RELAYS FOR INTERFERENCE MITIGATION IN WIRELESS NETWORKS

BY

ROHIT NAINI

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Electrical and Computer Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2010

Urbana, Illinois

Adviser:

Professor Pierre Moulin

ABSTRACT

Wireless links play an important role in the last mile network connectivity. In contrast to the strictly centralized approach of today's wireless systems, the future promises decentralization of network management. Nodes potentially engage in localized grouping and organization based on their neighborhood to carry out complex goals such as end-to-end communication. The quadratic energy dissipation of the wireless medium necessitates the presence of certain relay nodes in the network. Conventionally, the role of such relays is limited to passing messages in a chain in a point-point hopping architecture. With the decentralization, multiple nodes could potentially interfere with each other. This work proposes a technique to exploit the presence of relays in a way that mitigates interference between the network nodes. Optimal spatial locations and transmission schemes which enhance this gain are identified.

To my parents, for their constant support and encouragement

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my adviser, Prof. Pierre Moulin,for his guidance and support during the course of the project. I thank my family for always believing in and nurturing my abilities. I thank all my friends for being a source of inspiration and making the journey enjoyable. A special mention to my closest friend Mahati for always being there for me, through the ups and downs of graduate school.

I am grateful to the funding agency, DARPA, for supporting me and providing me with an opportunity to work closely with a pioneering team of PIs under the ITMANET project.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Communication networks are gaining increasing prominence in today's information age. With the ever increasing requirements in terms of data rates, volume of data transferred, and quality of network connectivity, among other performance measures, every aspect of network communication is being improved for efficiency. In the recent past, the last mile connectivity has drastically improved due to the incorporation of wireless links into networks. However, due to the nature of initial integration, wireless links are usually viewed as substitutes for wire. The interference from other simultaneous in-band transmissions is sometimes completely averted, as in some cellular systems, through centralized management. In other unregulated systems such as Wi-Fi, interference from simultaneous transmissions from other base stations is viewed as a malevolent effect and dealt with using robust design. The current work shows that the broadcast nature of wireless medium can be exploited by cooperatively handling interference across network nodes.

1.1 Motivation

Most wireless networks today are managed through a centralized agent, a router in Wi-Fi or the base station in cellular networks. Due to this centralization, there arises a need for careful deployment and maintenance. In addition, such a centralized network needs to be micro-managed during everyday operation. Any minor change in the network architecture, like an addition of a single cell to a cellular network, necessitates a complete overhaul of the system. In sharp contrast to this carefully monitored architecture, ad hoc networks rely on self-aware nodes with varying degrees of network cognizance. Such systems self-organize into localized structures and automatically establish a hierarchy among the nodes resulting in coordinated action.

This aspect of low maintenance coupled with rapid deployment makes such networks lucrative for both military and civilian applications. However, such ad hoc wireless networks have their own downside. Without centralized control, there arise situations where interference from spatially close network nodes outside the local cluster can affect communication within the cluster. One way to address the problem is to over-engineer the intra-cluster communication to handle the interference. Such an approach usually results in sub-optimal usage of the network resources. There is a need for an intermediate approach wherein network interference can be properly managed without requiring a centralized control. Our current work proposes such a technique to effectively manage network interference from nodes immediately beyond the local cluster using appropriately placed relay nodes within the cluster.

Our proposed scheme for interference mitigation is effective in scenarios where nodes are either present at optimal network locations or some network nodes are endowed with mobility within a certain region. If such autonomous mobility is available to nodes, it becomes important for each mobile node to operate at the best possible location subject to its localized restrictions. We investigate optimization strategies for node placement and relocation with due importance placed on the type of transmission scheme employed. We look at the simplest multi-terminal block incorporating elements of cooperation and interference, i.e., the single relay channel. We analyze the performance of a single relay channel with no extensive knowledge about transmissions external to the block. This is a suitable compromise between a point-point network structure and a highly centralized and demanding omniscient network architecture. We demonstrate that performance gains can be obtained using relay schemes like Compress-Forward which involve cooperation between nodes instead of seeking to individually combat interference at each receiver. These gains are further enhanced if certain freedom is available in terms of node mobility.

1.2 Related Work

Relays are an important mechanism through which end-end communication becomes feasible in a large network. Network routing is usually optimized through point-point relaying for best case end-end delays or number of hops.

Keeping in view that improving the performance using a single relay based multi-terminal block can significantly affect overall system performance, we look at the specific problem of a single relay channel (SRC). There are two parallel research areas which are related to the problem we consider. One is the network information theoretic perspective seeking to establish fundamental limits of multi-terminal communication. The most directly relevant work in this area addresses an interference channel with relays, which is dealt with in [1]. The work on interference forwarding is addressed by Dabora et al. in [2]. However, these works assume a complete knowledge of the codebooks of all transmitters at the relay which could easily break down in the case of larger systems. The Quantize-Forward approach taken by Avestimehr et al. in [3] considers only the part of the signal above the interference-noise floor for decoding. External interference is viewed as a negative effect on the relay based system which is an artifact of the coding scheme but not fundamental in nature. The other related research area is network optimization. Most optimization problems consider point-point relaying where the fundamental issue tackled is delays or number of hops, ignoring the multi-terminal capability of wireless nodes. A relevant work that considers relay selection using Amplify-Forward is presented in [4]. Yet another related work on algorithms for relay selection is presented by Vishwanath *et al.* in [5]. This work addresses relay selection in a stand-alone setting. We extend this problem of relay placement to the case where external interference is present. The resulting optimal solution can be very different because of interference cooperation between the relay and nodes downstream.

1.3 Outline

The thesis is organized as follows. Chapter 2 presents background material on the single relay channel, including modeling and description of certain achievable rate schemes used in this work. Chapter 3 introduces the channel models and the framework used to formalize the problem. Chapter 4 examines the relay placement and relaying schemes in the presence of a single external interferer and contrasts them with the case without external interference. We further extend the analysis to the more realistic setting wherein several interfering nodes are present in the immediate neighborhood and the relay has some knowledge about node locations and their transmit powers. Chapter 5 concludes the discussion summarizing the main observations based on our analysis and simulations and presents the chief contributions of our work.

