$ is the radial unit vector and $\bar{u}_\perp$ is the angular unit vector of circular motion. $R(t)$ is the function of the radius of the circular motion, while $v(t)$ represents velocity. The roll motion of the vehicle is influenced by the centripetal acceleration, therefore component $\bar{u}_R$ is considered in roll dynamics: $(I_{xx} + m\Delta h^2)\ddot{\phi} = mg\Delta h \phi + m \left(\ddot{R}(t) - \frac{v^2(t)}{R(t)} \right) \Delta h - B_i \sum_{i=1}^4 F_{susp,i}$ where Δh is the difference between the height of the center of gravity of the chassis and the roll center ($\Delta h = h_{CG} - h_M$), ϕ is the chassis roll angle, I_{xx} is the inertia of the chassis, B_i is the half-track and $F_{susp,i}$ are the vertical forces of the suspension. It can be seen that roll dynamics is influenced by $\ddot{R}(t) - \frac{v^2(t)}{R(t)}$ through Δh. In practice component $\ddot{R}(t)$ from the centripetal acceleration is ignored, therefore based on the expression $\frac{v^2(t)}{R(t)} = v(t)(\dot{\beta}(t) + \dot{\psi}(t))$, the formulation of roll dynamics is the following: $(I_{xx} + m\Delta h^2)\ddot{\phi} = mg\Delta h \phi + mv\Delta h(\dot{\beta} + \dot{\psi}) - B_i \sum_{i=1}^4 F_{susp,i}$. A possible way to minimize the chassis roll angle is the minimization of the height of the roll center h_M. Since h_M is divided into two parts, i.e., $h_M = h_{M,st} + \Delta h_M$, two performance criteria are formulated. According to performance z_2 the difference between the roll center and the center of gravity must be minimized and according to performance z_5 the dynamic displacement of the height of the roll center based on (5) must be minimized:

$$z_2 = \Delta h_M = \xi_2 t_z + \varepsilon_2 a_y; \qquad |z_2| \rightarrow min \tag{8}$$

$$z_5 = h_{CG} - h_{M,st}; \qquad |z_5| \rightarrow min \tag{9}$$

In the aspect of z_2 performance it can be established that the height of the roll center in steady state is determined by the suspension construction. Equation (9) shows that the vertical movement of the roll center is determined by t_z and a_y, where a_y is control input. It means that the minimization of the roll center is determined by the construction and control of the suspension simultaneously.

3./ Minimization of the half-track change: An additional important economy parameter is the half-track change. The lateral movement of contact point T is relevant from the aspect of tire wear [2], when the suspension moves up and down while the vehicle moves forward. Using an appropriate variable-geometry control these unnecessary movements can be eliminated. Based on (6) the performance criterion z_3 must be minimized. The inadequacy of this performance can be compensated for with performance z_6, which extends the half-track change to the nonlinear range:

$$z_3 = \Delta B = \xi_3 t_z + \varepsilon_3 a_y; \qquad |z_3| \rightarrow min \tag{10}$$

$$z_6 = \int\int |\Delta B| da_y dt_z; \qquad |z_6| \rightarrow min. \tag{11}$$

4./ Control input minimization: During the control tasks it is necessary to prevent large control input, because a_y has construction limits. Therefore the fourth performance focuses on the minimization of the input displacement:

$$z_4 = a_y; \qquad |z_4| \rightarrow min \tag{12}$$

Performances can be divided into two groups. Performances z_1, z_2, z_3 and z_4 can be expressed by linear combinations of the control input a_y and disturbance t_y. Vector $\mathcal{Z}_1$ includes the performances which are used in the control design:

$$\mathcal{Z}_1 = \begin{bmatrix} z_1 \ z_2 \ z_3 \ z_4 \end{bmatrix}^T \tag{13}$$

Performances z_5 and z_6 cannot be expressed by any linear combination and they are used only in the construction design. Vector $\mathcal{Z}_2$ includes the performances which are used in the construction design:

$$\mathcal{Z}_2 = \begin{bmatrix} z_5 \ z_6 \end{bmatrix}^T \tag{14}$$

The designed performances must be considered in both the control design and the construction design. In the control design the control input of the mechanism a_y affects the performance specifications. In the construction design the selected points B_z and D_z, which are vertical positions of points B and D, respectively, play an important role.

