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Presenting a bi-objective integrated model for production-distribution problem in a multi-level supply chain network

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Abstract

In this study, a bi-objective model for integrated planning of production-distribution in a multi-level supply chain network with multiple product types and multi time periods is presented. The supply chain network including manufacturers, distribution centers, retailers and final customers is proposed. The proposed model minimizes the total supply chain costs and transforming time of products for customers in the chain. The proposed model is in the class of linear integer programming problems. The complexity of the problem is large and in the literature, this problem has been shown to be NP-hard. Therefore, for solving this problem, two multi objective meta-heuristic approaches based on Pareto method including non-dominated Sorting Genetic Algorithm-II (NSGA-II) and non-dominated Ranking Genetic Algorithm (NRGA) have been suggested. Since the output of meta- heuristic algorithms are highly dependent on the input parameters of the algorithm, Taguchi method (Taguchi) is used to tune the parameters. Finally, in order to evaluate the performance of the proposed solution methods, different test problems with different dimensions have been produced and the performances of the proposed algorithms on the test problems have been analyzed.

Keywords: Supply chain management; Multi objective production-distribution planning problem; NSGA-II; NRGA; Taguchi method.

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1. Introduction

In today's world, industrial development and economic changes occur at an ever increasing rate in comparison with the past. Increasing customer expectations, improving and expanding global competition, force organizations to pay more attention to the customer satisfaction and investigate their logistic systems (Chopra and Meindle, 2004). Supply chain management has been a research area of increasing interest in recent years, to academics, consultants and business management (Cohen and Lee, 1988). In today's world, industrial development and economic changes occur at an ever increasing rate in comparison with the past. Increasing customer expectations and global competition rates force organizations to focus on improving the efficiency of the manufacturers by taking advantage of the immediate supplier's capability and technology. Moreover, Market globalization is forcing firms to make decisions that are more coordinated and integrated in order to be able to provide goods and services to the customer at a lower cost and with higher service levels (Thomas and Griffin, 1996). Decision-making increasingly happens at all levels of businesses, companies and organizations. There is a need for building theory and developing normative tools and methods for successful SCM (Lee and Kim, 2002). Most of the proposed models in the integrated supply chain management can be classified as the following: Integrated Buyer-Seller, Integrated Production-Distribution Planning, Integrated Production-Inventory Planning, and Location-Allocation Models. In the efficient designed production/distribution, products are produced and distributed at the right quantities, to the right customers, and at the right time, in order to minimize system wide costs while satisfying all required demands. Production and distribution models are operationally connected and closely related with each other. These two linked problems are considered as a production-distribution model in supply chain. To find an optimal solution for this problem, we need to propose an integrated model and a solution method that considers production-distribution characteristics simultaneously (Mohamed, 1999).

2. Literature review

The modeling and analysis of production–distribution systems in supply chain management has been an active area of research for many years. Erenguc et al. (1999) and Fahimnia et al. (2013) provided excellent reviews on supply chain management literature. Chen and Lee (2004) proposed a multi-product, multi-stage, and multi-period scheduling model for a multi-echelon supply chain network. The suggested model deals with uncertain market demands and product prices. Cohen and Lee (1988); Chandra and Fisher (1994) proposed a model for the integrated production-distribution problem. Within this problem, the produced number of products over time and the demand for each product are known at each retail outlet for each period of a planning horizon. Sabri and Beamon (2000) developed a multi objective supply chain management problem with production, distribution and demand uncertainty. Their model involves simultaneous strategic and operational planning. Chan et al. (2005) proposed a hybrid genetic algorithm for production and distribution problems in multi-factory supply chain models. Analytic hierarchy process (AHP) combined with genetic algorithm has been used for assigning jobs into suitable production plants and optimizing objective functions including total cost, fulfillment lead time, and equity of utilization ratios will be considered. Keskin and Uster (2007) consider a multi-product two-stage production-distribution system design problem (PDSD). They proposed a mixed-integer problem model for minimizing the total costs in the system. Their suggested model locates a fixed number of capacitated distribution centers with respect to capacitated suppliers and retail locations. They provide meta-heuristic procedures, including a population-based scatter search tabu search for solving the problem. For solving subproblems, they developed efficient construction heuristics and transshipment heuristics that are incorporated into the heuristic procedures. Kazemi et al. (2009) suggested a multi-agent system for solving a multi-product two-stage production /distribution system design problem (PDSD). They presented a mixed-integer problem formulation and meta-heuristic procedures for minimizing the total costs in the system and locating a fixed number of capacitated distribution centers with respect to the capacitated suppliers and retail locations. Jolai et al. (2011) suggested a multi-objective linear programming problem consisting of a manufacturer, with multiple plants, products, distribution centers, retailers and customers for integrating a production–distribution problem. They propose three meta-heuristics including: (1) a simple genetic algorithm; (2) particle swarm optimization (PSO) algorithm with a new fitness function; (3) improved hybrid genetic algorithm. Liu and Papageorgiou (2013) developed a multi objective mix integer linear programming approach for optimizing the total cost, total flow time, and total lost sale. The Ɛ-constraint and lexicographic minimax method were used to optimize considering cost, responsiveness and customer service level simultaneously. Kalaitzidou et al (2014) suggested a modeling framework based on a mixed integer linear programming problem for the design of supply chain networks and finding the optimal structure of the network considering market demand satisfaction and overall capital and operational cost minimization.

Sarrafha et al. (2015) suggested a bi-objective integrated procurement, production, and distribution problem of a multi-echelon supply chain network design. Objectives of the model are minimizing total chain's costs and minimizing the average tardiness of product to distribution centers considering a flow-shop scheduling problem in manufacturing part of supply chain. To solve the proposed model, they suggested a multi-objective biogeography based optimization (MOBBO) and a multi-objective simulated annealing (MOSA).

In this research, an integrated production planning-distribution model for designing four- level supply chain with multi product types and multi periods time is suggested for minimizing the total supply chain costs and the transfer time of the products to the customers. In this model, in the case of product shortage, backorder cost is considered. To solve the problem, two multi objective meta- heuristic algorithms, NRGA and NSGA-II, based on Pareto method is proposed. The remainder of this paper is organized as follows: problem definition and the detailed mathematical formulation are shown in section 3. The proposed solution method will be discussed in section 4. In section 5, the obtained optimization results will be analyzed. Finally, conclusion and suggestion of future research will be presented in section 6.

3. Problem definition

In this section, the production-distribution problem studied is defined more precisely, and parameters, variables, and assumptions are defined. A bi-objective integrated formulation for designing and optimizing a multi-level supply chain network is proposed.