CHAPTER 2

BACKGROUND

This chapter presents some material on relay channels [6, 7] which is essential to appreciate the results presented in this thesis. We look at the modeling of a discrete-memoryless single relay channel and give formal definitions for achievable rate regions, capacity characterization and other related aspects. The Cut-Set bound on network capacity is introduced in a generalized framework and specific interpretation is given for a single relay channel. A detailed discussion of Decode-Forward and Compress-Forward schemes for a relay channel is presented and the corresponding rate-region results are invoked in later chapters.

2.1 Single Relay Channel

A single relay channel is a multi-terminal network problem where a source intends to transmit a message to a destination node with help from an intermediate node called the relay. For a discrete-memoryless setting, at each time instant the received signals at both the relay and the destination node depend only on the signal transmissions at that time instant. Suppose we label the source as Node 1, the relay as Node 2 and the destination as Node 3 in the system. Let X_i^n be the signal transmitted and Y_i^n be the signal received at node i. Then the discrete memoryless relay channel is completely specified by the probability distribution $P_{Y_2Y_3|X_1X_2}(.)$. Figure 2.1 is a block diagram of the channel.

The problem of passing on a message $w \in W$ from the source to the destination is achieved through n instances of the channel use. In this setting, the source node needs to design a codebook of transmissions $X_1^n = \{X_{1i}, 1 \leq i \leq n\}$ corresponding to each message. The relay node needs to design its own codebook, which is a mapping from all previously received transmissions

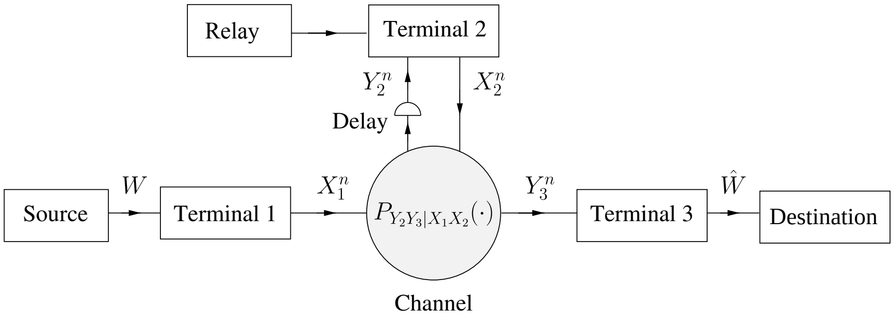

Figure 2.1: Single relay channel block diagram

Y_2^{t-1} to X_{2t}, which would help the destination node to interpret the original transmission from the source. The destination node, on the other hand, must design an appropriate decoding function which maps from the received signal Y_3^n to a decoded message symbol $\hat{w} \in W$. In this discrete memoryless formulation the joint probability distribution of the random variables involved factorizes as

$$\begin{aligned} P(w, x_1^n, x_2^n, y_2^n, y_3^n, \hat{w}) &= P(w)P(x_1^n|w) \\ &\times \left[\prod_{i=1}^{n} P(x_{2i}|y_2^{i-1}) P_{Y_2Y_3|X_1X_2}(y_{2i}, y_{3i}|x_{1i}, x_{2i})\right] \\ &\times P(\hat{w}|y_3^n) \end{aligned}$$

where $P(x_1^n|w)$,$P(x_{2i}|y_2^{i-1})$ and $P(\hat{w}|y_3^n)$ take only values 0 or 1, implying that they are deterministic mappings.

A rate R is said to be achievable on the relay channel if there exist suitable codebooks at the source and relay coupled with appropriate decoding function such that, for a sequence of size 2^{nR} codebooks, the decoding error probability $\Pr\{w \neq \hat{w}\} \to 0$ for n sufficiently large. In other words, given a sufficient block length n, the channel allows for a transmission of nR bits of information from the source to the destination via the relay with negligible probability of error. The capacity of the relay channel is defined as the supremum of all achievable rates.

2.2 Cut-Set Bound

In this section, we present the Cut-Set bound on capacity of discrete memoryless multi-terminal networks in a generalized setting and then apply it to a single relay channel. This bound is similar to the celebrated Ford-Fulkerson theorem on max-flow min-cut of commodity flows. However, information not being a commodity, the bound suggested by the theorem cannot always be achieved. We see that the actual achievability of the bound is contingent upon simultaneously finding suitable joint distributions for all the source nodes in the network. We state the theorem here and interpret it; proof can be found in Chapter 14 of [7].

Given a network of intercommunicating transceiver nodes 1,2,...,m, the set of rates $\{R^{ij}\}$ is said to be achievable if there exist encoding and decoding functions at nodes i and j respectively such that over a transmission blocklength of n, the probability of error in decoding nR^{ij} bits of information tends to 0. For any set of achievable rates $\{R^{ij}\}$ and a subset of nodes $S \subset \{1, 2, ..., m\}$, the following bound on the sum of achievable rates holds:

$$\sum_{i \in S, j \in S^c} R^{ij} \leq I(X^{(S)}; Y^{(S^c)} | X^{(S^c)}) \tag{2.1}$$

where S^c is the set of complementary nodes to S and the mutual information term is the conditional information over the entire set of transmissions in S with the entire set of receptions in S^c. Thus the total rate of flow of information across a network cut is bounded by the conditional mutual information across the cut.