The lateral vehicle model introduced in Section II contains wheel camber γ as input, which can be replaced by control input a_y according to (4). Note that there is a significant interaction between the longitudinal velocity and the lateral displacement. Consequently, it is necessary to design the tracking system, which is able to handle the changes of forward velocity v. The motion equation of the vehicle is constructed in a state-space representation form, in which the state vector $x = \begin{bmatrix} \dot{\psi}, \ \beta \end{bmatrix}^T$. The control input of the system is a_y on the front wheels $u = a_y$, while the front steering of the vehicle and t_z are handled as the disturbances of the system $w = \begin{bmatrix} \delta, \ t_z \end{bmatrix}^T$. The performance output of the model z consists of four equations according to the components of vector $\mathcal{Z}_1$. The measured output of the model is the yaw-rate, i.e., $y = \dot{\psi}$. The system matrices depend on the velocity of the vehicle nonlinearly. Assuming velocity is measured or estimated and using a scheduling variable $\rho = v$ the nonlinear model is transformed into an LPV model.

V. Integration of the control design and the construction

The goal of this paper is to integrate the control design and the variable-geometry construction. First the design of a control system based on performance $\mathcal{Z}_1$ is proposed and second the integrated design based on performances $\mathcal{Z}_1$ and $\mathcal{Z}_2$ is proposed.

A. Control design

The closed-loop interconnection structure, which includes the feedback relationship of the model P and controller K, is shown in Figure 2. The external signals v; δ; t_z; $\dot{\psi}_{ref}$ and w_n are the measured scheduling variable, the steering angle, the road disturbance, the reference input and the sensor noise, respectively. The control design is based on a weighting strategy in the closed-loop interconnection structure. Purpose of weighting functions W_p is to define the performance specifications in such a way that a trade-off is guaranteed between them. The purpose of the weighting function W_n is to reflect sensor noise, while W_d represents the steering angle and t_z. An unstructured uncertainty is modeled by connecting an unknown but bounded perturbation block Δ to the plant. The magnitude of input multiplicative uncertainty is handled by weighting function W_u.

