UILU-ENG-80-2002

CIVIL ENGINEERING STUDIES

STRUCTURAL RESEARCH SERIES NO. 472

THE COMMONALITY OF EARTHQUAKE AND WIND ANALYSIS

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A Technical Report of
Research Supported by the
NATIONAL SCIENCE FOUNDATION
under Grant No. ENV 75-08456
and
Grant No. ENV 77-07190

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ILLINOIS
at URBANA-CHAMPAIGN
URBANA, ILLINOIS
JANUARY 1980

BIBLIOGRAPHIC DATA SHEET	1. Report No. UILU-ENG-80-2002	2.	3. Recipient's Accession No.
4. Title and Subtitle The Commonality of Earthquake and Wind Analysis			5. Report Date January 1980
			6.
7. Author(s) Pedro J. Cevallos-Candau and William J. Hall			8. Performing Organization Rept. No. SRS No. 472
9. Performing Organization Name and Address University of Illinois at Urbana-Champaign Urbana, Illinois 61801			10. Project/Task/Work Unit No.
			11. Contract/Grant No. ENV 75-08456 ENV 77-07190
12. Sponsoring Organization Name and Address National Science Foundation Washington, D. C. 20550			13. Type of Report & Period Covered
			14.

15. Supplementary Notes

16. Abstracts

Earthquakes and wind loadings constitute dynamic effects that often must be considered in the design of buildings and structures. The primary purpose of this research study was to investigate the common features of general dynamic analysis procedures employed for evaluating the effects of wind and earthquake excitation. Another major goal was to investigate and develop a basis for generating response spectra for wind loading, which in turn would permit the use of modal analysis techniques for wind analysis in a manner similar to that employed for earthquake engineering. In order to generate wind response spectra, the wind loading is divided into two parts, a mean load treated as a static component and a fluctuating load treated as a dynamic component. The spectral representation of the wind loading constitutes a simple procedure for estimating the forces associated with the dynamic component of the gusting wind. Several illustrative examples are presented demonstrating the commonality.

17. Key Words and Document Analysis. 17a. Descriptors

Structural Dynamics
Earthquake Engineering
Wind Engineering
Response Spectrum
Modal Analysis

17b. Identifiers/Open-Ended Terms

17c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report) UNCLASSIFIED	21. No. of Pages 208
Release unlimited	20. Security Class (This Page UNCLASSIFIED	22. Price

ABSTRACT

Earthquakes and wind loadings constitute dynamic effects that often must be considered in the design of buildings and structures. The primary purpose of this research study was to investigate the common features of general dynamic analysis procedures employed for evaluating the effects of wind and earthquake excitation.

Another major goal was to investigate and develop a basis for generating response spectra for wind loading, which in turn would permit the use of modal analysis techniques for wind analysis in a manner similar to that employed for earthquake engineering. Random vibration techniques were applied for developing response spectra for wind loading. In order to generate wind response spectra, the wind loading is divided into two parts, a mean load that is treated as a static component and a fluctuating load that is treated as a dynamic component. The spectral representation of the wind loading constitutes a simple procedure for estimating the forces associated with the dynamic component of the gusting wind.

Several illustrative examples are presented to demonstrate the common application of modal analysis and response spectrum techniques for evaluating the effects of wind and earthquake excitation.

ACKNOWLEDGMENT

This report is based on the doctoral dissertation by Pedro J. Cevallos-Candau submitted to the Graduate College of the University of Illinois at Urbana-Champaign in partial fulfillment of the requirements for the Ph.D. degree. The study was directed by William J. Hall, Professor of Civil Engineering, and was supported by the National Science Foundation under Grants ENV 75-08456 and ENV 77-07190. The authors also would like to thank Dr. Nathan M. Newmark, Professor of Civil Engineering and in the Center for Advanced Study, Emeritus, for his inspiration and guidance in the early phases of this study. Appreciation also is extended to Dr. Y. K. Wen for his assistance in the beginning of this study and to Dr. T. Krauthammer for his continuous review and commentaries. Computer service support for the preliminary computation was provided in part by the Research Board of the Graduate College of the University of Illinois at Urbana-Champaign.

The authors would like to thank the following organizations for their permission to use figures published by them:

1. Portland Cement Association, Skokie, Illinois
2. American Society of Civil Engineers, New York, New York

Any opinions, findings,and conclusions or recommendations expressed in this report are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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NOTATION

A	Constant
Amp(ω)	Amplification factor
Amp_c	Amplification factor associated with a correction mode shape
Amp_ℓ	Amplification factor associated with a linear mode shape
B	Background component of the gust factor
b	Width of the building
be	Effective width of the building
b(z)	Width of the building as a function of height
C	Constant
C_c	Coefficient Eq. (5-15)
C_D	Drag coefficient
C_E	Earthquake participation factor
C_i	Participation factor
C_L	Lift coefficient
C_ℓ	Coefficient Eq. (5-14)
C_m	Added mass coefficient
C_{oi}	Amplification factor A.T.C. Eq. (5-4)
C_t	Surface friction drag coefficient
C_{vxm}	Coefficient A.T.C. Eq. (5-4)
C_Y	Correlation coefficient, horizontal direction
C_Z	Correlation coefficient, vertical direction
[C]	Generalized damping matrix
[c]	Damping matrix
D	Constant

D_{max}	Maximum amplification factor
D_i	Amplification factor
d	Depth of the building
E	Constant
F	Gust factor parameter
$\overline{F}$	Mean force
F'	Fluctuating force
F_{cx}	Forces associated with a correction mode
$F_{\ell x}$	Forces associated with a linear mode
F_x	Force at level x
F(x, t)	Forcing function
$F(\omega)$	Frequency representation of the force
$Fr(\omega)$	Random force (frequency domain)
$\overline{F}r(\omega)$	Mean component of random force (frequency domain)
$Fr'(\omega)$	Fluctuating component of random force (frequency domain)
$\tilde{F}$	Parameter Eq. (3-9)
f	Frequency
f(t)	Time component of forcing function
fr(t)	Random force (time domain)
$\overline{f}r(t)$	Mean random force (time domain)
fr'(t)	Fluctuating random force (time domain)
$\tilde{f}$	Parameter Eq. (3-10)
{f}	Vector of Forces
$F'_r\ \omega$	Power spectral density function of random force
G	Gust factor
$G_i(x)$	Geometrical component of response

$\overline{g}$	Peak factor
$H(\omega)$	Frequency transfer function
h	Height of the building
$h(t-\tau)$	Impulse function
$\overline{h}$	Average height of structures surrounding a given building
$J(\omega)$	Correlation transfer function
[K]	Generalized stiffness matrix
[k]	Stiffness matrix
$\overline{m}$	Average mass per unit length
m_x	Mass at level x
[M]	Generalized mass matrix
[m]	Mass matrix
N	Number of degrees of freedom
Nu	Cross correlation reduction function
P_{a-b}	Equivalent concentrated force at point a of span a-b
P_B	Background pressure
P_i	Concentrated force
P_o	Reference value of force (line pressure 1/2 ρ (V^2)
P_r	Effective pressure coefficient
P(z)	Distribution of pressure
$\overline{P}(z)$	Distribution of mean pressure
P'(z)	Distribution of fluctuating pressure
P_a	Value of distributed force at point a
p(x)	Geometrical component of forcing function
{P}	Vector of forces
Q_i	Generalized force

$\dot{q}_i$	Generalized displacement
$\ddot{q}_i$	First derivative of generalized displacement
q_i	Second derivative of generalized displacement
$\{Q_i\}$	Vector of generalized forces
R	Resonant component of gust factor
R_N	Reduction factor for cross correlation
R_u	Correlation function
$S(\omega)$	Power spectral density function of response
$S_c(\omega)$	Power spectral density function of correction function
$S_{pc}(\omega)$	Power spectral density function of a perfectly correlated force
S_r	Power spectral density function of effective pressure coefficients
$S_v(\omega)$	Power spectral density function of the fluctuating wind velocity
$S_{vv}(\omega)$	Power spectral density function of the vertical fluctuation of wind velocity
T	Period of vibration
$T_i(t)$	Time component of the response
t	Time
u	Relative displacement
$\{u\}$	Vector of relative displacement
V	Wind velocity
Vel	Velocity
V_c	Base shear associated with a correction mode
V_ℓ	Base shear associated with a linear mode
V_o	Base shear
V_r	Reference wind velocity
V_z	Distribution of wind velocity

$\overline{V}(z)$	Distribution of mean wind velocity
$V'(t)$	Distribution of fluctuating wind velocity
$\dot{V}(z, t)$	First derivative of wind velocity
V^*	Friction wind velocity
ν_{om}	Base shear, A.T.C. Eq. (5-2)
W	Weight of the structure
W_i	Weight at level i
$\overline{W}_i$	Coefficient A.T.C. Eq. (5-3)
w_{ij}	Weighting factor
$[W]$	Weight matrix
X_n	Model displacement
X_o	Reference value of response
X9x, t)	Response
$X(\omega)$	Frequency representation of displacement
$x(t)$	Displacement
$\overline{X}^2 (x, t)$	Mean square of the response
$\{x\}$	Vector of displacements
$\{\dot{x}\}$	Vector of velocities
$\{\ddot{x}\}$	Vector of accelerations
Y	Quantity Eq. (2-23)
Y_n	Quantity Eq. (2-23)
z	Height
z_d	Zero plane displacement Eq. (3-3)
z_o	Roughness length
z_r	Reference height

α	Exposure coefficient
β	Damping as a percent of critical
Γ_e	Numerator of participation factor for earthquake
Γ_i	Numerator of participation factor
Γ_ω	Numerator of participation factor for wind
γ_i	Participation factor modification factor
ΔX	Separation in X
ΔY	Separation in Y
ΔZ	Separation in Z
δ	Exponent Eq. (4-11)
θ_i	Response
λ	Exponent Eq. (4-11)
μ	Exponent Eq. (4-12)
ν	Apparent frequency
ξ	Parameter Eq. (3-15)
ρ	Density
σ	Standard deviation
σ_g	Standard deviation of peak factor
τ	Time
Φ	Matrix of mode shapes
ϕ	Mode shape
$\chi(\omega)$	Power spectral density function
ω	Circular frequency
$\{\rho\}$	Vector defined in Eq. (2-18)

CHAPTER 1

INTRODUCTION

1-1 Objectives of the Investigation.

Practically all buildings and structures are subjected to forces arising from natural hazards. Among those, earthquake and wind forces are of primary concern to the designer of building structures. As a result of their random properties earthquake and wind forces are difficult to predict and must be estimated on the basis of judgement, experience and statistical analyses.

In the design of buildings and structures, the geographic location plays an important role in the determination of the earthquake-wind hazard. There are zones known to be especially earthquake prone; there are locations where high winds may be the dominant parameter; there is also a third category where both the seismic and the wind hazard may be nearly equally important. The type of structure also plays a role in arriving to the design criteria. As a rule, structures with high natural periods of vibration are especially sensitive on an overall basis to wind loading; structures with intermediate to low periods of vibration are likely to experience strong lateral forces during an earthquake.

In dynamic problems there are two general types of analysis procedures; one is carried out in the time domain and the other is carried out in the frequency domain. Time domain analysis is preferred for highly transient short duration loadings, while frequency domain analysis is better suited for long duration and/or more steady state type loadings. In addition, loadings commonly are estimated through either deterministic or probabilistic procedures.

In the past, earthquake resistant design has been treated generally as a deterministic procedure wherein the forcing load is usually approximated by an equivalent static loading coefficient. More recently the loading coefficients have been determined on the basis of statistical analysis of a selected sample of acceleration time histories. Present code recommendations based on deterministic time domain analysis of earthquake base excitation permit three general procedures for evaluating the dynamic forces on structures. These procedures are: (1) equivalent lateral load procedure involving specification of a base shear coefficient, (2) modal analysis and response spectrum techniques, and (3) step by step integration of the time history.

Wind forces, on the other hand, constitute a more steady long duration loading, where frequency domain techniques are a more suitable procedure of analysis. In addition, specific problems of wind loading such as correlation of pressures, which are better treated by statistical means, and the stationary properties of the wind flow make random vibration theory the most appropriate procedure for evaluating the response of structures subjected to strong winds. Present code specifications for wind loading are derived partially through the use of standard random vibration theory.

The principal objectives of this research study were to investigate the relationship between dynamic analyses for earthquake and wind loading, to demonstrate the commonality between the analysis techniques and to develop response spectra for wind loading which could be employed with modal analysis techniques to compute the response of structures subjected to strong winds. In order to achieve clarity it is noted that wind response spectrum is presented as a plot of effective pressures and it is divided into two parts a mean pressure and a fluctuating effective pressure.

In addition, and in order to compare the effects of earthquake and wind loading, the base shear was used as a reference frame for determining which of the dynamic loadings, namely earthquake or wind, was the governing factor in design. Consequently, another goal of this investigation was to develop a simple procedure to evaluate the base shear associated with the wind loading.

The foregoing should not be construed as suggesting that the design criteria for earthquake and wind hazards are the same. Indeed quite different criteria commonly are involved. For example in the case of earthquake resistant design the approach may be one of accepting the possibility of some limited damage yet desiring to prevent serious damage or collapse. In the case of wind damage design the approach may be one of desiring to preclude undesirable building motion in strong gusting wind and to preclude localized building damage. The goal is to provide a structure which remains serviceable under all design wind loading, including lower level wind loadings that may occur often and routinely. In spite of these differences there are features common to the analysis of the building structure as a whole for both wind and earthquake excitation, and it is these approaches, centering around modal analysis and response spectrum techniques, which are the subject of this research investigation. Although it is believed the features (principles and techniques) of the commonality of approaches are clearly delineated herein, it is appreciated that design application may follow only after further study and after additional loading data for wind becomes available.

1-2 Background.

In the past few decades, with the worldwide population increase and concentration into metropolitan areas and the proliferation of man-made structures the consequences of natural disasters have become increasingly important. Natural hazards such as strong earthquakes and high winds have brought attention to dynamic loadings which in turn have led engineers to search for improved approaches for analysis and design.

Earthquake -- Although it is recognized that earthquakes are a random process which can best be evaluated through the use of statistical techniques, the deterministic analysis based on modal decomposition and response spectrum techniques is commonly used in earthquake resistant design. Modal analysis procedures, which are extensively discussed in the literature, can be found in such standard references as Newmark and Rosenblueth (1971), Clough and Penzien (1975), and Blume, Newmark and Corning (1961). Statistical analysis of acceleration records summarized as guide lines for the estimation of the response spectrum as a function of frequency and damping have been presented by Newmark and Hall (1969, 1973 and 1978), Hall, Mohraz and Newmark (1976), and Newmark, Blume and Kapur (1973).

Present approaches for earthquake resistant design can be divided into three areas. The first and most general procedure, step by step integration of the equations of motion, requires a formidable computational effort and is used only for complex problems. The second procedure consists of modal analysis and response spectrum techniques. This approach also requires a considerable amount of computation for evaluating the mode shapes of the structure and to combine the various modal contributions. The third approach is the base shear coefficient or equivalent lateral load procedure. In this case a base shear coefficient is specified as a percent of the total weight of the structure. Once the base shear is evaluated it is distributed to the various nodal points of the structure. All three procedures may include consideration of inelastic properties and soil structure interaction effects.

In the United States base shear coefficients procedures are specified by various code authorities including, for example, the Uniform Building Code (U.B.C.), the National Building Code (N.B.C.) and the American National Standards Institute (A.N.S.I.). These procedures, in some cases, present great variations from one code to another and between editions of the code.

More recently the Applied Technology Council (A.T.C. (1973)) has presented a comprehensive tentative specification for earthquake design of buildings. An attempt has been made to explicitly evaluate in a rational manner the various parameters that enter into the analytical procedure. Procedures presented in A.T.C. (1973) include consideration of response spectra for different types of structures and soil conditions, and the analytical techniques for computing earthquake response including inelastic properties of the structures, and soil-structure interaction effects. It seems fairly certain that the modern principles delineated in A.T.C. (1973) will find their way into building codes in the years ahead.

Wind -- Historically, concepts of isotropic turbulence have been used for studying wind forces, and wind loading has been customarily described utilizing statistical formulations. One of the techniques frequently used is to divide the turbulent wind flow in two parts. The first is a mean flow and the second a fluctuating flow. Therefore, the wind velocity is divided into a mean velocity which has a constant time history which is customarily treated as a static loading, and a fluctuating velocity which varies with time and which is treated as a dynamic loading.

Since the beginning of the century considerable effort has been devoted to the study of the velocity pressure relations and the evaluation of drag coefficients. Drag coefficients for wind flow obtained from experimental studies are reported throughout the literature.

In more recent time, the response of structures to the dynamic component of the wind loading usually has been treated as a stochastic process, with random vibration techniques being used to predict the behavior of structures under wind excitation. Different spectral representations for the dynamic component of the wind velocity have been developed by Davenport (1961), Simiu (1973), and Kaimal et. al.(1972). As a result of the application of such spectral representations and random vibration analysis, a gust response factor has been proposed by various investigators, including principally, Davenport (1967), Vellozi and Cohen (1968), Vickery (1971), and Simiu and Lozier (1975).

Earthquake and wind -- Extensive research has been undertaken in both earthquake and wind analysis as separate subjects. However, little research has been conducted in the area of similarities between the behavior of structures under both earthquake and wind excitations. Newmark (1966) proposed that since the equations of motion are practically the same for earthquake and wind vibrations, the procedure of analysis should be similar in both cases. Newmark points out that some of the experience and knowledge available in earthquake engineering could be advantageously used in the design of structures subjected to strong winds. Newmark and Hall (1968) suggested that it is possible to draw a diagram similar to the earthquake response spectrum for loading such as wind making use of the relationship between the response of dynamic systems to motion or to external loading and inertial loadings. More recently, Novack (1974) suggested that such concepts as soil structure interaction, developed for earthquake engineering, could be included in wind resistant design. These studies have suggested, for some time, the possibility that there are common analysis procedures that need to be explored in detail.

1-3 Scope of the Investigation.

In this dissertation the commonality of analysis techniques for earthquake and wind loading was investigated. Also techniques for developing of response spectra for the dynamic component of the gusting wind were studied intensely.

In chapter two a brief overview of structural dynamic methods for deterministic and stochastic analyses is presented. Special attention is devoted to the derivation and interpretation of the participation factors fundamental to a complete understanding of the modal analysis technique. Continuous and discrete systems are discussed, and rules for the computation of participation factors for the case of wind loadings (distributed loads acting on discrete systems) also are given. Power spectral density functions and input-output relations as well as mean square response and peak factors are described briefly for random vibration analysis. In every case an attempt is made to explain the physical meaning of the expressions and an attempt has been made to keep the mathematical complexity of the random vibration theory to a minimum. In addition, the commonality of the solution procedures for the equation of motion for earthquake excitation and a general dynamic loading is discussed. Moreover a solution technique for a general dynamic loading is presented as a function of the well known modal procedure employed for earthquake analysis. Finally, the similarity of deterministic and random vibration analysis is presented.

In chapter three, the most important physical properties of wind velocity, pressure, and velocity-pressure relationships are outlined. Special consideration is given to the wind-structure interaction properties such as drag coefficients and correlation of pressures. Also, the geometrical distribution of wind pressures on the various faces of the structure is discussed.

In chapter four, various procedures for the computation of response spectra for the dynamic component of the wind loading are presented. The result is a spectrum of effective pressures for the fluctuating component of the wind loading. The response spectra are calculated using the power spectral density functions proposed by Davenport (1961) and Simiu (1973). A simplified procedure is presented for arriving at the response spectrum for wind. This procedure includes specific recommendations for computing the spectral base lines (unamplified lines) and the amplification factors.

In chapter five, a deterministic procedure for the analysis of wind loadings is presented and demonstrated. Such procedure includes the computation of the participation factors and the use of the response spectrum derived in chapter four. In this chapter the following topics for wind analysis receive special attention:

Development of a simplified procedure to compute the lateral forces using the concept of the distribution of base shear.

A simplification of the deterministic procedure, using a first mode approximation, to obtain a gust response factor.

A comparison of the gust response factors computed using the response spectrum procedure with the gust response factors specified by various code authorities such as A.N.S.I. and the National Building Code of Canada (N.B.C.C.) and some independent authors such as Vickery (1971) and Simiu and Lozier (1975).

Finally, to demonstrate the applications of the response spectrum approach for wind loading and its commonality with earthquake analysis one simple building is analyzed for earthquake and wind excitations, and three for wind alone. Particular attention is given to the computation of base shear and the distribution of lateral forces in the structures for both earthquake and wind loading.

CHAPTER 2

OVERVIEW OF STRUCTURAL DYNAMICS

Only a brief description of the principles of structural dynamics employed in this investigation are presented in this chapter. For additional information on the subject, readers are referred to such standard sources as Newmark and Rosenblueth (1971), Clough and Penzien (1975), and Hurty and Rubinstein (1964).

2-1 Definition of the Equation of Motion.

The movement of a linear system with N degrees of freedom, as shown in Fig. 1, can be written in a general way as a set of coupled differential equations which include parameters such as time or frequency, geometry, mass, stiffness, and damping. The equation of motion for a lumped mass system is usually derived by using either the d'Alembert principle or equilibrium relations and has the following form:

$$[m]\,\{\ddot{x}\} + [c]\,\{\dot{x}\} + [k]\,\{x\} = F\,(z,t) \qquad (2\text{-}1)$$

where

$[m]$ is the mass matrix,

$[c]$ is the damping matrix,

$[k]$ is the stiffness matrix,

$\{x\}$ is the displacement vector,

$\{\dot{x}\}$ is the velocity vector,

$\{\ddot{x}\}$ is the acceleration vector,

$F(z,t)$ is the forcing function which is dependent on the time history or frequency content of the record and the geometric distribution of the forcing load.

2-2 Solution of the Equation of Motion.

In the solution of the equation of motion the forcing load F(z,t) is usually separated into two independent functions. As a result, the forcing function F(z,t) is separated into the product of two components. The first is a geometry dependent part, p(z), which is a function of the spatial distribution of the forcing load. The second, f(t), represents the variation of the load as a function of time or frequency. As a result of the separation of variables hypothesis, the forcing function may be expressed as follows:

$$F(z,t) = P_0\ p(z)\ f(t) \tag{2-2}$$

where

P_0 is a constant, or reference value, usually taken as the maximum value of the forcing load,

p(z) is the geometric distribution of the forcing load,

f(t) is the time history of the forcing load.

The response function, X(z,t) is separated into a set of geometric functions, $G_i(z)$, and a set of time history distributions $T_i(t)$. The response, then, is written as follows:

$$X\ (x,t) = X_0 \sum G_i(z)\ T_i(t) \tag{2-3}$$

where

X_0 is a constant, or a reference value of the response,

$G_i(z)$ is the geometric distribution of the response,

$T_i(z)$ is the time history of the response.

The solution procedure consists of finding a relationship between p(z) the input, and $G_i(z)$ the output geometric functions, and f(t) the input time history and $T_i(t)$ the output response function. In obtaining the response it is obvious that the input forcing load is modified by the resisting system with its inherent mass, stiffness and damping properties. The parameters p(z) and f(t) as well as the geometry and properties of the resisting system can be considered to be given quantities. On the other hand, the response obviously is a function of the resistance, or in one sense the resisting system can be thought of as a transfer function in relating loading and response. The response is of particular theoretical and practical interest, and in the following section attention will be focused especially on two aspects of the total response function, namely the participation factor one part of the response that accounts for the space distribution of the forcing load and the time dependent part of the response. The time dependent part is conveniently expressed for practical interpretation in the frequency domain.

Generalized coordinates -- In the solution of the equations of motion a series of operations is required. One of the techniques frequently used is the transformation of the equations of motion, written in Cartesian coordinates, into a new set of real variables known as generalized coordinates that will uncouple the system of equations. Langhaar (1962) gives the following definition for generalized coordinates: "If a mechanical system consists of a finite number of material points, its configuration can be specified by a finite number of real variables called generalized coordinates". To transform the equation of motion into the generalized coordinates the displacements are defined as a function of a set of vectors $\{\phi_i\}$, which are known as "mode shapes" and are dependent only on the geometrical configuration and structural parameters; and a new set of generalized time dependent coordinates q_i. This transformation is written as follows:

$$\{x\} = \sum_{i=1}^{N} \{\phi_i\}\, q_i \qquad (2\text{-}4)$$

Upon transformation to the new set of generalized coordinates the equation of motion will yield an eigenvalue problem of the form:

$$\phi_i'' + \lambda\phi_i = 0 \qquad (2\text{-}5)$$

Or expressed in a more convenient way:

$$[k - \omega_i^2\, m]\, \{\phi_i\} = 0 \qquad (2\text{-}6)$$

where

ω_i is the ith circular frequency of vibration, and $\{\phi_i\}$ is the ith normal mode of vibration.

With the help of the orthogonality properties of $\{\phi_i\}$, the equations of motion can be written in their uncoupled form in terms of the generalized parameters as follows:

$$[M]\, \{\ddot{q}\} + [C]\, \{\dot{q}\} + [K]\, \{q\} = \{Q\} \qquad (2\text{-}7)$$

where by definition

$[M]$ is the generalized mass matrix, $= \Phi^T\, [m]\, \Phi$

$[C]$ is the generalized damping matrix, $= \Phi^T\, [c]\, \Phi$

$[K]$ is the generalized stiffness matrix, $= \Phi^T\, [k]\, \Phi$

$\{Q\}$ is the generalized load, $= \Phi\{P\}$

Φ is the matrix of the vectors $\{\phi_i\}$.

It is important to note that the forcing load F(z,t) is transformed into a set of generalized loads, Q_i. The meaning of the generalized loads Q_i can be derived conceptually from the work done by the forcing load when the structure is vibrating in its ith natural mode. For lumped mass systems, the generalized load associated with the ith mode of vibration may be written as:

$$Q_i = P_o \sum_{j=1}^{N} \phi_{ij} P_j f(t) \tag{2-8}$$

where

j is the jth point of location, or nodal point.

As a result of the assumption that the time and the geometry can be separated into two independent functions, it is easy to see that the generalized load has a part, defined as Γ_i, which accounts for the geometric variations of the forcing function and a time dependent function f(t). The quantity Γ_i may be thought of as a measure of the extent to which the ith normal mode participates in synthesizing the total load of the structure. For discrete systems, Γ_i is written as follows:

$$\Gamma_i = P_o \sum_{j=1}^{N} \phi_{ij} P_j \tag{2-9}$$

for continuous systems, the summation is replaced by an integral and Γ_i becomes:

$$\Gamma_i = P_o \, 1/L \int_0^L \phi_i (z) \, p(z) \, dz \tag{2-10}$$

The factor 1/L is introduced in order to make Eq. (2-10) dimensionally consistent with Eq. (2-9). Therefore, the generalized force can be written in a general form as:

$$Q_i = \Gamma_i \, f(t) \tag{2-11}$$

There are also cases where a distributed load is applied on a lumped mass system. For those cases, the load should be concentrated at the discrete points of the structure. Guidelines for computing the equivalent concentrated loads are presented by Newmark and Rosenblueth (1971), and summarized as follows for the case of uniformly spaced discrete points:

Let h denote the distance between uniformly spaced points a, b, c, etc. (see Fig. 2), and p_a, p_b, p_c the value of the distributed load at points a, b, and c. Define P_{a-b} as the equivalent concentrated load at point a of span a-b, and P_{b-a} as the equivalent concentrated load at point b of span a-b.

For a polygonal approximation, the equivalent concentrated loads are:

$$P_{a-b} = \frac{h}{6} (2 \, p_a + p_b) \tag{2-12a}$$

$$P_{b-a} = \frac{h}{6} (2 \, p_b + p_a) \tag{2-12b}$$

The treatment of more complex cases as higher order approximations and nonequally spaced discrete points is found in Newmark (1943), Newmark and Rosenblueth (1971), and Salvadori and Baron (1952).

2-3 Deterministic analysis.

The deterministic analysis, based on modal decomposition and the response spectrum technique is widely used for engineering applications. The following derivations are based on the assumption that the equations of motion can be uncoupled and solved as a system of "equivalent" single degree of freedom systems that later are combined to obtain the complete response of the structure.

For convenience, the damping term is excluded from the equation of motion and is included in the response spectrum. This procedure is usually preferred because of the difficulties involved in the evaluation of the damping matrix. In addition the solution procedure is simplified because the system is now treated as a combination of undamped single degree of freedom systems. The equation of motion for the ith generalized coordinate is then simplified to:

$$\ddot{q}_i + \omega_i^2 q_i = Q_i/M_i \qquad (2\text{-}13)$$

The decoupled ith equation of motion can be solved using the Laplace transformation technique or any other suitable procedure. The solution yields the well known expression in terms of the participation factors C_i, which are a function of Γ_i, and the response function θ_i. The solution of the ith generalized coordinate then may be written as follows:

$$q_i = C_i \theta_i \qquad (2\text{-}14)$$

For linear systems, θ_i is obtained from the evaluation of the Duhamel integral. Therefore θ_i is defined as follows:

$$\theta_i = \frac{1}{\omega_{Di}\, m_i} \int_0^t \bar{e}^{\beta\, \omega_i (t-\tau)} \sin \omega_{Di}\, (t-\tau)\, f(\tau)\, d\tau \qquad (2\text{-}15)$$

where

ω_{Di} is the damped frequency of vibration = $\omega_i \sqrt{1 - \beta_i^2}$

In general, for transient excitation it is difficult to find a closed form solution for the response integral. Therefore numerical procedures such as Newmark-Beta-Method (Newmark, 1963) are commonly used.

Participation factors -- The participation factor C_i requires additional attention. C_i is the ratio of Γ_i, the geometric component of the ith generalized load (Eq. (2-9) and Eq. (2-10)) and M_i the generalized mass corresponding to the ith mode of vibration. Therefore C_i is written as follows:

$$C_i = \Gamma_i / M_i \qquad (2\text{-}16)$$

The quantity Γ_i can be derived from the work done by the forcing load when the structure is vibrating in its ith natural mode of vibration. Since Γ_i is dependent on the structure mode shapes and the geometric configuration of the forcing load, a different value of Γ_i must be expected for different types of structures and geometrical configurations of the forcing load. The generalized mass M_i is independent of the forcing load, and therefore it is a constant regardless of the type of excitation applied to the structure.

Newmark (1966) has proposed that the numerator of the participation factor, Γ_i, can be written in the following general form:

$$\Gamma_i = \{\phi_i\}^T [m] \{\rho\} \tag{2-17}$$

where

$\{\rho\}$ is a vector which depends on the geometry of the applied load and the mass distribution of the structure.

The vector $\{\rho\}$ can be easily calculated by Eq. (2-9) and Eq. (2-17) as follows:

$$\Gamma_i = P_o \sum_{j=1}^{N} \phi_{ij} P_j = P_o \{\phi_i\}^T \{P\} = \{\phi_i\}^T [m] \{\rho\}$$

and after some manipulation the following expression is obtained:

$$\{\rho\} = P_o [m]^{-1} \{P\} \tag{2-18}$$

where

$[m]^{-1}$ is the inverse of the mass matrix,

$\{P\}$ is the geometrical distribution of the forcing load applied at discrete points 1, 2, 3, .. , j,..N

For earthquake excitation the discrete forcing loads are dependent only on the mass of the structure and the ground acceleration. In this case the vector $\{\rho\}$ is reduced to a unit vector. It should be pointed out that since $\{\rho\}$ is independent of the mode shape $\{\phi_i\}$ it must be calculated only once during the solution procedure of any given vibration problem.

In the solution of a vibration problem the participation factor does not have a unique value. The numerical value of the coefficient C_i depends on the normalization procedure used for the computation of the ith mode of vibration. The variation in the numerical value of the participation factor is not surprising, because the participation factor has the mode of vibration in the numerator and the square of the mode of vibration in the denominator, therefore the normalization constant for C_i does not cancel out. Fortunately, this apparent inconsistency does not affect the solution of the problem because the quantity that must remain constant is the product $\{\phi_i\} \times C_i$, and it can be seen that the mode normalization constant, for this product is thus cancelled out. There are various ways of normalizing the modes of vibration. Some engineers normalize the modes of vibration in such a way that the resulting participation factors have a numerical value equal to one, whereas other analysts will prefer a set of modes of vibration that yield a unit generalized mass. The two approaches have different computational advantages and simplify the calculation of the response.

In general, summarizing the concepts presented in this section, the participation factor for earthquake base excitation is written as:

$$C_i = \frac{\{\phi_i\}^T [m] \{1\}}{\{\phi_i\}^T [m] \{\phi_i\}} \tag{2-19a}$$

and the participation factor for a general loading as:

$$C_i = \frac{\{\phi_i\}^T [m] \{\rho\}}{\{\phi_i\}^T [m] \{\phi_i\}} \tag{2-19b}$$

<u>Combination of modes</u> -- In the previous Section the solution of only one of the generalized coordinates was presented. To obtain a complete solution of the equation of motion it is necessary to combine and transform the generalized coordinates into the original set of Cartesian or material

coordinates. In order to attain this transformation Eq. (2-15) is substituted into Eq. (2-4) and the final form of the displacements written as a linear combination of the generalized coordinates, is given by the expression:

$$X_{(j)} = \sum_{i=1}^{N} \phi_{ij} C_i \theta_i \tag{2-20}$$

In earthquake engineering it is customary to define the maximum absolute value of θ_i as the amplification factor D_i. The value of D_i is usually given by the response spectrum which is a function of the natural frequency of vibration and the amount of damping present in the system. In general, the same response spectrum technique can be used for other types of forcing loads such as wind and blasting, as noted by Newmark (1966) and Newmark and Hall (1968). The displacements can now be rewritten as a function of the response spectrum as follows:

$$X_{(j)} = \sum_{i=1}^{N} \phi_{ij} C_i D_i \tag{2-21}$$

Since D_i is the maximum absolute value of the response of an equivalent single degree of freedom system, Eq. (2-21) represents an upper bound of the response given as the sum of the maximum absolute values of each modal contribution. This upper bound is conservative because it is highly improbable that all the maximum responses would occur at the same time. Consequently, other techniques for the combination of modal contributions normally are used.

It is shown by Goodman, Rosenblueth and Newmark (1955) and Rosenblueth (1956), that for structures exhibiting linear behavior with uncorrelated or statistically independent modes of vibration the expected numerical value of the response is the square root of the sum of the squared contributions associated with the various modes of vibration. Therefore the most probable value of the response can be written as follows:

$$X_{(j)probable} = \left[\sum_{i=1}^{N} (C_i \; \phi_{ij} \; D_i)^2 \right]^{1/2} \tag{2-22}$$

Rosenblueth (1956) has noted that the same technique could be used to calculate quantities other than displacements if the response of the desired quantity is equal to the sum of the responses of the natural modes, where each modal contribution is regarded as an independent, linearly damped, single degree of freedom system. If the particular desired quantity (for example base shear, story shear, overturning moment, stress, etc.) is designated as Y and the particular response of the nth mode of vibration as Y_n the maximum value of Y_n is given by:

$$Y_{max} = \sum_{n=1}^{N} | C_n \; Y_n \; D_n | \tag{2-23}$$

and the most probable value of Y as:

$$Y_{probable} = \left[\sum_{i=1}^{N} (C_n \; Y_n \; D_n)^2 \right]^{1/2} \tag{2-24}$$

Moreover, Rosenblueth (1956) observed that the condition of statistical independence of the normal modes is satisfied even for relatively close natural periods and that Eq. (2-24) may be expected to hold with sufficient accuracy in practically all cases of interest in design. However, in cases where the frequencies are very close, the use of Eqs. (2-22) or (2-24) may result in a large underestimation of the response. It is an accepted practice to include the cross-products of the responses associated with each pair of natural modes, in the computation of the total response, when the difference between the natural frequencies is less than ten percent. The ten percent limit is defined as follows:

$$\frac{\omega_j - \omega_i}{\omega_i} \leq 0.10 \tag{2-25}$$

where

$1 \leq i \leq j \leq N$, and

N is the number of modes used in the computation of the response.

If all the cross-terms are included, the equation for combining the modal contribution of the response is written as:

$$Y = \left[\sum_{i=1}^{N} (C_i \; Y_i \; D_i)^2 + \sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} Y_i \; Y_j \; C_i \; C_j \; D_i \; D_j \; w_{ij} \right]^{1/2} \tag{2-26}$$

where

w_{ij} is a weighting factor which varies between 0 and 1 according to the importance of the contribution of the cross-product.

In the cases where the cross-terms are included in the computation of the response, the resulting value will be somewhere between the values predicted by Eqs. (2-23) and (2-24). If the numerical value of the weighting factors, w_{ij}, are equal to one Eq. (2-25) will yield Eq. (2-23), whereas if all w_{ij} are equal to zero Eq. (2-25) would simplify into Eq. (2-24).

Procedures used to evaluate the importance of the cross-term's contributions vary from (1) considering w_{ij} equal to one if the difference between the frequencies ω_i and ω_j is less than ten percent and zero in any other case (ten percent procedure) to (2) evaluation of the weighting factor for each cross-term contribution.

Sometimes, for special structures, modes associated with high frequencies may be present in the computation of the response. In those cases the combination of the contributions of such higher modes requires special attention. If those high modes correspond to the unamplified high frequency region of the response spectrum (unamplified acceleration in earthquake analysis) that portion of the loading will behave just like a static load. Therefore, those modes should be combined in an algebraic manner and independent of the modes associated with the amplified region of the response spectrum.

In earthquake engineering only massive structures, as for example nuclear power plants, have significant contributions at such high frequencies (over 33 hertz). However in wind problems, most of the energy is associated with the low frequency range and the response spectrum (which will be developed in Chapter 4) converges to the unamplified high frequency line at frequencies as low as 2 to 5 hertz.

Buildings subjected to wind forces have modes with significant contributions even in the rigid region of the spectrum. In this case the modal contributions are divided into two groups. The first includes the modes with frequencies associated with the amplified region of the spectrum. The second group consists of the modes with frequencies corresponding to the rigid region of the spectrum. The first group of modal contributions is combined in a square root of the sum of the squares fashion. The second group represents a set of essentially static loads and its modal contribution should be added in an algebraic manner. The high frequency static contributions can be condensed into a single residual term. A procedure for carrying out this computation has been presented by Biswas and Duff (1979). Those high mode contributions become more critical for effects associated with the lower levels of the structure.

Forces and Base Shear -- Once the modal displacements, either absolute or relative, have been computed the lateral forces induced on the structure by these displacements can be easily calculated. The operations are performed for each mode of vibration and the forces are combined using any of the rules previously discussed.

In the beginning of this section it is established that the displacements associated with the nth mode of vibration are given by the following expressions:

for the absolute displacements:

$$X_n = \{\phi_n\} C_n D_n \tag{2-27a}$$

for the relative displacements:

$$U_n = \{\phi_n\} C_n D_n \tag{2-27b}$$

Since the displacements are known, their associated modal lateral forces can be calculated using the following statical relationship:

for the absolute displacements:

$$\{f\} = [k]\ \{X\} \tag{2-28a}$$

for the relative displacements:

$$\{f\} = [k]\ \{U\} \tag{2-28b}$$

As has been previously shown in Eq. (2-6), the mass and the stiffness matrixes are related by the characteristic equation:

$$[k]\ \{\phi_n\} = [m]\ \{\phi_n\}\ \omega_n^2 \tag{2-29}$$

The forces associated with the nth mode of vibration are calculated substituting Eqs. (2-27) and (2-29) into Eq. (2-28). Therefore the resulting forces vector is written as follows:

$$\{f_n\} = [m]\ \{\phi_n\}\ C_n\ D_n\ \omega_n^2 \tag{2-30}$$

where

$\{f_n\}$ is the vector of the lateral forces associated with the nth mode of vibration.

The modal base shear ν_{on} is the sum of the components of the modal forces vector $\{f_n\}$ If matrix notation is used, the sum is obtained by premultiplying Eq. (2-30) by a unitary row vector. Thus the base shear associated with the nth mode of vibration is written as:

$$\nu_{on} = \{1\}^T\ [m]\ \{\phi_n\}\ C_n\ D_n\ \omega_n^2 \tag{2-31}$$

2-4 Forces and Base Shear for a General Loading as a Function of the Earthquake Participation Factor.

Historically, for civil engineering structures, modal analysis procedures have been used to study the response of structures subjected to earthquake base excitation. Therefore it is convenient to express the solution of the equation of motion, for the case of a general load, in terms of the earthquake participation factors modified by a constant, γ_i, to account for the geometric differences of the forcing load.

For each mode of vibration the constant γ_i can be calculated by equating Eq. (2-19b), the participation factor for a general load, with Eq. (2-19a), the participation factor for earthquake base excitation, modified by the constant γ_i. This relation is written as follows:

$$\frac{\{\phi_i\}^T [m] \{\rho\}}{\{\phi_i\}^T [m] \{\phi_i\}} = \gamma_i \frac{\{\phi_i\}^T [m] \{1\}}{\{\phi_i\}^T [m] \{\phi_i\}}$$

Since the result of all four matrix triple products are scalars, it is possible to solve for γ_i, and the following expression is obtained:

$$\gamma_i = \frac{\{\phi_i\}^T [m] \{\rho\}}{\{\phi_i\}^T [m] \{1\}} \tag{2-32}$$

For earthquake base excitation γ_i has a numerical value equal to one. The participation factor for a general load can now be rewritten as a function of the earthquake participation factor as follows:

$$C_i = \gamma_i C_{E_i} \tag{2-33}$$

where

C_{E_i} is the participation factor for earthquake base excitation associated with the ith mode of vibration.

Therefore the expanded form of the participation factor for a general type of loading is:

$$C_i = \gamma_i \frac{\{\phi_i\}^T [m] \{1\}}{\{\phi_i\}^T [m] \{\phi_i\}} \tag{2-34}$$

Once the participation factor has been calculated the lateral forces and modal base shear can be found using the same procedures discussed in Section 2-3. It will be apparent later that the constant γ_i will appear in the expressions for lateral forces and modal base shear, but it will not be present in the equation for the distribution of the base shear.

