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ABSTRACT

Timely access to health services has become increasingly difficult due to demographic change and aging people growth. These create new heterogeneous challenges for society and healthcare systems. Congestion at acute hospitals has reached unprecedented levels due to the unavailability of acute beds. As a consequence, patients in need of treatment endure prolonged waiting times as a decision whether to admit, transfer, or send them home is made. These long waiting times often result in boarding patients in different places in the hospital. This threatens patient safety and diminishes the service quality while increasing treatment costs. It is argued in the extant literature that improved communication and enhanced patient flow is often more effective than merely increasing hospital capacity. Achieving this effective coordination is challenged by the uncertainties in care demand, the availability of accurate information, the complexity of inter-hospital dynamics and decision times. A hybrid simulation approach is presented in this paper, which aims to offer hospital managers a chance at investigating the patient boarding problem. Integrating 'System Dynamic' and 'Discrete Event Simulation' enables the user to ease the complexity of patient flow at both macro and micro levels. 'Design of Experiment' and 'Data Envelopment Analysis' are integrated with the simulation in order to assess the operational impact of various management interventions efficiently. A detailed implementation of the approach is demonstrated on an emergency department (ED) and Acute Medical Unit (AMU) of a large Irish hospital, which serves over 50,000 patients annually. Results indicate that improving transfer rates between hospital units has a significant positive impact. It reduces the number of boarding patients and has the potential to increase access by up to 40% to the case study organization. However, poor communication and coordination, human factors, downstream capacity constraints, shared resources and services between units may affect this access. Furthermore, an increase in staff numbers is required to sustain the acceptable level of service delivery.

1. Introduction

Patient boarding is a prevalent phenomenon within hospitals, especially within the emergency department (ED) [1–4]. A 'boarded patient' in the ED is described as a patient who must stay in the ED while awaiting transfer to an inpatient unit after a decision for hospitalization has been made [5,6]. This phenomenon is also identified as access blocked [7,8], exit block [9] and patient blocking [10], where a patient requires further inpatient care, however, a hospital bed within a reasonable time is unavailable. These patients, who have completed their treatments and are medically ready to leave the unit, are the main reason for bed-blocking and insufficient free beds [11]. This limits accepting new patients to the ED. Boarding problems occur in other units across the hospital, such as in the Intensive Care Unit (ICU), where critically ill patients require intensive care. However, they are placed in an alternative subspecialty unit [12,13].

Moreover, boarding occurs when delays in discharging patients who no longer require acute care services and are waiting for post-acute care outside the hospital setting [11]. In 2017, the Health Service Executive (HSE) in Ireland reported 201,977 bed days were classified as lost due to delays in discharges [15]. Furthermore, 12,201 patients waiting on trollies in the EDs or on additional beds placed in the corridors throughout the hospitals (Fig. 1). The red dotted curve estimates the boarding trend using polynomial regression of order 2 in time. The trend shows under the current system conditions; the boarding trend is growing very fast.

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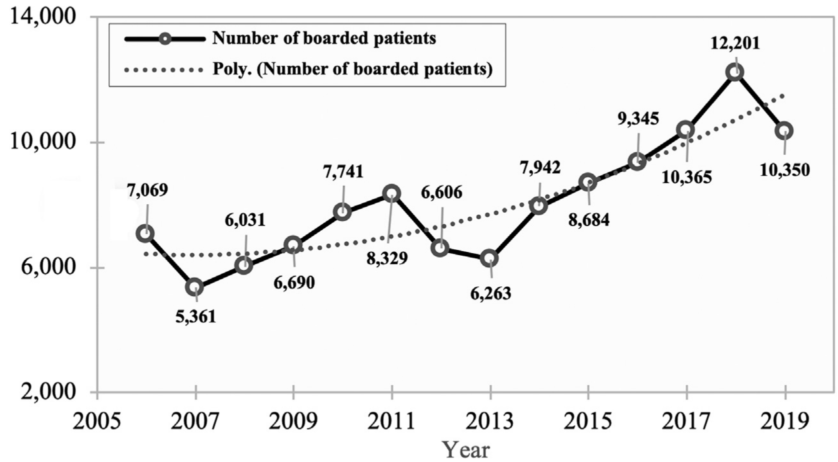

Fig. 1. The trend of the boarded patient in Ireland.
Source: Data source: [14].

Evidence increasingly confirms that boarding and admission delays are associated with a higher mortality rate of critically ill patients [12,16]. Patients boarded for less than 2 hours have a lower mortality rate than patients boarding 12 hours or more [17]. Boarding prolongs hospital stays and increases hospital occupancy, which in turn, exacerbates boarding [18,19].

Moreover, prolonged boarding creates additional stress on already suffering patients, families and staff [4]. These stressful atmospheres result in increased medical errors, risks, and decreased quality of care [20]. Moreover, boarding has adverse financial consequences due to the revenue lost from patients leave without being seen, as well as ambulance diversions [5]. The extra costs related to delayed transfers from ICUs also result in revenue losses [21].

There are many factors contributing to the boarding problem. Demographic changes, population growth, aging and increased life expectancy all contribute to the considerable increase in the number of emergency visits. This, in turn, increases the pressure on hospital EDs [22]. The demand for care services in Ireland has grown at a rate of 3.1% in patients over 65-years old and 4.2% in patients over 85-years old, between 2015 and 2016 [23]. The lack of inpatient beds for EDs and ICUs patients is recognized as one of the primary drivers of boarding [8,24]. Patients are experiencing delays in their transfers due to insufficient ward beds or understaffing [16]. Staff availability, staff skillset, as well as timely transferring patients, are other causes of boarding [25]. The lack of efficiency in the continuity of care through step-down and appropriate alternative care provisions also causes bed-blocking, which in turn exacerbates critical care resource shortages [11].

Ultimately, hospital management is looking for ways to reduce overcrowding and boarding patients [26]. The boarding problem is not only an ED-based problem, but it is an indication of the dysfunction of interrelated parts of a broader system. It is challenging to draw clearly defined boundaries around a unit in order to address this problem. Especially where different contributing factors affect each other. There is a need to adopt a system-wide approach to address patient boarding. This includes the investigation of multi-stage patient flows, throughout the entire process of care, as well as considering the interactions and interdependencies between various hospital units. Thus, given the complexity of the problem, it is not possible to use only a single method to study the problem in detail and at the same time allow a holistic system view.

Furthermore, healthcare systems are human-based systems that involve multiple stakeholders who interact with each other in complex ways. Due to the stochastic nature of the healthcare systems, its complex dynamics and interactions of their inputs, activities and outputs, healthcare managers require tools which can enable them to understand this complexity and thus to enhance their system performance. Over the years, research has shown that the use of modeling approaches significantly improves decision-making for healthcare management at various levels if successfully applied [27]. The following modeling approaches were utilized: Discrete Event Simulation (DES), System Dynamics (SD), Agent-Based Simulation (ABS), and Monte Carlo (MC) simulation. Research has indicated that the use of hybrid simulation would improve the capabilities of simulation solutions. Hybrid simulation, by combining two or more simulations, not only enables the symbiotic realization of the strengths of individual methods but also reduce the limitations of a single method [28]. The main restrictions of DES lie in its inability to capture the feedback dynamics related to the holistic structure of a system, a substantial amount of accurate data is required to build a model, and it is time-consuming to develop and run [29,30].

On the other hand, SD models fail to capture detailed complexities and individual movements through queues and activities within the system [31]. Morgan et al. [32] also discussed all the intended objectives of the study cannot be accomplished with a single simulation method. This is due to the model requiring several assumptions regarding system behavior. Therefore, a hybrid simulation provides a more realistic picture of the systems from different perspectives [33], which allows fewer assumptions and increased accuracy of outcomes, without oversimplifying some aspects [34].

This paper presents an integrated hybrid approach to investigate the impact of the boarding problem on patients' ability to access other units in the hospital while considering the intra-departmental and inter-departmental interactions in both up and down-stream hospital facilities. The proposed method integrates DES, SD, Design of Experiments (DoE) and Data Envelopment Analysis (DEA). Firstly, the DES component offers a better representation of the complexity of the process in detail, including patients flow, processes and underlying relationships with supporting units in the hospital. Secondly, the SD component captures the cross-boundary interactions between the hospital facilities. This represents a holistic view of the feedback between elements of the system. Finally, DoE and DEA are used to explore the most efficient system configuration according to a set of predefined key performance indicators (KPIs). It is also envisaged this effort will have a positive impact on the overall issue of patient boarding, within the hospital context.

The remainder of the paper is organized as follows. Section 2 reviews the extant literature, focusing on studies, which addressed the problem of boarding and overcrowding. The proposed methodology is discussed in Section 3. Section 4 presents a case study in the context of the Irish healthcare system. It provides a detailed application of the proposed approach from Section 3. Section 5 presents the findings and discusses the results. Finally, Section 6 concludes the paper.

2. Literature review

2.1. Solution strategies

Several improvement options are investigated in the literature in order to alleviate the patient boarding problem. These are mainly divided into ED-based and hospital-wide solutions. The improvements in the first category are mainly related to the increased capacity of rooms, beds and staff. While increasing the number of ED trollies and inpatient beds might seem to mitigate the boarding issue intuitively, many studies have reported that increased capacity is not only ineffective but also an expensive solution. The process of increasing the transfer rate of ED patients to inpatient beds (i.e., unlocking the access block from the ED to inpatient beds) would have a more significant impact on reducing the number of ED boarders, than the increase of medical staff or assessment cubicles [35,36]. Bed management also plays a critical role to improve transfer rate and patient flows [37,38]. As a function, bed management has several responsibilities including placing emergency and elective admissions into appropriate beds and enabling patient discharge and transfer by coordinating services which patients may require [39]. Bed management is widely used to reduce boarding and overcrowding in the ED. This can be achieved by classifying patients as soon as they enter the ED, which enables the staff to understand the require inpatient beds [37,40] and also predicting daily attendance [41]. Furthermore, improving patient flow from the ED to inpatient areas by considering both emergency and elective patient flows [38], and analyzing the impact of critical bed status on ED patient flow [42] supports to alleviate the problem.

Many hospitals have also started to use Acute Medical Units (AMUs) and Acute Surgical Units (ASUs) as alternative models of care for patients who present to EDs [43,44]. These units aim to restructure acute patient flow in the ED and provide better hospital services. The benefits of AMUs and ASUs are derived from streamlining of medical and surgical patients to a location, where they can be seen without delay. Moreover, these units are staffed by experienced consultants and multidisciplinary teams, which allow the patients to be comprehensively assessed and managed before being either discharged or transferred to the inpatient care setting [45]. If admission is required, this will occur within a defined period, and the patient is admitted to the most appropriate clinical area in the hospital. Therefore, these units operate as the interface between primary services and the downstream medical specialty wards. In terms of locations, these units are usually co-located with EDs, which eases access to diagnostic services such as radiology and enhances service delivery [46].

In the UK, Australia and New Zealand, many hospitals have implemented AMUs/ASUs or units with synonymous names. Recently, several studies reported significant improvements in service delivery levels such as decreased mortality rates [47], reduction in length of stay [48–50], reduced waiting times in EDs [51], decreased admission rates [24] and improved patient and staff satisfaction [52]. However, the introduction of these units without improving the streamlining of patient flow across the hospital may result in shifting the boarding issue to other parts of the system. This, in turn, could block access to appropriate clinical areas [53]. Therefore, some studies focus on improvement options outside the ED, especially in downstream units such as inpatient wards. Improving inpatient discharge timing by shifting the time of discharge to an earlier time in the day [26,54,55], can improve resources utilization and alleviate the boarding problem. Additionally, effective scheduling strategies can also be applied to manage the demand for hospital beds from competing admission sources. These strategies can be designed to offset unscheduled ED admission requests with elective arrivals and hence reduce boarding times [1]. For example, considering different strategies to schedule elective patients while reserving a fixed capacity for ED admissions [56]. Moreover, demographic data and clinical information can be used to estimate the likelihood of patient admission accurately. These predictions enable hospital managers to improve their estimation of required resources and the process of assigning beds to patients [25]. Finally, researchers indicate that a frequent assessment of boarders, effective communication and coordination between the ED and other related departments facilitate patient transfers and result in lower patient boarding [25, 57].

