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STATIC LOAD DEFLECTION TESTS OF BEAM-COLUMNS

By
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Approved by
N. M. NEWMARK

Final Report for the Period
1 July 1952 to 15 September 1953
to
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Contract No. AF 33(616)-170
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UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

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NOTATION

In the figures the following notation has been used.

P, the applied lateral load

P_e, the applied lateral load at the elastic limit of beam-column

M, the bending moment at a section

M_e, the bending moment at the elastic limit

δ, the center deflection of the beam-column

δ_e, the center deflection when the load P_e was applied

ε, the maximum fiber strain for beam-columns tested without an axial load. When an axial load was applied this strain, the flexural strain, was one-half of the difference between the maximum fiber strains.

ε_e, the fiber strain at the elastic limit of the material

I. INTRODUCTION

1. Object and Scope of the Investigation

This program is concerned with the determination of the structural parameters required to predict the response of frames and frame components to both static and dynamic loads. Thus far in the program emphasis has been placed on the experimental investigation of the properties of a frame component which includes the column of the frame and a portion of the floor or roof system framing into it. This column, plus the floor system restraint, is the beam-column specimen which is discussed in this report. Since the loading condition of interest at this time is a lateral load applied to the frame at the floor or roof level, a study is being made of the response of the beam-column specimens for lateral loading conditions.

In this report the results of a portion of a series of tests on beam-column specimens are summarized. In the series of tests discussed consideration has been given to the effect of the direction of the load application when the lateral load produces bending about one of the principal axes of the cross section. In addition, some information is included on the influence of axial loads on the response of the beam-column specimens.

In the last section of the report a brief summary has been included of the work that has been done on the effect of the orientation of the cross section with respect to the applied lateral load. Thus far in the program this problem has been analytically studied and the program for the experimental investigation has been planned.

2. Acknowledgment

This report is a final report of a research project conducted in the Engineering Experiment Station of the University of Illinois, Department of Civil Engineering, and sponsored by the Wright Air Development Center, Department of the Air Force, under Contract AF 33(616)-170.

The work constitutes a part of the structural research program of the Department of Civil Engineering under the general direction of N. M. Newmark, Research Professor of Structural Engineering. The research project was under the direct supervision of F. L. Howland, Research Associate in Civil Engineering. The research was performed by W. Egger, R. Mayerjak, and R. Munz, Research Assistants in Civil Engineering. In addition to the regular project personnel assistance was obtained from Capt. J. Frazer of the Army Corps of Engineers, Lt. W. Francy of the Navy, and 1st Lt. R. Grubaugh of the Air Force, who are stationed here to pursue a course of study in Structural Dynamics.

The author wishes to acknowledge the assistance of W. H. Munse, Research Associate Professor of Civil Engineering, G. K. Sinnamon, Research Assistant Professor of Civil Engineering and W. J. Hall, Research Associate in Civil Engineering in planning the program and in the interpretation of the test results.

3. Summary of the Results of the Investigation

Since the investigation of the parameters influencing the response of beam-column specimens has not been completed during the period covered by this report, only general observations can be stated.

However, in the next section of this report the results of the beam-column tests are described and the experimentally determined load-deflection and moment-strain relationships are presented in the figures.

From the load-deflection relationships it can be seen that, in the cases where no axial load was applied, the lateral load increased with the deflection until the failure condition was developed. The rate of increase of the lateral load was less than the rate for the original elastic loading condition. When an axial load was added, the rate of increase of the lateral load after the elastic limit was exceeded was found to be less than the rate for specimens tested without the axial load. When the axial load was large, as in the case of specimen A2 S 4 M 13.0, the lateral load decreased with increasing deflection soon after the maximum load was applied to the specimen.

During the past year some interest has been placed on determining the influence of an axial load, which is constant throughout the loading, on the basic response of the beam-columns. From the two tests mentioned in this report the axial load influence is realized in two ways: first, the axial load changes the moment-strain relationship (the moment-curvature relationship), the change depending on the shape of the cross section and on the magnitude of the axial load, and second, the axial load will affect the load-deflection relationship directly by contributing to the bending moment at any section along the beam. In the tests it was found that, if the axial load is small, the effect of the axial load on the moment-strain relationship may be slight but, the influence of the axial load on the load deflection relationship may be large particularly in the development of a failure condition. This study is continuing at this time both by means of experimental studies and also by analytical studies using the elementary theory of plasticity. This theory neglects the time effects and uses as a stress-strain relationship the stress-strain relationship similar to the results obtained from static tension tests. A rather complete discussion of this theory is presented in Seely and Smith, "Advanced Mechanics of Materials", second edition, Wiley, 1952.

In the series of tests discussed in this report only one case of shear interaction occurred. From this test it was found that the shearing forces noticeably change the moment-strain relationship and tend to reduce the capacity of the beam-column. Thus far, however, nothing has been done on the problem of combined flexure, shear, and axial load. At this time, W. J. Hall of the staff of this laboratory has been independently studying the shear problem and the results of this investigation will be made available for use in this program.

II. STATIC TESTS OF BEAM-COLUMNS

4. Introduction

As was mentioned in the first section of this report the beam-column tests have been performed to determine what variables influence the static response of beam-columns when the loads are applied statically. The information obtained from these tests provide, in addition, information for analytical studies which are made to deduce the interaction of these variables.

Thus far in this program only the interaction between the bending load and axial loads has been studied. In these studies the primary interest has been placed on the influence of the axial load on, first, the moment curvature relationship and, second, on the load-deflection relationship. In a certain sense these relationships are interconnected since, if the moment curvature relationship is known, the load-deflection relationship can be deduced for given boundary conditions. In the study conducted to date the specimens have been tested for bending about the principal axes of the cross section. Another phase of the program is a study of the effect of oblique loading conditions which result in nonsymmetrical bending. This latter problem will be discussed in the next section of the report.

In the remainder of this section of the report a brief description is presented of the test specimens, the test apparatus and instrumentation, the results and comparisons of the tests, and a brief summary of the general features of the test results.

The essential features of the tests of beam columns are summarized in Table I. In Table II, the detailed results of these tests are summarized. A brief summary of the yield stresses found for the materials used in the fabrication of the specimens, is in Table III.

5. Beam Column Test Specimens

The beam-column specimens tested in this portion of the program simulate a single column, pin connected at its base, from a structural steel frame. The floor or roof system framing into the column has been simulated by fastening a stub section to the center of the column section. Thus far in the program the stub connection has been a fully welded connection which provides full restraint to the column at this center location. A detail of the connection used throughout this portion of the program is shown in Fig. 1. As can be seen in this figure the connection used is considerably more rigid than the connection usually found in practice. However, this rigid detail insures that the inelastic response desired will occur outside of the detail. The stub also provides the location for applying the lateral load to the specimen.

In the axial load tests the axial load is applied through the end reactions in such a way that the load is essentially concentrically applied throughout the test of the specimen . In all of these tests the axial load was maintained at a constant value throughout the test.

To interpret the results of these beam-column tests it has been felt that definite information should be obtained of the mechanical properties of the column section. Not only are the average mechanical properties of interest but the distribution of these properties through the cross section are also of some importance. In order to determine the distribution of the mechanical properties, test coupons have been taken from the center section of each of the specimens. After the coupon block was removed, the ends of the specimen were rewelded. Since this weld is in the center of the fully restrained portion of the specimen it should not appreciably influence the response of the beam-column. From each of the coupon blocks two types of coupons are made. The first are standard one-half by one-quarter inch tension coupons which are tested statically to obtain the engineering stress-strain curve for tensile loading. Generally three coupons are made for each flange of the section and the web is subdivided to provide as many coupons as is possible. The results of these static tension tests have been summarized in Table III where the average yield stress is shown for various locations in the cross section of each of the beam-column specimens tested thus far.

The second set of coupons are used to determine the distribution of the relative hardness values as determined by the Rockwell hardness test. These hardness values cannot be used to determine the yield stress at a given point but do indicate how the yield stress varies throughout the section. In general the hardness surveys indicate the same distribution of yield stress as has been found from the tension coupons.

Throughout the series of tests performed to date three lengths of beam-column have been used. In the early phases of the tests, beams with a span of twelve and four feet were tested. It was found in these tests that, if the shearing forces were relatively low, there was no essential difference between the test results. When the shear forces became important marked, influences on the response occurred. However, for the tests performed thus far, the inclusion of the shear problem would complicate the study of the effect of axial load on the response and it was decided that the specimen length should be such that the shear forces would not influence the results.

In addition to the effect of shear on the response, it was noted that the specimens with the twelve foot span generally failed by lateral buckling at a relatively low maximum fiber strain. It was decided that beam-column specimens shoudl be of such a length that the lateral stiffness would be sufficiently large to prevent early lateral buckling. To accomplish this balance, a specimen length of approximately eight feet was used for the last part of the program.

6. Test Apparatus

The test apparatus can best be described by considering it in the following order: the end-reaction system, the lateral loading system, the center restraining system, and the axial loading apparatus. A complete test arrangement without the axial load apparatus is shown in Fig. 2.

The End Reaction System: In order to provide an end-reaction system which would provide the minimum of constraint to the ends of the beam-column specimens a knife edge system was designed and constructed. The general features of this system are shown in Fig. 3. In addition to the requirement of minimum end restraint the end reaction system is used to measure the vertical reaction by means of weigh-bars located in the supporting tie bars. The details of this measuring system will be described in the next section of this report.

The Lateral Loading System: During this series of tests two systems have been used to apply the lateral load to the test specimens.

In the first tests a compression jacking system was used. In this system the specimen was deformed by pushing down on the stub with a compression hydraulic jack. Since this system is relatively stable if the specimen does not twist, the procedure was used for the first six tests performed.

When the longer specimen with a 6 I 12.5 section was tested the lateral deflection became sufficiently large for the compression jacking system to upset. To overcome the danger of upsetting the jacking system when the specimens are axially loaded, the lateral loading system was modified to increase the stability of the system. The increased stability was obtained by using a tension jacking system which pulls the specimen downward. This system is shown in Fig. 4. This tension jacking system will be used in all future tests of the beam-column specimens.

The Center Restraining System: When the tension jacking system was added a small amount of lateral movement occured which can induce a lateral buckling failure. To prevent this failure from occurring at strains close to the elastic limit strain, a center restraining system was introduced. This system is shown in Fig. 5. The restraining system permits the specimen to move vertically downward by adding restraining forces through a roller-rail system. The forces introduced in this system do not seriously affect the test results since the applied loads were measured at the end reactions.

The Axial Loading Apparatus: For the specimens which were tested under combined bending and axial loads, special apparatus was required to apply the axial load and maintain it at a constant magnitude throughout the test. As shown in Fig. 6 the apparatus was essentially two cross beams at the end reactions which were linked by a tie rod system. The applied axial load was measured by means of dynamometers located in the tie bars. The axial load was applied by a hydraulic jack placed between the end reaction and the cross beam at that end. At the other end of the specimen the thrust was transferred directly from the cross beam to the specimen. In both cases the linkage between the cross beams and the end reaction plates was accomplished with knife edges which permit the specimen to deflect in the direction of the applied lateral load.

The overall stability of the axial loading apparatus has been obtained by making the cross beams in the form of a "U" such that the tie bars join the ends of the "U" at a point between the end reaction and the center of the specimen. The stability of the overall system has been increased further by using two tie bars, oriented in a vertical plane on each side of the specimen.

In order to support the axial loading system and to eliminate the vertical load on the knife edges, roller guides, which support the cross beams, were attached to the end reaction system. The point of support was placed so that the cross beams are supported along a line passing through the knife edge. In the tests performed to date, the strain distribution across the section of the beam-column indicates that the applied load was nearly axial, with some discrepancy resulting from the deflection of the specimen during the application of the axial load.

7. Instrumentation

In the static tests of the beam-column specimens the instrumentation used falls into three categories: first, the load measuring system, second, the determination of the maximum fiber strains and the strain distribution, and, third, the determination of the deflected shape of the specimens. Each of these categories will now be discussed in some detail.

Load Measuring System: To determine the loads applied to the beam column specimens two distinct load measuring and control systems were used. The first of these systems was the system required for measuring the applied lateral load. Because of the central restraining system and the extraneous forces existing in the tension jacking system, the applied lateral load cannot be measured accurately by the pressure in the hydraulic system. For the static tests the applied load was measured by measuring the magnitude of the end reaction. In order to measure the end reactions, weigh-bars were introduced into the tie bars which support the end reaction assembly. These weigh-bars are calibrated tension rods for which the strain is measured with a wire-resistance electrical strain gage system. The bridge circuits were carefully calibrated so that the indicated strain, as measured with a portable strain indicator, can be converted directly into the tie bar load. The two tie bars have been used in each end reaction assembly to provide information on when the lateral buckling mode of failure developes since this type of failure results in an unbalance in the end reactions.

In addition to the measurment of the lateral load, a measurment and control system for the axial load was required. As was mentioned before the applied axial load is measured by weigh bars introduced into the tie bars used to link the crossbeams in the axial load apparatus. These weigh-bars were the same as the weigh-bars used in the end reaction measuring system.

The major problem in the axial loading system was to maintain the applied axial load at a nearly constant magnitude. In the first axial load test this was done by controlling the pressure in the hydraulic system. However, it was found that this system was not sensitive enough and large fluctuations in the load occurred. To overcome this difficulty a null type system was constructed which provides the control of the load by summing the bridge outputs from the weighbars which form the load measuring system. Because four bridge circuits are linked together, the portable strain indicator became insensitive. However, the lack of sensitivity of the strain system can be overcome by using a null type system which depends only on a setting of the indicator and the use of the indicator galvanometer to determine if the system is balanced. The procedure used is as follows: The approximate desired magnitude of the axial load was applied to the specimen using the hydraulic pressure gage as an approximate measure of the load. When the desired load was applied the portable strain gage was balanced and the total strain in the dynamometers was read as a reference setting. At all other times during the test the indicator was set at this reading of strain and the axial load was adjusted by rezeroing the galvanometer needle which indicates the balance of the indicator circuit. By using this null system the sensitivity of the control system was improved since the galvanometer indicates slight unbalanced conditions which cannot be accurately determined by readings of the strain scale.

Strain Measurement System: Throughout the tests mentioned in this report the maximum fiber strains and the strain distribution through the cross section of the specimen has been determined in two ways. For the determination of the strain distribution and the maximum fiber strains, particularly when the strains were less than approximately 1.5 per cent, wire-resistance electrical strain gages (Type SR-4) were used. At the end of each of the loading intervals all of these strains were read.

To supplement the wire resistence gage readings and to determine the larger strains, frequent measurements of the maximum fiber strain were made with a mechanical strain gage.

Deflection Measuring System: In analyzing the test data and in determining the response of these specimens rather complete information was obtained of the changes in the deflected shape of the specimens as a function of the applied load. To provide the deflected shape of the specimen throughout the range in deflections obtained in these tests, two deflection measuring systems were used. The first of these systems was used to measure the deflected shape of the specimen when the deflections were small. In this system, Ames dials, mounted on a beam which was fastened to the central stub, were used to measure the deflections relative to a line through the stub and approximately parallel to the undeformed axis of the beam column.

When the deflections became large and when the lateral movements of the specimen produced noticeable rotation of the Ames dial system a precision level was used to measure the deflected shape of the specimen relative to the floor of the laboratory. In all of the tests performed thus far the agreement between the two measurement system has been satisfactory up to the point where the deflections were large or the errors introduced by the lateral deflections invalidated the Ames dial measurements.

8. Testing Procedure

For all of the tests described in this report the same method of controlling the test was used. The test was essentially controlled by the center deflection of the specimen. When the desired center deflection was obtained the lateral loading was stopped and, if necessary, the lateral load was permitted to decrease slightly until the deflection of the specimen stopped. When the test included combined bending and axial loads the axial load was maintained at a constant magnitude throughout the test. The readings of the load, deflections and strains were taken when the deflection of the specimen stopped. While these readings were being made frequent checks were made of the axial load magnitude and, if necessary, the load was adjusted to its proper magnitude.

In all of the tests discussed in this report the specimen was loaded until either the limit of the apparatus for side and vertical deflection was reached or until the applied lateral load was essentially zero. In the case of the axially loaded specimens the test was stopped when the axial load had to be decreased to prevent further deflection of the specimen.

9. Test Results for Specimen 60 S 3 1 7.5

The load-deflection and moment-strain relationships in dimensionless form are shown in Figs. 7 and 8 respectively. In Table II the results of this test have been summarized. For this specimen the failure was by lateral buckling which limited the load carrying capacity. After the load capacity had reached its maximum the deflection was continued until the lateral deflection reached the limit of the apparatus.

During this continued application of deflection the load capacity did not decrease. The final deflected shape of the specimen is shown in Fig. 9.