In cases where we are interested in the maximum achievable rate between a particular source node to a sink node, the problem reduces to identifying the minimum cut of mutual information across the network with source in S and the sink in S^c. Figure 2.2 depicts this concept for a single relay channel. The Cut-Set bound on maximum achievable rate R^{13} is given by

$$C \leq \max_{P(X_1, X_2)} \min \left[I(X_1; Y_2, Y_3 | X_2), I(X_1, X_2; Y_3) \right] \tag{2.2}$$

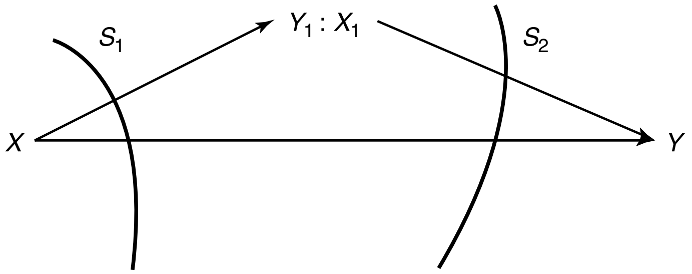

Figure 2.2: Illustration of Cut-Set bound for the single relay channel

2.3 Decode-Forward Scheme for the Relay Channel

The Decode-Forward technique is a commonly used mode of relay operation. This scheme provides encoding functions at both the source and the relay and a corresponding decoding function at the destination. The relay decodes the entire source message and then cooperates with the source over a subsequent transmission. The Decode-Forward scheme achieves the cut-set bound for a physically-degraded relay channel wherein the destination reception is strictly inferior to the relay in terms of additive Gaussian noise. Such a channel automatically entails decodability at the relay given that the destination decodes the source message. Therefore, for a degraded relay channel, no additional restrictions are imposed by the decode-forward formulation, thereby achieving capacity.

When compared to the Cut-Set bound, the Decode-Forward scheme is sub-optimal on the first cut, i.e., the broadcast cut in the system. Instead of the $I(X_1; Y_2, Y_3|X_2)$ we have for the broadcast cut, we have a more stringent bound given by $I(X_1; Y_2|X_2)$. The cooperation is achieved in blocks. The relay helps the destination by reducing its uncertainty about the source transmission in the previous block. The source in a given block transmits codes containing information about the new message and also cooperates with the relay on the previous message block.

The achievable rate region for a relay channel using the Decode-Forward scheme is given by

$$R = \max_{P(X_1,X_2)} \min\left[I(X_1; Y_2|X_2), I(X_1, X_2; Y_3)\right] \tag{2.3}$$

The Decode-Forward scheme usually performs very close to the Cut-Set bound in situations where the source-relay channel is stronger than the source-destination channel. In most practical cases of interest, the relay is usually somewhere in between the source-destination pair, making this condition true. The added simplicity of implementing the Decode-Forward scheme popularized it for general (not necessarily degraded) relay channels.

2.4 Compress-Forward Scheme for the Relay Channel

The Compress-Forward scheme relies on the relay node to provide information about the source transmission to the destination. Unlike in the case of a Decode-Forward scheme, the relay node does not have the capability to decode the entire source message in the block. Instead it sends a compressed version $\hat{Y_2}^n$ of its reception Y_2^n to the destination. The extent of this compression depends on the capacity of the downlink from the relay to the destination.

The destination node of the Compress-Forward scheme decodes based on Y_3^n, which contains the direct channel as well as the compressed version of Y_2^n. The achievability of this scheme is proved in [8]. The encoding is done in a block Markov fashion and the decoding is joint based on both receptions. The maximum achievable rate using this scheme is given by

$$R = \sup\left[I(X_1; Y_3, \hat{Y_2}|X_2)\right] \tag{2.4}$$

subject to the downlink channel constraint

$$I(X_2, Y_3) \geq I(Y_2; \hat{Y_2}|X_2, Y_3)$$

The Compress-Forward scheme usually performs well in cases where the relay location is quite close to the destination node, implying that the source-relay channel is not significantly better. In that case the availability of a compressed version of the relay reception helps the overall decoding at the destination node.

CHAPTER 3

CHANNEL MODEL AND FRAMEWORK

3.1 Channel Model

A discrete-time memoryless Gaussian channel model is employed throughout this work. Given a transmit sequence X^n and received sequence of symbols Y^n with discretized time as $i = 1, 2, ..., n$, the channel is characterized by

$$P_{Y^n|X^n}(y^n|x^n) = \prod_{i=1}^{n} P_{Y|X}(y_i|x_i)$$

where each $P_{Y/X}$ is Gaussian.

In the case of multiple sources, the model is extended such that each node receives a linear combination of all the transmissions airing at that instant with appropriate distance based attenuation with an additive Gaussian noise component corresponding to receiver noise.

It is further assumed that all nodes are capable of transmitting and receiving simultaneously. A quadratic path loss model is used. A further assumption is made about free space transmission. Given a pair of nodes i and j, the signal transmission depends only on distance d_{ij} between them

$$Y_j^n = F(d_{ij})X_i^n$$

where $F(x) = 1/x$ is a quadratic loss in energy.

Average transmit power limitations are imposed on each transmitting node. Given a node i, let the signal transmission be X_i^n; then we have

$$E[X_i^2] \leq P_i$$

3.2 Framework

We make the following assumptions:

- Relays are autonomously mobile in a restricted spatial region around the source destination.
- Relays act in complete coordination with the small block of nodes in the immediate neighborhood.
- There is a mechanism for exchange and update of codebooks within each block.
- The relays know the spatial locations and transmit powers of nodes lying immediately beyond the block but causing significant interference to transmissions within the block. Such nodes constitute the interference neighborhood of the block.
- Nodes much farther away with negligible interference on the block are treated as Gaussian noise and are accounted for using an increased received noise variance.
- All transmitting nodes in the network use Gaussian codebooks.

Figure 3.1 depicts an example ad hoc network block in the aforementioned framework.

3.3 Notation

We use the following standard notation throughout this work:

- Symbol sequence X_i^n is used for signal transmitted from node i.
- Y_i^n denotes the signal received at node i.
- Sum of received Gaussian noise and interference at node i is denoted by Z_i^n.
- The transmit power at node i is called P_i.
- The Euclidean distance between nodes i and j is denoted by d_{ij}.