In this problem, we consider an integrated supply chain network including: plants, distribution centers, retailers and customers both with specific locations. In the beginning, different products are produced in the different plants. In the second stage, for those plants and distribution centers there is a link between them; produced products are sent from plants to the distribution centers. In the third stage, for those distribution centers and retailers there is a link between them; produced products are shipped from distribution centers to the retailers. In the last stage, the final product is shipped from retailers to the final customers. In this model, allocation of different levels of supply chain is investigated in order to obtain optimal quantity of production, distribution, transportation and inventory holding and backorder level for minimizing the total chain costs, and transfer time of products to customers. Figure (1) indicates the proposed supply chain network.

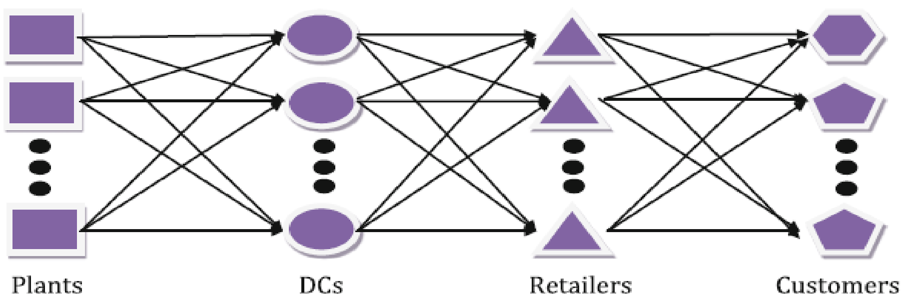

Figure 1. Four level supply chain network

The assumptions are:

- Four level supply chain including plants, distribution centers, retailers and final customers is considered.
- Decisions for multi- products and periods are given.
- Constraints of inventory holding capacity for the plants, distribution centers, retailers, and final customers are considered.
- Equal Transportation capacity is considered for all the levels of supply chain.
- Each manufacturer can produce all types and various kinds of products
- The production capacity is considered equal for all plants.
- In the case of product shortage, backorder cost is considered for the retailers.

3.1. Indexes and parameters

The indices and parameters are as follows:

p: index of manufactures (p=1,…,P)

d: index of distributers (d=1, … ,D)

r: index of retailers (r=1,…,R)

c: index of customers (c=1, …,C)

i: index of products (I=1, …, I)

t: index of time periods (t=1, …, T)

CD_c^{it} : Demand of customer c for the product i in period t.

SEC_p^{it} : Production preparation cost of product i at plant p in period t.

PC_p^{it} : Production cost of product i for manufacturer p in period t.

HC_p^{it} : Inventory holding cost of product i for manufacturer p in period t.

HC_d^{it} : Inventory holding cost of product i for distributer d in period t.

HC_r^{it} : Inventory holding cost of product i for retailer r in period t.

TC_{pd}^{it} : Purchase and transportation cost of product i from manufacturer p to distributer d in period t.

TC_{dr}^{it} : Purchase and transportation cost of product i from distributer d to the retailer r in period t.

TC_{rc}^{it} : Purchase and transportation cost of product i from retailer r to customer c in period t.

IC_p^{it} : Inventory holding capacity of product i in plant p in period t.

IC_d^{it} : Inventory holding capacity of product i in distributer d in period t.

IC_r^{it} : Inventory holding capacity of product i in retailer r in period t.

TCP_p^{dt} : Transportation capacity from manufacturer p to distributer d in period t.

TCP_d^{rt} : Transportation capacity from distributer d to retailer r in period t.

TCP_r^{ct} : Transportation capacity from retailer r to customer c in period t.

PCM_p^{it} : Maximum capacity of production capacity of product i for manufacturer p in the period t.

PCL_p^{it} : Minimum production capacity of product i for the manufacturer p in period t.

TI_{pd}^{t} : Product transportation time from manufacturer p to distributer d in period t.

TI_{dr}^{t} : Product transportation time from distributer d to retailer r in period t.

TI_{rc}^{t} : Product transportation time from retailer r to customer c.

BC_c^{it} : Shortage cost of product i for customer c in period t.

3.2. Decision variables

The decision variables are as follows:

PQ_p^{it} : Production quantity of product i by manufacture p in period t.

TQ_{pd}^{it} : Quantity of product i shipped from manufacturer p to the distributer d in period t.

TQ_{dr}^{it} : Quantity of product i shipped from distributer d to retailer r in period t.

TQ_{rc}^{it} : Quantity of product i shipped from retailer r to the customer c in period t.

IP_p^{it} : Inventory level of product i for manufacturer p in period t.

IP_d^{it} : Inventory level of product i for distributer d in period t.

IP_r^{it} : Inventory level of product i for retailer r in period t.

BL_c^{it} : Backorder level of product i for customer c in period t.

X_p^{it} : It is equal to one if product i is assigned to manufacturer p in period t, otherwise it is equal to zero.

Y_{pd}^{t} : It is equal to one if product i is produce by manufacturer p in period t, otherwise it is equal to zero.

Y_{dr}^{t} It is equal to one if manufacturer p is assigned to distributer d in period t, otherwise it is equal to zero.

Y_{rc}^{t} It is equal to one if retailer r is assigned to customer c in period t, otherwise it is equal to zero.

3.3. Objective function

In this section, a bi-objective integrated production-distribution mathematical model is proposed. The first objective is to minimize the total cost of the chain including: (1) production preparation cost of product, (2) production cost, (3) inventory holding cost in the production level, (4) purchase and transportation cost of products from manufacturers to the distributers, (5) inventory holding cost of products for distributers, (6) purchase and transportation cost of products from distributers to the retailers, and (7) purchase and transportation cost of products from retailers to the customers. The second objective is minimizing total transportation time related to transferring products from the manufacturers to the final customer in the chain. The objectives are conflicting objectives; decreasing transportation times can lead to increase in chain costs and vice versa.

