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Comparison between methods for calculating the volume of rock blocks

M T Carriero[1], A M Ferrero[2], M R Migliazza[1], G Umili[2]

[1]Department of Structural, Geotechnical and Building Engineering, Polytechnic of Turin, Corso Duca degli Abruzzi 24, 10129 Turin, Italy

[2]Department of Earth Sciences, University of Turin, via Valperga Caluso 35, 10125 Turin, Italy

mtcarriero@hotmail.it

Abstract. Many methods for calculating the volume of rock blocks have been developed in the last decades. The first attempts to estimate such crucial quantity produced analytical equations to calculate the mean and variance of volume, considering blocks created by three discontinuity sets with a certain spacing probability distribution. From then, the research community followed three kinds of approaches for calculating block volume: the fully analytical one (e.g., Palmstrøm's formula), the fully probabilistic one (e.g., Discrete Fracture Network generators), and the mixed one (e.g., In Situ Block Size Distribution). In this paper, a comparison among the different methods is presented, supported by numerical examples, highlighting their strengths and disadvantages in terms of reliability and repeatability.

1. Introduction

In rock masses, blocks are delimited by discontinuity planes and can have various shapes depending on the number and orientation of the discontinuities which form them. Therefore, the calculation of rock block volume is not trivial, and many attempts have been made to propose simple analytical methods.

One of the most commonly used is Palmstrøm's formula [1] for calculating the volume of a block created by the intersection of three discontinuity sets, based on the spacing values and the angles between pairs of sets.

Lopes and Lana [2] proposed an analytical solution developed for tabular, prismatic, and tetrahedral blocks. The solution is based on linear algebra and vectorial analysis concepts. It depends on discontinuity orientations, spacing, and block shape. A discontinuity plane with dip ϕ_n, dip direction θ_n, and spacing E_n can be represented by its director vector $\vec{\mu'_n} = (a_n, b_n, c_n)$, where $\left\|\vec{\mu'_n}\right\| = E_n$ and

$$\begin{bmatrix} a_n \\ b_n \\ c_n \end{bmatrix} = \begin{bmatrix} E_n \sin(\phi_n)\sin(\theta_n) \\ E_n \sin(\phi_n)\cos(\theta_n) \\ E_n \cos(\phi_n) \end{bmatrix} \tag{1}$$

They demonstrate that the volume V of a block created by three discontinuity sets can be calculated as:

$$V = \left|\left(\vec{\mu'_1} \times \vec{\mu'_2}\right) \cdot \vec{\mu'_3}\right| = |det(M)| \tag{2}$$

where

$$M = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

The triple product gives the volume of a six-face solid. Still, spatially these direction vectors may also define a solid with a different number of faces; therefore, the result could need to be multiplied by a constant.

Analytical methods give a deterministic value, which is usually considered as a mean value of the block volume when the discontinuity spacing is represented by its average measure, but actually, no evidence of the correctness of this assumption can be found in the literature.

The fully probabilistic methods aim to simulate the 3D reconstruction of a fractured rock mass volume: they consist of Discrete Fracture Network generators [i.e., 3,4]. 3DEC [5], as a 3-dimensional distinct code, is able to generate a virtual rock mass model as a system of blocks in a specific volume by defining the discontinuity sets in terms of their orientation, spacing, and persistence. It is possible, in this way, to obtain a statistical distribution of blocks having variable volume and shape depending on the statistical variability of the defined geometric parameters. Thus, the distribution of blocks is statistically representative of the actual rock mass structure only if the model boundaries, which represent fictitious, non-natural limits, do not alter the block volume distribution. This condition can be fulfilled by identifying the representative elementary volume (REV), which is defined as the minimum volume of the reference model so that the edges of the model do not influence the result.

Mixed methods also exist: many methodologies were developed to forecast the IBSD (In Situ Block Size Distribution) based on analytical solutions and coefficients obtained from DFN [6,7]. The IBSD represents the cumulative curve of the in situ blocks, and its construction considers the frequency distributions of spacing values.