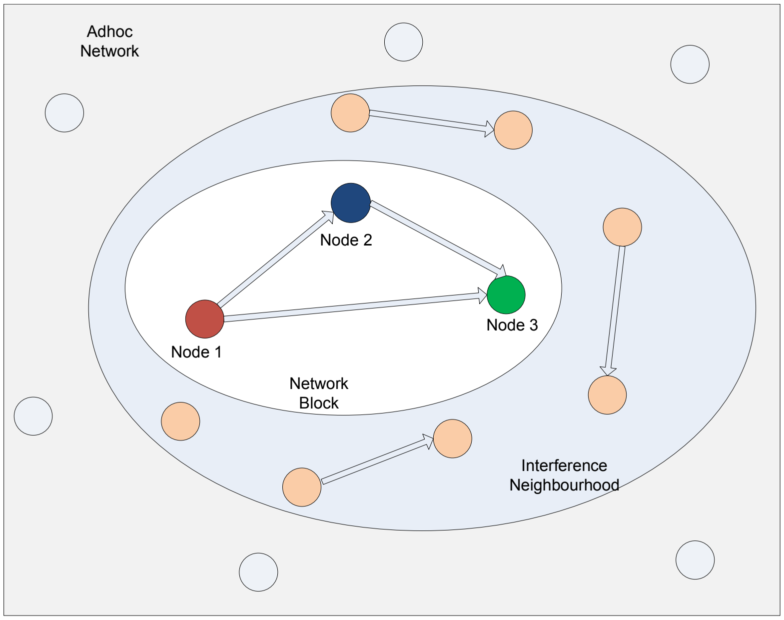

Figure 3.1: An example single relay channel block in the specified framework

- The signal attenuation in a link between node i and j is denoted by $a_{ij} = 1/d_{ij}$.
- Gaussian channel capacity is denoted by $C(SNR) = \frac{1}{2}\log(1 + SNR)$.

For the network shown in Figure 3.1,

$$\begin{aligned} Y_2 &= a_{12}X_1 + Z_2 && (3.1) \\ Y_3 &= a_{13}X_1 + a_{23}X_2 + Z_3 && (3.2) \\ E[X_i^2] &= P_i && (3.3) \\ E[Z_i^2] &= N_i \quad \forall\, i && (3.4) \end{aligned}$$

CHAPTER 4

SINGLE RELAY CHANNEL UNDER EXTERNAL INTERFERENCE

In this chapter we analyze the performance of a single relay channel in terms of maximum achievable rate between a source-destination node pair with assistance from a single relay. As a good starting point for analysis, we consider the case where only one external interfering node is present in the immediate neighborhood of the single relay channel block. This problem is akin to the comparison of performance of single relay channel under various relay techniques presented in [6]. One important difference lies in the fact that influence of external interfering transmissions is introduced into the analysis. The resulting solution is compared to the case where no external interference is present. Performance analysis and a qualitative comparison are carried out using three different metrics: the Cut-Set bound, Decode-Forward rate and the Compress-Forward rate.

Formulation We assume the following:

- The Source node 1 is located at point $(0, 0)$ and has a transmit power constraint of P_1.
- The destination node 3 is located at point $(d, 0)$.
- The relay node 2 moves as (rd, θ) where r lies in $[-2, 2]$ and θ lies in $[0, 2\pi)$.
- The relay has a transmit power constraint of P_2.
- The receiver noises (excluding interference) at nodes 2 and 3 are uncorrelated Gaussian with powers N_2^0 and N_3^0 respectively.
- We consider an external interfering node 4 within a circle of radius $2d$ around the source.

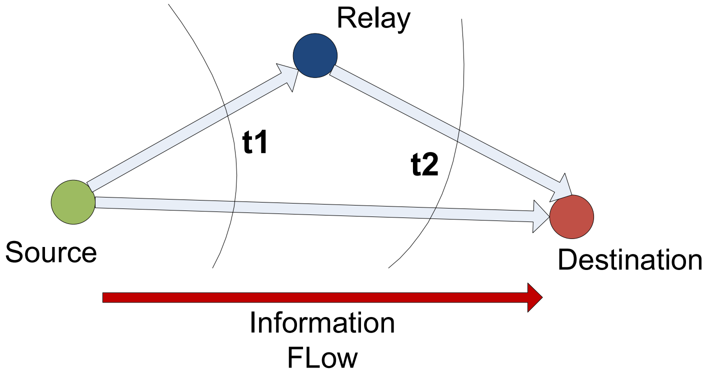

Figure 4.1: Constituent cuts of the Cut-Set bound

- The effect of interference for our purposes is completely characterized by a parameter ρ_N which is the correlation between interference-noise receptions at nodes 2 and 3.
- We make a comparison across schemes by varying the position of the relay within the region of interest.

$$E[X_1^2] = P_1,\ E[X_2^2] = P_2$$

$$E[Z_2^2] = N_2 = a_{42}^2 P_4 + N_2^0 \tag{4.1}$$

$$E[Z_3^2] = N_3 = a_{43}^2 P_4 + N_3^0 \tag{4.2}$$

where $a_{ij} = 1/d_{ij}$

$$\rho_N = \frac{E[Z_2 Z_3]}{\sqrt{E[Z_2^2]E[Z_3^2]}} \tag{4.3}$$

4.1 Cut-Set Bound

The Cut-Set bound of Sec. 2.2 is an upper bound to the maximum achievable rate between a source and a sink node. Figure 4.1 depicts the two cuts which constitute the bound for the relay channel. We use this bound here to help identify potential performance gains under appropriate relay placement. Applying the Cut-Set bound [7], we obtain

$$\begin{aligned} C &\leq \max_{P(X_1,X_2)} \min\left[I(X_1;Y_2,Y_3|X_2), I(X_1,X_2;Y_3)\right] && (4.4)\\ &\leq \max_{\rho} \min\left[I(X_1;Y_2,Y_3|X_2), I(X_1,X_2;Y_3)\right] && (4.5) \end{aligned}$$

where $\rho = \frac{E[X_1X_2]}{\sqrt{E[X_1^2]E[X_2^2]}}$ is the normalized correlation between X_1 and X_2.