$$\begin{aligned}Min\ Z_1 =& \sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{t=1}^{T} SEC_p^{it} \times X_p^{it} + \sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{t=1}^{T} PC_p^{it} \times PQ_p^{it} + \sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{t=1}^{T} HC_p^{it} \times IP_p^{it} \\ &+ \sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{i=1}^{I}\sum_{t=1}^{T} TC_{pd}^{it} \times TQ_{pd}^{it} + \sum_{d=1}^{D}\sum_{i=1}^{I}\sum_{t=1}^{T} HC_d^{it} \times IP_d^{it} + \sum_{d=1}^{D}\sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{t=1}^{T} TC_{dr}^{it} \times TQ_{dr}^{it} \\ &+ \sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{t=1}^{T} HC_r^{it} \times IP_r^{it} + \sum_{r=1}^{R}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{t=1}^{T} TC_{rc}^{it} \times TQ_{rc}^{it} + \sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{t=1}^{T} BC_c^{it} \times BL_c^{it}\end{aligned} \tag{1}$$

$$Min\ Z_2 = \sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{i=1}^{I}\sum_{t=1}^{T} TI_{pd}^{t} \times TQ_{pd}^{it} + \sum_{d=1}^{D}\sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{t=1}^{T} TI_{dr}^{t} \times TQ_{dr}^{it} + \sum_{r=1}^{R}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{t=1}^{T} TI_{rc}^{t} \times TQ_{rc}^{it} \tag{2}$$

st.

$$PCL_p^{it} \times X_p^{it} \le PQ_p^{it} \le PCM_p^{it} \times X_p^{it} \qquad \forall\, p,i,t \tag{3}$$

$$IP_p^{it} \le IC_p^{it} \qquad \forall\, p,i,t \tag{4}$$

$$\sum_{i=1}^{I} TQ_{pd}^{it} \le TCP_{pd}^{t} \times Y_{pd}^{t} \qquad \forall\, p,d,t \tag{5}$$

$$IP_d^{it} \le IC_d^{it} \qquad \forall\ d,i,t \tag{6}$$

$$\sum_{i=1}^{I} TQ_{dr}^{it} \le TCP_{dr}^{t} \times Y_{dr}^{t} \qquad \forall\ d,r,t \tag{7}$$

$$IP_r^{it} \le IC_r^{it} \qquad \forall\ r,i,t \tag{8}$$

$$\sum_{i=1}^{I} TQ_{rc}^{it} \le TCP_{rc}^{t} \times Y_{rc}^{t} \qquad \forall\ r,c,t \tag{9}$$

$$IP_p^{it} = IP_p^{it-1} + PQ_p^{it} - \sum_{d=1}^{D} TQ_{pd}^{it} \qquad \forall\ p,i,t \tag{10}$$

$$IP_d^{it} = IP_d^{it-1} + \sum_{p=1}^{P} TQ_{pd}^{it} - \sum_{r=1}^{R} TQ_{dr}^{it} \qquad \forall\ d,i,t \tag{11}$$

$$IP_r^{it} = IP_r^{it-1} + \sum_{d=1}^{D} TQ_{dr}^{it} - \sum_{c=1}^{C} TQ_{rc}^{it} \qquad \forall\ r,i,t \tag{12}$$

$$BL_c^{it} = BL_c^{it-1} + CD_c^{it} - \sum_{r=1}^{R} TQ_{rc}^{it} \qquad \forall\ c,i,t \tag{13}$$

$$\sum_{c=1}^{C} BL_c^{iT} = 0 \qquad \forall\ i \tag{14}$$

$$PQ_p^{it},\ TQ_{pd}^{it},\ TQ_{dr}^{it},\ TQ_{rc}^{it},\ IP_p^{it},\ IP_d^{it},\ IP_r^{it},\ BL_c^{it} \ge 0 \tag{15}$$

$$X_p^{it},\ Y_{pd}^{t},\ Y_{dr}^{t},\ Y_{rc}^{t} \in \{0,1\} \tag{16}$$

$$IP_p^{i0},\ IP_d^{i0},\ IP_r^{i0}, BL_c^{i0} = 0 \tag{17}$$

Constraint 3 indicates that if product i produced at manufacture p in period t then a production capacity is considered for product i at manufacture p. Constraint 4 states that the inventory level of product i at each period is limited by inventory capacity for manufactures. Constraint 5 means that if a link between two levels of the chain exists, then the amount of product i to be shipped in time period t from manufacturer p to the distribution center d is limited by transportation capacities. Constraint 6 shows that distribution center d capacity for holding the inventory of each unit of product i. Constraint 7 indicates that if a link between two levels of the chain exists, then the amount of product i to be shipped in time period t from distribution center d to the retailer r is limited by transportation capacities. Constraint 8 shows retailer r capacity for holding inventory of each unit of product i. Constraint 9 states that if a link between two levels of the chain exists, then the amount of product i to be shipped in time period t from retailer r to the customer c is limited by transportation capacities. Constraints 10, 11, and 12 state the balance equations of the products in the different levels of the chain. For example, in constraint 10, the balance equation of product i in the plant p is equal to the summation of the inventory of product i in the period t-1 and production of the product i in the period t minus the amount of transferred product i from manufacturer p to distributer d in the period t. Constraint 13 shows backorder cost for customer c. Constraint 14 means that backorder level in the last period for meeting customer's needs should be zero. Constraint 15 and 16 indicate that variables are non-negative and binary , respectively. Constraint 17 states the initial values of the inventory and the backorder. Integrated bi-objective distribution-production problem is a NP-hard problem and the optimal solutions are very difficult to obtain.

Large numbers of constraints and decision variables including considering binary variables and multi objective and conflicting objectives in the model are the factors that cause more complexity in the problem. Due to the complexity of the problem and the huge computational time that are required for solving the problems network model design of supply chain is a very complicated issue (Jolai et al., 2011). Therefore, a meta-heuristic algorithm has been proposed for solving the integrated production- distribution problem.

4. Solution methodology

Multi-objective problems are concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. Multi-objective optimization has been applied in many fields of science, including engineering, economics and logistics, where optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. In the multi-objective optimization problem, there does not exist a single solution that simultaneously optimizes each objective. In that case, the objective functions are said to be conflicting, and there exists a possibility for infinite number of optimal solutions.

In this paper, two multi objective algorithms based on Pareto have been suggested for solving the integrated production-distribution model. The proposed algorithms are called non-dominated Sorting Genetic Algorithm (NSGA-II) and non-dominated Ranking Genetic Algorithm (NRGA).

