CIVIL ENGINEERING STUDIES

STRUCTURAL RESEARCH SERIES NO. 352

AUTOMATIC ANALYZER FOR ITERATIVE DESIGN

by

Rebii Ertas

and

S. J. Fenves

A Technical Report
of a Research Program
Sponsored by
THE OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
Contract Nonr 1834 (03)
Project NR-064-183

UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS
SEPTEMBER, 1969

October 31, 1969

Martha Jo:

The gentleman below called to say they had received SRS Report entitled "Automatic Analyzer for Iterative Design" by Ertas and Fenves, the outside of which is labelled SRS 352, and the inside labelled SRS 349.

Sure enough, on page one of the Document Data in the back of the Report, it is labelled SRS 349. Now what do we do????

Mr. Laananen
Office of Naval Research (Code 439)
Department of the Navy
Washington, D.C. 20360
Tele. 202-696-2283

Barbara Reuter
201 EH - 3-3823 or 3821

P.S. The enclosed came in the mail. I presume you know what to do with it--
I don't!

AUTOMATIC ANALYZER FOR ITERATIVE DESIGN

by

Rebii Ertas

and

S. J. Fenves

A Technical Report
of a Research Program
Sponsored by
THE OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
Contract Nonr 1834(03)
Project NR-064-183

University of Illinois

Urbana, Illinois

September, 1969

ACKNOWLEDGEMENT

This report was prepared as a doctoral dissertation by Mr. Rebii Ertas and was submitted to the Graduate College of the University of Illinois in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering. The work was done under the supervision of Dr. Steven J. Fenves, Professor of Civil Engineering.

The authors wish to thank Dr. John W. Melin, Associate Professor of Civil Engineering, and Dr. Leonard A. Lopez, Assistant Professor of Civil Engineering for their assistance in using the POST executive system.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

A	=	Transformation matrix between joint and member quantities
$\bar{A}$	=	Transformation matrix of the modified structure
A_m	=	Transformation vector of member m
A_x	=	Member cross-sectional area
E	=	Modulus of elasticity
F	=	Joint flexibility matrix
$\bar{F}$	=	Joint flexibility matrix of the modified structure
I	=	Unit matrix
I_Z	=	Moment of inertia of cross-section with respect to Z-axis
J	=	Torsional constant for cross-section
k_m^*	=	Stiffness matrix of member m
$\bar{k}_m^*$	=	Stiffness matrix of the modified member M
K	=	Joint stiffness matrix
$\bar{K}$	=	Joint stiffness matrix of the modified structure
L_m	=	Length of member m
P	=	Joint actions of all joints

P_{ms}, P_{me}	=	Start and end forces of member m with respect to joint coordinate system
P_{ms}^*, P_{me}^*	=	Start and end forces of member m with respect to member coordinate system
P_s^*, P_e^*	=	Member-start and -end forces of all members
$\bar{P}$	=	Joint action vector of the modified structure
r_z	=	Radius of gyration of cross-section with respect to z-axis
R_m	=	Rotation matrix from member coordinate system of member m to joint coordinate system
T_m	=	Translation matrix from end to start terminal of member m
U	=	Joint displacements of all joints with respect to joint coordinate system
$\bar{U}$	=	Joint displacements of the modified structure
U_{ms}, U_{me}	=	Start and end displacements of member m with respect to joint coordinate system
U_{ms}^*, U_{me}^*	=	Start and end displacements of member m with respect to member coordinate system
V^*	=	Distortion of all members
V_m^*	=	Distortion of member m
x, y, z	=	Member coordinate axes
X, Y, Z	=	Global coordinate axes
K	=	Change in joint stiffness matrix
k_m^*	=	Change in member stiffness matrix

CHAPTER 1

INTRODUCTION

1.1 Object of Study

The advent of high-speed computers has provided the structural engineer with improved analytical capabilities to cope with the highly complex problem of structural synthesis. In seeking to exploit the full potential offered by the computer, structural theory has undergone an extensive reorientation from classical formulations to matrix methods of analysis. Meanwhile, considerable effort has been invested in developing programs for the analysis of highly refined models of structures subjected to various service conditions.

In spite of the valuable progress achieved, the ultimate goal of complete automation of optimum structural synthesis seems to be far from reality. In fact, such a possibility is often disputed on the grounds that the design factors involved are prohibitively high in number and quite diverse in nature. Consequently, the essential character of the present design process still remains iterative, in which each design is analyzed, evaluated, modified and reanalyzed repeatedly in order to obtain a satisfactory design.

The goal of this study is to investigate the possibility of integrating an automatic analyzer into an iterative design process more effectively by:

a) incorporating partial reanalysis techniques into the methods of solution, so that the performance of the current trial design can be determined by utilizing the information gained in the previous cycle of analysis; and

b) providing quantitative data pertaining to the rate of change of the response quantities of the structure due to modifications of various design parameters.

1.2 Background

There is a considerable body of literature on the matrix methods of analysis of skeletal structures. In this section it is intended to mention only a few of the works which are relevant to the present study.

a) Matrix Methods of Analysis

Initial contributions to the development of the matrix formulation of the flexibility method were made by Lang and Bisplinghoff(2), Langefors(3), and Wehl and Lansing(4) who added to the basic theoretical knowledge presented in an earlier paper by Levy(1).

Somewhat later, Levy suggested the use of a stiffness approach to the analysis of high-speed air frames(5). Turner, Clough, Martin and Topp(6) presented the first treatment of the stiffness method by deriving the stiffness matrices for various types of structural components.

It was shown in the same paper that the joint stiffness matrix of the structure can be obtained by superimposing the stiffness matrices of individual members.

A unified discussion of the both methods of analysis was presented by Argyris in 1960[7].

A detailed exposition of the subject by A. S. Hall and R. W. Woodhead in 1961[8], initiated a period of continuous publication of similar textbooks presenting up-to-date developments[9,10].

b) Solution Techniques

Various algorithms for the solution of the basic equations of the stiffness and flexibility method of analysis have been developed using the available techniques of linear algebra[11]. A brief discussion of related work are given in a recent paper by Spillers[12].

Kron[13] attempted to simplify the solution process for large, complex structures by tearing the interconnecting methods. The same approach has been explained by means of a straight-forward application of Housholder's modification formula[14]. Branin[15] formulated the basic ideas of Kron's approach by the orthogonal equations of a network. Later, Fenves and Branin[16] demonstrated the applicability of the network formulation to structural analysis.

c) Sensitivity Analysis

There seems to be no study reported so far for a general formulation of sensitivity functions for structural analysis. The papers written on optimization techniques employ the basic idea indirectly by using partial derivatives of design quantities. A direct reference to sensitivity coefficient was made by K. F. Reinschmidt, C. A. Cornell and J. F. Brotchie[17].

Sensitivity analysis has received considerable attention in control theory of dynamic systems. In particular, references 18 and 19 present a comprehensive discussion of the subject.

d) Computer Programs

STRESS[20] is generally accepted as the program having had the most pronounced effect on structural engineering. The development of the program took place on the basis provided by Fenves and Branin[16] in 1963. In this paper it was shown that the problem of elastic analysis of structures is just a particular case of the more general network theory of linear systems, and that the network topological formulation of structural analysis is well suited for programming on digital computers. Further additions and extensions were incorporated into the most recent version of STRESS[21] to be used in an on-line environment.

Utilizing the basic characteristics of STRESS language, a computer system for structural design, STRUDL[22] was developed in 1966. In addition to analysis, STRUDL has the capability of performing numerous design operations.

1.3 Assumptions and Limitations

The scope of this study is limited to the analysis, partial analysis and sensitivity analysis of linear elastic skeletal structures under static loads.

The term skeletal indicates a group of structures consisting of the following specific types: plane truss, plane frame, plane grid, space truss and space frame. This group of structures is composed of members that can be represented by their centroidal axis and analyzed as line elements.

It is assumed that the material of the frame is such that a linear relationship exists between the applied actions and the resulting displacements.

The overall deformation of the frame is assumed to be small enough so that the entire analysis can be based on the undisturbed configuration of the structure.

The loading is restricted to static loads which may be in the form of joint loads, member loads, member distortions, member end loads and support displacements.

1.4 Organization of Report

In Chapter 2, a brief review of the basic definitions of structural analysis is presented, in order to set the stage for subsequent discussion. Following a comparison of methods of structural analysis, particular attention is given to the stiffness formulation and related algorithmic considerations.

Chapter 3 starts with a brief discussion of the function of an automatic analyzer in an iterative design process. Then the algorithmic considerations involved in the reanalysis phase are discussed and formulations suitable for partial reanalysis are presented on the basis provided by the stiffness and the flexibility methods of analysis.

Chapter 4 starts with a discussion of the objective of sensitivity analysis and proceeds to argue that sensitivity coefficients can form a quantitative basis for helping the designer in his decision-making. Then the sensitivity functions are obtained as functional derivatives of the expressions derived for the stiffness formulation. Finally, design parameters of interest which can be included in sensitivity analysis are discussed.

The subject of Chapter 5 is the computer program developed on the basis of the formulations and algorithms discussed in the previous chapters. Following the description of the scope, implementation, input and output of the program, its logical and functional organization is presented.

In Chapter 6, a number of sample problems are considered, and the results provided by the computer program are examined to emphasize the main points developed in this study.

Chapter 7 presents a summary of the conclusions reached in the study and some suggestiong for further investigations.

CHAPTER 2

INITIAL ANALYSIS

In this chapter a brief review of basic definitions of structural analysis is presented in order to set the stage for subsequent discussion. Following a comparison of methods of structural analysis, particular attention is given to the stiffness formulation. Finally, algorithmic considerations in formulating an automatic analyzer based on the stiffness method are examined.

2.1 Structural Analysis

An elastic structure is essentially a deformable body having a specific configuration and boundary conditions. The objective of analysis is to predict the manner in which such a body, presented by an idealized model, behaves as a response to disturbances (loads) imposed on it. Structural analysis implies the complete description of this behavior by determining sufficient information from which stress and strain at any point of the structure can be calculated.

2.1.1 Basic Concepts

For purposes of analysis a skeletal structure may be conceptually decomposed into elements which are commonly referred to as members. The points where the elements terminate are the joints. There are two main advantages derived from this decomposition:

a) the analysis which involves integral or differential equations is localized to members; and

b) the analysis of the entire structure composed of an assemblage of members is reduced to a problem of finite mathematics.

Due to the fact that the complete pattern of stress and strain for each member can be determined if its terminal quantities are known, the analysis may be centered on the stress and strain resultants at the ends of the members (member terminal actions and displacements), and on the corresponding quantities assigned to the joints (joint displacements and reactions). The process of analysis is based on relations between the above quantities imposed by the geometry and topology of the structure. The mathematical basis for the formulation of these relations is provided by the three fundamental concepts of structural analysis, namely: equilibrium of the member end actions with the applied loads on the members themselves or at the joints; compatibility between member distortions and joint displacements; and the load-deformation characteristics of the members.

2.1.2 Methods of Analysis

The methods of analysis are generally classified according to the order in which the conditions of equilibrium and compatibility are applied. Methods in which the compatibility conditions are satisfied first give rise to equations of equilibrium and are called stiffness (equilibrium or displacement) methods, whereas, those in which the equilibrium conditions are used first lead to equations of displacement compatibility and are called flexibility (compatibility or force) methods[9].

In the flexibility method, a hyperstatic structure is first made statically determinate by relaxing the conditions of compatibility at a sufficient number of points. The modified structure is then analyzed, and the forces which must be applied at the points of discontinuity to produce compatibility of displacements are calculated. The analyzer (either the engineer or the computer program) is faced with the problem of determining the number and location of releases before the basic variables, i.e., the self-balancing force pairs of redundants, can be defined. While it is relatively easy to determine the number of releases required to make a given structure determinate[23], it is difficult to give general rules for choosing a good set of releases which will avoid an unstable primary structure, an excessive amount of work in the determinate analysis, or computational problems in restoring the compatibility of the releases.

In the stiffness method the basic unknowns are the joint displacements. Therefore, the number of equations to be solved is equal to the number of degrees of freedom of the structure. The displacement variables can be systematically assigned to the joints according to the type of the structure. In setting up the equations, a joint at a time is considered along with the members incident on it. Consequently, at each step one is concerned only with local topology and geometry, rather than with the complexity of the entire structure.

On the basis of the amount of computational work involved in solving the simultaneous equations, one may argue that the flexibility method is justified for structures with a degree of indeterminacy relatively small in comparison to its degree of freedom. However, even for such a case, a comparison based on the amount of work which has to be done in setting up the equations and the ease with which this work can be systematized supports the stiffness method. In the remainder of this study, the stiffness method will be used for the initial analysis, but modification techniques for reanalysis will be based on both methods.

2.2 Stiffness Formulation

2.2.1 Coordinate Systems and Transformation Matrices

The specification of action and displacement vectors and the expression of basic relations between them requires proper coordinate systems associated with member ends (member coordinate systems) and joints (joint coordinate systems). It is also necessary to define the linear operators for transformations from one system of coordinates to the other.

The geometric layout of the structure is described with reference to a single global coordinate system by specifying the coordinates of the terminal joints of each member. Joint coordinate systems are located at each joint, and have the same orientation as that of the global coordinate system. In the case of members, the orientation of the coordinate system is chosen such that the x-axis passes through the terminals of the member, and the y- and z-axes are generally taken as the principal axes of the member cross-section.

Although the member coordinate system located at one end of a member and the joint coordinate systems of the corresponding terminal joint share a common origin, they will, in general, have different orientations. Therefore a rotation matrix, R_m, is associated with each member for transformations between the member and joint coordinate systems. R_m is defined by the member terminal coordinates and the angle, β, between the member and global y-axes[8,20].

A translation matrix, T_m, must also be defined for each member in order to carry out the transformations between the member coordinate systems located at two ends of the member[8].

2.2.2 Member Relations

The load-deformation characteristics of a member, in the member coordinate system, can be condensed in the form of a member stiffness matrix, k_m^*, which relates the member distortion V_m^* to member end action P_{me}^* as

$$P_{me}^* = k_m^* V_m^* \tag{2-1}$$

The matrix k_m^* is a function of the elastic constants, shape, and the end release conditions of the member[8].

The relation between P_{me}^* and the member start actions P_{ms}^* is established by requirement of equilibrium as

$$P_{ms}^* + T_m P_{me}^* = 0 \tag{2-2}$$

Using Eq. (2-2) and contragredience[9] the member distortion V_m^* can be expressed as

$$V_m^* = U_{me}^* - T^t U_{ms}^* \tag{2-3}$$

where U_{ms}^* and U_{me}^* are the displacements of the start- and end-terminals of the member. Using Eq. (2-3) for V_m^*, Eq. (2-1) and (2-2) can be rewritten as

$$P_{me}{}^{*} = k_m{}^{*} \left(U_{me}{}^{*} - T_m{}^{t}\, U_{ms}{}^{*}\right) \tag{2-4a}$$

and

$$P_{ms}{}^{*} = -T_m k_m{}^{*} \left(U_{me}{}^{*} - T^{t}\, U_{ms}{}^{*}\right) \tag{2-4b}$$

Combining the last two equations in a single expression yields:

$$\begin{Bmatrix} P_{ms}{}^{*} \\ P_{me}{}^{*} \end{Bmatrix} = \begin{bmatrix} T_m k_m{}^{*} T_m{}^{t} & -T_m k_m{}^{*} \\ -k_m{}^{*} T_m{}^{t} & k_m{}^{*} \end{bmatrix} \begin{Bmatrix} U_{ms}{}^{*} \\ U_{me}{}^{*} \end{Bmatrix} \tag{2-4}$$

Eq. (2-4) relates the member terminal actions to the member terminal displacements, rather than the distortions.

The above quantities, expressed with reference to the member coordinate system, can be transformed to the joint coordinate system by means of the rotation matrix R_m as

$$\begin{aligned} P_{ms} &= R_m P_{ms}{}^{*} \\ U_{ms}{}^{*} &= R_m{}^{t} U_{ms} \\ P_{me} &= R_m P_{me}{}^{*} \\ U_{me}{}^{*} &= R_m{}^{t} U_{me} \end{aligned} \tag{2-5}$$

where quantities expressed in the joint coordinate system are distinguished as being not starred.

Using Eq. (2-5), Eq. (2-4) can be transformed to

$$\begin{Bmatrix} P_{ms} \\ \\ P_{me} \end{Bmatrix} = \begin{bmatrix} R_m & 0 \\ \\ 0 & R_m \end{bmatrix} \begin{bmatrix} T_m k_m^* T_m^t & -T_m k_m^* \\ \\ -k_m^* T_m^t & k_m^* \end{bmatrix} \begin{bmatrix} R^t{}_m & 0 \\ \\ 0 & R^t{}_m \end{bmatrix} \begin{Bmatrix} U_{ms} \\ \\ U_{me} \end{Bmatrix}$$

(2-6)

which expresses the relation given by Eq. (2-4) in the joint coordinate system.

2.2.3 Global Relations

As the individual members are assembled to make up the structure, continuity of the structure requires that:

$$\begin{aligned} U_{ms} &= U_i \\ U_{me} &= U_j \end{aligned} \tag{2-7}$$

where U_i and U_j are the displacements of the start joint, i, and end joint, j, of the member in question.