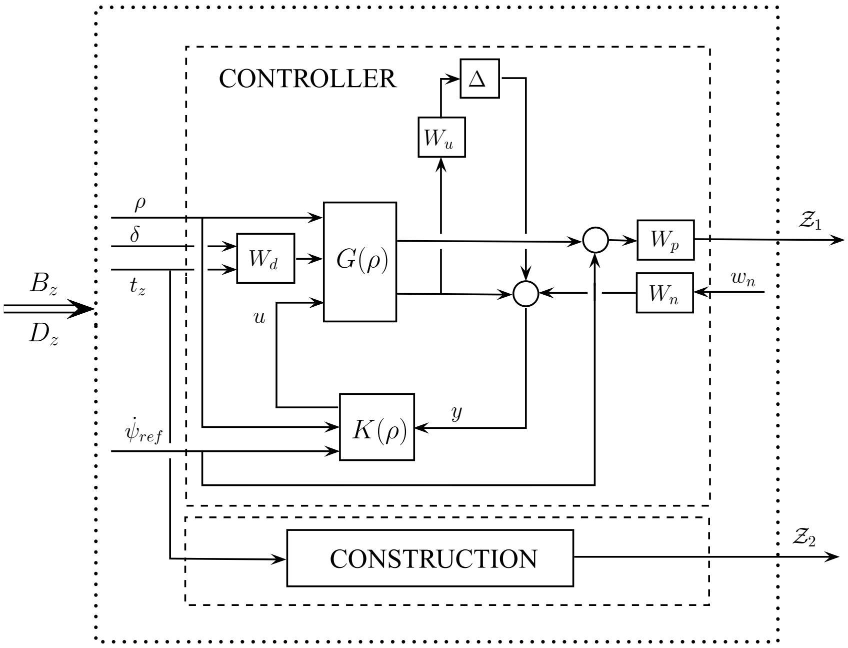

Fig. 2. Closed-loop interconnection structure

The control design is based on LPV methods. The advantage of these methods is that the controller meets robust stability and performance demands in the entire operational interval, since the controller is able to adapt to the current operational conditions, [13], [19]. The solution of an LPV problem is governed by the set of infinite dimensional LMIs being satisfied for all $\rho \in \mathcal{F_P}$, thus it is a convex problem. In practice, this problem is set up by gridding the parameter space and solving the set of LMIs that hold on the subset of $\mathcal{F}_{\mathcal{P}}$. The control design is based on the LPV method that uses parameter-dependent Lyapunov functions, see [19]. The induced $\mathcal{L}_2$ norm of parameter-dependent stable LPV systems with zero initial conditions is defined as

$$\inf_{K} \sup_{\Delta} \sup_{\|w\|_2 \neq 0, w \in \mathcal{L}_2} \frac{\|\mathcal{Z}_1\|_2}{\|w\|_2}. \tag{15}$$

where w is the disturbance, $\mathcal{Z}_1$ is the performance and Δ represents the unmodelled dynamics. The quadratic LPV performance problem is to find a parameter-varying controller in such a way that the resulting closed-loop system is quadratically stable and the induced $\mathcal{L}_2$ norm from the disturbance and the performances (15) is less than a predefined value.

The existence of a controller that solves the quadratic LPV performance problem can be expressed as the feasibility of a set of Linear Matrix Inequalities (LMIs), which can be solved numerically. When the LPV controller has been synthesized, the relationship between the state, or output, and the parameter $\rho = \sigma(x)$ is used in the LPV controller, such that a nonlinear controller is obtained.

The control design is performed in continuous-time, in which it is assumed that scheduling parameter ρ is known in continuous-time. Since ρ is measured only at sampling times, the realization of an LPV controller poses a problem. Instead of obtaining a fixed dependence of system matrices on ρ, the matrices are only known at discrete ρ values. The suitable sampling time must be selected according to the physical system; however, the real sampling time is modified by the implementation possibilities. A simple procedure applied in practice uses a zero-order hold method between sampling times.

B. Integrated design

In the integrated design the construction of the variable-geometry suspension must also be performed. It is important to emphasize that the control design and the construction design are not independent. The construction of the system influences the characteristics $\gamma(a_y, t_z)$, ΔB and Δh_M, which are also parts of the control design according to (13). The performances in (14) also affect the dynamics of the vehicle. Both B_z and D_z have bounds and they are determined by the construction of the wheel hub and the size of the tire: $B_z \in [B_{z,min}\ B_{z,max}]$, $D_z \in [D_{z,min}\ D_{z,max}]$.

In the integrated design the operator norms from inputs of the system to the performance outputs

$\mathcal{Z}_1$ have an essential role. $\mathcal{T}_{i,j}(B_z, D_z)$ is the operator, in which i is the input and j is the output of the system, see Figure 2. Weighting functions $W_{p,i}$, $i = [1; 2; 3; 4]$ represent the importance of the performances of $\mathcal{Z}_1$. These features have been considered in the design of LPV control. In the following the construction components are taken into consideration. Performances in $\mathcal{Z}_2$ also have weights, which are noted as $W_{p,j}$, $j = [5; 6]$. The integrated design is performed in three steps.