4.1. Non-dominated Sorting Genetic Algorithm (NSGA-II)

Non-dominated Sorting Genetic Algorithm (NSGA-II) is one of the most successful and widely used multi objective evolutionary algorithms that have been introduced by Deb et al. (2006). In the single objective problems, finding the solution is based on an objective, while in the multi objective problems there does not exist a single solution that simultaneously optimizes each objective, so there will be a set of optimal solutions that is called non-dominated solutions. The set of all efficient points to a multiple objective optimization problem is known as the efficient frontier. A solution is called non-dominated, Pareto optimal, Pareto efficient or no inferior, if none of the objective functions can be improved in value without degrading some of the other objective values. Without additional subjective preference information, all Pareto optimal solutions are considered equally good. Pareto-based algorithms are a new generation of the multi-objective algorithms which are mostly working in accordance with the domination concept. In a multi-objective model with m minimization objective functions, i.e. F(x) =[f (x), ..., fm(x)] subject to gi(x) ≤ 0, i =1, 2, ..., m, in which xεX is a n-dimensional vector that can get real, integer, or even Boolean value and X is the feasible region, domination concept is defined as follows

$$f_i(\overrightarrow{x_a}) \leq f_i(\overrightarrow{x_{a'}}),\ i = 1,\ 2,\ ...,\ m$$
$$\exists\ i \in \{1,\ 2,\ ...,\ m\} : f_i(\overrightarrow{x_a}) < f_i(\overrightarrow{x_{a'}})$$

According to these conditions, solution dominates solution a' under the simultaneous existence of the two mentioned conditions. Based on this definition, Pareto optimal front is called to a set of solutions that cannot dominate each other. This front has two main features which are known as 1) good convergence and 2) good diversity within the solutions of the Pareto front.

4.1.1. Initialization

Initial population size (nPop), crossover probability (Pc), mutation probability (Pm), and number of iterations (nIt) are required for starting the NSGA-II. Setting of the parameters' value is obtained using the Taguchi method.

4.1.2 .Chromosome structure

In this section, to represent solution, structure variables are used. Each structure in the created solutions is an expression of a feature of the solution. The structure of model variables is as follows in Figure (2) - Figure (5).

The structure of the zero and one model variables for the product manufacturing and assignment of different chain levels to each other is shown in Figure 2.

The structure of the problem variables in the manufacturer- distributer level for the product manufacturing and assignment of different chain levels to each other are shown in Figure 2.

The structure of the problem variables in the manufacturer- distributer level is an array with dimension of p, d, i, and t including: product quantity, product transferred quantity, and inventory level. An example of the mentioned structure is shown in Figure 3.

The structure of the problem variables in the distributer-retailer level is an array with dimension of d, i, r, and d including product quantity and product transferred quantity. An example of the mentioned structure is shown in Figure 4.

The structure of the problem variables in the retailer-customer level is an array with dimension of r, c, i, and t including product transferred quantity and inventory level. An example of the mentioned structure is shown in Figure 5.

$p \times d \times t$				$d \times r \times t$				$r \times c \times t$			
1	0	...	1	0	0	...	1	0	1	...	1

Figure 2. Structure of the solution in the allocation level in the period t

$p \times i \times t$				$p \times i \times t$				$p \times d \times i \times t$			
PQ_{111}	PQ_{121}	...	PQ_{PIT}	IP_{111}	IP_{121}	...	IP_{PIT}	TQ_{1111}	TQ_{1121}	...	TQ_{PDIT}

Figure 3. Structure of the solution in the manufacturer- distributer level in the period t

$d \times i \times t$				$d \times r \times i \times t$			
IP_{111}	IP_{121}	...	IP_{DIT}	TQ_{1111}	TQ_{1121}	...	TQ_{DRIT}

Figure 4. Structure of the solution in the distributer-retailer level in the period t

$r \times i \times t$				$r \times c \times i \times t$			
IP_{111}	IP_{121}	...	IP_{RIT}	TQ_{1111}	TQ_{1121}	...	TQ_{RCIT}

Figure 5. Structure of the solution in the retailer level in the period t

Application of meta-heuristics algorithm to constrained optimization problems and handling constraints is a challenging problem. Penalty function method is one of the most important approaches for solving the constrained optimization problems (Yeniay and Ankare, 2005). In fact, Penalty method transforms a constrained problem to an unconstrained one. It is calculated in the equation (18).

$$x = \begin{cases} f(x) & ; x \in \textit{feasible Region} \\ f(x) + P(x) & ; x \notin \textit{feasible Region} \end{cases} \tag{18}$$

In the above equation p(x) presents a penalty term. If no violation occurs, p(x) will be zero and positive otherwise. Since different constraints can have a different large degree of violation, it seems necessary to normalize the constraints. For example $g_i(x) \le b_i$ is normalized according to the equation (19).

$$P(x) = M \times Max\left\{\left(\frac{g(x)}{b} - 1\right), 0\right\} \tag{19}$$

In which $g(x)$ and M are the constraints that need to be normalized and a large Number (a large positive constant), respectively. Therefore, deviation from normalized constraints would be equal. Now, summation of these deviations is calculated easily and only a penalty parameter as a total penalty of constraints is added to the objective function.

4.1.3 .A fast non-dominated sorting approach

In order to sort a population of size N according to the level of non-domination, each solution must be compared with every other solution in the population to find if it is dominated.

This requires O(mN) comparisons for each solution, where m is the number of objectives. When this process is continued to find the members of the first non-dominated class for all population members, the total complexity is O(mN^2). At this stage, all individuals in the first non-dominated front are found. In order to find the individuals in the next front, the solutions of the first front are temporarily discounted and the above procedure is repeated. In the worst case, the task of finding of the second front also requires O(mN^2) computations. The procedure is repeated to find the subsequent fronts.

To get an estimate of the density of solutions surrounding a particular point in the population, we take the average distance of the two points on either side of this point along each of the objectives. This quantity i distance serves as an estimate of the size of the largest cuboid enclosing the point i without including any other point in the population (we call this the crowding distance). In Figure 6, the crowding distance of the i-th solution in its front (marked with solid circles) is the average side-length of the cuboid (shown with a dashed box).

Between two solutions with differing crowding distance we prefer the point with the lower density. Otherwise, if both of the points belong to the same front then we prefer the point which is located in a region with less crowding distance (Deb et al., 2000).

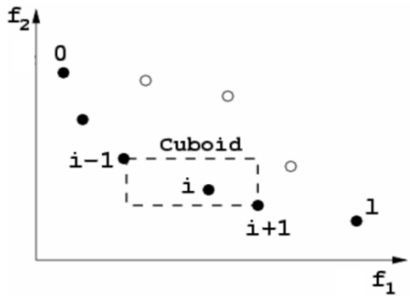

Figure 6. Crowding distance calculation (Deb et al., 2000)

4.1.4. Parent and selection strategy

The crowded tournament selection operator is used for parent population selection by applying crossover and mutation on them. This operator compares two solutions and chooses the better one (i). We assume that every individual i in the population has two attributes.