Using Eqs. (2-5) and (2-3), the member distortion V_m^* can be expressed in terms of the displacements of the terminal joints as

$$V_m^* = R_m^t\, U_j - T_m^t R_m^t\, U_i \tag{2-8a}$$

Observing the relation given by Eq. (2-8a), a row vector A_m may be introduced which is composed of submatrices defined as:

$$\begin{aligned} A_{m,i} &= -T_m^t R_m^t \\ A_{m,j} &= R_m^t \\ A_{m,\ell} &= 0 \qquad \text{for } \ell \neq i \neq j \end{aligned} \tag{2-8b}$$

Thus, the member distortion vector V_m^* can be related to the joint displacement vector U by

$$V_m^* = A_m U \qquad (2\text{-}8)$$

The row vectors A_m associated with all members may be combined to define a matrix A such that the total member distortions of all members, V^*, can be expressed by

$$V^* = AU \qquad (2\text{-}9)$$

Using Eq. (2-9) and contragredience, and noting that the subvectors V_m^* of V^* are expressed in member coordinate systems located at the end terminal of each member, the joint equilibrium equations can be written as

$$P = A^t P_e^* \qquad (2\text{-}10)$$

where P includes all joint loads and P_e^* is composed of individual subvectors P_{me}^* of all members.

From Eq. (2-1), the member distortions V^* is related to P_e^* by a diagonal matrix k^*, of individual member stiffness matrices k_m^*, as

$$P_e^* = k^* V^* \qquad (2\text{-}11)$$

Using Eqs. (2-9) and (2-11), Eq. (2-10) can be rewritten as

$$P = (A^t k^* A)\, U \qquad (2\text{-}12)$$

Eq. (2-12) represents a set of linear algebraic equations to be solved for the basic unknowns of the formulation, namely, the joint displacements U. Member distortions and member end forces can be found by substituting the results into Eq. (2-9) and (2-11). The matrix product

$$K = A^t k^* A \qquad (2\text{-}13)$$

is referred to as the joint stiffness matrix of the structure, and is composed of contributions from the individual member stiffness matrices k^*_m through the matrix A which contains the geometric and topological description of the structure.

2.3 Algorithmic Considerations

It is clear from the discussion of the methods of analysis in section 2.1.1 that the stiffness formulation with its systematic nature is well suited for computer programming. The entire process of setting up Eq. (2-12) can be a repetition of a single procedure which sums the contribution of each member to the joint stiffness matrix and the load vector. Such an approach automatically exploits the sparsity of $A^t k^* A$ and avoids the unnecessary computations involved in the formal matrix multiplication $A^t k^* A$.

The solution of the set of linear algebraic equations

$$KU = P \qquad (2\text{-}14)$$

constitutes the major algorithmic problem in developing an automatic analyzer based on the stiffness formulation. While the other phases of the program outlined in the preceding section call for rather straight-forward procedures, there is considerable challenge available in developing methods of solutions, with economic considerations such as minimum computation time and storage requirements as objectives. Efforts invested in fulfilling such objectives can be justified by the fact that the cost of analysis, especially in the case of large structures, depends mainly on the efficiency of the algorithm used for the solution of the equations, since it constitutes the major portion of the task performed by the analyzer.

The methods available for the solution of linear algebraic equations have been widely discussed and compared in the literature(11). These methods either directly solve the equations, or indirectly obtain the results by first computing the inverse of the coefficient matrix K^{-1}. The operations performed on the coefficient matrix K remain the same in both cases, and only those performed on P differ. Therefore, the efficiency of one scheme over the other depends on the number of columns of the actual load vector P relative to the unit matrix which is used if a full inversion is performed. In structural analysis, in most cases the number of columns in P does not approach the size of K, and therefore direct methods of analysis are generally preferred to inversion. However, it should be noted that there are certain advantages in having the inverse matrix, $F = K^{-1}$ available, as will be apparent from the next chapters.

Among the various algorithms available for the solution of Eq. (2-14) the Gauss elimination method seems to be the most popular one, due to its efficiency and simplicity for programming. The method requires two passes through the coefficient matrix K. In the forward pass, elimination is performed within the lower triangular portion of the coefficient matrix. In the second pass, back-substitution is performed with the upper triangular matrix obtained in the first pass. The basic idea of the algorithm can be expressed explicitly as factorizing K into a lower and upper triangular matrix as L and U, as:

$$LU = K \tag{2-15}$$

The Gauss elimination method, when used in conjunction with the stiffness formulation, is usually modified in order to exploit the symmetry and sparsity of the joint stiffness matrix K. Due to symmetry, the work in factorizing K is almost cut to half, since

$$L = U^t \tag{2-16}$$

In exploiting the sparsity, the objective is to work only with the non-zero terms of the coefficient matrix. The objective is partially achieved by working only within the band width which contains all the non-zero coefficients. It is possible to go one step further and exploit the sparsity within the band also. Whether such a refinement is justified or not depends on the additional indexing required within the band as compared to the savings it provides.

An additional feature of the joint stiffness matrix is that although its original degree of sparsity is unique for a given structure, the sparsity of the triangular matrices L and U is a function of the numbering of the joints and the topology of the structure. Therefore there is an optimum numbering scheme for the joints of a given structure which can lead to the most efficient solution[12]. A joint renumbering algorithm was not included in the program developed for this study.

CHAPTER 3

REANALYSIS

An automatic analyzer used in iterative design is required to perform multiple reanalyses as well as the initial analysis. In this chapter algorithmic considerations involved in the reanalysis phase are discussed, and formulations suitable for partial reanalysis are presented. The principal objective is to integrate the analyzer into the design process as effectively as possible.

3.1 Automatic Analyzer in Iterative Design

The present approach to the design of statically indeterminate structures is largely one of trial and error, consisting of repeated cycles of analysis and redesign. In such an iterative design procedure, the designer's problem is to evaluate the behavior of his trial design and to attempt to produce an improved one on the next cycle.

Automation in the design process can be achieved to some degree by means of a structural analysis program which determines the performance of a given structure under the effects imposed on it. Presently there are programs available for this purpose. However, such programs are formulated in such a way that they save the designer only from the burden of lengthy computations. This is mainly due to the fact that these programs have been conceived to be used in a single design cycle, rather than as an integral part of the entire design process. Consequently, such programs conform poorly to the basic requirements of the iterative design process, inasmuch as they do not store any information obtained in the previous cycle to be used for the next one. In each design cycle, the designer has to provide the complete description of the problem all over again, either by reentering the description or, at best, by using a file of the original description. Furthermore, the analyzer must internally reprocess the entire set of input data to set up again the equations to be solved.

3.2 General Algorithmic Considerations

Modifications can broadly be classified as those in intrinsic properties and those in extrinsic properties of the structure. The variables which define the geometry (joint coordinates) member properties (elastic constants, member cross-section properties) and topology (the manner in which the joints are connected to each other by members) fall into the first group, whereas the second group of modifications include the changes in loading on the structure.

The change in joint coordinates requires modification of the length of the members incident on the joint being shifted. Therefore the transformation matrices T_m and R_m, as well as the member stiffness matrix k_m^* assume new values to account for the change in the length of the member being effected. The changes in member properties and releases only cause the member stiffness matrix k_m^* to be modified.

Deviations from the original topology of the structure are introduced either by the addition and deletion of members, or changes in joint releases. Every new member has to be introduced with all the information necessary for its complete description, such as the designation of its terminal joints, elastic constants, cross-section properties, the rotation angle β, and release conditions. The new data can be processed internally by the same procedures as those used for the initial analysis. In the case of a member being deleted such precomputations are not required and the member is simply inactivated in the modification phase.

It should be noted that due to the discrete element idealization of the structure, it is possible to process the modification of intrinsic properties by implementing the local effect of each modification within the matrices R_m, T_m and k_m^*, which can then be used to update the overall response of the structure with the partial analysis techniques discussed in the next section.

The modification in extrinsic properties of the structure may either be explicit or implicit changes in loading. Explicit changes are those which are directly specified, whereas the implicit changes in loading are caused by the modifications in intrinsic properties. As an example of the latter case, the deletion of a member automatically eliminates any loads acting on that member.

3.3 Techniques of Reanalysis

Techniques for updating the response of a structure to reflect modifications in member properties, topology or geometry can be developed on the basis provided by both methods of analysis.

If the stiffness method is used for reanalysis, two approaches are possible. In one approach the basic formulation, Eq. (2-14), is transformed so as to yield more suitable algorithms. In the second approach, the original algorithm for solving the equations is modified so that the solution process is repeated only within a predefined region of interest to account for the modifications.

In the flexibility approach, the response of the structure can be modified by introducing releases in the region of modifications and then, on the basis of the given modifications, restoring the compatibility conditions. It should be noted that the application of the flexibility method at this stage of analysis does not involve the problems faced in the initial analysis, such as calculating the degree of indeterminacy and generating a primary structure, due to the fact that the structure to be modified acts as the "primary" structure and only the modified quantities are the redundants.

In the next sections, the formulations for partial reanalysis are first derived in their simplest form. The formulations derived are then investigated as to their limitations and possible extensions prior to further algorithmic considerations.

3.4 Modified Stiffness Formulation

3.4.1 Derivation

Due to any modification in the intrinsic properties of a structure, the joint stiffness matrix K is incremented by K, so that:

$$\bar{K} = K + \Delta K \qquad (3\text{-}1)$$

where $\bar{K}$ is the joint stiffness matrix associated with the modified structure.

The same notation "-" will be used in the remainder to distinguish the quantities of the modified structure from the corresponding quantities for the original structure.

Eq. (2-14) can be written for the modified structure as:

$$\bar{K}\,\bar{U} = \bar{P} \qquad (3\text{-}2)$$

or, using equation (3-1):

$$(K + \Delta K)\,\bar{U} = \bar{P} \qquad (3\text{-}3)$$

Premultiplying both sides of Eq. (3-3) by the joint flexibility matrix of the original structure, $F = (K)^{-1}$, yields:

$$(I + F\ \Delta K)\ \bar{U} = F\ \bar{P} \tag{3-4}$$

The expressions on both sides of Eq. (3-4) represent the joint displacements of the original structure subjected to the loading on the modified structure.

If the modifications represented by ΔK do not render the modified structure unstable, $(I + F\ \Delta K)$ is a product of two non-singular matrices, F and $\bar{K}$. Therefore $(I + F\ \Delta K)^{-1}$ exists and Eq. (3-4) can be transformed to

$$\bar{U} = (I + F\ \Delta K)^{-1}\ F\ \bar{P} \tag{3-5a}$$

Noting that $\bar{U}$ is related to $\bar{P}$ by:

$$\bar{U} = \bar{F}\ \bar{P} \tag{3-5b}$$

the modified joint flexibility matrix is obtained in terms of the previous joint flexibility matrix, F, and the joint stiffness change matrix, ΔK, as:

$$\bar{F} = (I + F\ \Delta K)^{-1}\ F \tag{3-6}$$

3.4.2 <u>Algorithm</u>

The above formulation has no limitation on the type of modification it can process. This can be seen more clearly if ΔK is expressed in terms of transformation and member stiffness matrices as

$$\Delta K = \bar{A}^t(\bar{k}^*)\ \bar{A} - A^t(k^*)\ A \tag{3-7}$$

Eq. (3-7) indicates that by updating A and k^* for changes in geometry and topology, or k^* only for modification in member properties, it is possible to obtain a matrix ΔK which represents all possible modifications.

Since only the terms associated with the terminal joints of the modified members in ΔK are nonzero, only the corresponding portion of A and k^* participate in the computation of ΔK. As in the case of creating the entire stiffness matrix, the formal matrix multiplication indicated need not be carried out, and only the contributions from each modified member needs to be computed and "plugged" into the appropriate positions corresponding to the terminal joints of the modified member.

The efficiency of an algorithm based on Eq. (3-6) can be improved by exploiting the sparsity of ΔK. For this purpose, ΔK and F are partitioned to distinguish the region of modifications and the rest of the structure, represented symbolically as:

$$\begin{aligned} \Delta K &= \begin{bmatrix} 0 & 0 \\ 0 & \Delta K_{22} \end{bmatrix} \\ F &= \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \end{aligned} \qquad (3\text{-}8)$$

where the subscript 1 refers to the unchanged region of the structure and subscript 2 is associated with the region of modification. With such a rearrangement the computation of $I + F\ \Delta K$ and its inverse can be expressed explicitly as:

$$I + F\ \Delta K = \begin{bmatrix} I_{11} & F_{12}\ \Delta K_{22} \\ 0 & F_{22}\ \Delta K_{22} + I_{22} \end{bmatrix} \qquad (3\text{-}9)$$

$$(I + F\ \Delta K)^{-1} = \begin{bmatrix} I_{11} & -F_{12}\ \Delta K_{22}\ (F_{22}\ K_{22} + I_{22})^{-1} \\ 0 & (F_{22}\ \Delta K_{22} + I_{22})^{-1} \end{bmatrix} \qquad (3\text{-}10)$$

where I_{11} and I_{22} are identity matrices of the appropriate size.

The total amount of computational work required by Eq. (3-6) depends on the size of ΔK_{22} and F. If F represents a structure of r joints and ΔK_{22} involves s joints, then F is an array-matrix of order (r x r), K_{22} and F_{22} are array-matrices of order (s x s), and F_{12} is an array-matrix of order {(r-s) x s}. The computation of $\bar{\bar{F}}$ by Eq. (3-6) requires $rs(\frac{r}{2} + 2s)$ submatrix multiplications. For one member being modified; s = 2, and the amount of computational work is r(r + 8) submatrix multiplications. If M members are processed one at a time the total number of submatrix multiplications is Mr(r + 8). If all M members, having no terminal joints in common are processed simultaneously, s = 2M, and the total amount of computational work is Mr(r + 8M) submatrix multiplications. The members entering into the modification may have several joints in common, so that $s \leq 2M$, and the computational work is correspondingly reduced. However, in general, processing one member at a time by Eq. (3-6) requires less computational work.

3.4.3 Modified Gauss Algorithm

As described in Section 3.3, an alternate approach to the reanalysis based on the stiffness method is to modify the original solution process. Following the idea of partitioning employed in the previous section, the modifications may be restricted in advance to a certain region of interest. In such a case $K = A^t k^* A$ itself is partitioned as:

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \tag{3-11}$$

where K_{22} is the part of the joint stiffness matrix associated with the region of interest.

Introducing Eq. (3-11) into Eq. (2-14) yields

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \tag{3-12}$$

Since all modifications induce changes only in K_{22}, the forward elimination can be performed outside the region of interest, so that Eq. (3-12) is transformed to

$$\begin{bmatrix} I & \tilde{K}_{12} \\ 0 & \tilde{K}_{22} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} \tilde{\mathbf{P}}_1 \\ \tilde{\mathbf{P}}_2 \end{Bmatrix} \tag{3-13}$$

where

$$\begin{aligned} \tilde{K}_{12} &= K_{11}^{-1} K_{12} \\ \tilde{K}_{22} &= K_{22} - K_{21} K_{11}^{-1} K_{12} \\ \tilde{\mathbf{P}}_1 &= K_{11}^{-1} \mathbf{P}_1 \\ \tilde{\mathbf{P}}_2 &= \mathbf{P}_2 - K_{21} K_{11}^{-1} \mathbf{P}_1 \end{aligned} \tag{3-13a}$$

Symbolically, the original analysis can be completed as

$$\begin{aligned} U_2 &= (\tilde{K}_{22})^{-1} \tilde{\mathbf{P}}_2 \\ U_1 &= \tilde{\mathbf{P}}_1 - \tilde{K}_{12} U_2 \end{aligned} \tag{3-14a}$$

Each time a modified structure is to be investigated, the rest of the forward elimination and the back-substitution can proceed from Eq. (3-13). When a modification represented by ΔK_{22} is introduced, the solution for the modified structure can be obtained as:

$$\begin{aligned} \bar{U}_2 &= (\tilde{K}_{22} + \Delta K_{22})^{-1} \tilde{\mathbf{P}}_2 \\ \bar{U}_1 &= \tilde{\mathbf{P}}_1 - \tilde{K}_{12} \bar{U}_2 \end{aligned} \tag{3-14b}$$

The smaller the region of modification, the more efficient the algorithm becomes, since the size of the condensed joint stiffness matrices K_{22} and K_{12} determines the additional computations in each cycle of analysis.

The major drawback of this approach is that the region of interest containing all possible modifications must be defined prior to the original analysis.

3.5 Modified Flexibility Formulation

3.5.1 Derivation

The method can be derived by considering a single imaginary member, having a stiffness matrix Δk_m^*, introduced to represent the modifications between any two joints. This "member" is treated as a part of the modified structure, released from it by cuts at its terminals. The objective is to restore the continuity of the structure by satisfying the conditions of compatibility of displacements between the structure and the imaginary member. For this purpose, first a set of unit actions are applied at the end of the imaginary member in its coordinate system, and the corresponding start actions determined by equilibrium. These member terminal actions are then transferred to the corresponding joints to account for the interaction between the original structure and the imaginary member. Under such a set of unit actions, the gap in the cuts, expressed with reference to the coordinate system associated with the member-end, is:

$$f_c^* = A_m F \, A_m^t + (\Delta k_m^*)^{-1} \tag{3-15}$$

which may be termed as the flexibility of the cut and its inverse, $k_c^* = (f_c^*)^{-1}$, as the stiffness of the cut.