2.2. Solution methods

Simulation paradigms, particularly DES and SD, are widely used to capture and understand the dynamics of a system. SD is mainly used at more strategic levels in order to gain insight into the interrelations between the different parts of the healthcare system [58]. However, SD models are less powerful in capturing the level of granularity and less flexible in modeling individual entities of the system [59].

Due to the dynamism and multi-disciplinary nature of the healthcare system, DES is a valuable tool in assisting healthcare managers in decision-making. It captures more of the detailed complexity [60]. DES has been proven to be a useful tool for process modeling and improvement [61]. Healthcare managers can apply DES to assess a systems current performance, predicting the impact of operational changes and examining the trade-offs between system variables [62]. Review studies on modeling and simulation methods within the healthcare context have reported a vast number of DES models for modeling service operations, capacity, process and workforce issues in different units in hospitals and clinics [63–66].

Furthermore, DES has been used to study patients boarding problems and their implications on patient experience. De Boeck et al. [67] explored the impact of boarding patients on the ED system performance and compared different priority policies for ED physicians decision. The tradeoff between increasing the physical capacity of the ED and reducing patient boarding times was examined in Khare et al. [35]. The impact of inpatient boarding on the ED's efficiency was also explored in Bair et al. [68]. The relationship between inpatient discharge times and ED boarding of admitted patients was presented by Powell et al. [26] using a cross-sectional computer model. Levin et al. [1] utilized DES in investigating the effect of bed demand of cardiology admission sources on the ED boarding. Pines et al. [5] applied DES and regression analysis to study the financial implication of ED boarding on the overall hospital revenues. The study of Roh et al. [69] developed a DES to understand the flow of mental health patients within the ED and to inpatient settings. Several scenarios designed to specify the percentage of increase in beds necessary in reducing mental health ED boarding times The model of Shi et al. [70] explored the operations within the inpatient wards and their relation to the ED in order to reduce boarding time. This study linked boarding times to the imbalance between the daily number of arrivals and discharges numbers, as well as a mismatch between the discharge timing and hourly arrival pattern. Furthermore, Mustafee et al. [71] used DES to investigate the bed management strategies in reducing bed blocking in specialized and integrated care units. The DES, in Crawford et al. [72], has been used to model patient pathways in an acute care hospital in order to investigate the effects of discharge timing on ED waiting and boarding time.

Recently, there has been increasing attention in using hybrid simulation to address healthcare system challenges [59,73]. Hybrid simulation is a form of mixing methodologies and is defined as a modeling approach, which combines two or more simulation methods (e.g., DES, SD and ABS) to model complex systems [74]. They potentially provide a more realistic picture of systems from different perspectives, while reducing the constraints of a single method. Furthermore, hybrid simulation provides improved insights into complex systems as they offer a holistic approach to system analysis [33].

Several studies discussed how various simulation methods can be combined in a hybrid model [59,74–76]. Different studies also used hybrid simulation within the context of healthcare. Hybrid simulation (i.e. DES-SD) was applied to improve outpatient scheduling [77]. DES was used to investigate the influence of a new scheduling approach on patient cycle time, while SD was used to understand the relationship between the scheduling system, patient demand and service capacity. Rohleder et al. [78] presented a hybrid simulation model in order to redesign and implement new healthcare facilities. They used DES to design a more useful set of facilities and made recommendations for resource changes. SD was also used to predict new patterns in demand and examine the possible adverse effects on a new system. Viana et al. [29] built a hybrid simulation to address the problem of the sexually transmitted infection chlamydia. DES was used to model the hospital outpatient clinic where patients get treated, and SD was used to model the infection process in the community. The impact of developing integrated patient pathways was explored by Zulkepli and Eldabi [79]. In this study, DES was developed to model assessment and intermediate care processes. While in turn, SD was used to capture the effect of patient readmission on the care process. Hybrid simulation has also been used to forecast healthcare demand [80]. The SD model simulates the continuing evolution of the population and DES generates patient arrival times and the prevalence of needs for service in the healthcare system.

More specifically, DES-SD simulation is used to improve the ED process. Ahmad et al. [81] used DES to model the complexities of the integrated ED system. In turn, SD was utilized in capturing the interdependency between the ED and other sub-units within a hospital. The impact of strategic changes on demand levels was examined by Bell et al. [82]. The variation in demand for care, in particular for unplanned demand, was simulated by SD. DES was built to represent patient activity through urgent care services. Similarly, Chahal et al. [83] used hybrid simulation to explore and evaluate the effect of a whiteboard on the workflow of an ED. The DES model was used to capture the detailed complexity of the ED's processes. The variation in ED performance in response to the whiteboard information flow was presented in the SD model. This hybrid model was able to capture the detailed operational level, as well as the impact of information flows throughout the ED process.

The combination of DES-ABS-SD has also been recognized as beneficial in analyzing hospital process and workflows [84]. In this study, DES was applied to represent processes, ABS and state charts were used to reflect individual behavior at the micro-level. On the other hand, SD was used to model abstract and continuous structures.

Furthermore, ABS and DES has been used to model emergency medical services (EMS) [85], radiology center [86], and analyze sustainable planning strategies for EMS [87], study patient choice and behavior in the healthcare system [88].

Data envelopment analysis (DEA) is a mathematical programming model, which is used to evaluate and compare the efficiency of decision-making units (DMUs) [89]. DEA overcomes many drawbacks of other performance evaluation approaches, such as ratio-based analysis (RBA), least-squares regression (LSR), total factor productivity (TFP) and stochastic frontier analysis (SFA). DEA is a non-parametric technique, which can provide a consistent benchmark for all inputs and outputs, predict the best performance or the most efficient relationships and identify individual inefficient units [90,91].

Additionally, DEA has proven to be a useful tool for evaluating the efficiency of hospitals [92] and in measuring performance efficiency within the healthcare system [93]. DEA can be applied to evaluate the simulation results and facilitate the search process [94]. However, few studies have examined the use of DEA in simulation analysis. The integration of DES and DEA has been used to improve the quality of care in ED by modeling different errors from nurses and technicians [95], analyzing ED efficiency from eight hospitals [96], allocating ED resources efficiently [97] and improving patient flow [98].

The use of hybrid models in healthcare modeling is growing. Research in modeling of healthcare systems illustrates that the simulation models if integrated appropriately, can be used in healthcare settings as an active decision support system for the management team. Without compromising patient safety, managers can practice decision-making in specific clinical situations and develop reasoning for new strategies.

3. Material and methods

The presented approach utilizes various methods of data collection to incorporate multiple perspectives, so a mixture of qualitative and quantitative has been engaged (Fig. 2). Interviews and observations have a qualitative nature. This has the significant benefit to understand and model the workflow (processes) in the healthcare facility. This form of data allows for the incorporation of practitioners' view to enrich understanding. On the other hand, quantitative data has a factual nature and depends on verifiable information. This type of data is collected from various sources, including the Hospital Information System (HIS) and local databases. In order to collect this data, institutional approval was granted from the Ethics Research Committee of the Technological University of Dublin. They confirmed that there were no ethical issues regarding the project. Furthermore, the anonymity of the participants and the confidentiality of data is maintained at all times and no private information will be disseminated.

The data analytics component helps to assess and analyze the patient volume, their severity mix and patterns of patient presentation. Descriptive analytics play a key role in the analysis of the historical data, which is used to capture insightful information from the data. The outputs of descriptive analytics are the probability distributions and parameters, which are required for simulation.

Conceptual modeling is a significant step in the building of a simulation model, and it is potentially the most significant stage in a simulation study. In order to modeling the underlying business processes, knowledge from the individuals directly involved in service delivery is required. The modeler uses various methods (e.g., interviews, direct observation and focus group) to get as much information as possible, without influencing or manipulating the problem definition. Since model conceptualization is an iterative process, it requires close interaction with

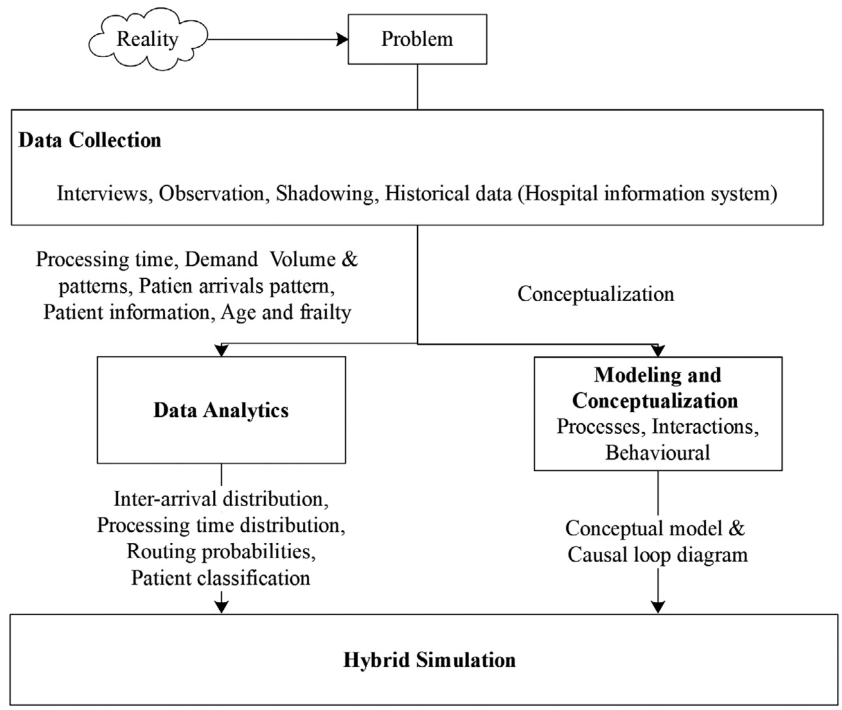

Fig. 2. Model formulation and understanding.

experts and practitioners in order to obtain holistic insights into the aspects of the system under scrutiny. There are two outputs of this component: a detailed process representation of the real system for DES and, a feedback conceptual model for SD. These processes are mapped into a conceptual model using one of the well-developed modeling languages such as a flowchart and or state chart diagram. This is where sub-processes and activities are identified. Feedback conceptualization is an essential activity for modeling and captures the dynamic behavior of the system. Key variables and factors can be identified through a series of interviews, focus groups and secondary data mined from the literature. Causal loop diagrams and sub-system diagrams are regularly used to captured feedback loops. A fundamental principle is modeling and conceptualization should be focused on a problem instead of a system and guided by a clear purpose and objectives.

Modeling patient flow across hospital units necessitates the integration of the flow between downstream and upstream facilities, which display a high degree of interdependency. Therefore, a hybrid SD-DES simulation model is developed to address the consequences of patient boarding problem in a hospital setting (Fig. 3). The upstream units represent the demand sources of the specific unit under investigation (e.g., ED, AMU, and ICU), while the downstream components model the patient disposition. Patient disposition refers to two cases in any patient boarding problem in a given hospital unit (i.e., Hospital unit *i*). Firstly, patients who finished their care episode in that unit and are waiting for beds in other downstream units. Secondly, patients have been boarded to that unit from other upstream units. In the proposed hybrid simulation, patient flow within the hospital unit with boarding problem is modeled using a DES. This allows simulating the unit activities and processes in detail, while considering resources interaction. Downstream and upstream hospital operations are explained in the SD model. This enables managers to envisage the impact of changes using feedback loops between the different activities. The two simulation models run simultaneously, and the information is exchanged between models in the runtime with a parallel interaction. This is enabled by AnyLogic software.

