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RIBBED SLABS UNDER CONCENTRATED LOADS

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Studies of ribbed slabs subjected to single and multiple concentrated loads were carried out on both one-span and two-span continuous structures. The ribbed slab was decomposed into a series of beam and slab elements. Joint forces of the decomposed structure were determined under the condition that forces and displacements of both elements at each joint be equal. A solution for the structure under the applied loads was obtained by substituting the calculated joint forces into the equilibrium equations of the individual elements.

The results obtained were internal moments and forces in the ribs due to the applied loads. Also calculated were the equivalent uniformly distributed loads on the ribbed slabs subjected to multiple loads such as might be found in parking garages.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page

Figure | Page

CHAPTER 1

INTRODUCTION

1.1 General

Reinforced concrete ribbed slabs, or one-way joist floors, are monolithic combinations of regularly spaced reinforced concrete ribs and slabs cast-in-place to form integral units with supporting beams or walls. Span lengths of this floor type can be longer than the others. The ribbed slab behaves like a one-way structure or slab, as its geometry indicates, under a uniformly distributed load. However, for the case of concentrated load, in which the problem is more complicated, the direct design method for one-way slabs is no longer applicable.

Ribbed slabs are one of the more suitable floor types for a parking garage, since they can be built with relatively long spans. Many such parking garages have already been constructed, but some of them have had serviceability problems, such as severe cracking and punching shear failures of the slab, and some have even collapsed (1). It might be claimed that such problems resulted from not using proper design criteria, since parking garages are subjected to wheel loads from vehicles which are concentrated, while most such structures are designed for distributed loads only.

Current design methods for ribbed slabs are not specific but are left to the judgement of engineers. The Building Code Requirements for Reinforced Concrete (ACI-318-71) (2), Section 8.8, has provided some limitations on the geometry of the cross section of one-way joist floor systems and the requirements for reinforcement, but it is very brief and specifies nothing about concentrated loads. A design handbook by Reese (3) has some examples of design calculations and many tables for various span lengths and loads. However, these tables are restricted to the calculation for uniformly distributed loads only. For concentrated loads (i.e., in parking garages) some building codes, such as the BOCA Basic Building Code (4), have specified using an equivalent uniformly distributed load (i.e., 50 psf for open parking structures) in lieu of a more accurate solution. The BOCA Code also requires consideration of a 2000 lb load distributed over an area 2.5 ft sq, but this requirement seldom governs and apparently is often ignored.

1.2 Previous Studies

Previous studies which are relevant to one-way joist floor systems have been concerned mainly with highway bridge structures such as orthotropic steel plate deck with stiffeners, composite steel girders with slabs, and composite precast prestressed concrete girders with slabs. Few studies exist about ribbed slabs and nothing has been found concerning concentrated loads. However, the ribbed slab structure is structurally similar to the girder-slab-highway bridge structure, which has been studied extensively. Numerous analytical and experimental studies of girder-slab bridges have been done at the Engineering Experiment Station, University of Illinois. Most of these investigations were concentrated on simple span bridges with five or six girders. The greatest number of girders tested, by Hondros and Marsh (5) in their studies of load distribution in composite girder-slab systems, was ten.

In analysis, different approaches and assumptions have been used in solutions of integrated girder-plate type structures. Some investigators looked upon the slab as being a series of members laid perpendicular to the girders and the resulting equivalent grillage structure was analyzed. On the other hand, some investigators considered the girders as stiffeners of the slab and the girder-slab structure was replaced by an orthotropic plate of equivalent stiffness. A numerical method was applied by Newmark and Siess (6), who considered the girder-slab structure as a plate supported on a series of girders with no interface shear forces. The girders and slab were then treated so that they deflected together with no separation. Composite action of the slab with the girders was taken into account by using a beam stiffness equivalent to the composite section stiffness of the girder and a portion of the slab. Then the assumed structure was analyzed by Newmark's moment distribution procedure (7). In-plane forces in the slab and axial forces in the girder cannot be taken into account in this procedure. However, this analysis is among the references listed in the present design manual of highway bridges (8).

Various numerical methods constitute very important and powerful approaches, especially when high-speed computers are available. Several numerical techniques, such as the finite difference and finite element methods, have become important in the study of girder-slab bridges. Chen, Siess, and Newmark (9) used the finite difference method to solve skewed girder-slab bridge problems. Gustafson and Wright (10) used a finite element procedure to solve similar problems. The results of both methods are in general agreement, but the latter procedure gave better solutions for the same mesh size. The finite element method is also applicable to the study of ribbed slabs. However, because of the geometry of the ribbed slab, a huge number of elements are required to obtain an accurate solution. Therefore, it might be an uneconomical procedure for this type of structure.

Sithichaikasem and Gamble (11), and Van Horn and Mortajemi (12), using the Goldberg-Leve folded plate theory (13), considered the girder-slab bridge deck as a series of slab and girder elements. At the joints between elements, there are unknown forces, which take into account the in-plane forces in the slab, and the biaxial bending and torsional moments in the girders, in addition to the forces considered in the analysis by Newmark and Siess. Solutions of the individual slab and girder elements were then obtained, with both bending and membrane theories being used for the slab elements. For the girder elements, solutions were obtained by integration of the equilibrium equations of a small element. Then compatibility at the joints between slab and girder elements was restored to obtain solutions for the bridge structure, in which T-beam action of the girder and slab was directly taken into account rather than being represented by some approximation.

The above studies were done on simple span bridges only. Wong and Gamble (14) extended the investigation to continuous bridges, using the same procedure. This method of analysis is one of the more rigorous procedures and so was selected as the basis of the analysis for the study of ribbed slabs subjected to concentrated loads.

1.3 Object and Scope

This study examines the general behavior of ribbed slabs subjected to a single concentrated load, and then investigates their characteristics under groups of concentrated loads. Since problems concerning concentrated loads are complicated, the characteristics of ribbed slabs under a single concentrated load will be studied first. After obtaining vital information about single loads, the ribbed slabs under multiple loads (i.e., as in parking garages) will be examined.

The ribbed slab is generally considered a flexure type structure; hence, bending moment is the most important internal force. Besides bending, shear and torsion may also be significant forces for this type of structure under concentrated loads. The investigation will be performed using loadings that result in maximum internal forces. Maximum deflections corresponding to the maximum moment loadings will also be presented. The relationships between the internal forces due to the applied load and the important parameters will be examined and reported. The various cross sections of ribbed slabs to be analyzed will be chosen such that the magnitudes of the parameters cover the reasonable range of current practice. Both simple-span and two-span continuous structures are to be investigated in this study.

The study considers the forces in the ribs due to various loadings, but does not take into account local stresses within the slabs which are caused by concentrated loads.

The information obtained from this study will lead to some recommendations for the design procedures of the ribbed slab under concentrated loads.

1.4 The Typical Ribbed Slab

The typical ribbed slab will be considered to consist of identical parallel ribs at equal spacings and slabs of uniform thickness. For this study, seventeen ribs will be used for the analysis. The ribs have a constant thickness rectangular cross section, whereas in practice they usually have a tapered cross section, which makes construction somewhat easier. The average width of the tapered rib might be used as the rib width in design calculations. The rib width is assumed constant along all spans, although the width is sometimes increased near the supports in current construction practice.

The rib spacing, b, is the distance between the centers of any pair of adjacent ribs, and slab thickness, t, is the total depth of the slab. Span length, a, is measured from center to center of the supports, and for the case of two-span ribbed slab, two equal spans is the typical case. The ribbed slabs are analyzed as being simply supported, while in practice the ribbed slab is usually cast monolithically with the supports. In the analysis, the interior support of the two-span ribbed slab is provided on the ribs only, while at the exterior ends of spans, both ribs and slabs are simply supported.

Figure 1.1 shows typical ribbed slabs, both one- and two-span, and also the cross sections of the tapered and equivalent rectangular ribs.

1.5 Selected Parameters

Parameters that influence the load carrying behavior of the ribbed slabs include both geometry of the structure and material properties. However, if the same materials are always used in both ribs and slabs, the remaining variables depend only upon the geometry of the ribbed slab. The important geometric variables are the span length, rib spacing, cross section of the ribs, and slab thickness.

The magnitude of bending moment is a direct function of span length in any flexure member. For the ribbed slab, not only the span length but also the rib spacing and slab stiffness, which is proportional to its thickness, affect the magnitude of bending in the ribs. So far, ribbed slabs generally have been designed to resist bending and shear only. In this study, torsional moments are found to be important, because of both the geometry of the structure and the nature of the loading. The torsional moment in a rib depends upon the torsional stiffness of the cross section as well as the flexure stiffness.

It can be concluded that the major parameters which may affect the load carrying characteristics of the ribbed slab are:

1. Span length, a;
2. Rib spacing, b ;
3. Flexural stiffness of slab, D;
4. Flexural stiffness of the rib, EI;
5. Torsional stiffness of the rib, GJ.

Material properties, as mentioned above, might also affect the load carying behavior of the ribbed slab, but throughout this study material properties are kept constant. For concrete, Poisson's ratio in the elastic range is almost constant, and in this analysis the value of 0.15 is used. The modulus of elasticity of concrete is considered to be 3,600,000 psi. The modulus of elasticity does not affect the magnitude of the internal forces due to applied load, but does affect the displacements. The effects of material properties on the behavior of ribbed slabs under applied loads consequently are not included in this study.

The above parameters can be reduced to three and made more general by converting to relative measures. The rib spacing can be considered relative to the span length, since it is reasonable that the width of the spacing should be compared to its span length. The flexural stiffness of the rib could reasonably be considered relative to the stiffness of the slab. And the torsional stiffness of the rib can be taken in terms of its flexural stiffness. Therefore, three nondimensional parameters are introduced as following:

1. b/a is rib spacing relative to span length (i.e., aspect ratio),
2. $H = \frac{EI}{aD}$ is flexural stiffness of the rib relative to that of the slab, and
3. $T = \frac{GJ}{EI}$ is torsional stiffness of the rib relative to its flexural stiffness.

These three dimensionless parameters and their relationships to the load carrying characteristics of the ribbed slab will be the focus of this study.

1.6 Notation

The letter symbols throughout this study are defined as following:

a	Span length of the ribbed slab, measured from center to center of supports
b	Rib spacing, measured from center to center of ribs
b_r	Width of the rib
h	Total height of the rib cross section (including the slab)

n	Number of loads on a ribbed slab in the span direction for the case of multiple loading
t	Thickness of slab
w_e	Equivalent uniformly distributed load for ribbed slabs
$D = \frac{Et^3}{12(1 - \mu^2)}$	Flexural stiffness of slab per unit width
E	Modulus of elasticity of material of the ribbed slab
F_G, F_S	Flexibility matrices for beam and slab elements, respectively
F^*	Flexibility matrix for the ribbed slab structure
F_I^*	Absolute flexibility matrix at the interior support for the two-span ribbed slab
G	Shear modulus of material of the ribbed slab
$H = \frac{EI}{aD}$	Flexural stiffness of the rib relative to that of the slab
I	Moment of inertia of the composite section of the rib and slab
J	Torsional constant of the cross section of the rib
$K = \delta \frac{EI}{nPa^3}$	Deflection coefficient
L_{kl}, L_{kr}	Displacement vectors at the left and right edges, respectively, of k element due to the applied load on that element

L^* — Displacement vector of the structure due to the applied load

L^*_{ix} — Displacement vector at the edge x of i element due to the applied multiple load on that element

M — Bending moment of the composite section of the rib and slab

M_B — Maximum bending moment of a simple beam with the same span length and load spacing as the ribbed slab

M_t — torsional moment of the rib

M_j — Moment per unit length around the immaginary joint, j, between the beam and slab elements

N_l, N_r — In-plane normal forces per unit length on joint j, on the left and right edges of the element

P — The applied concentrated load

P_{kl}, P_{kr} — Internal force vectors at the left and right edges of k element

P^* — Internal force vector of the ribbed slab structure

R_l, R_r — Vertical reactions per unit length on joint, j, on the left and right edges of element

R_I	Redundant reactive force vector at the interior support of the two-span ribbed slab
S_l, S_r	In-plane shearing forces per unit length on joint, j, on the left and right edges of element
$T = \frac{GJ}{EI}$	Torsional stiffness of the rib relative to its flexural stiffness
V	Shear force of the rib
V_B	Maximum shear of the beam loaded as for M_B
Δ	Absolute displacements at the interior support of the two-span ribbed slab due to the applied load, when the interior support is ignored
μ	Poisson's ratio of concrete
δ	Absolute value of the maximum deflection of a ribbed slab

CHAPTER 2

METHOD OF ANALYSIS

2.1 General

The analysis of reinforced concrete ribbed slabs subjected to a single or multiple concentrated load is based on the method of analysis which has been used in the studies of the effects of diaphragms in bridges with prestressed concrete I-beam girders by Sithichaikasem and Gamble (11). This method is possibly even better suited to the monolithic ribbed slab structure than to the composite bridge structure, since the method was derived assuming monolithic structures without joints.

In practice, a ribbed slab is usually cast in place monolithically with its supports, which are then partially fixed. Hence, the supporting conditions deviate somewhat from the restrictions of the method of analysis, which is limited to simply supported structures only. This limitation results from using of Fourier series in the analysis; in which the displacement function, internal forces, and external loads are put in terms of a sine series which satisfies boundary conditions of a simple support. However, the partially restrained supporting condition of the usual ribbed slab structure would influence the results of the analysis toward the conservative side, so far as the forces in the ribbed slab are concerned.

2.2 Assumptions

The assumptions in this analysis are concerned mainly with solving for the force-displacement relationships of plate and beam elements. Therefore, the basic assumptions for flexure theory of a medium-thick plate, membrane theory for a thin plate, and theory of biaxial bending combined with axial force and torsion of a beam will hold for this analysis. Since the ribbed slab is a reinforced concrete structure, the additional assumptions for material properties are as follows:

1. Concrete is homogeneous, isotropic, and elastic;
2. Poisson's ratio for concrete is equal to 0.15.

All supports are assumed to be unyielding.

2.3 One-Span Ribbed Slab

The method of analysis is initiated by decomposing the ribbed slab into a series of plate and beam elements. At joints between the elements, there are four unknown internal forces: in-plane normal force N, in-plane shearing force S, vertical reactive force R, and moment acting perpendicular to the joint M_j (see Fig. 2.1). Then solutions for the individual plate and beam elements under the conditions of the applied load and the internal forces along the joint are carried out separately.

The exact solution of the plate element is rather complicated, since it is the problem of bending of a plate under the combined action of the lateral load, edge moments, and forces in the middle plane. For this study, the bending theory and the plane stress theory for thin plates are applied independently. It is not extremely accurate, since bending moments in the plate are affected by the in-plane forces. However, this effect is negligible for plates with small deflections. Another source of error is found in solutions of the membrane theory since equations in the form of Fourier series do not satisfy the end support condition that membrane shear should be zero, but the effect is also negligible, as shown by Savern (15).

The flexibility matrix and displacement vectors at the left and right edges of the plate element are obtained by combining solutions of bending and membrane theories. The flexibility matrix and displacement vectors at the left and right edges of the beam element are determined by integration of the equilibrium equations of a small element of the beam (see Fig. 2.1), and then using the force-displacement relationships for solutions. The detailed equations for solutions for both plate and beam elements are discussed and reported in Ref. 11.

The internal forces can be determined by the compatibility conditions at the joints of the decomposed structure. In other words, the decomposed structure is assembled under the condition that displacements of the plate and beam elements at the same joint are equal.

By this argument the solutions can be obtained as follows:

F_G is the flexibility matrix of the beam element;

F_S is the flexibility matrix of the plate element;

L_{kl}, L_{kr} are the displacement vectors at the left and right edges, respectively, of element k due to the applied external load on that element;

P_{kl}, P_{kr} are the vectors of internal forces at the left and right edges of element k

Both the plate and beam flexibility matrices have size of 8 by 8, since each element is subjected to eight internal forces (see Fig. 2.1). The flexibility matrix of each element can be partitioned into four submatrices, as far as internal forces are concerned. For example, for an element k

$$F_k = \left[\begin{array}{c|c} F_{kll} & F_{klr} \\ \hline F_{krl} & f_{krr} \end{array}\right]$$

At any joint N, the right edge of the beam element i is connected to the left edge of the plate element j, or vice-versa. The displacement of the right edge (i.e., r edge) of the beam element i is:

$$F_{Grl}\, P_{il} + F_{Grr}\, P_{ir} + L_{ir}$$

and the displacement of the left edge of the plate element i + 1 = j is:

$$F_{Sll}\, P_{jl} + F_{Slr}\, P_{jr} + L_{jl}$$

For compatibility of joint N, the displacement at the right edge of element i must equal the displacement at the left edge of element j, and the force vectors of both elements at the edges of the common joint N must be the same, that is:

$$F_{Grl}\, P_{il} + F_{Grr}\, P_{ir} + L_{ir} = F_{Sll}\, P_{jl} + F_{Slr}\, P_{jr} + L_{jl}$$

and

$$P_{ir} = P_{jl} = P_N$$

where P_N is the internal force vector at joint N. Then we can write

$$F_{Grl}\, P_{il} + [F_{Grr} - F_{Sll}]\, P_N - F_{Slr}\, P_{jr} = L_{jl} - L_{ir}$$

By applying the above conditions for all the joints, then equations for the assembled structure are obtained as follows:

$$F^* P^* = L^*$$

where

F^* is the assembled flexibility matrix of the structure

P^* is the force vector at all joints

L^* is the applied load displacement vector

The internal forces at every joint are, therefore, obtained by solving the above equations. Internal forces in any slab and beam can be determined by substituting the joint forces into the equilibrium equations of the individual element. As far as the geometry of the structure is concerned, it is reasonable that the rib and slab interact somewhat as a T-beam, but there is no direct way to determine the effective width of the flange of each rib in this analysis. In order to find the effective T-beam moment, the T-beam action is evaluated for the condition that the sum of axial forces in the effective composite T-beam section is zero under pure bending. Then the composite bending moment of a rib including the interaction with the slabs is calculated.

2.4 Two-Span Ribbed Slab

Solutions of a two-span ribbed slab are determined by the unit load method, in which there are three steps in the calculations. First, the interior support is removed, and the structure becomes a simple span ribbed slab under the applied load. Second, to determine the redundant reactions (vertical reactions and twisting moments), unit loads and couples are applied to each rib at the interior support. Third, the solutions of the first and second steps are combined to yield the solution of the two-span ribbed slab by restoring the interior support points to zero deflection and zero rotation about the rib axes.

Solution of the first step is like that of a simple span ribbed slab subjected to externally applied load, so the calculation is similar to that described in Section 2.3. Besides internal forces, solution of this step also involves absolute displacements (deflections and rotations) at the interior support. In the second step, the simple span ribbed slab is subjected to unit loads and couples acting on every rib at the interior support location. Displacements are found using the procedure in Section 2.3, resulting in the absolute displacements at the interior support, as well as internal forces. Then, with the compatibility condition that there are no vertical displacements or rotation about the rib axis at the interior support of the structure, equations are formed as follows:

$$F_I^* R_I + \Delta = 0$$

where

F_I^* is the absolute flexibility matrix of the structure at the interior support;

R_I is the redundant reactive force vector at the interior support;

Δ is the absolute displacement vector at the interior support location due to the externally applied load, with the interior support removed.

By solving the above equations, the redundant reactions at the interior support line are determined.

Third step, solution of the two-span ribbed slab is obtained by combining the solution of the first step due to the externally applied load and that of the second step, using the calculated reactions instead of unit loads and couples. Then we have:

$$S^* = S_1 + S_2 R_I$$

where

S^* is the solution of a two-span ribbed slab subjected to the applied load;

S_1 is the solution of the first step, considering the applied load;

S_2 is the solution of the second step, considering the applied unit loads and couples.

This technique of applying redundant reactions instead of the interior support is not exactly correct, since the real support normally supports not only the ribs but also the slab. However, the inaccuracy affects only the portions of slabs close to the support and is negligible for the structure as a whole.

2.5 Multiple Loads

The analysis of highway bridges (11,14) was made with single loads only, and the superposition method was then used for the solution of truck loadings or any other combination of loads. For this study, multiple loads are considered (i.e., wheel loads of vehicles in parking garages). The superposition method is not practical, because of the great number of wheel loads possible in parking garages. Therefore, a direct solution for multiple loads is necessary.

In the analysis, all vehicles are assumed to have the same wheel spacings, to weight the same, and have the weight divided evenly among the four wheels, as is further discussed in Chapter 4. Starting with the method of analysis described in Sections 2.3 and 2.4, the only things to be changed for multiple loads are the displacement vectors at the edges of the elements due to the applied loads. Since the displacement vectors due to single loads are available, the displacement vectors for multiple loads equal the sum of displacement vectors at the corresponding edges due to all the externally applied loads on an element. For example, if there are M loads on an element i, the displacement vector at the edge x will be as follows:

$$L^*_{ix} = \sum_{N=1}^{M} L_{ixN}$$

where

L^*_{ix} is the displacement vector at the edge x of the element i due to M loads

L_{ixN} is the displacement vector at the edge x of the element i due to a single load N

Solutions of ribbed slabs due to multiple loads can then be obtained by the method of analysis in Sections 2.3 and 2.4, with the displacement vectors calculated by the above procedure.

2.6 Accuracy of the Analysis

The computer program was modified from the program developed by Sithichaikasem and Gamble (11). The accuracy of the computation depends largely on the number of terms of harmonics used in the calculation for the Fourier series type equations. It has been shown by Wong and Gamble (14) that the rate of convergence toward the correct solution for a simple-span bridge is slow after 5 harmonics. At 35 harmonics, 97 percent computational accuracy was obtained, comparing total composite moment of all beams with the static moment as calculated from the elementary beam theory. The computing work was carried out on the IBM 360/75 computer, using double precision arithmetic.

In this study, the computational accuracy of the solution for shear is also examined, comparing total shear at all supports in all ribs with the applied load. A simple span ribbed slab with H = 4.5, b/a = 0.052, and T = 0.100, and a two-span ribbed slab with parameters of the same magnitudes were taken as examples for showing the computational accuracy of solutions. Twenty harmonics were used for the simple span ribbed slab and 35 for the two-span.

The results of two loading cases, a single load P at midspan of center and edge ribs, are as follows:

Simple-Span Ribbed Slab

Load P at midspan of the center rib:

Total moment at midspan of 17 ribs = 0.2427Pa, 97.1 percent convergence

Total shear at supports of 17 ribs = 0.9588P, 95.9 percent convergence

Load P at midspan of the edge rib:

Total moment at midspan of 17 ribs = 0.2443Pa, 97.7 percent convergence

Total shear at supports of 17 ribs = 0.9652P, 96.5 percent convergence

Two-Span Ribbed Slab

Load P at midspan of the center rib:

Total moment at midspan plus one-half total moment at interior support of 17 ribs = 0.2408Pa, 96.3 percent convergence

Total shear at all supports of 17 ribs = 0.9864P, 98.6 percent convergence

Load P at midspan of the edge rib:

Total moment at midspan plus one-half total moment at interior support of 17 ribs = 0.2426Pa, 97.0 percent convergence

Total shear at all supports of 17 ribs = 0.9889P, 98.9 percent convergence

The static moment is 0.25Pa.

For the simple span ribbed slab, the example above shows that the computational accuracy for the solution of shear is slightly less than that of the solution for moment. A reason for this is that some shear forces are carried directly to the supports by the slab. The composite action between the ribs and slab is taken directly into account in the solution of moments. Some of the moment which is unaccounted for after 20 harmonics is resisted by bending in the slab elements, and most of the rest by the loaded rib.

For the two-span ribbed slab, results opposite those of the simple span are obtained. The computational accuracy of solution for moment is obtained by comparing total moment at midspan plus one-half of total moment at the interior support of all ribs (i.e., 17 ribs) with the static moment. Because of the additional dimension involved in this structure, the moments at midspan of the unloaded ribs are not all maximum values. The comparison to the simple beam moment is correct only if the total moments in all 17 ribs are considered. A better result in the computational accuracy for the solution for shear is obtained because only the ribs are supported at the interior support, where about 70 percent of the applied load is carried. Therefore, the amount of shear carried by the slabs is minimized, and a higher computational accuracy is obtained.

CHAPTER 3

BEHAVIOR OF RIBBED SLABS UNDER A SINGLE CONCENTRATED LOAD

3.1 General

This study has considered both one-span and two-span continuous ribbed slabs. A single concentrated load is the basic loading case for studying the general load distribution behavior of ribbed slabs. Various load locations were chosen to cover the possible range of interest, such as at midspan of center and edge ribs.

The structural behavior of ribbed slabs subjected to a concentrated load might be characterized by slab and rib actions. An interpretation of these actions might be made in two directions, relative to geometry of the structure. First, in the transverse direction (direction perpendicular to the rib), slab action is likely to take the form of an elastic support of the ribs, especially of the loaded rib. The ribs also act as elastic supports of a continuous one-way slab. Second, in the longitudinal direction (direction along the rib), the rib is somewhat like a beam on an elastic foundation as far as the transverse slab action is concerned. The slab also acts as a flange of a T-beam which has the rib as its web.

This investigation of the ribbed slab under concentrated loads is mostly concerned with the study of moment, shear, and torsion (such as their distributions, as influenced by various parameters). The important parameters in this study are H, b/a, and T, as discussed in Section 1.5. To examine the influence of these parameters on the load-carrying characteristics of both simple span and two-span continuous ribbed slabs, various sections were chosen for the analysis. The ribbed slabs analyzed have magnitudes of H varying from 2.0 to 9.0, b/a from 0.035 to 0.10, and T kept about constant at 0.100 (see Table 3.1). For generality and to help comparisions, the presentations of moment, shear, and torsion are in dimensionless forms as M/M_B, V/V_B, and M_t/M_B respectively.

3.2 Moment, Shear, and Torsion Diagrams

The study of moment, shear, and torsion diagrams due to single loads was carried out on both simple span and two-span continuous ribbed slabs with H = 4.5, b/a = 0.052, and T = 0.100. Two loading cases, for loads at midspan on the center and edge ribs, will be presented in this section.

3.2.1 Moment Diagrams

a. One-Span Ribbed Slab

Figure 3.1 shows moment diagrams of a few ribs due to the load at midspan on the center and edge ribs. The moment diagrams of all ribs, except the loaded rib and the first adjacent rib, are more or less parabolic curves which indicates that those ribs were subjected to a distributed load of some kind. These loads are transmitted from the loaded to the unloaded ribs by bending and shear forces in the slab. The slab also acts as an elastic support for the loaded rib, causing a portion of the moment diagram for the loaded rib between the support and the load to be concave upward. For the load at midspan on the center rib, the moment diagram of the first adjacent rib is also concave upward between the support and midspan, indicating greater load distribution occurred around midspan (which is close to the applied load). Instead of being concave, the moment diagram for the first adjacent rib due to the edge rib loading is more or less a straight line, indicating that this rib effectively is loaded only near midspan. As a result of the use of the Fourier series, the moment diagrams of the loaded ribs have a rounded curve instead of a sharp break at the point of maximum value.

The magnitudes of moments due to the edge rib loading are about twice those due to the center rib loading. The reason for this is that the load on the center rib is distributed to adjacent rib on both sides, which is impossible on the edge rib. For the ribbed slab analyzed in this section, the maximum moments of the loaded center and edge ribs are 0.056Pa and 0.125Pa, respectively, as compared to the simple beam moment of 0.25Pa.

b. Two-Span Ribbed Slab

The moment diagrams of the two-span ribbed slab are presented for two loading cases--loads at midspan on the center and edge ribs of one span. Midspan loading location is not for maximum moment, but it is the simplest location for studying general behavior of the two-span ribbed slab under a concentrated load and for making comparisons.

Figure 3.2 shows the moment diagrams of a two-span ribbed slab due to the above loadings. The general characteristics of these diagrams are similar to those of the simple span ribbed slab. For this particular ribbed slab, absolute values of the maximum positive and negative moments of the loaded edge rib are 0.094Pa and 0.042Pa respectively, and those of the loaded center rib are 0.051Pa and 0.017Pa. In comparison, the maximum positive and negative moments of a correspondingly loaded two-span beam are 0.203Pa and

0.094Pa respectively. It is clear that the maximum moments of the loaded ribs are significantly smaller than those of the beam, and for the same reasons as in the simple span ribbed slabs.

3.2.2 Shear Diagrams

a. One-Span Ribbed Slab

Shear diagrams of the unloaded ribs are more or less parabolic curves between the support and midspan, with maximum values at the supports, except that of the first adjacent rib which has high shear values on the portions close to midspan before it curves down to zero there. For the loaded rib, a distinctive high shear value was obtained on the portion close to the load, as would be expected. The shear diagram of this particular ribbed slab shows that shear is as high as 0.4P and 0.47P in the loaded center and edge ribs, respectively, whereas simple beam shear is constant at 0.5P all the way to the support. However, shear diagram of the loaded ribs is not like that of the simple beam, since for the edge rib loading, the maximum shears decrease sharply to a value about the same as those of the unloaded ribs; for the center rib loading, maximum shears decrease to a value somewhat less than those of the first few adjacent ribs, indicating good shear distribution. The maximum positive and negative shears of the loaded ribs are found a little distance away from midspan, and the portion of shear diagram connecting these two maximum shears is almost a straight line, which is a result of using Fourier series.

The shear diagrams also show some idea about how various ribs are affected when the load is applied at midspan on the center or edge rib. The maximum shear of the loaded rib decreases sharply, indicating that the rib is subjected to a significant downward load over the portion between the maximum positive and negative shear points, and to a relatively great upward reaction from the slab over the high shear portion. The upward load decreases rapidly from the point of maximum shear and continues as a small distributed load to the support, since shear increases slightly from the support to the high shear portion. Through slab action, the first adjacent rib is loaded downward by a load of the same magnitude as the upward load on the loaded rib (or by one-half of that if the loaded rib is the center one), although the force is distributed over a greater length than in the loaded rib. The rib is also subjected to some distributed upward load with a more or less parabolic shape with the maximum value at midspan. Loads on the other ribs also result from slab action and can be explained in the same way as above. Examples of these loads are sketched and shown in Fig. 3.3.

b. Two-Span Ribbed Slab

Shear diagrams of the two-span ribbed slab due to the load at midspan of the first span on the center and edge ribs are shown in Fig. 3.2. The general characteristics of the diagrams are similar to those of the simple span, as discussed above. In addition, the decreasing shear at the interior support results from the nature of the Fourier series, since the interior support was replaced by sets of vertical reactions and twisting moments on the ribs. Sketches showing an interpretation of the shear and torsional moment diagrams near the interior support are shown in Fig. 3.4. For the results of the particular ribbed slab analyzed in this section, the maximum positive and negative shears of the loaded center rib are 0.39P and 0.41P respectively, and those of the loaded edge rib are 0.44P and 0.50P (whereas the maximum positive and negative shears of a two-span prismatic beam are 0.41P and 0.59P). The difference between the maximum positive and negative shears of the loaded ribs is much smaller than that of a beam--especially in the loaded center rib. The effect might be attributed to the transverse shear distribution that reduces the shear at the support of the loaded rib. Slab action in the two-span ribbed slab is somewhat the same as that in the simple span, as far as shear diagrams are concerned.