As the structure is acted upon by the given loads $\bar{P}$ of the modified structure, a discontinuity

$$V_c^* = A_m F \bar{P} \tag{3-16}$$

is created between the imaginary member and the structure. In order to restore continuity, a set of forces

$$P_c^* = -\{A_m F A_m^t + (\Delta k_m^*)^{-1}\}^{-1} A_m F \bar{P} \tag{3-17}$$

is introduced at the cuts. The corresponding forces which have to be applied on the structure at the terminal joints of the imaginary member, expressed with reference to the joint coordinate system, are:

$$\Delta P = -A_m^t \{A_m F A_m^t + (\Delta k_m^*)^{-1}\}^{-1} A_m F \bar{P} \tag{3-18}$$

These additional joint loads, when combined with $\bar{P}$, produce the same joint displacements on the original structures as $\bar{P}$ itself would produce on the modified structure. Therefore, the joint displacements of the modified structure, $\bar{U}$, are:

$$\bar{U} = F[\bar{P} - A_m^t \{A_m F A_m^t + (\Delta k_m^*)^{-1}\}^{-1} A_m F \bar{P}] \tag{3-19}$$

Since $\bar{U}$ is also related to $\bar{P}$ by the joint flexibility matrix, $\bar{F}$, of the modified structure as:

$$\bar{U} = \bar{F} \bar{P} \tag{3-5b}$$

elimination of $\bar{P}$ from Equations (3-19) and (3-5b) yields

$$\bar{F} = F - F A_m^t \{A_m F A_m^t + (\Delta k_m^*)^{-1}\}^{-1} A_m F \tag{3-20}$$

for the modified joint flexibility matrix.

Eq. (3-20) was derived for a single imaginary member accounting for a local modification. In order to obtain a formulation to process a set of modifications involving more than one member simultaneously, Eq. (3-20) can be transformed to

$$\bar{F} = F - FA_s^{\,t}\,\{A_s F A_s^{\,t} + (\Delta k_s^{\,*})^{-1}\}^{-1}\,A_s F \qquad (3\text{-}20a)$$

where A_s includes the transformation vectors associated with the entire set of modified members, and the diagonal matrix $\Delta k_s^{\,*}$ is similarly composed of contributions from individual submatrices $\Delta k_m^{\,*}$ of the modified members.

3.5.2 Algorithm

In the formulation given by Eq. (3-20), the matrix

$$\Delta k_m^{\,*} = \bar{k}_m^{\,*} - k_m^{\,*} \qquad (3\text{-}21)$$

represents the change in the stiffness matrix of the member to be modified. It is conceivable that $\Delta k_m^{\,*}$ becomes singular, so that its inversion can not be carried out formally. Such a case may arise, for example, if either one of the $\bar{k}_m^{\,*}$ or $k_m^{\,*}$ matrices accounts for the shearing deformations but not the other. Since the term corresponding to axial deformation, $\Delta k_m^{\,*}$ (1,1), is not affected the corresponding pivot term in $\Delta k_m^{\,*}$ becomes zero. One way of bypassing this limitation is to compute directly the flexibility of the imaginary member on the basis of the change in member cross-section properties, rather than by inverting $\Delta k_m^{\,*}$.

In Eq. (3-20), the joint flexibility matrix F and the transformation vector A_m are expressed with reference to the coordinate systems just prior to the current modification. Therefore any modification in geometry which implies a change in reference coordinate systems cannot be handled by Eq. (3-20) directly. Consequently, modifications in geometry are best processed by means of the stiffness approach discussed in the previous sections.

The matrix within the bracket in Eq. (3-20), which is to be inverted, is the flexibility of the cut as given by Eq. (3-15). It can be expressed explicitly in terms of joint flexibility matrices of the terminal joints and the transformation matrices R_m and T_m of the imaginary member if the matrix multiplication $A_m F\, A_m^t$ is carried out formally. Thus for a member directed from joint j to joint i:

$$f_c^* = R_m^t\, F_{ii} R_m - R_m^t F_{ij} R_m T_m - T_m^t R_m^t F_{ji} R_m$$

$$+ T_m^t R_m^t F_{jj} R_m T_m + (\Delta k_m^*)^{-1} \qquad (3\text{-}22)$$

is the flexibility of the cut with reference to the member coordinate system. The matrix f_c^* is a single submatrix and its inverse is $k_c^* = (f_c^*)^{-1}$. Premultiplying by A_m^t and postmultiplying by A_m transforms k_c^* to joint coordinate system as

$$K_c = A_m^t\, k_c^*\, A_m \qquad (3\text{-}23)$$

The non-zero matrices of K_c can be expressed explicitly as:

$$
\begin{aligned}
K_c(i,i) &= R_m k_c^* R_m^t \\
K_c(i,j) &= -R_m k_c^* T_m^t R_m^t \\
K_c(j,i) &= -R_m T_m k_c^* R_m^t \\
K_c(j,j) &= R_m T_m k_c^* T_m^t R_m^t
\end{aligned}
\qquad (3\text{-}24)
$$

Only those columns and rows of F which are associated with joints i and j participate in the matrix multiplication $\bar{\bar{F}} = - F\, K_c\, F$.

It should be noted that only half of ΔF needs to be computed because of symmetry.

If the size of the joint flexibility matrix, F, is (r x r), and M modified members contribute non-zero terms to s joints in K_c, the amount of computational work involved in the inversion of the inner bracket and the matrix product FK_sF is $\frac{1}{2}(sr^2 + s^2r) + M^3$ matrix multiplications. In processing M members with no common terminal joints simultaneously, s = 2M, and the total number of matrix multiplications is $M(r + M)^2$. Using Eq. (3-20) for a single member requires $(r + 1)^2$ multiplications, and for processing M members one at a time $M(r + 1)^2$ multiplications. Therefore Eq. (3-20) is more efficient in processing a group of members one at a time.

CHAPTER 4

SENSITIVITY ANALYSIS

The primary objective of the iterative design process is to converge to a satisfactory solution within the design space defined by specified constraints. The activity on the part of the designer in fulfilling such an objective can broadly be divided in two phases. In the first phase, the performance of the current design is determined and checked against the design constraints. In the second phase, the choices of modifications to improve the current design is made. Analyses performed with the aid of the formulations presented in the previous chapters provide the necessary information for the first phase. Sensitivity analysis, which is formulated and presented in this chapter, yields results which can form a quantitative basis for the decision-making process of the second phase.

4.1 Objective of Sensitivity Analysis

The basic approach to the problem of sensitivity can be stated briefly as follows: starting from a reference solution of the equations describing the system under analysis, determine the trend of the solutions in some specified function space, i.e., imbed the reference solution in an appropriate parametric family to obtain a quantitative measure of the variation of the reference solution with respect to selected parameters. The results of this imbedding process are sensitivity functions which reveal the additional motion of the system in the vicinity of the reference solution caused by parameter variations. Mathematically, the sensitivity functions may be obtained by calculating functional derivatives of the original solution with respect to the parameters of interest.

There are two basic reasons to justify the interest in sensitivity analysis. First, in the physical realization of systems, uncontrolled parameter changes which are the consequence of uncertainties in component properties, component aging, environmental influences, etc., are constantly encountered. This means that no engineering device or system can be built so that its parameters will absolutely coincide with the parameters of its mathematical model. Should the system characteristics change significantly due to small parametric variations, it would be very difficult to produce or maintain the system physically. Sensitivity analysis, thus, represents a further connection between the mathematical model and the physical system. It enables the designer to apply the results from the mathematical model to the actual physical systems with far greater dependability.

Second, system design is generally guided by some method of successive parameter adjustments to achieve a pre-established criterion of performance. In such a design process, sensitivity analysis gives the designer information as to which parameter increments and in what order will best improve the system performance. Thus, the trial and error design procedure gets a clearer orientation, and the designer realizes the character of the influence of certain system design parameters.

Up to now there has not been any serious effort to formulate and employ sensitivity analysis as a design tool or as an augmentation of iterative design procedures in structural design. Such a delay is mainly due to the fact that the number of parameters which define the solution of a structure is prohibitively high. Therefore, the imbedding process which is the key to sensitivity analysis requires considerable computational work. However, such a restriction should no longer be decisive due to the analytical capability provided by digital computers. Since the additional information concerning the nature of a design will complement the designer's intuition and experience and thus refine his decision-making, the additional cost of an automated sensitivity analysis seems to be justified.

4.2 Sensitivity Functions

Sensitivity coefficients of the design variables can be obtained as functional derivatives of the expressions obtained for the stiffness formulation. Differentiating Eq. (2-14) yields:

$$dKU + KdU = dP \tag{4-1}$$

Rearranging Eq. (4-1), the differential of the joint displacements, dU, can be expressed as:

$$dU = FdP - FdK\,U \tag{4-2}$$

The joint displacement vector, U, is given in terms of the joint flexibility matrix, F, and the load vector P as:

$$U = FP \tag{4-3}$$

Substituting Eq. (4-3) into Eq. (4-2) yields:

$$dU = FdP - FdKFP \tag{4-4}$$

Eq. (4-4) indicates that the infinitesimal changes in U may be caused by similar changes in extrinsic or intrinsic properties of the structure, as represented by the load vector P and the joint flexibility matrix F, respectively.

The sensitivity to changes in extrinsic properties, i.e., the loading, is given by the first term of Eq. (4-4), i.e.;

$$dU = FdP \tag{4-4a}$$

For a particular component, ℓ, of a load associated with joint j, $P_{j\ell}$, the differential of the load vector is given by

$$dP = E_{j\ell}\, dP_{j\ell} \tag{4-5}$$

where $E_{j\ell}$ is an elementary vector with a 1 in the position corresponding to $P_{j\ell}$, all other components being zero. Therefore:

$$\frac{dU}{dP_{j\ell}} = FE_{j\ell} \tag{4-6}$$

represents the sensitivity of joint displacements with respect to $P_{j\ell}$. It can be seen from Eq. (4-6) that the sensitivity coefficients are simply the elements of the column of F associated with component ℓ of the joint j. Consequently the determination of sensitivities due to extrinsic properties is trivial once the joint flexibility matrix F is known, and will not be considered further in this study.

The sensitivity of U to intrinsic design parameters is given by the second portion of Eq. (4-4) as:

$$dU = -FdKFP \tag{4-4b}$$

For a constant loading, i.e., $dP = 0$, differentiating Eq. (4-3) yields:

$$dU = dFP \tag{4-7}$$

The sensitivity of the overall intrinsic response characteristics of the structure with respect to intrinsic parameters can be expressed by eliminating dU from Eq. (4-4b) and (4-7), to obtain

$$dF = -FdKF \tag{4-8}$$

In Eq. (4-8) the infinitesimal disturbances in the intrinsic properties are introduced through the matrix dK; the propagation of the effect of these disturbances throughout the structure is then given by the matrix product FdKF.

In order to obtain the sensitivity functions for member quantities V^* and $\mathbf{P}_e{}^*$, Eq. (2-9) and (2-10) are differentiated, resulting in:

$$dV^* = (dA)\, U + A\, (dU) \tag{4-9}$$

and

$$d\mathbf{P}_e{}^* = (dk^*)\, V^* + k^*\, (dV^*) \tag{4-10}$$

4.3 Algorithmic Considerations

The nature of sensitivity functions as revealed by the expressions of the last section is such that it is not possible to separate sensitivity analysis from direct analysis. This is actually to be expected, due to the fact that the source of these functions is the stiffness formulation. Consequently, preceding a sensitivity analysis one has to determine the overall response characteristics of the structure. In particular, in initial analysis, the method of solution of equations has to be chosen such that the joint flexibility matrix, F, is available for subsequent sensitivity analysis.

Having the results of initial analysis available, the sensitivity analysis can proceed first by determining the differential of the joint stiffness matrix, dK, which represents the infinitesimal disturbances in the intrinsic properties of the structure. The matrix dK can be expressed explicitly in terms of the member stiffness matrix, k^*, and the transformation matrix, A, by differentiating Eq. (2-13) to obtain:

$$dK = dA^t\ (k^*)\ A + A^t\ (dk^*)\ A + A^t\ (k^*)\ dA \qquad (4\text{-}11)$$

However, due to the high sparsity of matrices involved, it is not efficient to use Eq. (4-11) and perform the implied matrix multiplications in order to determine the matrix dK. Instead, the effect of infinitesimal disturbances can be decomposed and formulated separately for each member being influenced by the design parameter of interest. Thus the matrix dK can be obtained by superimposing the local sensitivity functions of the individual members.

The load-deformation characteristics of a single member, given by Eq. (2-6), can be rewritten as:

$$(A_m{}^t k_m{}^* A_m) = \begin{bmatrix} R_m & O \\ O & R_m \end{bmatrix} \begin{bmatrix} T_m k_m{}^* T_m{}^t & -T_m k_m{}^* \\ -k_m{}^* T_m{}^t & k_m{}^* \end{bmatrix} \begin{bmatrix} R_m{}^t & O \\ O & R_m{}^t \end{bmatrix} \qquad (4\text{-}12)$$

For the formulation of the local sensitivity functions, Eq. (4-12) needs to be differentiated with respect to the parameters defining the matrices R_m, T_m and $k_m{}^*$. The nature of these parameters is discussed in the next section.

Having the matrix dK and the results of the initial analysis, sensitivity coefficients of various response quantities can be obtained by evaluating the sensitivity functions derived in the last section. There are two major factors to be considered in improving the efficiency of this computation process. First, the sparsity of the matrix dK should be exploited in carrying out the matrix multiplications implied in the sensitivity functions. Second, the infinitesimal disturbances can be limited to a predefined region of interest. In such a case, only the joint flexibility matrix of the region of interest is required. This can be shown symbolically by differentiating the Eq. (3-14a) to obtain:

$$\begin{aligned} dU_2 &= dF_{22}\, P_2 \\ dU_1 &= -\tilde{K}_{12}\, dU_2 \end{aligned} \tag{4-13}$$

where

$$dF_{22} = -F_{22}dK_{22}F_{22} \tag{4-14}$$

4.4 Sensitivity Parameters

Intrinsic parameters of the structure for which sensitivity analysis can be carried out can be classified into two groups. The first group includes the elastic constants and member cross-section properties, whereas the coordinates of the joints constitute the second group of variables. The reason why the third group of intrinsic properties, namely topology, is left out is that it is not possible to introduce an infinitesimal change in topology.

4.4.1 Sensitivity to Member Properties

For the sensitivity of joint stiffness matrix to member properties, the first and third terms of the Eq. (4-11) drop out. Therefore, sensitivity with respect to a cross-section property s_m, of a member m is given

$$\frac{dK}{ds_m} = A_m^t \left(\frac{dk_m^*}{ds_m} \right) A_m \tag{4-15}$$

The available choice of the scalar variable s_m and the computation of $\frac{dK_m}{ds_m}$ depend on the type of the structure being analyzed. First, for linear structures, k_m^* is a linear function of the elastic constant, E. Assuming that Poisson's ratio remains constant, the sensitivity to E, with $s_m = E_m$, is

$$\frac{dk_m^*}{ds_m} = \frac{1}{E_m} k_m^* \tag{4-16}$$

Second, for trusses the member cross-sectional area, A_{xm}, can be chosen as the scalar s_m, thus obtaining:

$$\frac{dk_m^*}{ds_m} = \frac{1}{A_{xm}} k_m^* \tag{4-17}$$

Finally, in the case of moment resisting skeletal structures, the cross-section properties are not independent of each other. For a plane-frame member, for example, the moment of inertia, I_z, is related to the area, A_x, by

$$I_z = r_z^{\,2}\, A_x \tag{4-18}$$

It may be assumed that the radius of gyration r_z remains the same while proportionate changes in A_x and I_z are introduced. Such an assumption is quite reasonable for WF sections where the deviations in r_z from one section to another occur within very narrow limits. Therefore it is possible to identify each **WF** section by its area A_x, and seek sensitivity of response to a change from one **WF** section to another, using:

$$\left(\frac{dk_m^{\,*}}{ds_m}\right) = \begin{bmatrix} \frac{E}{L_m} & 0 & 0 \\ 0 & \frac{12\,E\,r_z^{\,2}}{L_m^{\,3}} & -\frac{6\,E\,r_z^{\,2}}{L_m^{\,2}} \\ 0 & -\frac{6\,E\,r_z^{\,2}}{L_m^{\,2}} & \frac{4\,E\,r_z^{\,2}}{L_m} \end{bmatrix} \tag{4-19a}$$

where $s_m = A_{mx}$.

Substituting the relation given by Eq. (4-18) into the last expression yields

$$\left(\frac{dk_m^{\,*}}{ds_m}\right) = \left(\frac{1}{A_x}\right) \begin{bmatrix} \frac{EA_x}{L_m} & 0 & 0 \\ 0 & \frac{12\,E\,I_z}{L_m^{\,3}} & -\frac{6\,E\,I_z}{L_m^{\,2}} \\ 0 & -\frac{6\,E\,I_z}{L_m^{\,2}} & \frac{4\,E\,I_z}{L_m} \end{bmatrix} \tag{4-19b}$$

Eq. (4-19b) indicates that $\left[\frac{dk_m^*}{ds_m}\right]$ can be obtained from k_m^* directly as

$$\left[\frac{dk_m^*}{ds_m}\right] = \frac{1}{A_x}\,(k_m^*) \qquad (4\text{-}19c)$$

The above derivation suggests that the matrix $\frac{dk_m^*}{ds_m}$ can generally be computed as if it were the stiffness matrix, k_{ms}^*, of a "sensitivity member" having a cross-sectional area

$$A_{xs} = \frac{dA_x}{ds_m} \qquad (4\text{-}20)$$

and a moment of inertia

$$I_{zs} = \frac{dI_z}{ds_m} \qquad (4\text{-}21)$$

For a WF section, the cross-section properties of the "sensitivity member" are

$$A_{xs} = 1 \quad \text{and} \quad I_{zs} = r_z^2 \qquad (4\text{-}22)$$

In the case of a rectangular cross-section, A_x and I_z can be expressed in terms of two dimensions, b and h, of the section, as:

$$A_x = bh$$
$$I_z = \frac{bh^3}{12} \qquad (4\text{-}23)$$

The question of sensitivity to one of the dimensions as the other remains constant can then be investigated. If, for example, $s_m = h$, then

$$A_{xs} = b$$
$$I_{zs} = \frac{bh^2}{4} \tag{4-24}$$

It is also possible to determine sensitivity of response with respect to cross-section properties of a group of members rather than a single one. For this purpose Eq. (4-15) has to be generalized as:

$$\frac{dK}{ds} = A^t \left[\frac{dk^*}{ds} \right] A \tag{4-25}$$

where $\frac{dk^*}{ds}$ is a diagonal matrix composed of non-zero sub-matrices $\frac{dk_m{}^*}{ds_m}$. The only requirement in using Eq. (4-25) is that the cross-section property, s, has to be identical for each member of the group.