To support simulation as a tool for experimentation, DoE is necessary. DoE allows the evaluation of scenarios to identify the most significant factors affecting the overall performance. DoE is a useful tool with many theoretical developments and practical applications in various fields [53].

The Taguchi method of DoE facilitates robust designs when selecting variables. This is achieved through applying different orthogonal arrays (OA) according to the number and level of parameters. The Taguchi DoE decreases experimental errors and increases both the efficiency and reproducibility of different experiments. It also considers two-way interaction factors which simplify the interpretation of results. This gives a better insight into the overall process analysis [99]. The results of DoE are then used as an input to DEA to evaluate and rank the best scenarios in improving an acute medical assessment unit (AMAU) performance. Due to the complexity and variety of measures in healthcare contexts, the output results of DoE are analyzed by DEA to measure the efficiency of different designs and recommend the most appropriate decision for the problem (Fig. 4).

4. Case study

The National Acute Medicine Program is a clinician-led initiative incorporating the Irish Health Service Executive's (HSE) Clinical Strategy and Programs Directorate, and the Royal College of Physicians of Ireland (RCPI), among others [100]. It has developed a framework to mitigate the pressure which Irish EDs face and to minimize the LOS. This is an effort to reduce overcrowding by introducing AMUs to work in parallel with the EDs. The framework aims to provide medical patients presenting to the ED, with a fast track to decisions regarding their treatment journey in the hospital. Patients who present in these units see a senior medical doctor (MD), who can make treatment decisions within almost one hour of admission.

However, shortly after opening, the boarding problem shifted from the EDs and to these units. Which, in turn, restricted access to the AMU by medical patients. Therefore, hospital managers and executives requested a formal assessment of this problem and

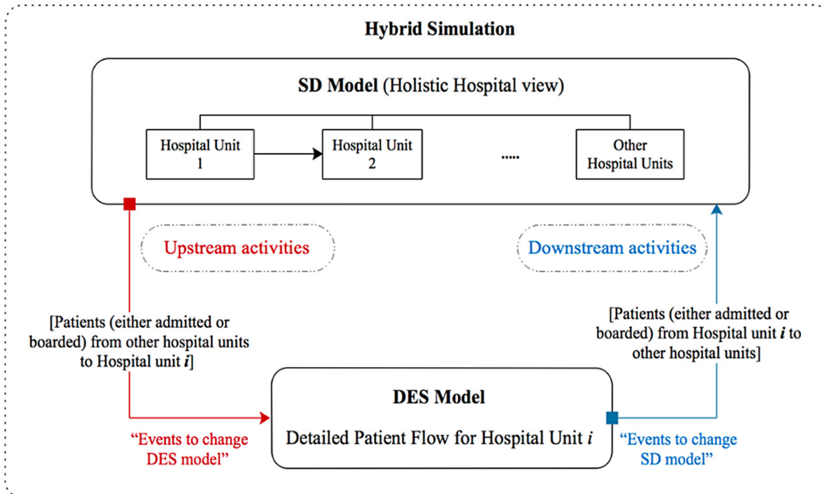

Fig. 3. Hybrid simulation.

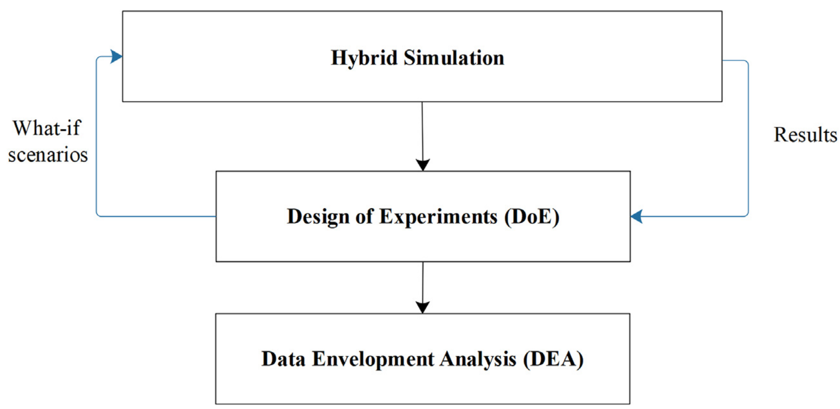

Fig. 4. Experimentation and decision-making.

on how these units may cope with the unpredicted increase in workload and demand. As a response, a project was carried out within the HSE to provide managers with a tool to investigate the boarding problem. This tool also enabled the assessment of a dedicated unit within a hospital and proposed a model to assist with resource planning.

4.1. Phase 1: Formulation and understanding

The partner hospital for this study has one of the busiest ED nationally that operates 24 h a day, seven days a week throughout the year. In 2012, 41,781 unscheduled adult patients visited this ED. This number increased by 9% in 2015 and has seen sustained growth in the above 65 age group, which has increased by 41% over the last six years. There is a higher acuity associated with this age group with a higher likelihood of admission, longer LOS and a higher incidence of both influenza and Noro-virus. This results in a considerable number of bed days lost due to isolation requirements.

The AMU is divided into two sub-units: the acute medical assessment unit (AMAU) and, the short-stay unit (SSU). The AMAU is considered to be the first gateway for acute medical patients who are referred from the ED, while the SSU is used by patients who need to be admitted to the hospital but their LOS is estimated to be less than five days. Patients can also be admitted directly to hospital clinical wards from the AMAU (Fig. 5). Recently, our partner hospital opened both an AMAU and SSU. The AMAU was opened as a discontinuous healthcare service which works as a 12-h' unit. It is open from 9 am to 9 pm, however, it only accepts patients up until 6 pm to allow beds to be released for the next day. The SSU also works as a short stay ward, on a 24/7 basis.

The AMAU is staffed by physicians, dedicated multidisciplinary medical and support teams. The only access to the AMAU is through the ED [53]. Patients are triaged in the ED and assigned a triage category, according to the Manchester Triage System (MTS) that uses a five-level scale for classifying patients per their care requirements [101]. The triage nurse usually contacts the AMAU consultant or registrar so that they can accept or reject the case. Patients routed to the AMAU are those medical patients who have been assigned a triage category of 2 or 3 (i.e., very urgent and urgent patients respectively), who do not require resuscitation or isolation facilities. The patients are only transferred to the AMAU if a trolley is available. Patients presented to these units will see a senior MD, who treats and discharge the patients within almost one hour of admission. The AMAU, SSU and ED share resources among them and share other resources with the hospital. When the AMAU was first introduced, there was an increase in the proportion of patients discharged within 24 h and also a decrease in LOS and overall medical bed day usage [102].

The capacities of the SSU and AMAU are 24 beds and 11 trolley spaces, respectively. While the SSU has 24 beds, only 12 of them are under the governance of an acute medical consultant. The remaining 12 beds are under the management of standard medical consultants in the hospital. The AMAU in this study faces two types of boarding problems: ED boarding and internal AMAU

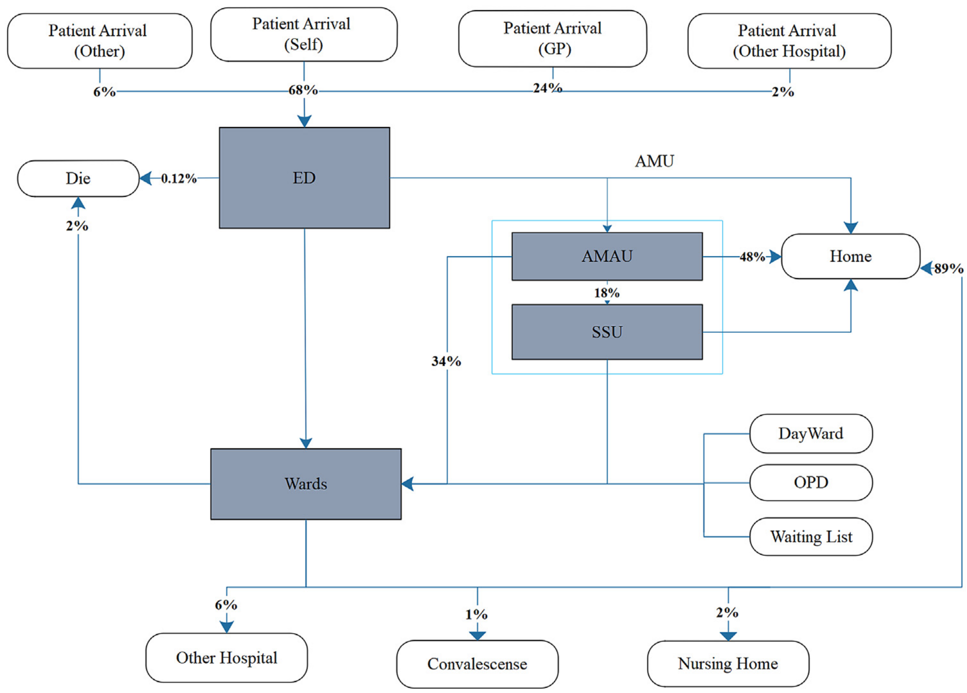

Fig. 5. Generic patient pathways through a hospital with the AMAU/SSU.

boarding. In the ED boarding case, patients occupied between one to six AMAU trollies for a maximum of 12 h. On the other hand, internal AMAU boarding occurs when patients from the AMAU require a hospital bed in another downstream unit (e.g., medical wards or SSU) for further treatment. As stated previously the AMAU opens from 9 am to 9 pm, and stop accepting new patients at 6 pm. Over six months, the number of medical patients presented to the ED with the triage categories 2 and 3 was 3753. However, only 40% of those patients accessed the AMAU with an average LOS (i.e., the total time from the patient entrance to the AMAU, until they exited the unit) of 4.45 h.

4.1.1. Data collection and analysis

The data collected for this project utilized both quantitative and qualitative data types. The quantitative data was collected from the historical ED logs, electronic patient records (EPRs) from the ED's IT system, and direct observation. The direct and indirect time per activity are not stored in their IT system and were collected from interviews and observations with staff. Furthermore, the qualitative data such as patient pathways and the process of conceptual modeling has been gathered through observation, interviews and focus groups. The sources of each data element are summarized in Table 1. This data has been de-identified, so the patient ID was replaced by a generated number. This enables the tracking of patient pathways without identifying them. Demographic data included only age and gender.

The sample data from all anonymized acute patients was gathered retrospectively for six months for patients that presented to the ED and AMAU between January 1st, 2014 and June 30th, 2014. A total of 20,493 de-identified patient records from ED and 1520 patient records from the AMAU was collected through the hospital's information system. This system is used by the staff (e.g., administrators, doctors, and nurses) to record data about each patient, through each stage of their care. All diagnostic and procedure types were considered and no exclusions were made. Patient records were analyzed and qualitative information about patients' arrival patterns, patient grouping and allocation were extracted. All days displayed high patient arrival numbers with the peak times being between 11 am and 3 pm. There was an hourly average in arrivals of 7.9. These patterns provide an overview regarding the demand volumes for services in the ED and different temporal scales for the patient arrival characteristics.

The hourly arrival data for the ED was consolidated by the hour of the day. This allowed the presentation of arrival rates that are required as inputs for the model. The impact of monthly and weekly variation have been smoothed. The inter-arrival times for each hour of the day was used to fit the exponential distribution using the maximum likelihood estimator (MLE). This analysis results in 24 different exponentially fitted distributions. Patients differ according to their medical complaints and the severity of their care needs. It was, therefore, essential to understand their different arrival patterns to reflect the overall characteristics and needs of various groups of patients.

