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Probabilistic Analysis of the Active Earth Pressure on Earth Retaining Walls for c-φ Soils According to the Mazindrani and Ganjali Method

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Probabilistic analysis of the active earth pressure on earth retaining walls for c-φ soils according to the Mazindrani and Ganjali method

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Abstract

The determination of the earth pressure coefficients (K) in geotechnical engineering is one of the most critical procedures in designing earth retaining walls. However, most earth pressure theories are made for either clay or sands, where the c-φ soils are the least analysed. In this paper, an analysis of the earth pressure for drained mixed soils based in Mazindrani and Ganjali (J Geotech Geoenviron Eng 123:110–112, 1997) theory was carried out. Earth pressure coefficients are generally used in a deterministic way and can represent designs under an inadmissible risk. Therefore, Reliability-based design arises as an essential tool to deal with soil variability as one of the main aspects of the geotechnical uncertainties. The influence of the soil variability in the active earth pressure for a c-φ soil was performed through probabilistic analysis concerning the K_a coefficient of variation (Cv) of both shear strength parameters. The sensitivity analysis shows a Cv in which the cohesion begins to have a more significant correlation with K_a than the friction angle. The results show an increase of the statistical K_a concerning the deterministic value as the soil variability and the soil slope (β) increase. Although the statistical value does not increase significantly, a statistical analysis on gravity walls and sheet pile walls in c-φ soils shows a significant probability of failure (p_f) increase. The p_f obtained through the c-φ variability can be considered inadmissible even if the required FS are met.

Keywords: Earth pressure coefficient, C-φ soils, Soil variability, Reliability-based design

Introduction

The determination of the active pressure coefficients (K_a) in geotechnical engineering is essential in designing earth retaining walls [24]. However, most earth pressure theories are made for clay or sands, where the c-ϕ soils are the least analysed. The soil lateral earth pressure is a function of K_a, unit weight, and the depth at which the pressure is required. Several authors have developed mathematical models to determine the earth pressure coefficient based on the limit equilibrium method (e.g., [4, 6, 8, 23, 29, 33]). However, these models were developed for sandy soils and did not consider the analysis of the cohesion (c) and the friction angle (ϕ) in mixed soils such as silts. The Mazindrani and Ganjali [22] theory has in mind the conditions of friction angle and soil cohesion. This equation is implemented for modeling mixed soil for drained conditions or effective stresses.

The Mazindrani and Ganjali [22] method arises for determining the earth pressure based on the limit equilibrium method. This method allows the consideration of mixed soils (silts) in the earth pressure coefficient calculation because it takes into account ϕ and c as implicit variables in the equation. However, the earth pressure coefficients are generally deterministic, representing designs under an inadmissible risk. Therefore, the reliability-based design is an essential tool to deal with the soil's uncertainties because soils are among the most variable materials in engineering [13].

There have been considerable advances in the soil variability characterisation and their influence in geotechnical designs [35]. Therefore, it was shown that soil properties variability is one of the main aspects of the geotechnical analyses uncertainties [18]. Soil shear strength properties and model uncertainties have been addressed by using different Factors of safety (FS) [16]. However, a geotechnical design with a high FS can have a high probability of failure (p_f) similar to a designed system with a low FS [21]. Therefore, the use of traditional methods in conjunction with probabilistic analyses has increased considerably in recent years [39]. The inclusion of probabilistic concepts can provide a better and more viable design method [3], thus reducing the uncertainties between the designs and the geo-structures' real behavior.

The soil variability influence in the active earth pressure for c-ϕ soil was performed according to probabilistic analyses. The probabilistic analyses consist of evaluating the K_a changes according to the coefficient of variation (Cv) of mixed soil shear strength parameters according to the Mazindrani and Ganjali [22] equation.

Mazindrani and Ganjali equation

Mazindrani and Ganjali [22] presented an analytical solution for the determination earth pressure based on the Rankine method. The lateral earth pressure coefficients are determined according to Eq. 1.

$$K_a = \frac{1}{cos^2\phi}\left\{\frac{2cos^2\beta + 2\left(\frac{c}{\gamma z}\right)cos\phi sin\phi -}{\sqrt{\left[4cos^2\beta\left(cos^2\beta - cos^2\phi\right) + 4\left(\frac{c}{\gamma z}\right)^2 cos^2\phi + 8\left(\frac{c}{\gamma z}\right)cos^2\beta sin\phi cos\phi\right]}}\right\} - 1 \quad (1)$$

where ϕ is the friction angle, c is the cohesion, γ is the unit weight, z is the vertical depth to any point on the back of the retaining structure and β is the backfill slope.

