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A METHOD FOR THE COMBINATION OF STOCHASTIC TIME VARYING LOAD EFFECTS

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CHAPTER 1

INTRODUCTION

1.1 Load Combination

Structural reliability theory has arisen from the need to account for the inherent variability of natural as well as man-made loads, together with the uncertainties in the strengths of structural members. The variability in natural loads may be seen, for example, in the varied intensity and duration of each storm or the magnitude of each earthquake at a given location. Live load in a building changes with a change in occupancy and extreme live loads may occur where large numbers of people are gathered during any one occupancy.

An important statistic for the designer to know is the maximum of a load during the prescribed design life. The distribution of the maximum for each load type can be obtained or extrapolated from observations of that load over a number of years. However, a description of a load in terms of the distribution of the maximum does not take account of how the load varies as a function of time. A more complete description of a load could be given in terms of a stochastic process rather than a random variable. This provides some information of how the load varies in time and is a realistic model of the physical processes.

The problem of "load combination" occurs when a number of different loads, all of which are time varying, act on a structure during the same time period. Design of a structure to withstand all the loads requires the estimation of the maximum of the combined load process. When the loads are modeled as stochastic processes the solution is desired in terms of the probability distribution of the maximum.

If the individual maxima were added, an extremely conservative estimate of the maximum load would result because it is highly unlikely that the maxima of all the processes occur simultaneously. Calculating the distribution of the maximum of the combination of the stochastic load (effect) processes is, however, complicated, particularly for the types of processes needed to model realistic loads.

Engineers in the design office generally rely on a code of practice when designing civil structures under the combined action of loads. In the past, probabilistic concepts were only used directly in codes to specify characteristic loads for use in the analysis, the value of the load being one which has only a small chance of being exceeded. Safety factors incorporated in the allowable stress were used to provide the margin of safety to ensure that the structure did not fail under the wide range of loads. For cases where the design required a combination of loads, a simple summation of the characteristic loads would result in a very conservative design. The code writing committees therefore provided simple rules (equations) to be followed in order to decrease the combined maximum. These safety factors and combination rules were based primarily on judgement. In a sense, the personalist concept (Bunge, 1982) of probability was indirectly being used as a basis for the safety factors in the codes. That is, the safety factors were based largely on the beliefs (subjective as they may be, although attained through much experience) of the members of the committee.

New codes are being proposed (e.g., Ellingwood, Galambos, MacGregor and Cornell, 1980), using the concept of limit states design, which have rational bases for obtaining load and resistance factors through the use of the mathematical theory of probability. Of course, much data is needed to compute the probability distributions of the random variables and due to a scarcity of some data it is sometimes necessary to use good judgement in making certain assumptions. As more data becomes available the distributions may be updated. The load and resistance factors will reflect more accurately the relative uncertainty associated with each of the variables.

Although buildings have generally behaved very well in the past (designed using allowable stress), new materials are becoming available and new facilities are being built with which we have little experience. Knowledge gained from present studies on probability based design and used in a rational way may help us to build, with confidence, these innovative structures, as well as economizing by improving the design in those situations where safety has been provided by ultra-conservatism.

A word of caution seems to be appropriate here. As in other complex techniques in the engineering discipline, the use of probability theory and the interpretation of the results should be accomplished with insight and as much understanding of the problem as possible. We should not attempt to get out of the analysis more than the input information and model allow us.

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The foregoing discussion briefly outlined the direction of the development of probabilistic methods as applied in structural codes. The problem of how to combine several load effects which vary with time is approached in an approximate and simplified manner in codes thus far. This load combination problem becomes more important when designing critical facilities which may be subjected to many time varying load processes.

The need therefore exists for the development of a general method which will allow the computation of the probability of failure of a structure over a given time period when a number of stochastic load processes are expected to occur. Since safety and damage may be of primary concern for many structures, the method should to be able to handle nonlinearities in the limit function, responses which may result from vibration of the structure, and the effects of correlation between load effects. It should be simple and flexible enough to allow the incorporation of new developments in reliability theory without too much difficulty.

1.3 Objectives of the Study

The objectives of this study are therefore to develop a method for the evaluation of the failure probability of structures undergoing the combined action of a number of stochastic load processes. Specifically, the Load Coincidence Method (Wen, 1977,1980a,1980b) will be extended for the general problem of load combination including vector processes crossing out of nonlinear safe domains, dynamic load combinations, and correlated effects.

The rules often suggested for combining stochastic loads for use in developing structural codes or in the design of nuclear power plants are compared with the results obtained from the load coincidence method to evaluate the type of error being introduced when using these rules for different risk levels.

A further objective is to give an appraisal and suggestions for improvements of present practice in structural codes with regard to reliability levels and internal consistency. Also the risk implications of the load and resistance factor format will be studied.

1.4 Organization

Chapter 2 summarizes the methods of modeling of static loads or load effects as random pulse processes and gives a review of results for the linear combination of stochastic load processes. The Load Coincidence method is briefly formulated.

Chapter 3 considers the problem of nonlinear combination, i.e., the crossing of vector pulse processes out of nonlinear safe domains. As in the case of time-invariant variables, an approximate method is to linearize the failure surface at a suitably chosen point. Exact crossing rates out of the linearized domain may be calculated. However, computational effort may be excessive. The load coincidence method on the other hand, is shown to be suitably versatile to handle efficiently nonlinear combinations for various limit states and load types.

CHAPTER 2

MODELING AND LINEAR COMBINATION OF STATIC LOAD EFFECT PROCESSES

2.1 The Poisson Pulse Process

The seemingly random occurrence of natural phenomena likely to cause stresses in a structure suggests that arrival times of these loads be points in a random point process. Beginning with the modeling of floor live loads (Peir and Cornell, 1973) most reported load models for loads which take on different magnitudes at different times (time varying) assume the form of a filtered Poisson process. The Poisson pulse process is a special form of this process and an efficient model for static load effects in combination studies. It is a convenient process for modeling a variety of loads which have independent arrival times within one process. The occurrence times of the loads are given by the points of a Poisson process having a mean rate of arrival of μ_d^{-1}, where μ_d is the mean duration of a load.

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Chapter 4 develops the method for combinations of processes consisting of dynamic structural responses and compares results with the "crossing rate" methods together with approximate combination rules used in structural codes.

Correlated dynamic effects are examined in Chapter 5. Correlation may exist between load events within one process or between two processes. Both these situations may be considered within the general framework of the load coincidence method.

Chapter 6 comments on present code formats for load combination and important considerations in developing new probability-based codes. An optimized design technique with reliability constraints is suggested, using the load coincidence method, for developing optimized codes and as a design decision tool.

1.5 Notation

d	duration of response
$f_X(x)$	probability density function
$F_X(x)$	probability distribution function
$g(x)$	limit state (performance) function
$L(t)$	reliability in (0,t)
N^+	number of crossings
p_{12}	conditional failure probability given coincidence
P[ ]	probability of [ ]
q	probability that a load is "on"
Q	varying loads in the ANSI code

r	threshold level
S_o	spectral density of white noise excitation
t	time
Y	random mean dynamic response
Z	total dynamic response given occurrence
α	load coincidence crossing rate
β	reliability index
γ	load factors
ζ	percent critical damping
λ	mean arrival rate of loads in process i
μ	mean load duration
ν	crossing rate
θ	lag time
ρ	correlation coefficient
σ	standard deviation
ϕ	standard normal density
Φ	standard normal distribution
ω	frequency of structure

CHAPTER 2

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The load pulses occur between two renewal points and may assume a number of shapes depending on the type of load being modeled. The most widely used shape is the rectangular pulse, but triangles, house shapes and sine waves have also been suggested. (Madsen, Kilcup and Cornell, 1979; Madsen, 1979). Successive loads have magnitudes (pulse heights) which are independent and identically distributed random variables. The pulse magnitude may, however, have a finite probability of being zero. The magnitude, in this case, has a mixed probability density function, with a discrete mass at zero and a continuous density for the other values of the variate.

The density and distribution functions for a rectangular pulse process are

$$\begin{aligned} f_S(s) &= (1-q)\ \delta(s) + qf_X(s) \\ F_S(s) &= (1-q)\ H(s) + qF_X(s) \end{aligned} \tag{2.1}$$

in which $\delta(s)$ = Dirac delta function, H(s) = step function, f_X and F_X are the conditional density and distribution functions given the magnitude is not zero. The real pulses (those with non-zero magnitude) have a mean occurrence rate given by $\lambda = q\,\mu_d^{-1}$ and the duration of the load pulse has an exponential distribution. Consecutive "on" or "off" times are possible with this model, which is completely characterized by the mean arrival rate λ of the pulses, the mean duration μ_d and the conditional density function $f_X(x)$. To include effects of load correlation the independence assumptions related to the occurrence time, intensity, and duration may be relaxed. This has been the subject of research in the reference by Wen and Pearce (1981), and is also considered in Chapter 5 of this study.

Some loads, such as live loads in buildings, are always "on". For such cases q = 1 and $\lambda = \mu_d^{-1}$ which means that $\lambda \mu_d = 1$. As the product $\lambda \mu_d$ becomes smaller the process becomes more sparse, i.e., the loads are infrequent or of short duration or both. If μ_d tends to zero while λ remains finite, the pulses become "spikes" and the result is a Poisson shock process (compound Poisson process). Sample functions of the Poisson rectangular pulse process are shown in Fig. 2.1. Derivations of the distribution functions of the magnitude at any time for other pulse shapes may be found in the references by Madsen, Kilcup and Cornell (1979) and Madsen (1979).

Other models which have been proposed result in similar sample functions to those above, but lack a certain flexibility. Ferry-Borges and Castanheta (1971) proposed a model in which each load history is described as a sequence of rectangular pulses of fixed duration. The pulse amplitudes are again independent and identically distributed. The durations of such are required to be either a multiple or a factor of one another, as shown in Fig. 2.2.

Madsen and Ditlevsen (1981) proposed an "on-off" Markov rectangular pulse process. It consists of independent exponentially distributed periods with constant random load, alternating with periods without load. The "off" periods are also exponentially distributed with the mean not necessarily the same as the "on" periods. The computation of the crossing rate of sums of these Markov processes is given in the above reference and is seen to be analytically complex.

2.2 Linear Combination of Load Effects Modeled as Poisson Pulse Processes

Prior research has been concerned mainly with the linear combination of load processes. Some results of this research will be reviewed briefly here, while extensions to nonlinear limit states and dynamic loads will be examined in more detail later in the text.

The distribution of the load magnitude is denoted $F_X(x_i)$ for the ith load process (see Eq. 2.1). The pulse process has a mean occurrence rate λ_i and the mean duration of a load pulse will be μ_i. For a Poisson square wave $\lambda_i \mu_i=1$, whereas for a sparse process the magnitude of the probability mass at zero will be $1 - \lambda_i \mu_i$. $F_{R_m}(r,t)$ is the distribution of the maximum of the combined processes during the interval 0,t.

2.2.1 Exact Solutions

Few exact solutions for the combination of stochastic load processes exist and these are limited to the linear combination of the simplest load types. As described briefly in Section 1.1, the maximum combined load usually determines the design load and therefore is the one of interest to the engineer or the code writing committee. In probabilistic terms, "solution" here means the computation of the distribution of the maximum of the combined process.

The study of the time dependent nature of floor live loads (Peir and Cornell, 1973) suggested they be modeled as a superposition of a shock process and a Poisson square wave (PSW) process. Hasofer (1974) obtained the distribution of the maximum of the sum of these processes in the form

$$F_{R_m}(r,T) = e^{-\lambda_2 T}\,\gamma(r,T) \tag{2.2}$$

$\gamma(r,T)$ is obtained as the solution to the Volterra integral equation

$$F(r,t) + \lambda_2 \int_0^t F(r,t-u)\,\gamma(r,u)\,du = \gamma(r,t) \tag{2.3}$$

where $F(r,t)$ is the distribution of the maximum during one pulse of the PSW process

$$F(r,\tau) = \int_0^r f_{X_2}(r-y)\,\exp\{-\lambda_1\tau[1 - F_{X_1}(y)]\}\,dy \tag{2.4}$$

The subscript 1 denotes the shock process and 2 the PSW process.

Bosshard (1975) used a Markov process model to compute the maximum of the sum of two PSW processes. The result is given in the form of an infinite sum which requires significant numerical computation.

Gaver and Jacobs (1981) make use of the Laplace transform method to get the transform of the maximum of the sum of a shock and PSW process.

The Laplace transform of the maximum distribution is given by

$$h_X(\xi) = \frac{M_X(\xi)}{1 - \lambda_2 M_X(\xi)} \tag{2.5}$$

where

$$M_x(\xi) = \int_0^r [\xi + \lambda_2 + \lambda_1 \{1 - F_{X_1}(r-y)\}]^{-1} F_{X_2}(dy) \tag{2.6}$$

Mean times to first passage are obtained as $E[T_r] = h_x(0)$. Results are given for specific distributions of the load magnitudes.

George (1981) uses the same technique to give results for shock and PSW processes whose occurrences are dependent on those of the other process.

The difficulties associated with computation of these exact solutions, together with their limitations as models for real loads makes them less attractive in analysis or design of real structures under combined loads. They do serve as good means of making comparisons for approximate solutions.

2.2.2 Approximate Methods

The maximum of the sum of N stochastic pulse processes wherein coincidence between processes may be neglected (e.g., shock processes or very sparse pulse processes) has the known distribution

$$F_{R_m}(r,t) = \exp - \left[\sum_{i=1}^{N} \lambda_i t\{1 - F_{X_i}(r)\} \right] \prod_{j=1}^{N} F_{X_j}(r) \tag{2.7}$$

Wen (1977) extended this result as an approximate method for solution of the maximum of the sum of pulse processes by considering the simultaneous occurrence of 2 or more loads. The result for two processes is

$$F_{R_m}(r,t) \simeq \exp\left[-t\{\lambda_1[1-F_{X_1}(r)]+\lambda_2[1-F_{X_2}(r)]+\lambda_{12}[1-F_{X_{12}}(r)]\}\right] \quad (2.8)$$

where λ_{12} is the mean rate of coincident loads and $F_{X_{12}}(r)$ is the distribution of the sum of the two random variables which are the magnitude of each process. It is the extension of this formulation to nonlinear limit states and dynamic load effects and its applications that are examined in this study.

Ferry-Borges and Castanheta (1971) modeled loads which vary with time by a sequence of independent and identically distributed pulses each of the same duration. Figure 2.2 shows a number of these sequences in the form required for their combination. That is, the ratios between pulse durations in successive processes are integer numbers. For combination, the processes are ordered according to decreasing pulse duration. Computation of the distribution of the maximum then requires the evaluation of the maximum of the $i+1^{th}$ process during one pulse of the i^{th} process, alternating with the convolution of the maximum of the $i+1^{th}$ process and the arbitrary point in time distribution of the i^{th} process, from i = n to 1. Breitung and Rackwitz (1978) developed an efficient algorithm which essentially performs an approximate ("fast"- see Lind, 1980) convolution to compute the distribution of the maximum of the sum of a number of these random load sequences. The algorithm is widely used in reliability calculations involving only random variables (see Appendix B).

Breitung and Rackwitz (1979) used the method of Laplace transform to obtain the transform of the upcrossing rate of the sums of renewal pulse processes with gamma distributed magnitudes, and filtered Poisson processes. The crossing rate is obtained by numerically inverting the Laplace transform using a suitable algorithm, and used to obtain an approximation to the extreme distribution of the sum process.

In keeping with second moment methods (Cornell, 1969; Ditlevsen, 1982) of reliability analysis, Der Kiureghian (1978,1980) proposed the approximate calculation of the first two moments of the extreme of the combined process from the point-in-time moments of the individual loads together with the mean arrival rate and duration of the pulses. The moments of the maximum R_m take the form

$$\begin{aligned} E[R_m] &= \sum_i E[X_i] + p[\sum_i \sigma^2_{X_i}]^{1/2} \\ \sigma^2_{R_m} &= q \sum_i \sigma^2_{X_i} \end{aligned} \tag{2.9}$$

where p and q are functions characterizing the stochastic fluctuations of the process.

Larrabee and Cornell (1979,1981) developed a method which calculates the crossing rate of the combined process from the crossing rates and arbitrary-point-in-time distributions of the individual processes. This is achieved through a number of convolutions

$$\nu_R(r) = \int_{-\infty}^{\infty} f_{X_2}(y)\ \nu_{X_1}(r-y)dy + \int_{-\infty}^{\infty} f_{X_1}(y)\ \nu_{X_2}(r-y)dy \tag{2.10}$$

The result is used in a general upper bound to the probability that a stochastic process exceeds a level r in the interval (0,T).

2.3 The Method of Load Coincidence

Since the method of load coincidence will be the basis for the analysis in this study, some details of this method are given in the following.

Consider the linear sum of two sparse Poisson pulse processes shown in Fig. 2.3. At times the sum process will consist only of a pulse from one or other of the individual processes, i.e., when one process is "on" and the other is "off". However, at some point in time both individual processes will be "on" and coincidence of two pulses will occur, the result being the sum of the pulse heights. Thus, the superposition of two processes, S_1 and S_2, consists of three components: the two individual processes without their coincident pulses and the third process consisting only of coincident events.

Let the event that each individual process does not exceed some level r in (0,t) be denoted E_1 and E_2 respectively and the event that the coincident process does not exceed r in (0,t) be E_{12}. The probability of E_1 occurring is the probability that the maximum of process 1 in (0,t) does not exceed r. This probability is derived as follows; let R = max{S(t) in 0,t}, N = number of renewals in (0,t).

$$P(R \leq r) = \sum_{k=0}^{\infty} P(R \leq r | N = k) \, P(N = k)$$

$$= \sum_{k=0}^{\infty} [F_S(r)]^{k+1} \frac{(\frac{t}{\mu_d})^k}{k!} e^{-t/\mu_d} \tag{2.11}$$

$$= F_S(r) \exp\{- \frac{t}{\mu_d} [1 - F_S(r)]\}$$

Substituting Eq. 2.1 into Eq. 2.11 the result is given in terms of the conditional distribution $F_X(r)$

$$\begin{aligned} F_R(r) &= [(1 - \lambda\mu_d) H(r) + \lambda\mu_d F_X(r)] \exp\{- \lambda t[1 - F_X(r)]\} \\ &\simeq \exp\{- \lambda t[1 - F_X(r)]\} \end{aligned} \tag{2.12}$$

(for large r)

The probability of the sum process not exceeding r in (0,t) is given by the probability of the intersection of the events E_1, E_2 and E_{12}. The events E_1 and E_2 are independent since the processes S_1 and S_2 are independent. E_{12} is positively correlated to E_1 and E_2 but an assumption of independence will be a conservative estimation of the probability of the intersection.

P(maximum of the sum does not exceed r in 0,t)

$$\begin{aligned} &= P[E_1 \cap E_2 \cap E_{12}] \\ &\simeq P[E_1]\, P[E_2]\, P[E_{12}] \\ &= \exp\left[- t\{\lambda_1[1-F_{X_1}(r)] + \lambda_2[1-F_{X_2}(r)] + \lambda_{12}[1-F_{X_{12}}(r)]\}\right] \end{aligned} \tag{2.13}$$

The combination of two Poisson point processes is also Poisson and the coincident process therefore has mean arrival rate given in Wen (1977) as

$$\lambda_{12} = \lambda_1 \lambda_2(\mu_{d_1} + \mu_{d_2}) \tag{2.14}$$

Monte Carlo simulation in the above reference has verified this occurrence rate for the coincident process. The occurrence rate of the individual processes should be modified to account for the coincidences

$$\kappa_1 = \lambda_1 - \lambda_{12} \tag{2.15}$$

The mean duration of the coincident pulses is also given by Wen as

$$\mu_{d_{12}} = \frac{\mu_{d_1} \mu_{d_2}}{\mu_{d_1} + \mu_{d_2}} \tag{2.16}$$

$F_{X_{12}}(r)$ is the distribution function of the sum of the amplitudes of the two pulses, given the coincidence.

The load coincidence formulation for the combination of n independent load effect processes may be written in the general form

$$P_f(t) = 1. - \exp[-\alpha t]$$

$$\alpha = \sum_{i=1}^{n} \kappa_i p_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \kappa_{ij} p_{ij} + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} \lambda_{ijk} p_{ijk} \tag{2.17}$$

$$\kappa_i = \lambda_i - \lambda_{ij} - \lambda_{ik} + \lambda_{ijk}$$

$$\kappa_{ij} = \lambda_{ij} - \lambda_{ijk}$$

where;

p_i = conditional probability of failure given the structure is subjected to load i only

p_{ij} = conditional probability of failure given the coincidence of loads i and j

λ_i = mean arrival rate of loads in process i

λ_{ij} = mean rate of coincidence of loads in processes i and j

λ_{ijk} = mean rate of coincidence of loads in processes i,j and k

κ_i = mean occurrence rate of individual loads without coincidence events

μ_j = mean duration of load in process j

The load coincidence method for linear combination of static load effects has been shown to provide good results for a wide range of parameters of the load processes (Wen, 1977,1980a). It is a conservative approximation under certain conditions as shown in Section 4.3. The effect of the coincidence term is dominant at low risk (high threshold) levels and therefore neglecting this term would introduce significant error. Results have also been very favorably compared with an exact solution for the combination of a Poisson square wave and a Poisson spike process. The point crossing method of Larrabee and Cornell (1978) generally gives results which are indistinguishable from those of the load coincidence method for static load effects.

CHAPTER 3

NONLINEAR COMBINATIONS OF STATIC LOAD EFFECTS

3.1 Introduction

The majority of the studies in load combinations thus far have dealt with linear limit states; i.e., the limit (performance) function g(X) is a linear function of the basic design variables (load effects and resistance). This implies a linear response of the structure and a linear resistance threshold. However, in many structural applications the limit state function is a nonlinear function of these variables. A nonlinear limit state may result for an elastic response of the structure as in the case of lateral torsional buckling of a steel beam-column. Nonlinear structural response must of course produce a nonlinear limit state function.

If we consider a given resistance and plot the limit surface as a function of the load effect variables, the problem may be visualized as an n-component stochastic pulse process crossing out of a nonlinear domain. This is the problem we will approach in this chapter without specific regard to the way in which the nonlinearity arises.

3.2 The Load Effect Model

Consistent with the model described in Section 2.1, each component process is modeled by one of the pulse processes used in linear combinations. To visualize the problem, Fig. 3.1a shows a perspective sketch of two component samples of a vector process and the time invariant limit surface. The mean arrival rate of the j^{th} component process is λ_j and the mean pulse duration is μ_j.

3.3 Mean Number of Outcrossings as Upper Bound

The probability that a general stochastic process exceeds a threshold in (0,t) (first excursion problem) has not yet been obtained analytically. An upper bound to this probability which provides a close bound at high threshold levels may be derived as follows

$$\begin{aligned} P_f &= P \text{ (at least one crossing occurs in } 0,t) \\ &= \sum_{j=1}^{\infty} P[N^+(r,t) = j] \\ &\leq \sum_{j=1}^{\infty} j \, P[N^+(r,t) = j] \\ &= E[N^+(r,t)] \\ &= \nu^+(r)t \end{aligned} \tag{3.1}$$

for a stationary process.

$N^+(r,t)$ = number of crossings in (0,t)

$\nu^+(r)$ = mean stationary crossing rate.

This upper bound has been used for bounding the failure probability of structures subjected to random excitation (Shinozuka, 1964; Veneziano, Grigoriu and Cornell, 1977). It is a general result and has recently been utilized in approximate solutions to the linear load combination problem (Larrabee and Cornell, 1978). As long as the crossing rate can be calculated for the specific response process, the result (Eq. 3.1) may be applied. The crossing rate was given for a continuous process by Rice (1944)

$$\nu^{+}(a) = \int_{0}^{\infty} (\dot{y}-a)\ f(a,\dot{y})\ d\dot{y} \tag{3.2}$$

$f(y,\dot{y})$ is the joint density function of Y and $\dot{Y}$.

3.4 Exact Crossing Rate for Poisson Pulse Processes

The above crossing rate may be used to compute an upper bound to the failure probability of a vector pulse process with nonlinear limit surface, as long as it can be calculated or closely approximated.

Breitung and Rackwitz (1982) have examined this problem and obtain the results for Poisson pulse processes and filtered Poisson processes. The exact solution of the total mean crossing rate for n independent component Poisson pulse processes is given by

$$\nu(D) = \sum \lambda_i \int_{R^n} P(X_i+\underline{x} \in D)\ P(X_i+\underline{x} \notin D)\ f_i^*(\underline{x})\ dx$$

$$f_i^*(x) = \delta(x_i) \prod_{j\neq i}^{n} f_j(x_j) \tag{3.3}$$

where $f_j(x_j)$ is the probability density of the amplitude of the jth component process and δ the Dirac delta. The first term in the integral is the conditional probability that the vector sum of the components remains inside the safe domain D at a renewal of component i, given the values of the other n-1 components. The second term is the complement of the first. When n is larger than 2 this multiple integral can become very costly. Under these circumstances an approximate solution of the crossing rate becomes necessary.

3.5 Approximation by Linearization

Consider first the problem of calculating the probability content of a nonlinear domain in time invariant reliability analyses. Hasofer and Lind (1974) first suggested a solution in the form of a reliability index which is defined as the smallest distance from the origin to the failure surface in some normalized space. This is a purely second moment solution.

When the distributions of the basic variables are known, an approximate solution is obtained by linearizing the safe domain at a suitable point on the failure surface. The success of this linearization suggests a similar technique for approximately estimating the crossing rate for a time varying load effect process. Linearizing the failure surface permits the relatively easy calculation of the mean crossing rate of the hyperplane rather than the complex nonlinear limit surface.

The crossing rate given by Eq. 3.3 is simplified for the case of Poisson renewal processes, with standard normal height distribution, crossing out of the hyperplane

$$\sum_{i=1}^{n} \alpha_i x_i - \beta = 0 \qquad (3.4)$$

where α_i are the direction cosines and β is the shortest distance from the origin to the plane. The mean crossing rate becomes

$$\nu(D_H) = \sum_{i=1}^{n} \lambda_i [\Phi(\beta) - \Phi(\beta,\beta,1 - \alpha_i^2)] \qquad (3.5)$$

As in the time invariant case it is necessary to know at which point the limit surface should be linearized in order to obtain as good an approximation as possible (Pearce and Wen, 1983). Breitung and Rackwitz (1982) have addressed this problem by investigating points in the time varying problem which are in several ways analogous to the linearization point used in the time independent problem (see Appendix A).

The first of these points is the point closest to the origin in a transformed space. The usual transformation is to a unit normal space. In the time dependent problem the analogous point is still that point closest to the origin. For time independent resistance variables and stationary load processes, this point will remain unchanged with time.

The second is the point of maximum mean rate of crossing the tangent hyperplane, suggested as being analogous to the point of maximum likelihood (Shinozuka, 1983) or maximum probability content outside the linearized safe domain in the time invariant case.

The third point is the point of maximum probability density on the surface. The analogous point for time varying load processes is suggested by Breitung and Rackwitz as the point of maximum local outcrossing defined by

$$\nu_L(D) = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \left[\begin{array}{l} \text{expected number of outcrossings} \\ \text{from D through surface area } \Delta S \\ \text{during one time unit.} \end{array} \right]$$

They conclude erroneously that there is no uniquely best point of linearization and that the latter point above is the generally superior point.

Appendix A demonstrates that these three points mentioned above coincide in the time invariant case and are the best points at which linearization should be performed only in the rotationally symmetric (uncorrelated) standard normal space.

When the general time dependent case is considered, even when the pulse distribution is transformed to unit normal, the symmetry is destroyed whenever the renewal intensities of all components are not equal. The above three points will then, in general, not coincide with the optimum point for linearization for time varying loads.

The optimum point is defined as that point which will yield the best approximation, through linearization, to the mean crossing rate out of the safe domain. This crossing rate is used to compute an approximation to the failure probability. Pearce and Wen (1983) have demonstrated that there is a unique optimum point for linearizing the safe domain and that it is a stationary point of the mean crossing rate out of the tangent hyperplane. Briefly, this may be shown as follows.

Let P^u be the exact crossing rate out of the safe domain and $\nu(x^o)$ the crossing rate out of the approximating hyperplane tangent at x^o. The error in the linearization is then given by

$$\varepsilon = |P^u - \nu(x^\circ)| \tag{3.6}$$

It is apparent that for largely convex safe domains (those in which the portion of the surface closest to the origin is convex and smooth), the crossing rate out of the tangent plane must be maximized for the error to be minimized, because the tangent approximation will underestimate the actual outcrossing rate. Conversely, when the limit surface of the safe domain is largely concave, the crossing rate out of the tangent plane overestimates the real crossing rate and must therefore be minimized for the error to be minimized.

The task of obtaining the local stationary points of the mean crossing rate is a nonlinear programming problem which may be costly for large dimensions. It also becomes increasingly difficult to locate the best point when there are more than two stationary points. In this case it seems that the stationary point of the mean crossing rate closest to the point of maximum local outcrossing will be the best point at which to linearize the domain. The programming problem for this case becomes increasingly difficult especially as the dimension increases.

Breitung and Rackwitz suggest that the point closest to the origin (Hasofer/Lind point) may produce sufficiently accurate results for the kind of limit surfaces found in structural mechanics. The examples in Fig. 3.2 show results of the crossing rate out of the approximating tangent hyperplane at the Hasofer/Lind point together with the point of stationary mean crossing rate and the point of maximum local crossing rate. For the convex safe domain in (a) (an ellipse), linearization of the surface will obviously yield a poor approximation to the mean crossing rate. However, as stated previously, the point of maximum mean outcrossing rate provides the sharpest lower bound, i.e., it is the optimum linearization point. In the special case of the ellipse, wherein the process on the minor axis experiences the smaller renewal intensity, and whose linearization at the point closest to the origin (Hasofer/Lind) is, of course, parallel to the major axis, only crossings by process 1 are possible. This explains the constant crossing rate with increasing renewal intensity rate in (a). This linearization is therefore very poor, but is certainly not generally so for all convex safe domains.