$$\begin{aligned} I(X_1;Y_2,Y_3|X_2) &= H(Y_2,Y_3|X_2) - H(Y_2,Y_3|X_1,X_2)\\ H(Y_2,Y_3|X_2) &= \frac{1}{2}\log\left(1-\rho^2 \begin{vmatrix} \frac{P_1}{rd} + \frac{N_2}{1-\rho^2} & \frac{P_1}{rd^2} + \frac{\rho_N\sqrt{N_2N_3}}{1-\rho^2} \\ \frac{P_1}{rd^2} + \frac{\rho_N\sqrt{N_2N_3}}{1-\rho^2} & \frac{P_1}{d} + \frac{N_3}{1-\rho^2} \end{vmatrix}\right)\\ H(Y_2,Y_3|X_1,X_2) &= \frac{1}{2}\log\left(\begin{vmatrix} N_2 & \rho_N\sqrt{N_2N_3} \\ \rho_N\sqrt{N_2N_3} & N_3 \end{vmatrix}\right) \end{aligned}$$

Combining the two terms, we have

$$I(X_1;Y_2,Y_3|X_2) = C\left(\frac{P_1(1-\rho^2)\left[\left(\frac{\sqrt{N_3}}{rd} - \frac{\sqrt{N_2}}{d}\right)^2 + 2(1-\rho_N)\frac{\sqrt{N_2N_3}}{rd^2}\right]}{(1-\rho_N^2)N_2N_3}\right) \tag{4.6}$$

$$\begin{aligned} I(X_1,X_2;Y_3) &= H(Y_3) - H(Y_3|X_1,X_2)\\ H(Y_3) &= \frac{1}{2}\log\left(\frac{P_1}{d^2} + \frac{P_2}{d_{23}^2} + 2\rho\frac{\sqrt{P_1P_2}}{d_{23}d} + N_3\right)\\ H(Y_3|X_1,X_2) &= \frac{1}{2}\log\left(N_3\right)\\ I(X_1,X_2;Y_3) &= C\left(\frac{\frac{P_1}{d^2} + \frac{P_2}{d_{23}^2} + 2\rho\frac{\sqrt{P_2P_3}}{d_{23}d}}{N_3}\right) \end{aligned}$$

where $d_{23} = d\sqrt{1+r^2-2r\cos\theta}$.

The actual choice of ρ depends on the network configuration being realized and is assumed to be known to both the source and the relay. The upper-bound corresponding to various locations of the relay coupled with the best choice of ρ are plotted for comparison with Decode-Forward and

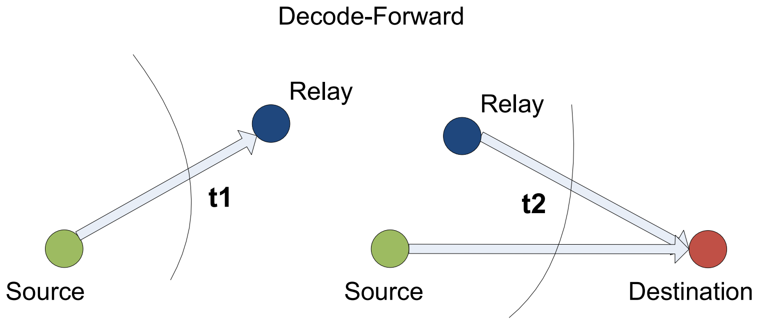

Figure 4.2: Decode-Forward scheme for the relay channel

Compress-Forward schemes. When the second cut (multiple-access) is not limiting, interference-noise correlation between the relay and destination receivers materializes as the $(1-\rho_N^2)$ term in the denominator of the first-cut (broadcast), leading to a performance gain in terms of network capacity.

4.2 Decode-Forward

The Cut-Set bound is tight in the case of a physically degraded relay channel. The Decode-Forward scheme discussed in the background section enables us to achieve this rate [8]. This scheme has gained popularity due to the ease of implementation and the fact that its rate is close to the Cut-Set bound when the relay node is in the neighborhood of the source-destination pair. This scheme may be used sub-optimally even in the case of a non-degraded relay channel. Figure 4.2 shows the two terms bounding the achievable rate in the Decode-Forward scheme. We show in this work that Decode-Forward performs poorly in the presence of external interference. The maximum possible rate using this relay scheme is given by

$$R \leq \max_{P(X_1,X_2)} \min\left[I(X_1;Y_2|X_2), I(X_1,X_2;Y_3)\right] \tag{4.7}$$

$$R \leq \max_{\rho} \min\left[I(X_1;Y_2|X_2), I(X_1,X_2;Y_3)\right] \tag{4.8}$$

where ρ is the correlation between X_1 and X_2

$$\begin{aligned} I(X_1;Y_2|X_2) &= H(Y_2|X_2) - H(Y_2|X_1,X_2) \\ H(Y_2|X_2) &= \frac{1}{2}\log\left(\frac{P1}{(rd)^2}(1-\rho^2)+N_2\right) \\ H(Y_2|X_1,X_2) &= \frac{1}{2}\log\left(N_2\right) \\ I(X_1;Y_2|X_2) &= C\left(\frac{P_1(1-\rho^2)\left[\frac{1}{(rd)^2}\right]}{N_2}\right) \end{aligned}$$

From the earlier calculation in the case of the Cut-Set bound, we have

$$\begin{aligned} I(X_1,X_2;Y_3) &= H(Y_3) - H(Y_3|X_1,X_2) \\ I(X_1,X_2;Y_3) &= C\left(\frac{\frac{P_1}{d^2}+\frac{P_2}{d_{23}^2}+\frac{2\rho\sqrt{P_2P_3}}{d_{23}d}}{N_3}\right) \end{aligned}$$

where $d_{23} = d\sqrt{1+r^2-2r\cos\theta}$.