1. Non-domination rank (r_i)

2. Local crowding distance (d_i)

That is, between two solutions with differing non-domination ranks, we prefer the point with the lower rank. Otherwise, if both of the points belong to the same front, then we prefer the point which is located in a region with a lesser number of points (Deb et al., 2000).

4.1.5. Crossover structure

During the iterations of the algorithm, to produce new offspring, uniform crossover operator is implemented. Generally, this method is used for those situations in which appropriate characteristics of genes are scattered throughout the chromosome (Bate and Jones, 2008). This crossover operator, some of genes swap within the chromosome of parents to produce offspring. Figure 7 illustrates a scheme of this operator graphically.

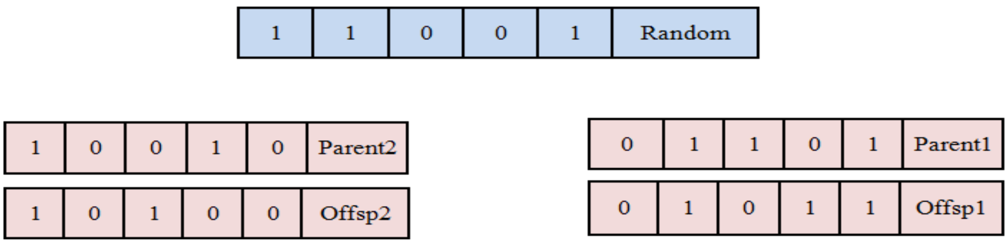

Figure 7. A sample of the uniform crossover operator for the manufacturer-distributer connection.

4.1.6. Mutation structure

A random mutation operator is used for this part. According to this operator, a matrix the same size as that part of the chromosome is generated and filed by numbers from [0,1]. Then, if the numbers generated by each gene are smaller than a certain value, the mutation is applied on the related gene and updates it. Figure 8 illustrates this operation.

X	1	0	0	1	1	…	0	1	0	1
Mask	0.5	0.5	0.5	0.5	0.5	…	0.5	0.5	0.5	0.5
Rand	0.55	0.23	0.87	0.98	0.41	...	0.62	0.05	0.49	0.91
X'	1	1	0	1	0	…	0	0	1	1

Figure 8. A sample of the mutation operator for the manufacturer-distributer connection.

4.1.7. Evaluation of children and creation of next generation

In this part of the algorithm, the population of parents and children are combined and a population twice the initial size of the population is formed. This combining of the solutions keeps the best solutions among the parents and children populations and elitism is also ensured. In this case, non-domination ranking is used so that each solution is evaluated based on its non-domination (Deb et al., 2000). Then fast non-dominated sorting approach and crowding distance are applied and the element of each population is ranked based on crowding distance and non-dominated, respectively (non-dominated fronts).

4.1.8. Stopping criteria

The last step of the genetic algorithm is stopping criteria. There are no specific stopping criteria of the multi-objective optimization problems. As a result, the algorithm stops when it reaches a maximum number of defined iterations.

4.2 .Non-dominated ranking genetic algorithm (NRGA)

A new multi-objective evolutionary algorithm is based on population and non-dominated ranking genetic algorithm (NRGA) was proposed by Al Jadaan et al. (2008). This successful algorithm was proposed for optimizing non-convex, discrete, and non-linear problems. The procedure is defined in such way that the better elements have a higher chance of reproduction and a higher chance for formation of the next generation. The flowchart of NRGA and NSGA-II algorithms has been shown in the Figure (9) (Al Jadaan et al., 2008).

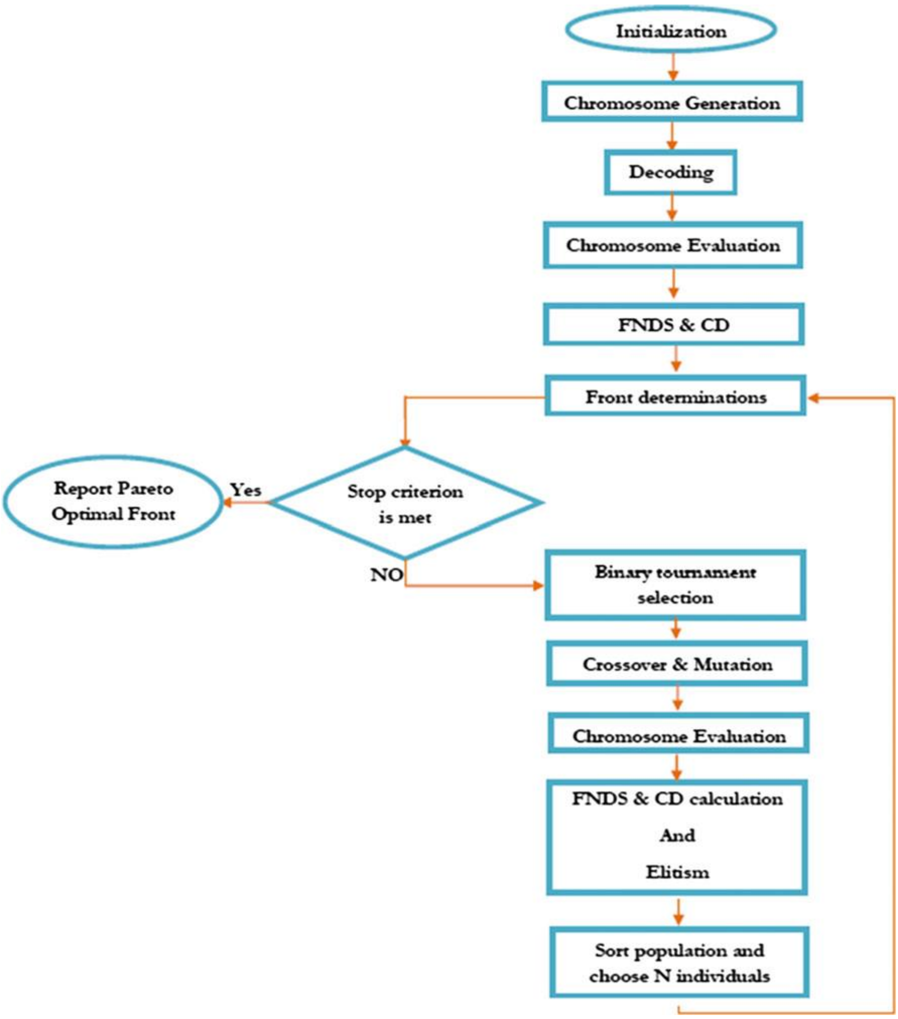

Figure 9. flowchart of the NRGA and NSGA-II algorithms

5. Experimental results

In this section, sample problems are generated and used for analyzing the results. Initially, Taguchi method is used for adjusting the parameter of the proposed model. The results are then compared and analyzed. After the algorithm's consecutive performances based on the values shown in Table 1, parameters are classified. Then parameter tuning is applied based on the Taguchi method. The results have been shown in Figure 10, Figure 11, and the highlighted parts of table 1. The 15 tests have been then implemented. Table 4 and table 5 show the sample problems and input parameters, respectively. All proposed algorithms have been developed with MATLAB software program.