In the first step the velocity domain is gridded for several points and at all of the selected grid points the operator norms in terms of $\mathcal{Z}_1$ are computed. The results of these computations represent the fulfillment of performance $\mathcal{Z}_1$ in the different suspension constructions:

$$J_{z,j}(B_z, D_z) = \|\mathcal{T}_{z,j}(B_z, D_z)\|, \quad j \in \{1, 2, 3, 4\}. \tag{16}$$

In the second step it is possible to compute the performances of $\mathcal{Z}_2$ in all constructions. Performances in $\mathcal{Z}_2$ have weights in terms of minimization of both the chassis roll angle and the half-track change. These performances must be normalized using the maximum value of the performances and they are multiplied with $W_{p,k}$ weights:

$$\begin{aligned} J_{z,k}(B_z, D_z) = W_{p,k} \cdot |z_k(B_z, D_z)|/z_{k,max} \\ k \in \{5, 6\}. \end{aligned} \tag{17}$$

The results of these computations represent the fulfillment of performance $\mathcal{Z}_2$ in the different suspension constructions.

In the third step these performance indexes are summarized, which results in a global cost for the fulfillment of $\mathcal{Z}_1$ and $\mathcal{Z}_2$ performances in the different suspension constructions. The global cost based on the performance indexes of the different suspension constructions is

$$\mathcal{J}_z(B_z, D_z) = \sum_j J_{z,j}(B_z, D_z) + \sum_k J_{z,k}(B_z, D_z). \tag{18}$$

Finally, it is necessary to find the construction parameters B_z and D_z with which $\mathcal{J}(B_z, D_z)$ can be minimized. The formulated optimization task is the following

$$\inf_{B_z \in \mathbf{B}} \inf_{D_z \in \mathbf{D}} \mathcal{J}_z(B_z, D_z) \tag{19}$$

The optimization algorithm for solving (19) is a subspace trust region method and is based on the interior-reflective Newton method.

VI. Simulation results

In this section the operation of the supervisory integrated system is illustrated through simulation examples. The control design is performed by using the Matlab/Simulink software, while the verification of the designed controller is performed by using the CarSim software.

In the example the integration of control and suspension construction design is illustrated through a vehicle maneuver simulation. The vehicle is traveling along a predefined road, while the variable-geometry suspension supports the driver to guarantee trajectory tracking. A typical E-class executive automobile is applied in the simulation with a human driver model. The mass of the 6-gear car is 1530 kg and its engine power is 300 kW, and it is fitted with independent double wishbone suspensions. The width of the track is $1605mm$ and the wheel-base is 3165 mm.

The optimization procedure (19) results in different optimal constructions and controllers as functions of the weighting strategies. The first integrated design variable-geometry suspension Sys_1 focuses on roll angle minimization according to performances z_2 and z_5 ($(B_z; D_z) = (350; 150)$), the second system Sys_2 minimizes the half-track change according to the performances z_3 and z_6 ($(B_z; D_z) = (450; 150)$), while the third integrated system Sys_3 actuates control input according to the minimization of performance z_1 ($(B_z; D_z) = (450; 250)$).

In a vehicle maneuver an uncontrolled vehicle is compared to controlled systems. All of the control systems are designed by the proposed integrated construction and control design method. In the simulation the driver generates the steering wheel angle, and the control system actuates the front wheel camber angles.

The vehicle is traveling along a road section and during the maneuver the velocity of vehicle changes significantly, see Figure 3(a). The driver is not able to perform trajectory tracking without the suspension control, therefore the reference yaw-rate signal is not tracked by the vehicle, see Figure 3(b). While the uncontrolled vehicle is not able to guarantee trajectory tracking and causes dangerous situations, all of the vehicles with variable-geometry suspension satisfy the tracking of the yaw-rate signal. Figure 3(c) shows the control inputs of the different systems. In system Sys_3 the minimization of the control input a_y is preferred, while in the other two systems larger control actuation is generated during the maneuver.