The cooperation term between X_1 and X_2 denoted by ρ still needs to be optimized to maximize the minimum of the two terms. However, one important difference from the Cut-Set bound is seen in the fact that there is no performance gain due to noise correlation between the relay and destination. In fact, both the multiple-access and broadcast terms in the rate region characterization take a hit because N_2 and N_3 terms in the denominators are constituted partly by external interference, thus decreasing the overall maximum achievable rate.

4.3 Compress-Forward

Compress-Forward is yet another relay scheme introduced in [8]. Unlike Decode-Forward, the relay does not attempt to decode the transmission from the source; instead, it just forwards its received signal after further encoding (compression) based on the capacity of its link to the destination. Figure 4.3 shows the high level idea of a Compress-Forward scheme. The achievable rate using Compress-Forward is given by

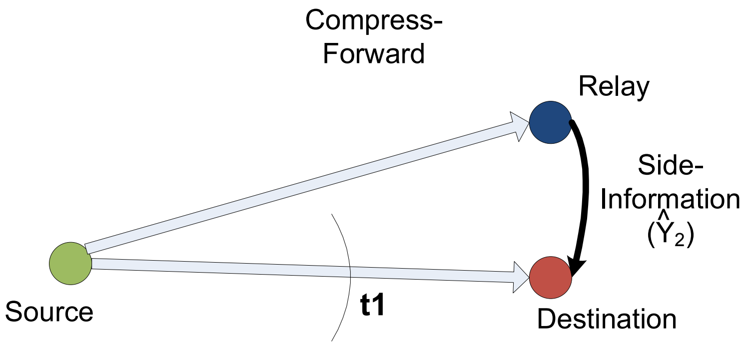

Figure 4.3: Compress-Forward scheme for the relay channel

$$R \leq I(X_1; \hat{Y_2}, Y_3 | X_2) \tag{4.9}$$

where $\hat{Y_2} = Y_2 + \hat{N_2}$ is a sufficiently compressed version of Y_2 subject to channel constraints to the destination given by

$$I(Y_2; \hat{Y_2} | X_2, Y_3) \leq I(X_2; Y_3) \tag{4.10}$$

For the given network configuration,

$$R = C\left(\frac{P_1 \left[\left(\frac{\sqrt{N_3}}{rd} - \frac{\sqrt{N_2}}{d} \right)^2 + 2(1-\rho_N)\frac{\sqrt{N_2 N_3}}{rd^2} + \frac{\hat{N_2}}{d^2} \right]}{N_2 N_3 (1-\rho_N^2) + \hat{N_2} N_3} \right) \tag{4.11}$$

where $\hat{N_2}$ is the minimum degradation to be added to Y_2 satisfying the inequality

$$\hat{N_2} \geq \frac{P_1 \left[\left(\frac{\sqrt{N_3}}{rd} - \frac{\sqrt{N_2}}{d} \right)^2 + 2(1-\rho_N)\frac{\sqrt{N_2 N_3}}{rd^2} \right] + N_2 N_3 (1-\rho_N^2)}{\frac{P_2}{d_{23}^2}} \tag{4.12}$$

where $d_{23} = d\sqrt{1 + r^2 - 2r\cos\theta}$.

We observe that the decorrelation term $(1 - \rho_N^2)$ which gives performance gain in the Cut-Set bound reappears in the Compress-Forward relay scheme. Upon simulation, the corresponding gain is easily seen in the maximum possible rate. The Compress-Forward scheme under certain capacity conditions on the relay-destination link, parallels a MIMO receiver with correlated noise at two antennas. Hence, the performance gain is to be expected from the noise correlation. The only additional complexity that the relay channel adds is that we have to do this decorrelation with a compressed version of one of the antenna receptions.

4.4 System with Multiple Interferers

Though the derivations in the previous section are for a case with single interferer, the analysis extends directly to the case with multiple interferers. However, we need to extend the definition of ρ_N to include effects from all interfering nodes. Supposing there are m interfering nodes labeled 4 to $m+4$ that cause significant interference to the single relay channel block, the corresponding definition of ρ_N changes to

$$\rho_N = \frac{E[Z_2 Z_3]}{\sqrt{E[Z_2^2]E[Z_3^2]}}, \tag{4.13}$$

$$E[Z_2^2] = N_2 = \sum_{i=4}^{m+4} \left(a_{i2}^2 P_i\right) + N_2^0,$$

$$E[Z_3^2] = N_3 = \sum_{i=4}^{m+4} \left(a_{i3}^2 P_i\right) + N_3^0$$

The rate regions for this case reflect the same trend as in the case of a system with a single interferer. However, the optimal relay location now becomes a more complicated choice dependent on the mobility constraints on the relay and the degree of correlation ρ_N between the receptions at the relay and the destination. The general rule of thumb is this: The higher the correlation, the better the performance, given the conditions that the capacity of the relay-destination and source-relay link is sufficient to exploit the noise correlation.

4.5 Simulations

We simulate the results in two separate cases, one where the relay is mobile in one dimension and is collinear with the source-destination pair. Comparison is made to the case where there is no external interference. It is observed that Decode-Forward is best suited for maximum rate in the absence of external interference and the optimum relay location is close to the midpoint of source-destination. In the case where an external interferer is present, Compress-Forward performs better and reflects gains expected in the Cut-Set bound. The location of the relay is a delicate balance between getting reception from the source and maximizing interference correlation with the destination.

A second simulation fortifying this observation of improved performance using Compress-Forward is shown by plotting a contour of achievable rates using Compress-Forward as the relay spatially relocates in a two-dimensional region around the source-destination. The maxima in the Cut-Set bound are directly reflected using the Compress-Forward scheme.