Uncertainty and soil variability

Soils are a natural variable material due to the formation processes and the continuous environmental changes that alter the external stresses, weathering, chemical reactions, the introduction of new substances, and human interventions [37]. Within a uniform geological layer, soil properties can be affected by inherent and epistemic uncertainties [14]. Inherent variability is the variation of soil properties from one place to another [37]. The epistemic uncertainty is related to the lack of knowledge about a variable, including the uncertainty in the measurements, in the data (limited information), and the model [14]. The uncertainty caused by the random behavior of the shear strength properties and the model's hypotheses can reduce the accuracy of the estimated bearing capacity, earth pressure coefficients, and slope stability analysis (e. g. [38])

Soil uncertainty is usually represented by the Standard deviation or the coefficient of variation (Cv). The latter is the ratio between the standard deviation over the mean. Table 1 presents some of the reported Cv values in the literature of the soil shear strength properties.

Reliability-based designs in geotechnical engineering

Probability theory and reliability analyses provide a rational framework to deal with the soil shear strength uncertainties [14]. Probabilistic analyses can be used through a deterministic model to perform a reliability-based design [20]. These designs are useful because it takes into account the soil variability for the evaluation of the probability of failure according to different failure mechanisms. However, structure failure can occur in their lifetime due to load changes [5]. One of the advantages of the reliability-based designs is that they can provide a margin for the designs according to a specific probability of failure, where all the model variables uncertainty can be taken into account [15].

According to Lacasse and Nadim [20], probabilistic analyses provide the following results:

- Probability of Failure (p_f)
- Reliability index
- Results sensitivity according to any changes in the parameters.
- An analysis of the parameters that can cause failure.

For reliability-based designs, several probability-based simulation approaches can be employed, including Monte Carlo, Point Estimates [31], First Order Reliability Method (FORM), and First Order Second Moment (FOSM) [32]. The Monte Carlo method is a sequence whose evolution is given by random events [17]. Monte Carlo is a powerful technique that applies to linear and non-linear problems and is the most used simulation method in geotechnical engineering [39]. However, it may require many simulations to provide reliable distribution of the output variable [14].

Table 1 Shear strength parameters coefficient of variation reported in the literature

Soil property	Coefficient of variation	Reference
Effective Friction angle (φ')*	2–13%	[19]
	3.7–9.3% sand	[30]
	7.5–10.1% clay	
	5–11% sand	[27]
	10–50% clay, silt	
	4–12% clay, silt	
Undrained Shear Strength (S_u)**	20–80%	[16]
	6–80% clay	[27]
	20–55% clay	[37]

* Results obtain of different drained Laboratory Test; **;Results obtain of different undrained Laboratory Test

The FOSM method uses the Taylor series to obtain the probability distribution of a function with several random variables [16]. One of the great advantages of the FOSM method is that it reveals the relative contribution of each variable to the general uncertainty in a clear and easily tabulated manner [2]. The purpose of considering the uncertainties due to the Soil Variability is to evaluate the impact that this variability generates in the structure design [37].

The reliability analysis focuses the probability of failure, as it is a more consistent and complete measure of safety because it is invariant to all mechanically equivalent definitions and incorporates additional information on uncertainty [26]. Reliability-based designs require a series of steps to define the different geotechnical properties and field characteristics that affect the probability of failure of geotechnical structures [39]. Reliability-based designs do not guarantee that the structure is immune to possible failures, but it does provide better decision-making tools [16]

Results and analysis

Earth pressure coefficient sensitivity analysis

A sensitivity analysis was performed on the Mazindrani and Ganjali [22] equation using the Spearman Correlation Coefficient (SCC), to evaluate the influence of the friction angle and cohesion variability (Fig. 1).

The Spearman Correlation Coefficient (SCC) is a method to conduct sensitivity analyses between the model input values and the output response. It is calculated using the ranking of the input values and not the actual values themselves, and it would render a value between 1 and − 1 [12]. The SSC provides a measure of the relationship between the input and output parameters. A positive correlation suggests that a high input value results in high output value and a negative correlation suggests that a high input value results in low output. The SCC quantifies how the input variables variability influences the output response dispersion [9]. Therefore, the SCC evaluates which variable exerts the most significant influence on the variation of the output parameters.