4.4.2 Sensitivity to Joint Coordinates

An infinitesimal movement in an arbitrary direction introduces infinitesimal variations in matrices R_m, T_m and $k_m{}^*$ of all members incident on the joint. Therefore, the computation of dK involves the evaluation of the full expression given by Eq. (4-11) for each incident member. The contribution to dK from each member incident on the joint of interest can be formulated by differentiating Eq. (4-12). Since the matrices R_m, T_m, and k_m^* are functions of the coordinates of the member terminal joints, it is necessary first to resolve the movement of the joint into components along the respective axes. If joint j is moved in a direction n_j defined by a set of direction cosines, cos θ_{xj}, cos θ_{yj} and cos θ_{xj}, the sensitivity of the joint stiffness matrix is obtained as:

$$\frac{dk}{dn_j} = \sum \frac{\partial k}{\partial \zeta_j} \cos \theta_{\zeta j} \tag{4-26}$$

$$\zeta_j = x_j,\ y_j,\ z_j$$

Eq. (4-26) requires that the differentiation of Eq. (4-12) be carried out for each ζ_j so that the sensitivity to a movement in the specified direction can be determined.

In order to present the formulation in a compact form, Eq. (4-12) will be rewritten as:

$$(A_m{}^t k_m{}^* A_m) = M_R M_k M_R{}^t \tag{4-27}$$

where

$$M_R = \begin{bmatrix} R & O \\ O & R \end{bmatrix}$$

and

$$M_k = \begin{bmatrix} T_m k_m{}^* T_m{}^t & -T_m k_m{}^* \\ -k_m{}^* T_m{}^t & k_m{}^* \end{bmatrix}$$

Differentiating Eq. (4-27) yields:

$$\frac{d(A_m{}^t k_m{}^* A_m)}{d\zeta_j} = \frac{dM_R}{d\zeta_j} M_k M_R{}^t + M_R \frac{dM_k}{d\zeta_j} M_R{}^t$$

$$+ M_R M_K \frac{dM_R{}^t}{d\zeta_j} \qquad (4\text{-}28)$$

The matrix $\frac{dM_R}{d\zeta_j}$ is composed of two submatrices $\frac{dR_m}{d\zeta_j}$. The matrix $\frac{dR_m}{d\zeta_j}$ is derived in Appendix A as:

$$\frac{dR_m}{d\zeta_j} = (\eta_{\alpha\zeta})_j R_m C_\alpha + (\eta_{\gamma\zeta})_j C_\gamma R_m \qquad (4\text{-}29)$$

where the scalars $\eta_{\alpha\zeta}$, $\eta_{\gamma\zeta}$ are functions of the terminal joint coordinates of member m, and C_α and C_γ are elementary transformation matrices.

As shown in Appendix A, differentiating M_k yields:

$$\frac{dM_k}{d\zeta_j} = \frac{(\Delta\zeta)_j}{L_m{}^2} [\,Z\,] \qquad (4\text{-}30)$$

where L_m is the length of the member and $(\Delta\zeta)_j$ is defined as

$$(\Delta\zeta)_j = \zeta_j - \zeta_i$$

i being the other terminal joint of the member. The matrix Z is explicitly expressed in terms of T_m, $k_m{}^*$ and $\frac{dk_m{}^*}{d\zeta_j}$ in Appendix A, as is the matrix $\frac{dk_m{}^*}{d\zeta_j}$.

Having obtained the components along the x, y, and z axes, the local sensitivity of a member m to an infinitesimal movement dn_j in the direction of n_j of its incident joint j can be expressed as:

$$\frac{d(A_m{}^t k_m{}^* A_m)}{dn_j} = \left\{ \left(\sum_\zeta \eta_{\alpha\zeta} \cos \theta_\zeta\right)_j R_m C_\alpha + \left(\sum_\zeta \eta_{\gamma\zeta} \cos \theta_\zeta\right)_j C_\gamma R_m \right\} M_K M_R{}^t + \left\{ \sum_\zeta \frac{(\Delta\zeta)_j}{L_m{}^2} \cos \theta_{\zeta j} \right\} M_R Z M_R{}^t + M_R M_K \left\{ \left(\sum_\zeta \eta_{\alpha\zeta} \cos \theta_\zeta\right)_j C_\alpha{}^t R_m{}^t + \left(\sum_\zeta \eta_{\gamma\zeta} \cos \theta_\zeta\right)_j R_m{}^t C_\gamma{}^t \right\} \quad (4\text{-}31)$$

The matrix $\frac{dK}{dn_j}$ is built up by the topological summation of the local sensitivities as expressed by Eq. (4-31) for all members incident on the joint of interest.

A direct generalization of the above is to seek the sensitivity to the movement of a group of joints in the same direction, n. All the members incident on one of these joints have to be processed to determine dK. For a member incident on two joints of the group, an equal infinitesimal movement of its two terminal joints in the same direction does not produce any infinitesimal variation in its matrices T_m, R_m and $k_m{}^*$. Therefore, there is no contribution to dK from such a member.

CHAPTER 5

COMPUTER PROGRAM

The subject of this chapter is the computer program developed on the basis of the formulations and algorithms discussed in the previous chapters.

5.1 Purpose and Scope

The program includes capabilities for initial analysis, reanalysis and sensitivity analysis for linear elastic skeletal structures. Initial analysis and reanalysis may be performed either to determine the intrinsic structural properties such as member lengths, member and joint stiffness and flexibility matrices, or, in addition, to compute responses such as joint displacements, member distortions, member end forces and reactions for given static loads. Sensitivity analysis furnishes information pertaining to the rate of change of the quantities determined in the initial or reanalysis phases to changes in specified design parameters.

The term "skeletal" indicates that only framed structures composed of members that can be represented by their centroidal axis and analyzed as line elements fall within the scope of the program. This group of structures includes the following specific types: plane truss, plane frame, plane grid, space truss and space frame.

5.2 Implementation

The programming system is conceived for use either in an on-line environment or to be run as a batch job. The present version is implemented in the latter mode on an IBM System 360 computer.

The entire program is stored on the disk storage unit and brought into the main computer memory for execution.

In addition to the capability of performing consecutive cycles of analysis for the same problem or different ones in a single run, a provision is implemented to store the quantities pertaining to the response of the current structure permanently on the disk to be retrieved and used in a later run.

The main body of the program is written in POST[24] which is a FORTRAN-like language with implied matrix operation, dynamic storage allocation and dynamic array dimensioning. In the case of large problems with storage requirements exceeding the primary memory capacity of the computer, the POST executive system has the capability of automatically performing temporary data storage and retrieval using secondary storage devices.

The important specific features of the POST language are noted below.

The variables are global to the entire program, and their name, type, mode and size have to be specified at the beginning of the program by DECLARE statements. The naming convention is such that either the abbreviated form or the full name may be used. Thus, NAME.OF.THE.VARIABLE, N.O.T.V., NA.O.T.VAR, NOTV represent the same variable. There are five types of variables; Elements, Vectors, Matrices, Sets of Subvectors and Arrays of submatrices. The mode of each variable can be either floating or integer.

In the present version of the program, input is accepted in free-format by the POST READ command. Similarly, the output is accomplished by using the WRITE command. In both input and output, the name of a variable precedes its value(s). The POST READ command continues reading names and values of variables until a RETURN statement is encountered in the data stream.

5.3 Input to the Program

The capacity of the program with respect to problem size depends on the size limitations of the POST executive system. The maximum size of a vector or matrix is limited by the page size, which is presently 4095 locations. The total amount of space available is dictated by the number of pages, which is set at 100 presently.

The description of a problem to the program consists of three blocks of data. The process descriptors are given in the first data block. The size descriptors are specified in the second data block. The structural data and loading data descriptors constitute the third data block. The third RETURN statement, specifying the end of the data pertaining to that problem, initiates the internal solution process. After the desired results of analysis are displayed, the program automatically returns to its initial state, i.e., ready to accept the subsequent set of data either for a completely different problem or for the next cycle of analysis of the same problem.

5.3.1 Process Descriptors

This group of variables provide information about the type of analysis, the assignment of permanent secondary storage and the desired selective output.

OPTION is an integer element which specifies the type of analysis, i.e., whether it is initial analysis, reanalysis or sensitivity analysis.

DISK.INPUT.OUTPUT is a two-element vector, the elements specifying whether input data are to be retrieved from or output data stored on the disk.

TABULATE.OUTPUT is a vector, each element of which requests a specific selective output.

All the process descriptors have to be specified in the first data block for each analysis.

5.3.2 Size Descriptors

For initial analysis the variables needed to define the size of the problem are:

NUMBER.MEMBERS
NUMBER.JOINTS
NUMBER.SUPPORTS
NUMBER.LOADS
NUMBER.MEMBER.LOADS
NUMBER.JOINTS.OF.INTEREST
NUMBER.STRUCTURE.TYPE

The first two variables imply that the joints and members are referred to by sequential identification numbers from one to N.M. or N.J.

N.J.O.I. specifies the size of the region of interest and is the number of free joints and released support joints present in the region of interest. NUMBER.STRUCTURE.TYPE is the code number for the appropriate structure type.

In case of reanalysis, only the increase in the number of members, loads or member loads need to be specified by giving updated values for the descriptors N.M., N.L. and N.M.L. No changes in the remaining size descriptors are accepted, as they would create an analysis problem of an entirely different nature. If these variables need to be changed, it is best to request a new initial analysis for the modified problem.

For sensitivity analysis, two size descriptors must be given. NUMBER.OF.SENSITIVITY.MEMBERS specifies how many members are to be included in the sensitivity analysis to determine the rate of change of response to the changes in the cross-section properties of these members. The number of joints which are given the same infinitesimal movement in an arbitrary direction for sensitivity analysis is specified by NUMBER.OF.SENSITIVITY.JOINTS.

5.3.3 Structural Data Descriptors

Geometry is specified in terms of joint coordinates given by the set JOINT.COORDINATES. Identification number of support joints are given in the vector SUPPORT.JOINTS and their release condition in the vector SUPPORT.RELEASES.

The interconnection of the members is described by the set MEMBER.INCIDENCE giving the starting and ending joint of each member. The vector MEMBER.BETA specifies the β-angle for all members. The presence of hinges in the members is indicated in the vector MEMBER.RELEASE.

The load-deflection characteristics of the members are specified in the set MEMBER.PROPERTIES.PRISMATIC for prismatic members. The array matrices FLEXIBILITY.GIVEN and STIFFNESS.GIVEN are used if member flexibility or stiffness matrices are input directly. For members with variable cross-sections, the corresponding data is given in MEMBER.PROPERTIES. VARIABLE and NUMBER.OF.VARIABLE.PANELS. Elastic constants associated with members are specified in the vectors E and G.

Identification numbers of the joints in the region of interest are given in vector JOINTS.OF.INTEREST.

For reanalysis, the structural data specified in terms of the above variables for initial analysis can be modified by giving the corresponding modified data under the variable names:

JOINT.COORDINATES.MODIFIED
MEMBER.INCIDENCE.MODIFIED
MEMBER.PROPERTIES.PRISMATIC.MODIFIED
FLEXIBILITY.GIVEN.MODIFIED
STIFFNESS.GIVEN.MODIFIED
MEMBER.PROPERTIES.VARIABLE.MODIFIED
NUMBER.OF.VARIABLE.PANELS.MODIFIED
E.MODIFIED
G.MODIFIED
MEMBER.RELEASE.MODIFIED
MEMBER.BETA.MODIFIED

For sensitivity to member cross-section properties, the identification numbers of the members entering the sensitivity analysis are specified in the vector SENSITIVITY.TO.MEMBERS. The cross-sectional properties of the "sensitivity member" defined in Section 4.3.1, is given in SENSITIVITY.MEMBER.PROPERTIES. For sensitivity to geometry, the joints which are given an infinitesimal movement are identified in the vector SENSITIVITY.TO.JOINTS. The direction of the movement is specified in DIRECTION.COSINES.

5.3.4 Loading Data Descriptors

The loading applied to the structure is described for initial analysis by:

JOINT.LOADS
SUPPORT.DISPLACEMENTS
MEMBER.LOAD.DISTORTIONS
MEMBER.LOAD.TYPES
MEMBER.LOAD.VARIABLES

In the reanalysis phase, the loading data defined by the above variables can be modified by giving the modified loading data in:

JOINT.LOADS.MODIFIED
SUPPORT.DISPLACEMENTS.MODIFIED
MEMBER.LOAD.TYPE.MODIFIED
MEMBER.LOAD.VALUE.MODIFIED

There is no loading data description for sensitivity analysis.

5.4 Output

The results of analysis are displayed in the form of printed output.

In the case of initial analysis and reanalysis, the available output quantities are:

MEMBER.LENGTH
MEMBER.STIFFNESS
MEMBER.ROTATION
MEMBER.END.FORCES
MEMBER.START.FORCES
MEMBER.DISTORTIONS
JOINT.STIFFNESS
JOINT.FLEXIBILITY
JOINT.DISPLACEMENTS
JOINT.REACTIONS

The results of sensitivity analysis are displayed in terms of:

SENSITIVITY.JOINT.FLEXIBILITY
SENSITIVITY.JOINT.DISPLACEMENTS
SENSITIVITY.JOINT.REACTIONS
SENSITIVITY.MEMBER.DISTORTIONS
SENSITIVITY.MEMBER.END.FORCES
SENSITIVITY.MEMBER.START.FORCES

5.5 Organization of the Program

5.5.1 Logical Organization

Logically, the program consists of three major phases: input, execution and output, as shown on the block diagram of Fig. 5.1.

Each cycle of analysis starts with the input of appropriate data, as described in the previous section. The process descriptors are always read from cards. The description of the remainder of the problem may be entirely in the form of punched cards or may be partially supplied from the disk.

In the execution phase, depending on the value of OPTION specified in the input phase, the program branches into initial analysis, reanalysis or sensitivity analysis.

In the last phase, the results are either displayed in the form of printed output or stored on the disk.

5.5.2 Functional Organization

Functionally, the program is organized into a number of major POST subroutines and several POST utility subroutines and FORTRAN subroutines linked to the main program.

The functional organization of the program is shown in Fig. 5.2(a). SUBROUTINE.INITIALIZE, which performs initializing and output of data, and SUBROUTINE.OUTPUT, which displays desired selective output, are common to all types of analyses.

Depending on the value of OPTION specified, the program branches to one of three main paths.

For initial analysis (Fig. 5.2(b)), the process consists of the following major steps:

1. Generation of an internal topological representation suitable for the subsequent algorithms (SUBROUTINE.INTERNAL.TOPOLOGY);
2. Computation of member lengths, stiffness and transformation matrices (SUBROUTINE.MEMBER.QUANTITIES);
3. Processing member loads to compute member fixed-end forces (SUBROUTINE.MEMBER.LOADS);
4. Modification of member stiffness matrices and member fixed-end forces due to member releases (SUBROUTINE.MEMBER.RELEASE);
5. Determination of effective joint loads from given joint loads and computed member fixed-end forces (SUBROUTINE.JOINT.LOADS);
6. Generation and solution of equations (SUBROUTINE.GENERATE.EQUATION and SUBROUTINE.SOLVE);
7. Determination of member actions and distortions and joints reactions from the joint displacements (SUBROUTINE.BACK.SUBSTITUTION).

For reanalysis, the following major steps are executed (see Fig. 5.2(c)):

1. Processing the specified modifications in structural data and updating the original data (SUBROUTINE.PROCESS.STRUCTURAL.MODIFICATION);
2. Processing the specified modifications in loading data and updating the original data (SUBROUTINE.PROCESS.LOADING.MODIFICATION);
3. Reanalysis performed by one of the following:
   a) Using Eq. (3-20) (SUBROUTINE.REANALYSIS.BY.FLEXIBILITY)
   b) Using Eq. (3-6) (SUBROUTINE.REANALYSIS.BY.STIFFNESS)
   c) Using Eq. (3-14b) (SUBROUTINE.SOLVE);
4. Determination of member actions, distortions and joint reactions (SUBROUTINE.BACK.SUBSTITUTION);

For sensitivity analysis the major functional steps are (see Fig. 5.2(d)):

1. Computation of local sensitivities to be assembled into dK (SUBROUTINE.SENSITIVITY.TO.GEOMETRY and SUBROUTINE.SENSITIVITY.TO.MEMBERS);
2. Computation of dF and determination of sensitivity of member quantities (SUBROUTINE.BACK.SUBSTITUTE.SENSITIVITY).

CHAPTER 6

ILLUSTRATIVE EXAMPLES

In this chapter, several examples are presented in order to emphasize the main points developed in this study. The numerical results reported were obtained by the use of the computer program discussed in Chapter 5.

In all of the examples, loads and forces are expressed in kips, moments in inch-kips, linear dimensions in inches, displacements in inches and rotations in radians.