5.1. Taguchi method

Since the output of the problems relies heavily on the proposed algorithms' parameters, the Taguchi method is used for adjusting these parameters. An advantage of the Taguchi method compared to the other methods of experimental design is that in addition to the cost, optimum tuned parameters are obtained in less time (Fraley et al., 2006). One of the most important steps of this method is a selection method of an orthogonal array in which estimates the effective changes in the mean response.

In this research, three level experiments have been identified as the best design. Considering Taguchi‘s standard Orthogonal array, L9 array has been selected as an appropriate experimental design for tuning the algorithm parameters. A statistical measure called the Signal to noise (S/N) ratio is considered for setting optimal parameter. This ratio involves means and deviations.

The considered response variable is Mean Ideal Distance (MID), a standard metric ratio for multi objective algorithms. Since this standard indicator is a "less is better" type, equation (20) is considered as its S/N ratio. Proposed meta-heuristic algorithm for each Taguchi experiment is performed and S/N ratio is calculated with Minitab 14.1 software. Experimental design and their L9 orthogonal arrays are shown in Tables (2) and (3).

$$S/N\ Ratio = -10\log\left(\frac{sum\left(y^2\right)}{n}\right) \tag{20}$$

Table 1. Factors and levels of the Algorithm's parameters

Algorithm	Parameters	Levels	Low (1)	Medium (2)	High (3)
NSGA-II	nPop (A)	25-75	25	50	75
	P_c (B)	0.7-0.9	0.7	0.8	0.9
	P_m (C)	0.1-0.3	0.1	0.2	0.3
	nIt (D)	50-100	50	75	100
NRGA	nPop (A)	25-75	25	50	75
	P_c (B)	0.7-0.9	0.7	0.8	0.9
	P_m (C)	0.1-0.3	0.1	0.2	0.3
	nIt (D)	50-100	50	75	100

Table2. Experimental design for the L9 orthogonal arrays L9 for NSGA-II

Run Order	Algorithm Parameters				Response Value of NSGA-II
	nPop	P_c	P_m	***nIt***	**MID**
1	1	1	1	1	127181807
2	1	2	2	2	159175480
3	1	3	3	3	107101043
4	2	1	2	3	57490695
5	2	2	3	1	63478881
6	2	3	1	2	69640193
7	3	1	3	2	33711338
8	3	2	1	3	42838449
9	3	3	2	1	46335901

Table. Experimental design for the L9 orthogonal arrays L9 for NRGA

Run Order	Algorithm Parameters				Response Value of NSGA-II
	nPop	P_c	P_m	***nIt***	**MID**
1	1	1	1	1	131872798
2	1	2	2	2	143311360
3	1	3	3	3	145587630
4	2	1	2	3	87661728
5	2	2	3	1	97230691
6	2	3	1	2	107552573
7	3	1	3	2	84071739
8	3	2	1	3	80507855
9	3	3	2	1	73480386

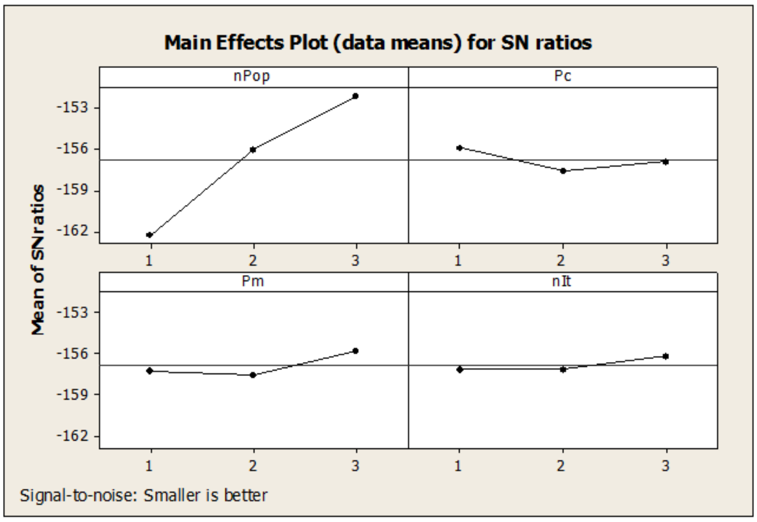

Figure 10. S/N ratio's plot of the parameters of NSGA-II

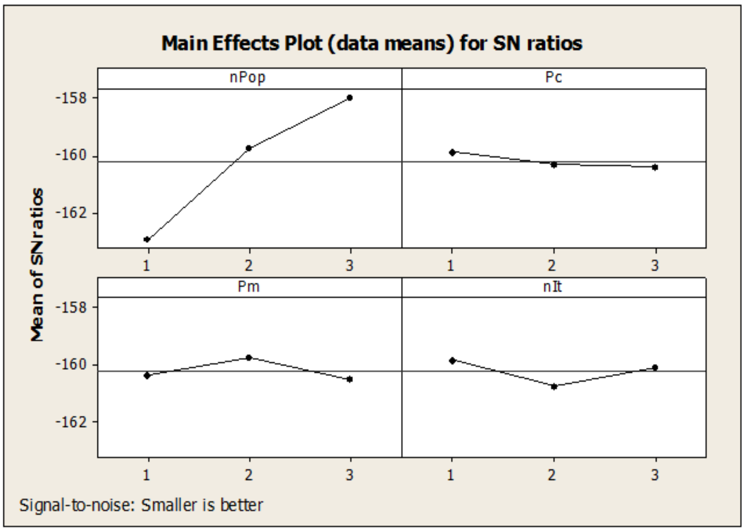

Figure 11. S/N ratio's plot of the parameters of NRGA

In the following, experiments are implemented on the 15 test problems and the solutions methods are compared. Generated test problems including the number of producers (P), the number of distribution centers (D), the number of retailers (R) and the number of clients (C) are different. Four product types and four time periods have been considered in this problem. The values are shown in table (4). For implementing the problems, input parameters are shown in table (5). In order to remove uncertainty, each problem is run three times and the average of means is considered as the final response variable. Indeed for solving the model, 90 problems have been run and analyzed.