The patients are clustered based on their triage category, which enables the differentiation of those patients who will be directed to the AMAU. Urgent patients (triage category 3) represent the largest group of new attendees to the ED annually (51% on average). They present to the hospital with a wide range of medical complaints.

As mentioned previously, the AMAU in this study faces two kinds of boarding issues: ED boarding and, internal AMAU boarding. In ED boarding, patients occupy between one to six AMAU trollies, for a maximum of 12 h. In internal boarding case, patients in the AMAU are waiting for inpatient ward beds or SSU beds in order to release AMAU trollies.

4.1.2. AMAU patient flow (DES model)

Upon arrival at the ED and registration, walk-in patients (self-referral or GP referral) remain in the waiting area to be triaged. When a patient's name is called, depending on the availability of triage staff, the patient is assessed by a triage nurse. Based on patient condition and triage assessment by MTS criteria, each patient is assigned a triage category. Then, based on their severity level, medical patients can be directed to either the ED or the AMAU. Medical patients are eligible for the AMAU path if they

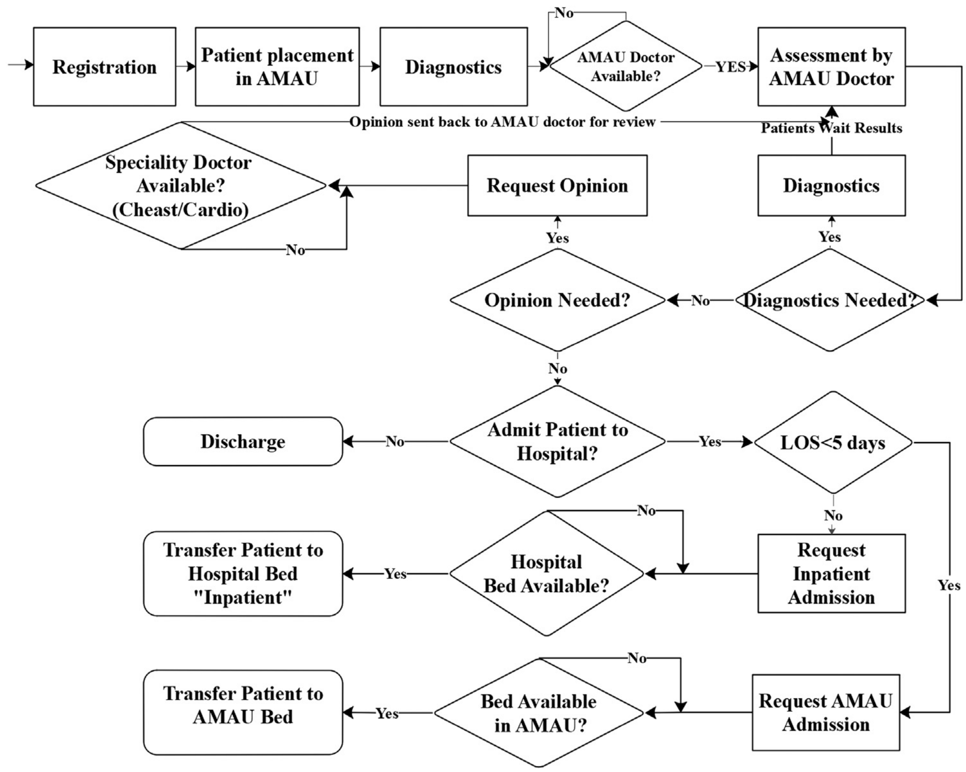

Fig. 6. Patient's flow in AMAU.

Table 1
The data sources.

Data	Source
• Patient arrival times, patient acuity, diagnosis and demographic data.	• Historical data gathered from the EPR, ED and AMAU logs.
• Activities: registration, triage, seeing a doctor, treatment.	• Historical data gathered from EPR.
• Duration of direct activities per patient.	• Observations, shadowing, and interviews and group discussion.
• Patient flow: pathways, routing probabilities, conceptual modeling.	• Historical data collected from EPR, interviews and observations.
• Human resource and non-human resource capacities: nurses, consultants, doctor, etc.	• Interviews and group discussion.
• Number of AMAU and ED boarders.	• AMAU logs.

arrive between 9 am and 6 pm and if they are assigned a triage category 2 or 3. Once these requirements are met, the triage nurse calls the AMAU's consultant to check trolley availability. The patient goes back to the ED path if a trolley is unavailable. The majority of patients in the AMAU are medical patients, which accounting 96% of the patients presented to the unit.

Following the triage process, a patient who is directed to the AMAU will be registered in the system, interviewed by a nurse, where their blood pressure and vitals are measured and recorded. Then they wait to be assessed by a doctor. Next, the AMAU doctor will discuss the case with the unit's consultant, who either asks for more tests, requests an opinion, or decides whether the patient needs to be admitted or discharged. These are the primary care stages, which are relevant for all AMAU's patients, whether they are discharged from or admitted to the hospital. Secondary patient stages are steps involved in the care of some, but not all patients, such as diagnostics (e.g., MRIs and CTs). The steps of the AMAU is depicted in the flowchart in Fig. 6.

4.1.3. Interdepartmental interaction (SD model)

The casual loop shown in Fig. 7 can be conceptualized with two main areas: The community area (i.e., outside of the hospital) and, the hospital area (i.e., the ED, AMAU, SSU and inpatient wards). Regarding the community, patients present to the ED from the hospital's surrounding catchment area and many are then discharged back to the community. The rate of patient arrivals depends on a variety of factors and the characteristics of the surrounding catchment area. The rate of discharge back to the community depends on patients' medical health condition. This is a function of the patients' average LOS in the hospital ward and bed occupancy. The factors affecting both the arrival rate (demand side) and the average LOS in hospital ward are not modeled in detail. Instead, they are modeled as exogenous variables. To reduce the pressure on the ED, medical patients can be dispatched to the AMAU pathway, subject to the AMAU's trolley availability. The majority of the patients (75%) which present to the ED are discharged back to the community after receiving their treatment. If further inpatient care is required, patients will wait in the ED for an inpatient bed. The rate of admission depends on both bed management and hospital bed occupancy. Due to the high bed occupancy of inpatients beds, patients that need inpatient care are boarded (delayed) in the ED while blocking ED trollies.

A similar situation occurs in the AMAU when patients require a further stay in the hospital. They should move to other downstream units such as the SSU or inpatient beds. To free-up the ED's blocked trollies, ED management transfers some patients to the AMAU overnight. This management practice just moves the ED bed-blocking partially from the ED to the AMAU. Consequently, the AMAU has an average of 3.2 blocked trollies due to the transferred ED boarded patients. The situation becomes worse when it is combined with the AMAU boarded patients. In turn, these limit access to the AMAU and subsequently increase the pressure on the ED.

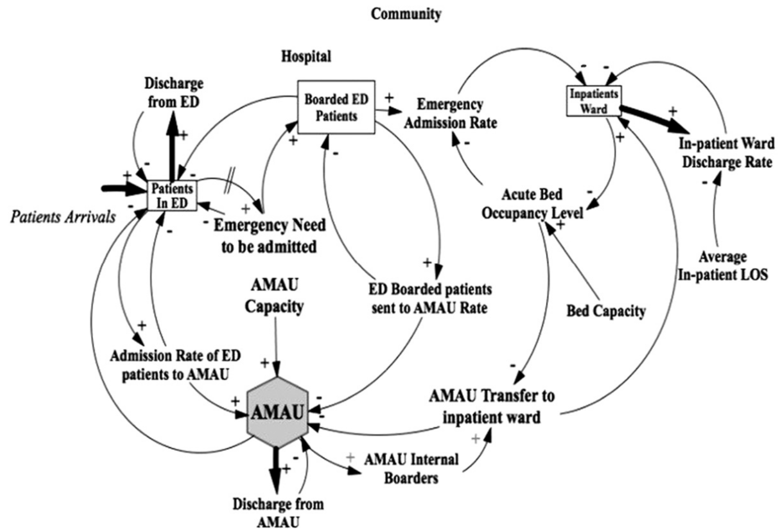

Fig. 7. A simplified causal loop diagram.

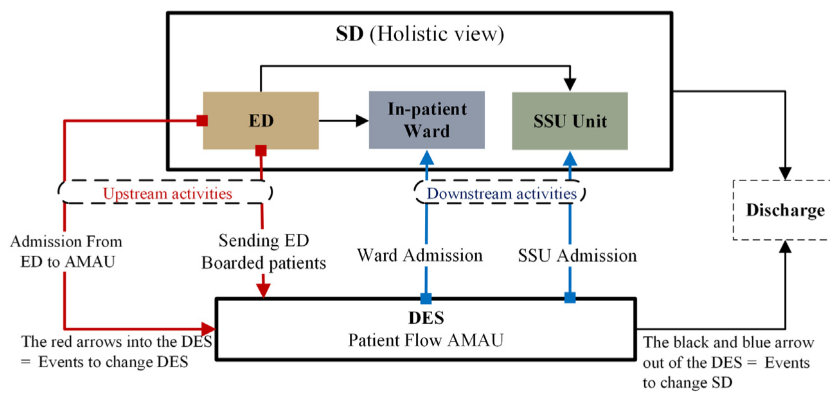

Fig. 8. The interaction between the DES and SD models. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4.2. Phase 2: Modeling complexity and dynamics

Analyzing the patient boarding problem in the AMAU necessitates integrating with downstream and upstream facilities which show high interdependency, in particular, the ED and inpatient wards. A better understanding of the problem and its implications can be accomplished when system integration is considered. Therefore, a hybrid SD-DES simulation model is developed to address the consequences of patient boarding in the AMAU (Fig. 8). Red arrows feed information into the DES from SD, blue arrows indicate change events from DES to SD, while black shows the flow of changes inside the SD.

The upstream component is the ED, which is the demand source of the AMAU. While the downstream components model the patient disposition. Patient disposition refers to two cases in AMAU: First, patients who are waiting for beds in other units to release AMAU trollies. Second, patients who been transferred to the AMAU overnight in order to free-up ED's blocked trollies. Patient flow in the AMAU is modeled using a DES to simulate the unit's activities and processes in detail, taking into account the interaction between resources. Downstream and upstream hospital operations are explained in the SD model, which enables managers to envisage the impact of changes using feedback loops between the different activities. Based on the analysis and conceptualization, a comprehensive simulation model for the ED and AMAU is constructed using the "*Any Logic 7 University Researcher*" simulation package. DES components include all detailed aspects of the AMAU and the SD to capture the dynamic complexity of the inter-departmental interactions. The two simulation models run simultaneously, and the information is exchanged from both models in the runtime with a parallel interaction. In this hybrid simulation model, DES interacts dynamically with the wider SD model. SD captures patients' arrivals to ED, as well as the interdependencies and relationships between capacities, along with the LOS for patients in various treatment units (e.g., SSU and ward). The outputs of SD provides the daily demand for the DES model in terms of the patients which are referred from the ED to the AMAU in order to complete their medical processes. This data becomes the parameters for the arrival distributions at the AMAU. Moreover, the DES used to model patient dispositions in the AMAU. In return, the main outputs of DES such as the number of patients admitted to the wards, and the number of patients admitted to the SSU are sent to the SD model automatically. Thus, continually changing elements represented by the SD cause changes in the discrete variables, and discrete variables cause changes in the continuous elements.

All model inputs are stored in a database attached to the simulation model. The model output is exported to an excel database for further analysis and validation. The simulation model also considers different types of medical staff, including nurses, senior house officers (SHOs), registrars, and consultants in the ED and AMAU. Furthermore, non-staff resources have also been included,

Table 2
Model validation.