6.1 Comparison of Reanalysis Methods

The efficiency of reanalysis techniques and the factors affecting the efficiency is to be investigated in this example.

The structure considered is a transmission tower analyzed as a space truss having 22 joints and 66 members, and subjected to two loading conditions. The configuration of the tower is shown in Fig. 6.1.

The structure was analyzed in the following manners:

1) No region of interest was defined. Only one cycle of analysis was performed to determine the member distortions, member forces, joint displacements and support reactions. The execution time was 40 seconds. If the tower

were analyzed in this manner after each set of modifications is introduced, the same execution time would be consumed for the solution of the problem.

2) The entire structure was defined as the region of interest. The initial analysis, which in this case determines the joint stiffness and flexibility matrices in addition to the member and joint response quantities, consumed 59 seconds. The 19 second increase as compared to the previous analysis is mainly due to the inversion of the joint stiffness matrix to obtain the joint flexibility matrix of the structure.

The four members indicated on Fig. 6.1, connecting the support joints to the adjacent free joints, were assigned different member properties. The modified tower was then reanalyzed by each of the three reanalysis techniques discussed in Chapter 3. The execution times were recorded as:

a) 23 seconds for the algorithm based on the modified flexibility formulation, Eq. (3-20)

b) 37 seconds for the algorithm based on the modified stiffness formulation, Eq. (3-6)

c) 32 seconds for the modified Gauss algorithm, Eq. (3-14).

3) Only the four free terminal joints of the four members to be modified were specified as the joints of interest. Due to the decrease in the size of the region of interest, the execution time for initial analysis dropped to

51 seconds. For the reanalysis of the modified structure the following execution times were recorded:

a) 10 seconds for the algorithm based on the modified flexibility formulation, Eq. (3-20)

b) 9 seconds for the algorithm based on the modified stiffness formulation, Eq. (3-6)

c) 11 seconds for the modified Gauss algorithm, Eq. (3-14).

It is seen from the recorded execution times that with full structure as the region of interest there is no saving to speak of, even when repeated reanalyses are made. With a reduced region of interest substantial savings in execution time can be achieved even for one cycle of reanalysis. It should be noted that the efficiency of each reanalysis technique relative to the others changes with a change in the size of the region of interest.

The general conclusion which can be drawn from this example is that the partial reanalysis techniques may lead to savings in analysis time if the region of interest is relatively small.

6.2 Uncertainties in Design Parameters

A two story, one bay concrete frame will be used to demonstrate the use of sensitivity analysis in getting "confidence limits" for the uncertainties in the actual rigidity of the structure due to cracking. The structure is subjected to two concentrated loads as shown in Fig. 6.2. All the members were assigned a cross-sectional area of 20 sq. inches, and moment of inertia of 100 inches^4.

The axial force, shear, and moment at both ends of each member are given in the first, second and third columns of Table 6.1. Inspection of these forces indicates that members 4 and 6, having the largest moment and shear, are more liable to crack than the other members. It was desired to check the effect of the cracking of members 4 and 6 on the member terminal forces. For this purpose a sensitivity analysis was performed with respect to the moment of inertia of members 4 and 6. The sensitivity coefficients, $\frac{d\mathbf{P}_m}{dI_{4,6}}$ are given in Table 6.1.

It may be assumed that, as a result of cracking, the moment of inertia of members 4 and 6 would be reduced by 50 per cent. Extrapolating from the sensitivity coefficients, the resulting percentage change in the member forces would be predicted as

$$\% \; \Delta \mathbf{P}_m = \frac{\frac{d\mathbf{P}_m}{dI_{4,6}} \, (.50 \; I_{4,6})}{\mathbf{P}_m} \qquad (6.2\text{-}1)$$

The results obtained using Eq. (6.2-1) are given in the last three columns of Table 6.1, and indicate that the internal force distribution is rather insensitive to changes in the rigidity of members 4 and 6. The largest percentage change was observed to be 1.44 per cent for the axial force of member 5.

This example is typical of the type of sensitivity analysis which may be performed whenever there are uncertainties about the parameters.

6.3 Sensitivity Analysis for Changes in Member Properties

It is essential for the designer to be able to choose modifications which would improve his design. It will be shown that the sensitivity coefficients provide the quantitative basis for such a decision.

The structure of this example is the plane truss shown in Fig. 6.3. All members were assigned a cross-sectional area of 8 sq. inches. Since the right support is released in the x-direction the structure is externally determinate. However, due to the additional diagonal member in the center panel, the truss is internally indeterminate to one degree. Modifications in the areas of the members incident on the support joints do not change the value of the member forces. Therefore, the discussion will be focused on the remaining members.

The member forces induced by the joint loads shown in Fig. 6.3 were determined by an initial analysis. Then a sensitivity analysis was performed to determine the sensitivities of the member forces to changes in the areas of the bars 1, 2, 3 and 4. The results obtained are presented in

Table 6.2. Member 1 will be used to interpret these sensitivity coefficients.

The member force associated with member 1 is given by the initial analysis as $P_1 = -21.87$ kips. The largest sensitivity coefficient for P_1, $\frac{dP_1}{dA_4} = -0.249$, indicates that P_1 is most sensitive to a change in the area of member 4. The negative sign of the coefficient indicates than an increase in A_4 will cause a change in P_1 by a negative increment. This means that a decrease in A_4 reduces P_1 in absolute value.

On the basis of similar interpretations of the sensitivity coefficients, it was decided to decrease P_1 by increasing A_3 and decreasing A_4 and A_{10} by 50 per cent, i.e.,

$$\Delta A_3 = 4.0 \text{ sq. in.}$$

$$\Delta A_4 = \Delta A_{10} = -4.0 \text{ sq. in.}$$

The axial force associated with member 1 of the modified structure, $\bar{P}_1$, was predicted by linear extrapolation from the sensitivity coefficients as:

$$\bar{P}_1 \simeq P_1 + \frac{dP_1}{dA_3}(\Delta A_3) + \frac{dP_1}{dA_4}(\Delta A_4) + \frac{dP_1}{dA_{10}}(\Delta A_{10})$$

Substituting the numerical values of the variables in the last expression yields

$$\bar{P}_1 \simeq -19.27 \text{ kips.}$$

The reanalysis carried out for the modified structure yielded

$$\bar{P}_1 = -19.48 \text{ kips.}$$

The comparison of the predicted values of $\bar{P}_1$ with its calculated value indicates that linear approximations using sensitivity coefficients may lead to reasonable predictions for the response quantities of the modified structure.

The predicted and calculated values of the other member forces are also given in Table 6.2.

It may be concluded that the trial and error procedure of iterative design gets a clearer orientation with the information provided by the sensitivity analysis.

6.4 Sensitivity Analysis for Changes in Geometry

In this example, the structure shown in Fig. 6.4 is used to demonstrate the use of sensitivity analysis with respect to joint coordinates in design. The structure is a symmetric plane frame subjected to symmetric loading. All members were assigned a cross-sectional area of 20 sq. in. and a moment of inertia of 100 inches^4. The member end moments determined by initial analysis are given in Table 6.3. The sensitivity of these member forces to the y-coordinate of the joints 2 and 4 were computed by sensitivity analysis and the results are given in Table 6.3.

The start moment, M_{1s}, and end moment, M_{1e}, of member 1 are selected to interpret the information provided by the sensitivity coefficients. The values of the moments M_{1s} and M_{1e} were given by the initial analysis as

$$M_{1s} = -289.2 \quad \text{inch-kip}$$

$$M_{1e} = -319.7 \quad \text{inch-kip}$$

Sensitivity analysis yielded

$$\frac{dM_{1s}}{dy_{2,4}} = 1.170 \quad \frac{\text{inch-kip}}{\text{inch}}$$

$$\frac{dM_{1e}}{dy_{2,4}} = 0.348 \quad \frac{\text{inch-kip}}{\text{inch}}$$

This means that if the joint 2 and 4 are given an infinitesimal movement in the positive y-direction, M_{1s} will change by a negative increment, and M_{1e} by a positive increment. Consequently, M_{1s} will increase whereas M_{1e} decreases. Therefore, assuming that the most efficient design is sought and the position of joint 2 is a design variable, it may be possible to induce a change Δy in y_2 and y_4 such that

$$\bar{M}_{1s} = \bar{M}_{1e} \tag{6.4-1}$$

A reasonable guess for such a Δy can be made first predicting M_{1s} and M_{1e} associated with the modified structure as

$$\bar{M}_{1s} = M_{1s} + \frac{dM_{1s}}{dy_{2,4}} \; (\Delta y)$$

$$\bar{M}_{1e} = M_{1e} + \frac{dM_{1e}}{dy_{2,4}} \; (\Delta y) \qquad (6.4\text{-}2)$$

Substituting Eq. (6.4-2) into Eq. (6.4-1) and rearranging yields

$$\Delta y = \frac{M_{1e} - M_{1s}}{\dfrac{dM_{1s}}{dy_{2,4}} - \dfrac{dM_{1e}}{dy_{2,4}}} \qquad (6.4\text{-}3)$$

Substituting the numerical values of the variables in Eq. (6.4-3), Δy was obtained as

$$\Delta y = \frac{30.52}{1.52} = 20.0 \text{ inch}$$

The y-coordinates of the joints 2 and 4 were then changed by Δy = 20.0 inch, and a reanalysis was performed to determine the response quantities.

The end and start moments of member 1 of the modified structure were observed to be

$$\bar{M}_{1s} = -307.2 \quad \text{inch-kip}$$

$$\bar{M}_{1e} = -310.1 \quad \text{inch-kip}$$

It was also noted that the change in the terminal moments of all members were such that the difference between the smallest and the largest moment became 313.8 - 307.2 = 6.6 inch-kips, compared to 362.7 - 289.0 = 73.7 inch-kip associated with the initial structure.

It may be concluded that reasonably close predictions can be made in terms of sensitivity coefficients for changes in the geometry of the structure.

6.5 Accuracy of Predictions by Sensitivity Analysis

It is intended here to investigate the accuracy of the predictions made for the modified response quantities by linear extrapolation using sensitivity coefficients.

The structure of this example is a tied arch consisting of a girder and an arch connected to each other by vertical hangers. The hangers are truss-members having resistance only for axial force. The dimensions and the support conditions are given in Fig. 6.5. A cross-sectional area of 10 sq. in. for all members, and a moment of inertia of 100 in^4 for the arch and the girder were used.

The structure was analyzed to determine the response quantities induced by a single concentrated vertical load acting at the midspan of the girder. The joint displacements were selected as the quantities of interest, and their numerical values are given in Table 6.4. It was observed that the critical components were the vertical displacements, u_{jy}. A sensitivity analysis was performed to predict the change in the vertical displacements of the joints, u_{jy}, due to a change in the area of the hangers, A_h, the moment of inertia of the arch, I_a, and of the girder, I_g, The results $\frac{du_{jy}}{dA_h}$, $\frac{du_{jy}}{dI_a}$, and $\frac{du_{jy}}{dI_g}$ are presented in Table 6.4. The moment of inertia of the arch was then increased 50 per cent to decrease the joint displacements as predicted by the sensitivity coefficients. The reanalysis of the structure yielded the results given in the last column of Table 6.4. If these values were predicted by

$$\bar{U}_{jy} \simeq U_{jy} + \frac{du_{jy}}{dI_a} (\Delta I_a)$$

the expected values of the vertical joint displacements would be those given in the seventh column of Table 6.4. Comparison of the predicted and the exact values indicates that there is some discrepancy between the two. This is due to the curvatures of the curves representing the relation between the joint displacements and I_a. The greater the curvature at the point corresponding to the initial analysis the more is the discrepancy.

CHAPTER 7

CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

7.1 Conclusions

In this study an automatic analyzer with the capability of performing initial analysis, reanalysis and sensitivity analysis for linear elastic skeletal structures was formulated. The principal objective was to integrate the analyzer in the iterative design process as effectively as possible.

The stiffness method of analysis was used for initial analysis. It was noted that:

1) The method with its systematic nature is well suited for computer programming;
2) The major portion of the total analysis time is consumed by obtaining and solving the joint equilibrium equations;
3) The entire process of setting up the equations is simply a repetition of a single procedure which sums the contribution of each member to the joint stiffness matrix;
4) Exploitation of the sparsity of the joint stiffness matrix provides considerable savings in the execution time.

The reanalysis capability of the analyzer was conceived to operate on a structure previously analyzed. Having the information pertaining to the response characteristics of the previous structure, a partial reanalysis can be performed for the modified structure. In the formulation of the reanalysis phase of the analyzer the following were noted:

1) Input data for reanalysis need to include only specified modifications in the intrinsic and extrinsic properties of the structure;
2) The modifications in the intrinsic properties can be processed by first updating the internal topological representation of the structure, and then implementing the local effect of each modification within the matrices R_m, T_m and k_m^*;
3) The modifications in extrinsic properties of the structure may either be explicit or implicit changes in loading. In either case the portion of the member fixed-end forces and the effective joint load vector which is effected by the modifications need to be updated;
4) The modifications can be processed by the same subroutines as used for initial analysis. Thus, the additional programming can be kept to a minimum;

5) Several solution techniques for reanalysis can be developed on the basis provided by the stiffness and the flexibility methods of analysis. Efficiency of these algorithms increases by restricting the modifications to a predefined region of interest;

6) In general, partial reanalysis is justified for a reasonable amount of modifications.

The sensitivity analysis provides information pertaining to the rate of change of the response quantities due to changes in design parameters. Such information forms a quantitative basis for the decision-making process of the designer, and the trial and error design procedure assumes a clearer orientation. Furthermore, linear approximations utilizing the sensitivity coefficients can predict the changes in response quantities due to specified modifications rather closely.

7.2 Suggestions for Further Study

The following can be suggested in order to initiate further study:

1) In spite of the anticipated problems, the possibility of incorporating the flexibility method into the initial analysis should be investigated. A study conducted for this purpose will undoubtedly provide detailed

insight into the basic characteristics of the flexibility method.

2) In the case of the stiffness method of analysis, an optimum numbering of the joints in order to exploit the sparsity of the joint stiffness matrix to the highest possible degree should be included in the program.

3) The possibility of incorporating synthesis procedures using the sensitivity coefficients into the automatic analyzer should be given consideration. Such a study is needed in order to achieve further automation in design.

4) The capabilities of the automatic analyzer should be extended

   a) To provide more detailed selective output, such as the response quantities at specified cross-sections subject to the combination of multiple loading conditions;

   b) To include elastic structures having finite elements;

   c) To determine dynamic response characteristics of a structure.

5) The present version of the computer program can be improved by:

a) Converting the input data to a completely problem-oriented format to provide a higher level of communication;

b) Implementing the system in an on-line environment to insure continuous interaction between the designer and the analyzer.

