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Chess Training and Mathematical Problem-Solving: The Role of Teaching Heuristics in Transfer of Learning

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Recently, chess in school activities has attracted the attention of policy makers, teachers and researchers. Chess has been claimed to be an effective tool to enhance children's mathematical skills. In this study, 931 primary school pupils were recruited and then assigned to two treatment groups attending chess lessons, or to a control group, and were tested on their mathematical problem-solving abilities. The two treatment groups differed from each other on the teaching method adopted: The trainers of one group taught the pupils heuristics to solve chess problems, whereas the trainers of the other treatment group did not teach any chess-specific problem-solving heuristic. Results showed that the former group outperformed the other two groups. These results foster the hypothesis that a specific type of chess training does improve children's mathematical skills, and uphold the idea that teaching general heuristics can be an effective way to promote transfer of learning.

Keywords: chess; heuristics; mathematics; STEM education; transfer

INTRODUCTION

Recently, pupils' poor achievement in mathematics has been the subject of debate both in the United States (Hanushek, Peterson, & Woessmann, 2012; Richland, Stigler, & Holyoak, 2012) and in Europe (Grek, 2009). The current market requires more graduates in Science, Technology, Engineering, and Mathematics (STEM) subjects than graduates in the humanities. In addition, STEM-related jobs have become more competitive in recent years and require high skilled employees. Pupils' low mathematical ability is a serious impediment to the satisfaction of job market demands both quantitatively (number of graduates in STEM subjects) and qualitatively (level of mathematical competences in graduates).

Since pupils' poor mathematical achievement has become an issue, policy makers

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and researchers have investigated several alternative methods and activities with the aim of improving the effectiveness of mathematical teaching. Teaching chess in schools is one of these activities. Chess has recently become part of the school curriculum in several countries. Several large studies and educational projects involving chess are currently ongoing in the United Kingdom, Spain, Turkey, Germany, and Italy, among other countries. Moreover, the European Parliament has expressed its favourable opinion on using chess courses in schools as an educational tool (Binev, Attard-Montalto, Deva, Mauro, & Takkula, 2011) and, similarly, the Spanish Parliament has approved the implementation of chess courses during school hours.

The chess community's common optimistic opinion is that chess practice increases academic performance because chess is an intellectually demanding and stimulating game. Several policy makers share the view that chess "makes children smarter" (Garner, 2012), but is this belief justified?

State of the literature

- Since extensive empirical evidence suggests that chess players tend to be more intelligent than the general population, chess instruction has been recently proposed as an educational tool able to enhance children's cognitive and academic abilities.
- Thus, several studies have been carried out to demonstrate (or refute) the benefits of chess instruction, especially with regard to children's mathematical abilities.
- Chess instruction seems to effectively boost children's mathematical abilities, but some doubt still remains on the goodness of such practice. In fact, many studies lack a proper experimental design.

Contribution of this paper to the literature

- According to several authors, chess instruction may increase children's mathematical skills, because playing chess helps to shape children's way of thinking particularly when facing mathematical problems. The present study is the first attempt – to the best of our knowledge – to test this hypothesis.
- Understanding whether teaching chess problem-solving heuristics helps children to solve mathematical problems is a question of interest not only for the field of education, but also for the psychological issue of transfer of skills.
- The present study offers reliable results, thanks to the large sample size and the use of multilevel modelling.

Research on chess in school

Several studies (Bilalić, McLeod, & Gobet, 2007; Doll & Mayr, 1987; Frydman & Lynn, 1992; Grabner, Stern, & Neubauer, 2007; Horgan & Morgan, 1990) have suggested that chess players are often more intelligent than the general population. The fact that chess players tend to have superior intellectual abilities has upheld the idea that chess practice can make people more intelligent. Nevertheless, these findings do not prove that the practice of chess promotes the application of chess skills in other domains. The aforementioned studies have a correlational design, and thus the causal direction of the relationship still remains uncertain (Gobet & Campitelli, 2002). There are three possible scenarios that could explain the empirical evidence collected: (a) the game of chess actually improves people's intellectual abilities; (b) those with better mental abilities are attracted to chess and become chess players, achieve better results and thus tend to play more; (c) there are intervening factors – such as motivation towards the task, the ability to consider several alternatives and to decide which is the best in a limited period of time – that lead subjects attending a chess course to have a better expression of both intellectual and chess abilities.