3.2.3 Torsional Moment Diagrams

a. One-Span Ribbed Slab

Torsion diagrams due to the load at midspan on the center and edge ribs are shown in Fig. 3.1. This is not the loading case for maximum torsion, but rather is for maximum bending moment. The location of load for maximum torsion is somewhere between the support and midspan, as shown by the influence line for torsion at the support (see Fig. 3.9).

When the loading is at midspan on the center rib, there is no torsion on the loaded rib, because of symmetry, and maximum torsion is obtained at the support of the second adjacent rib. For the first adjacent rib, torsion increases slightly from the support toward midspan, until close to the quarter point of span. There the rate of increase is a little greater and torsion reaches a maximum value somewhere between the quarter point and midspan, whereas the other ribs have maximum values at the support. The torsion diagrams of all ribs except the first adjacent rib are more or less of parabolic shape between the support and midspan.

Load at midspan on the edge rib results in maximum torsion at the support of the loaded rib. It is different from the case of the load on the center rib, since the edge rib has a free edge, and therefore is freer to deflect and rotate than the others. The maximum torsion at the support of the loaded edge rib is more than three times the maximum torsion caused by the center rib loading. All torsion diagrams for the edge rib loading are also nearly parabolic for the portion between the support and midspan.

b. Two-Span Ribbed Slab

Figure 3.2 shows the torsion diagrams of the ribs of a two-span ribbed slab due to a load at midspan on the center and edge ribs of one span. At the exterior support of the loaded span, the maximum torsions of both loading cases occur in the same manner as those in the simple span ribbed slab. At the interior support, the torsion diagrams are also affected by the Fourier series, as are shear diagrams (see Fig. 3.4 for a graphical interpretation). The general characteristics of the torsion diagrams for the two-span ribbed slab are similar to those of the simple span ribbed slab.

3.3 Moment Envelopes and Influence Lines for Moment at Midspan and Influence Lines for Shear and Torsion at Support

The studies of the moment envelopes and influence lines for moment at midspan and influence lines for shear and torsion at support were carried out for a ribbed slab with H = 4.5, b/a = 0.052, and T = 0.100, for both one- and two-span structures. Two loading cases were considered--one load at midspan of the center and edge ribs. The moment envelopes and influence lines of any ribbed slab will be similar in general form. Therefore, the study in this section is intended to show an example of the general configuration and characteristics for a particular ribbed slab, which could be taken as the reference for any ribbed slab.

3.3.1 Moment Envelopes and Influence Lines for Moments at Midspan

a. One-Span Ribbed Slab

Figure 3.5 shows influence lines for moments at midspan of various ribs due to a load moving on the center and edge ribs. These influence lines have exactly the same shape and magnitude as the moment diagrams (see Fig. 3.1) due to a load at midspan on the corresponding center and edge ribs. This phenomena is explained by the fact that the simple span ribbed slab obeys the Reciprocal Theorem just as well as a simple beam does. However, the slab action causes both the shape and magnitude of the influence lines to be different from those of a simple beam.

Moment envelopes of the loaded ribs and a few adjacent ribs are also shown in Fig. 3.5. The moment envelope for the loaded rib is similar to that of a simple beam, but the magnitude is considerably smaller, as discussed above for the influence lines.

b. Two-Span Ribbed Slab

For two-span ribbed slabs, the influence lines for moments at midspan of one span and at the interior support due to a load moving on the center and edge ribs are illustrated in Fig. 3.6. Moment envelopes of the center and edge ribs and a few adjacent ribs are also shown in Fig. 3.6. The general characteristics of the influence lines and moment envelopes of the loaded ribs are similar to those of a two-span prismatic beam. Furthermore, moment envelopes of the loaded ribs indicate that the load location for maximum positive moment is approximately 0.4 times the span length from the exterior support; for maximum negative moment at the interior support, the load locations for center and edge ribs are about 0.7 and 0.8 times the span length respectively. These locations for maximum positive and negative moments compare with 0.43 and 0.57 times span length in the case of a two-span prismatic beam. It is quite clear that load location for maximum positive moment of rib and beam are about the same. For maximum negative moment, the load location on a rib (especially a center rib) is much closer to the interior support than is true on a beam. The cause of the difference could be slab action--i.e., load distribution among ribs is better when the load is far from the support. Note that better load distribution also means better moment distribution and reduced magnitude of the maximum moment.

For the unloaded ribs, the influence lines for moment at midspan and moment envelopes are more or less parabolic curves, especially for the ribs beyond the first adjacent rib (see Fig. 3.6).

3.3.2 Influence Lines for Shear at Support

a. One-Span Ribbed Slab

Influence lines for shear at the support show the distribution of reaction shears in various ribs due to a load moving on the center and edge ribs. Figure 3.7 shows that shear distribution at the support due to load at any section on the center rib is better than that due load at the corresponding section on the edge rib. When the load is a certain distance from the support, the reaction shear of the loaded center rib is smaller than those of the first few adjacent ribs (see Fig. 3.7). It could be interpreted that shear can distribute transversely to adjacent ribs better than longitudinally to the support of the loaded center rib. For the edge rib, shear can distribute transversely only to one side, and it must transmit the shear longitudinally to the support. These characteristics result from the geometry of the structure.

Influence lines for shear at the support of the loaded ribs approximate hyperbolic curves. This results from the two dimensional effect of the structure (slab action). The difference between the hyperbolic curve and a straight line is the amount of shear that distributes transversely to adjacent ribs by slab action. Also, as the load moves closer to the support, transverse shear distribution is limited and a high reaction shear on the loaded rib is obtained. For unloaded ribs, the influence lines are the result of distributed shears.

Reaction shear is not the maximum shear under a single load, except for the loaded rib with the load very close to, or at, the support. However, reaction shear is important, because in case of multiple loading, the maximum shear is always the reaction shear.

b. Two-Span Ribbed Slab

Influence lines for shear at the exterior support and at the 0.95 span point (near but not at the interior support) due to a load moving on the center and edge ribs are shown in Fig. 3.8. The general characteristics of the influence lines for shear at the exterior support are similar to those of the simple span. For shears at 0.95 span, a portion of the influence line very close to the interior support of the loaded rib is affected by usage of the Fourier series. In addition, shear distributions among the ribs, both at the exterior support and at 0.95 span points, due to loads at various sections on the center and edge ribs are different from those in the simple span ribbed slab in that shears in the loaded ribs are usually the maximum shears. This could mean that shear distribution in the simple span ribbed slab is better. From Fig. 3.8, the load location for maximum shear at 0.95 span would be somewhere between 0.90 and 0.95 of the span for both loading cases. For the unloaded ribs, the characteristics of the influence lines for shear at 0.95 span are also similar to those of the simple span. Therefore, the affect of the Fourier series on shear around the interior support is confined to the loaded ribs only.

3.3.3 Influence Lines for Torsion at Support

a. One-Span Ribbed Slab

Influence lines for torsion at the support of the ribs due to a load moving on the center and edge ribs are shown in Fig. 3.9. For the load on the edge rib, the greatest maximum torsion is obtained in the loaded rib, and maximum torsions of the other unloaded ribs are smaller in proportion to the distance from the loaded one. The load location for maximum support torsion in the loaded rib is nearest to the support, and for unloaded ribs the location is successively farther from the support, in proportion to the distance from the loaded rib.

When the load moves on the center rib, there is no torsion in the loaded rib because of symmetry, as mentioned in Section 3.2.3, and the greatest maximum torsion is found on the first adjacent rib. In the portion of the influence line from the support to the load location for maximum torsion in the first adjacent rib, the rate of increase of torsion is great in the first adjacent rib; but for the other portion of the span, the rate of decrease of torsion is also great, resulting in smaller torsion on the first adjacent rib in this portion of the span (see Fig. 3.9). The characteristics of load locations on the center rib for maximum torsion in the unloaded ribs are similar to those of the edge rib loading, as discussed above.

The difference in load locations for maximum torsion at the support of various ribs result from a shear distribution phenomena--i.e., shear distribution is limited if the load is close to the support, as mentioned in Section 3.3.2. Therefore, for the unloaded ribs far from the loaded one, the load has to be some distance out on the span before the unloaded rib receives greatest effect of distributed shears causing maximum torsion.

b. Two-Span Ribbed Slab

Influence lines for torsion at the exterior support and at the 0.95 span point due to a load moving on the center and edge ribs are shown in Fig. 3.10. The general characteristics of the influence lines for torsion at both the exterior support and at 0.95 span are similar to those of the simple span, as discussed above. But the magnitude of torsion at 0.95 span is somewhat smaller than at the exterior support. Torsions at the exterior support of various ribs are very small when the load is on the other span.

3.4 Moment Distribution and Moment Envelope Across the Span

a. One-Span Ribbed Slab

When a load moves along the midspan section of a ribbed slab, maximum moments at midspan of all the loaded ribs are about the same, except that the first few near the edge rib have greater moments. Figure 3.11 shows the moment envelope at midspan of a ribbed slab with H = 4.5, b/a = 0.052, and T = 0.100. For this particular ribbed slab, moments in the edge, second, and third ribs are 1.89, 1.29, and 1.10 of that of the center rib, respectively. Moments in the second and third ribs can be reduced if the edge rib is made stiffer (i.e., a strong edge beam is provided). In Fig. 3.11, two cases of moment envelopes of the ribbed slab with stiffer edge ribs are also illustrated. First, the width of the edge rib is double that of a regular one, and the results of moments in the edge, second, and third ribs are 2.28, 1.07, and 1.02 of that of the center rib, respectively. Second, the width of the edge rib is triple that of a regular one, and result of moments in the edge, second, and third ribs are 2.44, 0.99, and 0.98 of that of the center rib. Hence, moments in all ribs can be evened out by providing a very strong edge beam, which has to carry a greater bending moment.

Figure 3.11 also shows moment distributions of various ribs due to a single load at their midspan. Since the Reciprocal Theorem holds for the simple span ribbed slab, the moment distribution curves also represent the influence lines for moments at midspan of the loaded ribs due to a load moving along midspan. For any ribbed slab, moment distributions due to a load at the same location will have similar shapes, but the magnitude of the distributed moments depends upon the important parameters discussed in Section 1.5. Therefore, the study in this section should be considered as a reference for a general ribbed slab. The affects of those parameters on moments will be discussed later.

b. Two-Span Ribbed Slab

Figure 3.12 shows moment envelopes and moment distributions at midspan of the first span of a two-span ribbed slab due to a load moving along the midspan, and also those of the coresponding negative moment at the interior support. The general behavior of moment envelope and moment distributions at midspan is similar to the simple span ribbed slab; for example, moments under the load at midspan of the edge, second, and third ribs are 1.84, 1.24, and 1.08 of that of the center rib, respectively. The moment envelope at the interior support due to the load at midspan is similar to those of the positive moment at the midspan. But the negative moments of the edge and second ribs, as compared to that of the center rib, are much greater than in the case of the positive moments. The results in Fig. 3.12 indicate that negative moments at the interior support of the edge, second, and third rib are 2.47, 1.53, and 1.06 of that of the center rib, respectively. Negative moment distributions of various ribs at the interior support due to midspan loads are also shown in Fig. 3.12. The distributed negative moment in the first adjacent rib relative to that of the loaded rib is considerably greater than the relative distributed positive moment of the same rib at midspan. Other characteristics of negative moment distributions in general are similar to those of the positive moments at midspan.

The affects of a stiffened edge rib or edge beam are not presented for the two-span ribbed slab. However, results similar to those of the simple span ribbed slab should be expected.

3.5 Effects of Torsional Stiffness on Moment, Shear, and Torsion

The affects of the parameter T on moment have been studied and reported in Ref. 5 and 11. Reference 11 showed that the moment of the loaded girder decreases as magnitude of T increases, but a significant affect was found only in the range of T from 0.0 to 0.20. In Ref. 5, test results from steel I-beam girder-slab bridges indicated that the affect of variation in torsional stiffness of steel I-beam is negligible.

For this study, the influence of the parameter T on moment, shear, and torsion are presented. The investigation was performed on simple span ribbed slabs with constant magnitudes of H = 2.0 and 4.0, b/a = 0.05 and 0.10, and with T varied from 0.063 to 0.161 (see Table 3.2). This is in the range of significant affects of T on moment as reported in Ref. 11. The study considered two loading cases (load at midspan on the center and edge ribs), and results of the analysis are shown in Fig. 3.13. The influence of T on the moments at midspan, and shear at the support of the loaded ribs are similar--moment and shear decrease as T increases. But the changes are not very significant in the range of T investigated, which is considered the practical range for conventional ribbed slabs. For example, the results for ribbed slabs with H = 4.0 and b/a = 0.05 show that the maximum moments of the center and edge ribs decreased 8.5 and 13.7 percent, respectively, as T increased 98.8 percent.

The influence of T on maximum torsional moment at the support due to the above loadings is opposite the affects on moment and shear. As T increases, the maximum torsion also increases quite significantly. The results of the same ribbed slabs with H = 4.0, b/a = 0.05 show that the mix mum torsion due to a load at midspan on the center and edge ribs increased 59.6 and 42.8 percent, respectively, as T increased 98.8 percent (see Fig. 3.13). However, in this study, the magnitudes of T used are approximately constant, because the influence of T on bending moment and shear are very small, as mentioned above. The influence of T on the torsion is significant, but the increased torsional strength accompanying increased torsional stiffness would be enough to compensate for the amount of the increased torsion. Furthermore, torsion is considered of secondary importance in the structure. In addition, the magnitude of T selected for this analysis (about 0.100) are considered common for general practice.

3.6 Effects of H and b/a on Moment, Shear, and Torsion

From the results of the studies of behavior of ribbed slabs under a concentrated load in previous sections, it might be concluded that the characteristics of moment, shear, and torsion distributions are somewhat changed according to different load locations on different ribs. In order to make the study in this section simple and yet reveal enough detail, solutions for two loading cases are considered--load at midspan on the center and edge ribs. These load locations give the lower and upper bound values of moment for any ribbed slab under a concentrated load. For shear and torsion, the center and edge ribs might be considered as being representative of the interior and exterior ribs. And as shown in Sections 3.3.2 and 3.3.3, the characteristics of the shear and torsion distributions also depend upon locations along the span; therefore, midspan is considered to be a suitable load location for studying the general behavior.

The study of the influence of H and b/a on moment distribution is confined to the midspan section, where maximum moments are obtained. Shear and torsion are studied at the support. The results of the ribbed slabs analyzed with parameters of various magnitudes, as shown in Table 3.1, are presented in this section.

3.6.1 Effects of H and b/a on Moment Distribution Across Span and Maximum Moment of the Loaded Rib

a. One-Span Ribbed Slab

The moment distributions among ribs of simple span ribbed slabs due to a load at midspan on the center and edge ribs depend upon the values of the parameters H and b/a. The results of the analysis show that the moment distribution is better for the ribbed slab with smaller H and b/a values, as shown in Fig. 3.14. Comparing graphs of constant b/a and various H values, and those of constant H and various b/a ratios, it is evident that the effect of b/a is somewhat greater than that of H. For a ribbed slab, the moment distribution due to the load on the center rib is better than that due to the load on the edge rib. Examples of two simple span ribbed slabs with H = 2.0, b/a = 0.035, and H = 9.0, b/a = 0.10 are as follows:

H = 2.0, b/a = 0.035

Load at midspan on the edge rib:

Maximum moment of the loaded rib = $0.0761Pa = 0.3044M_B$

Moment of the eighth adjacent rib (center rib) = $0.0041Pa = 0.0164M_B$

Load at midspan on the center rib:

Maximum moment of the loaded rib = $0.0366Pa = 0.1464M_B$

Moment of the eighth adjacent rib (edge rib) = $0.0043Pa = 0.0172M_B$

$H = 9.0$, $b/a = 0.10$

Load at midspan on the edge rib:

Maximum moment of the loaded rib = $0.1507Pa = 0.6028M_B$

Moment of the third adjacent rib (rib no. 4) = $0.0066Pa = 0.0264M_B$

Load at midspan on the center rib:

Maximum moment of the loaded rib = $0.0940Pa = 0.3760M_B$

Moment of the third adjacent rib (rib no. 6) = $0.0073Pa = 0.0292M_B$

From the above examples we can see that for the ribbed slab with H = 2.0, b/a = 0.035, the moment of the eight rib from the loaded edge rib (i.e., the center rib) is about 5.4 percent of that of the loaded rib, and the moment of the eighth rib from the loaded center rib (i.e., the edge rib) is about 11.8 percent of that of the loaded rib. For the ribbed slab with H = 9.0, b/a = 0.10, the moment of the third adjacent rib from the loaded edge rib (i.e., rib no. 4) reduced to about 4.4 percent of that of the loaded rib, and the moment of the third rib from the loaded center rib (i.e., rib no. 6) reduced to about 7.8 percent of that of the loaded rib. It is quite obvious that the moment distribution of the former ribbed slab is much better than that of the latter one, and the maximum moments of the corresponding loaded ribs of the second ribbed slab are much greater.

As a result of good moment distribution, maximum moment of the loaded rib is smaller than if moment distribution is not good. It may be concluded that the magnitude of the maximum moment of the loaded rib is a direct function of H and b/a, which control the moment distributions among ribs. The relationships of the maximum moments of the loaded ribs with H and b/a are shown in Fig. 3.23. As shown in Fig. 3.23, the effects of H and b/a on the maximum moment are not in the same order. Graphs of the maximum moment versus b/a, for several constant H values, are closer together and have steeper slopes than graphs of the maximum moment versus H. Therefore, it is evident that b/a has a greater effect on the maximum moment of the loaded ribs than H. Furthermore, as a smaller maximum moment of the loaded rib also means better moment distribution and vice versa, it is evident that b/a is a more important parameter in controlling moment distribution than H.

b. Two-Span Ribbed Slab

The study of moment distributions among ribs of two-span ribbed slabs concentrated at the sections at midspan of the first span and at the interior support. Positive and negative moment distributions due to a load at midspan of one span on the center and edge ribs are illustrated in Fig. 3.15 and 3.16. The results show that the general characteristics of moment distributions at midspan are similar to those of the simple span. For negative moment at the interior support, moment distribution characteristics are similar to those of the positive moment when load is on the edge rib. For a load on the center rib, the negative moment in the first adjacent rib is comparable to that of the loaded rib, as discussed in Section 3.4. The effects of H and b/a on moment distributions, both at midspan and the interior support, are similar to those noticed in the simple span ribbed slab--i.e., a better moment distribution is obtained in a ribbed slab with smaller H and b/a values.

Relationships of the maximum positive and negative moments of the loaded ribs versus H and b/a are also similar to those of the maximum moment of the simple span--i.e., maximum moment of the loaded ribs increases at a diminishing rate as H and b/a increase (see Fig. 3.24).

3.6.2 Effects of H and b/a on Shear Distribution Across Span and Maximum Support Shear

a. One-Span Ribbed Slab

Shear distributions at the support due to a load at midspan on the center and edge ribs of various ribbed slabs are illustrated in Fig. 3.17. The distribution diagrams show that maximum support shear did not occur in the loaded rib except in the case of a loaded edge rib with rather stiff ribs and larger b/a ratios. For the center rib loading, the maximum shear can be in any of the first few adjacent ribs, depending upon H and b/a, but never in the center rib. If it is an edge rib loading, the maximum shear is either in the loaded or first adjacent rib. The effects of H and b/a on shear distributions are similar to those on moment distribution--i.e., the smaller H and b/a values result in better shear distribution.

In this analysis, the smallest H and b/a are 2.0 and 0.035, and the greatest are 9.0 and 0.10, respectively. Comparisons of shear distributions at support due to a load at midspan on the center and ribs of these two ribbed slabs are as follows:

$H = 2.0$, $b/a = 0.035$

Load at midspan on the edge rib:

Shear of the loaded rib = $0.0703P = 0.1406V_B$

Maximum shear in the first adjacent rib (rib no. 2) = $0.0820P = 0.1640V_B$

Shear of the eighth adjacent rib (center rib) = $0.0129P = 0.0258V_B$

Load at midspan on the center rib:

Shear of the loaded rib = $0.0166P = 0.0332V_B$

Maximum shear in the third adjacent rib (rib no. 6) = $0.0369P = 0.0738V_B$

Shear of the eighth adjacent rib (edge rib) = $0.0130P = 0.0260V_B$

$H = 9.0$, $b/a = 0.10$

Load at midspan on the edge rib:

Shear of the loaded rib (maximum shear) = $0.2200P = 0.4400V_B$

Shear of the fourth adjacent rib (rib no. 5) = $0.0017P = 0.0034V_B$

Load at midspan on the center rib:

Shear of the loaded rib = $0.0750P = 0.1500V_B$

Maximum shear in the first adjacent rib (rib no. 8) = $0.1097P = 0.2194V_B$

Shear of the fourth adjacent rib (rib no. 5) = $0.0050P = 0.0100V_B$

Shear distributions in the above example are quite different; the former ribbed slab with load at midspan on the edge rib, shear in the eighth adjacent rib (i.e., center rib) is about 15.7 percent of the maximum shear, and with load at midspan on the center rib, shear in the eighth adjacent rib (edge rib) is as high as 35.2 percent of the maximum shear. In the second ribbed slab with a load at midspan on the edge rib, shear in the fourth adjacent rib (i.e., rib no 5) is reduced to about 0.8 percent of the maximum shear, and with load at midspan on the center rib, shear in the fourth adjacent rib (i.e., rib no 5) reduced to about 4.6 percent of the maximum shear. It is clear that shear distributions of the former ribbed slab are much better than that of the latter one.

The relationships of the maximum shear with changes in H and b/a are shown in Fig. 3.25. The graphs indicate that the maximum support shear due to a load at midspan on the center and edge ribs increases more or less linearly as b/a increases. The effect of H on the maximum shear is somewhat similar to that of b/a, but is less significant.

b. Two-Span Ribbed Slab

Shear distributions at the exterior support and at the 0.95 span location due to a load at one midspan on the center and edge ribs are shown in Fig. 3.18 and 3.19, respectively. Note that according to the sign convention used in this analysis (see Fig. 2.1), shears at 0.95 span are negative, but for convenience, the absolute values are used in Fig. 3.19. The characteristics of shear distributions, at both the exterior support and the 0.95 span position, are somewhat different from those of the simple span ribbed slabs, in that the loaded ribs for both loading cases carry the maximum support shears for nearly all values of the parameters H and b/a. The maximum shears near the interior support are generally about twice the shear forces at the exterior supports for the midspan loading cases considered.

The effects of H and b/a on the characteristics of the shear distributions are similar to those in the simple span ribbed slabs, i.e., smaller H and b/a values result in better shear distribution. The relationships of the maximum reaction shear with H and b/a, as can be seen in Fig. 3.26, are also similar to those in the simple span ribbed slabs, i.e., the maximum reaction shear increases as H and b/a increase.

3.6.3 Effects of H and b/a on Torsional Moment Distribution, and Maximum Torsion

a. One-Span Ribbed Slab

Torsional moment distributions at the support due to the same loadings used in the studies of moment and shear distributions are considered in this section. These loadings are not for maximum torsion, but they are simply for general study and comparison. The torsion distributions of selected ribbed slabs with various values of H and b/a are shown in Fig. 3.20. Note that the torsions due to the load at midspan on the edge rib are negative according to the sign convention, but the absolute values are used in Fig. 3.20. The effects of H and b/a on torsion distributions are such that the distribution is better for the smaller H and b/a. Maximum torsion due to the load on the edge rib is much greater than that due to the load on the center rib, because of the geometry of the structure.

The relationships of maximum torsions for both loadings as functions of H and b/a are shown in Fig. 3.27. The maximum torsion increases at a diminishing rate as H and b/a increase, in a manner similar to that described in Section 3.6.1, but the effect of H on the maximum torsional moment is somewhat greater than that on the maximum bending moment.

b. Two-Span Ribbed Slab

Torsion distributions at the exterior support and at the 0.95 span point due to a load at midspan of one span on the center and edge ribs are shown in Fig. 3.21 and 3.22. Note that torsions at the exterior support due to the load on the edge rib, and torsions at 0.95 span due to the load on the center rib are negative according to the sign convention used, but the absolute values are used in Figs. 3.21 and 3.22. The general characteristics of the torsion distributions, both at the exterior support and at 0.95 span, are similar to those in the simple span ribbed slabs. The effects of H and b/a on the torsion distribution are also similar to those in the simple span ribbed slabs, i.e., the smaller H and b/a values result in better torsion distribution.

The relationships of the maximum torsion with H and b/a are shown in Fig. 3.28. The maximum torsion increases as H and b/a increase in the manner similar to that in the simple span ribbed slabs.

3.7 Effects of H and b/a on Influence Lines for Torsional Moment at the Support, and the Maximum Torsion

We have learned that the load location for maximum torsion is uncertain for different ribbed slabs. Therefore, this section will show the influence lines for torsion at the support of various ribbed slabs. Since the general characteristics of the influence lines for torsion at the supports of one-and two-span are similar, as shown in Figs. 3.9 and 3.10, the presentation in this section will be made for the simple span ribbed slabs only. Two loading cases, single loads moving on the center and edge ribs, are considered. The influence lines of the ribs with the maximum torsion for each loading case will be compared for various values of H and b/a, i.e., the influence lines of the loaded edge rib, and the first adjacent rib (rib no. 8) due to the load on the center rib.

Figure 3.29 shows the influence lines of the ribs mentioned above for various ribbed slabs analyzed. The load locations for the maximum torsion at the support for each ribbed slab are a little different, for the two loading cases. This location is nearer to the support for the smaller H and b/a values. For the case of loading on the edge rib, load locations for the maximum torsion at the support of the ribbed slabs analyzed are between 0.15 and 0.35 span from the support; and for the case of loading on the center rib, these locations are in the range of 0.1 to 0.3 of the span. The maximum torsion due to the former loading is much greater than that due to the later loading.

The relationships of the maximum torsion with H and b/a are shown in Fig. 3.30, and they are similar to those due to a load at midspan on the center and edge ribs, as described in Section 3.6.3.

CHAPTER 4

RIBBED SLABS UNDER MULTIPLE LOADS

4.1 General

The study of the general characteristics of ribbed slabs subjected to multiple loads, especially in parking garage floors, is the purpose of this chapter, and both one- and two-span structures are investigated. Multiple loads are considered to be the wheel loads of several parked cars of equal weight, and each car having the weight divided evenly among the four wheels. The arrangements of the loads will be made in such way that worst conditions are obtained, even though cars might not be parked like that in general paractice. In fact, cars are movable loads, which might cause impact and repeated load effects. However, in this study, static load is considered most important, because car speed in a parking garage is very slow and not all cars move or brake at the same time.

In this study, it is assumed that the cars are parked perpendicular to the ribs, so that the maximum bending moment, shear, and torsional moment in the ribs are obtained. The length of wheel base and spread between wheels (or tread width) are taken as ten and five feet, respectively. These dimensions are based on the information by Burrage and Morgen (16), and Baker (17). The car spacing is based on a suggested minimum width of the stall in a parking garage of eight feet. Distance between rows of cars from wheel base to wheel base is taken as six feet (i.e., the cars are parked bumper-to-bumper).

The characteristics of ribbed slabs under multiple loads will be studied under two situations; first, the perimeter wheel loads are on the edge rib and second, the perimeter wheel loads are on the second rib. The system of loading considered in this chapter is shown in Fig. 4.1. The cross sections and span lengths of the ribbed slabs in Table 3.1 will be used for the analysis. The presentation of results will be in dimensionless forms in order to make the study more general, since some of the ribbed slabs analyzed have different span lengths and were subjected to different numbers of loads. The dimensionless quantities are M/M_B, V/V_B, and M_t/M_B, where M, V, and M_t are bending moment, shear, and torsional moment of a rib, respectively, and M_B and V_B are the maximum moment and shear of a simple span beam which has the same span length and is subjected to loads with the spacing as the ribbed slab. Values of various M_B and V_B are listed in Table 4.1 for ease of comparison with the results of ribbed slabs discussed in this chapter.

4.2 Moment, Shear, and Torsional Moment Diagrams

The studies of moment, shear, and torsional moment diagrams of the ribs of one- and two-span ribbed slabs are carried out on a ribbed slab which has parameters H = 4.5, b/a = 0.052, and T = 0.100. Loadings for maximum moment, shear, and torsion are considered, in which the loadings for maximum moment and shear are coincident. For two-span structure, loadings for maximum positive and negative moments also produce maximum shears at the exterior and interior supports, respectively.

The moment, shear, and torsional moment diagrams, for both one- and two-span ribbed slabs, due to loadings mentioned above, are illustrated in Fig. 4.2 and 4.3. The general configurations of the moment diagrams are similar to those of isolated beams subjected to a uniformly distributed load. This phenomena might be explained that the loaded ribs or slabs were subjected to groups of concentrated loads with rather closed spacings (i.e., three and five feet alternately). Furthermore, these concentrated loads were replaced by a Fourier series approximation, which tends to distribute the loads at the load locations. The slab also tends to spread out the loads as they are distributed from rib to rib. Therefore, the influence of a group of concentrated loads becomes similar to that of a distributed line load.

Shear diagrams for the ribs subjected directly to the loads have some sharp slope discontinuities in the diagrams at the load locations because of the concentrated load effects. For the ribs which are not directly under the loads the shear diagrams are more or less parabolic curves. For two-span structure, a small portion of the shear diagram near the interior support is affected by using Fourier series approximation; the maximum shear is found at a very short distance from the interior support (i.e., at the 0.95 span point) instead of the support.

Torsional moment diagrams for one- and two-span ribbed slabs are also shown in Figs. 4.2 and 4.3, respectively. The general characteristics of these diagrams are similar to those of shear diagrams for the ribs which are not directly under the applied loads, as mentioned above. Torsional moment diagrams for the two-span ribbed slabs due to loads on one span (i.e., loadings for maximum positive moment) or two-span symmetry (i.e., loading for maximum negative moment at the interior support) are also affected by use of the Fourier series at the interior support. But for two span asymmetric loading (i.e., loading for maximum torsional moment at the interior support of the center rib), the torsional moment diagram is unique as shown in Fig. 4.3. Because torques are applied to the rib in the opposite direction at the interior support by such a loading, the effect of using Fourier series is eliminated.

4.3 Multiple Loads for Maximum Moment and Shear

The study of moments of ribbed slabs subjected to multiple loads is focused on the maximum moments of the ribs, and the distributions of moments. The characteristics of moment distributions among ribs depend upon the properties of the ribbed slabs and the loading conditions. In this study, the results of the analysis for the ribbed slabs listed in Table 3.1 are considered to cover the normal range of the important parameters for ribbed slabs. The loadings are again arranged such that the perimeter loads are on the edge rib, and then on the second rib, as mentioned in Section 4.1. Note that the load patterns for maximum positive and negative moments are coincident with those for maximum shears for one- and two-span ribbed slabs, respectively.