10. Test Results for Specimen 20 S 3 1 7.5

In general, the 20 S specimen responded in the same manner as the longer specimen (60 S). Because of the shorter span and the resulting increase in lateral stability, the deformations noted were considerably larger than those for the longer specimen. Also it was possible to cause larger deformations without any indications of failure of the specimen. The final deflected shape of this specimen is shown in Fig. 10. In Figs. 7 and 8 are shown the dimensionless load-deflection and moment-strain relationships. From Table II it can be seen that the elastic limit moment was considerably lower than the moment determined for specimen 60 S 3 1 7.5. Referring to Table III it can be seen that this can be expected since the yield stress for the flange material is lower than the yield stress for the same location in the 60 S specimens.

11. Comparison of the Test Results for Specimen 60 S 3 1 7.5 and Specimen 20 S 3 1 7.5

In these specimens the only difference in their behavior should result from the higher shearing forces present in the shorter specimen. However, Fig. 7 shows quite clearly that the shearing forces did not result in any decided difference in the response and that the only differences probably resulted from the increased stability of the shorter specimen which permitted larger deformations.

12. Test Results for Specimen 60 S 6 1 12.5

The dimensionless load-deflection and moment-strain relationships for this specimen are shown in Figs. 11 and 12. The elastic limit values for the variables have been summarized in Table II. As in the case of the long 3 1 7.5 beam-column specimen the failure of this specimen was also by lateral buckling. However, in this case the buckling became so pronounced that the specimen moved from under the loading apparatus and resulted in complete unloading of the specimen, as shown in Fig. 13.

13. Test Results for Specimen 20 S 6 1 12.5

For this specimen the load-deflection and moment-strain relationships are shown in Figs. 11 and 12, respectively, where they may be readily compared with the relationships obtained for the specimen 60 S 6 1 12.5. Because of the greater stability of this specimen resulting from its shorter length the deformations were larger than those for the preceding specimen. However, the length was such that the shear stresses in the web of the section exceeded the shear yield point of the material and the inelastically stressed material eventually included the whole web of the test beam. The final failure of the specimen was by local buckling of the flanges. This local buckling was accompanied by some lateral deflection but did not result in an appreciable decrease in the capacity of the specimen. The final deflected shape of the specimen is shown in Fig. 14.

14. Test Results for Specimen 42 S 6 1 12.5

This was the first test of the 6 1 12.5 section subjected to combined bending and axial loads. From Table I the features of the test specimen are summarized. In Table II the test results have been summarized. In Fig. 11 the dimensionless load-deflection relationship for this specimen is shown. In this figure the load-deflection relationship has been compared with the relationship obtained for specimens 60 S 6 1 12.5 and 20 S 6 1 12.5.

In Fig. 15 the experimentally determined moment-strain relationship is shown. The flexural strain used in the moment-strain relationship is one-half of the difference between the top and bottom maximum fiber strains. In this figure the moment has been subdivided into the contribution of the axial and lateral loads to the bending moment. In Fig. 12 the dimensionless moment-strain relationship is shown in comparison with the moment-strain relationships for specimens 60 S 6 1 12.5 and 20 S 6 1 12.5. The failure of this specimen occurred by lateral buckling. The final deflected shape is shown in Fig. 16.

15. Test Results for Specimen 6Y0 S 6 1 12.5

This specimen was the first specimen in which the lateral load was applied in the weak direction. Because of the great lateral stability, complete failure of the specimen was not obtained before the usable limit of the apparatus was reached. Since this specimen was not severly damaged during the initial loading, it was reloaded in the opposite direction. The final deflected shape is shown in Fig. 17.

In the initial test of this specimen the load-deflection and moment-strain relationships were obtained. These relationships, in a dimensionless form, are shown in Figs. 18 and 19 respectively.

For the reload test only the load-deflection relationship was obtained. With this information the load-deflection relationship for the complete loading cycle was obtained and is shown in Fig. 20. From the figure it is obvious that the unloading curves are nearly at the same slope as the initial loading curve, decreasing slightly as the number of unloadings is increased. Also it can be seen that the initiation of a nonlinear load-deflection relationship for the reloading occurs at a smaller lateral load than was required in the initial loading.

16. Test Results for Specimen 2YO 6 1 12.5

As in the case of specimen 6YO 6 1 12.5 it was not possible to obtain sufficiently large deformations of the specimen to produce a decrease in its load carrying capacity. However, for this specimen the deflections were large enough to produce local buckling of the compression flanges. Also yielding occurred in the web of the section near the connection detail. The final deflected shape is shown in Fig. 21.

The load-deflection and moment-strain relationships, in a dimensionless form, are shown in Figs. 10 and 11.

17. Comparison of the Test Results for the 6 1 12.5 Beam-Columns

The tests performed thus far provide some information about the response of beam-columns with a 6 1 12.5 section, loaded to produce bending about either principal axis and the effect of an axial load for the case of bending about the strong axis. For the case of bending about the strong axis, (specimens 60 S 6 1 12.5 and 20 S 6 1 12.5). the most noticeable difference between the behaviors of the two specimens was in the magnitude of the shearing forces. In the short specimen, (20 S), these forces were sufficiently large for yielding to occur throughout the web of the specimen. From the moment-strain relationships shown in Fig. 12 the effect of the shear was to reduce the resisting moment for any particular magnitude of the maximum fiber strain. The remaining differences were generally determined by the lateral stiffness of the specimens which controlled when and how the failure developed.

The influence of an axial load on the response of a 6 1 12.5 beam with bending about the strong axis is shown in Figs. 11 and 12. In Fig. 11 the load-deflection relationships are shown for the three 6 1 12.5 beams. It can be seen that the effect of the axial load is to decrease the magnitude of the lateral load and to cause the lateral load to decrease when the center deflection became sufficiently large.

The moment-strain relationship does not present so concise an interpretation. In Fig. 12 it can be seen that, as expected, the moment-strain relationship for the axially loaded specimen lies beneath the relationship for specimen 60 S 6 1 12.5 during the initial portions of the inelastic action. However, when the maximum strain was greater than approximately four times the elastic limit strain the resisting moment for the axially beam-column was greater than the resisting moment for the 60 S specimen. This discrepancy can be explained by considering the mode of the failure of the specimens. In the case of specimen 60 S 6 1 12.5 the lateral buckling failure started to develope soon after the elastic limit was exceeded. This failure did not result in a decrease in capacity but did prevent the lateral load from increasing above the load obtained when the maximum strain was approximately three times the elastic limit strain. For the axially loaded specimen the lateral buckling mode of failure did not develope until the maximum strains were large, the increased stability of specimen 42 S 6 1 12.5 resulting from the decreased length of the specimen.

When the specimens were loaded in the weak direction the length of the specimen, (and thus the shear forces), had no apparent influence on the moment-strain relationship. There were, however, some discrepancies in the load-deflection relationships, shown in Fig. 18. The reason for these discrepancies is not yet known.

18. Test Results for Specimen 40 S 4 M 13.0

The dimensionless load-deflection and moment-strain relationships for this specimen are shown in Figs. 22 and 23, respectively. It should be noted that the moment capacity for this specimen is larger than would normally be expected for this particular cross section shape since wide flange and I cross sections generally have approximately a 15 to 20 per cent increase in moment capacity over the elastic limit moment. However, for this specimen the indicated moment capacity is approximately 40 per cent greater than the elastic limit moment. However, for this specimen the stress-strain curves obtained from static tension tests indicate that the material has been subjected to cold rolling or working during fabrication. In Table III the effects of this cold working are indicated by the relatively high yield stresses occurring throughout the cross section. The effects of this cold working, as reflected in the stress strain curves of the material, could be included in the theoretical predictions of the specimen response. The final failure of this specimen, as shown in Fig. 24, was by lateral buckling.

19. Test Results for Specimen 415 S 4 M 13.0

In this test the specimen was subjected to combined bending and axial load. For this test the control of the axial load was not sufficiently sensitive and the load fluctuated approximately ten per cent about its average value of 16.6 kips per sq. in. However, the test results were not apparently changed by these variations. From Table III it can be noted that the material properties for this specimen are approximately the same in both magnitude and distribution as the properties for specimen 40 S 4 M 13.0.

In Figs. 22 and 23 are shown the dimensionless load-deflection and moment-strain relationships, respectively. These relationships can be readily compared with the results for specimen 40 S 4 M 13.0.

In Fig. 25 the experimentally determined moment strain relationship is shown with the bending moment subdivided into the contribution of the axial load and the lateral load to the bending moment. The flexural strain used in the moment-strain relationship is one-half of the differences in the maximum fiber strains for the top and bottom faces.

For this specimen the failure was by lateral buckling which began to develop soon after the lateral load reached its maximum value. As the deformation of the specimen continued the lateral load gradually decreased and the sideward deflection of the specimen increased. When the sideward deflection became appreciable the lateral load decreased rapidly to zero and the deflection of the specimen was continued with the axial load only. After this occurred the deflection could be stopped only be decreasing the applied axial load. The final deflected shape is shown in Fig. 26.

20. Comparison of the Test Results for Specimens 40 S 4 M 13.0 and 41S S 4 M 13.0

In Figs. 22 and 23 the load-deflection and moment-strain relationships for these specimens are shown. In both of these figures the elastic limit load, moment, center deflection or strain was the value obtained for the test of specimen 40 S 4 M 13.0.

From Fig. 22 it can be seen that the major effect of the axial load was to reduce the maximum applied load and to cause the applied lateral load to decrease soon after the maximum load was applied. In addition, the initial elastic portion of the load-deflection relationship is considerably shorter for the axially loaded specimen. This slope is slightly less than the slope for the specimen without the axial load. Another difference is in the response of the specimens during the failure. Without the axial load the capacity of the specimen did not decrease during the development of the lateral buckling failure. When the axial load was added the combination of the lateral and sideward deflections resulted in rather marked decreases in the lateral capacity of the specimen.

The moment-strain relationships shown in Fig. 23 also reveal the influence of the axial load on the response of the specimen. The most noticeable feature is that the relationship for the axially loaded specimen lies well below the relationship for the case of no axial load.

21. Comparison of the Test Results

In comparing the results of the tests on the beam-column specimens certain features of the specimen response were of a general nature and warrant discussion in a general qualitative nature. For all of these tests which were performed on specimens loaded in the strong direction the final failure was by lateral buckling which developed after the elastic limit had been passed. In the case of the short specimens, (20 S 3 1 7.5 and 20 S 6 1 12.5), the lateral buckling was accompanied by, and possibly preceeded by, local buckling of the compression flanges. This local buckling for the 6 1 12.5 probably triggered the final lateral buckling by occurring in an antisymmetric manner. When the beam-columns were loaded in the weak direction it was not possible to deform the specimens sufficiently for a decrease in load carrying capacity even though the compression flanges, in the case of the shorter specimens, did buckle locally. This indicates, that for these tests and for the amounts of deformations obtained here, lateral buckling is the general mode of failure and that local buckling induces failure by providing a loss in symmetry of the cross section which results in the lateral buckling mode.

In addition to the character of the failure some comments can be made about the general form of the inelastically deformed regions of the beam columns. In this series of tests, the "plastic hinges" appearing exhibit certain characteristics which reflect the loading conditions. When the specimens were reasonably long and the effects of shear were small the hinges noted by means of whitewash were nearly perpendicular to the axis of the specimen and existed over an appreciable length of specimen. This is shown in Fig. 27-a. When the shearing forces increased the character of the hinge also changed. For large shear stresses present in specimen 20 S 6 1 12.5 the hinge exhibited a combination of the flexural yielding and of shear yielding in the regions of high moment. When the moment decreased the yielding became parallel to the axis of the specimen and occurred only in the web of the section. This is shown in Fig. 27-b. For the axial load tests run thus far the length of the specimens was such that shear forces were small. Thus the hinge formed in these specimens should be primarily flexural in character. In Fig. 27-c the hinge is shown. From this figure it can be seen that this hinge differed from the one obtained without the axial load (Fig. 27-a) by the increased penetration from the compression face.

In all of the cases mentioned above the hinge did not form at the face of the stub but occurred at some distance from this face. In general the location of the point of the hinge could be taken as approximately one half of the depth of the web from the face of the stub. The axial load tends to push the hinge point towards the face of the stub, the amount being dependent on the magnitude of the axial load.

Thus far in the program only two tests have been performed with combined axial and bending loads. In both of these tests the effect of the axial load on the load-deflection relationship was appreciable. As shown in Figs. 11 and 22 the axial load reduces the elastic stiffness of the beam-columns. This reduction in stiffness is accompanied by a decrease in the maximum load and a decay of the applied lateral load after the maximum load has been applied. The rate of decay is apparently a function of the magnitude of the axial load combined with the shape of the cross section and the mechanical properties of the material. The influence of the axial load is realized in two ways. First the moment strain relationship is affected by the axial load and, second, the axial load also contributes to the bending moments along the specimen. In Figs. 12 and 23 the effect of the axial load on the moment strain relationship is shown. In Fig. 23 the influence of the axial load is large while in Fig. 12, the influence of the axial load is small. Thus, for the 6 1 12.5 section, the axial load will be only slightly important for its influence on the moment-strain relationship if the axial load is small and within the range generally found in actual structures. However, the effect of the axial load on the load-deflection relationship may be large since the contribution of the axial load to the bending moment is a function of the deflections of the specimen, and, once the maximum moment capacity is approached the lateral load must be decreased for equilibrium of the beam-column.

From the load-deflection relationships it can be seen that, in the cases where no axial load was applied, the lateral load increased with the deflection until the failure condition was developed. The rate of increase of the lateral load was less than the rate for the original elastic loading condition. When an axial load was added, the rate of increase of the lateral load after the elastic limit was exceeded was found to be less than the rate for specimens tested without the axial load. When the axial load was large, as in the case of specimen 42 S 4 M 13.0, the lateral load decreased with increasing deflection soon after the maximum load was applied to the specimen.

In the future more detailed information on the influence of the axial load will be available. At this time some work is being done on the effect of the axial load on the moment-strain relationship for several types of cross section which will be representative of the columns which might be used in full scale structures. Also included in this study is the determination of the influence of an axial load on the load-deflection relationship.

III. NONSYMMETRICAL BENDING

During the past year an investigation was started on the response of beam-column type specimens to loads applied in a direction which results in bending about both of the principal axes of the section. Thus far the investigation has been concerned with the analytical treatment of the problem. From the results of the analytical study the testing plan has been arranged and the apparatus required for these tests has been designed.

From the analytical study it has become apparent that the effects of nonsymmetrical loading conditions can best be presented in the form of an interaction diagram which will relate the bending moments about the principal axes of the section as a function of the maximum fiber strain or the depth of penetration of the inelastically strained material. For the case of ideal plasticity, for which yielding occurs at a constant stress without strain hardening, as shown in Fig. 28, the moments about the principal axes are related to the properties of the plastic and elastic portions of the cross sectional area by the following relationships:

$$(1) \quad M_x = f_e \left[(I_x^e \cos \phi)/v_e - (I_{xy}^e \sin \phi)/v_e + Q_x^p \right]$$

$$(2) \quad M_y = f_e \left[(I_{xy}^e \cos \phi)/v_e - (I_y^e \sin \phi)/v_e + Q_y^p \right]$$

where:

ϕ = the angle between the neutral axis and the "x-x" axis

f_e = the yield stress of the material

v_e = the distance from the neutral axis to the depth of penetration of the inelastically strained material

I = the second moment of the area about the axis indicated by the subscript

Q = the first moment of the area about the axis indicated by the subscript

Superscript e refers to the area of the cross section which is elastic

Superscript p refers to the area of the cross section which is inelastically strained

With equations (1) and (2) the relationship between the bending moments can be determined as a function of the angle of rotation of the neutral axis. In Fig. 28 the interaction diagram for an idealized 6 B 15.5 section is shown. On this diagram the lines of impending inelastic response and the line for which the entire cross section is inelastically deformed is shown. In addition to these interaction lines, curves for which the angle of inclination of the neutral axis to the "x-x" axis remains a constant are shown. It should be noted that lines of constant slope on the interaction diagram refer to paths of constant direction of loading. From the diagram it can be seen that the inclination of the neutral axis can vary considerably during a test in which the load is applied in a constant direction.

Because of the rotation of the neutral axis as a function of the maximum fiber strain, it has been decided that, for the experimental investigation, the position of the neutral axis will be set at a definite position at the center of the specimen. At all other points along the axis of the specimen the position of the neutral axis will be determined by the direction of the load application, this direction being determined by the ratio of the loads required to maintain the position of the neutral axis at the center of the specimen. By using this method of testing the direction of the load application will vary throughout the test as can be seen by the variation of the angle of load application for a path of constant angle of inclination of the neutral axis as shown in the interaction diagram. The above testing method was selected because, by constraining the position of the neutral axis at the center of the specimen, the specimen can be loaded by a single jack pulling the specimen, the constraining forces being supplied by a guide system. To determine the loads applied to the specimen both the horizontal and vertical components of the end reactions will be measured. This procedure is considerably simpler than attempting to test the specimens under conditions of constant direction of load application since, if the direction of the loading were maintained at a constant value, some method would be required to have the loading position move with the specimen throughout the test. This adjustment would be required since the specimen deflects along a path differing from the line of action of the applied force. It is felt that, if the analysis can be confirmed for the condition of constrained position of the neutral axis, the analysis will also be valid for conditions of constant direction of load application.