To disentangle these possible explanations, several studies have tried to demonstrate the potential benefits of chess training on various cognitive abilities such as attention (Scholz et al., 2008), development of spatial concepts (Sigirtmac, 2012), general intelligence (Hong & Bart, 2007), and meta-cognition (Kazemi, Yektayar, & Abad, 2012). Other studies focused on academic variables, such as reading (Christiaen & Verhofstadt-Denève, 1981) and, mostly, mathematics. Recently, the positive influence that chess could exert on children's mathematical abilities has been investigated (Barrett & Fish, 2011; Kazemi et al., 2012; Sala, Gorini, & Pravettoni, 2015; Scholz et al., 2008; Trinchero, 2012). Some of these studies (Kazemi et al., 2012; Sala et al., 2015; Trinchero, 2012) deserve particular attention as they focused on the potential relationship between chess training and higher mathematical competencies, such as complex problem-solving, which are involved in OECD-PISA (Oecd, 2012) and IEA-TIMSS (Mullis & Martin, 2013) surveys. However, the aforementioned studies obtained controversial results and, in most cases, the research design was not appropriate (in particular, lack of randomization which might lead to subjects' self-selection) to prove the benefits of chess training on cognitive or academic skills (Gobet & Campitelli, 2006; Sala & Gobet, in press).

The problem of transfer in chess

Whether chess practice improves cognitive and/or academic skills in children raises an important theoretical and practical issue: The question of transfer of learning (Perkins & Salomon, 1994). Transfer of learning occurs when skills acquired in one domain generalize to other domains or fosters general cognitive skills. An important distinction regarding transfer is between *near-transfer*, where transfer occurs between closely related domains (e.g., driving two different types of car), and *far-transfer*, where transfer occurs between domains which are loosely related (Mestre, 2005), such as chess and mathematics. If the former it is believed to occur quite often, the latter seems to occur rarely (Donovan, Bransford, & Pellegrino, 1999; Gobet, 2015), because transfer seems to be a function of the extent to which two tasks have perceptual features and cognitive elements in common (Singley & Anderson, 1989; Thorndike & Woodworth, 1901).

These limitations regarding the phenomenon of transfer seem to apply to the field of chess too. In her classical study, Chi (1978) demonstrated that chess players' (both adults and children) memory for chess positions did not extend to digits recall. Chess players outperformed non-chess players at remembering chess positions, but no difference occurred with lists of digits. The same result was obtained in the study of Schneider, Gruber, Gold, and Opwis (1993). More recently, Unterrainer, Kaller, Leonhart, and Rahm (2011) found that chess players' planning abilities did not transfer to the Tower of London, a test assessing executive function and planning skills. Waters, Gobet, and Leyden (2002) chess players' perceptual skills did not transfer to visual memory of shapes. Finally, Bühren and Frank (2010) found that chess ability did not predict performance in the economic game known as beauty contest. In accordance with most of the literature on transfer, all these studies have shown that transfer is at best unlikely, and that chess players' special abilities are context-specific.

Why should chess practice improve children’s mathematical ability?

Research has shown that transfer of learning occurs only when domains share perceptual and/or conceptual features. Thus, transfer is more likely when the set of skills which are supposed to generalize are not domain-specific (Ericsson & Charness, 1994), which explains why chess masters do not show better abilities (compared to the general population) beyond the 64 squares. Nevertheless, if skills trained by practicing in a domain are general enough to be common to another domain, then transfer of learning can occur.

Learning chess basics may be such an activity. Primary school children – who have never played chess or novices with only a basic knowledge of the game – may benefit from the practice of chess, provided that this practice deals with contents which are shared by chess and mathematics domains. Chess is a game based on elements of quantitative and geometrical nature. Playing chess demands children to evaluate the interactions between arithmetical elements such as the values of the pieces and tempos (i.e., the number of moves needed to reach a desired configuration) in a geometrical space (the chessboard), and according to geometrical rules (the movement of the pieces). In other words, playing chess demands the use of basic arithmetical and geometrical abilities, such as adding and comparing the values of the pieces, and locating the pieces on the squares, and thus it is reasonable to suppose that playing chess trains to some extent the latter abilities (Scholz et al., 2008). Consistent with the "identical element theory" (Thorndike & Woodworth, 1901), playing chess is believed to improve basic mathematical abilities (such as simple arithmetic) because it is an activity dependent on those skills to a certain extent.