4.3.1 <u>One-Span Ribbed Slab</u>

For the one-span ribbed slab, bending moment is important at midspan, whereas the combination of shear and torsional moment are most important at the support. Loading for maximum moment also results in maximum shear, but torsional moment is small. However, the effect of torsional moment in combination with shear should be considered. Therefore, moment at midspan, and shear and torsional moment at the support due to the loadings for maximum moment are presented.

a. Moments at Midspan

Maximum moments at midspan of various ribs due to both loading cases are shown in Fig. 4.4 for comparison. For the loading with the perimeter loads on the edge rib, the maximum moment always occurred on the edge rib, and its magnitude was considerably greater than the moments in the other ribs. For the loading with the perimeter loads on the second rib, the maximum moment might be in the edge or an interior rib, depending upon properties of the ribbed slab, in which b/a has the major effect (i.e., the maximum moment is in an interior rib for all the ribbed slabs analyzed with b/a greater than 0.035). The maximum moment due to this loading is considerably smaller than that of the former loading. The results in Fig. 4.4 show that moment distributions are quite uniform, especially for ribbed slabs with small H and b/a values, subjected to loads of the latter case.

b. Shear and Torsional Moment at the Support

For a flexure member, maximum shear is usually another force to be considered; shear is as important as bending moment, especially for a member without web reinforcement. For ribbed slabs subjected to concentrated loads, not only shear but also torsion exists at the support, as mentioned above. As far as the combined shears and torsions are concerned, the most significant torsion due to these loadings is not the maximum torsion, but the torsion in the rib with the maximum shear. However, torsion in various ribs is shown for better understanding. Shear and torsion at the support due to both loading cases for maximum moment are shown in Figs. 4.5 and 4.6, respectively. The shear distributions are less uniform than moment distributions for both loading cases. The torsion distributions are different and difficult to compare with moment or shear distributions, because the sign for torsion depends upon the direction of rotation of the ribs about their longitudinal axes. Therefore, torsion in the ribs on the opposite sides of a load would have a tendency to have different signs.

The general characteristics of the effects of loading conditions, H, and b/a on both shear and torsion are similar to those on moments. For the loading with the perimeter loads on the edge rib, both maximum shear and torsion occurred in the edge rib. For the loading with the perimeter loads on the second rib, the torsion corresponding to the maximum shear is very small.

For example, maximum moment, shear and the corresponding torsion of two ribbed slabs with H = 2.0, b/a = 0.035, and H = 9.0, b/a = 0.10 are shown as follows:

H = 2.0, b/a = 0.035

Perimeter loads on the edge rib:

Maximum moment (in edge rib)	=	$0.3185M_B$
Maximum shear (in edge rib)	=	$0.4000V_B$
Corresponding torsion (in edge rib)	=	$-0.0226M_B$

Perimeter loads on the second rib:

Maximum moment (in edge rib)	=	$0.2579M_B$
Maximum shear (in second rib)	=	$0.3016V_B$
Corresponding torsion (in second rib)	=	$-0.0052M_B$

H = 9.0, b/a = 0.10

Perimeter loads on the edge rib:

Maximum moment (in edge rib)	=	$0.5685M_B$
Maximum shear (in edge rib)	=	$0.5737V_B$

Corresponding torsion (in edge rib) = $-0.0567M_B$

Perimeter loads on the second rib:

Maximum moment (in rib no. 6) = $0.4255M_B$

Maximum shear (in rib no. 7) = $0.4158V_B$

Corresponding torsion (in rib no. 7) = $0.0031M_B$

These are the results of the smallest and greatest maximum moments for each loading case among the ribbed slabs analyzed.

4.3.2 Two-Span Ribbed Slab

In the positive moment region only the bending moment is important, whereas at the exterior support the combination of shear and torsion are important. In the negative moment region, or at the interior support, bending moment, shear, and torsion can exist simultaneously in many combinations, depending on both the loading and the properties of the structure. The most important combinations could be considered as follows: First, the case of maximum moment plus maximum shear, and torsion, and second, the case of maximum torsion plus shear and moment. The first case will be discussed in this section, and the second case will be discussed later.

a. Moment at 0.44 Span and at the Interior Support

For a two-span ribbed slab, the maximum positive moment due to the applied load does not occur at midspan, but rather at a point close to midspan, as shown by the moment diagrams in Fig. 4.3. For this study, the moment at 0.44 of the span from the exterior support is considered to be the maximum positive moment. If the maximum moment occurs at any other section, the value of moment obtained at 0.44 span would close enough to the maximum moment for any practical purpose, because the moment diagrams show that the rate of change of moment is very small in that portion of the span (see Fig. 4.3).

The positive moment distribution at 0.44 span and the negative moment distribution at the interior support of various two-span ribbed slabs due to the loading cases for maximum positive and negative moments are shown in Figs 4.7 and 4.8, respectively. The general characteristics of the solutions for the positive moment are similar to those of the moment at midspan of the one-span ribbed slabs, as discussed previously. The effects of H and b/a on the negative moment at the interior support are similar to those on the positive moment at 0.44 span or the moment at midspan of the one-span ribbed slabs, in that a better moment distribution is obtained for the smaller H and b/a values. But the negative moment distributions at the interior support are a little less uniform than those of the positive moments. One reason for this might be that the negative moment is most affected by loads which are relatively close to the interior support, whereas the positive moment is affected by most of the loads in the span, as shown by the influence line for positive and negative moments at midspan and the interior support, respectively (see Fig. 3.6). Note that the distribution of loads near midspan is better than that of loads near the support.

The magnitudes of the maximum negative moments are greater than those of the maximum positive moments, and for a similar loading case, the maximum positive and negative moments usually occurred in the same rib. In addition, the negative moments at the interior support due to loadings for maximum positive moment are exactly one-half the maximum negative moments shown in Fig. 4.8.

b. Shear and Torsional Moment at the Exterior Support and at 0.95 Span

Shear and torsion distributions at the exterior support and at the 0.95 span position due to two loading cases for both maximum positive and negative moments are shown in Figs. 4.9 to 4.14. The values at the 0.95 span point are presented instead of at the interior support, because of the effects of Fourier series, as mentioned before. The general characteristics of the shear and torsion distributions, as shown in the figures mentioned above, are similar to those of the one-span ribbed slabs. For the loadings with the perimeter loads on the second rib, torsion is relatively small as compared to that due to the loading with the perimeter loads on the edge rib. The important torsions due to these loadings are those of the ribs with maximum shear at the exterior support, and with maximum shear and moment at the interior support.

The shear and torsion distributions at the exterior support due to loadings for maximum negative moment are similar to those due to loadings for maximum positive moment, except that the magnitudes are smaller. They are also very similar to the values for one-span structures discussed previously. Therefore, such shear and torsion distributions at the exterior support are not presented. At the 0.95 span location (or at the interior support for moment), the combination of negative moment, shear, and torsion, makes the problem more complicated than at the other end of the span. Shears at the 0.95 span point due to both loadings for maximum negative moment are considerably greater than those due to loadings for maximum positive moment. Negative moment at the interior support due to the loadings for maximum negative moment is twice that due to the loadings for maximum positive moment. The combined shear, torsion, and bending moment at the interior support due to loadings for maximum negative moment is the most important set of forces governing the design of ribbed slab structures.

For example, solutions of two ribbed slabs with H = 2.0, b/a = 0.035, and H = 9.0, b/a = 0.10, are shown as follows:

H = 2.0, b/a = 0.035

Loadings for maximum positive moment at 0.44 span

Perimeter loads on the edge rib:

Maximum positive moment (in edge rib)	=	$0.2563M_B$
Maximum shear at the exterior support (in edge rib)	=	$0.3611V_B$
Torsion at the exterior support (in edge rib)	=	$-0.0205M_B$
Maximum shear at 0.95 span (in edge rib)	=	$-0.4718V_B$
Torsion at 0.95 span (in edge rib)	=	$0.0158M_B$
Negative moment at the interior support (in edge rib)	=	$-0.1873M_B$

Perimeter loads on the second rib:

Maximum positive moment at 0.44 span (in edge rib)	=	$0.1992M_B$
Maximum shear at the exterior support (in rib no. 2)	=	$0.2667V_B$
Torsion at the exterior support (in rib no. 2)	=	$-0.0046M_B$
Maximum shear at 0.95 span (in rib no. 2)	=	$-0.3099V_B$
Torsion at 0.95 span (in rib no. 2)	=	$0.0034M_B$
Negative moment at the interior support (in rib no. 2)	=	$-0.1266M_B$

Loadings for maximum negative moment at the interior support

Perimeter loads on the edge rib:

Maximum negative moment (in edge rib)	=	$-0.3746M_B$
Maximum shear at 0.95 span (in edge rib)	=	$-0.6390V_B$

Torsion at 0.95 span (in edge rib) = $0.0102M_B$

Maximum shear at the exterior support (in edge rib) = $0.3314V_B$

Torsion at the exterior support (in edge rib) = $-0.0185M_B$

Perimeter loads on the second rib:

Maximum negative moment (in rib no. 2) = $-0.2531M_B$

Maximum shear at 0.95 span (in rib no. 2) = $-0.4001V_B$

Torsion at 0.95 span (in rib no. 2) = $0.0018M_B$

Maximum shear at the exterior support (in rib no. 2) = $0.2394V_B$

Torsion at the exterior support (in rib no. 2) = $-0.0040M_B$

H = 9.0, b/a = 0.10

Loadings for maximum positive moment at 0.44 span

Perimeter loads on the edge rib:

Maximum positive moment (in edge rib) = $0.4579M_B$

Maximum shear at the exterior support (in edge rib) = $0.5317V_B$

Torsion at the exterior support (in edge rib) = $-0.0521M_B$

Maximum shear at 0.95 span (in edge rib) = $-0.6502V_B$

Torsion at 0.95 span (in edge rib) = $0.0423M_B$

Negative moment at the interior support (in edge rib) = $-0.3158M_B$

Perimeter loads on the second rib:

Maximum positive moment (in rib no. 6) = $0.3353M_B$

Maximum shear at the exterior support (in rib no. 7) = $0.3809V_B$

Torsion at the exterior support (in rib no. 7) = $0.0042M_B$

Maximum shear at 0.95 span (in rib no. 7) = $-0.4608V_B$

Torsion at 0.95 span (in rib no. 7) = $-0.0027M_B$

Negative moment at the interior support (in rib no. 7) = $-0.2234M_B$

Loadings for maximum negative moment at the interior support

Perimeter loads on the edge rib:

Maximum negative moment (in edge rib)	=	$-0.6316M_B$
Maximum shear at 0.95 span (in edge rib)	=	$-0.8673V_B$
Torsion at 0.95 span (in edge rib)	=	$0.0290M_B$
Maximum shear at the exterior support (in edge rib)	=	$0.4880V_B$
Torsion at the exterior support (in edge rib)	=	$-0.0475M_B$

Perimeter loads on the second rib:

Maximum negative moment (in rib no. 7)	=	$-0.4468M_B$
Maximum shear at 0.95 span (in rib no. 7)	=	$-0.5994V_B$
Torsion at 0.95 span (in rib no. 7)	=	$-0.0068M_B$
Maximum shear at the exterior support (in rib no. 7)	=	$0.3449V_B$
Torsion at the exterior support (in rib no. 7)	=	$0.0053M_B$

4.3.3 Maximum Deflections

The maximum deflections of ribbed slabs, both one- and two-span subjected to multiple loads, resulted from loading for maximum positive moment. The absolute values of maximum deflection (i.e., in inches) of the ribbed slabs analyzed are shown in Tables 4.2 and 4.3. Since the ribbed slabs analyzed have different span lengths and cross section properties, the deflection coefficients in terms of the applied load, span length, and the flexural stiffness of the composite section are considered for the study of the general relationships involved. The deflection coefficient is as follows:

$$K = \delta \frac{EI}{nPa^3}$$

where

K = deflection coefficient

δ = absolute deflection

n = number of the applied loads in the span direction, considering only those loads located more than the rib depth from the support

For example, the relationships of the maximum deflection coefficients for one-span ribbed slabs under the loading with the perimeter loads on the edge rib with H and b/a are shown in Fig. 4.15. The coefficient K increases slightly as H increases. The relationships between the coefficient K and b/a are more or less linear with the values of K considerably increases as b/a increases.

4.4 Multiple Loads for Maximum Torsional Moment

Since torsional moments are significant, this section will focus on maximum torsions of various ribbed slabs due to the loadings arranged as shown in Fig. 4.1. Load patterns that maximize torsional moment in two-span ribbed slabs depends largely upon the absolute rib spacing, and slightly upon H and b/a. Therefore, for simplification, load patterns on two-span ribbed slabs with the same magnitude of rib spacing were taken to be the same (i.e., on those with b/a = 0.07, and 0.10). Besides torsional moment, the corresponding values of shear and bending moment (if any) will also be presented, since the combined torsion, shear, and bending moment in the rib is the important phenomena, as mentioned previously.

4.4.1 <u>One-Span Ribbed Slab</u>

a. Torsional Moment at the Support

The transverse torsional moment distributions at the support of various simple span ribbed slabs are shown in Fig. 4.16. Note that the absolute values of torsions are used in Fig. 4.16. The effects of H and b/a on the distributions are similar to those in the case of the moment distributions (i.e., the transverse torsion distribution is better for small H and b/a values). The analysis of the ribbed slabs showed that the parameter b/a has a greater effect on the torsion distribution than H.

For loading with the perimeter loads on the edge rib, the maximum torsion is always found in the edge rib, as was the maximum moment. For the other loading with the perimeter loads on the second rib, the maximum torsion is always in an interior rib. The magnitude of the maximum torsion in the latter loading case is significantly smaller than that of the former loadings. In comparison with the maximum moments at midspan of one-span ribbed slabs in Section 4.3.1 (a), the maximum torsional moments are about 9 to 11 percent of the maximum bending moments for the former loading case, and about 6 to 10 percent for the latter loading case.

b. Shear at the Support

The important shear at the support due to loadings in this section is not the maximum shear, but rather the value of shear corresponding to the maximum torsion in the rib. However, for better understanding, the transverse shear distribution to various ribs at the support are shown in Fig. 4.16. For the loading with perimeter loads on the edge rib, both maximum shear and torsion occur in the edge rib. Their magnitudes are not much different from the shear and torsion for the maximum moment loading case considered in Section 4.3.1. For the loading case with the perimeter loads on the second rib, shear is somewhat smaller but torsion is much greater than that due to loading for maximum moment.

For example, the maximum torsion and the corresponding shear at the support of two one-span ribbed slabs with H = 2.0, b/a = 0.035, and H = 9.0, b/a = 0.10, are shown as follows:

H = 2.0, b/a = 0.035

Perimeter loads on the edge rib:

Maximum torsional moment at the support (in edge rib) = $-0.0331M_B$

Corresponding shear at the support (in edge rib) = $0.3778V_B$

Perimeter loads on the second rib:

Maximum torsional moment at the support (in rib no. 8) = $-0.0208M_B$

Corresponding shear at the support (in rib no. 8) = $0.1404V_B$

H = 9.0, b/a = 0.10

Perimeter loads on the edge rib:

Maximum torsional moment at the support (in edge rib) = $-0.0600M_B$

Corresponding shear at the support (in edge rib) = $0.5694V_B$

Perimeter loads on the second rib:

Maximum torsional moment at the support (in rib no. 6) = $-0.0369M_B$

Corresponding shear at the support (in rib no. 6) = $0.2290V_B$

4.4.2 Two-Span Ribbed Slab

a. Torsional Moment at the Exterior Support and at the 0.95 Span Point

Torsion at the exterior support and at the 0.95 span section of various two-span ribbed slabs due to both cases of loading for maximum torsion are shown in Figs. 4.18 and 4.19, respectively. The magnitudes of maximum torsions at 0.95 span are somewhat smaller than those at the exterior support. The general characteristics of the transverse torsional moment distribution among ribs at the exterior support and at 0.95 span are similar to those in the simple span ribbed slabs, but the magnitudes of maximum torsions in the two-span slabs are slightly smaller than those in the one-span case. The effects of the loading conditions, H and b/a, on the characteristics of the transverse torsion distributions are similar to those in the simple span ribbed slabs, as discussed in Section 4.4.1(a).

b. Shear at the Exterior Support and the 0.95 Span Point, and Bending Moment at the Interior Support

Shear at the exterior support due to multiple loads for maximum torsional moment is shown in Fig. 4.20; shear and negative moment at the interior support are shown in Figs. 4.21 and 4.22, respectively. The transverse negative moment distributions in various ribs are fairly uniform, whereas shear and torsion distributions are not because of the loadings. The magnitudes of the negative moments are about the same as those due to the maximum positive moment loadings discussed in Section 4.3.1 (b).

The general characteristics of the transverse distributions of shear, both at the exterior support and at 0.95 span, are similar to those in the simple span ribbed slabs, as discussed above. The magnitudes of maximum shear due to loadings in this section are not much smaller than the maximum shear due to multiple loads for maximum positive moment. However, the important shear for loadings in this section is not the maximum shear, but rather the shear corresponding to the maximum torsional moment. Unfortunately for the structure, the maximum shear, both at the exterior support and at 0.95 span, is the shear accompanying the maximum torsional moment in the case where the perimeter loads are on the edge rib. Hence, for this loading case, the combined torsional moment, shear, and bending moment at (or near) the interior support could be very important. For the loading with the perimeter loads on the second rib, the shear corresponding to the maximum torsion is considerably smaller than the maximum shear. The maximum torsion due to this loading is much smaller than that due to the former loading. In order to minimize the effects of the combined torsional moment, shear, and bending moment, it may be worthwhile to consider restricting the loading so that the perimeter loads cannot go outside of the second rib.

For example, maximum torsional moment, and the corresponding shear and bending moment of two two-span ribbed slabs with H = 2.0, b/a = 0.035, and H = 9.0, b/a = 0.10, are shown as follows:

H = 2.0, b/a = 0.035

Perimeter loads on the edge rib (forces in edge rib):

Maximum torsion at the exterior support	=	$-0.0308M_B$
Shear at the exterior support	=	$0.3379V_B$
Maximum torsion at 0.95 span	=	$0.0262M_B$

Shear at 0.95 span = $-0.4473V_B$

Bending moment at the interior support = $-0.1897M_B$

Perimeter loads on the second rib (forces in rib no. 9):

Maximum torsion at the exterior support = $-0.0207M_B$

Shear at the exterior support = $0.1134V_B$

Maximum torsion at 0.95 span = $0.0200M_B$

Shear at 0.95 span = $-0.1789V_B$

Bending moment at the interior support = $-0.1056M_B$

H = 9.0, b/a = 0.10

Perimeter loads on the edge rib (forces in edge rib):

Maximum torsion at the exterior support = $-0.0567M_B$

Shear at the exterior support = $0.5254V_B$

Maximum torsion at 0.95 span = $0.0473M_B$

Shear at 0.95 span = $-0.6415V_B$

Bending moment at the interior support = $-0.3142M_B$

Perimeter loads on the second rib (forces in rib no. 6):

Maximum torsion at the exterior support = $-0.0371M_B$

Shear at the exterior support = $0.1908V_B$

Maximum torsion at 0.95 span = $0.0364M_B$

Shear at 0.95 span = $-0.3324M_B$

Bending moment at the interior support = $-0.2420M_B$

4.5 Variations of Maximum Moment, Shear, and Torsional Moment, with H and b/a

It has been observed that good moment, shear, and torsion distributions are the charactistics of a ribbed slab with small H and b/a; hence, as far as statics is concerned, decreased maximum moment, shear, and torsion could be expected in such a ribbed slab. One might also conclude that the maximum moment, shear, and torsion of a ribbed slab due to any multiple loading case are direct functions of the parameters H and b/a. The relationships of the maximum moment, shear, and torsion with H and b/a are presented in this section, and also the corresponding values of shear, torsion, and moment (if any).

4.5.1 <u>One-Span Ribbed Slab</u>

The relationships of maximum bending moment at midspan, maximum shear and the corresponding torsion at the support due to both cases of loadings for maximum moment with H and b/a are shown in Fig. 4.23. The maximum moments increase at a diminishing rate as b/a increases, and the effect of b/a is greater for the larger values of H. The maximum moments increase slightly as H increases, with the larger changes occurring when b/a is larger. For the ribbed slabs analyzed, the relationship between the maximum shear (at the support, and for both loading cases) and b/a is approximately a parabolic curve with the maximum value somewhere between b/a = 0.07 and 0.10. In the other words, the maximum shear increases as b/a increases until b/a = 0.07, and at b/a = 0.10 the maximum normalized shear is slightly smaller than at b/a = 0.07. The maximum shear also increases at a very small rate as H increases. The torsional moments corresponding to the maximum shear due to the loading with the perimeter loads on the second rib are negligible. For the loading with the perimeter loads on the edge rib, the torsional moments corresponding to the maximum shear are significant, and they increase as both H and b/a increase.

The effect of b/a on the torsional moment is greater for greater H values, and the effect of H is also greater for greater b/a values.

Figure 4.24 shows the relationships of the maximum torsional moments and corresponding shears at the support with H and b/a, for both maximum torsional moment loading cases. For loading with the perimeter loads on the edge rib, the characteristics of the maximum torsions and corresponding shear are similar to those of the maximum shear and corresponding torsions due to the same loading case. For loading with the perimeter loads on the second rib, the general characteristics of the maximum torsions are similar to those of the maximum torsions due to the former loading case, except that the maximum torsions at b/a = 0.10 are slightly smaller than those at b/a = 0.07, as also happened for the maximum shear values. The relationships of shear corresponding to maximum torsions in the latter loading case with H and b/a are more ambiguous. However, for a general description, it might be said that the magnitude of this shear is about 50 to 60 percent of the shear due to maximum moment loading, as discussed above.

4.5.2 Two-Span Ribbed Slab

For both cases of loadings for maximum positive moment at 0.44 span, the relationships of the maximum moment, maximum shear and the corresponding torsion at the exterior support with H and b/a are shown in Fig. 4.25; the relationships of maximum shear at 0.95 span, and the corresponding negative moment at the interior support and torsion at 0.95 span with H and b/a are shown in Fig. 4.26. For the two maximum negative moment loading cases at the interior support, the relationships of the maximum moment and the corresponding shear and torsion at 0.95 span with H and b/a are shown in Fig. 4.27.

For both maximum torsion loading cases at the supports, the relationships of the maximum torsion and corresponding shear at the exterior support with H and b/a are shown in Fig. 4.28; the relationships of the maximum torsion and corresponding shear at 0.95 span and negative moment at the interior support with H and b/a are shown in Fig. 4.29.

All these relationships of the moment, shear, and torsion with H and b/a are similar to those for the one-span ribbed slabs. But at 0.95 span, the maximum shear decreased as b/a changed from 0.07 to 0.10 with the response somewhat greater than that extent at the exterior support, or in the one-span ribbed slabs.

4.6 Effects of a Stiffer Edge Rib

The study of the effects of the stiffer edge ribs were considering the simple span ribbed slab with H = 4.5, b/a = 0.052, T = 0.100. The width of the edge rib was twice the regular width. The absolute flexural stiffness of the edge rib was then double of that of the regular edge rib, but the relative flexural stiffness H of the edge rib, composite with the slab, was 50 percent greater than that of the regular edge rib. For the torsional stiffness, both the absolute value and the relative value of T were 5.7 times that of the regular edge rib. Two types of multiple loads are considered; first, multiple loads for maximum moment, shear, and torsion as shown in Fig. 4.1 and second, a line load on the edge rib. For the second type of loading, a simulated line load (i.e., concentrated loads at every 2.5 ft) was used instead of the actual line load, since the method of analysis was prepared for concentrated loads only.

4.6.1 Multiple Loads for Maximum Moment, Shear, and Torsion

Two loading cases, perimeter loads on the edge and second ribs as shown in Fig. 4.1, are again considered. The results of the analysis are shown in Fig. 4.30 for both cases of loadings. The results of the regular ribbed slab are also shown in Fig. 4.30 for comparison.

For the case of loading with the perimeter loads on the edge rib, the maximum bending moment in the stiffened edge rib was about 40 percent more than that of the regular edge rib. The stiffer edge rib caused some reduction of the maximum moment in the adjacent ribs. The effect of the stiffer edge rib on the maximum shear is similar to that on the maximum moment (i.e., the maximum shear in the stiffer edge rib was about 23 percent more than that in the regular edge rib). The effect of the stiffer edge rib on the maximum torsion is greater than on the maximum moment and shear (i.e., the maximum torsion in the stiffer edge rib was about 100 percent more than that in the regular edge rib) because the torsional stiffness T had increased 5.7 times, whereas the flexural stiffness H increased 1.5 times, as mentioned above.

For the case of loading with the perimeter loads on the second rib, the maximum moment also occurred in the edge rib; and the maximum moment in the second and near adjacent ribs were reduced when the edge rib was stiffened. The effect of the stiffer edge rib on the maximum shear is similar to that on the maximum moment (i.e., the maximum shear in the stiffer edge rib increased and maximum shear in the adjacent ribs decreased). The maximum torsion for this loading is not affected by the stiffer edge rib, since it occurs in an interior rib. For the stiffer edge rib, torsion changed sign which means that the rotation in the stiffer edge rib occurred in the opposite direction to that of the regular edge rib for this loading (i.e., a ball placed over the first rib tends to roll toward the edge of the structure for the case of the regular rib and toward the interior with the stiffened rib).

4.6.2 Line Load on the Edge Rib and Across the Span

The study of line load on the edge rib in this section is intended to show the effects of possible line loads (such as a wall or dead load of the widened edge rib) on moment, shear, and torsion in the ribs. The results for both the stiffened edge rib and the regular edge rib are shown in Fig. 4.31. The line load produced significant values of moment and shear (i.e., greater than 10 percent of the maximum value) to rib no. 5, but the effect on torsion is significant to rib no. 8.

The effects of the stiffer edge rib are similar to those due to the loadings described in the previous section. The maximum moment, shear, and torsion in the edge rib were about 30, 20, and 120 percent, respectively, more than those of the regular edge rib. The reductions in the maximum moment, shear, and torsion in the second rib were about 20, 15, and 57 percent, respectively, of the maximum values in the regular edge rib. For the same reason as described in the previous section, the change of torsional moment is significantly greater than of bending moment or shear. The effects of the stiffer edge rib exist in only a few ribs near the loaded edge rib (i.e., to rib no 4 for bending and shear for the particular ribbed slab analyzed).

For the case of a line load across the span (parallel to the supports), an approximation was made by applying single concentrated loads on every rib (i.e., single loads at midspan on all ribs). This loading shows the effects of the regular edge rib and of the stiffer edge rib, as illustrated in Fig. 4.32. For the regular edge rib, the moment, shear, and torsion in an outer rib are slightly greater than in an inner rib due to the edge effects. In fact, moment and shear due to this loading should be about the same as the static values in a simple beam, but they are slightly smaller because of the computational inaccuracy, as discussed in Section 2.6. The effect of a stiffer edge rib is somewhat similar to the effect of a line load on the edge rib. Moment and shear are greater in the stiffer edge rib and smaller in the adjacent ribs, as compared to the regular edge rib case. Torsion increases in all ribs, with a significant value in the stiffer edge rib.

CHAPTER 5

DISCUSSION AND RECOMMENDATIONS

5.1 General

The results of the investigation (bending moment, shear, and torsional moment) were presented relative to the static moment and shear in simple beams. Since static torsion does not exist in the simple beam, torsional moment was reported relative to the simple beam bending moment.

The results are presented as functions of H and b/a. However, the different absolute values of b and a also have some influence on the results. For the case of single loads, this effect is negligible; for the case of multiple loads, small effects exist because of the different number of loads and loading locations relative to the rib (e.g., compare the maximum moments shown in Fig. 3.1 and 4.2 with the graphs of maximum moment in Fig. 3.23 and 4.23 respectively). However, the rib spacing b for this type of structure is in a very narrow range of about 2 to 3 feet, and for this reason the effect of the different absolute values of b and a for a particular H and b/a would be limited.

5.2 Discussion and Comparison

The discussion of the results of the investigation in this section will concern the results produced by multiple loads as described in Chapter 4. Since these multiple loads were considered to be wheel loads of vehicles in parking garages, the discussion will compare the results obtained from the analysis with the current design provisions for such structures; for example, the equivalent uniformly distributed load for the design of an open parking structure is specified to be 50 psf by Ref. 4.

For the purpose of comparison, the equivalent uniformly distributed loads for the results of this analysis will be evaluated. By considering the rib as an equivalent beam, then the equivalent uniformly distributed loads can be determined by the formulae as follows:

For One-Span Ribbed Slab:

Equivalent load based on the maximum moment:

$$w_e = \frac{8M_{max}}{a^2b}$$

Equivalent load based on the maximum shear:

$$w_e = \frac{2V_{max}}{ab}$$

For Two-Span Ribbed Slab:

Equivalent load based on the maximum positive moment:

$$w_e = \frac{512M_{max}}{49a^2b}$$

Equivalent load based on the maximum shear at the exterior support:

$$w_e = \frac{16V_{max}}{7ab}$$

Equivalent load based on the maximum negative moment:

$$w_e = \frac{8M_{max}}{a^2b}$$

Equivalent load based on the maximum shear at 0.95 span:

$$w_e = \frac{40V_{max}}{23ab}$$

where w_e is the equivalent uniformly distributed load per unit area.

The magnitudes of the concentrated wheel loads P are taken as 1000 lb for the calculation of the equivalent loads w_e in this section.

For both one- and two-span ribbed slabs, the case of loading with the perimeter loads on the edge rib resulted in significantly greater maximum moment, shear, and torsion in the edge rib than in the interior ribs, especially at the interior support of a two-span structure. However, the maximum forces in interior ribs due to this loading are nearly the same as the maximum values produced in interior ribs when the perimeter loads are on the second rib. Therefore, it might be said that the results from the investigation for the latter loading case could be considered as the reference for all the interior ribs; and the results due to the former loading case could be considered as the reference for the edge rib when loading is possible to load it directly.

a. Perimeter Loads on the Second Rib

The following discussion is concerned mainly with the results produced by the loading with the perimeter loads on the second rib, for both the one- and two-span structures described in Chapter 4. The equivalent loads w_e for these results calculated by the above formulae are approximately in the range of 30 to 40 psf, except that the equivalent loads based on the maximum shear at 0.95 span for the two-span structure are somewhat larger, at 40 to 50 psf. The equivalent loads calculated are listed in Tables 5.1 and 5.2. These equivalent loads suggest that the current specified design load of 50 psf for an open parking structure is adequate as long as significant area load reductions, as permited by Ref. 4, are not allowed. However, none of the building codes has mentioned any possibility of the effects of torsional moment in combination with shear, or shear and bending moment. The results of this investigation indicates that significant torsion exists simultaneously with shear and bending moment.

For one-span ribbed slabs, maximum torsion at the support (as shown in Table 5.3) is about 6 to 10 percent of the maximum bending moment. (Maximum torsion and maximum moment are caused by different loading patterns, as shown in Fig. 4.1.) The shear corresponding to the maximum torsion is generally not the maximum shear but is approximately 40 to 60 percent of the maximum shear, as mentioned in Chapter 4 (see Table 5.4).