For the concave safe domain in (b) (a hyperbola) the point of minimum mean outcrossing rate gives the closest upper bound as expected. The point of maximum mean outcrossing rate does, of course, give results which are extremely conservative as found by Breitung and Rackwitz. The Hasofer/Lind point actually gives better results in this example than the point of maximum local outcrossing.

3.6 Load Coincidence Method

The general formulation of the load coincidence method given in Eq. 2.17 is not restricted to linear limit states. For nonlinear limit states and processes of the kind shown in Fig. 3.1 the occurrence and coincidence rates are the same as in Eq. 2.17. The calculation of the conditional failure probabilities becomes more difficult. For some regularly shaped domains and certain load distributions the closed form solution of the conditional probabilities may be possible. However, where this is not possible some approximation to these probabilities is necessary, as is currently the practice for time invariant reliability problems.

The load coincidence method uses the Hasofer/Lind point in a slightly different way from that suggested by Breitung and Rackwitz to calculate the crossing rate. The term for the crossing rate is given as

$$\alpha = \kappa_1 p_1 + \kappa_2 p_2 + \lambda_{12} p_{12} \tag{3.7}$$

for two load processes. The occurrence rates of the individual pulses alone are given by κ_1 and κ_2; and of the coincident pulses by λ_{12}. The term p_i is the conditional probability that a single pulse will exceed the limit state in the direction of that component while no other load is present. The conditional probability that the vector sum of two component pulses, given coincidence, exceeds the limit state is p_{12}.

The calculation of this probability is generally achieved, for large dimensional nonlinear safe domains, through use of the Rackwitz-Fiessler algorithm (Appendix B). The algorithm locates the optimum linearization point (Hasofer/Lind point in unit normal space) and the approximate conditional probability of failure is given by

$$p_{12} = 1. - \Phi(\beta) \tag{3.8}$$

where β is the distance of the point from the origin.

For sparse load processes the Hasofer/Lind point will give very satisfactory results because it is unlikely that two pulses from one process coincide with one pulse from another, destroying the symmetry of the problem. The unsymmetrical nature arises when the renewal intensity of one process is very different from that of another and the process is dense. Many renewals of the one process may occur during just one occurrence of the other.

For processes which are always on, the coincidence rate for the load coincidence method is given by the sum of the individual renewal intensities. The conditional failure probability may be calculated at any instant of time as there is always coincidence. Calculation of the crossing rate by the product of the coincidence rate and the conditional failure probability implies that this probability at each renewal of the process with larger renewal intensity is independent of that at the previous renewal. Clearly this is not true if no renewal of the other process has occurred because its value is perfectly correlated until the next renewal point. The approximation by independent pulses is a conservative one and it retains the simplicity of the load coincidence method.

3.7 Examples and Comparisons

The first example is a very nonlinear, but not really practical, example. A two component renewal pulse process with unit normal pulse height distribution crosses out of a centered circular domain. In this particular example it is obvious that a single tangent hyperplane approximation will yield results very different from the actual circular domain. However, for this example, the exact probabilities are easily calculated because of the fact that the sum of squares of normal variates has a chi-squared distribution. The crossing rate obtained by the load coincidence method is then given by

$$\alpha = 2[1 - \Phi(r)](\lambda_1 + \lambda_2 - 2\lambda_{12}) + \lambda_{12}\, \chi^2(r^2) \tag{3.9}$$

where r is the radius of the circle and χ^2 is the chi-squared probability distribution function.

Results for $\lambda\mu = 1$ (always on) and $\lambda\mu = 0.5$ and 0.1 are shown in figs. 3.3 and 3.4, respectively, together with the exact crossing rates computed using Eq. 3.3. The load coincidence method gives increasingly better results as the threshold increases and as the processes become more sparse. When the radius is 3.0 and $\lambda\mu = 0.1$ no difference is visible between the exact result and that of the load coincidence method even for the very unsymmetrical case where $\lambda_2 \gg \lambda_1$.

A second, more practical, example is given wherein a beam-column is subjected to an axial load and end moments, both of which are modeled as Poisson pulse processes. The interaction curve for buckling is shown in Fig. 3.5 together with section sizes and properties. The curve (i.e., failure surface) is generally very close to linear, becoming noticeably nonlinear only when the column is very slender.

The pulse amplitudes are assumed log-normal and the Rackwitz/Fiessler algorithm using the principle of normal tail approximation is employed in order to estimate the conditional probability of failure given that pulses from the two load effect processes coincide. The safe domain in the original space is concave which leads us to expect a conservative estimate of the conditional probability of failure. However, the results show an unconservative value which is due to the transformation of the failure surface to unit normal space. The conservatism of the load coincidence method at low thresholds and less sparse processes is greater than the unconservative values of the probabilities and the overall effect is to give a conservative result for the crossing rate. This can be seen in Fig. 3.6 where the exact crossing rates are also plotted for comparison. For slightly higher thresholds and sparser processes the results become unconservative but still compare well with the exact crossing rates. The error on the unconservative side will not be greater than that produced by the transformation to normal space.

More complex examples with larger numbers of random components (load and resistance) are more easily handled with the load coincidence method. The conditional probabilities p_i, p_{ij} are calculated without much difficulty using the fast convolution technique of the Rackwitz-Fiessler algorithm or some method of nonlinear programming.

Random resistances are incorporated easily at the conditional failure probability level, using the above technique, rather than requiring the numerical integration of the total conditional probability over the distribution of the resistance. This does however imply that the resistance is independent from occurrence to occurrence of a pulse, which is an incorrect, but conservative, implication. Correlated load effects may also be considered and calculations of p_{ij} performed by making use of a suitable transformation as shown by Hohenbichler and

Rackwitz (1981).

3.8 Other Methods for Calculating p_{ij}

3.8.1 Multiple Checking Points and Systems Approach

Linearization at just one point on the failure surface may not approximate the safe domain sufficiently well for certain moderately curved domains or when a large number of failure modes have to be considered.

It is possible then to approximate the failure surface by the intersection of a number of tangent hyperplanes enclosing a polyhedral safe domain. The difficulty is to establish the points at which the planes are to be made tangential.

Ditlevsen (1982a) suggests this multiple point approximation for systems with more than one mode of failure, each mode being linearized. The reliability of the system is then given in the form of upper and lower bounds obtained from system reliability techniques. The hyperplanes are made tangent at the locally most dangerous point of each mode, i.e., the stationary point of the distance of the orthogonal projection point on the plane, from the origin.

3.8.2 PNET Method

Structures which fail in a ductile manner by forming plastic hinges may be analysed by the PNET method developed by Ang and Ma (1979). The method consists of two parts,

1. Identification of all significant modes

of collapse (failure mechanisms)

2. Synthesis of the probabilities of the individual collapse mode to obtain the system collapse probability.

In an earlier paper Wen (1980b) demonstrated the use and accuracy of this method incorporated in the load coincidence formulation.

3.9 Conclusions

The load coincidence method provides a relatively simple extension of some methods for time invariant reliability analysis to the very complex problem of vector pulse processes crossing out of nonlinear domains. Accuracy of the method is good especially for sparse processes, but does depend on the accuracy of the conditional failure probabilities and therefore on the shape of the domain.

Recently Grigoriu (1983) has proposed a method which approximates the limit surface by some polynomial function of the basic variables. Optimal estimates of the distribution of a control variable are obtained to compute necessary probabilities. A linearization approach is also proposed and said to be good for extension to time variant problems using Turkstra's rule (see Section 4.5.3). The methods have not yet been tested.

CHAPTER 4

MODELING AND COMBINATION OF DYNAMIC LOAD AND LOAD EFFECT PROCESSES

4.1 Introduction

The forces of nature have challenged the engineer since the first structures were erected. The dynamic loads applied by these forces have, in the extreme, caused havoc and destroyed whole communities. But the extreme loads are not the only ones causing damage. Combinations of less severe loads may be just as harmful.

Though predictions of environmental forces such as those due to earthquakes and storms are subjects of current research, the occurrence, intensity and fluctuation of such forces are largely random. The mathematical modeling of these loads is therefore best achieved by probabilistic means, and the variation in time requires the use of the theory of stochastic processes.

4.2 Intermittent Continuous Process for Loads and Load Effects

Dynamic loads are those loads which fluctuate with a time scale close to the natural period of the structure causing severe changes in the deformations and stresses. Examples of this type of loads are: ground motion caused by earthquakes; wind; water waves; pressure transients in nuclear reactors; shock or blast loads. Loads of this type which occur in nature do so intermittently with long periods where no such excitation, or excitation of such a low level to be unimportant, will occur. Therefore the sparse Poisson process is suitable for modeling the occurrence of such loads. However, in addition the loads also fluctuate within each occurrence and therefore a more complex model behavior than a rectangular pulse is required.

This is achieved by introducing a fluctuating process within the duration of the pulse. This is done differently for two load types: the zero-mean loads and the non-zero-mean loads.

Those loads, such as earthquakes and water waves, which have zero mean are modeled by a continuous stochastic process whose expected value is zero and variance function which is a measure of the magnitude of the random fluctuation. During the total load process, the intensity of the load will, in general, vary at each occurrence. The intensity is therefore a random variable. The spectral density of the random process, which is related to the mean square value, thus has a random fluctuation from occurrence to occurrence.

Loads such as wind pressure are composed of a mean or "static" component and a gusting or fluctuating component. The mean component, denoted Y and shown in Fig. 4.1, is allowed to vary at each load occurrence and is therefore a random variable. The fluctuating component is related to the mean component through its mean square value. That is, it is proportional to the mean. The total load is thus a superposition of a zero-mean continuous stochastic process X and a rectangular pulse of magnitude Y. Such a composite random process is hereafter referred to as an intermittent continuous process.

Any convenient process may be used to model the dynamic effect. The processes used in this study are the Gaussian processes. Many physical processes, such as a steady sea state, turbulence in wind, as well as ground acceleration can be approximately modeled by Gaussian processes. Without loss of generality, in this study excitation is modeled by Gaussian white noise for computational ease. More realistic models can be introduced without analytical difficulty. For example, a filtered white noise using a filter of the Kanai-Tajimi type is more representative of earthquake motion than a modulated white noise and can be used when analysing the response of a specific design to real ground motion.

4.3 The Load Coincidence Formulation

4.3.1 Load Coincidence Solution as Upper Bound

The exact solution of the failure probability in (0,t) of a system subjected to a stochastic load process requires a first passage probability formulation and is very difficult to compute. The upper bound in the form (see Eq. 3.1)

$$p_f(t) \leq p_f(0) + \nu(D)t \tag{4.1}$$

may be utilized in the reliability analysis. Consider a general Poisson pulse process; let p_c be the probability of failure during one occurrence, and λ the rate of occurrence. The probability of failure in (0,t) is then

$$\begin{aligned} p_f(t) &\leq p_f(0) + P \text{ (at least one crossing in } 0,t) \\ &= p_f(0) + [1 - \sum_{0}^{\infty} \{(1 - p_c)^k \frac{(\lambda t)^k}{k!} e^{-\lambda t}\}] \\ &= p_f(0) + 1 - \exp[-\lambda p_c t] \\ &\leq p_f(0) + \lambda p_c t \end{aligned} \tag{4.2}$$

This can be repeated for a combination of loads assuming the coincident process to be independent of the others. The condition for the inequality to hold is that λ_{12} must be a conservative mean coincidence rate, which it is (Wen, 1977).

The crossing rate ($\kappa_1 p_{c_1} + \kappa_2 p_{c_2} + \lambda_{12} p_{c_{12}}$) given by the load coincidence formulation therefore provides an upper bound to the failure probability in unit time, so long as the conditional probabilities p_{c_i} are computed exactly or estimated conservatively.

In fact, the Poisson nature of the pulse process implies that the Eq. 4.2(d) is a conservative value of the probability of failure in (0,t). For combined dynamic load effects modeled as intermittent continuous processes the load coincidence crossing rate is stated again as

$$\alpha = \sum_{i=1}^{n} \kappa_i p_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \kappa_{ij} p_{ij} + \cdots \qquad (4.3)$$

where now the p_i and p_{ij} are conditional probabilities that the maximum response during one load occurrence or coincidence will exceed the failure threshold. It is this probability, the probability of first passage during the excitation, that we need to compute.

4.3.2 Computation of the Conditional Probabilities

The failure (first passage) probability during one load occurrence or coincidence may be conservatively approximated from the crossing rate of the failure threshold by assuming crossings occur independently and are points in a Poisson process. Then the probability that no crossing occurs in (0,t) is

$$L(t) = \exp[-\nu(r)t] \qquad (4.4)$$

and the failure probability $p_f(t) = 1.-L(t)$. This result has been shown to be asymptotically exact as r increases to infinity. Use of this expression will be made because of its simplicity. Its accuracy will be investigated later. If the process is nonstationary then Eq. 4.4 becomes

$$L(t) = \exp[-\int_0^t \nu(r,\tau)d\tau] \tag{4.5}$$

For loads which can be represented as essentially stationary during one occurrence a Gaussian white noise is used to model the excitation. The spectral density S_o is constant over the frequency domain but is random from load to load.

The stationary variance of the response is

$$\sigma_x^2 = \frac{\pi S_o}{2\zeta\omega^3} \tag{4.6}$$

in which

ζ is the percent of critical damping.

ω is the natural frequency.

The response of a linear system to a Gaussian input is also Gaussian, and the crossing rate of level r is given by

$$\nu_r^+ = \int_0^\infty \dot{x}\, f_{X\dot{X}}(r,\dot{x})d\dot{x} \tag{4.7}$$

which for a stationary Gaussian process becomes

$$\nu_r^+ = \frac{1}{2\pi}\frac{\sigma_{\dot{x}}}{\sigma_x}\exp[\frac{-r^2}{2\sigma_x^2}] \tag{4.8}$$

The stationary variance of the derivative of the response is

$$\sigma_{\dot{x}}^2 = \frac{\pi S_o}{2\zeta\omega} \tag{4.9}$$

so that the crossing rate may be written

$$\nu_r^+ = \frac{\omega}{2\pi} \exp[\frac{-r^2\zeta\omega^3}{\pi S_o}] \tag{4.10}$$

When the load may be more closely approximated by a modulated white noise this is represented by

$$z(t) = \phi(t)\ n(t)$$

in which, n(t) is a white noise process, $\phi(t)$ is a multiplier function. The mean of z(t) is zero and its autocorrelation is given by

$$R_{ZZ}(t_1,t_2) = \phi(t_1)\ \phi(t_2)\ 2\pi S_o\ \delta(t_2 - t_1) \tag{4.11}$$

The statistics of the response, its variance and the variance of the derivative function may be obtained in closed form, through lengthy integrations, for certain multiplier functions (Lin, 1965). From the above information, the rate of crossing a given threshold r at time t is obtained as (Cramer and Leadbetter, 1967)

$$\begin{aligned} \nu(r,t) &= \frac{1}{\sqrt{2\pi}} \frac{\sigma_{\dot{x}}}{\sigma_x} \exp[\frac{-r^2}{2\sigma_x^2}]\ \{F\} \\ \{F\} &= \eta\ \Phi(\frac{\eta}{\sqrt{1-\rho^2}}) + \frac{\sqrt{1-\rho^2}}{\sqrt{2\pi}} \exp[\frac{-\eta^2}{2(1-\rho^2)}] - \frac{\eta}{2} \\ \eta &= \frac{\rho r}{\sigma_x} \\ \rho &= \rho(t) = \frac{\mathrm{Cov}[\dot{x}(t)\ x(t)]}{\sigma_{\dot{x}}(t)\ \sigma_x(t)} \end{aligned} \tag{4.12}$$

Then the probability of failure during the response of duration d is given by

$$p_1(r,d) = 1. - \exp[-\int_0^d \nu(r,t)dt] \tag{4.13}$$

At each occurrence of a pulse in one load process the "intensity" of the excitation, measured by the spectral density S_o, may differ from the previous or any other occurrence. Therefore, in the model, the measure of intensity is considered to be a random variable. In the case of a non-zero mean process the static component may be modeled by a random variable with the measure of intensity of the dynamic response in some way related to the static value. Therefore $p_1(r,d)$ as given above is a conditional probability, dependent on the spectral density S_o(which is related to the static component Y). For the non-zero mean case the unconditional probability of failure during a pulse of mean duration μ_d is

$$p_1(r) = \int_0^\infty [1 - \exp\{-\int_0^{\mu_d} \nu_r^+(t)|_{Y=y}\, dt\}]\ f_Y(y)dy \tag{4.14}$$

which for the stationary response is

$$p_1(r) = \int_0^\infty \left[1 - \exp\{\frac{-\mu_d\omega}{2\pi} \exp[\frac{-(r-y)^2\zeta\omega^3}{\pi S_o}]\}\right]\ f_Y(y)dy \tag{4.15}$$

The first excursion probability is, of course, dependent on the duration of the process. An approximation is used in the above two equations by evaluating the probability at the mean of the duration μ_d, instead of integrating over the probability density function. However, for stationary processes this was found to be a very close approximation and one which has negligible effect on the design life reliability.

When two loads coincide, since linear combination of Gaussian processes is again Gaussian, the crossing rate of the combined process can be computed without difficulty. The variance of the combined process is the sum of the variances of the individual processes and the unconditional failure probability is given by

$$p_{12}(r,\mu_d) = \int_0^\infty \left[1 - \exp\{\frac{-\mu_d \omega}{2\pi} \exp[\frac{-(r - y_1 - y_2)^2 \zeta\omega^3}{\pi(S_1 + S_2)}]\}\right] f_{Y_1}(y_1)\, f_{Y_2}(y_2)dy_1 dy_2 \tag{4.16}$$

The coincidence duration is given by

$$\mu_d = \frac{\mu_{d_1}\,\mu_{d_2}}{\mu_{d_1} + \mu_{d_2}} \tag{4.17}$$

The use of this duration embodies more than one approximation. The first is the evaluation of the probability at the mean value of the duration, as above. Another is that the initial conditions are not "at rest" conditions when the coincidence begins. The lag between the two loads should also be considered random, but here only the mean value is used. These are all mean value approximations and, as first order approximations of a smooth function, are considered reasonable and as a result the calculation procedure is simplified tremendously. The accuracy of this approximation is verified in the latter part of this chapter (Section 4.6.3).

Combining nonstationary responses requires additional approximations if a reasonable simplicity is to be retained in the evaluation of the conditional failure probabilities given coincidence.

Figure 4.2 shows a piecewise linear modulating function and the combination of two nonstationary modulated white noise processes. The lag time θ is a random variable. The variance of the combined process depends on the lag time. Lengthy integrations are required first to compute the variance of the individual response and its derivative and then again to account for all possible values of lag time θ, duration d, and the variation of the response intensity Y.

To simplify this, an equivalent stationary process of the same duration is defined, conditional on the value of the intensity Y. The expected number of crossings by the nonstationary process in μ_d for a single load is given by

$$E[N^+(\mu_d)] = \int_0^{\mu_d} \nu_r^+(\tau)\ d\tau \tag{4.18}$$

A conditional stationary crossing rate is then defined for the equivalent stationary process by

$$\nu_e\Big|_{Y=y} = \frac{1}{\mu_d} E[N^+(\mu_d)] \tag{4.19}$$

The combination of two such responses (in this case Gaussian) requires knowledge of the variances of the equivalent processes. The expression

$$\frac{\omega}{2\pi} \exp\left[\frac{-(r - y)^2}{2\sigma_e^2|_{Y=y}}\right] = \nu_e|_{Y=y} \tag{4.20}$$

for the conditional crossing rate of the equivalent stationary process enables us to calculate the required variance for each of the individual responses

$$\sigma_{e_i}^2|_{Y_i=y_i} = \frac{-\frac{1}{2}(r - y_i)^2}{\ln(\frac{2\pi}{\omega}\,\nu_{e_i}|_{Y_i=y_i})} \tag{4.21}$$

The variance of the combined response is the sum of the variances of the equivalent stationary responses, and the crossing rate for a non-zero-mean process is given by

$$\nu_{12}|_{y_1,y_2} = \frac{\omega}{2\pi} \exp\left[\frac{-(r-y_1-y_2)^2}{2(\sigma_{e_1}^2 + \sigma_{e_2}^2)}\right] \tag{4.22}$$

The approximation of the nonstationary processes by equivalent stationary processes is conservative when the two processes are combined, for a given duration of each load.

4.4 Crossing Rate Methods

Crossing rate methods are defined here as those methods which use the "arbitrary point in time" crossing rate obtained from Rice's equation in the calculation of the overall mean combined crossing rate. The combined crossing rate generally takes the form of the product of the probability of occurrence and the crossing rate at an arbitrary point in time. This contrasts with the load coincidence formulation which uses the product of the load arrival rate and the conditional failure probability of the structure given the occurrence and coincidence.

4.4.1 Point Crossing Method

The point crossing method (Larrabee and Cornell, 1979) was originally used in the study of combinations of renewal pulse processes to obtain a combined crossing rate as upper bound to the failure probability in unit time. Winterstein and Cornell (1980) extended the method for use in the combination of intermittent dynamic responses. Each response process is characterized by its first order probability density function $\hat{f}_X(x)$ and mean upcrossing rate function $\hat{\nu}_X(x)$, together with the mean arrival rate of pulses and the mean pulse duration.

The point crossing method has been shown to provide an upper bound to the crossing rate and therefore to the probability of failure. For two load effect processes with stationary responses the combined mean crossing rate is given by

$$\bar{\nu}_r = \int_{-\infty}^{\infty} \hat{\nu}_1(r-x)\ \hat{f}_2(x)dx + \int_{-\infty}^{\infty} \hat{\nu}_2(r-x)\ \hat{f}_1(x)dx \qquad (4.23)$$

which, for dynamic processes, becomes

$$\bar{\nu}_r = \lambda_1\mu_1(1 - \lambda_2\mu_2)\ \nu_1 + \lambda_2\mu_2(1 - \lambda_1\mu_1)\ \nu_2 + \lambda_1\lambda_2\mu_1\mu_2\nu_{12} \qquad (4.24)$$

where

ν_i = conditional crossing rate of process i given a pulse is "on".

ν_{ij} = crossing rate of a coincident process

$\lambda_i\mu_i$ = Prob(process i is "on")

$\lambda_1\lambda_2\mu_1\mu_2$ = Prob(coincidence occurs)

λ_i = mean arrival rate of process i.

μ_i = mean duration of pulse in process i.

4.4.2 Shinozuka´s Bound

Under the assumption of independence and stationarity of the load processes, Shinozuka and Tan (1981) express the overall average crossing rate in the form,

$$\bar{\nu}_r = P_1\nu_1 + P_2\nu_2 + P_{12}\nu_{12} \qquad (4.25)$$

P_i = probability that only load effect i is on.

P_{12} = probability that load effects 1 and 2 are on simultaneously.

ν_i, ν_{ij} are crossing rates defined for the point crossing method.

P_i and P_{ij} are calculated, for different load duration distributions, from queueing theory. For two load processes wherein the interarrival time between renewals and the load duration is exponentially distributed, the above two methods arrive at the same mean crossing rate, given in Eq. 4.24.

The combined mean crossing rate is conditional on the value of the measure of intensity (spectral density) of the load processes. When this intensity is uncertain, Winterstein models the processes as the product of a rectangular pulse process with constant (in time), but random (from sample to sample), intensity A, and a stationary continuous process X(t).

Both Winterstein and Shinozuka suggest the use of the Poisson assumption, with crossing rate given in the form of Eq. 4.24, to approximate the first passage probability for the combined process. Winterstein describes two ways in which the condition on the crossing rate is removed by integration over the distribution of the intensity A. The "direct" method removes the condition on the crossing rate (computes expected crossing rate with respect to A) and uses the expected rate as the crossing rate in the Poisson assumption for first passage probability. The "conditional" method calculates a conditional failure probability given the crossing rate and then evaluates the expected probability by integration. The conditional method was found to give better results for the situation modeled by Winterstein, in which A remains constant throughout the process, but for the purpose of comparison with the load coincidence formlation, the direct method is used here since a realistic model should allow intensity variation from occurrence to occurrence. The difference between the direct and conditional methods is shown in Fig. 4.3.

The shortcoming of the crossing rate (Shinozuka, or Winterstein) model is that, although the pulse intensity (or mean value) may be considered random, the way it is modeled only allows variation from process to process and not from pulse to pulse within one process. For example, an earthquake load process and a wind load process may have different intensities but each occurrence within the earthquake process will have the same intensity as the others using this model. The random variable A controls the value of all the pulses within one process.

The load coincidence model does allow variation of the pulse intensity within each process and is therefore a more physically satisfying model.

If the upper bound to the conditional failure probability is used in the load coincidence formulation (as the terms p_i, p_{ij}) the point crossing method (together with that of Shinozuka) and the load coincidence method give similar results. However, when the crossings occurring during each response pulse are assumed to be Poisson, and the conditional failure probability is computed using this assumption, considerable differences may arise for low thresholds and long load durations, and also for high frequency oscillators (Figs. 4.4 through 4.12). The methods converge for high threshold levels and short load duration when the period of the oscillator is large, say greater than 1.25 seconds. It may be worthwhile here to point out that the load coincidence method generally yields conservative results indicating that under these circumstances the crossing rate method may yield results which are unduly conservative.

Results are shown for stationary excitation in Figs. 4.4 through 4.12 for oscillators having natural frequencies of 3, 5 and 10 rad/sec and 5% of critical damping. Implied probabilities of failure given by use of the SRSS and Turkstra rule are also plotted. The methods of calculating these probabilities are briefly described in the following section. The Poisson assumption was used when evaluating the CDF of the maximum for use in the above two methods. Comparisons are made therefore, based on the relative magnitude of the probability given by each method, acknowledging that the Poisson assumption yields a conservative result in each case.

The graphs show how much more conservative the point crossing method may be. In fact, only for the shorter durations and lowest frequency considered is it less conservative than the SRSS value.

The differences between the methods may be explained by considering the level at which the Poisson assumption is made. The point crossing method calculates the mean crossing rate of the whole intermittent dynamic response process and then assumes crossings occur as a Poisson process over the whole design life. The load coincidence method assumes that crossings during one pulse or coincidence occur as a Poisson process and calculates the conditional probability of failure. Then, using the fact that the pulses do indeed arrive in Poisson fashion, it uses the conditional failure probability and the mean arrival rates to evaluate the crossing rate of this process.

In addition to the Poisson assumption being used at different times, two effects contribute to the sometimes significant difference between the methods. The first is that for a given mean response the difference between the methods increases as the threshold decreases. The difference may be very large at small thresholds because the crossing rate is large and if the duration is large as well, the upper bound to the conditional failure probability may be much greater than unity. The use of the Poisson assumption in calculating the conditional failure probability in the load coincidence method limits this probability to being less than unity. At large thresholds the two methods converge fairly rapidly.

The second effect is that the mean response level is a random variable, in this case modeled by a gamma variate. The unconditioning of the crossing rate or conditional failure probability is done by weighting the conditional value by the probability density at that level.

Figure 4.13 shows the conditional failure probability as a function of the mean Y of the response. The probability density of Y is also shown. The effect of the weighting of the failure probability by the density is to "spread" the differences between the two methods for the case where only a single value of the mean response is considered. The resulting differences are those shown in the Figs. 4.4 through 4.12.

4.5 Other Rules for Load Combination

Structural codes or recommendations use simpler and more approximate methods to account for the fact that it is unlikely that the maximum values of a number of loads will occur simultaneously. These methods of evaluating the maximum design load take the form of a set of rules to be applied by the designer. Essentially these rules reduce the problem of the combination of random processes to that of random variables. As a result, inaccuracies are introduced. The probability of failure implied by these rules is examined here for comparison with the results based on the more complete stochastic process models.

4.5.1 SRSS Rule

The square root of the sum of the squares (SRSS) rule has been extensively studied and recommended (e.g., by Mattu and Nureg committee) for combining two dynamic responses which coincide, but with random lag time. The rule states that for two load processes S_1 and S_2;

$$\max[S_1(t) + S_2(t)] = \sqrt{R_1^2 + R_2^2} \tag{4.26}$$

where

$$R_1 = \max[S_1(t)]$$

$$R_2 = \max[S_2(t)]$$

It apears to give reasonably conservative results for a wide range of combination situations. It is compared here with the load coincidence method for use in computing the design life maximum of a number of combined load processes.

For two processes the implied probability that the maximum R_m of the combined process exceeds a level r is given by

$$P(R_m > r) = 1. - \int_0^r F_{R_1}(\sqrt{r^2 - r_2^2})\ f_{R_2}(r_2)dr_2 \qquad (4.27)$$

where, for the case of dynamic response, we use

$$F_{R_j}(r_j) = \exp\left[- \lambda_j t\{1 - \exp[- \nu_j(r_j)\ \mu_j]\}\right] \qquad (4.28)$$

4.5.2 Load Reduction Factor

Some codes use a load reduction factor to reduce the total maximum of two or more combined time varying loads. For example, in the ACI-318 concrete code, a factor of 0.75 is used when two loads such as live and wind are together in a combination equation. It was found that a load reduction factor of 0.7 has a safe domain very similar to the SRSS domain for two loads and any necessary conclusions will be drawn from the results of the SRSS (Wen and Pearce, 1981b).

4.5.3 Turkstra's Rule

The developers of the new ANSI A58.1-1982 Minimum Design Loads have applied Turkstra's rule when considering combinations of time varying loads; i.e., other than dead load.

The rule states that the maximum of the sum of two or more random load processes is approximated by the sum of the maximum of process k and the arbitrary point-in-time values of the other processes, with k chosen to give the maximum of all n such combinations, n being the number of load processes. The advantage of this approximation is that it uses only a random variable representation of the load processes. However, the result is always an unconservative estimate of the maximum combined load. As the maximum value and the arbitrary point in time value are correlated, the evaluation of the implied probability is more involved. Some details of the calculations for static load effects are found in the paper by Wen (1980a).