The modal lateral forces are obtained replacing Eq. (2-34) into Eq. (2-30). Therefore the lateral forces are:

$$\{f_i\} = \gamma_i \frac{[m] \{\phi_i\} \{\phi_i\}^T [m] \{1\}}{\{\phi_i\}^T [m] \{\phi_i\}} D_i \omega_i^2 \tag{2-35}$$

In a similar way the modal base shear is calculated by substituting Eq. (2-34) into Eq. (2-31). Then the modal base shear is expressed as:

$$V_{oi} = \gamma_i \frac{(\{\phi_i\}^T [m] \{1\})^2}{\{\phi_i\}^T [m] \{\phi_i\}} D_i \omega_i^2 \tag{2-36}$$

Eq. (2-36) can be rewritten in a form similar to that proposed by A.T.C. (1973), Eq. (5-2), as follows:

$$\nu_{oi} = \overline{W}_i \, \gamma_i \, C_{oi} \tag{2-37}$$

where

γ_i accounts for the variation of the participation factor for a general load as a function of the earthquake participation factor.

$\overline{W}$ is given by A.T.C. (1973) Eq. (5-3). $\overline{W}$ is defined as a measure of the force synthesized by the ith mode of vibration and is given by the following expression:

$$\overline{W}_i = \frac{\left(\sum_{j=1}^{N} W_j \, \phi_{ij}\right)^2}{\sum_{j=1}^{N} W_j \, (\phi_{ij})^2} = \frac{(\{\phi_i\}^T \, [W] \, \{1\})^2}{\{\phi_i\}^T \, [W] \, \{\phi_i\}} \tag{2-38}$$

where

W_j is the weight of the structure at level j,

[W] is the matrix of the weights of the structure

C_{oi} is the amplification factor which is dependent on the frequency content and time history of the forcing loading. For earthquakes A.T.C. (1973) has defined the amplification factor as follows:

$$C_{oi} = \frac{D_i \, \omega_i^2}{g} \tag{2-39}$$

g is the acceleration of gravity.

Once the base shear is calculated, the factor γ_i is absorbed into the base shear coefficient and the equation for the distribution of forces becomes independent of the forcing load. Hence, the lateral forces are rewritten as a function of the modal base shear as follows:

$$\{f_i\} = \frac{[m]\ \{\phi_i\}}{\{\phi_i\}^T\ [m]\ \{1\}}\ \nu_{oi} \tag{2-40}$$

For the case of a diagonal mass matrix, Eq. (2-40) is similar to A.T.C. (1973) Eqs. (5-4) and (5-4a). In A.T.C. (1973) Eq. (5-4) the force at level x associated with the mth mode of vibration, F_{xm}, is given by:

$$F_{xm} = C_{vxm}\ \nu_{om} \tag{2-41}$$

where C_{vxm} accounts for the force distribution at different levels of the structure and is given by A.T.C. (1973) Eq. (5-4a)

$$C_{vxm} = \frac{W_x\ \phi_{xm}}{\sum_{i=1}^{N} W_i\ \phi_i\ m} \tag{2-42}$$

Therefore, it becomes apparent (regardless of the type of forcing load that is applied to the structure) that once the modal base shear is computed, the force distribution obeys the same relation for any type of forcing load applied to the structure. Moreover, if a linear mode shape is assumed, Eqs. (2-41) and (2-42) can be simplified to the following expression used in earthquake engineering for the distribution of base shear:

$$F_x = \frac{V_o\ W_x\ h_x}{\sum W_x\ h_x} \tag{2-43}$$

where

F_x is the lateral force at level x,

W_x is the weight at level x,

h_x is the height of the level above the base,

$\sum W_x h_x$ is the sum of all weights W multiplied by their respective heights h above the base and

V_o is the base shear force.

2-5 Random Vibration Analysis.

For completeness in this study and because wind loading is normally handled by random vibration procedures, it was felt desirable to present a brief description of these techniques and to demonstrate their commonality with deterministic analysis. One objective of this study was to demonstrate the practical applications of random vibration procedures and also to show that these techniques can be conveniently employed for generating response spectra. As will be presented in this section once the response spectrum has been evaluated the same modal analysis techniques usually employed in deterministic analysis may be used for random vibration analysis.

Before discussing the response of dynamic systems to random excitation it is convenient to review briefly the deterministic solution of a single degree of freedom system using the frequency response method. It should be noted that random vibration techniques and frequency response analysis are relatively unfamiliar to most structural engineers. The solution of vibration problems in the frequency domain requires the use of mathematical tools such as the Laplace transform and concepts of Fourier analysis. Normally Fourier analysis techniques are used to represent the forcing function in the frequency domain. If the forcing function is periodic such representation is attained by employing a Fourier series expansion, whereas if the forcing function is nonperiodic, as is generally the case in vibration problems, the frequency representation of the forcing load is obtained through the use of the Fourier integral.

Solution of a S.D.O.F. system in the frequency domain -- Only a summary of the most important points of the frequency response method is presented here. A complete and rigorous treatment of the method can be found in Hurty and Rubinstein (1964).

The equation of motion of a single degree of freedom oscillating system is written as:

$$m\ddot{x} + c\dot{x} + kx = f\,(t) \tag{2-44}$$

Eq. (2-44) has a time domain solution of the form

$$X(t) = \int_0^t h\,(t - \tau)\, f\,(\tau)\, d\tau \tag{2-45}$$

where

$$h(t - \tau) = \frac{1}{m\,\omega_D}\, e^{-\beta\omega\,(t-\tau)}\, \sin \omega_D\,(t-\tau)$$

This representation of the time domain response (as presented in Section 2-3) is commonly referred as the Duhamel integral.

To solve the equation of motion using the frequency response method, the first step is to represent the loading in the frequency domain by evaluating the Fourier transform of the forcing function f(t). This frequency representation of f(t) is defined as $F(\omega)$. Therefore $F(\omega)$ is expressed as follows:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt \tag{2-46}$$

In the frequency response method the output function or response $X(\omega)$ is also given as a function of frequency. Furthermore the relationship between the forcing function and the response is written as follows:

$$X(\omega) = H(\omega)\ F(\omega)\ \times\ \frac{1}{m\bar{\omega}^2} \tag{2-47a}$$

where

$H(\omega)$ is known as the frequency transfer function and defined as:

$$H(\omega) = \frac{1}{1 - \frac{\omega^2}{\bar{\omega}^2} + i\ 2\ \beta\ \frac{\omega}{\bar{\omega}}}$$

$\bar{\omega}$ is the natural frequency of the S.D.O.F. system

β is the damping as percent of the critical.

It can be shown, see Hurty and Rubinstein (1964), that $h(t-\tau)$ and $H(\omega)$ are a Fourier transform pair.

The relationship between the input force and the output force can be written as follows:

$$\tilde{F}(\omega) = |H(\omega)|\ F(\omega) \tag{2-47b}$$

A summary of the solutions for time and frequency domain are presented and compared in Table 1.

Random Vibration Techniques -- In the random vibrations techniques presented in this section, the level of statistics has been reduced to a minimum. Explanations are made in terms of familiar statistical concepts such as mean, variance and standard deviation. It is assumed that the random process can be characterized by two statistical descriptors which are its mean value and its variance or its standard deviation. Furthermore, any random process can be divided into two independent random processes, one is a mean process with zero variance, and the other is a zero mean process representing the variance of the original process. Moreover, the variance of a zero mean process equals its mean square value. Therefore the original random process is split into a mean process and a mean square process. It should be pointed out that the mean of the forcing function will yield a mean response and the mean square of the forcing function will yield a mean square response.

Solution of a S.D.O.F. Subjected to Random Input -- Now consider the same single degree of freedom system described by Eq. (2-44). In this case the forcing function is a random function defined as $F_r(t)$, then Eq. (2-44) is written as:

$$m\ddot{x} + c\dot{x} + kx = F_r\,(t) \tag{2-48}$$

Using the principle of superposition $F_r(t)$ can be divided into two parts. One $\overline{F}_r$ is the mean value of the forcing function and the other $F_r'(t)$ is the variation of $F_r(t)$ from the mean value $\overline{F}_r$. The forcing function is then written as:

$$F_r(t) = \overline{F}_r + F_r'\,(t) \tag{2-49}$$

The solution of Eq. (2-48) is now split into the solution of two equations, the mean response which is treated as a quasi-static problem and is given by the solution of:

$$m\ddot{x} + c\dot{x} + kx = \overline{F}_r \tag{2-50a}$$

which can be solved as

$$kx = \overline{F}_r \tag{2-50b}$$

and the mean square response can be obtained from the solution of the expression:

$$m\ddot{x} + c\dot{x} + kx = F_r' (t) \tag{2-51}$$

<u>Computation of the mean square response</u> -- The quantity $F_r' (t)$ is a random function having a zero mean. Hence, it represents the variance (as a function of time) of the random function $F_r(t)$. Consequently, the solution of Eq. (2-51) will consist in finding the mean square response as a function of the mean square forcing function. In order to obtain a solution it is necessary to represent the mean square of the forcing load in the frequency domain.

The frequency representation of the mean square can be derived from the Parseval theorem of Fourier analysis. Parseval's theorem can be written as follows (Spiegel 1972):

$$\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X}_1 (\omega)\, X_2 (\omega)\, d\omega \tag{2-52}$$

where

$x_1(t)$ and $X_1(\omega)$ are a Fourier transform pair, and $x_2(t)$ and $X_2(\omega)$ are another Fourier transform pair. In this case for the computation of the mean square

$x_1(t) = x_2(t) = x(t)$ and

$X_1(\omega) = X_2(\omega) = X(\omega)$

Therefore Eq. (2-52) can be written as:

$$\int_{-\infty}^{\infty} x^2 (t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, \bar{X}(\omega)\, d\omega = \frac{1}{\pi} \int_{0}^{\infty} |X(\omega)|^2\, d\omega \qquad (2\text{-}53)$$

Changing the limits of integration and dividing by 2T one obtains:

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2 (t)\, dt = \lim_{T \to \infty} \frac{1}{2\pi} \int_{0}^{T} \frac{|X(\omega)|^2}{T}\, d\omega \qquad (2\text{-}54)$$

The left hand side of Eq. (2-54) is precisely the definition of mean square.

The expression $|X(\omega)|^2/T$ is defined normally as the power spectral density function of x(t). This relation is written as follows:

$$\chi(\omega) = \frac{|X(\omega)|^2}{T} \qquad (2\text{-}55)$$

Therefore the mean square is equal to the integral along the frequency line of the power spectral density function divided by 2π. A graphical representation of this relation is shown in Fig. 3. It should be noted that the power spectral density function can be computed from statistical procedures, if one has sufficient data, by employing the autocorrelation function. Construction techniques for the power spectral density function employing the autocorrelation function are presented by Crandall and Mark (1963), Robson (1964), etc.

Computation of the response of a single degree of freedom system-- The solution of Eq. (2-51):

$$m\ddot{x} + c\dot{x} + kx = F_r'(t)$$

can be written using the frequency response procedure as follows:

$$X(\omega) = H(\omega)\, F_r'(\omega) \tag{2-56}$$

If each side of Eq. (2-56) is multiplied by its complex conjugate the following expression is obtained:

$$X(\omega)\, \overline{X}(\omega) = H(\omega)\, F_r'(\omega)\, \overline{H(\omega)\, F_r'(\omega)}$$

which can be rewritten as

$$|X(\omega)|^2 = |H(\omega)|^2\, |F_r'(\omega)|^2$$

If now we divide by T, the following expression is obtained:

$$\frac{|X(\omega)|^2}{T} = |H(\omega)|^2\, \frac{|F_r'(\omega)|^2}{T} \tag{2-57}$$

From Eq. (2-55) it can be seen that $\frac{|X(\omega)|^2}{T}$ is the power spectral density function of x(t), and $\frac{|F_r'(\omega)|^2}{T}$ is the power spectral density function of $F_r'(t)$. Therefore the power spectral density function of the response is the power spectral density function of the forcing function multiplied by the square of the frequency transfer function $|H(\omega)|^2$.

Therefore the mean square response can be written (using Eq. (2-55) and

and Eq. (2-57)) as follows:

$$\overline{X}^2\ (t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2\ (t)\ dt = \frac{1}{2\pi} \int_0^{\infty} |H\ (\omega)|^2\ F'_r\ (\omega)\ d\omega \qquad (2\text{-}58)$$

where

$|H\ (\omega)|^2$ is the square of the frequency transfer function and commonly known as the mechanical admittance function.

$F'_r(\omega)$ is the power spectral density function of $F'_r\ (t)$.

Root mean square response for multi degree of freedom systems. -- In Section 2-3 it has been shown that the ith decoupled equation of motion has the form:

$$\ddot{q}_i + 2\beta_i\ \omega_i\ \dot{q}_i + \omega_i^2\ q_i = C_i\ f(t) \qquad (2\text{-}59)$$

The decoupled equation of motion can be solved in the frequency domain as an equivalent single degree of freedom system. The steady state solution of the ith generalized coordinate is:

$$q_i(t) = \frac{C_i}{\omega_i^2} \qquad \frac{1}{1 - (\omega/\omega_i)^2 + i\ 2\beta\ \omega/\omega_i} \qquad (2\text{-}60a)$$

where

ω is frequency,

ω_i is the ith natural frequency of the system,

If the definition of mechanical admittance, Eq. (2-48) is now recalled, Eq. (2-60a) can be written in a more compact form as follows:

$$q_i(t) = \frac{C_i}{\omega_i^2} H_i\ (\omega)\ F'_r\ (t) \qquad (2\text{-}60b)$$

The response X(z,t) of a point z on the system at time t can be expressed in terms of the normal modes $\{\phi_i\}$ and the generalized coordinates $q_i(t)$, as previously defined in Eq. (2-4) as:

$$X_j(t) = \sum_{i=1}^{N} \phi_{ij}\, q_i(t) \tag{2-61}$$

Moreover, the mean square value of a zero mean response can be written as:

$$\overline{X^2}(z,t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} X^2(z,t)\, dt \tag{2-62}$$

To obtain an expression for the mean square response Eq. (2-60b) is substituted into Eq. (2-61) and then into Eq. (2-62) and the following result is obtained:

$$\overline{X^2(z,t)} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \sum_{i=1}^{N} \sum_{j=1}^{N} \phi_i\, \phi_j\, C_i\, C_j\, \frac{H_i(\omega)\, H_j(\omega)}{\omega_i^2\, \omega_j^2}\, (F_r'(t))^2 dt \tag{2-63}$$

The response can be simplified interchanging the order of integration and summation as follows:

$$\overline{X^2(z,t)} = \sum_{i=1}^{N} \sum_{j=1}^{N} \phi_i\, \phi_j\, C_i\, C_j \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{H_i(\omega)\, H_j(\omega)}{\omega_i^2\, \omega_j^2}\, (F_r'(t))^2 dt \tag{2-64}$$

The integral in the last expression requires further consideration. The following approximation may be used to get a more treatable solution.

$$\int_{-\infty}^{\infty} H_i(\omega)\, H_j(\omega)\, (F_r'(t))^2\, dt \simeq \int_{-\infty}^{\infty} |H_i(\omega)|\, |H_j(\omega)|\, (F_r'(t))^2\, dt \tag{2-65}$$

Thomson and Barton (1957) have shown that this approximation disregards phase relations, which will tend to result in a lower mean square value. Therefore, this is a conservative approximation.

With the help of Eq. (2-54) the forcing function $F_r'(t)$ is transformed into the frequency domain and the mean square response is written as:

$$\overline{X^2(z,t)} = \sum_{i=1}^{N} \sum_{j=1}^{N} \phi_i\, \phi_j\, \frac{C_i\, C_j}{\omega_i^2\, \omega_j^2}\, S_{ij}(\omega_i, \omega_j) \tag{2-66}$$

where

$S_{ij}(\omega_i, \omega_j)$ is the response function defined as:

$$S_{ij}(\omega_i, \omega_j) = \frac{1}{2\pi} \int_0^{\infty} |H_i(\omega)|\, |H_j(\omega)|\, F_r'(\omega)\, d\omega \tag{2-67}$$

A general forcing load may also have spatial correlations as is the case of wind loads which have correlation functions which vary with the frequency and the size of the structure. To account for this variation of the forcing load, a correlation function $|J(\omega)|$, is included in the forcing load. Now, the mean square value of the response function for a general type of forcing load may be written as:

$$S_{ij}(\omega_i, \omega_j) = \int_0^{\infty} |J_i(\omega)|\, |J_j(\omega)|\, |H_i(\omega)|\, |H_j(\omega)|\, F_r'(\omega)\, d\omega \tag{2-68}$$

where

$|J(\omega)|$ is the correlation function which accounts for the spatial variation of the forcing load ($|J(\omega)|$ is discussed in Chapter 3). If now in Eq. (2-66) the similar terms and cross-terms are separated the mean square response can be written as:

$$\overline{X^2 (z,t)} = \sum_{i=1}^{N} \phi_i^2 C_i^2 \frac{S_{ii} (\omega_i)}{\omega_i^4} + \sum_{i=1}^{N} \sum_{j=1}^{N} \phi_i \phi_j C_i C_j \frac{S_{ij} (\omega_i , \omega_j)}{\omega_i^2 \omega_j^2} \qquad i \neq j \tag{2-69}$$

It is important to note the similarity between Eq. (2-69) and Eq. (2-26) (the combination of modes for deterministic analysis). The same observations made in Section 2-3 for evaluating and ignoring the contribution of the cross-terms may apply here. In addition, it is noted that the products $|H_i(\omega)||H_j(\omega)|$ for $i \neq j$ are relatively small when compared with the same products for $i=j$. Moreover, terms with $i \neq j$ may be negative as well as positive depending on the product $\phi_i\phi_j C_iC_j$, while terms with $i=j$ are always positive. The contribution of the cross product terms to the mean square response will therefore be small. Based on these observations, it can be concluded that the cross-terms of the response could be neglected. This approximation has been extensively used in random vibration analysis. The simplified (approximate) form of the mean square response of the displacements may be expressed as:

$$\overline{X^2 (z,t)} = \sum_{i=1}^{N} \phi_i^2 C_i^2 \frac{S_{ii} (\omega_i)}{\omega_i^4} \tag{2-70}$$

2-6 Peak Factors

In engineering problems the maxima or peak values of the response are usually required. The techniques presented in Section 2-5 yield a mean square value of the response, but they do not allow a consideration of the maximum values.

The peak factor, defined as the ratio of the maximum value of the response to the root mean square value, provides a tool for assessing the maximum values of the response. Davenport (1961), using a statistical approach, has presented a derivation of the peak factor as a function of the equivalent frequency of vibration and the time interval over which the record has been averaged, as noted below.

The mean value of the peak factor, $\bar{g}$, and its standard deviation, σ_g can be computed employing the first crossing probability. For a narrow band process, Davenport has proposed the following expressions:

$$\bar{g} = \sqrt{2 \ell n \, \nu T} + \frac{0.577}{\sqrt{2 \ell n \, \nu T}} \qquad (2\text{-}71)$$

$$\sigma_g = \frac{\sqrt{6}}{\sqrt{2 \ell n \, \nu T}} \qquad (2\text{-}72)$$

where

ν is the equivalent or apparent frequency

$$\nu^2 = \frac{\int_0^\infty S(\omega)\, \omega^2 \, d\omega}{\int_0^\infty S(\omega)\, d\omega} \qquad (2\text{-}73)$$

T is the time interval over which the record has been averaged.

The range of values of $\bar{g}$ is generally between 2.5 to 4.5. In lieu of computing the first crossing probability, Vickery (1971) has suggested that $\bar{g}$ could be taken as 3.5 for most of the cases in design.

2-7 The Random Vibration Techniques Applied to the Response Spectra.

The same techniques used for deterministic analysis can be used for random vibration analysis if a suitable definition of a response spectrum function is attained. It will be shown that a response spectrum computed from random vibration techniques is very useful for practical applications.

There are some loadings in engineering, such as wind, that can be best described by statistical means. However the random vibration theory requires a great deal of computation and a rigorous mathematical treatment which makes random vibration techniques applicable only to a few highly idealized problems. The development of a response spectrum from random vibration will permit the treatment of more realistic problems including for example differences in the mass and stiffness distribution on the structure.

In order to remain consistent with the deterministic philosophy of analysis, the response spectrum should be calculated for an equivalent single degree of freedom system with various natural frequencies of vibration and amounts of damping. For random vibration analysis two types of response spectra can be calculated: a root mean square response spectrum and a peak or maximum response spectrum.

Root mean square response spectrum -- The root mean square response spectrum is given by the square root of the function $S_{ii}\ (\omega_i)/\omega_i^{\ 4}$, which represents the mean square value of the response. Such a function may be evaluated for various natural frequencies of vibration and amounts of damping. The root mean response spectrum for a given value of frequency and damping is defined as:

$$D_i \; (\omega_i, \beta) = \frac{S_{ii} \; (\omega, \omega_i, \beta)}{\omega_i^4} \tag{2-74}$$

where

ω_i is the natural or resonant frequency of vibration

β_i is the damping expressed as percent of critical S_{ii} $(\omega_i, \omega, \beta)$ is the power spectral density function of the response as defined in Eq. (2-68).

Peak response spectrum -- The peak response spectrum can be calculated by multiplying the root mean square spectrum, times the peak factor $\bar{g}$ given by Eq. (2-71).

$$D_{max_i} \; (\omega_i, \beta) = D_i \; (\omega_i, \beta) \times \bar{g} \tag{2-75}$$

This definition of a maximum value for an equivalent single-degree-of-freedom system is consistent with the philosophy of the response spectrum technique.

With this convenient definition of response spectra the modal forces and base shears can be computed by employing the same equations derived in Sections 2-3 and 2-4 for deterministic analysis.

The two spectra described in this section are calculated for wind loading in Chapter 4, using the power spectral density function of the wind velocity as input.

CHAPTER 3

PHYSICAL PROPERTIES OF THE WIND

3-1 Introduction

In order to study the behavior of structures subjected to strong wind loading it is necessary to review the principal physical properties of winds. Strong winds constitute a turbulent phenomenon with statistical descriptors being used to study and quantify the wind flow.

From an engineering point of view the most important properties of the wind are its velocity and intensity of turbulence, the velocity pressure relations and the distribution of pressures (associated with the wind) on the various faces of the structure. In addition, the aerodynamic properties of the structure also play an important role in the computation of the wind forces. Statistical formulations are used commonly for describing the fluctuation and intensity of the wind velocity, and some of the aerodynamic properties such as the correlation of pressures.

3-2 Wind Velocity.

The wind velocity is the most accepted parameter employed for classifying the various types of winds. Since the beginning of the century the measurement of the wind velocity has been a topic of study and concern among practicing engineers. With the invention of modern anemometers many wind records have been taken at different geographic locations, terrain roughness conditions and varying altitudes. Unfortunately those records have been averaged over different intervals of time; and various types of anemometers, having different response properties, have been employed.

The difference in the time interval for averaging the wind velocity presents serious problems. The numerical value of the average wind velocity has large variations depending on the chosen interval used for averaging the wind velocity. Drust (1960), and Deacon (1965), have presented procedures to normalize the wind velocity averaged over different time intervals to a uniform period of one hour. In this study, as generally is the case in wind engineering, the velocities are averaged over an interval of one hour. In other words and in order to be consistent with previous research all the computations are carried out in terms of hourly mean velocities.

3-3 The Wind Boundary Layer.

Moving along the surface of land or water wind develops a boundary layer in a manner similar to the boundary layer developed when a fluid flows over a rough plate. However, the thickness of the earth boundary layer is larger than those found in man-made aerodynamic bodies. In steady winds the thickness has been stated to be higher than 1000 ft. Statistical observations measured by Goddard (1935) and reported by Hoerner (1965) indicate that the boundary layer thickness (measured in feet) is on the order of 30 to 50 times the speed V (in ft/sec) above the layer. A.N.S.I. recommends the use of 900 to 1500 ft depending on whether or not the wind is blowing in open country or in the center of a large city.

Vertical Distribution of the Wind Velocity -- Inside the earth boundary layer the wind velocity is not constant. The wind velocity suffers some retardation near the ground because the terrain produces a friction drag which steadily decreases the wind velocity.

The effects of the friction drag decrease rapidly with the altitude through the thickness of the layer and become negligible outside of the boundary layer.

The altitude where the wind velocity is independent of the terrain friction drag is called the geostrophic altitude. The thickness of the boundary layer, or geostrophic altitude, is a function of the friction drag and consequently a function of the terrain roughness. Values of the boundary layer thickness for various exposure conditions are presented in Table 2. Therefore the wind velocity is largely dependent on the conditions of the terrain over which the wind flows.

Two models are generally employed for describing the vertical distribution of the wind velocity, namely the power law and the logarithmic law distributions.

The power law -- The power law is the model used in the majority of the present building codes such as A.N.S.I. and N.B.C.C. Early studies reported by Hoerner (1965) suggested that the wind velocity is proportional to the nth root of the altitude, n being somewhere between 2 to 7. This relationship may be stated generally as follows:

$$V(z) \propto \sqrt[n]{z} \qquad (3\text{-}1)$$

where

z is the altitude.

Recently Davenport has concluded that, for engineering applications, the mean wind velocity is well represented by the following power law:

$$V(z) = V_r \left(\frac{z}{z_r}\right)^{\alpha} \qquad (3\text{-}2)$$

where

z is the altitude,

z_r is a reference altitude taken as 33 ft (10 m),

V_r is the velocity at the reference altitude z_r,

α is a constant which depends on the conditions of the terrain.

The conditions of the terrain (exposure of the structure) are usually divided into three broad categories, i.e., A.N.S.I. gives the following classification:

Exposure A: Center of large cities and rough hilly terrain.

Exposure B: Rough wooded country, towns and city outskirts.

Exposure C: Flat open country, open flat coastal areas and grass land.

The values of the constant α for exposures A, B, and C are given in Table 2. Also, in Fig. 4 a plot of Eq. (3-2) for various values of α and their corresponding geostrophic altitude is presented.

Logarithmic law -- More recently, Simiu and Lozier (1975) on the basis of theoretical derivations and experimental measurements, have recommended the use of a logarithmic relationship for describing the vertical distribution of the wind velocity. The logarithmic distribution has the following form:

$$V(z) = 2.5\ V^*\ \ell n\ \left(\frac{z-z_d}{z_r}\right) \qquad (3\text{-}3)$$

where

$V(z)$ is the mean wind velocity at height z,

z_d is the zero plane displacement that should be taken as zero except for city centers where the smaller of 60 ft (20 m) or 0.75 $\overline{h}$ (where h is the average height of the surrounding area) must be taken.

z_o is the roughness length given in Table 3 for various terrain conditions.

V^* is the friction velocity defined as follows:

$$V^* = \frac{V(z_r)}{2.5 \ \ell n \left(\frac{z_r - z_d}{z_o}\right)}$$

z_r is a reference altitude usually taken as 33 ft (10 m),

$V(z_r)$ is the wind velocity at the reference altitude.

3-4 Velocity Pressure Relations.

In engineering problems the momentum model is generally accepted as a good representation of the velocity pressure relation. The momentum model assumes that the pressure is proportional to the square of the velocity plus an added mass term. The pressure $P(z)$ at a point p of elevation z on the surface of a building immersed in an unsteady flow can be expressed as:

$$P(z) = \overline{P}(z) + P'(z) \qquad (3\text{-}4)$$

$$P(z) = 1/2 \ \rho \ C_D \ (\overline{V}(z) + V'(z))^2 + C_m \ b(z) \ \mathring{V} \ (z,t) \qquad (3\text{-}5)$$

where

$\overline{P}(z)$ is the mean pressure,

$P'(z)$ is the fluctuating pressure,

$\overline{V}(z)$ is the mean velocity,

$V'(z)$ is the fluctuating velocity

$\mathring{V}(z,t)$ is the first time derivative of the velocity,

C_D is the drag coefficient,

ρ is the air density (0.00252 if the velocity is in ft/sec, or 0.00512 if the velocity is in mph),

C_m is the added mass coefficient,

b(z) is the width of the structure.

This expression of the momentum model for the velocity pressure relationship is accurate under the assumption that the transverse building dimension is small compared to the scale of the energy containing the eddies of turbulence (Simiu and Lozier, 1975). In addition, the assumption that the pressure is proportional to the square of the velocity is reasonable for the range of velocities usually found in ordinary wind flow. The importance of the added mass term has been analyzed by Vickery and Kao (1972), Kao (1971), Bearman (1972), and Petty (1972). All these authors, using wind tunnel tests, have concluded that for bluff bodies immersed in turbulent flow the added mass term is negligible. The velocity pressure relationship is therefore simplified to:

$$P(z) = 1/2\ \rho\ C_D\ \overline{V}^2(z)\ \left(1 + 2\frac{V'(z)}{\overline{V}(z)} + \left(\frac{V'(z)^2}{\overline{V}(z)^2}\right)\right) \qquad (3\text{-}6)$$

Further information from wind tunnel tests by Vickery and Kao (1972) has shown that the contribution of the quadratic ratio term $\left(\frac{V'}{\overline{V}}\right)^2$ is in the order of 3 to 5 percent. A more recent study by Soize (1978) indicates that this term may account for as much as 10 to 25 percent of the total pressure for structures smaller than 150 ft tall (50 m) located in city exposure. However, the influence of the quadratic ratio term decreases as the terrain friction decreases and the height of the structure increases. In this study the influence of the quadratic term in the velocity pressure relationship will be neglected.

3-5 Distribution of Wind Pressure, Drag Coefficients.

The drag coefficients and pressure distributions are quantities that must be determined experimentally. Drag coefficients are largely dependent on the shape and aerodynamic properties of the structure.

There are two types of drag coefficients: (1) the over-all drag coefficient which affects the whole structure, and (2) the local drag coefficients which are related to specific parts or portions of the structure.

A summary of drag coefficients for various types of structures are presented by Hoerner (1965), A.S.C.E. Wind Committee Report (1961) and Sachs (1972). Coefficients for the distribution of local pressures on roofs and walls can be found in A.N.S.I. and the A.S.C.E. Wind Committee Report (1961).

Horizontal distribution of the wind pressure -- For rectangular structures N.B.C.C. and A.N.S.I. have recommended the use of a drag coefficient of 0.80 and a lift coefficient of 0.50. In addition, A.N.S.I. recommends a lift coefficient of 0.60 for narrow structures in which the ratio height-width (h/b) or height-length (h/d) is larger than 2.5. For straight wind normal to the face of the structure a typical horizontal distribution of the wind velocity is presented in Fig. 5. It can be seen that at the windward face the pressures are larger in the center of the structure than in the corners of the building. At the leeward face, the pressures (actually suction or negative pressure) are more uniform and of smaller magnitude than those occurring in the windward face. At the lateral sides of the structure high pressures occur close to the windward edge, decreasing steadily to a minimum close to the leeward face of the structure.

Generally, in the computation of the along wind response of a structure, the variations of the horizontal distribution of wind pressures may be ignored and an equivalent uniform design value can be used. However, special attention should be given to the design of windows and cladding, principally close to the center and corners of the structure. Drag coefficients for the design of parts or portions of a building are given by A.N.S.I. and N.B.C.C.

Vertical distribution of pressure -- The vertical distribution of wind pressure on the windward face of the structure is proportional to the distribution of wind velocity which has been defined by Eq. (3-2). The mean wind pressure can be assumed proportional to the square of the wind velocity for the range of velocities considered in this study. Therefore the vertical distribution of mean wind pressure in the windward face of the structure is given by the following expression:

$$\overline{P}(z) = P_{z_r} \left(\frac{z}{z_r}\right)^{2\alpha} \tag{3-7a}$$

The vertical distribution of the fluctuating pressure is:

$$P'(z) = P_{z_r} \left(\frac{z}{z_r}\right)^{\alpha} \tag{3-7b}$$

In the leeward face of the structure suction rather than pressure occurs. The distribution of wind suction is somewhat more uniform than the distribution of wind pressure. It is recommended by N.B.C.C. and A.N.S.I. that a uniform distribution can be used. This distribution has a value equal to the pressure computed at the mid height of the structure employing the exponential law described by Eqs. (3-7a) and (3-7b).

A summary of the pressure and suction coefficients discussed in the preceding paragraphs is presented in Table 4 and illustrated in Fig. 6. These distributions of pressures will be used in Chapter 5 to compute the participation factors associated with wind loading.

3-6 Statistical Description of the Wind Turbulence.

Concepts of isotropic turbulence have been used to study and quantify wind forces. Therefore winds have been customarily described using statistical formulations.

The wind velocity is generally divided into a mean component and a fluctuating component having a zero mean. Thus, the wind velocity will produce two separate effects, one is a static effect associated with the mean component, and the other is a dynamic effect associated with the fluctuating component. Since the fluctuating effect has a zero mean, it is characterized by the second statistical descriptor, namely, the variance. The variance for a zero mean process is equal to the mean square value, i.e. the statistical treatment of the fluctuating component will yield the standard deviation of the process.

The fluctuating component of the wind is generally analyzed by employing random vibration techniques. Therefore statistical functions that characterize the fluctuating wind velocity must be developed. As has been discussed in Chapter 2, Section 2-5, those are the power spectral and the correlation functions.

Wind Power Spectral Density Function -- The power spectral density function is generally employed for representing the energy content of the wind fluctuation associated with a specific frequency. One of the first efforts to quantify the fluctuating component of the wind velocity was reported by Van der Hoven (1957), as cited by Davenport (1961). With the data obtained at Brookhaven, N.Y., over a large number of years, Van der Hoven was able to present the energy content of the wind turbulence over a wide range of frequencies. This spectrum is presented in Fig. 7.

For strong winds such as those associated with a wind storm, the wind velocity should be averaged over a shorter period of time. For those cases Davenport (1961) has proposed an energy spectrum in which the mean square values of the energy of the wind are given as a function of the wave length. Davenport's spectrum is defined as follows:

$$\frac{f\ S_v\ (f)}{C_t\ \overline{V}^2\ (z)} = \frac{4X^2}{(1 + X^2)^{4/3}} \tag{3-8}$$

where

f   is the frequency in hertz,

S_v (f) is the spectral energy at frequency f in ft/sec Hz,

$X = \dfrac{4000\ f}{\overline{V}\ (z)}$,

C_t is the dimensionless terrain frictional drag coefficient.

C_t has a range of values between 0.0005 for wind flow on sea surface to 0.050 for flow over an urban area. Values of the coefficient C_t, for various terrain conditions, are listed in Table 4.

Davenport's spectrum covers a range of wave lengths from 50 to 5000 ft, but can be extrapolated with some reliability for shorter wave lengths

(Sachs, 1972). A plot of Davenport's spectrum is shown in Fig. 8, it can be seen that the spectrum has a peak at a wave length of 2000 ft., showing that there is a high energy concentration in the low frequency range.

For a mean wind speed of 100 ft/sec (68 mph or 33 m/sec) the spectrum covers a frequency band between 0.02 and 2 hertz which includes the fundamental frequency of most structures. For this specific velocity, the spectrum has a peak at a frequency of 0.05 hertz.

Although the wind flow is mainly horizontal near the ground, in the upper regions the boundary layer may have vertical components of considerable value. A spectrum of vertical gustiness, which is strongly dependent on the height above the ground, has been proposed by Panofsky and McCorning (1960). The spectrum is defined as follows:

$$\frac{f\ S_{vv}\ (f)}{C_t\ \overline{V}^2\ (z)} = \frac{6\ \tilde{F}}{(1 + 4\ \tilde{F})^2} \qquad (3\text{-}9)$$

where

$\tilde{F}$ is the reduced frequency $f \times z/\overline{V}(z)$,

S_{vv} (f) is the vertical spectral energy at frequency f.

The vertical spectrum is important only for structures having a vertical degree of freedom such as suspension bridges and suspended cables.

More recently, Simiu (1973) has proposed a spectrum which accounts for both the horizontal and the vertical fluctuations of the wind velocity. Simiu's spectrum is expressed as follows:

$$\frac{f\ S_v\ (f)}{C_t\ \overline{V}^2\ (z)} = \frac{200\ \tilde{f}}{(1 + 50\ \tilde{f})^{5/3}} \qquad (3\text{-}10)$$

where

$$\tilde{f} = \frac{f(z - z_d)}{\overline{V}\ (z)}$$

The various representations of the wind power spectral density function discussed in this section are employed in Chapter 4 for computing the dynamic response associated with the fluctuating component of the wind flow.

Correlation of Wind Pressure. -- A wind gust should be at least as large as the structure for full effectiveness. The turbulent flow contains gusts of various sizes, many of these smaller in size than the structure, and therefore not completely effective over the whole area.

To compute the overall response of a structure a value of the equivalent pressure acting over the complete area should be estimated. Obviously, to assume that the equivalent pressure is equal to the largest peak pressure is a safe assumption, however it may yield results that are unrealistically high. Correlation functions are needed for the computation of the effective pressure coefficients described in Section 4-3 and Appendix A.

One of the principal parameters used to quantify the effective pressure is the size of the structure. Early work in aeronautical engineering suggested that the effective pressure decreases with an increase in the size of the structure. Another way to quantify the relative intensity of the pressure peaks is to measure the separation that exists between peaks of high intensity.

For some time, in aeronautical and wind engineering, the correlation of pressures has been used to describe the average or equivalent effect of the various pressure gusts acting on a structure. Generally, the correlation of pressures has been expressed in terms of the coherence functions (defined as the square root of the absolute value of the correlation). Various authors in aeronautical engineering and, more recently, Davenport in wind engineering have suggested that the coherence function for wind pressure is an exponential function which decreases with the frequency and separation of the wind flow and the size of the structure. Usually it is assumed that the coherence can be represented as an exponential function of the reduced frequency quantity, $\frac{f\Delta_z}{\overline{V}}$

where

f is the frequency,

Δ_z is the separation,

$\overline{V}$ is the mean wind velocity.

Therefore the coherence function in the vertical direction can be written as follows:

$$\text{Vertical Coherence} = \exp\{-C_z\left(\frac{f\Delta_z}{\overline{V}}\right)\} \qquad (3\text{-}11)$$

The quantity C_z is a constant that has to be determined experimentally. A plot of the vertical coherence function is presented in Fig. 9.

Similarly the coherence function in the horizontal direction is written as follows:

$$\text{Horizontal Coherence} = \exp\{-C_Y\left(\frac{\Delta_y}{\overline{V}}\right)\} \qquad (3\text{-}12)$$

Davenport has suggested that the horizontal and vertical correlations of wind pressure could be written as the square of the coherence functions as follows:

$$\text{Vertical Correlation} = \exp \left\{ \frac{-2\, C_Z\, |\, Z_1 - Z_2\, |\, f}{\overline{V}} \right\} \tag{3-13a}$$

$$\text{Horizontal Correlation} = \exp \left\{ \frac{-2\, C_Y\, |\, Y_1 - Y_2\, |\, f}{\overline{V}} \right\} \tag{3-13b}$$

After extensive wind tunnel testing, Davenport has recommended the use of the values $C_Z = 10$ and $C_Y = 16$.

In order to obtain the overall effect of the wind pressure it is necessary to combine the vertical and horizontal correlation functions. Davenport has proposed that such combination can be attained as follows:

$$R_u = \exp \left\{ \frac{-2\, f\, [C_Z^2\, |\, Z_1 - Z_2|^2 + C_Y^2\, |\, Y_1 - Y_2|\,]^{1/2}}{\overline{V}\, (z_1) + \overline{V}\, (z_2)} \right\} \tag{3-14}$$

Equation (3-14) with values of $C_Z = 10$ and $C_Y = 16$ will be used in Chapter 4 for computing the effective pressures acting on a structure.

Cross-correlation of Pressures -- In order to provide a complete picture of present knowledge in wind engineering, the cross correlation of pressures between the windward and leeward faces of the structure is presented. However this function will not be used in the present study for the computation of wind response spectrum. For deep structures, the distance between the windward and the leeward faces could be large. In those cases, the windward pressure and the leeward suction are not in phase. Therefore a reduction in the dynamic response of the structure can be expected.

It is easy to see that as the size of the structure increases (especially the distance between the windward and leeward faces) the cross-correlation of pressures acting on the opposite faces of the structure should decrease.

This effect will introduce a reduction in the overall response of the structure. Simiu and Lozier (1975) have suggested that, for regular rectangular structures, this reduction due the cross-correlation can be written as follows:

$$N_u = \frac{1}{\xi} - \frac{1}{2\xi^2} (1 - 2^{-2\xi}) \qquad (3\text{-}15)$$

and ξ is defined as

$$\xi = \frac{3.85 \; f \Delta x}{\overline{V}} \qquad (3\text{-}16)$$

and Δx is the smallest of 4h, 4b, or 4d (being h, b and d the dimensions of the building).

In addition, for computing the mean square response Simiu and Lozier (1975) have recommended the use of this reduction function only in the high frequency range, and to take $N_u = 1$ for frequencies smaller than ~ 0.9 of the natural frequency of vibration.

A simpler procedure to account for the cross-correlation of pressures, and also proposed by Simiu and Lozier (1975), is to reduce the equivalent pressure by the following factor:

$$R_N = \frac{(C_D^2 + 2N_u \; C_D C_L + C_L^2)^{1/2}}{C_L + C_D} \qquad (3\text{-}17)$$

This reduction coefficient (R_N) can have values as low as 0.71 when $C_D = C_L$ and $N_u = 0$, or be equal to one if $N_u = 1$. In most cases the coefficient R_N will decrease the dynamic pressure of the structure by as much as 10 to 20 percent. The cross correlation of pressures, Eq. (3-15) can be included in the computation of the response. However if the simplified form, Eq. (3-17) is employed, it could be applied directly in the computation of the participation factors.

3-7 Response Spectrum Formulation for Wind Loading.