However, some authors (Bart, 2014; Kazemi et al., 2012; Root, 2006; Sala et al., 2015; Trinchero, 2012) have proposed that chess can boost mathematical abilities not only due to the mathematical features which chess possesses, but also to several general heuristics that chess players use during games. According to these authors, some of these heuristics uphold planning behaviour and monitor taken decisions, and are similar to the ones used in mathematical problem-solving tasks. Thus, chess practice can be used to train general strategies which are useful to solving mathematical problems.

Heuristics can be defined as "methods for arriving at satisfactory solutions with modest amount of computations" (Simon, 1990, p. 11). Heuristics help to interpret and to manage complex situations (such as chess configurations and mathematical problems) by reducing the possible options, and therefore reducing cognitive load (Shah & Oppenheimer, 2008). During a chess game players cannot calculate all the legal (i.e., allowed by the rules) moves and variants (i.e., sequences of moves), because there are too many to be all analysed simultaneously. Similarly, a mathematical problem cannot be solved by trying every possible combination of data and operations. Mathematics and chess are domains where brute force strategies – i.e., searching methods calculating all the possible options – are often ineffective. Heuristics are needed to interpret situations, establish aims, select salient information, and reflect on and monitoring the consequences of decisions. This strategic behaviour is common both to chess and mathematical problem-solving.

The idea that teaching general heuristics is an effective method to train transferable abilities is not new in educational research (Feuerstein, 1980; Feuerstein, Feuerstein, Falik, & Rand, 2006; Halpern, 1998; Perkins & Grotzer, 1997; Shayer, 1999). For example, in a meta-analysis on methods of teaching mathematical problem-solving, Marcucci (1980) reported that teaching general problem-solving heuristics had a positive impact on pupils' mathematical problem-solving skills. However, learning and yielding abstract heuristics without a context of application may be difficult for primary school children.

In fact, Feuerstein et al. (2006) have claimed that several general cognitive functions – such as the "ability to understand the existence of a problem" and "planning behaviour" – are trainable by a set of activities acting as media. Media must have two features to be effective: (a) pupils must not be too familiar with the activities, because the latter are supposed to demand some cognitive effort, and thus to induce a state of attention in pupils; and (b) pupils must be familiar enough with the activities, in order to avoid excessive cognitive effort to perform the task. In other words, the pupil must perform a set of novel activities in order to stimulate the use of cognitive resources. Nonetheless, these activities must not be completely unrelated to the tasks pupils are habitually involved in.

Playing chess may provide such activities. It is a well-known board game, based on geometry and arithmetic (e.g., adding the value of the pieces), and involves planning and calculation, which are concepts primary school children are familiar with. It is also a demanding and compelling game, rich in new situations for pupils, such as the different movements of the pieces, and the concepts of check and checkmate. According to this perspective, chess is believed to activate *analogical transfer* (Gick & Holyoak, 1980) from chess to the mathematics domain. Children who have been taught how to efficiently solve chess problems may be able to find analogies with the mathematical problem-solving process. More precisely, it is possible that teaching chess problem-solving heuristics – provided that the latter are not too specific – helps to build a set of one-to-one correspondences between chess and mathematical methods to solve problems. In fact, when facing both chess and mathematical problems, children must be able (a) to recognise and interpret a situation, (b) consider only a few alternatives among the many possible ones, and (c) select an option and monitor its consequences. Thus, training the correct methods to manage and solve chess problems should generalize to mathematical domains, enhancing children's mathematical problem-solving ability.