4.5.4 Conclusions

It can be seen from Figs. 4.4 through 4.12 that, in general, the type of load combination rules used in codes and described above are not capable of giving consistent estimates of the combined maximum of a number of intermittent continuous processes.

The SRSS rule is generally a conservative method for large risk levels (low thresholds) but becomes an unconservative estimate at low risk (high threshold) levels due to the failure to properly account for the coincident load effect which dominates at high thresholds.

Turkstra's rule always gives an unconservative estimate of the combined maximum, the accuracy of which depends more on the load occurrence and duration characteristics than on the structural parameters.

4.6 Accuracy of the Conditional Failure Probability

In this chapter and the next, mention is often made of the conditional probability of failure during the occurrence or coincidence of dynamic loads. This may be stated as the probability that the maximum of the response process in (0,d) the duration of the response, exceeds the limit state; or, the probability that the time to first passage of the limit surface is less than d. Since the probability is an important element in the combination analysis, it is of importance to examine the accuracy of the method used in this study and its effect on the overall probability of combined load.

4.6.1 First Passage Studies

This problem of the time to first passage (crossing) of some level by a linear oscillator experiencing random vibration has been extensively addressed in the literature. Most studies have considered the case of a Gaussian white noise excitation because the response (x,x) is governed by a vector Markov process of dimension two. The joint transition density function is obtained as the solution of the Fokker-Planck equation (Lin, 1965).

Both numerical and approximate analytic approaches to the solution of the failure probability have been attempted with varying degrees of complexity and success. Simulation studies have also been undertaken for comparison purposes. A discussion of these solutions and a good list of references may be found in the works by Bergman (1979,1982) and Yang and Shinozuka (1971,1972).

No simple approximate analytical procedure exists whereby reasonably accurate results are achieved for a broad range of problems. Bergman (1979) points out that a solution is available for the case where the boundary conditions to the Fokker-Planck equation are natural, i.e., no finite bounds on the displacement are prescribed. Then, for an approximate solution to the first passage problem, the failure process is modeled as an intersection between an unbounded distribution and a deterministic threshold. Rice (1944) formulated the general equation for the rate of upcrossing of a threshold by a random process, the solution of which is easiest for the Gaussian process.

$$\nu(a) = \int_0^\infty (\dot{y}-a)\ f_{Y\dot{Y}}(a,\dot{y})d\dot{y} \tag{4.29}$$

This result may be used in the simplest approximation to the first passage distribution function by assuming that crossings of the level occur independently and are therefore points in a Poisson process. This solution is asymptotically exact as the threshold increases but for small thresholds is very conservative.

In a narrow-band response the peaks of the process tend to occur in clumps and are therefore not independent. It has thus been suggested that the first passage probability would be estimated more accurately if the crossings of the envelope of the process are considered independent. Using the crossing rate of the envelope process given in Cramer and Leadbetter (1967), the first passage probability thus obtained is still a conservative estimate, but less so than the Poisson assumption. Vanmarcke (1975) gives a slight improvement on the envelope crossing rate by noting that there can be envelope crossings that are not followed by process crossings.

Bergman (1979) has recently solved the Pontriagin-Vitt equation for the moments of time to first passage of the linear, and some nonlinear, oscillators subject to a white noise excitation, using a finite element technique. In subsequent studies (1982) he has obtained the numerical solution to the transient problem of the probability of failure of the oscillator subject again to white (or modulated white) noise excitation. The reciprocal of the expected time to first passage is a good approximation to the crossing rate though the probability so obtained using an exponential distribution is generally unconservative.

The results for dynamic combination in this study are all obtained using the assumption of Poisson crossings for the approximate conditional first passage probability, given the occurrence or coincidence of a load pulse. This is done because the probability is simply expressed in analytic form and because the probabilities of interest are connected with high threshold levels. The probability density function of the first passage time is also obtainable and is required for the calculations of the probabilities implied by the SRSS and Turkstra methods.

4.6.2 Errors in the Poisson Assumption

It is necessary to try to evaluate the errors which might occur for the design life failure probability when the Poisson assumption is used in the conditional first passage probabilities.

It is, at present, not feasible to incorporate the finite element solution in the load combination problem because of the number of times a solution is required for the case where the intensity and lag times are random variables.

A very close (and conservative) approximation to the first passage probability has been obtained empirically by Lutes, Chen and Tzuang (1980) by fitting curves to simulation results for different values of the percent of critical damping. The form of the solution for the failure probability is

$$P_f(t) = 1. - \exp[- \int_0^t \eta(\tau) d\tau] \qquad (4.30)$$

With the results obtained by Bergman, avoiding the need for simulation and providing a much broader data base, a still better empirical fit is now possible.

However, the result given by Lutes is used in this study to examine the overall effect of the Poisson assumption. Figure 4.14 shows a comparison of the failure probability of a linear structure subjected to a white noise excitation, for low thresholds, given by the Poisson assumption, Vanmarcke's approximation, Lutes' empirical fit, and Bergman's finite element solution.

Both Vanmarcke's approximation and Lutes' result have been used for the conditional probability in the load coincidence calculations and the comparison with the Poisson assumption is shown in Fig. 4.15. For the design life failure probability the difference between Vanmarcke's and Lutes' methods is not visible and the Poisson assumption provides sufficiently close yet conservative results.

4.6.3 Effects of the Mean Duration Approximation

In Section 4.3.2 mention was made of the use of the mean duration in calculating the conditional failure probabilities. It can be shown analytically that for a given mean (static) component, the use of the mean duration in the evaluation of the probability is conservative for exponentially distributed duration. The same can be done for the coincident loads assuming a gamma distribution for the coincident duration as it is no longer exponential.

To support this statement, without details of the analysis, a Monte Carlo simulation was performed. Two Poisson processes of mean occurrence rates λ_1 and λ_2 are first simulated. For each load occurrence (point in the Poisson process) an exponentially distributed duration, and a gamma distributed static component are generated. Due to the excessive cost of simulating continuous processes at each occurrence, the assumption of Poisson crossings is used to simulate the maximum of the dynamic response for the actual load and coincident durations. The maximum of the combined process is found and the procedure repeated for the required sample size. The simulation results (circles) are plotted in Fig. 4.16 together with the results obtained analytically (solid line) using the load coincidence method. The good comparison provides validity for the use of the mean duration (a first order approximation) in the computations.

4.7 Nonlinear Failure Surfaces

For failure surfaces defined by some nonlinear function of the basic variables the problem becomes one of a multi-dimensional continuous process exiting the safe domain. The conditional probability of failure given occurrence and coincidence can be evaluated using the Poisson approximation, if the outcrossing rate can be obtained.

The crossing rate out of the safe domain may be given by a generalization of the result given by Rice (1944)

$$\nu_D = \int_{\Gamma_D} d\underline{x} \int_0^\infty \dot{x}_n \, f_{\underline{x},\dot{x}_n}(\underline{x},\dot{x}_n)\, d\dot{x}_n \tag{4.31}$$

(Belyaev, 1969).

Veneziano, Grigoriu and Cornell (1977) have utilized this result and specialized the problem to Gaussian processes crossing out of certain regularly shaped domains.

The simplest approximation for the general nonlinear surface is to linearize it at just one point and approximate the real crossing rate by the rate of crossing the tangent hyperplane. The result for Gaussian processes is

$$\nu_D = \frac{1}{2\pi}\left(\sum_i^n \alpha_i^2 \, \sigma_{ii}^2\right)^{1/2} \exp(-\tfrac{1}{2}\beta^2) \tag{4.32}$$

The point at which the linearization should be performed is the point on the surface at which ν_D becomes stationary (either maximum or minimum, depending on the shape of the domain) and is found by an appropriate nonlinear programming technique.

If the domain is known to be better approximated by other shapes such as rectangles or circles, similar results are given for the crossing rates in Veneziano, et al., (1977).

4.8 Inelastic Material Behavior

Structures which undergo inelastic deformations (eg. hysteretic behavior) while subjected to random vibration have been treated by using the method of equivalent linearization (Wen, 1980a). Calculation of the first passage probability is achieved by use of the above-mentioned approximations for the equivalent linear systems. Accuracy of the results of the load coincidence formulation will depend on the accuracy obtainable through the linearization technique and the first passage approximation.

CHAPTER 5

CORRELATED LOAD EFFECTS

5.1 Introduction

The loads considered thus far have been modeled by independent processes. Many loads may show correlation between occurrence times, duration and intensity. An earthquake of large magnitude is usually longer than one of lesser magnitude. The same may be said for a storm. The wave height in a storm at sea is dependent on the intensity of the wind. In the design of nuclear power plants there has been much discussion as to whether a design basis accident load such as a large LOCA could be caused by an earthquake, thereby introducing dependence of the LOCA on earthquake occurrence and intensity. For example, Stevenson (1980) argues that the present practice of allowing only one pipe break to occur for a given earthquake is a non-rational constraint. For if the earthquake could cause a pipe break, even when the piping is specifically designed to resist seismic movements, then it would be quite possible for more than one break to occur considering the amount of piping present in the plant. If no dependence is considered and the LOCA and earthquake are assumed independent then the probability of simultaneous occurrence is so small as to be ignored for usual design basis extreme loads.

We note that even for piping designed to resist an earthquake, there is still a small probability of a break occurring and that this probability and the resulting combined loads ought to be examined.

An event which certainly can be triggered by an earthquake is the release of pressure from a Safety Relief Valve (SRV) in a Boiling Water Reactor (BWR), producing a high probability of the effects of the two loads being combined.

This type of dependent load effect may be modeled with correlated stochastic processes in order to solve for the maximum of combined processes.

Motivation for proposing correlated models is given by the need for a realistic treatment of the dynamic effects experienced by important structures under natural hazards, in particular, Nuclear Power plants (Schwartz, Ravindra, Cornell and Chou, 1980; Ravindra, 1980).

5.2 Correlated Processes

Correlation amongst static load effect processes, in terms of occurrence time, duration and intensity, has been considered for linear combinations of rectangular pulse processes in Wen and Pearce (1981). For dynamic load effects the correlation between individual responses is considered here by assuming that the mean values Y of the response processes exhibit dependent behavior. When the process has a mean of zero, dependence is assumed between the measures of intensity (spectral density) of the loads.

In the following, the additional modeling required for dynamic load effect processes is described. The models describing the correlation structure, and much of the analysis, generally follows that in Wen and Pearce (1981). Two general types of dependence will be considered; (1) Within-load dependence in which correlation exists only between the parameters of one load type, (2) Between-load dependence in which the parameters of one load process are correlated with those of another.

The form of the load coincidence equation remains essentially the same as in Eq. 2.17 though now the occurrence rates and conditional probabilities will be modified to account for the correlation. They will not always be separable as in the independent case.

It is convenient to define the conditional distribution of the maximum response during the pulse of duration d, given the mean value Y, as

$$S(r|y) = \exp\{-\frac{d\omega}{2\pi}\exp[\frac{-(r-y)^2\zeta\omega^3}{\pi S_o}]\} \quad (5.1)$$

and for the coincident loads

$$S(r|y_1,y_2) = \exp\{-\frac{d\omega}{2\pi}\exp[\frac{-(r-y_1-y_2)^2\zeta\omega^3}{\pi(S_1 + S_2)}]\} \quad (5.2)$$

Except for the case of individual pulses with duration-intensity dependence, the expected value of $S(r|y)$ with respect to the duration d, will be approximated by $S(r|y)$ evaluated at the expected duration.

5.3 Within-Load Dependence

Loads from two simultaneously occurring processes are assumed to be statistically independent of each other, but occurrence times, duration and intensity of loads within one process may be correlated.

5.3.1 Duration-Intensity Correlation

Certain natural loads exhibit correlation between their duration and intensity, e.g., intense storms will last longer than minor ones. The occurrence times are assumed independent and therefore the individual occurrence rates κ_i remain unchanged from Eq. 2.17. However, since a longer duration implies a larger coincidence rate and a higher probability of maximum response, the coincident crossing rate $\lambda_{ij}p_{ij}$ will change, but the two terms may not be considered separately. The maximum dynamic response depends on the duration and therefore the individual conditional failure probability will also be altered.

The duration and intensity of the static component are assumed to be jointly normal, for computational ease, although the duration is usually assumed exponential. This approximation should have negligible effect when sparse processes are considered.

The failure probability, given the occurrence, for a load without coincidence is

$$P_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [1. - S(r|y)] \, f_{D|Y}(d|y) \, f_Y(y) dd \, dy \qquad (5.3)$$

The mean crossing rate for the coincident pulses is given, for bivariate normal duration and intensity, by

$$\nu_{12}p_{12} = \lambda_1\lambda_2\mu_1[1. - \int_0^\infty L(r|y_1)\ M_1 f_{Y_1}(y_1)\ dy_1] + \lambda_1\lambda_2\mu_2[1. - \int_0^\infty L(r|y_2)\ M_2 f_{Y_2}(y_2)\ dy_2] \quad (5.4)$$

in which

$$L(r|y_1) = \int_0^\infty S(r|y_1,y_2)\ f_{Y_2}(y_2)\ dy_2$$

$$M_1 = 1 + \rho_{D_1,Y_1} \frac{\sigma_{D_1}}{\sigma_{Y_1}} \left(\frac{y_1 - \mu_{Y_1}}{\mu_{D_1}}\right) \quad (5.5)$$

$\rho_{d,y}$ is the correlation coefficient between D and Y.

The results of this dependence on the non-exceedence probability (NEP) are plotted on type-I extreme probability paper together with the independent case (Figs. 5.2 through 5.8). The values of the parameters used are E[Y] = 1. and $\sigma_Y = 0.3$

The duration-intensity dependence yields a probability, of not exceeding a certain level, smaller than the independent case, ie., a higher maximum combined response is expected. Use of an independent assumption means that an unconservative estimate of the maximum is obtained. However, this difference is small, e.g., at high thresholds the ratio of the dependent failure probability to the independent is only about 1.3 for an almost perfect correlation (ρ = 0.99) Therefore this correlation may not affect a design significantly.

5.3.2 Intensity Dependence

For example, the intensity of pressure transients, in nuclear systems, caused by discharge of a relief valve due to buildup of pressure may depend on the intensity of the previous transient.

This dependence between load pulses in a single process is achieved by imbedding a Gauss- Markov sequence in the pulse process. The value of the mean component of one pulse is dependent on the value of the mean of the previous pulse as follows

$$Y_{k+1} = \rho\, Y_k + \sqrt{1 - \rho^2}\, V_k \tag{5.6}$$

Y_k is the mean of the k^{th} pulse in the process. V_k is an independent normal variate with $E[V] = E[Y]\sqrt{(1-\rho)/(1+\rho)}$ and $\sigma_V = \sigma_Y$. The correlation structure is completely specified by the coefficient ρ.

Let the maximum response in a single pulse, the sum of the static and dynamic components, be Z_i for the i^{th} pulse. Then the conditional probability distribution of Z_2 given Z_1 is

$$L(Z_2|Z_1) = P(Z_2 \le r|Z_1 \le r) = \frac{F_{Z_2Z_1}(r,r)}{F_{Z_1}(r)} \tag{5.7}$$

in which

$$F_{Z_1}(r) = \int_0^\infty S(r|y_1)\, f_{Y_1}(y_1)\, dy_1$$

$$F_{Z_2Z_1}(r,r) = \int_0^{\infty}\int_0^{\infty} S(r|y_1)\, S(r|y_2)\, f_{Y_2Y_1}(y_2,y_1)\, dy_2 dy_1 \tag{5.8}$$

where $S(r|y)$ is the probability that the maximum response is less than r given that the load intensity is y (Eq. 5.1).

The joint density is derived from the product of a conditional density and a marginal density. The conditional density is obtained from the relationship given in Eq. 5.6

$$f_{Y_2|Y_1}(y_2|y_1) = \frac{1}{\sqrt{1-\rho^2}} f_{V_1}\left(\frac{y_2 - \rho y_1}{\sqrt{1-\rho^2}}\right)$$

$$f_{V_1}\left(\frac{y_2 - y_1}{\sqrt{1-\rho^2}}\right) = \frac{1}{\sqrt{2\pi}\,\sigma_Y} \exp\left\{-\frac{1}{2}\left[\frac{y_2 - \rho y_1 - (1-\rho)\mu_Y}{\sqrt{1-\rho^2}\,\sigma_Y}\right]^2\right\} \tag{5.9}$$

$$f_{Y_2Y_1}(y_2,y_1) = \frac{1}{2\pi\,\sigma_Y^2\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2}\left[\left(\frac{y_2 - \rho y_1 - (1-\rho)\mu_Y}{\sqrt{1-\rho^2}\,\sigma_Y}\right)^2 + \left(\frac{y_1 - \mu_Y}{\sigma_Y}\right)^2\right]\right\}$$

Considering the Poisson character of the occurrence of load pulses, the probability of the maximum in T years of a single process of non-coincident pulses not exceeding a threshold r, is given approximately by (Wen and Pearce, 1981)

$$P(R_1 \le r) \approx \frac{F_{Z_1}(r)}{L(Z_2|Z_1)} \exp\{-\kappa_1 t[1 - L(Z_2|Z_1)]\} \tag{5.10}$$

The coincident pulse sequence is also treated as Markovian and an equivalent one-step correlation coefficient is computed from the parameters of the individual processes and the coincidence rate.

The non-exceedance probability $P(R_{12} \leq r)$ of the coincident process is then obtained in a manner similar to that above and the combined NEP may then be given by the product

$$NEP = P(R_1 \leq r)\ P(R_2 \leq r)\ P(R_{12} \leq r) \tag{5.11}$$

For correlation coefficients less than 0.9 there is no visible difference between the dependent and independent cases. As the coefficient goes from 0.9 towards 1.0 the difference increases at low thresholds but the independent results are always conservative. For perfect correlation (ρ = 1.0) all the pulses in the process have the same static component and the exceedance probability is greatly reduced at low thresholds (Figs. 5.2 to 5.8).

5.3.3 Occurrence Dependence (Clustering)

For example, safety relief valves are triggered in clusters in order to discharge the pressure buildup in a BWR containment. Clusters of tornadoes may be spawned by a single storm.

Such clustered processes are modeled by a point process of the Bartlett-Lewis type in Wen and Pearce (1981). In that study the loads were considered to be static rectangular pulses. However, the addition of a dynamic response will have no effect on the relative results since clustering has significant effect only on the coincidence rate and therefore the conclusions may be drawn from that first study. These are that the overall effect of clustering is slight and produces a lower lifetime combined maximum.

5.4 Between Load Dependence

Times of occurrence or the intensity of a load may be affected by a load of a different type producing dependence effects between load processes.

5.4.1 Intensity Dependence

The intensity of natural dynamic loads such as wind, wave and surge, or wind and snow in a single storm, may be quite correlated for short times after their arrival. For such loads their occurrence may also be highly correlated, but in this first section the occurrence times and durations are assumed to be independent.

The correlation structure of the intensity of the loads, considerindg two processes, is described by conditional auto- and cross-correlation functions given the occurrence of a load pulse. The intensities of the pulses are then sampled from a continuous vector process (see Fig. 5.1) with correlation matrix (for stationary processes)

$$\begin{bmatrix} R_{11}(\tau) & R_{12}(\tau) \\ R_{21}(\tau) & R_{22}(\tau) \end{bmatrix} \qquad (5.12)$$

e.g., $R_{12}(\ )$ = cross-corrrelation function betweeen the processes; τ = occurrence time difference.

The within-load intensity correlation is accounted for approximately using the Gauss-Markov result given in section 5.3.2 with an equivalent correlation coefficient

$$\rho_i = \frac{[R_{ii}(\frac{1}{\lambda_{ii}}) - E^2(Y_i)]}{\sigma^2_{Y_i}} \tag{5.13}$$

Since the maxima of the individual processes are now correlated, a Gumbel type B bivariate extreme value distribution is utilized to find the distribution of the combined non-coincident maximum.

Using Eq. 5.13 and the joint distribution of Y_1 and Y_2 (a bivariate Gausian is used here) one can obtain the conditional exceedance probability of the maximum response given coincidence as

$$F^*_{12}(r) = \int_0^\infty \int_0^\infty [1 - S(r|y_1,y_2)]\ f_{Y_1Y_2}(y_1,y_2)\ dy_1 dy_2 \tag{5.14}$$

in which $S(r|y_1,y_2)$ = probability of maximum response given y_1 and y_2. The distribution of the maximum response due to the coincident process is

$$F_{R_{12}}(r) = \exp[-\lambda_{12}\ t\ F^*_{12}(r)] \tag{5.15}$$

The maximum due to the noncoincident and the coincident parts of the combined process are then assumed independent to compute the overall maximum.

Between-load intensity correlation can have a large effect on the maximum of combined intermittent dynamic load processes, as shown in Figs. 5.2 through 5.8. For extremely sparse processes ($\lambda\mu$ very small) the coincidence term is negligible and the main effect is that of within-load correlation, which causes a lower combined maximum at medium and low threshold levels (Figs. 5.2 and 5.3).

As the product $\lambda\mu$ increases (the processes are less sparse), coincidence is more likely to occur and the coincident term begins to become dominant. The exceedance probability in turn is greatly increased, as compared to independent loads, to the extent that at high thresholds it is larger by a factor of 65 (Figs. 5.7 and 5.8).

5.4.2 Occurrence Clustering among Loads

This type of clustering is usually due to some type of causal effect that one "parent" load may have. The loads will be clustered around some common point in time (Fig. 5.9) therefore significantly increasing the likelihood of coincidence. In this section, to examine this clustering effect, the intensity and duration of the loads are assumed to be independent and only occurrence times are correlated. It is recognized that for loads such as wind and wave there may well be correlation between the intensities and durations of the two processes and this can be incorporated in the model (Wen and Pearce, 1981).

The derivation of the coincidence rate analysis is achieved through use of a conditional occurrence rate function details of which may be found in Cox and Lewis (1972) (therein called "cross intensity"). To show the effect of the correlation an example is given here of an idealized transient load due to discharge of a pressure buildup from a safety relief valve in a Boiling Water Reactor. The SRV load may occur independently as pressure builds up in the vessel or may be triggered by an earthquake with a random delay (lag) time. The SRV load effect is idealized as an impulse response of random amplitude while the earthquake is modeled as a Gaussian shot noise (modulated white noise). The probability that an earthquake triggers an SRV load is P_s, which will depend on the intensity distribution of the earthquake.

For exponentially distributed earthquake duration and lag time the probability of coincidence of the SRV with the earthquake is

$$P(\theta \leq d_e) = \frac{\mu_e}{\mu_e + \mu_\theta} \tag{5.16}$$

μ_e and μ_θ are the mean values of the earthquake duration and SRV lag time. The total coincidence rate of the two processes (including the independent SRV) is

$$\lambda_{es} = \lambda_e \lambda_s (\mu_e + \mu_s) + \lambda_e P_s \left(\frac{\mu_e}{\mu_e + \mu_\theta}\right) \tag{5.17}$$

λ_e, λ_s = occurrence rate of earthquakes and independent SRV loads. μ_s is the mean SRV load duration. The first term is the coincidence rate due to the independent SRV and the second term is that triggered by the earthquake. Depending on the parameters considered, the second term may dominate the coincidence rate.

The failure probability conditional on coinciding events is the probability that the sum of a Gaussian random process and an impulse response of random amplitude exceeds a threshold r. This may be interpreted as the probability of the random process exceeding a time-dependent threshold of deterministic shape (impulse response) but random amplitude.

The crossing rate of the time-dependent threshold $g(t) = r - u(\hat{t})$, where $\hat{t} = t - \theta$, by the nonstationary Gaussian process, Fig. 4.2, is

$$\nu(t) = \frac{\sigma_{\dot{x}}}{\sigma_x}\sqrt{1-\rho^2}\;\phi\left[\frac{u(\hat{t}) - r}{\sigma_x}\right]\{2\phi(G) - G + 2G\Phi(G)\}$$

$$G = \frac{\left[\dot{u}(\hat{t}) - \frac{\sigma_{\dot{x}}}{\sigma_x}\rho\{u(\hat{t}) - r\}\right]}{\sigma_{\dot{x}}\sqrt{1-\rho^2}} \qquad (5.18)$$

$$u(\hat{t}) = \frac{A}{\omega_d}\sin\omega_d\hat{t}\;e^{-2\zeta\omega\hat{t}} \qquad ; \qquad \hat{t} \geq 0$$

$$\rho(t) = \frac{Cov[x\dot{x}]}{\sigma_x\sigma_{\dot{x}}}$$

For the piecewise linear modulation function used here, it is possible, through lengthy integrations, to obtain the variances and covariance of the response process and its derivative analytically.

The conditional probability of failure is then given by

$$p_f(\mu_e) = \int_0^{\infty}\{1. - \exp[-\int_0^{\mu_e}\nu|_{S_o}(t)\,dt]\}\,f_{S_o}(s)\,ds \qquad (5.19)$$

in which the mean earthquake duration and SRV lag time, together with the mean amplitude are used as first-order approximations.

The probabilities of lifetime maximum combined earthquake and SRV response are compared in Fig. 5.10 for independent processes and for the correlated processes. The exceedance probability increases significantly due to the clustering effects. When both the intensity and the occurrence times are correlated among load processes, the effect may produce very much higher failure probabilities at high thresholds.

5.5 Conclusions

We have shown that the load coincidence method is well suited for the treatment of correlated load processes of the intermittent continuous type. Results of the non-exceedence probability of combinations of these processes show similar trends to those found for static Poisson pulse processes. Serious consequences may arise if certain correlated effects are neglected in a design, the most important ones being those due to intensity dependence between loads and also occurrence clustering between loads.

CHAPTER 6

APPLICATION TO STRUCTURAL CODES

6.1 Introduction

6.1.1 Code Formats and Load Combinations

There is agreement amongst a growing number of engineers that because of the random nature of loads and the scatter in the data of material strengths, a probabilistic treatment provides a consistent means whereby new structural codes may be developed. The trend is to propose a limit states design methodology with load and resistance factors provided to ensure a specified level of component reliability. The main reason for this proposal is that a code should account for all design situations including different types and numbers of loads, geographical location, intended structural use, and different materials and this is best accomplished by a number of factors rather than by one global factor.

The selected formats in two codes are shown here as examples of partial factor limit states checking equations. The designer is given a number of load combinations (checking equations) together with the load factors and characteristic values of the loads. These loads are substituted, in turn, into the load combination equations and the most severe load combination is the one that governs the design.

The National Building Code of Canada (NBCC) has adopted the format

$$\phi R \geq \alpha_D D + \gamma\psi[\alpha_L L + \alpha_Q Q + \alpha_T T] \tag{6.1}$$

where D,L,Q,T are the dead, live, wind or earthquake, and deformation loads respectively. The α are the load factors, ϕ is the resistance factor, γ the importance factor (usually = 1.0), and ψ is the combination factor.

$$\psi = \begin{cases} 1.0 \text{ for one load} \\ 0.7 \text{ for two loads} \\ 0.6 \text{ for three loads} \end{cases}$$

The proposed format for the Load and Resistance Factor Design of steel structures in the U.S.A. is;

$$\phi R \geq \gamma_D D + \gamma_i Q_i + \sum_{j \neq i}^{n} \gamma_j Q_j \tag{6.2}$$

where the γ are the load factors, Q_D is the dead load, Q_i is the maximum lifetime value of one of the variable loads and the Q_j are the arbitrary point in time values of the other variable loads.

Both of these formats recognize the fact that when loads which vary with time act on a structure, it is extremely improbable that their maximum values will occur simultaneously.

When the nominal values of the loads are obtained from the statistics of the maxima it is necessary to apply a load combination factor to reduce the combined load. This is what is done in the NBCC with the factor ψ.

The approach taken by the developers of the LRFD specifications is to make use of Turkstra's rule (section 4.5.3) (Turkstra, 1970). Combination of the loads requires summation of the maximum value of one load with the arbitrary point in time value of the other loads. In the actual design process only the characteristic (usually a percentile) values of the maxima will be used so that the load factor on a load which should take on its point in time value in a combination will be less than 1.0 while that on a load taking its maximum value will be greater than one.

These two methods of dealing wih the combination of loads which vary with time are simplified methods thought necessary for use in structural codes due to the way in which design loads are specified in today's codes. In the first case the designer actually applies the reduction factor, whereas in the second Turkstra's rule is used by the code-writing committee and the designer merely applies the partial load factor specified.

An extensive study was undertaken by Ellingwood, Galambos, MacGregor and Cornell (1980) in developing the latest code requirements for minimum loads in buildings (ANSI A58.1-1982). We remark here on some observations and questions raised in that study in light of the methodology developed herein and the findings obtained so far.

6.1.2 Code Calibration

The purpose of code calibration (Siu, Parimi and Lind, 1975) could be described as avoiding the introduction of levels of reliability less than the least acceptable in current practice.

This requires the evaluation, through probabilistic modeling and analysis, of the current implied reliability (termed notional reliability because it is not absolute) present in the codes. Ellingwood, et al., obtained these values, for different load combinations, in the form of reliability indices.

It was found that the reliability indices implied by current design equations for combinations including earthquake loads were lower than those including wind loads, which in turn were lower than those including just live loads. It was decided to retain this practice in the new specifications.

6.1.3 Adolescence

We should note that, although probabilistic methods have been used throughout the development of the new specifications (ANSI A58), in the end, compromises were deemed necessary to ensure a smooth transition for engineers and for the safety of the structures.

This may, in part, be put down to the fact that, though structural reliability theory has been termed a healthy adolescent, (Cornell, 1982), it still is in its adolescence. The new specifications are a natural step along the way to becoming an adult.

We believe that the load coincidence method of combining time varying loads will provide understanding and a tool for growth.

6.2 Code Optimization

The objective of a design is twofold; (1) to achieve an acceptable level of safety, and (2) to do so at the least possible cost. Ravindra and Lind (1973) wrote that "structural code optimization is the search for a rational balance between society's investment in structures and the reliability achieved." Ideally, this can be done by considering costs associated with construction and the consequence of failure, with the design variables being chosen such that an optimum can be reached in terms of risk-benefit trade-off.