The value of actuation a_y is influenced significantly by the appropriate selection of $W_{p,i}$. The half-track changes in the front wheels are shown in Figure 3(d). Using Sys_2 system the half-track change can be reduced. The minimum value of the half-track change and control input a_y are located at different $(B_z; D_z)$. As a result Sys_2 has minimal half-track change values, but it has the largest control input values - and vice versa. In the aspect of these two performances integrated variable-geometry control Sys_1 guarantees a balance. The roll angle of the chassis is shown in Figure 3(e). Since the minimal $\mathcal{J}(350, 150)$ cost and $\mathcal{J}(450, 150)$ cost at Sys_1 are close to each other, the roll angles of chassis are close to each other at Sys_1 and Sys_2. In the case of Sys_3, the roll angle increases because of the increase of $|h_{CG} - h_{M,st}|$.

VII. Conclusion

The paper has proposed the simultaneous design of robust control and the construction of a variable-geometry suspension in order to enhance vehicle stability. While the driver performs a maneuver by using the steering wheel, an autonomous control system modifies the camber angles of the front wheels in order to improve road stability. A control-oriented model of a double wishbone suspension is formulated. There is a trade-off between the control design and the variable-geometry suspension construction, therefore an optimization criterion which contains both the performances of the suspension construction and the performances of control design is formulated. The control design is based on robust LPV methods, in which both performance specifications and model uncertainties are taken into consideration. In the simulation examples, the effectiveness of the variable-geometry suspension is illustrated.

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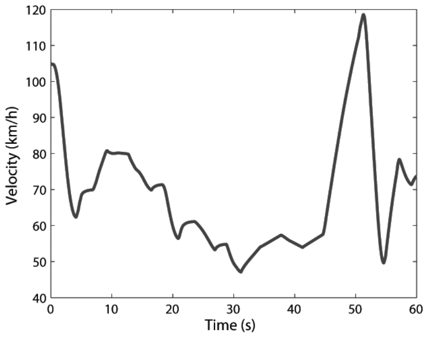
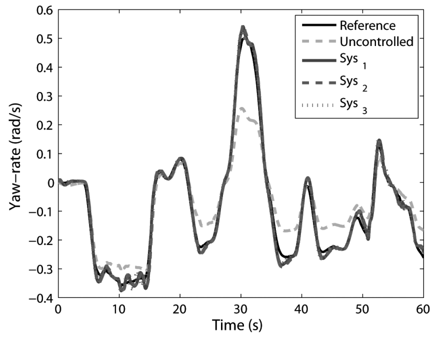
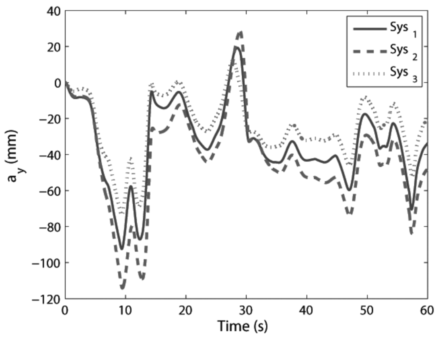


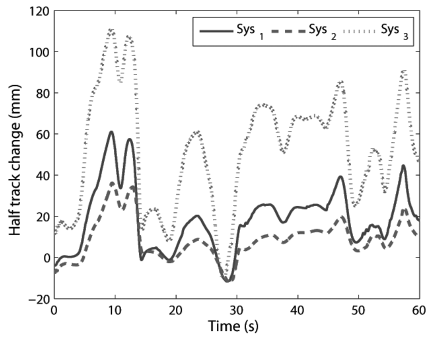
(a) Velocity of vehicle



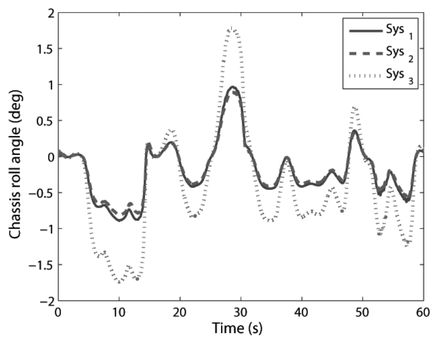
(b) Yaw-rate of vehicle



(c) Input a_y



(d) Half-track change



(e) Roll of chassis

Fig. 3. Operation of the controlled suspension system

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