Run	LOS (Min)	Patient access	Discharge home	Admitted
1	279.77	1749	795	954
2	276.39	1697	765	932
3	277.20	1708	803	905
4	275.65	1746	811	935
5	277.52	1735	770	965
6	271.96	1668	753	915
7	276.00	1688	740	948
8	280.79	1703	739	964
9	277.42	1700	748	952
10	275.23	1704	745	959
Summary statistics				
Mean	277	1710	767	940
Stdev.	2.44	25.92	27.04	18.24
Half CI	1.74	18.52	19.32	12.97
LB	275.05	1691.48	747.68	952.97
UB	278.53	1728.52	786.32	927.02
Actual	267.6	1520	734	786
% Diff	3.4	12.5	4.4	19.5

such as beds, physical spaces, and diagnostic rooms. Access to diagnostic services and inpatient care has also been modeled. The model tracks the individual patients through their journey in the hospital in order to quantify the amount of care required and for their interactions with the staff.

To reduce the model development cycle time and to increase confidence in the results, verification and validation was executed throughout the development phases of the simulation model. After each development phase, the model was verified and validated in conjunction with completed phases. For the verification process, the model logic is verified to ensure that patients follow the correct care path as expected. This was achieved by tracing and debugging functions besides the visual tracking of patients. It was also achieved through checking intermediate output values such as queue lengths and waiting times between processes. The conceptual model was documented and validated by circulating the document among the ED and AMAU senior consultants and senior nursing staff (more details of the model is provided in Appendix A).

The final results of the simulation model were validated using the two following techniques: face validation and, comparison testing. Face validation is performed by interviewing the ED and AMAU senior consultants and nursing staff in order to validate the final results of the simulation model. The verification and validation were carried out through all the development phases of the model. Visual tracking was used to verify the model logic to ensure that patients follow the correct care path as expected and are going to the expected units. Before running the simulation model, the following Key Performance Indicators (KPIs) were used for the AMAU: the average of AMAU LOS, and patient access (i.e., the total number of patients enter the AMAU). The main reason for choosing patient access as one of the main KPIs is that increasing patients access to the AMAU can mitigate overcrowding in the ED [53]. The second approach was comparison testing, which is achieved by comparing the output of the simulation model with the real output of the system under identical input conditions. A sample output of selected KPIs for the AMAU is given in Table 2. Prior to the application of this approach, the number of replications was determined using both graphical and confidence interval methods [103,104] (more details are provided in Appendix B).

The model runs for a twenty-four week period with a warm-up period of eight weeks. The model is also run ten times for each scenario. Each run has different random seed checks to ensure that the results have an accurate reflection and precisely represent the AMAU. Then, the average of ten runs is reported for the discussion of the results.

Table 3
Simulation results for a baseline (95% confidence intervals).

KPIs	Baseline scenario
Avg. LOS(Minutes)	277 ± 1.74
Avg. Patient access	1710 ± 18.52
Avg. Number of discharged patients	767 ± 19.32
Avg. Number of admitted wards	460 ± 7.92
Avg. Number of admitted SSU	480 ± 10.06

4.3. Phase 3: Experimentation and decision-making

Different scenarios including boarding experiments, and stock and flow interventions, which impact the boarding problem, were tested. The DEA model is then used to rank the scenarios and choose the best strategy to alleviate the problem.

5. Findings and results discussion

The simulation results of the baseline show that on average 1710 patients had access to the AMAU unit over 24 weeks. In the current situation, the AMAU works 9h/5days a week; from 9 am to 6 pm. According to the baseline, only 40% of the medical patients in the ED could access the AMAU. However, the goal of the ED's managers is to increase the flow to the AMAU to absorb most if not all medical patients who attend to the ED during its opening hours. The average LOS of the baseline – presented in Table 3 – is currently 277 min and 1710 patients did access to the AMAU. Of that 1710 patients, 767 patients were discharged to home, while 460 and 480 patients moved to inpatient wards and the SSU, respectively.

5.1. Boarding experiments

In the AMAU, there are three types of patients. The first type is medical patients who are in the process of treatment in the unit. The second type is the medical patients who have completed their treatment in the unit and are waiting to be admitted to an appropriate inpatient bed. This type of patient is referred to as "internal AMAU boarders". The third category is the "ED boarders" who are clinically unnecessarily transferred to the AMAU in order to free up ED trollies. Several scenarios were designed to investigate the boarding problem in the AMAU and identify the factors which contribute to LOS and patients' access to this unit. Two factors are examined in this experiment ED boarders and internal AMAU boarders (Table 4). The value of 'YES' to the ED boarders means allowing the 'ED boarded' patients

Table 4
Boarding scenario design.

	Allow ED boarders	Allow internal AMAU boarders
Base scenario	Yes	Yes
Scenario 1	No	Yes
Scenario 2	Yes	No
Scenario 3	No	No

Table 5
Simulation results for boarders scenarios (95% confidence intervals).

KPIs	Base	Scenario 1		Scenario 2		Scenario 3	
		O/P[a]	(%)	O/P[a]	(%)	O/P[a]	(%)
Avg. LOS (Minutes)	277	272 ± 2.9	−1.8	251 ± 1.5	−9.3	286 ± 1.8	3.2
Avg. Patient access	1710	2152 ± 49.0	25.9	2337 ± 57.1	36.7	3052 ± 13.4	78.5

a. O/P: simulation output; % percentage of increase/decrease relative to the base scenario.

Table 6
Scenarios stock and flow factor analysis.

Scenarios	Stock factors		Flow factors		KPI
	Number of SSU beds	Number of ward beds	LOS SSU	LOS ward	Patient access
Base	24	520	105	162	1710
1	18	390	79	122	1678
2	18	520	105	162	1189
3	18	650	131	203	724
4	24	520	131	122	1350
5	24	650	79	162	1795
6	30	650	105	122	1865

to be transferred to the AMAU. On the other hand, the value 'YES' for the 'internal AMAU boarders' means allowing the AMAU admitted patients to be boarded in the unit when they waited for an inpatient bed. The value 'NO' for both factors corresponds to the immediate transfer of patients downstream. This value can be seen as a relaxation of the capacity constraint in the downstream hospital facilities.

Scenario one assumes constant inpatient beds availability for ED patients, in order to prevent ED boarders from being transferred to the AMAU. This scenario resulted in a significant increase in patient access to the AMAU by 25.9% (i.e., from 1710 to 2152 patients). This improvement is attributed to the increase in the availability of AMAU trollies as a result of less waiting time. However, there is an insignificant reduction in the LOS of AMAU patients (−1.8% of the baseline). The second scenario (Table 5) seems to be more efficient than the first one, regarding a patient access increase (from 1710 patients to 2337 patients) and reduction in LOS (9.3%). Scenario 3 is the ideal situation for the system, in which there is no ED boarding and internal boarding. In this scenario, there is a noticeable increase of 78.5% in patient access. However, LOS has not been significantly improved, and on the contrary, it was slightly increased (3.2%). The substantial increase in patient access can partially explain this. There are two ways to achieve this scenario: either increase bed capacity (stock interventions) or, reduce LOS of patients in downstream facilities/units (flow interventions). This will be explored in Section 5.2.

A thorough analysis of the AMAU queues (Fig. 9) shows that the majority of patients are waiting for transfer, (from the ED to the AMAU namely 'Patient pass'), and waiting to be seen by a consultant. The results have raised the need for further experimentations to identify the most significant factors which determine the performance of the AMAU. Therefore, a DoE along with multivariate analysis was conducted and presented in Section 5.3.

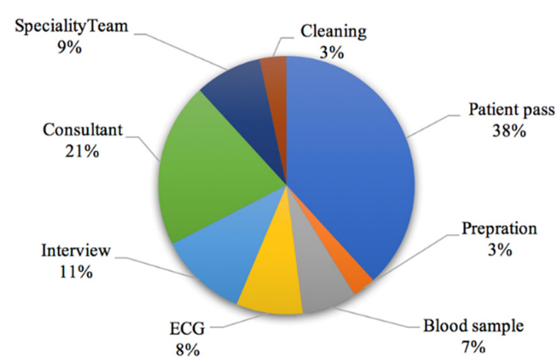

Fig. 9. Percentage of waiting times during different procedures in AMAU.

5.2. Stock/flow interventions

In this analysis, various scenarios are examined to explore the impact of stock and flow factors identified in the SD level on patient access to the AMAU defined in the DES model. Stock factors investigated are the number of beds in SSU and number of inpatient ward beds. Through discussion with stakeholders, the flow factors included in these experiments are the average LOS in both the SSU and inpatient ward. Table 6 summarizes the scenarios used to examine their impact.

The evaluation of these scenarios is presented in Fig. 10. Results show that the SSU beds are positively linearly correlated to the patient access, with $R^2 = 42.9\%$ (Fig. 10a). The results confirm that more SSU beds facilitate in reducing the internal boarding and therefore increases the patient access. However, the capacity expansion of the inpatient ward has insignificant linear effect with $R^2 = 1.6\%$ (Fig. 10b). The outcomes of this capacity expansion policy in the model reveal that increasing inpatient bed capacity only provides a temporary solution, and its impact is insignificant over longer periods [11]. When the inpatient bed capacity is increased, simply more patients are admitted. Therefore, the effect of that capacity expansion can only offer a temporary solution. As soon as the additional capacity is used up, the situation gradually deteriorates again. Capacity expansion is an example of stock interventions, which have time-limited effects and may even stimulate more demand.

On the other hand, the average LOS in both the SSU beds and inpatients beds are examples of flow interventions. Fig. 10c

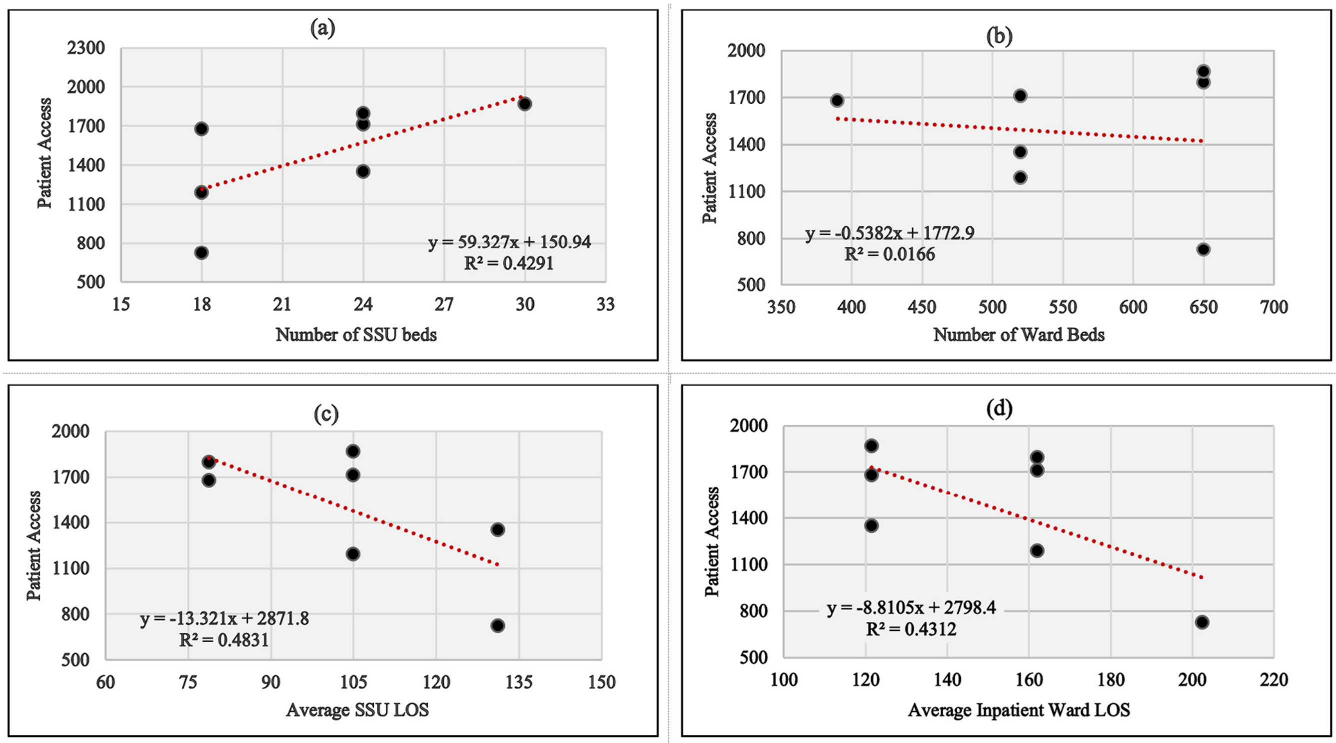

Fig. 10. Scatter plots of the stock and flow factors against the patient access to the AMAU.

and d show a negative linear relationship between patient access and both average LOS in SSU ($R^2 = 48.3\%$) and inpatient ward ($R^2 = 43.1\%$). The results showed that flow interventions can be more effective in increasing patient access, and are more likely to be effective in improving healthcare system performance than implementing stock interventions separately. Creating new channels to reduce emergency admissions and the average LOS can make a significant improvement.