This clearly shows that performance gains can be obtained in network relaying by taking due care of the interference signal at intermediate nodes rather than each node treating interference in a stand-alone fashion.

4.5.1 No External Interference

We run simulations in the case where there are no nodes external to the single relay channel within the specified interference neighborhood. This serves as a control for comparison in the presence of interfering nodes. The simulation results are shown in Figure 4.4.

As expected, the Decode-Forward scheme performs well in the region where the relay lies between the source-destination pair. It is further observed that Compress-Forward is inferior in most of the region of interest.

4.5.2 With External Interference

In the presence of an external interference source, it is seen that the overall rate region collapses. However, the Cut-Set bound plot shows that there exists a potential to harness the decorrelation gain in regions where the

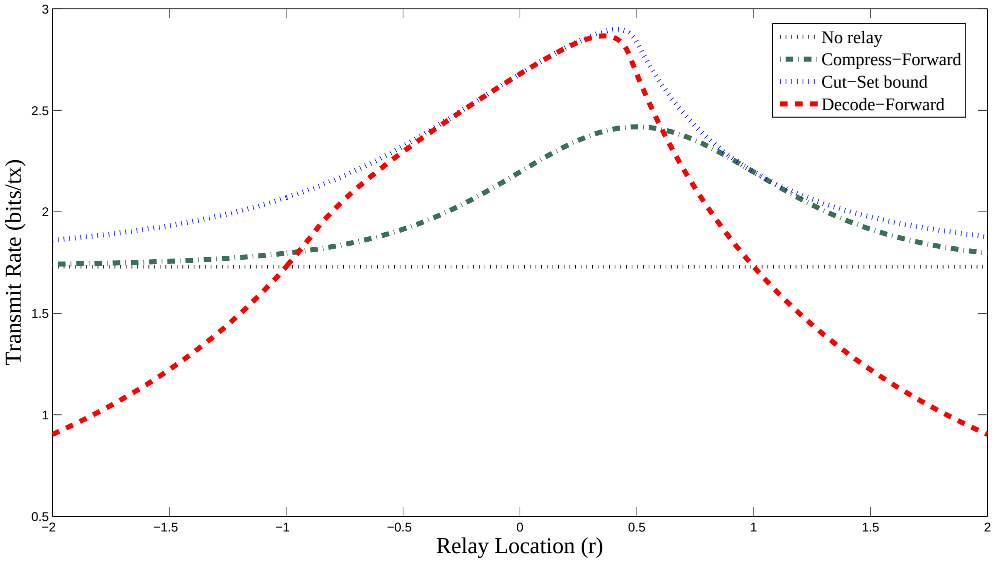

Figure 4.4: Plot comparing various schemes when relay is collinear with source destination pair under no external interference (source is at $r = 0$ and destination is at $r = 1$)

interference-noise correlation is high between the relay and the destination. The corresponding plot is shown in Figure 4.5.

4.5.3 Contour Plot with Multiple Interferers

We simulate a scenario where there are multiple sources of interference in the neighborhood of the block and observe the existence of a global optimum in the outer bound of achievable rates characterized by the Cut-Set bound. It is noted that this maximum corresponds to the location of highest noise correlation with a non-limiting broadcast cut. Furthermore, the corresponding maximum is also observed for the case of the Compress-Forward rate. The simulation plot is shown in Figure 4.6.

4.5.4 Relay Scheme Separation Regions

Simulations to determine the spatial regions wherein one relay scheme outperforms the other are performed. External interferers are present in the neighborhood as shown in the simulation plot. In this scenario, it is ob-

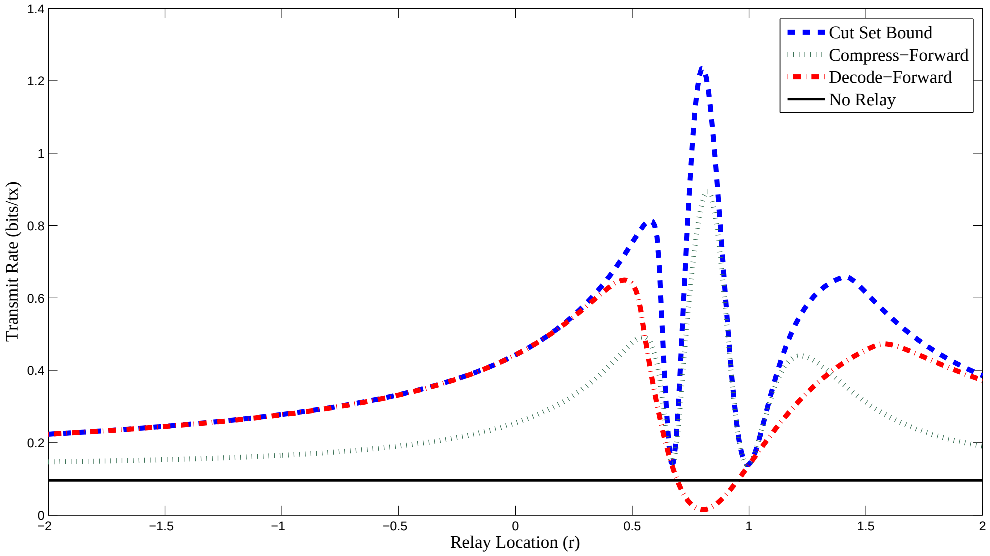

Figure 4.5: Plot comparing various schemes when relay is collinear with source-destination pair under an external interferer at $r = 0.8$, θ=$\pi/50$, with source at $r = 0$ and destination at $r = 1$