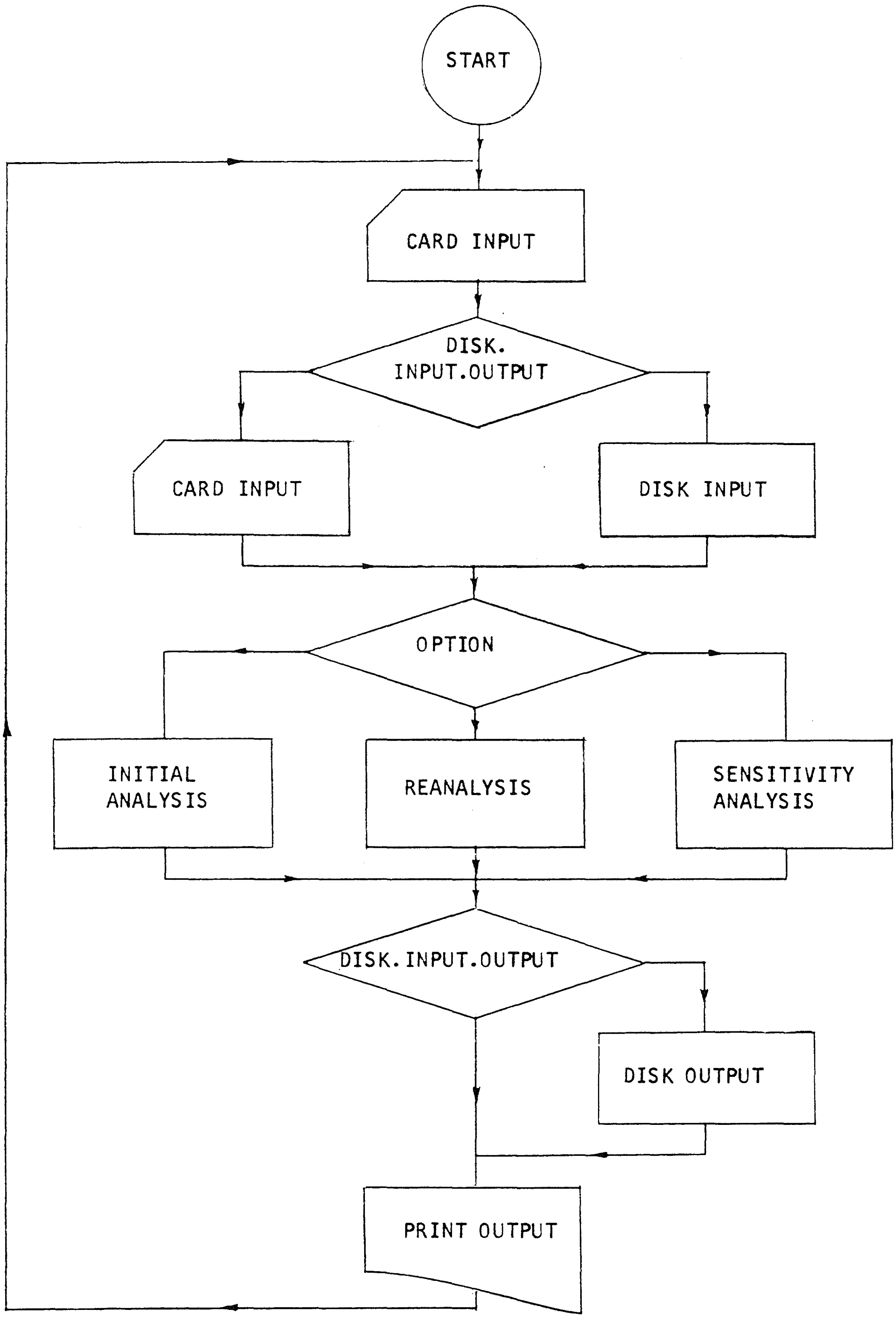

Fig. 5.1 - Logical Organization of the Program

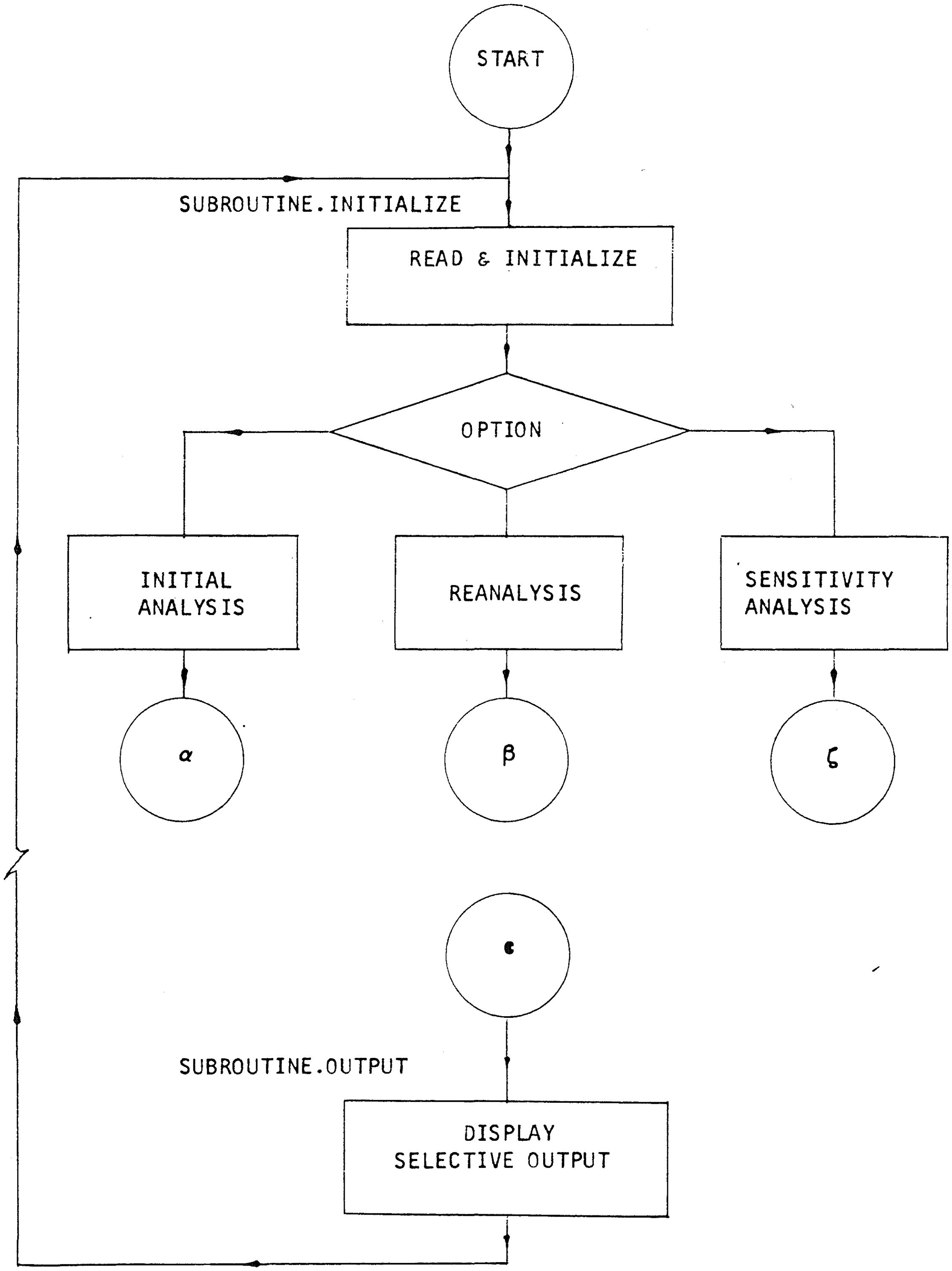

Fig. 5.2a - Functional Organization of the Program

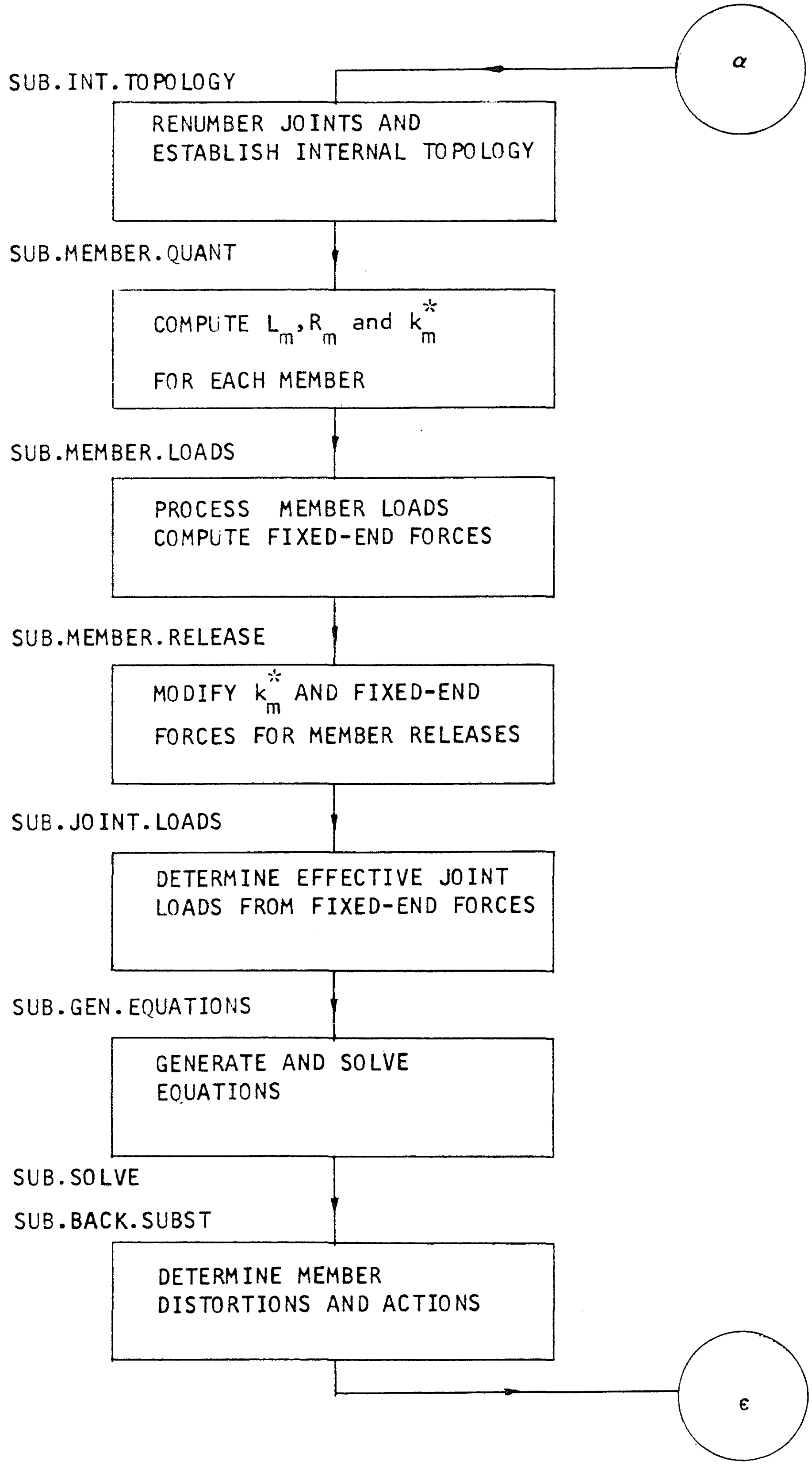
α
SUB.INT.TOPOLOGY
RENUMBER JOINTS AND ESTABLISH INTERNAL TOPOLOGY
SUB.MEMBER.QUANT
COMPUTE L_m, R_m and k^*_m FOR EACH MEMBER
SUB.MEMBER.LOADS
PROCESS MEMBER LOADS COMPUTE FIXED-END FORCES
SUB.MEMBER.RELEASE
MODIFY k^*_m AND FIXED-END FORCES FOR MEMBER RELEASES
SUB.JOINT.LOADS
DETERMINE EFFECTIVE JOINT LOADS FROM FIXED-END FORCES
SUB.GEN.EQUATIONS
GENERATE AND SOLVE EQUATIONS
SUB.SOLVE
SUB.BACK.SUBST
DETERMINE MEMBER DISTORTIONS AND ACTIONS
ε

Fig. 5.2b

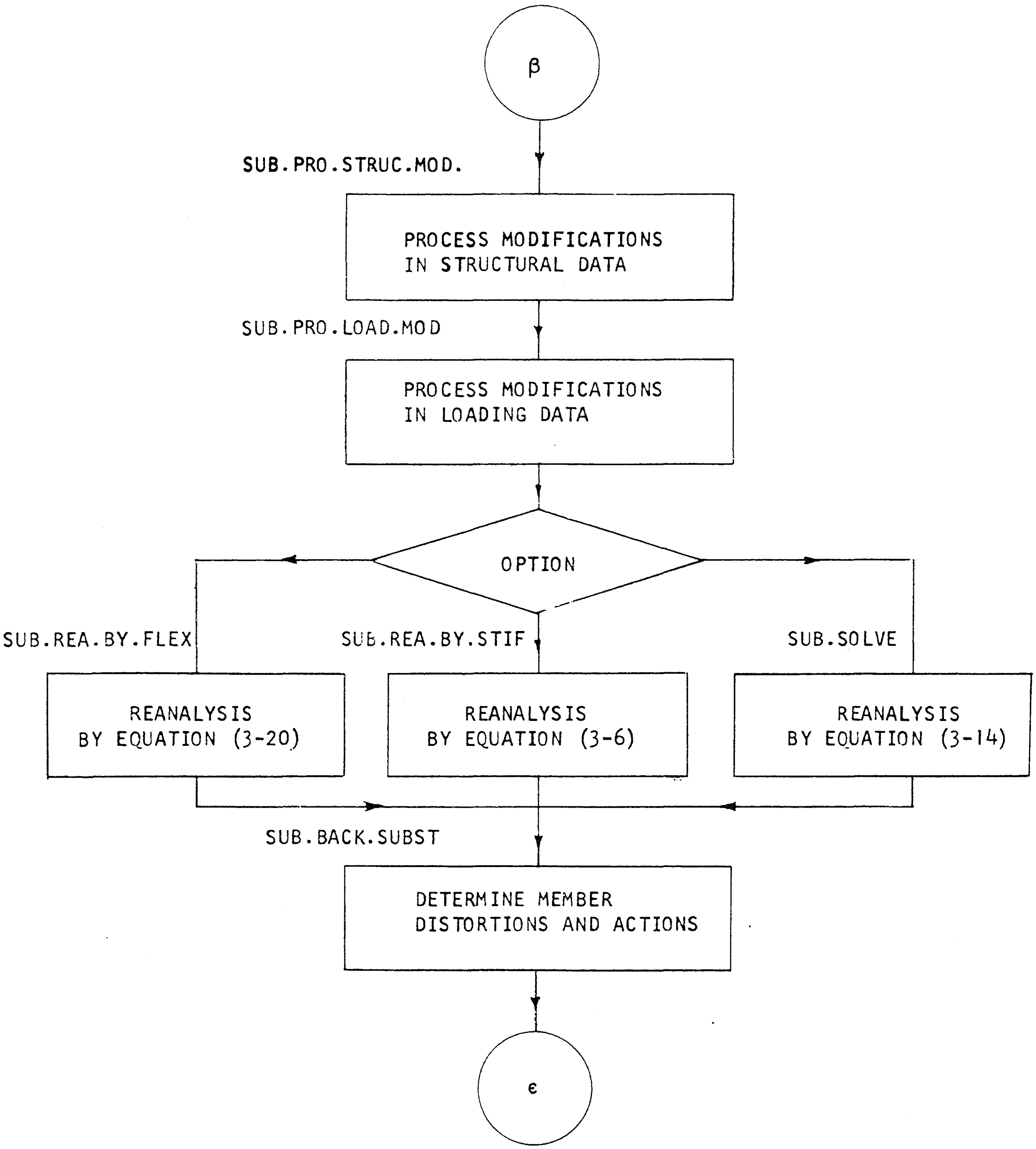
β
SUB.PRO.STRUC.MOD.
PROCESS MODIFICATIONS IN STRUCTURAL DATA
SUB.PRO.LOAD.MOD
PROCESS MODIFICATIONS IN LOADING DATA
OPTION
SUB.REA.BY.FLEX
SUB.REA.BY.STIF
SUB.SOLVE
REANALYSIS BY EQUATION (3-20)
REANALYSIS BY EQUATION (3-6)
REANALYSIS BY EQUATION (3-14)
SUB.BACK.SUBST
DETERMINE MEMBER DISTORTIONS AND ACTIONS
ε

Fig. 5.2c

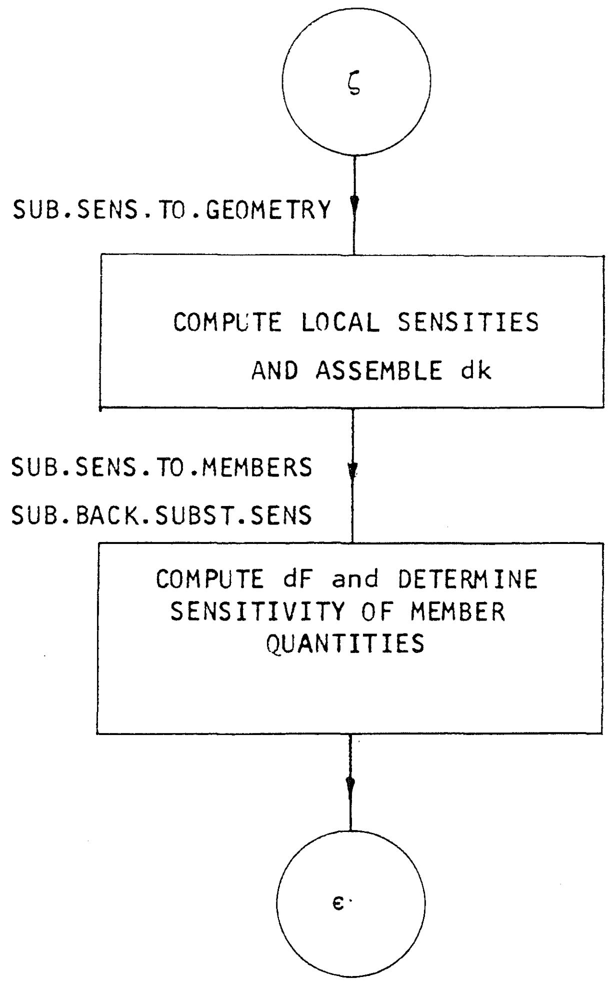
ζ
SUB.SENS.TO.GEOMETRY
COMPUTE LOCAL SENSITIES
AND ASSEMBLE dk
SUB.SENS.TO.MEMBERS
SUB.BACK.SUBST.SENS
COMPUTE dF and DETERMINE
SENSITIVITY OF MEMBER
QUANTITIES
ε

