Challenging Swampland Conjectures

in Exotic Corners of the Landscape

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Prüfen von Swampland Conjectures

in Exotischen Bereichen des Landscape

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Zusammenfassung

Effektive Feld-Theorie erlaubt uns die Physik unserer Welt auf unterschiedlichen Energie-Niveaus zu beschreiben. Umgekehrt muss eine effektive Theorie, mit der wir unsere Welt auf alltäglichen Skalen darstellen, bei hohen Energien mit der Quantengravitation vereinigt werden. Dies ist nicht uneingeschränkt möglich. Deshalb möchte das Swampland-Programm die Voraussetzungen unter welchen eine effektive Theorie mit der Gravitation gekoppelt werden kann eruieren.

Diese Bedingungen werden "Swampland Conjectures" genannt. Sie geben klare Vorhersagen und begrenzen die effektiven Theorien, welche über das Standardmodell hinaus in Betracht gezogen werden sollten. Zum Beispiel soll ein langlebiges de-Sitter (dS) Universum schlicht unmöglich sein. In Anti-de-Sitter (AdS) ist die kosmologische Konstante an die Massen eines Turms von Teilchen gekoppelt, sodass der Energiebereich in welchem die effektive Theorie valide ist im Grenzfall des flachen Raumes verschwindet. Allerdings sind Swampland Conjectures generell nicht mathematisch beweisbar. Nur wenige konnten bislang rigoros bewiesen werden, und auch diese nur in speziellen Bereichen der String Theorie. Die meisten Swampland Conjectures beziehen Inspiration und Evidenz von der String Theorie und Schwarzen Löchern.

String Theorie, eine natürliche Theorie der Quantengravitation, hat eine große Zahl an Vakua mit vielen gut verstandenen effektiven Theorien. Dies stellt eine riesige Datenmenge dar, mit welcher Swampland Conjectures überprüft werden können. Unglücklicherweise sind die am besten verstandenen Vakua nicht unbedingt eine repräsentative Menge. Mit ähnlichen Konfigurationen von BPS D-Branen und p-Form Flüssen sowie perturbativen Beiträgen besteht die reelle Gefahr, dass manche Swampland Conjectures nur ein Produkt der selektiven Datenauswahl sind.

Die Arbeit, welche in dieser Dissertation präsentiert wird, testet die dS und AdS Swampland Conjectures in bislang weniger gut untersuchten Bereichen der String Theorie. Mit non-BPS Branen und Exotischen String Theorien versuchen wir Hindernisse, welche dS Vakua verhindern, zu umgehen. Immer wenn wir einen Schritt näher an dS kommen, tauchen stattdessen neue Probleme auf. Demzufolge wird die dS Swampland Conjecture auch in diesen exotischen Bereichen des String Landscape bestätigt. Schließlich untersuchen wir nicht-perturbative Beiträge und erkennen, dass die AdS Swampland Conjectures um log-terme ergänzt werden müssen. Zusammengefasst bestätigen wir die Swampland Conjectures bis auf log Korrekturen auch in exotischen Bereichen der String Theorie.

Abstract

Effective field theories are the way physics describes the world at different energies. Conversely, this means that any effective theory of our universe should couple to quantum gravity at high energies. Realizing that this is not always possible, the swampland program tries to delineate the conditions under which a low energy theory can be consistently completed with gravity in the UV.

These conditions are called swampland conjectures. They give real predictions and bounds on the effective theories we should consider in beyond the standard model physics. For instance, it is conjectured that a long-lived de Sitter (dS) vacuum is simply impossible. In Anti-de Sitter (AdS), the magnitude of the cosmological constant is related to the mass of a tower of states, so that the energy cutoff of the effective theory goes to zero as we approach flat space. However, as the name "conjecture" already implies, these statements are not in general mathematically proven. Only a very few conjectures have been rigorously proven, and even then only in special subsectors of string theory. The bulk of the swampland conjectures takes inspiration and evidence from string theory and black hole physics.

String theory as a natural theory of quantum gravity has a huge number of vacua, with many well characterized effective theories. This provides an enormous data set that swampland conjectures can be tested on. Unfortunately, the best understood vacua of string theory are not necessarily a representative set. With similar setups, only using standard D-branes, p-form fluxes and perturbative contributions, there is a danger that the swampland conjectures are a product of the lamppost effect.

This is the motivation for the work presented in this thesis. We investigate the dS and AdS swampland conjectures in less explored regimes of string theory, in order to escape the lamppost. We introduce non-BPS branes and consider exotic string theories to try and get around various obstructions to finding dS vacua. Always we observe that while we do manage to circumvent obstructions, new problems appear. This confirms the no-dS conjecture also in exotic corners of the string landscape. Finally we consider non-perturbative contributions and find that here the AdS swampland conjectures have to be corrected by log-terms. In summary, we find that even in strange and new corners of string theory, and up to quantum log-corrections, the swampland conjectures still hold.

Acknowledgements

What do I do when my love is away?
Does it worry you to be alone?
How do I feel by the end of the day?
Are you sad because you're on your own?

No, I get by with a little help from my friends.

The Beatles

The COVID-19 pandemic has hit the world hard for the last 16 months, and continues to do so as I write. With the spread of vaccinations, a temporary semblance of normality has returned locally, but the global changes to communication and collaboration are ongoing. Starting an academic career in this space of online talks and seminars, of zoom conferences and empty offices, is a hard change of pace to the close and personal conferences I was lucky enough to experience during the first half of my PhD. I want to thank everybody who makes these circumstances better and easier to live through.

First I want to thank Ralph Blumenhagen, for his support and kindness. He always has an open door and an open ear for all his students. The leadership he provides while leaving space for personal development leads to a very productive and healthy scientific environment. In this spirit I want to extend my thanks to Dieter Lüst and the entire string group in Munich for the friendly and supportive atmosphere. A particular thank you also to Michael Haack for his helpful and personable attitude.

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Chapter 1

Introduction

Modern physics is almost unreasonably effective in describing our universe. Quantum field theory (QFT) and the Standard Model of particle physics (SM) predict physics on subatomic scales to frankly astonishing precision [1], unifying forces and fundamental particles in quantum gauge theory. On the opposite end of the spectrum, physics on cosmological scales is governed by General Relativity. Although much weaker than the subatomic forces, its long range and independence of charges makes gravity the dominant force over long distances. The standard model of cosmology, ΛCDM, parametrizes the cosmological observations in a very simple model of general relativity and provides a good account for large scale structures, cosmic microwave background and the accelerated expansion of our universe.

We can describe these two very different scales through different theories because physics allows us to decouple different energy regimes. At large energies (small length scales), the gravitational interaction is much weaker than the strong or electro-weak interactions and can be safely ignored when thinking about e.g. atomic processes. Conversely, on large length scales the electro-weak force drops off quickly and the strong force is confining, which leaves gravity as the dominant force. For cosmological purposes, gravity is the only relevant force. This is called Wilsonian perspective, and means that the physics at a given energy can be described by effective theories without detailed knowledge of the physics at other energies.

However, there are clear signs that these two effective models cannot simply fuse to a complete description of our universe. Apart from the many parameters that have to be determined experimentally, the SM does not account for the neutrino mass [2]. While this can be accommodated fairly easily, it increases the number of parameters that can't be explained by the theory. Other problems like the Higgs hierarchy problem are harder to solve. The precision calculations and measurements of QED, a sector of the SM, also signals its own incompleteness. The g-2 experiment at Fermilab that has measured the anomalous magnetic moment of the muon claims a 4.2σ discrepancy with the SM [3]. This falls just short of discovery but nevertheless strongly suggests new physics beyond the Standard Model.

The ΛCDM falls equally short of completeness. Both dark matter and dark energy have failed to be directly detected yet, and the tiny value of the cosmological constant presents yet another hierarchy problem. The biggest problems however arise when combining particle physics and cosmology. Not only does the standard model have no place for dark matter, it actually predicts a cosmological constant many orders of magnitude too large [4, 5].

The most severe problems arise when trying to describe gravity similarly to the forces in QFT. The natural way to formulate a quantum theory of gravity is to describe its effect as mediated by a spin-2 boson, the graviton. Unfortunately, the quantum theory of such a particle is non-renormalizable. The point-particle interaction of fundamental particles results in UV divergencies. To consistently define a full QFT of gravity one would need to introduce an infinite amount of parameters, losing all predictability along the way. Another approach is needed. It should quantize gravity analogously to the other fundamental forces, while at low energies decoupling gravity from the quantum theory, reproducing general relativity and the standard model.

The most successful candidate for such a theory is string theory. While it started as an attempt to understand the strong force and the many hadronic particles - this is now understood as bound states of quarks in QCD - it was later realized that the quantization of a 1-dimensional object, a string, includes excitations that can be identified with the graviton. Because of the extended nature of the string, the divergences of the point-particle theory are absent. While the string length should be larger than the Planck length so quantum gravity effects on the string itself can be safely ignored, as long as the string length is sufficiently smaller that the resolution of current colliders we would see excitations of the string as point particles in experiments. Strings can naturally propagate as closed loops or open strings, with closed strings containing the gravitational sector. Open strings have been found to end on D-branes, which are higher-dimensional dynamical objects, and contain gauge theories. Fermions can be described by the supersymmetric extension to superstring theory.

String theory is fundamentally very predictive. The string length is the only free parameter from which all other parameters like masses and couplings can in principle be derived. Unfortunately, string theory also predicts something that effectively destroys the predictability. The number of dimensions for string theory cannot be chosen to be the observable $3+1$ space-time dimensions. Instead it turns out the only number of dimensions in which superstring theory is consistent, the critical dimension, is 10. While attempts to understand non-critical string theory are ongoing [6–8], the usual approach is to compactify the extraneous directions. Essentially, the 6 dimensions to be removed are curled up on a volume small enough so the energy needed to resolve it is far beyond experimental reach. The prototypical example of compactification is Kaluza-Klein (KK) theory [9]. The details of the compactification manifold, usually taken to be a Calabi-Yau (CY) manifold, substantially dictate the parameters of the 4D effective theory. For a given manifold, variations of the shape appear as massless scalars in 4D which are called moduli. To trust the compactification, the scales of the theory have to be ordered as

$$M_{\mathrm{SM}} \ll M_{\mathrm{mod}} < M_{\mathrm{KK}} \ll M_{\mathrm{s}} < M_{\mathrm{pl}} \,. \tag{1.0.1}$$

Since there is a huge number of CY manifolds, not to mention general 6D compact manifolds, having to compactify introduces a huge parameter space. Additionally, there are typically $\mathcal{O}(100)$ moduli which have to be removed in some fashion since we do not find them in experiments. This can be achieved by adding fluxes, non-trivial expectation values of p-form field strengths, and wrapping D-branes on the compact manifold. There are many ways to add fluxes and branes to any given CY manifold, further increasing the parameter space.

This enormous parameter space, the huge number of 4D string theory vacua, is called the string theory landscape. A lot of effort has gone into exploring the landscape in the past, but without a clear vacuum selection mechanism it is a lot like searching the metaphorical needle in the haystack. Only even if the needle has the size of an atom and the haystack is the size the universe, we have a much better chance of randomly stumbling across the needle than the correct string vacuum. Of course the search for our string vacuum should not be random, but it still gives us a good idea of how daunting the task is.

So as long as we do not know a vacuum selection mechanism which tells us where in the landscape to look, the top-down approach seems hopeless. But the many scans for the standard model and classification attempts of the landscape have not left us empty handed. We now have a huge set of compactifications with the corresponding effective field theories, and it seems that the low energy theories coming from string theory are actually rather restricted in some sense compared to all possible EFTs. So much so that C. Vafa proposed that the theories which can be consistently coupled to quantum gravity are a measure zero subset of all possible quantum field theories [10], kickstarting the swampland program. Introduced as a contrast to the landscape, the swampland delineates the space of QFTs which seem consistent at low energies but cannot be coupled to gravity in the UV. The goal of the swampland program is to find the features that distinguish theories in the landscape from the swampland. While most of the evidence for these conjectures comes from string theory, the swampland program claims to make statements that hold for any theory of quantum gravity. This claim can be supported by supplementing semi-classical black hole arguments. Furthermore, the idea of string universality has been put forward, asserting that string theory actually contains all relevant aspects of quantum gravity, a concept also known as the string lamppost principle [11, 12].

Still, where black hole arguments are lacking the evidence for swampland conjectures usually comes from a rather narrow set of theories. Although many string theory vacua have been explored, most arise from rather similar setups. These are supersymmetric Calabi-Yau orientifold compactifications, with standard BPS D-branes and fluxes and at string tree-level, where the theory is under best calculational control. So even with so much data, we risk introducing a bias which could misinform some conjectures.

Certainly the most contentious swampland conjecture concerns the cosmological constant. While observations [13, 14] tell us that our universe is in a phase of accelerated expansion, best described by a de Sitter (dS) space, it has been near impossible to construct reliable dS vacua in string theory. There do exist proof of principle constructions of dS from string theory, but a fully fledged model where all parameters are under good control is still lacking. This has led to the proposal that dS is in fact in the swampland [15]. While there is certainly space for other theoretical explanations for the observations, these are also heavily restricted by experimental bounds [16].

In this work we address the one-sidedness of evidence for some swampland conjectures by challenging them outside of the usually considered string vacua. After introducing the concepts we need in part I, we will look at three different string theory setting in part II which all promise potential for violating swampland conjectures, and see that string theory always enforces the swampland conjectures in a roundabout way.

We start out from the basics of string theory in chapter 2, introducing T-duality and D-branes as well as the conformal field theory (CFT) description of string theory. Equipped with the relevant supergravity (SUGRA) effective actions, we take a closer look at compactifications and flux vacua. We close the first chapter with examples of string vacua, in particular reviewing dS constructions.

The swampland program and the relevant conjectures will be introduced in chapter 3. The no-dS conjecture is complemented by the trans-Planckian censorship conjecture, which provides a microscopic principle for the former. They essentially restrict the form of the scalar potential in string compactifications so no dS theory can arise. Concerned with the opposite sign of the cosmological constant, the Anti-de Sitter (AdS) conjectures predict the breakdown of the effective theory in the flat space limit by preventing scale separation of the compact space from the EFT. Finally we discuss the emergence proposal as an alternative point of view to distance related swampland conjectures, where towers of states become light at infinite distance in field space, as is the case for one of the AdS conjectures.

The first unusual setting to test the no-dS conjecture contains non-BPS branes. While these are usually unstable, in chapter 4 we find certain non-BPS branes in type IIA that can be stabilized by orientifolds. With these it is possible to circumvent a no-go theorem for tree-level dS vacua, and a toy model shows that positive minima are indeed present. While these branes are not restricted by the usual tadpole conditions, they do add a non-trivial K-theory charge to the compactification. Recalling that this has been shown to lead to anomalies on probe branes, we conclude that the no-dS conjecture implies cancellation of K-theory charge on these models, and vice versa.

We take a step further away from conventional string theory in chapter 5 and introduce exotic string theories to challenge the no-dS conjecture again. Originally conceived by C.Hull [17–20], they are time-like dualities of the usual type II string theories. Exotic strings form a web of theories sporting combinations of multiple time-like directions, Euclidean D-branes and/or Euclidean strings. While it is clear that many potential pathologies arise from extra times, they also allow for dS vacua in flux compactifications. We manage to construct minimally phenomenological models, where we only require the gauge sector to be ghost free. However, it turns out that the only models where this is possible do not allow for dS after all. Even in such strange string theories, the no-dS conjecture seems to hold.

Chapter 6 concerns the dS constructions of KKLT and the LVS. While the uplift to a final dS vacuum is still somewhat questionable, the intermediate AdS minimum found in both constructions also violates swampland conjectures. The thing setting these vacua apart from the usually studied theories are quantum effects that become important in the minimum and actually balance against tree-level contributions to stabilize the minima. Keeping this in mind we find that the quantum effects have to be accounted for as log-corrections in the swampland conjectures. Similar corrections to the no-dS conjecture are actually present in the TCC. We close that chapter with a note on the emergence proposal in KKLT.

Finally, we end the thesis in chapter 7 with some finishing remarks. The thesis is based on the following publications:

[21] *"A Note on the dS Swampland Conjecture, Non-BPS Branes and K-Theory"*
R. Blumenhagen, M. Brinkmann and A. Makridou,
Fortsch.Phys. 67 (2019) 11, 1900068.

[22] *"dS Spaces and Brane Worlds in Exotic String Theories"*
R. Blumenhagen, M. Brinkmann, A. Makridou, L. Schlechter and M. Traube
JHEP 06 (2020) 077.

[23] *"Quantum Log-Corrections to Swampland Conjectures"*
R. Blumenhagen, M. Brinkmann and A. Makridou,
JHEP 02 (2020) 064.

The following publications were also product of my research at the Max Planck Institute for Physics in Munich:

[24] *"KKLT and the Swampland Conjectures"*
R. Blumenhagen, M. Brinkmann, D. Kläwer, A. Makridou and L. Schlechter
PoS CORFU2019 (2020) 158.

[25] *"Small Flux Superpotentials for Type IIB Flux Vacua Close to a Conifold"*
R. Álvarez-García, R. Blumenhagen, M. Brinkmann and L. Schlechter
Fortsch.Phys. 68 (2020) 11-12, 2000088.

Part I

String Theory and the Swampland Program

The first part of the thesis is dedicated to collecting the knowledge needed as a backdrop for the results presented in the second part. Most of this part is standard textbook material, and where not stated separately we will follow the material presented in [26, 27].

We shall start from the basic beginnings, stating the string theory action and discussing how the super-string theories arise. The mass spectrum of the string states shows a symmetry under compactification, which is introduced as T-duality. Accepting T-duality as a feature of string theory forces us to introduce further dynamical objects, D-branes. In type IIA (IIB) superstring theories, only the even (odd) D-branes are allowed. This follows both from the presence of closed string R-R fluxes which act as sources for D-branes, as well as the K-theory charge which the branes carry.

As an interlude we will look at the theory on the world-sheet. This 2D CFT is highly constrained by its symmetries, so much so that the operator algebra completely defines the theory. Introducing boundary states to describe open strings, we find that the open string partition function is dual to the exchange of a closed string between two D-branes. This can be used to determine the D-brane tension.

We will then move on to the low energy limit of string theory, giving the supergravity actions of type II string theories. Finally we will describe compactifications of string theory. Orientifolds and fluxes allow us to describe effective theories in 4D, with constructions of de Sitter vacua marking the end of the first chapter.

For the rest of this part, we will introduce the swampland program. “Everything goes” is a statement one often hears about string theory, but this could not be further from the truth. Effective field theories have to satisfy a number of conditions to allow for a UV completion coupled to gravity, conditions that have been stated as swampland conjectures. The swampland program has grown to include a healthy number of conjectures, of which we will only introduce those that will be used in the work presented in part two.

Chapter 2

String Theory and Flux Compactification

2.1 String theory 101

String theory in its most basic idea is the physics of an extended object replacing the point particle of particle physics. A string-like object moving through space-time defines bosonic string theory. Its action is simply the area of the world-sheet Σ swept out by the movement of the 1-dimensional string in space-time (also called target space). Parametrizing the world-sheet by $\sigma^\alpha = (\sigma, \tau)$, the world-sheet position in space-time is given by $X^\mu(\sigma, \tau)$. Then with the spacetime metric $g_{\mu\nu}(X)$ and the world-sheet metric $h_{\alpha\beta}$, the area is given by the Polyakov action

$$S_P = -\frac{1}{4\pi\alpha'} \int_\Sigma \mathrm{d}^2\sigma \sqrt{-\det(h)}\, h^{\alpha\beta}\, \partial_\alpha X^\mu\, \partial_\beta X^\nu\, g_{\mu\nu}\,. \tag{2.1.1}$$

The factor α' is related to the string length scale as $\ell_s = 2\pi\sqrt{\alpha'}$ and the string mass scale as $M_s = 1/\sqrt{\alpha'}$. Classically, this action has a global space-time Poincaré symmetry, a local reparametrization symmetry on the worldsheet as well as a local invariance under Weyl rescaling of the world-sheet metric. The latter symmetry is broken during quantization, leading to the Weyl anomaly. Cancelling the anomaly requires the target space of the bosonic string to have a fixed number of space-time directions, called the critical dimension $d_c = 26$.

The classical movement of the string is given by the movement of its center of mass on the one hand and excitations of the string on the other. These excitations, describing standing waves of the string, factorize into left- and right-moving parts

$$X(\sigma, \tau) = X_R(\tau - \sigma) + X_L(\tau + \sigma)\,. \tag{2.1.2}$$

This suggests using light-cone coordinates $\sigma_\pm = \tau \pm \sigma$ on the word-sheet. Expanding the string position functions into Fourier modes gives the mode operators of the theory, denoted α_i ($\overline{\alpha}_i$) for the right- (left-)movers. The precise form and relation

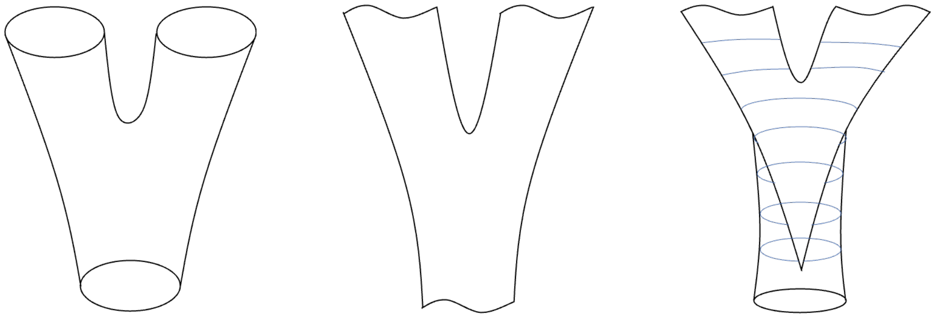

Figure 2.1: Different 2-to-1 string scattering diagrams. Closed strings interact by merging (left), so a purely closed string theory is possible. The open string interaction connects string end-points to other end-points, so an open string can always close as seen on the right.

between the left- and right-movers is determined by the boundary conditions. For illustration, let us write out the expansion of the right-moving closed string

$$X_R^\mu(\sigma_-) = \frac{1}{2}x^\mu + \frac{\alpha'}{2}p^\mu\sigma_- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\,\alpha_n^\mu\, e^{-in\sigma_-}\ , \tag{2.1.3}$$

with the center of mass position x^μ and momentum p^μ.

A string can intuitively take two different forms. It is either closed with periodic boundary conditions, or open with a choice of Neumann or Dirichlet boundary conditions at either side. The boundary conditions at the end of the open string correlate the left- and the right-movers. While the component of the momentum normal to the world-sheet boundary has to vanish in Neumann boundary conditions, Dirichlet conditions fix the location of the string end-points in space-time. We will see later that the open string boundary conditions also force the introduction of further higher-dimensional dynamical objects called D-branes. Closed strings contain the gravity sector of the theory, while the open strings give rise to gauge theories. While closed strings alone can form a complete theory on their own, open strings interact with each other through the joining of two ends. Two open strings may join at their end to produce a new open string, but there is nothing to stop an open string from interacting with itself, producing a closed string from open string interactions. In this way a theory with open strings must always include closed strings as well. The string interactions are visualized in fig. 2.1.

In the low energy regime (compared to the string scale), the quantized string excitations become bosonic point particles in target space with mass given by the excitation level. Unfortunately the lowest level can be found to have imaginary mass, resulting in a tachyonic state.

This basic theory of an extended object is lacking in two ways. First, phenomenologically we expect to encounter fermions in target space. Second, the tachyonic state signals an instability of the entire theory. This instability can mean that the theory is in fact not a fundamental theory but a saddle point of another fundamental theory [28].

Both issues can be solved by adding fermions to the world-sheet theory, interpreting them as superpartners of the coordinate fields. The critical dimension of the superstring is reduced to $d = 10$, and naively the tachyonic ground state is still present. However, the theory is only consistent if the spectrum is supersymmetric, since a gravitino is present. This can be achieved by a projection introduced by Gliozzi, Scherk and Olive (GSO) [29], which leads to $\mathcal{N} = 2$ space-time supersymmetry. The necessity of these projections can also be seen from modular invariance of the CFT. Since there is only the bosonic tachyon, it must always be projected out. The GSO projection removes chiral or antichiral fields from the left- and right-moving excitations independently, and can thus be taken symmetrically or anti-symmetrically. The resulting theories are the type IIA and type IIB superstring. The massless bosonic closed string spectra of the type II string theories is presented in table 2.1. The NS-NS sector with the graviton G, dilaton Φ and Kalb-Ramond field B_2 is universal, while the R-R sector containing p-form fields C_p depends on the theory. The mixed NS-R and R-NS sectors give rise to space-time fermions.

	NS-NS	R-R
IIA:	G, Φ, B_2	C_1, C_3
IIB:	G, Φ, B_2	C_0, C_2, C_4

Table 2.1: Bosonic massless spectrum of closed type II string theories.

The world-sheet is usually assumed to be oriented. This assumption can be removed by projecting onto states symmetric under world-sheet parity transformations. World-sheet parity reverses the σ-direction, equivalently exchanging $X_R \leftrightarrow X_L$. Projecting type II theories by only the world-sheet parity singles out type IIB theory, as type IIA corresponds to the anti-symmetric choice of GSO. The resulting unoriented string is the type I superstring with $\mathcal{N} = 1$ SUSY. Combining the world-sheet parity transformation with space-time $\mathbb{Z}_2$ symmetries defines more general constructions called orientifolds. The fixed points of the space-time involution defines an orientifold plane (O-plane). In the case of pure world-sheet involution the O-plane is space-time filling. We shall come back to orientifolds in the discussion of compactifications.

For completeness let us now introduce the remaining known superstring theories, the heterotic string. These theories contain only oriented closed strings. Interestingly, the left- and right-moving modes are almost completely independent, and there is no obstruction against setting them up in different theories. Using the bosonic string for the left-movers and the superstring for the right-movers, realizes a chiral $\mathcal{N} = 1$ theory in target space. The mismatch of critical dimensions is resolved by compactifying the bosonic string on a 16-dimensional lattice. For consistency, this lattice can only be an $E8 \times E8$ or an $SO(32)$ gauge lattice. We will not go deeper into heterotic theory here, as we will only be concerned with type I and type II theories.

The heterotic string theories complete the list of the five known string theories,

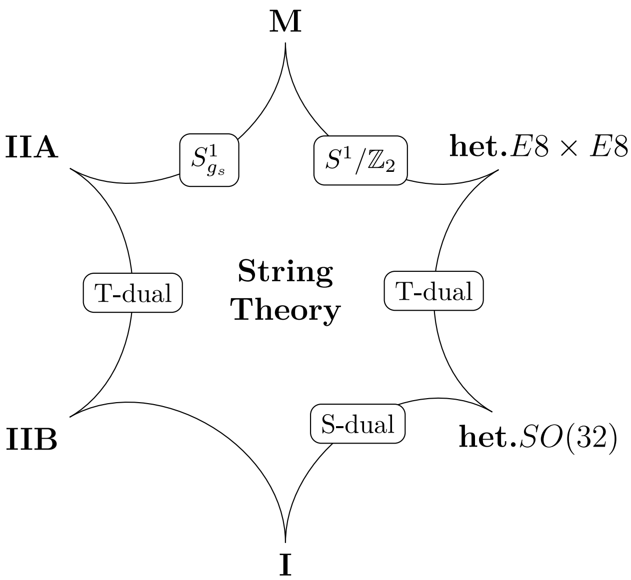

Figure 2.2: The duality web of the five known superstring theories and the 11-dimensional M-theory. All of these theories are thought to be various limits of string theory, instead of truly different theories.

which are known to be connected through a web of dualities depicted in fig. 2.2. Additionally to the 10-dimensional theories, there is the 11-dimensional M-theory thought to complete the picture. M-theory compactified on a circle is the strong coupling limit of type IIA. Adding a $\mathbb{Z}_2$ orbifold to the circle which M-theory is compactified on is dual to the strong coupling limit of the heterotic $E8 \times E8$ theory. S-duality between type I and the heterotic $SO(32)$ theory is also a weak/strong coupling duality. The T-duality between the heterotic or the type II theories relates the theories through circle compactification. We will use it to argue for the presence of D-branes in the following section.

2.2 T-duality and D-branes

After considering a theory for 2-dimensional fundamental strings, it is a straightforward thought to consider even higher dimensional objects which we shall call D-branes. It is a little bit surprising then that this is not only an option, but D-branes necessarily appear in a theory with open strings. In particular, the end of an open string can satisfy Dirichlet boundary conditions along some directions, in which case the position of the string end-point must remain fixed in these directions. The hyperplane on which the string can end is precisely a D-brane. A Dp-brane denotes a $(p+1)$-dimensional D-brane with p space-like directions.

It remains to be seen that this hyperplane is indeed a dynamical object, and why one cannot just demand Neumann boundary conditions for all directions in order to avoid introducing D-branes. The latter question will be answered by introducing T-duality, while the former is necessitated by its coupling to gravity. Intuitively, an open string with both ends on a single D-brane may pinch off and leave the D-brane as a closed string. This interaction contains a graviton being emitted by the D-brane. There cannot be any truly rigid objects coupling to gravity, so the D-brane must indeed be dynamical. Note that this is not the case for O-planes, which do not couple to the string but rather describe fixed points of the background.

2.2.1 T-duality

Since the critical dimensions of both the bosonic as well as the superstring theories are much larger than 4, it is clear that the superfluous dimensions must be compactified. The myriad of ways to compactify 6 dimensions is what gives rise to the vast string landscape, as well as many interesting features of string theory. Most notably, through compactification the five superstring theories exhibit dualities that connect all of them, which suggests a parent theory in 11 dimensions called M-theory. T-duality in particular describes the equivalence of strings propagating on circles with different radii.

The mass spectrum of closed strings compactified on a circle comes with quantized momentum and integer winding contributions. For the closed bosonic string on S^1, the mass spectrum is given by

$$\alpha' m^2 = \alpha' \left(\left(\frac{M}{R} \right)^2 + \left(\frac{LR}{\alpha'} \right)^2 \right) + 2(N_L + N_R - 2)\,, \tag{2.2.1}$$

with winding number L, quantized momentum M/R, and radius R. $N_{L/R}$ are the level of left- and right-moving excitations on the string. Clearly this formula is invariant under the inversion of the radius $R \to \frac{\alpha'}{R}$ and simultaneous exchange of winding and momentum numbers $M \leftrightarrow L$.

The mode expansion of the closed string on S^1 replaces the free momentum in the compact direction by the quantized momentum and winding numbers so the compact coordinate expressions become

$$X_L(\sigma_+) \sim \left(\alpha' \frac{M}{R} + LR \right) \sigma_+\,, \qquad X_R(\sigma_-) \sim \left(\alpha' \frac{M}{R} - LR \right) \sigma_-\,. \tag{2.2.2}$$

Note the opposite signs for the left- and right-movers. Exchanging momentum and winding due to T-duality then amounts to flipping the sign of the right-moving component of the compact coordinate.

This operation does not leave the physical states invariant, thus it is not a symmetry of the theory. The Hilbert space however is mapped to itself, the spectrum and interactions of the theory are invariant. This is the hallmark of dualities. In this case, T-duality provides a way to describe the theory at small radius by a different theory at large radius.

The closed bosonic string is self-dual under T-duality, but this cannot be true for the open string with Neumann boundary conditions on the circle. The momentum along the compact circle must still be quantized, but the open string has no conserved winding number which could exchange places with it. Conversely, a similar statement holds for the open string with Dirichlet boundary conditions. Here the string has a meaningful winding number, but no conserved momentum on the circle. Indeed one finds that in order to exchange momentum and winding numbers, T-duality of the open string changes the boundary conditions simultaneously. Therefore one expects to encounter both boundary conditions in general.

T-duality remains present also when introducing fermions in the superstring theories, where it contributes to the web of dualities connecting the five superstring theories. However due to the sign exchange of the right-moving sector, the type IIA and B theories are T-dual to each other. We shall see in the next section what this means for D-branes in these theories. In summary, when accepting T-duality as a feature of string theory, one is left with no choice but to include D-branes in the theory.

2.2.2 D-branes

Let us begin with a short discussion of the open string tachyon. As we have mentioned, the presence of a tachyon indicates an instability of the theory. While the closed string tachyon of bosonic string theory corresponds to a fundamental instability, the open string tachyon only signals an instability of the open string sector. In particular, the open string tachyon signals the instability of D-branes.

An open string with both ends on the same D-brane may pinch off and move away from the D-brane as a closed string state, indicating a decay of the D-brane into closed string radiation. This is difficult to show directly, however using Witten's open string field theory it has been shown that the energy of the tachyonic maximum equals the D-brane tension [30–32].

An open string can also end on different D-branes. If the branes are far apart, the distance which the string is stretched across contributes to the mass of the open string states. Coincident branes on the other hand do not change the mass spectrum. In this case one can assign additional degrees of freedom to the string ends. These so-called Chan-Paton charges label which of the coincident branes the string ends on. The stringy interaction of these Chan-Paton charges gives rise to nonabelian gauge theories.

Moving on to supersymmetric type II theories, one finds that half of the D-brane spectrum is stabilized by supersymmetry. In general, adding D-branes to the theory breaks supersymmetry. The set of D-branes which is stable breaks only half of the supersymmetry, a property which is called BPS. The remaining $\mathcal{N} = 1$ SUSY is sufficient to remove the open string tachyon from the spectrum. There are several ways to deduce the spectrum of stable BPS D-branes. It is already clear from T-duality that type IIA and B must have complementary spectra. We will now look at the charges carried by the D-branes and their coupling to p-form fields from the string spectrum to deduce the stable D-branes.

2.2.3 R-R vs K-theory charges

In 10D, a $(p+1)$-dimensional Dp-brane couples electrically to a $(p+1)$-form gauge field, and magnetically to a $(7-p)$-form field. In other words, the massless excitations of the closed string which are higher form gauge fields couple to D-branes of certain dimensions. Such fields arise from the R-R sector, with a 1-form and a 3-form field in type IIA and a 0-form, a 2-form and a 4-form in type IIB. Finding out which D-branes couple in this way may point us toward the stable branes. Indeed, the charges carried by the D-branes must be conserved. This prevents the branes from decaying. Moreover, the coupling to the D-branes induces tadpoles for the R-R form fields. These tadpoles violate charge conservation, and have to be cancelled. This leads to conditions on the number of fluxes, Dp-branes and Oq-planes.

For type IIA, the Dp-branes that can couple to the present fields are $p \in \{0, 2, 4, 6\}$. One can introduce the D8-brane as well to complete the even numbered spectrum. This brane would couple to a 9-form field, which has no dynamical degrees of freedom and thus does not show in the R-R string spectrum.

The type IIB form fields couple to Dp-branes for $p \in \{-1, 1, 3, 5, 7\}$. The D(-1)-brane is an object localized in space as well as time, also called D-instanton. Seeing as all the odd number are present, one can also introduce a space-time filling D9-brane. This spectrum of odd p is indeed T-dual to the even spectrum of type IIA.

The conserved charges of gauge theories are usually classified by cohomology classes of gauge field configurations, called R-R charges in this context. It turns out that R-R charges are a little too restrictive to fully describe the charges carried by D-branes. In particular, cohomology classes do not account for the addition or subtraction of vector bundles. However we have seen that stacking D-branes "adds" gauge fields on the world-volume by adding Chan-Paton labels. Similarly, the oppositely charged $\overline{\text{D}p}$ anti-branes annihilate with the corresponding Dp-brane when they are stacked on top of one another, "subtracting" the gauge fields. In other words, we are looking for equivalence classes of gauge fields, such that e.g. adding a Dp-$\overline{\text{D}p}$ brane pair which can annihilate completely does not change the equivalence class. The mathematical setup for this is K-theory [33].