5.3. Multivariate analysis of variance

An efficient DoE can reduce the experimental effort and simplifying the task of interpreting the results by identifying the factors which are essential for each response. Also, DoE helps to specify the different effects on the responses and factor interaction [105] by using seven different factors, with three levels (i.e., L27). The Taguchi orthogonal array (OA) factorial designs provide the possibility to consider a subset of combinations of multiple factors run at different numbers of levels. OAs are balanced to ensure that all levels of all factors are considered equally. Table 7 summarizes the factors used and designs develop the DoE.

Parameters are divided into two groups: 1. Controllable ones, which are directly observable such as the number of porters or trollies and, 2. Uncontrollable ones, which require statistical inference such as an average number of ED boarders. Two responses which were used in this experiment, include the average LOS in the AMAU and the average number of patients accessing the AMAU. In these designs, for example, scenario one means there is one porter, nine trollies, and one consultant while one trolley is blocked for 4 h due to the transferring of patients from the ED to the AMAU overnight. Also, the average waiting time for accessing ward beds and average waiting time for accessing SSU beds, (in order to release AMAU trolley) is one hour.

In order to test the significance of the input parameters, MANOVA was used. MANOVA is very similar to ANOVA for testing the significant differences between two or more groups of participants. However, MANOVA is appropriate when the study involves more than one criterion variable. In this study, MANOVA analysis is performed on responses in order to indicate the influence or significance of input parameters on the AMAU KPIs. The MANOVA table for all responses and variable interactions are presented in Table 8.

By considering p-value = 5% and within the selected L27's range, the results indicate the number of porters, consultants, trollies, ED boarding time, the time waiting for SSU and ward beds, are significantly related to the LOS and patient access. However, the average number of blocked trollies in the ED boarding case by p-value = 0.5783790 is insignificant. This result shows that the blocking time of trollies has a higher impact on the KPIs than the number of blocked trollies. Also, two-way interactions applied to the data in the investigation of interactions among the independent variables. According to the results, the number of porters is significantly related to LOS and patient access. The number of porters also has a significant interaction ($p < 0.05$) with the number of consultants and the number of trollies which implies the effect of trollies and consultants to improve KPIs depends on the number of porters. In addition, a significant interaction ($p < 0.05$) were found between the number of consultants and trollies. However, according to p-values, no other interactions have been seen between other factors.

Table 7
A description of DoE variables and responses with L27 simulation designs.

Factors		
Type	Variable Description	Levels
Uncontrollable	P_1: Avg. ED boarding time	3L: 4, 8 and 12
	P_2: Avg. number of blocked trollies in ED boarding	3L: 1, 3 and 6
	P_3: Avg. waiting time for inward beds (i.e., Internal boarding)	3L: 1, 2 and 3
	P_4: Avg. waiting time for SSU beds (i.e., Internal boarding)	3L: 1, 2 and 3
Controllable	X_1: Number of porters	3L: 1, 2 and 3
	X_2: Number of trollies	3L: 9, 11 and 16
	X_3: Number of consultants	3L: 1, 2, and 3
Responses Variables (Predictors): Y_1: Avg. LOS in AMAU Y_2: Avg. Patient access		

L27 Simulation Designs

	Controllable Variables			Uncontrollable Variables				KPIs	
Design	X_1	X_2	X_3	P_1	P_2	P_3	P_4	LOS	Patient access
1	1	9	1	4	1	1	1	272 ± 1.86	2239 ± 34.82
2	1	9	1	4	3	2	2	305 ± 1.98	2118 ± 31.89
3	1	9	1	4	6	3	3	332 ± 2.63	1984 ± 13.79
4	1	11	2	8	1	1	1	260 ± 1.46	2200 ± 8.67
5	1	11	2	8	3	2	2	290 ± 1.07	2112 ± 11.06
6	1	11	2	8	6	3	3	323 ± 2.42	2047 ± 18.26
7	1	16	3	12	1	1	1	265 ± 3.47	2386 ± 15.77
8	1	16	3	12	3	2	2	295 ± 2.98	2354 ± 25.39
9	1	16	3	12	6	3	3	329 ± 1.14	2307 ± 15.34
10	2	11	1	12	1	2	3	297 ± 3.31	1846 ± 26.53
11	2	11	1	12	3	3	1	282 ± 5.88	1931 ± 11.63
12	2	11	1	12	6	1	2	267 ± 3.28	1954 ± 13.78
13	2	16	2	4	1	2	3	290 ± 2.15	2488 ± 6.95
14	2	16	2	4	3	3	1	273 ± 3.08	2493 ± 16.09
15	2	16	2	4	6	1	2	257 ± 4.32	2496 ± 18.87
16	2	9	3	8	1	2	3	287 ± 6.04	1873 ± 22.15
17	2	9	3	8	3	3	1	269 ± 2.55	1955 ± 10.79
18	2	9	3	8	6	1	2	253 ± 2.40	1989 ± 14.86
19	3	16	1	8	1	3	2	300 ± 3.34	2395 ± 19.68
20	3	16	1	8	3	1	3	287 ± 2.32	2422 ± 11.33
21	3	16	1	8	6	2	1	266 ± 1.84	2421 ± 10.84
22	3	9	2	12	1	3	2	285 ± 2.28	1550 ± 15.57
23	3	9	2	12	3	1	3	271 ± 1.68	1629 ± 10.16
24	3	9	2	12	6	2	1	255 ± 2.94	1627 ± 10.27
25	3	11	3	4	1	3	2	287 ± 1.35	2422 ± 17.36
26	3	11	3	4	3	1	3	272 ± 2.14	2436 ± 19.40
27	3	11	3	4	6	2	1	254 ± 1.51	2453 ± 15.93

Table 8
MANOVA for LOS and patient access and two-way interactions between different factors.

Factors	D.F.	Pillai	Approx. F	Num. D.F.	Den D.F.	Pr. (>F)
X_1 (Porter)	1	0.97173	68.75	2	4	0.0007992 ***
X_2 (Consultant)	1	0.98223	110.52	2	4	0.0003159 ***
X_3 (Trolley)	1	0.99816	1082.95	2	4	3.398e-06 ***
P_1 (ED boarding time)	1	0.99709	685.88	2	4	8.453e-06 ***
P_2 (Number of blocked trollies in ED boarding)	1	0.23949	0.63	2	4	0.5783790
P_3 (Waiting for ward beds)	1	0.99388	324.68	2	4	3.748e-05 ***
P_4 (Waiting for SSU beds)	1	0.99441	355.63	2	4	3.127e-05 ***
X_1:X_2	1	0.86957	13.33	2	4	0.0170112 *
X_1:X_3	1	0.98366	120.38	2	4	0.0002671 ***
X_1:P_2	1	0.31838	0.93	2	4	0.4646111
X_1:P_3	1	0.24724	0.66	2	4	0.5666508
X_2:X_3	1	0.90862	19.89	2	4	0.0083500 **
X_2:P_2	1	0.56800	2.63	2	4	0.1866280
X_2:P_3	1	0.46795	1.76	2	4	0.2830789
X_2:P_4	1	0.44654	1.61	2	4	0.3063215
X_3:P_2	1	0.56393	2.59	2	4	0.1901581
X_3:P_3	1	0.02828	0.06	2	4	0.9442465
X_3:P_4	1	0.04237	0.09	2	4	0.9170541
P_1:P_2	1	0.01317	0.03	2	4	0.9738276
P_1:P_3	1	0.12482	0.29	2	4	0.7659392
P_5:P_6	1	0.34373	1.05	2	4	0.4306851

Significance codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1.

Table 9
The score for each scenario by output-oriented BCC model.

Rank	DMU/Design	Score	Rank	DMU/Design	Score
1	D27	**1.301238**	15	D1	1
2	D18	1.289810	16	D8	0.986507
3	D15	1.244203	17	D9	0.966728
4	D4	1.158426	18	D17	0.961707
5	D21	1.093559	19	D2	0.946221
6	D24	1.083322	20	D5	0.925746
7	D12	1.082793	21	Baseline	0.921769
8	D20	1.081651	22	D23	0.905874
9	D14	1.072572	23	D6	0.897512
10	D7	1.070592	24	D3	0.886189
11	D19	1.069948	25	D11	0.842808
12	D13	1.067622	26	D16	0.836520
13	D26	1.053364	27	D10	0.808481
14	D25	1.048632	28	D22	0.785705

After completing DoE and analyzing different scenarios and results, the designs are used as an entry for the DEA model to rank the scenarios and choose the best strategy.

5.4. DEA results

The ultimate objective of this study is to see the trade-off between scenarios and determine the most efficient scenario. Therefore, DEA is used to rank the scenarios and select the most efficient one. DEA, as a mathematical approach, is one of the most robust methodologies used to evaluate the efficiency of decision-making units (DMU), as it considers multiple inputs and outputs. Also, due to the complexity and variety of measures in a healthcare context, DEA is considered as a useful method to provide a valid model for decision-making [89]. More details about different DEA models is provided in Appendix C. In constant returns to scale (CCR) and variable returns to scale (BCC) DEA models, DMU is efficient if $w_0 = 1$ and DMUs with $w_0 < 1$ consider inefficient. In some cases, there is a possibility that multiple DMUs reach $w_0 = 1$. Therefore, to overcome this problem, the super-efficiency technique is used by ranking efficient DMUs. The best DMU has the highest super-efficiency score [106] baseline and 27 scenarios are considered as DMUs in the DEA model. As the focus of this study is on the outputs of the model, so the output-oriented DEA models are applicable. The output-oriented BCC performs better compared with other models; it is thus chosen for ranking. It is essential to fix the parameters of the problem with the structure of the model before using the model to calculate efficiencies and rank scenarios. The output-oriented BCC model maximizes the outputs of the model (larger the better type). However, in this model, the LOS should be smaller. Therefore, equation one is applied to transform larger the better type to smaller the better type (e.g., LOS) and vice versa [107].

$$x = \frac{\max(x) - x}{\max(x) - \min(x)} \tag{1}$$

Eq. (1) normalizes the data and gives the values between 0 and 1 and also change their type. Now, by using the output-oriented BCC model on the standardized data, it is possible to calculate efficiencies of different DMUs. The initial results showed multiple DMUs reach $w_0 = 1$, so to solve this problem and discriminate among efficient DMUs, the super-efficiency technique is applied to rank them (Table 9). The results for current and optimal scenarios show that the current design of the AMAU has a lower performance than 20 defined DMUs.