In order to obtain the representation of the wind dynamic problems as a combination of single degree of freedom systems, the response spectrum procedure could be employed. This deterministic representation may be achieved in the following way:

The pressure as defined in Eq. (3-6) is written as follows:

$$P(z) = 1/2\ \rho\ C_D\ \overline{V}^2\ (z) + \rho\ C_D\ \overline{V}\ (z)\ V'\ (y,\ z,\ t) \qquad (3\text{-}18)$$

It can be seen that the first term is a static force, and the second represents a dynamic force. Therefore the governing equation of motion (Eq. (2-1)) can be written as follows:

$$[m]\ \{\ddot{x}\} + [c]\ \{\dot{x}\} + [k]\ \{x\} = \overline{F} + F' \qquad (3\text{-}19)$$

where

$$\overline{F} = \int_0^b 1/2\ \rho\ C_D\ \overline{V}^2\ (z)\ dy$$

$$F' = \int_0^b \rho\ C_D\ \overline{V}\ (z)\ V'\ (y,\ z,\ t)\ dy$$

Moreover if the horizontal distribution of the wind pressure is assumed to be uniform the forces can be written as follows:

$$\overline{F} = 1/2\ \rho\ b\ C_D\ \overline{V}^2\ (z) \tag{3-20a}$$

$$F' = \rho\ b\ C_D\ V\ (z)\ V'\ (y,\ z,\ t) \tag{3-20b}$$

Now the distributed forces can be discretized at the nodal points of the structure. In this way we obtain the following force vectors:

$$\overline{F} \simeq \{\overline{F}\} \tag{3-21a}$$

$$F' \simeq \rho\ V_o\ \{F'\}\ \ f(y,\ z,\ t) \tag{3-21b}$$

where

V_o is a reference value of velocity,

$f(y, z, t)$ is a function of the correlation of wind pressure.

Therefore Eq. (3-19) can be divided into two equations

$$[k]\ \{x\} = \{\overline{F}\} \tag{3-22a}$$

$$[m]\ \{\ddot{x}\} + [c]\ \{\dot{x}\} + [k]\ \{x\} = \rho\ V_o\ \{F'\}\ f(y,\ z,\ t) \tag{3-22b}$$

The solution of Eq. (3-22a) is obtained from statics. However, the solution of Eq. (3-22b) may be solved in terms of the generalized coordinates (as discussed in Chapter 2) as follows:

$$\ddot{q}_i + 2\omega\beta\ \dot{q} + \omega^2\ q = C_i\ \rho\ V_o\ f\ (y,\ z,\ t) \tag{3-23}$$

where

$$C_i = \frac{\{\phi_i\}^T\ \{F'\}}{\{\phi_i\}^T\ [m]\ \{\phi_i\}}$$

Finally the solution of the fluctuating component of the wind loading can be written as a combination of the modal contributions as follows:

$$\{x\} = \sum_{i=1}^{N} \{\phi_i\} \; C_i \; \frac{P_{i\ eff}}{\omega_i^2}$$

The quantity P_{eff} will be computed in the following chapter as a response spectrum of effective pressures.

CHAPTER 4
WIND RESPONSE SPECTRA

4-1 Introduction.

In the last fifteen years there has been an increasing interest in the study of the commonality of earthquake and wind analysis procedures. One of the principal points addressed by various investigators has been the need to develop response spectra for wind loading. The ability to develop response spectra for wind excitation is attractive from two standpoints. First, it would provide a convenient basis for comparing, measuring, and expressing the wind loading. Second, it would enable one to interpret and understand the behavior of structures subjected to wind loading by using, in part, the experience and knowledge accumulated through more than 30 years of research in earthquake and wind engineering. In addition, a design response spectrum for wind loading makes it possible to develop a common analysis procedure for earthquake and wind loadings. Such a procedure may in certain situations, considerably simplify the analysis and design of structures especially those built in areas where both natural hazards (earthquake and strong wind) are likely to occur and are of significance.

In this chapter, during the development of wind response spectra, the physical meaning of the quantities involved is explained and the relationships to their common parameters occurring in earthquake engineering are presented. It is noted that wind response spectrum is divided into two parts, a mean component and a fluctuating component. The fluctuating component of wind response spectra is computed using random vibration techniques. Therefore these calculations are carried out in the frequency domain as discussed in Chapter 2.

4-2 Basis for Wind Response Spectra.

For the computation of wind response spectra, the pressures are divided into two components. One is a mean component which is associated with the mean wind velocity. The other is a fluctuating component which accounts for the gusting variations of the wind velocity over the mean value. Normally the mean component is applied statically to the structure, and the fluctuating component is treated as a dynamic load. The total solution is given by the superposition of both the mean and the fluctuating components.

The purpose of wind spectra is to represent the effects of wind fluctuating loading in a clear and practical manner common in philosophy, but not parallel numerically, to that used for earthquake base excitation. In the most direct sense wind response spectra can be employed to present the effects of wind loading as effective or amplified pressures.

To obtain a spectral representation of the wind loading a procedure common in concept to the earthquake response spectra is developed in this section. Such a procedure should satisfy the following conditions:

1) The procedure should be accurate over a large range of frequencies and relatively high values of damping (2 to 10 percent of critical).

2) The procedure should be clearly stated and practical. Consequently, it should be developed through the use of a few principal parameters that in turn can be presented in graphical, tabular or analytical form.

3) The physical meaning of the quantities involved should be retained during the mathematical treatment of the problem.

4) The resulting spectra should be presented, or have the potential of representation, in a form similar to the tripartite logarithmic plot employed for the graphical description of deterministic problems.

Historical background -- In one of the earliest papers addressed to wind spectral representation Newmark (1966) suggested that the response of structures subjected to wind excitation could be treated in a manner similar to earthquake analysis if the corresponding participation factors and response spectra could be specified. Newmark recommended that the participation factors for general loading could be calculated employing the following expression (previously discussed in Chapter 2 Eq. (2-19a)):

$$C_i = \frac{\{\phi_i\}^T [m] \{\rho\}}{\{\phi_i\}^T [m] \{\phi_i\}} \qquad (4\text{-}1)$$

For approximating the mean component of the response spectrum, Newmark recommended the use of a long sinusoidal pulse with a period of 30 seconds to 1 minute. To account for the fluctuating component of the wind velocity he suggested the use of a smoothed response spectrum of a typical wide-band earthquake having the following peak ground motion values 1g, 48 in/sec and 36 in, shifted down in the frequency scale by a factor of about 10 to 15 in order to account for the fact that the large energy contribution of the wind fluctuation is in the low frequency range. The response spectrum as proposed by Newmark is shown in Fig. 10. In this example a mean pressure of 10 psf is presented as P_1 and an unamplified fluctuating pressure of 15 psf as P_2. The amplification factors are typical values for a two percent of damping earthquake response spectrum. As will be apparent later in this chapter, the general shape of the spectrum is quite accurate. However, no specific directions for the computation of the response spectrum are presented.

As illustrated in Fig. 10, the quantities employed for drawing the response spectrum are pressure and pseudo impulse, which are acceleration and velocity multiplied by the mass and divided by a unit of area.

More recently, based on the study of pressure time data and experience Newmark and Hall (1968) presented another sketch of a possible response spectrum for wind pressures. It is pointed out that the construction of a response spectrum is made possible by employing the relations that exist between the forcing loading and the intertia force (mass times acceleration). This spectrum is shown in Fig. 11; again the shape of the spectrum is correct but no rules for the computation of such spectrum are presented.

Basis for the computation of response spectra -- The response spectrum has known behavior at the very high and very low frequencies. For earthquake base excitation, the amplified ordinates of the spectrum should converge to the maximum ground acceleration for high frequencies and to maximum ground displacement in the low frequency range. In general, in the high frequency range the system behaves as a rigid body and the force (mass times acceleration) becomes a static force. In the low frequency range (when the frequency goes to zero) the system presents a rigid body motion degree of freedom. This relationship is illustrated in Fig. 12.

Force response spectra as can be expected for wind loading would converge to the unamplified force or pressure in the high frequency range. However, in the low frequency range as a result of the law of conservation of momentum the amplified line of the spectrum should converge to the unamplified impulse at zero frequency.

Computation of response spectra can be summarized as (1) evaluation of the spectral base lines, (2) computation of a set of amplification factors for the various regions of the spectra, and (3) plotting the response spectra. For wind loading, these steps are discussed in the following paragraphs.

Evaluation of the base line -- A base line of unamplified pressures can be computed as the static response of a rigid system. Hence by evaluating the response of a system having a large freqency the value of the base line in the high frequency range can be obtained. This terminal, or base line pressure, is the line pressure $1/2\ \rho V^2$ multiplied by an effective pressure coefficient (based on geometry and exposure generally) which accounts for the reduction in wind force associated with the differences in correlation of the fluctuating wind flow.

Computation of the amplification factors -- A set of amplification factors can be computed for the various regions of the spectra. For wind loading the amplification factors are computed in the acceleration control region (pressure), and in the velocity control region. In order to maintain the meaning of the amplification factor as a function of frequency and damping, and independent of the geometry, the correlation of pressures is not included in the computation of the amplification factors. In other words the amplification factors are computed for a perfectly correlated wind flow.

Plotting wind response spectra -- To draw wind response spectra, Newmark (1966) and Newmark and Hall (1968) have recommended the use of the quantities pressure and pseudo impulse. Such parameters are related to the generally used acceleration and velocity by multiplying the latter by a unit mass and dividing by a unit of area. Thus the quantities plotted are pressure in psf and pseudo impulse per unit of area psf x sec. Note then pseudo impulse per unit area has the units of pressure divided by frequency. Moreover if the spectrum is plotted for a unit mass then the quantities pressure and pseudo velocity can be used.

The familiar tripartite logarithmic paper can be employed to plot wind response spectra. However, the tripartite paper prepared for the representation of earthquake response spectra must have the scales modified. In order to be consistent with the scales usually found in tripartite paper for seismic effects, one must plot either pressure divided by the acceleration of gravity (g) and pseudo impulse, or pressure and pseudo impulse multiplied by the acceleration of gravity (g). In both cases special care should be taken with the units and the frequency scale should be adjusted accordingly.

It also is possible to develop a new tripartite paper tailored to satisfy wind response relationships. These scales will represent the relations $P = \omega V$ and also $P = 1/2 \rho V^2$. The examples presented in Section 4-5 are plotted in this "pressure-velocity-frequency" scale.

4-3 Evaluation of Wind Response Spectra.

Although the idea of wind response spectra is similar to that used in earthquake engineering, wind loading presents two additional problems.

First, the loading is divided into a mean and a fluctuating component; and second, the computations associated with the fluctuating component should include the correlation functions presented in Chapter 3. The numerical procedures employed for the evaluation of the various parameters occurring in wind response spectra are discussed in Appendix A.

Mean response spectra -- The mean response spectra is associated with the mean wind velocity. Moreover, since the time history of the mean wind velocity has been defined to yield an almost static behavior, the mean component of the response spectra can be assumed to be a static force. Hence, it has an amplification factor equal to one. In addition, the mean wind flow is assumed to be perfectly correlated. Thus, no reduction in the mean force associated with the correlation functions should be expected.

Fluctuating wind spectra -- As has been described in Chapter 3, pressures associated with gusts which are large enough to envelop the complete structure will produce significant dynamic effects in the structure. On the other hand, short gusts can produce high localized pressures which are not as important in the overall behavior of the structure as they are for the design of parts or portions of the buildings. To account for the size of the gusts and their relative importance in the overall dynamic response, a correlation function for wind pressure, as defined in Chapter 3, is necessary.

Commonly, the response spectrum is defined as the response of a single degree of freedom damped system. The computation is carried out for oscillators having various values of natural frequency of vibration and percent of critical damping.

Since the fluctuating component of the wind force has been described by employing a statistical formulation, its response spectrum should be computed using the random vibration techniques presented in Chapter 2. Therefore, the mean square value of the response spectrum as defined by Eq. (2-68) is expressed as follows:

$$S(\omega) = \frac{1}{2\pi} \int_0^\infty |J(\omega)|^2 \; |H(\omega)|^2 \; S_V(\omega) \; d\omega \qquad (4\text{-}2)$$

where

$|J(\omega)|^2$ is the correlation function for wind pressures,

$|H(\omega)|^2$ is the frequency transfer function,

$S_V(\omega)$ is the input power spectral function for wind.

$|H(\omega)|^2$ has been derived in Chapter 2, and $|J(\omega)|^2$ and $S_V(\omega)$ are discussed in Chapter 3.

As discussed in the previous section, three steps are required for evaluating wind response spectra. These steps are (1) estimation of the base line, (2) evaluation of the amplification factors and (3) plotting of the response spectra. The first two points are addressed in this section, however the last will be discussed in Section 4-5. The theory behind the evaluation procedure and the numerical techniques required are discussed in Appendix A.

Estimation of the base line -- For wind loading a general procedure for evaluating the force or acceleration base line is based on the computation of the response of the forcing load on a rigid system (zero period). For such systems the amplification associated with the dynamic properties of the system is equal to one. For a random vibration problem, which is computed in the frequency domain, the base line could be estimated by setting the function $|H\ (\omega)|^2$ in Eq. (4-2) equal to one (this means a rigid system). In this case, Eq. (4-2) is simplified into the following expression:

$$S_r = \frac{1}{2\pi} \int_0^\infty |J\ (\omega)|^2\ S_v\ (\omega)\ d\omega \qquad (4\text{-}3)$$

where

S_r is the mean square pressure response (pressure that leads to a given response) of a rigid system.

The value of the base line is then obtained by computing the square root of Eq. (4-3) as follows:

$$P_r = \sqrt{S_r} \qquad (4\text{-}4)$$

Numerical integration techniques are required for the evaluation of Eq. (4-4). For the case of wind loading the spectral base line is a function of the shape of the structure and the correlation of wind pressures. The results of the numerical evaluation of Eq. (4-4), employing the expression for the correlation of pressures discussed in Chapter 3, Eq. (3-14), are presented in Figs. 13 and 14 and Appendix A. The coefficients presented in these Figures should be interpreted as the fraction of the line pressure $1/2\ \rho\ V^2$ associated with the fluctuating component of the wind velocity that will be felt by a rigid body system. In other words, the product of the line pressure times the effective pressure coefficients will give an estimate of the effective unamplified pressure. This concept is similar to the idea of effective values employed in computing the base lines for earthquake response spectra.

Amplification factors -- Generally for vibration problems, the amplification factors are defined as the ratio of the maximum response to the spectral base line.

In deterministic analysis the amplification factors for a representative sample of records are computed. Then, employing statistical techniques, a mean value and a standard deviation of the amplification factors are usually specified. In the case of random vibration analysis, the power spectral density function is usually employed to reflect the "average value" (in the mean square sense) of a sample of records. Therefore the resulting values of the amplification factors have already been averaged and no further statistical treatment is required. The amplification factors associated with the wind gust fluctuations can be evaluated for a perfectly correlated system. A perfectly correlated system is chosen in order to maintain the definition of amplification factor as a quantity independent of the geometrical configuration of the loading. Otherwise it will be impossible to compute an amplification factor applicable to all types of structures, regardless of their geometrical and material properties. In other words, the amplification factor is kept only as a function of the dynamic properties of the structure. This definition of amplification factor is consistent with that currently employed in earthquake engineering.

In order to obtain an expression for the amplification factors the correlation function $|J(\omega)|^2$ in Eq. (4-2) is set equal to one (this means a perfectly correlated system). Therefore the mean square response of a perfectly correlated zero mean random procedure can be evaluated from the following expression:

$$S_{pc}(\omega) = \frac{1}{2\pi} \int_0^\infty |H(\omega)|^2 S_v(\omega)\, d\omega \tag{4-5}$$

where

$S_{pc}(\omega)$ is the mean square response for a perfectly correlated system.

For computing the amplification factors, Eq. (4-5) is divided by its terminal value. The terminal value is the value of Eq. (4-5) when the frequency approaches infinity. In computing the terminal value of Eq. (4-5) note that the frequency transfer function $|H(\omega)|^2$ (see Chapter 2) is equal to 1.0 for ω equal to infinity. Thus the amplification factor can be computed from the following expression:

$$\text{Amp}(\omega) = \left[\frac{S_{pc}(\omega)}{\int_0^\infty S_v(\omega)\, d\omega} \right]^{1/2} \tag{4-6}$$

Amplification factors for Davenport's power spectral representation of the wind fluctuation, Eq. (3-8), with a wind velocity of 100 ft /sec, and various values of damping are presented in Fig. 15. It can be seen that a peak occurs at a frequency of 0.04 hertz (T = 25 sec). At very low values of frequency (smaller than 0.04 hertz) the amplification factor decays to values smaller than one. In the range of frequencies between 0.04 hertz to 8 hertz the amplification factor decays to roughly one third of the maximum value. For frequencies greater than 8 hertz, the amplification factor decays slowly, approaching one at a very high value of frequency.

The spectrum for wind loading is a function of the wind velocity, frequency and length of gusts. For this reason it is possible to represent the amplification factors for any value of wind velocity as a function of the wave length (Vel/f in ft). In this form a single curve for each value of damping will cover the complete range of wind velocities. This spectrum of amplification factors, covering a large range of wind velocities is presented in Fig. 16. The spectrum reaches a maximum amplification at a wave length of 1500 to 2000 ft. For wave lengths smaller than 1500 ft the amplification decays in an almost linear manner (in a log-log plot) suggesting that the amplification can be expressed as an equation of the form:

$$\text{Amplification} = A \left(\frac{Vel}{f}\right)^{-\delta} \qquad (4\text{-}7)$$

where

A and δ are constants to be evaluated.

This exponential representation of the amplification is evaluated in section 4-6.

At the very low range of wave lengths the amplification converges to a value of one, independent of the amount of damping present in the system. These low values of the wave length represent either a low wind velocity or a high frequency rigid system.

It is important to note that the maximum value of the amplification factors for wind fluctuation, as presented in Table 5, are numerically similar to those proposed by Newmark and Hall (1978) for horizontal ground motion earthquake response spectrum. Those values are listed in Table 6. In Fig. 17 earthquake and wind maximum amplification factors are presented as a function of damping. For the purpose of comparison four curves are presented in Fig. 17. These are the mean and the mean plus one standard deviation acceleration amplification factors for earthquakes, and the wind pressure amplification factors computed employing the representations for the fluctuating wind velocity proposed by Davenport (1963) and Simiu (1970).

It can be seen in Fig. 17 that the amplification factors obtained from Davenport's representation of the wind fluctuation are similar to the mean plus one standard deviation acceleration amplification factors for earthquake response spectra. The amplification factors obtained from Simiu's representation of the wind fluctuation have numerical values that are smaller than those predicted by Davenport's spectrum and are somehwere between the mean and the mean plus one standard deviation earthquake acceleration amplification factors.

In Fig. 18 it is shown that the wind pressure maximum amplification factors can be represented with an equation of the form:

$$\text{Amplification}_{max} = \sqrt{1 + \frac{A}{\beta}} \qquad (4\text{-}8)$$

where

β is the percent of critical damping,

$A = 0.25$ for Davenport's representation,

$A = 0.17$ for Simiu's representation.

This representation of the maximum amplification follows the form of the narrow band approximation employed by various investigators in wind engineering for computing gust factors.

Similarly, for earthquake engineering, empirical equations for the amplification factors have been developed as a function of critical damping. A set of equations proposed by Newmark and Hall (1978) are presented in Table 7.

4-4 Special Considerations for Wind Response Spectra.

Influence of the correlation functions -- The product of the amplification factor times the effective pressure base line is in most cases an upper bound approximation of the response, but it may become extremely high especially for low damped systems. In such cases a correction factor is necessary (See Appendix A where the steps for the evaluation of the response spectra are described). For high frequency systems, the amplification converges to one and the behavior of the structure is essentially static. In this range of high frequencies the product of the amplification times the effective pressure, as discussed in Appendix A, is a good representation of the response. However, for the low and middle range of frequencies this product could be unreasonably high and a correction factor should be applied. The correction is necessary because the spectrum as given by Eq. (4-2) is a coupled function of frequency and geometry. Therefore the uncoupled solution (base line times amplification) is accurate only in the high frequency range.

The mean square of the correction factor can be evaluated by dividing the response integral Eq. (4-2) by the amplification factor Eq. (4-6) and the effective pressure coefficient Eq. (4-4). The correction factor is then expressed as follows:

$$S_c\ (\omega) = \frac{S\ (\omega) \int_0^\infty S_v\ (\omega)\ d\omega}{S_r\ S_{pc}\ (\omega)} \qquad \text{(4-9a)}$$

The correction function can be evaluated by taking the square root of Eq. (4-9a).

$$\text{Correction} = \sqrt{S_c\ (\omega)} \qquad \text{(4-9b)}$$

The numerical evaluation of Eq. (4-9b) is presented in Fig. 19 for 1, 2 and 5 percent of critical damping. The correction factor is plotted as a function of the reduced frequency fh/V. It can be seen that this factor is a coupled function of the dynamic and the geometric parameters of the structure. There are two additional points related to the correction function that should be discussed:

(1) The spectrum was normalized in such a way that the product of base line times amplification factor is accurate only in the high frequency range. Therefore the correction function should approach one for a large value of fh/V, but does not have to converge to one in the low range of fh/V.

(2) The function has two inflection points that could be used as control points for the construction of the pressure line of the spectrum. Equations for computing those points are presented in Table 11. The procedure described in this chapter for the computation of wind response spectrum is presented graphically in Fig. 20 and in the form of a flow chart in Fig. 21.

<u>The percentile level</u> -- In dynamic problems such as earthquake and wind the loading is specified by statistical means. Therefore tools such as the interval of confidence (percentile) are necessary to define the range of validity over which the values are applicable.

The percentile levels are generally defined by the number of standard deviations required to attain the desired probability level. For a normal process the mean represents a value which is larger than 50 percent and smaller than 50 percent of the sampled population. On the other hand the mean plus one standard deviation will guarantee that 81.4 percent of the samples will be smaller than the mean plus σ value.

The computation and the meaning of the standard deviation varies for deterministic and probabilistic problems. There is a difference between the probability level in deterministic analysis and the number of standard deviations required in a random vibration problem. In a zero mean random process only the value of the standard deviation, namely the root mean square, can be assessed. However the response spectrum technique usually requires a maximum value of the response which should be calculated by employing additional statistical procedures.

For earthquake response spectra the percentile level is employed to select the interval of confidence of the amplification factors. Also, σ levels commonly are used to assess the values of the base lines when records are not available.

In wind response spectra, the σ level is required for the computation of the fluctuating component. Since wind is treated as a zero mean random process only an average value (in the mean square sense) can be computed. This average, for the fluctuating response, covers both the base lines and the amplification factors.

For random vibration problems the maximum should be assessed from the root mean square value. The evaluation of the maximum value of the response for a random vibration problem was first treated by Rice (1945) in electrical engineering. In order to evaluate the maximum value of a random vibration response, Rice developed the concept of crossing a certain level line with a positive slope. More details about this procedure can be found in Rice (1945), Clough and Penzien (1975), and Crandall and Mark (1963). More recently, Davenport (1961) applied the concept to wind engineering and derived an expression for the so called peak factors which have been discussed in Section 2-6.

For wind, the equation of the peak factors, Eq. (2-71) yields values which in most cases are between 3 to 4. Vickery (1971) has proposed the use of a universal peak factor equal to 3.5. The use of a single value for the peak factor simplifies the computation of the response spectra. In this study a peak factor equal to 3.5 is used in the complete range of frequencies.

Moreover, since the peak factors apply to the computation of the complete fluctuating response the 3.5 coefficient can be included either in the amplification factors or in the effective pressure coefficients. In this study it was found convenient to include the peak factor in the computation of the effective pressure coefficients.

Comparison of procedures employed for computing earthquake and wind response spectrum -- A comparison of the procedure generally employed for computing earthquake response spectrum and the procedure proposed in this study for evaluating wind response spectrum is illustrated in Fig. 22. In this flow chart for the two loadings we start with a selected sample of time histories. The left branch which represents the earthquake design response spectrum computation starts with a single degree of freedom oscillator where the response values for a large number of frequencies are computed. This operation also transforms the response to the frequency domain. The next step is to compute the maximum value for each frequency, in this manner a set of maximum response functions corresponding to the input time histories is obtained. The following step involves the smoothing of each of the response functions for the various amplification regions. Finally the smoothed response functions are normalized with their respective base lines to obtain amplification factors which are then statistically treated to obtain design amplification factors.

For wind, the input time histories are first transformed to the frequency domain through the use of the Fourier transform. This process involves integration and the result is a smooth power spectral density function. A set of power spectral density functions computed for each time history is averaged and a single power spectral density function is employed for computing the response. The average power spectral density function together with the wind correlation functions are employed to compute the response of a linear oscillator. Now the maximum values for each frequency can be computed employing the first crossing probability to obtain an expected maximum value of the response. From this last step the base lines, amplification factors and regions of amplification can be inferred.

4-5 Plotting Wind Response Spectra.

Wind response spectra can be plotted in the same standard tripartite paper employed for the graphical representation of dynamic systems subjected to base excitation. In earthquake engineering, besides frequency or period, the response spectrum has three ordinates which represent acceleration (A), pseudo velocity (V) and displacement (D). In addition, there are certain relations between the quantities represented in the response spectrum defined as follows:

$$V = D\omega$$

$$A = V\omega = D\omega^2$$

where ω is the natural circular frequency of vibration $2\pi f$.

For wind, as discussed in Section 4-2, only two of the ordinates, besides frequency, have physical meaning. The velocity ordinate now represents impulse (mass times velocity), whereas the ordinate of acceleration in the earthquake response spectrum is replaced by a measure of the effective pressure associated with a certain wind velocity and a specific frequency of the structure. The earthquake displacement ordinate does not have an equivalent in the wind response spectrum.

The spectrum of effective wind pressures can be computed at any height of the structure (because the wind velocity varies with height) but it is advantageous to draw the response spectrum for the wind velocity at the reference altitude (33 ft or 10 m). The vertical variation of the wind velocity is included in the computation of the participation factors.

Drag coefficients are taken as one for the computation of the response spectrum. The drag coefficients together with the vertical distribution of the wind velocity and wind pressure are included in the computation of the participation factors.

Steps to draw wind response spectra -- The steps necessary for drawing wind response spectra can be summarized as follows:

1) Draw the mean wind velocity and wind pressure $1/2\ \rho\ V^2$. These lines represent the unamplified response of a single degree of freedom system subjected to a perfectly correlated mean wind flow.

2) Obtain the base lines for the fluctuating component by multiplying the mean wind pressure times the effective pressure coefficient presented in Figs. 13 and 14. In order to maintain the relation $P = 1/2\ \rho\ V^2$, the velocity base line is modified by multiplying the mean wind velocity times the square root of the effective pressure coefficient. These reduced base lines represent the wind pressure and the effective wind velocity that will be felt

by a rigid body associated with the fluctuating component of the turbulent wind flow.

3) Amplify the fluctuating base line to account for the dynamic effects of the wind. In the velocity region of the spectrum a single amplification factor is used, whereas in the pressure region more than one control point should be employed to obtain an accurate representation of the dynamic pressures. From the study of Figs. 16 and 19 it can be concluded that the following values of frequency can be used as control points for the pressure region: V/1500, where velocity is in ft/sec computed employing the reference velocity at 33 ft, and the two inflection points of the correction function, points A and B. Equations to compute the control frequencies A and B are presented in Table 11. The first point, V/1500 is the wave length where the amplification function has a maximum value (see Fig. 16). The control point A is the frequency where the correction function has a value equal to one, and the control point B is the frequency where the correction function has a minimum value. The latter two points are taken from study of Fig. 19 and parametric analyses.

4) Combination of the mean and the fluctuating components of the spectrum.

Together with the recommended control points, the following guide lines can be used for drawing the pressure line of the fluctuating response spectrum:

1) Use the maximum pressure amplification (given in Table 5) up to a frequency equal to V/1500.

2) There is a smooth decay in amplification from the frequency V/1500 to the control frequency A.

3) In the range of frequencies between control points A and B the amplification decays sharply approaching 1 at control point B. Values for the pressure amplification factors for control frequency V/1500, A and B are given in Table 8. A schematic representation of the procedure is presented in Fig. 23.

Examples -- Two examples are given to illustrate the procedure recommended for the construction of wind response spectra.

Example 1. Draw the wind response spectrum for a 1200 ft x 200 ft x 200 ft building. The velocity at the top is 148 ft /sec, the reference velocity at 33 ft is 42 ft /sec, the structure is located in the center of a city and the damping is two percent of critical.

Step 1. Mean base line.

Mean pressure = $1/2\ \rho\ V^2 = 1/2\ (.0024)\ (148)^2 = 27$ psf.

Mean velocity = 148 ft /sec.

Step 2. Fluctuating base line.

From Fig. 14, the effective pressure coefficient is 0.33.

Fluctuating pressure = 0.33 x 27 = 9 psf.

Fluctuating velocity = $\sqrt{0.33} \times 148 = 85$ ft /sec.

Step 3. Amplification factors.

The control points for the pressure region are:

V/1500 = 42/1500 = 0.028 hertz.

From Table 11:

$A = 0.020\ (148)\ (1200)^{-0.67} = 0.026$ hertz

Ignore the first point 0.028 hertz

$B = 0.75\ (148)\ (1200)^{-0.67} = 0.96$ hertz.

The values of the amplification factors for these control frequencies are presented in Table 9. The velocity amplification factor is (from Table 5) 2.74. The plot of the mean and fluctuating spectrum is presented in Fig. 24.

Example 2. A second example is presented in Fig. 25. Here the response spectrum is presented for a 150 ft x 150 ft x 150 ft building. The velocity at the top is 65 ft /sec, the reference velocity at 33 ft is 38 ft /sec, the structure is located in the center of a city and the damping is one percent of critical.

Step 1.

Mean pressure = $1/2\ (.0024)\ (65)^2 = 5$ psf.

Mean velocity = 65 ft /sec.

Step 2.

Fluctuating pressure = 0.98 x 5 = 4.90 psf.

Fluctuating velocity = $\sqrt{0.98} \times 65 = 64.5$ ft /sec.

Step 3.

V/1500 = 38/1500 = 0.026 hertz.

$A = 0.027\ (65)\ (150)^{-0.71} = 0.050$ hertz.

$B = 1.31\ (65)\ (150)^{-0.71} = 2.428$ hertz.

The values for the amplification factors for one percent of damping at the control points are shown in Table 10. The amplification factor for the velocity control region as presented in Table 5 is 3.53.

Plotting wind spectra in orthogonal paper. -- Since the knee of the spectrum (velocity pressure lines intersection) occurs at a very low frequency which is not likely to exist in most of the common civil engineering structures, only the pressure line of the spectrum needs to be constructed. It can be plotted on any kind of paper, but a five cycles semi-logarithmic or a 3 x 5 logarithmic paper will cover a large range of frequencies. The procedure now is similar to before, the only difference is that only the pressure line has to be constructed. The same examples discussed for the tripartite plot will be now presented in a 3 x 5 log-log paper. Example 1 is illustrated in Fig. 26 and example 2 in Fig. 27.

4-6 Empirical Representation of Wind Response Spectra.

In most cases wind response spectra can be represented by the use of an empirical equation of the following general form:

$$\text{Response Pressure} = A \ (fh/V)^{-\delta} \ h^{\lambda} \tag{4-10}$$

where

A is a constant which depends on the damping ratio and the wind exposure of the structure,

δ is a constant which depends on the damping,

λ is a constant which depends on wind exposure,

h is the height of the building in feet,

f is the frequency of vibration,

V is the velocity at the top of the structure.

In order to evaluate this empirical representation of the wind response spectra two functions are developed. One includes the amplification and the correction functions and the other the effective pressure coefficients.

The amplification times the correction function is expressed as follows:

$$\text{Amplification} = A\,(fh/V)^{-\delta}\,h^{\lambda} \qquad (4\text{-}11)$$

Similarly the effective pressure coefficients can be written as follows:

$$P_r = E\,(h)^{-\mu} \qquad (4\text{-}12)$$

where

E and μ are constants which depend on the wind exposure.

It is difficult to find an analytical expression that will cover the whole range of frequencies. However, it is possible to develop a set of empirical equations for the range where the fundamental frequency of vibration of most structures is likely to occur.

The approximation that was developed in this study covers the range of values of frequency between 0.1 to 3 hertz, and is intended for structures 50 to 500 ft tall.

To compute the effective pressure coefficients, the aspect ratio was taken as 1 for a 50 ft building and as 5 for a 500 ft structure. The values in between are interpolated between the noted values as illustrated in Fig. 28. It is believed that this formula constitutes a reasonable approximation.

Moreover, the approximation is good for values of $f < 4\ V/h$. This range of $fh/V < 4$ covers the values where the fundamental frequency of most structures are likely to occur. In order to demonstrate the validity of the last statement, the fundamental natural frequency can be approximated by the following expression:

$$f = 10/n \tag{4-13}$$

where

n is the number of stories.

In addition, if an average value of 12 feet per story is taken, Eq. (4-13) can now be rewritten as follows:

$$f = 120/h \tag{4-14}$$

where

h is the height of the building in feet.

Therefore the ratio fh/V, for the velocity in ft/sec, can be written as $120/V$. The latter result shows that for a design wind velocity of 30 ft/sec, which is very low, the ratio fh/V is equal to 4. Therefore the approximation can be used for most of the common engineering problems.

Expressions for the effective pressure coefficients are shown in Table 12. These are presented for three of the most common wind exposures (as defined in Chapter 3) and for two levels of σ.

The amplification factors for the range of frequencies $f < 4\ V/h$ or $f < 3$ hertz are presented in Table 13 for values of damping of 1, 2, 5, and 10 percent of the critical. Estimated values of the coefficients A and δ as a function of the critical damping are presented in Figs. 28 and 29.

These approximations are employed in Chapter 5 for deriving simple expressions for gust factors and base shear coefficients.

CHAPTER 5

COMPUTATION OF FORCES AND DISPLACEMENTS

5-1 Introduction.

In the previous chapters the necessary concepts for performing dynamic analyses of structures subjected to strong winds have been presented. Modal analysis procedures as applicable to wind and earthquake engineering are discussed in Chapter 2. In Chapter 3 the physical properties of turbulent wind are presented, and in Chapter 4 the basis for computing response spectra for wind loading are developed. Finally, the remaining step necessary for the evaluation of forces and displacements associated with strong winds is to combine the modal analysis procedures and the response spectra.

In order to achieve these goals, the following topics are presented herein:

1) Application of modal analysis procedures to evaluate wind forces and displacements in structures. Special consideration is given to the computation of participation factors.

2) Development of simple procedures for the computation of wind forces.

3) Comparison of results obtained using the procedures presented in this study with full-scale measurements.

5-2 Modal Analysis Procedures.

As a result of the separation of variables hypothesis the dynamic forcing function has been separated into a geometric and a time or frequency dependent function. The former is associated with the computation of the participation factors. The latter, however, is known to be associated with the amplification factors and the response spectra which have been presented in Chapter 4.

Once the response spectrum has been obtained, the next step in the evaluation of forces and displacements is to calculate the participation factors. The participation factors for wind loading, are dependent on the geometry of the structure, the drag coefficients, the mode shapes and the distribution of wind pressure on the various faces of the structure. The distribution of pressures for the mean and the fluctuating components of the wind velocity have been discussed in Section 3-5. The drag coefficients recommended for the design of buildings are listed in Table 4.

It should be noted that the correlation functions have been included in the computation of the response spectra. Therefore, they do not require further consideration for the evaluation of the participation factors. However, the reduction associated with the cross-correlation of pressures in the windward and the leeward faces of the structure, Eq. (3-17), could be included in the computation of the participation factors if so desired.

Modal analysis procedures will yield results that are only as accurate as the assumptions made for the modelling (idealization) of the forcing load and the structural properties. The modelling of the geometrical component of the forcing load (wind pressure distributions) has been presented in Chapter 3.

Participation factors -- Once the geometrical configuration of the forcing loading has been obtained, the participation factors can be evaluated by employing the modal analysis equations presented in Chapter 2. The participation factor for a distributed load has been defined as follows:

$$C_i = \frac{1/L \int_0^L \phi_i (x)\ P_o\ p(x)\ dx}{M_i} \tag{5-1}$$

In the most common case the wind loading is specified as a distributed force. On the other hand, the mode shapes are generally evaluated for a lumped model and only the values of the mode shapes on the nodal points of the structure are usually available. Therefore the computation of the participation factors for wind loading is, in most cases, a mixed problem of a distributed force acting on a discrete structure.

There are two possible approaches for the evaluation of the participation factors. The first is to compute an algebraic representation or a numerical interpolation of the mode shapes, and then to perform the integration of a quantity consisting of the mode shape times the force distribution. This operation can be accomplished employing any interpolation procedure to obtain a representation of the mode shape function. The second procedure consists of evaluating an equivalent concentrated force at each nodal point of the structure. This operation can be achieved by using integration procedures for the forcing load as described in Section 2-2, Eqs. (2-12a) and (2-12b). In this case the participation factors can be evaluated using the equation for discrete force systems Eq. (2-19b).

$$C_i = \frac{\{\phi_i\}^T\ [m]\ \ \{\rho\}}{\{\phi_i\}^T\ [m]\ \ \{\phi_i\}} \tag{5-2}$$

A summary of the computation procedure for wind loading as well as a comparison with earthquake modal analysis is presented in the following page:

	Earthquake	Wind
Problem Identification	Ground motion parameters	Wind velocity (V), mean pressure (P_o), structure exposure.
Mean Force	N. A.	$\overline{F} = P_o \times b \left(\frac{z}{h}\right)^{2\alpha}$ Discretize $\overline{F}$ at notes $\{\overline{F}\}$
Dynamic Force	$\{F\} = [m] \{\ddot{X}_G\}$	$F' = P_o P_r b \left(\frac{z}{h}\right)^{\alpha}$
Dynamic Response	$\{u\} = \sum C_i \phi_i D_i$	$\{X\} = \sum C_i \phi_i \frac{Peff_i}{\omega_i^2}$
Participation Factor	$C_i = \frac{\{\phi\}^T [m] \{1\}}{\{\phi\}^T [m] \{\phi\}}$	$C_i = \frac{\{\phi\}^T \{F'\}}{\{\phi\}^T [m] \{\phi\}}$
Amplification Factor	Obtain from response spectrum, A, V, and D	Obtain from response spectrum P_{eff}
Modal Forces	$\{f\} = \{\phi\}^T [m] C_i A_i$	$\{f'\} = \{\phi\}^T [m] C_i Peff_i$
Modal Base Shear	$\nu_{oi} = \{1\}^T \{\phi\}^T [m] C_i A_i$	$\nu_{oi} = \{1\}^T \{\phi\}^T [m] C_i Peff_i$
Distribution of Base Shear	$\{f\} = \frac{\nu_{oi} [m] \{\phi\}}{\{\phi\}^T [m] \{1\}}$	$\{f'\} = \frac{\nu_{oi} [m] \{\phi\}}{\{\phi\}^T [m] \{\phi\}}$
Combination Mean + Dynamic Components	N. A.	$\{F\} = \{\overline{F}\} + \{f'\}$

<u>Example</u> -- A three story, three degree of freedom building presented by Blume, Newmark and Corning (1961) was studied as an example to compare earthquake and wind modal analysis procedures. The geometry of the building together with the mass and stiffness properties are illustrated in Fig. 37.

The building was assumed to be at the center of a city and the damping was taken as 2 percent of critical. The wind is assumed to flow normal to the 60 ft x 42 ft face. The response spectrum for a 60 ft/sec wind velocity at the top of the building and two percent of damping is shown in Fig. 38. For earthquake El Centro N-S component is employed. The steps employed in this computation have been summarized in the previous table.

Mass Matrix

$$[m] = \begin{bmatrix} 1860 & & \\ & 1860 & \\ & & 3720 \end{bmatrix} \quad \text{lbs.}$$

Stiffness Matrix

$$[k] = \begin{bmatrix} 568 & & \\ & 1704 & \\ & & 2272 \end{bmatrix} \quad \text{K/inch.}$$

$$\omega_1^2 = 152.688 \rightarrow f_1 = 1.97 \text{ hertz}$$

$$\omega_2^2 = 610.753 \rightarrow f_2 = 3.93 \text{ hertz}$$

$$\omega_3^2 = 1832.258 \rightarrow f_3 = 6.81 \text{ hertz}$$

$$\{\phi_1\} = \begin{Bmatrix} 4 \\ 2 \\ 1 \end{Bmatrix}$$

$$\{\phi_2\} = \begin{Bmatrix} -1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{\phi_3\} = \begin{Bmatrix} 1/3 \\ -5/3 \\ 1 \end{Bmatrix}$$

Earthquake

Mean

N.A.

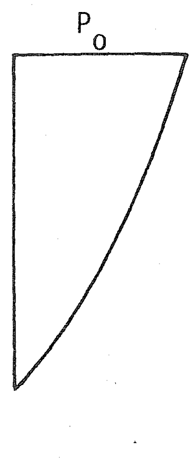
N.A.

Fluctuating

N.A.

Wind

Mean

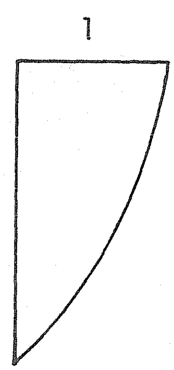


$$P_o = 1/2\ \rho\ C_D\ V^2 = 1/2\ (0.0025161)\ (60)^2$$

$$P_o = 4.53 \text{ psf}$$

$$\{\bar{F}\} = \begin{Bmatrix} 1834 \\ 3001 \\ 1879 \end{Bmatrix} \quad \text{lbs.}$$

Fluctuating



$$\{F'\} = \begin{Bmatrix} 447.08 \\ 798.18 \\ 621.41 \end{Bmatrix}$$

Earthquake

Participation Factors

$$\frac{\{\phi\}^T [m] \{1\}}{\{\phi\}^T [m] \{\phi\}}$$

C_1 = 8/22 =0.36

C_2 = 2/4 = 0.50

C_3 =0.67/4.90 =0.14

<u>Nodal Forces</u>

C_n [m] ϕ * A_n

$$\{f_1\} = .36 \times 1860 \begin{Bmatrix} 4 \\ 2 \\ 2 \end{Bmatrix} * A_1$$

$$\{f_2\} = .5 \times 1860 \begin{Bmatrix} -1 \\ 1 \\ 2 \end{Bmatrix} * A_2$$

$$\{f_3\} = .14 \times 1860 \begin{Bmatrix} .33 \\ -1.67 \\ 2 \end{Bmatrix} * A_3$$

$$\{f_1\} = \begin{Bmatrix} 2705.00 \\ 1352.00 \\ 1352.00 \end{Bmatrix} * A_1$$

$$\{f_2\} = \begin{Bmatrix} -930\ .00 \\ 930\ .00 \\ 1860.00 \end{Bmatrix} * A_2$$

$$\{f_3\} = \begin{Bmatrix} 84.35.00 \\ -422.61.00 \\ 506.12.00 \end{Bmatrix} * A_3$$

Wind

Participation Factors

$$\frac{\{\phi\}^T \{F'\}}{\{\phi\}^T [m] \{\phi\}}$$

C_1 = 4005.77/22 = 182.02

C_2 = 972.51/4 = 243.13

C_3 = -559.86/4.90 = - 114.26

<u>Nodal Forces</u>

C_n [m] ϕ * $Peff_n$

$$\{f_1'\} = 182.02 \times \frac{1860}{1860} \begin{Bmatrix} 4 \\ 2 \\ 2 \end{Bmatrix} * Peff_1$$

$$\{f_2'\} = 243.13 \times \frac{1860}{1860} \begin{Bmatrix} -1 \\ 1 \\ 2 \end{Bmatrix} * Peff_2$$

$$\{f_3'\} = -114.26 \times \frac{1860}{1860} \begin{Bmatrix} .33 \\ -1.67 \\ 2 \end{Bmatrix} * Peff_3$$

$$\{f_1'\} = \begin{Bmatrix} 728.26 \\ 364.13 \\ 364.13 \end{Bmatrix} * Peff_1$$

$$\{f_2'\} = \begin{Bmatrix} -243.02 \\ 243.02 \\ 486.04 \end{Bmatrix} * Peff_2$$

$$\{f_3'\} = \begin{Bmatrix} -38.17 \\ 190.93 \\ -229.03 \end{Bmatrix} * Peff_3$$

From Response Spectrum

$A_1 = 0.60$ g.