Consider a stack of n D9-branes and $\overline{n}$ $\overline{\text{D9}}$-branes with gauge bundles $E = U(n)$, $E' = U(\overline{n})$ on their world-volumes. Let us denote this system as (E, E'). The system can annihilate completely if the gauge bundles are topologically equivalent $E \sim E'$ which requires $n = \overline{n}$. This setup has no tadpoles and cancelling R-R charge. Then the configuration should be in the same equivalence class as the vacuum $(E, E') \sim 0$. Also, any other configuration of branes and anti-branes (E, F) should be equivalent to the addition of branes and anti-branes with identical vector bundles H,

$$(E, F) \sim (E \oplus H, F \oplus H)\,. \tag{2.2.3}$$

This is the definition of the K-theory $K(X)$, which gives a group structure on equivalence groups of pairs of vector bundles over the spacetime manifold X. For Dp-branes in flat space, the relevant gauge bundles are those independent of the position on the brane, i.e. gauge bundles over the normal directions. There is a subtlety that the rank of these gauge bundles is assumed to be equal, which is the case when the fields fall sufficiently at infinity. This restricted K-theory is called $\widetilde{K}(X)$. The sufficient falloff also means that we can add a point at space-like infinity, compactifying the flat normal space into a sphere. Following the scheme of this D9-brane setup, the K-theory charges of Dp-branes in flat space are

$$\widetilde{K}\left(S^{9-p}\right) = \begin{cases} \mathbb{Z}\,, & p \text{ odd} \\ 0\,, & p \text{ even} \end{cases} \,. \tag{2.2.4}$$

The interpretation of this is that the number $n - \overline{n}$ of coincident (anti-)branes is $\mathbb{Z}$-valued for p odd, but only the vacuum configuration is allowed for p even. This is what we expect for the type IIB theory which includes the D9-brane we started the discussion with.

For type IIA, we do not expect the D9-brane to be stable. It is however still possible to construct BPS type IIA branes as bound states of unstable D9-branes. Since the brane can decay by itself, we have to describe this setup in a different way [34]. The K-theory $K^{-1}(X)$ of the unstable D9-branes describes pairs (E, α) of a $U(N)$ vector bundle E and an automorphism α on E. The vector bundle is again the Chan-Paton bundle on the D9-brane, while the automorphism is related to the tachyon vacuum manifold and the construction of stable D-brane bound states. An element of $K^{-1}(X)$ is called elementary if the automorphism can be continuously deformed to the identity, which translates to the D9-brane carrying no lower-dimensional D-brane charge. These configurations can be created from and decay into the vacuum. Elements (E_i, α_i) of $K^{-1}(X)$ are defined up to the addition of elementary pairs (F_i, β_i)

$$(E_1 \oplus F_1, \alpha_1 \oplus \beta_1) \sim (E_2 \oplus F_2, \alpha_2 \oplus \beta_2)\,, \tag{2.2.5}$$

such that the D-brane configurations are defined up to D9-branes which can annihilate directly to the vacuum. The K-theory charges of type IIA Dp-branes in flat space are then given by

$$K^{-1}\left(S^{9-p}\right) = \begin{cases} \mathbb{Z}\,, & p \text{ even} \\ 0\,, & p \text{ odd} \end{cases} \,. \tag{2.2.6}$$

It is an interesting fact that $K^{-1}(X) \sim K(X \times S^1)$, which is actually the construction first proposed by Witten [33]. This gives the type IIA branes an even codimension, but the physical motivation for this extra circle remains mysterious. It has been argued that this construction could be related to M-theory.

The type I case is again very similar to type IIB, as D9-branes are stable. In fact, cancellation of the orientifold tadpole means the type I vacuum contains 32 D9-branes. These support an $SO(32)$ gauge group, and additional D9-branes add further $SO(N)$ bundles. Then the D-brane charges take values in real K-theory $KO(X)$. While conceptually very similar to $K(X)$, the orientifold changes the outcome quite dramatically. The spectrum of Dp-branes in type I string theory is given by $KO(S^{9-p})$, and is presented below in table 2.2.

The only truly stable, BPS branes in type I are the D1, D5 and D9-branes with $\mathbb{Z}$-valued K-theory charges. The non-BPS D(-1), D0, D7 and D8 configurations are

p	-1	0	1	2	3	4	5	6	7	8	9
$KO(S^{9-p}) \sim$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Table 2.2: Dp-brane spectrum of type I string theory.

stabilized by the presence of the orientifold [35]. The tachyon is then not removed by a GSO, but by the orientifold projection. However, the orientifold can only remove the tachyon of open strings with both ends on the same brane, such that two non-BPS branes on top of each other remain unstable. Thus their K-theory charge is $\mathbb{Z}_2$-valued. These non-BPS D-branes in type I will be the subject of the first result in part II.

2.3 Conformal field theory

Before moving on to the more phenomenological discussion of SUGRA and compactifications, let us review another important yet rather formal aspect of string theory. The world-sheet theory is a conformal field theory (CFT) in 2D, which is extremely constrained by its algebra. For more details on this extensive topic we refer to textbooks [36, 37]. It is useful for the CFT discussion of string theory to Wick rotate $\sigma_\pm = (\tau \pm \sigma) \to -i\tau \pm \sigma$ and define Euclidean coordinates on the complex plane,

$$z = e^{\tau - i\sigma}\,, \qquad \overline{z} = e^{\tau + i\sigma}\,. \tag{2.3.1}$$

With these coordinates, a Fourier expansion maps to a Laurent series with matching coefficients. The string action is then given by

$$S = \frac{1}{2\pi\alpha'} \int \mathrm{d}^2 z\, \partial X^\mu(z, \overline{z})\, \overline{\partial} X_\mu(z, \overline{z})\,. \tag{2.3.2}$$

The string position $X(z, \overline{z})$, which is the fundamental bosonic degree of freedom in the previous discussion, does not have a definite scaling dimension. The scaling dimension is defined for homogeneous rescaling of a conformal primary field $\phi(z, \overline{z})$ under dilation $z \to \lambda z$, $\lambda \in \mathbb{R}$, such that $\phi(\lambda z, \lambda \overline{z}) = \lambda^{-(h+\overline{h})}\phi(z, \overline{z})$ with h, $\overline{h}$ called conformal dimension and $h + \overline{h}$ the scaling dimension of ϕ. A field is called primary if it is well-behaved under conformal transformations. For the CFT, the better degrees of freedom are the derivatives of the bosons. Not only do these have a definite scaling dimension, the equations of motion $\partial\overline{\partial}X = 0$ imply that they are (anti-) holomorphic fields. With closed string boundary conditions, the mode expansion of the derivatives is

$$\Big(\partial X^\mu\Big)(z) = i\sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}} \alpha_n^\mu\, z^{-n-1}\,, \qquad \Big(\overline{\partial} X^\mu\Big)(\overline{z}) = i\sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}} \overline{\alpha}_n^\mu\, \overline{z}^{-n-1} \tag{2.3.3}$$

where the center of mass momentum defines the oscillator zero modes $\alpha_0^\mu = \overline{\alpha}_0^\mu = \sqrt{\alpha'/2}\, p^\mu$. These are primary fields with conformal dimension 1, with respect to the energy momentum tensor

$$T(z) = -\frac{1}{\alpha'} : \partial X(z) \cdot \partial X(z) : , \qquad \overline{T}(\overline{z}) = -\frac{1}{\alpha'} : \overline{\partial} X(\overline{z}) \cdot \overline{\partial} X(\overline{z}) : \tag{2.3.4}$$

with $a \cdot b = a^\mu b_\mu$. Here the normal ordering is defined as the regular part of the OPE. It is simpler to work with the modes α_n, where the normal ordered product of two fields χ and ϕ is defined as

$$(: \chi\phi :)_n \equiv \sum_{k>-h_\chi} \chi_{n-k}\phi_k + \sum_{k\leq -h_\chi} \phi_k \chi_{n-k} \tag{2.3.5}$$

with the conformal dimension $h = 1$ for ∂X. Then the energy momentum operator is defined as

$$T(z) = \sum_{n\in\mathbb{Z}} L_n \, z^{-n-2} , \qquad L_n = \frac{1}{2} \sum_{m\in\mathbb{Z}} : \alpha_{n-m} \cdot \alpha_m : \; . \tag{2.3.6}$$

Working with the expansion modes is very powerful in CFT, because the algebra generated by the conformal symmetry is infinite dimensional. Therefore the theory is highly constrained and all information lies already in the algebra. The expansion modes of the boson satisfy a current algebra

$$[\alpha^\mu_m, \alpha^\nu_n] \;=\; [\overline{\alpha}^\mu_m, \overline{\alpha}^\nu_n] \;=\; m \, \eta^{\mu\nu} \, \delta_{m,-n} \, , \qquad [\alpha^\mu_m, \overline{\alpha}^\nu_n] \;=\; 0 \tag{2.3.7}$$

and the statement that a (holomorphic) field ϕ is primary is equivalent to the algebra

$$[L_m, \phi_n] \;=\; \Big((h_\phi - 1)m - n\Big)\phi_{m+n} \quad \Rightarrow \quad [L_m, \alpha^\mu_n] \;=\; -n \, \alpha^\mu_{m+n}. \tag{2.3.8}$$

The energy momentum modes themselves obey the Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m-n} + \frac{c}{12}(m^3 - m)\delta_{m,-n} \tag{2.3.9}$$

with central charge c. This shows that the energy momentum tensor itself is in general not a primary field! The central charge term is a product of the Weyl anomaly. The energy momentum tensor is only globally well-behaved for vanishing total central charge, where the algebra shows that the energy momentum tensor has conformal dimension $h_T = 2$. This is also reflected by the fact that the zero mode of the energy-momentum tensor does not transform identically under the conformal transformation from the world-sheet cylinder to the complex plane. Rather, one finds $(L_0)_{\text{cyl.}} = (L_0)_{\text{plane}} - \frac{c}{24}$. Vanishing of the total central charge is a strong condition on the theory, that allows to determine the critical dimension of string theory. Each bosonic field, and equivalently each target space dimension, contributes additively with $c = 1$.

The Weyl anomaly plays a role also in the BRST quantization of the string theory. One finds that a (b, c) ghost system must be introduced to account for the gauge anomaly. These ghosts are chiral, fermionic primary fields with conformal dimension $h_b = 2$, $h_c = -1$. It turns out that the system of b and c ghost contributes a central charge of $c = -26$ to the theory, which means that the critical number of bosons/target space dimensions is also 26. Alternatively, the critical dimension can be deduced in light-cone quantization by calculating the anomalous algebra of Lorentz generators.

For closed world-sheet fermions, the boundary conditions are $\psi^\mu(\tau, \sigma + 2\pi) = \pm\psi^\mu(\tau, \sigma)$. In complex coordinates this translates to Laurent expansions with integer or half-integer valued modes

$$\psi^\mu(z) = \sum_{r\in\mathbb{Z}+s} b_r^\mu z^{-r-\frac{1}{2}}\,, \qquad \overline{\psi}^\mu(\overline{z}) = \sum_{r\in\mathbb{Z}+s} \overline{b}_r^\mu \overline{z}^{-r-\frac{1}{2}}\,. \tag{2.3.10}$$

The Neveu-Schwarz (NS) sector has anti-periodic boundary conditions and corresponds to $s = \frac{1}{2}$. The modes with periodic boundary conditions make up the Ramond (R) sector with $s = 0$. The modes satisfy the algebra

$$\{b_r^\mu, b_s^\nu\} = \eta^{\mu\nu}\delta_{r,-s}\,. \tag{2.3.11}$$

Fermions are primary fields of conformal dimension $h_\psi = \frac{1}{2}$. The supersymmetric energy-momentum tensor is just the sum of bosonic and fermionic contributions, with the latter given by

$$T^{(f)}(z) = \frac{1}{2} : \psi(z)\cdot\partial\psi(z) :\,, \qquad L_n^{(f)} = \frac{1}{2}\sum_{r\in\mathbb{Z}+s}\left(r + \frac{n}{2}\right) : b_{-r}\cdot b_{n+r} :\,. \tag{2.3.12}$$

These fermions have central charge $c = \frac{1}{2}$. In a supersymmetric setting there are as many fermionic fields as there are bosons, leaving a total central charge of $\frac{3}{2}$ per dimension. In addition to the (b, c) ghost system from the bosonic symmetries, the super-Weyl transformation must be fixed by another ghost system (β, γ) with bosonic statistics and conformal dimension $(\frac{3}{2}, -\frac{1}{2})$ respectively. The (β, γ) system contributes $c = 11$ to the total theory, fixing the critical dimension of the superstring to $D = \frac{2}{3}(26 - 11) = 10$.

The closed string Hilbert space is generated by the bosonic and fermionic oscillators. Positive modes, or rather the modes ϕ_n with $n > -h_\phi$, are considered annihilation operators while the modes $\phi_{n\leq h_\phi}$ are considered creation operators. Asymptotic in- and out-states $|\phi\rangle$, $\langle\phi|$ of a chiral primary field $\phi(z)$ are defined on the complex plane as

$$|\phi\rangle = \phi_{-h_\phi}|0\rangle = \lim_{z\to 0}\phi(z)|0\rangle\,, \qquad \langle\phi| = \langle 0|\phi_{h_\phi} = \langle 0|\lim_{z\to\infty}\phi(z)z^{2h}\,, \tag{2.3.13}$$

with $|0\rangle$ the vacuum. The vacuum state of the bosonic and NS sectors is unique, corresponding to a zero spin ground state. The R sector contains the fermionic zero mode b_0 which (anti-)commutes with all other modes. Thus the R ground state is degenerate, $|0\rangle \sim b_0^\mu|0\rangle$, and forms a spinor representation of $SO(d-1, 1)$.

The closed string partition function is computed by considering the theory on a torus. Define the CFT torus partition function as

$$\mathcal{Z}(\tau_1, \tau_2) = \mathrm{Tr}_{\mathcal{H}}\left(e^{-2\pi\tau_2 H}\, e^{+2\pi\tau_1 P}\right) \tag{2.3.14}$$

with Hamiltonian H, momentum operator P and torus modulus $\tau = \tau_1 + i\tau_2$, and taking the trace over the CFT Hilbert space $\mathcal{H}$. The different signs in the exponent are due to the Wick rotation implicit in describing the string by a Euclidean CFT. Then the cylinder Hamiltonian and momentum operators being given by

$$H_{\text{cyl.}} = (L_{\text{cyl.}})_0 + (\overline{L}_{\text{cyl.}})_0 \,, \qquad P_{\text{cyl.}} = i(L_{\text{cyl.}})_0 - i(\overline{L}_{\text{cyl.}})_0 \tag{2.3.15}$$

and defining $q = e^{2\pi i\tau}$, the CFT partition function is given by

$$\mathcal{Z}(\tau) = \text{Tr}_{\mathcal{H}} \left(q^{L_0 - \frac{c}{24}} \, \overline{q}^{\overline{L}_0 - \frac{\overline{c}}{24}} \right) . \tag{2.3.16}$$

Calculating the partition function of a free boson, one finds

$$\mathcal{Z}^{\text{torus}} = \frac{1}{2\pi\sqrt{\alpha'\tau_2}} \frac{1}{|\eta(\tau)|^2} \tag{2.3.17}$$

with the Dedekind η-function $\eta(\tau)$.

2.3.1 Boundary CFT

Up to this point we have only discussed closed strings, where the only viable boundary conditions are periodic for bosonic and periodic or anti-periodic for fermionic fields. Turning now to open strings, the boundary conditions can be Neumann $\partial_\sigma X^\mu = 0$ or Dirichlet $\partial_\tau X^\mu = 0$, and can be chosen separately for each string end-point and target-space dimension. These boundary conditions relate left- and right-moving excitations, leaving one independent sector. As before, the good fields to work with from a CFT perspective are derivatives ∂X, $\overline{\partial} X$. Instead of the entire complex plane, the open string CFT lives on the upper half plane with the world-sheet boundaries mapped to the positive and negative real line. The boundary conditions translate to the oscillator modes as

$$\begin{aligned} \alpha_n - \overline{\alpha}_n &= 0 \qquad \text{(Neumann)} \\ \alpha_n + \overline{\alpha}_n &= 0 \qquad \text{(Dirichlet)} \,. \end{aligned} \tag{2.3.18}$$

The total momentum of the string, corresponding to the zero mode oscillators, is given by $p_0 = \frac{1}{2}\alpha_0 = \frac{1}{2}\overline{\alpha}_0$ and clearly has to vanish for Dirichlet boundary conditions. The open string end-points with Dirichlet conditions are therefore fixed. Mixed boundary conditions can only be satisfied for half-integer mode numbers.

The one-loop partition function of the open string is computed from the cylinder, with Wick-rotated, Euclidean time compactified in a loop. The modulus t parametrizes the circumference of the cylinder, and with $q = e^{-2\pi t}$ the partition function is given by the trace over the opens string Hilbert space

$$\mathcal{Z}_{\text{op}}(t) = \text{Tr}_{\mathcal{H}_{\text{op}}} \left(q^{L_0 - \frac{c}{24}} \right) . \tag{2.3.19}$$

Taking the energy contribution of the distance $Y^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$ stretched between two Dirichlet loci into account, the partition function for the bosonic open string

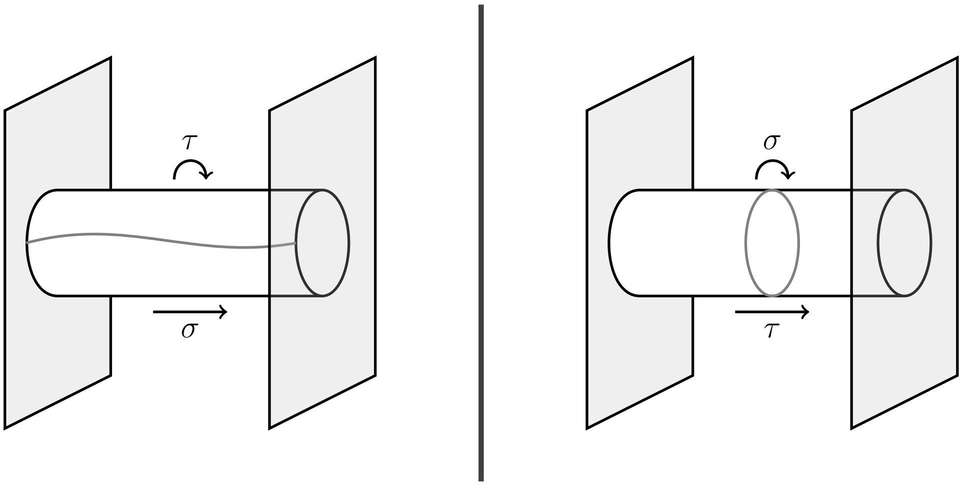

Figure 2.3: The open string loop diagram (left) can be interpreted as a tree-level closed string exchanged between two D-branes (right).

can be calculated to be

$$\begin{aligned} \mathcal{Z}^{(DD)}(t) &= e^{-\frac{t}{2\pi\alpha'}Y^2}\frac{1}{\eta(it)}\,, \\ \mathcal{Z}^{(NN)}(t) &= \frac{1}{\sqrt{2\pi\alpha'}}\frac{1}{\eta(it)}\,, \\ \mathcal{Z}^{(\text{mixed})}(t) &= \sqrt{\frac{2\eta(it)}{\vartheta_4(it)}} \end{aligned} \tag{2.3.20}$$

with mixed boundary condition leading to the Jacobi ϑ-function $\vartheta_4(it)$ thanks to the half-integer modes.

Now as shown in fig. 2.3, the open string 1-loop can be reinterpreted as a closed string being exchanged between two D-branes as. This duality between tree-level closed string and loop-level opens string exchanges the worldsheet directions $(\tau,\sigma)_{\text{open}} \leftrightarrow (\sigma,\tau)_{\text{closed}}$. In the closed string picture, the boundary is a coherent state of closed string states. The boundary emits a closed string which travels for a time π, i.e. the length of the open string, and gets absorbed by the second boundary. This process is nothing but the overlap of boundary states $|B\rangle$ with a closed string propagating in between,

$$\langle \Theta B|\, e^{-\pi l H_{\text{closed}}}\, |B\rangle = \tilde{\mathcal{Z}}(l)\,, \tag{2.3.21}$$

with Θ the CPT operator needed to reorient the second boundary and the closed string Hamiltonian H_{closed}. The tilde indicates that the calculation is done in the closed string channel, and is related to the open string partition function. This is known as open-closed duality. In particular, the cylinder is parametrized by the relation between its circumference and its length. Keeping in mind that we assume a fixed string length of 2π for the closed string and π for the open string, the modular parameters of the two sectors are

$$\tau_{\text{open}} = 2it\,, \qquad \tau_{\text{closed}} = \frac{i}{l}\,, \tag{2.3.22}$$

so the boundary state overlap and the open string partition function are identified through the modular transformation $t = \frac{1}{2l}$.

The boundary states must satisfy gluing conditions corresponding to the appropriate boundary conditions (2.3.23),

$$\begin{aligned} (\alpha_n^\mu + \overline{\alpha}_{-n}^\mu)\,|B\rangle &= 0 \qquad \text{(Neumann)}\,, \\ (\alpha_n^\mu - \overline{\alpha}_{-n}^\mu)\,|B\rangle &= 0 \qquad \text{(Dirichlet)}\,. \end{aligned} \tag{2.3.23}$$

Sorting the boundary conditions for ease of notation, such that the first $(p+1)$ directions $X^0, ..., X^p$ have Neumann and the rest have Dirichlet boundary conditions, we can collectively write down the gluing conditions

$$\left(\alpha_n^\mu + S^\mu_{\ \nu}\,\overline{\alpha}_{-n}^\nu\right)|B\rangle = 0 \tag{2.3.24}$$

by introducing the diagonal matrix $S = (\eta_{\alpha\beta}, -\delta_{ij})$ with -1 for the Dirichlet and $\eta_{\mu\nu}$ for the Neumann directions. Solutions to the gluing conditions are the boundary states

$$|B\rangle = \frac{1}{\mathcal{N}} \exp\left(-\sum_{n\in\mathbb{N}} \frac{1}{n} \alpha_{-n}^\mu S_{\mu\nu} \overline{\alpha}_{-n}^\nu\right)|0\rangle\ . \tag{2.3.25}$$

The normalization can be determined by calculating the overlap explicitly, performing the modular transformation and comparing it to the open string partition function (2.3.20). The normalization of a Dp-brane state turns out to be

$$\mathcal{N}^{-1} = 2^{-6}(4\pi^2\alpha')^{(12-p)/2}\,. \tag{2.3.26}$$

Finally the brane tension can be determined by the coupling of the brane to the graviton,

$$\langle V_g|B\rangle = -\mathcal{N}^{-1}\,\langle 0|\epsilon_{\mu\nu}S^{\mu\nu}|0\rangle = -\mathcal{N}^{-1}\epsilon_{\mu\nu}S^{\mu\nu}V_{p+1} \overset{!}{\sim} -T_p\epsilon_{\mu\nu}S^{\mu\nu}V_{p+1} \tag{2.3.27}$$

with the regularized Dp-brane volume V_{p+1}. The brane tension is proportional to the overall normalization of the boundary state.

2.4 Supergravity actions

Often it is not necessary or even feasible to consider the complete string theory. Instead one uses the Wilsonian approach and considers only the low energy effective theory. Here we mean that the energies are low compared to the string scale, while still far above the usual high energy physics scale. It very nicely turns out that the massless sector of superstring theory is supergravity. In particular, the effective action of the bosonic massless type II string excitations in 10D can be compactly written as

$$S[\text{IIA/B}] = S_{\text{NS}} + S_{\text{R}}[\text{A/B}] + S_{\text{CS}}[\text{A/B}]\,. \tag{2.4.1}$$

While the NS-NS part is the same for type IIA and IIB theories, the Chern-Simons (CS) and R part differ between type IIA and IIB. The massless fields are the metric G, dilaton Φ, and the 2-form Kalb-Ramond field strength $H_3 = \mathrm{d}B_2$ in the NS sector, and the even/odd R-R field strengths in the respective Ramond sectors. The gravitational coupling in 10D is denoted by κ_{10}. The various contributions are given by

$$\begin{aligned}
S_{\mathrm{NS}} &= \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{|\det G|}\; e^{-2\Phi}\left[\mathcal{R} + 4(\nabla\Phi)^2 - \frac{1}{2}|H_3|^2\right] ,\\
S_{\mathrm{R}}[\mathrm{A}] &= -\frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{|\det G|}\left[\frac{1}{2}|F_2|^2 + \frac{1}{2}|\tilde{F}_4|^2\right] ,\\
S_{\mathrm{R}}[\mathrm{B}] &= -\frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{|\det G|}\left[\frac{1}{2}|F_1|^2 + \frac{1}{2}|\tilde{F}_3|^2 + \frac{1}{4}|\tilde{F}_5|^2\right] ,\\
S_{\mathrm{CS}}[\mathrm{A}] &= -\frac{1}{4\kappa_{10}^2}\int B_2\wedge F_4\wedge F_4 ,\\
S_{\mathrm{CS}}[\mathrm{B}] &= -\frac{1}{4\kappa_{10}^2}\int B_2\wedge F_3\wedge F_5 .
\end{aligned} \tag{2.4.2}$$

These actions are given in string frame, which is related to Einstein frame by a field redefinition of the metric $G^E = e^{-\Phi/2}G$. The field strengths are defined as $H_3 = dB_2$ and $F_p = dC_{p-1}$, and $\tilde{F}_p = F_p - H_3\wedge C_{p-3}$. The kinetic term of the n-form F_n is defined as

$$|F_n|^2 = \frac{1}{n!}\,G^{i_1 j_1}\ldots G^{i_n j_n}\,F_{i_1\ldots i_n}\,F_{j_1\ldots j_n} . \tag{2.4.3}$$

The 5-form flux in type IIB has to be self-dual $F_5 = *F_5$, which must be imposed separately. The fermionic action and interaction terms that complete the theory are uniquely defined by supersymmetry.

D-branes are described in the effective theory by the Dirac-Born-Infeld (DBI) action, and another CS action. The D-brane CS action describes the coupling of the brane to R-R fluxes, and is involved in anomaly cancellation and supersymmetry conservation on BPS branes. The DBI action on the other hand involves the couplings of the NS fields to the brane world-volume Σ_{p+1}. Setting the Kalb-Ramond B_2 field to zero for simplicity, the Dp-brane effective action is given by

$$\begin{aligned}
S_{\mathrm{DBI}} &= -T_p\int_{\Sigma_{p+1}}\mathrm{d}^{p+1}\xi\sqrt{-g}e^{-\Phi(X)}\left[1+\frac{1}{4}(2\pi\alpha')^2|F|^2+\ldots\right] ,\\
S_{\mathrm{CS}} &= -\mu_p\int_{\Sigma_{p+1}}\left(C_{p+1} + F\wedge C_{p-1} + \ldots\right) ,
\end{aligned} \tag{2.4.4}$$

with F the field strength of the gauge theory on the brane. Both actions are only the first terms of an expansion. The DBI action is really the integral over $\mathcal{L}_{\mathrm{DBI}} \sim \sqrt{-\det(G+F)}$, and the full CS action involves Pontryagin classes of curvature forms. The first terms of the expansions will suffice for this thesis.

2.5 Compactifications

Until now we have not addressed the elephant in the room, arguably both a blessing for and a failing of string theory: the critical dimension. It is clear that any theory related to our world should, at low enough energies, be a $(3,1)$-dimensional theory. Contrary to the hope or even expectation of early string theorists, we have seen that superstring theory is only self-consistent in 10 dimensions. In order to describe our world, we must get rid of these extra dimension by compactifying them. More precisely, we assume that the target-space factorizes into a $(3,1)$-dimensional, noncompact space representing our observed universe, and a 6-dimensional compact space. Furthermore the size of the compact space is assumed to be small enough, so that the energies needed to probe it are beyond our experimental reach at this time. In this way, the critical dimension can be fulfilled in the UV string theory while also satisfying our 4D IR observations. However the lower-dimensional physics depends on the geometry of the compact space, dramatically reducing the predictive power of string theory.

The manifolds usually chosen for compactifications are usually chosen to be Calabi-Yau (CY), and conserve some supersymmetry in 4D. This is on the one hand phenomenologically motivated, as a supersymmetric extension of the standard model solves problems like the hierarchy problem of the Higgs mass. However with recent progress in accelerator physics failing to produce evidence for low energy supersymmetry, the SUSY breaking scale gets shifted to higher and higher energies and one might wonder if it might already be broken at the compactification scale [38–41].

The other reason we use CY compactifications is the computational advantage they bring with them. Calabi-Yau manifolds are Ricci-flat, so assuming a constant dilaton they are solutions to the 10D SUGRA action for the metric. They are also Kähler, carrying a complex structure $\mathcal{I}$, $\mathcal{I}^2 = -1$ and a Kähler metric G. The Kähler metric is locally derived from a Kähler potential K

$$G_{\alpha\overline{\beta}} = \partial_\alpha \overline{\partial}_{\overline{\beta}} K \tag{2.5.1}$$

in local complex coordinates z^α, $\partial_\alpha = \partial/\partial z^\alpha$ and $\overline{z}^{\overline{\beta}}$, $\overline{\partial}_{\overline{\beta}} = \partial/\partial \overline{z}^{\overline{\beta}}$. The Kähler potential is not unique, as adding (anti-)holomorphic functions $f(z)$, $\overline{f}(\overline{z})$ leaves the metric invariant. The associated fundamental Kähler form is then given by

$$J = iG_{\alpha\overline{\beta}}\, \mathrm{d}z^\alpha \wedge \mathrm{d}\overline{z}^{\overline{\beta}} = i\partial\overline{\partial}K\,. \tag{2.5.2}$$

While the existence of a Ricci-flat metric was proven by Yau [42], actually finding this metric is a very difficult undertaking. Only for the simplest examples, the 2-dimensional torus and K3, are the metrics explicitly known [43, 44]. However, in general and in particular for the 6-dimensional CY 3-folds, the metric can only be approximated numerically [45–47]. In practice, an equivalent definition of CY surfaces is more useful. In fact, any compact Kähler manifold with vanishing first Chern class is Calabi-Yau. Since Chern classes relate to topological properties, it is not necessary to construct a flat metric in order to apply this definition.

With the complex structure, the p-forms can be classified by holomorphicity. This means replacing de Rham cohomology $H^n_{(\mathrm{dR})}$ by Dolbeault cohomology classes $H^{p,q}_{(\mathrm{D})}$, $p+q=n$. The dimensions of the Dolbeault cohomology classes are called Hodge numbers $h^{p,q}$. The Hodge numbers of a complex manifold can be efficiently presented as a Hodge diamond, for example a complex manifold with 3 complex dimensions (a 3-fold) has the following Hodge diamond:

$$\begin{array}{ccccccc} & & & h^{0,0} & & & \\ & & h^{1,0} & & h^{0,1} & & \\ & h^{2,0} & & h^{1,1} & & h^{0,2} & \\ h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\ & h^{3,1} & & h^{2,2} & & h^{1,3} & \\ & & h^{3,2} & & h^{2,3} & & \\ & & & h^{3,3} & & & \end{array} \tag{2.5.3}$$

The Calabi-Yau condition highly constrains the Hodge numbers. In fact, only two Hodge numbers $h^{1,1}$ and $h^{2,1}$ remain as parameters. Complex conjugation and Hodge duality imply vertical and horizontal symmetry of the Hodge diamond, $h^{p,q} = h^{q,p} = h^{3-p,3-q}$. The points of the diamond are fixed to $h^{3,0} = 1$, while the edges vanish. This means there is a unique, nowhere vanishing holomorphic $(3,0)$-form, called Ω_3. The Hodge diamond of a Calabi-Yau 3-fold takes the simple form

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{2.5.4}$$

Upon compactification, the fields of the 10D theory are described as effective 4D fields plus contributions from the compact directions. The compact part takes values in cohomology, over which the 6D part of the action can be integrated to give the 4D effective action. This can be thought of as the 10D fields, e.g. the deformations of the background metric δG^{MN}, having legs on the 4D space $\delta G^{\mu\nu}$, on the compact CY space $\delta G^{\alpha\overline{\beta}}\omega_{\alpha\overline{\beta}}$, or mixed $\delta G^{\alpha\nu}\omega_\alpha$, $\delta G^{\mu\overline{\beta}}\overline{\omega}_{\overline{\beta}}$. From the 4D perspective, the first field is a tensor field, in this case the 4D metric, the second is a scalar and the mixed fields are vectors. In this example, the forms take values in $\omega_{\alpha\overline{\beta}} \in H^{1,1}$, $\omega_\alpha \in H^{1,0}$, and $\overline{\omega}_{\overline{\beta}} \in H^{0,1}$. However the dimension of the latter two spaces is zero on CY spaces, so there can be no vectors arising from the compactification of the metric. The constrained CY Hodge numbers thus also constrain the fields that can appear in 4D.

Taking a closer look at the scalar fields arising from integrating out the compact metric, we expand the deformations of the Kähler metric into a basis of harmonic $(1,1)$-forms b^a, $a \in \{1, ..., h^{1,1}\}$

$$\delta G_{\alpha\overline{\beta}} = \sum_{a=1}^{h^{1,1}} \tilde{t}^a (b^a)_{i\overline{\jmath}} \tag{2.5.5}$$

with real scalar fields $\tilde{t}^a$. These deformations correspond to deformations of the Kähler structure. To preserve the Calabi-Yau structure, the deformations must close on the space of Kähler metrics. The values of $\tilde{t}^a$ which respect this are called the Kähler cone. In general backgrounds, the excitations of the Kalb-Ramond field δB on the compact space also correspond to harmonic $(1,1)$-forms. Combining these with the Kähler deformations defines the complex scalar Kähler moduli t^a

$$(i\delta G_{\alpha\overline{\beta}} + \delta B_{i\bar{j}}) = \sum_{a=1}^{h^{1,1}} t^a (b^a)_{i\bar{j}} \,. \tag{2.5.6}$$

Above we have missed one kind of field arising from the 10D metric. The unique holomorphic $(3,0)$-form can be combined with a basis χ^b, $b \in \{1, ..., h^{1,2}\}$ of $H^{2,1}$ to describe $(2,0)$-deformations $\delta G_{\alpha\beta}$ of the Calabi-Yau, and similarly for $(0,2)$ deformations. Using the Kähler metric to raise and lower indices $\omega^{\alpha}{}_{\overline{\beta}} = G^{\alpha\overline{\gamma}}\omega_{\overline{\gamma}\overline{\beta}}$, the deformations can be expanded as

$$\overline{\Omega}_{\alpha\beta\rho}\delta G^{\rho}{}_{\overline{\gamma}} = \sum_{b=1}^{h^{2,1}} U^b (\chi^b)_{\alpha\beta\overline{\gamma}} \tag{2.5.7}$$

The complex scalars U_b are called complex structure moduli.

The massless scalars arising from compactification are called moduli. Their vacuum expectation value (vev) describes the background for the 4D theory. In addition to the moduli described above, the other massless 10D fields contribute to the moduli. In particular, the axio-dilaton S is universally present. Altogether, the moduli give rise to couplings and background in 4D. The number of moduli is given by the Hodge numbers of the Calabi-Yau, and they are typically organized in $\mathcal{N} = 2$ super-multiplets together with the other massless states. In particular, in type IIA there are $h^{2,1} + 1$ complex structure moduli organized in hyper multiplets and $h^{1,1}$ Kähler moduli in vector multiplets. In type IIB, one conversely finds $h^{2,1}$ complex structure moduli in vector multiplets while here the $h^{1,1} + 1$ Kähler moduli are organized in hyper multiplets. The spectra are summarized in tables 2.3 and 2.4.

The reversal of the roles of the Hodge numbers and multiplets between the spectra of IIA and IIB is a result of Mirror symmetry. This symmetry relates type IIA string theory compactified on a Calabi-Yau $\mathcal{M}$ with $(h^{1,1}, h^{2,1}) = (p, q)$ to type IIB on another Calabi-Yau $\mathcal{W}$ with reversed Hodge numbers (q, p), exchanging the Kähler and complex structure moduli in the process.

$\mathcal{N} = 2$ multiplet	Multiplicity	Moduli
Gravity	1	
Vector	$h^{1,1}$	Kähler
Hyper	$h^{2,1} + 1$	Complex Structure

Table 2.3: Massless spectrum of type IIA compactifications.

