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> The vast majority of students who cannot solve story problems are not restricted by their ability to read, but by their limited understanding of mathematics. They are frequently proficient in computation but are unable to identify which operation should be used in a particular context. The more complex the context (such as rate problems, problems which require more than one operation, or problems with extraneous information), the more difficulty students have. It would not be difficult to determine whether a student's problem is due to reading difficulties or to a limited understanding of mathematics.
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> Whatever the source of the student's difficulty, the most inappropriate response is to use "low verbal" approaches to instruction and the statement of problems. . . . Not only has this approach been shown to be ineffective, it denies students the opportunity to read in mathematics (Chambers, 1986, pp. 137-138).

The relation between reading and mathematics should be of particular interest to Michigan teachers. Mathematical problem solving, like reading, is a process that requires not only skill in recognizing symbols but also strategic knowledge applied within a particular context or for a particular purpose. Introduction of the new Michigan Education Assessment Program (MEAP) Essential Skills Mathematics Test has made many teachers aware of the similarities between the reading process and mathematical thinking. Teachers who are teaching the Michigan Definition of Reading can use the same techniques to help their students become better mathematical problem solvers.

Mathematics education has recently been redefined. The primary document is *Curriculum and Evaluation Standards for School Mathematics* prepared by the National Council of Teachers of Mathematics (NCTM). The Standards emphasize problem solving, reasoning and communication. They also recommend a new approach to instruction, calling for a reversal of the old pattern of teaching math facts before application. "Instead of the expectation that skill in computation should precede word problems, experience with problems helps develop the ability to compute" (National Council of Teachers of Mathematics, 1989, p. 9).

The Michigan Essential Goals and Objectives for Mathematics Education reflect the influence of the NCTM Standards. Consequently, a substantial proportion of the MEAP *Essential Skills Mathematics Test* is devoted to problem solving, including the knowledge of appropriate strategies as well as the application of strategies and computation skills to word problems. Because students have traditionally found word problems to be more difficult than number sentences, both students and teachers are finding the new mathematics test challenging. Contrary to popular belief, it is not usually the reading component that makes story problems difficult—if by reading we mean the traditional definition of decoding text.

Word problems are an ideal opportunity to begin applying the Michigan Definition of Reading to mathematics because it is constructing meaning from the text rather than decoding the text that makes word problems difficult. To construct meaning in reading a child need not have mastered all prerequisite decoding skills in order to convert the text from symbols to words. She must, however, activate prior knowledge, including associations for vocabulary and situation. Then, she must select and apply appropriate reading strategies on the basis of her purpose for reading and the type of text. As she moves through the text, she continually monitors meaning. Finally, the skilled reader checks the outcome or perceived meaning against her expectations, asking "Does that make sense?" Constructing meaning in mathematics requires similar skills.

This paper examines briefly several research studies that highlight some of the interesting "constructing meaning" requirements related to word problems.

Vocabulary: Words used by students in everyday conversation often take on special, more restricted, meanings when used in a mathematical context. Students who have not had an opportunity to discuss the differences in meaning related to context are likely to be confused when they encounter these terms in word problems. To illustrate, Durkin and Shire (1991) present the following list of words whose mathematical meaning is quite different from their everyday meaning.

> above, altogether, angle, as great as, average, base, below, between, big, bottom, change, circular, collection, common, complete, coordinates, degree, difference, different, differentiation, divide, down, element, even, expand, face, figure, form, grid, high, improper, integration, leaves, left, little, low, make, match, mean, model, moment, natural, odd, one, operation, overall, parallel, path, place, point, power, product, proper, property, radical, rational, real, record, reflection, relation, remainder, right, root, row, same, sign, significance, similar, small, square, table, tangent, times, top, union, unit, up, value, volume, vulgar, and discreet mathematics (p. 74).

Imagine the possibilities of the following assignment:

> Write a story to illustrate the differences between fashion coordinates and mathematical coordinates. You may wish to use the idea that fashion coordinates change with the four seasons and that mathematical coordinates are located within one of four quadrants.

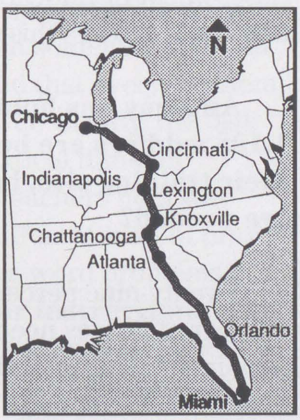
Imagine also the benefit for students who have explored the idea that the context in which the word appears determines which meaning they should use. In their investigation of this issue, Earp and Tanner (1980) found that sixth-graders accurately decoded 93% of the mathematical words in their textbook but could explain the meaning of only 50% of those words. When the words were presented again within the context of the textbook sentence in which they first appeared, students managed to define an additional 8%. When the words were presented again in rewritten sentences that provided even stronger contextual clues, students were able to define an additional 15%. Earp and Tanner associate the students' lack of understanding with the absence of opportunities to "talk" mathematics in most classrooms and recommend more discussion time devoted to mathematical ideas.