$A_2 = 0.70$ g.

$A_3 = 0.55$ g.

From Response Spectrum

$Peff_1 = 9.53$ psf

$Peff_2 = 8.38$ psf

$Peff_3 = 8.31$ psf

$$\{f_1\} = \begin{Bmatrix} 1623.00 \\ 811.50 \\ 811.50 \end{Bmatrix} \qquad \{f_1'\} = \begin{Bmatrix} 6938 \\ 3469 \\ 3469 \end{Bmatrix}$$

$$\{f_2\} = \begin{Bmatrix} -651.00 \\ 651.00 \\ 1302.00 \end{Bmatrix} \qquad \{f_2'\} = \begin{Bmatrix} -2038 \\ 2038 \\ 4076 \end{Bmatrix}$$

$$\{f_3\} = \begin{Bmatrix} 46.39 \\ -232.44 \\ 278.37 \end{Bmatrix} \qquad \{f_3'\} = \begin{Bmatrix} -1902 \\ 1585 \\ -317 \end{Bmatrix}$$

5-3 Simple Procedures to Estimate Wind Reponse.

For cases of preliminary analysis, at least two simplified procedures can be developed. These procedures include a new version of the gust response factor, and a procedure based on the computation and distribution of the base shear; both topics are presented and discussed in this Section. The empirical representation of the wind response spectra developed in Section 4-6 is employed for the derivation of both simplified procedures.

The gust factor -- For some time the results of studies of the response of structures to wind loading have been presented in the form of gust factors in contrast to the use of response spectra as discussed earlier herein. The gust factor is defined as the ratio of the total displacement of the structure to the mean displacement of the structure, usually evaluated at the highest level of the building. The gust factor can be considered to be an overload coefficient which multiplies the mean pressure to obtain the design forces for the structure. The mean force times the gust factor is intended to account for both the mean and fluctuating components of the wind loading. Various authors such as Davenport (1967), Vellozi and Cohen (1968), Vickery (1970) and Simiu and Lozier (1975) have presented different versions of this approach. In some cases, these procedures can differ by as much as 100 percent.

The gust factor usually represents a load which is the mean force plus 3 to 4 times the contribution of the fluctuating component of the wind; this combination yields values of total load that are generally between 1.8 to 2.5 times the mean value. However for the cases of structures subjected to high wind turbulence; such as small structures in the center of a large city, the gust factor may lead to total loading values on the order of 3 to 3.5 times the mean force.

For the computation of gust factors the following assumptions are made:

1) The response of the structure in the fundamental mode is dominant for both mean and fluctuating responses.

2) The masses and the stiffness of the structure are uniformly distributed.

3) The response can be calculated using a linear mode shape.

4) The peak factor can be taken as 3.5 times the fluctuating load (an average between 3 and 4).

The mean displacement associated with the first mode of vibration is given by:

$$\text{Mean displacement} = \frac{1}{2 + 2\alpha} \, \phi(x) \, \frac{P_o}{M\omega^2} \qquad (5\text{-}3)$$

Similarly the fluctuating displacement is given by:

$$\text{Fluctuating displacement} = \frac{1}{2 + \alpha} \; \phi\,(x) \; \frac{P_o\, P_r\, \text{Amp}}{M\,\omega^2} \qquad (5\text{-}4)$$

As has been discussed before, the gust factor is the ratio of the total displacement to the mean displacement multiplied times the peak factor. Thus the gust factor can be written as follows:

$$\text{Gust Factor} = 1 + 3.5\,\frac{(2 + 2\alpha)}{(2 + \alpha)} \quad P_r\ \text{Amp} \qquad (5\text{-}5)$$

The gust factor has been customarily computed in the past by employing a narrow band approximation, in such case it is usually represented by an equation of the following form:

$$G = 1 + \overline{g}\,\sqrt{B + R} \qquad (5\text{-}6)$$

where

B is the background component which represents static behavior (as for a rigid building),

R is the resonant component associated with the vibration,

$\overline{g}$ is the peak factor (taken as 3.5 in this study).

In the proposed equation for the gust factor, Eq. (5-5) the background component, B, is represented by the quantity $(P_r)^2$, and the resonant component, R, is given by $(\text{Amp}^2 - 1) \times (P_r)^2$, such that the relation $P_r \times \text{Amp} = \sqrt{B + R}$ is satisfied. The gust factor can be expressed in a simpler form as follows:

$$\text{Gust factor} = 1 + F \qquad (5\text{-}7)$$

The factor F (the latter part of Eq. (5-7)) is presented in Table 14 for three wind exposures and values of damping equal to 1, 2, and 5 percent of critical, as a function of the wind velocity at the top of the structure, the height and the fundamental frequency of vibration.

Example -- As an example the gust factor is computed for a 325 ft x 100 ft x 100 ft building. The structure is in the center of a city and the velocity at the top of the building is 85 ft /sec. The damping is assumed to be 2 percent of critical and the fundamental frequency of vibration is taken as 0.5 hertz.

From Eq. (5-7) and Table 14, the gust factor is:

$$\text{Gust factor} = 1 + 7.42(325 \times 0.5/85)^{-0.36}\,(325)^{-0.28} = 2.16$$

The gust factor also can be computed employing Eq. (5-5) as follows:

From Table 12

$P_r = 2.05 \times (325)^{-0.40} = 0.20$

From Table 13

$\text{Amp} = 0.90 \times (0.5 \times 325/85)^{-0.36} \times (325)^{0.12} = 1.43$

For city exposure, $\alpha = 0.35$

$(2 + 2 \times \alpha)/(2 + \alpha) = 1.15$

Gust factor = $1 + 3.5 \times 1.15 \times 1.43 \times 0.20 = 2.15$

The gust factor for the same example computed using the procedure recommended by the National Building Code of Canada is 2.35.

A comparison of the results obtained with the simple gust factor equation proposed in this study and those reported in the literature by various authors (as computed by the writer from the equations and charts presented in the references cited in Table 15)is illustrated in Table 15, where three buildings, located in city and open country exposures, are presented. The gust factors are given for 1 and 2 percent of critical damping. The dimensions and properties of the buildings studied in these examples are given in Table 16.

It can be seen in Table 15 that the gust factors computed using the simple equations developed in this study are in good agreement with the values predicted by various procedures reported in the literature.

<u>Distribution of base shear</u> -- The second simple procedure is based on the distribution of base shear. First, the base shear is computed and distributed using a linear fundamental mode shape. Secondly, a correction function, that accounts for the contribution of the higher modes, is applied.

<u>The two mode procedure</u> -- The two modes approach is based on the fact that the combination of the unamplified modal contributions should satisfy the static relation:

$$[m]\ \{1\} = \sum_{i=1}^{n} C_i\ [m]\ \{\phi_i\} \tag{5-8}$$

which for a diagonal mass matrix can be written in a simpler way as follows:

$$\{1\} = \sum_{i=1}^{n} C_i\ \{\phi_i\} \tag{5-9}$$

A practical and useful procedure can be developed assuming that the computation can be carried out employing only two modes, namely the fundamental mode of vibration and a second mode which represents the contribution of the higher modes. For two modes, Eq. (5-9) can be written as:

$$\{1\} = C_1\ \{\phi_1\} + C_2\ \{\phi_2\} \tag{5-10}$$

In addition if the participation factor of the second mode is conveniently defined to have a value of one the second mode can be written as:

$$\{\phi_2\} = \{1\} - C_1 \{\phi_1\} \tag{5-11}$$

It will be shown in the examples presented in this Chapter that the two modes procedure yields results that are in close agreement with those obtained by modal analysis techniques.

Distribution of base shear -- The procedure recommended for computing the forces associated with wind loading, using the two mode approach, is divided into four steps:

1) Computation of the forces associated with the mean component of the response.

2) Computation of the forces associated with the background part of the fluctuating component.

3) Distribution of the forces associated with the resonant part of the fluctuating component using a linear mode shape rule.

4) Correction to include the resonant response of the higher modes.

These steps are discussed in detail in the following paragraphs.

Step one -- The vertical distribution of the average pressure is given by the Eq. (3-7a) as follows:

$$P(x) = P_o \left(\frac{x}{h}\right)^{2\alpha} \tag{5-12}$$

where

P_o is the average pressure at the top of the building ($1/2\ \rho\ C_D\ V^2$). The mean forces at the various levels of the structure can be computed using the integration formulas given by Eqs. (2-12a) and (2-12b).

Step two -- The background pressure is given by the following expression:

$$P_B = P_o \, P_r \, (\frac{x}{h})^{\alpha} \tag{5-13}$$

where

P_r is the effective pressure coefficient given in Table 12.

Step three -- The resonant base shear for a linear mode shape can be computed as follows:

$$V_\ell = \Gamma_w \, C_E \, (Amp_\ell - 1)$$

$$\Gamma_w = \frac{b \, h^2 \, P_o \, P_r}{2 + \alpha}$$

$$C_E = \frac{\sum m_x \, h_x}{\sum m_x \, h_x^2}$$

where

V_ℓ is the base shear associated with a linear mode

m_x is the mass at level x,

h_x is the height of level x over the base of the building,

b is the width of the structure,

h is the height of the structure,

Amp_ℓ is the amplification factor associated to the linear mode shape which can be computed by employing the equations presented in Table 13. For this computation the fundamental frequency of vibration can be approximated as $f = 120/h$ (as discussed in Section 4-6).

Once the resonant base force has been computed, it can be distributed using the familiar expression, Eq. (2-43):

$$F_{\ell x} = V_\ell \frac{m_x h_x}{\sum m_x h_x}$$

The equation for the distribution of forces can be written in a simpler way as follows:

$$F_{\ell x} = C_\ell m_x h_x \qquad (5\text{-}14a)$$

where

$$C_\ell = \frac{\Gamma_w (Amp_\ell - 1)}{\sum m_x h_x^2} \qquad (5\text{-}14b)$$

<u>Step four</u> -- The forces associated with the higher modes can be computed as follows:

$$F_{cx} = \frac{V_c (1 - C_E h_x) m_x}{\sum m_x - C_E \sum h_x m_x}$$

where

V_c is the resonant base force associated with the higher modes and is given by the equation:

$$V_c = \left(\frac{bh P_o P_r}{1 + \alpha} - \Gamma_w C_E\right) (Amp_c - 1)$$

Amp_c is the amplification factor for the higher modes, which can be computed employing a frequency that is three times the frequency associated with the linear mode.

The forces associated with the higher modes can be written in a simpler way as follows:

$$F_{cx} = C_c (1 - C_E h_x) m_x \tag{5-15a}$$

where

$$C_c = \frac{V_c}{\sum m_x - C_E \sum h_x m_x} \tag{5-15b}$$

Example -- As an example, a ten story, eight degree of freedom building, is analyzed employing the distribution of base shear technique. The dimensions of the structure are presented in Fig. 34. The masses and stiffness of the columns are presented in Table 28. The building is assumed to be in the center of a city and has a damping equal to one percent of critical. The drag coefficient is taken as 1.3. Computations are presented for a wind velocity of 85 ft /sec at the top of the building in the E-W direction.

Step one.

$P_o = 1/2 \times 1.3 \times .0024 \times (85)^2 = 11.84$ psf.

The forces associated with the mean pressure are listed in the third column of Table 17.

Step two.

From Table 12

$P_r = 7.17\ (114)^{-0.40} = 1.078$

$P_B = 1.078 \times 11.84 = 12.76$ psf.

The forces associated with the background component are presented in the fourth column of Table 17.

Step three.

$\Gamma_w = 40 \times (114)^2 \times 12.76/2.35 = 2822620$

$\sum m_x h_x = 1417$

$\sum m_x h_x^2 = 120274$

$C_E = 1417/120274 = 0.012$

From Table 12 and taking f = 120/114 the amplification factor is:

$Amp_\ell = 1.12 \; (1.05 \times 114/85)^{-0.42} \; (114)^{0.12} = 1.75$

$C_\ell = 2822620 \; (1.75-1)/120274 = 17.55$

The forces associated with the fluctuating component of the linear mode are listed in the fifth column of Table 17.

Step four.

$Amp_c = 1.12 \; (3 \times 1.05 \times 114/85)^{-0.42} \; (114)^{0.12} = 1.10$

$V_c = (114 \times 40 \times 12.76/1.35 - 0.012 \times 2822620) \times (1.10-1) = 923$

$C_c = 923/(20.91 - 0.012 \times 1417) = 237$

The forces associated with the higher modes (the second term representation) are presented in Table 17. To complete the solution the four contributions are added. The total forces are presented in the last column of Table 17. The total forces are illustrated in Fig. 36, where the solution obtained by normal modal analysis procedures is also shown. It can be seen that the two solutions are in good agreement.

For the purpose of further simplifying the calculations Tables 18 to 26 have been prepared to present the effective wind pressures. These tables are similar to those presented in A.N.S.I. (Tables 5A, 5B and 5C) for computation of the basic wind pressures, and represent the wind pressure as a function of height and wind velocity with a drag coefficient equal to one.

The tables are computed for the three wind exposures described in Chapter 3, namely center of cities, suburban areas and open country. For each exposure three tables are presented for 1, 2 and 5 percent of critical damping. The wind velocities presented in the top row of the tables are those that will be read from the recurrence maps presented in A.N.S.I.. For each combination of height and velocity two entries are given. The upper figure, which is also the larger, is the total pressure (mean plus fluctuating), and the lower value is the fluctuating pressure. The tables reflect a drag coefficient equal to one. In other words the basic wind pressures should be multiplied by the drag coefficient.

5-4 Examples.

In order to demonstrate the application of modal analysis and the simplified approaches developed in Section 5-3 three examples are presented. The examples are three typical buildings which have been designed and constructued. These are a 25 story concrete building, a 10 story steel frame building and a 3 story building.

25 story building -- An effort was made to study a typical structure that should have most of the characteristics found in the practice and not covered by the assumptions made for the simplified procedures. Those characteristics are:

1) Different values of masses and stiffness at the various levels of the structure.

2) Presence of taller columns in the first floor.

3) Heavy service floors at the higher levels of the structure.

Those conditions were found in a reinforced concrete building presented by Blume, Newmark and Corning (1961). The structure is a 325 ft x 100 ft x 175 ft rectangular building whose plan and elevations are shown in Fig. 31. The masses and the stiffness of the columns are listed in Table 27. The wind forces were calculated assuming that the structure is constructed in the center of a city with a wind velocity of 85 ft /sec at the top of the building and a damping of two percent of critical. The wind is assumed to act normal to the 325 ft x 100 ft face. In addition, to simplify the computations, the drag coefficient was taken as 1.3 and the lift coefficient as zero. The response spectrum for this structure, for a wind velocity of 85 ft /sec and 2 percent of critical damping is presented in Fig. 32. The story forces computed using the three procedures described in this section are presented in Fig. 33.

10 story building -- A ten story, 8 degree of freedom building, as studied and reported by Nielsen (1968), was also analyzed. The geometry of the building is shown in Fig. 34. The building was assumed to be in the center of a city and the damping was taken as one percent of critical. The response spectrum for a 85 ft/sec wind velocity at the top of the building is shown in Fig. 35. The wind is assumed to flow in the E-W direction. The stiffness and mass values, as reported by Nielsen are listed in Table 28. The story forces computed employing nodal analysis and the two simplified procedures are shown in Fig. 36.

3 story building -- A three story, three degree of freedom building presented by Blume, Newmark and Corning (1961) was studied as a last example. The geometry of the building together with the mass and stiffness properties are illustrated in Fig. 37. The building was assumed to be at the

center of a city and the damping was taken as two percent of critical. The wind is assumed to flow normal to the 60 ft x 42 ft face. The response spectrum for a 60 ft/sec wind velocity at the top of the building and two percent of damping is shown in Fig. 38. The forces computed for this structure are shown in Fig. 39. The three examples presented in this Section have been computed by employing modal analysis techniques as described in Chapter 2 and the response spectrum developed in Chapter 4. These techniques are similar to the modal analysis procedures employed in earthquake engineering. Computations of the earthquake forces employing modal analysis procedures for the 25 and 3 story buildings can be found in Blume, Newmark and Corning (1961).

5-5 Comparison with Full-Scale Measurements.

In an effort to present the accuracy of the various procedures developed in this study the results are compared with the few full-scale measurements reported in the literature.

Davenport, Hogan and Vickery (1970) have reported measurements of the resonant displacement at the top of the John Hancock Center in Chicago, Il. Observations were made under two wind conditions: July 15, 1970 and July 17, 1970. The mean velocities at the top of the structure were 59 ft /sec and 42 ft /sec respectively. The measured damping for the first mode of vibration is reported as 0.4 percent of critical and the fundamental natural frequency of vibration as 0.21 hertz. The principal dimensions of the building are presented in Table 29.

For the computation of displacements, a linear mode shape (x/h), can be assumed. For this mode the resonant displacement at the top of the building can be computed as follows:

The resonant displacement is given by the equation:

$$\text{Displacement} = \frac{\phi_i(x)\ C_i\ P_o\ \sqrt{R}}{\omega_i^{\ 2}}$$

for a mode (x/h)

$$C_i = \frac{1/L \int_o^L (\frac{x}{h})^{\alpha}\ (\frac{x}{h})\ dx}{1/L \int_o^L (\frac{x}{h})^2\ \overline{m}\ dx}$$

and the following value is obtained for the participation factor:

$$C_i = \frac{1.3}{\overline{m}}$$

The following values for the average density and average width of the building are reported:

γ = 10.2 lbs /ft.^{3}

be = 196 ft.

Therefore the mass per linear ft can be written as:

$\overline{m}$ = (10.2 x 196)/32.2 = 62.09 lbs sec/ft.

Finally, for a frequency of 0.21 hertz and multiplying by 12 to obtain the result in inches, the displacement at top of the building is written as follows:

$$\text{Displacement at top} = 0.14\ P_o\ \sqrt{R}$$

where

P_o is the mean wind pressure $1/2\ \rho\ C_D\ V^2$,

R is the resonant contribution of the response.

It is interesting to note that the values of the displacements at the top of the building are insensitive to the form of the assumed mode shape.

For example, the deflected shape of a cantilever beam subjected to uniform loading also could be used as an approximation of the fundamental mode shape. In this case the displacement at the top of the building can be computed by employing a procedure similar to that employed before. The mode shape has the following form:

$$\phi_i(x) = 1/3\ (x^4 - 4x + 3)$$

the participation factor is given by the evaluation of the integral:

$$C_i = \frac{\int_0^1 1/3\ (x^4 - 4x + 3)\ (x)^{\alpha}\ dx}{\int_0^1 \bar{m}\ [1/3\ (x^4 - 4x + 3)]^2\ dx}$$

For the cantilever mode shape the following expression for the displacement at the top of the building is obtained:

$$\text{Displacement at top} = 0.135\ P_o \sqrt{R}$$

The values of P_o for wind velocities of 42 and 59 ft/sec are presented in Table 30.

It should be pointed out that the assumed form of the pressure distribution may have a large influence in the numerical value of the participation factor and thereby, in the response. For this example the same results have been analyzed in the literature by employing a linear distribution for the pressure. This distribution leads to a participation factor equal to one (which gives a displacement that is 30% lower than that calculated employing a pressure distribution of the form $(x/h)^{0.35}$). On the other hand, the same studies in the literature have used a distribution of pressures

$(x/h)^{0.35}$ for the computation of the resonant factor R. Therefore, this points out the important fact that one must be careful in interpreting results presented which in some cases may have been computed by employing inconsistent assumptions.

The resonant contribution can be computed employing either numerical integration or the simple equations developed in Chapter 4. The amplification factors computed by numerical integration for a frequency of 0.21 hertz and 0.4 percent of critical damping were found to be 1.37 for a velocity of 42 ft /sec and 1.70 for a wind velocity of 59 ft /sec. The resonant contributions computed by employing numerical integration are presented in Table 31.

The amplification factor also can be computed employing Eq. (4-11) with the coefficients given in Figs. 25 and 26. For 0.04 percent of damping and city exposure the amplification can be written as follows:

$$\text{Amp} = 1.5\ (fh/Vel)^{-0.50}\ h^{0.12}$$

The amplification factors computed using the last equation are 1.48 for a velocity of 42 ft /sec, and 1.75 for a wind velocity of 59 ft/sec, which are close to the above values computed by numerical integration.

The resonant response computed using numerical integration is presented in Table 30 for the two fundamental modes of vibration discussed in this section. In addition, in Fig. 40 the results obtained from numerical integration for both the linear and the cantilever mode are compared with the experimental measurements. It can be seen that the computed values are in reasonable agreement with the observations reported.

As a second example, the experimental measurements of the gust factor for a building located at Delft, Holland, reported by van Koten (1967) were studied. The building has a height of 150 ft, a length of 240 ft, and a width of 36 ft. The structure consists of a steel frame with prefabricated concrete slabs. The mean velocity at the top of the building was 45 ft /sec during the experimental period. The fundamental frequency is reported to be 0.7 hertz, but no values of damping are presented.

The gust factor computed by employing the results reported by van Koten, has a value of 2.63 for a peak factor of 3.5. Since measurements of damping are not available the gust factor was computed for 1 and 2 percent of critical. The values computed employing Eq. (5-7) were 2.59 for one percent, and 2.34 for 2 percent of damping. It can be seen that the gust factors predicted by Eq. (5-7) are in good agreement with the experimental results presented by van Koten.

5-6 Comparison of Earthquake and Wind Hazard.

Earthquake and wind hazard are dependent on the geographic location and the properties of the building. One possible procedure for the comparison of the earthquake and wind hazard is the evaluation of the base shear forces. Many parameters such as size, frequency and exposure are present on the computation of the base shear.

A distinction should be made between Code requirements and the actual behavior of the structure. Both earthquake and wind Code specifications are based on simplified approximations. Usually earthquake requirements are presented in the form of base shear coefficients. On the other hand, wind loading is generally specified as a distributed pressure varying with the altitude of the building.

Comparison of the effects of earthquake and wind loading on structures should be divided into two categories: the first involving Code requirements, the second concerning the behavior of the structure.

The procedures to compare Code requirements (useful for determining design loads) are relatively simple and will be discussed here briefly. On the other hand, the comparison of the behavior of structures under both types of loadings requires a detailed study and was not covered in this investigation.

For preliminary analysis, the base shear can be computed by employing the Code specifications for both earthquake and wind. Since earthquake Code provisions are generally specified in the form of base shear coefficients and wind Code provisions as a distributed pressure, one must calculate either the base shear coefficients or the base forces for both loading conditions. In both cases the mass or weight of the structure has to be estimated.

For convenience the base shear was chosen as the parameter of comparison, because earthquake loading is specified in this manner, whereas no base force is specified for wind. For wind the base shear coefficient can be computed by multiplying the distributed pressures times the exposed area and then dividing by the weight of the structure:

$$V_w = \frac{P \times \text{Area}}{W}$$

As an example the seismic and wind provisions of the latest edition of the Uniform Building Code were studied. The city of Chicago was chosen as the geographic location for this example. In this case, the base shear coefficients were computed for a set of buildings having the same cross section but varying altitudes. The period of vibration was assumed to be the number of stories divided by ten and an average value of 12 ft per story was taken. The computed base shear coefficients for earthquake and wind are shown in Fig. 41. For wind, two cross sections were studied. The first is a 100 ft x 100 ft having a weight of 15 lbs /ft ^{3} and the second is a 200 ft x 200 ft with a weight of 10 lbs /ft ^{3}.

CHAPTER 6

SUMMARY

A procedure for the computation of along overall wind response of buildings and structures has been developed and presented. Such an approach is common to the techniques currently employed for the evaluation of forces and displacements arising from earthquake base excitation.

The results presented in this study can be divided into four broad categories as follows:

1) The development of a basic methodology for the computation of response spectra employing random vibration techniques. Specifically, in this study response spectra have been developed for wind loading by the noted technique.

2) The analytical treatment of the equation of motion to obtain a solution in terms of participation factors for wind excitation.

3) Development of simplified methods of analysis for wind loading.

4) Comparison of wind and earthquake loadings and analysis procedures.

The development of wind response spectra covered the three basic steps generally employed for the computation of earthquake response spectra, namely: 1) evaluation of the effective base parameters (in this case pressure and velocity) to account for the various sizes and intensities of the wind gusts; 2) computation of the amplification factors associated with the wind dynamic (fluctuating) loading; 3) development of a suitable scale for the spectral representation of wind loading (similar to the tripartite plot).

There are various significant similarities between earthquake analysis techniques and the proposed procedure for analysis of wind loading:

1) The general polygonal shape and the numerical values of the amplification factors for both spectra are similar.

2) The equations for the distribution of base shear are the same. This fact permits the development of simple approximate procedures for the determination of which hazard (earthquake or wind) is the dominant parameter in design. In addition, this similarity in the distribution of base shear will enable the determination of the seismic resistance of a building that has been designed to sustain wind loading and vice versa.

3) Once the participation factors have been evaluated, the modal analysis techniques required for computation of the response associated with both loadings are basically similar.

On the other hand, there are also differences between both loadings and spectral representations. Among these the more significant are the following:

1) Wind forces are separated into a mean force which is treated as a static load, and a fluctuating force which represents the dynamic component of the loading.

2) The range of frequencies of the amplified region of the fluctuating wind response spectra is considerably lower than the typical range of frequencies present in earthquake response spectra. This fact indicates that flexible structures with large periods of vibration are more sensitive to wind loading, whereas stiff structures are

likely to be more sensitive to earthquake forces, (with exception of long period waves).

3. The amplified pressure ordinate for wind response spectra converges to its terminal value (base line) at a frequency which is relatively low when compared with the 33 hertz recommended for earthquake response spectra. For this reason special provisions should be taken in combining high modes where loading contributions will be more of a static rather than of a dynamic type. This fact also suggests that the best combination of modal contributions for wind loading may be the algebraic sum in contrast to the square root of the sum of the squares currently used in earthquake engineering and random vibration computation.

The hyperbolic equations derived in Section 5-4 for the computation of gust factors are simple, easy to use and provide a technique for a quick estimate of the wind loading in cases of preliminary design. The gust factors computed using the equations are in good agreement with the results predicted by various gust factor procedures available in the literature.

The two mode procedure discussed in Section 5-4 is an accurate tool of analysis (when compared to modal techniques) for the determination of wind forces on structures.

The set of tables for the basic effective pressure are innovative in the sense that the damping has been used as one of the explicit parameters permitting, in this manner, more flexibility to the designer.

A procedure based on the computation of the base shear has been employed as the technique for comparing wind and seismic effects upon buildings. This procedure takes into consideration the geographic location and the configuration of the building. The same approach can be used to evaluate the seismic resistance of a building that has been designed to sustain wind forces and vice versa.

Finally a comparison of the results obtained using the procedures for wind loading developed in this study with full-scale measurements, shows values that are believed to be reasonable within the framework of the approximations involved, and the accuracy of the statistical functions employed as input, for the computation of wind response spectra.

It is appreciated that this research investigation involves only analysis techniques and that the procedures developed and the computations presented in this study are valid only for the elastic range. However, an effort should be made to obtain a better understanding regarding loading and resistance. It is believed that more and more accurate records of wind velocity and wind pressure should be obtained. In addition, other parameters such as cross wind vibrations and possibly soil structure interaction as well as moderate inelastic behavior should be included in wind resistant design.

Comparison of earthquake and wind loading suggest that both natural hazards need consideration in design. In some cases one or the other may be the controlling parameter, while in other cases both may be of nearly equal importance. However, even if the analysis techniques are similar for earthquake and wind loading, the design criteria and detailing requirements may be different. For example, earthquakes constitute strong events which may never or seldom occur during the expected life of the structure. On the other hand, moderate wind loading may frequently occur, and even strong wind condtions may be expected to occur several times during the life span of the building. Moreover, little effort in the detailing of windows, roofs, and cladding may produce a substantial improvement in the overall wind resistance of the structure, whereas a ductile and redundant structural system should be provided to withstand moderate seismic loading and to prevent structural collapse and loss of human life in the event of a strong earthquake.

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TABLE 1 COMPARISON OF TIME DOMAIN AND FREQUENCY DOMAIN SOLUTIONS FOR LINEAR SYSTEMS

	Input	Linear System Transfer Function	Output
Time Domain	$f(t)$	$h(t-\tau)$	$x(t) = \int_0^t h(t-\tau)f(\tau)d\tau$
Frequency Domain	$f(\omega)$	$H(\omega)$	$X(\omega) = H(\omega)\ F(\omega)$

TABLE 2 COEFFICIENT α FOR VARIOUS WIND EXPOSURES RECOMMENDED BY VARIOUS AUTHORS AND CODES

Exposure	ANSI	NBCC	Vickery (1970)
A	0.33	0.40	0.35
B	0.22	0.28	0.22
C	0.14	0.14	0.14

TABLE 3 SUGGESTED VALUES OF z_o FOR VARIOUS TYPES OF EXPOSURE*

Type of Exposure	Coastal[a]	Open	Outskirts of Towns, Suburbs	Center of Towns	Centers of Large Cities
z_o (meters)	0.005-0.01	0.03-0.10	0.20-0.30	0.35-0.45	0.60-0.80

[a] Applicable to structures directly exposed to winds blowing from open water.

* After Simiu and Lozier (1975)

TABLE 4 RECOMMENDED DRAG AND LIFT COEFFICIENTS

Exposure	C_D	C_L	$C_L \times (0.5)^{2\alpha}$
A	0.80	0.50	0.31
B	0.80	0.50	0.34
C	0.80	0.50	0.41

TABLE 5 SPECTRUM AMPLIFICATION FACTOR
FOR ELASTIC WIND RESPONSE

Damping % of Critical	Velocity	Pressure (Davenport Input)	Pressure (Simiu Input)
1%	3.53	5.10	4.24
2%	2.74	3.66	3.08
5%	2.24	2.60	2.10
10%	1.64	1.87	1.64

TABLE 6 SPECTRUM AMPLIFICATION FACTORS
FOR HORIZONTAL ELASTIC RESPONSE*

Damping, % Critical	One Sigma (84.1%)			Median (50%)		
	A	V	D	A	V	D
0.5	5.10	3.84	3.04	3.68	2.59	2.01
1	4.38	3.38	2.73	3.21	2.31	1.82
2	3.66	2.92	2.42	2.74	2.03	1.63
3	3.24	2.64	2.24	2.46	1.86	1.52
5	2.71	2.30	2.01	2.12	1.65	1.39
7	2.36	2.08	1.85	1.89	1.51	1.29
10	1.99	1.84	1.69	1.64	1.37	1.20
20	1.26	1.37	1.38	1.17	1.08	1.01

* After Newmark and Hall (1978)

TABLE 7 EQUATIONS FOR SPECTRUM AMPLIFICATION FACTORS FOR HORIZONTAL MOTION*

Quantity	Cumulative Probability, %	Equation
Acceleration	84.1 (One Sigma)	$4.38 - 1.04 \ln \beta$
Velocity		$3.38 - 0.67 \ln \beta$
Displacement		$2.73 - 0.45 \ln \beta$
Acceleration	50 (Median)	$3.21 - 0.68 \ln \beta$
Velocity		$2.31 - 0.41 \ln \beta$
Displacement		$1.82 - 0.27 \ln \beta$

* After Newmark and Hall (1978)

TABLE 8 RECOMMENDED AMPLIFICATION FACTORS FOR THE PRESSURE LINE OF ELASTIC WIND SPECTRUM

Damping % of Critical	Vel/1500	Frequency Vel/h	15 Vel/h
1%	5.36	5.10	1
2%	4.21	3.66	1
5%	3.25	2.60	1
10%	2.43	1.87	1

TABLE 9 AMPLIFIED PRESSURES FOR EXAMPLE 4-1

Control Frequency Hertz	Amplification Factor	Amplified Pressure psf
V/1500 = 0.028	4.21	37.89
A = 0.026	3.66	15.41
B = 0.960	1	4.21

TABLE 10 AMPLIFIED PRESSURES FOR EXAMPLE 4-2

Control Frequency Hertz	Amplification Factor	Amplified Pressure psf
V/1500 = 0.026	5.36	26.26
A = 0.050	1.10	24.09
B = 2.428	1	4.90

TABLE 11 CONTROL FREQUENCIES A AND B FOR DRAWING THE PRESSURE LINE OF WIND RESPONSE SPECTRUM

WIND EXPOSURE		1% Damping	2% Damping	5% Damping	10% Damping
City	A	$0.027vh^{-0.71}$	$0.020vh^{-0.67}$	$0.015vh^{-0.60}$	$0.011vh^{-0.54}$
	B	$1.31\ vh^{-0.71}$	$0.75\ vh^{-0.67}$	$0.30\ vh^{-0.60}$	$0.12\ vh^{-0.54}$
Suburban	A	$0.017vh^{-0.57}$	$0.013vh^{-0.50}$	$0.009vh^{-0.40}$	$0.005vh^{-0.31}$
	B	$0.84\ vh^{-0.57}$	$0.48\ vh^{-0.50}$	$0.17\ vh^{-0.40}$	$0.06\ vh^{-0.31}$
Open Country	A	$0.013vh^{-0.50}$	$0.010vh^{-0.42}$	$0.006vh^{-0.30}$	$0.004vh^{-0.19}$
	B	$0.62\ vh^{-0.50}$	$0.36\ vh^{-0.42}$	$0.13\ vh^{-0.30}$	$0.04\ vh^{-0.19}$

TABLE 12 EFFECTIVE PRESSURE COEFFICIENTS

WIND EXPOSURE	α COEFFICIENT	Pr 1 σ	Pr 3.5 σ
City	0.35	$2.05\ h^{-0.40}$	$7.17\ h^{-0.40}$
Suburban Areas	0.22	$0.76\ h^{-0.27}$	$2.67\ h^{-0.27}$
Open Country	0.14	$0.27\ h^{-0.14}$	$0.95\ h^{-0.14}$

TABLE 13 AMPLIFICATION FACTORS

WIND EXPOSURE	1% Damping	2% Damping	5% Damping	10% Damping
City	$1.12 \left(\frac{fh}{v}\right)^{-0.42} h^{0.12}$	$0.90 \left(\frac{fh}{v}\right)^{-0.36} h^{0.12}$	$0.70 \left(\frac{fh}{v}\right)^{-0.30} h^{0.12}$	$0.58 \left(\frac{fh}{v}\right)^{-0.26} h^{0.12}$
Suburban Areas	$0.93 \left(\frac{fh}{v}\right)^{-0.42} h^{0.18}$	$0.77 \left(\frac{fh}{v}\right)^{-0.36} h^{0.18}$	$0.59 \left(\frac{fh}{v}\right)^{-0.30} h^{0.18}$	$0.48 \left(\frac{fh}{v}\right)^{-0.26} h^{0.18}$
Open Country	$0.82 \left(\frac{fh}{v}\right)^{-0.42} h^{0.21}$	$0.69 \left(\frac{fh}{v}\right)^{-0.36} h^{0.21}$	$0.54 \left(\frac{fh}{v}\right)^{-0.30} h^{0.21}$	$0.43 \left(\frac{fh}{v}\right)^{-0.26} h^{0.21}$

TABLE 14 PARAMETER F FOR COMPUTING GUST FACTORS

WIND EXPOSURE	1% Damping	2% Damping	5% Damping
City	$9.23 \left(\frac{fh}{v}\right)^{-0.42} h^{-0.28}$	$7.42 \left(\frac{fh}{v}\right)^{-0.36} h^{-0.28}$	$5.77 \left(\frac{fh}{v}\right)^{-0.30} h^{-0.28}$
Suburban Areas	$2.73 \left(\frac{fh}{v}\right)^{-0.42} h^{-0.09}$	$2.26 \left(\frac{fh}{v}\right)^{-0.36} h^{-0.09}$	$1.73 \left(\frac{fh}{v}\right)^{-0.30} h^{-0.09}$
Open Country	$0.83 \left(\frac{fh}{f}\right)^{-0.42} h^{0.03}$	$0.70 \left(\frac{fh}{v}\right)^{-0.36} h^{0.03}$	$0.55 \left(\frac{fh}{v}\right)^{-0.30} h^{0.03}$

TABLE 15 COMPARISON OF GUST FACTORS COMPUTED BY VARIOUS PROCEDURES

			Eq. 5-7	Vickery (1971)	Vellozi and Cohen (1968)	Davenport (1967)
Building 1	City	1%	2.60	2.67	2.59	2.57
		2%	2.35	2.66	2.59	2.57
	Open	1%	2.00	1.83	1.72	1.74
		2%	1.85	1.83	1.72	1.72
Building 2	City	1%	2.59	2.16	1.80	2.28
		2%	2.28	2.12	1.80	2.10
	Open	1%	2.38	1.97	1.56	1.43
		2%	2.14	1.88	1.54	1.78
Building 3	City	1%	2.31	2.26	1.63	2.48
		2%	2.05	2.09	1.61	2.08
	Open	1%	2.44	2.45	1.58	2.03
		2%	2.20	2.18	1.52	1.95

TABLE 16 PROPERTIES OF BUILDINGS STUDIED FOR THE COMPUTATION OF GUST FACTORS

	Building 1	Building 2	Building 3
Height	150'	500'	1200'
Cross section	150' x 150'	200' x 200'	200 x 200
Frequency	1 HERTZ	0.2 HERTZ	0.1 HERTZ
Velocity Open	110 ft hec.	130 ft hec.	150 ft hec.
Velocity City	65 ft hec.	95 ft hec.	130 ft hec.

TABLE 17 FORCES AT LEVELS FOR EXAMPLE-2 SECTION 5-3

level	mass	h	P mean	P background	P linear	P correction	P total
1	2.38	14	1527	3475	585	265	5872
2	2.38	28	2498	4464	1170	212	8344
3	2.38	42	3321	5151	1754	158	10384
4	2.38	56	4064	5700	2339	104	12207
5	2.38	70	4754	6164	2924	51	13893
6	2.38	84	5400	6571	3509	-3	15477
7	2.38	98	6468	7444	4093	-56	17949
8	4.24	114	3727	4127	8483	-209	16138

Units:
mass: kps/g
h: ft.
force: lbs.