Optimization over the whole data (design) space is an enormous task. One of the major difficulties is, of course, putting a value on the loss of a human life due to structural failure. At the next level, a design can be based on an acceptable level of probability of failure (limit state being exceeded). For example, what has been attempted by the ANSI study is such a risk-based code without direct consideration of cost. However, an allowable risk of failure can be satisfied by an infinite number of designs, but the desired one is the one which is an optimum in terms of yielding minimum cost. Calibration with present practice may put some constraint on cost and therefore may not be an optimum.

The present format in structural codes (Section 6.1.1) obviously has limitations in achieving the target reliability in all the designs because one equation attempts to take account of a wide range of structural types and loads. For a given design situation (structure type, geographic area) one of the combination equations is meant to dictate what resistance is required. That is, knowing the loading situation one should be able to recognize which will be the determining equation for the design. This combination equation carries with it an implied reliability which the component should attain. This certainly is true when one combination clearly dominates the others, e.g., wind is the major force to consider and there is little earthquake risk.

However, when two combinations give similar results (e.g., D+L+W and D+L+E) the implied reliability is undefined in the current procedure and is obviously lower than under any particular load combination. We say this because one combination gives a value of 2.5 for the reliability index while the other gives a value of 1.75, yet the design is the same for both.

No combination including both wind and earthquake appears in the specifications because, though both may occur at one site during the 50 year design life, they have small likelihood of occurring simultaneously at combined values exceeding the maximum of one load (Turkstra and Madsen 1980). However, the exclusion of the one load is necessarily an unconservative step, most significant when the two combinations are similar in value.

This is demonstrated by making use of the load coincidence method in the design of a member and the evaluation of the consequent reliability for different combinations. The procedure first requires the stochastic process modeling of the loads (D,L,W,E) at a given location so that the extreme value distributions match those given in the ANSI study (Ellingwood, et al., 1980) for the design life of 50 years.

In reality, of course, a structure (member) will be subjected to all the load processes over the design life, rather than a given code specified load combination. The reliability of a member with given resistance is therefore calculated for the total combined process using the load coincidence method. To obtain the best design which satisfies the reliability constraint, a simple one dimensional search is utilized until the minimum resistance is obtained. Having obtained the required design, its reliability for different combinations of the loads is again calculated using the load coincidence method.

Results of the reliability indices for the different combinations are shown in Table 6.1 for different ratios of the varying loads (L,W,E) to the dead load (D). The optimum (minimum) resistance, R, is also given as a ratio of the dead load. The target reliability index for the total load combination is 2.33 (failure probability = 0.01). The index implied by each combination differs most from the target index when two combinations show similar design requirements. This effect may be serious for situations where many combinations of numerous loads are prescribed, such as in the design of nuclear power plants.

Table 6.1 Reliability Index for Different Load Ratios and Combinations

R/D	D	L/D	W/D	E/D	β_{D+L}	β_{D+L+W}	β_{D+L+E}
3.21	1.0	.34	0.5	.45	4.35	3.09	2.37
3.06	1.0	.34	0.5	.40	4.22	2.90	2.40
2.94	1.0	.34	0.5	.35	4.10	2.74	2.46
2.92	1.0	.34	0.5	.34	4.09	2.70	2.50

The values of the loads given in the ANSI specifications are the characteristic 50 year extreme values and, as such, are random variable representations of the loads. It has been suggested (Larrabee 1978) that due to the second moment procedure for calculation of the load and resistance factors, a load combination method which uses only a random variable representation of the loads is required for reliability analysis. This is found in the method described in Section 4.5.3 known as Turkstra's rule. Turkstra's method was used by Ellingwood, et al., in developing the new ANSI specifications.

An arbitrary point in time statistic as well as a lifetime maximum statistic is required for each load. Only extreme characteristic values are used in the code, which explains the live load factor of 0.5 when combined with wind load ($\gamma_W = 1.3$).

However, as has been found in this study as well as in Wen (1980a), Turkstra's rule yields unconservative results for the maximum in (0,t) of the combined process and for certain load types and probability levels this unconservatism can be quite serious. The load coincidence method gives a more realistic treatment of the problem yet computationally is still efficient. Therefore it can be used in the formulation of a risk-based building code as exemplified in the foregoing. This can be done so that the optimum resistance is obtained for the given loads and desired reliability. Having obtained the required resistance, the load and resistance factors, consistent with current code formats, can then be evaluated. These nominal loads are taken from the distribution of the maxima in the 50 year design period. Therefore, second moment methods (togeher with some full distribution information) may be applied, using the extreme distributions of the loads (as in ANSI) to calculate the load factors.

Summarizing: given a design life target reliability, the load coincidence method enables the calculation of the required reliability for any given combination of loads; and combined with a second moment analysis, permits the computation of load factors which are more internally consistent than the current formulation and yield a design of prescribed reliability against all loads.

6.3 Choice of the Target Reliability

The discrepancies between the reliability indices implied by current practice for combinations of dead and live loads, and those including either wind or earthquake loads have been mentioned earlier. The question is how are these differences to be interpreted and can we tolerate such low reliability for members subjected to earthquakes?

The committee developing the new specifications decided to retain these values of the reliability index for the new codes so as not to upset present practice when insufficient data is available to suggest radical changes. This seems to be the prudent thing to do right now. However there are some pressing questions in light of this which do need attention.

One statistic given to back up this decision is that buildings appear to be surviving well thus far. It is true that successes are documented but it should be pointed out that statistically speaking we do not have sufficient experience with earthquake loads to back up the statement.

It has been suggested that the computed reliability is an apparent reliability due to mitigating effects which do not occur due to live loads, e.g., live loads act directly on the members, whereas the lateral forces of earthquake and wind act on the whole structure. The way the analysis was performed in the study by the NBS would not allow these differences to affect the final result unless the statistics of the earthquake forces were erroneous. The study took no account of how the load was applied, but considered only a number of ratios of the load effects.

Another suggested reason for the lower reliability is the use of the load reduction factor of 0.75 when combining D+L+W or D+L+E (Galambos, et al., refer to this as the one third increase in allowable stress). The implication is that the resulting value of the load is an unconservative estimate of the maximum of the combined loads, and the factor would have to be increased. To demonstrate that this is not so, we compare the failure probability implied by using the LRF and the load coincidence methods. The loads considered are the combination of dead, live and wind. The extreme (50 year maximum) statistics are those used in the ANSI study. The stochastic process models are computed such that the extreme values match those of the 50 year maximum used above. The mean occupancy for the live load process is 8 years and the mean occurrence rate of significant wind is 10 per year. The result is plotted in Fig. 6.1 showing the conservative use of the LRF of 0.75. Numerical integration was used with full distribution information, for the computation of the probabilities. The SRSS rule for three loads gives similar results to the load reduction factor procedure with a factor of about 0.6. A factor of 0.75 will therefore always be more conservative than the SRSS. The conservatism depends on the mean occurrence rate λ of each of the loads when the conditional failure probability given occurrence is independent of the duration (i.e., static loads). For dynamic loads the LRF is conservative for all λ at medium and large failure probabilities.

It is important that research is continued to evaluate the accuracy of the earthquake statistics and the reliability calculated in the ANSI study before an optimizing risk-benefit study can be initiated.

A quite plausible explanation for the reduced reliability is our willingness to accept damage due to rare natural events such as extreme winds and earthquakes. We are not, however, willing to accept the same kind of damage due only to some excessive live load. This is evidenced by the public reaction to failures due to both of these causes.

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 Summary

The problem of evaluating the probability that a structure becomes unsafe under a combination of loads, over a given time period, is addressed. The loads are modeled by stochastic pulse processes, characterized by the mean occurrence rate and the mean duration of the pulses and by a description of the variation of the load within a pulse. Static loads (effects) are represented by pulses with some specified shape. Dynamic loads are modeled by a zero-mean Gaussian process, superimposed on a rectangular static component for those loads with non-zero mean.

Exact solutions to the load combination problem are, in general, difficult rare and an upper bound solution given by the mean number of crossings in (0,t) is usually sought.

The load coincidence method (Wen 1977) is extended to problems with both nonlinear limit states and to dynamic responses, including the case of correlated dynamic responses.

Linearizing a nonlinear limit state is a common way of obtaining reliability estimates for time invariant problems. This same technique is investigated for time-varying combination porblems, with emphasis on selecting the linearization point.

The load coincidence method is formulated also for dynamic combinations and compared with other methods, namely the upcrossing rate methods. Use of simpler rules (SRSS and Turkstra) for load combination is also compared with the results from the more complete stochastic process models.

Correlated effects amongst dynamic loads are examined to see how results differ from correlated static loads, and to demonstrate which types of load dependencies are most important, i.e., affect the exceedance probabilities the most.

Application of the load coincidence method to code development is briefly discussed.

7.2 Conclusions

Conclusions from this study may be summarized as follows;

1. There is a uniquely optimum point at which a nonlinear limit state should be linearized for approximate evaluation of the crossing rate out of the domain. The point is the stationary point of the mean crossing rate out of the tangent hyperplane, and is found by a suitable nonlinear programming algorithm. For a concave safe domain the point is a local minimum, whereas

for a convex safe domain it is a local maximum.

2. The load coincidence method may be easily applied to the case of a nonlinear limit state. When the conditional failure probabilities can be obtained exactly the results are very good especially for sparse processes, i.e, they give close, conservative estimates of the overall failure probability. When the exact conditional failure probabilities are unobtainable, approximate convolution by the Rackwitz/Fiessler algorithm may yield sufficiently accurate results. However, depending on the shape of the domain and the probability distributions of the basic variables, it is not always possible to say whether the estimate is conservative or not.

3. New techniques (e.g., the multiple checking point approach; polynomial fitting) for evaluating the probability content of a nonlinear domain can be easily incorporated in the load coincidence method.

4. The "crossing rate" methods, which use the crossing rate of the dynamically fluctuating process to calculate the expected number of crossings in (0,t), tend to be overly conservative in the evaluation of the failure probability over the prescribed design life, particularly for long load duration, short period structures, and low to medium failure thresholds.

5. Use of the conservative "Poisson approximation", for the conditional first passage probability given occurrence or coincidence of a load, does not have significant effect on the overall lifetime failure probability.

6. Use of the mean duration in the computation of the conditional failure probability is a reasonable first order approximation and greatly reduces computation time and effort. Its accuracy has been carefully examined by analysis as well as Monte Carlo Simulation and found to be very good.

7. The load combination rules (SRSS, LRF, Turkstra) applied in codes and code development are not consistent in their estimation of the combined maximum load for intermittent continuous processes. The SRSS is affected more by the structural parameters and is generally conservative for the high and medium risk levels, while Turkstra's rule is always unconservative and is affected more by the load process characteristics (λ, μ).

8. The dynamic process model used is suited for modeling correlated effects both within and between loads. The analysis is more involved than for correlated static loads but the trends in the results, for the non-exceedance probability of combined loads, are similar.

9. The most significant correlation effects are seen for loads which have intensity dependence between processes and for occurrence clustering between processes. At medium to high thresholds these effects are especially important, and a combination of the two could be even more serious.

10. The present practice of neglecting certain loads in combination equations because of the small probability of their simultaneous occurrence with other loads in the combination, is unconservative. However, the unconservatism is only significant when the two loads are equally important in causing load effects on the structure or when a large number of loads needs to be considered.

11. The load coincidence method can be well utilized in the development of probability based structural codes, by obtaining an optimum design for a given target reliability constraint under a set of real loads. Reliability indices may then be calculated for any given combination of loads and advanced second moment methods used to obtain load and resistance factors.

12. The questions arising from the difference in the reliability of members designed only for live loads and those designed for a combination of live and earthquake loads seems to imply that more research is needed on the accuracy of the earthquake data

before we can be sure that present practice is as it appears.

13. The load coincidence formulation has proved to be a relatively simple, versatile and accurate method for the evaluation of the failure probability of a structure under the combined action of a number of time varying loads.