For two-span ribbed slabs, maximum torsion at the exterior support is about 6 to 10 percent of the maximum negative moment at the interior support (see Table 5.5). The shear corresponding to this maximum torsion is usually in the range of 40 to 50 percent of the maximum shear at the exterior support (see Table 5.6). Maximum torsion near the interior support (i.e., at 0.95 span) is about 6 to 9 percent of the maximum negative moment at the interior support (see Table 5.7). The shear corresponding to this maximum torsion is about 40 to 60 percent of the maximum shear at the 0.95 span location (see Table 5.8); and the corresponding negative moment is about 40 to 55 percent of the maximum negative moment at the interior support (see Table 5.9).

Note that torsion accompanying the maximum shears and bending moments are generally negligible.

b. Perimeter Loads on the Edge Rib

The equivalent loads based on maximum moment and shear values calculated by the above formulae, for both one- and two-span, are mostly in the range of 40 to 50 psf. However, the equivalent loads based on the maximum shear at the 0.95 span location of the two-span structure is greater and varies from about 55 to 80 psf. These equivalent loads are somewhat greater than the often specified design load of 50 psf. According to these results, it might be said that if loading directly on the edge rib is possible, a stronger edge rib or edge beam is needed. Furthermore, considerable torsion occurred simultaneously with maximum shear at the exterior support (i.e., about 7 to 11 percent of the maximum positive moment, see Table 5.10); and some torsion occurred with maximum shear and bending moment at or near the interior support (i.e., about 3 to 5 percent of the maximum negative moment, see Table 5.11).

The other important combination of forces at the support is due to loading for maximum torsion. For one-span ribbed slabs, the maximum torsion is about 9 to 11 percent of the maximum bending moment (see Table 5.3). The shear corresponding to this maximum torsion is about the same as the maximum shear (i.e., about 95 to 100 percent, see Table 5.4). For two-span structures, maximum torsion at the exterior support is about 7 to 9 percent of the maximum negative moment at the interior support, and shear corresponding to this maximum torsion is about the same as the maximum shear at the exterior support (i.e., about 95 to 100 percent). Maximum torsion at the 0.95 span location is about 6 to 7 percent of the maximum negative moment at the interior support (see Table 5.7); shear corresponding to this maximum torsion is about 70 to 75 percent of the maximum shear at 0.95 span, and the corresponding negative moment is about 50 percent of the maximum negative moment (see Tables 5.8 and 5.9). Note that the above combinations of the results are also shown graphically in Chapter 4 for both cases of loading. According to the results discussed above, if loading on the edge rib is possible, the effects of the above combinations of shear and torsion, or shear, torsion and bending moment ought to be taken into consideration.

5.3 General Remarks

On the basis of the results of the investigation as discussed in the previous sections, most of the problems which have occurred in in-service parking garage ribbed slabs (i.e., Ref. 1) probably did not result from use of an inadequate design live load (if 50 psf or more was used). The trouble may have resulted from various other causes which have not been included in this study, for example:

1. Restrained shrinkage effects in concrete (i.e., tensile stress reduces shear strength of reinforced concrete).
2. Repeated load effects on the fatigue strength of the concrete, especially in the plate elements, in the lower stories of a ramp parking structure.
3. Temperature change effects, especially when the effects of temperature and shrinkage are additive.

5.4 Recommendations

The following recommendation for the design of ribbed slabs subjected to concentrated loads (as in parking garages) are based on the results of this investigation:

1. When possible, loading on the edge rib should be avoided so as to eliminate the large-bending moment, shear, and torsional moment in that edge rib.
2. If the edge rib is to be loaded, a stiffened edge beam should be provided (i.e., double the width and reinforcement).
3. In so far as the calculated moments and shears are concerned, the equivalent design load of 50 psf is adequate for interior ribs, unless a very heavy line load is applied to the edge rib.
4. The increase in the allowable shear stress for ribs allowed by the ACI Building Code (2) should not be permitted, unless the effects of potential torsional moments are considered.

CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1 General Outline of the Investigation

The analytical study presented in this report is concerned with the bending moments, shears, and torsional moments in the ribs of one-span and two-span continuous ribbed slabs subjected to concentrated loads.

All ribbed slabs analyzed consisted of seventeen identical ribs spaced uniformly with the cross section of the rib constant along the span. The range of the parameters considered is 0.035 to 0.10 for b/a, and 2.0 to 9.0 for H, and a nearly constant T of 0.100. The supports were considered to be nondeflecting and to provide complete restraint against torsional rotation of elements about their longitudinal axes, but to provide zero flexural restraint.

Solutions were obtained by a numerical procedure based on the Goldberg-Leve folded plate theory (13). The studies considered two cases:

a. Solutions involving single concentrated loads, mainly with loadings at midspan on the center and edge ribs, and
b. Multiple loadings as in parking garages.

The loading systems were set, as shown in Fig. 4.1, to obtain maximum bending moment, shear, and torsional moment in the ribs. In addition, the effects of various loads (including a line load) on the stiffened edge rib were also examined. The presentations were made using dimensionless quantities (i.e., M/M_B, V/V_B, and M_t/M_B) for most of the results.

6.2 General Conclusions

The following conclusions for this investigation are believed to be applicable to all simply supported one-span and two-span continuous ribbed slabs which have at least 17 ribs and values of parameters H, b/a, and T in the range considered in this study.

For Single Loads

1. The moment, shear, and torsion distributions are better for small H and b/a values.
2. The maximum moment, shear, and torsion increase as H and b/a increase, but in the range b/a = 0.07 to 0.10 the maximum shear decreases slightly, and b/a has somewhat greater effect than H.
3. The maximum moments at midspan in the center and edge ribs of one-span ribbed slabs are about $0.15M_B$ to $0.38M_B$, and $0.30M_B$ to $0.60\ M_B$, respectively.
4. The maximum shear occurs near the applied load, and the magnitude can be as great as a simple beam shear (i.e., 0.5P).
5. A stiffer edge rib reduces the maximum moments in the ribs near the edge rib to about the same as in the other interior ribs, but increases the maximum moment in the edge rib.
6. The load locations for maximum torsional moment at the support of one-span ribbed slabs are about 0.15 to 0.35 of the span length from the support for the edge rib loading, and 0.1 to 0.3 of the span length for the center rib loading.

For Multiple Loads:

1. The maximum moment at midspan of the simple span ribbed slabs is about $0.32M_B$ to $0.57M_B$ for the case of the perimeter loads on the edge rib, and $0.26M_B$ to $0.38M_B$ when the perimeter loads are on the second rib.
2. The maximum shear at the support of the simple span ribbed slabs is between $0.40V_B$ and $0.59V_B$ for the case of the perimeter loads on the edge rib, and between $0.30V_B$ and $0.45V_B$ for the case of the perimeter loads on the second rib.
3. The maximum torsional moment at the support of the simple span ribbed slabs varies from about $0.033M_B$ to $0.060M_B$ for the case of the perimeter loads on the edge rib, and $0.021M_B$ to $0.037M_B$ for the case of the perimeter loads on the second rib.
4. The maximum positive moment at the 0.44 span location of two-span ribbed slabs is from $0.26M_B$ to $0.46M_B$ for the case of the perimeter loads on the edge rib, and from $0.20M_B$ to $0.33M_B$ for the case of perimeter loads on the second rib.
5. The maximum negative moment at the interior support varies from $0.38M_B$ to $0.63M_B$ for the case of the perimeter loads on the edge rib, and from $0.25M_B$ to $0.45M_B$ for the case of the perimeter loads on the second rib.
6. The maximum shear at the exterior support is about $0.36V_B$ to $0.59V_B$ for the case of the perimeter loads on the edge rib, and from $0.27V_B$ to $0.38V_B$ for the case of the perimeter loads on the second rib.

7. The maximum shear at the 0.95 span location of the two-span structure is about $0.64V_B$ to $1.00V_B$ for the case of the perimeter loads on the edge rib, and from $0.40V_B$ to $0.70V_B$ for the case of the perimeter loads on the second rib.
8. The maximum torsion at the exterior support is about $0.031M_B$ to $0.057M_B$ for the case of the perimeter loads on the edge rib, and from $0.021M_B$ to $0.037M_B$ for the case of the perimeter loads on the second rib.
9. The maximum torsion at the 0.95 span location varies between $0.026M_B$ and $0.047M_B$, and from $0.020M_B$ to $0.036M_B$ for the case of the perimeter loads on the edge and second ribs, respectively
10. The maximum moment, shear, and torsion increase as H and b/a increase, but in the range of b/a = 0.07 to 0.10 the maximum shear decreases somewhat.
11. The influence of b/a on the maximum moment and shear is somewhat greater than that of H; they have about the same influence on the maximum torsion.
12. The maximum deflection coefficient K increases more or less linearly with b/a; the coefficient K is nearly independent of H.
13. The effects of a line load on the edge rib are significantly transmitted to only a few adjacent ribs.
14. A stiffer edge rib (double the width) carries more internal forces, and reduces those in a few adjacent ribs, as compared to the regular edge rib.
15. When the perimeter loads are limited to the second rib, the

equivalent uniformly distributed loads calculated based on the maximum moment and shear are between 30 and 50 psf for both one- and two-span ribbed slabs.

16. If the perimeter loads are on the edge rib, the equivalent loads based on the maximum moment and shear in the edge rib are about 40 to 55 psf for one-span ribbed slabs, and 40 to 80 psf for two span. The equivalent loads for interior ribs would be in the same range as those due to the perimeter loads on the second rib.

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Table 3.1

DIMENSIONS AND PARAMETERS OF THE
VARIOUS RIBBED SLABS STUDIED

b/a	H	T	t (in.)	b_r (in.)	h (in.)	b (in.)	a (ft)
0.035	2.0	0.102	4.0	7.0	19.8	24.0	57.0
	4.0	0.100	3.5	7.4	21.8	24.0	57.0
	6.0	0.098	3.0	7.2	21.7	24.0	57.0
	9.0	0.097	2.5	6.9	21.1	24.0	57.0
0.05	2.0	0.100	4.0	6.8	18.5	30.0	50.0
	4.0	0.102	3.0	6.6	17.8	30.0	50.0
	6.0	0.104	2.5	6.4	17.2	30.0	50.0
	9.0	0.104	2.5	7.0	19.4	30.0	50.0
0.052	4.5	0.100	3.0	6.3	17.7	25.0	40.0
0.07	2.0	0.098	4.0	6.6	17.3	36.0	42.8
	4.0	0.103	3.5	7.2	18.8	36.0	42.8
	6.0	0.100	3.0	7.0	18.7	36.0	42.8
	9.0	0.102	2.5	6.8	18.1	36.0	42.8
0.10	2.0	0.099	4.0	6.1	15.7	36.0	30.0
	4.0	0.102	3.5	6.6	17.7	36.0	30.0
	6.0	0.098	3.0	6.4	16.9	36.0	30.0
	9.0	0.099	2.5	6.2	16.3	36.0	30.0

Table 3.2

PARAMETERS AND DIMENSIONS OF THE RIBBED SLABS FOR THE STUDY OF THE EFFECTS OF TORSIONAL STIFFNESS ON MOMENT, SHEAR, AND TORSION

b/a	H	T	t (in.)	b_r (in.)	h (in.)	b (in.)	a (ft)
0.05	2.0	0.075	4.0	6.0	18.8	30.0	50.0
		0.100	4.0	6.8	18.5	30.0	50.0
		0.146	4.0	8.0	17.8	30.0	50.0
0.05	4.0	0.081	3.0	6.0	18.2	30.0	50.0
		0.102	3.0	6.6	17.8	30.0	50.0
		0.126	3.5	8.0	19.8	30.0	50.0
		0.143	3.0	7.6	17.2	30.0	50.0
		0.161	3.0	8.0	17.0	30.0	50.0
0.10	4.0	0.063	3.5	5.4	17.8	36.0	30.0
		0.102	3.5	6.6	17.7	36.0	30.0
		0.117	3.5	7.0	16.8	36.0	30.0

Table 4.1

MAXIMUM SIMPLE BEAM MOMENT AND SHEAR VS. SPAN

Span Length (ft)	M_B (K-ft)	V_B (K)
40.0	49.5	5.0
57.0	101.0	7.0
50.0	77.5	6.0
42.8	56.5	5.0
30.0	28.0	4.0

Table 4.2

MAXIMUM DEFLECTIONS OF ONE-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM MOMENT

Loading	H	δ (in.)			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads	2.0	0.7343	0.6152	0.4570	0.1686
on the Edge	4.0	0.5557	0.7341	0.3474	0.1310
Rib	6.0	0.5926	0.8549	0.3707	0.1411
	9.0	0.6947	0.5792	0.4298	0.1655
Perimeter Loads	2.0	0.5856	0.4712	0.3540	0.1235
on the Second	4.0	0.4370	0.5586	0.2675	0.0956
Rib	6.0	0.4624	0.6496	0.2845	0.1026
	9.0	0.5391	0.4400	0.3304	0.1210

Table 4.3

MAXIMUM DEFLECTIONS OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM POSITIVE MOMENT

Loading	H	δ (in.)			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads	2.0	0.5272	0.4409	0.3283	0.1226
on the Edge	4.0	0.4002	0.5283	0.2504	0.0958
Rib	9.0	0.5034	0.4191	0.3118	0.1224
Perimeter Loads	2.0	0.4124	0.3320	0.2495	0.0878
on the Second	9.0	0.3813	0.3119	0.2352	0.0886
Rib					

Table 5.1

EQUIVALENT UNIFORMLY DISTRIBUTED LOADS FOR ONE-SPAN RIBBED SLABS

Loading	Calculation Based On	w_e^* (psf)							
		H = 2.0				H = 9.0			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10	b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	M_{max}	40	40	38	41	42	43	42	47
	V_{max}	49	46	42	46	54	50	46	51
Perimeter Loads on the Second Rib	M_{max}	32	31	30	31	33	37	33	35
	V_{max}	37	35	35	33	40	38	32	37

* The equivalent load w_e based on the magnitude of the concentrated wheel loads P = 1000 lb.

Table 5.2

EQUIVALENT UNIFORMLY DISTRIBUTED LOADS ON TWO-SPAN RIBBED SLABS

Loading	Calculation Based On	w_e^* (psf)							
		H = 2.0				H = 9.0			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10	b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	M_{max} (at 0.44 Span)	41	40	40	43	46	44	44	50
	V_{max} (at Exterior Support)	44	48	45	48	56	54	49	54
	M_{max} (at Interior Support)	47	44	42	45	54	51	48	52
	V_{max} (at 0.95 Span)	68	64	60	56	81	73	68	67
Perimeter Loads on the Second Rib	M_{max} (at 0.44 Span)	33	31	31	31	33	33	33	36
	V_{max} (at Exterior Support)	38	37	33	34	41	40	37	39
	M_{max} (at Interior Support)	32	31	31	32	34	34	35	37
	V_{max} (at 0.95 Span)	43	41	42	40	48	47	48	47

* The equivalent load w_e based on the magnitude of the concentrated wheel loads P = 1000 lb.

Table 5.3

MAXIMUM TORSIONS AT THE SUPPORT OF ONE-SPAN RIBBED SLABS

Loading	H	$M_{t_{max}}$ (% M_{max})			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	2.0	10	9	9	9
	9.0	11	11	10	11
Perimeter Loads on the Second Rib	2.0	8	7	7	6
	9.0	10	10	9	9

Table 5.4

SHEARS ACCOMPANYING THE MAXIMUM TORSIONS AT THE SUPPORT OF ONE-SPAN RIBBED SLABS

Loading	H	V (% V_{max})			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	2.0	94	97	98	100
	9.0	97	97	98	100
Perimeter Loads on the Second Rib	2.0	47	45	57	53
	9.0	46	43	58	53

Table 5.5

MAXIMUM TORSIONS AT THE EXTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS

Loading	H	$M_{t_{max}}$ (% Neg. M_{max})			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on The Edge Rib	2.0	8	8	7	8
	9.0	9	9	9	9
Perimeter Loads on the Second Rib	2.0	8	7	6	6
	9.0	10	9	8	8

Table 5.6

SHEARS ACCOMPANYING THE MAXIMUM TORSIONS AT THE EXTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS

Loading	H	V (% V_{max} at Exterior Support)			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Ribs	2.0	94	96	96	98
	9.0	96	97	97	99
Perimeter Loads on the Second Rib	2.0	41	39	49	48
	9.0	41	37	50	50

Table 5.7

MAXIMUM TORSIONS AT THE 0.95 SPAN LOCATION
IN TWO-SPAN RIBBED SLABS

Loading	H	$M_{t_{max}}$ (% Neg. M_{max})			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	2.0	7	7	6	6
	9.0	7	7	7	7
Perimeter Loads on the Second Rib	2.0	8	6	6	6
	9.0	9	9	8	8

Table 5.8

SHEAR ACCOMPANYING THE MAXIMUM TORSIONS AT THE 0.95
SPAN LOCATION IN TWO-SPAN RIBBED SLABS

Loading	H	V (% V_{max} at 0.95 Span)			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	2.0	70	73	74	74
	9.0	69	72	74	74
Perimeter Loads on the Second Rib	2.0	45	51	54	57
	9.0	42	48	53	55

Table 5.9

NEGATIVE BENDING MOMENTS ACCOMPANYING THE MAXIMUM TORSIONS AT THE 0.95 SPAN LOCATION IN TWO-SPAN RIBBED SLABS

Loading	H	M (% Neg. M_{max})			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	2.0	51	50	51	50
	9.0	50	50	50	49
Perimeter Loads on the Second Rib	2.0	40	52	53	55
	9.0	41	53	54	54

Table 5.10

TORSIONS ACCOMPANYING THE MAXIMUM SHEARS AT THE EXTERIOR SUPPORT IN TWO-SPAN RIBBED SLABS

Loading	H	M_t (% Pos. M_{max})			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	2.0	8	7	7	8
	9.0	11	11	10	11

Table 5.11

TORSIONS ACCOMPANYING THE MAXIMUM SHEARS AT THE 0.95 SPAN LOCATION IN TWO-SPAN RIBBED SLABS

Loading	H	M_t (% Neg. M_{max})			
		b/a = 0.035	b/a = 0.05	b/a = 0.07	b/a = 0.10
Perimeter Loads on the Edge Rib	2.0	3	3	3	3
	9.0	4	4	4	5

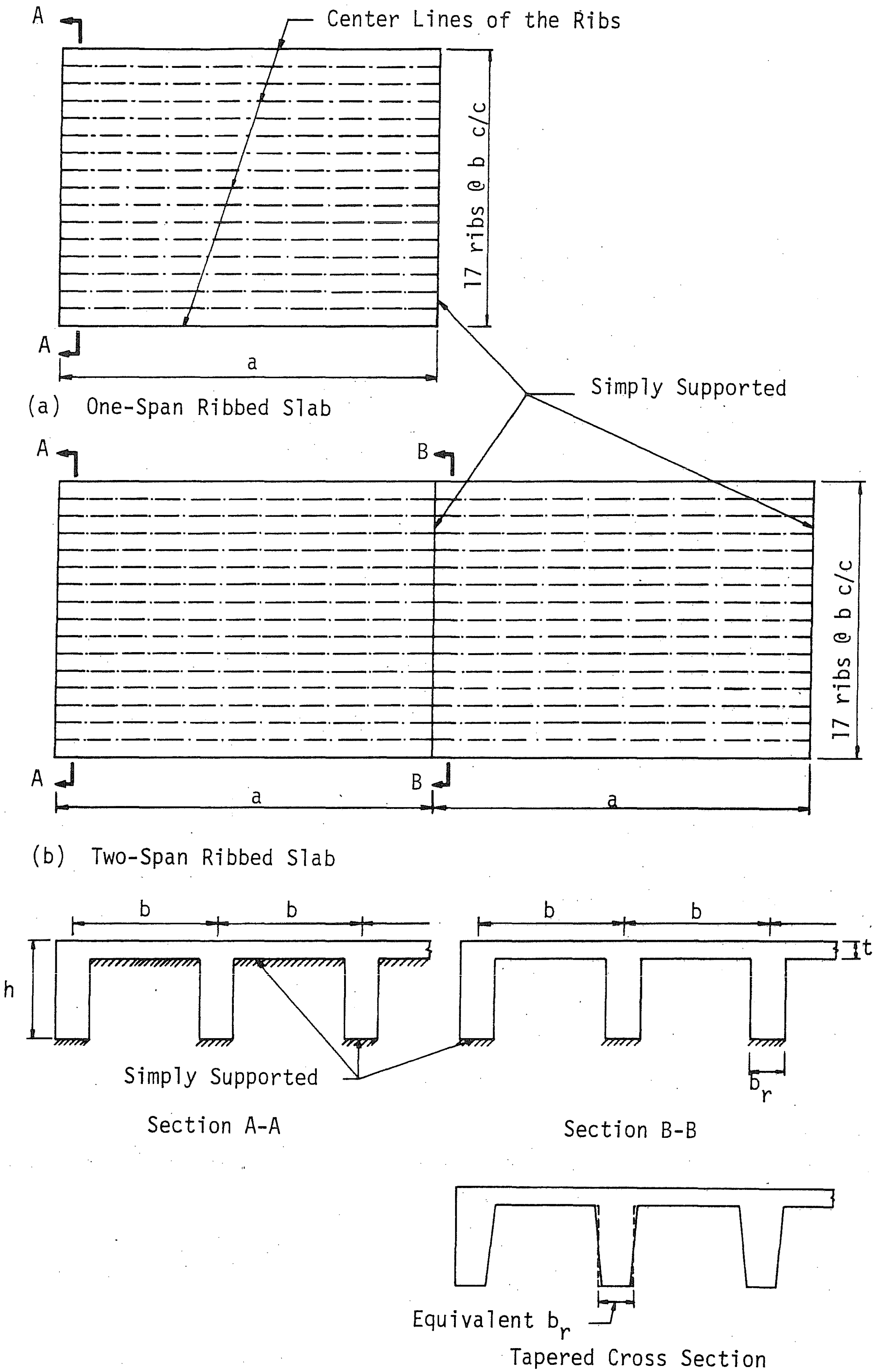
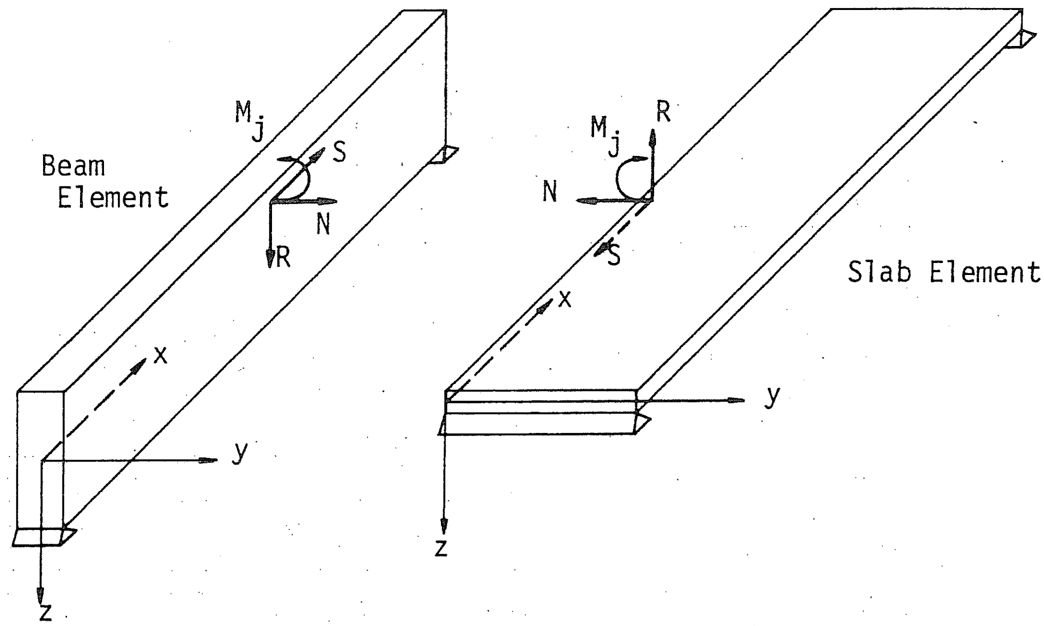
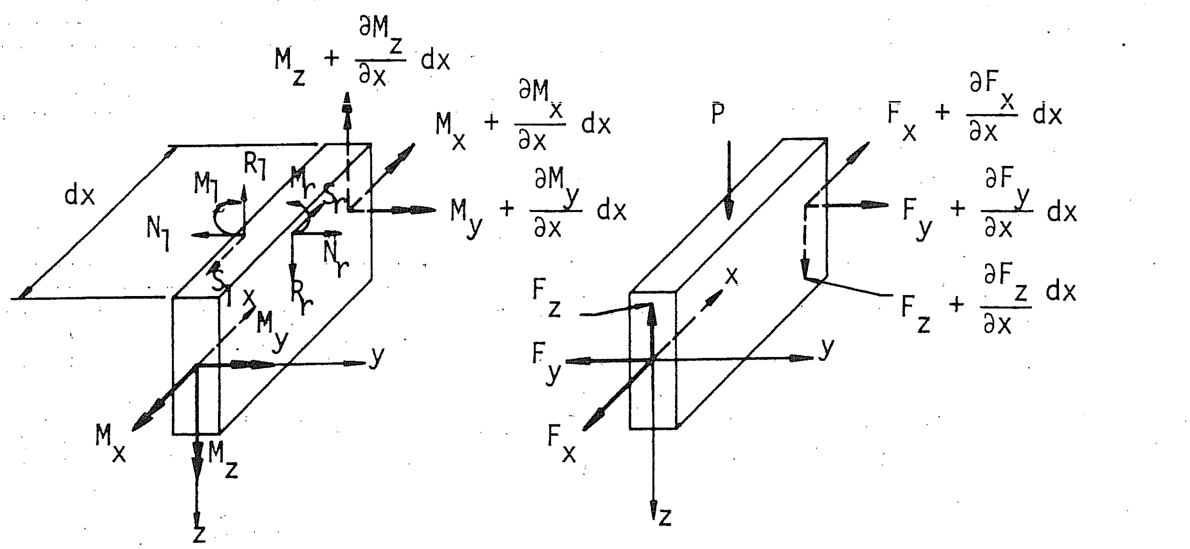