TABLE I

Comparison of the Beam-Column Specimens

Specimen Number	Test Section (1)	Direction of Bending (2)	Span Between Reactions	Connection Detail (3)	Axial Load (4)
60 S 3 I 7.5	3 I 7.5	"x-x"	12 ft.-2 in.	"A"	NONE
20 S 3 I 7.5	3 I 7.5	"x-x"	4 ft.-2 in.	"A"	NONE
60 S 6 I 12.5	6 I 12.5	"x-x"	12 ft.-2 in.	"A"	NONE
20 S 6 I 12.5	6 I 12.5	"x-x"	4 ft.-2 in.	"A"	NONE
42 S 6 I 12.5	6 I 12.5	"x-x"	8 ft.-2 in.	"A"	10.6 ksi
6Y0 S 6 I 12.5	6 I 12.5	"y-y"	12 ft.-2 in.	"A"	NONE
2Y0 S 6 I 12.5	6 I 12.5	"y-y"	4 ft.-2 in.	"A"	NONE
40 S 4 M 13.0	4 M 13.0	"x-x"	8 ft.-2 in.	"A"	NONE
415 S 4 M 13.0	4 M 13.0	"x-x"	8 ft.-2 in.	"A"	16.6 ksi

(1) and (2) Notation from the AISC Steel Construction Handbook.
(3) A detail drawing of this connection is shown in Fig. 1
(4) This is the nominal axial load in kips per square inch based on the cross section areas from the AISC Steel Construction Handbook.

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TABLE II

Summary of the Beam-Column Test Results

Specimen Number	Elastic Limit Load kips	Max. Load kips	Elastic Limit Bending Moment in.-kips	Max. Bending Moment in.-kips	Elastic Limit Center Deflection in.	Max. Center Deflection in.	Mode of Failure
60 S 3 I 7.5	2.12	2.73	73.7	92.8	1.12	11.0	Lateral
20 S 3 I 7.5	6.0	13.25	60.0	132.5	0.175	4.72	Lateral
60 S 6 I 12.5	7.06	9.44	240	322	0.71	7.34	Lateral
20 S 6 I 12.5	26.20	52.25	262	526	0.123	3.62	Local
42 S 6 I 12.5	8.73	13.85	207	332	0.30	2.44	Lateral
6YO S 6 I 12.5	1.59	2.89	54.0	108.4	1.37	14.2	None Obtained
2YO S 6 I 12.5	5.37	13.5	53.7	135.0	0.276	6.31	Local
40 S 4 M 13.0	9.62	15.8	212	348	0.61	4.28	Lateral
415 S 4 M 13.0	6.09	8.25	170	226	0.55	3.24	Lateral

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TABLE III

Summary of the Yield Stresses Obtained from Tension Coupons
(The yield stress is based on a 0.2 percent offset)

Specimen Number	Location of the Coupon in the Cross Section			
	Near Tips of the Flanges (Av. of 4 Coupons)	Center of Flange (Av. of 2 Coupons)	Junction of Flange and Web (Av. of 2 Coupons)	Center of Web
60 S 3 I 7.5	39,200 psi	37,300 psi	- - - - -	34,900 psi
20 S 3 I 7.5	26,600 psi	35,700 psi	- - - - -	36,800 psi
60 S 6 I 12.5	39,100 psi	38,000 psi	46,800 psi	42,700 psi
20 S 6 I 12.5	39,600 psi	37,800 psi	48,000 psi	42,300 psi
42 S 6 I 12.5	48,200 psi	39,200 psi	52,700 psi	46,600 psi
6Y0 S 6 I 12.5	45,500 psi	46,800 psi	51,000 psi	- - - - -
2Y0 S 6 I 12.5	48,900 psi	37,600 psi	50,800 psi	- - - - -
40 S 4 M 13.0	57,100 psi	42,600 psi	67,500 psi	66,000 psi
415 S 4 M 13.0	58,200 psi	38,300 psi	69,400 psi	59,500 psi

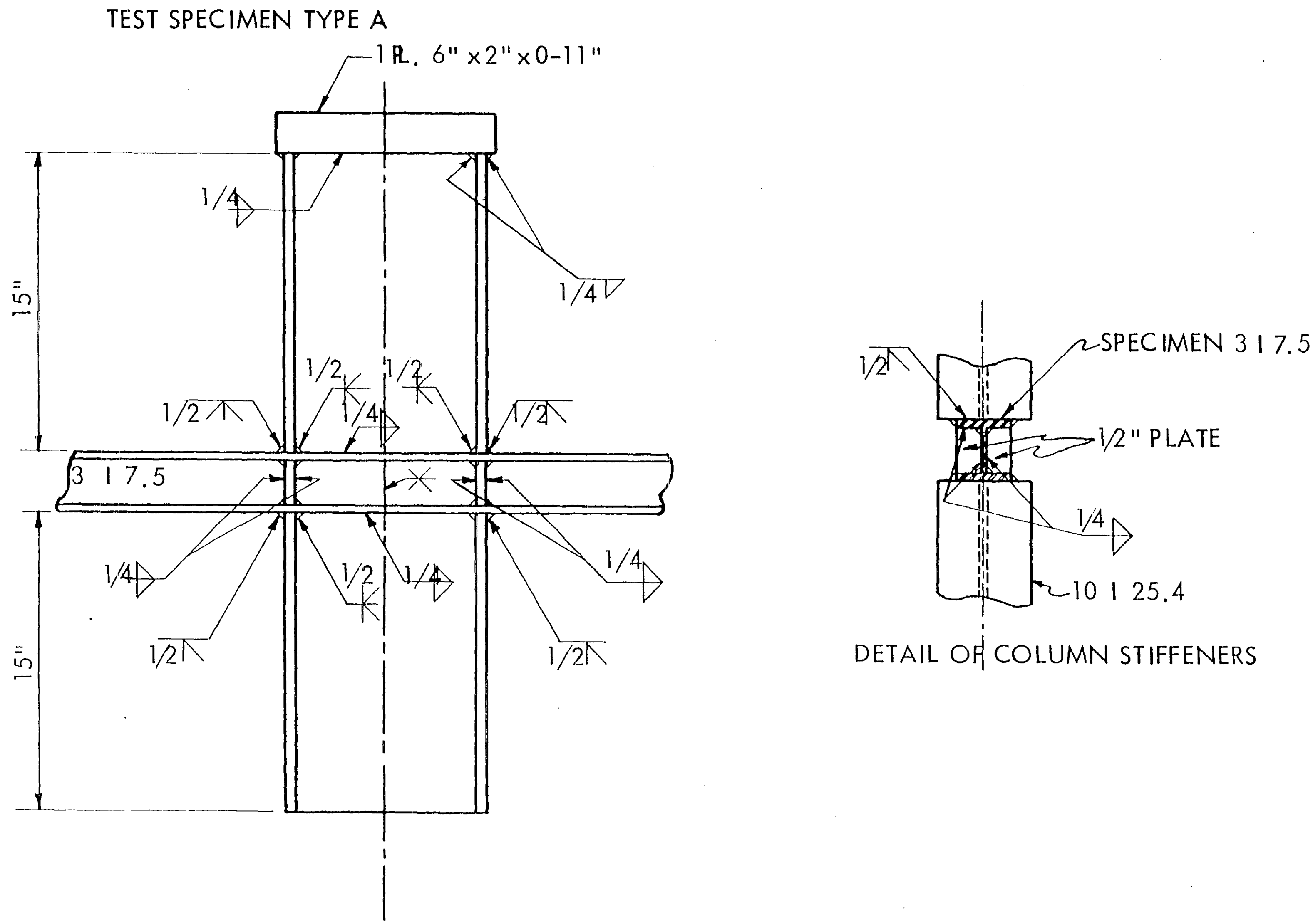


FIGURE 1. DETAIL OF THE STUB CONNECTION

SCALE 1 1/2 = 1'-0"

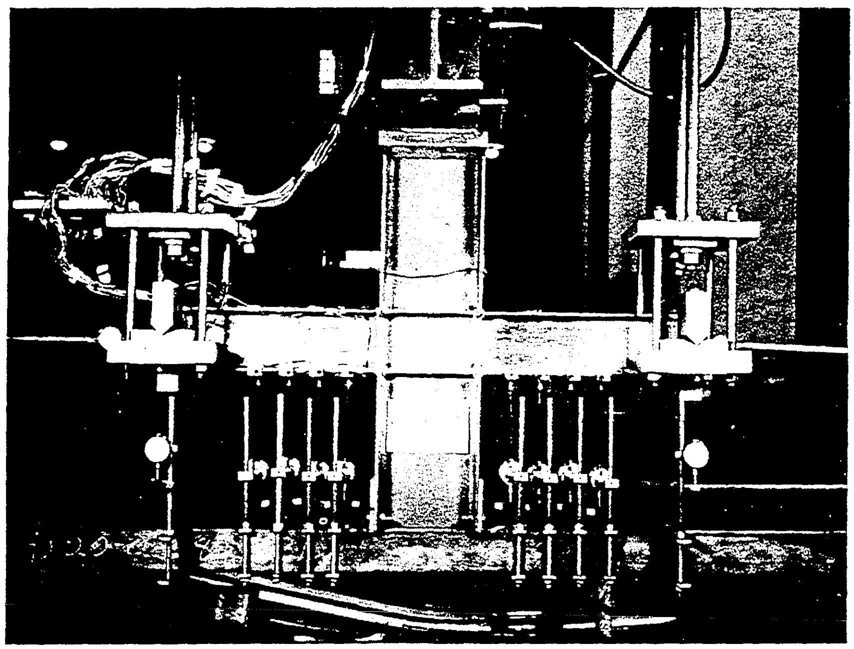


Fig. 2. Assembled Test Apparatus

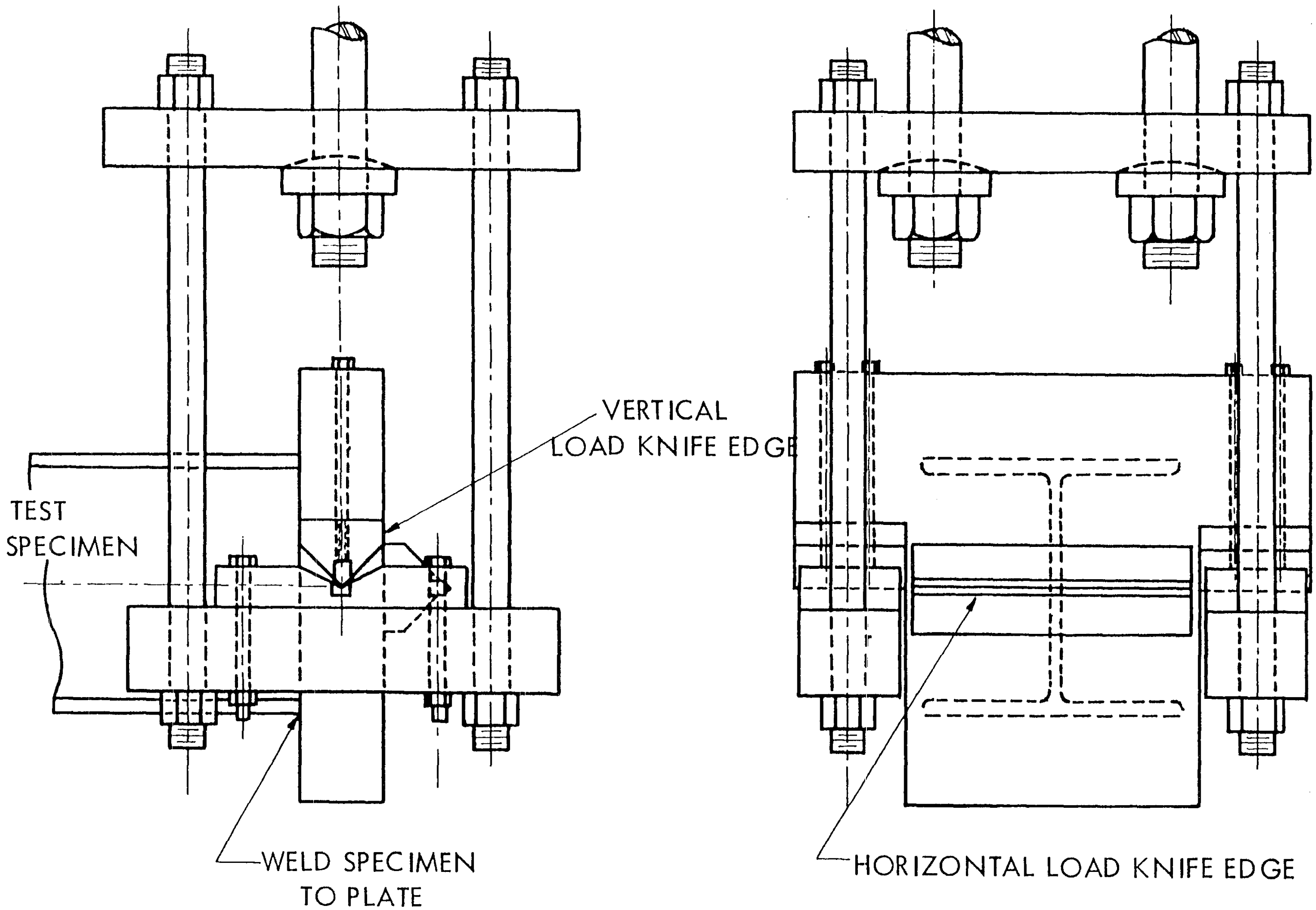


FIGURE 3. END REACTION SYSTEM   END ASSEMBLY   SCALE 1/4" = 1"