FIGURES

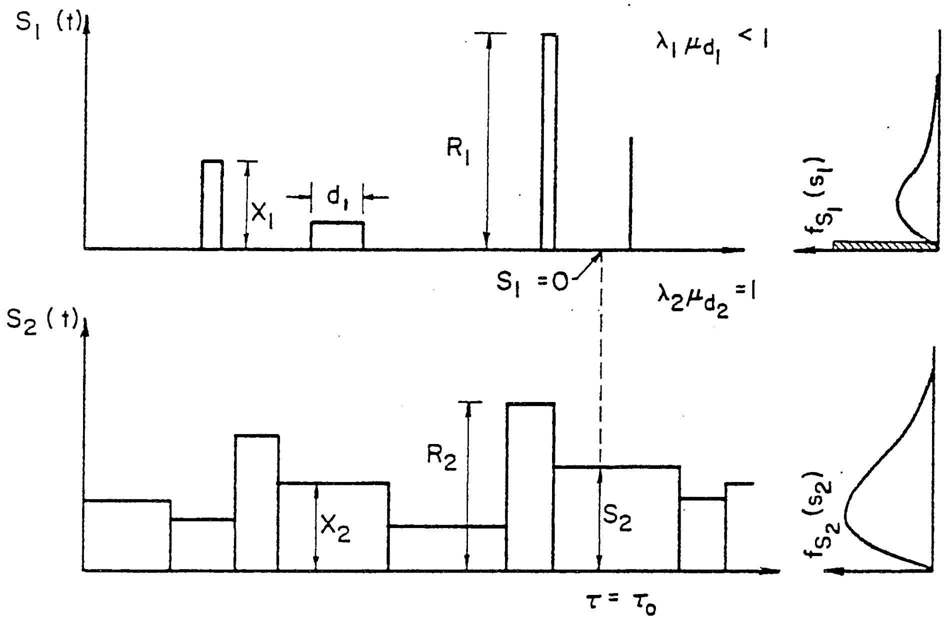


Fig. 2.1 Poisson Rectangular Pulse Processes

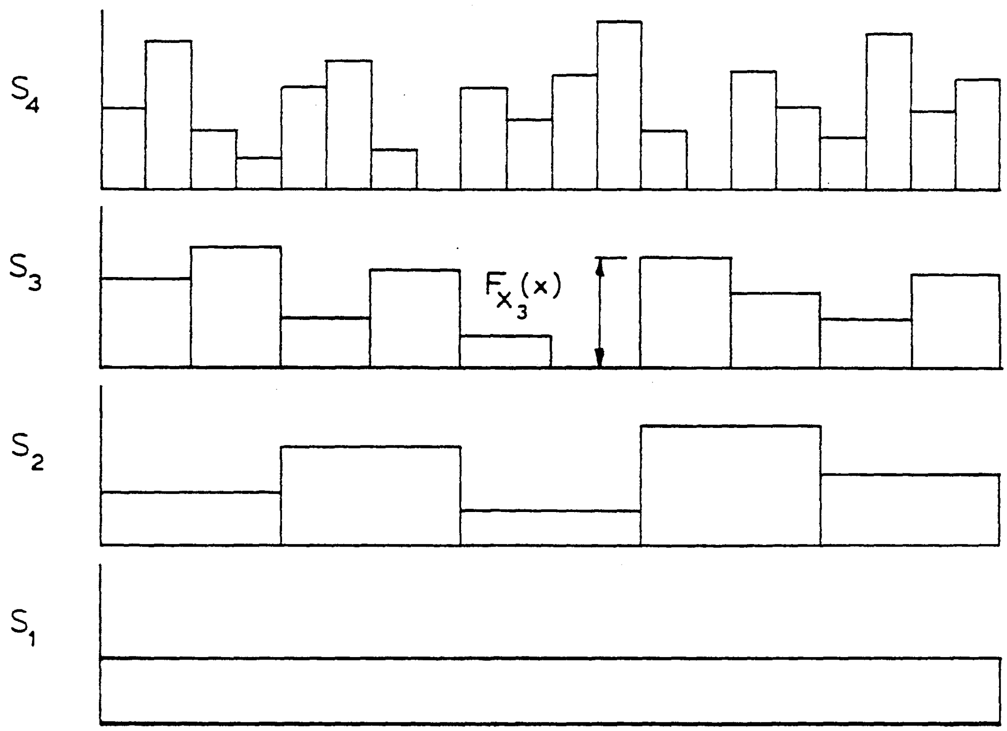


Fig. 2.2 Combination of Ferry-Borges Processes

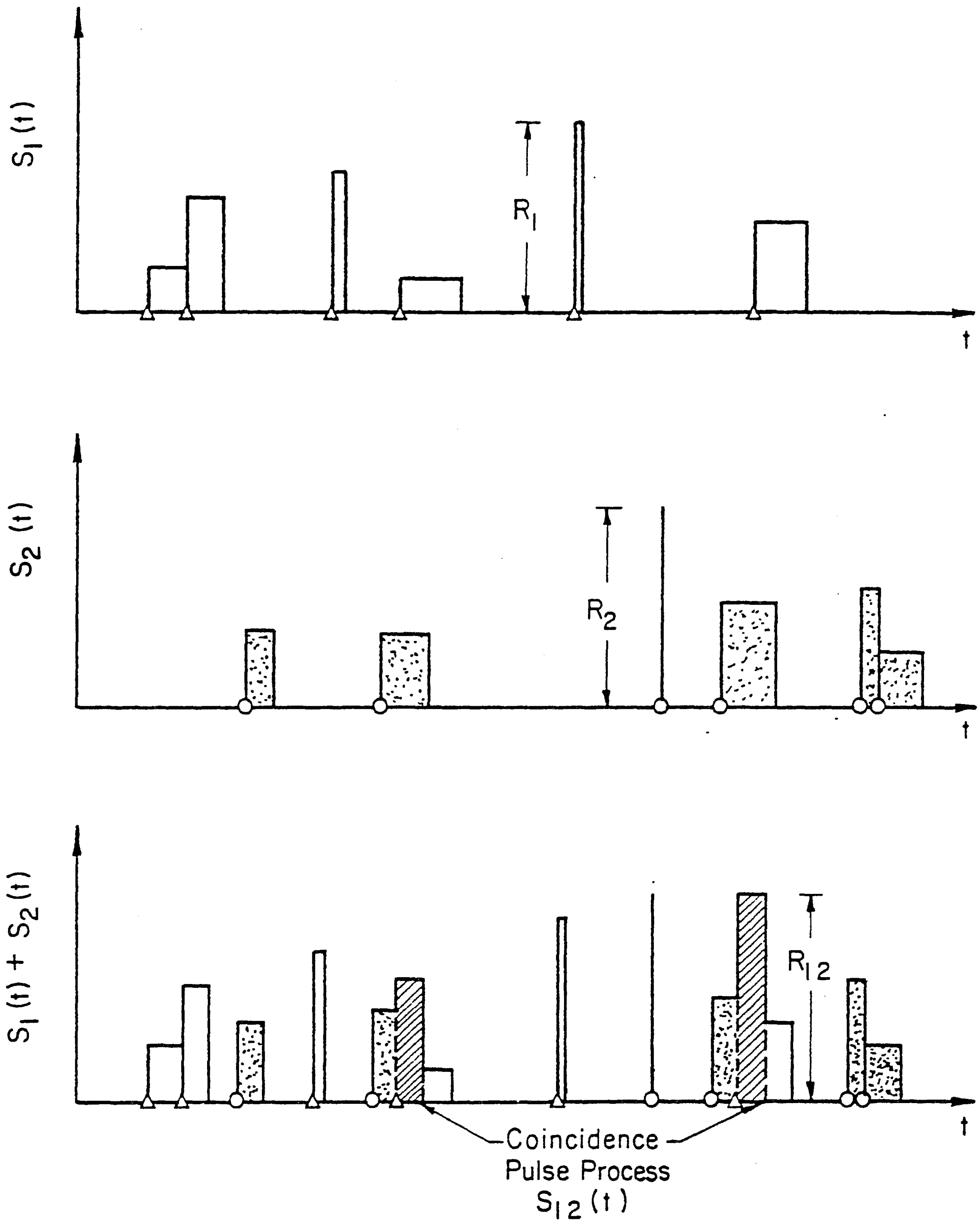


Fig. 2.3 Combined Sparse Poisson Pulse Processes

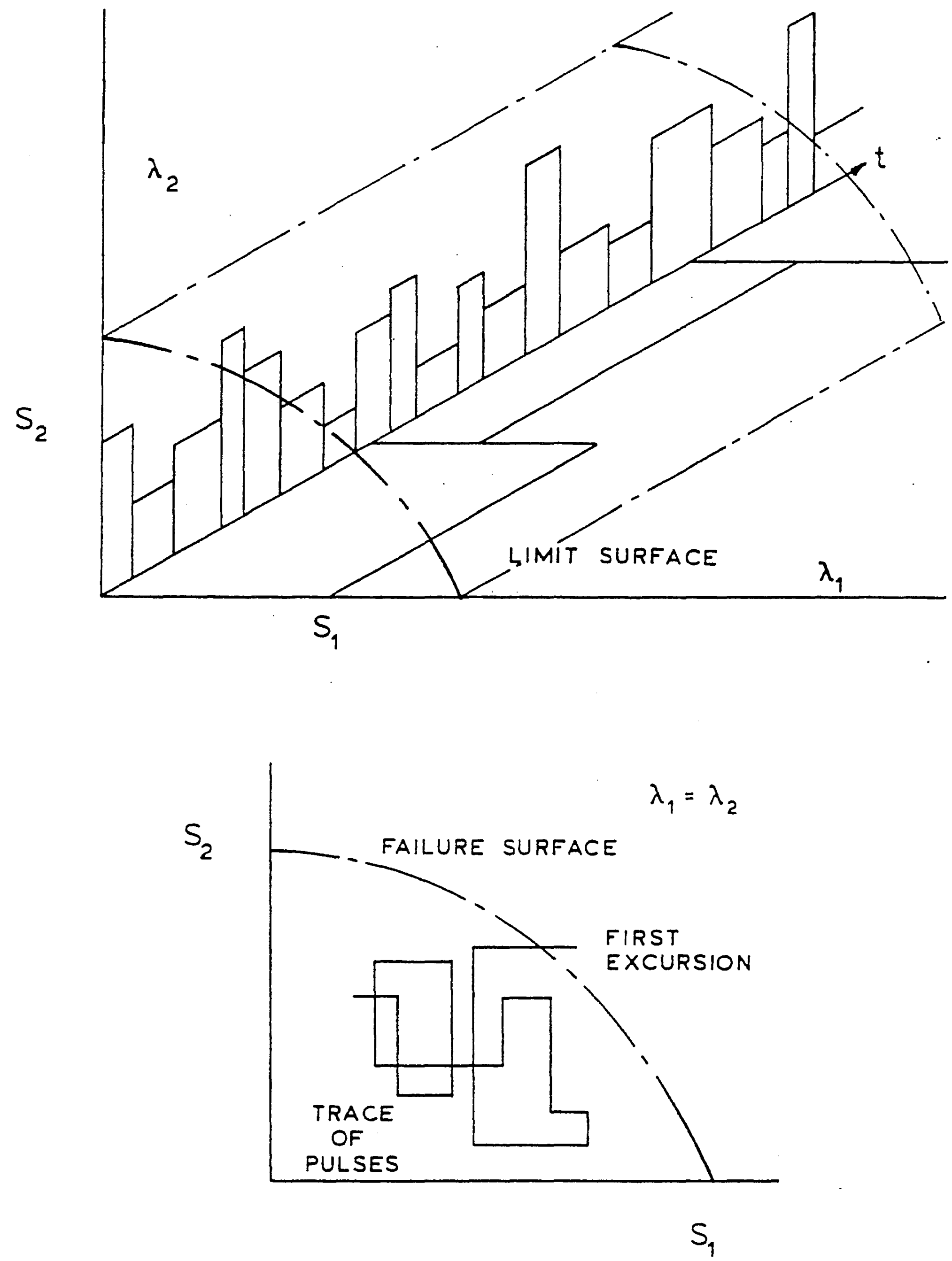


Fig. 3.1 Vector Pulse Process with Nonlinear Domain

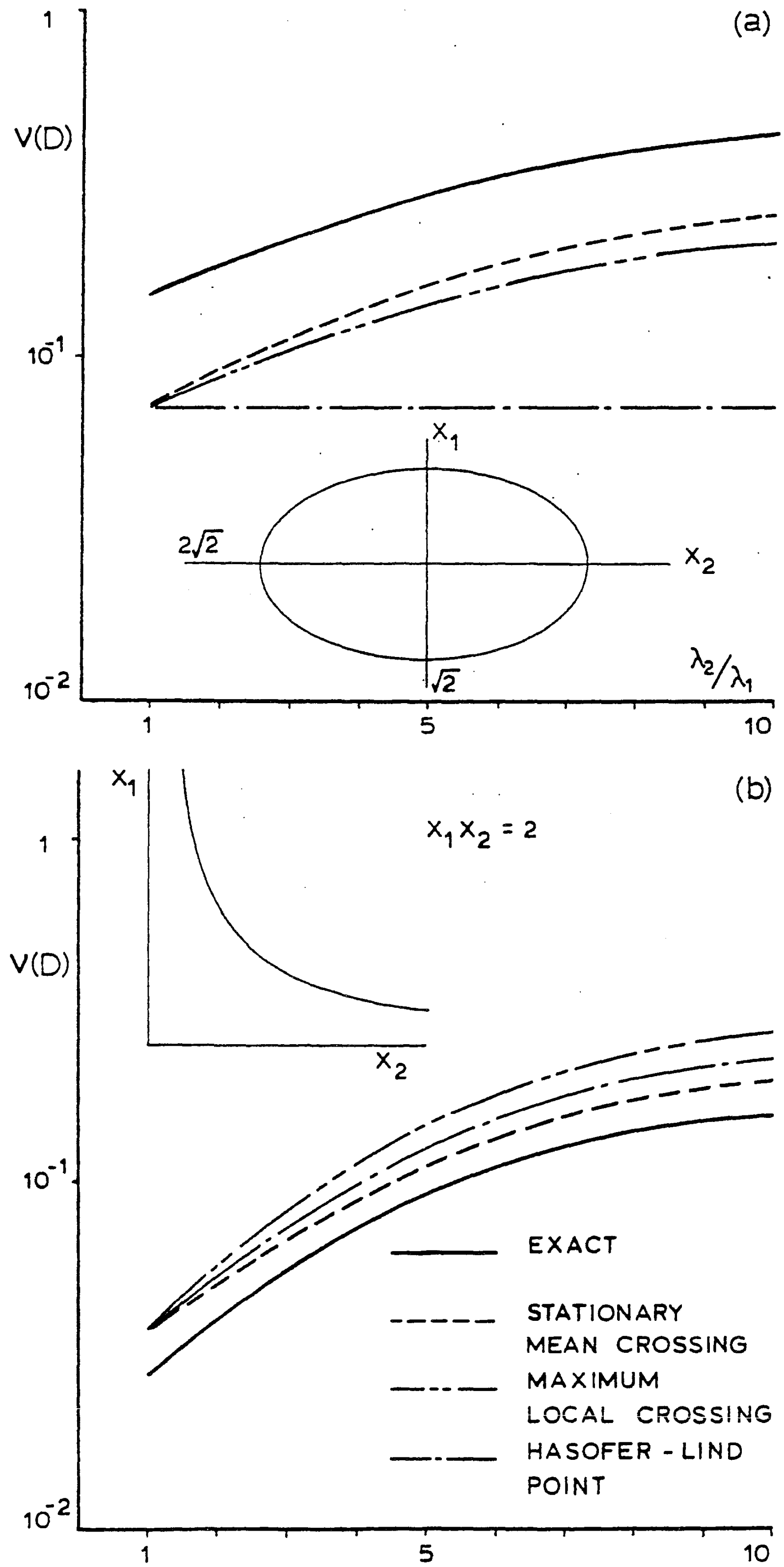


Fig. 3.2 Crossing Rate Comparisons for 3 Linearization Points

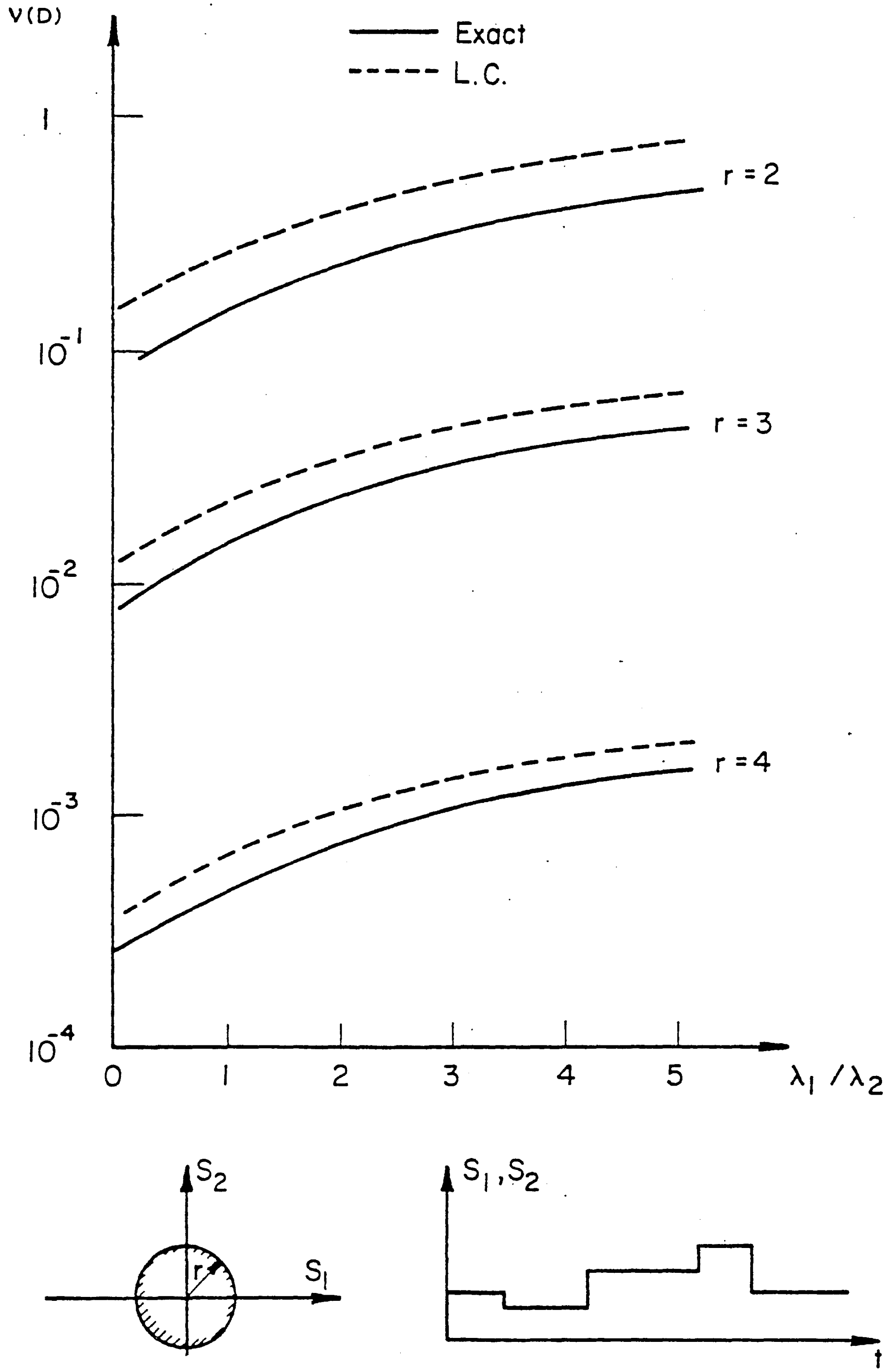


Fig. 3.3 Vector Poisson Square Wave - Circular Domain

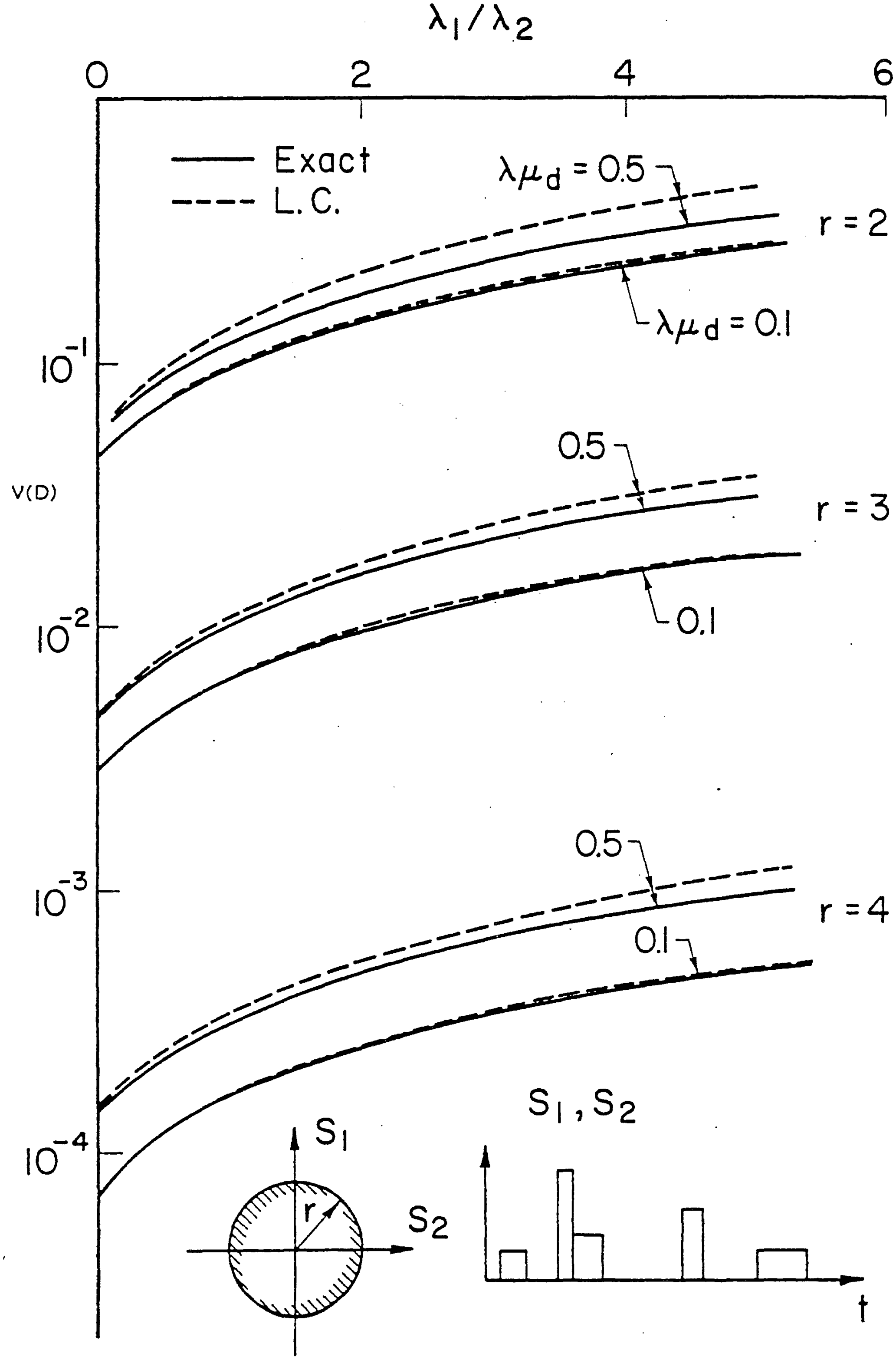


Fig. 3.4 Sparse Poisson Process - Circular Domain

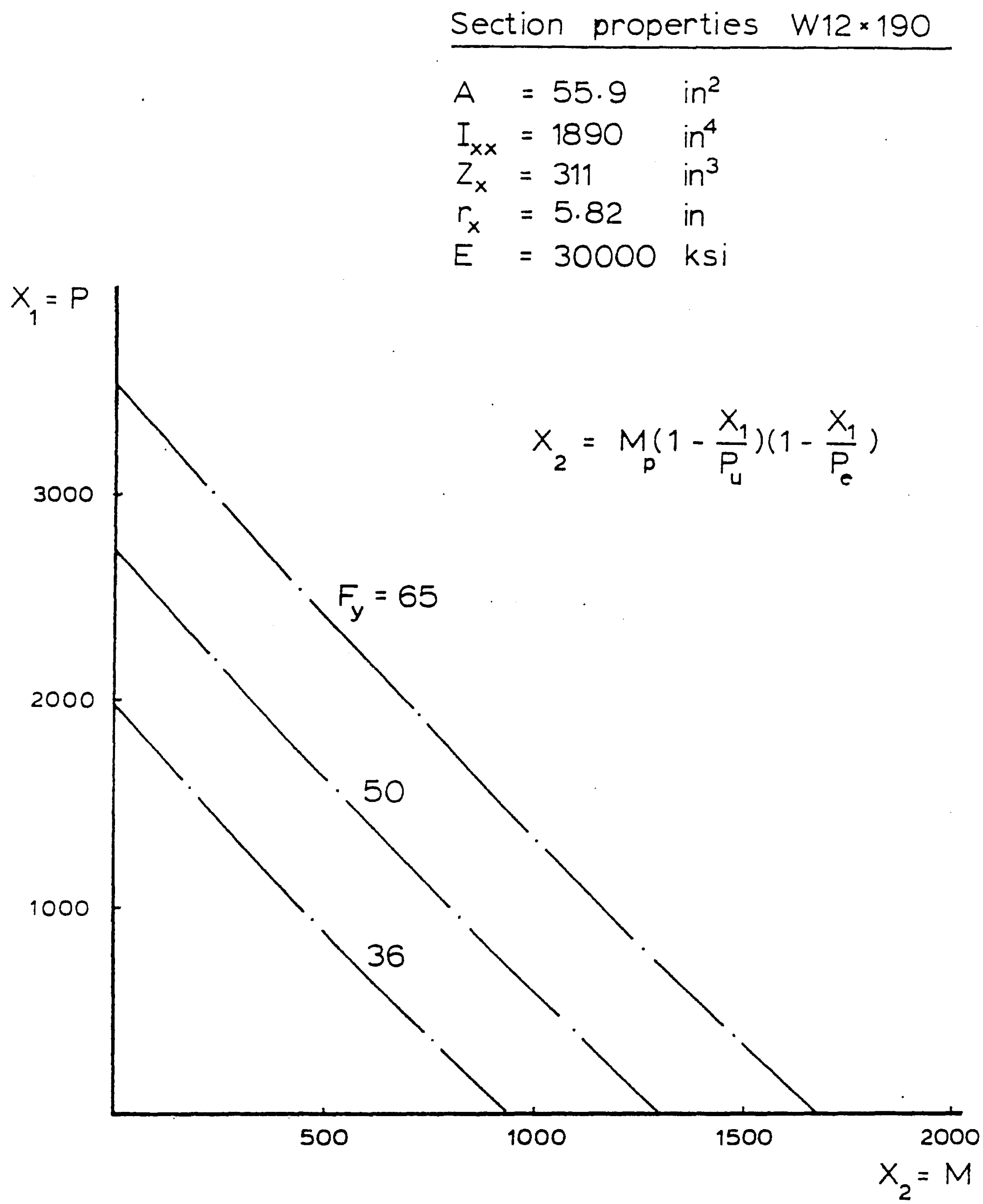


Fig. 3.5 Interaction Curve for Beam Column

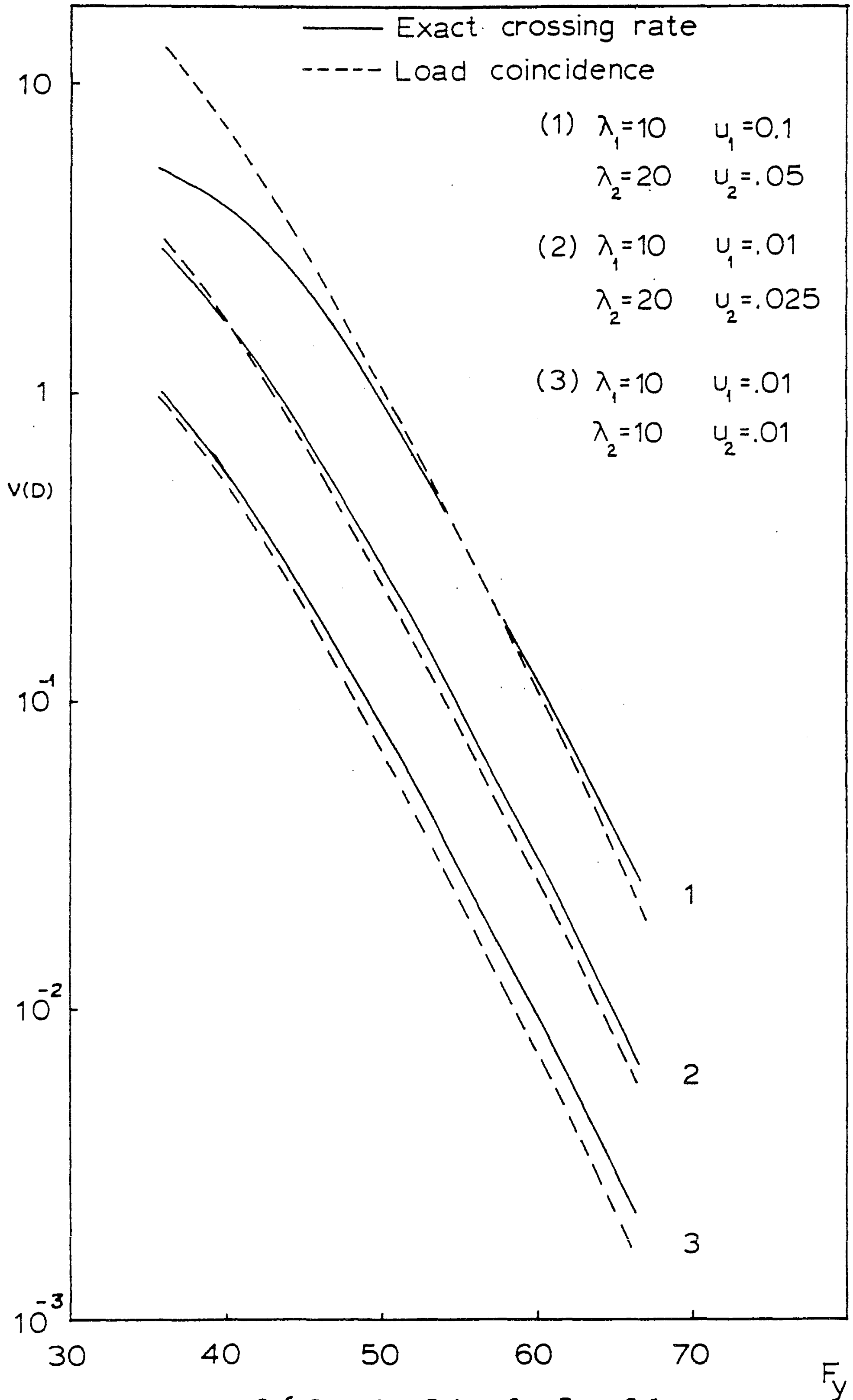


Fig. 3.6 Crossing Rates for Beam Column

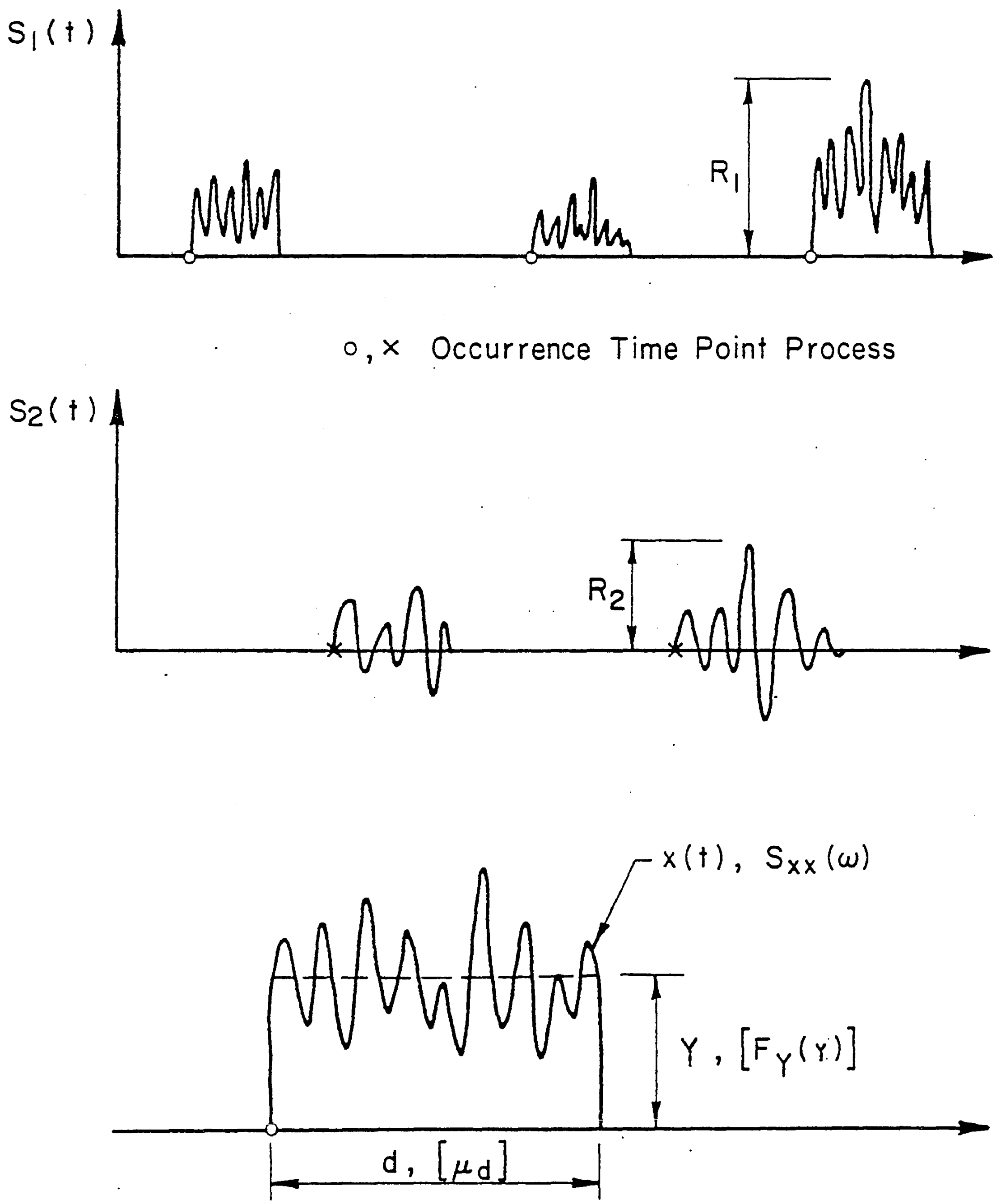
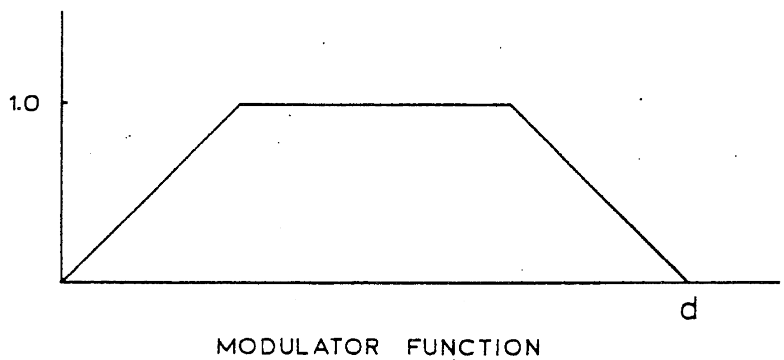