$\mathcal{N}=2$ multiplet	Multiplicity	Moduli
Gravity	1	
Vector	$h^{2,1}$	Complex Structure
Hyper	$h^{1,1}+1$	Kähler

Table 2.4: Massless spectrum of type IIB compactifications.

The moduli space, i.e. the field space of Kähler and complex structure moduli, factorizes as the complex structure moduli don't interact with the rest. A powerful fact about Calabi-Yau spaces is that both factors are themselves Kähler spaces. This allows to describe the field space using two Kähler potentials.

In type IIB, the complex structure deformations mix the holomorphic 3-form with the $(2,1)$-forms. This is reflected in the Kähler potential for the complex structure moduli space, which is given by

$$K^{\mathrm{cs}} = -\log\left(i\int \Omega_3 \wedge \overline{\Omega}_3\right) . \tag{2.5.8}$$

Moreover, the complex structure space is actually special Kähler, such that it is completely defined by the periods of the Calabi-Yau and a prepotential. However this will not be necessary for the present discussion.

The Kähler moduli are related to the size of the Calabi-Yau. Concretely, the real part of type IIB Kähler moduli correspond to 4-cycle volumes. The respective Kähler potential including the universal axio-dilaton $S = C_0 + ie^{-\phi}$ reads

$$K = -\log(S+\overline{S}) - 2\log(\mathcal{V})\,, \qquad \mathcal{V} = \frac{1}{3!}\int_{\mathcal{M}} J\wedge J\wedge J\,, \tag{2.5.9}$$

with the overall volume $\mathcal{V}$ of the Calabi-Yau $\mathcal{M}$ given by the integral over the Kähler form J, which is directly related to the Kähler moduli.

Typically, Hodge numbers can be of order $\mathcal{O}(100)$, resulting in a huge amount of 4D massless scalar fields. At this stage, the potential for the moduli is completely flat, leaving coupling constants and background structure of the low energy theory unfixed. This can be solved by adding fluxes to the compactification, giving a potential to the moduli and stabilizing them at finite values. This happens through a flux induced superpotential, which also breaks part of the supersymmetry to $\mathcal{N}=1$. As we have seen, fluxes also play a role in tadpole cancellation conditions, which means that in general we have to introduce D-branes and O-planes to cancel flux contributions. So before going on to review flux compactifications, we shall take a look at compactifications on orientifolds.

2.5.1 Orientifolds

As we have mentioned, an orientifold projection involves a world-sheet parity transformation Ω and additional $\mathbb{Z}_2$ space-time symmetries. The additional projection is denoted σ in type IIB and $\overline{\sigma}$ in IIA. The prototype orientifold is the construction of the type I string from IIB, with σ the identity. The following discussion follows the review [48].

In general, an orientifold will break supersymmetry completely. We will consider special cases where supersymmetry is partially preserved, which fixes the orientifold projection to $\Omega\sigma(-1)^{F_L}$ or $\Omega\overline{\sigma}(-1)^{F_L}$ for type IIA or IIB respectively. Here σ $(\overline{\sigma})$ is an (anti-) holomorphic involution acting on the compact space and F_L is the space-time fermion number in the left-moving sector. The action on the Kähler form J and holomorphic 3-form Ω_3 is defined as

$$\begin{aligned}
&\text{IIA:} \qquad \overline{\sigma}(J) = -J\,, \qquad \overline{\sigma}(\Omega_3) = \overline{\Omega}_3\,, \qquad \text{(O6-plane)}\,,\\
&\text{IIB:} \qquad \sigma(J) = J\,, \qquad \begin{cases} \sigma(\Omega_3) = -\Omega_3\,, & \text{(O3/O7-plane)}\,,\\ \sigma(\Omega_3) = \Omega_3\,, & \text{(O5/O9-plane)}\,. \end{cases}
\end{aligned} \tag{2.5.10}$$

The cohomology of the Calabi-Yau is split into invariant eigenspaces of the involution, $H^{p,q} \to H^{p,q}_+ \oplus H^{p,q}_-$. The involution of type IIB acts as the identity on cohomology

$$\sigma(H^{p,q}) = H^{p,q}\,, \tag{2.5.11}$$

so all subspaces split into invariant eigenspaces. The orientifold breaks part of the supersymmetry, reorganizing the fields into $\mathcal{N}=1$ multiplets as shown in table 2.5.

$\mathcal{N}=1$ multiplet	Multiplicity: O3/O7	O5/O9
gravity	1	1
vector	$h^{2,1}_+$	$h^{2,1}_-$
chiral	$h^{2,1}_- + h^{1,1} + 1$	$h^{2,1}_+ + h^{1,1} + 1$

Table 2.5: Massless spectrum of Type IIB orientifold.

On the other hand, the action of the anti-holomorphic involution of type IIA on the cohomology is

$$\overline{\sigma}(H^{p,q}) = H^{q,p}\,, \tag{2.5.12}$$

indicating that while $(1,1)$-forms are still mapped to themselves and take values in $H^{1,1}_\pm$ eigenspaces, the $(3,0)$ and $(2,1)$-forms are exchanged with their complex conjugates. The $\mathcal{N}=2$ vector multiplets split directly according to their eigenvalues, while the hyper multiplets can form linear combinations such that half of the scalars are even under the orientifold. The reorganized fields are presented in table 2.6.

The moduli space is still parametrized by the Kähler and complex structure moduli, which form chiral multiplets now. The moduli space of orientifold compactifications turns out to be a Kähler space as well.

$\mathcal{N}=1$ multiplet	Multiplicity
gravity	1
vector	$h_+^{1,1}$
chiral	$h_-^{1,1}+h^{2,1}+1$

Table 2.6: Massless spectrum of Type IIA orientifold.

2.5.2 Flux compactification

We have seen that the moduli space of Calabi-Yau and orientifold compactifications are characterized by Kähler potentials. However the potential for the massless fields is flat, which means that many background values of the lower dimensional theory are not fixed. Conversely, the massless scalars would appear as additional long-distance forces, which are phenomenologically excluded. Additionally, the moduli correspond to deformations of the compactification space. A flat potential means that there is no energy cost associated with these deformations, so the compactification is not very stable.

Adding non-vanishing R-R and H_3 background fluxes can stabilize the moduli to finite values, while also breaking supersymmetry to a more phenomenologically viable $\mathcal{N}=1$. Similarly to charge quantization, p-fluxes on compact cycles Σ_p have to satisfy a quantization condition,

$$\frac{1}{\ell_s^{p-1}}\int_{\Sigma_p} F_p \in \mathbb{Z}\,, \tag{2.5.13}$$

which thanks to the Bianchi identity $\mathrm{d}F_p=0$ depends only on the cohomology of F_p. In addition to the Kähler potential of the previous sections, the fluxes induce a superpotential W. Adding fluxes has the drawback of inducing a tadpole, which can be cancelled by D-branes and O-planes. As we have seen, this breaks supersymmetry to $\mathcal{N}=1$, however the fluxes themselves generically break supersymmetry completely. Conveniently, this can be handled by explicit supersymmetry breaking terms in the $\mathcal{N}=1$ theory. More dramatically, the added energy density on the compact space backreacts on the geometry, giving the compactification space curvature. In other words, the manifold is no longer Calabi-Yau. In order to keep on using the Kähler prescription for the moduli space, one must assume that the backreaction is small. Usually this is achieved by taking the large volume limit while keeping the fluxes fixed, whereby the backreaction goes to zero. This procedure is called dilute flux limit.

The dynamics of a $\mathcal{N}=1$ theory is governed by the scalar F-term potential V generated by the discrete fluxes. This is given in terms of the Kähler potential K and the holomorphic superpotential W as

$$V=\frac{M_{\rm pl}^4}{4\pi}e^K\left(G^{I\bar{J}}D_IWD_{\bar{J}}\overline{W}-3|W^2|\right)\,. \tag{2.5.14}$$

Here $G^{I\bar{J}}=(\partial_I\overline{\partial}_{\bar{J}}K)^{-1}$ is the inverse Kähler metric, and the sum runs over all the moduli. We will usually omit units and constant factors and concentrate on the functional form. The Kähler covariant derivative is defined as

$$D_I W = \partial_I W + K_I W\,, \tag{2.5.15}$$

with $K_I = \partial_I K$. The scalar potential describes the potential energy carried by the fluxes on the compact background. The background is described by the moduli, which will want to minimize the scalar potential. In this way, the moduli receive a potential through the fluxes. Moreover, the scalar potential gives the background energy density for the 4D space, giving an effective cosmological constant.

In type IIB, the odd fluxes F_1, $F_3 - H_3$, and $F_5 - H_3 \wedge C_2$ are present in the 10D action (2.4.2). However, a Calabi-Yau supports neither 1-cycles, nor 5-cycles, so only the 3-form flux can contribute to the background. The corresponding superpotential was found by Gukov, Vafa and Witten [49] to be an integral over the complexified 3-form flux $G_3 = F_3 - iSH_3$ and the holomorphic 3-form

$$W_{\mathrm{IIB}} = \int_{\mathcal{M}} \Omega_3 \wedge G_3\,. \tag{2.5.16}$$

This can be expressed in terms of the periods of the Calabi-Yau, and depends only on the axio-dilaton and the complex structure moduli. The superpotential in type IIA is more involved and has been worked out in [50]. Here the superpotential depends on both complex structure and Kähler moduli, and since even cycles are present on CYs all the fluxes contribute. The tree-level superpotential is given by a sum

$$\begin{aligned} W_{\mathrm{IIA}} &= W^Q + W^K\,, \\ W^Q &= \int_{\mathcal{M}} \Omega_C \wedge H_3\,, \\ W^K &= e_0 + \int_{\mathcal{M}} J_c \wedge F_4 - \frac{1}{2}\int_{\mathcal{M}} J_c \wedge J_c \wedge F_2 - \frac{m_0}{6}\int_{\mathcal{M}} J_c \wedge J_c \wedge J_c\,. \end{aligned} \tag{2.5.17}$$

Similarly to the Gukov-Vafa-Witten superpotential, W^Q consists of the 3-form flux and the complex structure moduli which are encoded in Ω_C. The second contribution W^K couples the complexified Kähler form J_c, and thus the Kähler moduli, to the even fluxes.

A general result is that supersymmetric vacua have a covariantly flat superpotential $D_I W = 0$ for all moduli I. With this condition, the scalar potential (2.5.14) simplifies to $V = -3\, e^K |W|^2 \leq 0$, so supersymmetric minima must be Minkowski or AdS. Another interesting feature of many type IIB models is the no-scale structure. Since the superpotential does not depend on the Kähler moduli, $\partial_A W = 0$ and the scalar potential contains

$$e^K \left(G^{A\overline{B}} K_A K_{\overline{B}} |W|^2 \; - 3|W|^2 \right)\,, \tag{2.5.18}$$

with $A, \overline{B}$ running only over the Kähler moduli. If $G^{A\overline{B}} K_A K_{\overline{B}} = 3$, the contributions cancel out and the scalar potential simplifies to

$$V_{\mathrm{no-scale}} = e^K D_M W D_{\overline{N}} \overline{W}\,. \tag{2.5.19}$$

Now $M, \overline{N}$ run over all moduli except the Kähler moduli, which are not fixed by the potential. These models are called no-scale models. Clearly, supersymmetric solutions are necessarily Minkowski. Because of the no-scale structure, SUSY-breaking contributions of Kähler moduli $D_T W \neq 0$ cancel against the $-3|W|^2$ terms, and so one also finds non-SUSY vacua with vanishing cosmological constant.

While the superpotential only receives non-perturbative corrections, the Kähler potential is not exact in α'. This means also the no-scale structure is in general broken at higher loop order.

Moduli are considered stabilized if they lie in a minimum of the scalar potential. The curvature around the minimum gives the moduli an effective mass, which may be integrated out, exchanging the dynamical moduli for their expectation values. These solutions are called flux vacua. The number of flux vacua is famously enormous, as in addition to the large number of possible compactifications, we have also introduced a many possible flux configurations to each of them. This is known as the string landscape. The vastness of this solution space has given string theory the reputation of being unpredictive, since the landscape is so large that one might think that "anything goes". While large swaths of the landscape have been explored [51–57], the bottom-up approach to finding a phenomenological IR theory has proven to be very difficult. Statistical approaches [58–63] and more recently machine learning techniques have been applied [64–67], but still finding the exact vacuum corresponding to our universe among this huge number of possibilities seems almost hopeless.

However, the idea that "anything goes" is not quite true. The swampland program, which we shall introduce in the next chapter, is based on the premise that quantum gravity cannot be coupled to all theories, or conversely that there are features of effective theories that cannot be found when integrating out quantum gravity. Before moving on to the swampland program, let us take a closer look at some classes of flux vacua. First we shall introduce two typical classes of tree-level solutions, the type IIA DGKT models [68] and Freund-Rubin models [69]. Finally, we will look at two archetypical constructions of dS in string theory with different types of quantum corrections, KKLT [70] and LVS [71, 72].

Type IIA orientifolds with D6-branes and fluxes (DGKT)

Because the type IIA superpotential contains all the moduli, these models potentially provide a mechanism to stabilize all the real moduli at tree-level. The axions typically vanish in the minimum. A particularly interesting class of type IIA flux vacua with O6-planes and D6-branes was proposed by DeWolfe, Giryavets, Kachru and Taylor (DGKT) [68]. Not only did they demonstrate that indeed all moduli can be stabilized by fluxes, they found an infinite family of supersymmetric AdS vacua with parametric control.

In particular, the tadpole conditions usually providing an upper bound to the number of fluxes only concern the NS H_3-flux and the F_0-flux. However, the R-R fluxes are free parameters and can in principle be chosen arbitrarily large. This corresponds to arbitrarily large volume and small string coupling.

As a typical example of DGKT-like models let us consider type IIA orientifolds with fluxes on an isotropic six-torus. Here one has three chiral superfields $\{S, T, U\}$ whose real parts are defined as

$$\tau = r_1 r_2 \,, \qquad s = e^{-\phi} r_1^3 \,, \qquad u = e^{-\phi} r_1 r_2^2 \,. \tag{2.5.20}$$

The axions do not play any role and are stabilized at vanishing value. The Kähler potential is given as

$$K = -3\log(T + \overline{T}) - 3\log(U + \overline{U}) - \log(S + \overline{S}) \,. \tag{2.5.21}$$

Now we turn on just R-R fluxes and H_3-form flux so that the flux induced superpotential reads

$$W = i f_0 T^3 - 3 i f_4 T + i h_0 S + 3 i h_1 U \,. \tag{2.5.22}$$

Then there exist both supersymmetric and non-supersymmetric AdS minima. For instance in the supersymmetric vacuum, the saxionic moduli are stabilized at

$$\tau = \kappa \frac{f_4^{\frac{1}{2}}}{f_0^{\frac{1}{2}}} \,, \qquad s = \frac{2\kappa}{3} \frac{f_4^{\frac{3}{2}}}{f_0^{\frac{1}{2}} h_0} \,, \qquad u = 2\kappa \frac{f_4^{\frac{3}{2}}}{f_0^{\frac{1}{2}} h_1} \tag{2.5.23}$$

with $\kappa = \sqrt{5/3}$. For the non-supersymmetric minima only the numerical prefactors change. The effective masses of the moduli all scale in same way as

$$m^2_{\rm mod} \sim -\Lambda \sim \frac{f_0^{\frac{5}{2}} h_0 h_1^3}{f_4^{\frac{9}{2}}} M^2_{\rm pl} \,. \tag{2.5.24}$$

The two KK scales determining the size of the compactification are

$$m^2_{{\rm KK},1} = \frac{M_s^2}{r_1^2} = \frac{f_0^{\frac{3}{2}} h_0 h_1}{f_4^{\frac{7}{2}}} M^2_{\rm pl} \,, \qquad m^2_{{\rm KK},2} = \frac{M_s^2}{r_2^2} = \frac{f_0^{\frac{3}{2}} h_1^2}{f_4^{\frac{7}{2}}} M^2_{\rm pl} \,. \tag{2.5.25}$$

Since the F_4-flux is not constrained by the tadpole condition, we can choose $f_0, h_0, h_1 = O(1)$ and $f_4 \gg 1$. Then both the volume is large, suppressing stringy corrections, and the string coupling $g_s = e^{\phi} \ll 1$ is small.

Freund-Rubin models and geometric fluxes

The next tree-level models to look at are Freund-Rubin type vacua [69]. In these types of models, the backreaction of the background is integral to the construction. In the original work, one actually starts with fluxes on a flat space and observes that the backreaction of the flux leads to “spontaneous” compactification of the space spanned by the fluxes. The prototypical example for this kind of vacua is the $\mathrm{AdS}_5 \times S^5$ solution with 5-form flux on the S^5. Let us now consider the effective description of this model.

Recall the 10D effective SUGRA action governing the dynamics of the metric and a form field C_{n-1}

$$S \sim M_s^8 \int d^{10}x \sqrt{|G|} \left(e^{-2\phi} R - \frac{1}{2} |F_n|^2 \right) , \tag{2.5.26}$$

where the dilaton is set to a constant and the H_3-flux is set to zero. For the R-R 4-form of interest, in addition one has to impose the self-duality relation $F_5 = \pm\star F_5$ by hand and change the prefactor of $|F_n|^2$ to 1/4. The resulting equation of motion for the metric reads

$$R_{ij} - \frac{1}{2} g_{ij} R \;=\; \frac{1}{2(n-1)!} \left(F_{i\,k_2 \ldots k_n} F_j^{k_2 \ldots k_n} - \frac{1}{2n} g_{ij} F_{k_1 \ldots k_n} F^{k_1 \ldots k_n} \right) \tag{2.5.27}$$

and for C_{n-1}

$$\partial_i \left(\sqrt{|G|}\, F^{i\,k_2 \ldots k_n} \right) = 0 \,. \tag{2.5.28}$$

We can write the first relation (2.5.27) as a matrix equation $\mathbf{R} = \mathbf{T}$. For the metric we make the ansatz $\mathrm{AdS}_5 \times S^5$. For a self-dual five-form flux we can then solve (2.5.28) simply by choosing F_5 to satisfy the Bianchi identity $dF_5 = 0$. This is the case for

$$F_5 = f E^1 \wedge \ldots \wedge E^5 - f \mathcal{E}^1 \wedge \ldots \wedge \mathcal{E}^5 \tag{2.5.29}$$

with constant f and with the E^i, $\mathcal{E}^i$ the 5-beins of AdS_5 and S^5 respectively. Choosing the same curvature radius α for the AdS_5 and S^5 factors, the Ricci scalar vanishes and the left hand side of (2.5.27) becomes

$$\mathbf{R} = \begin{pmatrix} -\frac{4}{\alpha^2}\eta & 0 \\ 0 & \frac{4}{\alpha^2}\delta \end{pmatrix} . \tag{2.5.30}$$

The right hand side then is

$$\mathbf{T} = \begin{pmatrix} -\frac{f^2}{4}\eta & 0 \\ 0 & \frac{f^2}{4}\delta \end{pmatrix} . \tag{2.5.31}$$

Therefore, for $\alpha = 4/f$ the equations of motion for a nontrivial 5-flux are satisfied by the $\mathrm{AdS}_5 \times S^5$ metric. The curvature is determined by the flux $f \in \mathbb{Z}$ and the radius of the compact and non-compact spaces is equal.

Defining $\rho = R/M_{\mathrm{pl}}$ as the radius of the S^5 in Planck units, the string scale and the 5D Planck scale are related as $M_{\mathrm{pl}}^8 = M_s^8 \rho^5$. Going to Einstein-frame and performing dimensional reduction of the 10D type IIB Einstein-Hilbert term and the kinetic term for the 5-form flux on the fluxed S^5, one obtains the 5D effective potential

$$V \sim M_{\mathrm{pl}}^5 \left(-\frac{1}{\rho^2} + \frac{f^2}{\rho^5} \right) . \tag{2.5.32}$$

The AdS minimum is at $\rho_0^3 = 5f^2/2$, where the cosmological constant is $\Lambda \sim -\rho_0^{-2} M_{\rm pl}^2$. The mass of the modulus ρ can be determined as

$$m_\rho^2 = G^{\rho\rho}\partial_\rho^2 V\Big|_0 \sim \frac{M_{\rm pl}^2}{\rho_0^2} \tag{2.5.33}$$

with the metric on the moduli space $G_{\rho\rho} \sim \rho^{-2}$. Therefore, the mass of the ρ modulus scales in the same way as the geometric KK scale.

This seems to be a generic feature for models where curvature terms are relevant for moduli stabilization. In the framework of 4D flux compactifications this is described by turning on so-called geometric fluxes. A typical example of this kind is presented below. As before, the $\Lambda \to 0$ limit is reached at infinite distance in field space. In these Freund-Rubin type scenarios, there is no dilute flux limit and the KK scale is of the same order as the moduli mass scale. The same feature appears for the non-geometric type IIB flux models presented in [73, 74].

As a typical simple model of compactification with geometric fluxes we consider the isotropic torus as before, now with geometric fluxes ω_0, ω_1 in the superpotential

$$W = f_6 + 3f_2T^2 - \omega_0 ST - 3\omega_1 UT\,. \tag{2.5.34}$$

The axions vanish again in the minimum. Then the saxions are stabilized in a supersymmetric AdS minimum at

$$\tau = \frac{1}{3}\frac{f_6^{\frac{1}{2}}}{f_2^{\frac{1}{2}}}\,, \qquad s = 2\frac{f_2^{\frac{1}{2}} f_6^{\frac{1}{2}}}{\omega_0}\,, \qquad u = 2\frac{f_2^{\frac{1}{2}} f_6^{\frac{1}{2}}}{\omega_1} \tag{2.5.35}$$

and receive masses that scale as

$$m_{\rm mod}^2 \sim -\Lambda \sim \frac{\omega_0\,\omega_1^3}{f_2^{\frac{1}{2}} f_6^{\frac{3}{2}}} M_{\rm pl}^2\,. \tag{2.5.36}$$

In this case the two KK scales are

$$m_{\rm KK,1}^2 = \frac{\omega_0\,\omega_1}{f_2^{\frac{1}{2}} f_6^{\frac{3}{2}}} M_{\rm pl}^2\,, \qquad m_{\rm KK,2}^2 = \frac{\omega_1^2}{f_2^{\frac{1}{2}} f_6^{\frac{3}{2}}} M_{\rm pl}^2 \tag{2.5.37}$$

which does not scale like the moduli masses. Recall however that the backreaction of the fluxes is integral to these models. And indeed, the geometric fluxes in the denominator also cancel when taking the backreaction onto the metric into account [75], and parametrically one finds $m_{\rm KK}^2 \sim m_{\rm mod}^2$. Therefore, in such models with geometric flux there is no parametric separation of the KK scale and the moduli masses, the same behavior that occurs for Freud-Rubin compactifications.

2.5.3 dS constructions in string theory

At the end of a phenomenological compactification we could want to find a positive cosmological constant. Although formulating a sensible quantum theory in dS has been notoriously difficult [76–78], it is the wide spread understanding of the cosmology community that our universe currently is in an expanding, dS-like phase [79]. And while the difficulties in finding controlled string theory setups with at least a metastable dS vacuum have led to a dedicated no-dS swampland conjecture [15, 80, 81], there have been some successful constructions. In particular, we will consider the well-known constructions by Kachru, Kallosh, Linde, Trivedi (KKLT) [70] and the Large Volume Scenario (LVS) originally proposed by Balasubramanian, Berglund, Conlon, and Quevedo [71].

These dS constructions are in their base form cooking recipes, taking various elements of string compactifications and arguing that in a particular combination the amalgamation should give a dS minimum. Both rely on different quantum corrections to first find an AdS vacuum, which is then uplifted to dS by a small positive contribution. The original works should be seen as a proof of concept rather than full string theory models. There has been much recent work on verifying if the elements that go into the constructions work together as intended [82–102].

KKLT

The starting point of KKLT is a type IIB flux compactification. As we have seen, the complex structure moduli and the dilaton can be stabilized by fluxes. The only contribution left after integrating out these massive modes is a constant flux superpotential W_0. It is assumed that W_0 is very small and negative, and indeed both statistical methods [103] and recently proposed algorithmic approaches [25, 104, 105] have shown that this is not hard to achieve in a generic compactification.

For simplicity, only a single Kähler modulus $T = \tau + i\theta$ is taken into account. A nontrivial potential for T is generated by adding a non-perturbative term to the superpotential. This can be understood as the contribution of a gaugino condensate on a stack of D-branes or D3-instantons. In either case, the potentials take the form

$$W = W_0 + Ae^{-aT}\,, \qquad K = -3\log(T + \overline{T})\,. \tag{2.5.38}$$

The Pfaffian A may depend on the moduli, but as long as it does not depend on the Kähler modulus we may take it to be constant. Indeed, KKLT assumes that A and a are both real constants. A vanishing axion $\theta = 0$ minimizes the potential, and a supersymmetric minimum $D_T W = 0$ can be found for solutions of the transcendental equation

$$W_0 = -Ae^{-a\tau_0}\left(1 + \frac{2}{3}a\tau_0\right)\,. \tag{2.5.39}$$

The scalar potential (2.5.14) is negative in this minimum, indicating that we have found the intermediate AdS vacuum

$$V_{\mathrm{AdS}} = -a^2 A^2 \frac{e^{-2a\tau_0}}{6\tau_0}\,. \tag{2.5.40}$$

With very small W_0, the minimum is also close to zero. By adding a small positive contribution, one may hope to uplift the minimum to a positive value while retaining the local minimum. In KKLT, the positive contribution comes from a $\overline{\text{D3}}$-antibrane. To keep the uplift small, both out of phenomenological reasons and so the minimum is not flattened out, the $\overline{\text{D3}}$-brane is confined to a warped throat. Then it contributes directly to the scalar potential with $\delta V_{\overline{\text{D3}}} = D/\tau^3$ with a constant D determined by the number of anti-branes and the warp factor in the throat. Finally, the total KKLT scalar potential close to the minimum is given as

$$V_{\text{KKLT}} = \frac{aA^2}{6\tau^2}e^{-2a\tau}(3+a\tau) + \frac{aAW_0}{2\tau^2}e^{-a\tau} + \frac{D}{\tau^3}\,. \tag{2.5.41}$$

For $(A, a) \sim \mathcal{O}(1)$ and small W_0 and D it is then straightforward to find a dS minimum.

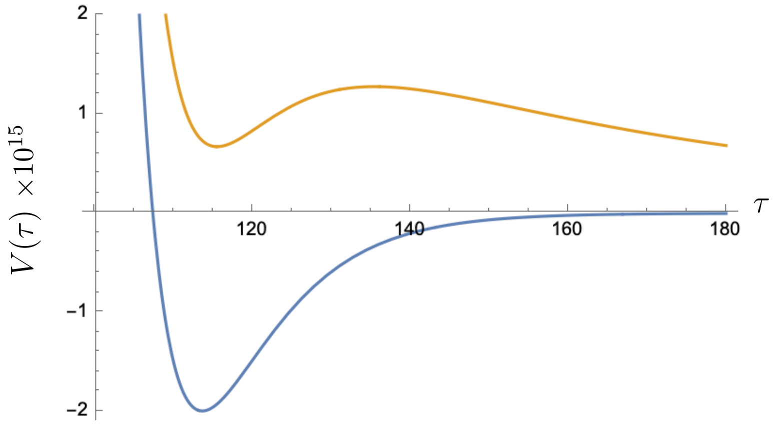

Figure 2.4: Plot of the KKLT potential before (blue) and after (yellow) the uplift to dS with parameters $A = 1$, $a = 0.1$, $W_0 = -10^-4$ and $D = 4{*}10^{-9}$. The potential is rescaled by 10^{15}.

Let us make some remarks about the status of KKLT. While the string theoretic validity of the uplift is still not settled, with the physics in the warped throat introducing new subtleties investigated e.g. in [90, 97], the AdS vacuum has proven robust under rather intense scrutiny. Its full ten-dimensional description has been analyzed in a series of recent papers [88, 89, 93–95, 106–108] converging to the conclusion that the 4D effective KKLT description captures the main aspects of this vacuum. Furthermore, the small W_0 which was controversial in the past, has recently been shown to be a valid assumption both in the large volume regime [104] as well as close to a conifold [25, 105], where one finds the large warping needed for the subsequent uplift.

The Large Volume Scenario

While KKLT used non-perturbative D-brane effects and large warping to arrive at a dS minimum, the large volume scenario [71] adds perturbative α'-corrections to balance the quantum and tree-level contributions without the need for large warping. The setup is somewhat more complicated, starting from a "swiss-cheese" type Calabi-Yau with two Kähler moduli, one big T_b and one small T_s. As the name indicates, the large Kähler modulus controls the overall size of the compactification manifold, while the small one controls the "holes" of the Calabi-Yau, such that the total volumes scales like $\mathcal{V} \sim \tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}}$. The complex structure moduli and the dilaton are again stabilized by fluxes, contributing only a fixed, but not necessarily small superpotential W_0. With the first α'-corrections included, the super- and Kähler potential read

$$W = W_0 + Ae^{-aT_s}\,, \qquad K = -2\log\left(\tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} + \frac{\xi}{2}\mathrm{Re}(S)^{\frac{3}{2}}\right)\,. \tag{2.5.42}$$

The α' corrections go with $\xi = \frac{\chi(\mathcal{M})\zeta(3)}{2(2\pi)^3}$, where $\chi(\mathcal{M})$ is the Euler number of the Calabi-Yau $\mathcal{M}$ and $\zeta(3) \approx 1.202$ is Apéry's constant. As before, the axions can be set to zero. The relevant terms to the scalar potential are

$$\begin{aligned} V &= e^{K_{\mathrm{CS}}}\frac{g_s}{2}\left(\frac{|aA|^2\sqrt{\tau_s}e^{-2a\tau_s}}{\mathcal{V}} - \frac{W_0|aA|\tau_s e^{-a\tau_s}}{\mathcal{V}^2} + \frac{\xi W_0^2}{g_s^{\frac{3}{2}}\mathcal{V}^3}\right) \\ &= \lambda\frac{\sqrt{\tau_s}e^{-2a\tau_s}}{\mathcal{V}} - \mu\frac{\tau_s e^{-a\tau_s}}{\mathcal{V}^2} + \frac{\nu}{\mathcal{V}^3}\,. \end{aligned} \tag{2.5.43}$$

Here K_{CS} is the Kähler potential of the complex structure moduli, and we have collected all the prefactors in the second line. The potential has a non-supersymmetric AdS minimum. Solving the minimum condition $\partial_{\mathcal{V}}V_{\mathrm{LVS}} = 0$ one finds

$$\mathcal{V} = \frac{\mu}{\lambda}\sqrt{\tau_s}e^{a\tau_s}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right) \tag{2.5.44}$$

and from $\partial_{\tau_s}V_{\mathrm{LVS}} = 0$ one obtains

$$\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)\left(\frac{1}{2} - 2a\tau_s\right) = (1 - a\tau_s)\,. \tag{2.5.45}$$

Now, one proceeds by working in the perturbative regime $a\tau_s \gg 1$, in which case the two relations can be solved analytically, yielding the values of the moduli in the LVS minimum

$$\tau_s^0 = \left(\frac{4\nu\lambda}{\mu^2}\right)^{\frac{2}{3}}\,, \qquad \mathcal{V}^0 = \frac{\mu}{2\lambda}\sqrt{\tau_s^0}\,e^{a\tau_s^0}\,. \tag{2.5.46}$$

However, plugging this back into the potential (2.5.43) one gets zero, indicating that the scalar potential close to the minimum is suppressed compared to the no-scale potential for the complex structure moduli. This is called extended no-scale structure. Therefore, to find the actual non-vanishing value of the potential in the LVS minimum, one has to compute the next order in $1/\tau_s$ [109]. The only approximation we did is in the solution to (2.5.45). Thus there will be a correction to τ_s^0, which at leading order is a just a shift by a constant $\tau_s^0 \to \tau_s^0 + c/a$, which one can show to be positive. The value of the cosmological constant will then be

$$V_0 \sim -\frac{3c\lambda^2 e^{-3c}}{\mu a\,\tau_s^0}\,e^{-3a\tau_s^0}\left(1 + O(\tfrac{1}{\tau_s})\right) \tag{2.5.47}$$

which is indeed negative. The LVS AdS vacuum can then be uplifted to dS by $\overline{\text{D3}}$ anti-branes in a similar fashion to KKLT. The status of LVS is much less understood than the status of KKLT, both due to the higher complexity but also the higher flexibility of the model. However, the AdS vacuum seems to be similarly robust [110] while the uplift may be of questionable validity.

Chapter 3

The Swampland Program

The vast string landscape has given string theory the reputation of lacking predictability. Indeed, in a theory with easily over 10^{500} vacua, how can one expect to learn specific truths about our universe, which corresponds to a single vacuum that we have not even been able to identify yet?

However, with the large amount of vacua discovered and characterized in the past, string theory has one of the largest theoretical datasets imaginable. Identifying structures in this set of vacua is the aim of the swampland program. It turns out that not every low energy EFT can be coupled to gravity at high energies. This space of UV-inconsistent theories has been called the swampland [10, 111, 112] to contrast the string landscape. Indeed, asserting string theory/quantum gravity at high energies can severely constrain the low energy theory. These constraints are formulated as swampland conjectures.

Aiming beyond string theory vacua, the swampland program truly wants to set a bound for all quantum theories of gravity, not only string theory (although the Lamppost Principle [11, 12] argues that there is actually no difference). This can be achieved by complementing the evidence from string theory by semi-classical black hole arguments. However, the swampland constraints are generally hard to prove mathematically. Swampland conjectures are expected to become trivial as one decouples gravity $M_{\text{pl}} \to \infty$, and are often formulated as bounds on the spectrum or field ranges of EFTs or restrictions on the form of the scalar potential. We shall introduce only the conjectures relevant to this thesis. For a more comprehensive overview see e.g. the reviews by Palti and Graña et al. [113, 114] or the lecture notes by Valenzuela et al. [115].

3.1 Relevant swampland conjectures

3.1.1 AdS swampland conjectures

One of the most powerful tools of string theory, the AdS/CFT correspondence, allows to make very precise statements about string theory on AdS. In fact, swampland constraints on the symmetry of the EFT, more precisely that no global symmetries are possible in a theory coupled to quantum gravity, have been mathematically proven in this context [116–118].

The two conjectures we will look at are much less rigorous, and aim to bound the mass spectrum of the EFT living in AdS space by the cosmological constant. The first conjecture we will discuss is the AdS distance conjecture [119]. It states that the flat space limit is at infinite distance in field space, and the EFT breaks down when approaching this boundary. In particular, there is a tower of massive, previously integrated out states which becomes light as the AdS radius grows. Eventually the new light states dominate the theory, and the EFT no longer accurately describes the theory. The states in question are usually the KK tower, but it is not excluded that a different tower may play the role [120]. This behavior of towers of states becoming light is common for large field distances in string theory, and has led to many distance conjectures [112, 121–126]. The second conjecture relates only a single state, the lightest modulus, to the AdS radius [108].

AdS distance conjecture

Quantitatively, the AdS distance conjecture (ADC) [119] (see also [127]) states that for an AdS vacuum with negative cosmological constant Λ, the limit $\Lambda \to 0$ is at infinite distance in field space and that there will appear a tower of light states whose masses scale as

$$m_{\text{tower}} = c_{\text{AdS}}\, |\Lambda|^{\alpha} \tag{3.1.1}$$

for some constant c_{AdS} of order one and $\alpha > 0$. Moreover, for supersymmetric AdS vacua a stronger version of the AdS distance conjecture was claimed, namely that in this case $\alpha = 1/2$. Assuming that the tower of states is just the KK tower, the strong ADC generalizes earlier Maldacena-Nuñez type obstructions [128] for scale separated type II AdS flux vacua without negative tension object and rephrases them as a swampland conjecture.

In string theory, where the relevant states are the KK tower, the conjecture implies that there is no parametric scale separation between the compact space and the AdS space. This means that it is not really sensible to speak of a lower-dimensional theory, as the internal space is accessible to the EFT states. Another implication of the conjecture is the infinite distance of the flat limit, which indicates that there is no smooth path from AdS to dS in field space.