TABLE 18 EFFECTIVE VELOCITY PRESSURES FOR CITY EXPOSURE AND 1% DAMPING

Height (ft)	Basic Wind Velocity (mph) 50	60	70	80	90	100	110	120	130	140
50	3	4	6	9	12	15	19	24	29	34
	1	2	3	5	7	9	12	15	19	24
100	4	6	9	12	16	21	26	32	39	47
	2	3	5	7	9	13	16	21	26	31
150	4	7	10	14	19	25	31	39	47	56
	2	3	5	8	11	15	19	24	30	36
200	5	8	12	17	22	29	36	44	54	64
	2	4	6	9	12	17	22	27	34	41
250	6	9	13	18	25	32	40	49	60	71
	2	4	7	10	14	18	23	30	37	44
300	6	10	15	20	27	35	44	54	65	78
	3	5	7	11	15	20	25	32	39	48
350	7	11	16	22	29	37	47	58	70	83
	3	5	8	11	16	21	27	34	42	51
400	7	11	17	23	31	40	50	62	75	89
	3	5	8	12	16	22	28	35	44	53
450	8	12	18	25	33	42	53	65	79	94
	3	6	9	12	17	23	29	37	46	56
500	8	13	19	26	35	44	56	69	83	99
	3	6	9	13	18	24	31	39	48	58
550	9	14	20	27	36	47	58	72	87	104
	3	6	9	13	19	25	32	40	49	60
600	9	14	21	28	38	49	61	75	91	108
	4	6	10	14	19	25	33	41	51	62
650	9	15	21	30	39	51	64	78	94	112
	4	6	10	14	20	26	34	43	53	64
700	10	15	22	31	41	53	66	81	98	117
	4	7	10	15	20	27	35	44	54	65
750	10	16	23	32	42	54	68	84	101	121
	4	7	10	15	21	28	36	45	55	67
800	10	16	24	33	44	56	70	87	104	124
	4	7	11	16	21	28	37	46	57	69
850	11	17	25	34	45	58	73	89	108	128
	4	7	11	16	22	29	37	47	58	70
900	11	17	25	35	46	60	75	92	111	132
	4	7	11	16	22	30	38	48	59	72
950	11	18	26	36	48	61	77	94	114	135
	4	7	11	17	23	30	39	49	60	73
1000	12	18	27	37	49	63	79	97	117	139
	4	8	12	17	23	31	40	50	62	75

TABLE 19 EFFECTIVE VELOCITY PRESSURES FOR CITY EXPOSURE AND 2% DAMPING

Height (ft)	Basic Wind Velocity (mph) 50	60	70	80	90	100	110	120	130	140
50	2	4	5	7	10	13	16	20	24	29
	0	1	2	3	5	6	8	11	13	17
100	3	5	7	10	14	18	22	27	33	39
	1	2	3	5	7	9	12	15	18	22
150	4	6	9	13	17	22	27	33	40	48
	1	2	4	6	8	11	14	18	22	27
200	5	7	11	15	19	25	31	38	46	55
	2	3	4	7	9	12	16	20	25	30
250	5	8	12	16	22	28	35	43	51	61
	2	3	5	7	10	13	17	22	27	33
300	6	9	13	18	24	30	38	47	56	67
	2	3	5	8	11	14	19	24	29	35
350	6	10	14	19	26	33	41	50	60	72
	2	4	6	8	12	15	20	25	31	38
400	7	10	15	21	27	35	44	54	65	77
	2	4	6	9	12	16	21	26	33	40
450	7	11	16	22	29	37	46	57	69	81
	2	4	6	9	13	17	22	28	34	41
500	7	12	17	23	31	39	49	60	72	86
	2	4	7	10	13	18	23	29	36	43
550	8	12	18	24	32	41	51	63	76	90
	3	4	7	10	14	19	24	30	37	45
600	8	13	19	26	34	43	54	66	79	94
	3	5	7	10	14	19	25	31	38	46
650	9	13	19	27	35	45	56	69	83	98
	3	5	7	11	15	20	26	32	39	48
700	9	14	20	28	37	47	58	71	86	102
	3	5	8	11	15	20	26	33	41	49
750	9	15	21	29	38	48	60	74	89	105
	3	5	8	11	16	21	27	34	42	51
800	10	15	22	30	39	50	62	76	92	109
	3	5	8	12	16	22	28	35	43	52
850	10	16	22	31	40	52	64	79	95	112
	3	5	8	12	17	22	28	36	44	53
900	10	16	23	32	42	53	66	81	98	116
	3	6	9	12	17	23	29	36	45	54
950	11	17	24	33	43	55	68	84	100	119
	3	6	9	13	17	23	30	37	46	55
1000	11	17	25	34	44	56	70	86	103	122
	3	6	9	13	18	23	30	38	47	56

TABLE 20 EFFECTIVE VELOCITY PRESSURES FOR CITY EXPOSURE AND 5% DAMPING

Height (ft)	Basic Wind Velocity (mph) 30	60	70	80	90	100	110	120	130	140
50	2	3	4	6	8	11	13	16	20	23
	0	0	0	1	2	3	4	5	7	9
100	3	4	6	9	12	15	19	23	28	33
	0	0	1	2	3	5	7	9	11	14
150	3	5	8	11	14	18	23	28	34	40
	0	1	2	3	5	6	8	11	14	17
200	4	6	9	13	17	21	27	32	39	46
	0	1	2	4	5	7	10	13	16	19
250	5	7	10	14	19	24	30	36	44	52
	1	2	3	4	6	8	11	14	17	21
300	5	8	11	16	21	26	33	40	48	57
	1	2	3	5	7	9	12	15	19	23
350	6	9	12	17	22	29	35	43	52	61
	1	2	3	5	7	10	13	16	20	25
400	6	9	13	18	24	31	38	46	56	66
	1	2	4	6	8	11	14	18	22	26
450	6	10	14	19	26	33	40	49	59	70
	1	2	4	6	8	11	15	18	23	28
500	7	10	15	21	27	34	43	52	63	74
	1	3	4	6	9	12	15	19	24	29
550	7	11	16	22	28	36	45	55	66	78
	1	3	4	7	9	12	16	20	25	30
600	8	12	17	23	30	38	47	57	69	81
	2	3	5	7	10	13	17	21	26	32
650	8	12	17	24	31	40	49	60	72	85
	2	3	5	7	10	13	17	22	27	33
700	8	13	18	25	32	41	51	62	75	88
	2	3	5	7	10	14	18	22	28	34
750	9	13	19	26	34	43	53	65	78	92
	2	3	5	8	11	14	18	23	29	35
800	9	14	20	27	35	44	55	67	80	95
	2	3	5	8	11	15	19	24	29	36
850	9	14	20	28	36	46	57	69	83	98
	2	3	6	8	11	15	19	24	30	36
900	9	15	21	28	37	47	59	72	86	101
	2	4	6	8	12	15	20	25	31	37
950	10	15	22	29	38	49	61	74	88	104
	2	4	6	9	12	16	20	26	32	38
1000	10	15	22	30	40	50	62	76	91	107
	2	4	6	9	12	16	21	26	32	39

TABLE 21 EFFECTIVE VELOCITY PRESSURES FOR SUBURBAN EXPOSURE AND 1% DAMPING

Height (ft)	Basic Wind Velocity (mph) 50	60	70	80	90	100	110	120	130	140
50	5	8	12	17	23	29	37	46	56	66
	2	4	6	9	12	17	22	27	34	41
100	7	11	16	22	29	37	47	58	70	84
	3	5	8	12	16	21	28	35	43	52
150	8	12	18	25	33	43	54	67	81	97
	3	6	9	13	18	25	32	40	49	60
200	9	14	20	28	37	48	60	74	89	107
	4	7	10	15	20	27	35	44	54	66
250	9	15	22	30	40	51	65	80	96	115
	4	7	11	16	22	29	38	47	58	71
300	10	16	23	32	43	55	69	85	103	122
	4	8	12	17	23	31	40	50	62	75
350	11	17	24	34	45	58	73	89	108	129
	5	8	12	18	25	32	42	53	65	78
400	11	18	26	35	47	61	76	94	113	135
	5	8	13	19	26	34	44	55	67	82
450	12	18	27	37	49	63	79	97	118	141
	5	9	13	19	27	35	45	57	70	85
500	12	19	28	38	51	65	82	101	122	146
	5	9	14	20	27	36	47	59	72	87
550	12	20	29	40	53	68	85	104	126	150
	6	9	14	21	28	37	48	60	74	90
600	13	20	30	41	54	70	88	108	130	155
	6	10	15	21	29	38	49	62	76	92
650	13	21	30	42	56	72	90	111	134	159
	6	10	15	22	30	39	51	64	78	95
700	14	21	31	43	57	74	92	113	137	163
	6	10	16	22	31	40	52	65	80	97
750	14	22	32	44	59	75	95	116	140	167
	6	10	16	23	31	41	53	66	82	99
800	14	22	33	45	60	77	97	119	144	171
	6	11	16	23	32	42	54	68	83	101
850	15	23	33	46	61	79	99	121	147	175
	6	11	16	24	32	43	55	69	85	102
900	15	23	34	47	62	80	101	124	149	178
	7	11	17	24	33	44	56	70	86	104
950	15	24	35	48	64	82	103	126	152	181
	7	11	17	24	33	44	57	71	87	106
1000	15	24	35	49	65	83	104	128	155	184
	7	11	17	25	34	45	58	72	89	107

TABLE 22 EFFECTIVE VELOCITY PRESSURES FOR SUBURBAN EXPOSURE AND 2% DAMPING

Height (ft)	50	60	70	80	90	100	110	120	130	140
	Basic Wind Velocity (mph)									
50	5	8	11	15	20	26	33	40	48	58
	1	3	4	6	9	12	16	20	25	31
100	6	10	14	20	26	33	42	51	62	73
	2	4	6	9	12	16	21	27	33	40
150	7	11	16	23	30	38	48	59	71	84
	3	5	7	10	14	19	25	31	38	46
200	8	13	18	25	33	43	53	65	79	93
	3	5	8	12	16	21	27	34	42	51
250	9	14	20	27	36	46	58	71	85	101
	3	6	9	13	17	23	30	37	46	55
300	9	15	21	29	38	49	61	75	91	108
	4	6	9	14	19	25	32	40	49	59
350	10	15	22	31	40	52	65	79	96	113
	4	6	10	14	20	26	33	42	51	62
400	10	16	23	32	42	54	68	83	100	119
	4	7	10	15	21	27	35	43	53	64
450	11	17	24	34	44	57	71	87	104	124
	4	7	11	16	21	28	36	45	55	67
500	11	18	25	35	46	59	73	90	108	129
	4	7	11	16	22	29	37	47	57	69
550	12	18	26	36	48	61	76	93	112	133
	5	8	12	17	23	30	38	48	59	71
600	12	19	27	37	49	63	78	96	115	137
	5	8	12	17	23	31	40	49	61	73
650	12	19	28	38	50	65	81	99	119	141
	5	8	12	18	24	32	41	51	62	75
700	13	20	29	39	52	66	83	101	122	145
	5	8	13	18	25	32	42	52	64	77
750	13	20	29	40	53	68	85	104	125	148
	5	9	13	19	25	33	42	53	65	78
800	13	21	30	41	54	70	87	106	128	151
	5	9	13	19	26	34	43	54	66	80
850	14	21	31	42	56	71	89	108	130	155
	5	9	14	19	26	35	44	55	68	81
900	14	22	31	43	57	72	90	111	133	158
	5	9	14	20	27	35	45	56	69	83
950	14	22	32	44	58	74	92	113	136	161
	6	9	14	20	27	36	46	57	70	84
1000	14	23	33	45	59	75	94	115	138	164
	6	9	14	20	28	36	46	58	71	86

TABLE 23 EFFECTIVE VELOCITY PRESSURES FOR SUBURBAN EXPOSURE AND 5% DAMPING

Height (ft)	Basic Wind Velocity (mph) 50	60	70	80	90	100	110	120	130	140
50	4	6	9	13	17	22	27	34	40	48
	0	0	1	3	4	6	9	11	14	18
100	5	9	12	17	22	28	35	43	52	61
	1	2	3	5	7	10	13	17	21	26
150	6	10	14	20	26	33	41	50	60	71
	1	3	4	7	9	12	16	20	25	31
200	7	11	16	22	29	37	46	56	67	79
	2	3	5	8	11	14	18	23	28	34
250	8	12	17	24	31	40	49	60	72	86
	2	4	6	8	12	16	20	25	31	38
300	8	13	19	25	33	43	53	64	77	91
	2	4	6	9	13	17	22	27	33	40
350	9	14	20	27	35	45	56	68	82	97
	3	4	7	10	13	18	23	29	35	43
400	9	14	21	28	37	47	59	72	86	101
	3	5	7	10	14	19	24	30	37	45
450	10	15	22	30	39	49	61	75	89	106
	3	5	8	11	15	20	25	31	39	47
500	10	16	23	31	40	51	64	78	93	110
	3	5	8	11	15	20	26	33	40	48
550	10	16	23	32	42	53	66	80	96	114
	3	5	8	12	16	21	27	34	41	50
600	11	17	24	33	43	55	68	83	99	117
	3	6	8	12	17	22	28	35	43	51
650	11	17	25	34	44	56	70	85	102	121
	3	6	9	13	17	22	29	36	44	53
700	11	18	25	35	46	58	72	88	105	124
	4	6	9	13	18	23	29	37	45	54
750	12	18	26	36	47	59	74	90	108	127
	4	6	9	13	18	24	30	38	46	55
800	12	19	27	37	48	61	76	92	110	130
	4	6	9	14	18	24	31	39	47	57
850	12	19	27	37	49	62	77	94	113	133
	4	6	10	14	19	25	32	39	48	58
900	13	20	28	38	50	64	79	96	115	136
	4	7	10	14	19	25	32	40	49	59
950	13	20	29	39	51	65	80	98	117	138
	4	7	10	14	20	26	33	41	50	60
1000	13	20	29	40	52	66	82	100	119	141
	4	7	10	15	20	26	33	41	51	61

TABLE 24 EFFECTIVE VELOCITY PRESSURES FOR OPEN COUNTRY EXPOSURE AND 1% DAMPING

Height (ft)	Basic Wind Velocity (mph) 50	60	70	80	90	100	110	120	130	140
50	8	13	19	27	36	46	58	71	86	103
	3	5	9	13	18	24	31	39	49	60
100	10	16	23	32	43	55	69	85	103	123
	4	7	11	16	22	30	39	49	60	73
150	11	18	26	36	48	61	77	95	115	137
	5	8	13	18	25	34	44	55	68	82
200	12	19	28	39	51	66	83	102	124	147
	5	9	14	20	28	37	47	60	74	89
250	13	20	29	41	54	70	88	108	131	156
	6	10	15	22	30	39	51	63	78	95
300	13	21	31	43	57	73	92	113	137	163
	6	10	16	23	31	41	53	67	82	99
350	14	22	32	44	59	76	96	118	142	170
	6	11	16	24	33	43	55	69	86	104
400	14	23	33	46	61	79	99	122	147	176
	7	11	17	25	34	45	57	72	89	107
450	15	23	34	47	63	81	102	125	152	181
	7	12	18	25	35	46	59	74	91	110
500	15	24	35	49	65	83	105	129	156	186
	7	12	18	26	36	47	61	76	94	113
550	16	25	36	50	66	85	107	132	159	190
	7	12	19	27	37	49	62	78	96	116
600	16	25	37	51	68	87	110	135	163	194
	7	13	19	27	38	50	64	80	98	119
650	16	26	38	52	69	89	112	137	166	198
	8	13	20	28	38	51	65	81	100	121
700	17	26	38	53	70	91	114	140	169	202
	8	13	20	29	39	52	66	83	102	123
750	17	27	39	54	72	92	116	142	172	205
	8	13	20	29	40	53	67	84	104	126
800	17	27	40	55	73	94	118	145	175	209
	8	14	21	30	40	53	68	86	105	128
850	18	28	40	56	74	95	119	147	178	212
	8	14	21	30	41	54	70	87	107	129
900	18	28	41	56	75	97	121	149	180	215
	8	14	21	30	42	55	71	88	100	131
950	18	28	41	57	76	98	123	151	183	218
	8	14	22	31	42	56	71	90	110	133
1000	18	29	42	58	77	99	124	153	185	220
	9	14	22	31	43	56	72	91	111	135

TABLE 25 EFFECTIVE VELOCITY PRESSURES FOR OPEN COUNTRY EXPOSURE AND 2% DAMPING

Height (ft)	Basic Wind Velocity (mph) 50	60	70	80	90	100	110	120	130	140
50	8	12	18	24	32	42	52	64	77	91
	2	4	6	10	13	18	24	30	37	46
100	9	15	21	29	39	50	62	76	92	109
	3	6	9	13	18	24	31	39	48	58
150	10	16	24	33	43	55	69	85	102	121
	4	7	10	15	20	27	35	44	54	65
200	11	18	26	35	47	60	75	91	110	131
	4	7	11	16	23	30	38	48	59	71
250	12	19	27	37	49	63	79	97	117	138
	5	8	12	18	24	32	41	51	63	76
300	13	20	28	39	52	66	83	101	122	145
	5	8	13	19	26	34	43	54	66	80
350	13	20	30	41	54	69	86	105	127	151
	5	9	14	20	27	35	45	56	69	83
400	14	21	31	42	56	71	89	109	131	156
	6	9	14	20	28	37	47	58	72	86
450	14	22	32	43	57	73	92	112	135	161
	6	10	15	21	29	38	48	60	74	89
500	14	22	32	45	59	75	94	115	139	165
	6	10	15	22	30	39	50	62	76	92
550	15	23	33	46	60	77	97	118	142	169
	6	10	16	22	30	40	51	64	78	94
600	15	23	34	47	62	79	99	121	146	173
	6	11	16	23	31	41	52	65	80	96
650	15	24	35	48	63	81	101	123	149	176
	7	11	16	23	32	42	53	67	82	98
700	16	24	35	49	64	82	103	126	151	180
	7	11	17	24	32	43	54	68	83	100
750	16	25	36	49	65	84	104	128	154	183
	7	11	17	24	33	43	55	69	85	102
800	16	25	37	50	66	85	106	130	157	186
	7	11	17	25	34	44	56	70	86	104
850	16	26	37	51	67	86	108	132	159	189
	7	12	18	25	34	45	57	71	87	105
900	17	26	38	52	68	88	109	134	161	191
	7	12	18	26	35	46	58	72	89	107
950	17	26	38	53	69	89	111	136	163	194
	7	12	18	26	35	46	59	73	90	108
1000	17	27	39	53	70	90	112	137	165	196
	7	12	18	26	36	47	60	74	91	110

TABLE 26 EFFECTIVE VELOCITY PRESSURES FOR OPEN COUNTRY EXPOSURE AND 5% DAMPING

Height (ft)	Basic Wind Velocity (mph) 50	60	70	80	90	100	110	120	130	140
50	6	11	16	21	28	36	45	55	66	78
	0	1	3	5	7	10	14	18	23	28
100	8	13	19	26	34	43	54	66	79	94
	2	3	5	8	11	15	20	26	32	39
150	9	15	21	29	38	48	60	73	88	104
	2	4	7	10	14	19	24	30	37	45
200	10	16	23	31	41	52	65	79	95	112
	3	5	8	11	16	21	27	34	41	50
250	11	17	24	33	43	55	69	84	101	119
	3	6	9	12	17	23	29	36	45	54
300	11	18	25	35	46	58	72	88	105	125
	4	6	9	13	18	24	31	39	47	57
350	12	18	26	36	47	60	75	92	110	130
	4	6	10	14	19	25	33	41	50	60
400	12	19	27	37	49	63	78	95	114	134
	4	7	10	15	20	27	34	42	52	63
450	13	20	28	39	51	64	80	98	117	138
	4	7	11	15	21	28	35	44	54	65
500	13	20	29	40	52	66	82	100	120	142
	4	7	11	16	22	29	36	45	56	67
550	13	21	30	41	53	68	84	103	123	146
	5	8	12	17	22	29	38	47	57	69
600	14	21	30	42	55	69	86	105	126	149
	5	8	12	17	23	30	39	48	59	70
650	14	22	31	42	56	71	88	107	129	152
	5	8	12	17	24	31	39	49	60	72
700	14	22	32	43	57	72	90	109	131	155
	5	8	13	18	24	32	40	50	61	74
750	15	22	32	44	58	74	91	111	133	158
	5	8	13	18	25	32	41	51	62	75
800	15	23	33	45	59	75	93	113	136	160
	5	9	13	19	25	33	42	52	64	76
850	15	23	33	45	60	76	94	115	138	163
	5	9	13	19	26	33	43	53	65	78
900	15	24	34	46	61	77	96	117	140	165
	5	9	14	19	26	34	43	54	66	79
950	15	24	34	47	61	78	97	118	142	168
	6	9	14	20	26	35	44	55	67	80
1000	16	24	35	47	62	79	98	120	144	170
	6	9	14	20	27	35	45	55	68	81

TABLE 27 MASS AND STIFFNESS FOR 25 STORY BUILDING(*)

	MASS Kps	STIFFNESS 10^3 Kps/in.
25	492	0.3
24	3082	4.4
23	4175	5.3
22	2973	7.2
21	2957	7.3
20	2474	8.7
19	3170	10.1
18	3170	10.1
17	3215	13.3
16	3259	14.1
15	3259	14.1
14	3299	16.3
13	3436	19.5
12	3436	19.5
11	3465	21.0
10	3693	21.0
9	3443	21.0
8	3555	23.2
7	3820	31.8
6	3820	31.9
5	3848	33.4
4	4097	39.8
3	4097	39.5
2	4368	42.4
1	4635	28.1

* After Blume, Newmark and Corning (1961)

TABLE 28 MASS AND STIFFNESS FOR 8 STORY BUILDING(*)

	STIFFNESS Kps/in.	MASS Kps sec^2/in.
1	4005	2.38
2	3483	2.38
3	3606	2.38
4	3493	2.38
5	3914	2.38
6	3373	2.38
7	3045	2.38
8	2409	4.24

* After Nielsen (1968)

TABLE 29 PRINCIPAL PROPERTIES OF JOHN HANCOCK CENTER

Height	
Cross Section:	
Base	165 x 265 ft.
Top	100 x 160 ft.
Frequency E-W (Observed)	0.21 hertz
Damping E-W (Observed)	0.4%

TABLE 30 MEAN PRESSURES AND DISPLACEMENTS FOR JOHN HANCOCK CENTER

		July 25	July 27
Velocity at top		59 ft/sec	42 ft/sec
P_o		4.14 psi	2.16 psi
Mean	Linear mode	0.51 inch	0.26 inch
Deflection	Cantilever mode	0.49 inch	0.25 inch

TABLE 31 RESONANT RESPONSE AT TOP OF THE JOHN HANCOCK CENTER

Velocity	Deflection at top of Structure (inches)		
ft/sec	Linear Mode	Cantilever Mode	Measured
42	0.031	0.030	0.017
59	0.079	0.076	0.042

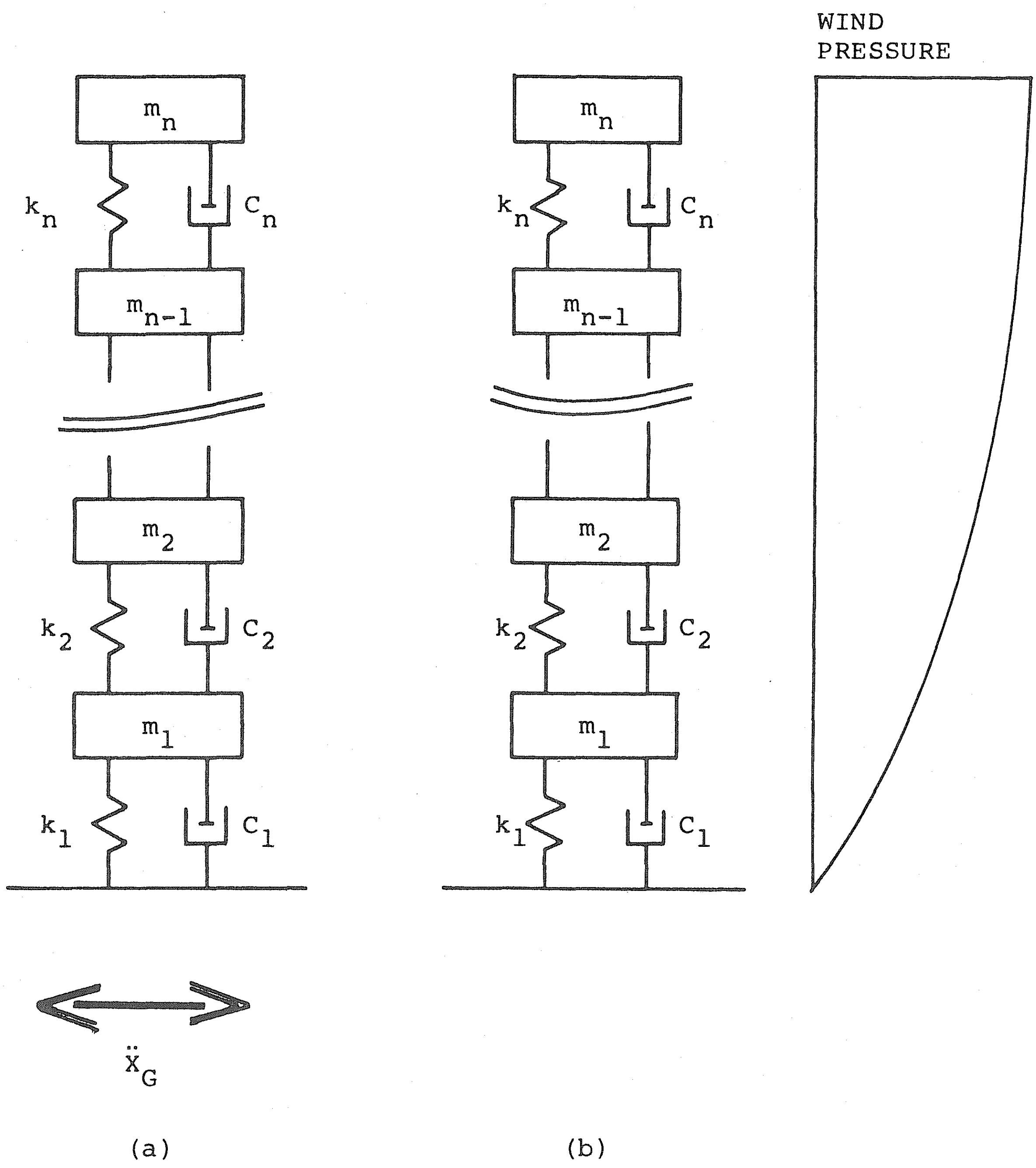


FIG. 1 TYPICAL IDEALIZED BUILDING

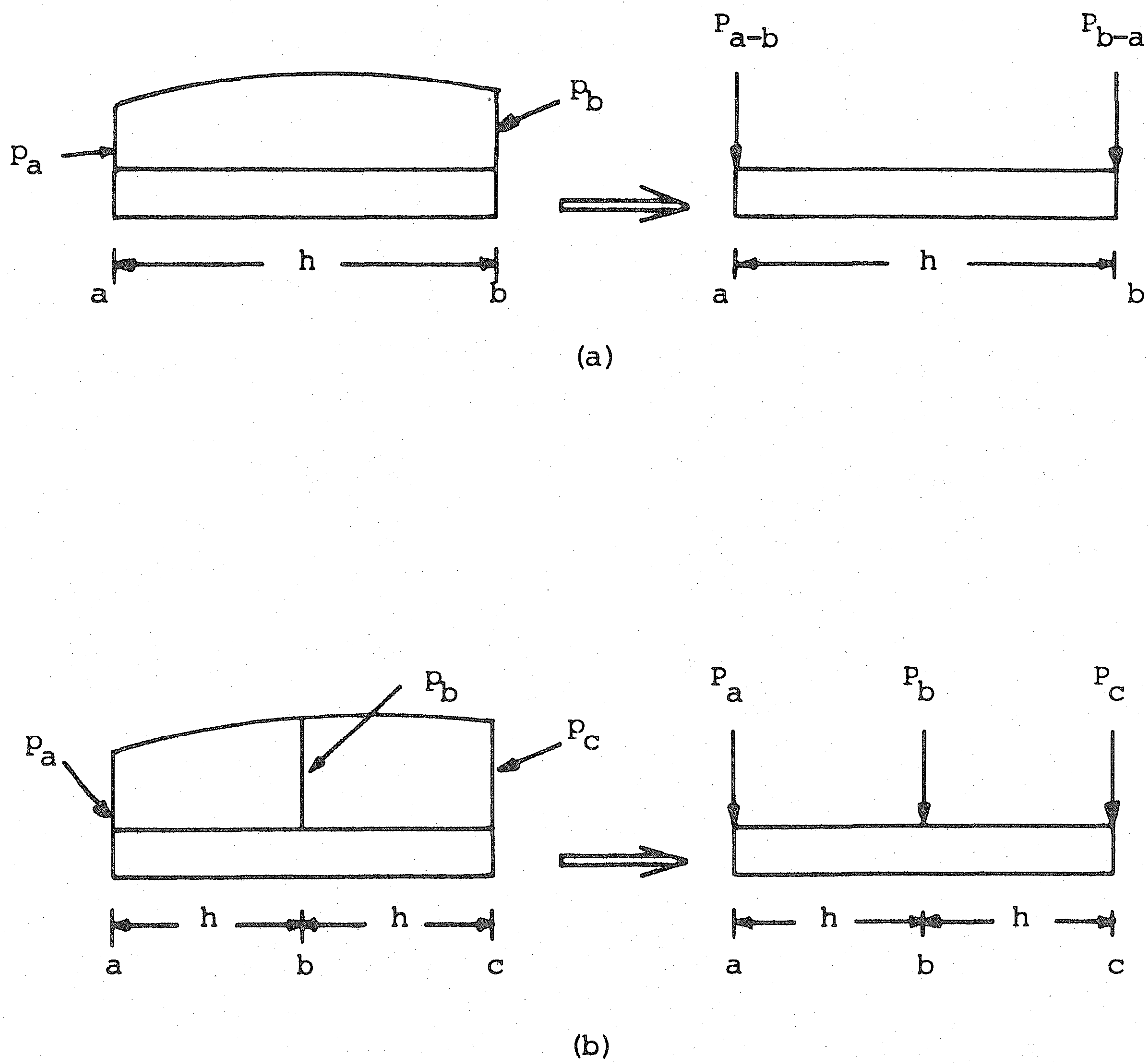
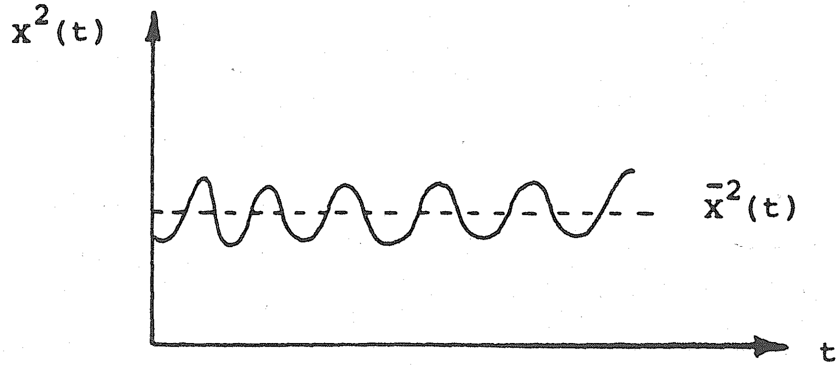
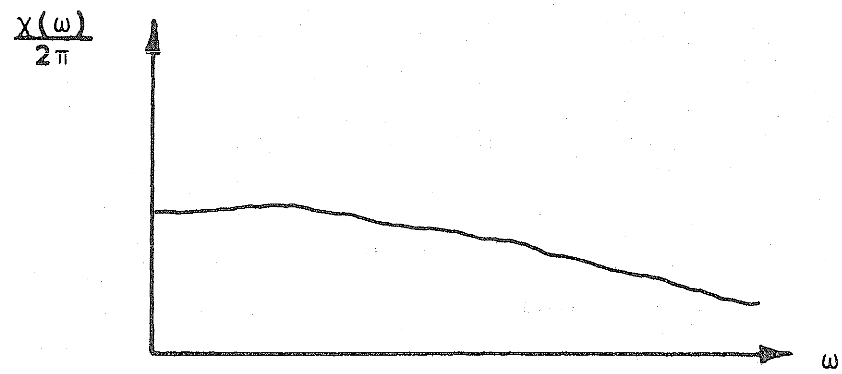


FIG. 2 EQUIVALENT CONCENTRATED FORCES



$$\bar{x}^2(t) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$$



$$\bar{x}^2(t) = \frac{1}{2\pi} \int_{0}^{\infty} \chi(\omega)\, d\omega$$

FIG. 3 RELATIONSHIP BETWEEN THE MEAN SQUARE AND THE POWER SPECTRAL DENSITY FUNCTION

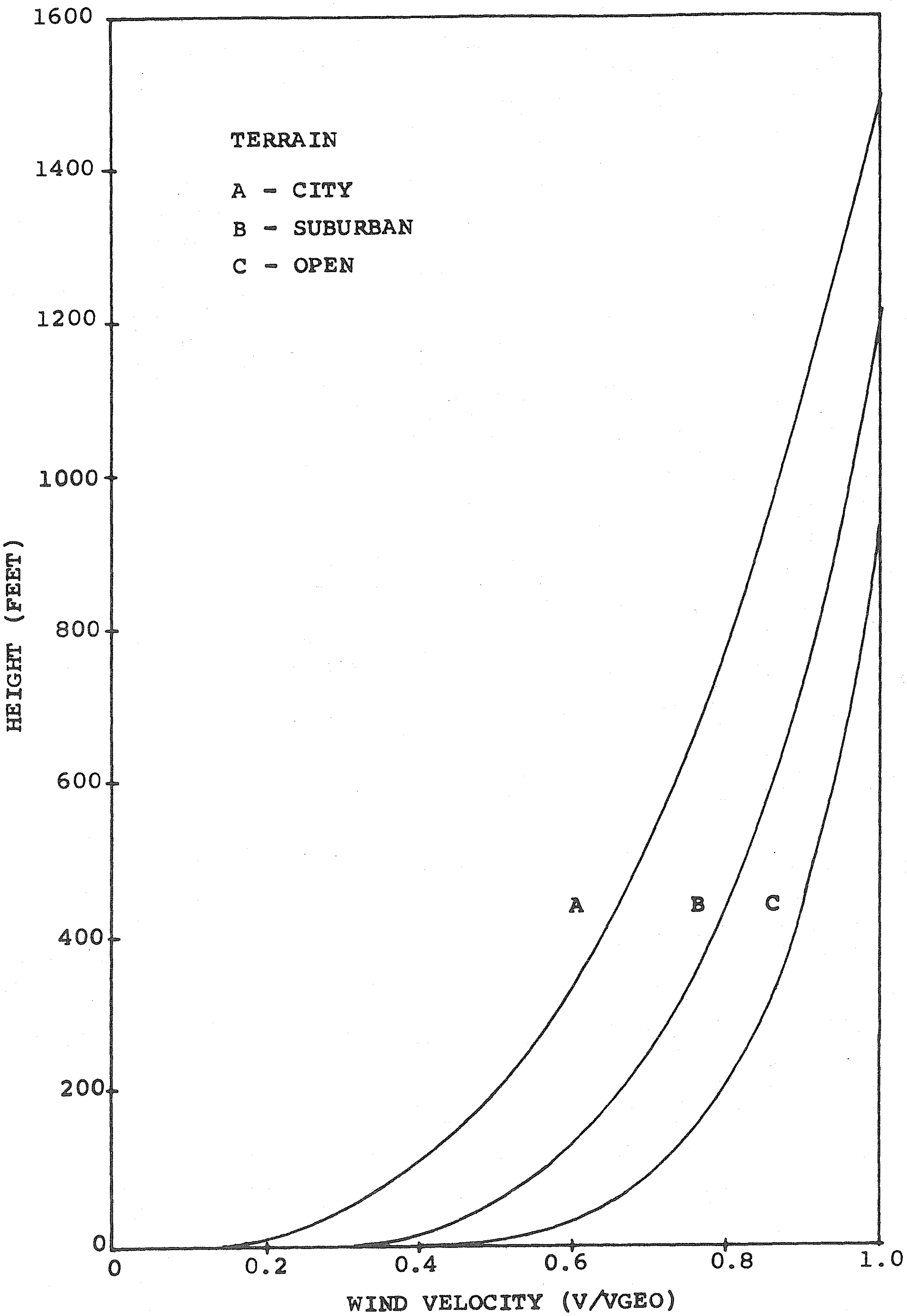


FIG. 4 VERTICAL DISTRIBUTION OF THE WIND VELOCITY

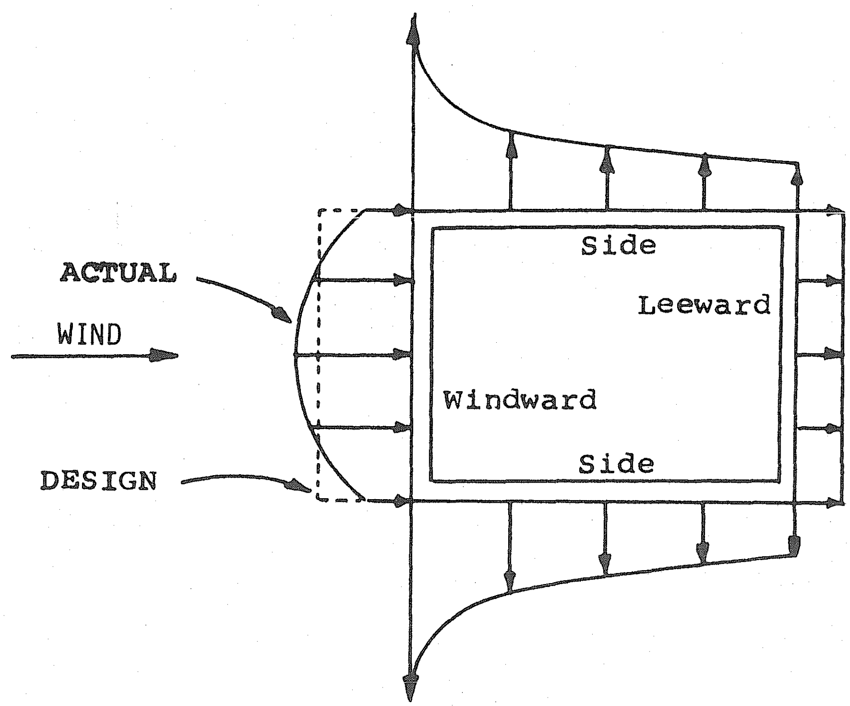


FIG. 5 HORIZONTAL DISTRIBUTION OF WIND PRESSURE

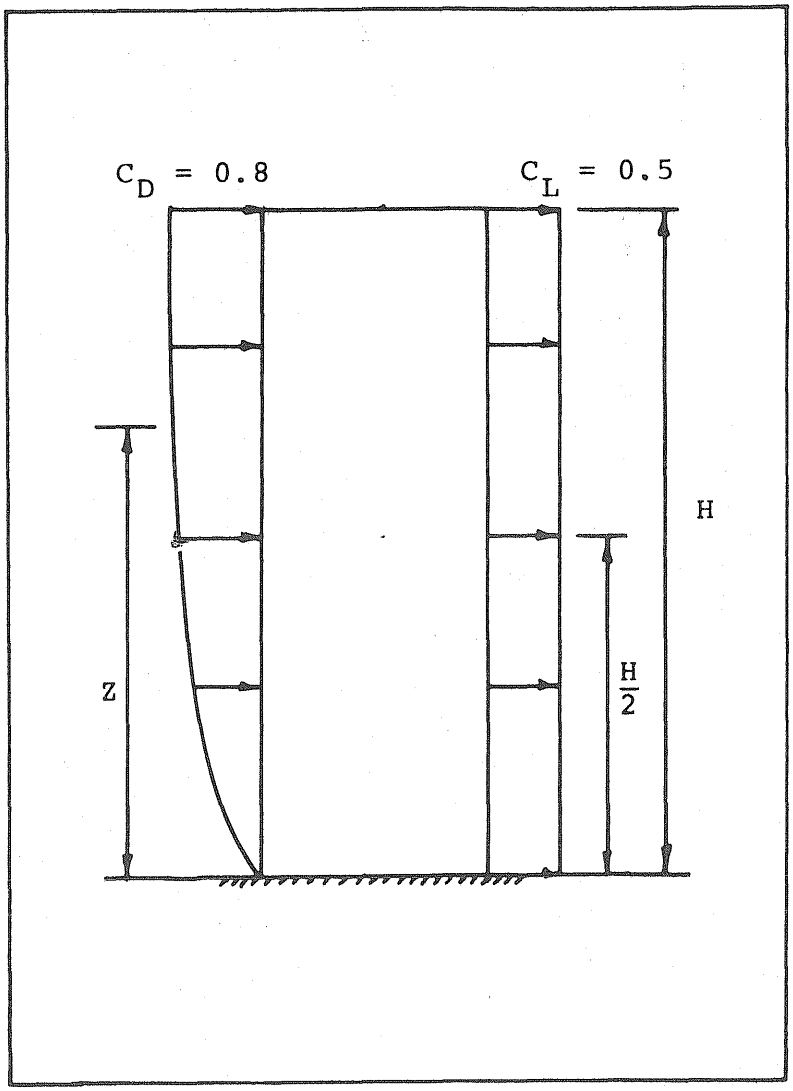


FIG. 6 DRAG COEFFICIENTS AND VERTICAL PRESSURE DISTRIBUTIONS (AS RECOMMENDED BY N.B.C.C.)