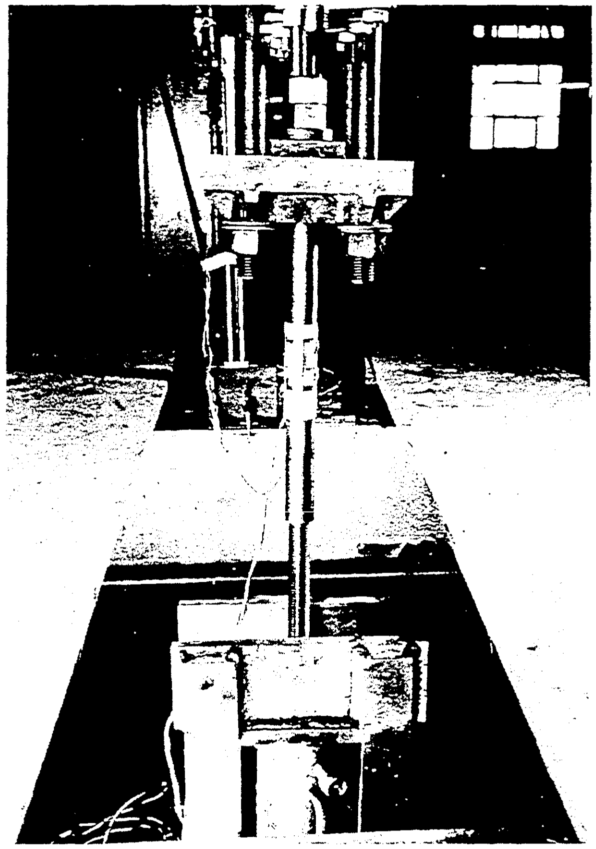


Fig. 4. Tension Jacking System

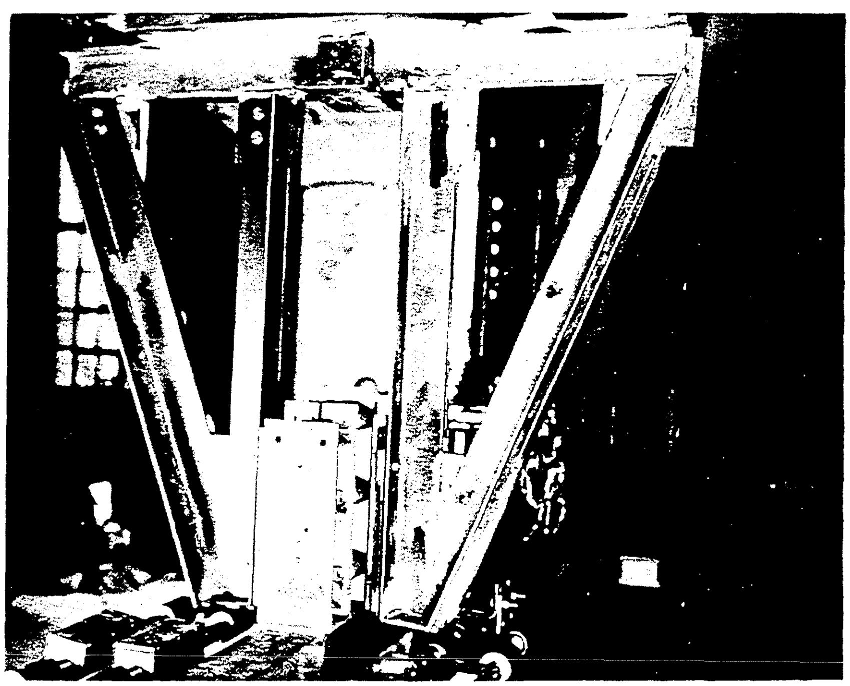


Fig. 5. Center Restraining System

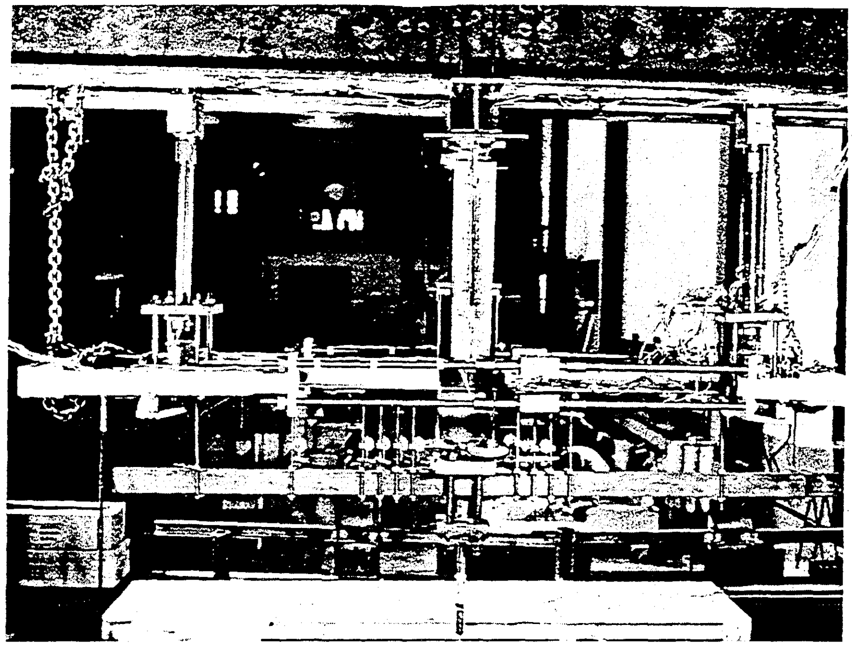


Fig. 6-a. Assembled Axial Load Apparatus

Fig. 6-b. Axial Load Jacking System

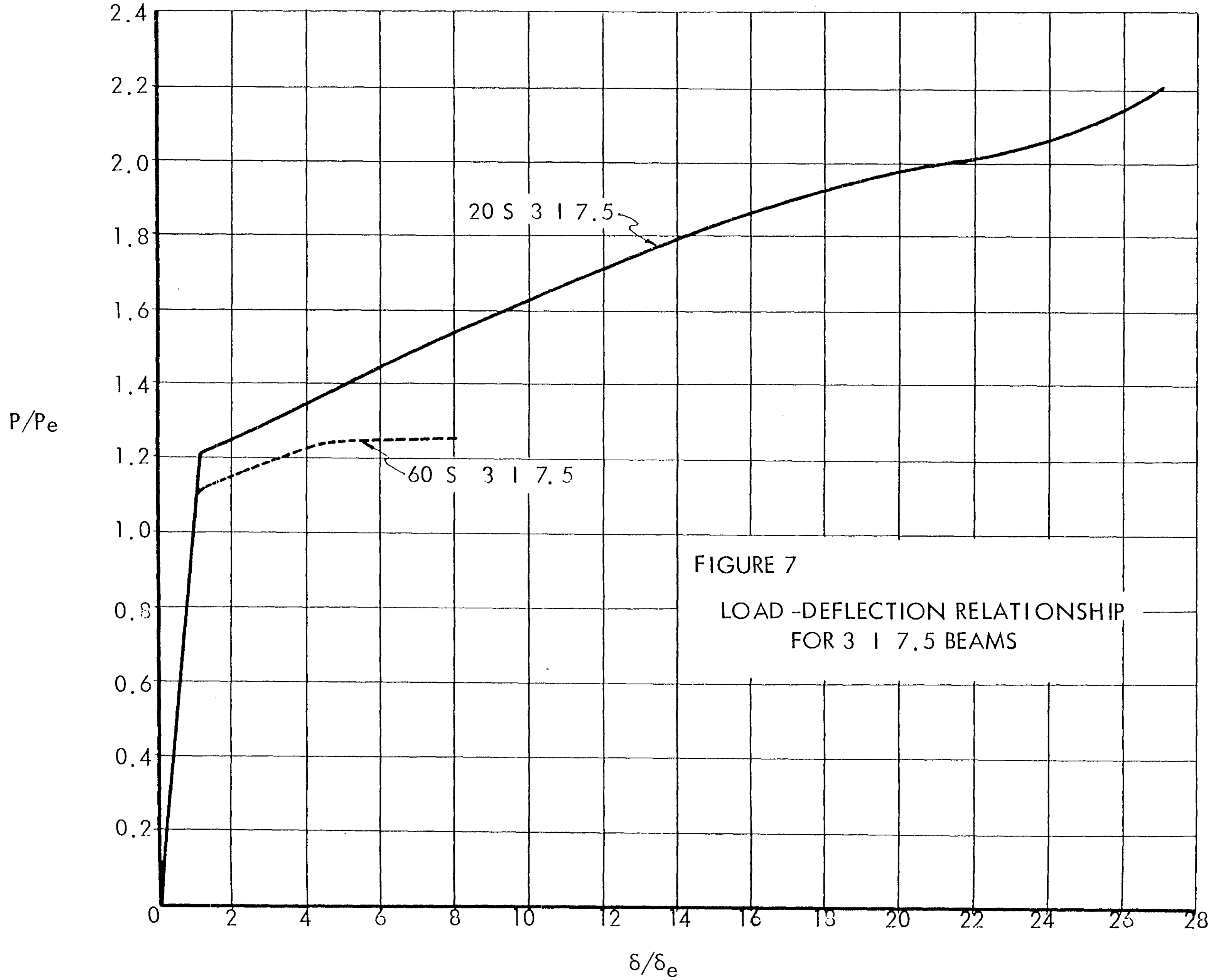


FIGURE 7

LOAD-DEFLECTION RELATIONSHIP FOR 3 I 7.5 BEAMS

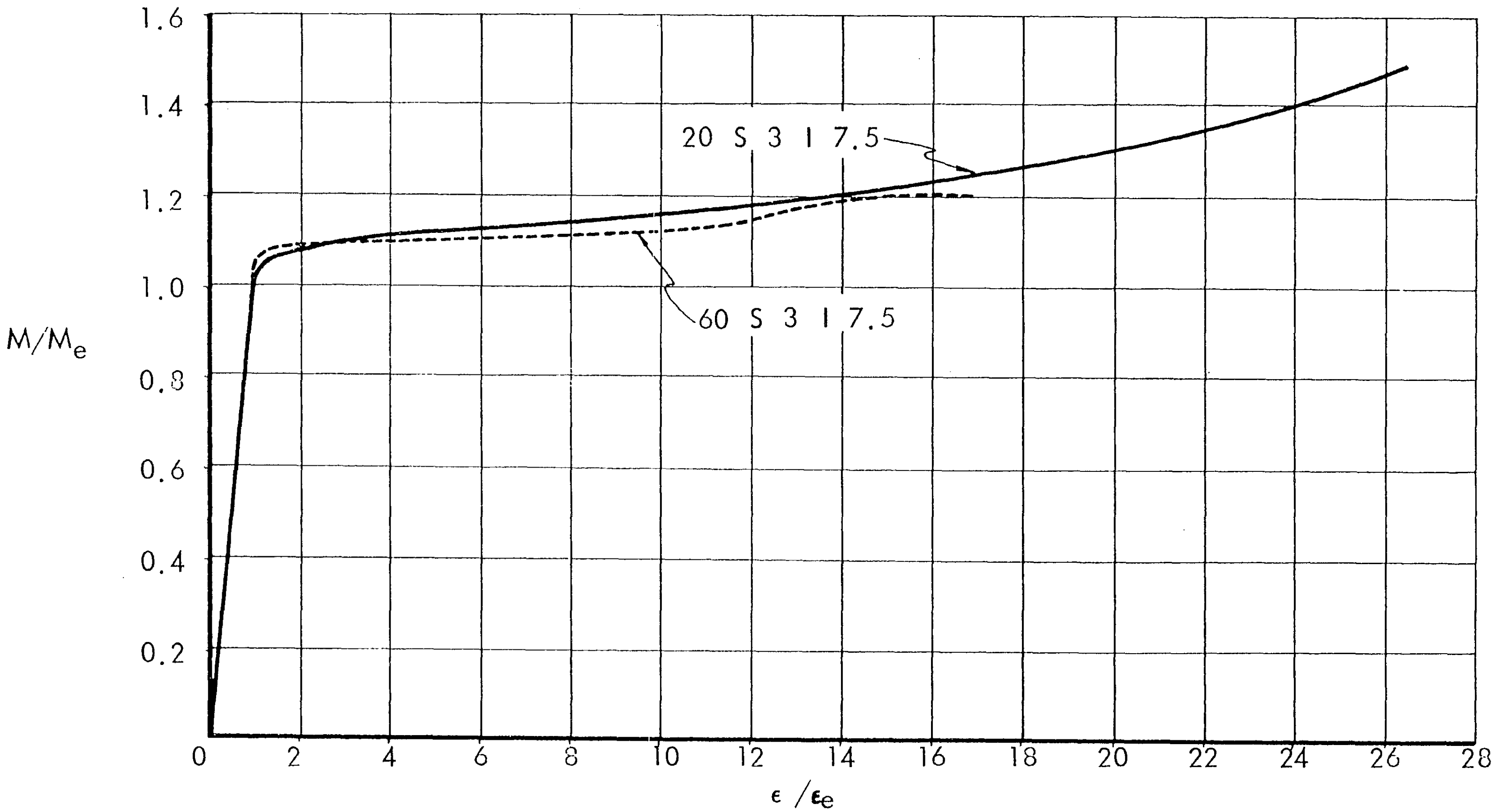


FIGURE 8. MOMENT-STRAIN RELATIONSHIPS FOR 3 I 7.5 BEAMS

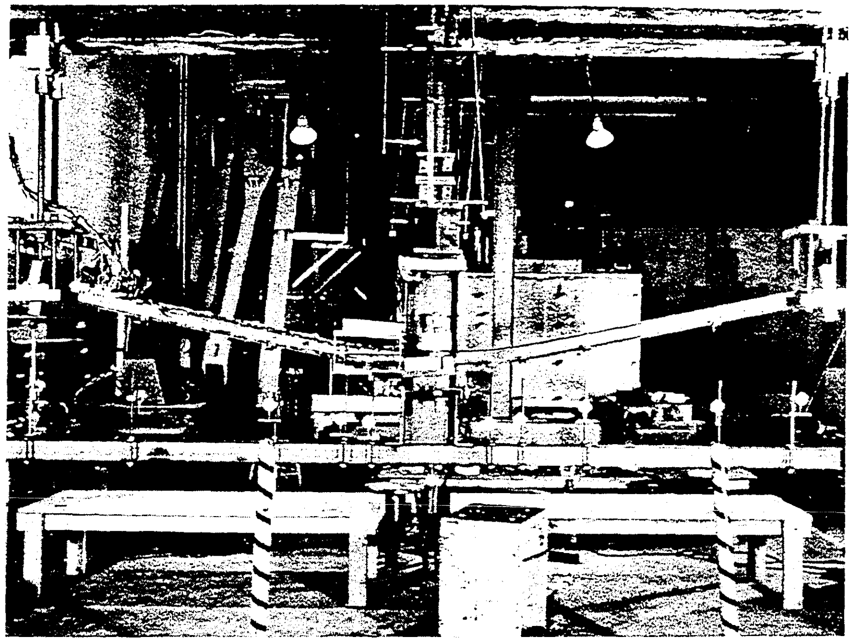


Fig. 9-a. Deflected Shape of Specimen 60 S 3 I 7.5

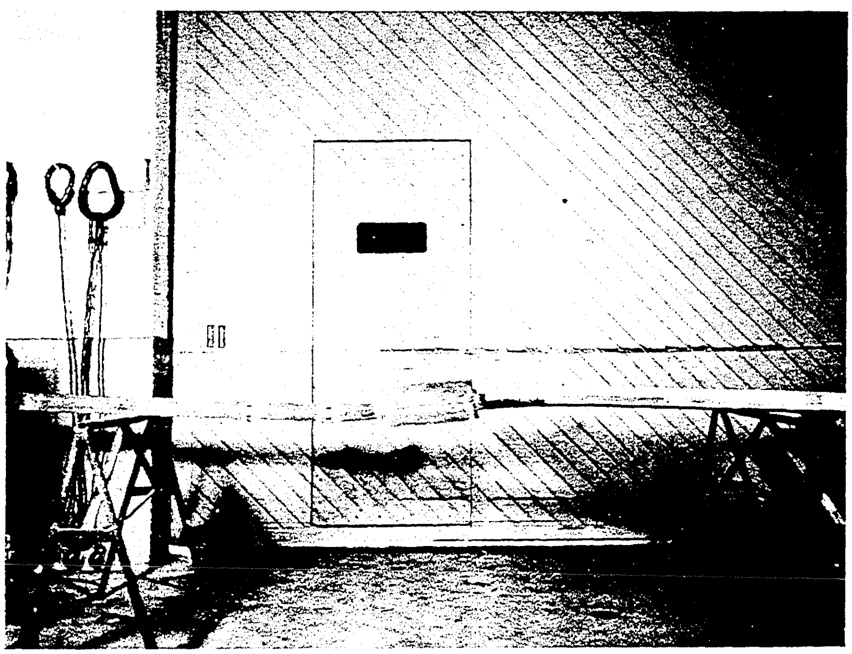


Fig. 9-b. Plan View of Specimen 60 S 3 I 7.5

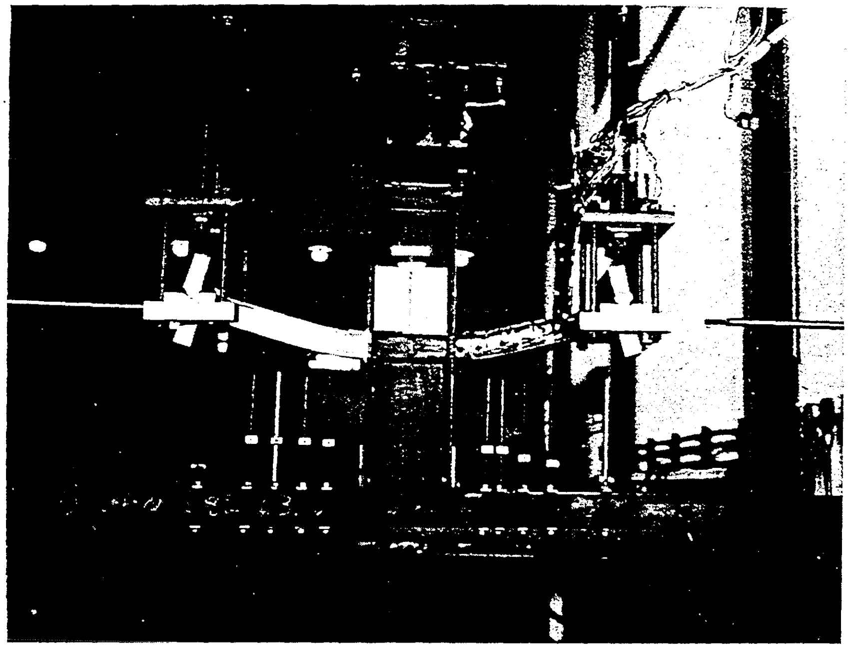


Fig. 10-a. Final Deflected Shape of Specimen 20 S 3 I 7.5