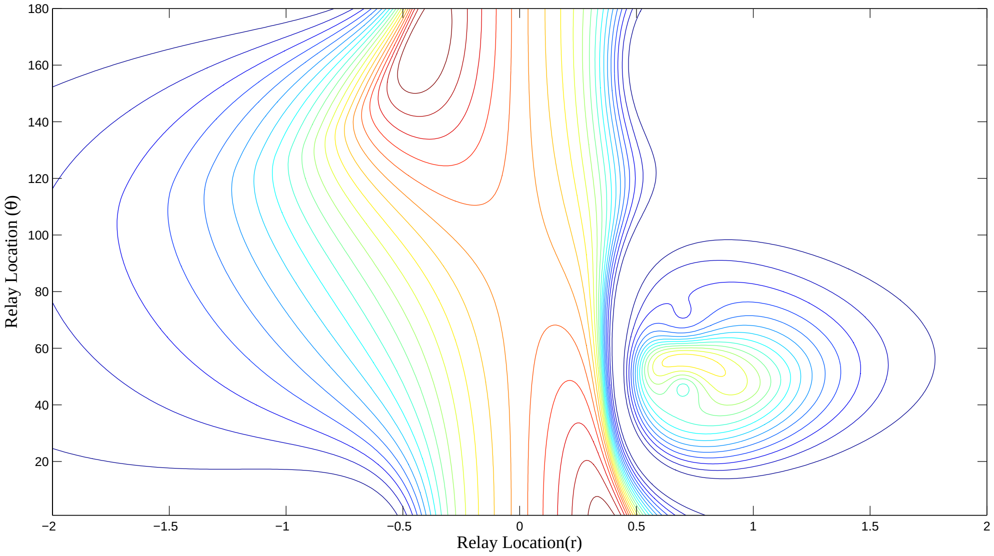

Figure 4.6: Contour plot showing Cut-Set bound as a function of relay location with three external interferers in the neighborhood $r = [1.5, 0.7, 0.7]$, $\theta = [45, 72, 162]$, with the source at $r = 0$ and destination at $r = 1$

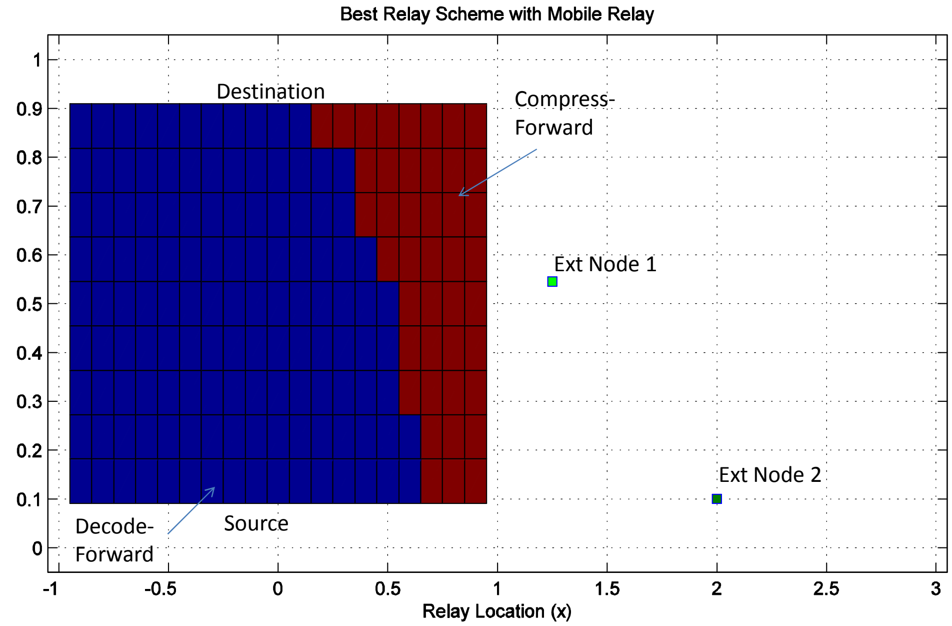

Figure 4.7: Plot showing the best relay scheme with mobile relay with restrictions and multiple external interferers

served that Compress-Forward performs better in regions of higher interference correlation corresponding directly to proximity to the interfering nodes. Figure 4.7 shows the corresponding plot with separation regions.

CHAPTER 5

CONCLUSION

In this work, we have demonstrated that a relay can play an important role in dealing with interference and boosting the capacity region of small network blocks. If the appropriate relaying scheme is used to handle interference in conjunction with acquiring the best possible spatial location, performance gains can be obtained. The Compress-Forward scheme is usually limited to situations where the relay is close to the destination, but we show that it enhances performance even when the relay is distant from the destination node in the presence of external interference. This work demonstrates the fact that relay based interference mitigation can be extremely useful when the system is marred by heavy interference and the relay has a good correlation of the interfering signal with the final receiver.

REFERENCES

[1] O. Sahin, E. Erkip, and O. Simeone,"Interference channel with a relay: Models, relaying strategies, bounds," in *Information Theory and Applications Workshop*, Feb. 2009, pp. 90-95.

[2] R. Dabora, I. Maric, and A. Goldsmith,"Relay strategies for interference-forwarding," in *Proceedings of 2008 IEEE Information Theory Workshop*, Porto, Portugal, May 2008, pp. 46-50.

[3] A. S. Avestimehr, S. Diggavi, and D. Tse,"A deterministic model for wireless relay networks and its capacity," in *2007 IEEE Information Theory Workshop on Information Theory for Wireless Networks*, July 2007, pp. 6-11.

[4] I. Krikidis, J. Thompson, and S. McLaughlin,"Relay selection issues for amplify-and-forward cooperative systems with interference," in *IEEE WCNC 2009*, April 2009, pp. 1-6.

[5] C. K. Lo, S. Vishwanath, and R. W. Heath Jr., "Relay subset selection in wireless networks using partial decode-and-forward transmission," in *IEEE Transactions on Vechicular Technology*, vol. 58, pp. 692-704, Feb. 2009.

[6] G. Kramer, "Topics in multi-user information theory," *Foundations and Trends in Communication and Information Theory*, vol. 4, no. 4-5, pp. 265-444, 2007.

[7] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, New York, NY: Wiley-Interscience, 1991.

[8] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, pp. 572-584, Sept. 1979.