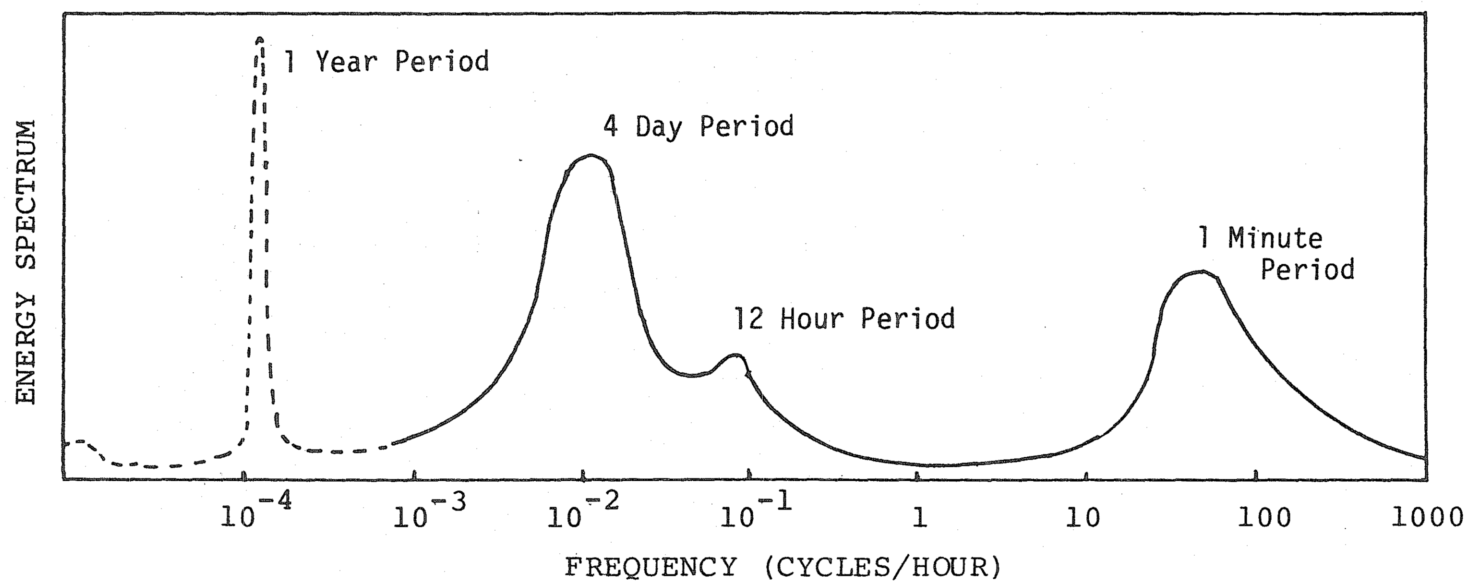


FIG. 7 SPECTRUM OF HORIZONTAL WIND SPEED NEAR THE GROUND FOR AN EXTENSIVE FREQUENCY RANGE (AFTER VAN DER HOVEN [1957])

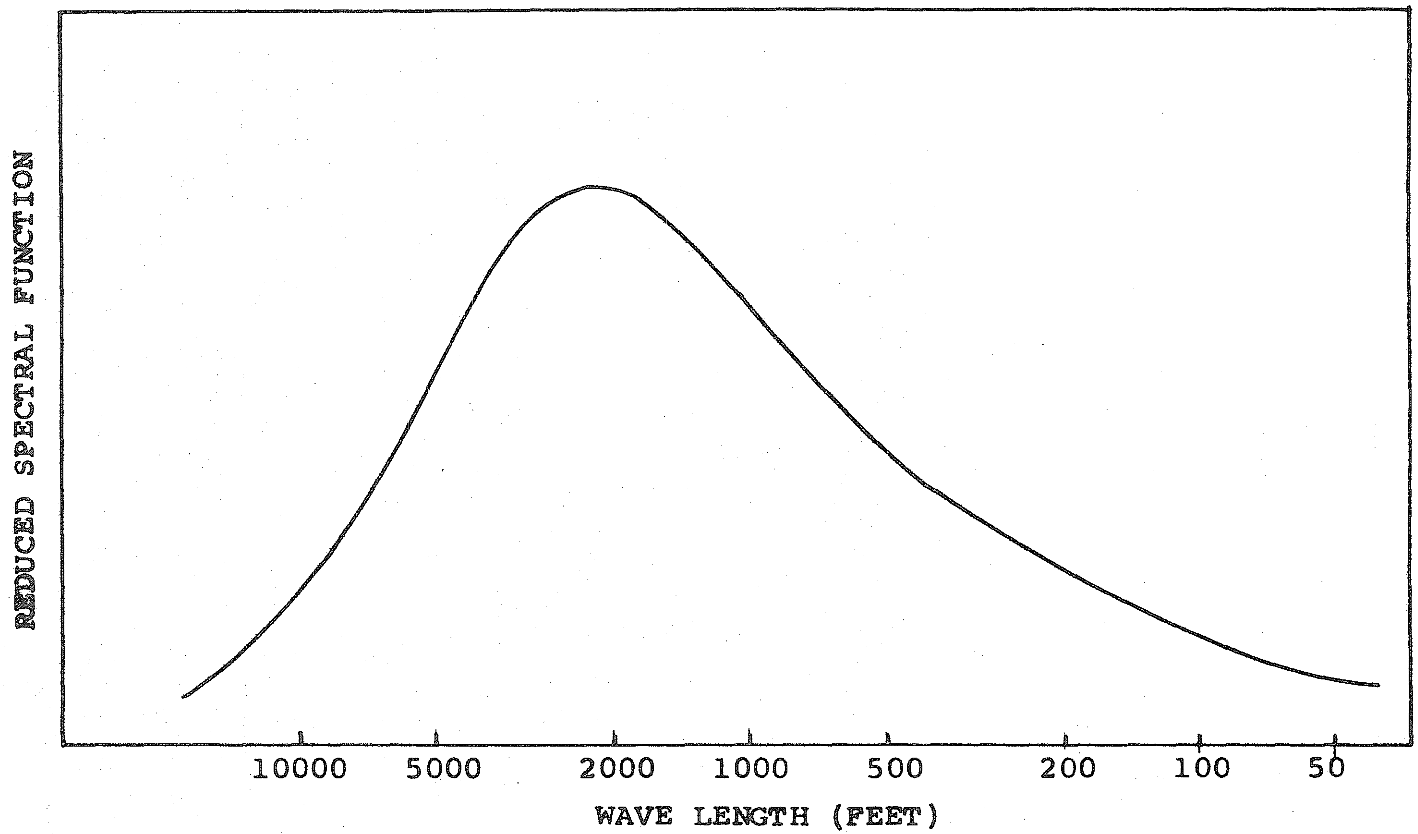


FIG. 8 SPECTRUM OF HORIZONTAL GUSTINESS IN HIGH WINDS (AFTER DAVENPORT [1961])

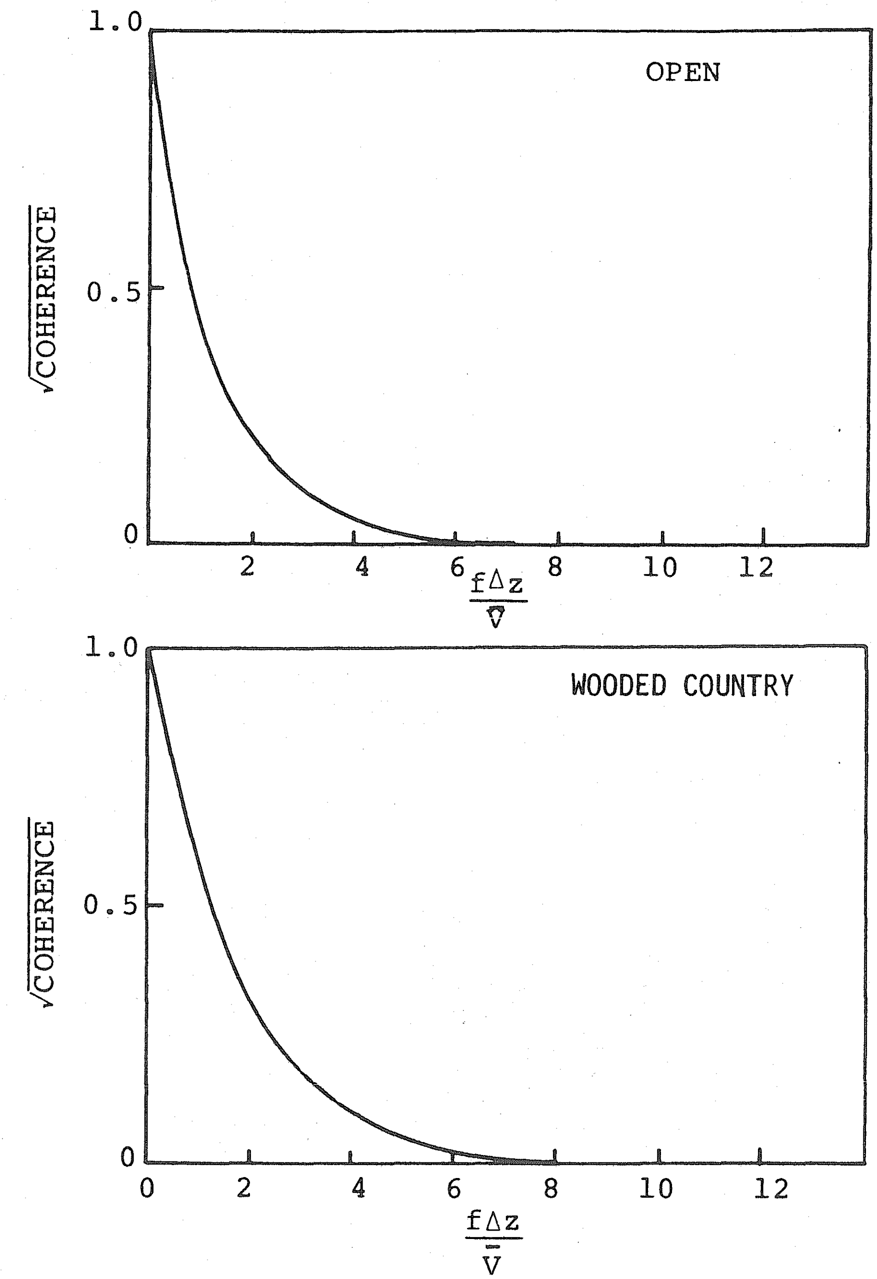


FIG. 9 ABSOLUTE VALUE OF THE CORRELATION (COHERENCE) OF WIND SPEED IN VERTICAL DIRECTION (AFTER DAVENPORT [1961])

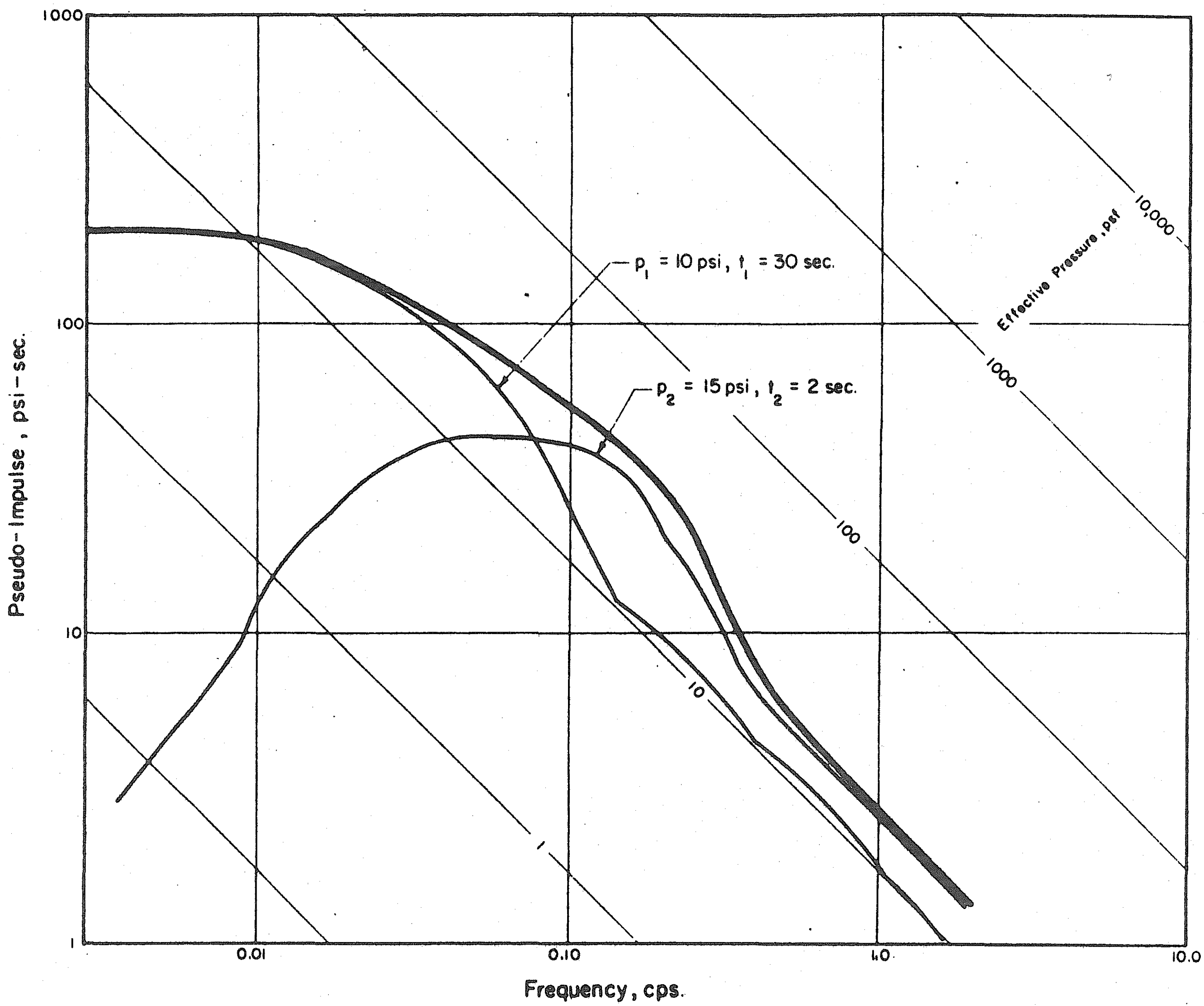


FIG. 10 COMBINED EFFECTIVE PRESSURE SPECTRUM FOR WIND LOADING (AFTER NEWMARK [1966])

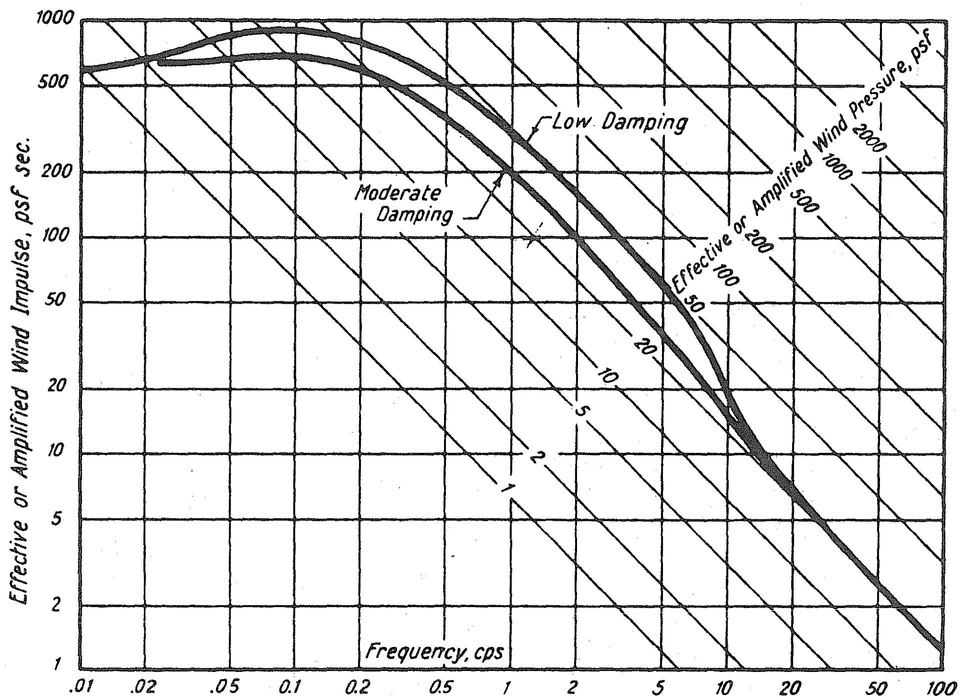


FIG. 11 POSSIBLE SKETCH OF WIND RESPONSE SPECTRUM (AFTER NEWMARK AND HALL [1968])