The authors gather evidence from various tree-level string models, for which a well-understood 10D uplift is accessible. Additionally, they argue through holographic evidence for the lack of scale separation that the bound on the KK tower should be generically true. However, they admit that DGKT-type models only satisfy the weak ADC. Additionally, the non-perturbative vacua of KKLT and the LVS violate the ADC. We shall investigate this point in chapter 6.

AdS/moduli scale separation conjecture

The AdS/moduli scale separation conjecture (AM-SSC) [108] also concerns separation of scales. Here instead of a tower of states, only a single, specific state has its mass bounded by the cosmological constant. The proposal is that the lightest modulus of non-vanishing mass has to satisfy

$$m_{\text{mod}}\, R_{\text{AdS}} \leq c\,, \tag{3.1.2}$$

where c is an order one constant and $R_{\text{AdS}}^2 \sim -\Lambda^{-1}$ the size of AdS. A strong version of this conjecture says that this relation is saturated, i.e. $m_{\text{mod}} \sim R_{\text{AdS}}^{-1}$.

The main argument for this conjecture is an analysis of the uplift from AdS vacua to dS via $\overline{\text{D3}}$-branes, as utilized in the dS constructions discussed above. Indeed, any small SUSY-breaking effect would lead to a metastable dS minimum. The AM-SSC is then a corollary of the no-dS conjecture which we will review next.

3.1.2 no-dS conjecture

The arguably most controversial swampland conjecture is the no-dS conjecture (often simply called the dS swampland conjecture) [15, 80, 81]. Giving a bound on the scalar potential, it states that

$$|\nabla V| \geq \frac{c}{M_{\text{pl}}} \cdot V \qquad \text{or} \qquad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_{\text{pl}}^2} \cdot V\,, \tag{3.1.3}$$

where c, c' are order one numbers and $\min(\nabla_i \nabla_j V)$ is the minimal eigenvalue of the Hessian matrix. The bound clearly forbids local minima with a positive value, and furthermore the curvature of any extremum must be steep enough that thermal effects in dS are not enough to stabilize the maximum/saddle-point. Qualitatively, any metastable dS vacuum is conjectured to belong to the swampland.

The no-dS conjecture is motivated by the difficulty to construct concrete examples of dS in string theory, with all moduli stabilized under perturbative control. Recall that the constructions we reviewed are little more than toy models, and while the no-dS conjecture sparked a large amount of work to review and pick apart these dS constructions, the final verdict is not yet clear. In sections 4 and 5 we shall find more evidence for this conjecture.

The idea that dS is not a part of the string landscape is not new to the conjecture, see e.g. [129] for a review, and dS has long been known to present difficulties for our understanding of quantum theories [76]. Indeed, recently it has been argued that a positive cosmological constant is incompatible with S-matrix theories [77, 78]. Still, it is important to acknowledge the fact that our universe is expanding, which implies that we are at least in a dS-like cosmological phase. The theoretical conjecture seems to be in disagreement with observations.

3.1.3 Trans-Planckian censorship conjecture

It has been recently suggested that a more "global" version of the no-dS conjecture might be the more general statement. In [130] an underlying quantum gravity reason for the no-dS conjecture was proposed, namely the so-called trans-Planckian censorship conjecture (TCC). It proposes that sub-Planckian fluctuations must stay quantum and should never become classical in an expanding universe with Hubble constant H

$$\int_{t_i}^{t_f} dt\, H < \log\left(\frac{M_{\rm pl}}{H_f}\right) . \tag{3.1.4}$$

Two points are to be emphasized here, namely that this conjecture is not local (as it involves an initial and final time), and the appearance of a logarithm on the right hand side. For a monotonically decreasing positive potential, one can derive a global version of the no-dS swampland conjecture from the TCC

$$\left\langle \frac{-V'}{V} \right\rangle\bigg|_{\phi_i}^{\phi_f} > \frac{1}{\Delta\phi}\log\left(\frac{V_i}{M}\right) + \frac{2}{\sqrt{(d-1)(d-2)}} , \tag{3.1.5}$$

where $\left\langle \frac{-V'}{V} \right\rangle\Big|_{\phi_i}^{\phi_f}$ denotes the average of $-V'/V$ in the interval $[\phi_i, \phi_f]$ between initial and final field configurations. Here $V < M < M_{\rm pl}$ and M is a mass scale that can be lower than the Planck-scale. For the asymptotic case $\phi_i, \phi_f \to \infty$, $\Delta\phi \to \infty$ the TCC has the same form as the no-dS conjecture.

In general, the TCC does allow for meta-stable dS vacua with a short lifetime $\tau < -\ln(H_\Lambda)/H_\Lambda$ bounded by the Hubble constant H_Λ. This is of course much less restrictive than the no-dS conjecture, and might be a resolution of the tension between observation and theory. The log-corrections are particularly interesting for the further course of this thesis, as we shall find similar corrections to swampland conjectures in section 6.

3.2 The emergence proposal

In a sense orthogonal to the swampland distance conjectures, it has been observed [131–134] that infinite distances in field space associated with towers of states getting light can actually be understood as arising from integrating out states which have become lighter than the quantum gravity cutoff. The natural cutoff of a gravitational theory itself has been shown to scale with the number of states in the EFT. This is called the species scale [135]. In other words, the emergence proposal claims that the 1-loop contribution to the moduli field metric, arising from integrating out states that are lighter than the species scale, is proportional to the tree-level metric.

Say one has an effective theory in D dimensions that has a tower of states with masses $m_n = n\Delta m(\phi)$, with a degeneracy of states at each mass level that scales like n^K. Note that the mass depends on the value of a modulus field ϕ. If $N_{\rm sp}$ of these states become lighter than the species scale

$$\Lambda_{\rm sp} = \frac{\Lambda_{\rm UV}}{{N_{\rm sp}}^{\frac{1}{D-2}}}, \tag{3.2.1}$$

they impose a one-loop correction to the field space metric of the field ϕ. Here the UV cutoff $\Lambda_{\rm UV}$ is often chosen to be the Planck scale but could in principle be lower. The number of species are given by

$$N_{\rm sp} = \sum_{n=1}^{\Lambda_{\rm sp}/\Delta m} n^K \approx \left(\frac{\Lambda_{\rm sp}}{\Delta m}\right)^{K+1} . \tag{3.2.2}$$

The latter two relations can be inverted to give

$$\Lambda_{\rm sp} = \left(\Lambda_{\rm UV}\right)^{\frac{D-2}{D+K-1}} \left(\Delta m\right)^{\frac{K+1}{D+K-1}} , \qquad N_{\rm sp} = \left(\frac{\Lambda_{\rm UV}}{\Delta m}\right)^{\frac{(K+1)(D-2)}{D+K-1}} . \tag{3.2.3}$$

Then the one loop-correction to the field space metric for the modulus ϕ in D dimensions can be written as

$$G^{\rm loop}_{\phi\phi} \sim \frac{\Lambda_{\rm sp}^{D+K-1}}{M_{\rm pl}^{D-2}} \frac{\left(\partial_\phi \Delta m(\phi)\right)^2}{\left(\Delta m(\phi)\right)^{K+3}} . \tag{3.2.4}$$

The emergence proposal states that this loop correction to the moduli space metric is proportional to the tree-level metric, $G^{\rm loop}_{\phi\phi} \sim G^{\rm tree}_{\phi\phi}$.

Part II

Challenging the Swampland with Exotic Ingredients

It is unfortunately in the nature of swampland conjectures to be evidence-driven, as proving statements about quantum gravity first requires us to truly know what quantum gravity is. The aim of the work presented in this part was to test swampland conjectures for boundaries of validity or weaknesses.

As one of the more impactful, while also less rigorously motivated statements, the no-dS conjecture is the target of the first two chapters. We will try to circumvent the conjecture by introducing increasingly unorthodox objects and theories, but will always hit roadblocks that strengthen the no-dS conjecture.

The first objects we investigate are non-BPS $\widehat{\mathrm{D}}$-branes. In type IIA theory, there is a theorem that there can be no tree-level flux vacua with positive cosmological constant. Crucially, the theorem holds for Dp-branes with $p \leq 6$. This is fine, since Calabi-Yau manifolds have no 5-cycles which a D8 would wrap. However, non-BPS $\widehat{\mathrm{D}}7$-branes could provide a loophole in the theorem. The orientifold can stabilize these branes, and indeed we find dS vacua. There will be a non-trivial K-theory charge on the compactification, which while not contributing to a tadpole, has been argued to be anomalous via probe branes. Thus we establish a connection between cancellation of K-theory charge and the no-dS conjecture.

Then we shall consider even more unusual theories. Exotic string theories are theories that arise from usual string theory by a web of T- and S-dualities, where the T-duality is also performed along time-like directions. The resulting theories exhibit various combinations of Euclidean strings and D-branes, as well as multiple time directions. We will try to make sense of phenomenology in these strange settings, and construct dS brane worlds with phenomenologically viable gauge sectors. However it will turn out that the O-planes which cancel the tadpole arising from these branes also remove the field combinations that give rise to the dS in the first place. In the end, we find that we can either have dS in exotic string theories, or a phenomenologically viable gauge sector, but not both.

In the third chapter, we will scrutinize the AdS swampland conjectures from the perspective of non-perturbative vacua. We will see that the relevance of quantum effects in these vacua forces us to admit log-corrections to swampland conjectures. These corrections are reminiscent of corrections found in the TCC, which is also motivated by quantum arguments.

Chapter 4

dS Vacua from Non-BPS Branes

One of the few proven statements about the swampland has to be the dS no-go theorem by Hertzberg, Kachru, Taylor and Tegmark [136]. In a classical flux vacuum of type IIA string theory, with standard ingredient of such a theory, there cannot be any (meta-) stable dS vacua. For us, the crucial word of the statement is "standard", meaning fluxes, BPS D-branes and O-planes. We will try to circumvent the no-go theorem by introducing decidedly non-standard ingredients, specifically non-BPS $\widehat{\text{D}}$-branes.

Stable non-BPS branes exist in type I superstring compactifications, where single non-BPS branes can carry a topologically conserved K-theory charge. In contrast to R-R charges, these K-theory charges have no Bianchi identity which forces the total charge to vanish on a compact space. However there have been indirect arguments using probe branes and particles in favor of K-theory charge cancellation [137]. Through T-duality we identify the non-BPS brane spectrum of type IIA orientifolds. Assuming then that the K-theory charge does not cancel, we find that we can indeed circumvent the dS no-go theorem. Thus we can link the vanishing of K-theory charges to the no-dS conjecture: if K-theory charges are allowed to persist on compact spaces, the conjecture is disproven. However previous arguments in favor of K-theory charge cancellation rather leave us to conclude that the no-dS conjecture stands firm.

4.1 Type IIA orientifolds with fluxes and branes

Let us briefly review the specific set-up that we have in mind. In type IIA orientifolds with intersecting D6-branes, NS-NS and R-R fluxes stabilize all closed string moduli at string tree-level. As these type IIA orientifolds are T-dual to type I theories which contain stable non-BPS branes, also here there are non-BPS branes which are stable.

4.1.1 Equivariant homology of type IIA orientifolds

We consider an orientifold projection of a Calabi-Yau threefold X given by $\Omega\overline{\sigma}(-1)^{F_L}$, with the world-sheet parity transformation $\Omega : (\tau, \sigma) \to (\tau, -\sigma)$, the left-moving space-time fermion number F_L and the anti-holomorphic involution $\overline{\sigma}$ acting as

$$\overline{\sigma} : J \to -J \,, \qquad \overline{\sigma} : \Omega_3 \to \overline{\Omega}_3 \tag{4.1.1}$$

on the Kähler form J and the covariantly constant holomorphic 3-form Ω_3.

The fixed point of the space-time involution $\overline{\sigma}$ defines the location of O6-planes, whose tadpoles need to be cancelled by introducing stacks of (generally intersecting) D6-branes. Intersecting D6-brane models have been extensively studied, reviewed e.g. in [48, 138]. The $\mathcal{N} = 1$ supersymmetric spectrum on such a compactification can be organized by the equivariant cohomology groups. The space-time involution $\overline{\sigma}$ acts on the cohomology groups by

$$\overline{\sigma} : H^{p,q} \mapsto H^{q,p}, \tag{4.1.2}$$

such that (1,1)-forms are invariant while (3,0) and (2,1)-forms are exchanged with their complex conjugate. Thus $H^{1,1}$ can be split into equivariant subspaces $H^{1,1}_{\pm}$ corresponding to their eigenvalues under $\overline{\sigma}$. Of forms compactified on cycles with other cohomologies, only linear combinations with eigenvalues ± 1 survive the projection. For the SUGRA multiplets, this means that the $h^{2,1}$ ($\mathcal{N} = 2$) complex structure hypermultiplets of type IIA are reduced to $\mathcal{N} = 1$ chiral multiplets U, and the universal hypermultiplet on $\Sigma_3 \in H^{3,0} \oplus H^{0,3}$ is similarly reduced to a chiral multiplet S. The $h^{1,1}$ vector multiplets making up the type IIA Kähler moduli on the other hand split into $h^{1,1}_+$ vector multiplets V and $h^{1,1}_-$ chiral multiplets T under the orientifold. The massless spectrum is listed in table 4.1.

$\mathcal{N} = 1$ multiplet	State	Cohomology
chiral	$U = \int_{\Sigma_3} \Omega_c$	$\Sigma_3 \in H^3_+(X)$
chiral	$T = \int_{\Sigma_2} J_c$	$\Sigma_2 \in H^2_-(X)$
vector	$V = \int_{\Sigma_2} C_3$	$\Sigma_2 \in H^2_+(X)$

Table 4.1: Massless spectrum of Type IIA orientifold.

The tree-level Kähler potential encoding the kinetic terms for the moduli is

$$K = -\log\left(\frac{4}{3}\int J \wedge J \wedge J\right) - 2\log\left(\int \mathrm{Re}(\Omega_c) \wedge \star\mathrm{Re}(\Omega_c)\right) \tag{4.1.3}$$

with $\Omega_c = e^{-\phi}\mathrm{Re}(\Omega_3) + iC_3$ and $J_c = J + iB$. As mentioned, the S, U and T moduli can be fixed purely by introducing fluxes in type IIA. The NS-NS three-form flux $H = dB$ fixes the complex structure moduli U and S, while the Kähler moduli T are stabilized by R-R fluxes F_0, F_2, F_4, and F_6. The Gukov-Vafa-Witten type superpotential [49] is given by

$$W = i\int_X \Omega_c \wedge H + \int_X e^{iJ_c} \wedge \mathcal{F} \,, \tag{4.1.4}$$

where $\mathcal{F}$ represents the sum over all even R-R fluxes. Expanding the forms in $H^2_\pm$ and $H^3_\pm$ one finds that the orientifold even fluxes take values in cohomology as listed in table 5.6.

Flux	Cohomology
H	$H^3_-(X)$
$\{F_0, F_2, F_4, F_6\}$	$\{H^0_+, H^2_-, H^4_+, H^6_-\}$

Table 4.2: Cohomology groups of orientifold even fluxes.

Note that since the total volume form J^3 is odd under the involution, these are also those fluxes that can give non-vanishing contributions to the superpotential.

4.1.2 Non-BPS $\widehat{\mathrm{D}}$-branes

As we have seen earlier in section 2.2.3, an orientifold projection can enable the existence of stable non-BPS branes. This corresponds to torsional K-theory groups [33, 139]. The stable, non-BPS branes of the type I superstring are listed in table 4.3 [140]. As these branes only couple to the NS-NS sector, there is no GSO projection to remove the tachyon, and no tadpole cancellation conditions arise. Instead, the open string NS-tachyon is projected out by the world-sheet parity transformation Ω. However this is only true for an open string starting and ending on the same brane, leaving the tachyon stretched between two non-BPS branes intact and the system unstable. In other words, two non-BPS branes stacked onto each other are unstable and decay to the vacuum. Hence the K-theory group is $\mathbb{Z}_2$. The branes of interest to us are the 4D space-time filling $\widehat{\mathrm{D}}7$ and $\widehat{\mathrm{D}}8$ branes, as we do not want to break 4D Lorentz invariance.

non-BPS brane	Tension	K-theory
$\widehat{\mathrm{D}}8$	$\sqrt{2}\,T_{\mathrm{D8}}$	$KO(S^1) = \mathbb{Z}_2$
$\widehat{\mathrm{D}}7$	$2\,T_{\mathrm{D7}}$	$KO(S^2) = \mathbb{Z}_2$
$\widehat{\mathrm{D}}0$	$\sqrt{2}\,T_{\mathrm{D0}}$	$KO(S^9) = \mathbb{Z}_2$
$\widehat{\mathrm{D}}(-1)$	$2\,T_{\mathrm{D}(-1)}$	$KO(S^{10}) = \mathbb{Z}_2$

Table 4.3: Stable non-BPS branes for the Type I superstring.

For concreteness, let us consider a $T^6 = (T^2)^3$ compactification of the type I string with complex and real coordinates z_i, $\bar{z}_i$ and $z_i = x_i + iy_i$, $i \in \{1, 2, 3\}$. We can relate that to the type IIA string by a threefold T-duality along the three y_i directions. The anti-holomorphic involution is then characterized by $z_i \to \bar{z}_i$ or $y_i \to -y_i$. It is now a simple matter of reading off the non-BPS brane configuration in type IIA after T-duality.

For example, a $\widehat{\mathrm{D}}8$ brane localized at $y_3 = 0$ in type I is T-dualized to a $\widehat{\mathrm{D}}7$ brane in type IIA, localized at $y_1 = y_2 = 0$ and wrapping the third torus. The

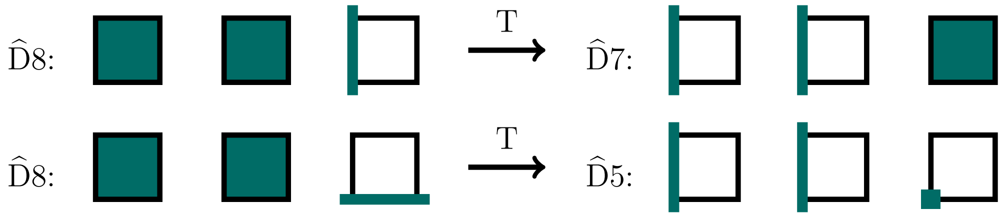

Figure 4.1: A $\widehat{\mathrm{D}}$8-brane in type I wrapping the first two tori is T-dual to either a $\widehat{\mathrm{D}}$7 or a $\widehat{\mathrm{D}}$5-brane in type IIA, depending on the initial configuration.

four-cycle wrapped by the brane is odd under the involution, so it is in the $H_4^-(X)$ homology. The type I $\widehat{\mathrm{D}}$8-brane could instead be localized at $x_3 = 0$, in which case the T-dual is a $\widehat{\mathrm{D}}$5-brane wrapping only the H_2^+ two-cycle spanned by (x_1, x_2). These two configurations are shown in fig. 4.1.

The same logic applies to the type I $\widehat{\mathrm{D}}$7-brane configurations, revealing the stable non-BPS brane configurations and their homology class for type IIA orientifolds shown in table 4.4. Generally, the initial type I branes could also wrap more general one-cycles within the tori. These configurations T-dualize to branes with a non-trivial line bundle. As these fluxed non-BPS branes do not contribute to our results, we shall not consider them. To find the complete spectrum of stable non-BPS branes, one has to determine the corresponding K-theory groups, which is outside the spectrum of this analysis.

Type I	Type IIA	Homology
$\widehat{\mathrm{D}}$8	$\widehat{\mathrm{D}}$7	$H_4^-(X)$
	$\widehat{\mathrm{D}}$5	$H_2^+(X)$
$\widehat{\mathrm{D}}$7	$\widehat{\mathrm{D}}$4	$H_1^+(X)$
	$\widehat{\mathrm{D}}$6	$H_3^-(X)$
	$\widehat{\mathrm{D}}$8	$H_5^+(X)$

Table 4.4: Stable (non-fluxed) non-BPS branes for the Type IIA orientifolds.

Let us now determine the type IIA branes which are most relevant for the further analysis. On a Calabi-Yau, the lack of homological one- and five-cycles means that the $\widehat{\mathrm{D}}$4 and $\widehat{\mathrm{D}}$8-branes have no support. And occupying the same odd (co-) homology group as the H-flux, the $\widehat{\mathrm{D}}$6 branes are in danger of developing a Freed-Witten anomaly.

The $\widehat{\mathrm{D}}$5 brane on the other hand lives in a lower, even homology and are hence safe from this anomaly. A $\widehat{\mathrm{D}}$7 brane is in principle also prone to the Freed-Witten anomaly, but can be protected by wrapping it on a four-cycle that does not contain an odd homological three-cycle, e.g. a del-Pezzo surface. An added benefit of del-Pezzo surfaces is their rigidity, which means that by wrapping the D6-branes also on rigid, $\overline{\sigma}$-even three-cycles one can keep the non-BPS and the BPS branes separated. The interaction between them will then only give a one-loop contribution to the closed string moduli potential, which is subdominant in the flux-stabilized vacuum.

Summing up, the new ingredients we can add for the purpose of circumventing the dS no-go theorem in type IIA orbifolds are the non-BPS $\widehat{\mathrm{D}}5$ and $\widehat{\mathrm{D}}7$ branes, wrapped on appropriate rigid cycles.

4.2 Failure of the dS no-go theorem

Let us now introduce the dS no-go theorem in full and see what changes with the introduction of non-BPS branes. We will then analyze the scalar potential of a toy model with a single non-BPS brane on a fluxed type IIA orientifold to confirm the existence of a dS vacuum under these assumptions.

4.2.1 The no-go theorem and non-BPS branes

The no-go theorem of [136] shows that any classical type IIA flux potential with $D6$-branes and $O6$-planes satisfies the condition

$$\frac{M_{\rm pl}^2}{2}\left(\frac{\nabla V}{V}\right)^2 \geq \frac{27}{13} \tag{4.2.1}$$

for $V > 0$, so dS vacua cannot exist. This lower bound arises from manipulating the general form of the scalar potential arising from dimensional reduction and subsequent transformation to Einstein frame. Tracking the scaling of the various flux and brane contributions with the universal modulus $s = \mathrm{Re}(S) = e^{-\phi}\mathrm{vol}^{1/2}$ and the size modulus $t = \mathrm{vol}^{1/3}$, and including the non-BPS branes found in the previous section, the scalar potential takes the form

$$V = \frac{A_H}{s^2 t^3} + \sum_{p\,\mathrm{even}} \frac{A_{F_p}}{s^4 t^{p-3}} + \frac{A_{D6}}{s^3} - \frac{A_{O6}}{s^3} + \frac{A_{\widehat{D}5}}{s^3 t^{\frac{1}{2}}} + \frac{A_{\widehat{D}7} t^{\frac{1}{2}}}{s^3}\,. \tag{4.2.2}$$

The coefficients are semi-positive placeholders for (generally quite complicated) functions of the other moduli. The great realization leading to the no-go theorem is that a combination of derivatives reproduces the scalar potential up to some additional terms,

$$t\frac{\partial V}{\partial t} + 3s\frac{\partial V}{\partial s} = -9V - \sum_p \frac{pA_{F_p}}{s^4 t} - \frac{1}{2}\frac{A_{\widehat{\mathrm{D}}5}}{s^3 t^{\frac{1}{2}}} + \frac{1}{2}\frac{A_{\widehat{\mathrm{D}}7} t^{\frac{1}{2}}}{s^3}\,. \tag{4.2.3}$$

Without non-BPS branes, it is clear that an extremum $\partial_t V = \partial_s V = 0$ cannot be positive $V \leq 0$ and so no dS vacuum can arise. The positive sign in front of the $\widehat{\mathrm{D}}7$-brane term however nullifies this argument, and there remains a chance that a dS minimum may exist.

4.2.2 A toy model

The failure of the no-go theorem does not necessarily indicate that a dS vacuum can indeed exist. There could still be factors conspiring to keep a dS minimum from actually appearing. To prove a contradiction to the no-dS conjecture we will consider a simple SUGRA model with a non-BPS $\widehat{\text{D}}$7-brane, and see that a dS minimum generically exists.

The model we consider is a toroidal STU model with $T = T_i$ and $S = U_i$, $i \in \{1, 2, 3\}$ identified. While one can think of it as an effective model after integrating out the other moduli, rendering the "fluxes" real parameters, we shall consider integer fluxes here. Indeed, if this already allows enough tuning to find dS minima, then adding more freedom should only allow for more such vacua.

The Kähler potential for our toy model is

$$K = -3\log(T + \bar{T}) - 4\log(S + \bar{S}) \tag{4.2.4}$$

and the flux superpotential is given by

$$W = -4ihS + f_6 + 3if_4T - 3f_2T^2 - if_0T^3\,. \tag{4.2.5}$$

Further simplifying by setting $f_6 = f_2 = 0$, the axions $\mathrm{Im}(S)$ and $\mathrm{Im}(T)$ vanish in the minimum of the scalar potential. The remaining saxions $s = \mathrm{Re}(S)$ and $t = \mathrm{Re}(T)$ feel the scalar potential

$$V_F = \frac{h^2}{8s^2t^3} + \frac{f_0^2t^3}{32s^4} + \frac{3f_4^2}{32s^4t} - \frac{f_0h}{4s^3}\,, \tag{4.2.6}$$

with the exact scaling predicted in (4.2.2). Note that while no branes have been explicitly added yet, the last term scales like a D6-brane contribution. This is because the flux combination hf_0 takes part in the D6-brane tadpole cancellation condition, which must be implicitly satisfied when writing the scalar potential as an F-term of the SUGRA toy model.

At this point, we expect an AdS minimum as we have not yet added the non-BPS brane contribution. Indeed, the potential has a supersymmetric flux-scaling type vacuum [74] at

$$s_0 = \sqrt{\tfrac{20}{27}}\,\frac{f_4^{\frac{3}{2}}}{f_0^{\frac{1}{2}}h}\,, \qquad t_0 = \sqrt{\tfrac{5}{3}}\,\frac{f_4^{\frac{1}{2}}}{f_0^{\frac{1}{2}}}\,, \qquad V_0 \approx -0.059\,\frac{f_0^{\frac{5}{2}}h^4}{f_4^{\frac{9}{2}}}M_{\mathrm{pl}}^4\,, \tag{4.2.7}$$

where we require $t_0 \gg 1$ and $e^{-\phi_0} = s_0/t_0^{3/2} \sim (f_0f_4^3)^{1/4}/h \gg 1$ for perturbative stability. In general, this holds true for large enough flux f_4.

Finally we add by hand the single non-BPS $\widehat{\text{D}}$7-brane contribution with the appropriate scaling

$$V = V_F + \frac{A_{\widehat{\text{D}}7}t^{\frac{1}{2}}}{s^3}\,, \qquad A_{\widehat{\text{D}}7} = \frac{3\sqrt{2}}{16}\,, \tag{4.2.8}$$

where the coefficient is determined by the tension through dimensional reduction. Choosing the fluxes to be $h = 3$, $f_0 = 2$ and $f_4 = 23$, the model now has a dS minimum at perturbatively reasonable values of the moduli.

$$s_0 \approx 105.17\,, \quad t_0 \approx 6.54\,, \quad V_0 = 4.87\cdot 10^{-9}M_{\mathrm{pl}}^4\,. \tag{4.2.9}$$

The minimal eigenvalue of the Hessian matrix in this minimum is found to be positive $min(\nabla_i\nabla_jV)|_{(s_0,t_0)} \approx 3.711\cdot 10^{-8}M_{\mathrm{pl}}^2$, which together with the plot fig. 4.2 makes it clear that we have indeed found a minimum, and not a saddle point.

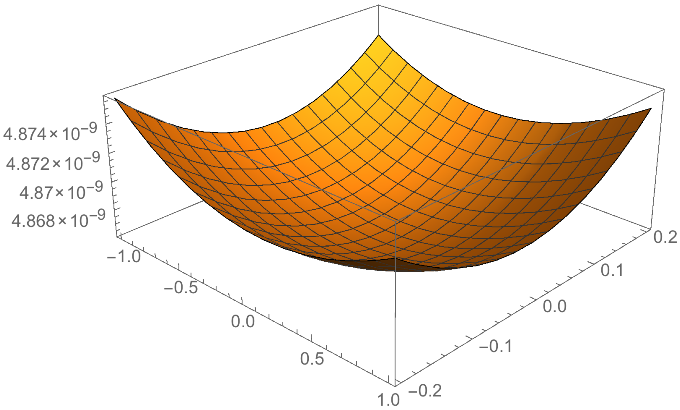

Figure 4.2: Plot of the potential $V\left(s_0+x+y, t_0+0.02(x-y)\right)$ in the range $|x| \leq 1$ and $|y| \leq 0.2$ with fluxes $h=3$, $f_0=2$, $f_4=23$.

4.3 Discussion

The result of the toy model seems to indicate that non-BPS $\widehat{\mathrm{D}}7$-branes not only invalidate the no-go theorem, but truly allow for dS vacua to appear. Of course this is not a true string theory compactification, but rather a string-inspired SUGRA model with a hard supersymmetry breaking sector added by hand. However, a full model should not behave too differently from the usual D-brane story. The non-BPS branes should not backreact heavily on the geometry as long as the string coupling is small, as their tension scales in the same way as BPS branes. Furthermore, separating the non-BPS brane from the D6-branes by wrapping them on rigid cycles leaves the contributions from their attraction/repulsion to the tree-level scalar potential subleading.

These stable dS minima are in stark violation of the no-dS conjectures. The evidence is however on the conjecture's side, suggesting that something is wrong with our setup. In particular, nontrivial K-theory charges on compact spaces have already been shown to result in pathologies on probe-brane theories [137]. The obvious suggestion here is to demand cancellation of these charges analogously to the tadpole conditions of R-R charges.

A trivial $\mathbb{Z}_2$ charge means that there must be an even number of non-BPS $\widehat{\mathrm{D}}7$-branes wrapping a four-cycle in $H_4^-(X)$. However the open string tachyon stretched between two different $\widehat{\mathrm{D}}7$-branes survives the orientifold, signaling the annihilation of the branes. If one of them carries an additional line-bundle, we expect them to decay to non-BPS $\widehat{\mathrm{D}}5$-branes that may decay further. The positive contribution to the no-go theorem will disappear in any case, and so will the chance for a dS minimum. In other words, the no-dS conjecture promotes the statement that K-theory charges must cancel on compact spaces.

Chapter 5

dS Spaces and Brane Worlds in Exotic String Theories

We have seen that simply introducing non-standard brane configurations to the string theory does not suffice to circumvent the no-dS conjecture. In this chapter, we shall go one step further and allow for non-standard space-time configurations by considering the exotic string theories pioneered by C. Hull [17–20]. These theories naturally exhibit de Sitter solutions, but are filled with pathologies like ghosts and time-like loops. Arguing that some pathologies might be acceptable, as long as the observable gauge sector is ghost free, we shall try to find a solution combining a dS vacuum with a ghost-free brane world. However, the branes which allow for a viable gauge sector will be exactly those related to the orientifold projections which discard the dS vacua.

We shall begin by reviewing Hull's exotic string theories and how they arise from usual type II string theory through generalized T-duality and S-duality. Exotic string theories will be classified by the signature of space-time, as well as the world-volume signatures of fundamental strings and D1/D2-branes. An alternative viewpoint was given in [141], where it was shown that the geometry close to negative tension branes in regular type II theory can be described by exotic string theories with Euclidean fundamental strings. This will later allow us to extrapolate the tension of branes in exotic theories.

The pathologies of exotic theories and their consequences will be discussed next. On the one hand, exotic theories generically contain closed string ghost fields. It turns out that at least the massless ghosts can be removed by an orbifold projection. On the other hand, ghosts of the closed string sector are what allows us to write down dS vacua in the first place. It has been argued that these ghosts are just a result of the perturbative approach and are resolved in the full UV-complete theory. We chose to stay agnostic on the ultimate fate of closed string ghosts, in order to continue with our investigation. However unlikely it may seem, the closed string quantum gravity sector is not well tested experimentally, and may still break unitarity or other generic assumptions at high energies. The open string gauge sector on the other hand has been experimentally probed to extreme precision, and there is little to no room for deviations from usual gauge theory. At the very least, the open sector must be ghost-free and unitary.

Another issue is that time-like compactification can produce infinitely many arbitrarily light states. These compactifications are inevitable, as we start with multiple times and want to end up in 3+1 dimensions. An abundance of ultra-light states spoils the Wilsonian approach to the compactified effective theory. While we have no solution for this problem, we find that at least for the ghost-free brane sectors of theories with Euclidean fundamental strings the issue remains absent.

The exotic nature of these theories is quite mind-bending in low-energy thought experiments, but fortunately the CFT fundamentals of string theory are robust enough to describe these theories abstractly. We continue extending CFT techniques to more general space-time and world-volume signatures, expanding on the pioneering work by [141]. The subtle differences in the formulae lead to the appearance of complex phases. In some states, this manifests as a negative norm and subsequently ghosts. For open strings, the complex phase in the boundary CFT amplitudes allows us to constrain the D-brane spectrum to those with real tension.

Finally we discuss the phenomenology of exotic D-brane theories. The negative brane approach supplies their effective actions. This complementary method verifies the spectrum found in the CFT computation and agrees with the results of [19]. Additionally, the effective action allows us to determine the phenomenologically viable brane-world theories, i.e. ghost-free and with a (3,1)-dimensional subspace. While such brane worlds do exist, tadpole cancellation requires the addition of O-planes. For theories with Euclidean fundamental strings, the O-planes are precisely those required to project out all massless ghosts of the 10D theory, eliminating also the loophole for dS in these theories. Lorentzian exotic strings do seem to admit a ghost-free brane sector with potential for dS compactifications. However, here we find infinitely many ultra-light states appearing also in the brane sector, invalidating any 4D effective action.

5.1 Time-like dualities and exotic superstring theories

Ordinarily, T-duality is described as the equivalence of two string theories compactified on a spacial circle under inversion of the radius and exchange of winding and momentum modes. The non-compact theories are just the large/small radius limit of the same theory. Similarly, S-duality identifies the strong coupling limit of one theory with the weak coupling limit of another.

In the late 90s C. Hull [17, 18] showed that also time-like circles give rise to T-dual theories. By applying successive duality transformations, the usual type II theories with $(9, 1)$ signature give rise to a bouquet of string theories with exotic signatures and branes. This net of exotic theories with various signatures $(10-p, p)$ have similar supergravity actions to the type II actions, but some kinetic terms come with the opposite sign.

These ghost have been argued to be an artifact of the low energy limit and perturbative description, and the full non-perturbative theory whose limits are the various (exotic) type II theories is supposedly well-behaved. On the other hand, the method of reaching the exotic theories via time-like circle compactifications could introduce true pathologies. We shall assume in the following that the former is the case, and that exotic theories are consistent theories in the UV.

5.1.1 The zoo of type II$^{\alpha\beta}$ theories

We want to explore the net of exotic theories. We employ the notation introduced in [141], labeling type IIA/B$^{\alpha\beta}_{(10-p,p)}$ theories by the space-time signature $(10-p,p)$ and two signs $\alpha, \beta \in \{+,-\}$. The usual $(9,1)$ signature with 9 space- and 1 time-like direction may be omitted for better readability. The sign α indicates whether the fundamental string of the theory has a Lorentzian $(+)$ or Euclidean $(-)$ signature. In the same way, the second sign β concerns the signature of the D1/D2 branes. Not all branes in a theory will necessarily have the same (even/odd) signature. To keep with the usual naming scheme for D-branes, a Dp-brane has a $(p+1)$-dimensional world-volume, which does not necessarily contain the time direction(s). For this classification, we call all space-times and world-volumes with odd (even) numbers of time-like directions Lorentzian (Euclidean). We will summarily denote the class of theories with Lorentzian (Euclidean) fundamental strings by IIA/B$^{\mathrm{L}}$ (IIA/B$^{\mathrm{E}}$). In this notation, the usual type II theories are IIA^{++} and IIB^{++}.