The different methods will be briefly compared in the following sections through examples, highlighting their strengths and disadvantages in terms of reliability and repeatability.

2. Analytical methods for calculating block volume

A simple case is considered to compare Palmstrøm's and geometric methods: a block is created by the intersection of three discontinuity sets (K1, K2, K3), each characterized by a spacing value (table 1).

The dip of K1 varies from 1° to 90°, while K2 and K3 have a fixed orientation and are mutually perpendicular. Therefore, when the value of the dip of K1 is 90°, the three sets are perpendicular. With this configuration, the two methods give the same results, even changing spacing values: we considered three cases in which spacing is equal for the three sets (S = S1 = S2 = S3) and one case in which spacing values are mutually different. This fact is shown in figure 1, where a single trend for each case, obtained by varying the dip of K1, is plotted.

Table 1. Orientation and spacing of the three sets.

K1		K2		K3	
Orientation [dip/dip dir]	Spacing S1 [m]	Orientation [dip/dip dir]	Spacing S2 [m]	Orientation [dip/dip dir]	Spacing S3 [m]
[1:90]/000	0.5	90/090	0.5	00/000	0.5
[1:90]/000	1	90/090	1	00/000	1
[1:90]/000	1.5	90/090	1.5	00/000	1.5
[1:90]/000	2	90/090	2	00/000	2
[1:90]/000	1.2	90/090	0.7	00/000	2.1

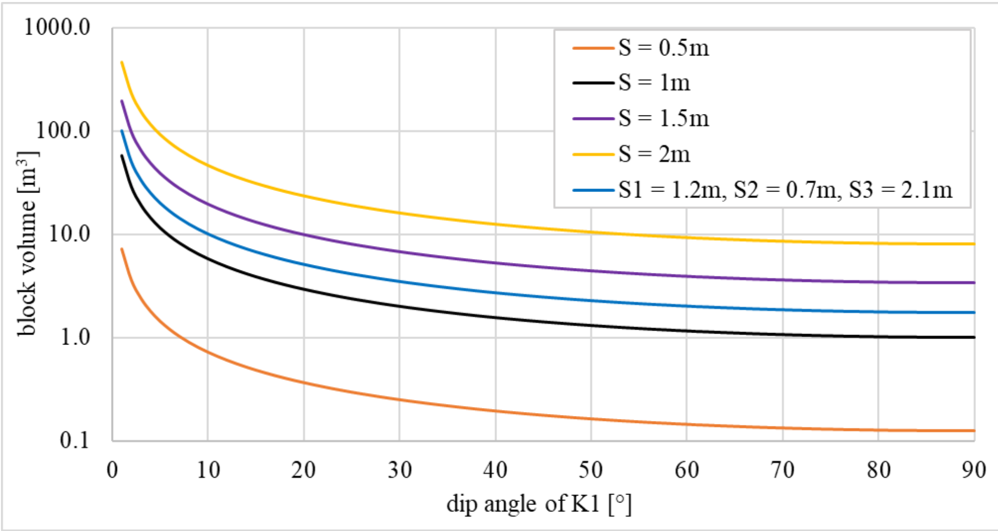
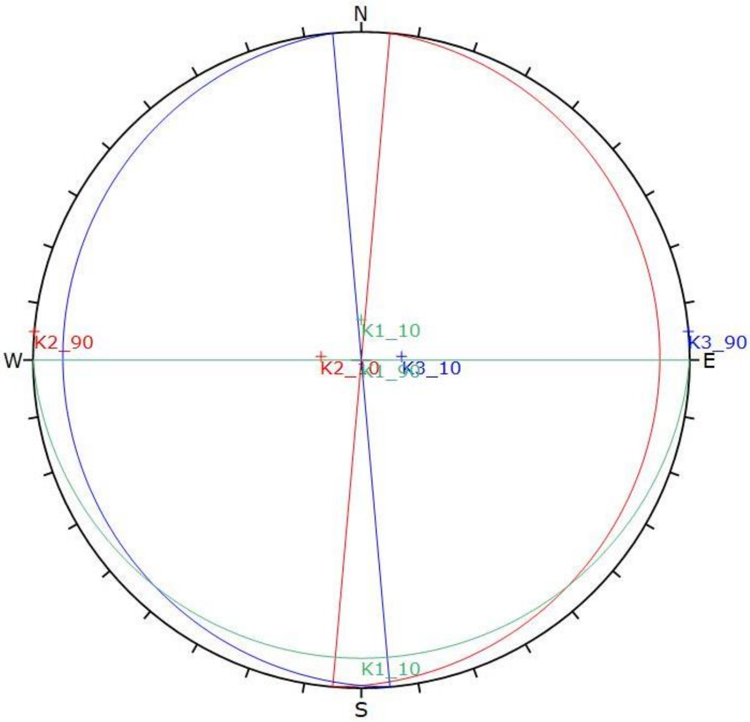