FIG. 1.1 TYPICAL RIBBED SLABS AND CROSS SECTIONS



Forces at the Junction of Beam and Slab Elements



Internal Moments and Joint Forces

Internal Forces and the Applied Load

Small Element of Beam

FIG. 2.1 INTERNAL FORCES AT THE JUNCTION OF BEAM AND SLAB ELEMENTS, AND ON A SMALL ELEMENT OF BEAM

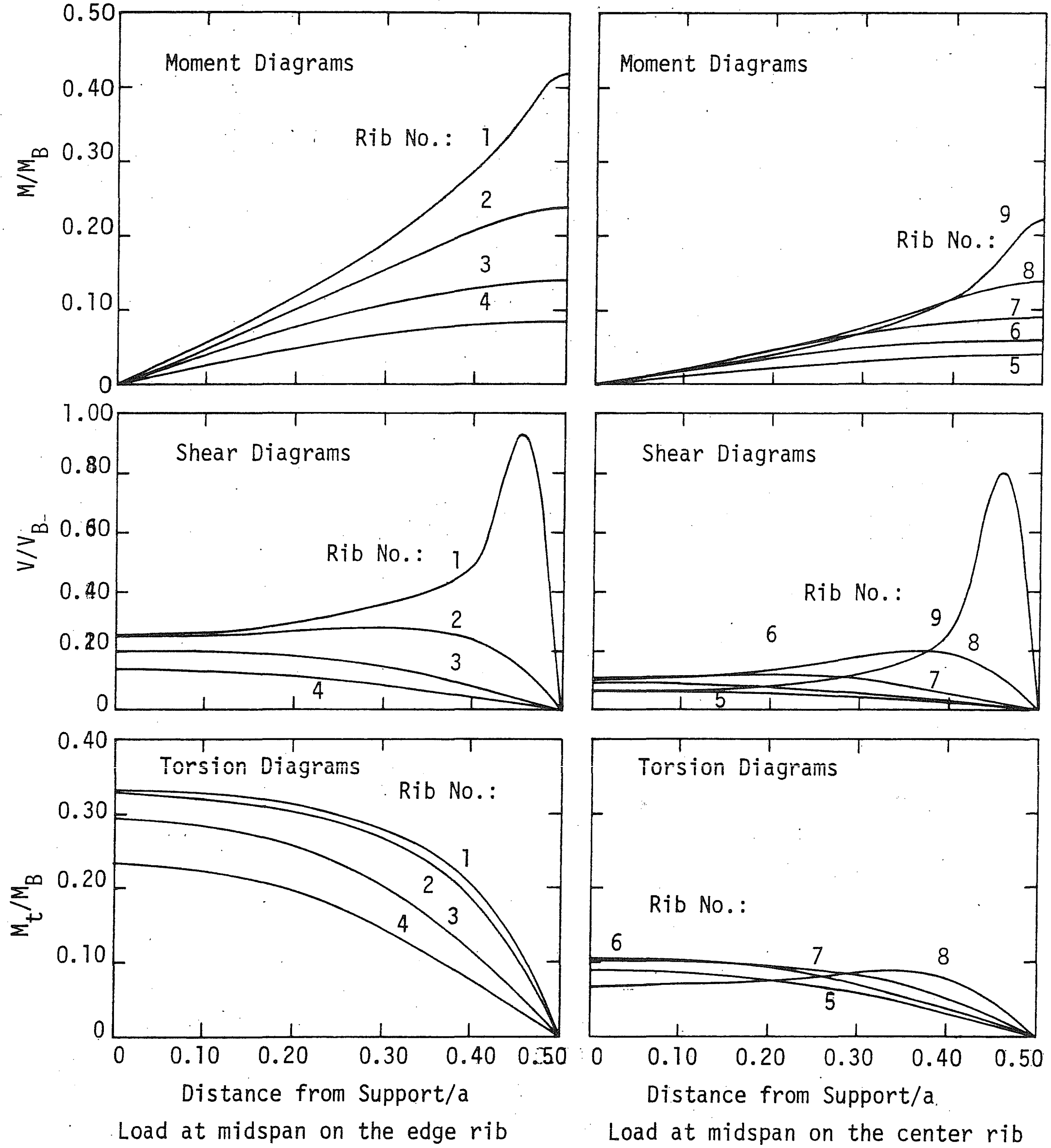
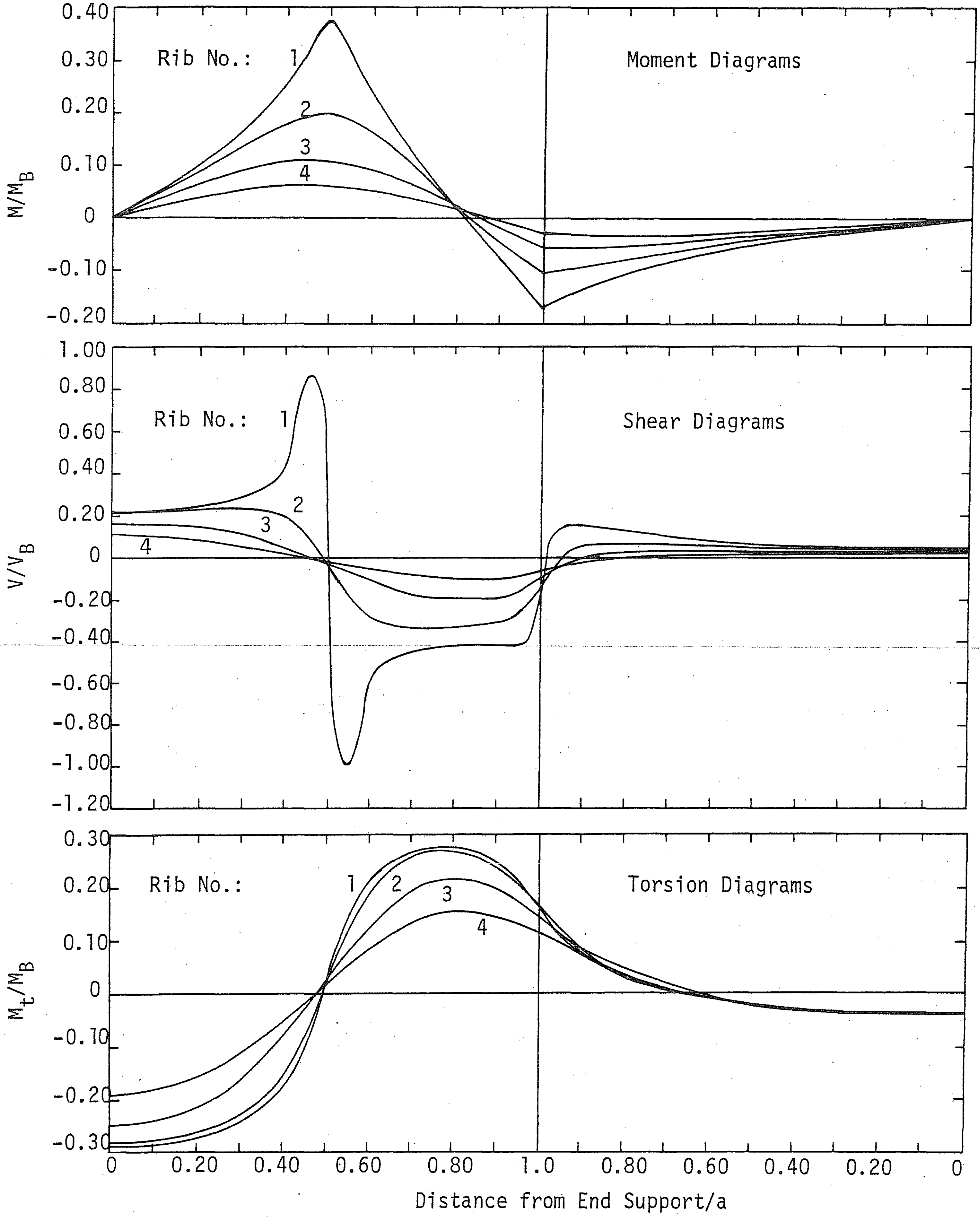
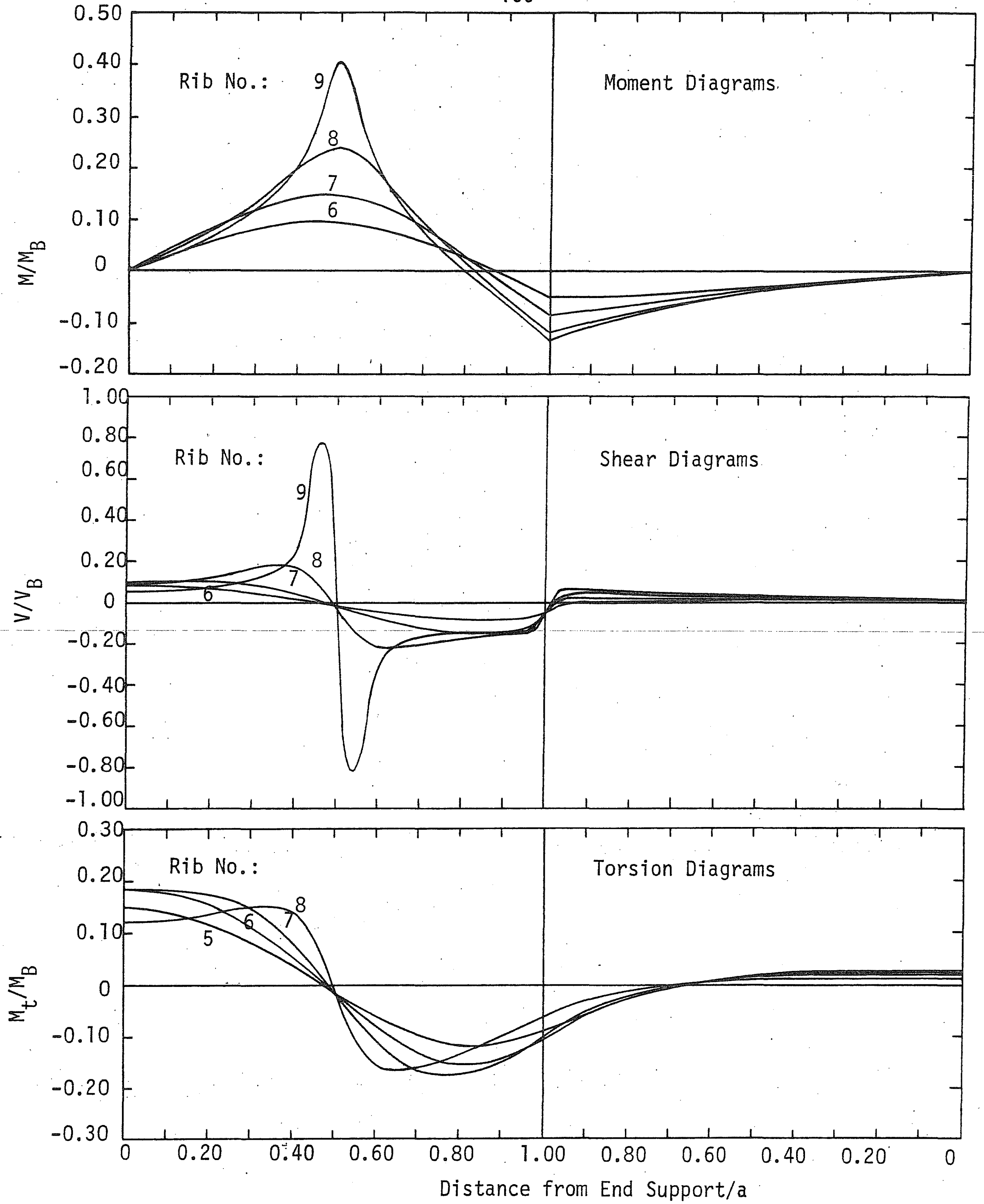


FIG. 3.1 MOMENT, SHEAR, AND TORSION DIAGRAMS FOR A ONE-SPAN RIBBED SLAB DUE TO A SINGLE LOAD AT MIDSPAN ON THE EDGE AND CENTER RIBS



(a) Load at Midspan on the Edge Rib

FIG. 3.2 MOMENT, SHEAR, AND TORSION DIAGRAMS FOR A TWO-SPAN RIBBED SLAB DUE TO A SINGLE LOAD AT ONE MIDSPAN ON THE EDGE AND CENTER RIBS



(b) Load at Midspan on the Center Rib

FIG. 3.2 (CONT.)

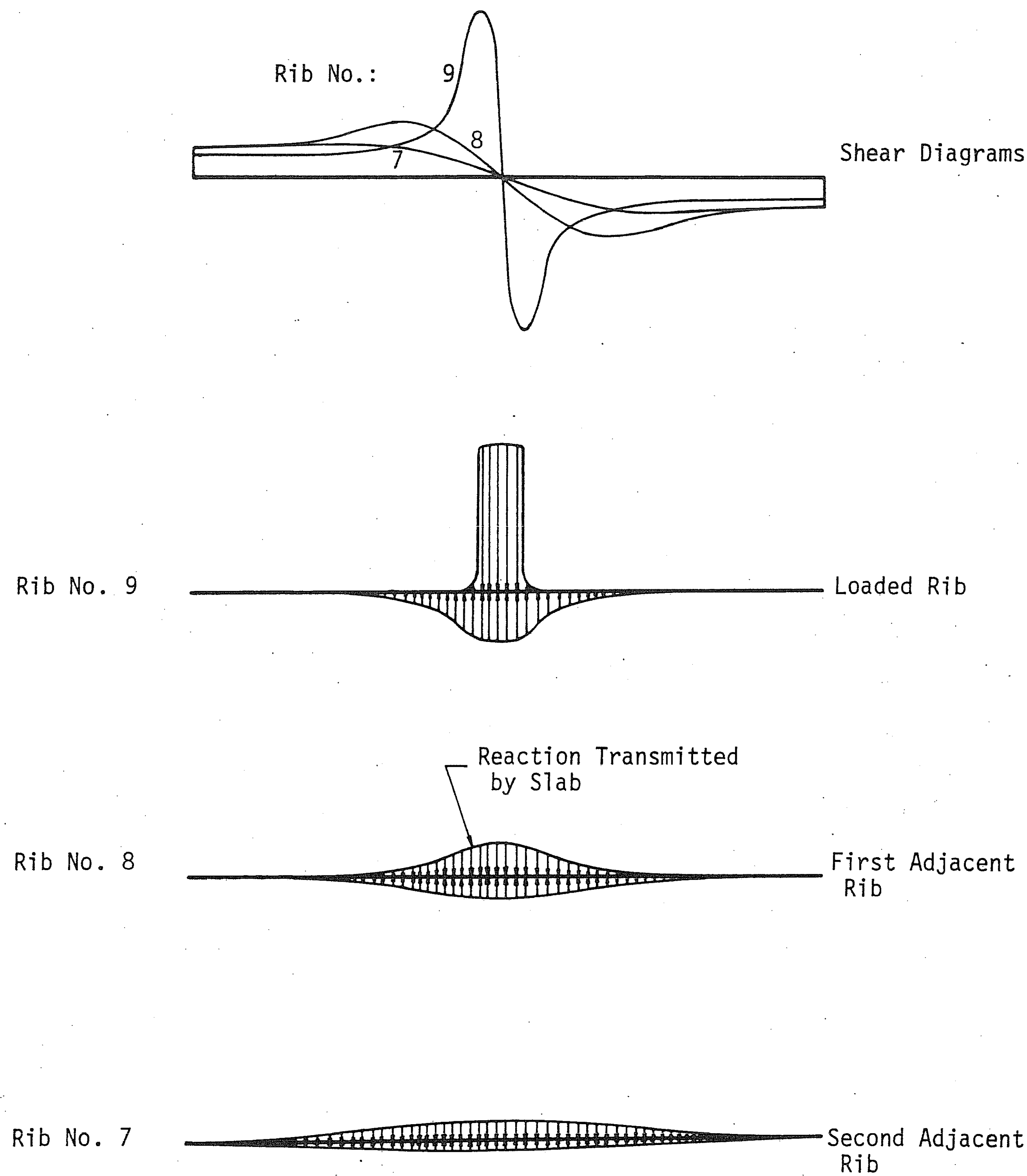
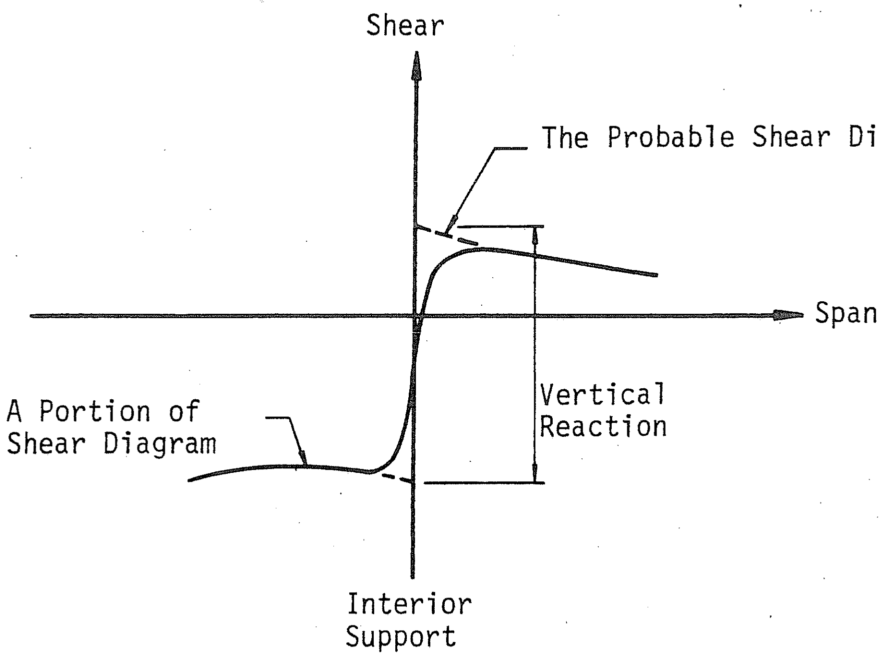
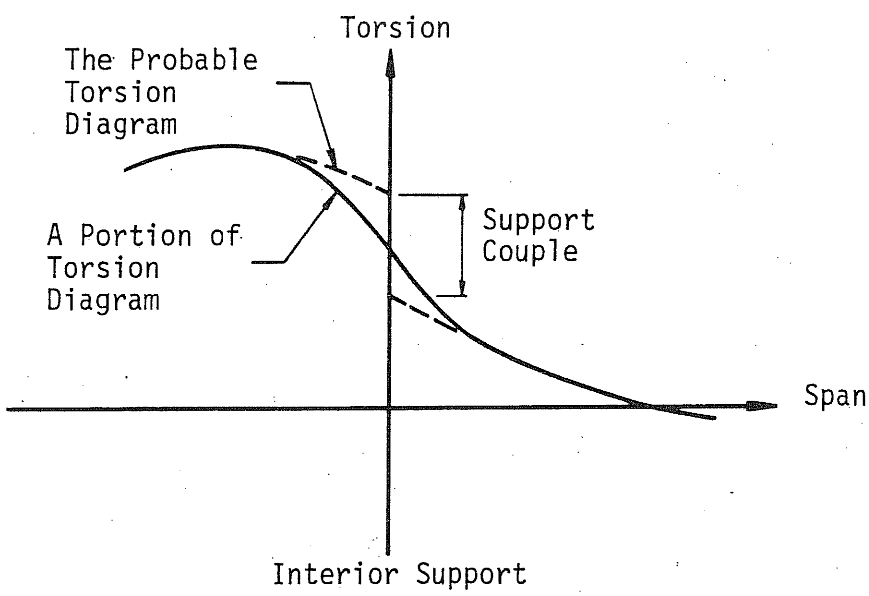


FIG. 3.3 THE INTERPRETATION OF THE LOADING EFFECTS ON THE RIBS



(a) A Portion of a Shear Diagram and Its Interpretation near the Interior Support



(b) A Portion of a Torsion Diagram and Its Interpretation near the Interior Support

FIG. 3.4 THE INTERPRETATION OF REACTIONS AND COUPLES AT THE INTERIOR SUPPORT

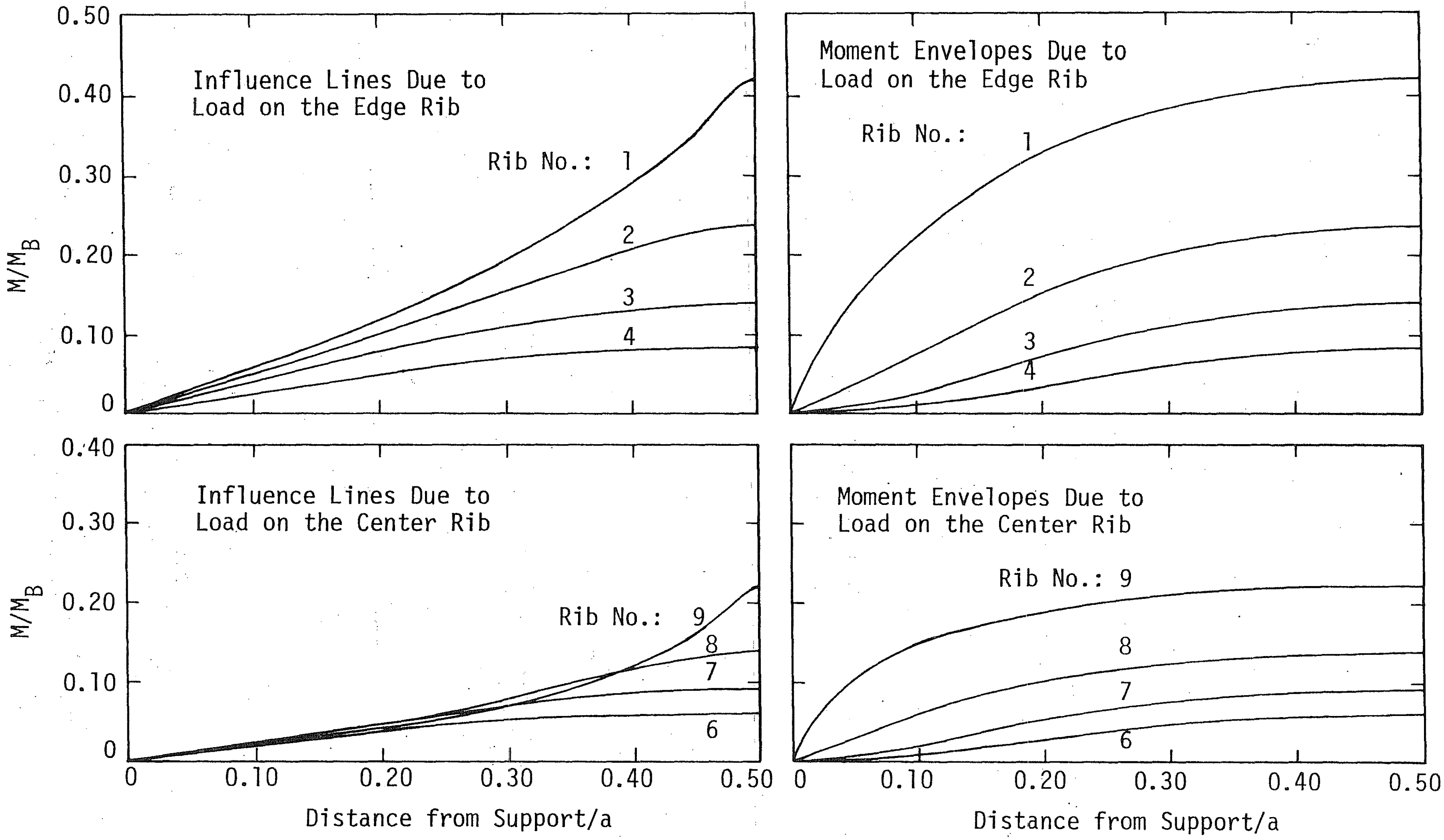


FIG. 3.5 INFLUENCE LINES FOR MOMENT AT MIDSPAN AND MOMENT ENVELOPES FOR A ONE-SPAN RIBBED SLAB DUE TO A LOAD MOVING ON THE EDGE AND CENTER RIBS

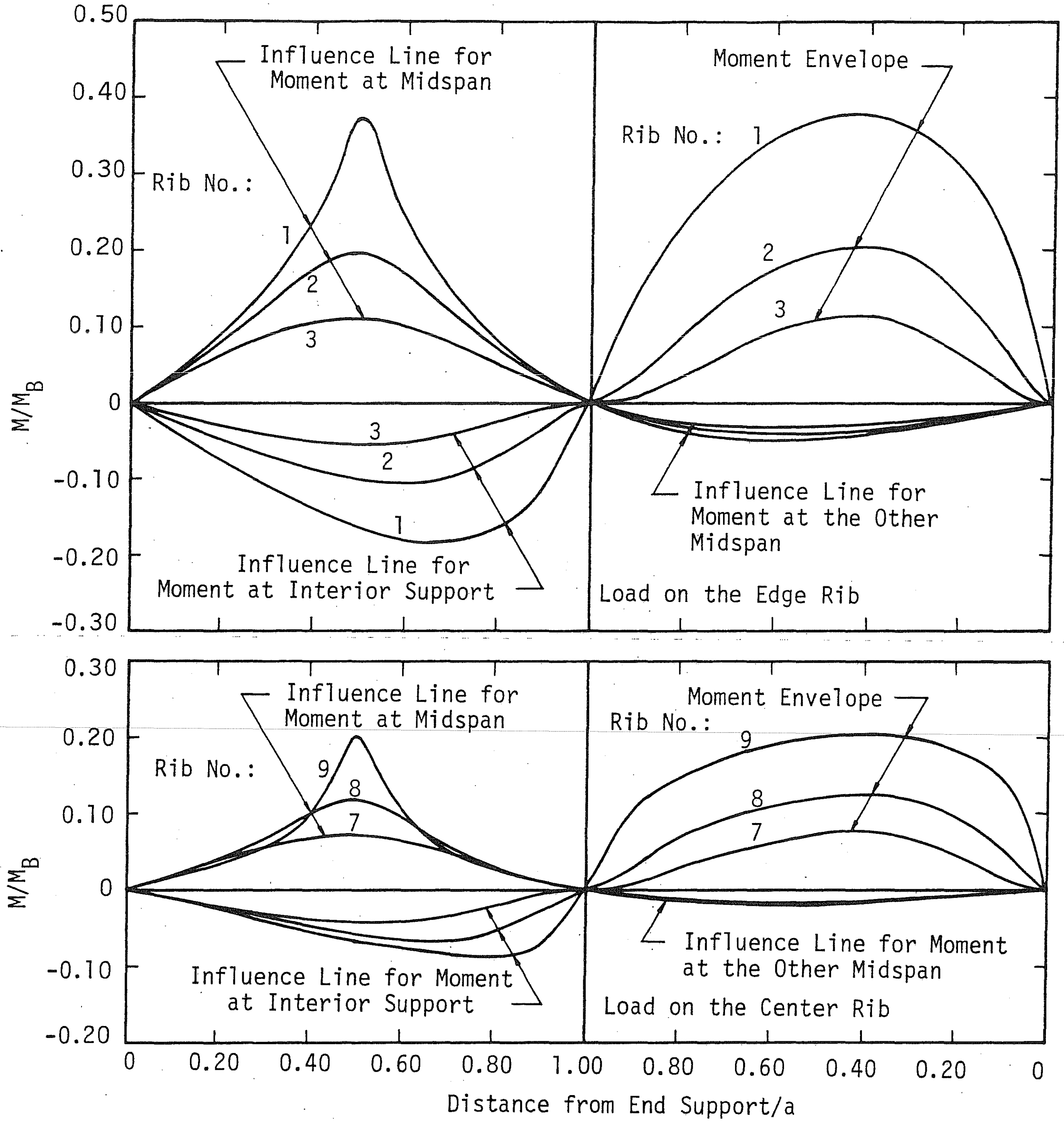


FIG. 3.6 INFLUENCE LINES FOR MOMENT AT MIDSPAN, AND MOMENT ENVELOPES FOR A TWO-SPAN RIBBED SLAB DUE TO A LOAD MOVING ON THE EDGE AND CENTER RIBS

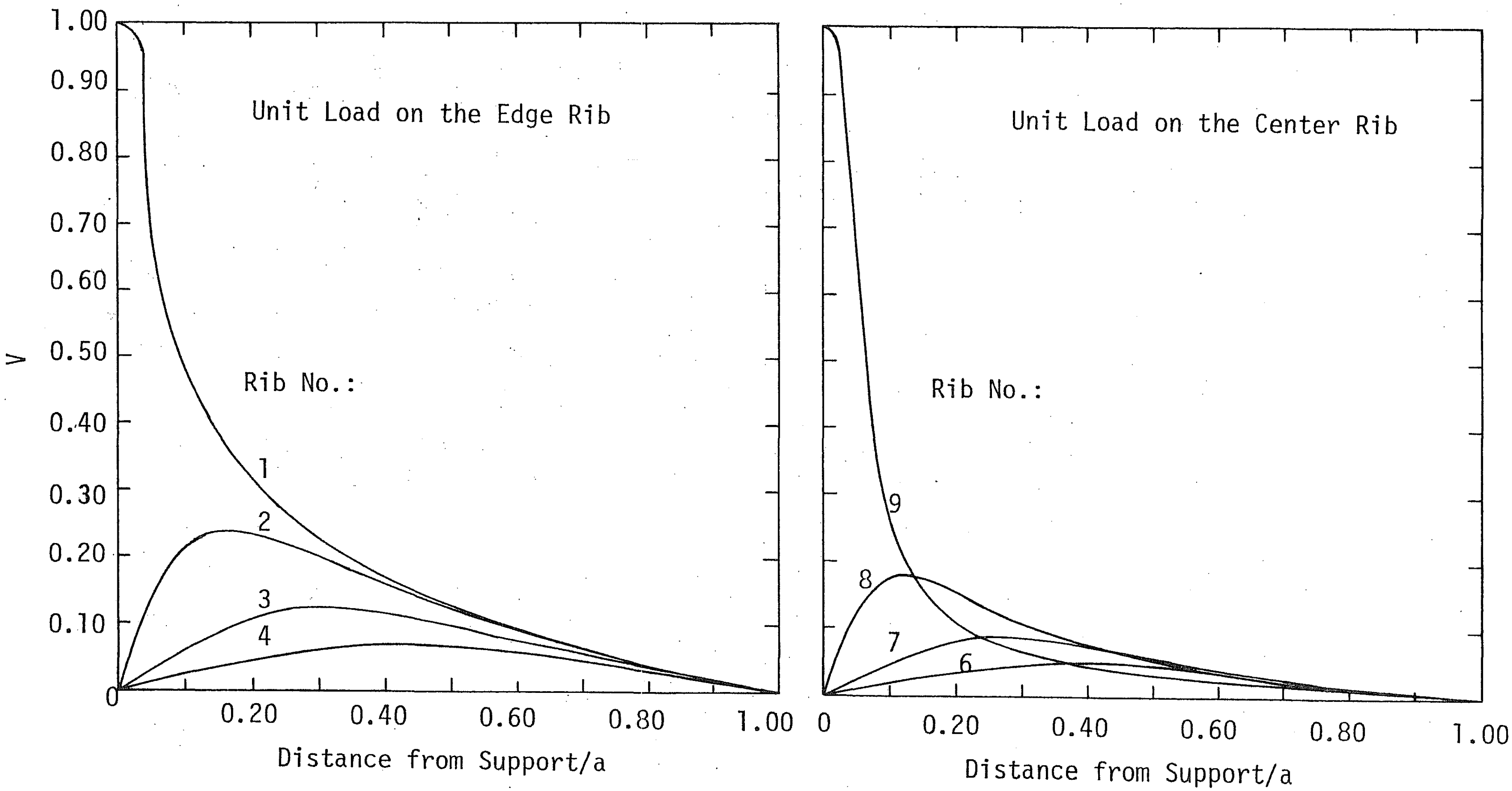


FIG. 3.7 INFLUENCE LINES FOR SHEAR AT THE SUPPORT OF A ONE-SPAN RIBBED SLAB DUE TO A LOAD MOVING ON THE EDGE AND CENTER RIBS

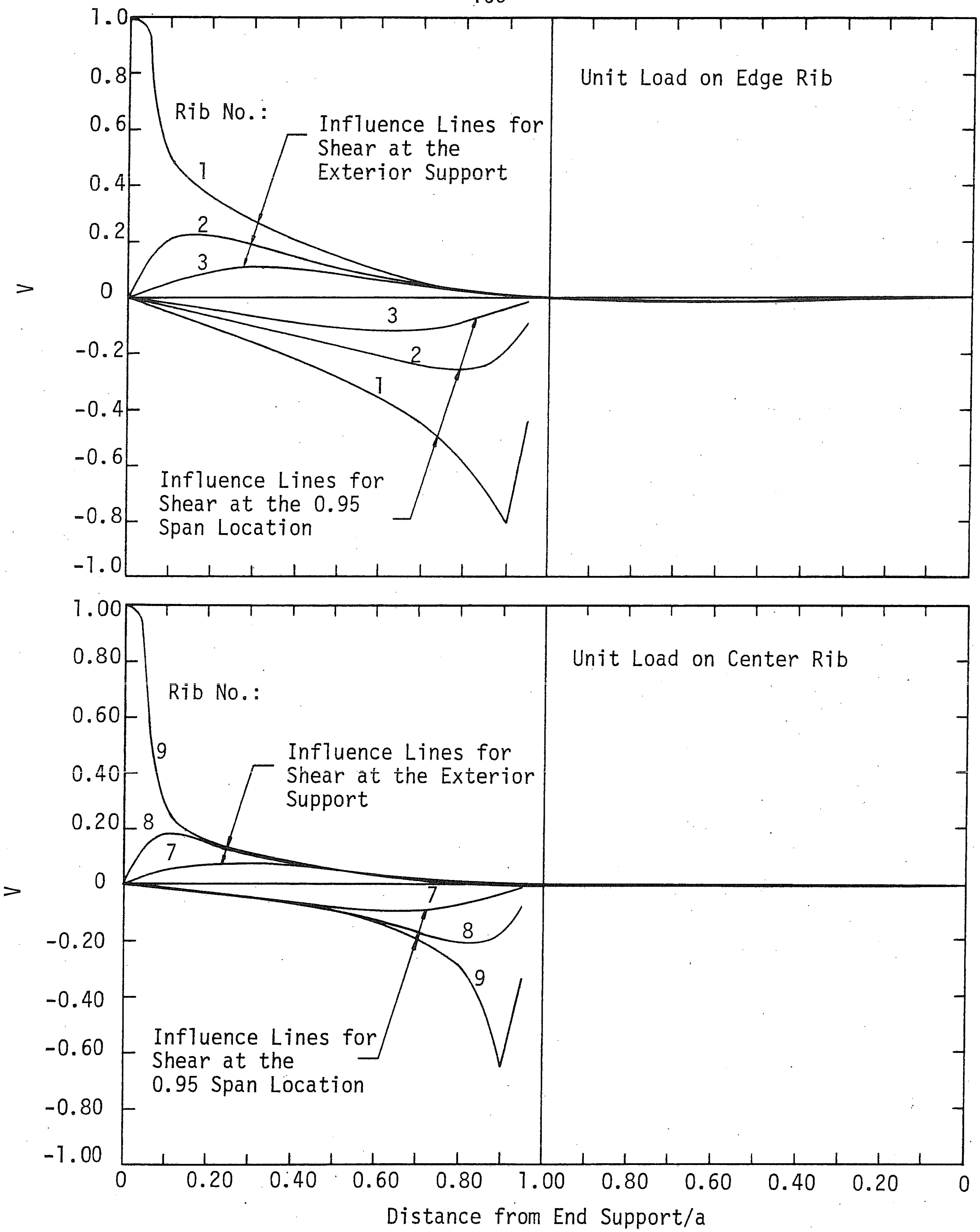


FIG. 3.8 INFLUENCE LINES FOR SHEAR AT THE EXTERIOR SUPPORT AND AT THE 0.95 SPAN LOCATION OF A TWO-SPAN RIBBED SLAB DUE TO A LOAD MOVING ON THE EDGE AND CENTER RIBS