The present study

To evaluate the role of chess heuristics in promoting children's mathematical problem-solving skills, we compared the effectiveness of two different types of chess training in enhancing mathematical problem-solving abilities. One group was taught chess by experienced chess instructors, and one group was taught chess by school teachers, who had been previously taught chess to children. Both groups were explicitly asked to follow a precise didactic program (see Method for more details). However, only the chess instructors were asked to teach specific heuristics to solve chess problems, whereas the school teachers were not provided any specific instruction regarding the use of problem-solving heuristics. Thus, the two groups received the same treatment in terms of contents of the lessons, and the only difference was the teaching approach. In addition, the performance of the two treatment groups was compared to a control (do-nothing) group, in order to check for possible developmental or testing effects.

Our hypothesis is that transfer of learning from chess to mathematical problem-solving can occur only if chess practice is able to effectively train chess problem-solving heuristics. The mere exposure to chess practice is not likely to effectively activate any transfer of problem-solving skills. Starting from this assumption, the hypothesis of the study is that the group run by the chess instructors will both the school teachers' group and the control group in mathematical problem-solving scores.

METHOD

Participants

Fifty-three third, fourth and fifth grade classes from 20 different schools were randomly assigned to three groups (Table 1):

- Chess training performed by chess instructors.
- Chess training performed by school teachers.
- Control group, attending regular school activities.

Nine-hundred-and thirty-one third, fourth and fifth graders took part in the study. The mean age of the sample was 8.59 years (S.D. 0.75). The three groups did not significantly differ in terms of mean age (F (2, 930) = 2.706, ns) and in terms of mean pre-test mathematical abilities (F (2, 930) = 0.754, ns).

Table 1. The three groups in the study

Group	N of Classes	N of Participants
Chess instructors	18	320
School teachers	12	220
Control	23	391

Materials and procedure

The study duration was approximately six months (from December 2013, to May 2014). All the three groups performed two tests (see below) one week before the chess intervention, and the same two tests one week after the conclusion of the course. The pre-tests and post-tests consisted of the same items in order to guarantee comparability of the results and to exclude any difference in the items difficulty. The two tests administered at the beginning and at the end of the study were:

- Test of mathematical problem-solving ability: Mathematical problem-solving ability was assessed by seven OECD-PISA items (Oecd, 2012). These items consisted of mathematics-related tasks which were not directly linked to the regular school curriculum. On the contrary, OECD-PISA items focus on the ability to solve problems set in real-life situations. It must be noticed that OECD-PISA items are calibrated on students aged 15. For this reason, the items that were selected had contents which the participants could deal with (e.g., problems involving only whole numbers, and with easily understandable instructions). The items were therefore quite difficult to be solved by children, and we expected the participants to perform relatively poorly. Nonetheless, the test was able to evaluate the mathematical problem-solving ability of the children beyond their curriculum-based knowledge. The score range for this test was 0 – 7.
- Test of chess ability: Chess ability was assessed by a 12-items test designed by the authors and used in other studies (see Trinchero, 2012 for more details). Those who declared to be unable to play chess did not perform the chess pre and/or post-test. The score range for this test was 0 – 18.

The classes in the two experimental groups received chess lessons during school hours, whereas the classes of the control group participated in their regular lessons. The chess lessons were based on a method (SAM protocol, for details see Trinchero, 2012) especially designed for 7 – 11 year-old children which has already been used in several previous studies (Sala et. al., 2015; Trinchero, 2012). The mean duration of the training was 14.00 hours (S.D. 2.69) for the chess instructors' group, and 15.17 hours (S.D. 6.35) for the school teachers' group (t (538) = 2.924, p = .011). Along with in-presence chess lessons, the chess instructors' group and the school teachers' group pupils had the opportunity to play (mainly at home) a computer-assisted training (CAT) on the Web, a game providing 12 levels of chess training (for more details see Trinchero, 2012). CAT training was not mandatory, but highly recommended. The time of use of CAT was recorded for every participant. The mean time of use of CAT was 4.76 hours (S.D. 5.75) for the chess instructors' group, and 5.17 hours (S.D. 5.87) for the school teachers' group (t (538) = 0.818, p = .414).