Fig. 5.2d

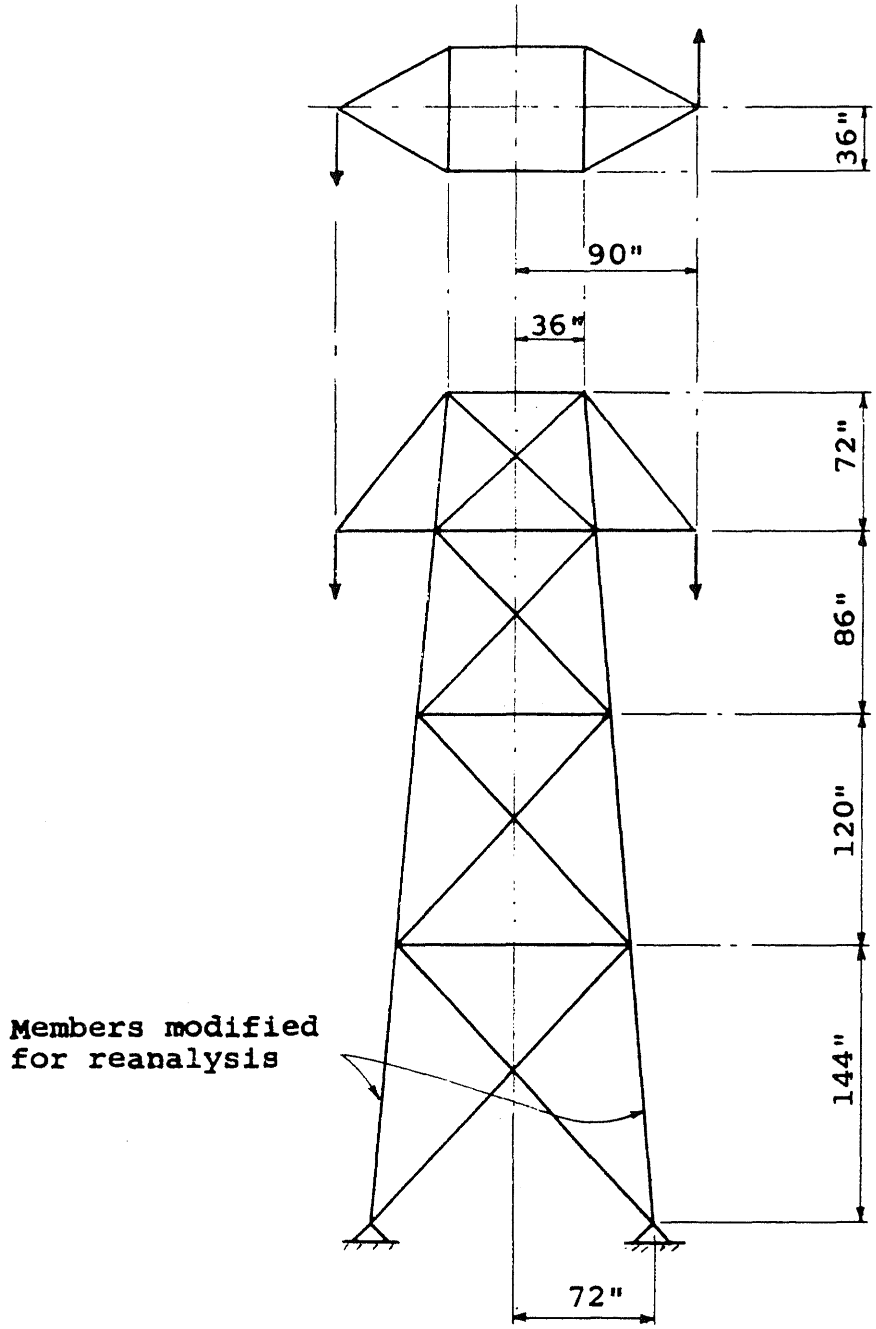

Fig. 6.1 - Transmission Tower

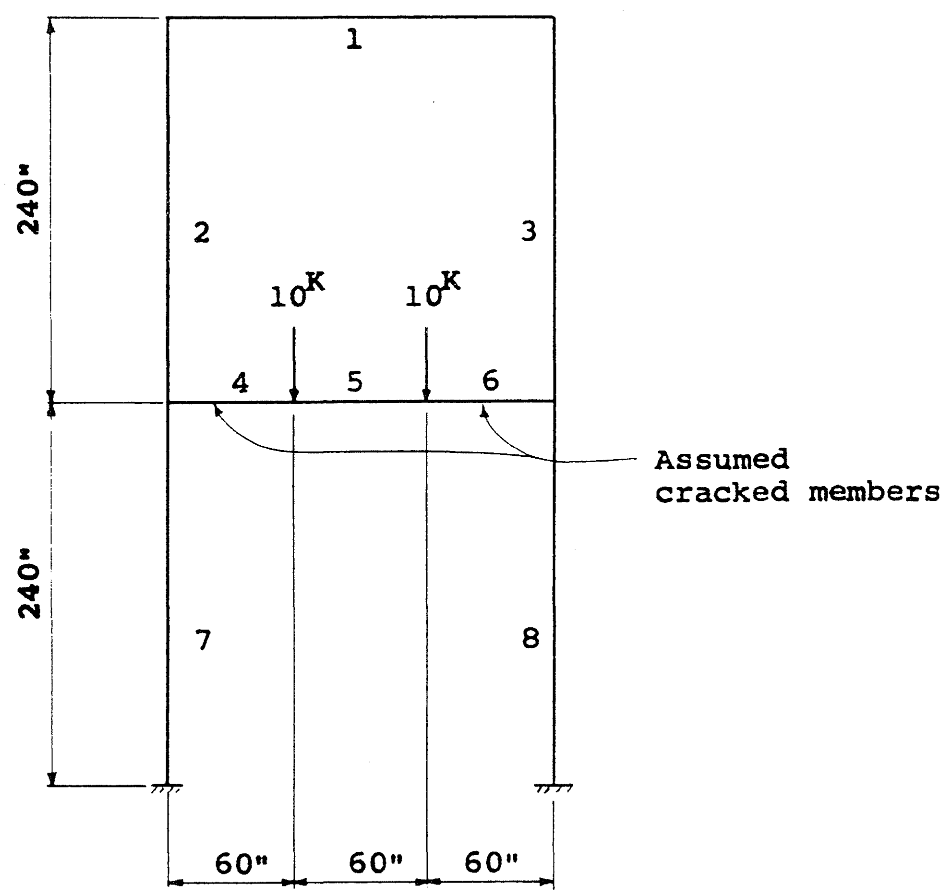

Fig. 6.2 - Concrete Frame

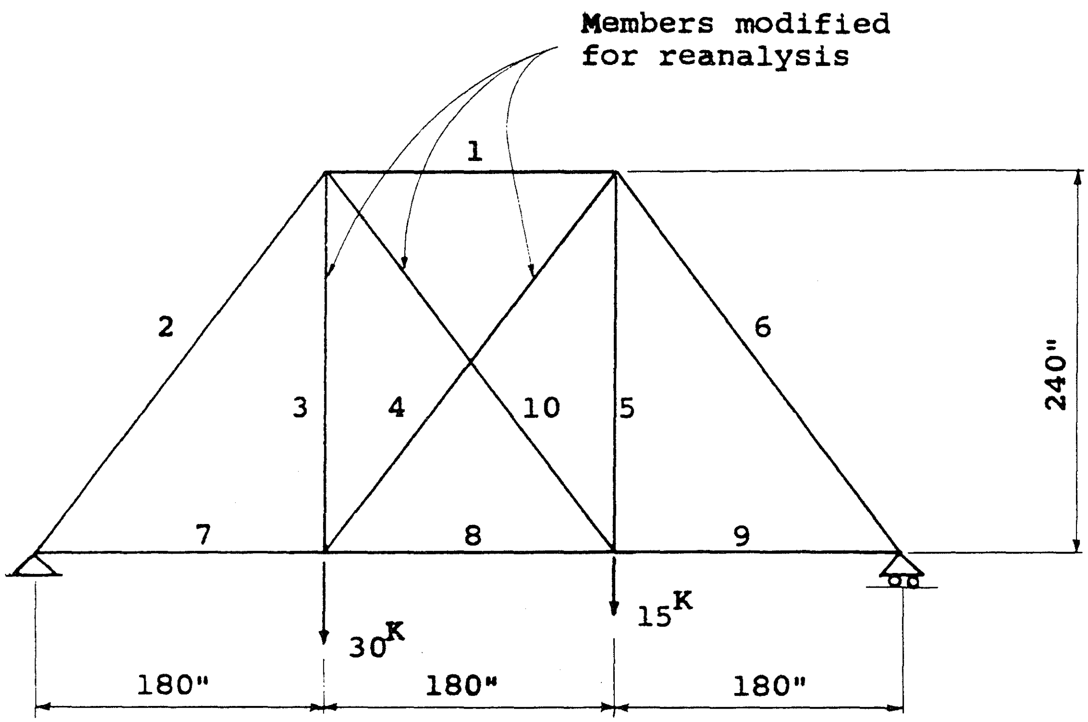

Fig. 6.3 - Plane Truss

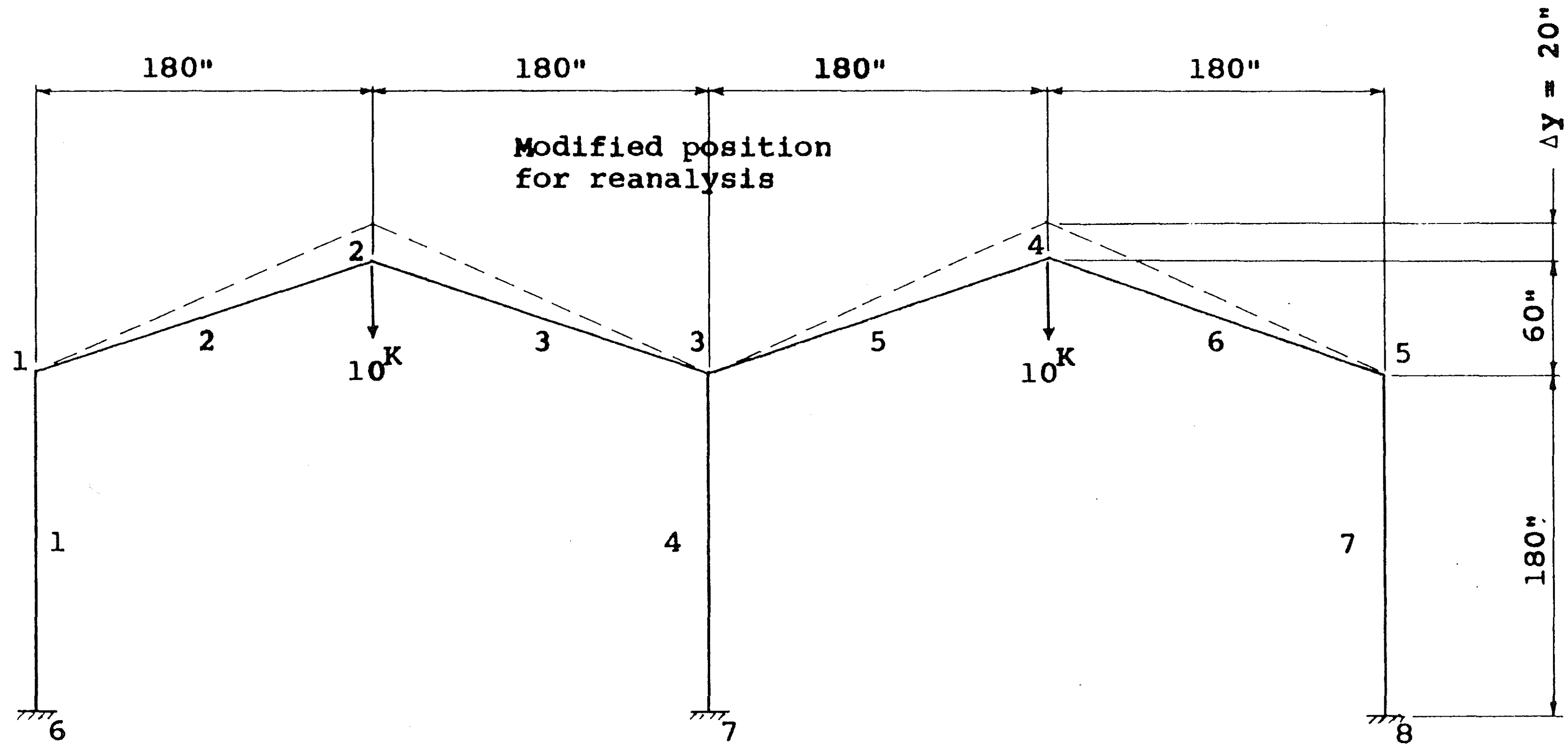

Fig. 6.4 - Gable Frame

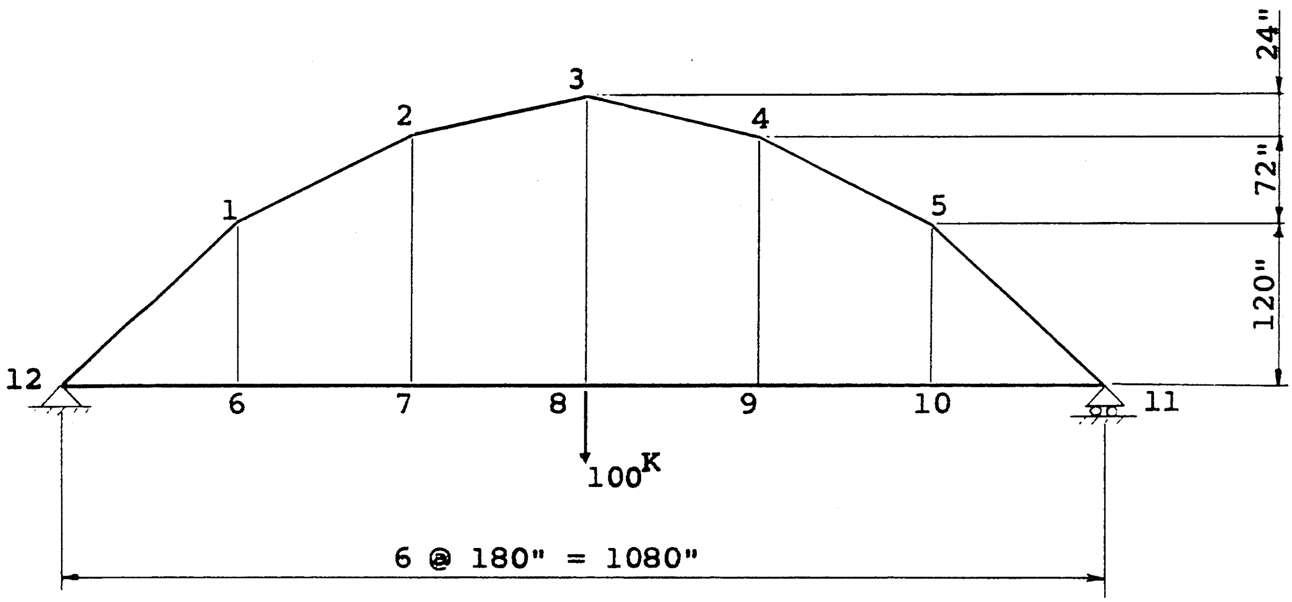

Fig. 6.5 - Tied Arch

Member	Terminal	Member Forces, P_m			Sensitivity Coefficients, $\frac{dP_m}{dI_{4,6}} \times 10^{-2}$			% Change in Member Forces Due to 50% Change in $I_{4,6}$		
		Axial Force	Shear	Moment	Axial Force	Shear	Moment	Axial Force	Shear	Moment
1	Start	6.95	0.00	317.8	-0.076	0.000	-3.161	-.54	.00	-.50
1	End	-6.95	0.00	-317.8	0.076	0.000	3.161	-.54	.00	-.50
2	Start	0.00	-6.95	-317.8	0.000	0.069	3.161	.00	-.50	-.50
2	End	0.00	6.95	-1350.0	-0.000	-0.069	13.436	.00	-.50	-.50
3	Start	0.00	6.95	317.8	-0.000	-0.069	-3.161	.00	-.50	-.50
3	End	0.00	-6.95	1350.0	0.000	0.069	-13.434	.00	-.50	-.50
4	Start	2.98	100.00	2940.0	-0.015	0.000	-29.230	-.25	.00	-.50
4	End	-2.98	-100.00	3059.0	0.015	-0.000	29.260	-.25	.00	.48
5	Start	2.98	0.00	-3059.0	-0.086	0.000	-29.240	-1.44	.00	.50
5	End	-2.98	0.00	3059.0	0.086	-0.000	29.260	-1.44	.00	.50
6	Start	2.98	-100.00	-3059.0	0.026	0.000	-29.260	-.44	.00	.48
6	End	-2.98	100.00	-2940.0	0.026	0.000	29.230	-.44	.00	-.50
7	Start	100.00	-9.93	-1589.0	0.000	0.099	15.800	.00	-.50	-.50
7	End	-100.00	9.93	-794.6	-0.000	-0.099	7.901	.00	-.50	-.50
8	Start	100.00	9.93	1583.0	0.000	-0.099	-15.800	.00	-.50	-.50
8	End	-100.00	-9.93	794.6	-0.000	0.099	-7.902	.00	-.50	-.50

TABLE 6.1 Results of Analysis for Concrete Frame

Member m	Initial Structure Member Forces P_m	SENSITIVITY COEFFICIENTS $\frac{dP_m}{dA_1}$	$\frac{dP_m}{dA_3}$	$\frac{dP_m}{dA_4}$	$\frac{dP_m}{dA_5}$	$\frac{dP_m}{dA_8}$	$\frac{dP_m}{dA_{10}}$	Modified Structure Predicted $\bar{P}_m$	Calculated $\bar{P}_m$
1	-21.87	-.171	.289	-.249	.150	.093	-.113	-19.27	-19.47
3	20.83	-.228	.386	-.332	.201	.124	-.150	24.31	24.03
4	11.46	.285	-.482	.414	-.251	-.155	.188	7.14	7.46
5	10.83	-.228	.386	-.332	.201	.124	-.150	14.31	14.03
8	11.87	-.171	.289	-.249	.150	.093	-.113	14.47	15.00
10	5.20	.285	-.482	.414	-.251	-.155	.188	.88	1.21

TABLE 6.2 Results of Analysis for Plane Truss

Member	Terminal	Initial Analysis Moment, M_m	Sensitivity of Moment, $\frac{dM_m}{dy_{2,4}}$	Reanalysis Moment, $\bar{M}_m$
1	Start	-289.2	-1.170	-307.2
1	End	-319.7	.348	-310.1
2	Start	319.7	-.348	310.1
2	End	355.6	-2.210	313.3
3	Start	-355.6	2.210	-313.3
3	End	-362.6	2.580	-313.8
4	Start	-0.0	.000	0.0
4	End	-0.1	.000	0.0
5	Start	362.7	-2.580	313.7
5	End	355.6	-2.210	313.3
6	Start	-355.6	2.210	-313.3
6	End	-319.5	.348	-310.2
7	Start	319.5	-.348	310.2
7	End	289.0	1.170	307.3