Figure 1. Trends of volumes calculated with Palmstrøm's and geometric method (coincident for the cases in Table 1) by varying spacing.

Then, to investigate the possible effects of orientation, a series of cases was considered: the idea was to have a block formed by three sets oriented so that two of the angles among sets are identical, while the third is the biggest one. Dip values of the three sets were varied so that angles change but maintain these properties. In figure 2, the limits of the configuration are reported.



	Color	Dip	Dip Direction	Label
User Planes				
2	red	10	95	K2_10
3	red	90	95	K2_90
4	blue	10	265	K3_10
5	blue	90	265	K3_90
6	green	10	180	K1_10
7	green	90	180	K1_90

Plot Mode	Pole Vectors
Vector Count	0 (0 Entries)
Hemisphere	Lower
Projection	Equal Area

Figure 2. Scheme of the configuration: plotted planes represent the limits within which dip values were made vary.

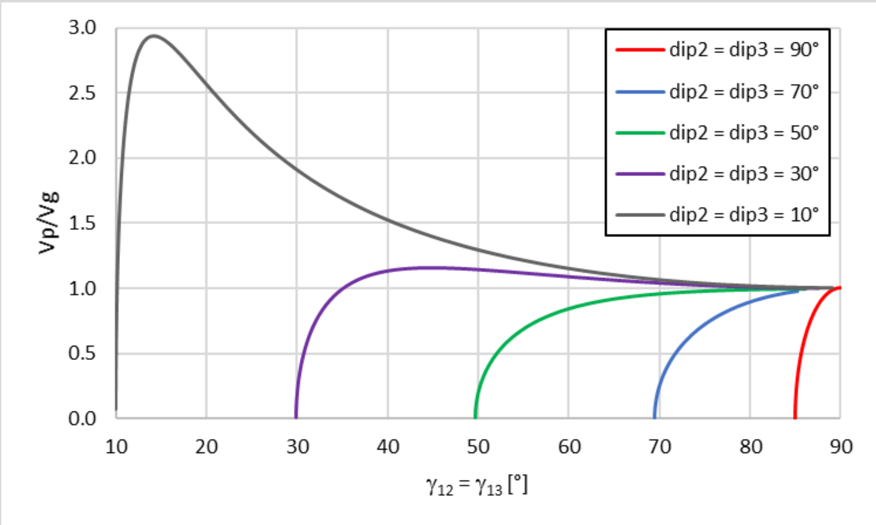


Figure 3. Ratios between volumes Vp and Vg by varying angles between sets.

Figure 3 shows how the ratio between volumes calculated with Palmstrøm's (Vp) and geometric (Vg, Equation 2) methods changes by varying the angles between sets ($\gamma_{12} = \gamma_{13} < \gamma_{23}$). One can desume that Palmstrøm's formula is valid only in a few particular configurations, but in general, its result underestimates or overestimates the block volume.

3. Block volume distribution

The virtual rock mass model (figure 4) was created with 3DEC considering a cubic volume affected by the presence of three discontinuities sets with orientation and spacing values reported in table 2. The number of joints for each set was chosen large enough in a way that the entire model was affected by the joints. The origin of the Cartesian coordinate system was located in the center of the model, which was set as the point of origin of each joint set.