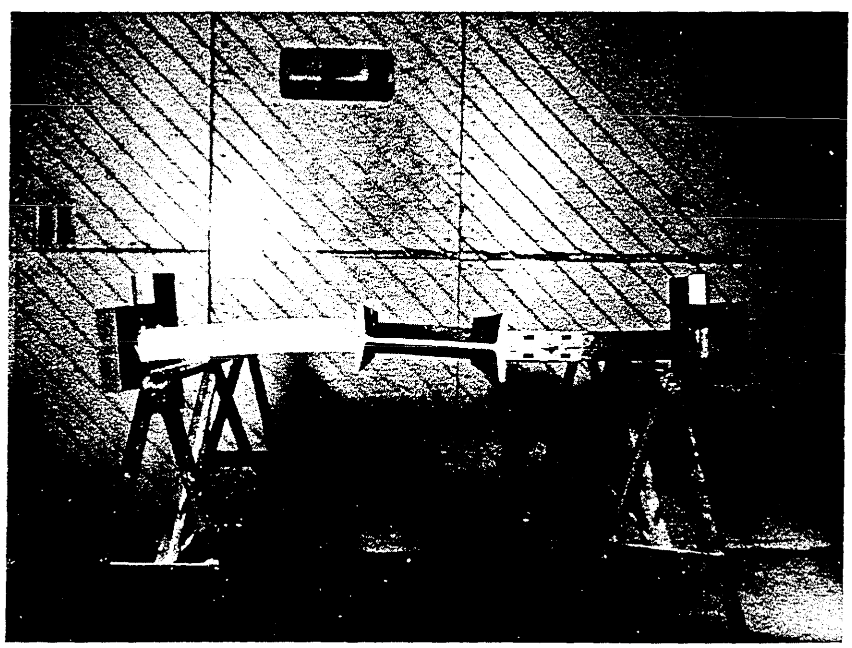


Fig. 10-b. Plan View of Specimen 20 S 3 I 7.5

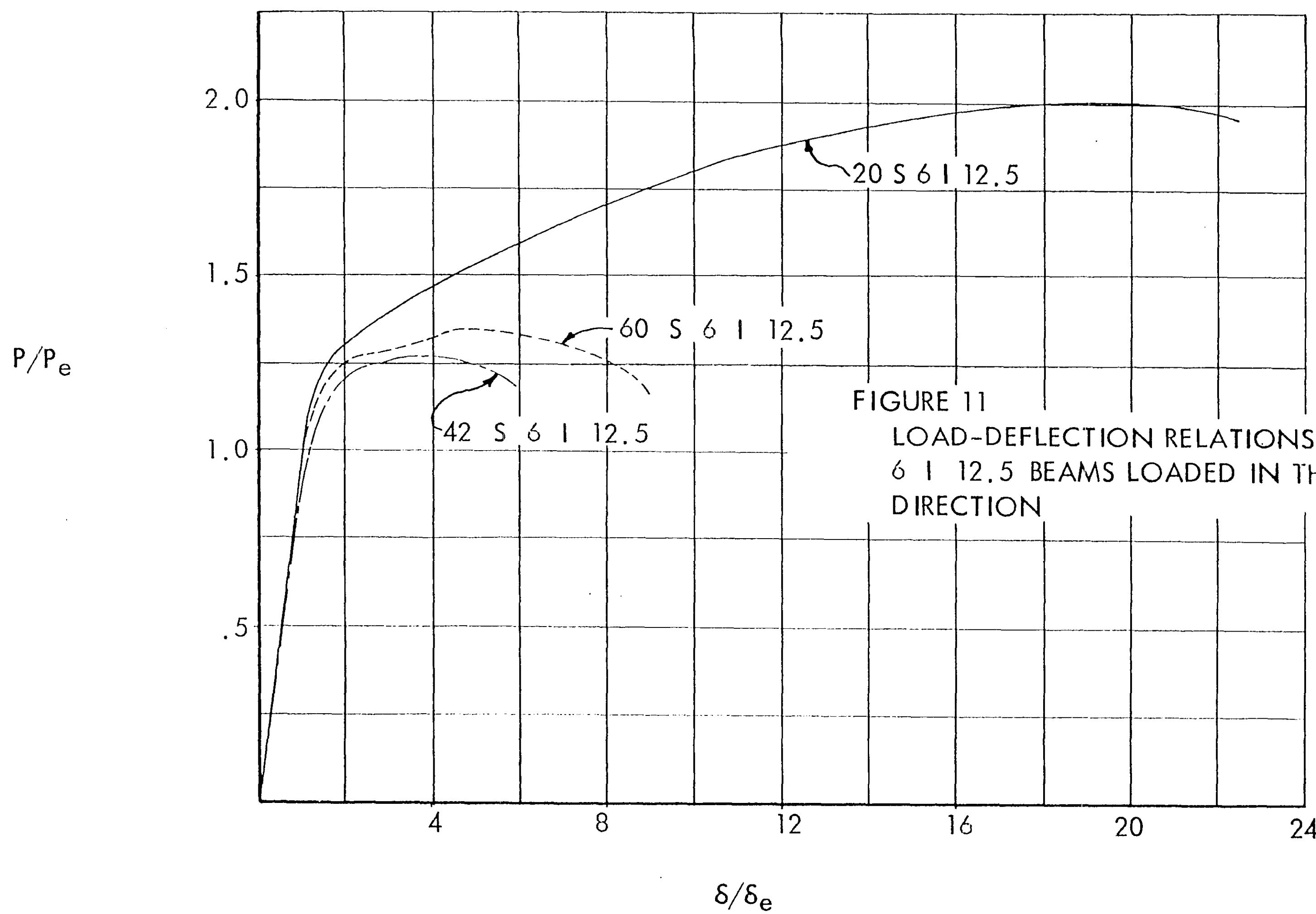


FIGURE 11
LOAD-DEFLECTION RELATIONSHIP FOR 6 I 12.5 BEAMS LOADED IN THE STRONG DIRECTION

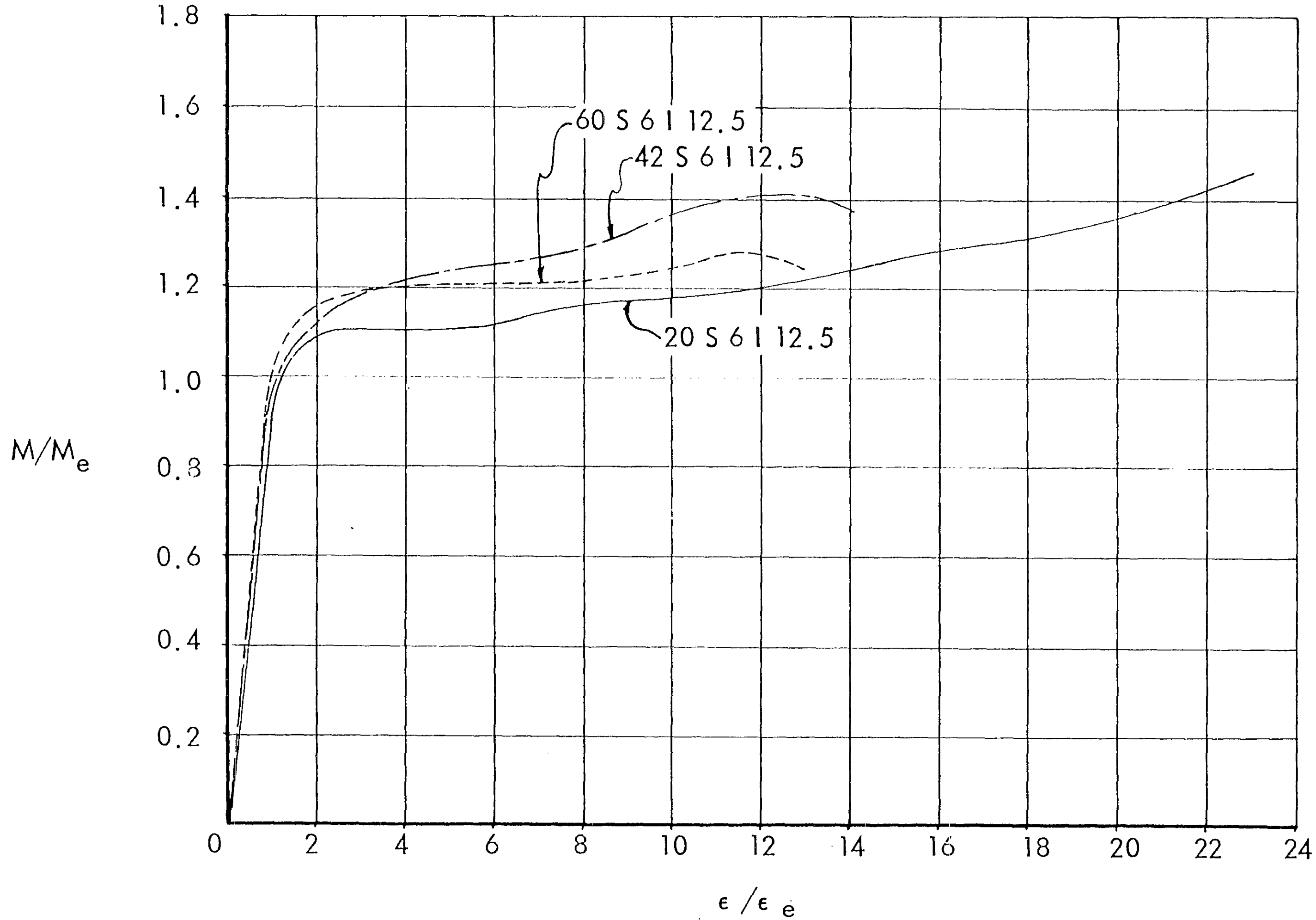


FIGURE 12
MOMENT STRAIN RELATIONSHIP FOR THE
6 I 12.5 BEAMS LOADED IN THE STRONG DIRECTION

RESTRICTED

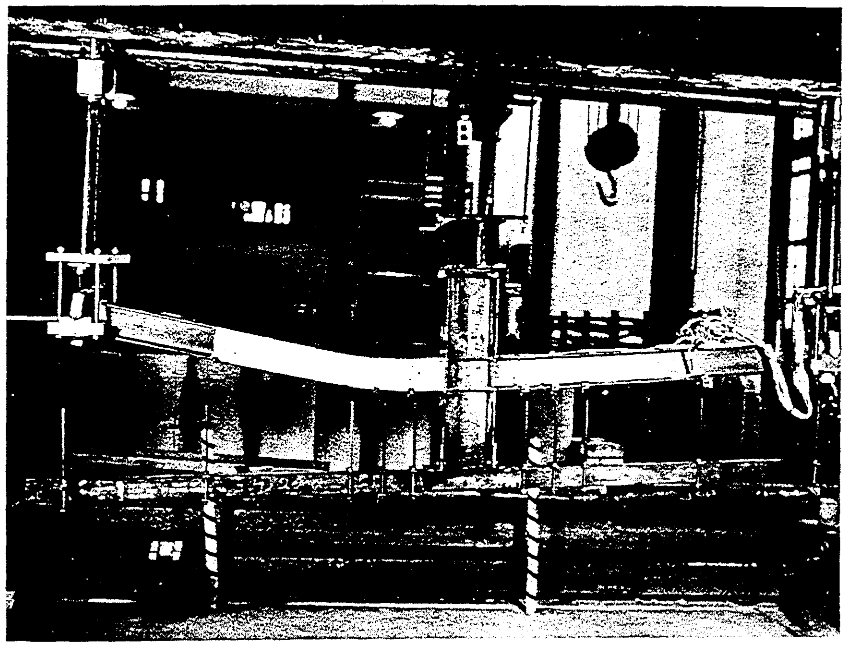


Fig. 13-a. Final Deflected Shape of Specimen 60 S 6 I 12.5

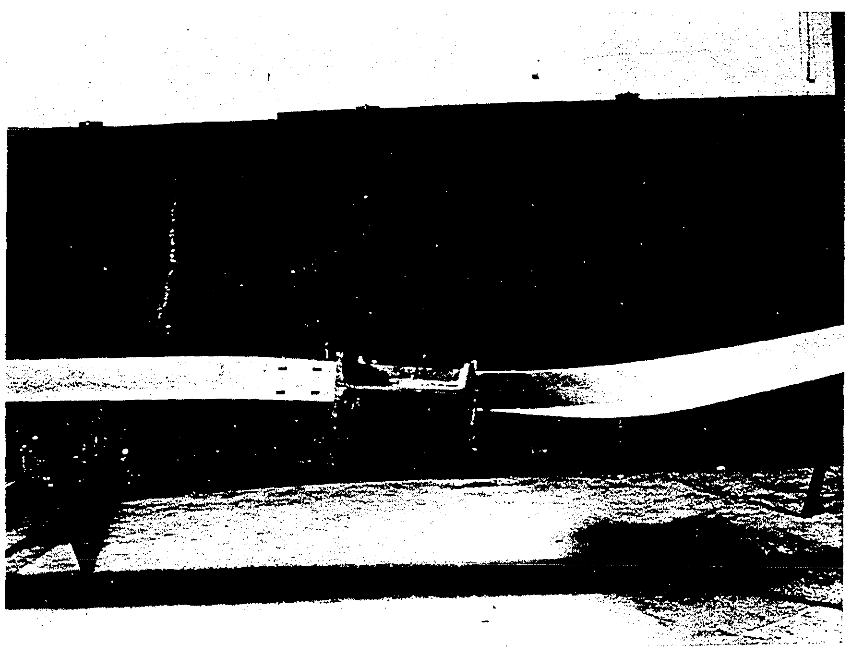


Fig. 13-b. Plan View of Specimen 60 S 6 I 12.5

Fig. 14-a. Final Deflected Shape of Specimen 20 S 6 I 12.5

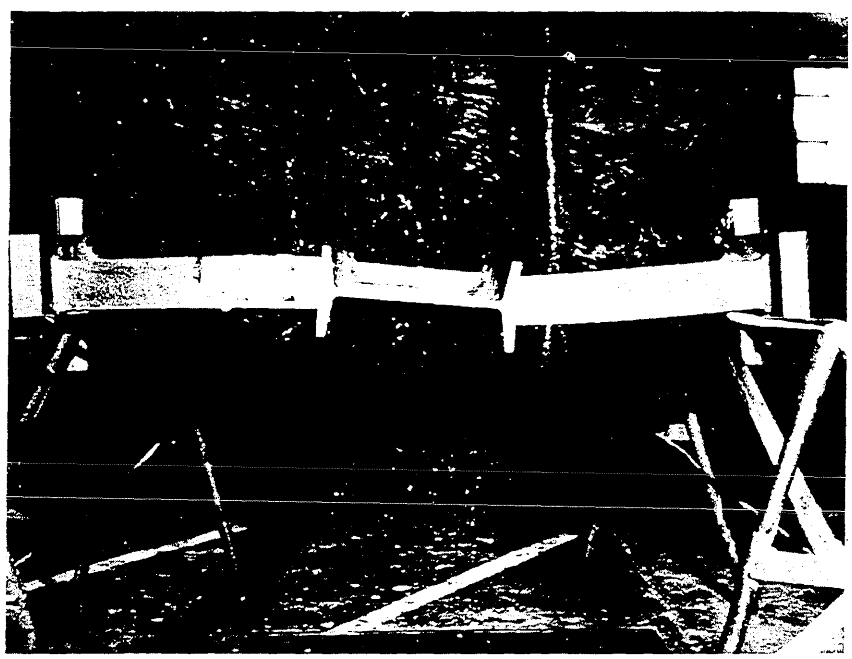
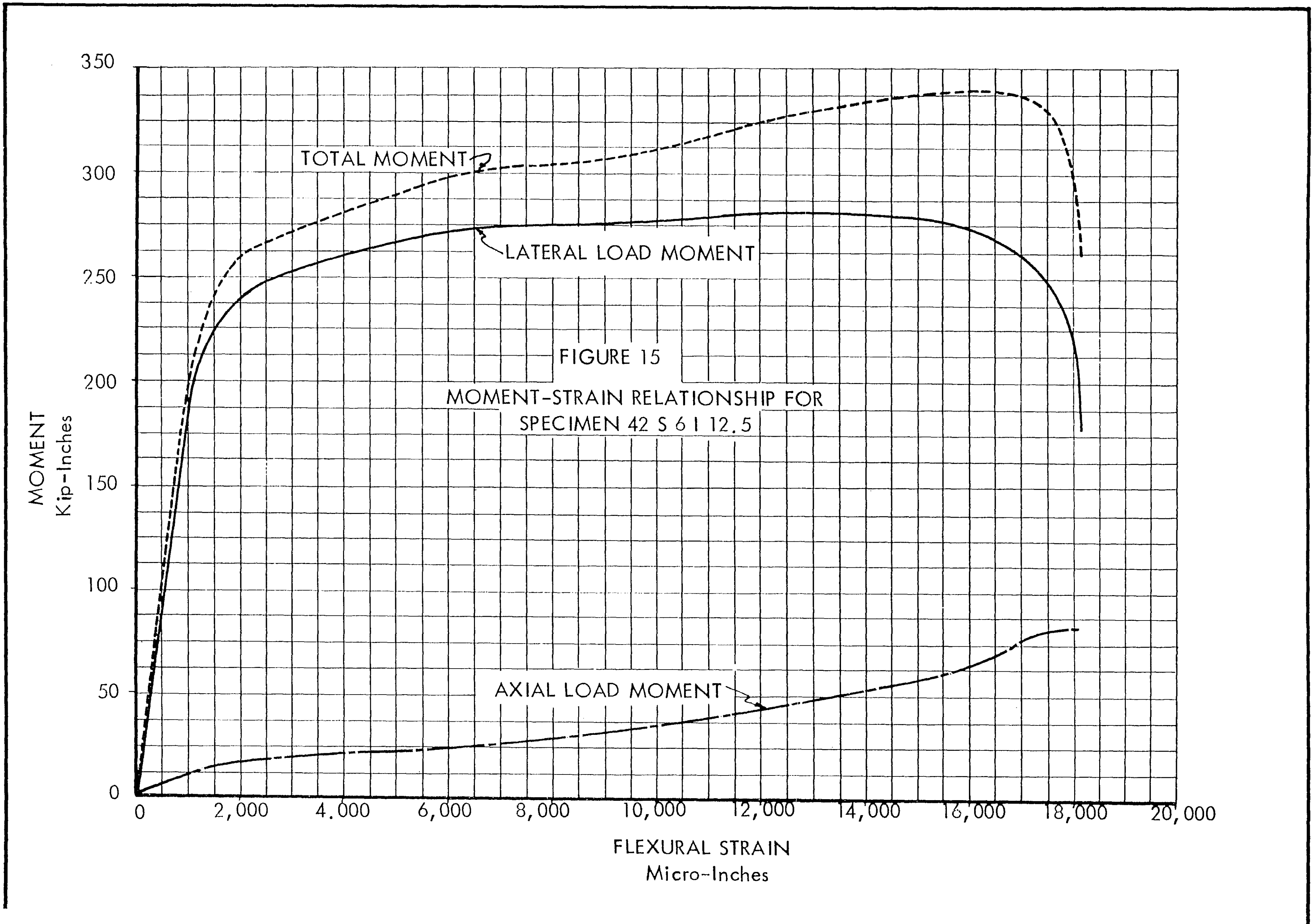