Let us begin by considering the effect of time-like T-duality on D-branes. As usual, T-duality exchanges Dirichlet and Neumann boundary conditions. Space-like T-duality adds or subtracts one dimension from the D-branes present in the theory, thus switching from type IIA to B and vice versa. However, all usual string theories have Neumann boundary conditions in the time direction, giving the Dp-branes a Lorentzian $(p,1)$ signature. This means a time-like T-dual of regular type II theories can only have Euclidean branes of one dimension less, i.e. (IIA^{++} $\leftrightarrow$ IIB^{+-}) and (IIB^{++} $\leftrightarrow$ IIA^{+-}). Space-like T-duality then not only relates the regular theories (IIA^{++} $\leftrightarrow$ IIB^{++}) but also the two new exotic theories (IIB^{+-} $\leftrightarrow$ IIA^{+-}).

Having found a complete set of theories under T-duality, let us apply S-duality to the type IIB theories. The type IIA theories are S-dual to two M-theory variants with Lorentzian or Euclidean M2-branes. We shall not be concerned with the exotic M-theories here, for more information see [18, 141]. Taking the strong coupling limit of type IIB theories, S-duality exchanges F1$\leftrightarrow$D1 and NS5$\leftrightarrow$D5, leaving D3 invariant. This leaves the regular IIB^{++} theory self-dual, while the Euclidean D-branes of type IIB^{+-} get exchanged with the Lorentzian fundamental string and NS5-brane. We end up with a Euclidean fundamental string and Lorentzian D1-branes, so we denote the theory by IIB^{-+}. The other objects in the theory are the Lorentzian D5- and Euclidean NS5- and D3-branes.

With a new IIB^{-+} theory in the duality net, we can continue applying T-dualities. The alternating signatures of the D-branes mean that the dual of type IIB^{-+} compactified on a space-like circle can only be a theory on a time-like circle! Otherwise, the Euclidean D3-brane with signature $(4,0)$ would give rise to a Euclidean $(3,0)$ D2-brane, while the Lorentzian $(1,1)$ D1-brane is T-dual to a Lorentzian $(2,1)$ D2-brane. Both cannot be true, as reversing the duality cycle would imply that all theories must contain both Euclidean and Lorentzian branes, which we do not want for the regular type II^{++} theories.

For a signature-changing T-duality, this problem disappears. The T-dual to the Euclidean $(4,0)$ D3 remains the Euclidean $(3,0)$ D2, as the signature change applies to the direction the brane loses under duality. Not originally wrapping the signature-changing direction, the Lorentzian $(1,1)$ D1-brane adds a leg on the new time direction and results in a Euclidean $(1,2)$ D2-brane with two time-like directions. With Euclidean D2-branes and one more time-like direction, this new theory is of type $\mathrm{IIA}^{--}_{(8,2)}$.

This pattern of signature-changing duality continues for all theories with Euclidean fundamental strings. Their D-brane spectra have alternating signatures, and each T-duality inverts the pattern. Limited only by the extreme cases of pure time- or space-like spacetime, the resulting pattern is as follows:

$$\mathrm{IIA}^{-+}_{(10,0)} \leftrightarrow \mathrm{IIB}^{-+}_{(9,1)} \leftrightarrow \mathrm{IIA}^{--}_{(8,2)} \leftrightarrow \mathrm{IIB}^{--}_{(7,3)} \leftrightarrow \mathrm{IIA}^{-+}_{(6,4)} \leftrightarrow \ldots \leftrightarrow \mathrm{IIA}^{--}_{(0,10)}\,. \tag{5.1.1}$$

Following the arrows to the right (left) corresponds to T-dualizing along a space-like (time-like) direction. As we have now found two more type IIB^{-+} theories with different signatures, we can S-dualize back to theories with Lorentzian fundamental strings. The resulting theories are just type IIB^{+-} theories with corresponding signature, which T-dualize in the same way.

The complete duality net (except M-theories) are then four classes of type II^{L} theories, the regular type II theories and their time-like T-duals, in signatures $(9,1)$, $(5,5)$ and $(1,9)$. On the type II^{E} side, T-duality changes the signature of space-time and organizes the theories in a spiral between the extreme signatures. All theories and their duality relations are organized in fig. 5.1.

5.1.2 Exotic type $\mathrm{II}^{\alpha\beta}$ supergravities

The 10D SUGRA actions of exotic string theories have been deduced in the original papers by C. Hull [17, 18] and condensed to a compact form in [141]. The actions split into contributions from NS-NS, R-R and Chern-Simons sectors

$$S[\mathrm{IIA/B}^{\alpha\beta}] = S^{\alpha\beta}_{\mathrm{NS}} + S_{\mathrm{R}}[\mathrm{A/B}]^{\alpha\beta} + S_{\mathrm{CS}}[\mathrm{A/B}]\,. \tag{5.1.2}$$

While the NS-NS part is the same for type IIA and IIB theories but changes with α, β, the CS part is independent of α, β but differs between type IIA and IIB. The

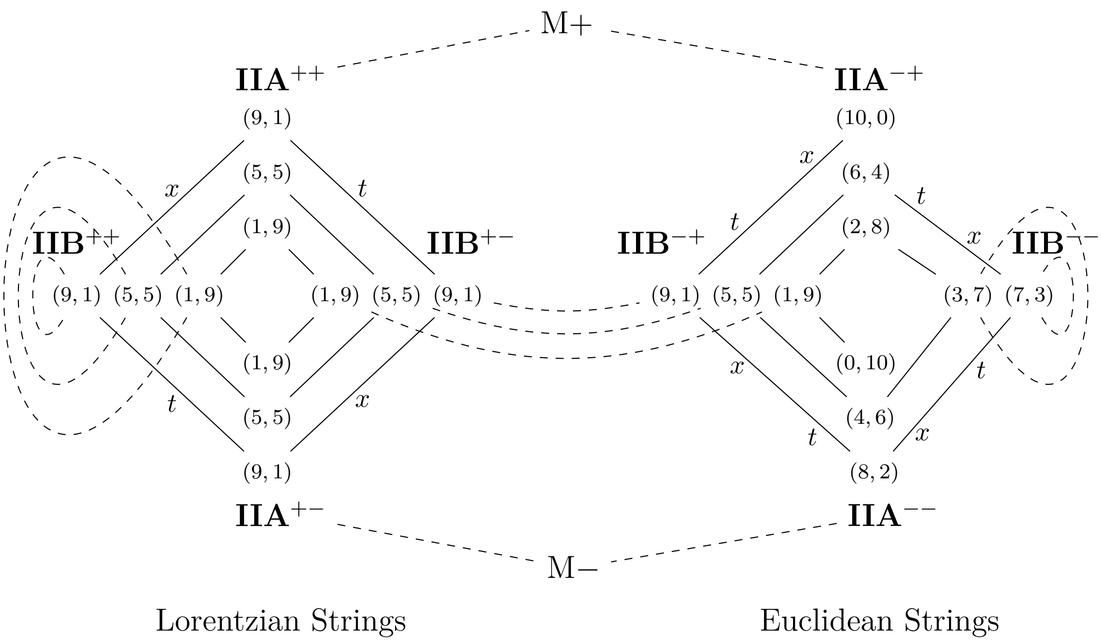

Figure 5.1: The duality web of exotic string theories with type II^L theories on the left and type II^E theories on the right. T-dualities are represented by solid lines, and S-dualities by dashed lines. The x (t) labels indicate dualities arising from compactification on a spatial (time-like) circle. M-theory limits of the various type IIA theories are indicated without giving details. *(Diagram adopted from [141])*

various contributions are given by

$$\begin{aligned} S_{\rm NS}^{\alpha\beta} &= \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{|\det G|}\; e^{-2\Phi}\Big[\mathcal{R} + 4(\nabla\Phi)^2 - \frac{\alpha}{2}|H_3|^2\Big]\,, \\ S_{\rm R}[\mathrm{A}]^{\alpha\beta} &= -\frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{|\det G|}\left[\frac{\alpha\beta}{2}|F_2|^2 + \frac{\beta}{2}|\tilde{F}_4|^2\right]\,, \\ S_{\rm R}[\mathrm{B}]^{\alpha\beta} &= -\frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{|\det G|}\left[\frac{\alpha\beta}{2}|F_1|^2 + \frac{\beta}{2}|\tilde{F}_3|^2 + \frac{\alpha\beta}{4}|\tilde{F}_5|^2\right]\,, \\ S_{\rm CS}[\mathrm{A}] &= -\frac{1}{4\kappa_{10}^2}\int B_2\wedge F_4\wedge F_4\,, \\ S_{\rm CS}[\mathrm{B}] &= -\frac{1}{4\kappa_{10}^2}\int B_2\wedge F_3\wedge F_5\,. \end{aligned} \tag{5.1.3}$$

Here the field strengths are $H_3 = dB_2$ and $F_p = dC_{p-1}$, and $\tilde{F}_p = F_p - H_3\wedge C_{p-3}$. As usual, the (anti-)self-duality of $\tilde{F}_5 = (\alpha\beta)\star\tilde{F}_5$ has to be additionally required in type IIB. The Romans mass F_0 of the type IIA theories is set to zero for simplicity. Note that the action only implicitly depends on the signature $(10-p, p)$.

The sign of the kinetic terms, which determines whether a field is a ghost, is governed by two effects. The overall sign of the terms in (5.1.3) are influenced by

α and β. Writing this first sign as $\kappa^{(\alpha\beta)}$, the other effect is the combination of signs in the inverse metrics multiplying the field strengths

$$\mathcal{L}_{\text{kin}} \sim -\kappa^{(\alpha\beta)}\sqrt{|G|}\,|F_n|^2 = -\frac{\kappa^{(\alpha\beta)}}{n!}\sqrt{|G|}\,G^{i_1 j_1}\dots G^{i_n j_n}\,F_{i_1\dots i_n}\,F_{j_1\dots j_n}\,. \tag{5.1.4}$$

For $\kappa^{(\alpha\beta)} = +/-1$, an odd/even number of time-like indices indicates a ghost. While ghosts are usually undesirable, as mentioned their presence here is responsible for the existence of dS solutions. For example, the prototype $\mathrm{AdS}_5 \times S^5$ solution of type IIB with F_5 flux generalizes with multiple times to $\mathrm{AdS}_{5-m,m} \times dS_{5-n,n}$, where $m+n$ is odd. In fact, the many arguments for the no-dS swampland conjecture assume that all kinetic terms have the usual sign. Naturally, further investigation is required to determine whether a full flux compactification of an exotic string theory indeed admits meaningful dS minima, but at least the no-go theorems are not expected to hold in the presence of ghosts.

The equations of motion for the exotic SUGRA theories reveal the brane spectra for these theories [19,20]. The D-branes coupling to R-R fields in the various exotic theories are given in tables 5.1 and 5.2. The additionally present NS-NS branes as well as pp-waves etc. are given in [19].

Theory	D0	D2	D4	D6	D8
$\mathrm{IIA}^{-+}_{(10,0)}$	$(1,0)$	-	$(5,0)$	-	-
$\mathrm{IIA}^{++}_{(9,1)}$	$(0,1)$	$(2,1)$	$(4,1)$	$(6,1)$	$(8,1)$
$\mathrm{IIA}^{+-}_{(9,1)}$	$(1,0)$	$(3,0)$	$(5,0)$	$(7,0)$	$(9,0)$
$\mathrm{IIA}^{--}_{(8,2)}$	$(0,1)$	$(3,0)$,$(1,2)$	$(4,1)$	$(7,0)$,$(5,2)$	$(8,1)$
$\mathrm{IIA}^{-+}_{(6,4)}$	$(1,0)$	$(2,1)$,$(0,3)$	$(5,0)$,$(3,2)$,$(1,4)$	$(6,1)$,$(4,3)$	$(5,4)$
$\mathrm{IIA}^{++}_{(5,5)}$	$(0,1)$	$(2,1)$,$(0,3)$	$(4,1)$,$(2,3)$,$(0,5)$	$(4,3)$,$(2,5)$	$(4,5)$

Table 5.1: Brane spectrum of $\mathrm{IIA}^{\alpha\beta}$ theories.

SUGRA solutions with mirrored signatures differ from the shown theories only by an overall minus sign, also mirroring the world-volume signature of the branes. For the type IIB theories admitting a D(-1)-brane, the dual D9-brane is also expected to exist. Note also that T-duality implies the existence of a $(9,0)$ D8-brane in type $\mathrm{IIA}^{+-}_{(10,0)}$ [19]. We shall come back to a more detailed description of exotic D-branes later.

5.1.3 CFT description of exotic strings

While in a target-space approach the multiple times and Euclidean nature of the exotic string theories may seem strange, in the CFT all that changes are some factors. Switching space-time or target-space signatures amounts to minus signs in the sigma model, and in the CFT some factors of i appear. We shall review and

Theory	D(-1)	D1	D3	D5	D7
$IIB^{++}_{(9,1)}$	-	$(1,1)$	$(3,1)$	$(5,1)$	$(7,1)$
$IIB^{+-}_{(9,1)}$	$(0,0)$	$(2,0)$	$(4,0)$	$(6,0)$	$(8,0)$
$IIB^{-+}_{(9,1)}$	$(0,0)$	$(1,1)$	$(4,0)$	$(5,1)$	$(8,0)$
$IIB^{--}_{(7,3)}$	-	$(2,0)$,$(0,2)$	$(3,1)$,$(1,3)$	$(6,0)$,$(4,2)$	$(7,1)$,$(5,3)$
$IIB^{++}_{(5,5)}$	-	$(1,1)$	$(3,1)$,$(1,3)$	$(5,1)$,$(3,3)$,$(1,5)$	$(5,3)$,$(3,5)$
$IIB^{+-}_{(5,5)}$	$(0,0)$	$(2,0)$,$(0,2)$	$(4,0)$,$(2,2)$,$(0,4)$	$(4,2)$,$(2,4)$	$(4,4)$
$IIB^{-+}_{(5,5)}$	$(0,0)$	$(1,1)$	$(4,0)$,$(2,2)$,$(0,4)$	$(5,1)$,$(3,3)$,$(1,5)$	$(4,4)$

Table 5.2: D-brane spectrum of $IIB^{\alpha\beta}$ theories.

extend the closed Euclidean string construction of [141] to open strings, in order to prepare for the later discussion of boundary states.

As usual, the action for a free boson is given by the Polyakov action

$$S_b = \frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{\det g}\, g^{ab}\,\eta_{\mu\nu}\,\partial_a X^\mu \partial_b X^\nu\,, \tag{5.1.5}$$

where now the world-sheet metric g is gauge fixed to the flat Euclidean metric $g_{\sigma_1\sigma_1} = g_{\sigma_2\sigma_2} = 1$. Light cone coordinates are chosen such that

$$\sigma_\pm = \sigma_1 \pm i\sigma_2\,, \qquad \partial_\pm = \frac{1}{2}(\partial_{\sigma_1} \mp i\partial_{\sigma_2})\,. \tag{5.1.6}$$

We now choose convenient mode expansions, starting with the closed string sector. The goal of the mode expansion will be to simplify the mode algebra as much as possible. In this framework the oscillator modes will behave as in the usual string theories. The zero modes will be solely responsible for the changes in the physics. The mode expansion of the closed string sector is given by

$$X^\mu(\sigma_1,\sigma_2) = x^\mu + \alpha' p^\mu \sigma_1 + \sqrt{\frac{\alpha'}{2i}}\sum_{n\neq 0}\left(\frac{\alpha_n^\mu}{n}e^{-n\sigma^+} + \frac{\overline{\alpha}_n^\mu}{n}e^{-n\sigma^-}\right)\,, \tag{5.1.7}$$

with an extra i in the normalization of the oscillator terms, so that the mode algebra becomes

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}\,, \qquad [\alpha_m^\mu, \alpha_n^\nu] = [\overline{\alpha}_m^\mu, \overline{\alpha}_n^\nu] = m\,\delta_{m,-n}\,\eta^{\mu\nu} \tag{5.1.8}$$

for $m, n \neq 0$. Moreover, one has as usual $[\alpha_m^\mu, \overline{\alpha}_n^\nu] = 0$, and the oscillators α_m^μ, $\overline{\alpha}_m^\mu$ commute with the zero modes x^μ and p^μ. The open string sector can be expanded into modes in a similar fashion.

To arrive at this standard mode algebra, we have effectively rescaled the standard oscillator modes $\hat{\alpha}_n$ by a factor of $\sqrt{i}$. As a consequence, one needs to be very careful when computing overlaps of states $\langle\phi^1|\phi^2\rangle$. Indeed, taking the general definition of the conjugate $(\phi_n)^\dagger = (\phi^\dagger)_{-n}$ for a field ϕ in Euclidean CFT into account, the rescaling leads to phase factors, as some of the fields won't be purely real anymore. On the one hand, in this paper we are mostly concerned with partition functions where these phases do not matter as one simply counts the number of states at each level. On the other hand, in the boundary state overlaps that will give us the D-brane theory, due to loop-channel tree-channel equivalence the (suitably generalized) CPT operator Θ has to remove these factors. These two facts make this basis very useful for our computations.

The (time-like) T-duality arguments suggest that a change of the target-space signature does not affect the critical dimension of the string theory. Let us check this explicitly on the world-sheet. The critical dimension can be read off in light cone gauge, e.g. for the bosonic string, by checking for anomalies of the $SO(p,q)$ Lorentz symmetry. The world-sheet metric is gauge-fixed to $h_{\alpha\beta} = \eta_{\alpha\beta}$ and we introduce space-time light cone coordinates $X^+ = 1/\sqrt{2}\,(X^0 + X^1)$, $X^- = 1/\sqrt{2}\,(X^0 - X^1)$, where we singled out one time and spatial direction X^0, X^1. The target-space metric is $\eta_{+-} = \eta_{-+} = -1$ for the light cone, $\eta_{ab} = -\delta_{ab}$ for $a, b = 1, \ldots, p-1$ remaining time directions and $\eta_{mn} = \delta_{mn}$ for the $m, n = 1, \ldots, q-1$ spatial directions.

We now consider the open string with Neumann boundary conditions. The remaining gauge freedom is fixed by setting $X^+(\sigma,\tau) = x^+ + p^+\tau$. Using the Virasoro constraint equation $\eta_{\mu\nu}(\dot{X}^\mu \pm X'^\mu)(\dot{X}^\nu \pm X'^\nu) = 0$ to express the oscillator modes of X^- in terms of the transverse modes yields

$$\alpha_n^- = \frac{1}{p^+}\left(\frac{1}{2}\sum_{k=-\infty}^{\infty} :\eta_{ij}\,\alpha_{n-k}^i \alpha_k^j: \; - a\,\delta_{n,0}\right) \tag{5.1.9}$$

with i, j running over the transverse directions and for simplicity setting $\alpha' = 1/2$. The modes still satisfy a "transverse" Virasoro algebra

$$\left[p^+\alpha_m^-, p^+\alpha_n^-\right] = (m-n)\,p^+\alpha_{m+n}^- + \left(\frac{D-2}{12}(m^3 - m) + 2am\right)\delta_{m+n}\,, \tag{5.1.10}$$

and have commutation relations with the transverse oscillator modes

$$\left[\alpha_n^i, p^+\alpha_k^-\right] = n\,\alpha_{n+k}^i\,. \tag{5.1.11}$$

The only relevant appearance of the space-time metric is in commutation relations $[\alpha_m^\mu, \alpha_n^\nu] = k\,\eta^{\mu\nu}\delta_{m+n,0}$. We can use these commutation relations and follow the standard computation for the potentially anomalous commutator $[J^{i-}, J^{j-}]$ of Lorentz generators $J^{\mu\nu}$. Doing so we find

$$\left[J^{i-}, J^{j-}\right] = 0 \quad \Leftrightarrow \quad D = 26,\; a = 1 \tag{5.1.12}$$

but no additional constraints on the number of time respectively spatial dimensions. Hence Lorentz symmetry $SO(p,q)$ is preserved for a total of $p + q = 26$ space-time dimensions for the bosonic string, and analogously for 10 total dimensions for the superstring.

To calculate the low energy effective action and determine for instance the sign of the kinetic terms, one also needs to know the normalization of the corresponding vertex operators. In fact in [141] the normalization of the metric and B-field vertex operators have been determined to be

$$\begin{aligned} |V_G(p)\rangle &= \quad \epsilon_{\mu\nu}\, \alpha^\mu_{-1}\overline{\alpha}^\nu_{-1}|p\rangle\,, \\ |V_B(p)\rangle &= -ib_{\mu\nu}\, \alpha^\mu_{-1}\overline{\alpha}^\nu_{-1}|p\rangle\,. \end{aligned} \tag{5.1.13}$$

Thus, working with the modes $\alpha^\mu, \overline{\alpha}^\mu$ and treating them in the same way as the usual oscillators in string theory makes it evident that the B-field is a ghost.

Next we want to define the closed and open string partition functions. The unusual convention used for the mode expansion introduces a factor of i in the Hamiltonian

$$\begin{aligned} H &= \int_0^{2\pi} \frac{\mathrm{d}\sigma}{2\pi\alpha'}\Big((\partial_+X)^2 + (\partial_-X)^2\Big) = -i\left(L_0 + \overline{L}_0 - \tfrac{c}{12}\right), \\ L_0 &= i\frac{\alpha' p^2}{4} + \sum_{n>0}\eta_{\mu\nu}\,\alpha^\mu_{-n}\alpha^\nu_n \end{aligned} \tag{5.1.14}$$

and similarly for $\overline{L}_0$. The second term of L_0 is just the number operator which has non-negative integer eigenvalues. In contrast to the usual case, the zero mode contribution of the first term is purely imaginary. The momentum P which generates σ_2 translations on the other hand is given by

$$P = -i\int_0^{2\pi} \frac{\mathrm{d}\sigma}{2\pi\alpha'}\Big((\partial_+X)^2 - (\partial_-X)^2\Big) = -(L_0 - \overline{L}_0)\,. \tag{5.1.15}$$

In this case the normal ordering constant cancels out.

As a consequence the torus and cylinder amplitudes receive additional factors of i. Defining $q = e^{2\pi i(\tau_1 + i\tau_2)}$, the torus partition function can be written in the usual way as

$$Z^{\text{torus}} = \mathrm{Tr}(e^{-2\pi i\tau_2 H - 2\pi i\tau_1 P}) = \mathrm{Tr}\left(q^{L_0 - \frac{c}{24}}\,\overline{q}^{\overline{L}_0 - \frac{c}{24}}\right)\,. \tag{5.1.16}$$

Note that due to the missing Wick rotation for the Euclidean CFT, the coefficient in front of the Hamiltonian is $-2\pi i$ instead of the usual -2π. But this factor gets multiplied by the additional factor of $-i$ in the Hamiltonian (5.1.14), such that the final expression for the partition function is still the usual one. Evaluating the amplitude for a single target-space direction

$$Z^{\text{torus}} = \frac{e^{i\pi/4}}{\sqrt{4\pi\alpha'\tau_2}\,|\eta(\tau)|^2} \tag{5.1.17}$$

we find a complex phase, reproducing the result of [141].

Now we turn to the open string cylinder amplitude characterized by the circumference t of the cylinder,

$$Z^{\mathcal{C}}(t) = \mathrm{Tr}\left(e^{-2\pi i t H_{\text{open}}}\right) = \mathrm{Tr}\left(e^{-2\pi t(L_0 - \frac{c}{24})}\right)\,. \tag{5.1.18}$$

Again considering only a single direction of either NN or DD type, the open string partition functions can be evaluated to be

$$Z^{\mathcal{C}(\mathrm{NN})}(t) = \frac{e^{-i\pi/4}}{\sqrt{2\alpha' t}\,\eta(it)}\,, \qquad Z^{\mathcal{C}(\mathrm{DD})}(t) = e^{-\frac{it}{2\pi\alpha'}Y^2}\frac{1}{\eta(it)}\,. \tag{5.1.19}$$

The additional factor of $e^{-i\pi/4}$ in the Neumann-Neumann case arises from the analytic continuation of the Gaussian integral for the zero mode. The total distance between the Dirichlet loci is defined as $Y^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$. As usual, the open string (loop-channel) cylinder amplitude is closely related to the (tree-channel) overlap of boundary states

$$\tilde{Z}(l) = \langle \Theta B|\, e^{2\pi i l H_{\mathrm{closed}}}\,|B\rangle = \langle \Theta B|\, e^{-2\pi l(L_0+\overline{L}_0-\frac{c}{12})}\,|B\rangle\;, \tag{5.1.20}$$

with l the length of the cylinder formed by the closed strings exchanged between the boundaries. We will later construct the appropriate boundary states in order to characterize D-branes.

Let us now discuss the inclusion of fermions on the Euclidean world-sheet. The action for a free fermion is

$$S_f = \frac{i}{4\pi}\int \mathrm{d}^2\sigma\sqrt{\det g}\;\eta_{\mu\nu}\,\overline{\Psi}^\mu\,\gamma^\alpha\partial_\alpha\Psi^\nu\,, \tag{5.1.21}$$

where the 2×2 matrices γ^α satisfy the Clifford algebra with respect to the world-sheet metric $g_{\alpha\beta}$

$$\{\gamma^\alpha,\gamma^\beta\} = 2g^{\alpha\beta}\,\mathbb{1}_2\,. \tag{5.1.22}$$

Moreover, in the Euclidean case the conjugate $\overline{\Psi}^\mu = \Psi^\mu C$ is defined with the charge conjugation matrix C such that

$$(\gamma^\alpha)^{\mathsf{T}} = C\gamma^\alpha C^{-1}\,,\quad C^{\mathsf{T}} = C^\dagger = C^{-1} = C\,. \tag{5.1.23}$$

Choosing the Pauli matrices as a representation for the Euclidean Clifford algebra, the charge conjugation matrix is uniquely determined to be

$$\gamma^0 = \begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}\,,\qquad \gamma^1 = \begin{pmatrix} 0 & -i\\ i & 0\end{pmatrix}\,,$$
$$C = \begin{pmatrix} 0 & -1\\ -1 & 0\end{pmatrix} = -\gamma^0\,. \tag{5.1.24}$$

In light-cone coordinate, the equations of motion are

$$\partial_-\Psi^\mu_+ = \partial_+\Psi^\mu_- = 0 \tag{5.1.25}$$

with the usual (anti-)holomorphic solutions $\Psi^\mu_+ = \Psi^\mu_+(\sigma^+)$ and $\Psi^\mu_- = \Psi^\mu_-(\sigma^-)$, which can be expanded as

$$\Psi^\mu_+ = \sqrt{-i}\sum_r b^\mu_r\, e^{-2\pi i r\sigma^+}\,,\qquad \Psi^\mu_- = \sqrt{-i}\sum_r \overline{b}^\mu_r\, e^{-2\pi i r\sigma^-}\,. \tag{5.1.26}$$

As in the bosonic case, the factor $\sqrt{-i}$ ensures that the mode algebra takes the usual form

$$\{b_r^\mu, b_s^\nu\} = \delta_{r,-s}\,\eta^{\mu\nu}\,, \qquad \{\bar{b}_r^\mu, \bar{b}_s^\nu\} = \delta_{r,-s}\,\eta^{\mu\nu} \qquad \{b_r^\mu, \bar{b}_s^\nu\} = 0\,. \tag{5.1.27}$$

The energy momentum tensor is obtained by the Sugawara construction, resulting in the explicit expression for the zero mode

$$L_0 = \sum_{r\geq 1/2} \left(r + \tfrac{1}{2}\right) \eta_{\mu\nu}\, b_{-r}^\mu\, b_r^\nu\,. \tag{5.1.28}$$

5.2 Ghosts, orbifolds and compact dimensions

Fundamentally, ghosts are states of negative norm. Hilbert spaces with these kinds of states cannot be interpreted in a probabilistic manner, and unitarity is generally violated in such theories. While allowing some ghosts may be necessary to allow for dS vacua, massless or light ghosts are not phenomenologically viable.

A standard occurrence of ghosts are Faddeev-Popov ghosts in gauge theories. These are introduced to fix overcounting of states due to gauge invariance. Conversely, a standard procedure to get rid of ghost states is to gauge extra symmetries on the world-sheet, hence introducing new (b, c) ghost systems that change the critical central charge of the theory and cancel the contributions of the problematic ghosts. A famous example is the $N = 2$ superstring with a critical central charge of $c = 6$ and a four-dimensional target-space of signature $(2, 2)$ or $(0, 4)$ [142]. Due to the extra gauge symmetry, more target-space directions can be gauged away. However, we do not want to change the critical dimension. We can therefore only allow the usual gauge invariances leading to a critical central charge of 26 (15) for the bosonic (super) string theory. This means there will be a singled out, distinctive time direction and the corresponding bc (and $\beta\gamma$) ghost system.

What we can do however is project out the massless/light ghosts by other means, e.g. by orientifolds. This will not change the number of space-time dimensions, at the cost of potentially breaking 10D diffeomorphism symmetry to a subgroup. However this can be on the compact space, and is nothing unusual in string theory. We will analyze which orientifolds are sufficient to project out massless ghosts for each exotic theory.

5.2.1 Ghosts for the Lorentzian string

The Lorentzian fundamental strings in the $\text{IIA}^\text{L}/\text{IIB}^\text{L}$ theories can be quantized in complete analogy to the usual IIA and IIB (super-) strings with signature $(9, 1)$. This means that the mode algebra for the bosonic fields X^μ reads

$$[\alpha_m^\mu, \alpha_n^\nu] = m\, \eta^{\mu\nu}_{(10-p,p)}\, \delta_{m,-n}\,, \tag{5.2.1}$$

where $\eta^{\mu\nu}_{(10-p,p)}$ denotes the flat metric of signature $(10-p, p) \in \{(9, 1), (5, 5), (1, 9)\}$. In the following let us denote the space-like directions and the single universal time direction with indices $m, n, ...$ and the additional new time-like directions by $a, b, ...$. Note that the universal time and one space direction can be gauged away as usual. Then for instance the off-diagonal graviton states

$$|V_G^{st}(0)\rangle = \epsilon_{\mu a}\alpha^m_{-1}\tilde{\alpha}^a_{-1}|0\rangle \tag{5.2.2}$$

have negative norm (for $\langle 0|0\rangle = 1$) and give physical ghosts that cannot be gauged away. Note that the graviton modes in purely space-like $|V_G^{ss}(0)\rangle = \epsilon_{\mu\nu}\alpha^m_{-1}\tilde{\alpha}^n_{-1}|0\rangle$ and time-like polarizations $|V_G^{tt}(0)\rangle = \epsilon_{ab}\alpha^a_{-1}\tilde{\alpha}^b_{-1}|0\rangle$ have positive norm.

In the NS-NS sector of the superstring, one only has to replace the X^μ by their fermionic superpartners ψ^μ and the logic goes through analogously. In the R-R sector, there is the distinction between the IIA/B^{++} and the IIA/B^{+-} theories, where the latter carry a wrong overall sign for the kinetic terms of the massless R-R fields. This can be taken care of in the world-sheet theory by flipping by hand the overlap between the R-R ground states

$$\langle 0|0\rangle^{+-}_{\rm RR} = -\langle 0|0\rangle^{++}_{\rm RR}\,. \tag{5.2.3}$$

Let us now see how one can remove these massless ghost states. Following the usual recipe for performing an orbifold in string theory, the untwisted sector is projected to invariant states and a twisted sector must be introduced. In the IIA/B$^{+-}_{(9,1)}$ theory, the ghost R-R fields can be projected out by performing an orbifold by $(-1)^{F_L}$. To avoid the appearance of new massless ghosts in the $\mathbb{Z}_2$ twisted sector, one can combine this action with a half-shift $S : X \to X + \pi R$ along a compactified spatial direction.

For the IIA/B$^{++}_{(5,5)}$ theories, physical ghosts are related to four extra time-like directions. These ghosts can be removed by taking the quotient by a $\mathbb{Z}_2$ reflection $I_4 : x^a \to -x^a$ along these four directions. Similarly, the ghosts in IIA/B$^{++}_{(1,9)}$ are removed by I_8, reflecting the eight extra time-like coordinates. Finally, the massless ghosts of IIA/B$^{+-}_{(5,5)}$ and IIA/B$^{+-}_{(1,9)}$ are projected out by $I_4(-1)^{F_L}$ and $I_8(-1)^{F_L}$, combining the previous reasoning. These results are summarized in figure 5.2.

Eventually, we are of course interested in compactifications of the exotic string theories to 4D with signature $(3, 1)$, so in theories with multiple time-like directions some of them will need to be compactified. The standard problem of compact time-like dimensions are closed time-like curves which violate causality. Since the orbifolds project out massless excitations in these directions, one might naively think that the quotient theories are safe. However, we will see that compact time dimensions in exotic string theories lead to further complications.

For the IIA/B$^{+-}_{(9,1)}$ theory, the orbifold by $(-1)^{F_L}$ removes all ghost fields from the untwisted sector. In case of the IIA/B$^{++}_{(5,5)}$ theories however, even though the massless mixed graviton modes $|V_G^{(ev,odd)}(0)\rangle = \epsilon_{ma}\alpha^m_{-1}\tilde{\alpha}^a_{-1}|0\rangle$ are projected out, for non-vanishing momentum/energy the linear combination

$$|V_G^{(ev,odd)}(p^n, e^b)\rangle = \epsilon_{ma}\alpha^m_{-1}\tilde{\alpha}^a_{-1}|p^n, e^b\rangle - \epsilon_{ma}\alpha^m_{-1}\tilde{\alpha}^a_{-1}|p^n, -e^b\rangle \tag{5.2.4}$$

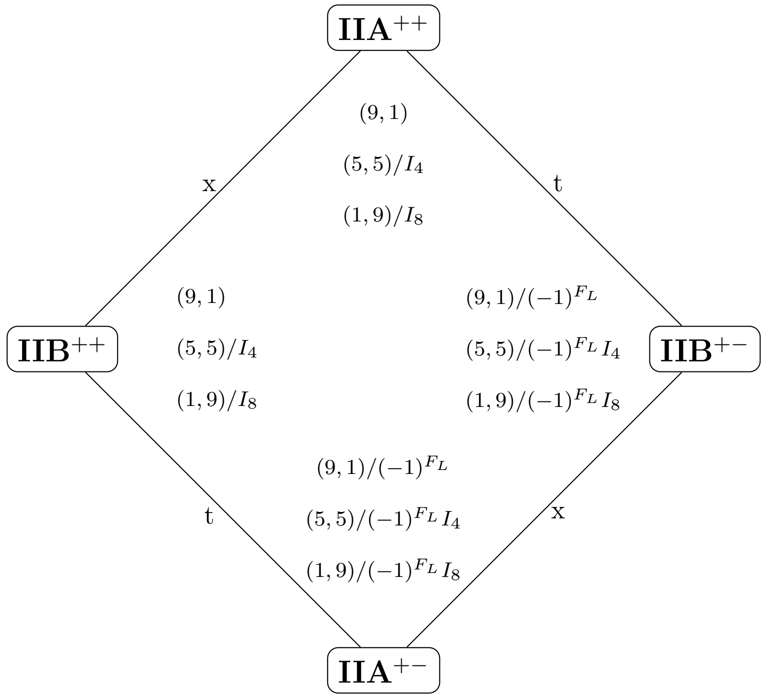

Figure 5.2: Orbifold projections that remove the massless ghosts for Lorentzian theories. New ghosts in twisted sectors can be avoided by combining these actions with a shift along a spatial direction.

remains in the spectrum. Here p^n, $m, n \in \{0, 1, \dots, 5\}$ denote the usual energy $p^0 = E$ and space-like momenta, and e^b, $a, b \in \{6, \dots, 9\}$ denote the extra time-like energies. The upper index pair (ev, odd) indicates the behavior of the left and right moving part under the $\mathbb{Z}_2$ operation. Therefore there are still massive ghosts in the string spectrum, the $\mathbb{Z}_2$ projection does not remove all ghosts.

The on-shell condition for such a state is

$$E^2 + \sum_a (e^a)^2 = \sum_i (p^i)^2 + m^2 \,, \tag{5.2.5}$$

where m is the mass of the state. We interpret this condition such that for a state of mass m with momenta p^i the total energy can be distributed among all the time-like energies such that this quadratic relation is satisfied [143]. Only E is the energy that we have access to. Note that while for negative E we have an interpretation in terms of anti-particles with positive E, the additional energies e^a can be both positive and negative.