As previously mentioned, the theoretical contents and the activities of the chess lessons were the same for the two treatment groups: Movements of the pieces, castling and promotion rules, check and checkmate, tactics and games. However, the school teachers were not provided with any specific instruction about the chess problem-solving heuristics to teach to the children, whereas chess instructors were. These heuristics dealt with chess-related problems like finding the shortest path to reach a square or to capture a piece, checking or checkmating in simple tactical or endgame situations, and evaluating short variants. All the heuristics that chess instructors used in their lessons aimed to help pupils to:

- recognise and interpret game situations (e.g., tactical positions, endgames, checkmate configurations) by focusing on relevant cues;
- narrow down the *candidate moves*, which are the moves to consider during analysis;
- select a move (or a variant), monitor its consequences, and change the move

if necessary.

An example of how to solve a chess problem using a simple heuristic is shown in Figure 1. As chess instructors are familiar with teaching such heuristics in their regular lectures, it was easy for the instructors involved in the experiment to comply with this request.

Finally, both the chess instructors and the school teachers were blind to the aim of the experiment. They were told that the study focused on the potential benefits of chess practice for children's mathematical skills, but they were not told anything about the role of teaching heuristics, and they did not know that the two experimental groups received two different treatments.

RESULTS

Since our data were nested – i.e., the participants were from 20 different schools – three multilevel linear models were run to estimate the variability in mathematical problem-solving ability between schools. Multilevel linear modelling was also used to reduce the risk of Type I error.

Main analysis

The descriptive statistics of the three groups are summarized in Table 2.

A multilevel linear model was performed in order to evaluate the role of group,

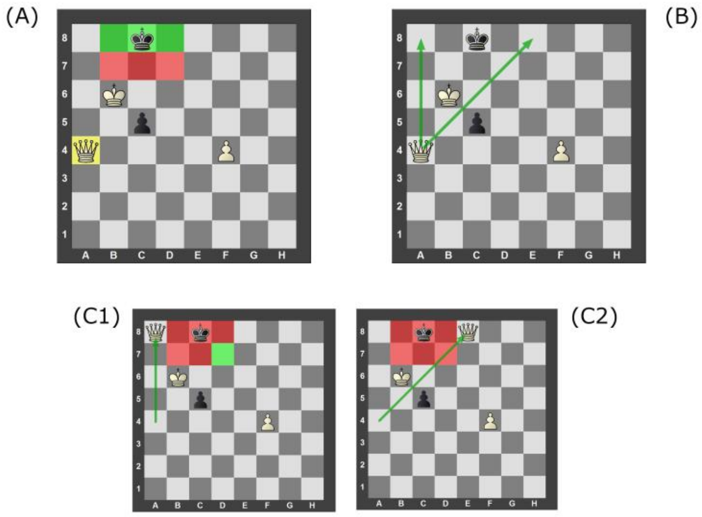

Figure 1. Example of a heuristic used to solve a chess problem

Panel A (White to move) shows the cues to correctly interpret the position. The presence of the white Queen and the black King's lack of space (the green squares are safe, whereas the red ones are not) suggest to look for the checkmate and to ignore the two pawns, because they do not have any significant role in this situation. Panel B shows the selection of the two candidate moves. The legal moves for White are 26, so the pupils have to adopt a criterion to analyse only a few possible moves. The criterion is choosing one of the two moves (green arrows) in order to control the green squares in Panel A. Panel C1 and C2 show how to monitor the consequences of each of the two moves. The pupils are asked to check for the squares through which the black King may escape. The move Queen a4-a8 (Panel C1) is wrong because it loses the control of d7 (green square), while Panel C2 shows the correct move (Queen a4-e8).

Table 2. Descriptive statistics of mathematical problem-solving and chess scores in the three groups

Group	Pre-test maths score	Post-test maths score	Pre-test chess score	Post-test chess score
Chess instructors	1.44 (1.12)	2.00 (1.27)	2.96 (4.58)	8.75 (4.83)
School teachers	1.51 (1.13)	1.74 (1.04)	1.51 (3.40)	6.02 (4.55)
Control	1.54 (1.14)	1.69 (1.23)	2.14 (3.71)	3.27 (4.52)

Note. Standard deviations are shown in brackets.

age and pre-test chess scores, in determining the gain in the score of mathematical problem-solving test (difference between post-test and pre-test scores; dependent variable). The model showed a significant effect of group (fixed factor), no significant effect of age (fixed covariate), no significant effect of pre-test chess scores (fixed covariate), and no significant effect of school of provenance (random factor, $var(u_{0j}) = 0.051$, $p = .105$) either. The model is shown in Table 3.