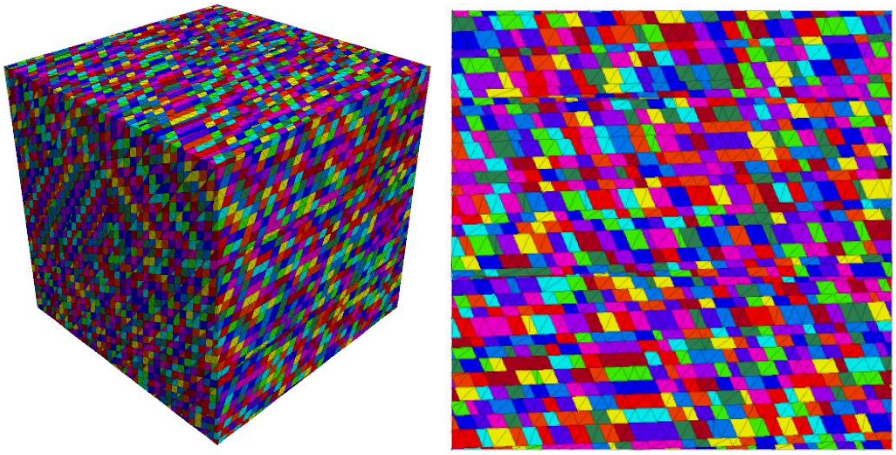


Figure 4. Three-dimensional model and block faces in case of 30% standard deviation on spacing.

To identify the REV, a first series of geometrical reconstruction was carried out by varying the cube edge length between 5m and 50 m (table 3) and by keeping the dip and dip direction and spacing values constant and equal to their average value (table 2, combination 0). Therefore, the distributions of volumes of the rock blocks for different REV values were built.

By analyzing cumulative frequency distributions of the block volumes for each edge length, it is possible to note as the curves tend to coincide as the dimension increases (figure 5). Considering a REV reached if at least 80% of the volume of the blocks fall into the same modal class, a dimension of the virtual rock mass of 35 m was chosen for the following analyses.

Table 2. Orientation and spacing of the three sets.

Combination	K1		K2		K3	
	Orientation [dip/dip dir]	Spacing [m]	Orientation [dip/dip dir]	Spacing [m]	Orientation [dip/dip dir]	Spacing [m]
0	86/180	1	24/185	1	70/120	1
1	86/180	1 (std 0.1)	24/185	1 (std 0.1)	70/120	1 (std 0.1)
2	86/180	1 (std 0.2)	24/185	1 (std 0.2)	70/120	1 (std 0.2)
3	86/180	1 (std 0.3)	24/185	1 (std 0.3)	70/120	1 (std 0.3)

Table 3. Cube dimension and rock block volumes for Combination 0.

Edge length [m]	Rock Block Volume					
	Maximum [m^3]	Minimum [m^3]	Average value [m^3]	Standard Deviation [m^3]	Modal value [m^3]	Modal Frequency [%]
5	1.37	2.68E-05	0.61	0.51	1.0	29.41
10	1.37	3.03E-05	0.90	0.54	1.4	46.95
15	1.37	2.89E-05	1.03	0.49	1.4	59.74
20	2.16	3.39E-05	1.10	0.45	1.4	67.66
25	2.74	5.82E-05	1.15	0.43	1.4	73.20
30	2.74	1.02E-04	1.19	0.40	1.4	77.24
35	2.74	8.94E-05	1.24	0.33	1.4	82.01
40	2.74	7.47E-05	1.23	0.36	1.4	82.43
45	2.74	1.68E-04	1.24	0.34	1.4	84.30
50	2.74	2.51E-04	1.26	0.32	1.4	85.78