Fig. 14 b. Plan View of Specimen 20 S 6 I 12.5


TOTAL MOMENT
LATERAL LOAD MOMENT
AXIAL LOAD MOMENT
FIGURE 15
MOMENT-STRAIN RELATIONSHIP FOR
SPECIMEN 42 S 6 I 12.5
MOMENT
Kip-Inches
FLEXURAL STRAIN
Micro-Inches
0
50
100
150
200
250
300
350
2,000
4,000
6,000
8,000
10,000
12,000
14,000
16,000
18,000
20,000

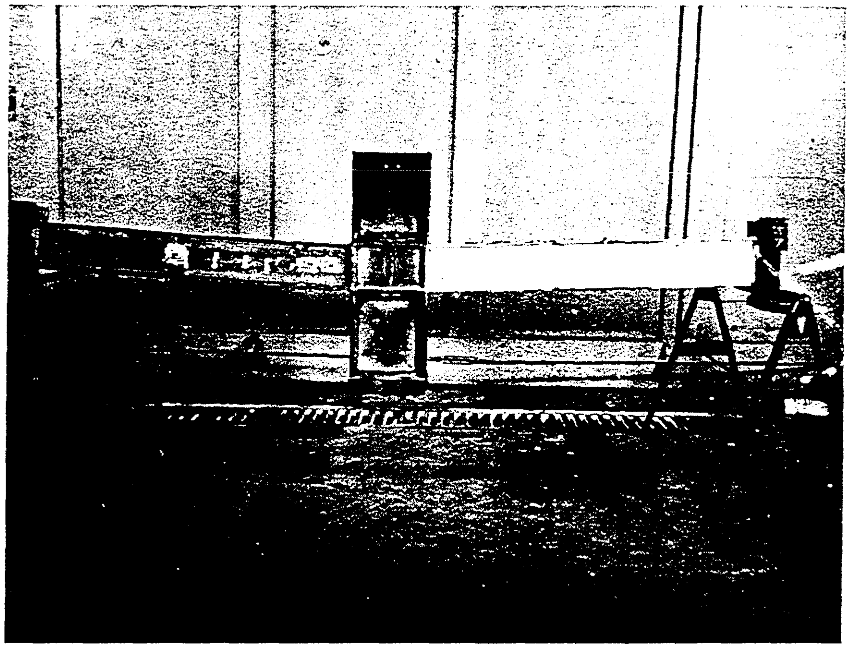


Fig. 16-a. Final Deflected Shape of Specimen 42 S 6 I 12.5

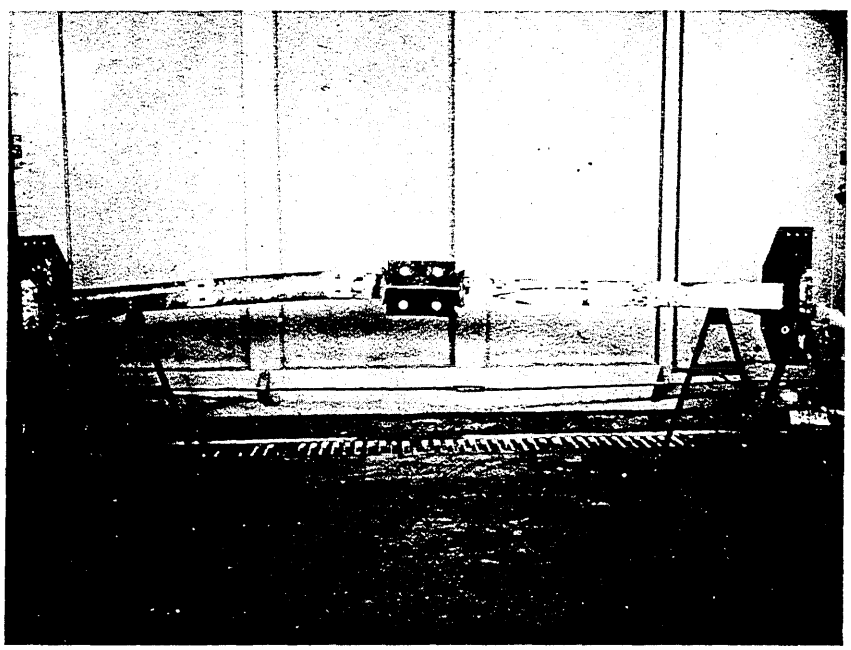


Fig. 16-b. Plan View of Specimen 42 S 6 I 12.5

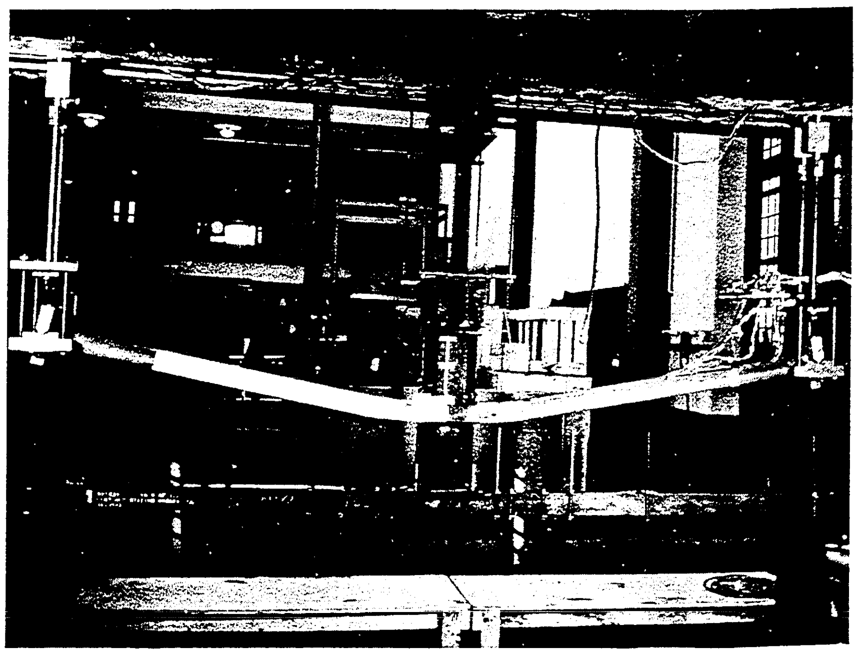


Fig. 17. Final Deflected Shape of Specimen 6YO S 6 I 12.5

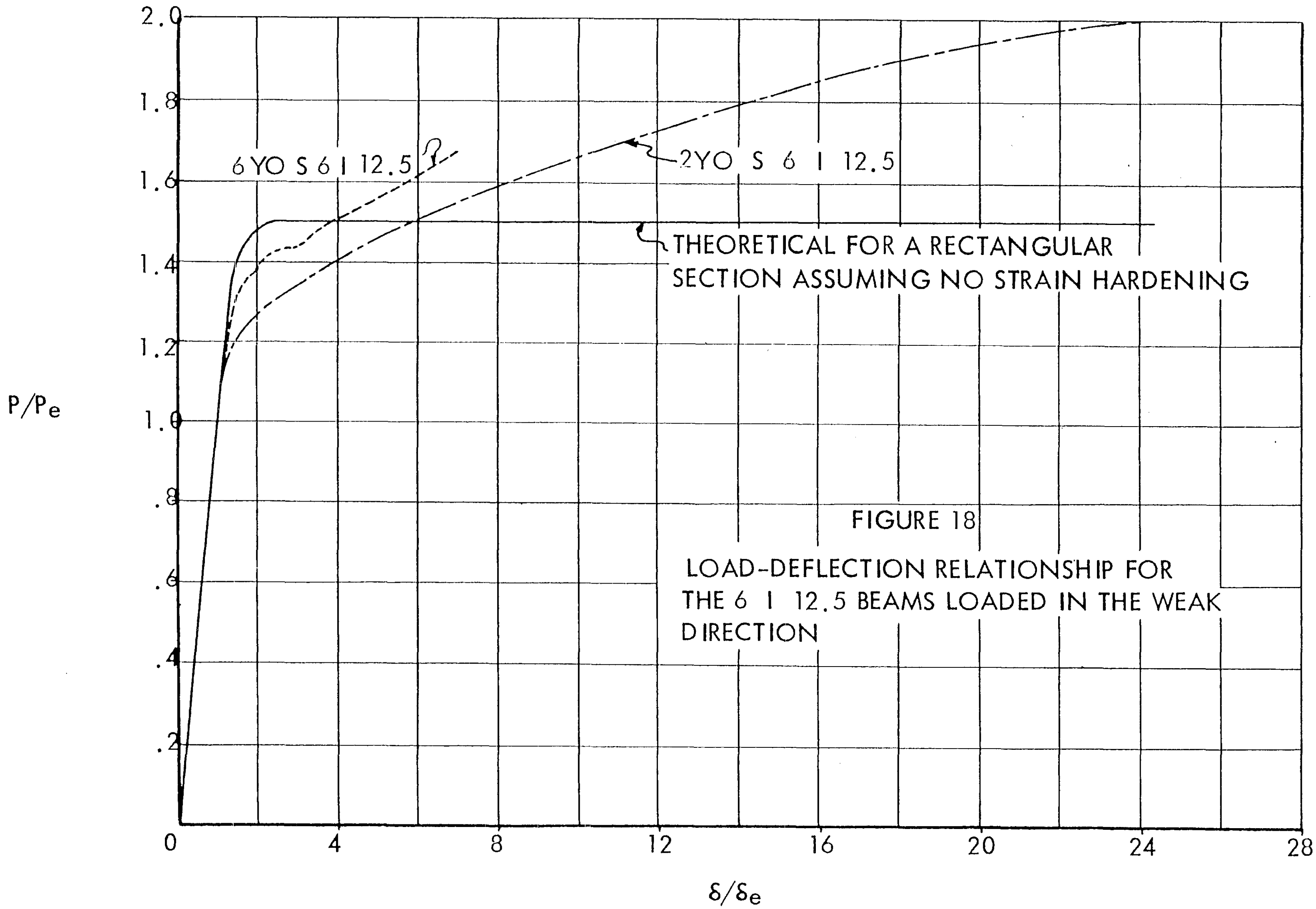


FIGURE 18

LOAD-DEFLECTION RELATIONSHIP FOR THE 6 I 12.5 BEAMS LOADED IN THE WEAK DIRECTION

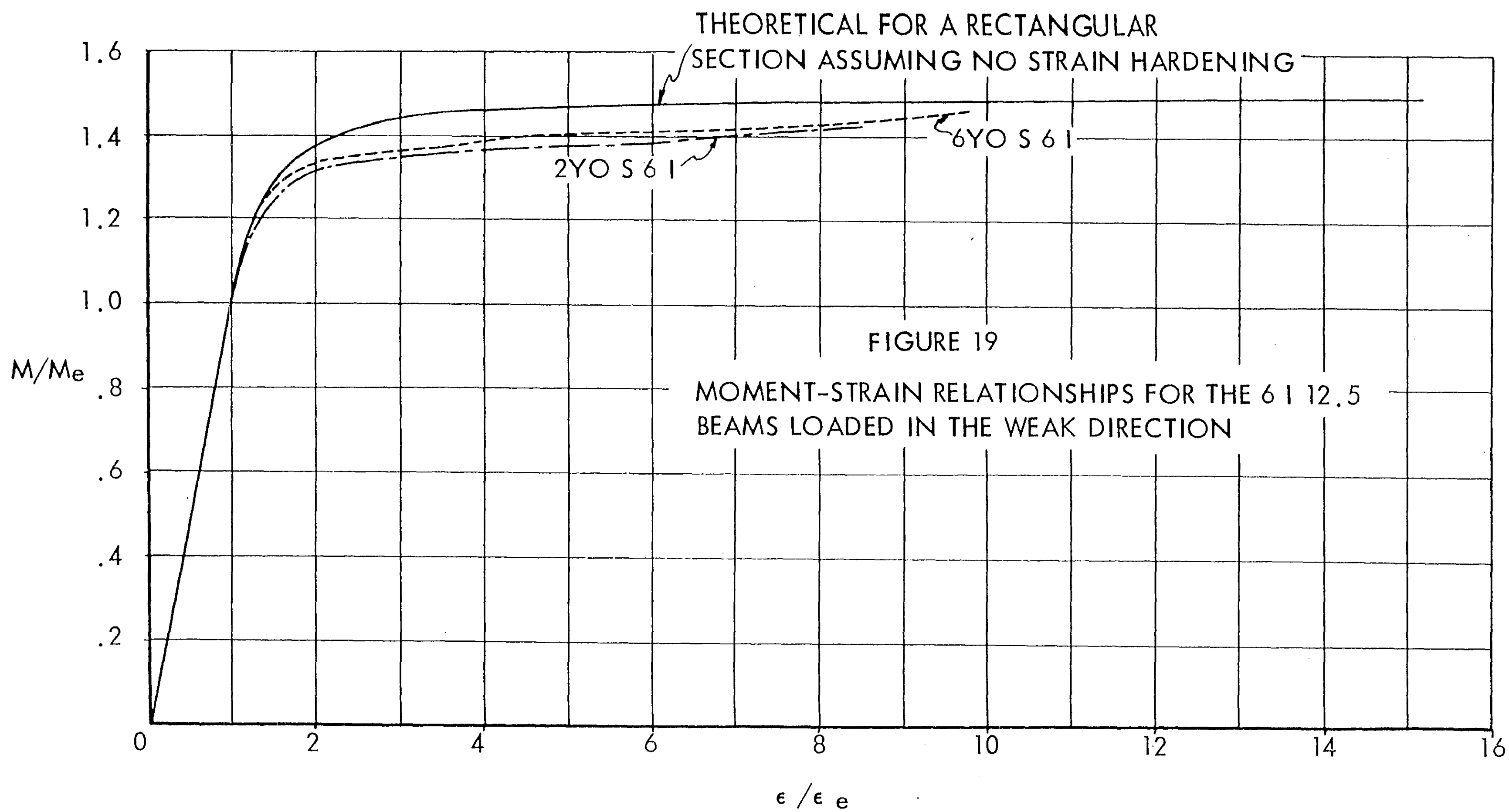
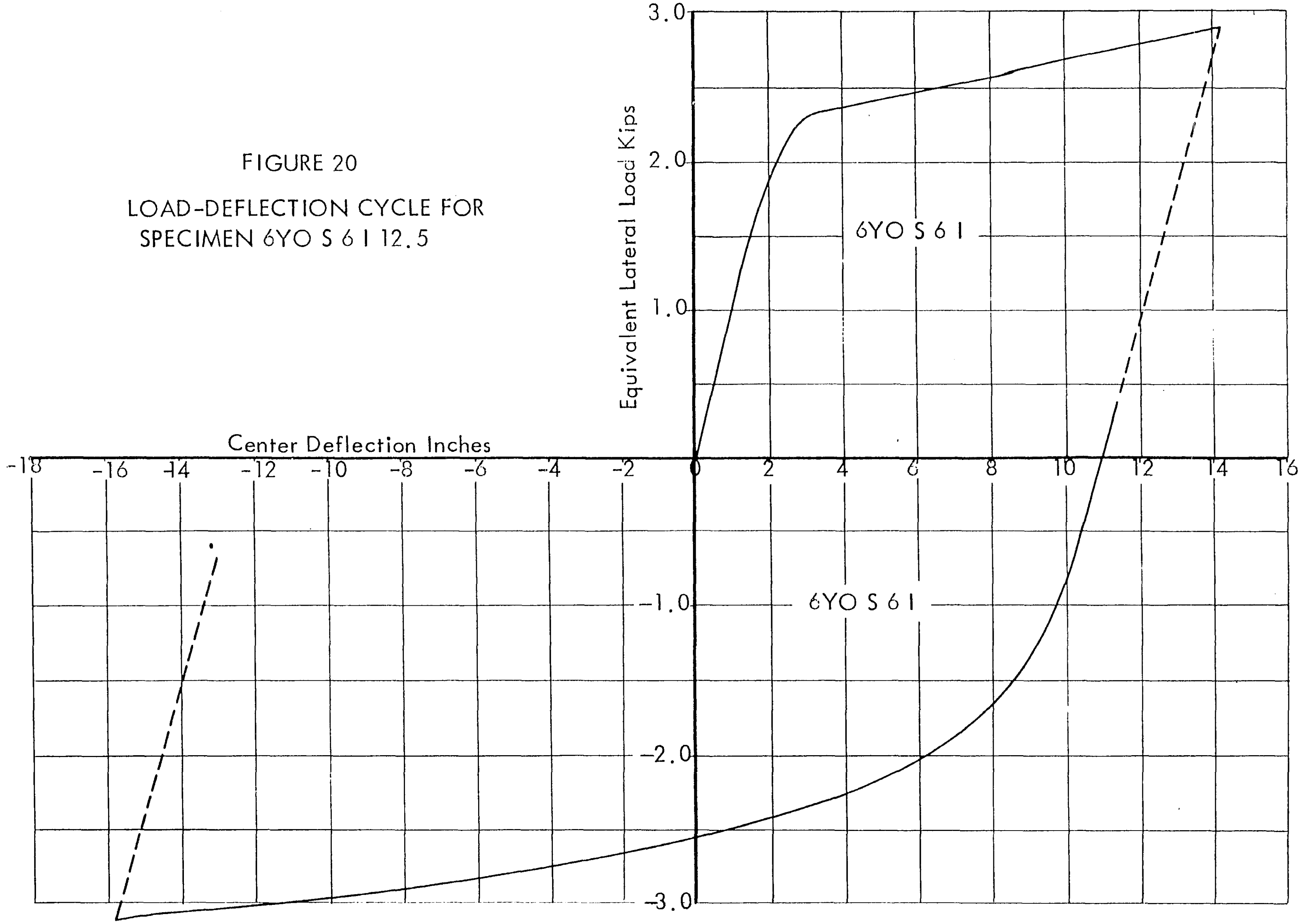