Fig. 4.1 Intermittent Continuous Process



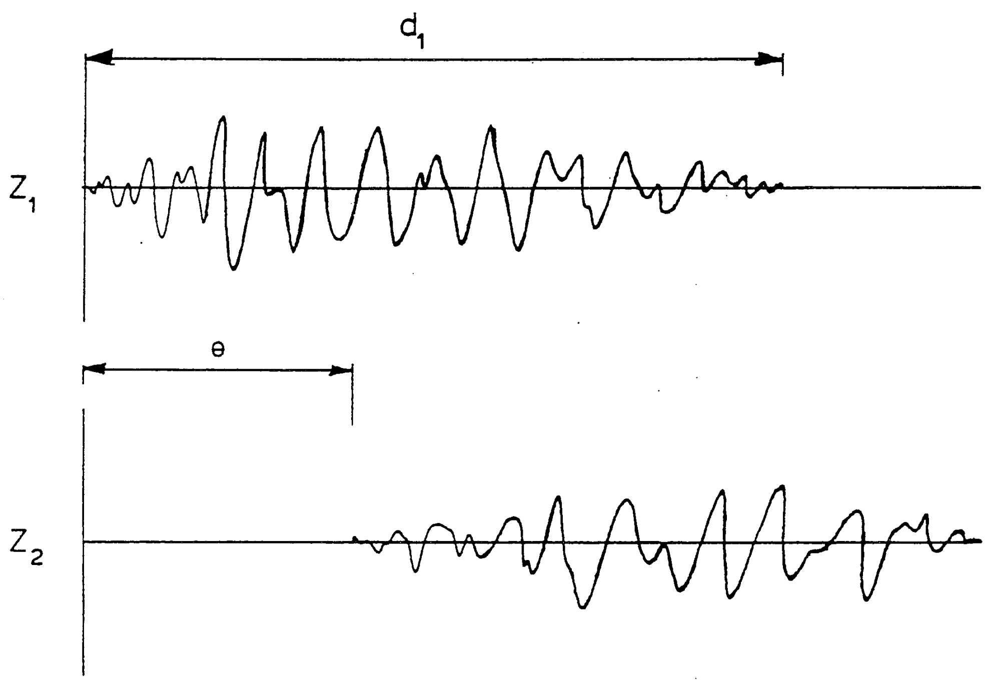


Fig. 4.2 Combined Nonstationary Processes

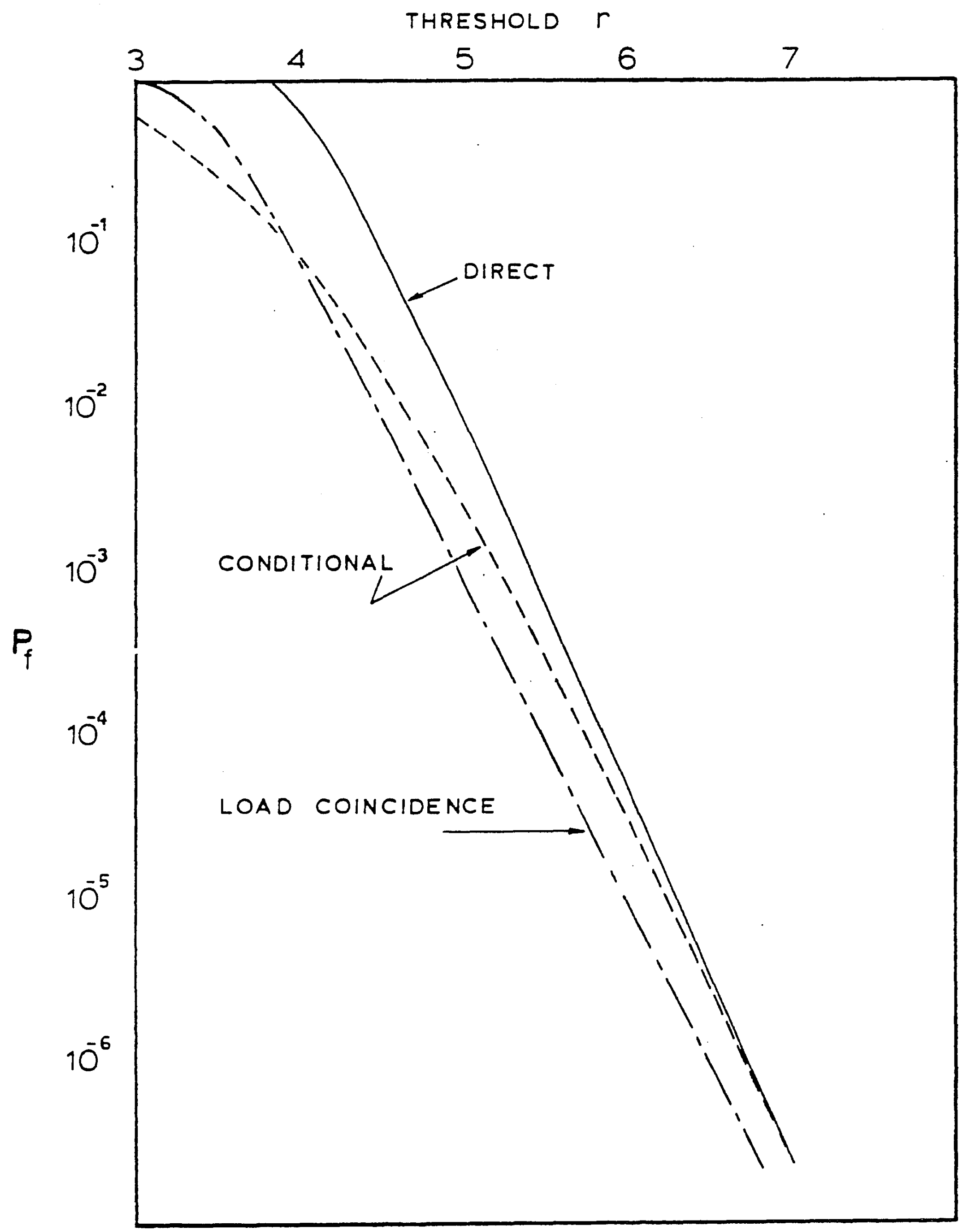


Fig. 4.3 Point Crossing Method - Direct and Conditional

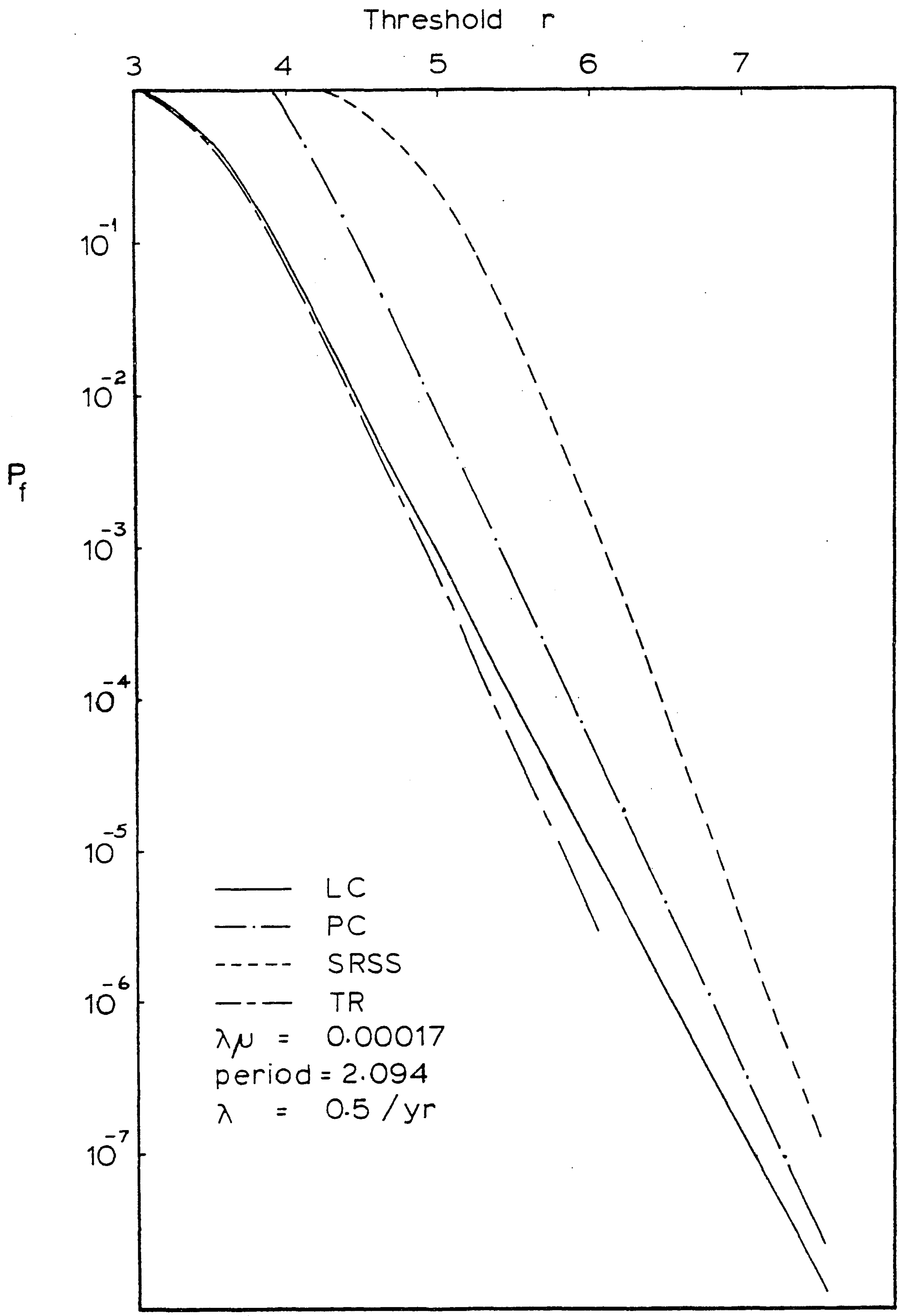


Fig. 4.4 Design Life Exceedance Probabilities

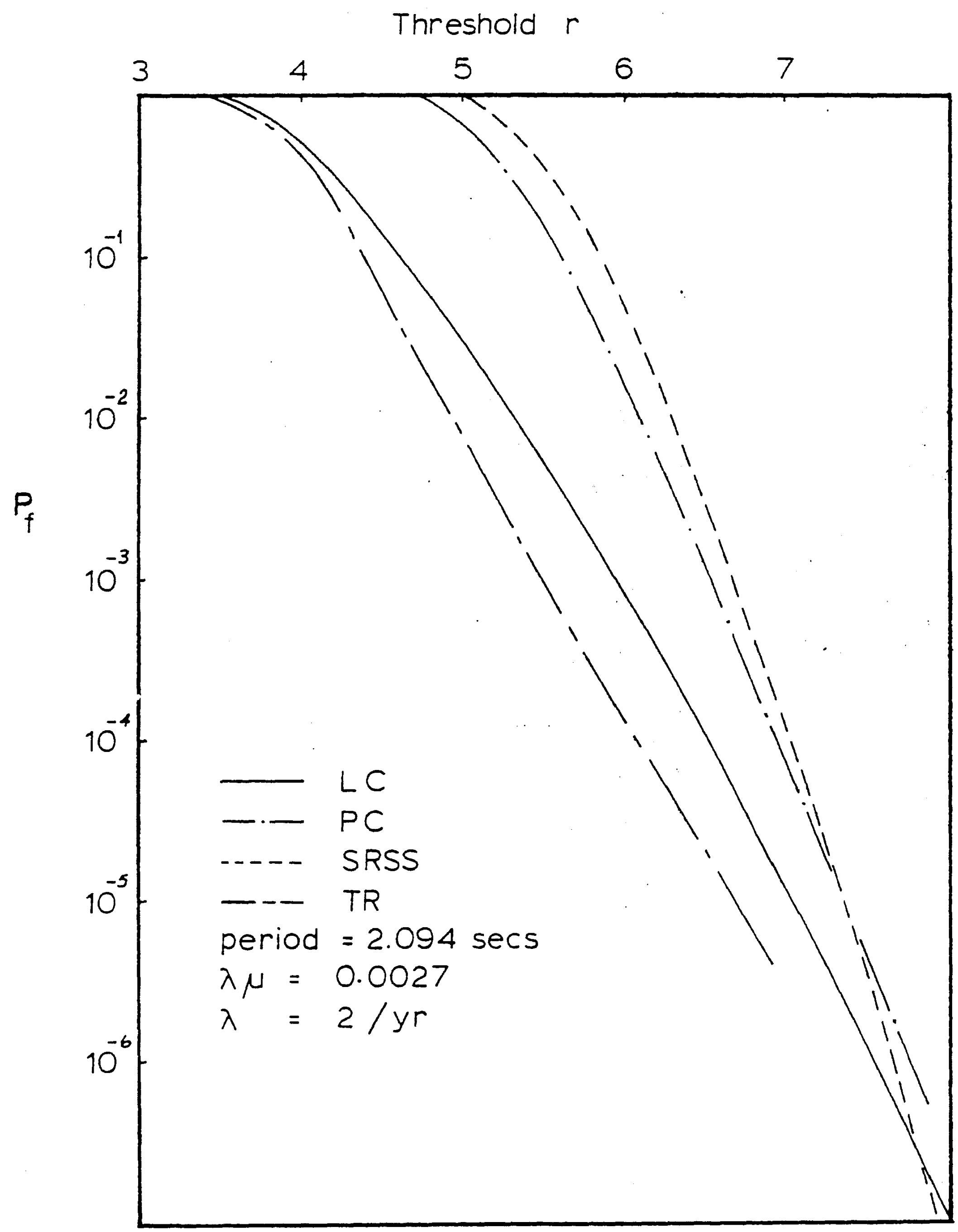


Fig. 4.5 Design Life Exceedance Probabilities

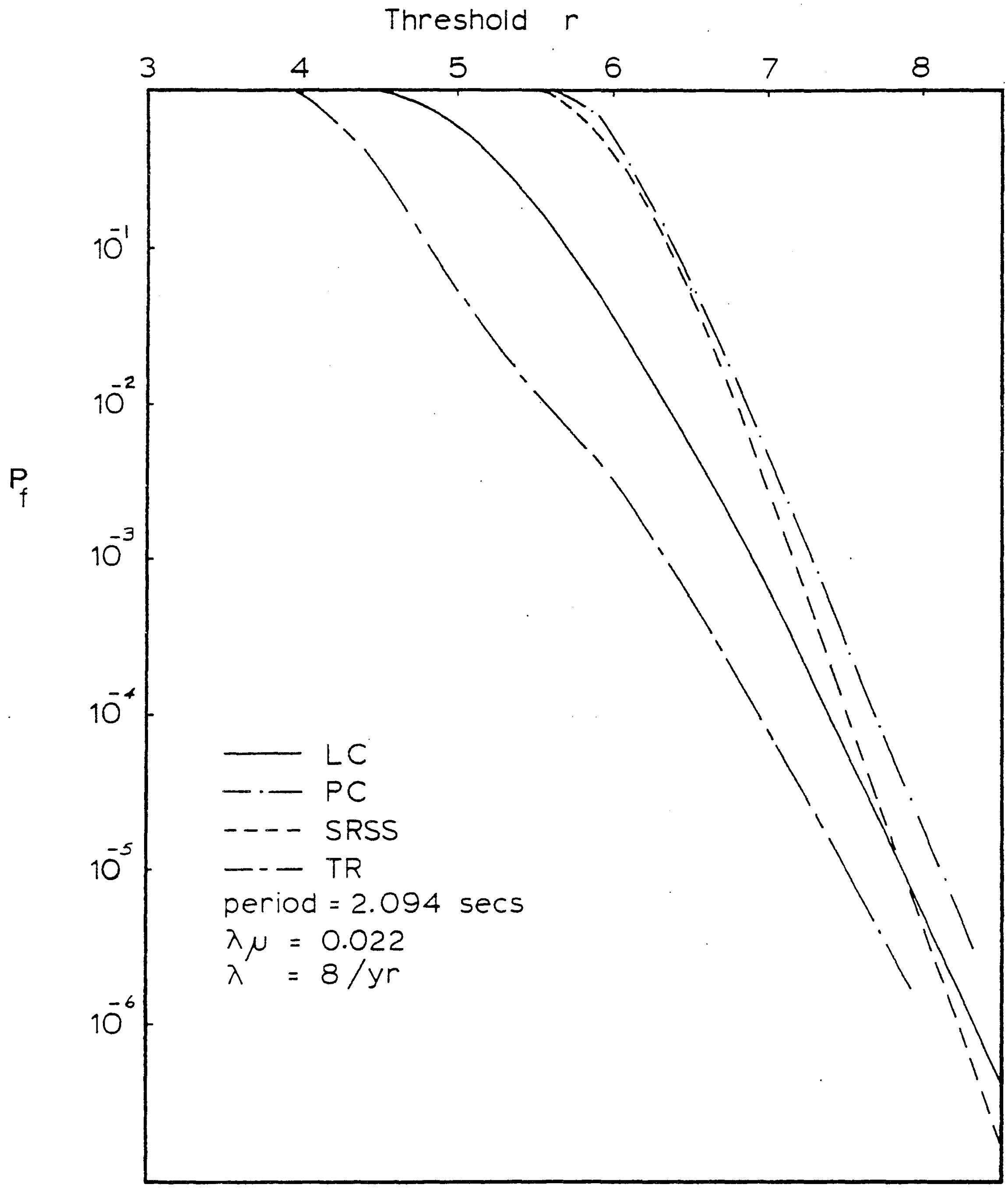


Fig. 4.6 Design Life Exceedance Probabilities

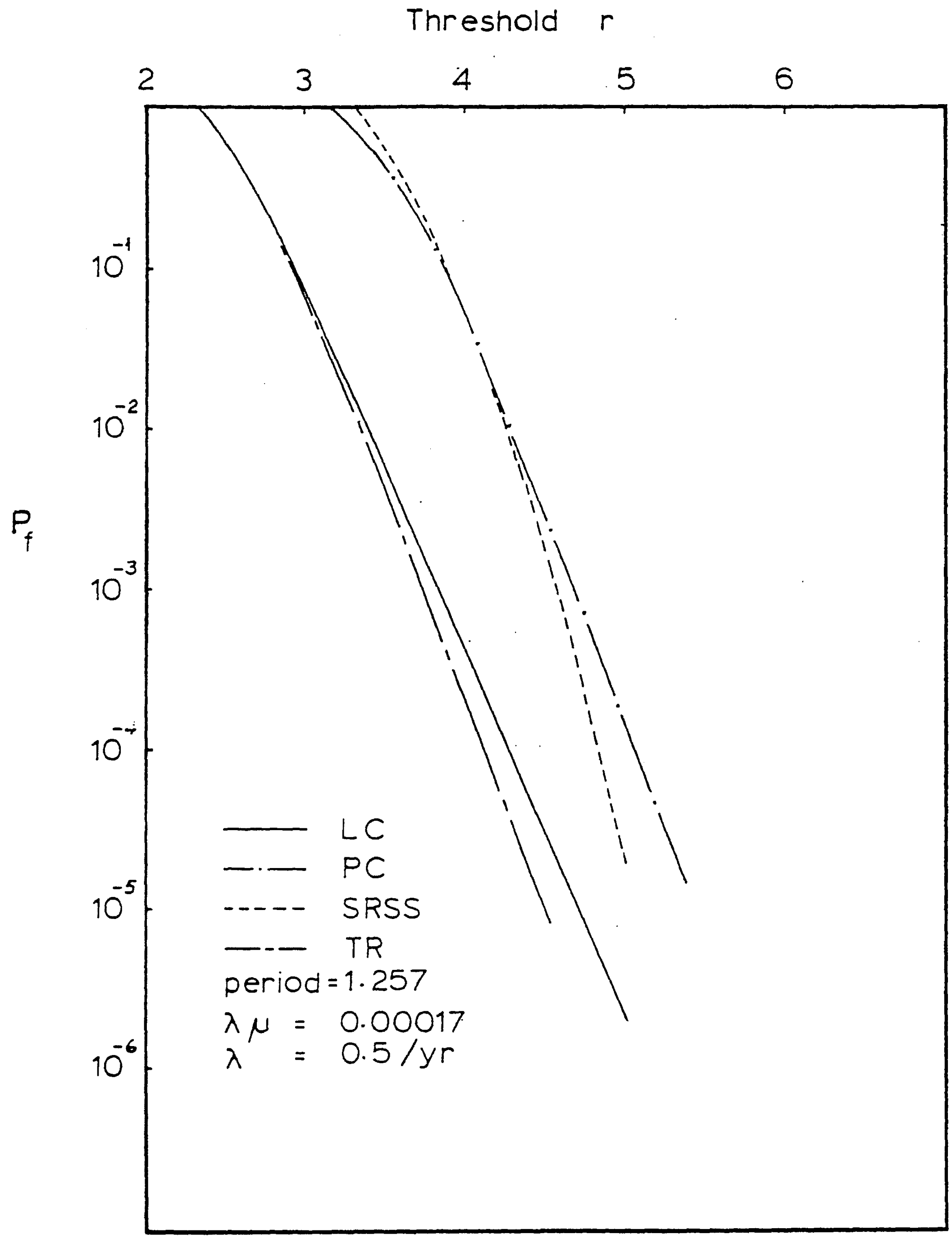


Fig. 4.7 Design Life Exceedance Probabilities

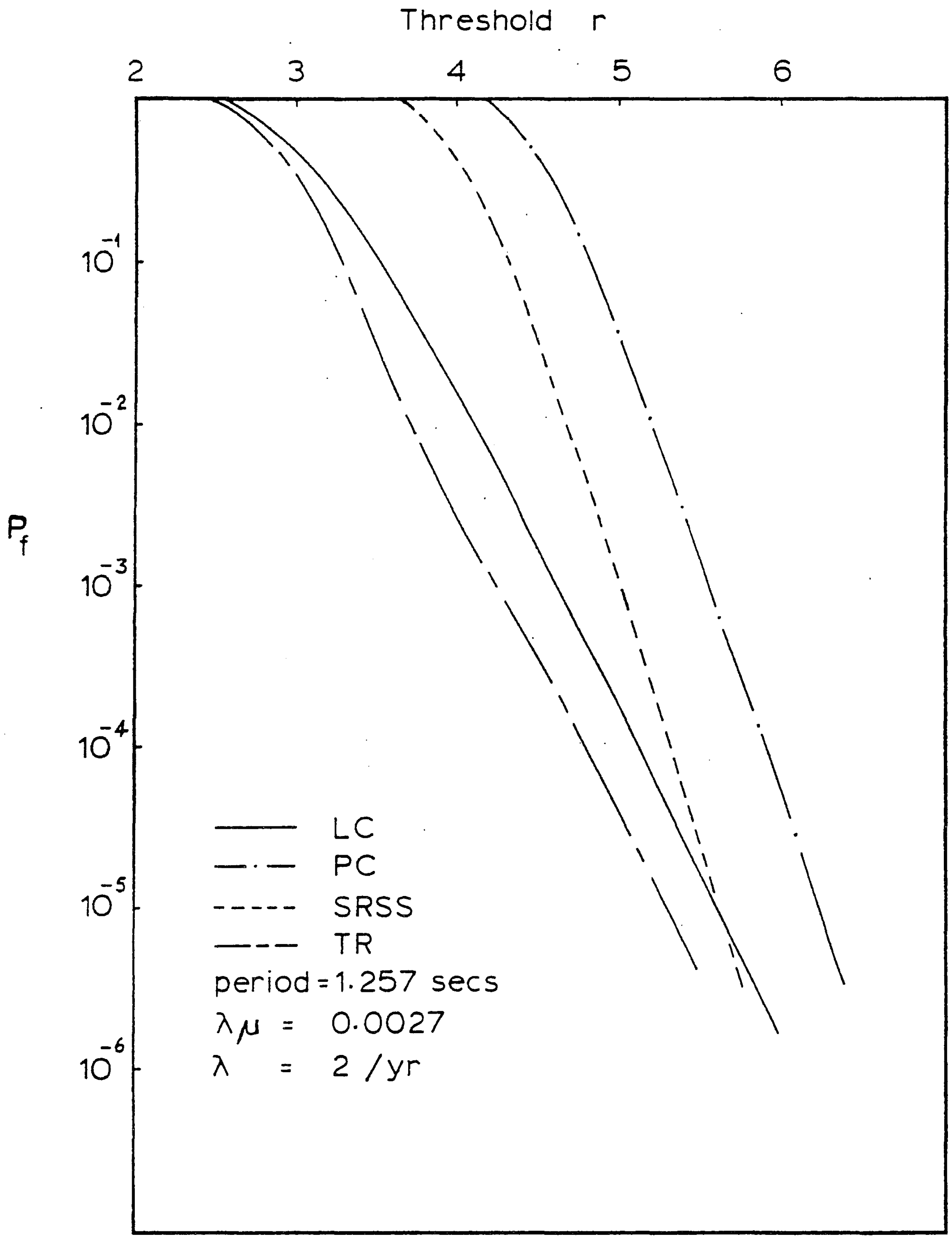


Fig. 4.8 Design Life Exceedance Probabilities

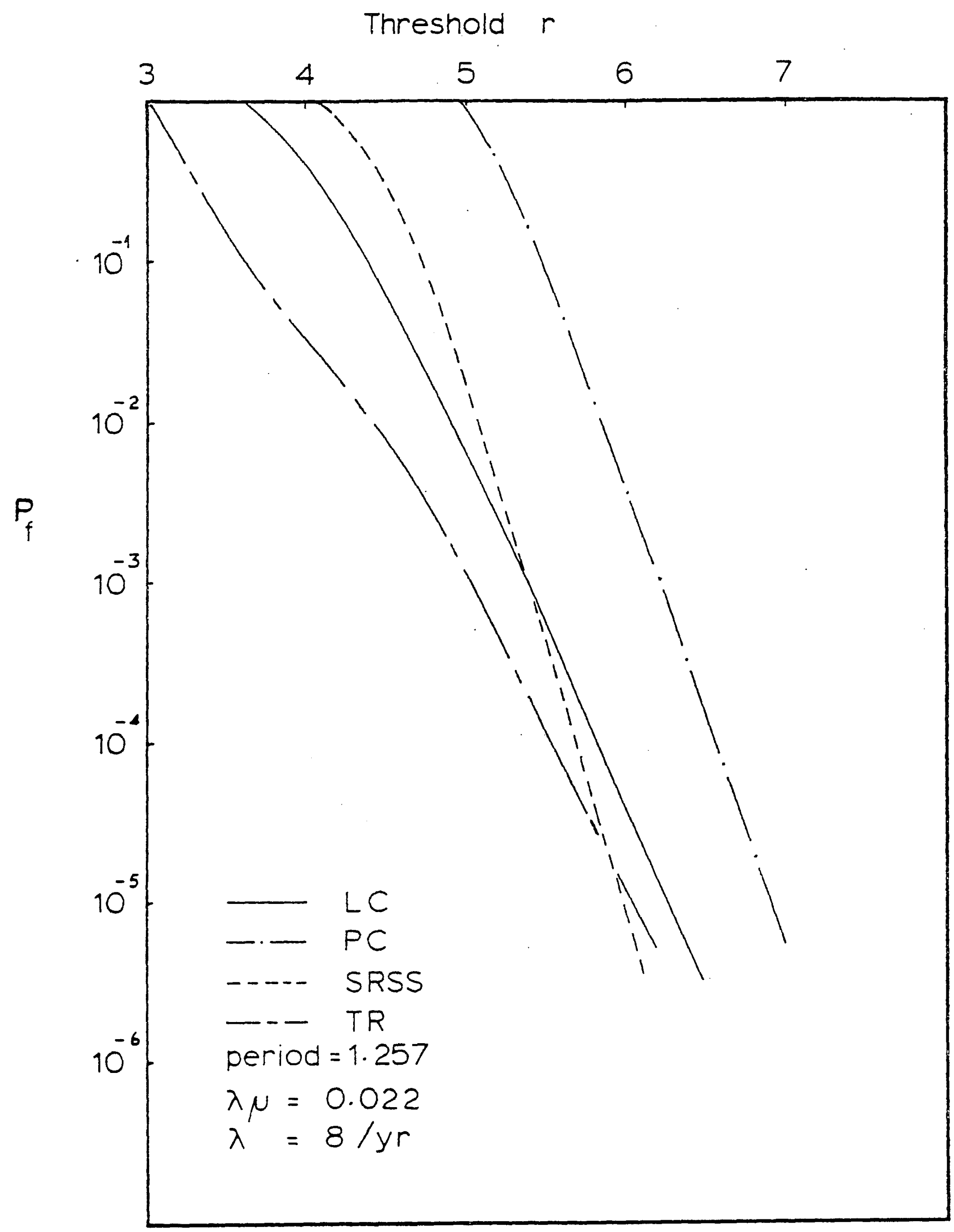


Fig. 4.9 Design Life Exceedance Probabilities

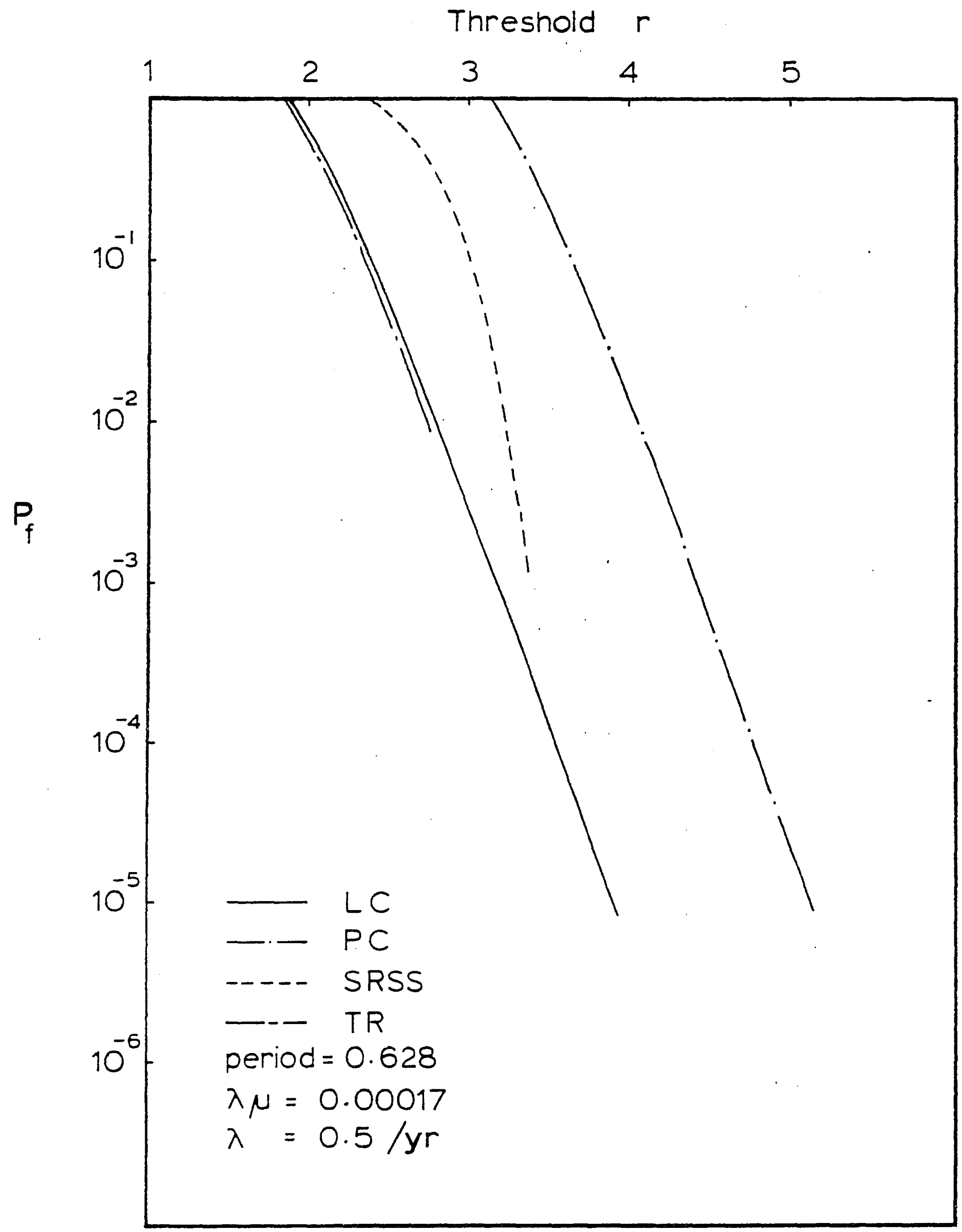


Fig. 4.10 Design Life Exceedance Probabilities

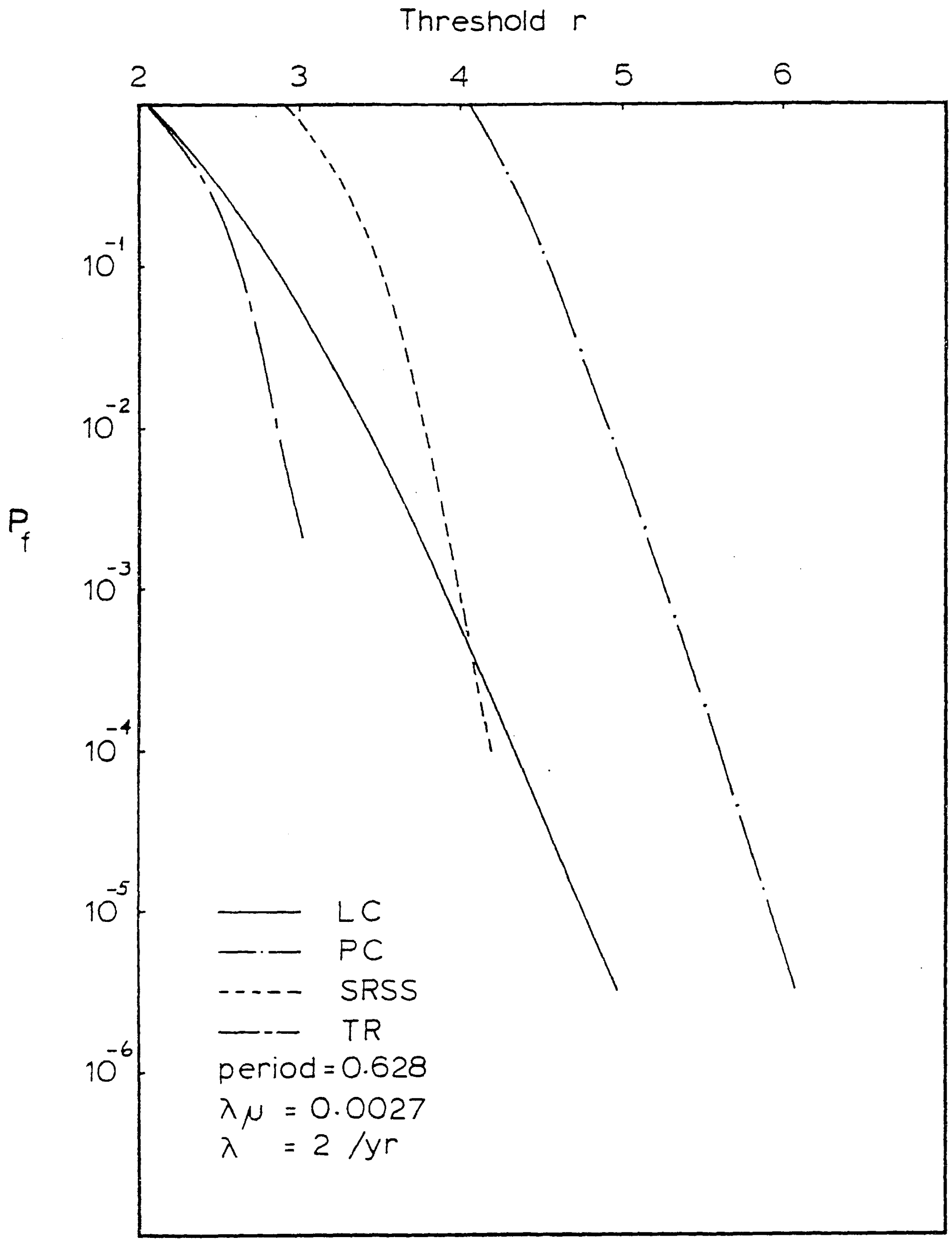


Fig. 4.11 Design Life Exceedance Probabilities

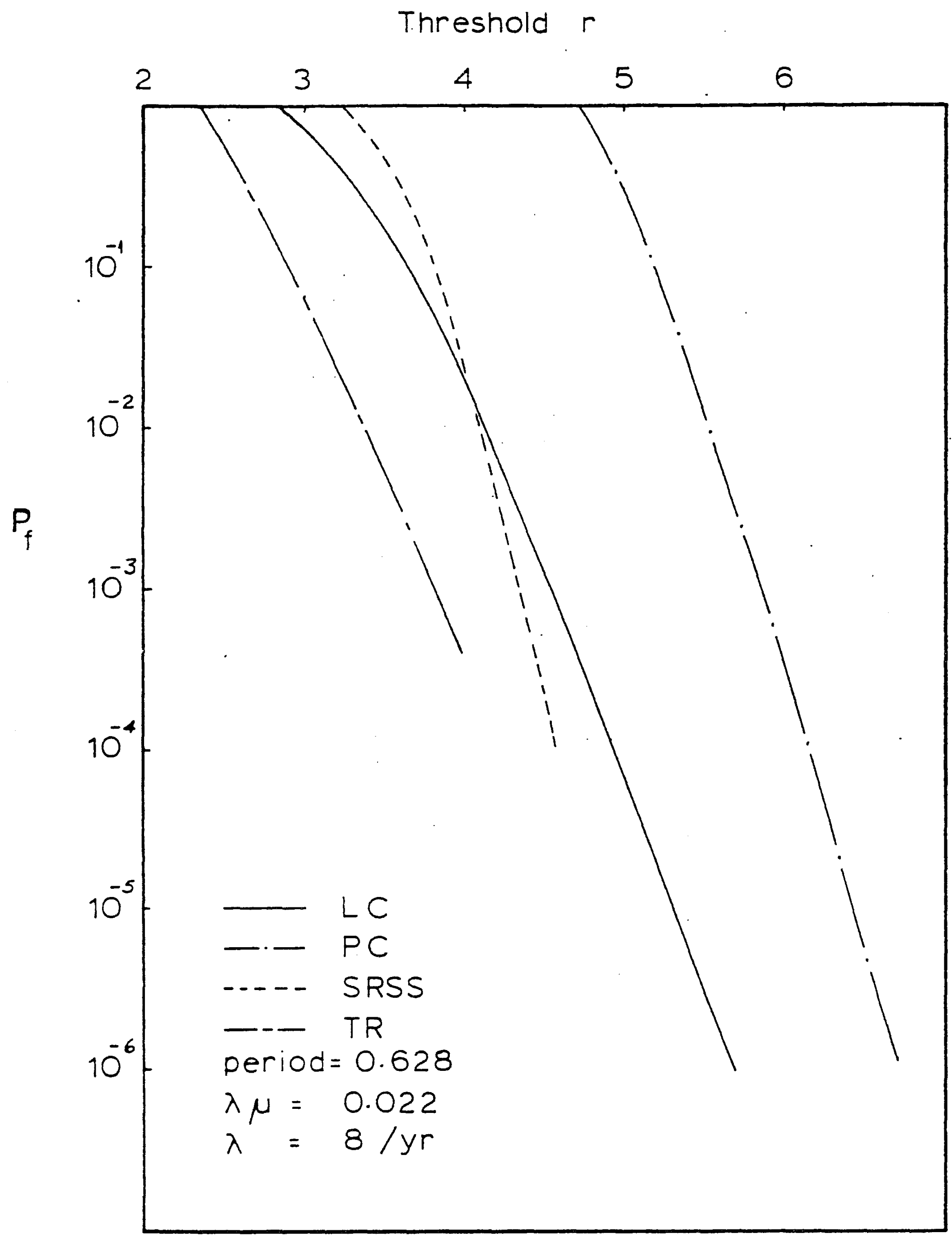


Fig. 4.12 Design Life Exceedance Probabilities

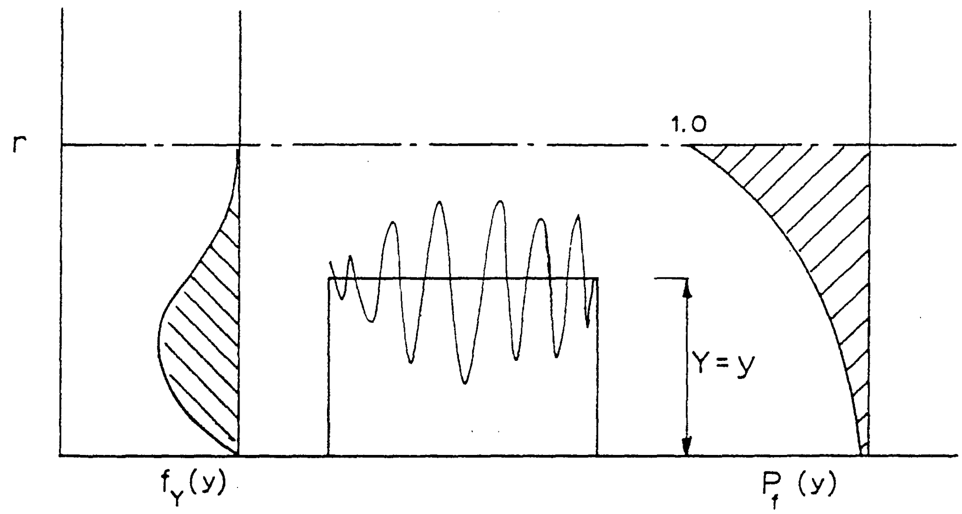


Fig. 4.13 Conditional Failure Probability $P_f(y)$ and Density $f_Y(y)$

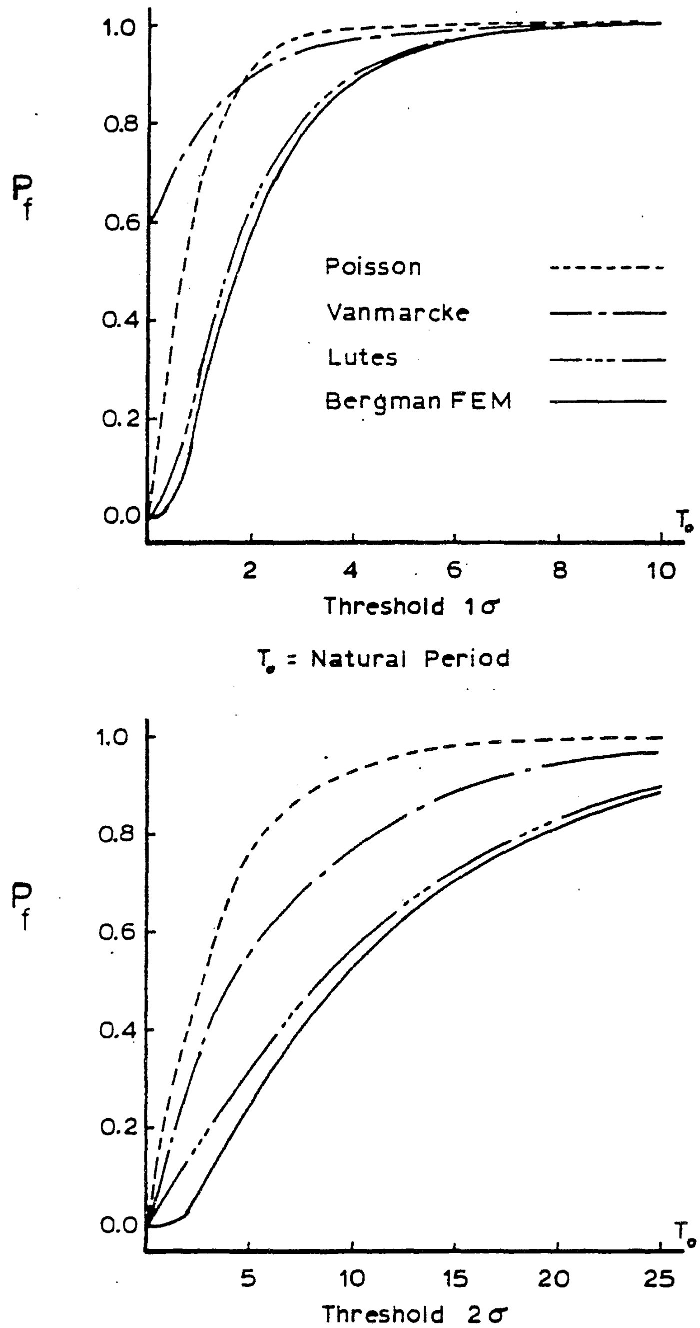


Fig. 4.14 First Passage Probability Solutions

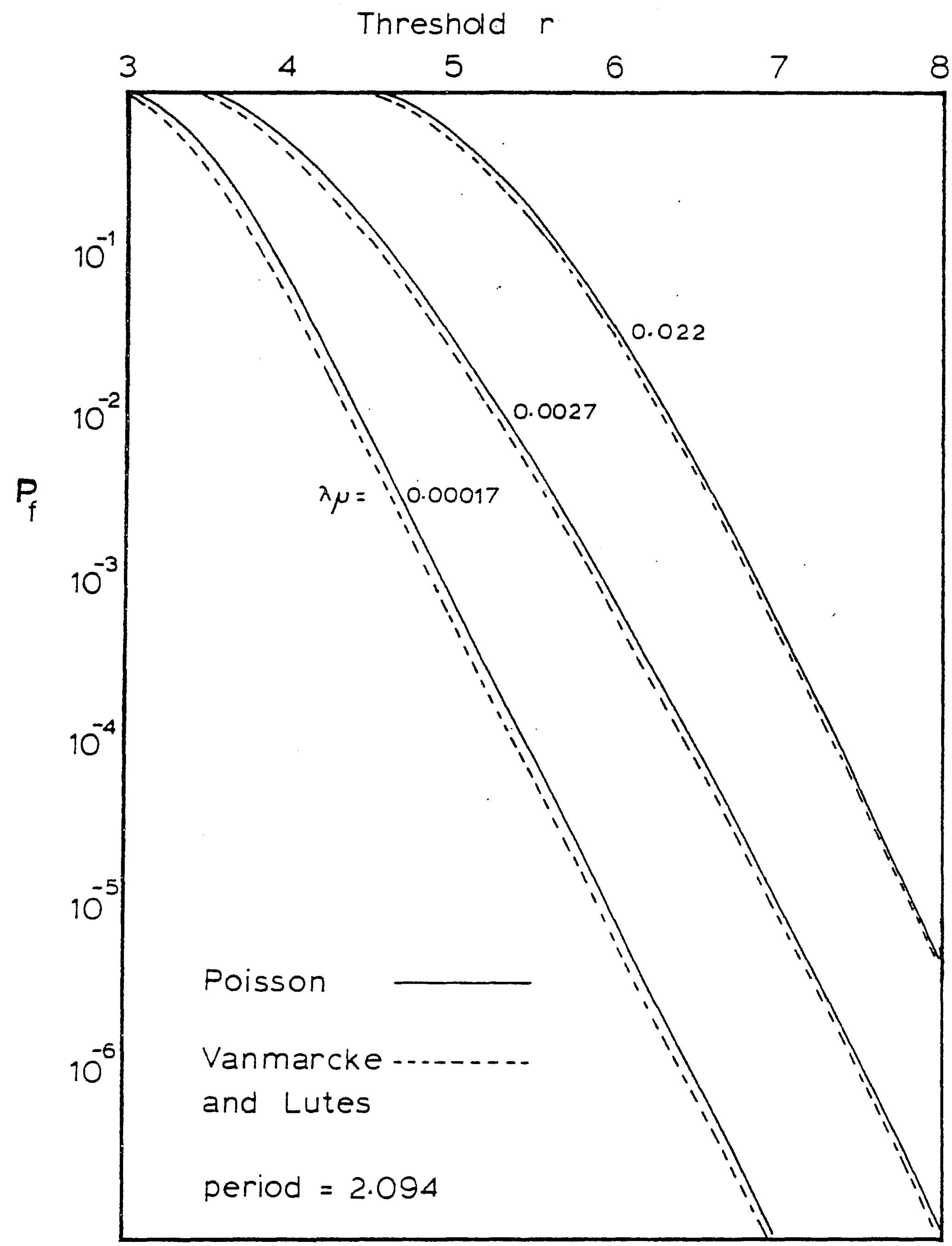


Fig. 4.15 Errors in Overall Failure Probabilities using Poisson Assumption

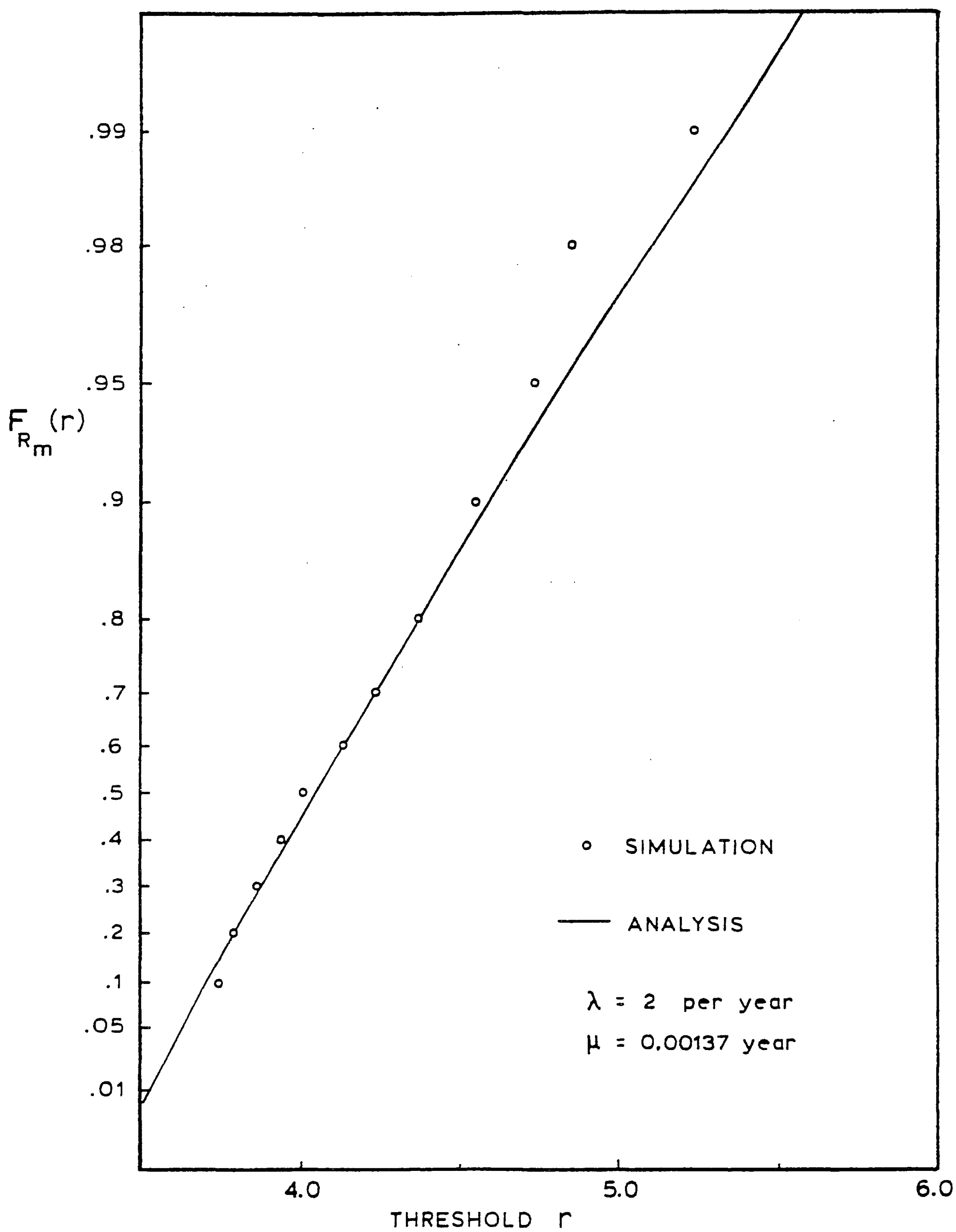


Fig. 4.16 Simulation Checking Mean Duration Assumption

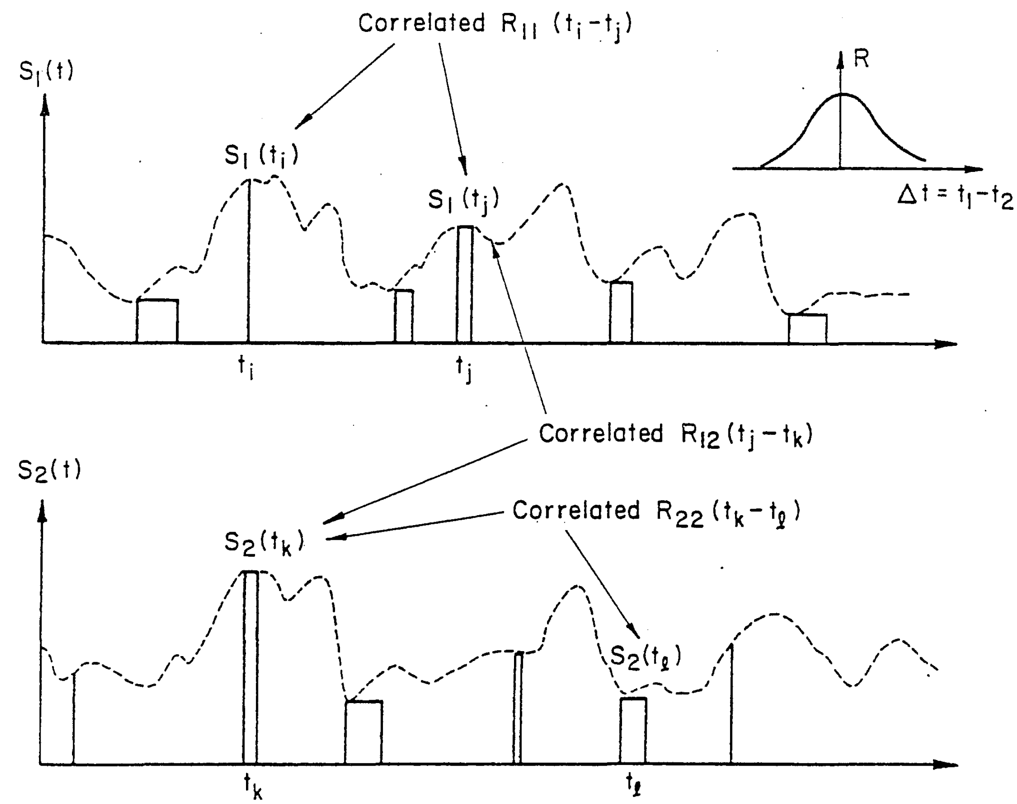


Fig. 5.1 Between Process Intensity Correlation Model

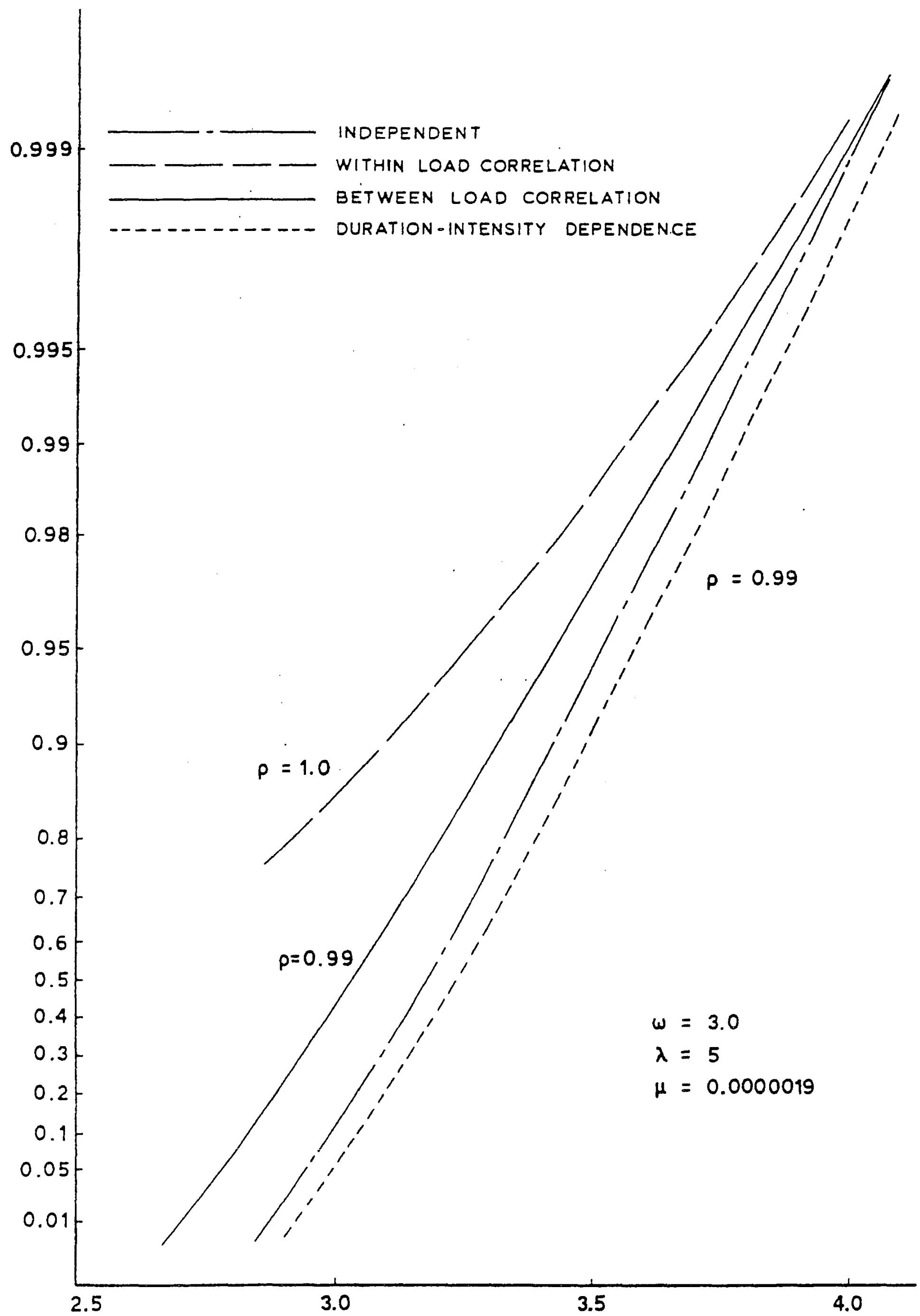


Fig. 5.2 Design Life Non-exceedance Probabilities

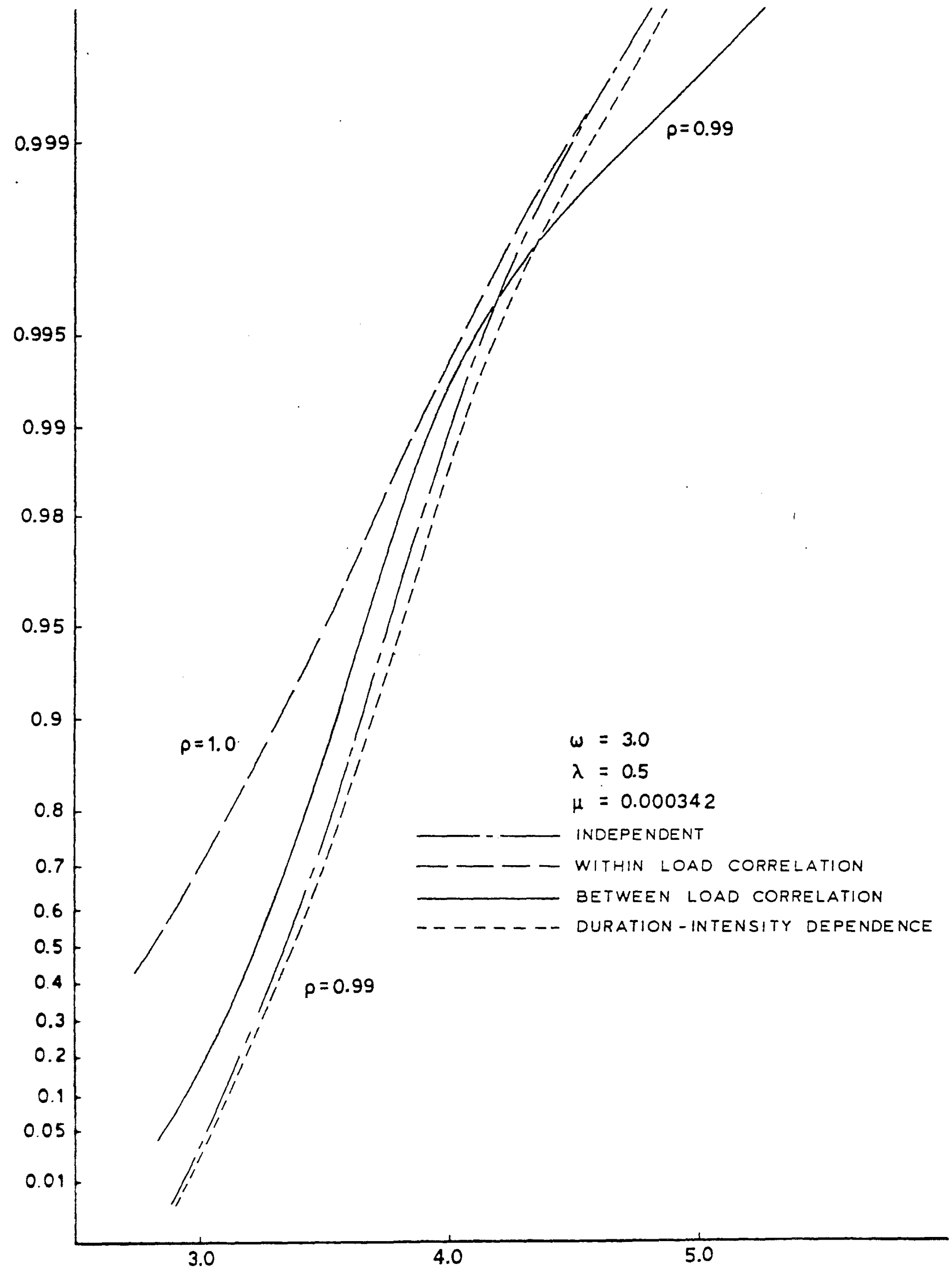


Fig. 5.3 Design Life Non-exceedance Probabilities

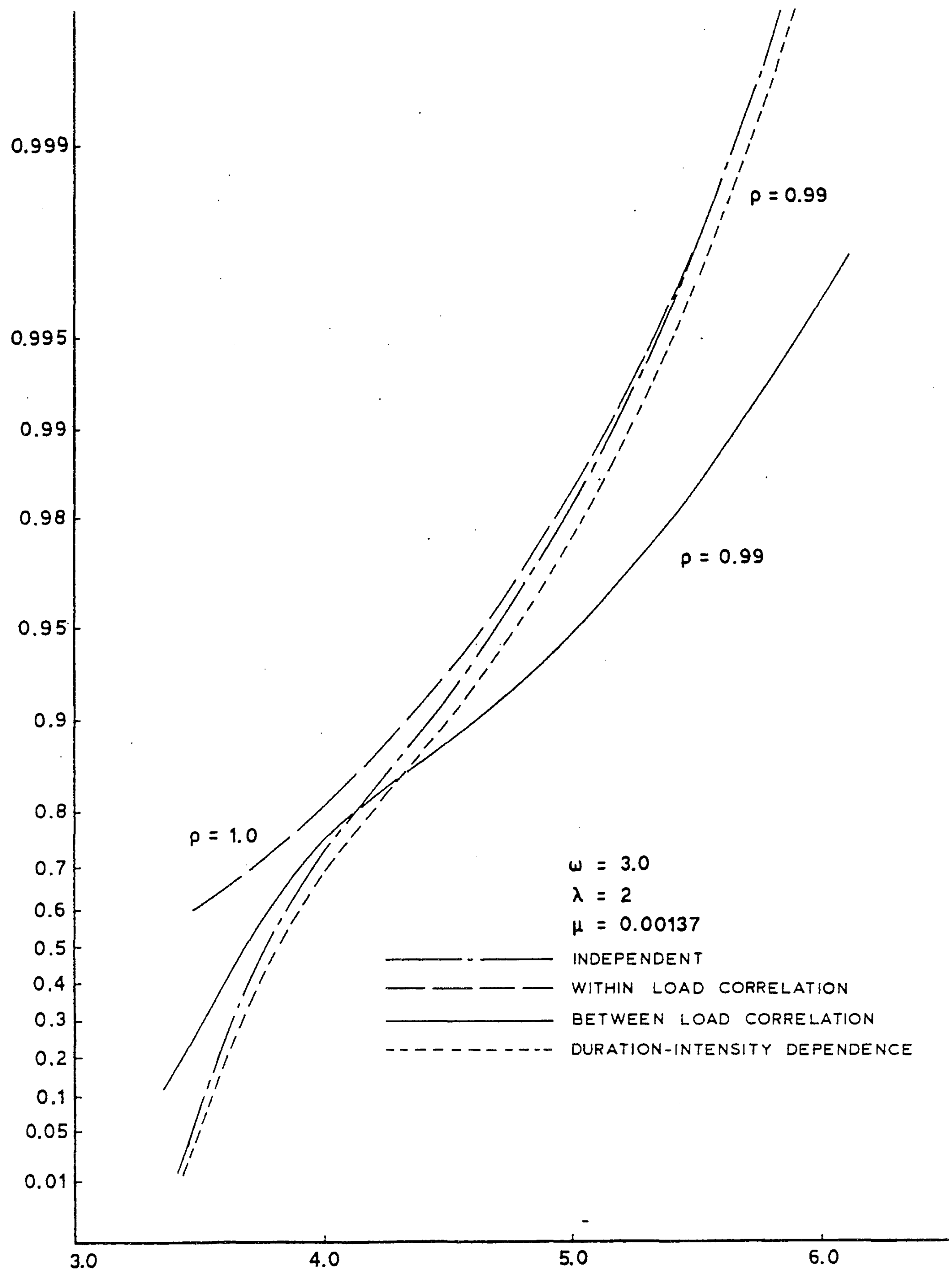


Fig. 5.4 Design Life Non-exceedance Probabilities

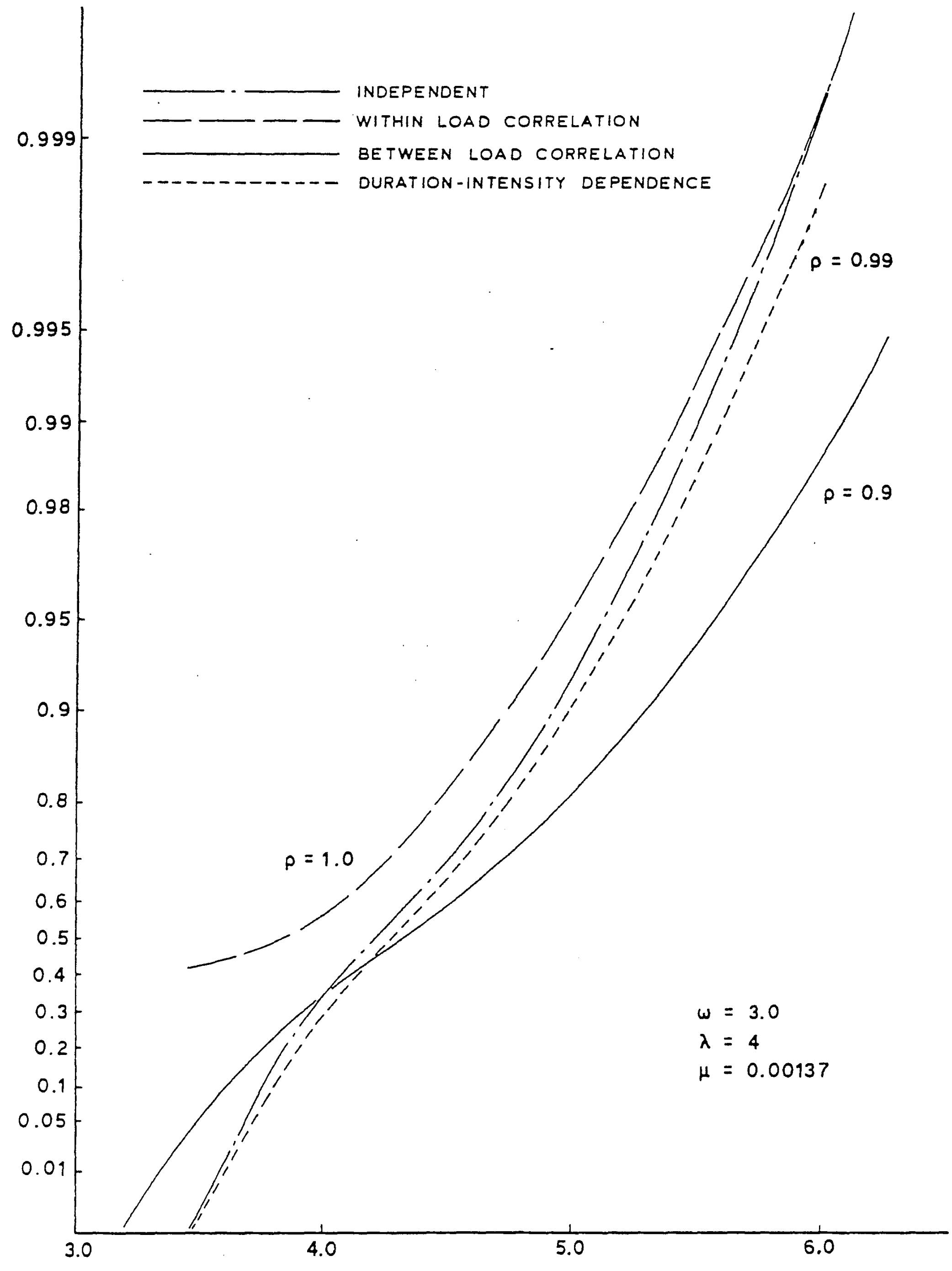


Fig. 5.5 Design Life Non-exceedance Probabilities

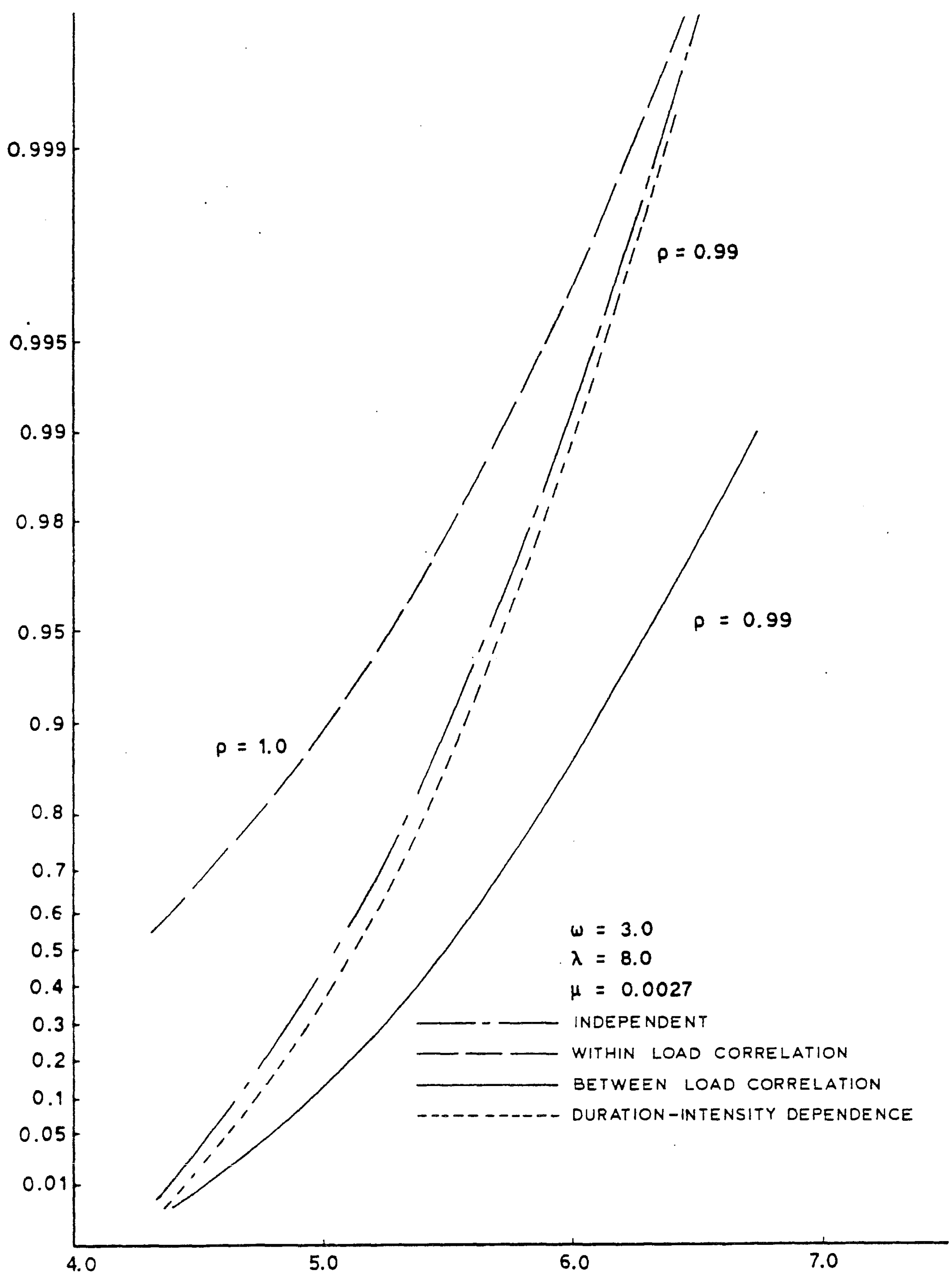


Fig. 5.6 Design Life Non-exceedance Probabilities

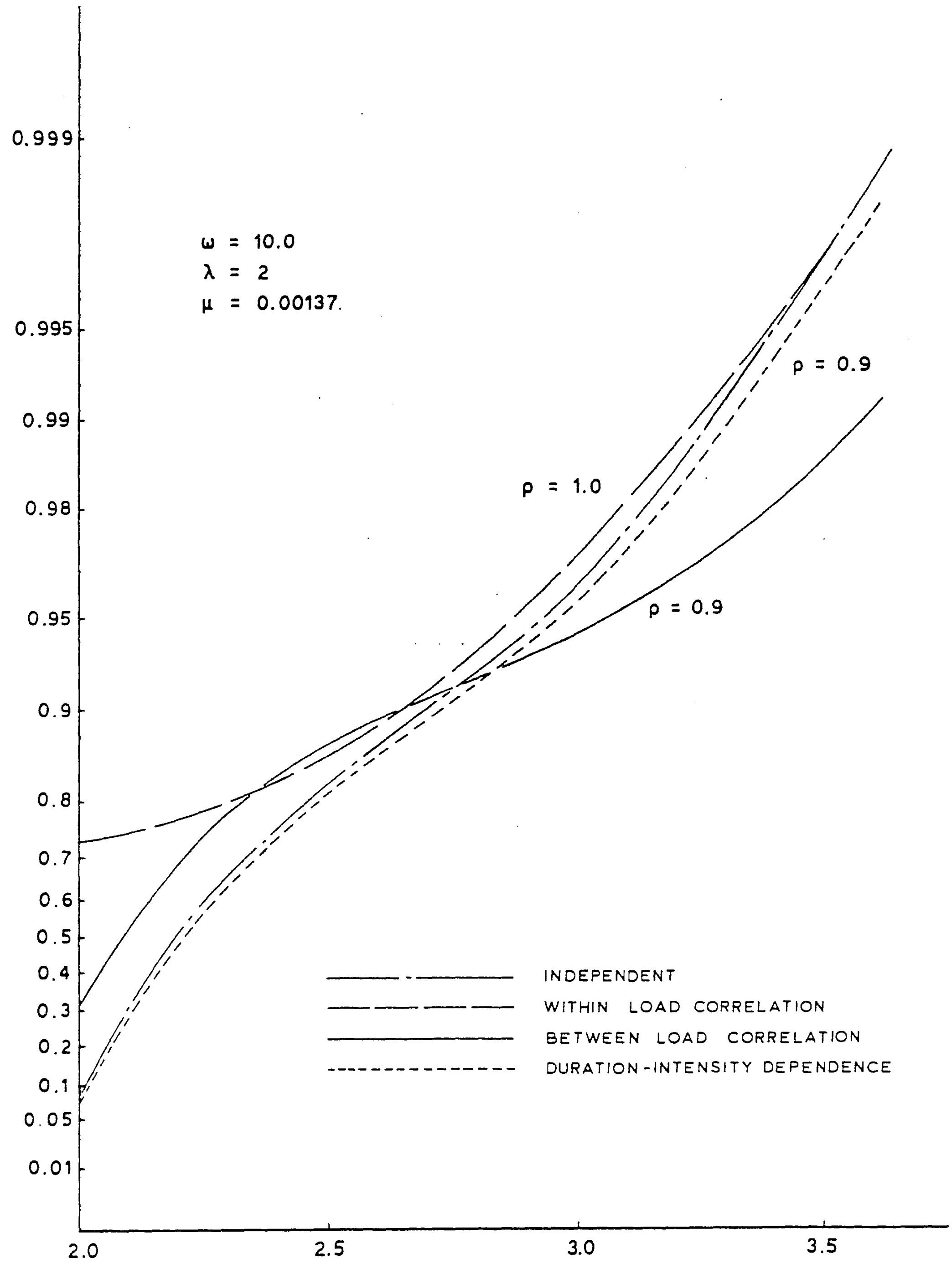


Fig. 5.7 Design Life Non-exceedance Probabilities

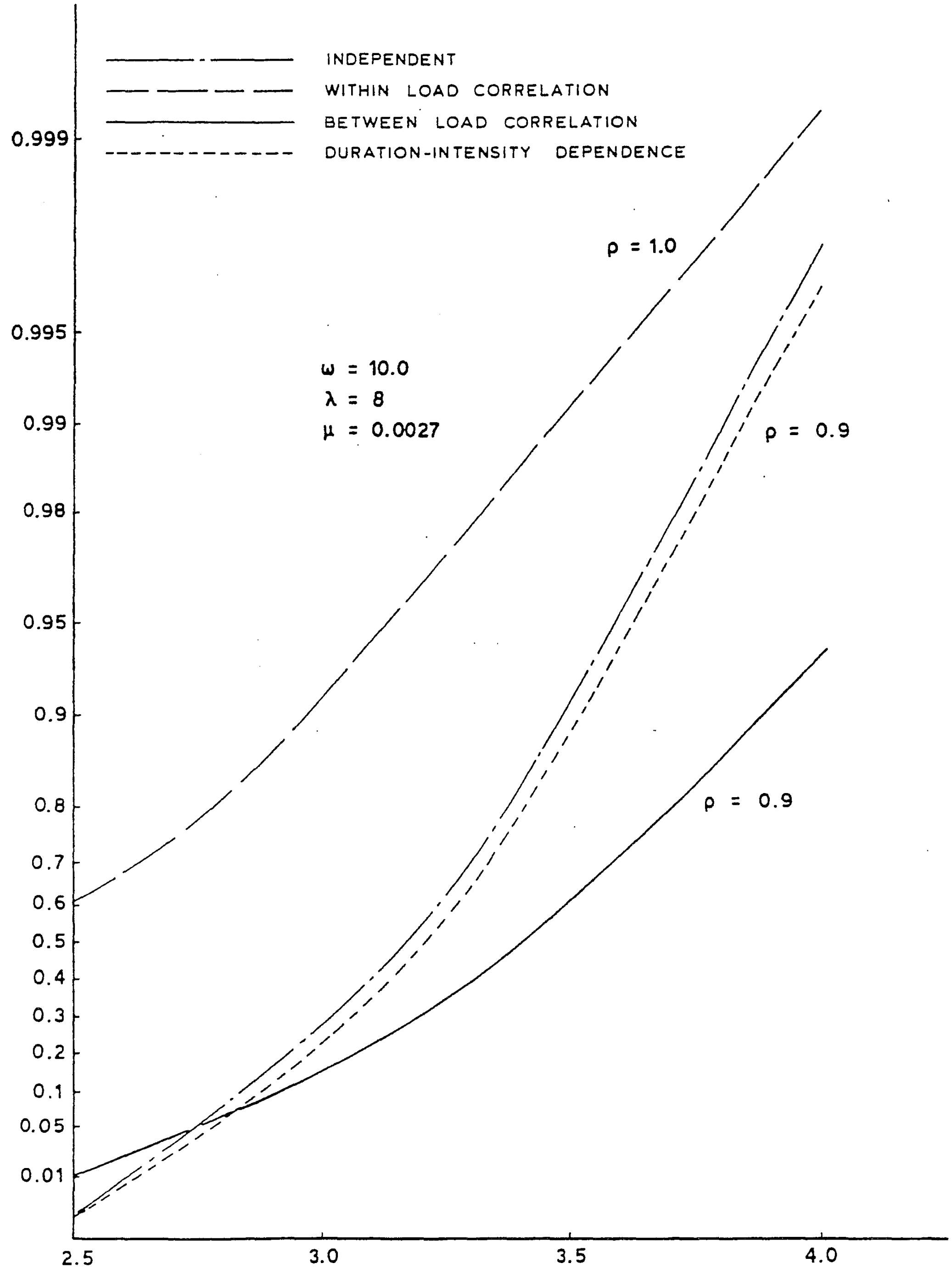
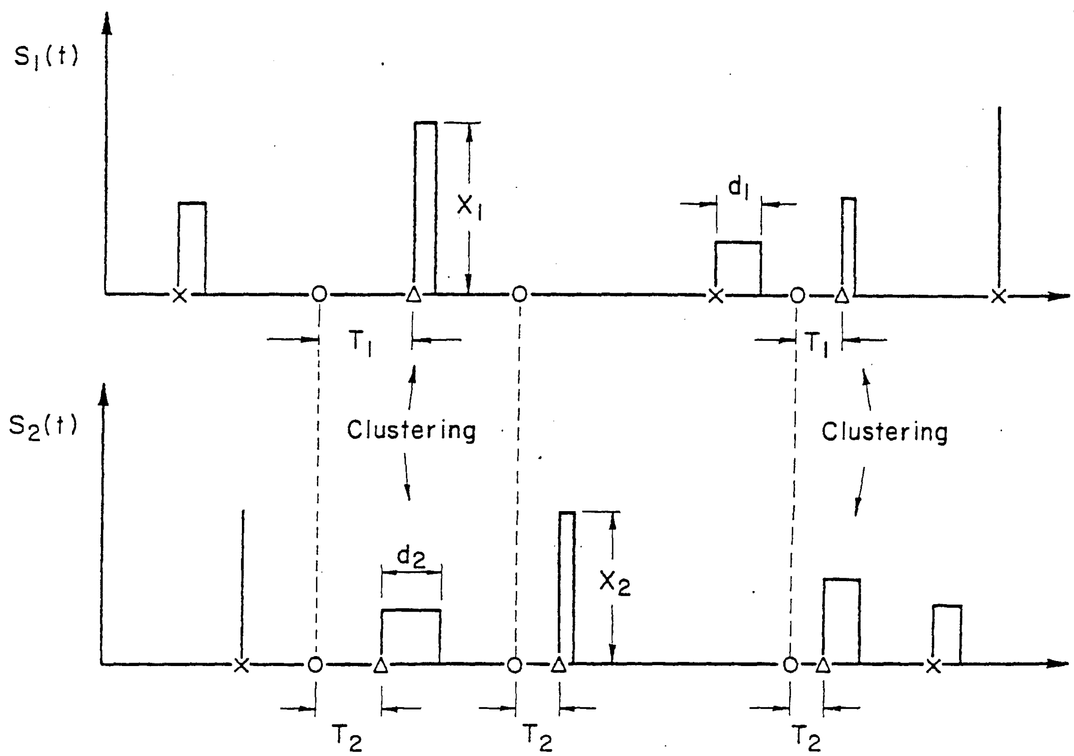


Fig. 5.8 Design Life Non-exceedance Probabilities



o Parent (generating) point process

Δ Delayed point process

x Noise process

T_i (Random) delay time

X_i Intensity given occurrence

d_i duration given occurrence

Fig. 5.9 Model for Occurrence Clustering

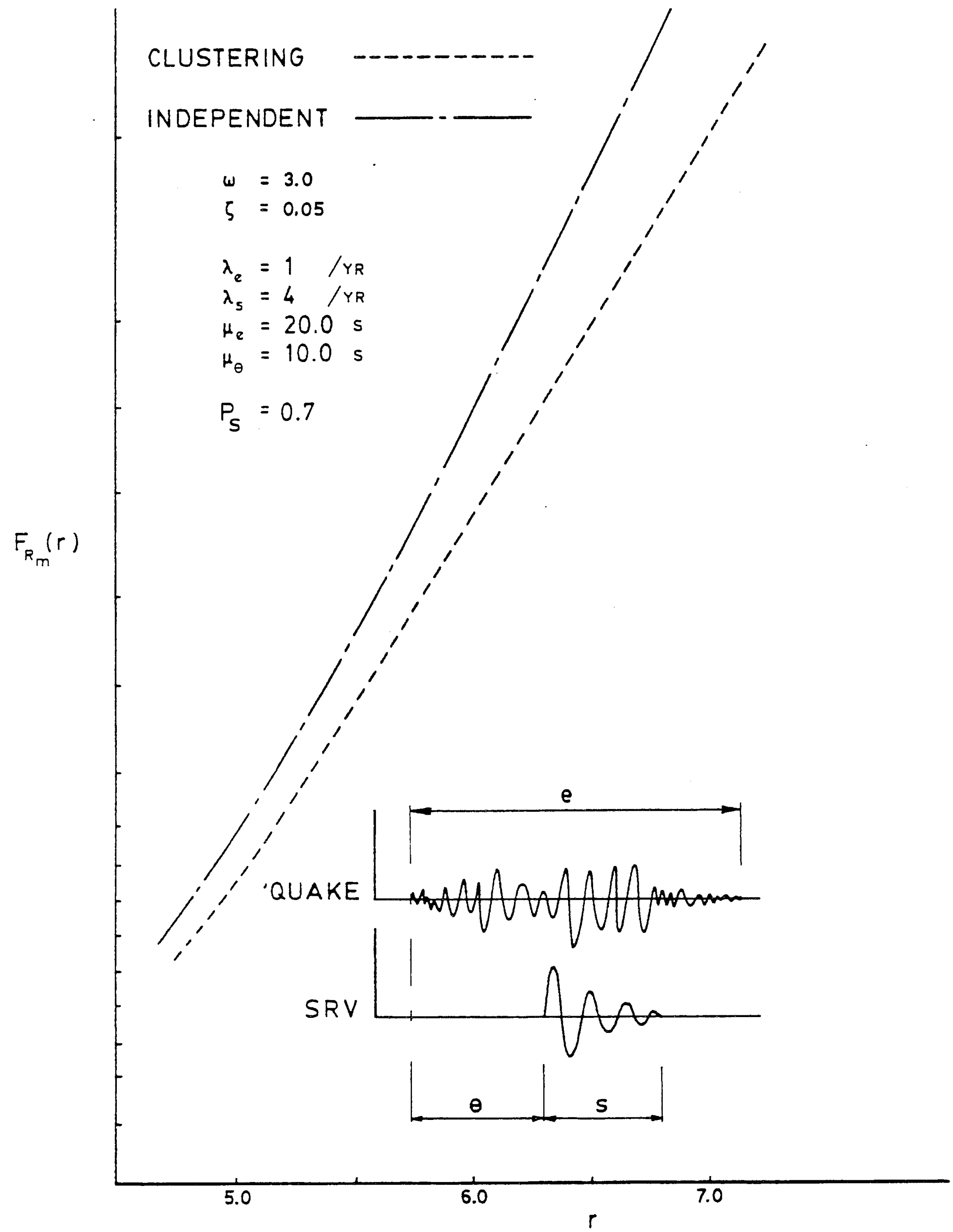


Fig. 5.10 Combined Nonstationary Earthquake and SRV Response

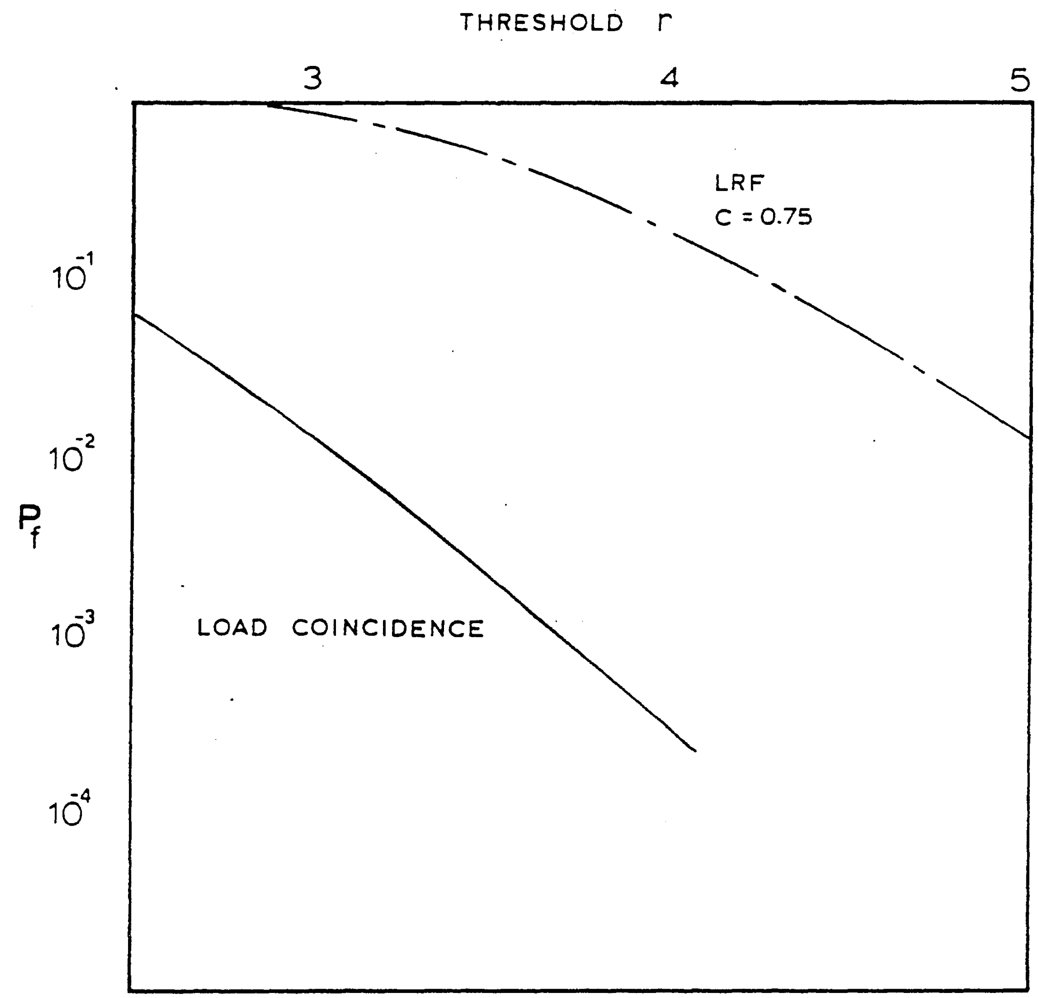


Fig. 6.1 Exceedance Probabilities of LRF and Load Coincidence

APPENDIX A

TIME INVARIANT RELIABILITY MEASURES

The well known Hasofer/Lind reliability index is a measure of the reliability of a component when only second moment information of the basic design variables is available.

The failure surface (limit state) of the compnoent is given in the form

$$g(X_1,X_2,X_3,\ldots,X_n) = 0 \tag{A.1}$$