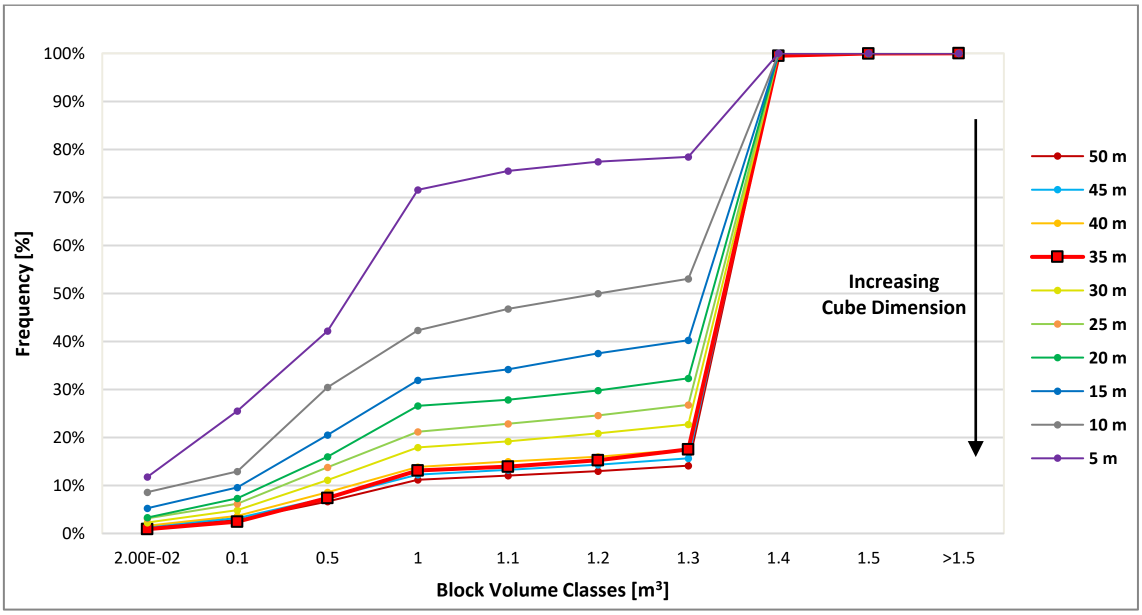


Figure 5. Cumulative Frequency Distributions for different REV values.

A following series of analyses was carried out to assess the influence of spacing variability on the volume of blocks. For this purpose, dip and dip direction angles were considered constant, while the spacing was defined by a fixed average value and a variable standard deviation (0, 10%, 20%, and 30%) (Combinations 0-3 of table 2). In this way, four volume distributions were obtained (figure 6).