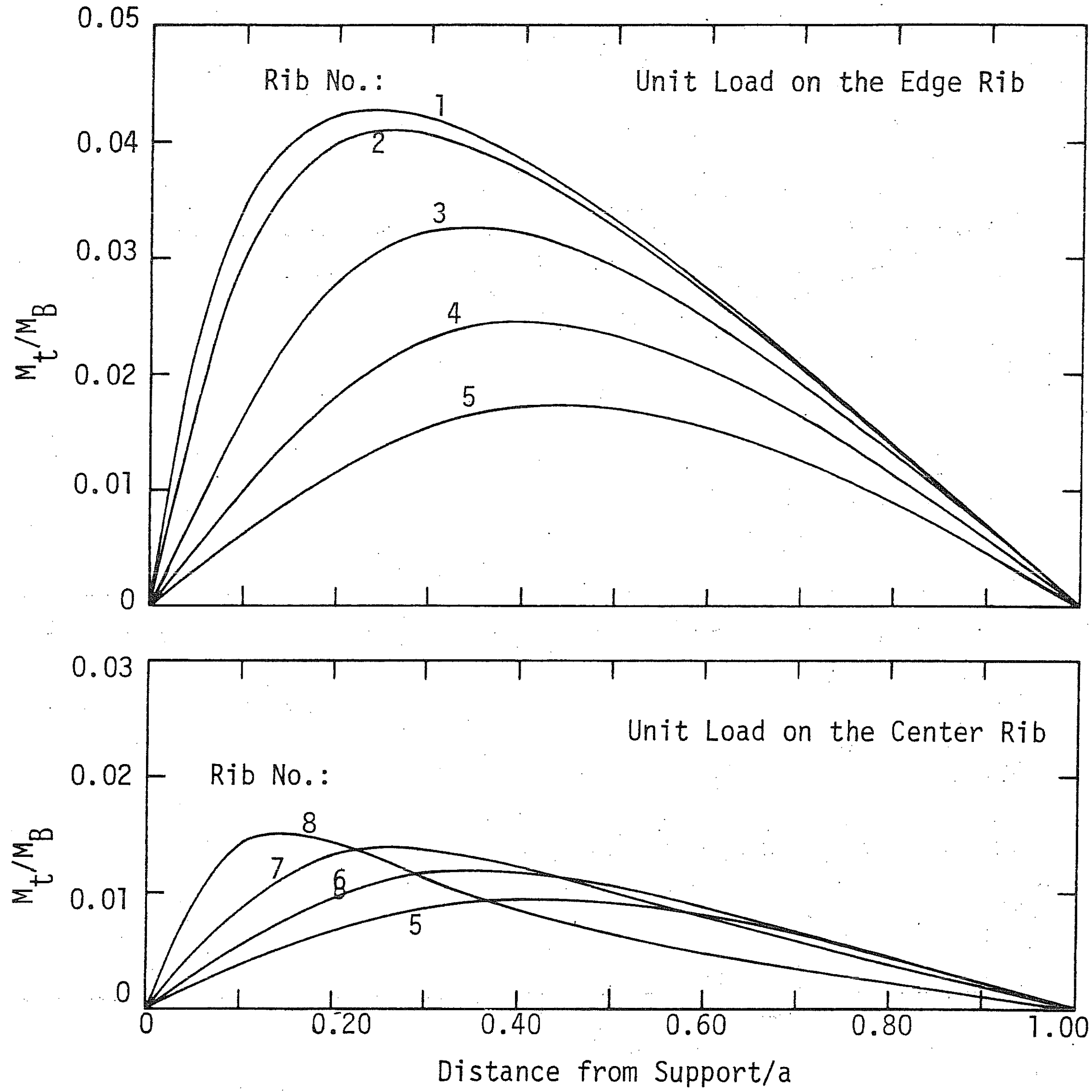


FIG. 3.9 INFLUENCE LINES FOR TORSION AT THE SUPPORT OF A ONE-SPAN RIBBED SLAB DUE TO A LOAD MOVING ON THE EDGE AND CENTER RIBS

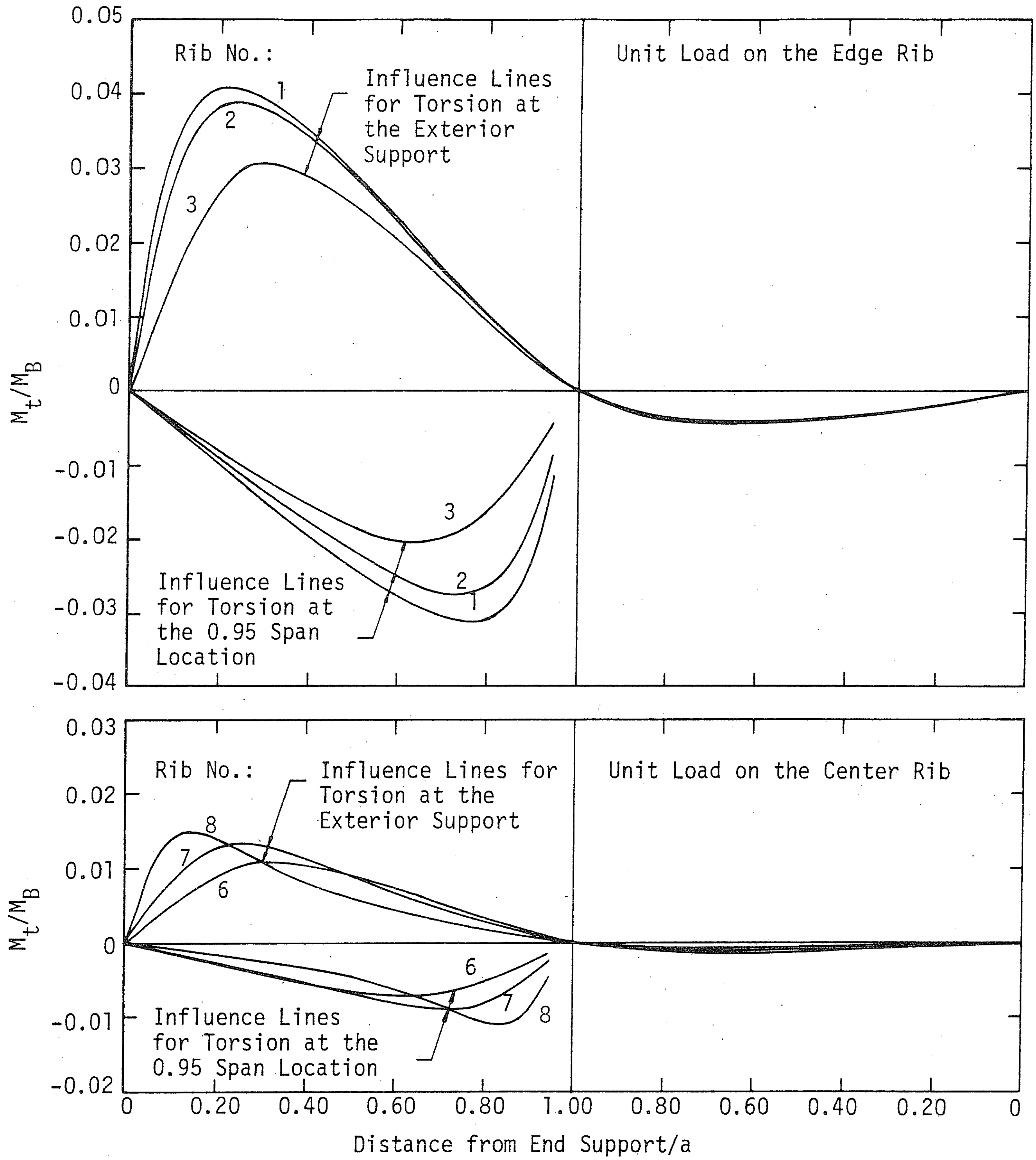


FIG. 3.10 INFLUENCE LINES FOR TORSION AT THE EXTERIOR SUPPORT AND AT THE 0.95 SPAN LOCATION OF A TWO-SPAN RIBBED SLAB DUE TO A LOAD MOVING ON THE EDGE AND CENTER RIBS

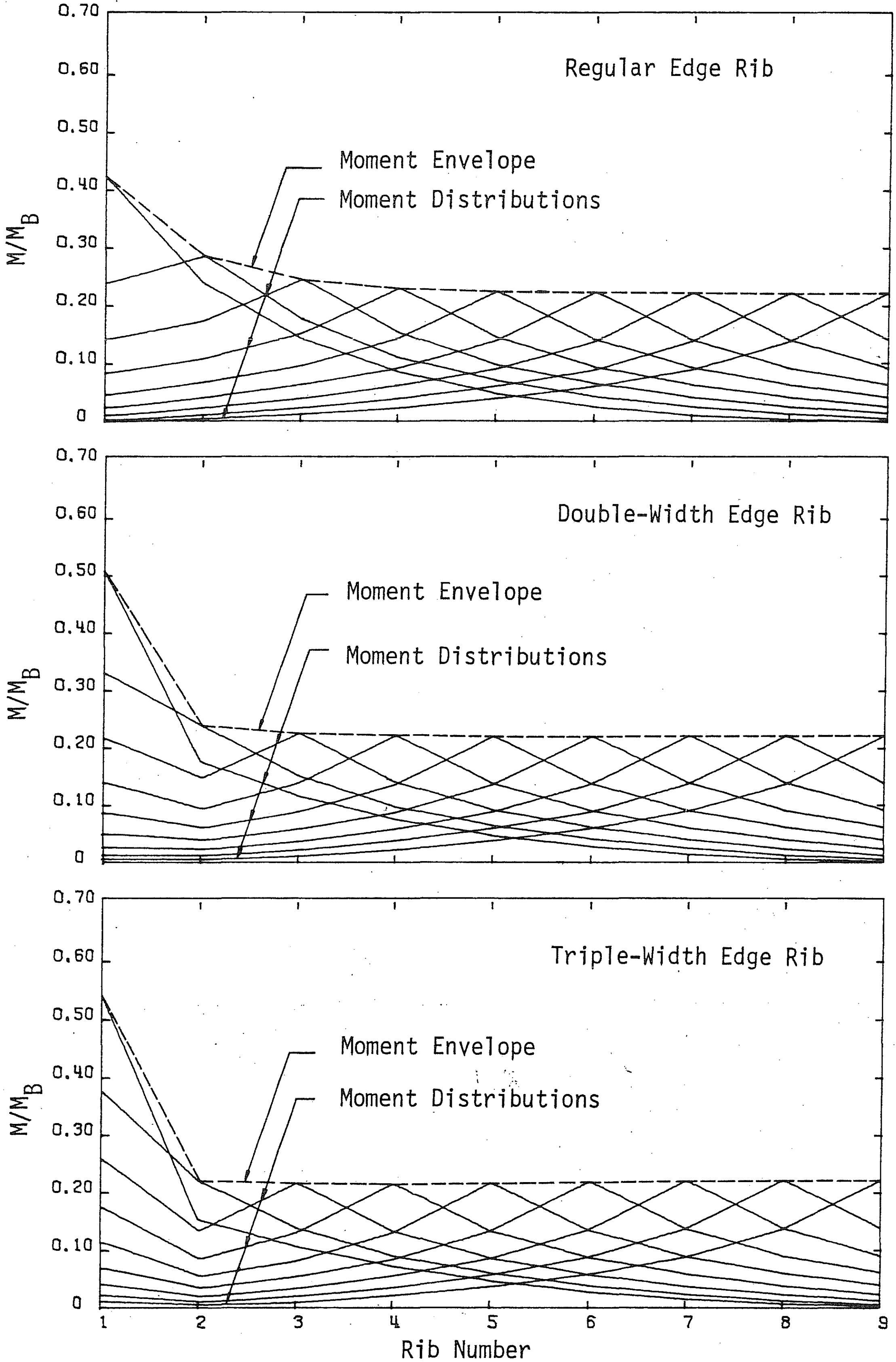


FIG. 3.11 MOMENT ENVELOPES AND MOMENT DISTRIBUTIONS AT MIDSPAN OF VARIOUS RIBS OF A ONE-SPAN RIBBED SLAB WITH THE REGULAR AND STIFFENED EDGE RIBS DUE TO A LOAD MOVING ALONG MIDSPAN

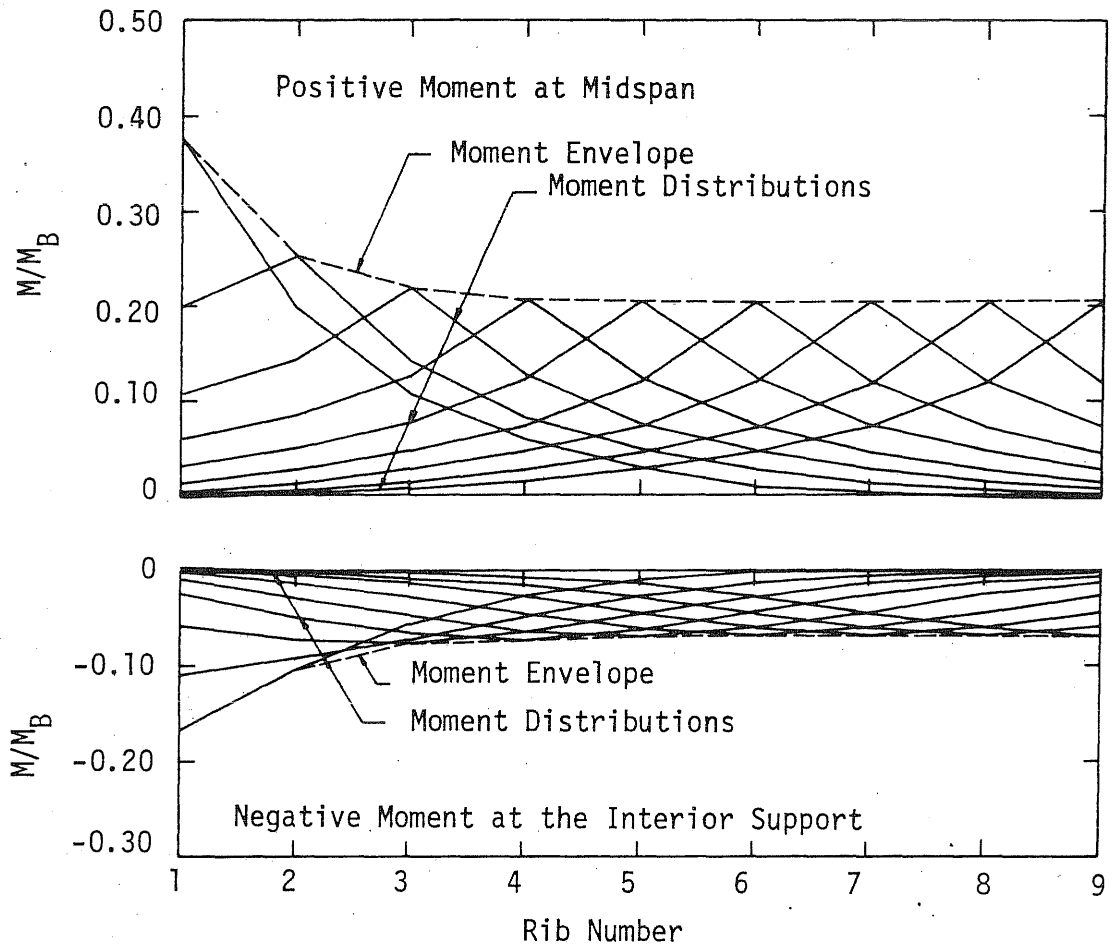


FIG. 3.12 POSITIVE AND NEGATIVE MOMENT ENVELOPES AND MOMENT DISTRIBUTIONS AT MIDSPAN AND AT THE INTERIOR SUPPORT OF VARIOUS RIBS OF A TWO-SPAN RIBBED SLAB DUE TO A LOAD MOVING ALONG MIDSPAN

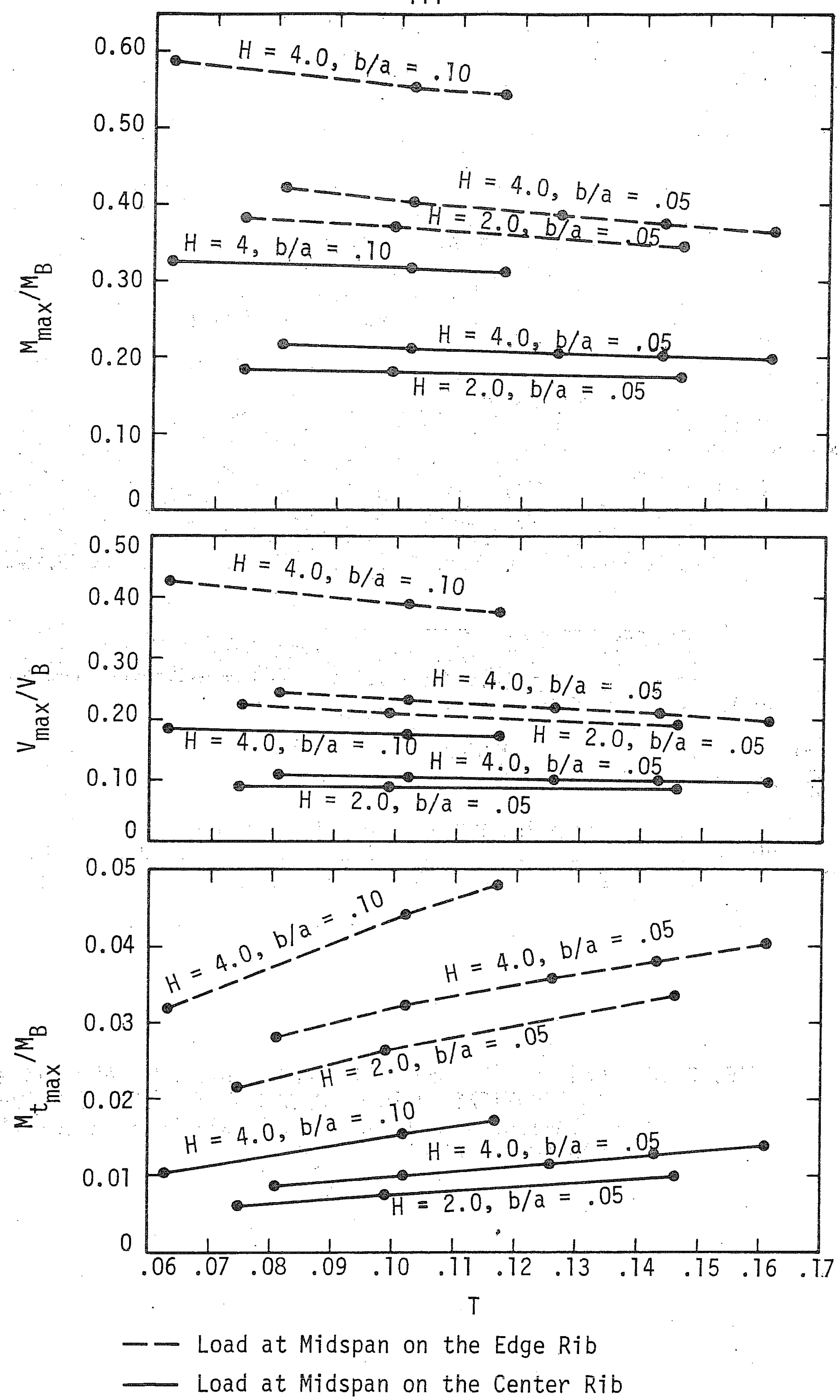


FIG. 3.13 EFFECTS OF TORSIONAL STIFFNESS ON MOMENT, SHEAR AND TORSION

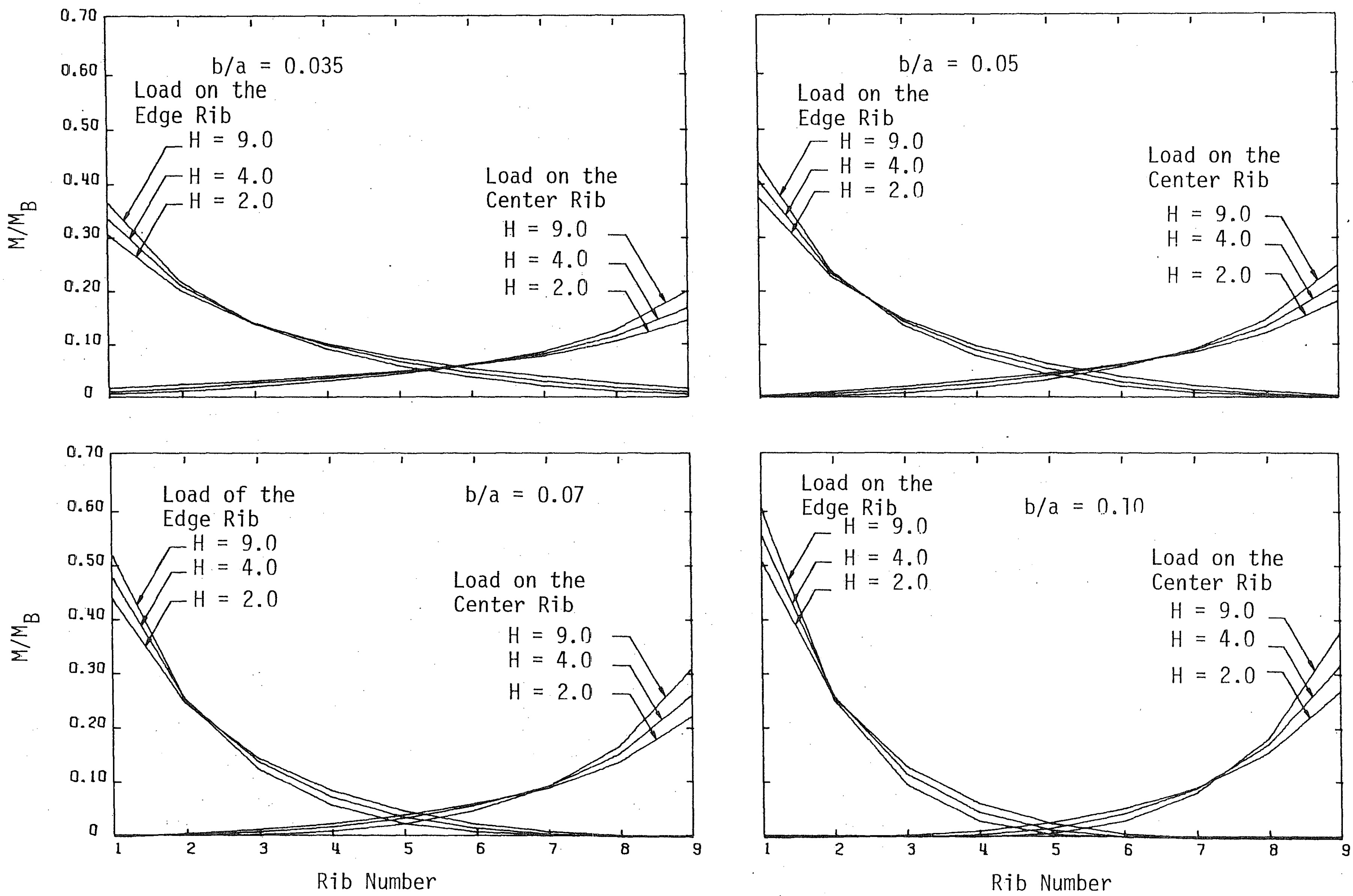


FIG. 3.14 MOMENT DISTRIBUTIONS AT MIDSPAN OF ONE-SPAN RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

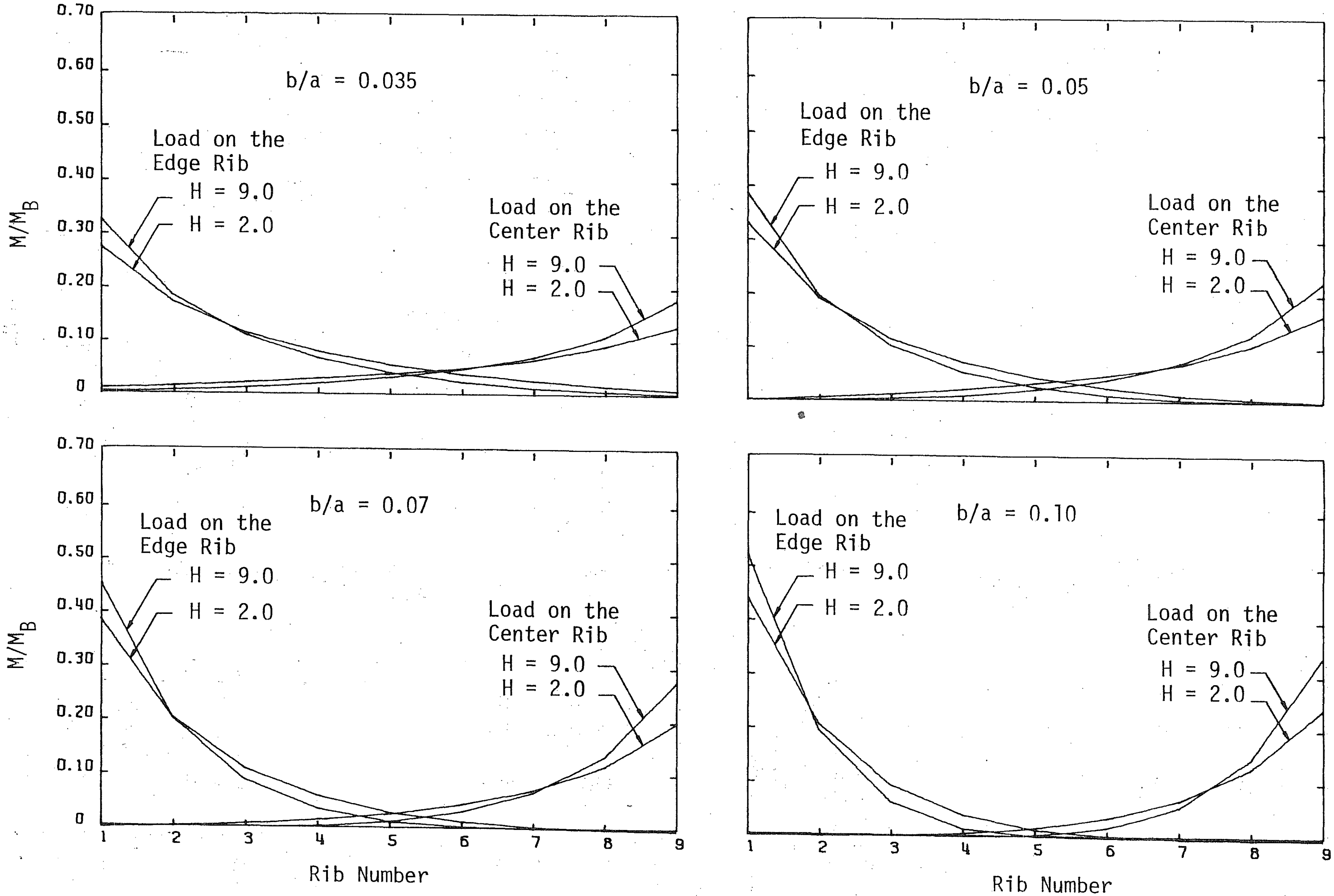


FIG. 3.15 POSITIVE MOMENT DISTRIBUTIONS AT MIDSPAN OF TWO-SPAN RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN OF THE EDGE AND CENTER RIBS