This surface divides the design space into two regions; a safe region in which $g(X)>0$ and a failure region in which $g(X)<0$. To obtain the Hasofer/Lind reliability index β_{HL}, the limit surface is transformed into a space of standardized variates

$$U_i = (X_i - \mu_{x_i})/\sigma_{x_i} \tag{A.2}$$

In this transformed space β_{HL} is the minimum distance from the origin to the limit surface

$$\beta_{HL} = \min\sqrt{\underline{u}^T\,\underline{u}} \tag{A.3}$$

For those cases in which the probability distribution of the basic variables is known or may be assumed, a more formal approach may be taken. If the limit state is linear and the variables are Gaussian, the reliability may be calculated exactly

$$L(\underline{X}) = \Phi(\beta)$$
$$\beta = \sum_i \alpha_i u_i^* \qquad (A.4)$$

α_i are the direction cosines; u_i^* are the coordinates of a point on the surface in standardized space.

In general, the limit surface will be a nonlinear function of the load and resistance variables, and the variables will not necessarily be Gaussian. For systems with high reliability or for relatively flat failure surfaces, a linearization of the surface will allow computation of the reliability with good approximation.

Let the basic variables X_i have distributions F_i and be independent. For correlated variables the reader should see Hohenbichler and Rackwitz (1981). Transform the variables and the limit surface to unit normal space.

Transformation: $\Phi(y_i) = F_i(x_i)$

Limit surface : $g(F^{-1}[\Phi(y_i)]) = 0$ (A.5)

Consider the point $\bar{y}^o = (y_1^o, y_2^o, \ldots, y_n^o)$ on the surface and linearize the surface at this point. The plane, tangent to the limit surface at this point has the equation

$$\sum c_i(y_i - y_i^o) = 0$$

where (A.6)

$$c_i = \left.\frac{\partial g(\underset{\sim}{y})}{\partial y_i}\right|_{y_i^o} = \frac{\partial g(\underline{x})}{\partial x_i} \frac{\phi(y_i^o)}{f_i(x_i^o)}$$

The tangent plane approximates the true failure surface and also divides the space into a safe and a failure region as shown in Fig. A.1. Define the distance from the origin to the orthogonal projection point on the plane as $\beta(\bar{y}^o)$, Fig. A.2. The approximate reliability given by the linearization is

$$L(\bar{y}^o) = \Phi[\beta(\bar{y}^o)] \tag{A.7}$$

Define the true reliability as R. The error produced by the linearization is

$$\varepsilon = |R - L(\bar{y}^o)| \tag{A.8}$$

This error represents the integral of the probability density over the shaded region in Fig. A.1. The optimum linearization point is that point which minimizes the error ϵ. Note that this is true for any distribution functions for which $L(\bar{y}^o)$ may be readily calculated, and not necessarily only for Gaussian variables.

From Eqs. A.7 and A.8 we deduce that the optimum point is obtained by finding the minimum of the local stationary points of $\beta(\bar{y}^o)$, where

$$\begin{aligned} \beta(\bar{y}^o) &= \sum_i \alpha_i \, y_i^o \\ \alpha_i &= \frac{c_i}{[\sum_j c_j^2]^{1/2}} \end{aligned} \tag{A.9}$$