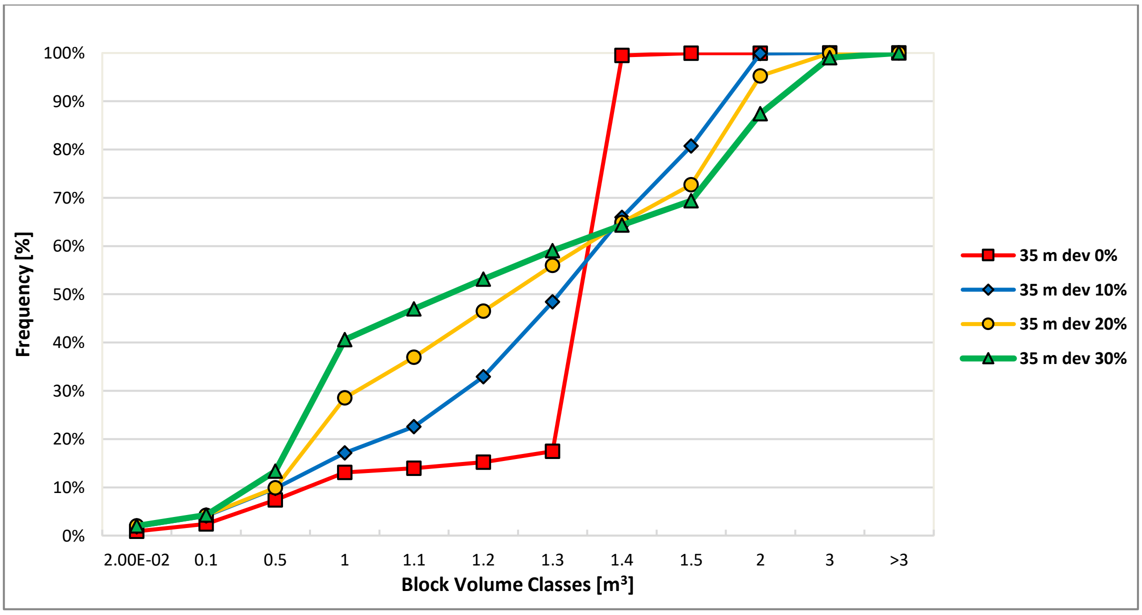


Figure 6. Cumulative Frequency Distributions for Combinations of table 2.

Table 4. 3DEC cubic model dimension and rock block volumes obtained for Combinations of table 2.

Edge length [m]	Combination	Rock Block Volume					
		Maximum [m^3]	Minimum [m^3]	Median value [m^3]	Standard Deviation [m^3]	Modal value [m^3]	Modal Frequency [%]
35	0	2.74	8.94E-05	1.35	0.33	1.4	82.01
	1	3.51	1.51E-04	1.31	0.41	2.0	19.13
	2	3.33	1.49E-04	1.25	0.51	2.0	22.54
	3	5.15	1.05E-04	1.15	0.67	1.0	27.24

4. Relationship between analytical values and DFN results

The main limitation of the analytical methods is the lack of statistical meaning of the deterministic output [8]. As described in [9], the spacing of a discontinuity set can be considered a continuous random variable; therefore, its cumulative distribution function, denoted as $F(x) = P\{X \leq x\}$, defines the probability that a given spacing value X is less than x.

A set of 100,000 input spacing values was randomly generated from a gaussian distribution with a mean equal to 1 and standard deviation equal to 10% of the mean (Combination 1): the corresponding cumulative distribution was obtained. The same was done for all the discontinuity sets and the other combinations. Volume values were then calculated with Equation 2 (table 5). Figure 7 shows the comparison between cumulative frequency distributions obtained with 3DEC and the analytical method considering the different combinations in table 2.

Table 5. Block volumes obtained with the analytical method for Combinations of table 2.

Combination	Spacing Standard Deviation [m]	Maximum [m^3]	Minimum [m^3]	Volume(50%) [m^3]	Standard Deviation [m^3]
0	0.0	1.37	1.37	1.37	0
1	0.1	3.92	0.62	1.37	0.88
2	0.2	9.31	0.21	1.37	2.48
3	0.3	16.73	0.04	1.37	4.65