TABLE 6.3 Results of Analysis of the Gable Frame

Joint	Initial Structure Joint Displacement			Sensitivity Coefficients			Modified Structure Joint Displacement	
j	U_{jx}	U_{jy}	U_{jz}	$\frac{dU_{jy}}{dA_h}$	$\frac{dU_{jy}}{dI_a}$	$\frac{dU_{jy}}{dI_g}$	Predicted $\bar{U}_{jy}$	Calculated $\bar{\bar{U}}_{jy}$
1	-3.67	5.35	-.003	.020	-2.765	-3.011	3.97	4.23
2	-.94	-1.65	-.066	.040	.415	.506	-1.45	-1.48
3	.18	-10.54	.000	-.129	4.713	5.010	-8.18	-8.64
4	1.30	-1.65	.066	.045	.414	.505	-1.45	-1.48
5	4.03	5.35	.003	.020	-2.766	-3.013	3.97	4.24
6	.06	5.35	-.002	.076	-2.762	-3.014	3.97	4.23
7	.12	-1.66	-.066	.127	.420	.502	-1.46	-1.49
8	.18	-10.59	.000	.323	4.700	5.021	-8.24	-8.69
9	.24	-1.66	.066	.127	.418	5.002	-1.46	-1.48
10	.30	5.35	.002	.076	-2.763	-3.015	3.97	4.23
11	.36	.00	-.046	.000	.000	.000	.00	.00
12	.00	.00	.046	.000	.000	.000	.00	.00

TABLE 6.4 Results of Analysis for Tied Arch

APPENDIX

DERIVATION OF DIFFERENTIAL MATRICES

A.1 Derivation of Eq. (4-29)

The position of the member coordinate system (x, y, z) relative to the joint coordinate system (X, Y, Z) can be specified by means of the two angles α and ϕ, as indicated in Figure A.1. The third angle, β, specifying the rotation of the member about its own x-axis is assumed to be unchanged. The transformation from the member coordinate system to the joint coordinate system can be performed by rotating the member coordinate system (x, y, z) first around its z-axis by an angle α and then around its y-axis by an angle ϕ. Representing the first rotation by a matrix R_α and the second one by a matrix R_ϕ, the rotation matrix, R_m, can be expressed as:

$$R_m = R_\phi R_\alpha \tag{A.1-1}$$

where

$$R_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.1-2}$$

and

$$R_\phi = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \tag{A.1-3}$$

Differentiating Eq. (A.1-1) yields

$$dR_m = dR_\phi R_\alpha + R_\phi dR_\alpha \qquad \text{(A.1-4)}$$

Differentiating Eq. (A.1-2) yields

$$dR_\alpha = R_\alpha C_\alpha d\alpha \qquad \text{(A.1-5)}$$

where,

$$C_\alpha = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Differentiating Eq. (A.1-3) yields

$$dR_\phi = C_\phi R_\phi d\phi \qquad \text{(A.1-6)}$$

where,

$$C_\phi = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Substituting Eq. (A.1-5) and (A.1-6) into Eq. (A.1-4) results in:

$$dR_m = C_\phi R_m d\phi + R_m C_\alpha d\alpha \qquad \text{(A.1-7)}$$

The angles α and ϕ can be related to the end joint, e, and the start joint, s, coordinates by

$$\sin \alpha = \frac{\Delta y}{[(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2]^{1/2}} \qquad \text{(A.1-8)}$$

and,

$$\sin\phi = \frac{\Delta Z}{[(\Delta X)^2 + (\Delta Z)^2]^{1/2}} \tag{A.1-9}$$

where

$$\begin{aligned} \Delta X &= X_e - X_s \\ \Delta Y &= Y_e - Y_s \\ \Delta Z &= Z_e - Z_s \end{aligned} \tag{A.1-10}$$

The differential of the angles α and ϕ can be obtained from Eqs. (A.1-8) and (A.1-9) as:

$$d\alpha_\zeta = \eta_{\alpha\zeta} d\zeta \tag{A.1-11}$$

and

$$d\phi_\zeta = \eta_{\phi\zeta} d\zeta \tag{A.1-12}$$

where

$$\zeta = X,\ Y,\ Z$$

and

$$\eta_{\alpha X} = (-1)^t \frac{-(\Delta X)\ (\Delta Y)}{L_m^2\ [(\Delta X)^2 + (\Delta Y)^2]^{1/2}}$$

$$\eta_{\alpha Y} = (-1)^t \frac{[(\Delta X)^2 + (\Delta Z)^2]^{1/2}}{L_m^2}$$

$$\eta_{\alpha Z} = (-1)^t \frac{-(\Delta Y)\ (\Delta Z)}{L_m^2 [(\Delta X)^2 + (\Delta Z)^2]^{1/2}} \tag{A.1-13}$$

$$\eta_{\phi X} = (-1)^t \frac{-(\Delta Z)}{(\Delta X)^2 + (\Delta Z)^2}$$

$$\eta_{\phi Y} = 0.$$

$$\eta_{\phi Z} = (-1)^t \frac{\Delta X}{(\Delta X)^2 + (\Delta Y)^2}$$

In Eq. (A.1-13), if the infinitesimal disturbance is associated with the end joint:

$$t = 2$$

otherwise:

$$t = 1$$

The differential of the angles α and β with respect to a coordinate axis ζ of joint j can be expressed as:

$$d\alpha_{\zeta j} = (\eta_{\alpha\zeta})_j d\zeta_j \tag{A.1-14}$$

and,

$$d\phi_{\zeta j} = (\eta_{\phi\zeta})_j d\zeta_j \tag{A.1-15}$$

Substituting Eq. (A.1-14) and (A.1-15) into Eq. (A.1-7) yields

$$\frac{dR_m}{d\zeta_j} = (\eta_{\phi\zeta})_j C_\phi R_m + (\eta_{\alpha\zeta})_j R_m C_\alpha \tag{4-29}$$

A.2 Derivation of Equation (4-30)

In the member coordinate system, the member terminal displacements are related to the member terminal forces by

$$M_k = \begin{bmatrix} T_m k_m^* T_m^t & -T_m k_m^* \\ -k_m^* T_m^t & k_m^* \end{bmatrix} \qquad (A.2\text{-}1)$$

where, for a prismatic member having six force and displacement components,

$$k_m^* = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ & \frac{12EI_z}{L^2} & 0 & 0 & 0 & \frac{-6EI_z}{L^2} \\ & & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ & \text{Symmetric} & & \frac{GJ}{L} & 0 & 0 \\ & & & & \frac{4EI_y}{L} & 0 \\ & & & & & \frac{4EI_z}{L} \end{bmatrix}$$

and

$$T_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -L & 0 & 1 & 0 \\ 0 & L & 0 & 0 & 0 & 1 \end{bmatrix}$$

For a member, m, connecting the joints i and j the derivative of the length L_m with respect to ζ_j is given by

$$\frac{dL_m}{d\zeta_j} = \frac{\Delta\zeta_j}{L_m} \tag{A.2-2}$$

where

$$\zeta_j = X_j,\ Y_j,\ Z_j$$

and

$$\Delta\zeta_j = \zeta_j - \zeta_i$$

Using Eq. (A.2-2), the derivatives of the matrices k_m^* and T_m can be expressed as:

$$\frac{dk_m^*}{d\zeta_j} = -\left(\frac{\Delta\zeta_j}{L_m^2}\right)\begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{36EI_z}{L^3} & 0 & 0 & 0 & \frac{12EI_z}{L^2} \\ & & \frac{36EI_y}{L^3} & 0 & \frac{12EI_y}{L^2} & 0 \\ & & & \frac{GJ}{L} & 0 & 0 \\ & \text{Symmetric} & & & \frac{4EI_y}{L} & 0 \\ & & & & & \frac{4EI_z}{L} \end{bmatrix}$$

(A.2-3)

and

$$\frac{dT_m}{d\zeta_j} = \frac{\Delta\zeta_j}{L_m^2}\,[T_m - I_m] \tag{A.2-4}$$

where I_m is a unit matrix of appropriate order.

Using Eq. (A.2-2), (A.2-3) and (A.2-4), the derivative of the Eq. (A.2-1) can be obtained as:

$$\frac{dM_k}{d\zeta_j} = \frac{\Delta\zeta_j}{L_m^2}\,\{M_1 + M_1^t + M_2\} \tag{A.2-5}$$

where

$$M_1 = \begin{bmatrix} I_m & I_m \\ 0 & 0 \end{bmatrix} M_k$$

and

$$M_2 = \left(\frac{L_m^2}{\Delta\zeta_j}\right) \begin{bmatrix} T_m \dfrac{dk_m^*}{d\zeta_j} T_m^t & -T_m \dfrac{dk_m^*}{d\zeta_j} \\ -\dfrac{dk_m^*}{d\zeta_j} T_m^t & \dfrac{dk_m^*}{d\zeta_j} \end{bmatrix}$$

Letting

$$Z = M_1 + M_1^t + M_2$$

Eq. (4-30) is obtained as

$$\frac{dM_k}{d\zeta_j} = \frac{\Delta\zeta_j}{L_m^2}\,Z \tag{4.30}$$

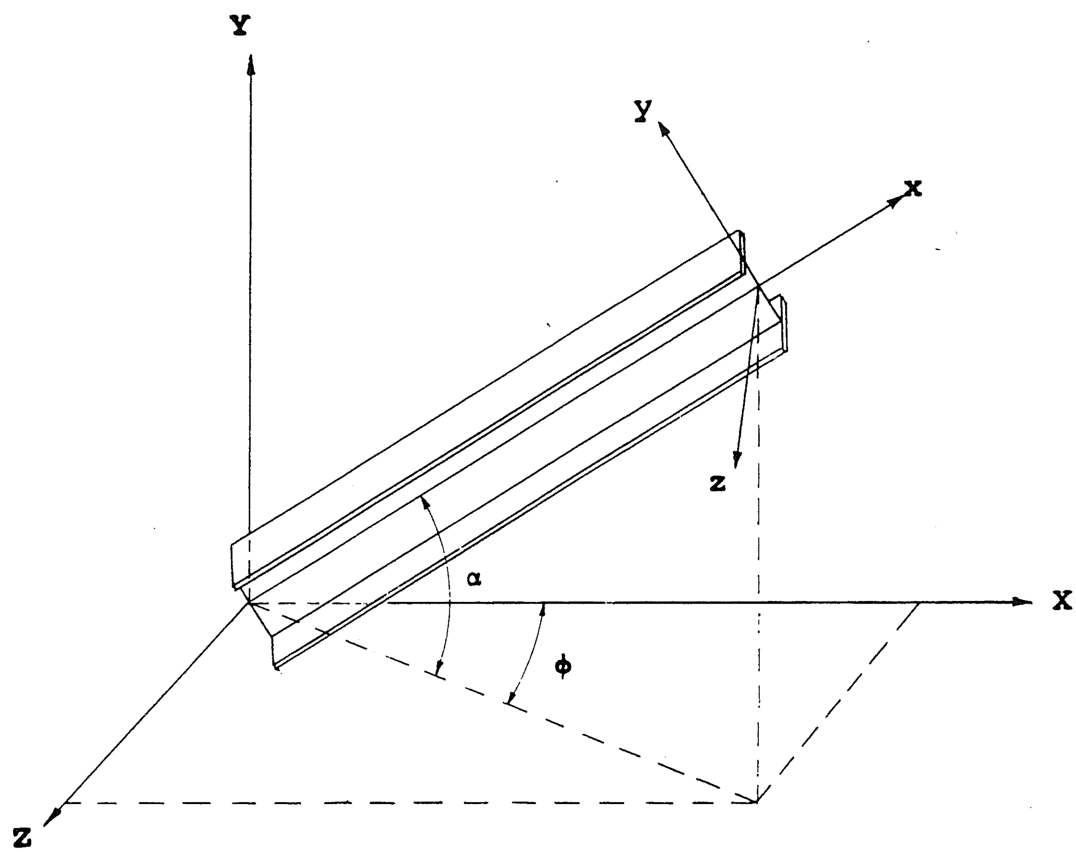

Fig. A.1 - Coordinate Systems

LIST OF REFERENCES

1. Levy, S., "Computation of Influence Coefficients for Aircraft Structures with Discontinuities and Sweepback," J. Aeronaut. Sci., Vol. 14, No. 10, pp. 547-560, October, 1947.

2. Lang, A. L., and Bisplinghoff, R. L., "Some Results of Sweptback Wing Structural Studies," J. Aeronaut. Sci., Vol. 18, No. 11, pp. 705-717, November, 1951.

3. Langefors, B., "Analysis of Elastic Structures by Matrix Transformation with Special Regard to Semimonocque Structures," J. Aeronaut. Sci., Vol. 19, No. 7, pp. 451-458, July, 1952.

4. Wehle, L. B. and Lansing, W., "A Method for Reducing the Analysis of Complex Redundant Structures to a Routine Procedure," J. Aeronaut. Sci., Vol. 19, No. 10, pp. 677-684, October, 1952.

5. Levy, S., "Structural Analysis and Influence Coefficients for Delta Wings," J. Aeronaut. Sci., Vol. 20, No. 7, pp. 449-454, July, 1953.

6. Turner, M. J., Clough, R. W., Martin, H. C. and Topp, L. J., "Stiffness and Deflection Analysis of Complex Structures," J. Aeronaut. Sci., Vol. 23, No. 9, pp. 805-824, September, 1956.

7. Argyris, J. H. and Kelsey, S., Energy Theorems and Structural Analysis, Butterworth Scientific Publications, London, 1960.

8. Hall, A. S. and Woodhead, R. W., Frame Analysis, John Wiley and Sons, Inc., New York, 1961.

9. Livesley, R. K., Matrix Methods of Structural Analysis, Pergamon Press, New York, 1964.

10. Martin, H. C., Introduction to Matrix Methods of Structural Analysis, McGraw-Hill Book Co., Inc., New York, 1966.

11. Faddeeva, V. N., Computational Methods of Linear Algebra, translated by C. D. Benster, Dover Publications, Inc., New York, 1959.

12. Spillers, W. R., "Techniques for Analysis of Large Structures," J. of Structural Division, ASCE, Vol. 94, No. ST11, November, 1968, pp. 2521-2534.

13. Kron, Gabriel, Diakoptics, McDonald, London, 1963.

14. Householder, A. S., "A Survey of Some Closed Methods for Inverting Matrices," J. Society for Industrial and Applied Mathematics, Vol. 5, No. 3, September, 1957.

15. Branin, F. H., Jr., "The Relation Between Kron's Method and the Classical Method of Network Analysis," The Matrix and Tensor Quaterly, Vol. 12, March, 1962.

16. Fenves, S. J. and Branin, F. H., Jr., "Network-Topological Formulation of Structural Analysis," J. of Structural Division, ASCE, Vol. 89, No. ST4, August, 1963, pp. 483-514.

17. Reinschmidt, K. F., Cornell, C. A., and Brotchie, J. F., "Iterative Design and Structural Optimization," J. of Structural Division, ASCE, Vol. 92, No. ST6, December, 1968, pp. 281-318.

18. Tomovic, R., Sensitivity Analysis of Dynamic Systems, McGraw-Hill Book Co., Inc., New York, 1963.

19. Radanovic, L., Sensitivity Methods in Control Theory, Pergamon Press, 1966.

20. Fenves, S. J., Logcher, R. D., Mauch, S. P. and Reinschmidt, K. F., "STRESS: A User's Manual," MIT Press, Cambridge, Mass., 1964.

21. University of Illinois, Department of Civil Engineering, "STRESS: A User's Manual," Urbana, Illinois, 1968.

22. Logcher, R. D. and Sturman, G. M., "STRUDL--A Computer System for Structural Design," J. of Structural Division, ASCE, Vol. 92, No. ST6, December, 1966, pp. 191-221.

23. Morice, P. B., Linear Structural Analysis, The Ronald Press Company, New York, 1959.

24. Melin, J. W., "POST--Problem Oriented Subroutine Translator," J. of the Structural Division, ASCE, Vol. 92, No. ST6, December, 1966.

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*
University of Illinois, Urbana, Illinois
Department of Civil Engineering

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE

AUTOMATIC ANALYZER FOR ITERATIVE DESIGN

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Progress: June 1968, August 1969

5. AUTHOR(S) *(First name, middle initial, last name)*
Ertas, Rebii
Fenves, Steven J.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August, 1969	147	35

8a. CONTRACT OR GRANT NO.
N000 14-67-A-0305-0010

b. PROJECT NO.
Navy 4-0305-0010

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
Structural Research Series
No. SRS 349

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

Qualified requesters may obtain copies of this report from DDC.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
Structural Mechanics Branch, Office of Naval Research, Department of the Navy

13. ABSTRACT

An automatic analyzer with the capability of performing initial analysis reanalysis and sensitivity analysis for linear elastic skeletal structures is formulated. For initial analysis the stiffness method of analysis is used. Partial reanalysis techniques developed on the basis of the flexibility and stiffness methods are presented. Quantitative data pertaining to the rate of change of the response quantities of the structure due to modifications of various design parameters are provided by sensitivity analysis. The sensitivity functions are obtained as functional derivatives of the expressions derived for the stiffness formulation. The parameters of interest which can be included in sensitivity analysis are discussed.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Structural Analysis Structural Design Sensitivity Analysis Computer Programming						