Figure 1 shows that, for the case evaluated here, the friction angle Cv used were 5%, 10%, and 15%, while the cohesion Cv ranged from 0 to 60%. The results show that cohesion is an important property in the K_a uncertainty with SCC values that tend to be highly negative (close to − 1) as the cohesion Cv increases. However, it can also be seen that for a less variable cohesion (low Cv), the K_a variability is mainly dependent on ϕ.

Figure 1 also shows that as the variability of the cohesion Cv increases, a "balance point" between the uncertainty contribution of both parameters is reached. As the cohesion Cv increases from the balance point, it becomes the parameter that contributes the most to the K_a uncertainty.

Soil variability influence in the lateral earth pressure

A gradual increase of Cv on each of the input parameters in the Mazindrani and Ganjali [22] equation was performed to determine the influence of the c-ϕ variability in K_a. Increasing Cv implies increasing the standard deviation or decreasing the geotechnical property mean of the Probability Density Function (PDF). In this case, the mean was kept constant while the standard deviation was gradually increased. The probabilistic evaluation of K_a was performed using a Normal and Log-Normal PDF for both

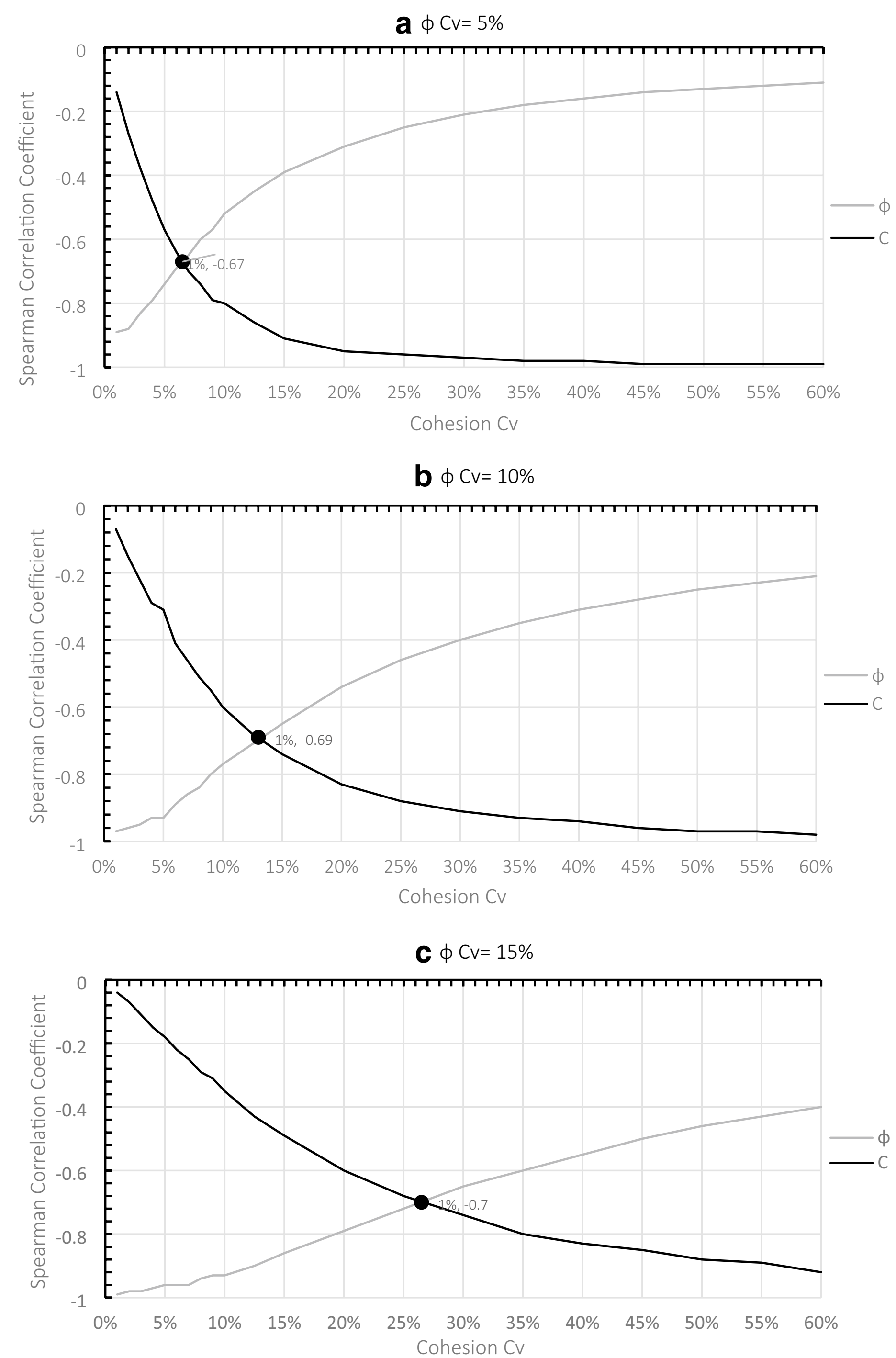

Fig. 1 K_a Spearman Correlation Coefficient for **a** φ Cv = 5%, **b** φ Cv = 10% and **c** φ Cv = 15% according to the Mazindrani and Ganjali [22] equation

properties because these are the most used functions in the literature (e. g. [1, 2, 5, 10, 11, 25, 28, 34, 37, 40]).