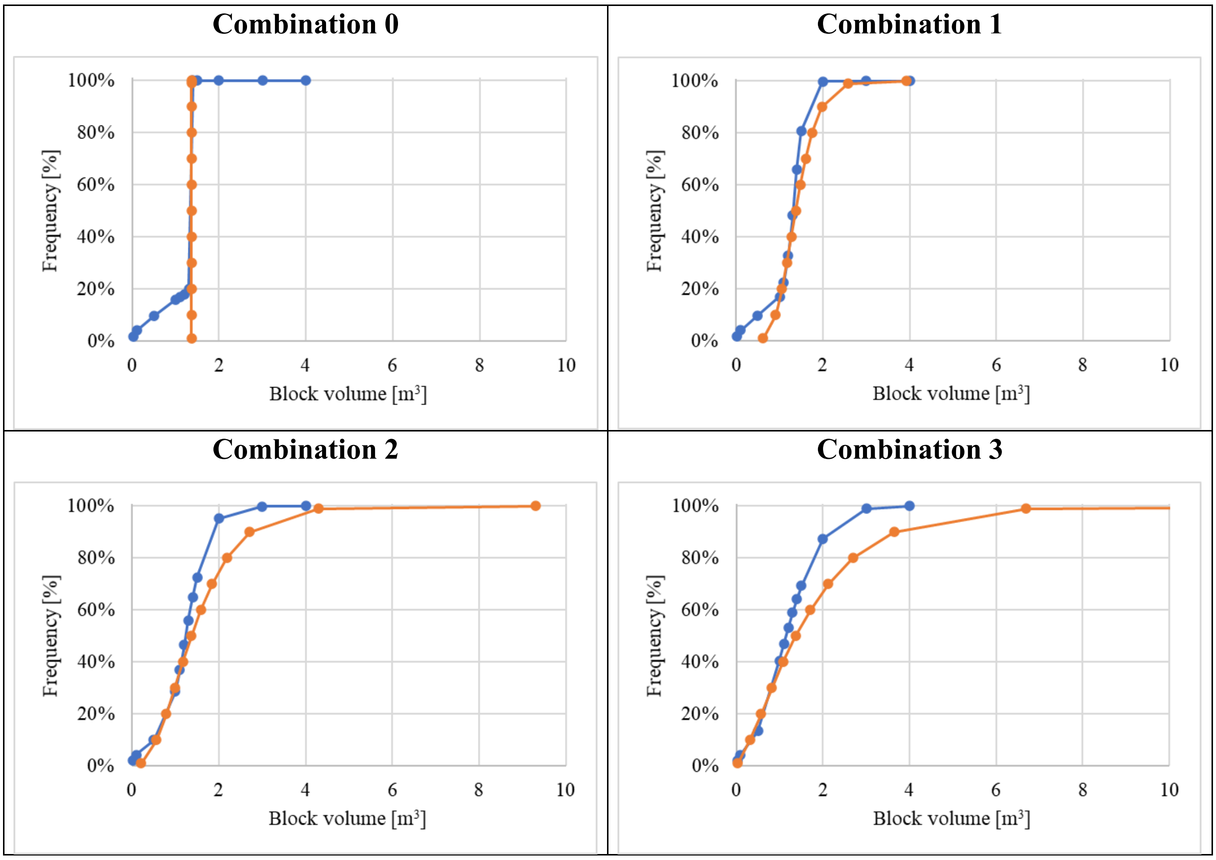


Figure 7. Comparison between cumulative frequency distributions obtained with 3DEC (blue) and analytical method (orange) considering the combinations in table 2

By comparing the obtained cumulative distributions, it is possible to make the following considerations. In Combination 0, the curves overlap almost entirely: the differences in the ends depend on the finite dimension of the 3DEC model. In particular, model boundaries produce a truncation effect that generates blocks smaller than the expected ones: this determines the differences in the frequency range 0-20%.

In Combination 1, there is still a very good agreement between curves, and the truncation effect is less evident. In Combinations 2 and 3, the overlapping parts reduce, particularly above 60% frequency.

In general, the differences in the curves increase more as the standard deviation increases. This fact may be due also to the different statistical distribution used for the spacing data extraction: Gaussian for the geometric approach and uniform for 3DEC.

5. Conclusions

A statistically robust block volume distribution assumes a fundamental role: it associates to each value in the volume range a probability of not being exceeded. Since volume depends on orientation and spacing variability, sampled data of each discontinuity set should contain a statistically sufficient number of measurements and be representative of the variability in the considered rock mass, independently of the method that will be used for creating the IBSD. Therefore, the larger the maximum spacing, the greater the considered area must be to allow enough measurements to be made.

The analytical methods' deterministic nature does not allow for taking into account the natural variability of block volumes. IBSD represents the evolution in the description of block volume aleatory nature.

The great advantage of using DFN generators for creating IBSD is their capability of recreating the entire rock mass structure, visually representing it in 3D. However, when using a DFN generator, the REV value should always be identified and considered for choosing the model size. This step is fundamental for avoiding boundary effects and blocks' artificial truncation.

The described approach for creating the curve starting from the geometrical method is more time-consuming since a Monte Carlo simulation and a vectorial calculation are required. Still, it could be much cheaper than buying a commercial DFN code.

Although it is necessary to investigate further the relationship between the IBSD built with 3DEC and the one created with the proposed approach, this simulation is a first attempt to assess the reliability and the statistical meaning of block volumes calculated based on data sampled in geostructural surveys.

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