Since group was the only significant effect, the model with the intercept and group as fixed factors were compared to the model with only the intercept. The former proved to be significantly better than the latter (AIC = 3240.785 and AIC = 3254.657, respectively; $\chi(2) = 13.872$, $p = .002$). Finally, the pairwise comparisons showed that the chess instructors' group outperformed both the school teachers' group ($p = .010$) and the control group ($p < .001$), whereas the school teachers' group and the control group showed no significant difference ($p = 1.000$) in terms of gain in mathematics scores (Table 4).

Additional multilevel linear models

Two multilevel models (school of provenance as random factor), one for each experimental group, were performed to evaluate the role of CAT, post-test chess scores and hours of training in determining the gain in the score of mathematical problem-solving test.

Regarding the chess instructors' group, post-test chess scores and hours of training positively affected the gain in mathematical scores, whereas CAT time of use did not. The comparison between the model with the intercept and the two significant effects (post-test chess scores and hours of training), and the model with only the intercept showed that the former was significantly better than the latter (AIC = 1099.720 and AIC = 1117.230, respectively; $\chi(2) = 17.510$, $p < .001$). Regarding the school teachers' group (school teachers' group), neither post-test chess scores, nor hours of training, nor CAT time of use significantly affected the gain in mathematical scores. In both cases the effect of school of provenance (random factor) was not significant ($p = .487$ and $p = .280$, respectively). The two models are summarized in Table 5 and Table 6. Finally, the data can be downloaded from www.edurete.org/data.

Table 3. Multilevel model of the improvement in mathematical problem-solving performance in the three groups

Variable	Coefficient	St. error	t	p-value
Intercept	-0.235	0.682	-0.345	.730
Chess instructors	0.336	0.139	2.417	.016
School teachers	-0.055	0.152	0.362	.717
Control	0[a]	-	-	-
Age	0.042	0.080	0.525	.600
Pre-test chess scores	0.009	0.014	0.643	.520

[a] *The parameter is set to zero because it is redundant.*

Table 4. Mean mathematical problem-solving scores gains in the three groups

Group	Maths score gain[a]	Adjusted maths score gain[b]	Cohen's d[c]
Chess instructors	0.56	0.60	0.50
School teachers	0.23	0.15	0.20
Control	0.15	0.15	0.13

[a] *The maths score gains are the difference between post-test and pre-test maths scores.*
[b] *Values with the two covariates (age and pre-test chess scores) being partialled out.*
[c] *Cohen's ds are calculated by using the formula* $d = (Y_{post} - Y_{pre}) / SD_{pre}$*, where* Y_{post} *and* Y_{pre} *are post-test and pre-test maths score respectively, while* SD_{pre} *is the standard deviation of pre-test maths scores.*

Table 5. Multilevel model of the improvement in mathematical problem-solving performance in the chess instructors' group

Variable	Coefficient	St. error	t	p-value
Intercept	-1.426	0.572	-2.493	.013
Post-test chess scores	0.065	0.015	4.333	< .001
Hours of training	0.095	0.039	2.436	.015
CAT time of use	0.007	0.012	0.583	.560

Table 6. Multilevel model of the improvement in mathematical problem-solving performance in the school teachers' group

Variable	Coefficient	St. error	t	p-value
Intercept	-0.207	0.319	-0.649	.516
Post-test chess scores	0.015	0.020	0.750	.453
Hours of training	0.007	0.019	0.368	.713
CAT time of use	0.018	0.014	1.286	.200

DISCUSSION

Our study aimed to evaluate whether chess practice improves children's mathematical problem-solving skills, and to compare two different types of training. The results suggest that chess practice can enhance problem-solving abilities in children, but only if chess training conveys problem-solving heuristics to pupils. Furthermore, the improvement observed in this study occurred only after approximately 15 hours of training.