Table 4. different levels in the proposed supply chain problem.

Test Problem Number	P	D	R	C
1	2	2	4	5
2	2	3	5	8
3	4	6	8	10
4	5	8	12	15
5	8	10	12	17
6	10	12	15	20
7	12	15	18	25
8	15	18	20	25
9	15	20	24	30
10	18	22	25	35
11	20	25	30	40
12	22	28	33	45
13	25	30	35	45
14	25	33	38	48
15	30	35	40	50

Table 5. distribution of the parameters for the proposed problems.

Parameter	Distribution	Parameter	Distribution
CD_c^{it}	Uniform(500,1000)	TC_{pd}^{it}	Uniform(10,15)
TC_{rc}^{it}	Uniform(5,8)	HC_d^{it}	Uniform(10,15)
TT_{pd}^{t}	Uniform(50,100)	TC_{dr}^{it}	Uniform(8,10)
PC_{pit}	Uniform(20,30)	SEC_p^{it}	Uniform(5,10)
HC_r^{it}	Uniform(10,15)	BC_{rc}^{it}	Uniform(8,15)
HC_p^{it}	Uniform(5,10)	TT_{rc}^{t}	Uniform(20,30)
TT_{dr}^{t}	Uniform(30,50)		

5.2. Comparing the results

In the following, standard criteria is presented for evaluating a multi-objective algorithm with Pareto approach. Unlike single-objective optimization, multi-objective optimization modeling involves two main criteria to maintain the diversity of the solutions and convergence to the Pareto set Pareto solutions (Deb et al., 2000). In this section, four comparing criteria for evaluating multi-objective optimization algorithm are presented.

5.2.1. Maximum Spread or Diversity

Equation 21 shows the calculation equation of this indicator.

$$D = \sqrt{\sum_{j=1}^{m} \left(\max_i f_i^{\,j} - \min_i f_i^{\,j} \right)^2} \tag{21}$$

The presented bi-objective model, this measure is equal to the Euclidean distance between the two boundary solutions in the objective space. The larger this measure the better (Zitzler and Thiele, 1998).

5.2.2. Spacing

Spacing criteria was proposed by Schott (1999) in which the relative distance of the sequential responses is calculated based on equation (22).

$$S = \sqrt{\frac{1}{|n-1|} \sum_{i=1}^{n} \left(d_i - \bar{d} \right)^2} \tag{22}$$

In which $\bar{d} = \sum_{i=1}^{n} \frac{d_i}{|n|}$ and, $d_i = \min_{k \in n \,\Lambda\, k \neq 1} \sum_{m=1}^{2} \left| f_m^i - f_m^k \right|$.

Minimum distance is equal to the sum of absolute difference between the measured values of the objective functions between the i th response and the response of the final non-dominated. It is noteworthy that this distance measure criterion is different from the minimum Euclidean distance.

5.2.3. Number of Pareto Solution (NOS)

The NOS measure represents the number of Pareto optimal solutions that can be found in each algorithm. In the issue of the multi-objective Pareto-based approach, one of the objectives is looking for the closer fronts to the origin of coordinates (Zitzler and Thiele, 1998).

After defining the standard criteria for comparing multi-objective problems based on Pareto, in table (6), measuring criteria for the generated test problems have been calculated. In Figure (12), the performances of proposed algorithms based on the four criteria have been depicted graphically. Then, the algorithms have been studied based on their outputs by statistical method and using analysis of variance. Figure (13) shows the statistical performance of the algorithms.

Table 6. Computational results and NRGA and NSGA-II comparison's criteria

Num	Proposed NSGA-II				Proposed NRGA			
	Spacing	***Diversity***	***NOS***	***MID***	***Spacing***	***Diversity***	***NOS***	***MID***
1	346961.8	2034125.4	6	16481106.6	321123.3	3609306.2	8	**98604607.6**
2	656002.8	3350446.6	5	54284355.8	1022883.1	5861412.9	7	**151912620.5**
3	1063232.6	6510197.3	3	322961078.6	3241877.8	21168876.6	8	**566415975.3**
4	5387461.1	17516987.1	5	859736315.2	3713872.3	29086974.7	6	**1158343306.2**
5	5410610.4	34217857.9	5	1361842884.6	2646189.7	47419477.4	10	**1385221398.9**
6	2084269.6	26278012.6	4	1954259316.8	7383551.4	51972722.1	7	**2046630681.4**
7	1010092	55472716.4	4	3408262287.6	7727795.5	54086873.9	5	**3597830157.8**
8	15966437.5	66191840.3	4	4337068746.9	7863225.9	78029553.3	9	**4615723919.8**
9	10521244.2	71899126.1	7	6058396658.2	7920878.8	77846277.6	5	**6468365992.3**
10	12640884.7	116933832.1	5	6939421513.4	14435384.2	74320413.4	5	**7344731465.1**
11	13272228.3	135287322.3	3	9829726653.1	25761265.5	182648766.1	8	**9882487338.7**
12	5072148.9	81667428.1	5	11567116447.4	14805384.6	116789064.2	8	**12018047245.2**
13	12177289.5	47818868.1	3	13443103257.7	11983618.8	97864653.4	3	**13809691604.8**
14	1143672.2	93846889.2	3	16984988003.1	1106424.4	85446477.2	3	**17476631880.1**
15	17797851.7	165027066.5	7	18480515025.5	17150775.1	171439608.6	10	**18863607488.3**
Sum	**104550387.3**	924052716	69	**95618163651**	127084250.4	**1097590458**	**102**	**99484245682**