FIGURE 19

MOMENT-STRAIN RELATIONSHIPS FOR THE 6 I 12.5 BEAMS LOADED IN THE WEAK DIRECTION

FIGURE 20

LOAD-DEFLECTION CYCLE FOR SPECIMEN 6YO S 6 I 12.5



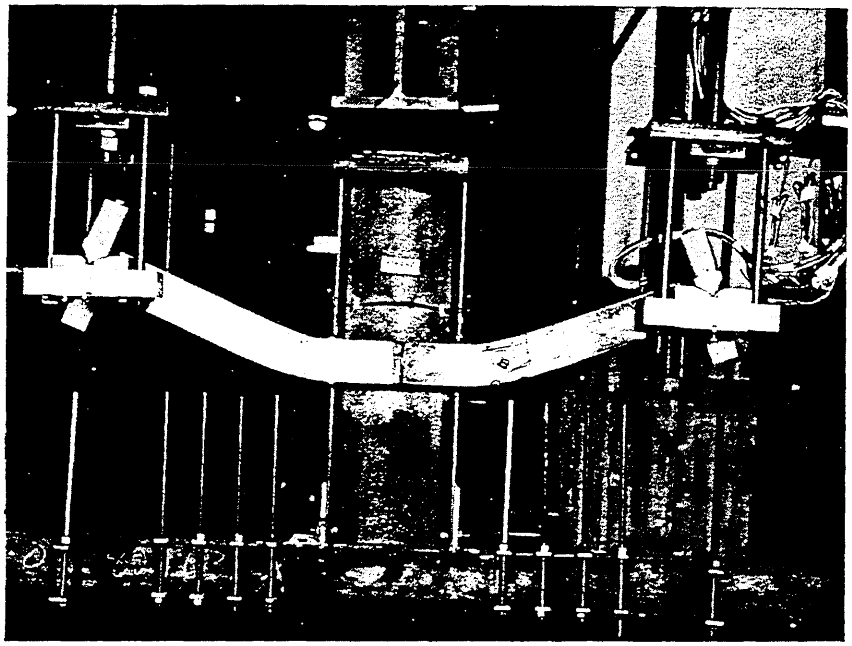


Fig. 21. Final Deflected Shape of Specimen 2YO S 6 I 12.5

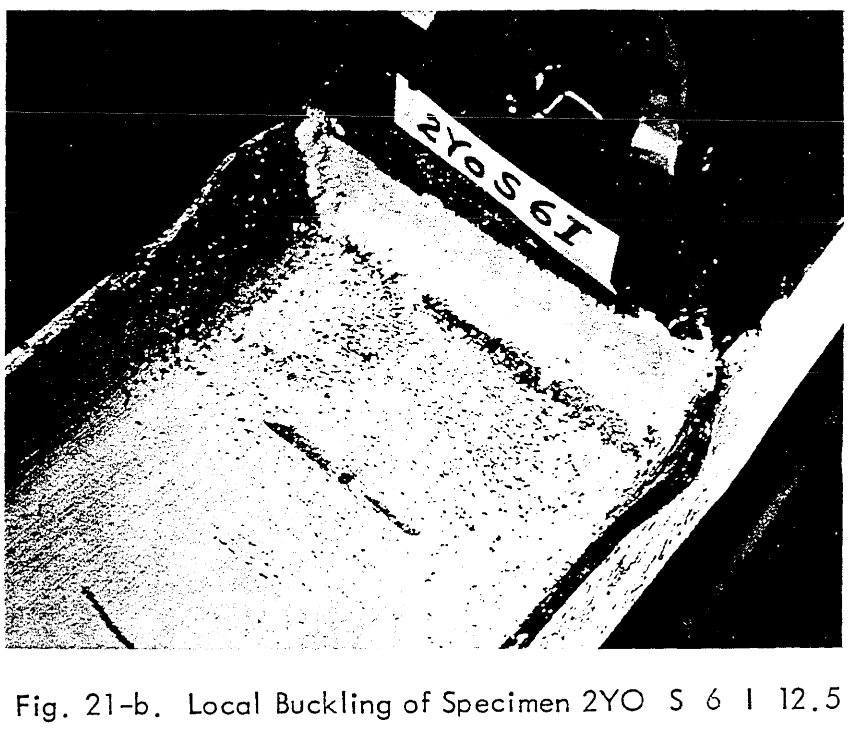


Fig. 21-b. Local Buckling of Specimen 2YO S 6 I 12.5

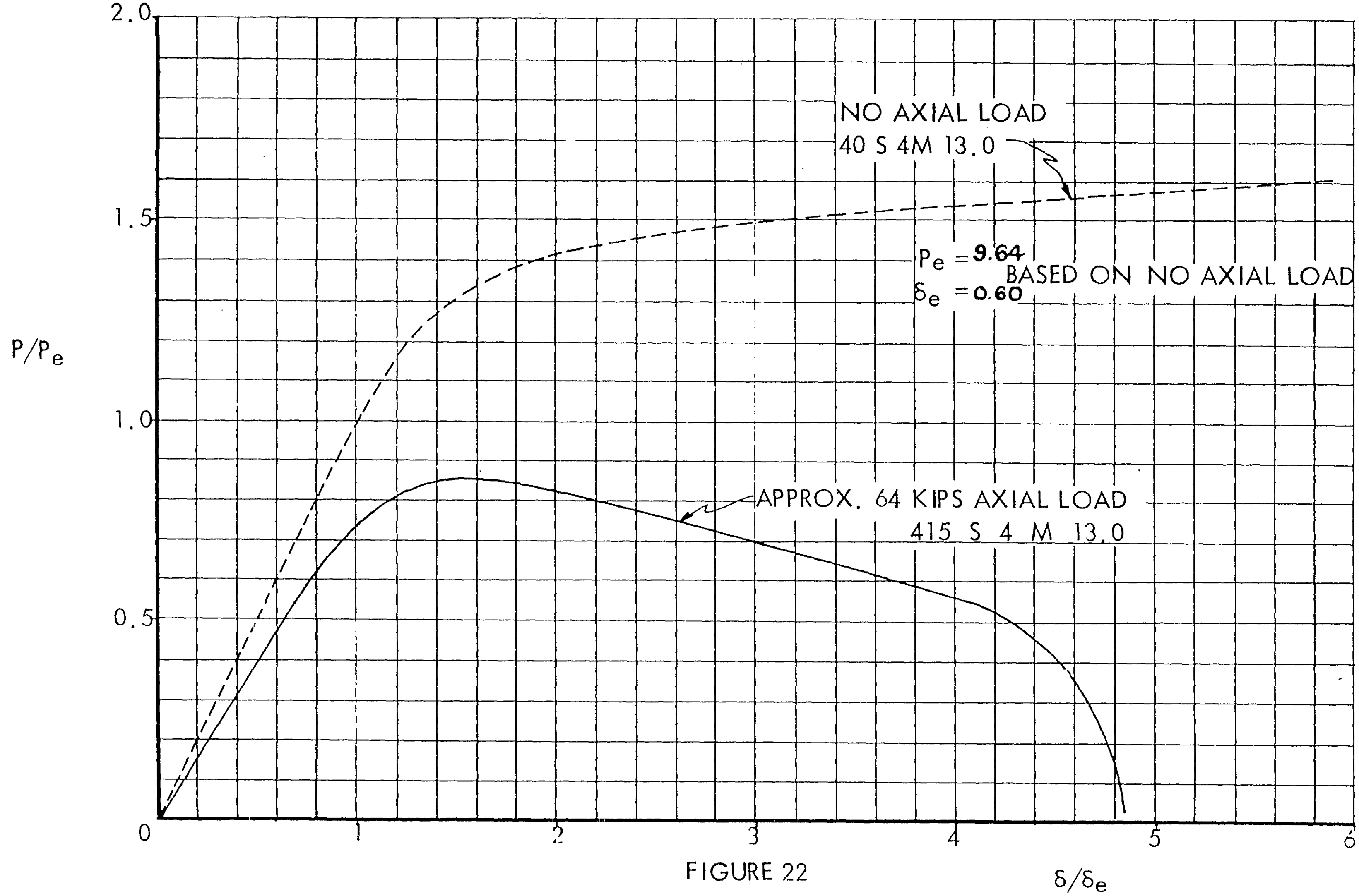


FIGURE 22

LOAD-DEFLECTION RELATIONSHIP
FOR THE 4 M 13.0 BEAMS

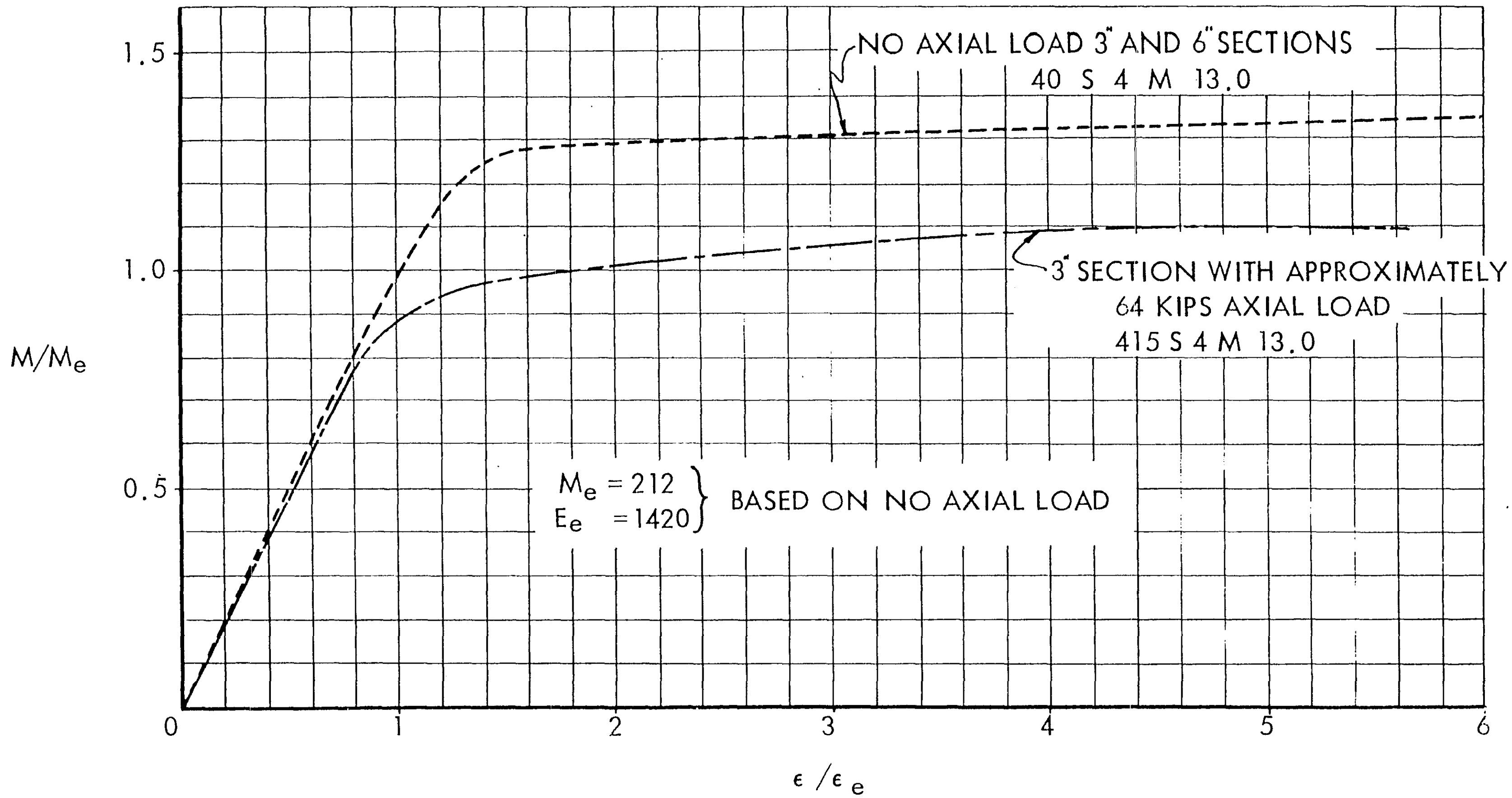


FIGURE 23

MOMENT-STRAIN RELATIONSHIPS
FOR THE 4 M 13.0 BEAMS

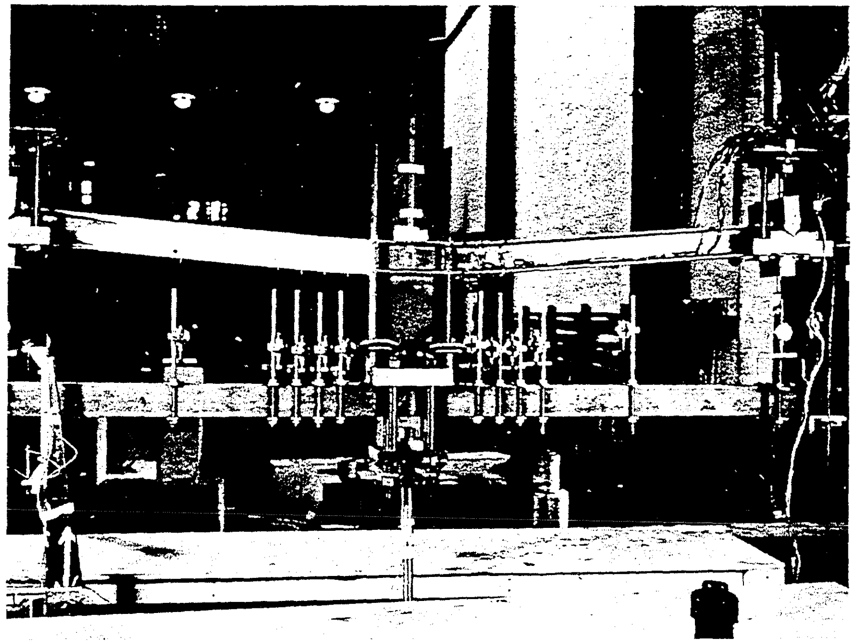


Fig. 24-a. Final Deflected Shape of Specimen 40 S 4 M 13.0

Fig. 24-b. Plan View of Specimen 40 S 4 M 13.0

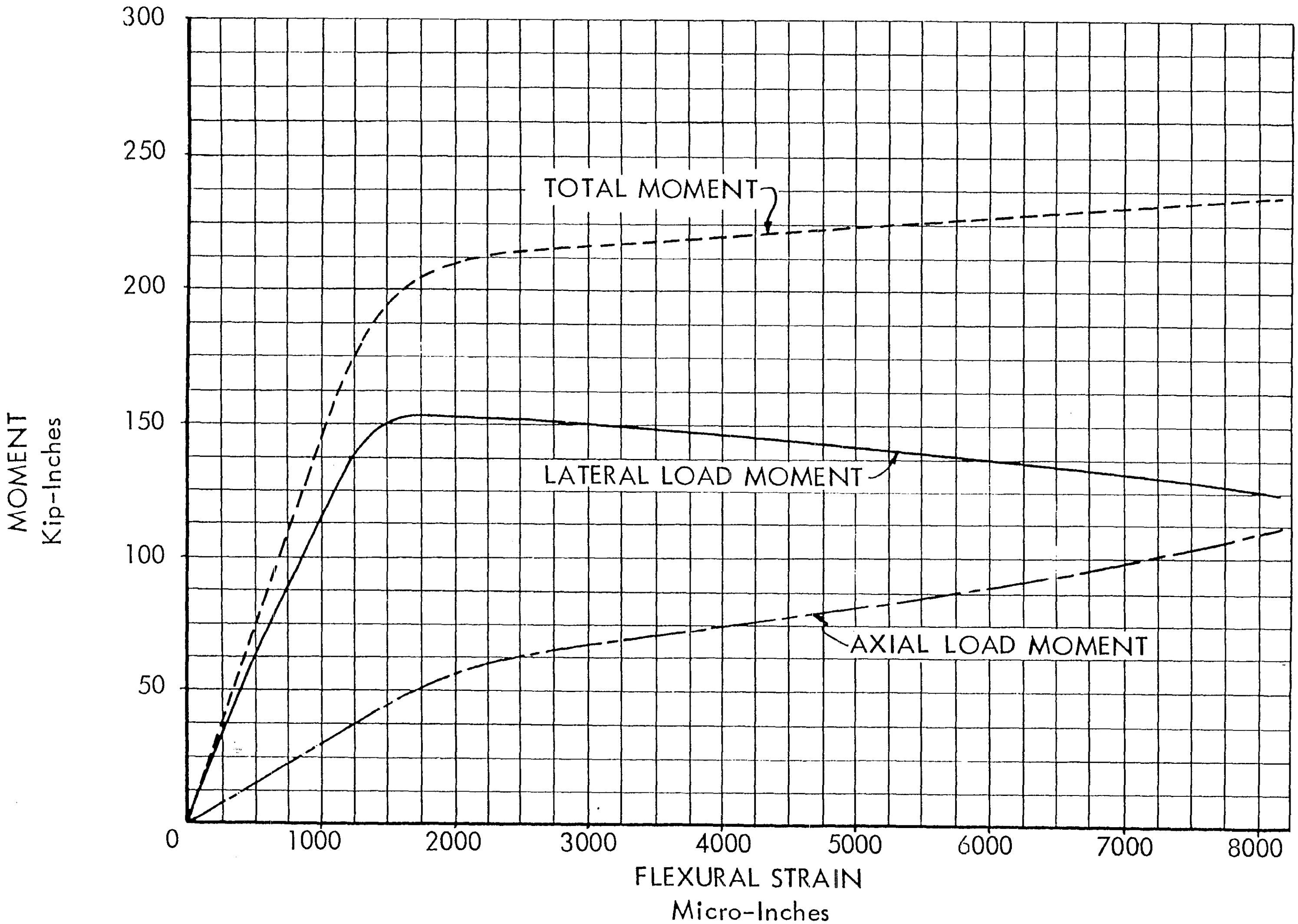


FIGURE 25 MOMENT-STRAIN RELATIONSHIP FOR SPECIMEN 415 S 4 M 13.0

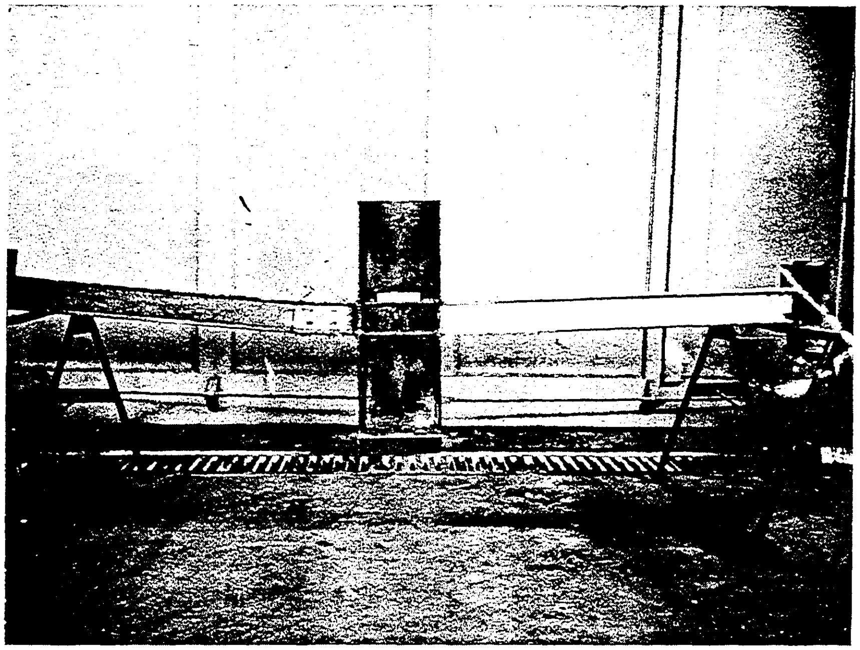