Normalization between the deterministic and statistical values of K_a was conducted to evaluate how the soil variability influences its value, as is shown in Eq. 2. The K_a normalization consists of dividing the earth pressure coefficient obtained for each Cv through a Monte Carlo Simulation (MCS) by the earth pressure coefficient obtained from the deterministic analysis

$$\boldsymbol{K}_{Normalized} = \frac{K_{Statistical}}{K_{Deterministic}} \tag{2}$$

According to the results in Fig. 2, it was shown that the cohesion variability does not present important changes in the magnitude of the normalized $K_{a.}$ Therefore, Cv(Cohesion) = 40% and Cv(ϕ) of 0–40% were used to obtain the normalized $K_{a,}$ as is shown in Fig. 3. The effect of the changes of the backfill slope (β) in K_a was also evaluated according to a backfill slope/friction angle ratio of β/ϕ = 0; β/ϕ = 0.4 and β/ϕ = 0.8.

Figure 3 shows an upward trend of the normalized K_a for both Normal and Lognormal ϕ PDF. The normal PDF shows higher K_a variations compared with the Lognormal PDF. Thus, it can be concluded that from a statistical point of view, the Normal function generates more conservative results of K_a values. However, it is shown that for a Cv > 30% for a normal PDF and β/ϕ = 0.8, K_a decreases. The above is due to the generation of unrealistic values in the MCS when β > ϕ, which affects the K_a output function and decreases the mean statistical value. Therefore, the Normalized K_a's decrease implies that for β/ϕ = 0.8, the K_a's probabilistic analysis is limited to low ϕ variability. The higher the β/ϕ ratio, the less admissible the statistical modeling is for the ϕ variability.

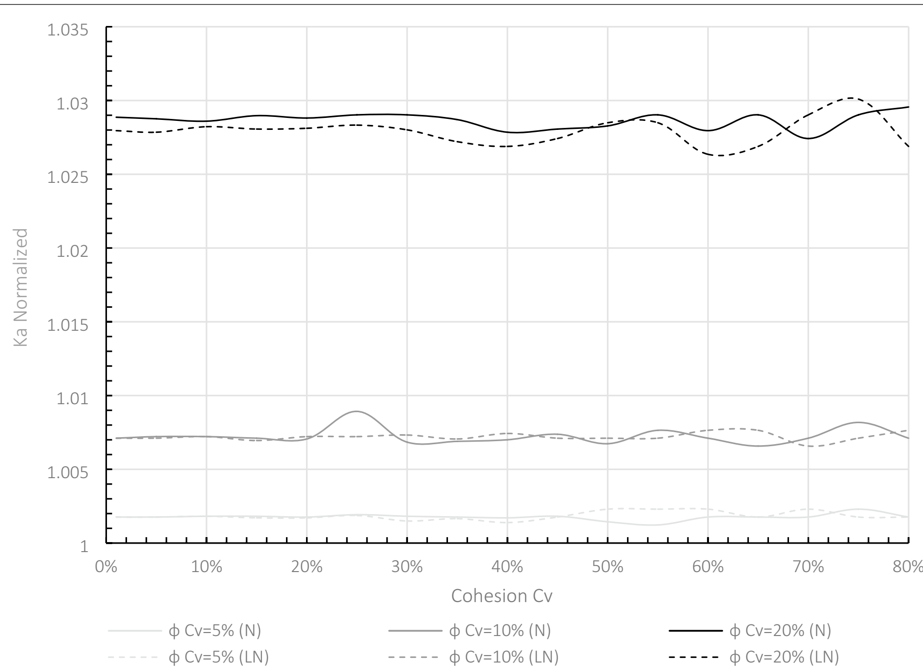

Fig. 2 Normalized Ka changes according to the cohesion Coefficient of variation for normal(N) and Lognormal (LN) PDF for φ = 25° and c = 17 kPa