Cue words: Many teachers try to boost students' problem solving skills by teaching so-called "cue words" that will help the student select the correct operation to solve a word problem. Although cue words may sometimes help, they are no substitute for teaching students to understand the problem context. Stockdale (1991) analyzed word problems in the three best-selling textbook series of 1985 for grades 3 through 6 and found that, on average, only 30% of the word problems included useful cue words. Further, specific cue words often referred to different operations in different problems. Cue words are almost never found in the kinds of interesting problems that students encounter in print every day—the kinds of problems that motivate sustained problem solving effort. Sometimes called "direct quote" problems, these are questions generated in response to quotes from newpapers, almanacs, popular magazines, or books such as the *Guinness Book of World Records*. Here's an example from *USA Today*.

The Quote:

Madden to Miami

John Madden's Maddencruiser arrived in Miami Wednesday afternoon after a 1,460-mile, 27 hour journey from Chicago. The party of five made just one overnight stop in Cinncinnati. There was no respite Tuesday; they traveled all night. Here's the route:



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The Problem: *Do you think that Madden observed the speed limit all the way? Explain.*

Readability: It is difficult to apply readability formulas to word problems for three reasons: word problems are usually very brief segments of text; they may contain 3- or 4-syllable technical terms that are instructionally appropriate for students at the grade level; and there is no method for evaluating the "readability" of illustrations, tables, or graphs which are often an integral part of the problem statement. (See Fry, 1977.) As an alternative to readability formulas, MEAP relied on the judgement of experienced classroom teachers to determine item readability. As the math test was developed, every test item was reviewed by 50 to75 teachers experienced at that grade level. Their instructions always asked "Is the wording clear and unambiguous? If not, please write in your suggestions for change." Although the wording of many items was revised to match teachers' suggestions, the revisions seldom produced the expected improvement in scores.

In order to investigate the impact of readability on test scores, researchers created a 15-item story problem test. The test was then modified to reflect three different readability levels: below Grade 4, Grade 4 to Grade 6, and above Grade 6. Readability at each level was varied by two different methods: either by adjusting vocabulary or by adjusting sentence structure and length. Then, the six forms of the test were administered to over 1000 children in Grades 3 to 6. "There was no effect of readability level on problem difficulty—not even the hint of an effect" (Paul, Nibbelink & Hoover, 1986). Problem difficulty was clearly the result of the mathematical understanding required rather than readability.

Reading strategically: Earp (1970) noted long ago that the special nature of verbal arithmetic problems requires the deliberate adoption of strategies such as reading at a slower rate than narrative text, re-reading, and adopting an aggressive attitude—questioning the text and paying special attention to special uses of common words. Students must not only be made aware of the need to vary their reading strategies but also need opportunities to practice the application of reading strategies to mathematical text.

Reading accuracy: Hollander (1991) suggests that teachers' insistence on a precisely correct reading of word problems should be modified in light of changing views of the reading process and the meaning of miscues. She observed that students who were the most accurate oral readers were frequently the least successful at solving word problems and notes that reading miscues are less a cause for worry than evidence of the students' efforts to get at the meaning. A sampling of her guidelines for helping students solve word problems illustrates the issues.

- Ability to read mathematical text with a high degree of accuracy does not necessarily lead to the successful solution of verbal problems and may, instead, interfere with problem solving.
- Excessive reference to the text may be an indication that the reader is having serious problems with comprehension.
- Students should be actively helped to develop awareness of the necessity for either total or partial rereading and, if necessary, rewording of verbal arithmetic problems until each problem becomes comprehensible . . . Metacognitive awareness of these translation techniques should be developed.

Miscue analysis can help the teacher determine whether a student's error is the result of inaccurate decoding, misinterpretation of vocabulary, selection of an inappropriate reading strategy or mathematical operation, or faulty computation. Beyond these types of errors, teachers must also be concerned with the student's ability to construct meaning and the difficulties that may be imposed by prior knowledge.

Constructing meaning: Solving word problems is very much a reading task—if by reading you mean constructing meaning, but "for most students, school mathematics is a habit of problem-solving without sense-making; one learns to read the problem, to extract the relevant numbers and operation to be used, to perform the operation, and to write down the result—without ever thinking about what it all means" (*Reshaping School Mathematics*, 1990, p. 32). What needs emphasis is the notion of making sense of the text.

Try this problem:

There are 26 sheep and 10 goats on a ship. How old is the captain?

More than 75% of the French students queried produced a numerical answer, most frequently 36. American students are not exempt. In 1983, the National Assessment of Educational Progress asked 8th graders this question:

An army bus holds 36 soldiers. If 1,128 soldiers are being bussed to their training site, how many busses are needed?