DMU 27 is the best with the efficiency score of 1.30. It is ranked as the DMU with the best overall performance. In this scenario, the number of AMAU trollies did not change. However, two porters and two consultants were added. Six trollies are blocked for 4 h while in the current situation between 1 to 6 AMAU trollies are blocked for a maximum of 12 h each. Also, waiting time for accepting a patient to the inward or SSU are 2 and 1 h, respectively. This scenario results in an 8.2% decrease in the LOS (from 276.8 to 253.9) and 43.42% increase in patient access (from 1710 to 2452.6). Scenario 18 with an efficiency equal to 1.28 ranks second, which leads to a reduction in the LOS by 8.5% and 16.3% increase in patient access. To compare scenario 18 with scenario 27, the number of porters is two, and the number of trollies decreases from 11 to 9. Also, six AMAU trollies are blocked for 8 h while waiting for inpatient ward beds is 1 h. Therefore, the blocking time of trollies plays a critical role and improving waiting times for SSU beds are effective than improving waiting for ward beds (i.e., internal boarding case) to DMUs efficiency. Furthermore, in both scenarios decreasing the ED boarding times and internal boarding times as well as improving staffing levels, is more efficient than increasing trolley capacity in comparison with other scenarios. It can be concluded that improving the flow between up and downstream units, as well as increasing the porter and the consultant staff, enhances the patient access more than increasing the number of AMAU trollies.

5.5. The managerial implication of study

Due to the complex and systemic causes for patient flow delays, simple local solutions are found to be ineffective. This study, firstly, offers useful insights to hospital managers in order to better comprehend the underlying dynamic interactions between the different elements of patient flow and enhance their understanding of their systems. Thus, such models are instrumental in assessing various flow delays, and possible interventions. They are also essential in finding the main causes of poor patient flow and offering solutions to support patient flow management. This study also highlights the benefit of focusing on care processes and considering their interdependency at various care stage. This is achieved using system-wide approaches to improve patient flow delays, through the continuum of care. In other words, removing barriers to improve the patient flow process requires including all the relevant departments and their interdependencies, rather than examining the role of each department individually. Therefore, it is recommended the hospital administrators and healthcare planners attempt to provide solutions, which consider consecutive steps in the care process, in order to ensure continuous care delivery. Finally, the insights from this study provide opportunities for future research and practice. It may guide future researchers and hospital managers in adopting hospital-wide approaches and exploring the relationship between downstream and upstream departments. There also should be a focus on shared resources, services between units, enhancing communication and coordination across the hospital in order to successful patient flow improvement.

6. Conclusion

Fundamentally, boarding patient problem has two interrelated issues. The first issue concerns considerable delays that occur when transferring patients to their appropriate medical care units. This matter has become a silent feature of overcrowded EDs, where patients wait in areas which are not designated for clinical care such as treatment rooms, corridors, chairs, or on trollies. The second issue is related to the inappropriate placement of admitted patients. This has a direct relationship to the overcrowding that is increasing the pressures to meet the national targets. Therefore, due to the aforementioned constraints, decisions to move patients to other areas which are not suitable for patient conditions can be argued. Although the ED's boarding problem has received considerable attention in the literature recently, the problem in other hospital units has not been adequately addressed.

This paper presents a system-wide approach to examine the boarding problem in order to improve the flow of patients across hospital units. A hybrid system of DES, SD, DoE and DEA is developed to provide a comprehensive understanding of the dynamic relationships between hospital units and their interdependencies. SD component captures the interactions and feedback dynamics between the AMU and connected units effectively (i.e., ED and medical wards). The DES element enables the identification of bottlenecks, which impede system performance. Incorporating DoE and DEA allows unit managers to evaluate the efficiency of a diversity of capacity, staffing and patient flow scenarios. The proposed methodology provides a practical guide for users of hybrid system that can help understand and model the complexity of healthcare systems.

This study also highlights that the patient flow, as a property of the entire care system, can only be truly optimized at the system level. Indeed, flow improvement requires the design and evaluation of interventions in different parts of the system. Therefore, to support patient flow improvement, hospital administrators and healthcare planners should view the whole care process while considering many interactions between patients, clinicians, different services and resources. Results also indicate focusing on improving the linkages and connections between different care process steps, provide new opportunities to alleviate the patient flow bottlenecks without more investment in expanding capacity.

Unnecessary delays in discharging patients from inpatient wards do increase the number of boarders in connected upstream units (i.e., units competing for inpatient beds such as ED, AMU, and ICU). Improving the process of patient discharge (on-time discharge of patients and the reduction of patient LOS), can accelerate transfer rates between hospital units and minimize the inappropriate placement of patients. Sustaining efficient flow of patients requires adequate staffing levels to deal with the upsurge in the rate of patients.

CRediT authorship contribution statement

Leila Keshtkar: Conceptualization, Methodology, Software, Analysis, Writing - original draft. **Wael Rashwan:** Conceptualization, Methodology, Software, Data collection, Validation, Writing - review & editing. **Waleed Abo-Hamad:** Supervision, Investigation, Writing - review & editing. **Amr Arisha:** Project administration.

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Appendix A. Model documentation

A.1. Objectives

A.1.1. Purpose of the model

This study presents an integrated hybrid approach to investigate the consequences of patient boarding in the AMAU and improve the flow of patients across hospital units.

A.1.2. Model outputs

There are two main KPIs for this study:

1- Average patient LOS (i.e., the total time from patients' entrance to the unit until their exits the unit).

2- Average patient access (i.e., the total number of patients enter the AMAU).

A.1.3. Experimentation aims

Several scenarios were designed to investigate the boarding problem in the AMAU by considering the interdependency between downstream and upstream facilities, in particular, ED and inpatient wards. These including baseline scenario, boarding experiments, stock and flow intervention scenarios; and the combination of first group scenarios with adding different resources. Moreover, to support simulation as a tool for experimentation, the DoE was used for the evaluation of scenarios and identify the most significant factors affecting the overall performance. Finally, DEA was applied to evaluate and rank the best scenarios in improving AMAU performance.

A.2. Logic

A.2.1. Base model overview diagram

Fig. 5 demonstrations the generic patient pathways through the hospital with the AMAU/SSU which is the broad conceptual model for the simulation. Fig. 6 also present the patient's flow in AMAU while Fig. 7 present a simplified causal loop diagram of the SD model.

A.2.2. Base model logic

SD basic model logic:

First, patients arrive at the ED. Then, patients are triaged and assigned a triage category according to the MTS. Patients routed to the AMAU are those medical patients triaged as category 2 or 3 (i.e., very urgent and urgent patients respectively) who do not require resuscitation or isolation facilities. The triage nurse usually contacts the AMAU consultant or registrar so that they can accept or reject the case. These patients only transferred to the AMAU if a trolley is available. Otherwise, patients should stay in the ED to finish their treatment. All the patients from the ED finally will be discharged to the home or transfer to the inpatient setting. In this model inpatient wards also receive elective patients and patients from the AMAU. In the last stage, patients after completing their treatment in the inpatient setting will leave the hospital.

DES basic model logic:

After transferring the patients from ED, patients presented in the AMAU will get to see a senior medical doctor, who should be able to treat and discharge patients, within a specific period of admission. AMAU acts as the first gateway for acute medical patients referred from the ED, while the SSU is used by patients who need to be admitted to the hospital but their estimated length of stay is below a certain threshold. Patients from AMAU after completing different processes can be discharge home, sent to SSU or admitted directly to hospital clinical wards.

A.2.3. Scenario logic

In the AMAU, there are three types of patients. The first type is medical patients who are in the process of treatment in the unit. The second type is the medical patients who completed their treatment in the unit and waiting to be admitted to an appropriate inpatient bed. This type of patient is referred as "internal AMAU boarders". The third category is the "ED boarders" who are clinically unnecessarily transferred to AMAU in sake to free up ED trolleys. Several scenarios were designed to investigate the boarding problem in the AMAU. First group scenarios consider as "boarding experiments" which mainly focus to explore the effect of existing and removing different types of boarding on KPIs. Second group scenarios, consider as "stock/flow interventions" which investigate the effect of changing the number of beds in SSU and number of inpatient ward beds as well as changing LOS in both the SSU and inpatient ward on KPIs. Third group scenarios, combine the first group scenarios with adding different resources.

A.2.4. Components

A.2.4.1. DES.

Entities:

The main entities of the model are patients that arrive in the ED from the community and each of them is assigned a set of attributes such as a triage category and clinical groups (medical or nonmedical).

In this model, patients flow through the system from ED (in the SD model) and medical patients (triage category 2 and 3) will send to AMAU. These patients pass different care stages while resources are identified and assigned to them during their journey. Next, after finishing their treatment in the AMAU, they will leave the unit to home, SSU or inpatient wards.

List of activities:

–Transfer to AMAU: Porter transfers the patient from ED to AMAU.
–Registration: Patient is registered and seize a trolley.
–Preparation in AMAU
–Assessment by AMAU doctor
–Different test: Some patients require to do the different test (e.g., blood test, ECG, X-ray).
–Consultant: The results from the previous stage will discuss by a consultant. The consultant will decide the patient may require a further text (e.g., MRI, CT) or should assess by the specialty team as well. Some patients also discharge to home directly.
–Specialty team stage
–Decision: Decision regarding discharge destination.
–Discharge: Patient will discharge to home, SSU or wards.
–Cleaning

List of resources:

Registrar, AMAU nurses, senior house officers (SHOs), AMAU consultant, porter, cleaners and AMAU trollies.

Schedules:

AMAU opening hours: 6 am to 9 pm.
Patient access hours to AMAU: 6 am to 6 pm.

List of queues:

Seize a trolley: First in first out.
Renege: If a trolley is not available.
Registration: First in first out.
Preparation in AMAU: First in first out.
Assessment by AMAU doctor: First in first out.
Different test (e.g., blood test, ECG, X-ray): First in first out.
Results of a test: First in first out.
Consultant: First in first out.
Specialty team stage: First in first out.
Decision: First in first out.
Accept to ward: First in first out.
Accept to SSU: First in first out.

Entry/Exit Points:

Entry point: Patients are sent from ED (in the SD model) to AMAU
Exit Points: Home, SSU and Wards.

A.2.4.2. SD.

Stocks: Stocks (i.e., state variables) are the accumulations that characterize the system's state.

–Patients in Triage: Accumulation of the patients after their arrival in order to continue the care in the ED or AMAU.
–Patients in ED: Those patients that are not medical patients with triage category 2 or 3, as well as the patients renege from the AMAU to ED are accumulated in this stock.
–ED patient waiting for the ward admission: Store the patients from ED to admit to the wards.
-AMAU patient waiting SSU bed: Patients from AMAU accumulate in this stock in order to receive a bed in SSU.
–AMAU patient waiting for the ward admission: Patients from AMAU accumulate in this stock in order to receive a bed in the inpatient setting.
–Scheduled elective waiting: Elective patient baulk in this stock in order to receive a bed in the wards.
–Patients in SSU: Patient baulk in SSU for a specific period to complete their treatment.
–Patients in the wards: Patient baulk in the wards for a specific period to complete their treatment.
–ED patient waiting for AMAU admission: Patients from ED accumulate in this stock in order to receive a bed in the wards.

Flows: Flow variables are rates or control variables that can change the state (i.e., the stocks) of the system.