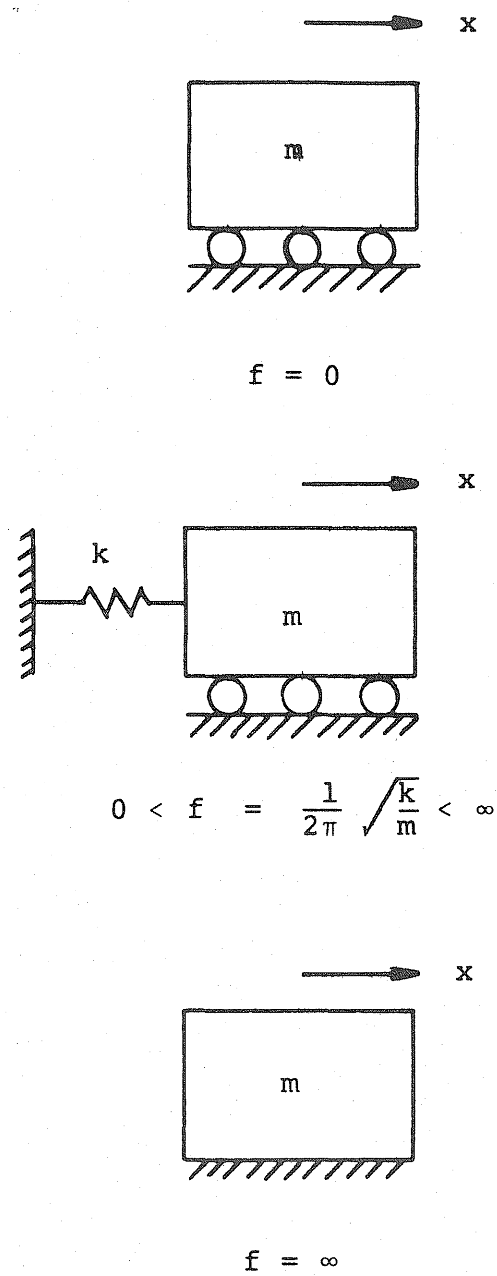


FIG. 12 FREQUENCIES AND THEIR CORRESPONDING BOUNDARY CONDITIONS

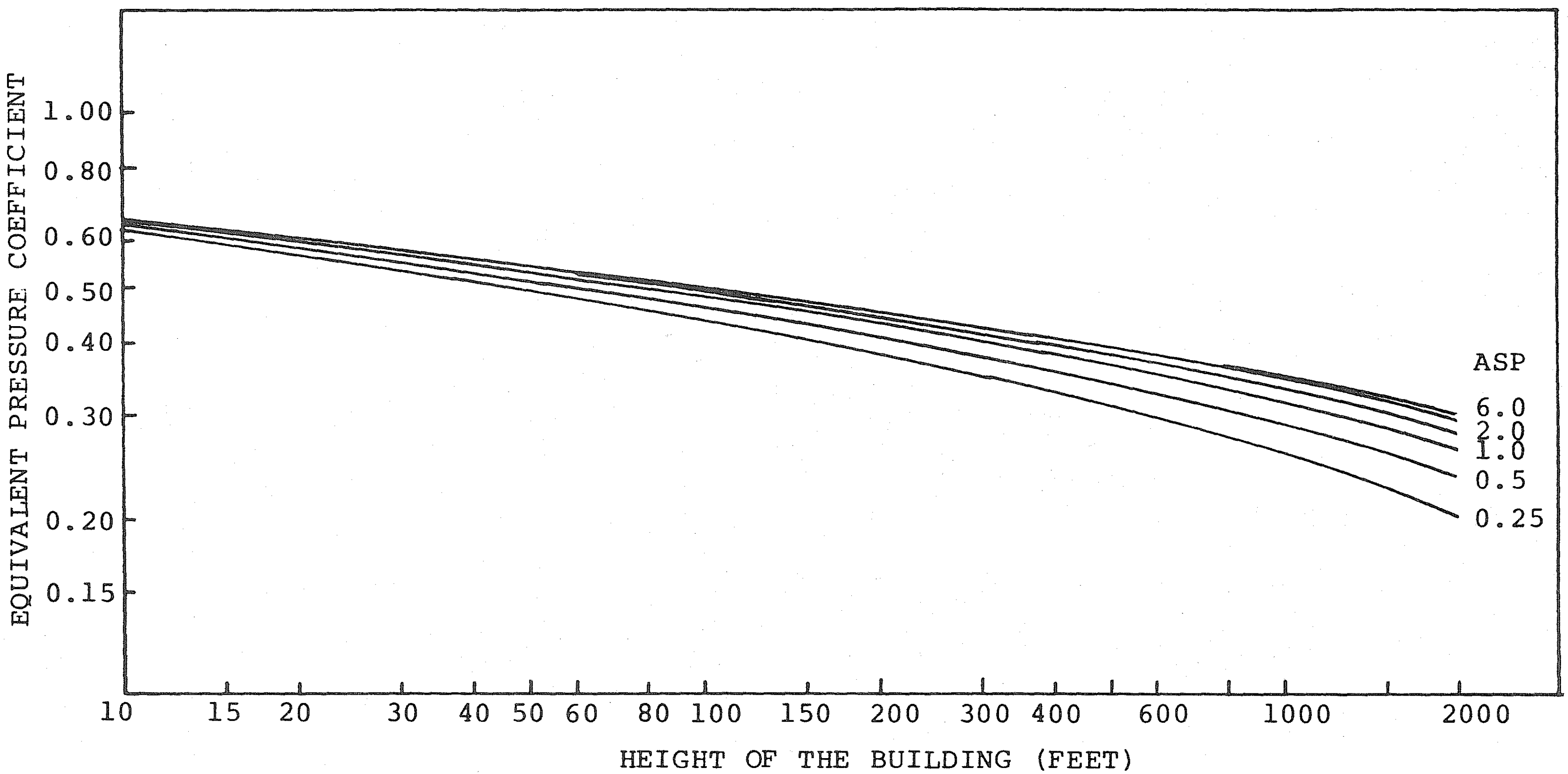


FIG. 13 EQUIVALENT PRESSURE COEFFICIENTS FOR OPEN EXPOSURE

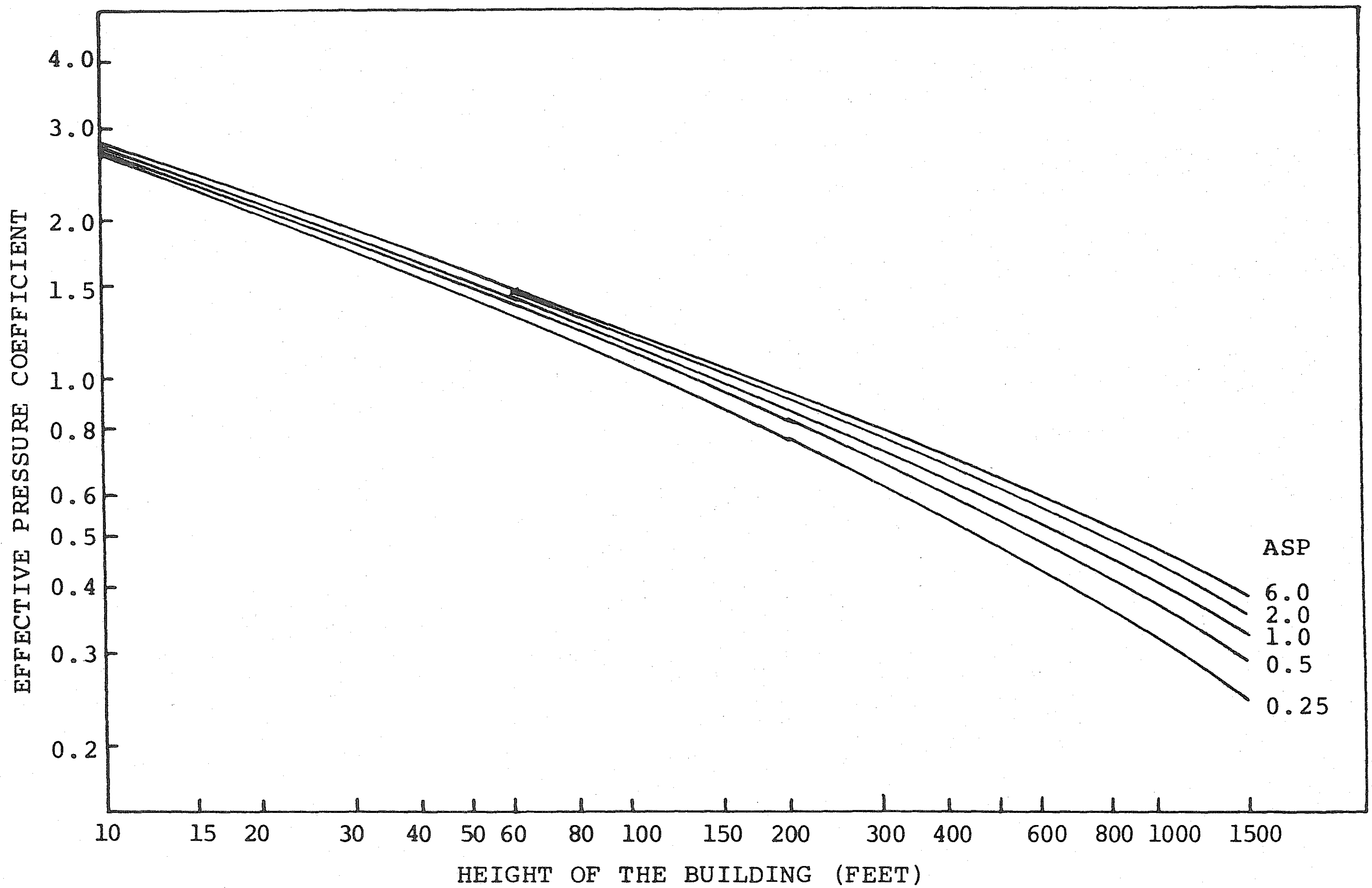


FIG. 14 EQUIVALENT PRESSURE COEFFICIENTS FOR CITY EXPOSURE

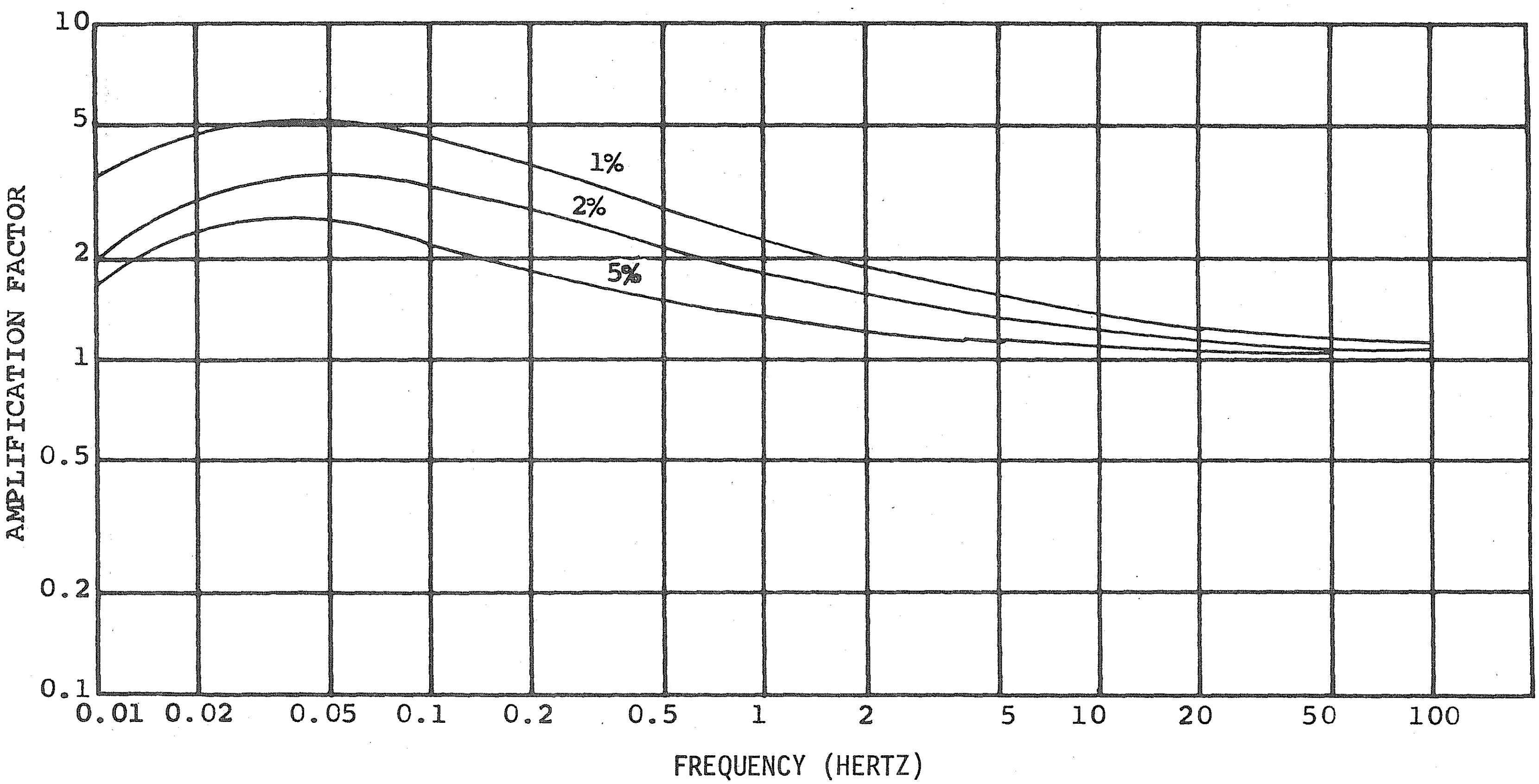


FIG. 15 WIND AMPLIFICATION FACTOR FOR A WIND VELOCITY OF 100 FT/SEC

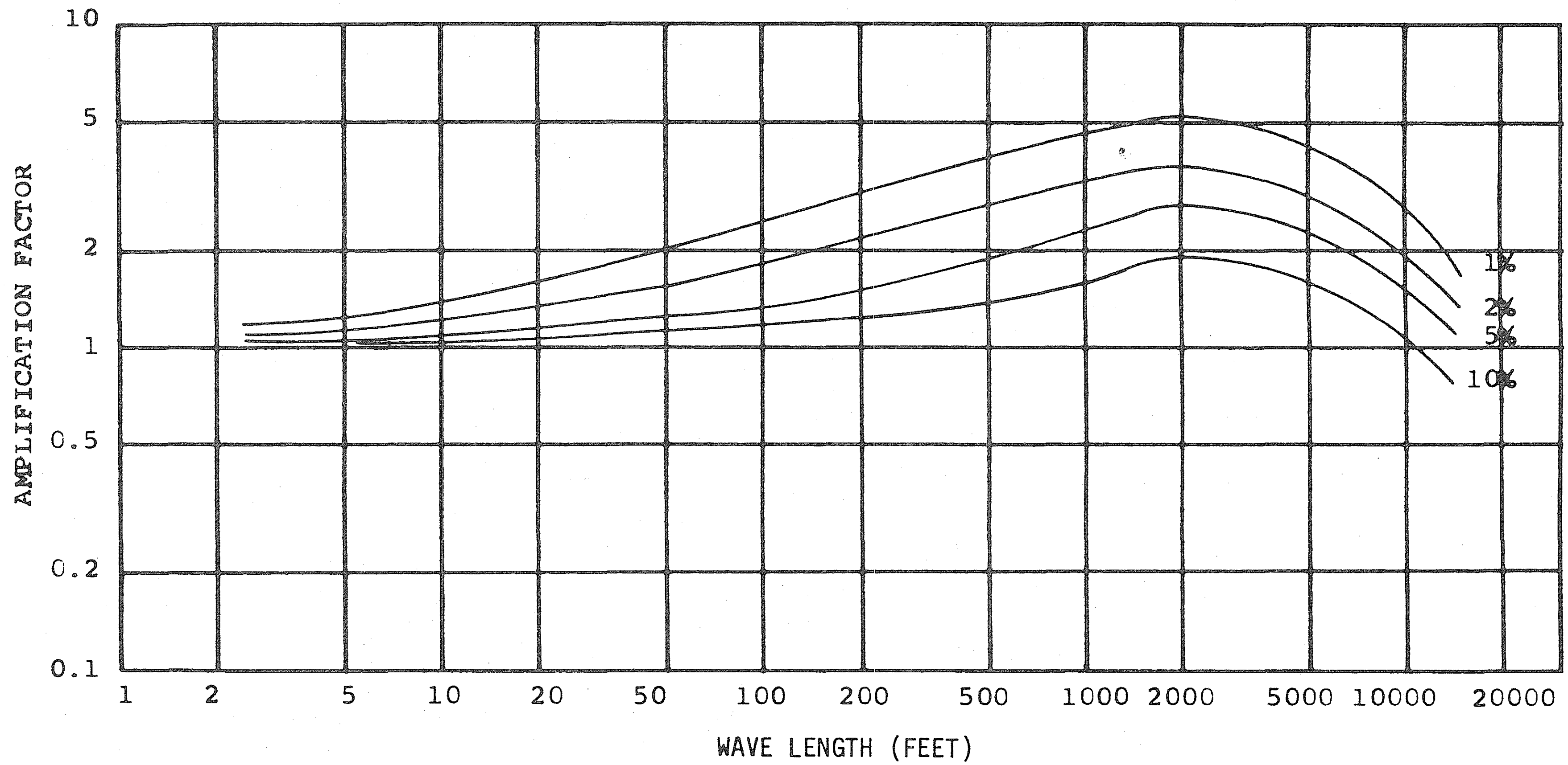


FIG. 16 GENERALIZED WIND AMPLIFICATION FACTORS

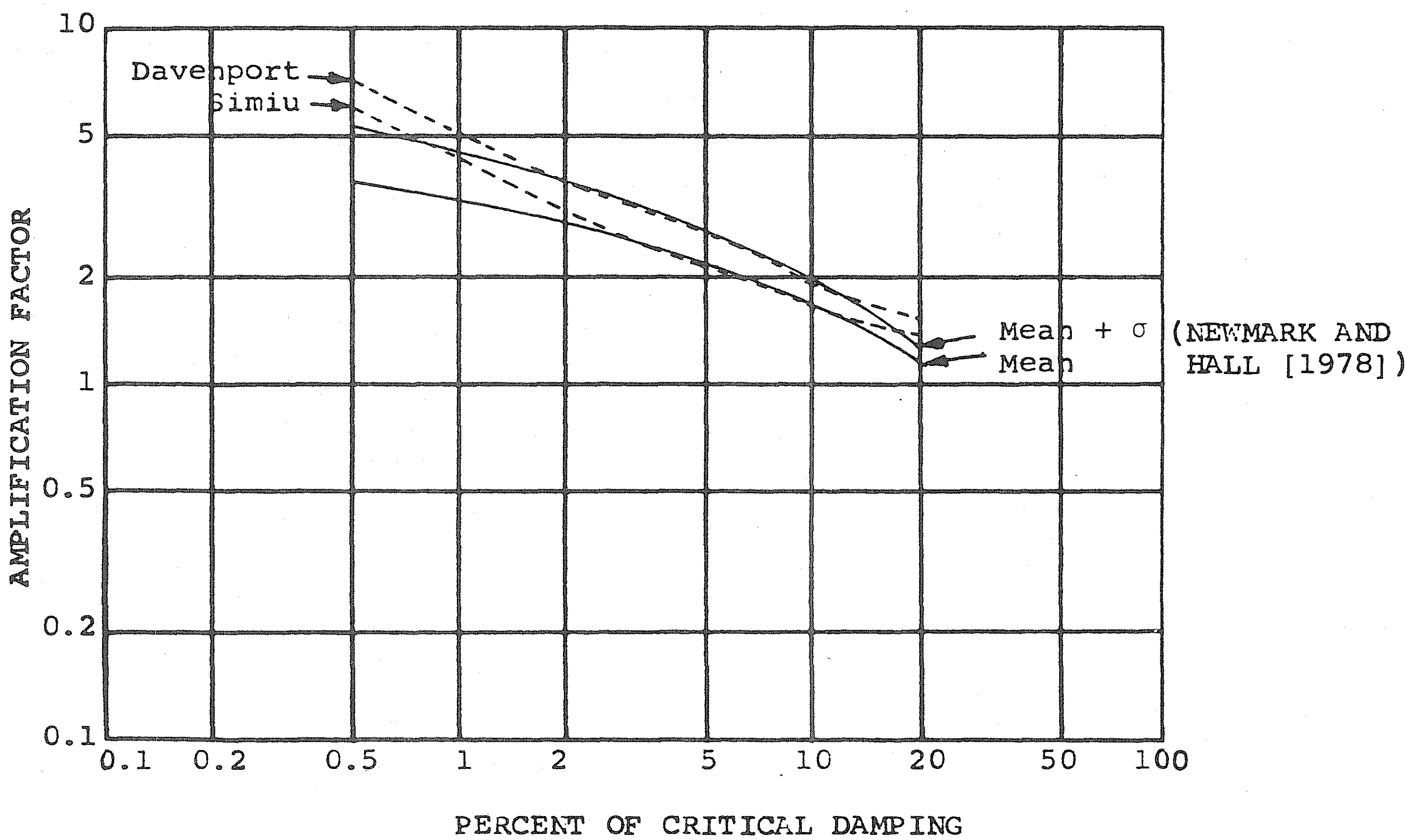


FIG. 17 WIND AND EARTHQUAKE AMPLIFICATION FACTORS

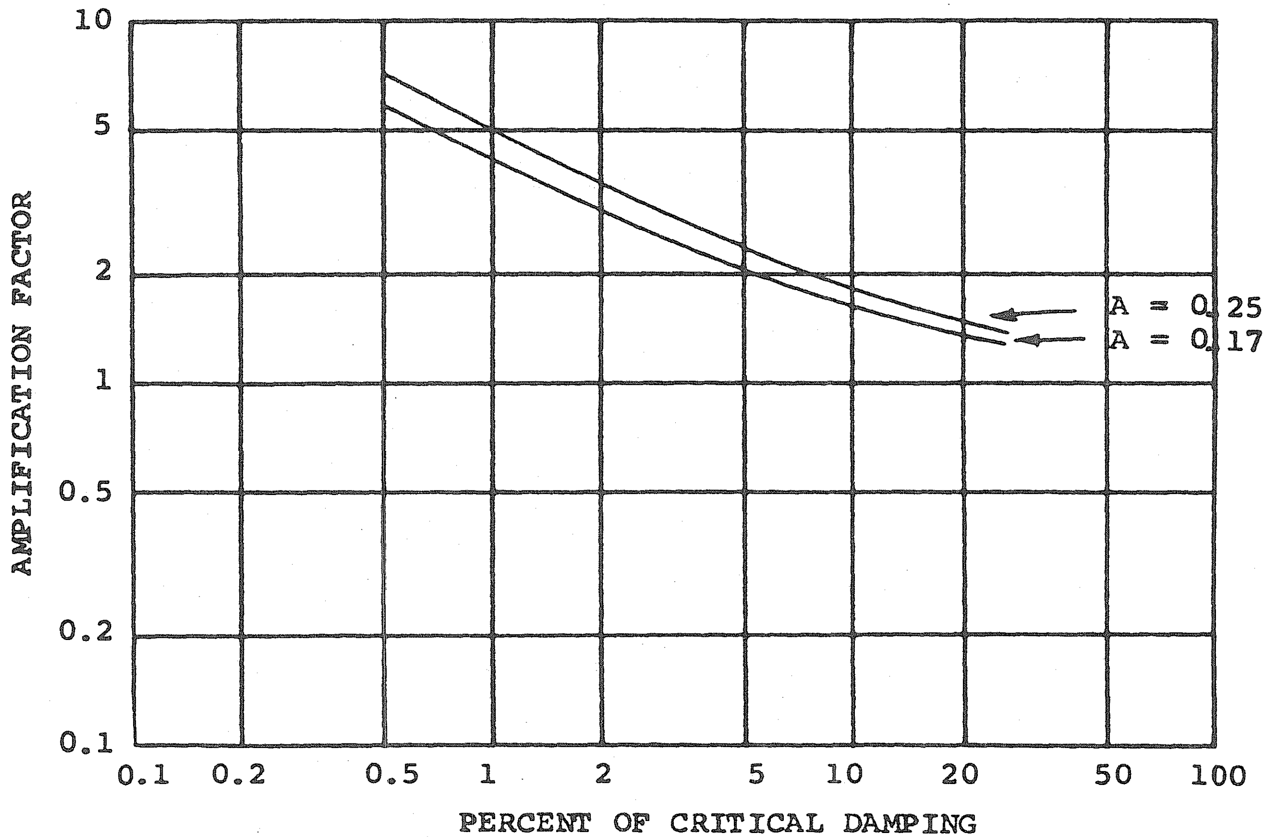


FIG. 18 WIND MAXIMUM AMPLIFICATION FACTOR AS A FUNCTION OF DAMPING

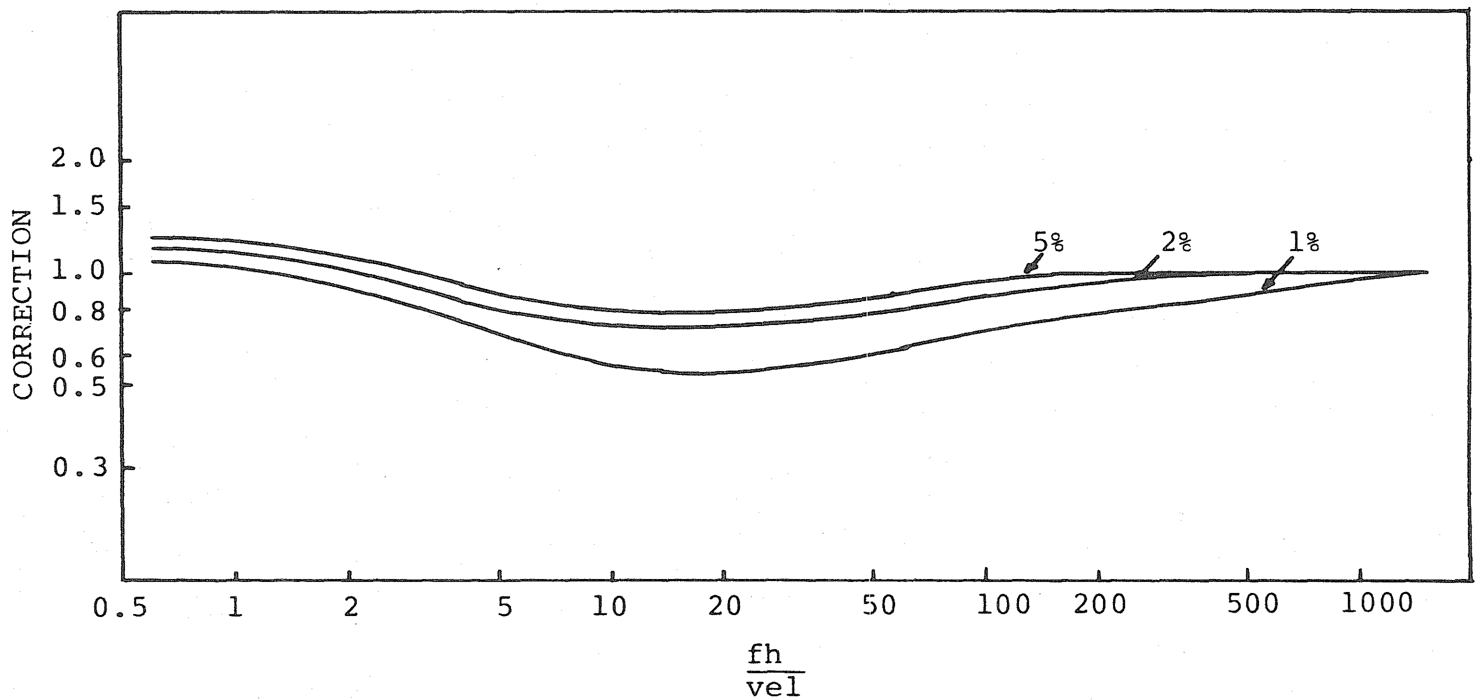


FIG. 19 CORRECTION FUNCTION

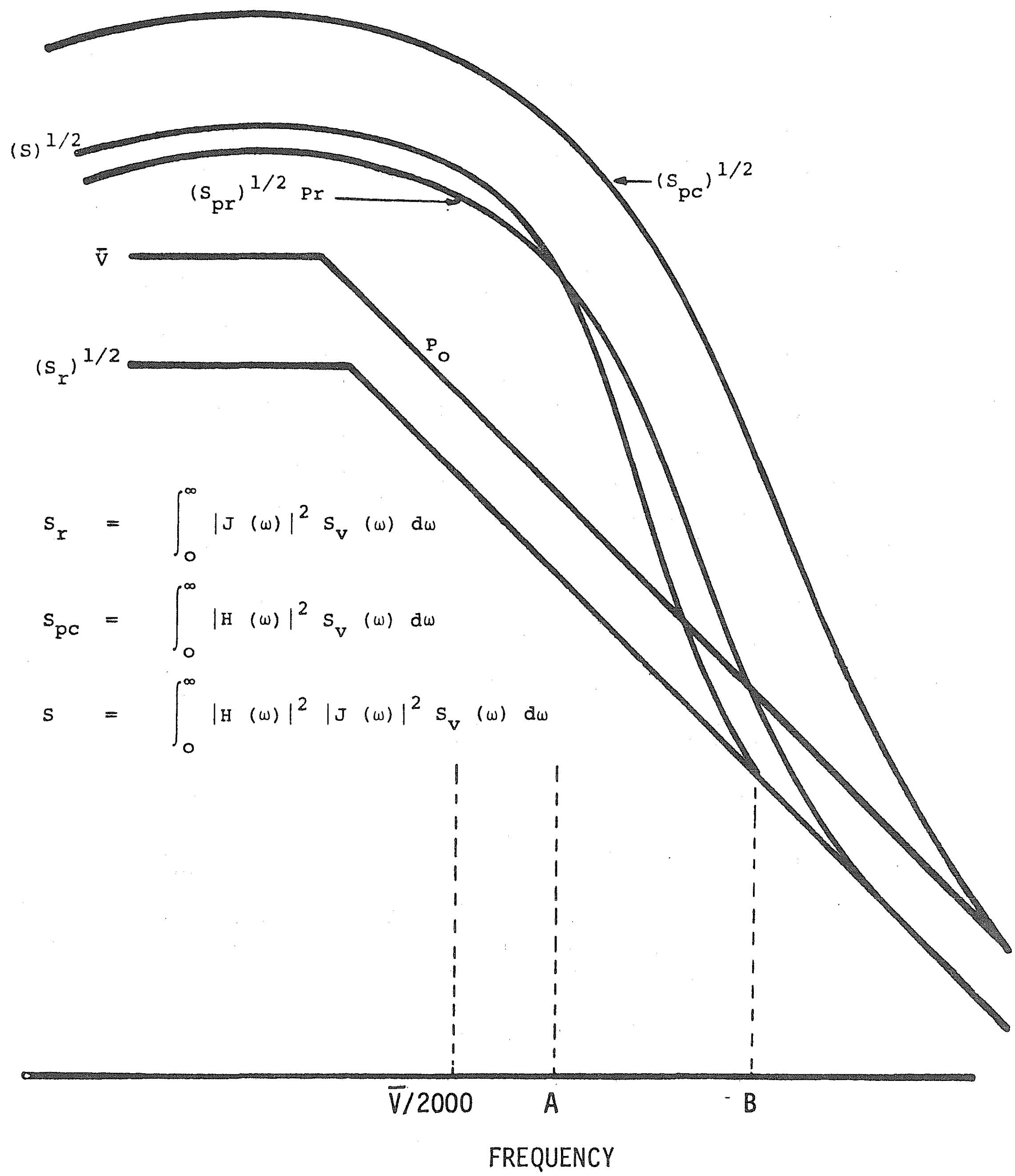


FIG. 20 COMPUTATION OF WIND RESPONSE SPECTRUM

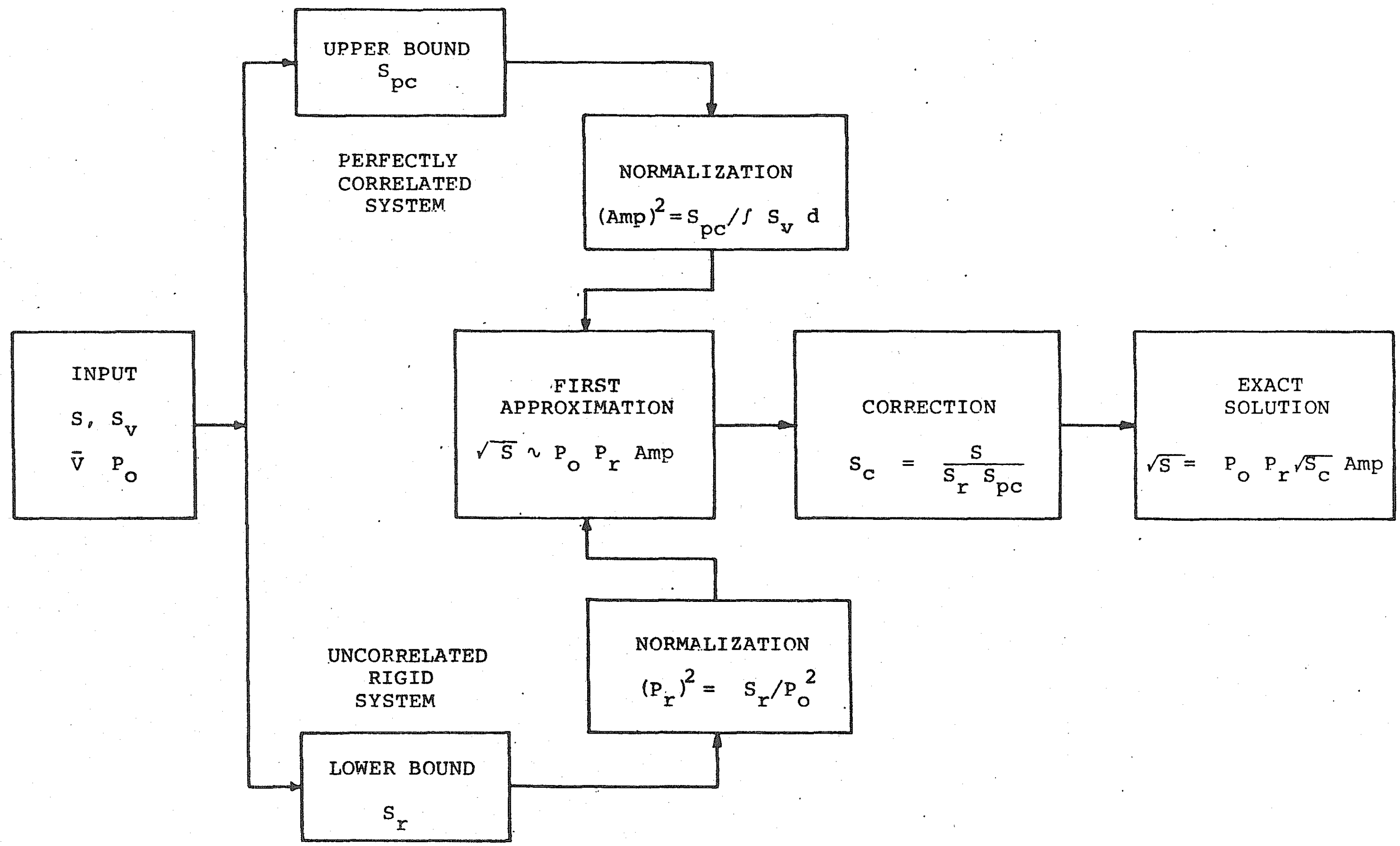


FIG. 21 STEPS TO COMPUTE DESIGN RESPONSE SPECTRUM

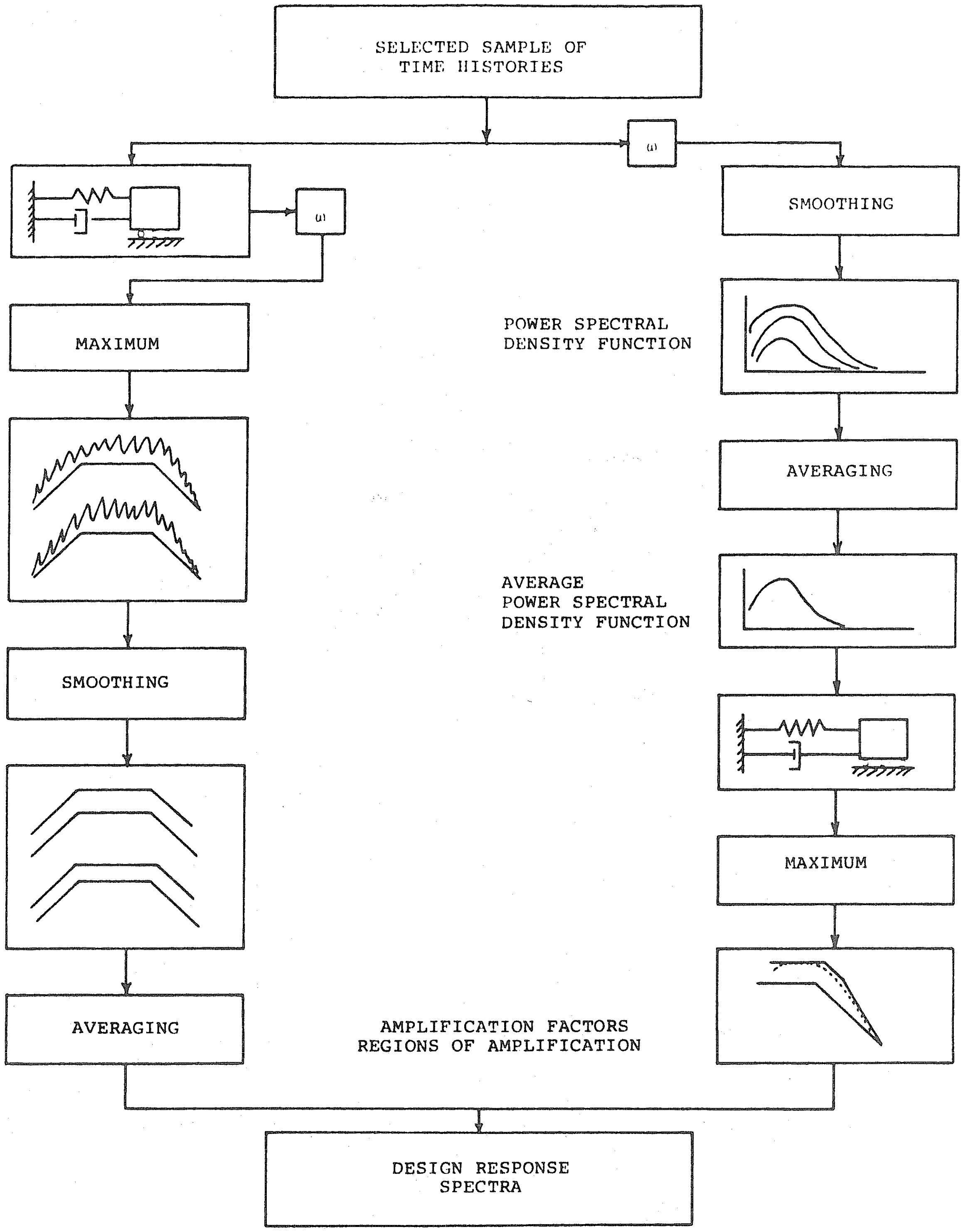


FIG. 22 COMPARISON OF COMPUTATION OF EARTHQUAKE AND WIND DESIGN RESPONSE SPECTRUM

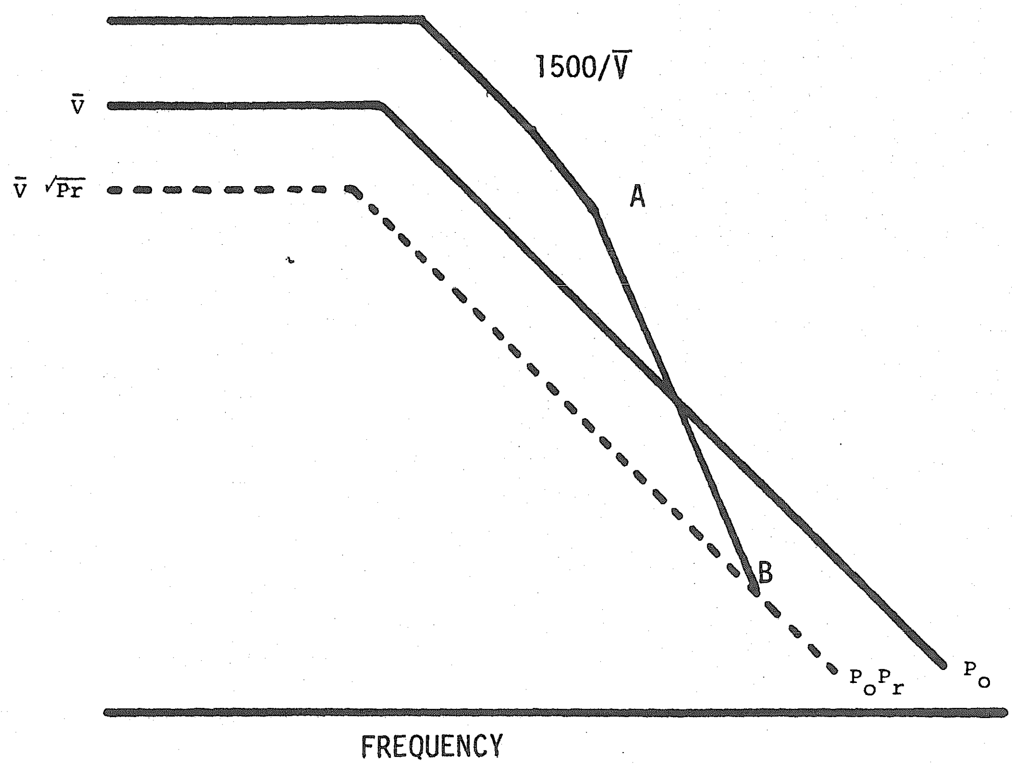


FIG. 23 SCHEMATIC REPRESENTATION OF WIND RESPONSE SPECTRUM

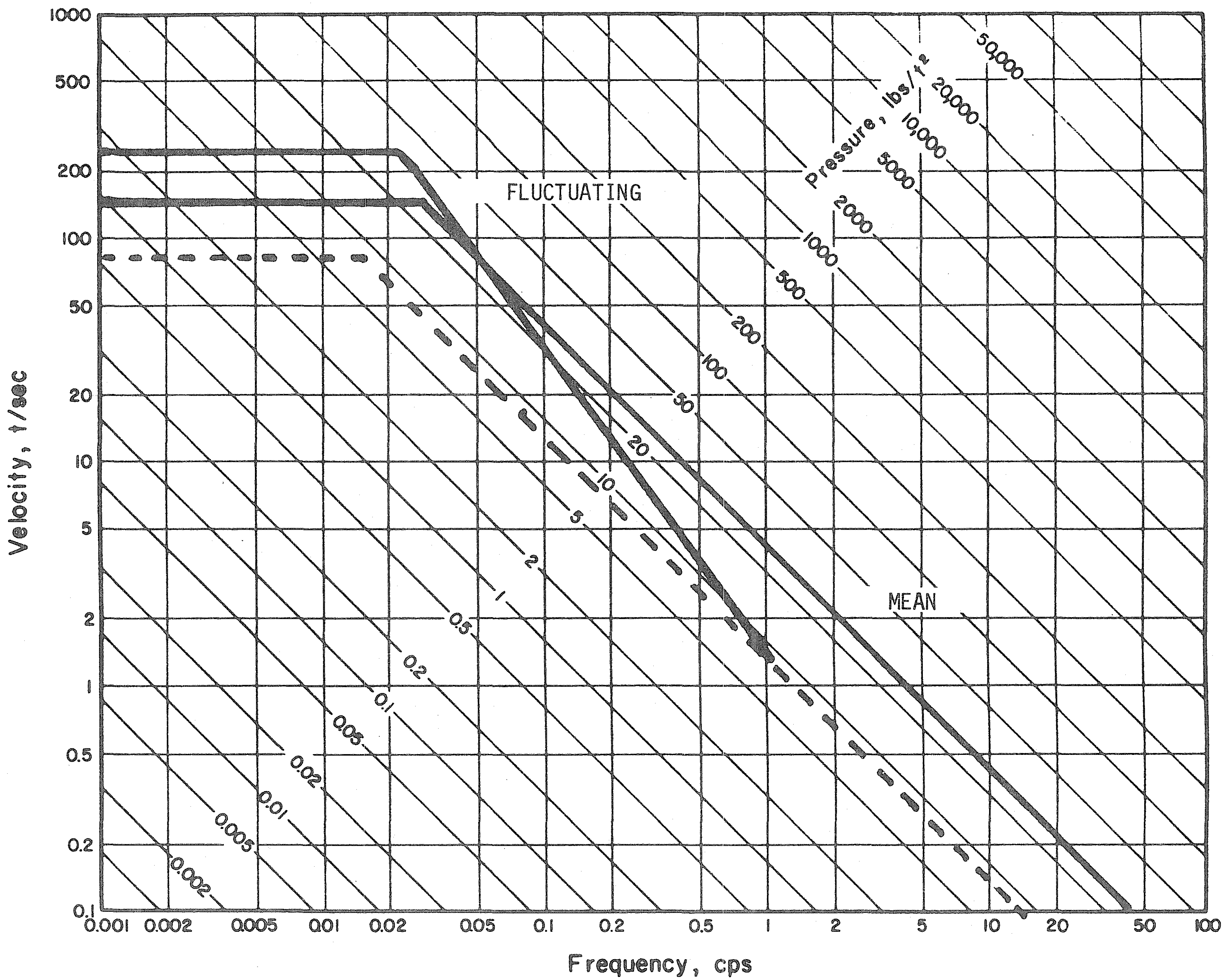


FIG. 24 RESPONSE SPECTRUM FOR 1200 x 200 x 200 FT BUILDING

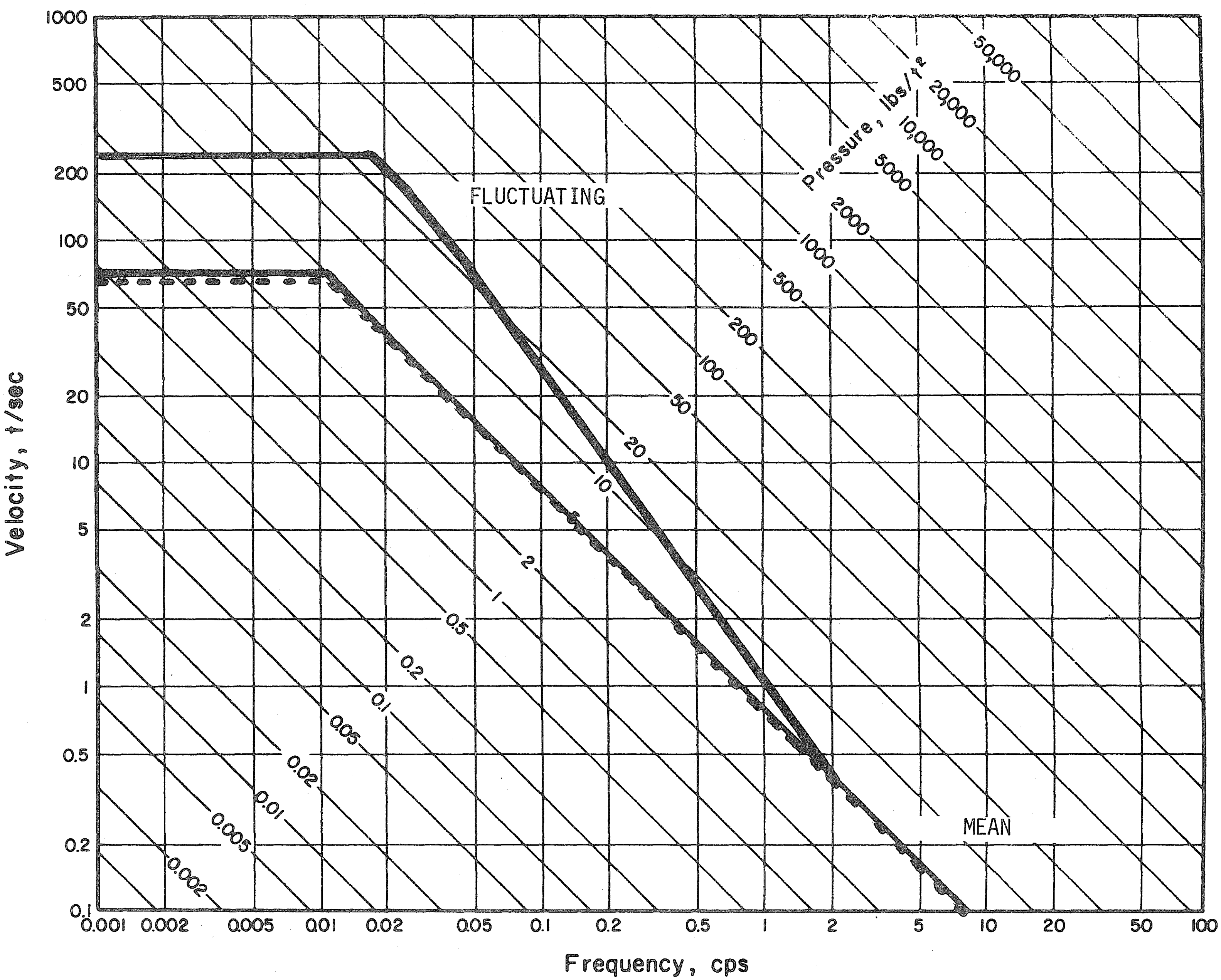


FIG. 25 RESPONSE SPECTRUM FOR 150 x 150 x 150 FT BUILDING

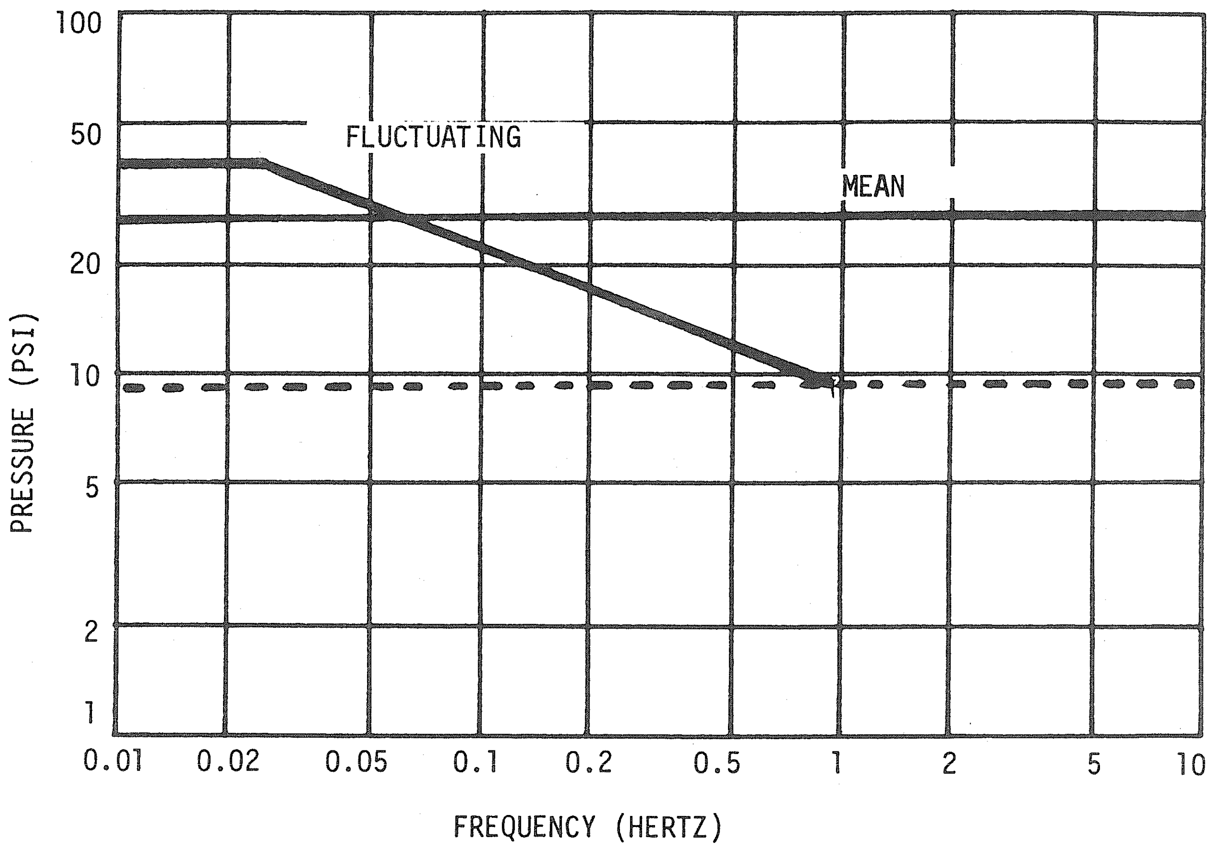


FIG. 26 RESPONSE SPECTRUM FOR 1200 x 200 x 200 FT BUILDING

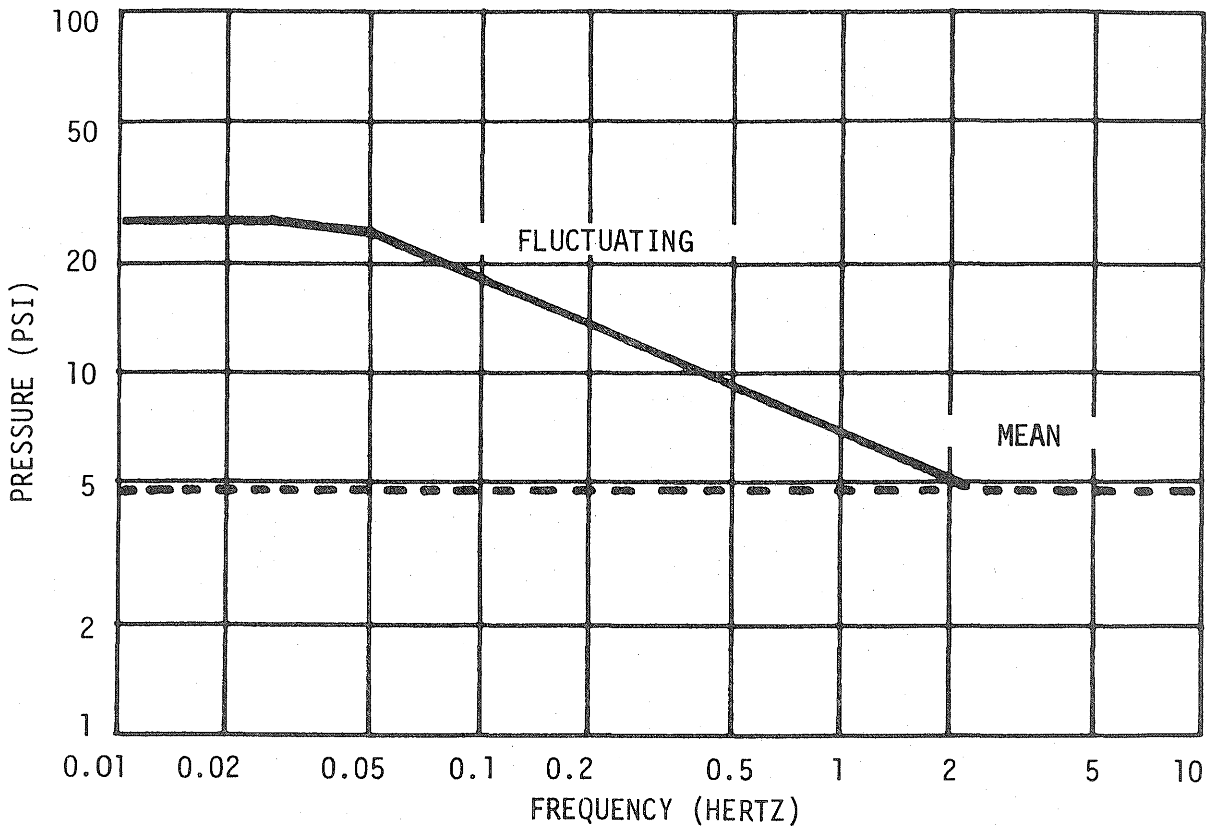


FIG. 27 RESPONSE SPECTRUM FOR 150 x 150 x 150 FT BUILDING

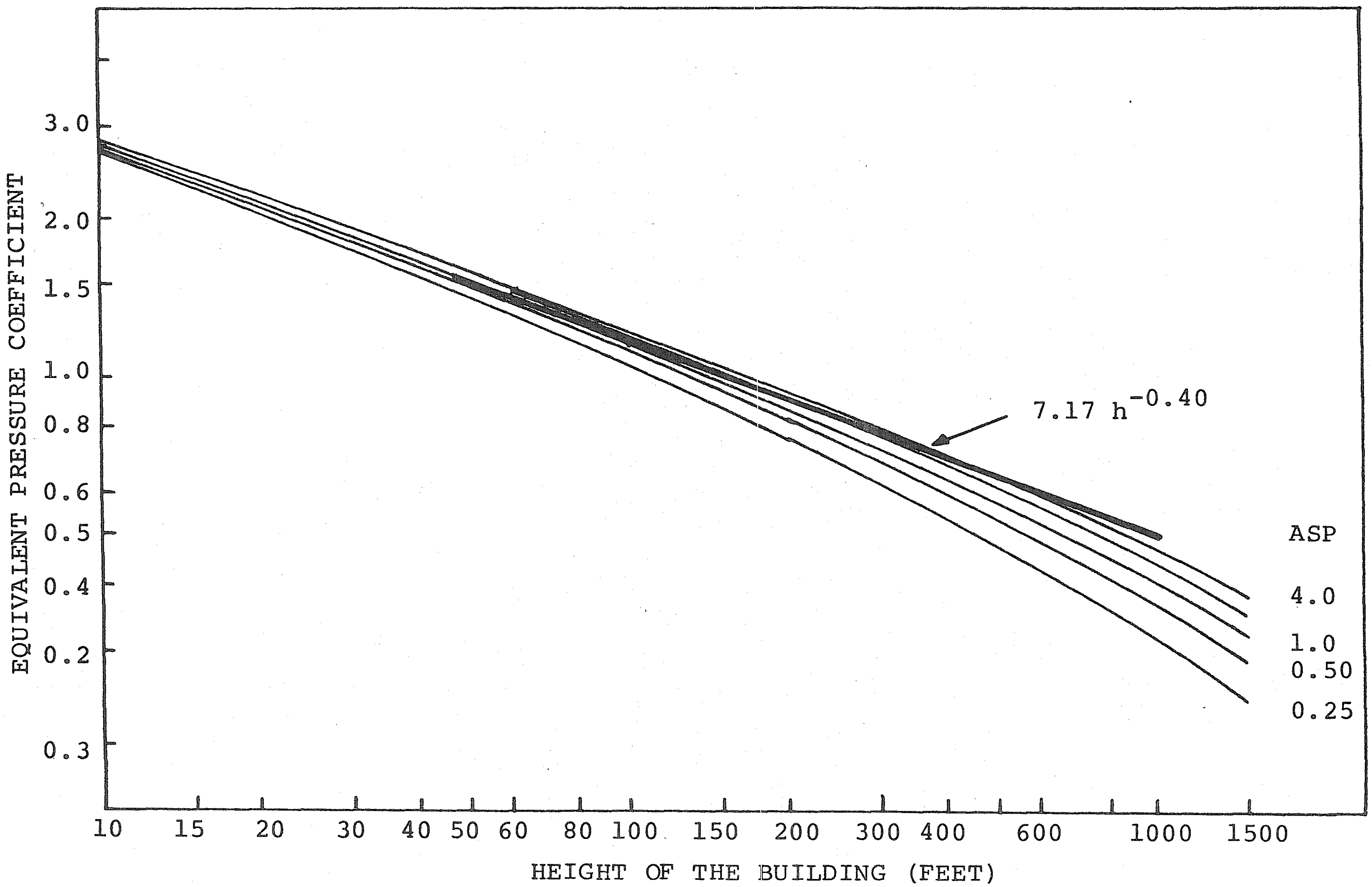


FIG. 28 INTERPOLATION OF EQUIVALENT PRESSURE COEFFICIENT

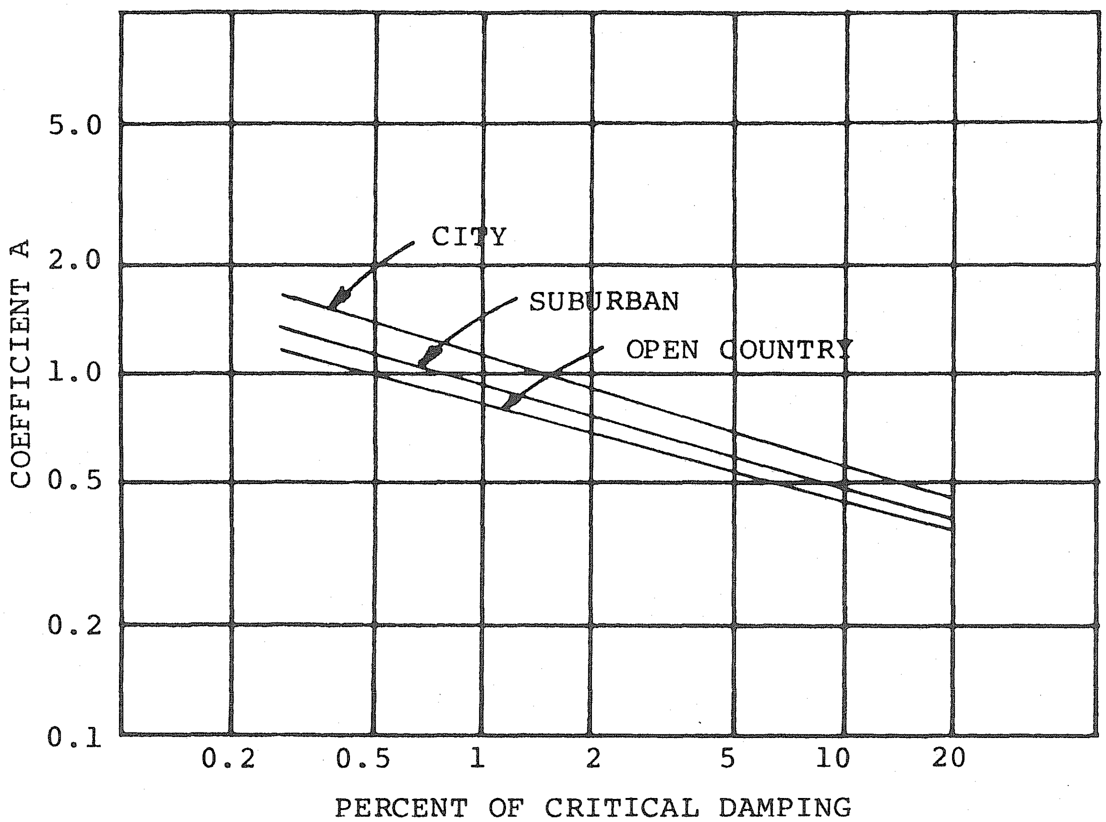


FIG. 29 COEFFICIENT A AS A FUNCTION OF WIND EXPOSURE AND PERCENT OF CRITICAL DAMPING

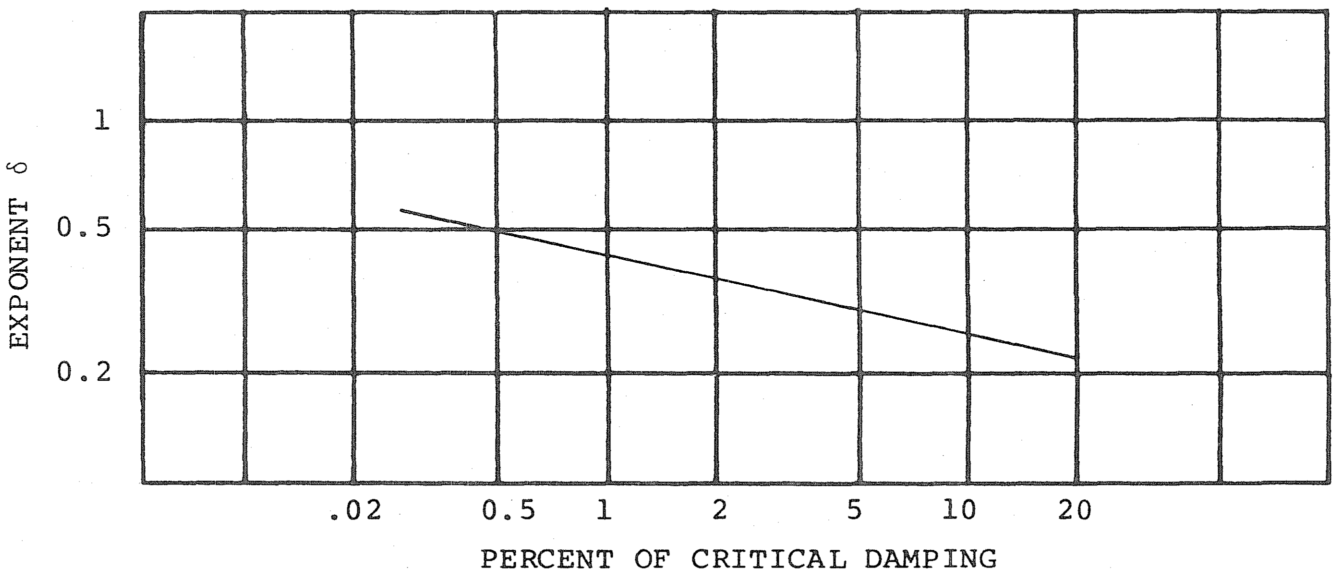


FIG. 30 EXPONENT δ AS A FUNCTION OF PERCENT OF CRITICAL DAMPING

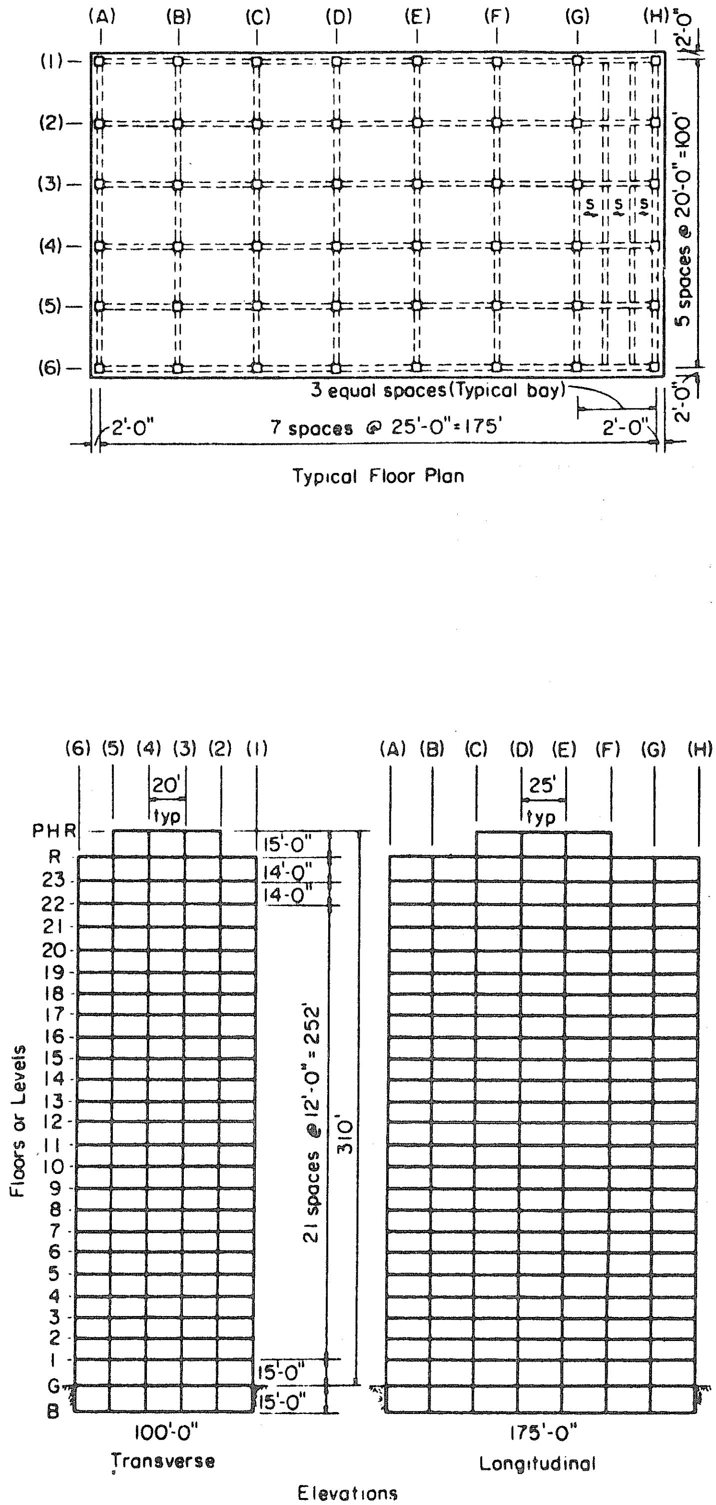


FIG. 31 PLAN AND ELEVATION OF 25 STORY BUILDING (AFTER BLUME, NEWMARK AND CORNING [1961])