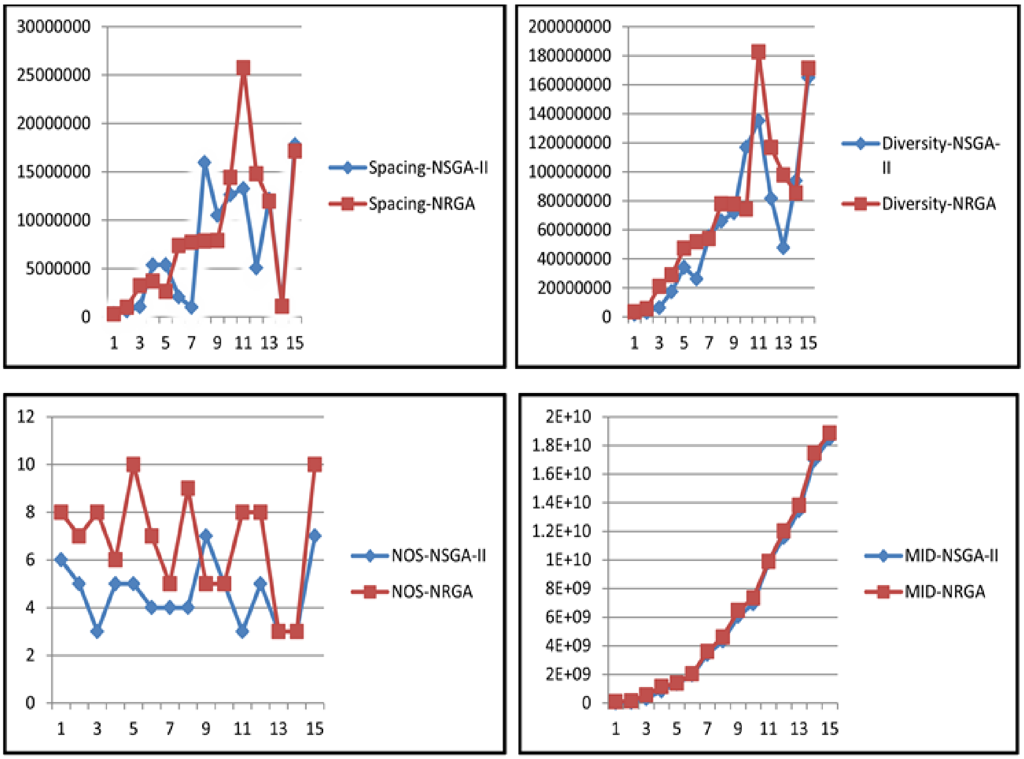

Figure 12. Graphical plots of NRGA and NSGA-II algorithms based on their comparison's criteria

As shown in the above table, MID and Spacing Diversity indexes have lower values than desire value. Also NOS and Diversity have higher values than desire level.

As shown in the bottom row of the table (6) Spacing and MID criteria in the algorithm NSGA-II algorithm and NOS and Diversity criteria in the algorithm NRGA have better performance. Statistical analysis and t-test has been used for investigating and comparing the problem more precisely. P-values and test results have been shown in table 7. Confidence intervals have been plotted in Figure 13. Therefore, statistical output indicates that there is difference between the algorithms only in NOS criteria; in other criteria the algorithms are quite capable of competing.

Table 7. Results of the statistical analysis

Metric	P-Value	Test Results
Diversity	0.545	**H0 is not rejected**
Spacing	0.544	**H0 is not rejected**
MID	0.912	**H0 is not rejected**
NOS	0.004	**H0 is rejected**

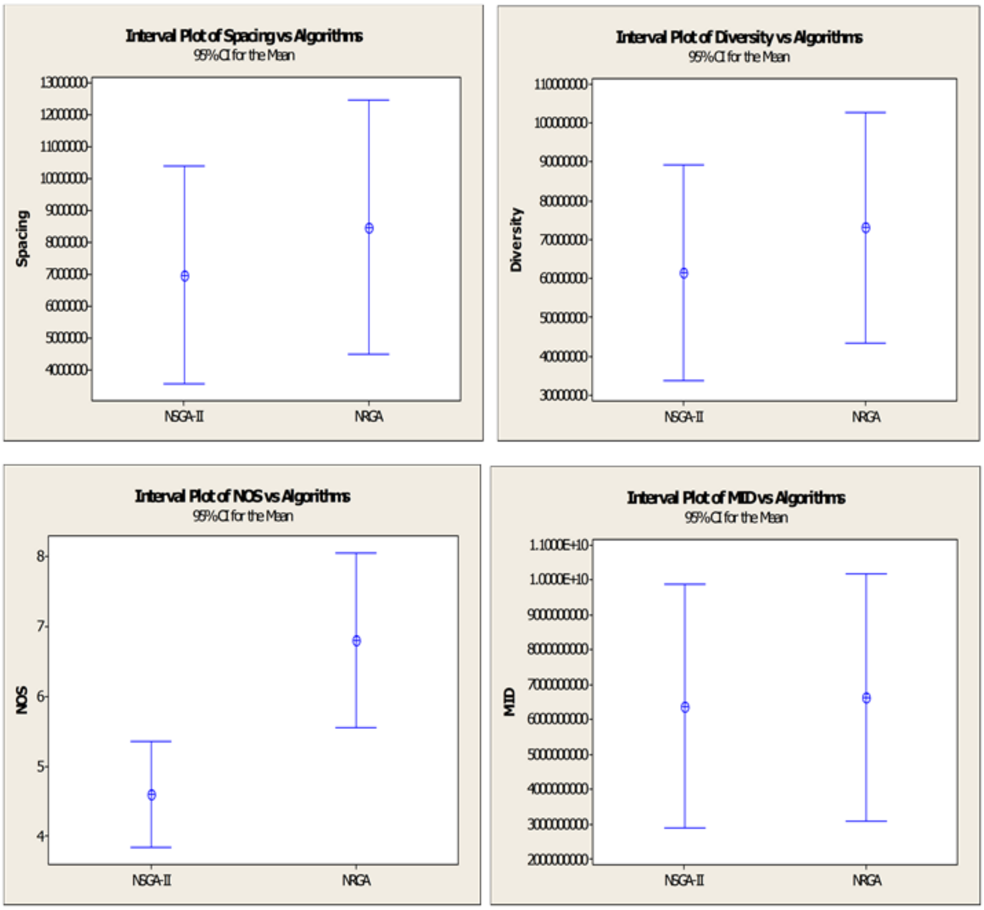

Figure 13. Interval plots of the NRGA and NSGA-II based on defined criteria.

6. Conclusion and Future works

In this research, an integrated production planning-distribution model for designing four- level supply chain with multi product types and multi periods has been presented. In the model, backorder cost has been considered in the case of product shortage. In addition to minimizing the total supply chain costs, the transfer time of the products to the customers has been also minimized.

Since the bi-objective production-distribution problem is NP-Hard problem two multi objective meta- heuristic algorithms have been developed for solving the problem. These two algorithms, NRGA and NSGA-II, have been created based on the Pareto method and their performance has been compared.

Selecting the algorithm's parameter is a very critical task so the Taguchi method has been used for tuning parameter. Finally, statistical analysis has been used in order to choose the most efficient method among presented models. Suggestions for the future research are listed as follows:

- Considering some parameters as Fuzzy parameters such as demand, production capacity and storage, as well as costs to make the problem more realistic.
- Considering discount for price of products as all unit and incremental.

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