The children in the school teachers group did not show any significant improvement in mathematical problem-solving mean scores, which supports the idea that the mere exposure to chess training is ineffective. In fact, previous chess knowledge (assessed by pre-test chess scores) did not significantly affect the participants' mathematical improvement, suggesting that the simple knowledge of some chess-related notion (e.g., knowing how to move the pieces) does not help children to solve mathematical problems. Moreover, the two treatment groups (i.e., the school teachers' group and the chess instructors' group) differed from each other not only for the scores in mathematical problem-solving, but also for the variables affecting their performance. The improvement of mathematical problem-solving ability in the school teachers' group was not related to the duration of the training, and not to the post-test chess scores either. On the contrary, the performance of the group of chess instructors' was positively influenced by the two latter variables. These results suggest that in the chess instructors' group some chess-related ability generalized to the mathematical domain, whereas the school teachers' group did not show any transfer of learning.

Another point of interest was that CAT time of use did not have any noticeable effects on mathematical performance. CAT activities were not mandatory and mostly autonomously performed at home by the pupils. Thus, it seems possible to state that the pupils' motivation to play chess did not play any significant role in affecting the score in mathematical problem-solving ability. Therefore, if motivation had been an active cause of pupils' mathematical performance, the most motivated pupils – playing CAT games more than the least motivated – would have proved to be the best achievers. However, they did not.

Significance of this study

Consistent with previous research (Hattie, 2009; Marcucci, 1980), the results of the study corroborate the idea that problem-solving is – to some extent – a context-independent skill, which is both possible to train and transfer. However, it must be remembered that cognitive transfer is a limited phenomenon, both in terms of time and extent. Chess training seems to be useful to strengthen several mathematics abilities – such as problem solving – at the beginning of their development, when the abilities to be trained are not too specific, and hence there is still an overlap between the two domains. Nonetheless, given that problem-solving expertise in a specific domain is – to a large extent – a context-dependent skill, it is possible that chess practice helps to develop some general heuristics and habits of mind (Costa & Kallick, 2009) able to facilitate the acquisition of further and more complex competences, such as mathematical problem-solving. Chess training seems to be effective even if children's mathematical problem-solving ability is assessed using an extremely difficult test for children in the age group tested. This fact suggests that children who are trained to think strategically – i.e., interpreting situations, narrowing down alternatives, and taking decisions and monitoring consequences – tend to face more effectively mathematical problems otherwise almost inaccessible for primary school students.

Limitations of the study

The present study has a limitation which is worth mentioning. Chess instructors' expertise might be a confounding variable. Chess instructors are able to play chess at a more advanced level than school teachers. This may explain why the pupils trained by the chess instructors achieved better results in the test of chess ability than the pupils trained by the school teachers. Thus, the vaster knowledge of chess instructors could explain the better performance of the chess instructors' group in mathematical problem-solving as well. However, teachers' knowledge of their fields does not seem to be an important factor in determining pupils' academic achievement. For instance, Hattie (2009) reported that the effect of teachers' knowledge of the subject on pupils' outcome is small. Similarly, Ahn and Choi (2004) found a very low correlation between teachers' knowledge of mathematics and pupils' achievement in mathematics.

That said, it must be noticed that the difference between the chess instructors and the school teachers may be not only quantitative – i.e., the former know more about chess than the latter – but also qualitative – i.e., the former teach chess differently than the latter. For example, expert teachers – like the instructors involved in the study – are more used to adopt subject-specific routines in their lessons. These automatisms allow expert teachers' to free their working memory and attention (Leinhardt & Greeno, 1986), and thus focus on other important aspects of teaching – such as feedback to pupils. Furthermore, thanks to the mental resources spared, chess instructors can adopt a problem-solving stance to their lectures, which is an important feature in effective teaching (Hattie, 2003). Thus, we cannot rule out the possibility that the positive effect we found was due – at least partly – to the expertise of the chess instructors.

Recommendations for future research

Further research is needed in order to expand our understanding of how chess positively affects children's mathematical skills. First of all, longitudinal studies are needed to evaluate how long chess training can be a tool to enhance mathematical skills, and how long the benefits of chess training last. Given that cognitive transfer can occur only when there is an overlap between domains, it is yet to be assessed when the overlap between mathematics and chess domains is bound to finish. Chess training benefits in mathematics probably tend to diminish over the years until they completely disappear. Knowing the amount of time during which chess instruction helps to boost mathematical problem-solving skills is needed to plan effective and efficient chess trainings in schools. To our knowledge to date, no peer-reviewed studies involving the relationship between mathematics and chess abilities have been conducted for more than one year. It is also essential to identify what are the most beneficial chess activities to develop children's mathematical problem-solving skills. If chess practice can enhance those skills, by teaching generalizable heuristics, it is reasonable that some chess activities provide no educational benefit at all. For instance, memorizing long opening variants is a necessary step for a chess player, but it is not likely to be a useful activity for any educational achievement.