Fig. 26-a. Final Deflected Shape of Specimen 415 S 4 M 13.0

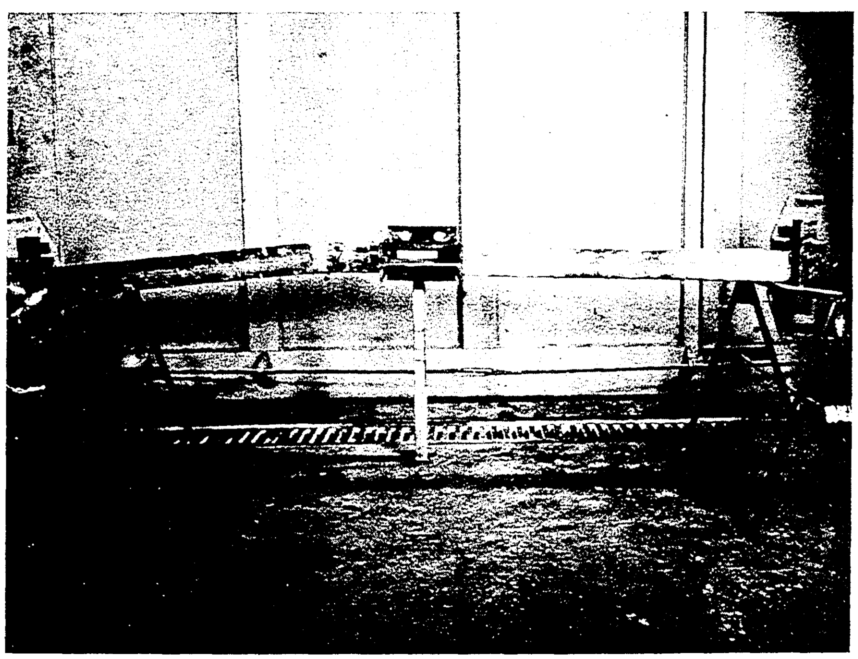


Fig. 26-b. Plan View of Specimen 415 S 4 M 13.0

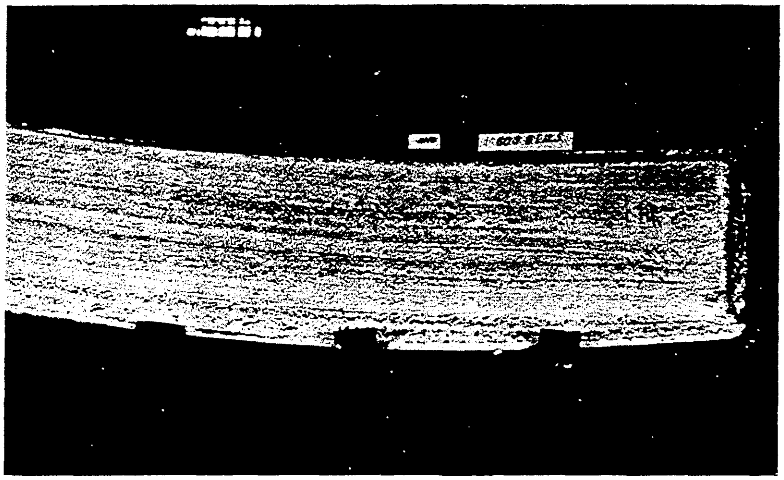
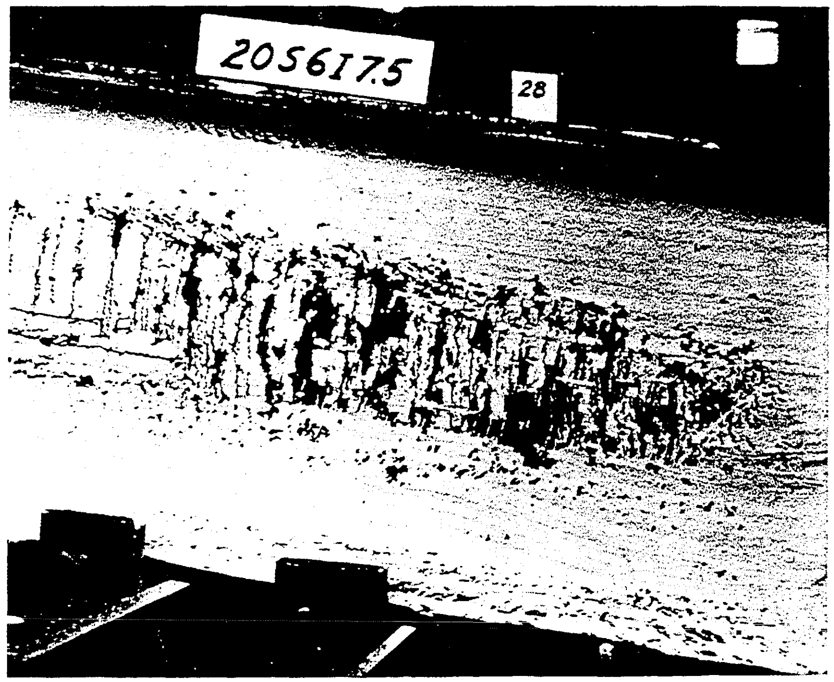


Fig. 27. Typical Flexural Hinge for the 6 I 12.5 Section

Fig. 27-b. Typical Shear Hinge for the 6 I 12.5 Section



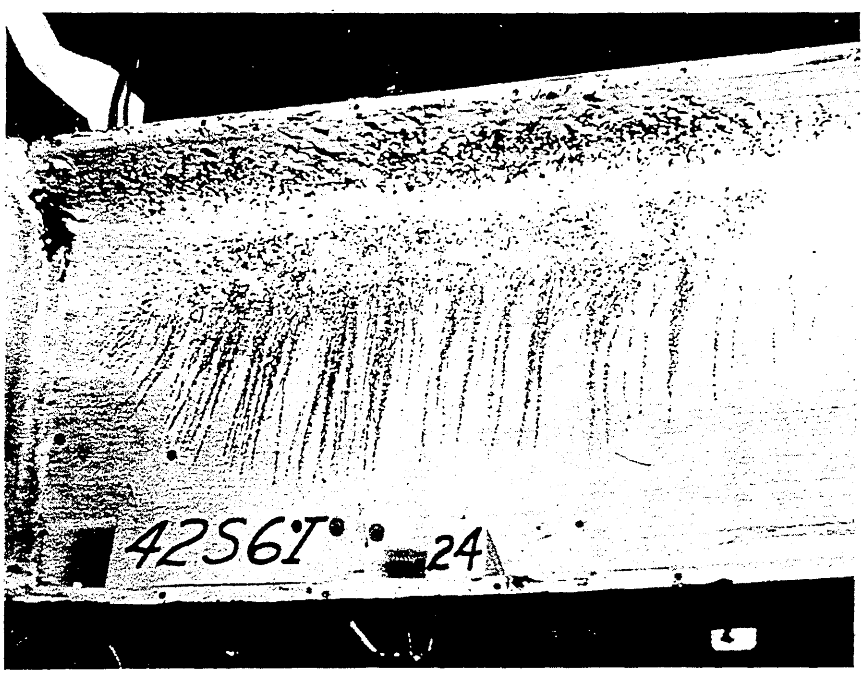


Fig. 27-c. Typical Hinge Resulting from Combined Bending and Axial Load

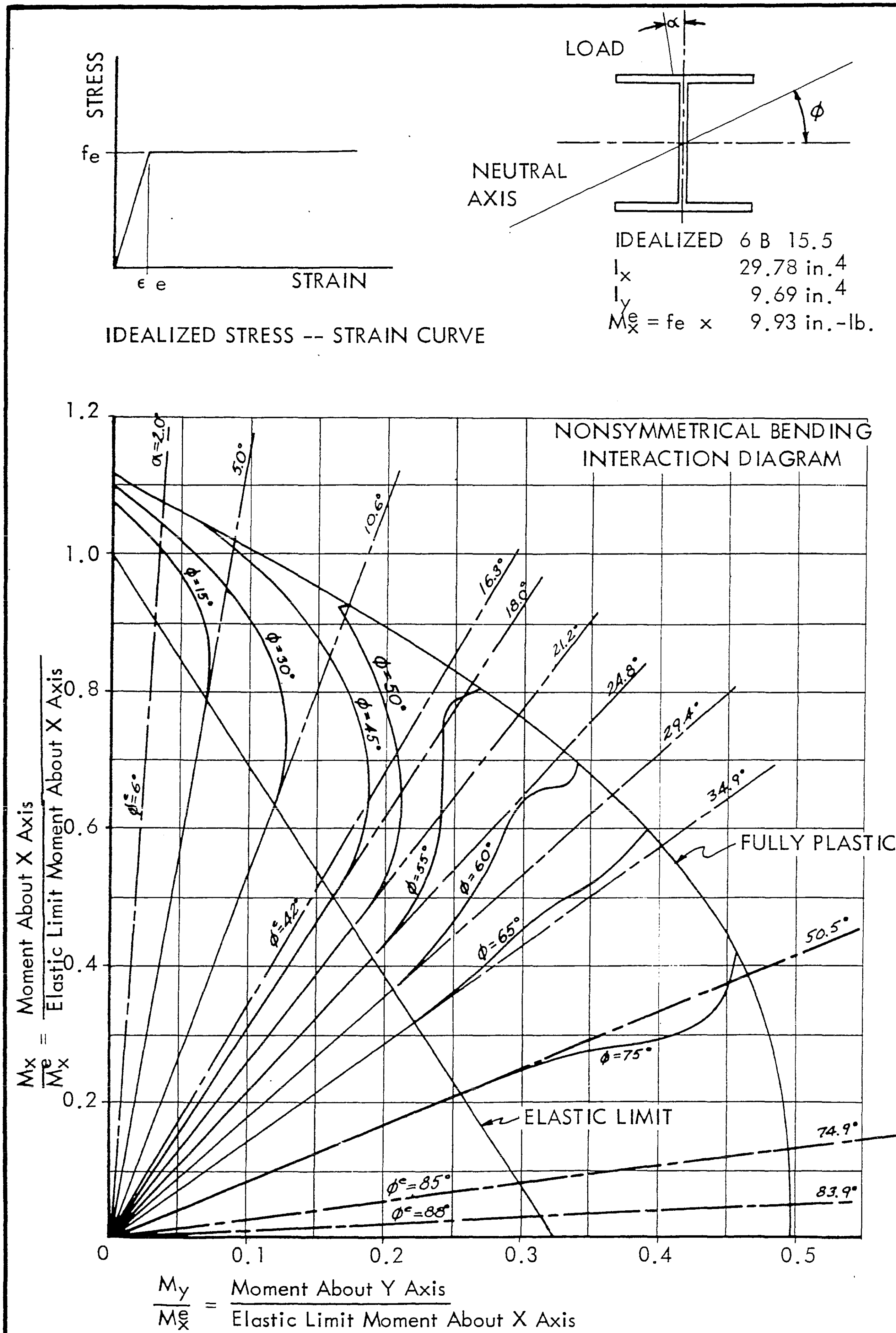

STRESS
fe
ϵ e
STRAIN
IDEALIZED STRESS -- STRAIN CURVE
LOAD
α
ϕ
NEUTRAL
AXIS
IDEALIZED 6 B 15.5
Ix 29.78 in.4
Iy 9.69 in.4
Mx e = fe x 9.93 in.-lb.
NONSYMMETRICAL BENDING
INTERACTION DIAGRAM
FULLY PLASTIC
ELASTIC LIMIT
Mx/Mx e = Moment About X Axis / Elastic Limit Moment About X Axis
My/Mx e = Moment About Y Axis / Elastic Limit Moment About X Axis

FIGURE 28