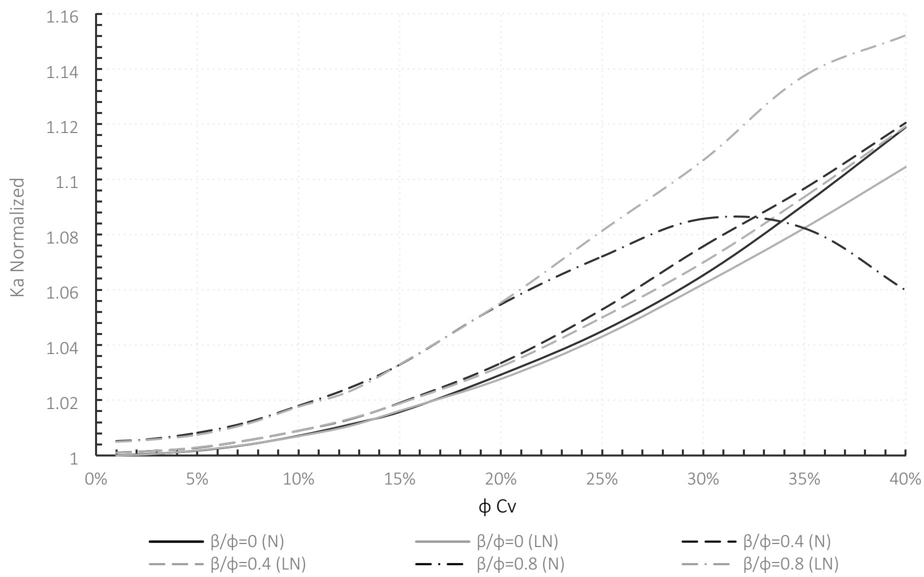

Fig. 3 Normalized Ka changes according to the friction angle Coefficient of variation for normal (N) and Lognormal (LN) PDF (mean $\varphi = 25°$ and $c = 17$ kPa)

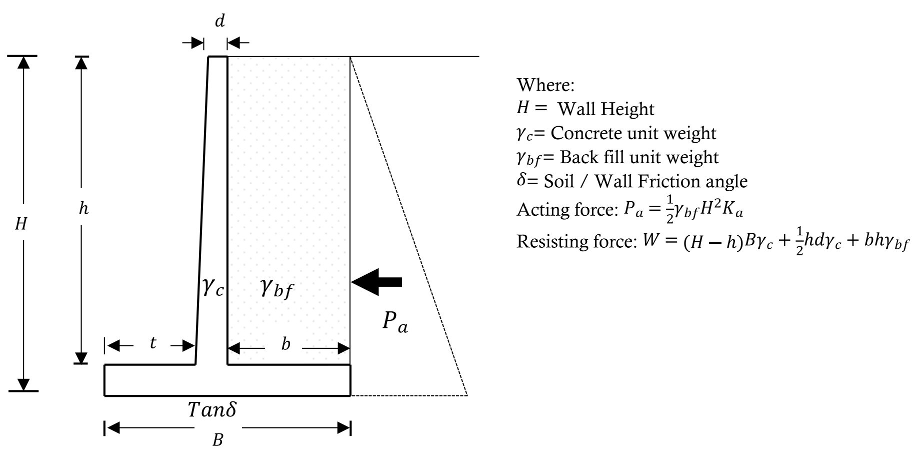

Fig. 4 Characteristics of the cantilever retaining wall analyzed

Illustrative examples

Probability of failure estimation for a cantilever wall

To evaluate the increase in K_a based on the increase of the Cv values, an assessment of the Sliding Factor of Safety (FS_{sl}—Eq. 3) and the probability of failure (p_f) for a cantilever retaining wall was carried out (Fig. 4). The evaluation is based on a deterministic design that initially reached an $FS_{sl} = 1.5$. The variation in K_a, as shown in Fig. 3, was used to determine the variation of the aforementioned deterministic FS. The probability of failure was evaluated using the First Order Second Moment (FOSM) simulation method for a c -ϕ soil following the normal PDF according to Eq. 4.