Twenty-nine percent wrote that the number of busses needed was 31, remainder 12, even though that answer doesn't make sense as a response to the question "How many busses are needed?" Only 23% gave the correct answer (Janvier, 1990).

Why do students make these kinds of errors? Because their prior knowledge overrides the information in the text. In the context of school mathematics, word problems always have answers. One correct answer per problem. The answer is almost always a number obtained by somehow manipulating the numbers that appear in the problem statement.

Here's another example:

There are 125 sheep and 5 dogs in a flock. How old is the shepherd?

When presented with this problem, more than three school children in four responded confidently with a numerical answer (Reusser, 1986). Listen to a typical student thinking out loud:

> One hundred twenty-five plus 5 is one hundred thirty. That's too big. One hundred twenty-five minus five gives one hundred twenty. Still too big. One hundred twenty-five divided by 5 makes 25. That works! I think the shepherd is 25 years old. *(Reshaping School Mathematics, 1990)*

Obviously, the student quoted above worked very hard to construct meaning but was misled by the conviction (prior knowledge) that any problem presented has meaning even if it is not apparent to the student; that by manipulating all of the numbers, you can somehow arrive at a correct answer; and that every problem has a "correct" answer. Clearly, the task of creating mathematical thinkers requires that we as teachers help students to acquire a different kind of prior knowledge regarding word problems.

Implications for instruction: Students at all levels need more practice reading mathematics text strategically.

> Most mathematics classes are organized around the material in a textbook, which students are seldom encouraged to read and make sense of. For the most part, the text becomes a source of homework problems rather than a source of information and ideas, leaving knowledge in the hands of the teacher to be transmitted to students rather than empowering students to seek information for themselves. (Lappan & Schram, 1989, p. 17).

Mathematics texts are seldom included in classroom reading materials, but there are many excellent commercial materials available at all grade levels.

Reading strategies that are helpful with expository text can also be helpful in mathematics. Reciprocal teaching, SQ3R, QAR, Mapping and DRTA can all be applied to mathematics text. Think Alouds are particularly useful for modeling problem solving. In addition to reading strategies, students need experience using common problem solving strategies such as finding patterns, guess-and-check, solving a simpler problem, working backwards, and others included in the *Michigan Essential Goals and Objectives for Mathematics Education.* Finally, teachers must model strategic thinking for their students in mathematics just the way they do in reading. Let students listen to your thinking as you solve a problem when you don't know the answer in advance. Make problem solving a classroom priority and topic of frequent discussion. When given an answer and asked to write a related problem, students' creations tend to be complex and interesting. Student problems and other written assignments including math-related reports, essays, stories, or lab reports can also provide expository text for classroom discussion.

The goal is to help students to graduate from brief statements of word problems with a single correct answer to the interesting but "inconsiderate" problems encountered in daily life.

> The non-routine problem situations envisioned in [NCTM] standards are much broader in scope and substance than isolated puzzle problems, which provide contexts for using particular formulas or algorithms but do not offer opportunities for true problem solving. Real-world problems are not ready-made exercises with easily processed procedures

and numbers. Situations that allow students to experience problems with "messy" numbers of too much or not enough information or that have multiple solutions, each with different consequences, will better prepare them to solve problems they are likely to encounter in their daily lives. (National Council of Teachers of Mathematics, 1989, p. 76)

As described in the *Michigan Essential Goals and Objectives*, both reading and mathematics are defined as a process of constructing meaning; both require the student to know, select and apply appropriate strategies for extracting meaning from text; and both recognize the importance of developing the student's metacognitive awareness of the thinking s/he is doing. Strategies developed for reading instruction can and should be applied in mathematics. Similarly the current emphasis on application of mathematics to "real-life" problems will require more "reading to do," since real problems tend to be ill-defined, requiring further research into their causes, and investigation of the constraints on possible solutions. As mathematics instruction becomes increasingly problem-centered and discourse-based, Michigan students will have an advantage as reading and mathematics instruction overlap.

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Mathematics teachers are reading:

Curriculum and Evaluation Standards for School Mathematics. (1989). National Council of Teachers of Mathematics (NCTM), 1906 Association Drive, Reston, VA 22091, 703-620-9840, Fax: 703-476-2970. 258 pp. $25

Everybody Counts: A Report to the Nation on the Future of Mathematics Education. (1989). National Academy Press, 2101 Constitution Avenue, N.W., Washington, DC 20418, 1-800-624-6242. 114 pp. $7.95

Reshaping School Mathematics: A Philosophy and Framework for Curriculum. (1990). National Academy Press, 2101 Constitution Avenue, N.W., Washington, DC 20418, 1-800-624-6242. 76 pp. $7.95

Professional Standards for Teaching Mathematics. (1991). National Council of Teachers of Mathematics (NCTM), Department E.,1906 Association Drive, Reston, VA 22091, 703-620-9840, Fax: 703-476-2970. 96 pp. $25