Another important issue which should be addressed by future studies is the cognitive processes underlying transfer of learning from chess to the mathematics domain. Up to now, only two studies assessed both mathematical and cognitive abilities (Scholz et al., 2008; Kazemi et al., 2012). The former found no effect of chess on concentration abilities in low-IQ (70-85) children, but positive effects on some arithmetical tasks. On the contrary, the latter found a significant effect of chess training on both mathematical and meta-cognitive abilities. This outcome corroborates the hypothesis that chess induces children to think on their own thinking processes, and to regulate them, in order to achieve goals, such as finding checkmate combinations and solving mathematical problems. Chess may be able to increase meta-cognitive skills in children, and in turn these latter may increase children's mathematical problem-solving ability (Desoete & Roeyers, 2003; Kramarski & Mevarech, 2003; Lucangeli & Cornoldi, 1997; Veenman, Van Hout-Wolters, & Afflerbach, 2006).

Finally, future studies should control for the socio-economic status (SES) of the pupils involved in chess-related activities. SES is a well-known factor affecting children's mathematical skills (Michael, 2008). The random assignment to the three groups guaranteed the equivalence *between* the groups, and thus the observed group difference in mathematics scores was not due to the potential effect of the participants' SES. However, SES could have influenced children's scores in mathematical problem-solving ability *within* the chess instructors' group – the only one showing a significant improvement. Possibly, the observed beneficial effect of chess on the participants' performance in mathematical problem-solving ability was somehow related to SES (e.g., only the children who were from families with high SES obtained good results). Thus, even if this does not invalidate the positive role of the game of chess, knowing the nature of the relationship, if any, between SES and chess benefits on children's mathematical problem-solving skill would be useful for educators. It would assist when evaluating whether chess training is useful for children from the general population, or only for a specific group.

CONCLUSIONS

Chess seems to be an effective tool to promote mathematical problem-solving ability in primary school children, but only if the teaching includes chess problem-solving heuristics. These heuristics help novice chess players to recognise and interpret game situations, to reduce the moves to analyse, and to play correct moves without overloading players' cognitive system. Analogously to chess positions, many mathematical problems cannot be solved by children through using brute force strategies (or oversimplified algorithms), but by using efficient heuristics. The latter allowed youngsters to correctly interpret mathematical problems, to recognise cues, and to link the latter to the correct solution(s). Chess practice may be a medium (in the sense of Feuerstein et al., 2006) able to shape children's way of approaching problem-solving tasks, compatibly with Costa and Kallick's (2009) concept of habit of mind. Chess, beyond being a game based on arithmetical and geometrical relationships, also seems to provide a context of application for training children's problem-solving heuristics, which may be generalizable enough to transfer from chess to mathematics domain.

The present study supports the position that teaching general heuristics is an effective way to foster the phenomenon of transfer of learning, and hence improve pupils' problem-solving abilities. Nevertheless, it must be remembered that transfer of learning is limited to the extent to which two domains share elements in common, and thus teaching general heuristics cannot be the only approach to adopt in order to develop high-level skills in students. To develop expertise, a large amount of context-specific practice is necessary, and learning context-specific heuristics is needed too.

Finally, further research is necessary to shed some light on the cognitive mechanisms underpinning the transfer of learning from chess to the mathematics domain. Some researchers have claimed that chess practice improves children's meta-cognitive ability, which in turn enhances mathematical problem-solving skills. To date, only one study (Kazemi et al., 2012) assessing the effects of chess practice on meta-cognitive and mathematical abilities has been carried out, with promising results. Further research is needed to replicate the study and to extend the search to other cognitive abilities, which could link chess practice to improvements in mathematical problem-solving ability in children.

AUTHORS' NOTE

The two authors equally contributed to the manuscript.

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