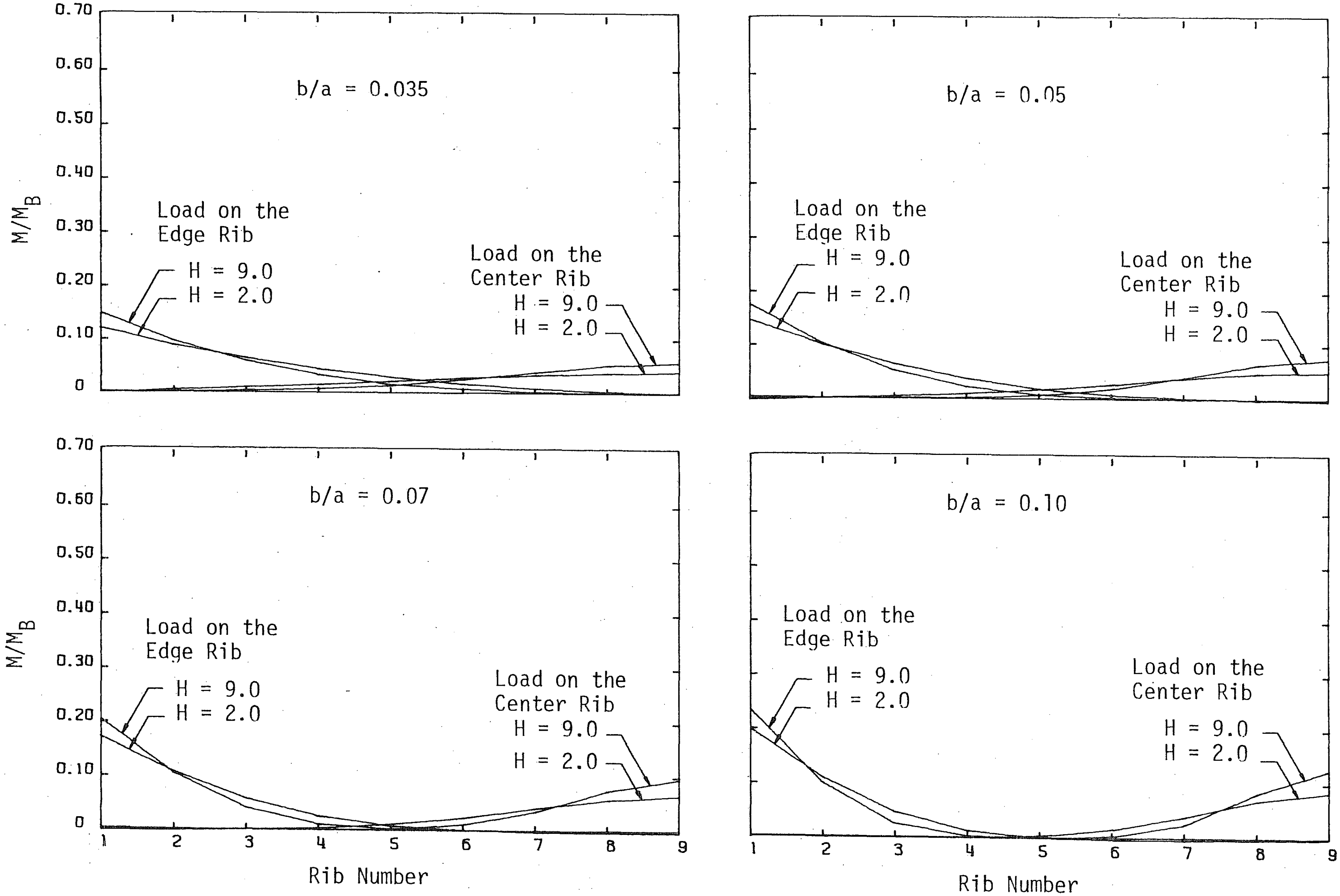


FIG. 3.16 NEGATIVE MOMENT DISTRIBUTIONS AT THE INTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

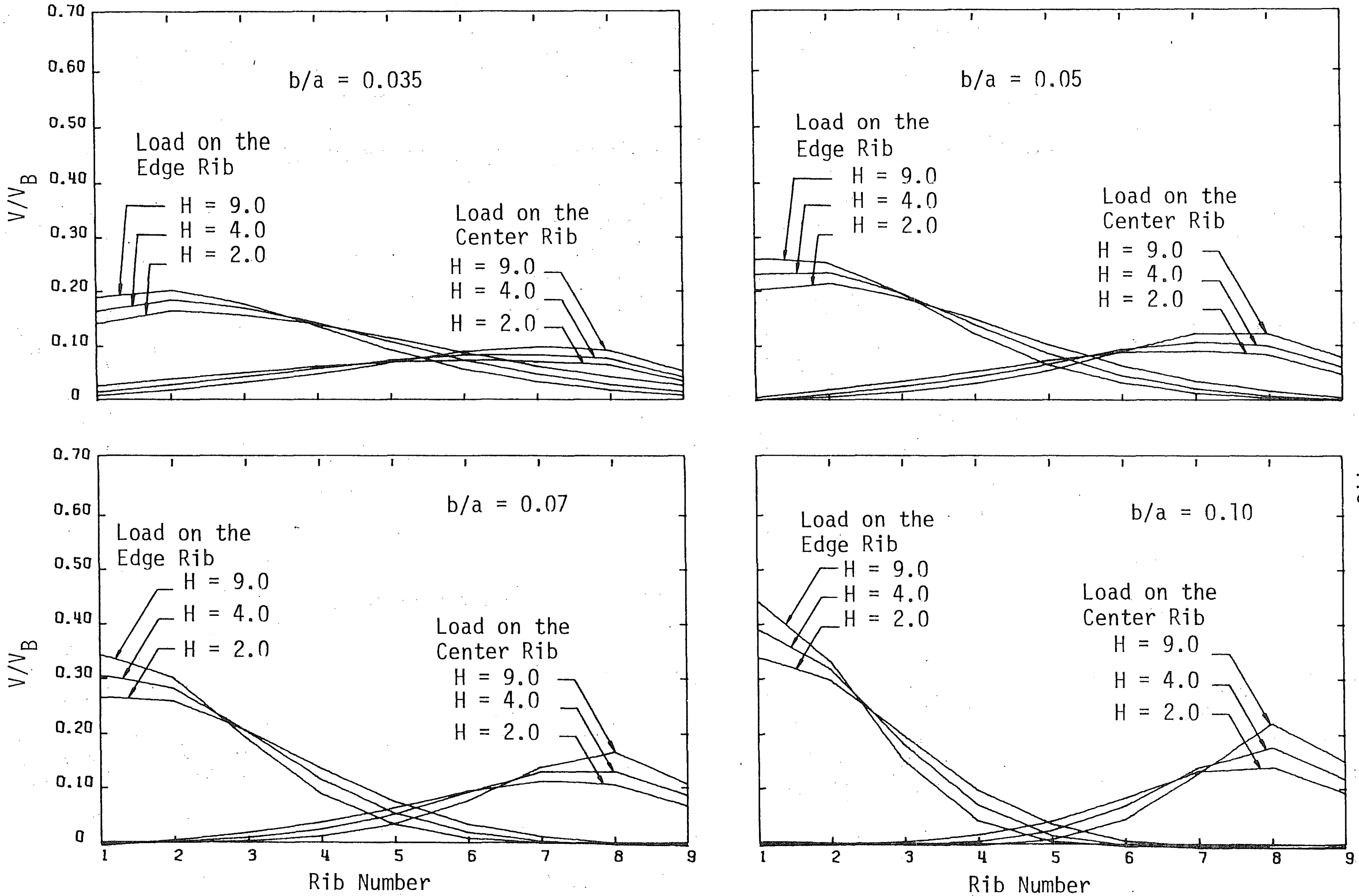


FIG. 3.17 SHEAR DISTRIBUTIONS AT THE SUPPORT OF ONE-SPAN RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

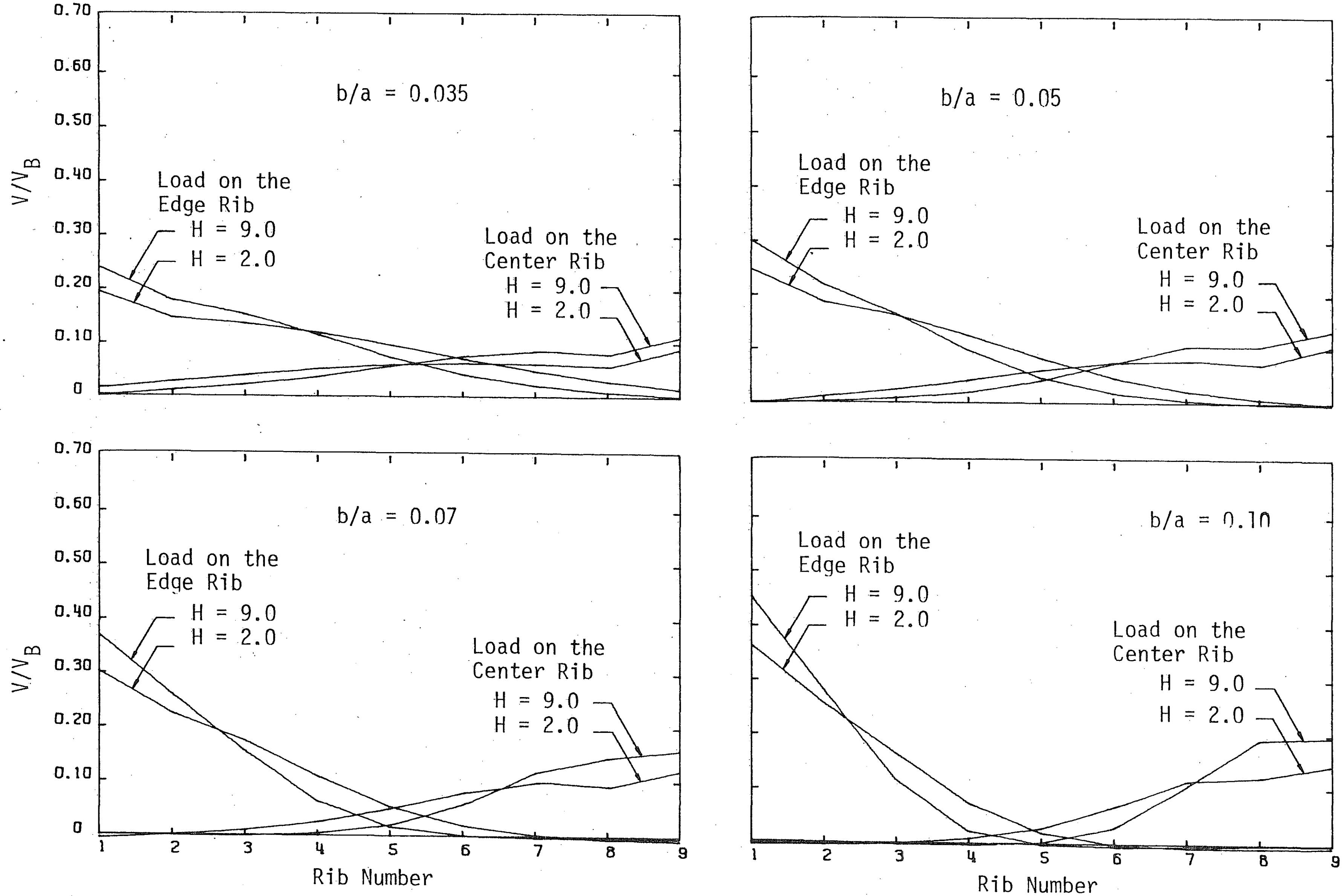


FIG. 3.18 SHEAR DISTRIBUTIONS AT THE EXTERIOR SUPPORT OF THE LOADED SPAN OF TWO-SPAN RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

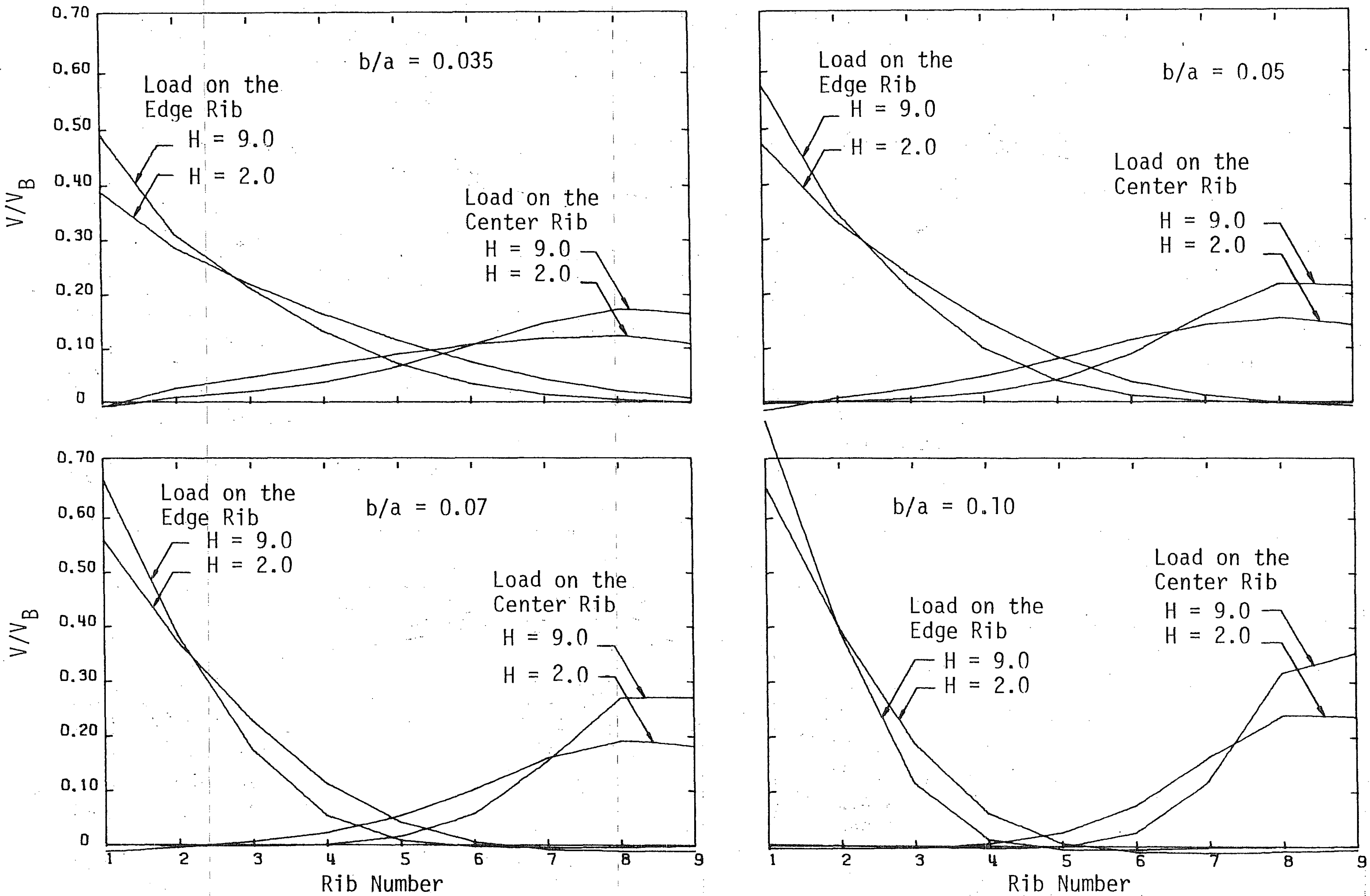


FIG. 3.19 SHEAR DISTRIBUTIONS AT THE 0.95 SPAN LOCATION OF THE LOADED SPAN OF TWO-SPAN RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

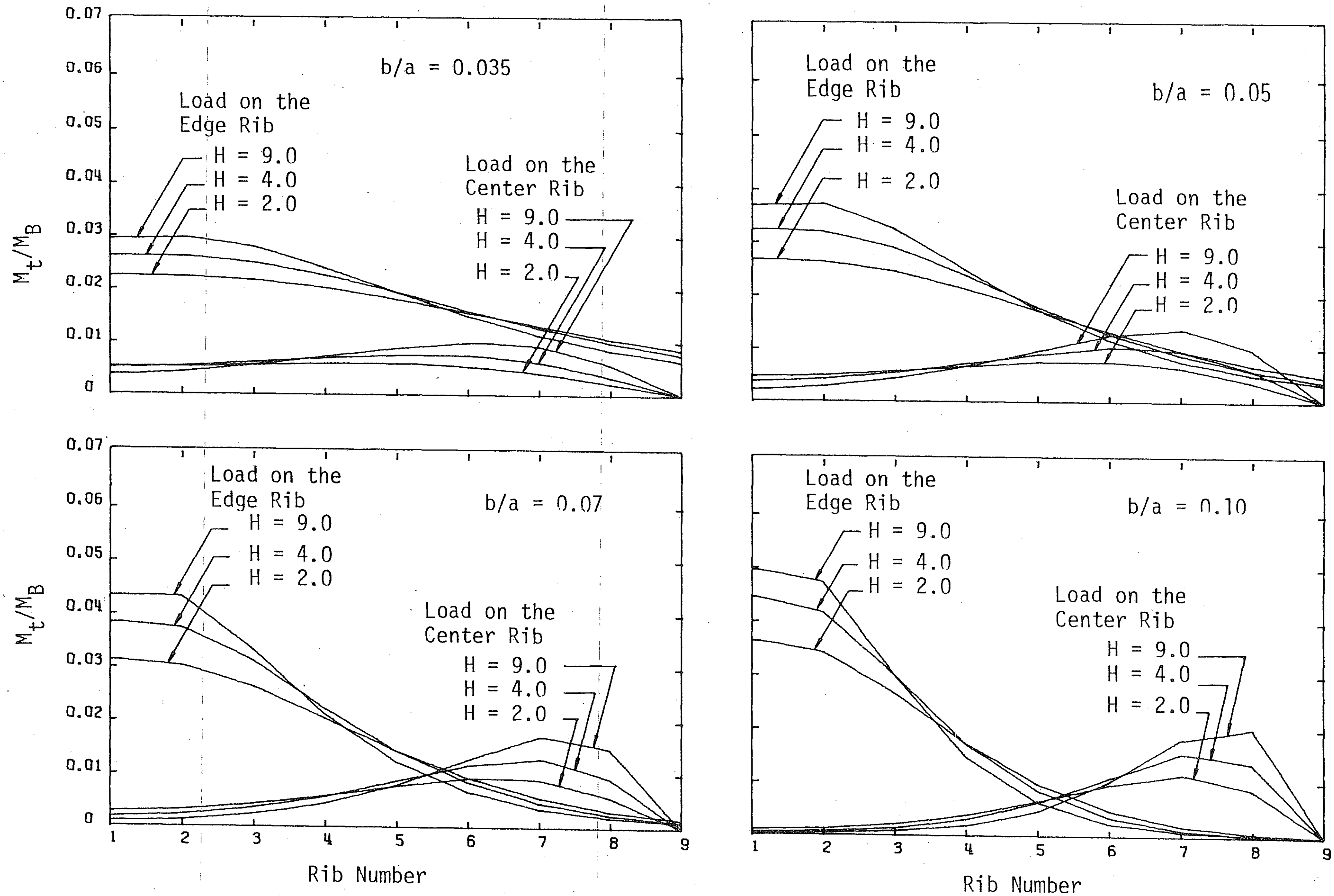


FIG. 3.20 TORSIONAL MOMENT DISTRIBUTIONS AT THE SUPPORT OF ONE-SPAN RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

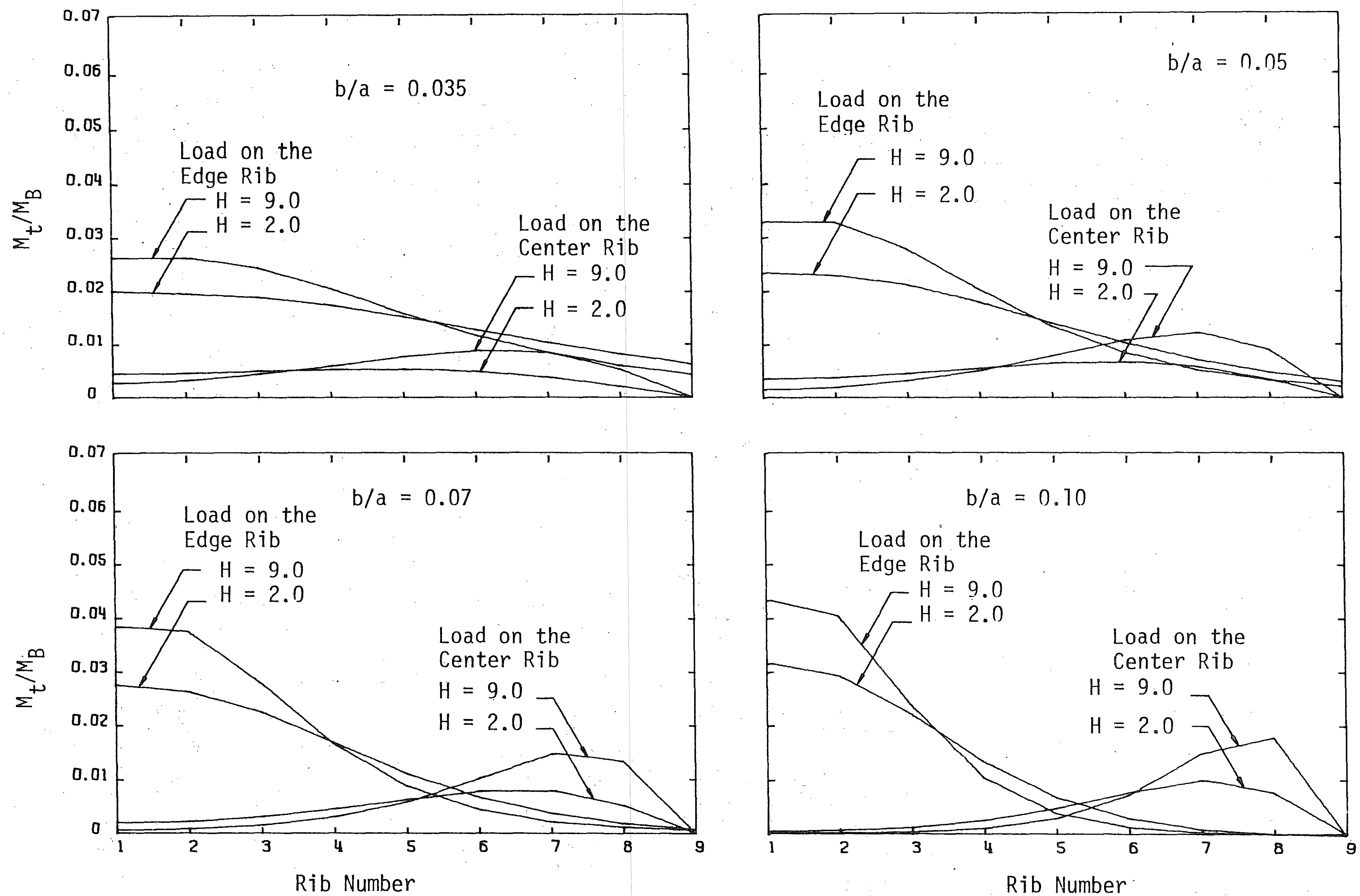


FIG. 3.21 TORSIONAL MOMENT DISTRIBUTIONS AT THE EXTERIOR SUPPORT OF THE LOADED SPAN OF TWO-RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

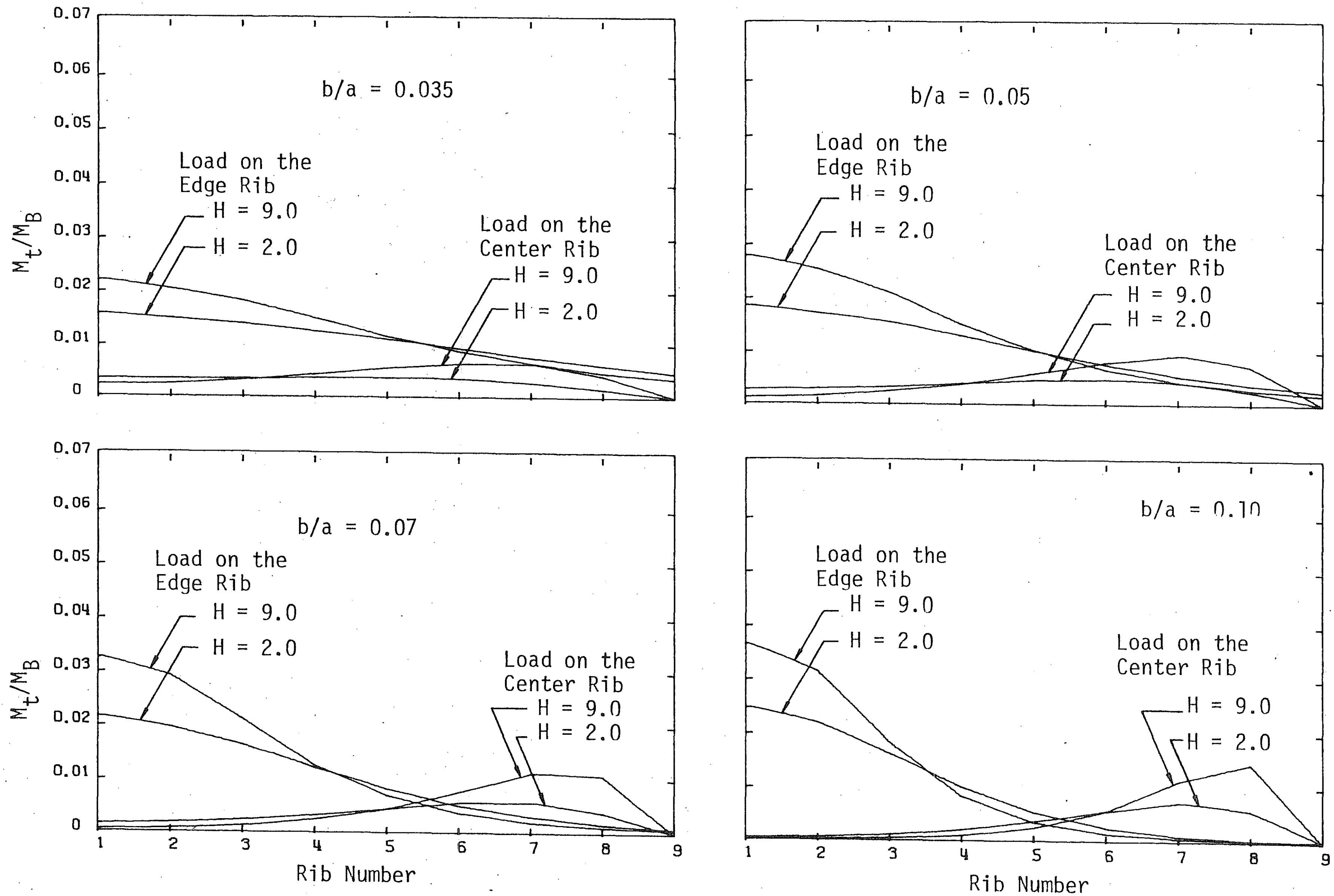
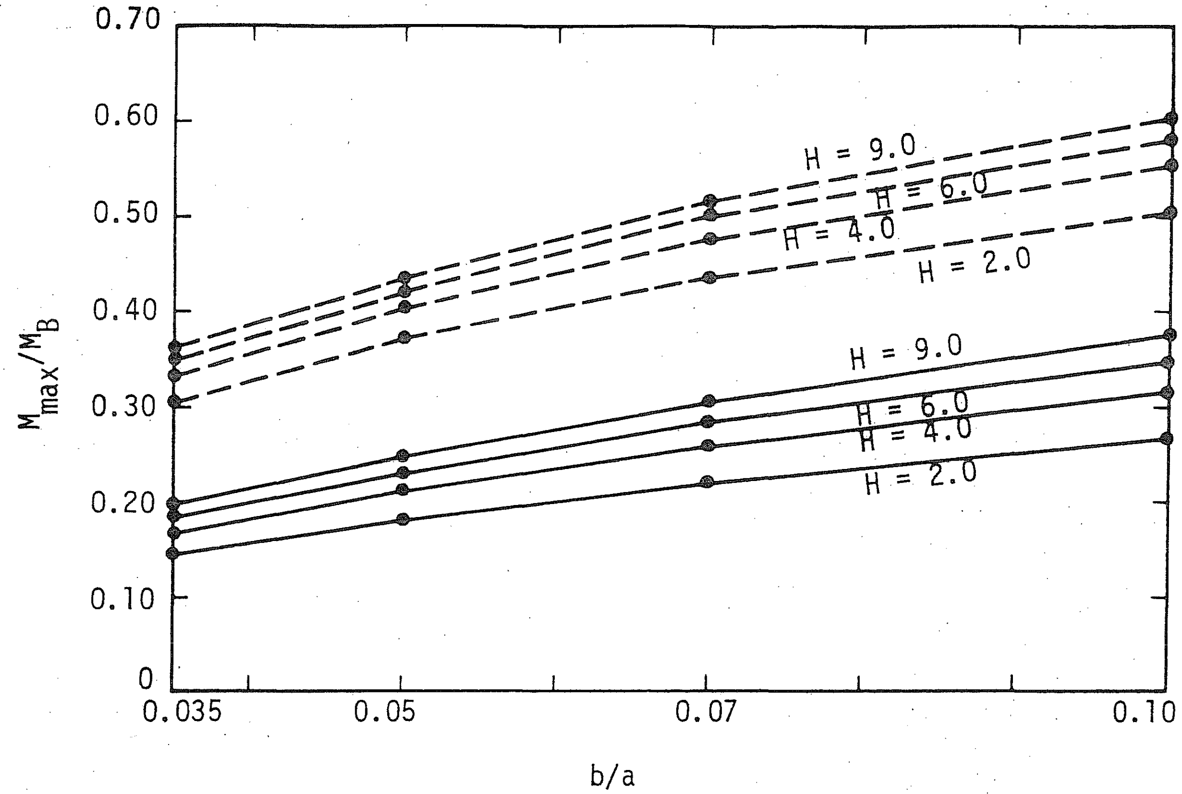


FIG. 3.22 TORSIONAL MOMENT DISTRIBUTIONS AT THE 0.95 SPAN LOCATION OF THE LOADED SPAN OF TWO-SPAN RIBBED SLABS DUE TO SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS



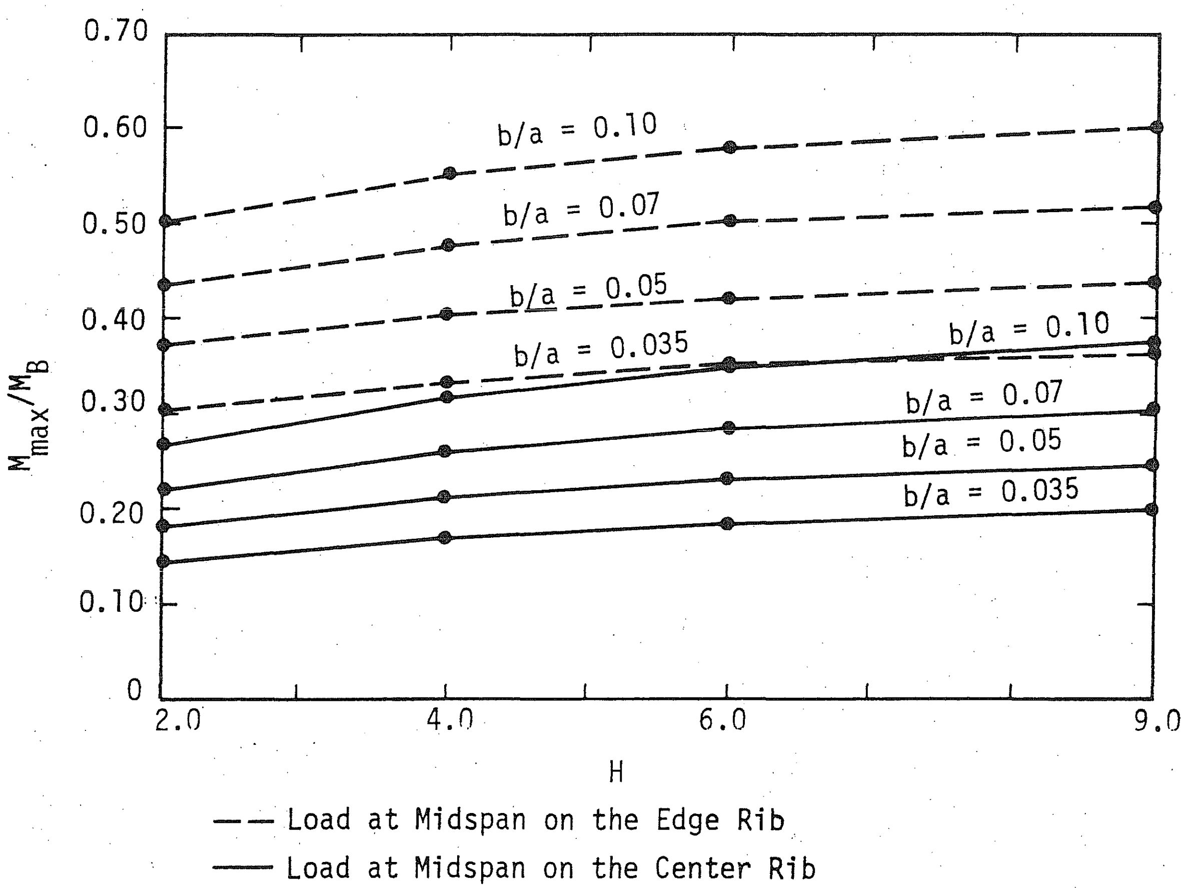


FIG. 3.23 VARIATIONS OF MAXIMUM MOMENT WITH H AND b/a FOR ONE-SPAN RIBBED SLABS UNDER SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