$$FS_{Sl} = \frac{Wtan\delta}{P_a} \quad (3)$$

$$Pf = P(FS_{Sl} \leq 1) \quad (4)$$

Figure 5 shows that as the Cv of the angle of friction increases, the FS decreases, and thus the probability of failure increases. The most critical changes in p_f and FS are present for a ϕ Cv > 20%, which may represent unrealistic coefficients of variations for soils with the same grain size distribution as described in the literature. However, the results show that for FS close to 1.5, The obtained p_f may not be admissible according to the US Army Corps of Engineers, which recommends that earth retaining walls meet a $p_f \leq 0.1\%$ [36].

Probability of failure estimation for a sheet pile wall

Similar to what was done for the cantilever wall, an evaluation of the probability of failure (p_f) and Factor of Safety (FS) against overturning was undertaken for a continuous sheet pile wall according to Eq. 5.

$$FS = \frac{\sum Acting\ moments}{\sum Resistant\ moments} \quad (5)$$

The earth pressure diagrams and the acting moments act about point O, as shown in Fig. 6, using the Simplified Hansen method. This method was selected as is one of the simplest earth pressure balance methods for sheet pile walls.

For the evaluation, the structure was required to comply with a global Factor of Safety against overturning ≥ 3,0 [7] regardless of its conditions. It was of particular interest to assess how the variability present in the soil would affect the probability of failure of a design that initially met an adequate deterministic Factor of Safety.

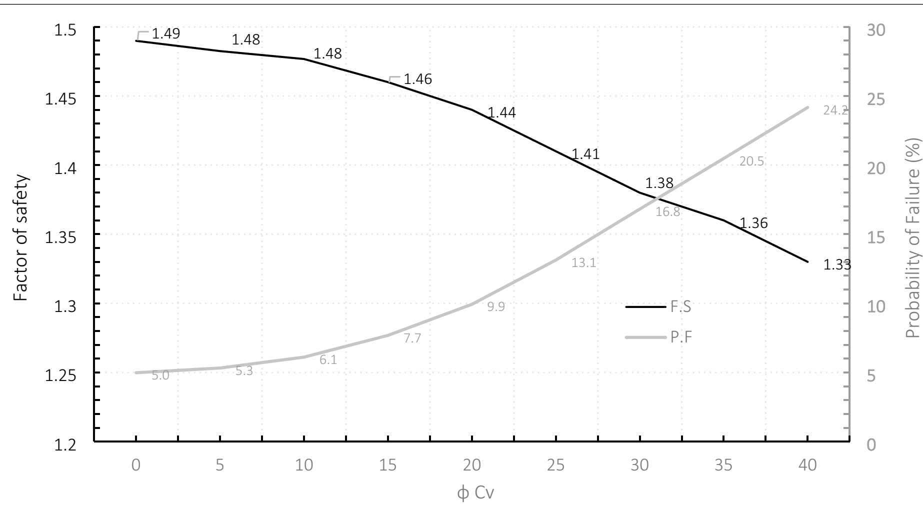

Fig. 5 Influence of friction angle variability in the factor of safety and probability of failure of a cantilever retaining wall for β/ φ = 0 (mean φ = 25° and c = 17 kPa)

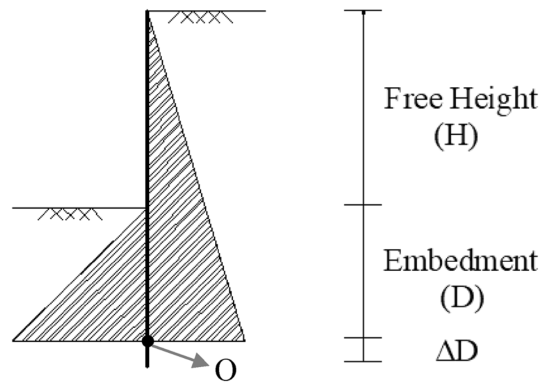

Fig. 6 Hansen earth pressure balance method for the design of sheet pile walls

Ultimately, it was observed that despite some designs comply with a deterministic Factor of Safety, when considering the soil variability and evaluating its probability of failure, they do not comply with the minimum probability of failure recommended by the USACE [36].