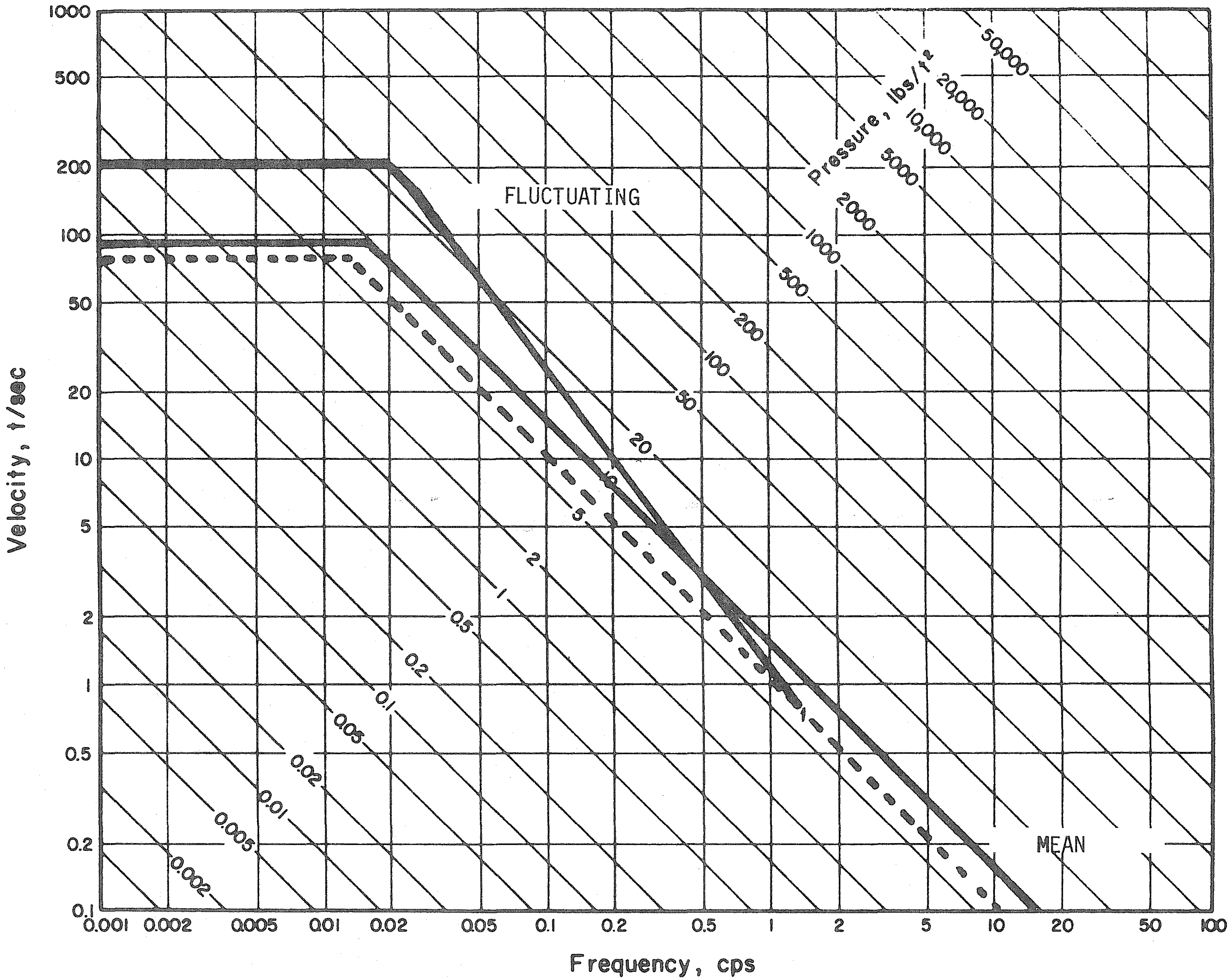


FIG. 32 RESPONSE SPECTRUM FOR 25 STORY BUILDING

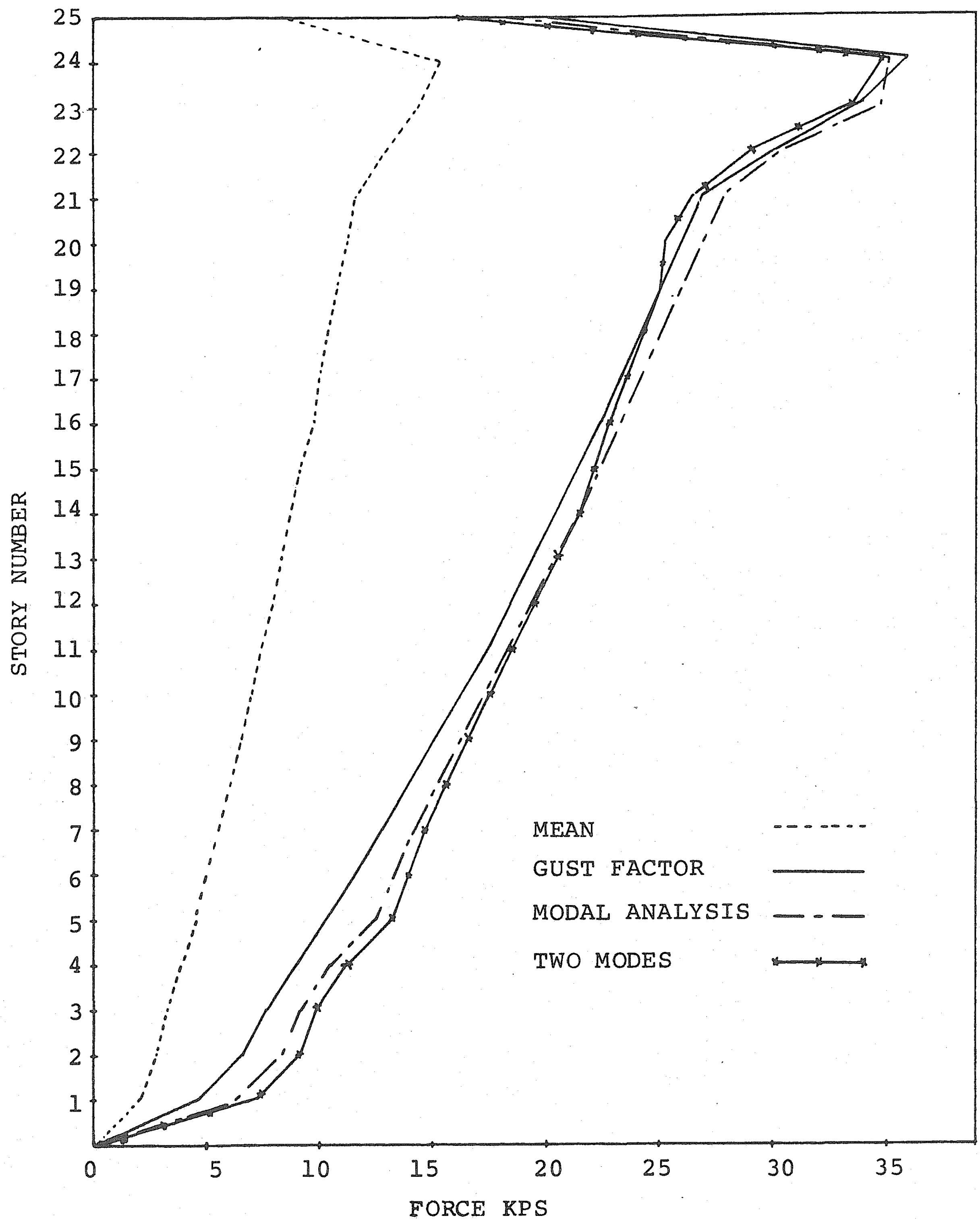


FIG. 33 STORY SHEAR ENVELOPE FOR 25 STORY BUILDING

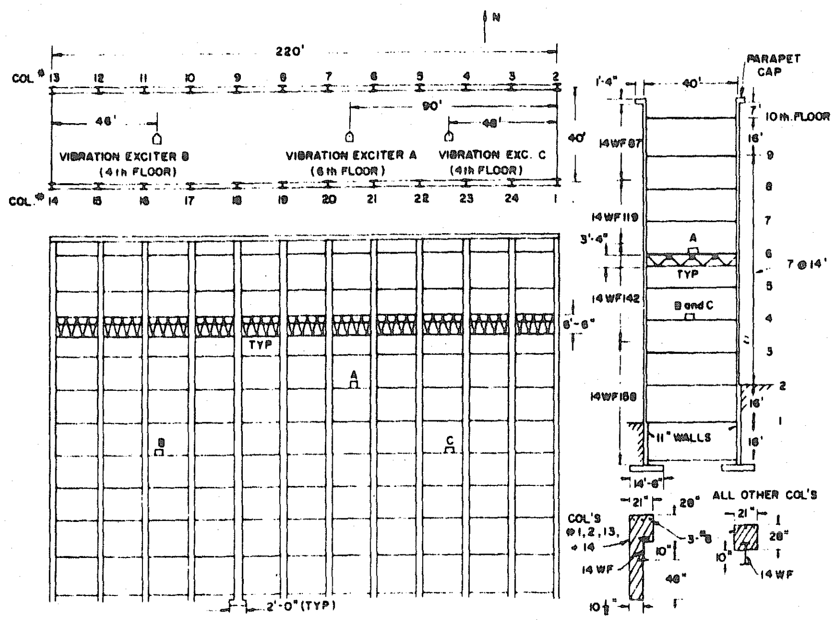


FIG. 34 PLAN AND ELEVATION OF 10 STORY BUILDING (AFTER NIELSEN [1968])

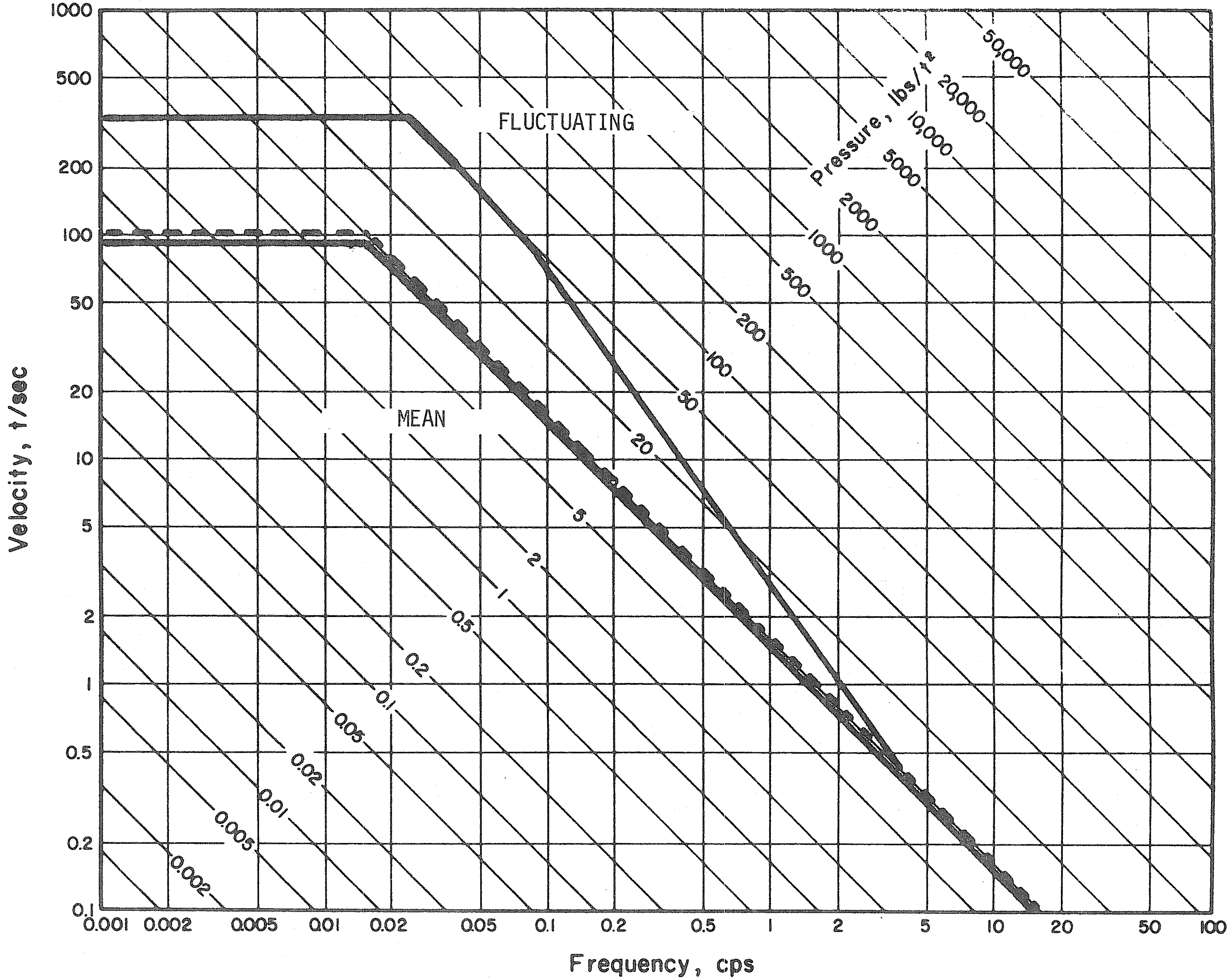


FIG. 35 RESPONSE SPECTRUM FOR 10 STORY BUILDING

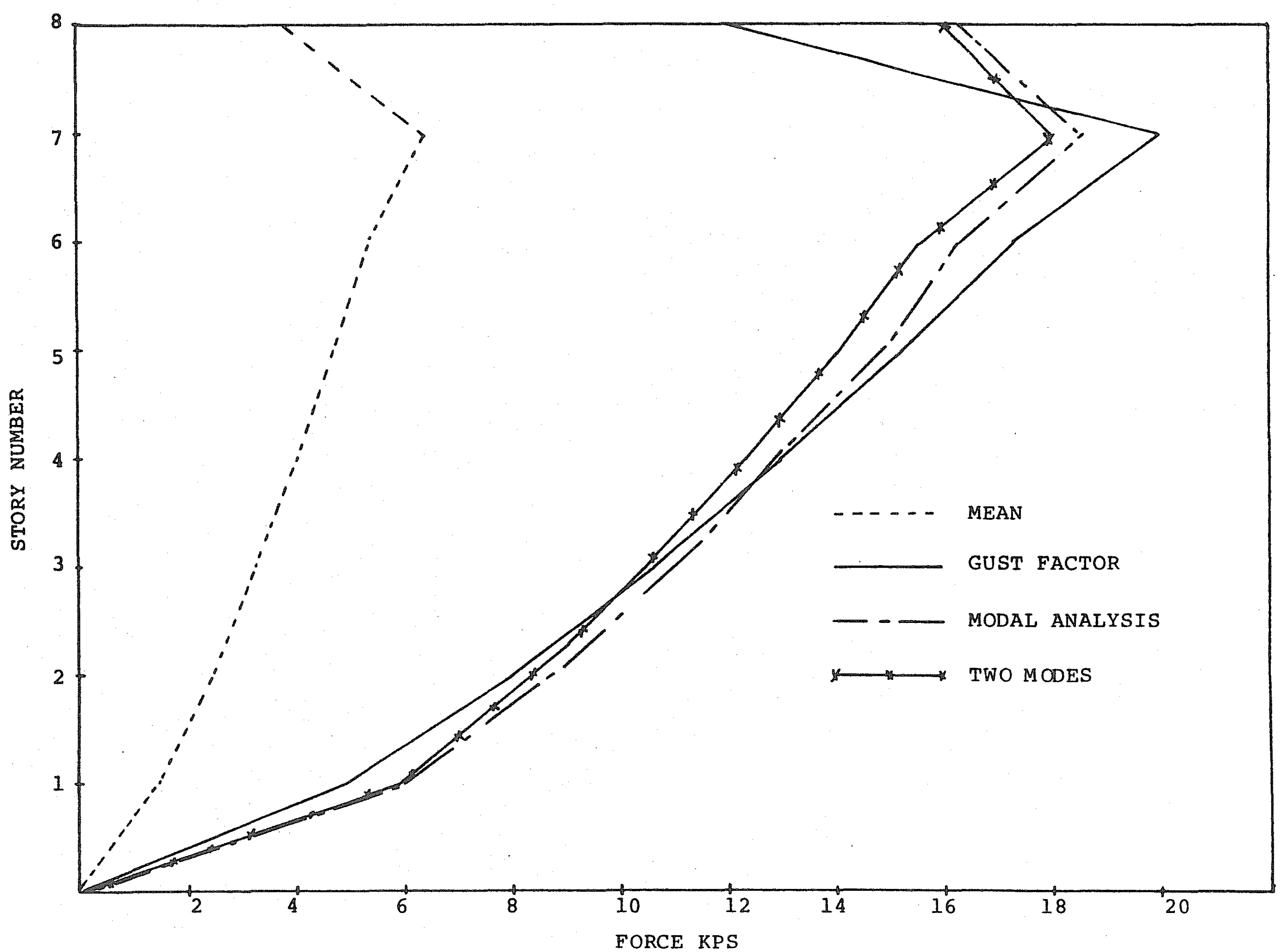
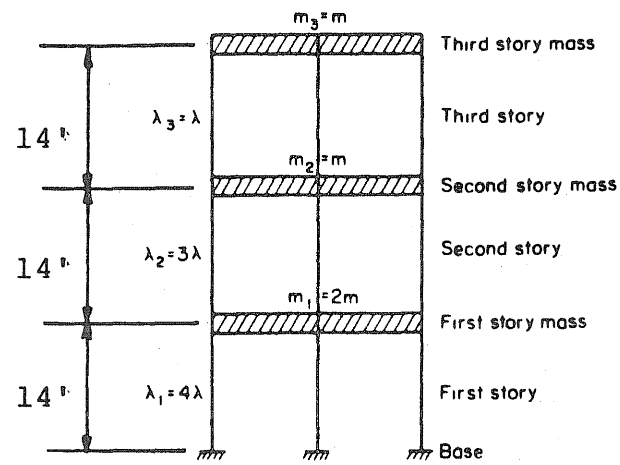
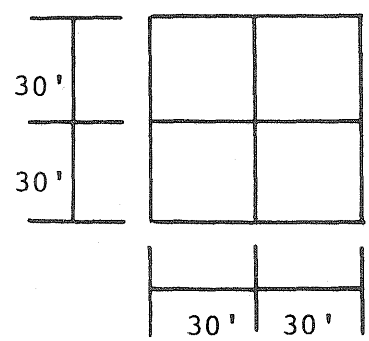


FIG. 36 STORY SHEAR ENVELOPE FOR 10 STORY BUILDING





$m = 0.93 \text{ k sec}^2/\text{in.}$

$\lambda = 284 \text{ k/in.}$

FIG. 37 PLAN AND ELEVATION FOR 3 STORY BUILDING (AFTER BLUME, NEWMARK AND CORNING [1961])

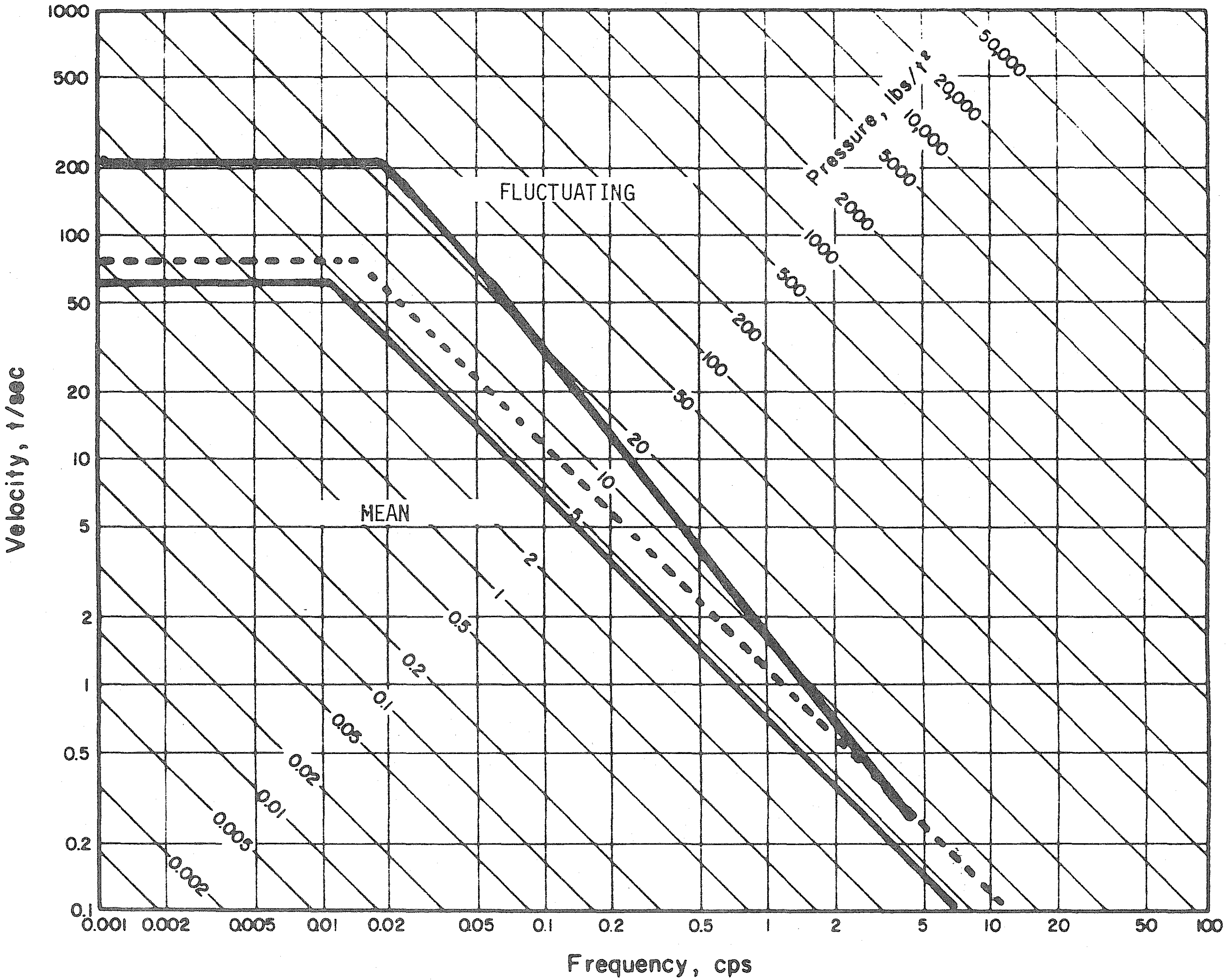


FIG. 38 RESPONSE SPECTRUM FOR 3 STORY BUILDING

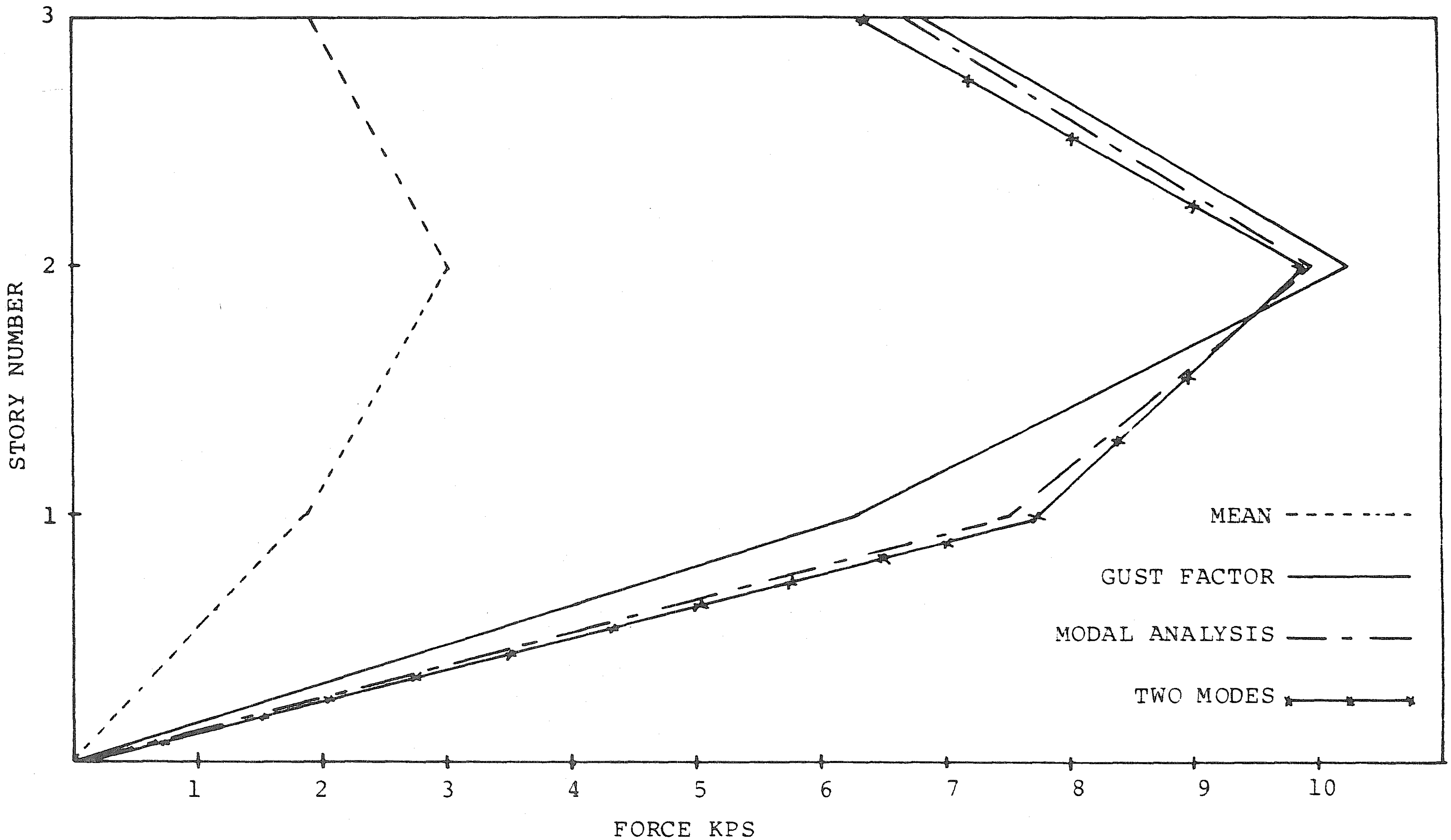


FIG. 39 STORY SHEAR ENVELOPE FOR 3 STORY BUILDING

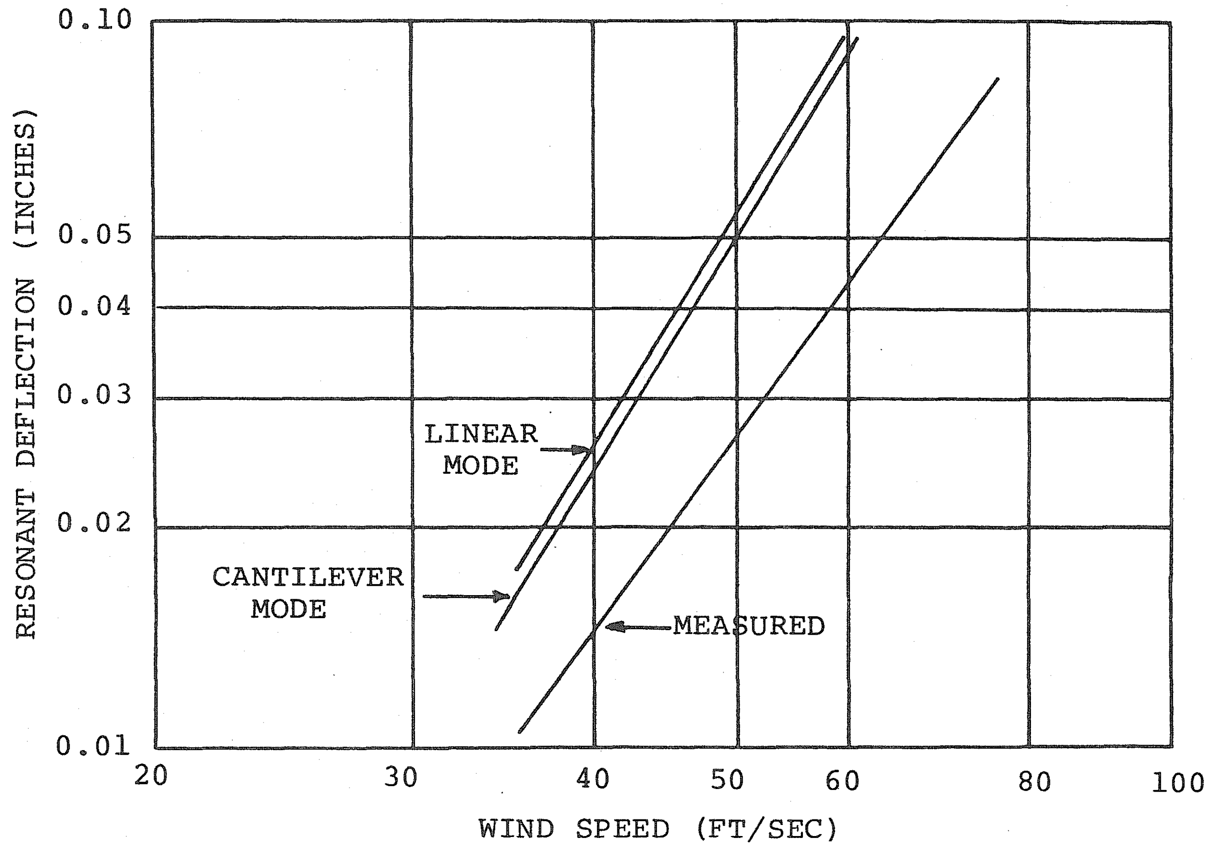


FIG. 40 COMPARISON OF PREDICTED AND OBSERVED VALUES OF EAST-WEST RESONANT DEFLECTION

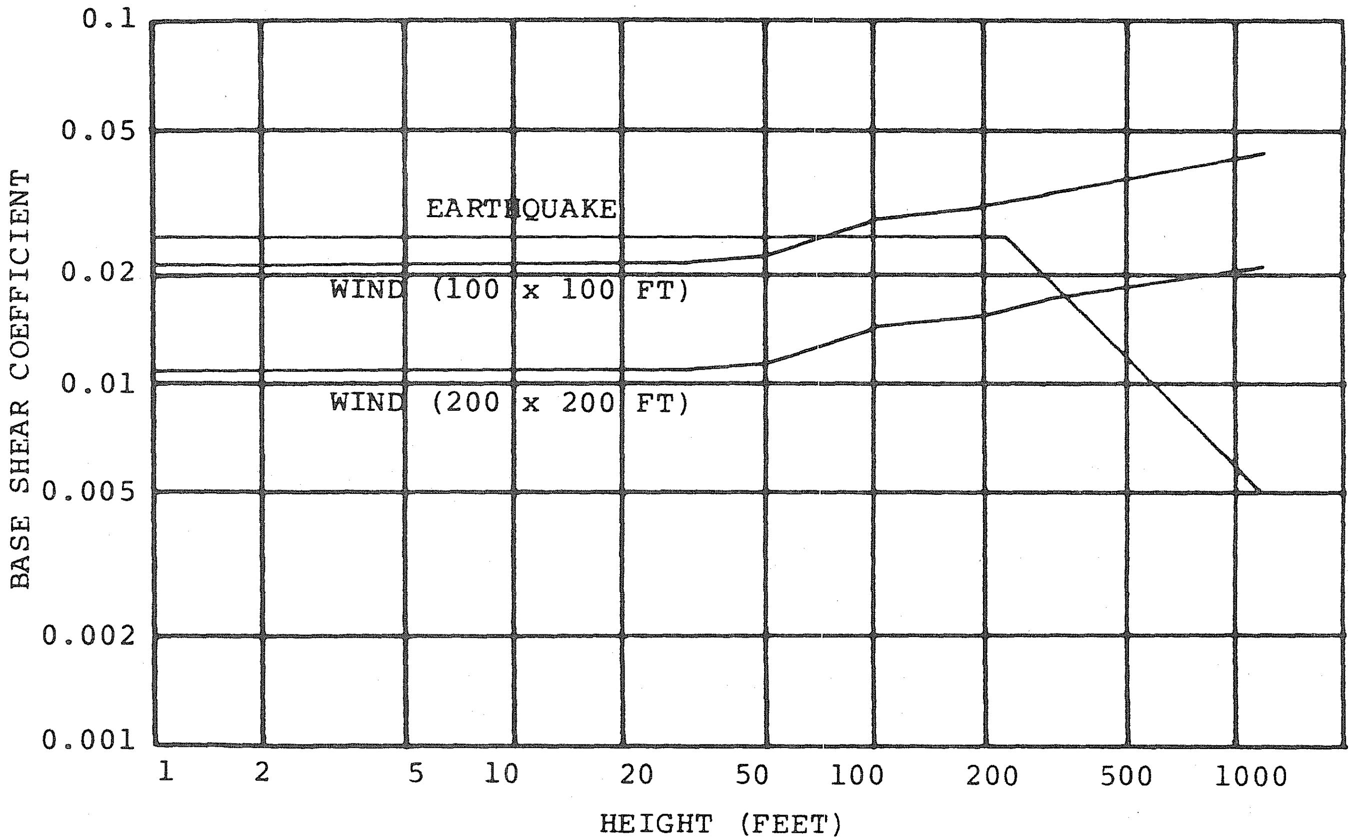


FIG. 41 COMPARISON OF U.B.C. WIND AND EARTHQUAKE BASE SHEAR COEFFICIENTS

APPENDIX A EVALUATION OF RESPONSE SPECTRA

A-1 Introduction.

For evaluating the parameters associated with wind response spectra, namely equivalent pressure coefficients, amplification factors and correction functions, numerical integration techniques were employed.

The response integral for the mean square response as defined by Eq. (2-68) has the following form:

$$S(\omega) = 1/2\pi \int_0^{\infty} |J(\omega)|^2 \; |H(\omega)|^2 \; S_i(\omega)\, d\omega \qquad \text{(A-1)}$$

where

$S(\omega)$ is the mean square response as a function of frequency,

$S_i(\omega)$ is the power spectral density function of the input function,

$|J(\omega)|^2$ represents the correlation of the input loading. For a perfectly correlated load $|J(\omega)|^2 = 1$.

$|H(\omega)|^2$ is the frequency transfer function Eq. (2-48).

In random vibration a narrow band approximation has been used generally for obtaining results of practical application. However, the narrow band approximation does not give good results when the natural frequency of vibration and the damping are relatively high. The narrow band approximation is extensively discussed in the literature (see, for example, Newmark and Rosenblueth (1975)).

A-2 General Procedure for Computing Response Spectra Employing Random Vibration Techniques.

In this study a new procedure for the evaluation of the response integral Eq. (A-1) was developed. This procedure is based on the premise that some bounds of the response integral have an actual physical meaning which can be related to the spectral base lines and amplification factors employed in the deterministic analysis of dynamic loading.

The evaluation technique developed in this study can be summarized as follows:

1) A lower bound of the response can be computed as the static response of a rigid building. For a rigid building having an infinite frequency of vibration the function $|H(\omega)|^2$ Eq. (2-48) is equal to one. Therefore the lower bound of the response is given by the expression:

$$A = \frac{1}{2\pi} \int_0^\infty |J(\omega)|^2 \; S_i(\omega) \; d\omega \tag{A-2}$$

2) An upper bound of the dynamic response can be computed by assuming that the pressures (forces) are perfectly correlated. A perfectly correlated pressure implies that the function $|J(\omega)|^2$ is equal to one. Therefore an upper bound of the response is given by:

$$B = \frac{1}{2\pi} \int_0^\infty |H(\omega)|^2 \; S_i(\omega) \; d\omega \tag{A-3}$$

Moreover a dynamic amplification factor can be obtained by dividing Eq. (A-3) by its terminal value as follows:

$$C = \frac{B}{1/2\pi \int_0^\infty S_i(\omega) \; d\omega} \tag{A-4}$$

The denominator of Eq. (A-4) is just the terminal value of Eq. (A-3), or in other words, the value of Eq. (A-3) when the natural frequency of the system approaches infinity.

3) A correction function can be evaluated to interpolate between A and B as follows:

$$D = \frac{S(\omega)}{A \times B} \quad \text{or} \quad D = \frac{S(\omega)\ 1/2\pi \ {}_0\int^{\infty} S_i(\omega)\, d\omega}{A \times C} \tag{A-5}$$

The products A x B x D or A x C x D, depending on the definition of D, give the exact value of the response integral Eq. (A-1).

It can be seen that any two of the quantities A, B or D can be arbitrarily defined, but the third parameter is enforced by the evaluation of the response integral Eq. (A-1).

For dynamic problems it is convenient to arbitrarily define the quantities A and C because it is possible to find physical meaning for both quantities. Indeed, A is the equivalent base line or terminal line of the response spectrum. On the other hand, C has a meaning similar in concept to the amplification factors of deterministic analysis.

It is convenient to define A and C in a manner such that at a very high frequency (over 100 hertz) the effects of the correction function are negligible and the following relation is satisfied:

$$S(\omega) = A \times C$$

Moreover since for a high frequency system the amplification function, C, approaches one the response is further simplified as follows:

$$S(\omega) = A$$
$$\omega \to \infty$$

Therefore, the value of A can be taken as the base line of the response spectrum. On the basis of the above definition of A and C, D must converge to 1 at the high frequency, but does not converge to 1 in the low frequency range. The advantage of this normalization is that the amplified bounds of the response spectra will converge to the base line A in the high frequency range.

The advantages of employing this evaluation procedure for the computation of the spectra are the following:

1) Represents the exact response.
2) A physical meaning can be associated with some of the parameter.
3) Permits a graphical representation and a simple computation similar to the familiar earthquake response spectra.

A-3 Evaluation of Effective Pressure Response Spectra for Wind Loading.

The procedure described in the previous Section will be employed now to evaluate the response spectra for wind loading.

Equivalent pressure coefficients -- In order to compute the equivalent frequency coefficients, P_r, as the root mean square of a rigid system the frequency transfer function has by definition a numerical value equal to one. In this case, the mean square response, Eq. (A-1), is reduced to the following expression:

$$A = S_r = 1/2\pi \int_0^\infty |J(\omega)|^2 \, S_v(\omega) \, d\omega \qquad \text{(A-6)}$$

where

S_v is the power spectral density function of the wind velocity, $|J(\omega)|^2$ is the aerodynamic transfer function that will be evaluated herein.

The effective pressure coefficients as defined in Section 4-3 are given by the square root of Eq. (A-6).

To obtain the numerical value of Eq. (A-6), the aerodynamic transfer function $|J(\omega)|^2$ has to be evaluated. The function $|J(\omega)|^2$ represents an average value of the intensity of the fluctuating pressure over the complete face of the structure. Therefore the square of the equivalent pressure coefficients can be written as follows:

$$S_r = \frac{1}{2\pi} \int_0^{\infty} S_v(\omega) \int_0^h \int_0^h \int_0^b \int_0^b R_u \, dy \, dy' \, dz \, dz' \, d\omega \qquad \text{(A-7)}$$

The functions R_u and $S_v(\omega)$ have been defined in Chapter 3. Comparison of Eq. (A-6) and Eq. (A-7) shows that the quantity $|J(\omega)|^2$ is defined as follows:

$$|J(\omega)|^2 = \int_0^h \int_0^h \int_0^b \int_0^b R_u \, dy \, dy' \, dz \, dz'$$

Simiu and Lozier (1975) have presented a procedure based on Montecarlo integration techniques to carry out the four fold integration. Once the four fold geometrical integral has been evaluated, the integration along the frequency line can be accomplished by employing Simpson's rule or any other one dimensional integration technique.

In this study the procedure proposed by Simiu and Lozier was employed for computing the geometrical integral (four fold integral in y,y' z and z') and then Simpson's rule was used for the integration on the frequency line. The operation was repeated for a large number of cases for various values of altitude, aspect ratio and types of exposure conditions, namely center of cities and open country. The effective pressure coefficients can be computed by taking the square root of Eq. (A-7). These results have been presented in Figs. 13 and 14.

Amplification factor -- The numerical evaluation of the amplification factors is simpler than that required for the equivalent pressure coefficients because the correlation function is defined to be equal to one and the four fold integration is eliminated.

The amplification factors have been defined in Section 4-3 as the square root of Eq. (A-4). For wind, Eq. (A-4) is written as follows:

$$C = (\mathrm{Amp}\ (\omega)\ )^2 = \frac{\int_0^{\infty} |H(\omega)|^2 \ S_V(\omega)\ d\omega}{\int_0^{\infty} S_V\ (\omega)\ d\omega} \qquad \text{(A-8)}$$

The numerator of Eq. (A-8) represents the response of a perfectly correlated single degree of freedom. The denominator of Eq. (A-8) is the terminal value of the numerator. In this manner the amplification factors will be always equal to or larger than one, converging to one in the high frequency range. The denominator has to be evaluated only once during the computation of the amplification factors. On the other hand the numerator should be calculated for a large sample of oscillators with various frequencies of vibration and excited by different levels of wind velocities.

The numerical techniques required for the evaluation of the amplification factors are reduced to only one dimensional numerical integration along the frequency line which was carried out by employing the Simpson's rule procedure. Results of the evaluation of the square root of Eq. (A-8) have been presented in Fig. 16.

Correction function -- The evaluation of the correction function involves a combination of both the effective pressure coefficients and the amplification factors. The correction function as defined by Eq. (A-5) has the following form for wind input:

$$S_c(\omega) = \frac{\int_0^\infty |J(\omega)|^2 \; S_v(\omega) \; |H(\omega)|^2 \, d\omega \; \int_0^\infty S_v(\omega) \, d\omega}{\int_0^\infty |J(\omega)|^2 \; S_v(\omega) \; d\omega \; \int_0^\infty S_v(\omega) |H(\omega)|^2 d\omega} = D \qquad \text{(A-9)}$$

where

$\int_0^\infty |J(\omega)|^2 \, S_v(\omega) |H(\omega)|^2 \, d\omega$ is the total response given by Eq. (A-3),

$\int_0^\infty S_v(\omega) \, d\omega$ is the denominator of the amplification factor Eq. (A-8),

$\int_0^\infty |H(\omega)|^2 \, S_v(\omega) \, d\omega$ is the numerator of the amplification factor Eq. (A-8),

$\int_0^\infty |J(\omega)|^2 \, S_v(\omega) \, d\omega$ is the effective pressure coefficient Eq. (A-7).

It is easy to see that the effects of the exposure are canceled out and the correction function is expected to be a function only of the frequency, velocity, damping and geometric properties of the structure. During the computation of the correction function it was found that it has a strong dependency with the damping and the ratio f x h/V. However, the aspect ratio has a small effect in the correction function and this dependency was ignored. The results of the evaluation of the square root of Eq. (A-9) have been presented in Fig. 19.