In the rotationally symmetric unit normal space it can be shown (CIRIA, 1977) that the optimum linearization point will coincide with the point on the surface closest to the origin (Hasofer/Lind point). It is easy to see that this point is also the point of maximum probability density on the surface.

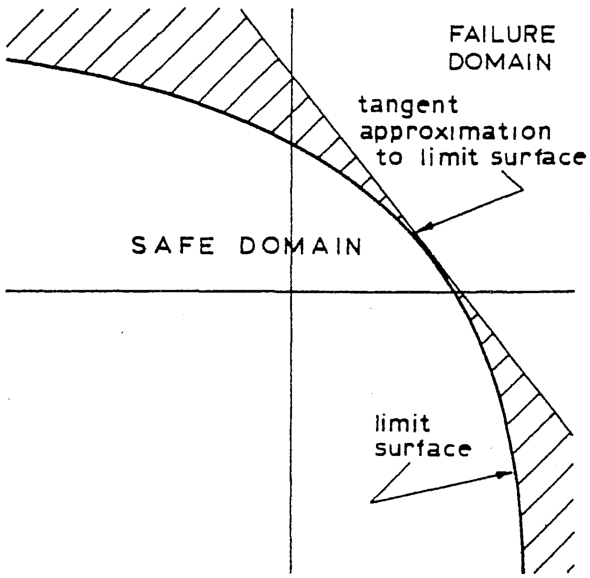


Fig. A.1 Linearized Limit Surface Dividing Safe and Failure Regions

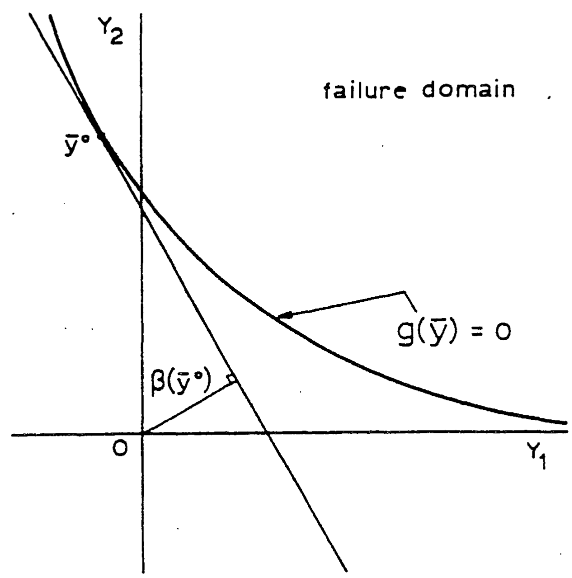


Fig. A.2 Definition of $\beta(\bar{y}^{\circ})$

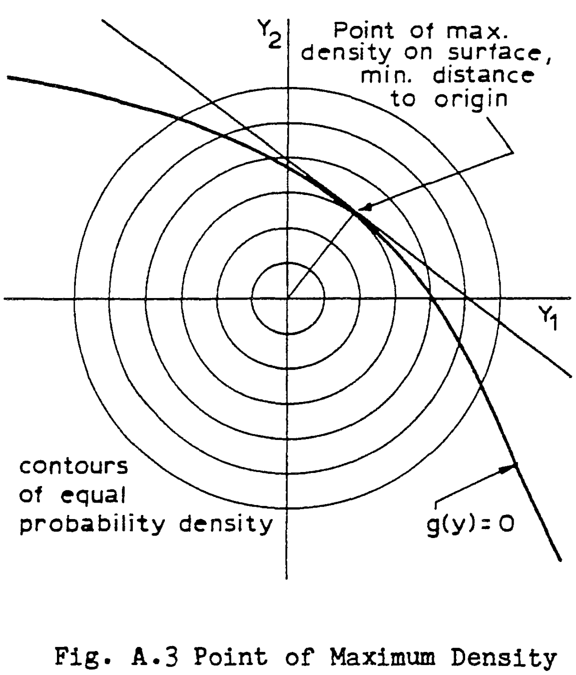


Fig. A.3 Point of Maximum Density in Normal Space

APPENDIX B

THE RACKWITZ/FIESSLER ALGORITHM

For an n-dimensional reliability problem it is necessary to evaluate an n-dimensional integral of the joint density function of the basic variables over the safe domain. The transformation to normal space and the approximation of the n-dimensional integral over the safe domain by a single integral for a linearized domain has been termed fast convolution (Lind, 1980; Ditlevsen, 1980). The procedure used in fast convolution is the Rackwitz/Fiessler algorithm.

The basic variables, generally non-normal, and the limit state function are as described in Appendix A.

The algorithm takes advantage of the fact that, in normal space, the optimum linearization point coincides with the point on the surface closest to the origin.

Instead of transforming the total space of basic variables into normal space (not an easy task), the linearized safe set of the transformed formulation space may be determined by calculations in the original space by using the principle of normal tail approximation (Ditlevsen, 1982; Hohenbichler and Rackwitz, 1981).

Given a point x^* on the surface, the mean and standard deviation of the equivalent normal are computed from

$$\Phi\left(\frac{x_i^* - \mu_i}{\sigma_i}\right) = F_i(x_i^*)$$

$$\frac{1}{\sigma_i}\,\phi\left(\frac{x_i^* - \mu_i}{\sigma_i}\right) = f_i(x_i^*) \qquad \text{(B.1)}$$

The point x, where the tangent plane at x^* cuts the failure surface is calculated. The new point x then takes on the role of x^* and the procedure repeated to provide a sequence of points. When x is sufficiently close to x^* the sequence has converged and the point of linearization has been computed. The reliability is then simply calculated for the linearized safe domain.

The algorithm may be given in the following steps:

a. Set initial point $X_i^* = \overline{X}_i$

b. Compute equivalent normal statistics from Eqs. B.1

c. compute partial derivatives $\partial g/\partial x_i$ at the point x_i^*.

d. Compute direction cosines

$$\alpha_i = \frac{\frac{\partial g}{\partial x_i}\,\sigma_i^N}{\left[\sum\left(\frac{\partial g}{\partial x_i}\,\sigma_i^N\right)^2\right]^{1/2}}$$

e. Now $x_i^* = \bar{x}_i^N - \alpha_i\,\beta\,\sigma_i^N$

f. substitute X^* in $g(X_i^*) = 0$ and solve for β.

g. calculate new point X_i^* on surface, from e.

h. GOTO b., and loop until β converges.

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