Mazindrani and Ganjali [22] formulations were used to obtain active and passive earth pressures considering the backfill slope angle. The variation of the p_f for $\beta/\phi = 0$; $\beta/\phi = 0.4$, and $\beta/\phi = 0.8$ was evaluated according to Eq. 6, and the results are shown in Fig. 7.

$$Pf = P(FS \leq 1) \tag{6}$$

Figure 7 shows that, as the backfill slope increases for sheet pile walls, the probability of failure decreases. The above is because sheet pile walls with higher slopes require greater embedment depths for a FS = 3.0. The increase of the soil slope requires a greater passive pressure area, which significantly increases K_p for small wall embedment length (D). Therefore, p_f is dependent on relatively small changes of K_p, which is more noticeable in the face of β/ϕ increases. The influence of the D increase according to the soils slope changes as presented in Fig. 8.

Figure 7 shows that for low ϕ Cv, a cohesion Cv < 20% meets with an admissible pf < 0.1%, as suggested by USACE [36]. However, as the ϕ variability increases, the probability of failure increase as well. Also, the Lognormal PDF renders lower values of the probability of failure in the evaluated model.

For c-ϕ soils, using an FS = 3.0 in the deterministic designs of sheet pile walls might not meet the maximum admissible value of the probability of failure suggested by the literature. Therefore, for a soil with high variability, it is essential to consider the influence of each parameter variability in the active earth pressure coefficient. The above is evidenced in the overall results, where it is shown that a well-performed deterministic design may have an inadmissible probability of failure.

Conclusions

The influence of the variability of the drained friction angle and the cohesion on the active earth pressure coefficient was estimated using the formulation proposed by Mazindrani & Ganjali [22] for c-ϕ soils. The influence is determined through the