–List of inflows with the equation:

ED patient arrival	*Arrival rate/time scale*
Renegade rate to ED	*Number Renege*
Patient in triage to ED	*(Patient in triage * (1-AMAU Admit Fraction))/(Triage time* Time Scale)*
Ward admission rate from ED	*Patients in ED * ED Admit Fraction/(Patient Time In ED* Time Scale)*
Admission to ward	*Min (Waiting Ward Admission /Delay Time to Move Patient, Available Beds)*
SSU Admission	*Min (Waiting SSU Admission, SSU Available Beds)*
AMAU to SSU admission rate	*Min (AMAU Await SSU Bed, SSU Available Beds — SSU Admission)*
AMAU to ward rate	*Min (AMAU Await ward Bed, (Available Beds-Ward Admission Rate))/Delay Time Ward*
The actual elective admission rate	*Min (Min (Desired Elective Admission Rate, Available Beds-Ward Admission Rate-AMAU to Ward Rate), Scheduled Elective Waiting)*
Schedule admission rate	*Desired Elective Admission Rate/(24* Time Scale)*
Other ED admission	*Daily Other ED Admission/(24* Time Scale)*

–List of outflows with the equation:

ED discharge rate	*Patients In ED*(1- ED Admit Fraction)/(Patient time in ED* Time scale)*
Sending patients to AMAU	*Admission AMAU*
SSU discharge rate	*Patient in SSU/(LOS SSU* Time scale)*
Ward discharge rate	*Patient in Ward/(LOS Ward* Time scale)*

Feedback loops:
Bed occupancy, waiting for the ward admission and waiting for SSU admission.

Constants/auxiliary variables:

Arrival Rate	*Table Function Arrivals*
Week hour	*((t/timescale)%168)*
AMAU Admit Fraction	*AMAU Fraction*AMAU Opened*
T	*time ()*
SSU Available Beds	*Max (0, SSU bed Capacity-Patient In SSU)*
LOS SSU	*base SSU LOS + Change in SSU LOS* base SSU LOS*
LOS ward	*base Ward LOS + Change in Ward LOS*base Ward LOS*
Ward Available beds	*Max (bed Capacity-Patient in Ward, 0)*
Daily Other ED Admission	*Monthly Other ED Admission /30*

Graphical functions/lookup tables: N/A

Source and sinks: A source represent the inflow that comes from the cloud. Sinks are the outflows that go to the cloud.
–List of sources:
ED arrival, renegade patients, scheduled patient in the wards and other ED daily admission.
–List of sinks:
Discharge patient from ED, discharge patient from SSU, discharge patient from the ward and elective cancellation.

A.3. Data

A.3.1. Data sources

Table 1 provides the details of the elements of data sources.

A.3.2. Input parameters

Input parameters for DES model summarized in the Table A.1.

A.4. Experimentation

A.4.1. Initialization

The model runs for twenty-four weeks' period with a warm-up period of 8 weeks. This period was found enough so that the model achieves a steady-state condition. No initial conditions are used for the DES model.

Initial values for SD model:

Patient in ward	400
Scheduled elective patients	200

A.4.2. Run length

All point estimates are based on the average of 10 replications of a model run (More details provided in Appendix B). Each run has different random seed.

A.5. Implementation

A.5.1. Software or programming language

DES and SD models are developed using AnyLogic 7 university.

A.5.2. Random sampling

All pseudo-random number generator was based on the default random number generator. The default random number generator is an instance of the Java class Random, which is a Linear Congruential Generator (LCG).

A.5.3. Model execution

One replication takes 1.89 min.

A.5.4. System specification

The model was run on iMac, with a 3.1 GHz intel Core i5 processor and 16 GB of memory and Macintosh HD.

A.6. Code access

Models are available on request. (See Fig. A.1).

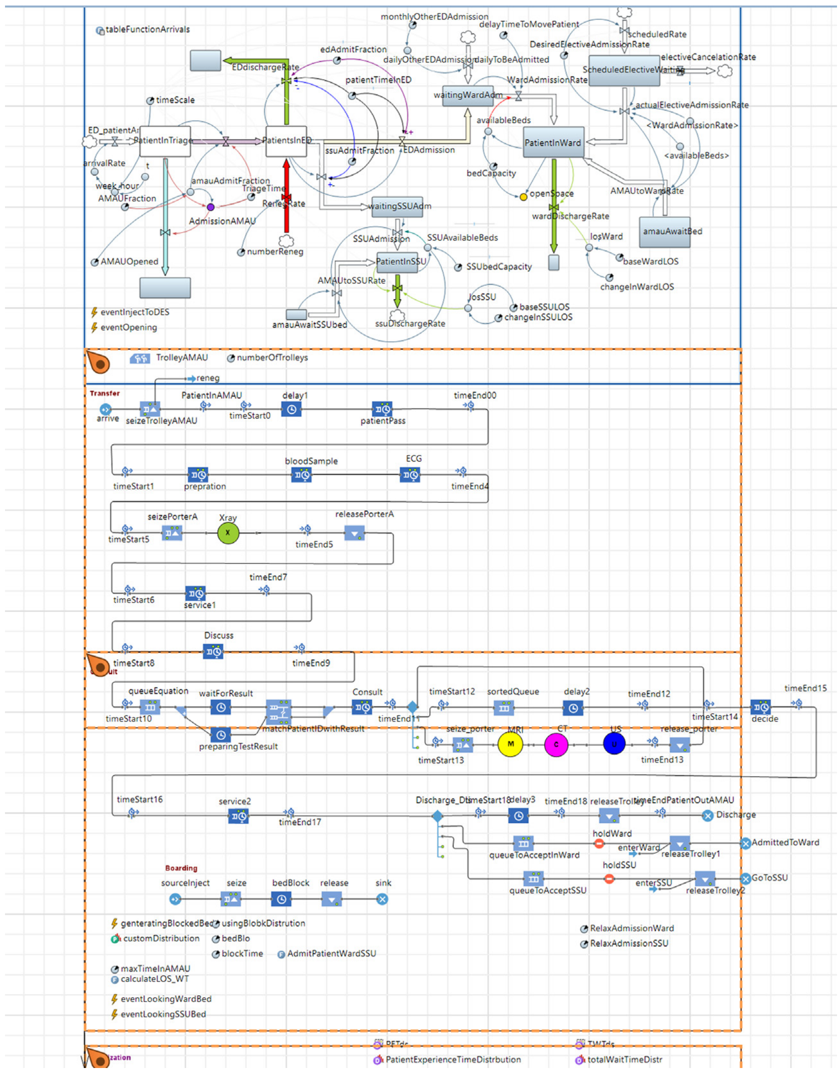

Fig. A.1. A screenshot of hybrid simulation model in Anylogic software.

Table A.1
DES model input parameter.

	Activity	Distribution	Min	Most likely	Max
Processing Time (min) AMAU	Registration	Triangular distribution	1	5	7
	Transfer patient	Triangular distribution	3	5	10
	Preparation	Triangular distribution	5	10	15
	Interview	Triangular distribution	20	30	50
	Discussion	Triangular distribution	5	10	15
	Consult	Triangular distribution	10	15	20
	Specialty team	Triangular distribution	90	180	300
	Consult Decision	Triangular distribution	5	10	15
	Exit	Triangular distribution	1	5	7
	Clean	Triangular distribution	1	5	7
Radiology	Radiology (X-ray)	Triangular distribution	30	40	60
	Radiology (MRI)	Triangular distribution	10	30	40
	Radiology (CT)	Triangular distribution	10	30	40
	Radiology (US)	Triangular distribution	10	30	40
Lab	Blood test	Triangular distribution	5	10	20

Appendix B. Finding the number of replications

To determine the number of replications both graphical and confidence interval methods were applied. The graphical method was used to provide a plot of the cumulative mean of the simulation output. As more simulation replications are performed, the cumulative mean graph converts to a flat line (Fig. B.1).

For a more accurate estimation of the number of replications, the confidence interval was calculated for the initial number of replications (Table B.1). As shown in Table B.1, the mean of confidence intervals becomes narrower as more simulation replications are performed. Therefore, ten replications were estimated.

Table B.1
Results from 11 replications for patient experience time.

				Significance level 5.0% confidence interval		
		Cumulative. mean	Standard	Lower	Upper	%
Replication	Result LOS	Average	Deviation	Interval	Interval	Deviation
1	279.77	279.77	n/a	n/a	n/a	n/a
2	276.39	278.08	2.390	256.61	299.55	7.72%
3	277.2	277.79	1.765	273.40	282.17	1.58%
4	275.65	277.25	1.794	274.40	280.11	1.03%
5	277.52	277.31	1.558	275.37	279.24	0.70%
6	271.96	276.42	2.589	273.70	279.13	0.98%
7	276	276.36	2.369	274.16	278.55	0.79%
8	280.79	276.91	2.696	274.66	279.16	0.81%
9	277.42	276.97	2.528	275.02	278.91	0.70%
10	275.23	276.79	2.446	275.04	278.54	0.63%
11	273.35	276.48	2.542	274.77	278.19	0.62%

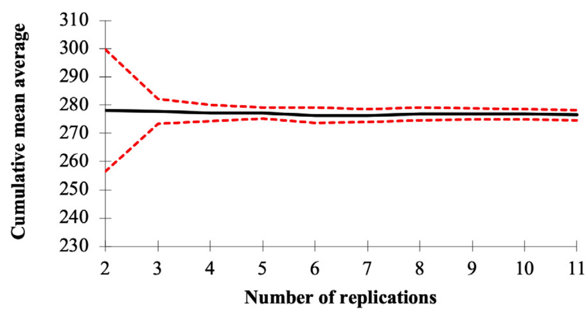

Fig. B.1. The plot of cumulative mean and 95%confidence interval.

Appendix C. Different DEA models

Various DEA models have been used in the healthcare context including CCR and BCC models as the classic forms of DEA [93], which are defined as follows. The CCR model presents a constant return to scale:

$$\max w_0 = \sum_{r=1}^{s} u_r y_{r0} \tag{2}$$

Subject to:

$$\sum_{i=1}^{m} v_i x_{i0} = 1 \tag{3}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1, 2, \ldots, n \tag{4}$$

$$u_r, v_i \geq 0, r = 1, 2, \ldots, s \text{ and } i = 1, 2, \ldots, m \tag{5}$$

where $r \in \{1, \ldots, s\}$ is the output index, and s is total the number of outputs; $i \in \{1, \ldots, m\}$ is the input index and m is the total number of inputs; y_{rj} is the rth output for the jth DMU; x_{ij} is the ith input for the jth DMU; u_r is the weight associated with the rth output; v_i is the weight associated with the ith input; w_0 is the relative efficiency of DMU_0, which is the DMU under evaluation; and y_{r0} and x_{i0} are respectively the outputs and inputs for DMU_0.

To consider variable return to scale, BCC model was developed by the addition of free variable C_0.

$$\max w_0 = \sum_{r=1}^{s} u_r y_{r0} + C_0 \tag{6}$$

subject to:

$$\sum_{i=1}^{m} v_i x_{i0} = 1 \tag{7}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + C_0 \leq 0 \quad j = 1, 2, \ldots, n \tag{8}$$

$$u_r, v_i \geq 0, r = 1, 2, \ldots, s \text{ and } i = 1, 2, \ldots, m \tag{9}$$

In the BCC model the use of data that present negative values are possible, which can happen in stochastic simulation models. Also, this model in contrast to CCR model is invariant to the application of linear transformations to the inputs and outputs values [106].

Both BCC and CCR model can be input or output-oriented. The input-oriented model is used to minimize inputs while keeping the outputs at their current levels. The output-oriented model intent to maximize outputs while using no more than the observed amount of any inputs [97].

To achieve sufficient discrimination of the DMUs in a traditional DEA model, the number of DMUs should satisfy the following equation:

$$n \geq Max\{(ms), 3(m+s)\} \tag{10}$$

where m is the total number of inputs and s is total the number of outputs; n is the total number of DMUs [108]. In CCR/BCC models DMU is efficient if $w_0 = 1$ and DMUs with $w_0 < 1$ consider inefficient. In some cases, there is a possibility that multiple DMUs reach $w_0 = 1$. Therefore, to overcome this problem, the super-efficiency technique is used by ranking the efficient DMUs. In fact, the best DMU has the highest super-efficiency score [106].

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