Let us now consider a Lorentzian string on a time-like torus of radius R. As for a space-like compactification, the time-like momentum (i.e. energy) gets quantized along the compact direction and leads to a mass contribution, resulting in a KK tower of massive states. Similarly, the winding modes contribute to the mass so that in total we find the on-shell condition

$$E^2 + \sum_b \left[\left(\frac{m_b}{R} \right)^2 + \left(\frac{n_b R}{\alpha'} \right)^2 \right] = \sum_i (p^i)^2 + \frac{2}{\alpha'}(N + \overline{N} - 2a) \tag{5.2.6}$$

with $a = 1/2$ for the superstring and winding and momentum numbers satisfying the level-matching condition $\sum_a m_a n_a = \overline{N} - N$. For $R > \sqrt{\alpha'}$ it is tempting to identify a UV cutoff with the Kaluza-Klein scale $\Lambda_{\rm UV} = 1/R$ that we assume to be only a few orders of magnitude below the string scale. Let us analyze this on-shell condition in the IR regime $|p| < \Lambda_{\rm UV}$.

In the massless sector $N = \overline{N} = 1/2$, a non-vanishing time-like KK/winding mode $(m_a, n_a) \neq (0,0)$ already lies outside the IR regime. Thus all the light on-shell states that we have access to are frozen in the extra time directions and feature $e^a = 0$. Then together with the $\mathbb{Z}_2$ projections there are no light ghosts left, so it seems that we are safe. However, for the tower of massive string excitations $N = \overline{N} > 1/2$ their contribution to the right hand side of (5.2.6) can be balanced against KK/winding contributions. Therefore, these massive excitations combine with time-like KK/winding modes to appear as extremely light particles from a 4D perspective. As already observed in [141], even for irrational values of the radius there will always be integers $N, \overline{N}, m_a, n_a$ such that their 4D mass lies below any cutoff. Relatedly, there exist kinematically allowed scattering processes like

$$\begin{aligned} &|V_{m_1=0}(p_1^m, e_1^a = 0)\rangle + |V_{m_2=0}(p_2^m, e_2^a = 0)\rangle \\ &\qquad \longrightarrow |V_{m_3>0}(p_3^m, e_f^a)\rangle + |V_{m_4>0}(p_4^m, -e_f^a)\rangle \end{aligned} \tag{5.2.7}$$

with the extra energies in the final state $e_f^a \neq 0$. Thus, the ultralight states with $N = \overline{N} > 1/2$ do not decouple in the scattering amplitudes of massless states with $N = \overline{N} = 1/2$. We can summarize these findings by saying that the dimensionally reduced 10D Lorentzian supergravity actions cannot be considered as Wilsonian effective actions of a 4D theory.

5.2.2 Ghosts for the Euclidean string

One new aspect of the quantization of the Euclidean string is that factors of $i = \sqrt{-1}$ appear at various places. For instance, the mode algebra for the bosonic fields X^μ now reads

$$[\hat{\alpha}_m^\mu, \hat{\alpha}_n^\nu] = -i\, m\, \eta^{\mu\nu}_{(10-p,p)}\, \delta_{m,-n}\, . \tag{5.2.8}$$

As a consequence, the diagonal graviton/B-field states $|V_G^{ss}(0)\rangle$ and $|V_G^{tt}(0)\rangle$ have negative norm and the off-diagonal ones $|V_G^{st}(0)\rangle$ positive norm (for $\langle 0|0\rangle = 1$). However, this is not consistent with the normalization of the Einstein-Hilbert term for the Euclidean string SUGRA actions (5.1.3). This can be remedied by choosing the correct normalization of the vertex operators. The graviton gets an extra factor of $-i$, rendering its norm positive, while the B-field remains a ghost. Of course the time-like ghosts from the previous section also remain in the spectrum.

Now we investigate whether there also exist $\mathbb{Z}_2$ operations that can mod out all the massless ghost fields for the Euclidean exotic type II$^{\rm E}$ string theories. Let us start with the IIB$^{-+}_{(9,1)}$ theory, which is the S-dual of the IIB$^{+-}_{(9,1)}$ theory. By looking at its SUGRA action (5.1.3) we see that H_3, F_1, F_5 have the wrong sign of the kinetic terms and F_3 the usual sign. These are precisely the p-form fields that are respectively odd and even under the world-sheet parity transformation Ω, and indeed the S-dual of $(-1)^{F_L}$ is known to be Ω. Therefore, the orientifold $\mathrm{IIB}^{-+}_{(9,1)}/\Omega$ has no ghost fields in the closed string sector. Depending on whether the orientifold projection has fixed loci or acts freely (after combining it again with a shift symmetry), there will be a twisted sector in the form of appropriate D-branes that need to be introduced to cancel the R-R tadpole of the O-plane. This open string sector can host additional ghosts. We will come to this point in section 5.3.

Now by successively applying spatial T-dualities we can find the orientifold projections for all the $\mathrm{IIA/B}^{-,\beta}_{(10-p,p)}$ theories. After one T-duality one gets $\mathrm{IIA}^{--}_{(8,2)}$ with the projection ΩI_1, where I_1 reflects the new additional time-like coordinate. The corresponding branes are D8-branes localized at a point in the new time-like direction. Another T-duality leads to the $\mathrm{IIB}^{--}_{(7,3)}/\Omega I_2(-1)^{F_L}$ orientifold, etc. All the resulting quotients are shown in the right hand part of figure 5.3. T-dualizing instead along the time-like direction, we find the appropriate orientifold quotient to be $\mathrm{IIA}^{-+}_{(10,0)}/\Omega\tilde{I}_1(-1)^{F_L}$, where $\tilde{I}_1$ is a reflection along the space-like direction that was created by T-dualizing.

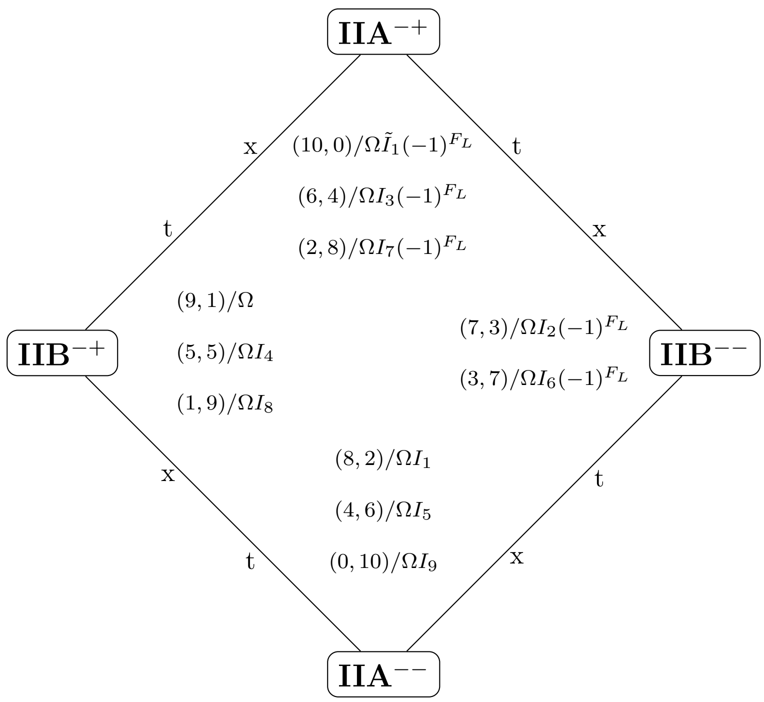

Figure 5.3: Orientifold projections removing the massless ghosts for Euclidean theories.

Another new aspect of the Euclidean theories is that the tower of string excitations has imaginary squared mass $m^2 \sim i$. Moreover, since under T-duality a space-like circle maps to a time-like one and vice versa, the winding modes contribute with the opposite sign as the KK modes. Thus, the on-shell relation for a compactification on a torus T^D of radii R_k with metric $\eta_k = \pm 1$ now reads

$$E^2 - \sum_{k=1}^{D} \eta_k \left[\left(\frac{m_k}{R_k} \right)^2 - \left(\frac{n_k R_k}{\alpha'} \right)^2 \right] = \sum_i (p^i)^2 - i\, \frac{2}{\alpha'} (N + \overline{N} - 2a) \tag{5.2.9}$$

with the level-matching condition $\sum_k \eta_k m_k n_k = N - \overline{N}$. In contrast to the Lorentzian string, here the KK/winding modes can never cancel against the string oscillations. However, for mixed space- and time-like compactifications the KK mode contribution can cancel up to arbitrary precision against the winding mode contribution, leading again to the conceptual problem of interpreting the dimensional reduction of the 10D Euclidean supergravity actions as Wilsonian effective theories. As for the Lorentzian string, these ultra-light modes do not decouple in string scattering amplitudes.

5.3 D-branes in exotic string theories

Although the perturbative approach to the exotic string theories is full of pathologies that are not yet completely understood, it has been argued that these are just artifacts of the perturbative approach and will get resolved in the full theories (as they are all dual to the original type IIA/B superstrings). On the other hand it could also well be that these pathologies are a result of having compact time directions (at least at intermediate stages) and that these are simply not allowed in any reasonable physical theory. In the latter case, there would be no point in further pursuing these ideas.

However, we would not want to miss a potentially interesting new aspect of string theory, as applying dualities has often led to new insights into the theory. Moreover, it is at least appealing that despite conceptual pathologies, the formalism per se seems to go through. Thus, in the following we still take a positive attitude and further investigate the open sector of exotic string theories. The question we are posing is whether a pathological closed string sector with ghosts that admits dS solutions, can nevertheless host a viable effective D-brane theory that by itself obeys the usual requirements for a consistent quantum field theory, i.e. is ghost free and unitary. This subsector could then be considered to be the Standard Model, whose quantum aspects we have direct experimental access to. Should we find a ghost-free gauge sector in a theory with closed string ghosts, our pragmatic approach would open a window for dS in string theory.

We will begin this section by discussing a map found in [141] between the usual theories and Euclidean exotic theories through the near-brane geometry of negative branes. This shall allow us to deduce the tension and signature of D-branes in exotic theories by mapping the DBI-action to exotic theories and matching the kinetic terms to the SUGRA action. The results will be confirmed by the exotic boundary CFT.

5.3.1 Exotic string map from negative tension branes

Negative $\mathfrak{D}$-branes are extended BPS objects carrying the opposite R-R-charge and tension from D-branes [144, 145]. The geometrical backreaction of these negative tension objects was studied in [141]. In a surprising twist, it was found that the negative branes are surrounded by a bubble of space-time with a different signature metric. This allows to determine a map from the usual type II actions to exotic theories of type II^{E}.

Considering the black brane geometry of a negative $\mathfrak{D}p$-brane, one finds a naked singularity forming a finite size bubble around the brane. Here the curvature becomes singular and the harmonic function H describing the geometry crosses zero. The metric of the initial background $\mathrm{d}s^2 = H^{-\frac{1}{2}}\mathrm{d}s^2_{p+1} + H^{\frac{1}{2}}\mathrm{d}s^2_{9-p}$ becomes complex as H becomes negative inside the bubble. Considering the bubble as an interface, one can analytically continue the background as performed in [141] to

$$\begin{aligned} \mathrm{d}s^2 &= \omega^{-1}\bar{H}^{-\frac{1}{2}}\mathrm{d}s^2_{p+1} + \omega\bar{H}^{\frac{1}{2}}\mathrm{d}s^2_{9-p}\,, \\ e^{-2\phi} &= \omega^{p-3}g_s^{-2}\bar{H}^{\frac{p-3}{2}}\,, \\ F_{p+2} &= -g_s^{-1}\mathrm{d}\bar{H}^{-1}\wedge\Omega_{p+1}\,, \end{aligned} \tag{5.3.1}$$

with $\bar{H} \equiv -H$ positive inside the bubble and the complex factor $\omega = \pm i$, with the sign depending on which direction $H = 0$ is run around in the complex plane. Ω_{p+1} is the volume form on the negative brane worldsheet Σ_{p+1}, and g_s the asymptotic value of the string coupling.

The complex metric can be made real by a Weyl transformation, getting rid of imaginary factors up to an overall sign

$$\mathrm{d}s^2 = -\bar{H}^{-\frac{1}{2}}\mathrm{d}s^2_{p+1} + \bar{H}^{\frac{1}{2}}\mathrm{d}s^2_{9-p}\,. \tag{5.3.2}$$

The dilaton profile can be made real as well by an appropriate field redefinition. Comparing with the initial background outside the interface, the directions parallel to the brane pick up a minus sign, dynamically shifting the space-time signature from $(9,1)$ to $(10-p,p)$. The negative brane giving rise to the interface has signature $(p,1)$, but inside the bubble it looks like a $(1,p)$-brane in an exotic theory. Moving mutually BPS D-branes through the interface one can probe the theory inside the bubble. In the exotic theory notation, outside the bubble we started with the regular type II^{++} theories. The theory inside the bubble depends only on p:

$$p \text{ even}: \ \text{IIA}^{-(-)^{\frac{p}{2}}}_{(10-p,p)}\,, \qquad p \text{ odd}: \ \text{IIB}^{-(-)^{\frac{p-1}{2}}}_{(10-p,p)}\,. \tag{5.3.3}$$

Choosing the different sign for the Weyl transformation leads to a $(p, 10-p)$ space-time signature, where directions transverse to the brane pick up a sign flip. This indicates a symmetry between space-times with mirror space-time (and brane) signatures,

$$\text{IIA}^{\alpha\beta}_{(10-\text{p},\text{p})} \leftrightarrow \text{IIA}^{\alpha(-\beta)}_{(\text{p},10-\text{p})}\,, \qquad \text{IIB}^{\alpha\beta}_{(10-\text{p},\text{p})} \leftrightarrow \text{IIB}^{\alpha\beta}_{(\text{p},10-\text{p})}\,. \tag{5.3.4}$$

With (5.3.1) one can now define a map between the regular and exotic theories. Since the metric determinant comes in a square root $\sqrt{|\det G|}$, we begin by defining the transformation of the vielbein determinant. This allows us to avoid branch cut issues from the start. The vielbein determinant $\det e^a_\mu = \sqrt{|\det G_{(9,1)}|}$ transforms with one factor of $\omega^{-1/2}$ ($\omega^{+1/2}$) for each direction parallel (transverse) to the negative brane. The vielbein itself transforms accordingly, and consistently with (5.3.1). Finally, the dilaton profile is simply redefined to get back to the standard form.

$$\begin{aligned}
\det e^a_\mu &\to \omega^{-p} \det e^a_\mu\,, \\
e^a_\mu = \left(e^a_\parallel, e^a_\perp\right) &\to \omega^{\frac{1}{2}} \left(\omega^{-1} e^a_\parallel,\, e^a_\perp\right)\,, \\
G_{(9,1)} &\to \omega\, G_{(10-p,p)}\,, \\
e^{-\phi} &\to \omega^{\frac{p-3}{2}} e^{-\phi}\,, \\
(C_{p+1})|_{\Sigma_{p+1}} &\to -(C_{p+1})|_{\Sigma_{p+1}}\,.
\end{aligned} \tag{5.3.5}$$

It is important to note that the vielbein does not map to the vielbein of the new theory. To make this clear, let us write the new metric of signature $(10-p,p)$ explicitly as

$$G_{(10-p,p)} = \left(\omega^{-1} e^a_\parallel,\, e^a_\perp\right) \eta_{(9,1)} \left(\omega^{-1} e^a_\parallel,\, e^a_\perp\right)^\mathsf{T} = \left(e^a_\parallel,\, e^a_\perp\right) \eta_{(10-p,p)} \left(e^a_\parallel,\, e^a_\perp\right)^\mathsf{T}\,. \tag{5.3.6}$$

Since the original metric is given in terms of the original vielbeins as

$$G_{(9,1)} = \left(e^a_\parallel,\, e^a_\perp\right) \eta_{(9,1)} \left(e^a_\parallel,\, e^a_\perp\right)^\mathsf{T}\,, \tag{5.3.7}$$

the actual vielbeins of the two theories are exactly the same. This also makes the second and third line of (5.3.5) consistent. Also the measures are then both equivalently given by the original vielbein determinant, such that the measure factor simply maps in the same way as the vielbein determinant

$$\begin{aligned}
\sqrt{|G_{(9,1)}|} &= \det e^a_\mu = \sqrt{|G_{(10-p,p)}|}\,, \\
\sqrt{|G_{(9,1)}|} &\to \omega^{-p}\sqrt{|G_{(10-p,p)}|}\,,
\end{aligned} \tag{5.3.8}$$

indeed avoiding branch cuts. The maps (5.3.5) can be shown to map the closed string sectors of the SUGRA actions according to

$$\mathrm{IIB} \to \mathrm{IIB}^{-(-)^{\frac{p-1}{2}}}_{(10-p,p)}\,, \qquad \mathrm{IIA} \to \mathrm{IIA}^{-(-)^{\frac{p}{2}}}_{(10-p,p)}\,. \tag{5.3.9}$$

5.3.2 D-brane spectrum from the exotic string map

We shall now apply this map to the DBI action of the usual type II^{++} theories, in order to deduce the effective action of the type II^{E} D-branes. Note that the negative tension $\mathfrak{D}p$-brane should be considered just as a nice tool to identify the correct map from regular to exotic theories as in (5.3.9). In the following, we will call this brane the defining $\mathfrak{D}p$-brane.

The usual DBI+CS action for a Dq-brane (in IIA/IB^{++} string theories) can be expanded as:

$$\begin{aligned} S_{\text{DBI+CS}} =& - T_q \int_{\Sigma_{q+1}} \mathrm{d}^{q+1}x \, \sqrt{|g|}\, e^{-\phi} \left[1 + \frac{1}{4}(2\pi\alpha')^2 F_{\mu\nu}F^{\mu\nu} + ...\right] \\ &+ \mu_q \int_{\Sigma_{q+1}} [C_{q+1} + F \wedge C_{q-1} + ...] \end{aligned} \tag{5.3.10}$$

where F denotes the gauge field strength on the brane and C_q are the bulk R-R p-forms. For a BPS Dq-brane the tension is the same as the R-R charge, $T_q = \mu_q > 0$.

The above action is ghost-free, since the gauge kinetic term has the expected overall minus sign. When one performs the mapping to the exotic string theories, there are two places in the above action where factors of i (or signs) will arise. The first is the relative sign between the two terms in the DBI part: since $|F|^2$ contains two inverse metric factors, it is clear that under (5.3.5) it will pick up a minus factor. This relative sign change happens always, regardless of the number of dimensions that change signature or the dimension of the Dq-brane. The second place is the overall sign of the DBI part due to the rescaling of the dilaton as well as the rescaling of the measure. The factor coming from the dilaton depends on the total number p of space-time dimensions that change sign (the dimension of the defining $\mathfrak{D}p$-brane), while the rescaling of the measure now depends on the position (number of parallel and transverse dimensions $n_\parallel$, $n_\perp$) of the Dq-brane in the signature-changing space-time directions. Since the topological CS term does not contain dilaton or metric factors, the only change there can come from the transformation of the R-R form C_{q+1} which will not be fully determined here.

It is worth noting that even though we are dealing with factors of i, all of them nicely cancel out for BPS configurations, giving at most an overall sign change. Here BPS means that the Dq-brane is supersymmetric relative to the defining $\mathfrak{D}p$-brane. The requirement for a Dq-brane to be BPS can be translated to the condition

$$n_\perp + (p+1) - n_\parallel = 0 \mod 4\,. \tag{5.3.11}$$

As a consequence, the DBI action for a BPS Dq-brane in the exotic theory can only take one of the two forms

$$S_{\text{DBI}} = \begin{cases} -T_q \int \mathrm{d}^{q+1}x \, \sqrt{|g|}\, e^{-\phi}\Big[1 - \frac{1}{4}(2\pi\alpha')^2 F_{\mu\nu}F^{\mu\nu} + ...\Big] \pm \mu_q \int [C_{q+1} + ...] \\ +T_q \int \mathrm{d}^{q+1}x \, \sqrt{|g|}\, e^{-\phi}\Big[1 - \frac{1}{4}(2\pi\alpha')^2 F_{\mu\nu}F^{\mu\nu} + ...\Big] \pm \mu_q \int [C_{q+1} + ...]\,. \end{cases} \tag{5.3.12}$$

Notice that the relative sign in front of the kinetic term of the gauge field changed in both cases. This reflects the generic sign change reported in [20] for all D-branes in Euclidean exotic string theories. In addition, our methods allow us to determine the sign of the overall normalization, i.e. the tension.

In the upper case the overall sign is the usual minus. The action is of the same form as the usual DBI+CS action, with the significant difference that the sign in front of the gauge kinetic term is now altered. Hence, the gauge field comes with a kinetic term of the wrong overall sign so that this brane sector is not ghost-free.

The physical interpretation of the second action is different. Here the gauge kinetic term comes with the usual negative sign, so the gauge sector is ghost-free. However, the first term in the bracket now carries a relative negative sign with respect to the usual case. This term corresponds to the physical tension of the brane. Therefore, ghost-free exotic Dq-branes have negative tension.

We will now move forward and present a comprehensive classification of the BPS branes that appear in the Euclidean exotic string theories. We start with the regular type IIB theory and consider a (Lorentzian) defining $\mathfrak{D}p$-brane, with p odd. Then the map (5.3.5) gives the exotic Euclidean IIB theory

$$\mathrm{IIB}^{++}_{(9,1)} \longrightarrow \mathrm{IIB}^{-(-)^{\frac{\mathrm{p}-1}{2}}}_{(10-\mathrm{p},\mathrm{p})} \tag{5.3.13}$$

and the corresponding mirror theories (5.3.4). Next, we introduce all possible Lorentzian Dq-branes which are mutually BPS (5.3.11) in the original type IIB theory and map them via the exotic string map (5.3.5) to the corresponding Dq-brane in the exotic IIB$^{-,\beta}$ theory. Hence, $(p+1)$ is the number of space-time directions x_i which will change signature, while the signature of the other $(9-p)$ directions y_j stays the same.

Let us mention again that we denote by $n_\parallel$ the number of dimensions along the signature changing x_i's, and $n_\perp$ the number of dimensions along the y_i's, with $n_\parallel + n_\perp = q+1$. A D$q$-brane in the exotic string theory is denoted as D$q^{(s,t)}_{(10-p,p)}$, where the pair (s,t) adds up to $q+1$ and indicates the signature of the brane.

Then applying the map to the metric g on Dq, the measure picks up a factor of $\omega^{-1/2}$ for each signature changing direction, and a factor of $\omega^{1/2}$ for the others. The metric and the dilaton transform exactly as in (5.3.5). It is then straightforward to determine how the DBI action for the Dq-brane transforms

$$\begin{aligned} S_{\mathrm{DBI}} \to &- T_q \int \mathrm{d}^{q+1}x\, \sqrt{|g|}\, \omega^{\frac{n_\perp - n_\parallel}{2}} \omega^{\frac{p-3}{2}}\, e^{-\phi} \left[1 - \frac{1}{4}(2\pi\alpha')^2 F_{\mu\nu}F^{\mu\nu} + ...\right] \\ &= -\omega^{\frac{p+n_\perp - n_\parallel - 3}{2}} T_q \int \mathrm{d}^{q+1}x\, \sqrt{|g|}\, e^{-\phi} \left[1 - \frac{1}{4}(2\pi\alpha')^2 F_{\mu\nu}F^{\mu\nu} + ...\right] , \end{aligned} \tag{5.3.14}$$

which allows to read-off the tension of the brane in the exotic string theory. We note that depending on the position of the Dq-brane, we might get branes of the same dimension which nevertheless have different tensions. As long as all branes of the same dimension are either Lorentzian or Euclidean, it is still consistent with our general framework. This is of course satisfied automatically.

The combined set of negative branes in type II^{++} and their mutually BPS branes classifies all possible D-branes in type II$^{\mathrm{E}}$ theories. These spectra are consistent with the previous classification reviewed in tables 5.1 and 5.2, only extending them by the missing D8 and D9 branes. Additionally the exotic string map allows us to give specify the tensions of the branes.

Let us present an illustrative example. We pick a defining $\mathfrak{D}3$-brane and present all the consistent branes in the corresponding exotic theories, namely IIB$^{--}_{(7,3)}$ and its "mirror" IIB$^{--}_{(3,7)}$. Either the first four directions parallel to the branes will get their signature reversed, leading to a $(7,3)$ space-time, or the six perpendicular directions, leading to a $(3,7)$ space-time. Then we successively consider D1-, D3-,…, D9-branes and put them in relatively BPS positions to the defining $\mathfrak{D}$3-brane. The BPS condition for $p = 3$ reduces to $n_\perp - n_\parallel = 0 \mod 4$. The result is summarized in table 5.3.

Starting with the D1-brane, there is only one mutually BPS position with $n_\perp = n_\parallel = 1$. With only the time-like direction parallel to the negative brane, the type IIB^{++} D1-brane maps to a Euclidean D1-brane with signature $(2,0)$ in type IIB$^{--}_{7,3}$. Recall that we mark D-branes with superscripts for their signature, and subscripts for the theory they reside in. The Euclidean D1-brane just discussed would be a D1$^{(2,0)}_{(7,3)}$. The tension of this D-brane is positive, implying that the gauge field on the brane is a ghost field.

Similarly analyzing the higher dimensional Dq-branes fills out the rest of table 5.3. In the "brane positioning" column of the table, we denote by the superscript whether a direction is space-like (s) or time-like (t) in the $(7,3)$ theory. The subscript denotes the same for the "mirror" $(3,7)$ theory. As expected, the signatures (Lorentzian/Euclidean) of the branes alternate. There are 7 different BPS configurations allowed. Out of these, 3 have negative tension and are therefore ghost-free. We should note here that the table only includes the overall sign of the brane tension, as the precise value is irrelevant for the present discussion. Let us also stress that while there exist negative tension (ghost-free) D3-, D5- and D7- branes, not all are of this type. The arrangement of the branes in space-time plays a crucial role here.

Dq	$n_\perp$	$n_\parallel$	k	Tension	Brane positioning										IIB$^{--}_{(7,3)}$	IIB$^{--}_{(3,7)}$	Brane
					0^s_t	1^t_s	2^t_s	3^t_s	4^s_t	5^s_t	6^s_t	7^s_t	8^s_t	9^s_t	Brane	Brane	Type(E/L)
D1	1	1	1	+	✓	-	-	-	✓	-	-	-	-	-	D1$^{(2,0)}_{(7,3)}$	D1$^{(0,2)}_{(3,7)}$	E
D3	0	4	0	−	✓	✓	✓	✓	-	-	-	-	-	-	D3$^{(1,3)}_{(7,3)}$	D3$^{(3,1)}_{(3,7)}$	L
D3	2	2	1	+	✓	✓	-	-	✓	✓	-	-	-	-	D3$^{(3,1)}_{(7,3)}$	D3$^{(1,3)}_{(3,7)}$	L
D5	3	3	1	+	✓	✓	✓	-	✓	✓	✓	-	-	-	D5$^{(4,2)}_{(7,3)}$	D5$^{(2,4)}_{(3,7)}$	E
D5	5	1	2	−	✓	-	-	-	✓	✓	✓	✓	✓	-	D5$^{(6,0)}_{(7,3)}$	D5$^{(0,6)}_{(3,7)}$	E
D7	4	4	1	+	✓	✓	✓	✓	✓	✓	✓	✓	-	-	D7$^{(5,3)}_{(7,3)}$	D7$^{(3,5)}_{(3,7)}$	L
D7	2	6	2	−	✓	✓	-	-	✓	✓	✓	✓	✓	✓	D7$^{(7,1)}_{(7,3)}$	D7$^{(1,7)}_{(3,7)}$	L
D9				*No consistent D9-brane configuration*											-	-	-

Table 5.3: Brane spectrum of IIB$^{--}_{(7,3)/(3,7)}$ theories from a defining $\mathfrak{D}$3-brane.

The type IIB$^{--}_{(3,7)/(7,3)}$ brane spectrum of table 5.3 is still incomplete. Because of the space-time mirror symmetry, a defining $\mathfrak{D}$7-brane also gives branes in the same theories. For example, there is an additional D1$^{(0,2)}_{(7,3)}$-brane with negative tension arising from that sector.

We can now perform the classification also for $p = 1, 5, 7, 9$. In appendix A.1 we present the brane spectra of exotic type IIB$^{\mathrm{E}}$ theories. In a similar fashion, we computed the D-brane spectra of the exotic Euclidean type IIA theories in the various consistent signatures. We present the results in appendix A.2. Note that there is one major difference to the type IIB case. Recall that the Euclidean type IIA space-time mirrors are $\mathrm{IIA}^{--}_{(10-p,p)} \leftrightarrow \mathrm{IIA}^{-+}_{(p,10-p)}$. While for type IIB the space-time mirror theories are of the same type, in type IIA the space-time mirror also affects the type of the theory, in particular whether the branes are Euclidean or Lorentzian.

Before searching the D-brane bouquet for phenomenologically viable solutions, we return to CFT and validate our probe-brane results through boundary solutions.

5.3.3 D-brane spectrum from exotic boundary CFT

Let us construct boundary states for the Euclidean string. Our analysis follows that of [37, 146–148] for Lorentzian strings. For the moment we assume also a purely Euclidean space-time and include the effects of the target-space metric signature later. The boundary conditions are unaffected by the signature of the world-sheet. Despite the now Euclidean signature we will think of the coordinate $\sigma_1 \in (0, l)$ as the time coordinate and $\sigma_2 \in (0, \pi)$ as the space component. The conformal map exchanging the open and closed channels as well as the Neumann and Dirichlet gluing conditions are exactly the same as in the Lorentzian case. Defining a matrix $S_{\mu\nu} = \pm\eta_{\mu\nu}$, with the + sign for Neumann directions and the − sign for Dirichlet directions, the non-zero mode conditions are given by

$$\left(\alpha^\mu_n + S^\mu{}_\nu\, \overline{\alpha}^\nu_{-n}\right) |B\rangle = 0\,. \tag{5.3.15}$$

As usual the solution to these gluing conditions is

$$|B\rangle = \frac{1}{\mathcal{N}} \exp\left(-\sum_{n=1}^{\infty} \frac{1}{n} \alpha^\mu_{-n}\, S_{\mu\nu}\, \overline{\alpha}^\nu_{-n}\right) |0\rangle\,. \tag{5.3.16}$$

The normalization can be determined through the tree-channel loop-channel equivalence (5.1.20) and turns out to be

$$\mathcal{N}_N = (\alpha')^{1/4} e^{i\pi/8}\,, \qquad \mathcal{N}_D = (\alpha')^{-1/4}\, e^{-i\pi/8} \tag{5.3.17}$$

for Neumann and Dirichlet type boundaries respectively.

We can then read off the tension of a D-brane by combining the single direction contributions into a total cylinder amplitude of two parallel $d = p + 1$ dimensional branes in D space-time dimensions. The combined normalization for this set-up is

$$\mathcal{N}^{-1} = 2^{\frac{D-2}{4}}\, e^{\frac{i\pi}{8}(D-2d)} (4\pi^2\alpha')^{\frac{1}{4}(D-2d-2)}\,. \tag{5.3.18}$$

Then the tension of the branes is determined by the coupling of the boundary state to a graviton with polarization $\epsilon_{\mu\nu}$

$$\langle V_g | B\rangle = -\frac{1}{\mathcal{N}} \langle 0|\, \epsilon_{\mu\nu} S^{\mu\nu}\, |0\rangle = -\frac{1}{\mathcal{N}}\, \epsilon_{\mu\nu} S^{\mu\nu}\, V_{d+1} \stackrel{!}{=} -T_d\, \epsilon_{\mu\nu} S^{\mu\nu}\, V_{d+1}\,, \tag{5.3.19}$$

so that the tension is given by the normalization of the boundary state as $T_d = \mathcal{N}^{-1}$. We require the tension to be real, so that the normalization of the boundary state also has to be real. Inserting $D = 10$ into (5.3.18), we see that there are exactly three cases $d \in \{1, 5, 9\}$ fulfilling this condition. These are d-dimensional D-branes with tension

$$T_d = \pm 2^2 (4\pi^2 \alpha')^{(4-d)/2} , \tag{5.3.20}$$

with the minus sign for $d \in \{1, 9\}$ and the plus sign for $d = 5$. The analogue discussion for fermionic boundary states gives the same consistent result for the normalization.

By absorbing the changes induced by the signature change into the mode expansion, the results of most calculations are essentially the same as for the Lorentzian string. The only difference resides in the zero mode contribution. As we will be concerned with branes wrapping various amounts of time dimension, let us consider a $\mathrm{D}^{(s,t)}_{(10-p,p)}$-brane that fills t time and s space dimensions in a $\mathbb{R}^{10-p,p}$ target-space with p time and $10 - p$ space dimensions. Thus the system we are concerned with consists of

- $N_t = t$ time dimensions with Neumann boundary conditions,
- $D_t = p - t$ time dimensions with Dirichlet boundary conditions,
- $N_s = s$ space dimensions with Neumann boundary conditions,
- $D_s = 10 - p - s$ space dimensions with Dirichlet boundary conditions.

In the analysis so far all directions were assumed to be space-like. Let us now analyze what changes in case some of the directions become time-like. First, recall that the oscillator part of the boundary state (5.3.16) involves the matrix $S_{\mu\nu}$. For a $\mathrm{D}^{(s,t)}_{(10-p,p)}$-brane this takes the form

$$S = \begin{pmatrix} \mathbb{1}_{N_s} & & & \\ & -\mathbb{1}_{N_t} & & \\ & & -\mathbb{1}_{D_s} & \\ & & & \mathbb{1}_{D_t} \end{pmatrix} . \tag{5.3.21}$$

Thus, we see that the oscillators of a space-like N/D direction contribute to the boundary state like a time-like D/N direction. However, these signs in $S_{\mu\nu}$ cancel when computing the overlap.

Now let us consider the zero mode contribution, where some phase factors appeared from the zero mode integrals. Changing the signature replaces p^2 by $-p^2$ in the Gaussian integral, so that for a Neumann boundary condition this phase is

$$\begin{aligned} \mathcal{N}^{-2}_{N,\text{space}} &\propto \int_0^\infty \mathrm{d}p \; e^{-\pi i p^2} = e^{-i\pi/4} , \\ \mathcal{N}^{-2}_{N,\text{time}} &\propto \int_0^\infty \mathrm{d}p \; e^{\pi i p^2} = e^{i\pi/4} . \end{aligned} \tag{5.3.22}$$

Similarly, for a Dirichlet direction the exact same integrals appear in the overlap of the zero modes of the boundary states, only for the inverse normalizations

$$\begin{aligned} \mathcal{N}_{D,\mathrm{space}}^2 &\propto \int_0^\infty \mathrm{d}p\, e^{-\pi i p^2} = e^{-i\pi/4}\,, \\ \mathcal{N}_{D,\mathrm{time}}^2 &\propto \int_0^\infty \mathrm{d}p\, e^{\pi i p^2} \;\;= e^{i\pi/4}\,. \end{aligned} \tag{5.3.23}$$

This implies that in changing the signature, the only effect on the normalization of the boundary state is a change of the phase factor such that

$$\begin{aligned} \arg\left(\mathcal{N}_{N,\mathrm{space}}\right) &= \arg\left(\mathcal{N}_{D,\mathrm{time}}\right), \\ \arg\left(\mathcal{N}_{N,\mathrm{time}}\right) &= \arg\left(\mathcal{N}_{D,\mathrm{space}}\right). \end{aligned} \tag{5.3.24}$$

Effectively this means that the formula for the normalization (5.3.18) holds in all signatures, one just has to adjust the phase factor as

$$T_{(10-p,p)}^{(s,t)} = 2^2 e^{\frac{i\pi}{4}(5+t-p-s)} (4\pi^2\alpha')^{\frac{1}{2}(4-s-t)}\,. \tag{5.3.25}$$

Note that we have simply replaced $d \to \tilde{d} = d + D_t - N_t = p + s - t$ in the phase factor to account for the additional phases. This formula is now valid for all branes in Euclidean world-sheet theories.