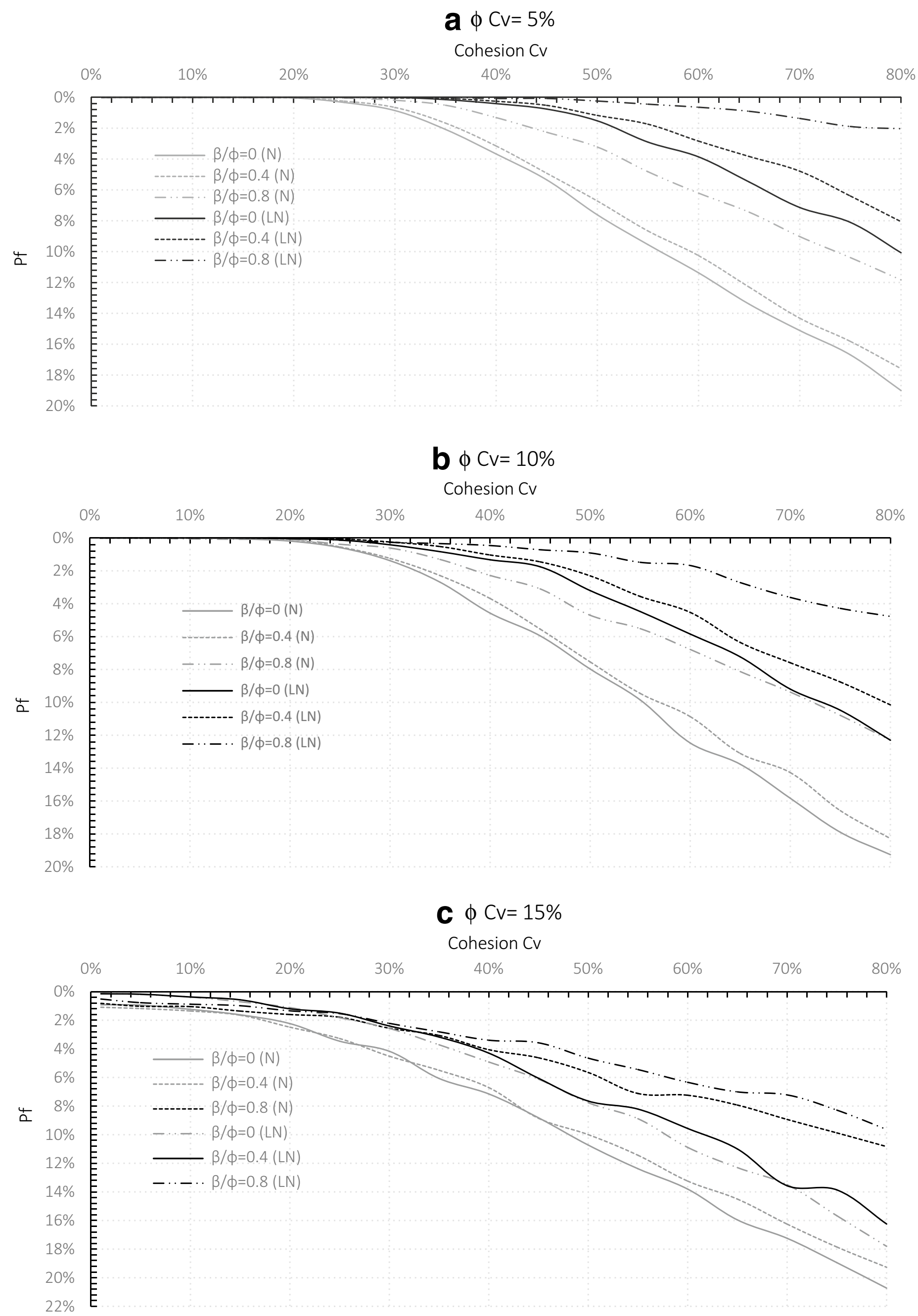

Fig. 7 Influence of cohesion Cv on sheet pile walls probability of failure for **a** φ Cv = 5%, **b** φ Cv = 10% and **c** φ Cv = 15%

relationship between the statistical and the deterministic coefficient. The results show an increase in the statistical K_a coefficients concerning the deterministic value as the soil variability is high. The statistical/deterministic ratio increases as the backfill slope increase.

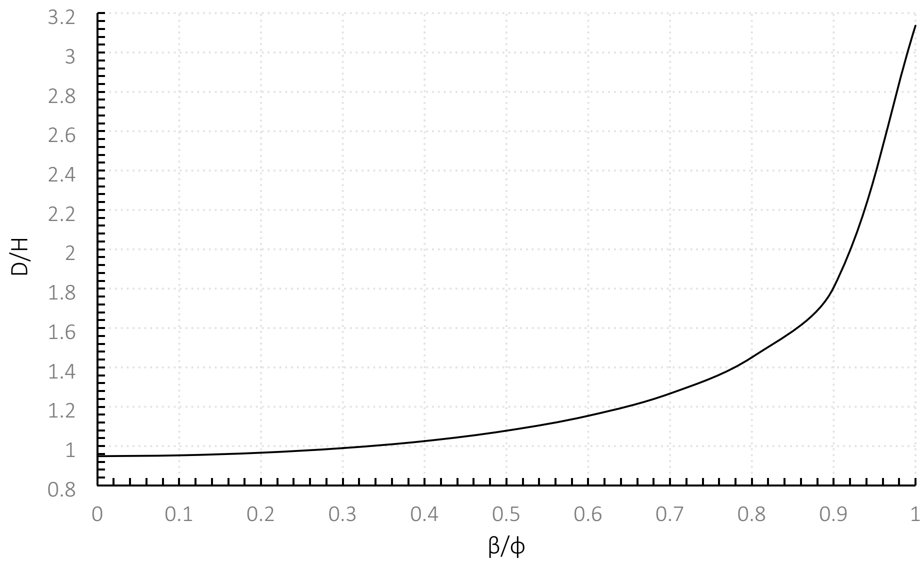

Fig. 8 Influence of the slope changes (β/φ) in the D/H ratio for sheet pile walls for an overturning F.S = 3.0

The sensitivity analysis using the Spearman's Correlation Coefficient (SCC) of the K_a shows a Cv for which the cohesion begins to have a more significant influence in the K_a variability than the friction angle. However, the K_a magnitude is mainly dependent on the friction angle variability, where the cohesion Cv does not change the deterministic value substantially. Therefore, it can be concluded that SCC is not an indicator of the magnitude of the output variable; however, it shows the soil variability's influence in the property's correlation.

Although the statistical analyses show a low increase in K_a for the c-ϕ variation, the probability of failure shows the importance of considering the soil variation. Even if the required Factor of Safety is met, the probability of failure may be inadmissible.

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Authors' contributions
JO contributed to the conception of the paper, drafted and performed the numerical analyses. JCV contributed the paper's conception, complementary numerical analyses and revised the manuscript. JPO contributed to the conception of the paper and revised the paper manuscript. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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