Taking now into account that the tension is real only for $\tilde{d} \in \{1, 5, 9\}$, it is straightforward to iterate all possible (real) branes for a given space-time signature. The result completely agrees with the categorization by the exotic string map.

As a final remark we note that in our derivation the constraints for the allowed D-branes followed directly from the normalization factor. We have not discussed the GSO projections in the fermionic sector, but as usual the constraint on even/odd dimension of the branes follows directly from the Clifford algebra of the fermionic zero modes. This computation does not change in the Euclidean case so that the D-branes obtained from the bosonic normalization are also GSO invariant.

5.3.4 Orientifolds of Euclidean strings

In this short section, we continue the CFT discussion to take a closer look at orientifold projections of the Euclidean exotic superstring theories. Once again the calculation strongly resembles the usual one, presented for example in [37, 149]. Here we only show that in the computation of the loop-channel Klein-bottle and Möbius strip amplitudes, the same phase factors appear as for the corresponding annulus amplitude.

Thus, let us consider a single bosonic direction $X(\sigma_1, \sigma_2)$. The orientifold projection $\Omega : (\sigma_1, \sigma_2) \to (\sigma_1, -\sigma_2)$ and its combination with the reflection $I_1 : X \to -X$ acts on the modes as

$$\Omega\, \alpha_n\, \Omega^{-1} = \overline{\alpha}_n\,, \qquad (\Omega I_1)\, \alpha_n\, (\Omega I_1)^{-1} = -\overline{\alpha}_n\,. \tag{5.3.26}$$

Moreover, we choose the action of Ω on the vacuum as $\Omega\,|0\rangle = |0\rangle$. Recall that the Klein bottle amplitude is defined as

$$Z^{\mathcal{K}}_{\Omega} = \mathrm{Tr}\left(\Omega\, q^{L_0 - c/24}\, \overline{q}^{\overline{L}_0 - c/24}\right) = \mathrm{Tr}_{\mathrm{sym}}\left(e^{-4\pi t(L_0 - c/24)}\right) . \tag{5.3.27}$$

The non-zero mode contribution again agrees with the usual result, while the zero modes contribute a phase due to the additional factor of i in the Gaussian integral. Thus for a single boson we get

$$Z^{\mathcal{K}}_{\Omega} = \frac{e^{-i\pi/4}}{\sqrt{\alpha' t}} \frac{1}{\eta(2it)} . \tag{5.3.28}$$

The Klein Bottle amplitude for the orientifold projection ΩI_1 does not receive any zero mode contribution so that one obtains

$$Z^{\mathcal{K}}_{\Omega I_1} = \mathrm{Tr}\left(\Omega I_1\, q^{L_0 - c/24}\, \overline{q}^{\overline{L}_0 - c/24}\right) = e^{\frac{i\pi}{24}} \sqrt{2} \sqrt{\frac{\eta(2it)}{\theta_2(2it)}} . \tag{5.3.29}$$

Turning to the open string sector, the action of the orientifold on the modes is

$$\Omega\, \alpha^{\mu}_{n}\, \Omega^{-1} = \pm(-1)^n \alpha^{\mu}_{n} , \tag{5.3.30}$$

with $(+)$ for NN and $(-)$ for DD boundary conditions. Again the non-zero modes agree with the usual expressions. Because the DD sector receives no zero mode contributions in the open string channel, the Möbius strip amplitude is as usual

$$Z^{\mathcal{M}(DD)} = e^{\frac{i\pi}{24}} \sqrt{2} \sqrt{\frac{\eta(it + \frac{1}{2})}{\theta_2(it + \frac{1}{2})}} . \tag{5.3.31}$$

The NN amplitude receives an additional phase from the Gaussian integral so that

$$Z^{\mathcal{M}(NN)} = e^{\frac{i\pi}{24}} \frac{e^{-i\pi/4}}{\sqrt{2\alpha' t}} \frac{1}{\eta(it + \frac{1}{2})} . \tag{5.3.32}$$

Therefore, both the former annulus amplitudes and these additional non-oriented one-loop amplitudes differ from the usual ones for Lorentzian signature by the same relative phases. The next step is to introduce the corresponding crosscap states satisfying the usual crosscap gluing conditions and allowing the description of the amplitudes in tree-channel. Moreover, one can add the contributions from the world-sheet fermions. The only difference to the standard case is again the appearance of the same phases as for the D-brane boundary states.

Performing now a full orientifold projection ΩI_{9-p} of the Euclidean type IIA/B superstring theories, the tadpole cancellation conditions go through as usual, and the Op-planes will have tension

$$T_{Op} = -2^{p-4} T_{Dp} . \tag{5.3.33}$$

As already shown in figure 5.3 there will be extra factors of $(-1)^{F_L}$ in certain cases, however this does not affect the tension. Introducing time-like directions has the same effect on the phase of the tension as for the corresponding boundary states. To cancel the tadpole induced by the orientifold projection one can introduce stacks of Dp-branes on top of the orientifold planes, or vice versa.

5.4 Ghost free brane worlds

We will now finally discuss the phenomenological viability of exotic brane worlds. Recall that we are looking for a ghost-free gauge sector in a theory with closed string ghosts, such that dS solutions are possible.

5.4.1 Brane worlds in type II$^{\mathrm{E}}$

We shall begin by scanning the Euclidean exotic D-brane bouquet for ghost-free D-branes with a $(3,1)$ subspace. These ghost-free branes are

$$\begin{aligned} &\text{type IIB}^{\mathrm{E}}: \ \mathrm{D}9^{(9,1)}_{(9,1)}, \quad \mathrm{D}7^{(7,1)}_{(7,3)}, \quad \mathrm{D}5^{(5,1)}_{(5,5)}, \quad \mathrm{D}3^{(3,1)}_{(3,7)}, \\ &\text{type IIA}^{\mathrm{E}}: \ \mathrm{D}8^{(8,1)}_{(8,2)}, \quad \mathrm{D}6^{(6,1)}_{(6,4)}, \quad \mathrm{D}4^{(4,1)}_{(4,6)}. \end{aligned} \tag{5.4.1}$$

In the following, we discuss this class of branes in more detail, as they share a couple of common features.

First, all these branes have in common that they are space-filling, but localized in the extra time-like directions. For instance, as can be seen from table 5.3, the $\mathrm{D}7^{(7,1)}_{(7,3)}$ brane is localized in the t_2 and t_3 directions and longitudinal along $s_0, t_1, s_4, \ldots, s_9$. Compactifying the extra time-like directions and all space-like directions except the three large ones that are to make our world, an open string ending on the brane will have KK modes along the compact space-like directions and winding modes in the compact time-like directions. As a consequence, employing (5.2.9) the mass spectrum of such an open string reads

$$E^2 = \sum_i (p^i)^2 + \sum_s \left(\frac{m_s}{R_s}\right)^2 + \sum_t \left(\frac{n_t R_t}{\alpha'}\right)^2 - \frac{i}{\alpha'}\,(N-a)\,, \tag{5.4.2}$$

where the indices $s(t)$ indicate space(time)-like directions. Therefore, for these particular branes both KK and winding modes contribute positively to the right hand side of (5.4.2). This is the same behavior as for D-branes in the usual type IIA/IIB theories. This implies that in contrast to closed strings, such D-branes do not have the problem of an infinite number of open string modes becoming arbitrarily light.

Being localized in the extra time-like directions, the transversal deformations of the D-branes in (5.4.1) will be ghosts. On a torus such deformations will exist but on a more general background they can be absent, if the brane wraps a rigid cycle. There will certainly exist massive open string ghosts, but they are expected to kinematically decouple from the massless open string states below a cutoff Λ_{UV}. Whether also the ultra-light closed string states decouple is a more intricate question. Since they couple gravitationally, they are expected to decouple in the large Planck-mass limit. However, there are in principle infinitely many such states, so it is not a trivial question whether they will have a negligible overall effect on the low-energy scattering of massless open string modes.

The second common feature of the ghost-free D-branes in (5.4.1) is that they are all directly related to the orientifold projections discussed in section 5.2.2 and summarized in fig. 5.3. If transversal directions of a D-brane are compactified, its R-R charge has to be cancelled, a feature known as tadpole cancellation. Therefore, one is forced to introduce also oppositely charged objects in the backgrounds. These are the orientifold planes constructed in section 5.3.4. They arise by performing an orientifold projection $\Omega I_\perp$, where $I_\perp$ reflects the coordinates transversal to the brane. As an example consider $\mathrm{D}7^{(7,1)}_{(7,3)}$, whose related orientifold quotient is $\mathrm{IIB}^{--}_{(7,3)}/\Omega I_2(-1)^{F_L}$, where I_2 reflects the two extra time-like coordinates. However, this is precisely the orientifold projection that removes all the 10D massless ghosts in the closed string sector. The same behavior persists for all ghost-free branes listed in (5.4.1).

Summarizing, the required orientifold projection that allows us to introduce these branes in the given background in the first place, is also the orientifold that we encountered in figure 5.3 which projects out all the massless ghosts appearing in the exotic 10D supergravity actions. Keeping in mind that the de Sitter solutions of exotic string theories were only possible precisely because of the existence of massless ghosts in 10D closed string action, dS vacua will very likely not be possible. Therefore, in Euclidean exotic string theories with in general multiple times, there is a strong anti-correlation between the presence of a phenomenologically viable D-brane (gauge theory) sector and the existence of dS solutions.

5.4.2 Brane worlds in type II^{L}

Let us now consider the Lorentzian exotic string theories and analyze whether ghost-free D-branes can be introduced there. Here the $\mathbb{Z}_2$ projections from figure 5.2 that project out the massless 10D ghosts are not orientifold projections but just $\mathbb{Z}_2$ orbifolds.

Thus, first we investigate what kind of $\mathbb{Z}_2$ projections the various kinds of such theories actually admit. Then we can analyze whether there exist orientifolds that support D-branes with a ghost-free kinetic term for the gauge field while still potentially admitting de Sitter solutions in the closed string sector.

We will be interested in $\Omega I_{m,n}$ orientifolds for the type $\mathrm{IIA/B}^{(+,\beta)}_{(10-p,p)}$ theories, where $I_{m,n}$ denotes the reflection of m space-like and n time-like directions. As is known already for the usual type II^{++} theories there appears a subtlety in the Ramond sector of the theory.

For all theories of signature $(q,p) \in \{(9,1),(5,5),(1,9)\}$ the Clifford algebra $\{\Gamma^A,\Gamma^B\} = 2\,\eta^{AB}$ has similar properties, as $q-p = 0 \bmod 8$. Here, $\eta^{AB} = \pm 1$ for space/time-like directions. Let us recall some of the salient properties of the Γ-matrices. In the following Γ^a denote space-like directions and Γ^α time-like ones. All Γ-matrices are unitary, if they satisfy the hermiticity conditions $(\Gamma^a)^\dagger = \Gamma^a$, $(\Gamma^\alpha)^\dagger = -\Gamma^\alpha$. Moreover, one can define the chirality operator

$$\Gamma^{10} = \prod_A \Gamma^A \tag{5.4.3}$$

which anti-commutes with all Γ^A, is Hermitian $(\Gamma^{10})^\dagger = \Gamma^{10}$ and satisfies $(\Gamma^{10})^2 = 1$. One can choose the Γ-matrices to be purely imaginary in which case Γ^{10} is real. In this representation, a Majorana spinor is real.

The Ramond ground state in both the left- and the right-moving sector is a Majorana-Weyl spinor in 10D, thus it is chiral and real. Type IIB has two spinors of the same chirality and type IIA two spinors of opposite chirality. These spinors of positive and negative chirality are denoted as usual by S^+ and S^-.

The reflection along a single space-like direction x^a acts on the spinors as

$$I^a : S \to i\Gamma^{10}\Gamma^a S\,, \tag{5.4.4}$$

guaranteeing $\{I^a, \Gamma^a\} = 0$ and $[I^a, \Gamma^B] = 0$ (for $a \neq B$) . This operation is Hermitian, real and changes the chirality of the spinor. Moreover, it satisfies $(I^a)^2 = 1$. The reflection along a time-like direction can also be chosen to be Hermitian but then it becomes purely imaginary

$$I^\alpha : S \to \Gamma^{10}\Gamma^\alpha S\,. \tag{5.4.5}$$

Thus, I^α is Hermitian, imaginary, changes the chirality and satisfies $(I^\alpha)^2 = 1$. Let us now consider a general $\mathbb{Z}_2$ reflection

$$I_{m+n} = \prod_{i=1}^{m} I^{a_i} \prod_{j=1}^{n} I^{\alpha_j} \tag{5.4.6}$$

along m space-like and n time-like directions. Its action on a spinor is summarized in table 5.4.

$I_{m,n}$	action on spinor
m even, n even	$S^\pm \to S^\pm$
m odd, n odd	$S^\pm \to iS^\pm$
m odd, n even	$S^\pm \to S^\mp$
m even, n odd	$S^\pm \to iS^\mp$

Table 5.4: Action of I_{m+n} reflection on spinors.

Using that $\{I^A, I^B\} = 2\delta^{AB}$, one can show that for $m+n = 2k$ or $m+n = 2k+1$ the square of I_{m+n} on the Ramond ground state is $I^2_{m+n} = (-1)^k$. We are interested in the consistent orientifold projections of type ΩI_{m+n} for the type IIB/IIA$^{+,\beta}$ string theories. Requiring that the full orientifold projection squares to +1 one obtains the admissible possibilities listed in table 5.5 for the type IIB/IIA string. Here, as usual the factor $(-1)^{F_L}$ is introduced to compensate for $(\Omega I_{m+n})^2 = -1$.

Now we may analyze whether orientifolds of type II$^{+,\beta}$ can support D-branes of at least signature $(3,1)$ and without gauge field ghosts while still admitting dS-type solutions in the closed string sector. The type II$^{+,\beta}_{(1,9)}$ theories can be dismissed right

Type IIA/B$^{\alpha\beta}$	orientifold
Type IIB^{++}	$\Omega I_{2m,2n} \left[(-1)^{F_L}\right]^{m+n}$
Type IIB^{+-}	$\Omega I_{2m-1,2n-1} \left[(-1)^{F_L}\right]^{m+n-1}$
Type IIA^{++}	$\Omega I_{2m-1,2n} \left[(-1)^{F_L}\right]^{m+n-1}$
Type IIA^{+-}	$\Omega I_{2m,2n-1} \left[(-1)^{F_L}\right]^{m+n-1}$

Table 5.5: Admissible orientifold projections.

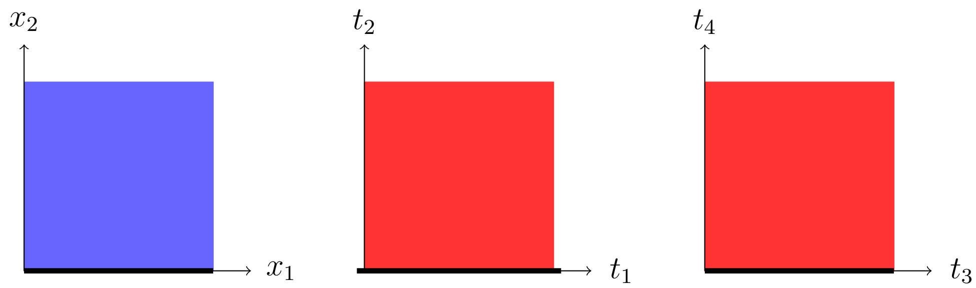

Figure 5.4: The six-torus of signature $(2, 4)$ with O6-plane along (x_1, t_1, t_3). The first torus is space-like, the other two are compact time-like directions.

away, as they do not have at least three space-like directions. Moreover, type $\mathrm{II}^{++}_{(9,1)}$ are just the usual type II theories for which the no-dS swampland conjecture is supposed to hold. The type $\mathrm{II}^{+-}_{(9,1)}$ theories only contain Euclidean D-branes that cannot support a gauge theory in $(3, 1)$ dimensions. Thus we are left with the type $\mathrm{II}^{+,\beta}_{(5,5)}$ theories.

Let us have a closer look at the type $\mathrm{IIA}^{++}_{(5,5)}$ theory. This theory still contains Lorentzian fundamental strings so that the CFT is like the usual type IIA theory, only the signature changes from $(9, 1)$ to $(5, 5)$. All D-branes have positive tension and the usual sign of the kinetic term for the gauge field. Of course, this implies that the time-like components of the gauge field A^μ are ghosts.

As we have seen, it is the $\mathbb{Z}_2$ projection I_4 or $I_4(-1)^{F_L}$ reflecting the four extra time-like directions that removes all closed string ghosts from the action. Clearly, this is not an orientifold so that it could well be that e.g. an orientifold $\Omega I_3(-1)^{F_L}$ with O6-planes and corresponding D6-branes gives a ghost-free gauge theory, while still allowing closed string ghosts.

For concreteness, let us consider a compactification on a six torus T^6. Let us denote the two compact space- and the four compact time-directions as $\{x_1, x_2; t_1, t_2, t_3, t_4\}$. Therefore, we can group the six-coordinates in three pairs $\{(x_1, x_2), (t_1, t_2), (t_3, t_4)\}$ and choose the orientifold projection to be of type $\Omega I_{1,2}(-1)^{F_L}$ reflecting the three coordinates $\{x_2, t_2, t_4\}$. This leads to an O6-planes parallel to the plane $\{x_1, t_1, t_3\}$. This is shown in figure 5.4. The induced tadpole can be cancelled by D6-branes on top of the O6-planes. Note that these D6-branes are Lorentzian in the sense that there is an odd number (namely three) of longitudinal time-like directions.

Moreover, the 4D gauge field on these D6-branes has the usual kinetic term and is ghost-free. However, in this toroidal example the Wilson-lines along the $\{t_1, t_3\}$ directions and the deformations of the brane in the $\{t_2, t_4\}$ directions will be ghosts in the effective 4D theory. However, for more general internal spaces (something like a CY of signature $(2, 4)$) these open string moduli could be avoided if the D6-branes wrap a rigid 3-cycle.

In order to see whether dS vacua are in principle possible, let us investigate which flux components survive the orientifold projection. For this purpose we recall the general result about the cohomological classification of the orientifold even fluxes shown in table 5.6.

Flux	Cohomology
H	$H^3_-(X)$
$\{F_0, F_2, F_4, F_6\}$	$\{H^0_+, H^2_-, H^4_+, H^6_-\}$

Table 5.6: Equivariant cohomology groups of orientifold even fluxes.

Let us look at F_2, for which the flux

$$F_2 = f\, dx_1 \wedge dt_2 \tag{5.4.7}$$

is in H^2_- and therefore survives the orientifold projection. Now, since F_2 is supported along one space-like and one time-like leg, the kinetic term of this two-form flux

$$|F_2|^2 \sim g^{i_1 i_2}\, g^{j_1 j_2}\, (F_2)_{i_1 j_1}\, (F_2)_{i_2 j_2} \tag{5.4.8}$$

has the opposite sign to the usual one. For the other fluxes one finds similar ghost-like components, as well. Therefore, this type IIA model features fluxes with the wrong sign of their kinetic terms. Then the usual dS no-go theorem does not apply and dS vacua might be possible. Since we will see below that these exotic orientifolds have other problems, it is beyond the scope of this paper to work out in detail a dS model on a fully fledged "CY" space. At least we can state that up to this point, the existing no-go theorems are not immediate obstructions for dS solutions with a ghost-free massless gauge field on the brane.

Finally let us recall that for the ghost-free D-branes in Euclidean exotic theories, the KK and winding modes were such that they contributed like a positive mass squared m^2 to the right hand side of the on-shell relation (5.4.2). This is different for the D-branes in Lorentzian exotic theories here. For such a D-brane the on-shell condition now reads

$$\begin{aligned} E^2 + \sum_{t_\parallel} \left(\frac{m_t}{R_t}\right)^2 + \sum_{t_\perp} \left(\frac{n_t R_t}{\alpha'}\right)^2 \\ = \sum_i (p^i)^2 + \sum_{s_\parallel} \left(\frac{m_s}{R_s}\right)^2 + \sum_{s_\perp} \left(\frac{n_s R_s}{\alpha'}\right)^2 + \frac{1}{\alpha'}\,(N-a)\,, \end{aligned} \tag{5.4.9}$$

where the indices $(s/t)_{\parallel}$ and $(s/t)_{\perp}$ indicate space-/time-like directions parallel and perpendicular to the D-brane world-volume. Therefore, here both time-like KK and time-like winding modes contribute always to the left hand side of this relation. As for the closed string, these time-like modes can cancel against oscillatory modes yielding infinitely many arbitrarily light open string modes. This questions the role of the Wilsonian effective gauge theory action on the D-brane and seems to be a general problem with a potential phenomenological application of orientifolds of Lorentzian exotic superstring theories (with multiple times).

5.5 Discussion

Driven by the possibility of finding dS solutions in supergravity theories with more than one time direction, we investigated the web of exotic string theories and their brane sectors. Despite being able to get rid of massless ghosts in the 10D theory by performing suitable orientifold projections, we found that compactifications of time-like directions lead to an unsatisfactory infinite tower of arbitrarily light states in the lower dimensional effective theories. One can ask if this is a generic behavior of time-like compactifications. In particular there exist pseudo-Riemannian analogs of Calabi-Yau manifolds with reduced holonomy (see the article of H. Baum in [150]) which would partially preserve supersymmetry upon compactifying exotic string theories on them. This could potentially provide the necessary obstruction for the towers of light states.

Although the closed string sector seems to show unavoidable pathologies, there could still be a phenomenologically consistent brane sector coupling to those unusual quantum gravity theories. After all, experiments today don't test quantum gravity, hence we adopted an agnostic point of view and merely asked for a well behaved massless open string sector. We took a perturbative point of view - after all, the dS solutions arise for the leading order supergravity actions. The exotic string map [141] between negative D-branes in usual type II theories and Euclidean string theories allowed us to deduce the D-brane spectra of exotic string theories. We found that in Euclidean theories, branes with negative tension are ghost-free. Expanding on prior work also by [141], we then constructed a CFT description of closed and open Euclidean exotic string theories. This confirmed the D-brane spectra deduced by probe branes and the exotic string map. In total, we compiled a complete list of all allowed D-branes of real tension in exotic string theories with all possible metric signatures. Finally we identified in every exotic theory a phenomenologically consistent D-brane without massless ghosts and supporting a $(3,1)$-signature subspace.

A nice feature of the Euclidean string theories is that the phenomenologically viable branes are localized in the extra time directions. Therefore the transversal deformations of the branes are ghosts, and can be absent if the brane wraps a rigid cycle. Also, the mass spectrum of a D-brane localized in the extra time-like directions avoids the ultralight modes problem. Hence D-branes in type II^{E} theories seem to be promising theories for a viable brane world scenario. However, the tadpole of these branes is cancelled precisely by those orientifold projections which also discard the massless closed string ghosts. In the end, the only viable D-branes also remove the loophole for dS in Euclidean exotic theories.

In the case of Lorentzian strings, the only exotic theories with viable brane worlds are the type IIA/$B^{++}_{5,5}$ theories. The Lorentzian closed string ghosts are projected out by orbifolds, so there is hope that viable D-branes with appropriate orientifold planes exist, while keeping closed string ghosts around. Indeed this is the case, as we demonstrated for a toroidal example. However here the D-branes wrap compact time dimensions, and the infinite tower of arbitrarily light states appears again. An effective description of low energy physics seems impossible with this tower present. Once again, we find no trustable brane-world sector which admits potential dS vacua.

Of course we cannot claim to have given a viable interpretation/description of quantum gravity with multiple time-like directions. While the formalism of conformal field theory and supergravity appears to go through for such theories, the conceptual interpretation still remains elusive and we have not much to add to that. We also left open a couple of admittedly important and interesting technical questions, the most pressing of which is the role of supersymmetry. Related to this is the question of general compactifications on manifolds with pseudo-Riemannian metrics that go beyond the toroidal case.

Chapter 6

Quantum Vacua and the AdS Conjectures

Twice now we have tried to introduce new variables into string compactifications, in order to open loopholes in the no-dS conjecture and twice have the loopholes proven to be unstable. Neither non-BPS branes nor even exotic signatures of both strings and branes have led us to doubt the swampland conjectures. If anything, the way how every time a loophole opens another obstruction falls into place, should strengthen our confidence in the no-dS conjecture.

However, there have been string theory constructions of dS vacua around for some time as well. These almost algorithmic constructions, most notably KKLT [70] and LVS [71], promise dS solutions of type IIB string theory without even introducing any unusual ingredients at all. Rather, flux compactifications are combined with non-perturbative or higher order effects in warped regions in the Calabi-Yau to balance the contributions to the scalar potential in such a way, that the result features a small positive minimum. These solutions seem to openly defy the no-dS swampland conjecture. Not only that, but the intermediate steps feature AdS solutions that also violate the AdS swampland conjectures.

Various steps of these dS constructions have been under heavy scrutiny recently [82–102]. However, it has proven to be difficult to isolate any particular shortcoming in the AdS minimum construction. Its full ten-dimensional description has been analyzed in a series of recent papers [88,89,93–95,106–108] converging to the conclusion that the 4D effective KKLT description captures the main aspects of this vacuum. The uplift to a dS vacuum is more subtle and new aspects of the validity of the effective field theory in the warped throat have been investigated in [90,97]. In summary, while the validity of the dS vacua is still an open question, the AdS vacua seem to be true counterexamples to the AdS swampland conjectures.

However what we said above is not completely true. While there are no truly exotic or unusual ingredients needed to construct these dS vacua, the balance of quantum and tree-level contributions to the scalar potential does set them apart from the otherwise studied, completely tree-level flux compactifications.

We shall investigate the quantum nature of these AdS vacua, and see that AdS swampland conjectures hold only up to additional log-terms. The origin of these terms can be seen to stem from the non-perturbative contributions, suggesting that they are quantum corrections to the swampland conjectures. Indeed, similar terms have appeared also in the TCC and can thus to be expected as corrections to the no-dS conjecture.

6.1 AdS swampland conjectures

As we will explicitly test the swampland conjectures reviewed in chapter 3, let us begin with a quick recap. The AdS/moduli scale separation conjecture (AM-SSC) [108] states that in an AdS minimum one cannot separate the size of the AdS space and the mass of its lightest mode. Quantitatively, the proposal is that the lightest modulus of non-vanishing mass has to satisfy

$$m_{\text{mod}}\, R_{\text{AdS}} \leq c \tag{6.1.1}$$

where c is an order one constant and $R^2_{\text{AdS}} \sim -\Lambda^{-1}$ the size of AdS. A strong version of this conjecture says that this relation is saturated, i.e. $m_{\text{mod}} \sim R^{-1}_{\text{AdS}}$.

A simple, enlightening example is once again the 5-form flux supported $\text{AdS}_5 \times S^5$ solution of the type IIB superstring. There, the sizes R_{AdS} and R_{S^5} of AdS_5 and S^5 are equal and both related to the 5-form flux. The lightest modulus mass scales as $m_{\text{mod}} \sim R^{-1}_{S^5}$ and saturates relation (6.1.1).

In the spirit of the swampland distance conjecture, one might also expect a tower of states to satisfy some mass bound. Indeed, the AdS distance conjecture (ADC) [119] states that for an AdS vacuum with negative cosmological constant Λ, the limit $\Lambda \to 0$ is at infinite distance in field space and that there will appear a tower of light states whose masses scale as

$$m_{\text{tower}} = c_{\text{AdS}}\, |\Lambda|^{\alpha} \tag{6.1.2}$$

for some constant c_{AdS} of order one and $\alpha > 0$. Moreover, for supersymmetric AdS vacua a stronger version of the AdS distance conjecture was claimed, namely that in this case $\alpha = 1/2$. In the following, we shall consider the KK tower only, leaving open the possibility of other towers appearing.

Let us again consider the prototype example of $\text{AdS}_5 \times S^5$. Having $\Lambda \sim -R^{-2}_{\text{AdS}}$, we are interested in R_{AdS} becoming large. Then the radius of S^5 also becomes large and the KK modes on S^5 scale as

$$m_{\text{KK}}(S^5) \sim \frac{1}{R} \sim |\Lambda|^{\frac{1}{2}}\,. \tag{6.1.3}$$

Therefore, these KK modes constitute the tower of states for the (strong) AdS distance conjecture with $\alpha = 1/2$.

Let us also state the no-dS conjecture and the TCC. The (refined) no-dS conjecture [15, 80, 81] states that

$$|\nabla V| \geq \frac{c}{M_{\rm pl}} \cdot V, \qquad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_{\rm pl}^2} \cdot V \tag{6.1.4}$$

where c is of order one, $\min(\nabla_i \nabla_j V)$ is the minimal eigenvalue of the Hessian matrix and c' is also of order one.

The trans-Planckian censorship conjecture (TCC) [130] presents a more "global" version of the no-dS conjecture. For a monotonically decreasing positive potential, the TCC states that the average of the no-dS conjecture $\left\langle \frac{-V'}{V} \right\rangle \Big|_{\phi_i}^{\phi_f}$ over the decreasing interval $[\phi_i, \phi_f]$ is bounded from below by

$$\left\langle \frac{-V'}{V} \right\rangle \Bigg|_{\phi_i}^{\phi_f} > \frac{1}{\Delta\phi} \log\left(\frac{V_i}{M}\right) + \frac{2}{\sqrt{(d-1)(d-2)}} \,. \tag{6.1.5}$$

Here $V < M < M_{\rm pl}$ and M is a mass scale that is lower than the Planck-scale. Note the usual constant on the right and the additional log-term, which is suppressed for large field ranges.

In the framework of the swampland distance conjecture it has been observed that the infinite distance can be understood as emerging from integrating out the appearing tower of light states [131–134]. In quantitative terms, the emergence proposal claims that the 1-loop contribution to the moduli field metric, arising from integrating out a tower of states that are lighter than the natural cutoff of the effective theory, is proportional to the tree-level metric.

For a D-dimensional theory with a tower of massive states $m_n = n\Delta m(\phi)$ with degeneracy n^K, whose mass gap depends on a modulus ϕ, the one loop-correction to the field space metric for the modulus can be written as

$$G_{\phi\phi}^{\rm loop} \sim \frac{\Lambda_{\rm sp}^{D+K-1}}{M_{\rm pl}^{D-2}} \frac{\Big(\partial_\phi \Delta m(\phi)\Big)^2}{\Big(\Delta m(\phi)\Big)^{K+3}} \,. \tag{6.1.6}$$

The species scale cutoff $\Lambda_{\rm sp} = \Lambda_{\rm UV} {N_{\rm sp}}^{-\frac{1}{D-2}}$ can be rewritten in terms of the mass gap $\Lambda_{\rm sp} = N_{\rm sp}^{1/(K+1)} \Delta m$.

6.2 Tree-level vacua

Before we turn to the well-known quantum AdS vacua, let us check whether already the better understood tree-level flux vacua comply with the AdS conjectures.

The constructions of the string vacua we consider start with an assumed Ricci-flat background, additionally equipped with fluxes and instantons. Then one looks for minima of the low energy effective action in which the moduli are stabilized in a controlled regime. In order to determine the true KK scale in these vacua one has to solve an eigenvalue problem for fluctuations around the background. For that purpose one really has to use the fully backreacted metric. This is often not possible and one hopes that a naive estimate using the initial background plus some control arguments give already a good estimate. However, that the backreaction can be essential for seeing some precise cancellations for models with geometric flux was nicely demonstrated in [75].

6.2.1 Type IIA flux models

The best understood examples of AdS minima in string theory are type IIA and type IIB flux compactifications on Calabi-Yau manifolds. In type IIA one can stabilize all closed string moduli via R-R and H_3 form fluxes. As reviewed in section 2.5.2, classes of such concrete models have first been analyzed in [68] and have been called DGKT models. More flux models of this type were considered recently in [151].

In DGKT-like models, one has a dilute flux limit that implies that the KK scales can be made parametrically larger than the masses of the moduli. Let us now recall the isotropic six-torus toy model presented earlier. The effective masses of the moduli all scale in same way as

$$m^2_{\rm mod} \sim -\Lambda \sim \frac{f_0^{\frac{5}{2}} h_0 h_1^3}{f_4^{\frac{9}{2}}} M^2_{\rm pl}\,. \tag{6.2.1}$$

The two KK scales determining the size of the compactification are

$$m^2_{\rm KK,1} = \frac{M_s^2}{r_1^2} = \frac{f_0^{\frac{3}{2}} h_0 h_1}{f_4^{\frac{7}{2}}} M^2_{\rm pl}\,, \qquad m^2_{\rm KK,2} = \frac{M_s^2}{r_2^2} = \frac{f_0^{\frac{3}{2}} h_1^2}{f_4^{\frac{7}{2}}} M^2_{\rm pl}\,. \tag{6.2.2}$$

Since the F_4-flux is not constrained by the tadpole condition, we can choose $f_0, h_0, h_1 = O(1)$ and $f_4 \gg 1$. Then both the volume is large, suppressing stringy corrections, and the string coupling $g_s = e^{\phi} \ll 1$ is small.

In this regime the KK scales are parametrically larger than the moduli masses and one has

$$m^2_{{\rm KK},i} \sim |\Lambda|^{\frac{7}{9}}\,, \tag{6.2.3}$$

thus satisfying the ADC with $\alpha = 7/18$, both in the supersymmetric and non-supersymmetric case.

In the limit $\Lambda \to 0$ some fluxes have to become infinite implying that also some of the moduli become infinite. Therefore, $\Lambda \to 0$ is reached at infinite distance in field space. As far as we can tell, all AdS flux models of this type studied in [68,151] satisfy the relation

$$m_{\rm mod} \sim |\Lambda|^{\frac{1}{2}} \tag{6.2.4}$$

between the mass of the lightest modulus and the cosmological constant. This includes also the non-supersymmetric models. Therefore, for DGKT models the

AM-SSC is satisfied. However, as also claimed in [119], the relevant supersymmetric DGKT vacua do not satisfy the strong version of the ADC but only its weak form with $\alpha < 1/2$, which is also satisfied for the non-supersymmetric ones.

6.2.2 Geometric fluxes and Freund-Rubin models

Another well known class of AdS minima are Freund-Rubin [69] backgrounds discussed in section 2.5.2. The standard example is the 5-form flux supported $\mathrm{AdS}_5 \times S^5$ solution of the type IIB superstring.

Recall that with $\rho = R/M_{\mathrm{pl}}$ the radius of the S^5 in Planck units, the 5D effective potential is given

$$V \sim M_{\mathrm{pl}}^5 \left(-\frac{1}{\rho^2} + \frac{f^2}{\rho^5} \right) . \tag{6.2.5}$$

Here $f \in \mathbb{Z}$ is the quantized 5-form flux and the first term is the contribution of the internal curvature. The AdS minimum is at $\rho_0^3 = 5f^2/2$, where the cosmological constant is given by $\Lambda \sim -\rho_0^{-2} M_{\mathrm{pl}}^2$. The mass of the modulus ρ can be determined as

$$m_\rho^2 = G^{\rho\rho} \partial_\rho^2 V \Big|_0 \sim \frac{M_{\mathrm{pl}}^2}{\rho_0^2} , \tag{6.2.6}$$

with the metric on the moduli space $G_{\rho\rho} \sim \rho^{-2}$. Therefore, the mass of the ρ modulus scales in the same way as the geometric KK scale. On the level of 4D flux compactifications these models are described by geometric fluxes. Let us also recall the masses found in the isotropic torus toy model in section 2.5.2. The saxions receive masses that scale as

$$m_{\mathrm{mod}}^2 \sim -\Lambda \sim \frac{\omega_0\, \omega_1^3}{f_2^{\frac{1}{2}} f_6^{\frac{3}{2}}} M_{\mathrm{pl}}^2 . \tag{6.2.7}$$

In this case the two KK scales are

$$m_{\mathrm{KK},1}^2 = \frac{\omega_0\, \omega_1}{f_2^{\frac{1}{2}} f_6^{\frac{3}{2}}} M_{\mathrm{pl}}^2 , \qquad m_{\mathrm{KK},2}^2 = \frac{\omega_1^2}{f_2^{\frac{1}{2}} f_6^{\frac{3}{2}}} M_{\mathrm{pl}}^2 , \tag{6.2.8}$$

which satisfy $m_{\mathrm{KK},1}^2 \sim m_{\mathrm{mod}}^2/\omega_1^2$ and $m_{\mathrm{KK},2}^2 \sim m_{\mathrm{mod}}^2/(\omega_1\omega_2)$, contrary to our expectation that the masses scale in the same way. However it was shown in [75] that taking the backreaction of the fluxes onto the metric into account, the geometric fluxes in the denominator of the KK masses also cancel and parametrically one indeed finds $m_{\mathrm{KK}}^2 \sim m_{\mathrm{mod}}^2$. The same feature appears for the non-geometric type IIB flux models presented in [73, 74]. As before, the $\Lambda \to 0$ limit is reached at infinite distance in field space. Therefore, irrespective of supersymmetry, these models satisfy both the AM-SSC and the strong ADC.

6.2.3 Generic scaling of moduli masses

We will now provide a simple argument why for classical flux compactifications the moduli masses are generally expected to scale like $|\Lambda|^{\frac{1}{2}}$. A generic contribution to the flux induced scalar potential scales like $V = A\exp(-a\phi)$, where ϕ is a canonically normalized modulus. Such terms balance against each other so that the cosmological constant is expected to also behave as $\Lambda \sim -\exp(-a\phi)$. Similarly, the masses around the minimum will be given by

$$m_{\rm mod}^2 \sim \partial_\phi^2 V \sim e^{-a\phi} \tag{6.2.9}$$

so that $m_{\rm mod} \sim |\Lambda|^{1/2}$ is to be expected for a generic tree-level flux compactification.

6.3 Non-perturbative AdS vacua

In this section, we investigate the two AdS swampland conjectures for the KKLT and the LVS. These vacua are genuinely non-perturbative, in the sense that tree-level contributions are balanced against non-perturbative effects.

6.3.1 The KKLT AdS vacuum

Let us first consider the KKLT AdS minimum [70] for the single Kähler modulus $T = \tau + i\theta$. Here τ measures the size of a 4-cycle and θ is an axion. We recall here the relevant details from the original paper and section 2.5.3.

After stabilizing the complex structure and axio-dilaton moduli via three-form fluxes, the effective Kähler- and superpotential of KKLT is defined by

$$K = -3\log(T + \overline{T})\,, \qquad W = W_0 + Ae^{-aT}\,. \tag{6.3.1}$$

Here $W_0 < 0$ is the value of the flux induced superpotential in its (non-supersymmetric) minimum and the second term in W arises from a non-perturbative effect like a D3-brane instanton or gaugino condensation on D7-branes. The resulting scalar potential after freezing the axion reads

$$V_{\rm KKLT} = \frac{aA^2}{6\tau^2}e^{-2a\tau}(3 + a\tau) + \frac{aAW_0}{2\tau^2}e^{-a\tau}\,. \tag{6.3.2}$$

The supersymmetric AdS minimum of this potential is given by the solution of the transcendental equation

$$A(2a\tau + 3) = -3W_0 e^{a\tau} \tag{6.3.3}$$

and leads to a negative cosmological constant

$$\Lambda = -\frac{a^2A^2}{6\tau}e^{-2a\tau}\,. \tag{6.3.4}$$

In view of the ADC, we first observe that $\Lambda \to 0$ means $\tau \to \infty$, which is at infinite distance in field space. Note that on an isotropic manifold the naive geometric KK scale can be expressed as

$$m_{\rm KK} \sim \frac{1}{\tau} \tag{6.3.5}$$

and hence is exponentially larger than any scale $|\Lambda|^\alpha$ expected from the ADC (6.1.2).

However, one has to keep in mind that for the KKLT setup, one needs an exponentially small W_0 in some kind of warped throat. That this is possible was argued for a long time only on a statistical basis (see [103] for a review). An algorithm to produce such small W_0 setups in a warped region was found by the author and collaborators in [25], simultaneously published with the independent work by [105] who also pioneered the approach at large volume [104].

For an intuitive look into KK scales we shall use simpler models instead. We claim that a hierarchically small W_0 in a warped throat requires that the background becomes highly non-isotropic so that the naive estimate of the KK scale (6.3.5) is not satisfied for the lightest KK or winding modes. In [23] we argue with a toroidal example that already the unwarped case requires a non-isotropic background. This example runs into some control issues with radii shrinking beyond the string length, but it already features the scaling of light KK modes $m \sim \exp(-a\tau)$.

Another option for a KKLT scenario was proposed in [97], namely that a superpotential involving the complex structure modulus Z governing the appearance of a conifold singularity can also generate an exponentially small value for W_0. If $|Z| \ll 1$ the three-cycle of the conifold becomes very small and locally the geometry is described by a Klebanov-Strassler(KS) throat. For our purpose we only need a couple of relations. First the superpotential in the minimum is given by

$$|W_0| \sim |Z| \sim \exp\left(-\frac{2\pi h}{g_s f}\right) , \tag{6.3.6}$$

where f, h are F_3 and H_3 fluxes supporting the strongly warped KS throat. It was shown in [97] that there exists a tower of light KK modes localized close to the tip of the conifold with masses

$$m_{\rm KK}^2 \sim \frac{1}{y_{\rm UV}^2} \left(\frac{|Z|}{\mathcal{V}}\right)^{\frac{2}{3}} M_{\rm pl}^2 , \tag{6.3.7}$$

where $\mathcal{V} = \tau^{3/2}$ denotes the warped volume of the threefold. Note that one must have $\mathcal{V}|Z|^2 \ll 1$ in the strongly warped throat. Moreover, $y_{\rm UV}$ denotes the length of the warped KS throat before it goes over to the bulk Calabi-Yau manifold. In the limit that the throat just fits into the Calabi-Yau volume one can relate $y_{\rm UV}$ to the other quantities as

$$y_{\rm UV} \sim -\log\left(\frac{|Z|}{\mathcal{V}}\right) \tag{6.3.8}$$

(see [97] for further details). It was also found that the mass scale of these KK modes is of the same order as the mass of the complex structure Z. Thus, one is still at the limit of control of the utilized effective theory. In that respect, this scenario is better controlled than the toroidal model mentioned before.

Now using again the KKLT minimum condition (6.3.3) we get $y_{\rm UV} \sim a\tau$ and can express this exponentially small KK scale as

$$\begin{aligned} m_{\rm KK}^2 &\sim \frac{|W_0|^{\frac{2}{3}}}{a^2\tau^3} M_{\rm pl}^2 \sim \left(\frac{(2a\tau+3)^2}{a^8\tau^8}\right)^{\frac{1}{3}} \left(\frac{a^2 e^{-2a\tau}}{\tau}\right)^{\frac{1}{3}} M_{\rm pl}^2 \\ &\sim \left(\frac{1}{\log^2(-\Lambda)} - \frac{2}{\log^3(-\Lambda)} + ...\right) |\Lambda|^{\frac{1}{3}} M_{\rm pl}^2 \,. \end{aligned} \tag{6.3.9}$$

Up to the log-term this satisfies the ADC with $\alpha = 1/6$. A more accurate approximation would be $y_{\rm UV} \sim a\tau + \frac{1}{2}\log\tau$ but this leads to $\log\log|\Lambda|$ terms, which we are neglecting here. Also note that $\mathcal{V}|Z|^2 \sim \tau^{7/2}\exp(-2a\tau)$, which for large τ is indeed much smaller than one. Therefore, stabilizing the Kähler modulus via KKLT is self-consistent with using the effective theory in the warped throat.

In summary, for the better controlled strongly warped throat

$$m_{\rm KK}^2 \sim \frac{1}{\tau^2}\frac{e^{-\frac{2}{3}a\tau}}{\tau^{\frac{1}{3}}} \sim \frac{1}{\log^2(-\Lambda)}|\Lambda|^{\frac{1}{3}} \,. \tag{6.3.10}$$

The masses scale exponentially with τ and feature log-corrections. Up to these corrections, the warped throat scenario only satisfies the ADC with $\alpha = 1/6$ while in the toroidal case the strong ADC is satisfied.

It is also known that the effective mass of the Kähler modulus τ turns out to be much smaller than the naive KK-scale (6.3.5), in fact it is the lowest mass scale in the problem. This is then the relevant scale for the AM-SSC. In the minimum of the potential one can determine

$$m_\tau^2 = K^{T\overline{T}}\partial_\tau^2 V\Big|_0 = \frac{a^2A^2}{6\tau}(2+5a\tau+2a^2\tau^2)\,e^{-2a\tau} \tag{6.3.11}$$

which indeed contains the desired factor $\exp(-2a\tau)$. Thus, one obtains the relation

$$m_\tau^2 = -(2+5a\tau+2a^2\tau^2)\,\Lambda\,. \tag{6.3.12}$$

Now, neglecting $\log\log$-corrections, for large $\tau \gg 1$ one can invert (6.3.4)

$$a\tau = -b_1\log(-\Lambda) + b_0 \tag{6.3.13}$$

with b_1 and b_0 positive constants of order one. Thus, one can express m_τ^2 as

$$m_\tau^2 = -\Big(c_2^2\log^2(-\Lambda) + c_1\log(-\Lambda) + c_0\Big)\,\Lambda \tag{6.3.14}$$

with $c_2 > 0$. After reintroducing powers of the Planck scale and working in the limit $\Lambda \to 0$, we can express the mass in the intriguing way

$$m_\tau \sim -c_2\log\Big(-\tfrac{\Lambda}{M_{\rm pl}^2}\Big)\,\Big|\Lambda\Big|^{\frac{1}{2}}\,. \tag{6.3.15}$$

Note that $|\Lambda| < M_{\rm pl}$ is required for the effective theory to be controllable. Moreover, in the limit $\Lambda \to 0$ the mass scale still approaches zero.

Therefore, in comparison to the (classical) AM-SSC there appears a logarithmic correction. We propose

$$m\, R_{\rm AdS} \leq c \log(R_{\rm AdS} M_{\rm pl}) \tag{6.3.16}$$

to be the quantum generalization of the AM-SSC. This is a weaker bound than the classical version (6.1.1) so that a slight (log type) scale separation between the internal space and the light mode is allowed.

Similarly, as shown in (6.3.10) we also found log-corrections to the ADC. Therefore we summarize that for quantum vacua like KKLT, where a non-perturbative contribution is balanced against a tree-level one, it seems that there appears a logarithmic correction to the result for simple perturbative vacua.

6.3.2 The large volume AdS vacuum

Let us analyze another prominent example, namely the large volume scenario (LVS). Recall that here one has a swiss-cheese Calabi-Yau threefold with a large and a small Kähler modulus, τ_b and τ_s. The precise definition can be found in [71, 72]. We have reviewed the LVS in section 2.5.3.

In the perturbative regime $a\tau_s \gg 1$, the moduli in the LVS minimum take values

$$\tau_s^0 = \left(\frac{4\nu\lambda}{\mu^2}\right)^{\frac{2}{3}}, \qquad \mathcal{V}^0 = \frac{\mu}{2\lambda}\sqrt{\tau_s^0}\, e^{a\tau_s^0}\,. \tag{6.3.17}$$

Because of the extended no-scale structure, to find the actual non-vanishing value of the potential in the LVS minimum one has to compute to next order in $1/\tau_s$ [109]. There will be a correction to τ_s^0, which is a shift by a positive constant $\tau_s^0 \to \tau_s^0 + c/a$. The value of the cosmological constant will then be

$$\Lambda \sim -\frac{3c\lambda^2 e^{-3c}}{\mu a\, \tau_s^0}\, e^{-3a\tau_s^0}\left(1 + O(\tfrac{1}{\tau_s})\right)\,. \tag{6.3.18}$$

The lightest modulus in the game is $\mathcal{V}$, whose mass can be determined by first integrating out τ_s and taking the second derivative of the effective potential with respect to $\mathcal{V}$ (see also [110]). After solving $\partial_{\tau_s} V = 0$, we can write the effective potential as

$$V_{\rm eff}(\mathcal{V}) = \frac{1}{\mathcal{V}^3}\left(\nu + \frac{\mu^2}{\lambda}\tau_s(\mathcal{V})^{\frac{3}{2}}\left(g(\mathcal{V})^2 - g(\mathcal{V})\right)\right) \tag{6.3.19}$$

with

$$g(\mathcal{V}) = 2\left(\frac{1 - a\tau_s(\mathcal{V})}{1 - 4a\tau_s(\mathcal{V})}\right) = \frac{1}{2}\left(1 - \tfrac{3}{4a\tau_s(\mathcal{V})} + \ldots\right). \tag{6.3.20}$$

Here τ_s depends implicitly on $\mathcal{V}$. Now, using that at leading order $\partial\tau_s/\partial\mathcal{V} \approx (a\mathcal{V})^{-1}$ we realize that the leading order term (in $1/\tau_s$) again cancels so that

$$m_{\mathcal{V}}^2 = K^{\mathcal{V}\mathcal{V}}\partial_{\mathcal{V}}^2 V_{\text{eff}}\Big|_0 \sim \frac{\lambda^2}{\mu a\,\tau_s^0} e^{-3a\tau_s^0}\left(1 + O(\tfrac{1}{\tau_s})\right). \tag{6.3.21}$$

Therefore, for the LVS AdS minimum we have found the relation

$$m_{\mathcal{V}}^2 \sim |\Lambda|\left(c_0 + \frac{c_{-1}}{\log(-\Lambda)} + \ldots\right), \tag{6.3.22}$$

which means that in the limit $\Lambda \to 0$ the LVS satisfies the strong (classical) AM-SSC. However, also for LVS there will be subleading log-corrections. In contrast to KKLT, here the first two coefficients are vanishing i.e. $c_2 = c_1 = 0$ which presumably is due to the extended no-scale structure and the perturbative stabilization of $\mathcal{V}$. For LVS one can have $W_0 = O(1)$ so that the naive estimate for the KK scale my well be justified

$$m_{\text{KK}}^2 \sim \frac{1}{\mathcal{V}^{\frac{4}{3}}} \sim \frac{1}{\tau_s^{\frac{2}{3}}} e^{-\frac{4}{3}a\tau_s} \sim \frac{1}{\log^{\frac{2}{9}}|\Lambda|}|\Lambda|^{\frac{4}{9}}. \tag{6.3.23}$$

This result is similar to the KK modes (6.3.10) for the KKLT model in the warped throat. Thus, the ADC again receives extra quantum log corrections and $\alpha = 2/9$.

6.4 Quantum vacua and other swampland conjectures

In this section we discuss the implications and relations of the log-correction to other swampland conjectures. First, following a similar reasoning as in section 6.2.1, we provide a general argument for the appearance of such corrections.

6.4.1 Origin of log-corrections

To see the origin of the log-corrections consider a typical non-perturbative contribution to the scalar potential, which in canonically normalized variables takes the following double-exponential form

$$V \sim A\, e^{-c\phi}\, e^{-(be^{a\phi})} + V_{\text{others}}. \tag{6.4.1}$$

The other corrections can be perturbative or non-perturbative, depending on the nature of the model. If moduli stabilization occurs such that the first term balances the terms in V_{others}, the size of the first one is expected to set the scale for the potential and the masses in the minimum. Computing its second derivative with respect to ϕ one gets

$$m^2 \sim \partial_\phi^2 V \sim \left(c^2 + 2abc\, e^{a\phi} - ba^2\, e^{a\phi} + (ab)^2 e^{2a\phi}\right) V. \tag{6.4.2}$$

Inverting (6.4.1) one can write

$$e^{a\phi} \sim -\frac{1}{b}\log\left(\frac{V}{A}\right) = -b_1 \log|V| + b_0 \tag{6.4.3}$$

so that

$$m^2 \sim -\Big(c_2^2 \log^2(|V|) + c_1 \log(|V|) + c_0\Big)\, V\,. \tag{6.4.4}$$

Observe that these terms take a very similar form to what we found for KKLT in (6.3.14). One can well imagine that for a full model the potential will be more complicated so that, like in LVS, also further subleading corrections $\log^{-n}(|V|)$ ($n \geq 1$) will appear.

Thus, we conclude that the logarithmic corrections are genuinely related to the appearance and relevance of non-perturbative effects in the scalar potential. In the moment that such genuinely non-perturbative vacua exist in string theory, the AdS swampland conjectures are expected to receive log-corrections.

6.4.2 Trans-Planckian censorship conjecture

If the AdS swampland conjectures receive such corrections, it is natural to expect that also the no-dS swampland conjecture will be changed. Computing the first derivative of (6.4.1), a naive guess would be

$$|\nabla V| \geq V\Big(c_1 \log\left(|V|\right) + c_2\Big)\,. \tag{6.4.5}$$

In contrast to the AdS swampland conjectures this relation is supposed to hold not only at a specific point in field space (namely the minimum) but at every point. It remains to be seen whether such a strong local bound really makes sense. In any case, it is remarkable that the right hand side could vanish for $V = \exp(-c_2/c_1)$, thus potentially allowing dS vacua. In this spirit, utilizing quantum effects to generate stable dS vacua has been discussed in e.g. [152, 153].

The TCC [130] has been proposed as a more general version of the no-dS conjectures, and stems from inherently quantum arguments about sub-Planckian fluctuations staying sub-Planckian in an expanding Universe. Note that the log-term inherent to the TCC bound (6.1.5) mirrors our naive guess above.

Let us check that a potential of the generic form

$$V(\phi) = A e^{-c\phi}\, e^{-(b e^{a\phi})} \tag{6.4.6}$$

indeed satisfies this averaged no-dS swampland conjecture. Note that for $a, b, c > 0$ this potential is indeed positive and monotonically decreasing, so the prerequisites for the TCC are satisfied. For the average value we can directly compute

$$\begin{aligned} \left\langle \frac{-V'}{V} \right\rangle \Bigg|_{\phi_i}^{\phi_f} &= \frac{1}{\Delta\phi}\int_{\phi_i}^{\phi_f} d\phi \left(c + ab\, e^{a\phi}\right) = c + \frac{b}{\Delta\phi}\left(e^{a\phi_f} - e^{a\phi_i}\right) \\ &> c - \frac{b}{\Delta\phi} e^{a\phi_i} > c + \frac{1}{\Delta\phi}\log\left(\frac{V_i}{A}\right)\,. \end{aligned} \tag{6.4.7}$$

This has precisely the form (6.1.5) so that we can state that non-perturbative contributions to the scalar potential induce the log-corrections in the TCC derived no-dS swampland conjecture (6.1.5). Moreover, we observe that for the three terms in the KKLT potential (6.3.2), one gets the parameters $c \in \{\sqrt{8/3}, \sqrt{2/3}\}$ which both satisfy $c \geq \sqrt{2/3}$, the value appearing in the original TCC bound (6.1.5).

We consider this connection to the TCC as further evidence for the appearance of log-corrections in the various swampland conjectures.

6.4.3 Emergence for KKLT

Finally, we comment on the emergence proposal. Before we come to the KKLT model let us first consider tree-level flux compactifications.

For compactification on S^5 one needs to take heed of the degeneracy of KK modes of mass $m_n = n\Delta m = nM_{\rm pl}/\rho$. This is given by the dimensionality of the space of harmonic functions of homogeneous degree n, which for the 5-sphere goes as n^4. Applying our general result (6.1.6) for the one-loop correction to the field space metric and setting it equal to the tree-level metric $G^{\rm tree}_{\phi\phi} \sim \rho^{-2}$ we obtain $\Lambda^8_{\rm sp} = M^3_{\rm pl}(\Delta m)^5 = M^8_{\rm pl}/\rho^5$. Taking the relation between the string scale and the D dimensional Planck scale into account it follows $\Lambda_{\rm sp} \sim M_s$. We expect that this relation will appear for all tree-level flux compactifications so that the true UV cutoff of these models is simply the string scale.

Let us now analyze the implications of the emergence proposal in the KKLT setting for the strongly warped throat.

As shown in [97], in the scenario where the small value of W_0 is generated by a strongly warped throat there exists a tower of highly red-shifted KK modes localized at the tip of the throat with masses

$$\Delta m_{\rm KK} \sim \frac{|Z|^{\frac{1}{3}}}{\tau^{\frac{1}{2}} y_{\rm UV}}\,, \tag{6.4.8}$$

where Z denotes the conifold (complex structure) modulus and $y_{\rm UV}$ is the length of the KS throat before it reaches the bulk Calabi-Yau. It was argued in [97] that these KK modes are lighter than the cutoff of the effective theory and that their one-loop contribution corrects the second (subleading) term in the Kähler potential

$$K = -3\log(T+\overline{T}) + c\frac{|Z|^{\frac{2}{3}}}{(T+\overline{T})}\,. \tag{6.4.9}$$

Using the general relation (6.1.6) and setting this one-loop correction equal to the Kähler metric $G_{\tau\overline{\tau}}$ (second term) one finds for the species scale

$$\Lambda^3_{\rm sp} \sim \frac{|Z|}{\tau^{\frac{3}{2}} y_{\rm UV}} M^3_{\rm pl}\,. \tag{6.4.10}$$

Since the first term in the above Kähler potential is also present in the unwarped case, we expect it to emerge from integrating out the tower of heavier bulk KK modes (6.3.5) with mass scale $\Delta m_{\rm KK,h} \sim 1/\tau$.

In the limit where the throat just fits into the warped Calabi-Yau volume, one can determine the cutoff $y_{\rm UV}$ as

$$y_{\rm UV} \sim -\log\left(\frac{|Z|}{\tau^{\frac{3}{2}}}\right) . \tag{6.4.11}$$

Now we stabilize the Kähler modulus τ via KKLT which gives the relations

$$|Z| \sim |W_0| \sim \tau e^{-a\tau}\,, \qquad y_{\rm UV} \sim \tau \tag{6.4.12}$$

so that the species scale can be expressed as

$$\Lambda_{\rm sp}^3 \sim \frac{e^{-a\tau}}{\tau^{\frac{3}{2}}} M_{\rm pl}^3\,. \tag{6.4.13}$$

This is reminiscent of the dynamically generated mass scale $\Lambda_{\rm SQCD}$ of the SYM theory that undergoes gaugino condensation. This scale is usually given by $\Lambda_{\rm SQCD}^3 = e^{-a/g^2} M^3$, where M denotes a UV cutoff scale. Noting that $g^{-2} \sim \tau$ we can write the KKLT cutoff as

$$\Lambda_{\rm sp}^3 \sim e^{-\frac{a}{g^2}} (g\, M_{\rm pl})^3 \sim \Lambda_{\rm SQCD}^3\,. \tag{6.4.14}$$

Thus the cutoff of the KKLT model is the scale at which the implicitly assumed gaugino condensation of the confining gauge theory occurs, while the true UV cutoff of the gauge theory itself is not simply the Planck scale but rather $M \sim \Lambda_{\rm UV} \sim g M_{\rm pl}$ as suggested by the weak gravity conjecture.

6.5 Discussion

In lieu of full trust in the dS vacua at the end of the KKLT and LVS constructions, we have investigated the intermediate AdS vacua and how they stand with respect to the AdS scale separation and distance conjectures. To this end, we have identified the relevant towers of light states and seen that realizing an exponentially small W_0 for the KKLT model results in a large hierarchy between different KK scales. Driven by confidence in the consistency of the aforementioned AdS vacua, we proposed log-corrections to the tree-level AdS swampland conjectures. Extending our reasoning, we expect similar log-corrections to the no-dS swampland conjecture. These might be in the same spirit as the log-corrections that were found for the "average" of the no-dS swampland conjecture in the recently proposed TCC.

Additionally, we analyzed the consequences of imposing the emergence proposal. For tree-level flux compactifications we found that the cutoff scale is simply the string scale. For the KKLT model, it is remarkable that both proposed scenarios for generating an exponentially small W_0 lead to a cutoff scale reminiscent of the dynamically generated scale for the condensing SYM theory. It is certainly encouraging that our observations seem to fit well within the broader swampland picture.

Chapter 7

Conclusions

In this thesis we have discussed different tests of swampland conjectures in three regions of the string landscape. Let us review the results of these tests and finish with an outlook.

Non-BPS branes

By including non-BPS $\widehat{\text{D}}$-branes into type IIA orientifold compactifications, we were able to circumvent a no-go theorem for dS in tree-level type IIA flux vacua. While non-BPS branes are usually not stable, the orientifold may actually project out the open string tachyon of a single brane. This means single non-BPS branes are indeed stable, and since they do not carry R-R charges they are not bounded by tadpole constraints either. Altogether there is no apparent obstruction to add a single $\widehat{\text{D}}$7-brane to the theory, which indeed circumvents the no-go theorem. This doesn't yet prove that one can actually find a dS minimum. We therefore checked that for a simple toroidal *STU* model a positive minimum does indeed appear. We conclude that adding non-BPS $\widehat{\text{D}}$-branes apparently enables us to violate the no-dS swampland conjecture.

However, the non-BPS brane does carry a K-theory charge. Since there is no tadpole generated by this charge, there is no associated cancellation condition. However, a non-trivial K-theory charge on compact space has also been associated with global gauge anomalies on the world-volume of probe branes. While not a consistency condition from first principles, it is reasonable to require that stable D-branes do not grow spurious anomalies in the presence of these new objects. This suggests that K-theory charges should also cancel on compact spaces.

Our result then links the no-dS swampland conjecture to the vanishing of K-theory charges. If the no-dS conjecture is true, then K-theory charges must necessarily vanish in type IIA string orientifolds. If on the other hand K-theory charges have to vanish, then we have another set of evidence for the no-dS conjecture.

Exotic string theories

Exotic string theories can be thought of as arising from the usual type II string theories by T-dualizing along time-like directions. Through a web of T- and S-dualities, a whole family of exotic theories with strings and D-branes of various world-sheet and target space signature can be constructed. Although the time-like T-duality induces apparent pathologies in the exotic theories, most prominently ghost states, the full non-perturbative theory was argued to be consistent.

Taking an agnostic approach about the fate of the gravity sector, we accepted closed string ghosts as quantum gravity effects and only demanded the open string gauge sector to be ghost free. Crucially, with closed string ghosts and flux contributions it is possible to construct dS minima. With these exotic dS minima at hand, we searched for D-brane configurations with ghost-free gauge sectors. Although such brane configurations do exist, we found that the O-planes which have to be introduced for tadpole cancellation are precisely those that also project out the closed string ghosts, destroying the possibility for dS vacua in the process.

The result of this foray into exotic string theories therefore reveals that whenever the gauge sector, which has been probed very precisely in experiments, is assumed to be ghost-free, the no-dS conjecture holds even in exotic string theories.

Non-perturbative vacua

Finally we investigated the status of the AdS vacuum of non-perturbative vacua. The KKLT and LVS constructions use quantum effects to balance against the tree-level potential, with intermediate AdS vacua that seem to violate some swampland conjectures. While the uplift to dS is still an open problem, the intermediate AdS minima have been thoroughly investigated recently and no problems have been found. In consequence, either these swampland conjectures are wrong or something else is happening.

Indeed, the quantum nature of these vacua induces log-corrections that have to be taken into account in the swampland conjectures. A simple demonstration of the exponential structure of non-perturbative contributions to the scalar potential solidifies this point. The log-corrections are reminiscent of the trans-planckian censorship conjecture, which exhibits the exact form we would naively expect for log-corrections to the no-dS conjecture.

Outlook

We have set out to challenge swampland conjectures, and observed that every time we found a loophole in the swampland conjectures a new obstacle appeared. Perhaps most surprising were the exotic theories, where of the many possible D-branes only those were ghost-free where the associated O-planes forbid the dS vacua from appearing. Altogether, it now seems somewhat less likely that the swampland conjectures are just a product of the lamppost of perturbative string theory.

However, the lamppost is not completely torn down yet. Our work has been with critical string theory on flux compactifications that preserve some supersymmetry in mind. Although steps toward non-supersymmetric compactifications [41] or non-critical string theories [8, 154] are constantly being made, the control granted by supersymmetry is hard to let go.

Furthermore the cosmological constant question is far from being settled. The tests of KKLT and LVS have shown no destructive faults so far, and the log-corrections to the swampland bound suggested by our work and the TCC do in principle allow for short-lived dS vacua. On the other hand, interesting connections between the no-dS conjecture and quantum breaking of dS [155, 156] suggest that there may be deeper principles forbidding positive cosmological constants in S-matrix theories of any kind [77]. This could suggest that a more general description of string theory or quantum gravity in general is needed, which ideally includes non-perturbative effects in a unified way. There is much yet to be understood, as Edward Witten remarked at the end of this years strings conference:

> "What is string theory?
> It is amazing to know so much about a theory,
> yet feel one has so little idea what it really is."

Appendix A

Exotic D-brane Spectra

A.1 Table of branes in exotic IIB theories

Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free	Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free
	D(-1)	$D(-1)^{(0,0)}_{(9,1)}$	E	−	✓		D(-1)	$D(-1)^{(0,0)}_{(1,9)}$	E	−	✓
	D1	$D1^{(1,1)}_{(9,1)}$	L	−	✓		D1	$D1^{(1,1)}_{(1,9)}$	L	−	✓
$IIB^{-+}_{(9,1)}$	D3	$D3^{(4,0)}_{(9,1)}$	E	+	-	$IIB^{-+}_{(1,9)}$	D3	$D3^{(0,4)}_{(1,9)}$	E	+	−
	D5	$D5^{(5,1)}_{(9,1)}$	L	+	-		D5	$D5^{(1,5)}_{(1,9)}$	L	+	−
	D7	$D7^{(8,0)}_{(9,1)}$	E	−	✓		D7	$D7^{(0,8)}_{(1,9)}$	E	−	✓
	D9	$D9^{(9,1)}_{(9,1)}$	L	−	✓		D9	$D9^{(1,9)}_{(1,9)}$	L	−	✓

Table A.1: Brane spectrum of mirror $IIB^{-+}_{(9,1)/(1,9)}$ theories.

Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free	Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free
	D1	$D1^{(2,0)}_{(7,3)}$	E	+	-		D1	$D1^{(0,2)}_{(3,7)}$	E	+	-
		$D1^{(0,2)}_{(7,3)}$	E	−	✓			$D1^{(2,0)}_{(3,7)}$	E	−	✓
	D3	$D3^{(3,1)}_{(7,3)}$	L	+	-		D3	$D3^{(1,3)}_{(3,7)}$	L	+	-
$IIB^{--}_{(7,3)}$		$D3^{(1,3)}_{(7,3)}$	L	−	✓	$IIB^{--}_{(3,7)}$		$D3^{(3,1)}_{(3,7)}$	L	−	✓
	D5	$D5^{(4,2)}_{(7,3)}$	E	+	-		D5	$D5^{(2,4)}_{(3,7)}$	E	+	-
		$D5^{(6,0)}_{(7,3)}$	E	−	✓			$D5^{(0,6)}_{(3,7)}$	E	−	✓
	D7	$D7^{(5,3)}_{(7,3)}$	L	+	-		D7	$D7^{(3,5)}_{(3,7)}$	L	+	-
		$D7^{(7,1)}_{(7,3)}$	L	−	✓			$D7^{(1,7)}_{(3,7)}$	L	−	✓

Table A.2: Brane spectrum of mirror $IIB^{--}_{(7,3)/(3,7)}$ theories.

Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free
$IIB^{-+}_{(5,5)}$	D(-1)	$D(-1)^{(0,0)}_{(5,5)}$	E	+	-
	D1	$D1^{(1,1)}_{(5,5)}$	L	+	-
	D3	$D3^{(2,2)}_{(5,5)}$	E	+	-
		$D3^{(0,4)}_{(5,5)}$	E	−	✓
		$D3^{(4,0)}_{(5,5)}$	E	−	✓
	D5	$D5^{(3,3)}_{(5,5)}$	L	+	-
		$D5^{(1,5)}_{(5,5)}$	L	−	✓
		$D5^{(5,1)}_{(5,5)}$	L	−	✓
	D7	$D7^{(4,4)}_{(5,5)}$	E	+	-
	D9	$D9^{(5,5)}_{(5,5)}$	L	+	-

Table A.3: Brane spectrum of $IIB^{-+}_{(5,5)}$.

A.2 Table of branes in exotic IIA theories

Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free	Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free
$IIA^{-+}_{(10,0)}$	D0	$D0^{(1,0)}_{(10,0)}$	E	−	✓	$IIA^{--}_{(0,10)}$	D0	$D0^{(0,1)}_{(0,10)}$	L	−	✓
	D2	*No consistent D2-brane configuration*					D2	*No consistent D2-brane configuration*			
	D4	$D4^{(5,0)}_{(10,0)}$	E	+	-		D4	$D4^{(0,5)}_{(0,10)}$	L	+	-
	D6	*No consistent D6-brane configuration*					D6	*No consistent D6-brane configuration*			
	D8	$D8^{(9,0)}_{(10,0)}$	E	−	✓		D8	$D8^{(0,9)}_{(0,10)}$	L	−	✓

Table A.4: Brane spectrum of mirror $IIA^{-+}_{(10,0)}$ and $IIA^{--}_{(0,10)}$ theories.

Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free	Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free
$IIA^{--}_{(8,2)}$	D0	$D0^{(0,1)}_{(8,2)}$	L	−	✓	$IIA^{-+}_{(2,8)}$	D0	$D0^{(1,0)}_{(2,8)}$	E	−	✓
	D2	$D2^{(1,2)}_{(8,2)}$	E	−	✓		D2	$D2^{(2,1)}_{(2,8)}$	L	−	✓
		$D2^{(3,0)}_{(8,2)}$	E	+	-			$D2^{(0,3)}_{(2,8)}$	L	+	-
	D4	$D4^{(4,1)}_{(8,2)}$	L	+	-		D4	$D4^{(1,4)}_{(2,8)}$	E	+	-
	D6	$D6^{(5,2)}_{(8,2)}$	E	+	-		D6	$D6^{(2,5)}_{(2,8)}$	L	+	-
		$D6^{(7,0)}_{(8,2)}$	E	−	✓			$D6^{(0,7)}_{(2,8)}$	L	−	✓
	D8	$D8^{(8,1)}_{(8,2)}$	L	−	✓		D8	$D8^{(1,8)}_{(2,8)}$	E	−	✓

Table A.5: Brane spectrum of mirror $IIA^{--}_{(8,2)}$ and $IIA^{-+}_{(2,8)}$ theories.

Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free	Theory	Dp	Branes	Type(E/L)	Tension	Ghost-free
$\text{IIA}^{-+}_{(6,4)}$	D0	$\text{D0}^{(1,0)}_{(6,4)}$	E	$+$	-	$\text{IIA}^{--}_{(4,6)}$	D0	$\text{D0}^{(0,1)}_{(4,6)}$	L	$+$	-
	D2	$\text{D2}^{(0,3)}_{(6,4)}$	L	$-$	✓		D2	$\text{D2}^{(1,2)}_{(4,6)}$	E	$-$	✓
		$\text{D2}^{(2,1)}_{(6,4)}$	L	$+$	-			$\text{D2}^{(3,0)}_{(4,6)}$	E	$+$	-
	D4	$\text{D4}^{(1,4)}_{(6,4)}$	E	$-$	✓		D4	$\text{D4}^{(4,1)}_{(4,6)}$	L	$-$	✓
		$\text{D4}^{(3,2)}_{(6,4)}$	E	$+$	-			$\text{D4}^{(2,3)}_{(4,6)}$	L	$+$	-
		$\text{D4}^{(5,0)}_{(6,4)}$	E	$-$	✓			$\text{D4}^{(0,5)}_{(4,6)}$	L	$-$	✓
	D6	$\text{D6}^{(4,3)}_{(6,4)}$	L	$+$	-		D6	$\text{D6}^{(3,4)}_{(4,6)}$	E	$+$	-
		$\text{D6}^{(6,1)}_{(6,4)}$	L	$-$	✓			$\text{D6}^{(1,6)}_{(4,6)}$	E	$-$	✓
	D8	$\text{D8}^{(5,4)}_{(6,4)}$	E	$+$	-		D8	$\text{D8}^{(4,5)}_{(4,6)}$	L	$+$	-

Table A.6: Brane spectrum of mirror $\text{IIA}^{-+}_{(6,4)}$ and $\text{IIA}^{--}_{(4,6)}$ theories.

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