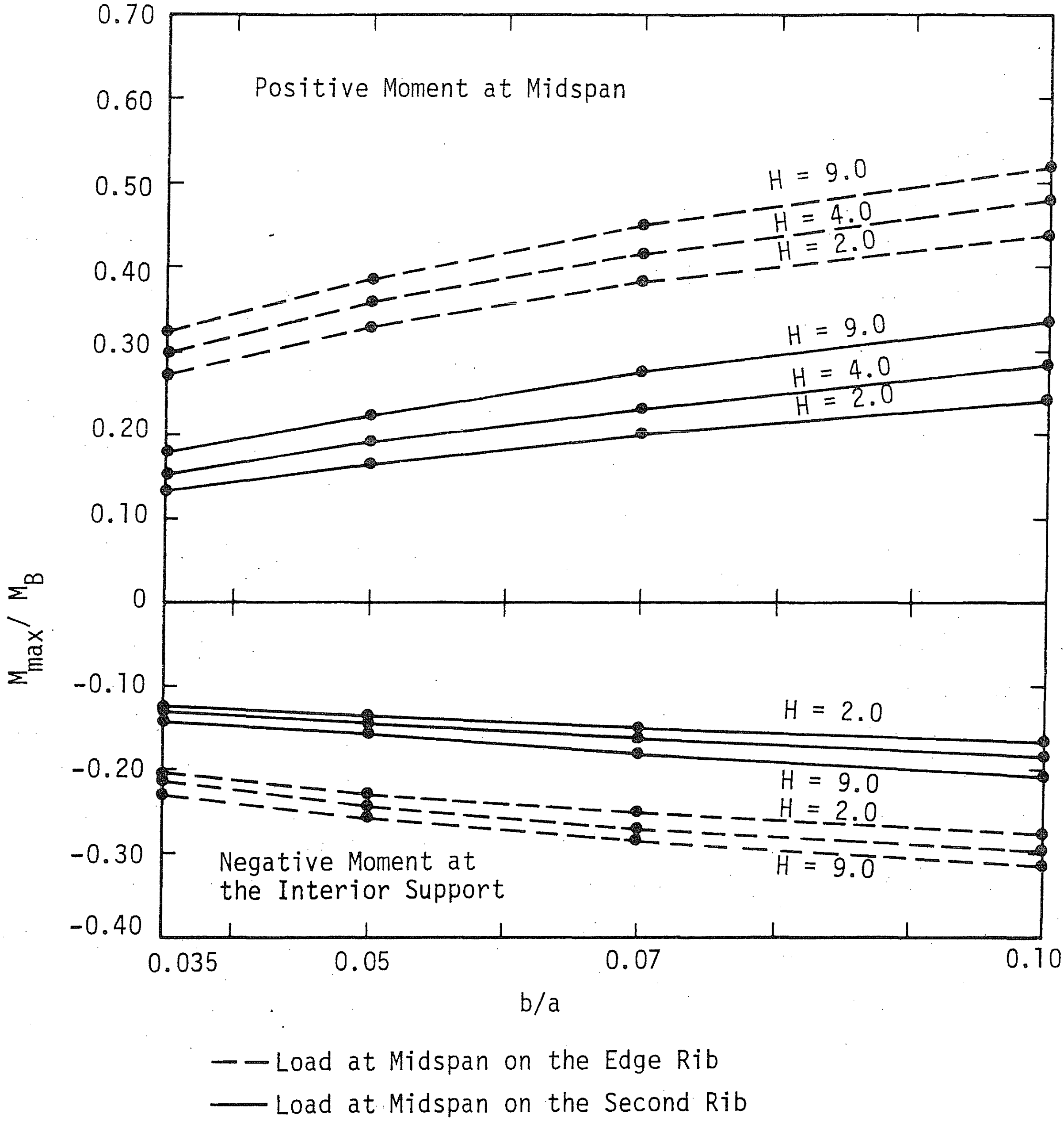


FIG. 3.24 VARIATIONS OF MAXIMUM POSITIVE AND NEGATIVE MOMENTS WITH H AND b/a FOR TWO-SPAN RIBBED SLABS UNDER SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

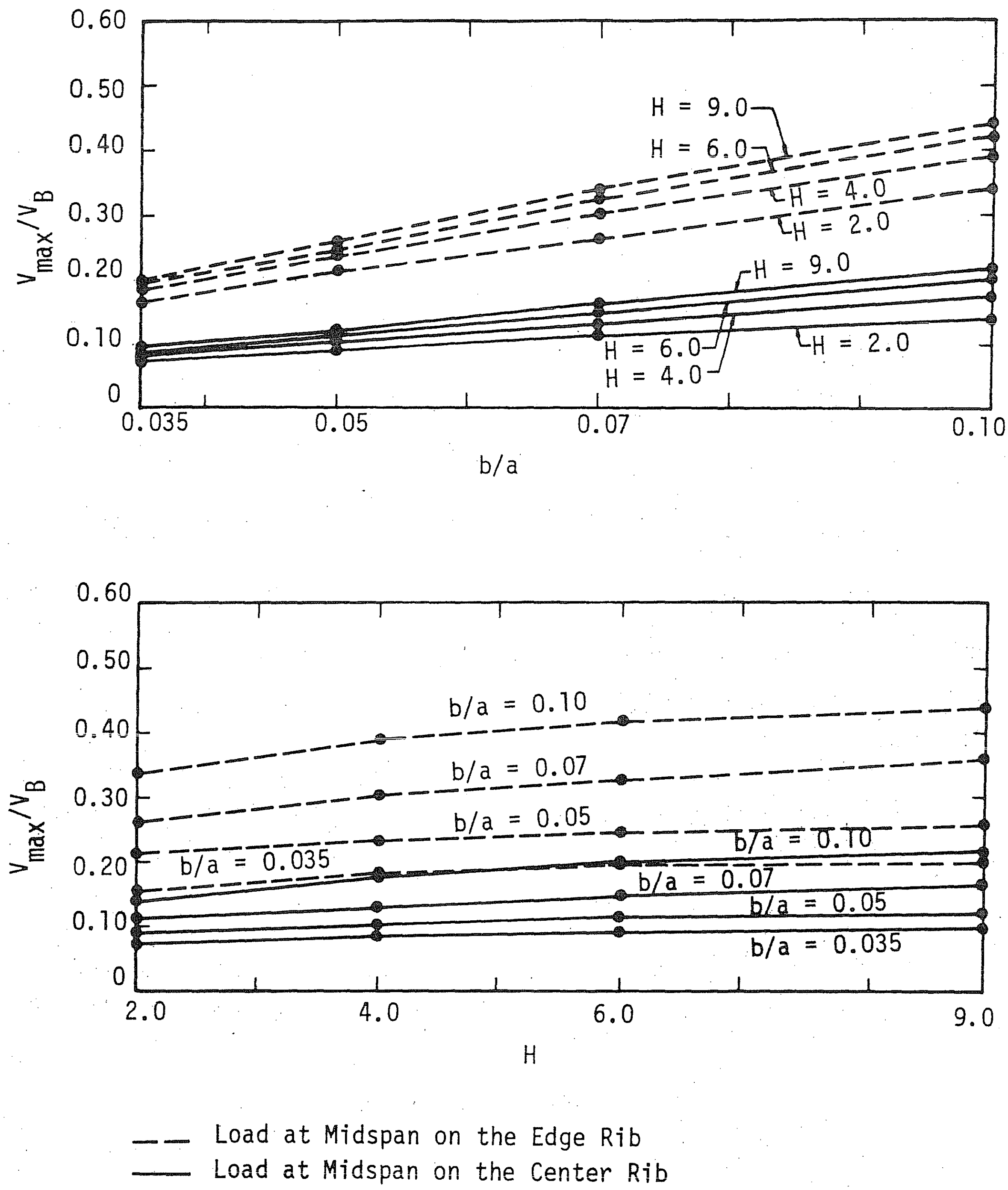


FIG. 3.25 VARIATIONS OF MAXIMUM SHEAR AT THE SUPPORT WITH H AND b/a FOR ONE-SPAN RIBBED SLABS UNDER SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

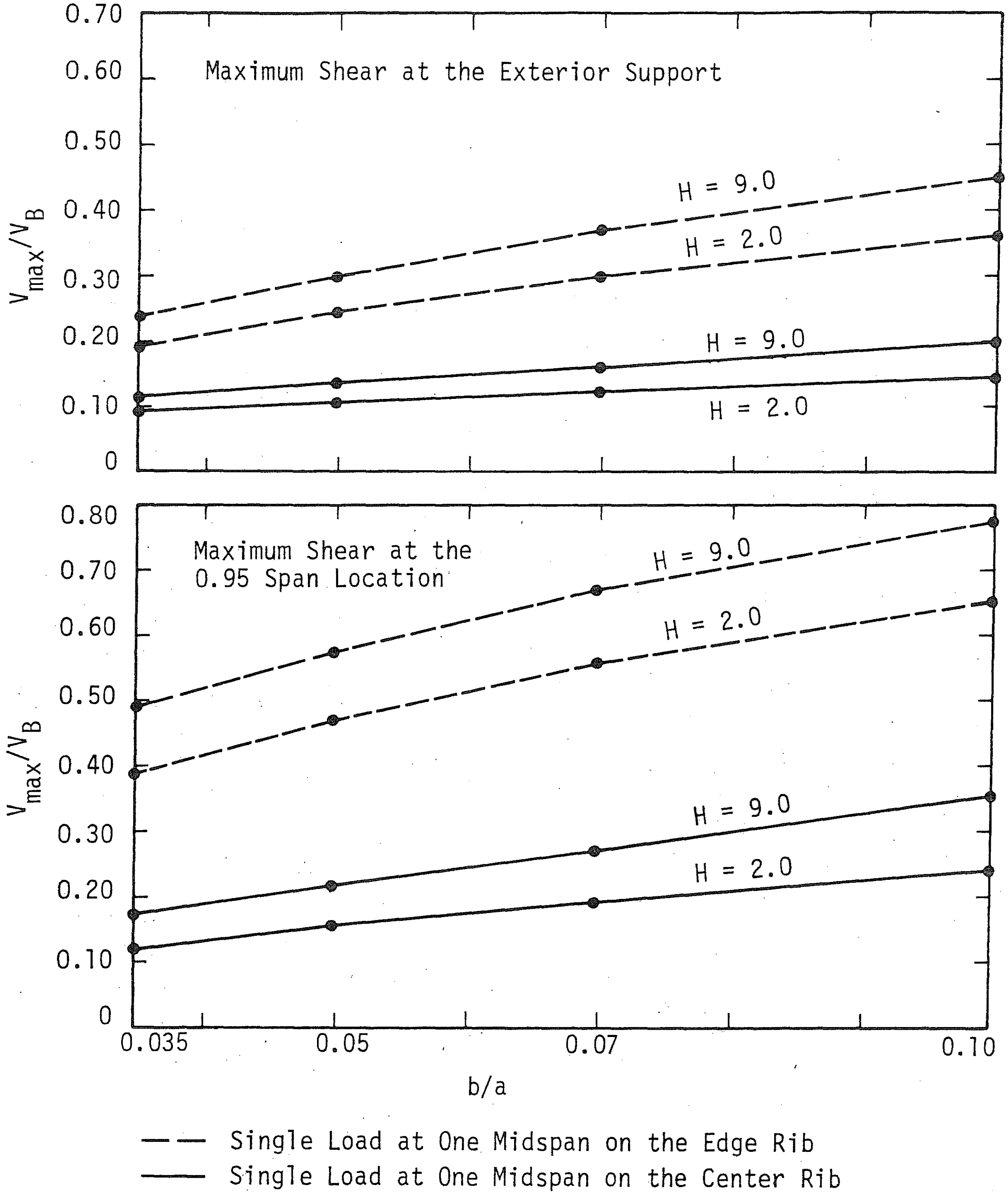


FIG. 3.26 VARIATIONS OF MAXIMUM SHEAR AT THE EXTERIOR SUPPORT AND 0.95 SPAN LOCATION WITH H AND b/a FOR TWO-SPAN RIBBED SLABS UNDER SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

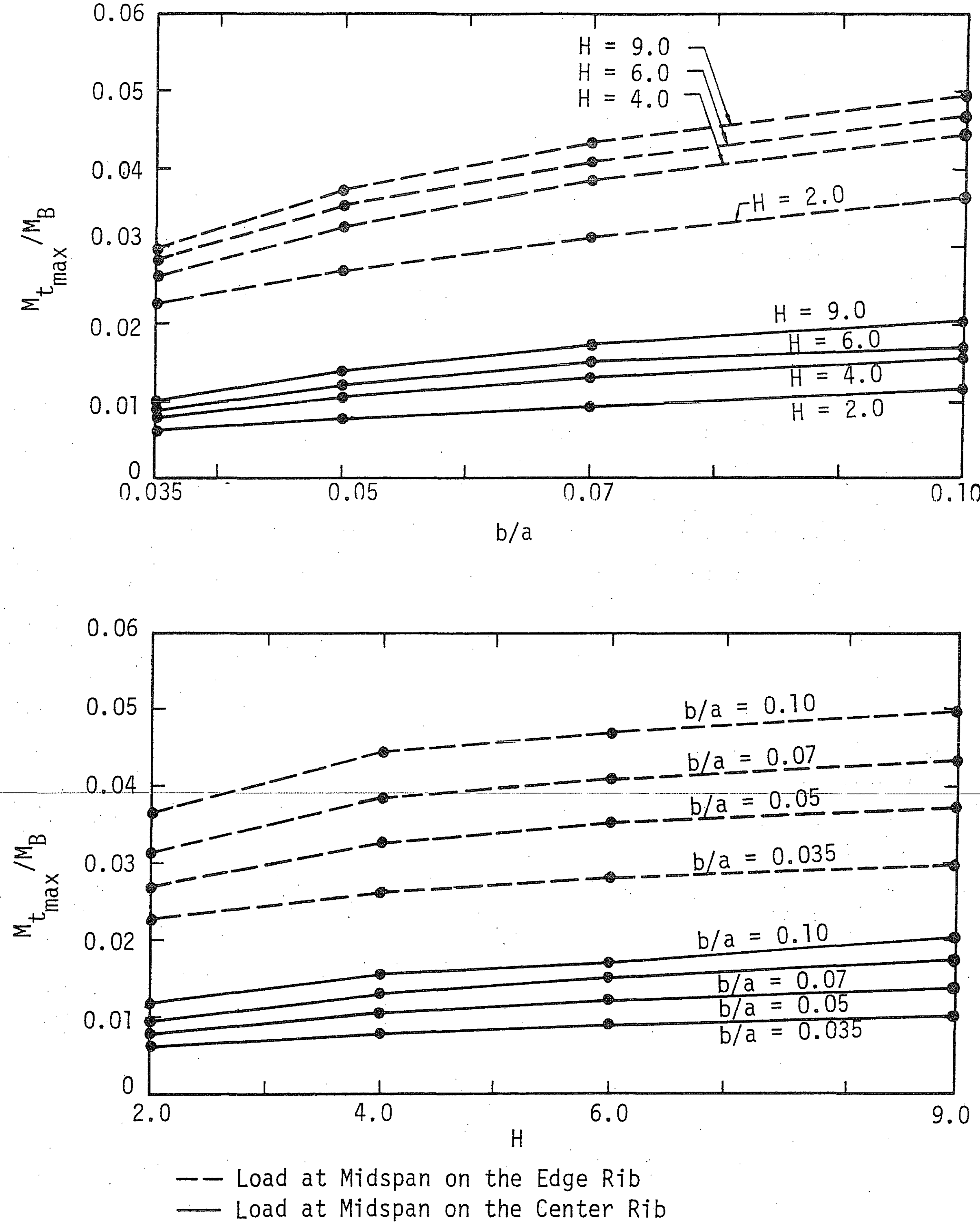
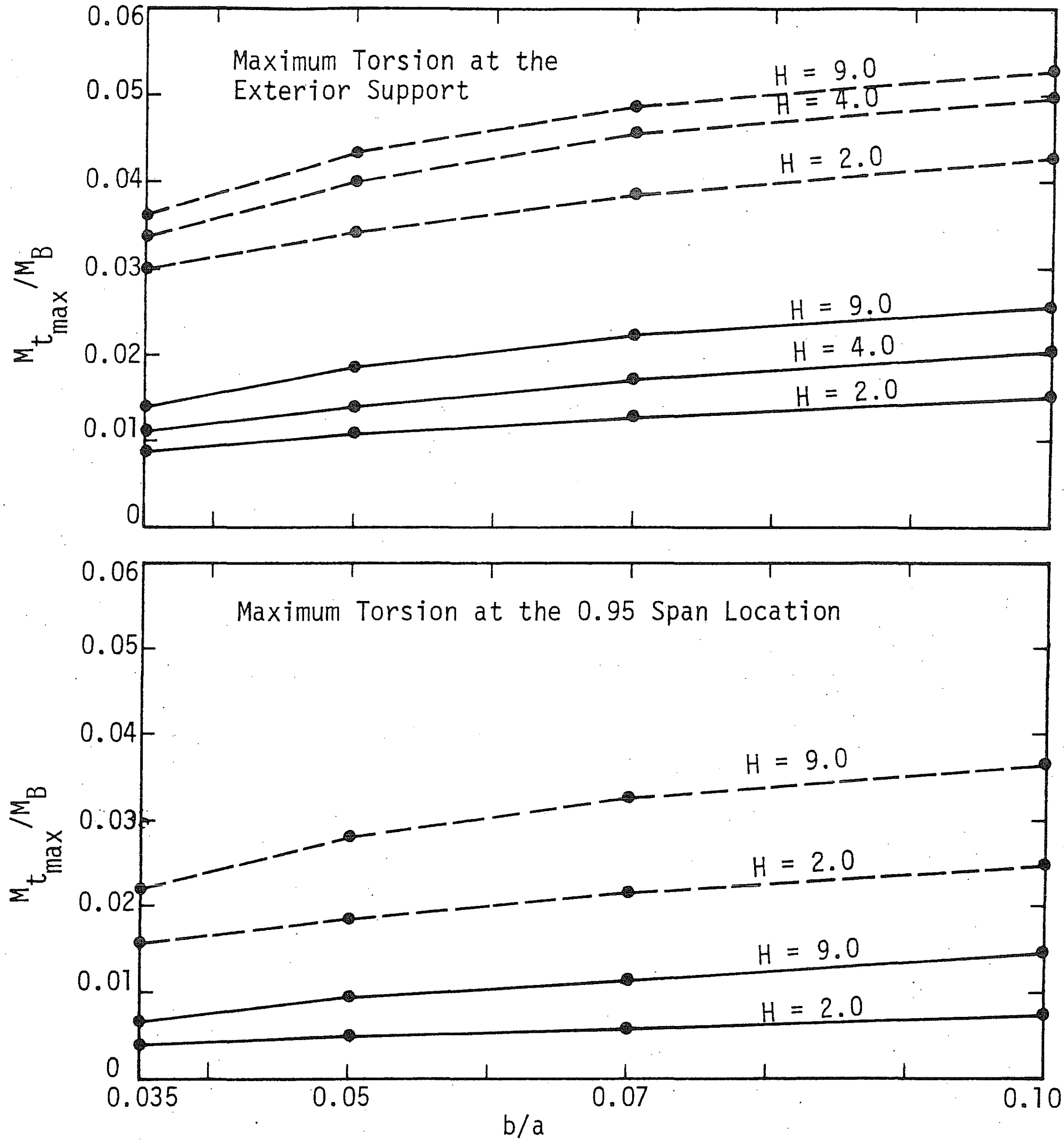


FIG. 3.27 VARIATIONS OF MAXIMUM TORSION AT THE SUPPORT WITH H AND b/a FOR ONE-SPAN RIBBED SLABS UNDER SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS



— — Load at Midspan on the Edge Rib

—— Load at Midspan on the Center Rib

FIG. 3.28 VARIATIONS OF MAXIMUM TORSION AT THE EXTERIOR SUPPORT AND 0.95 SPAN LOCATION WITH H AND b/a FOR TWO-SPAN RIBBED SLABS UNDER SINGLE LOADS AT MIDSPAN ON THE EDGE AND CENTER RIBS

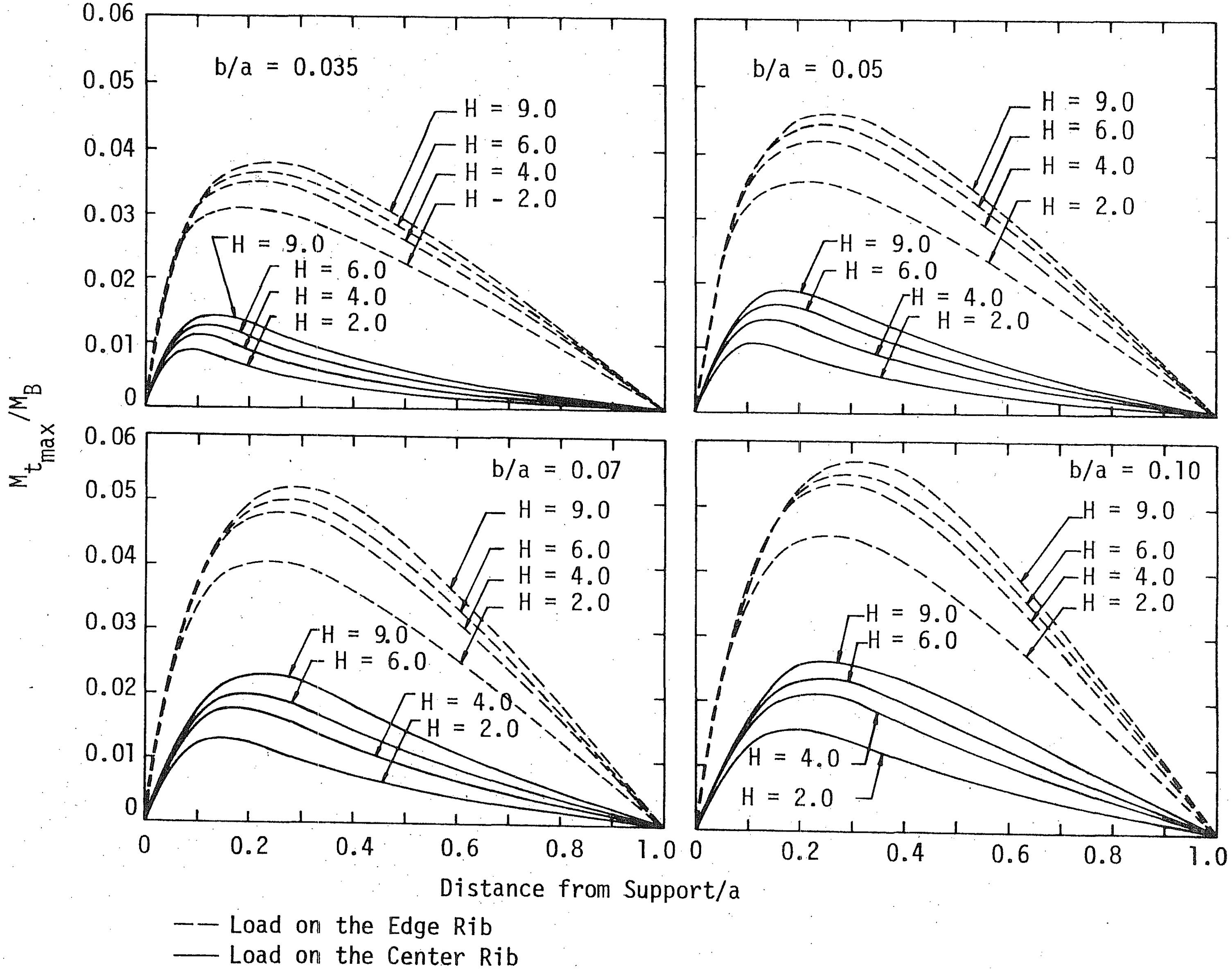


FIG. 3.29 INFLUENCE LINES FOR TORSION AT THE SUPPORT OF VARIOUS ONE-SPAN RIBBED SLABS DUE TO A LOAD MOVING ON THE EDGE AND CENTER RIBS

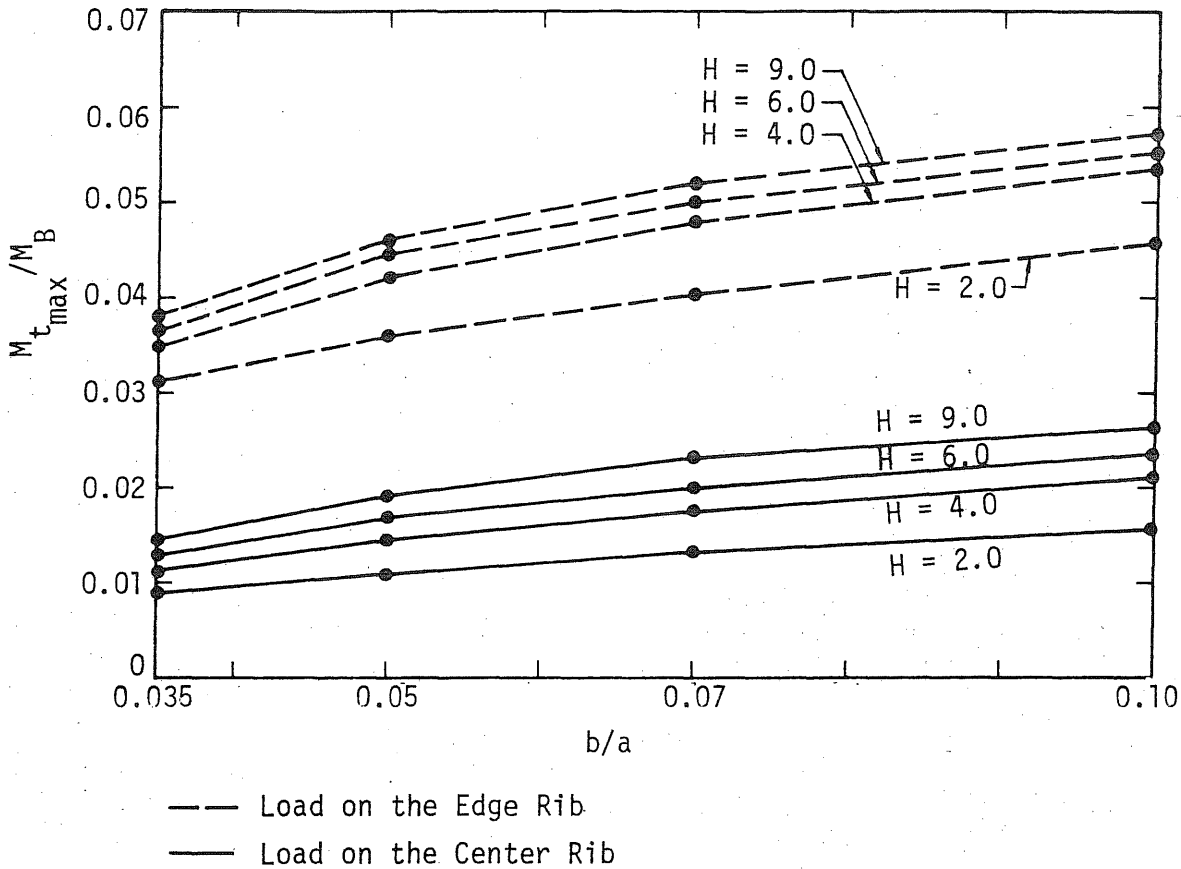
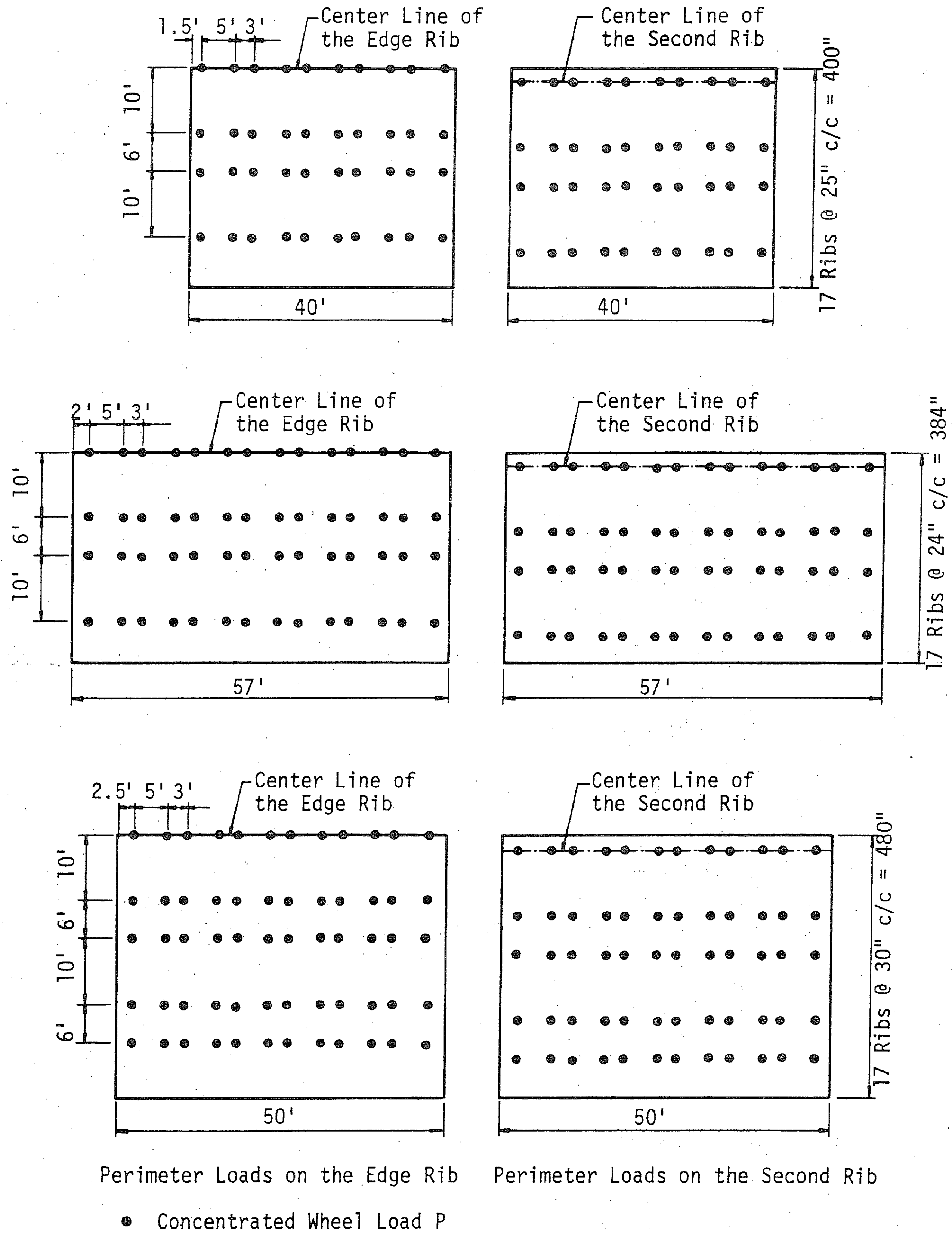
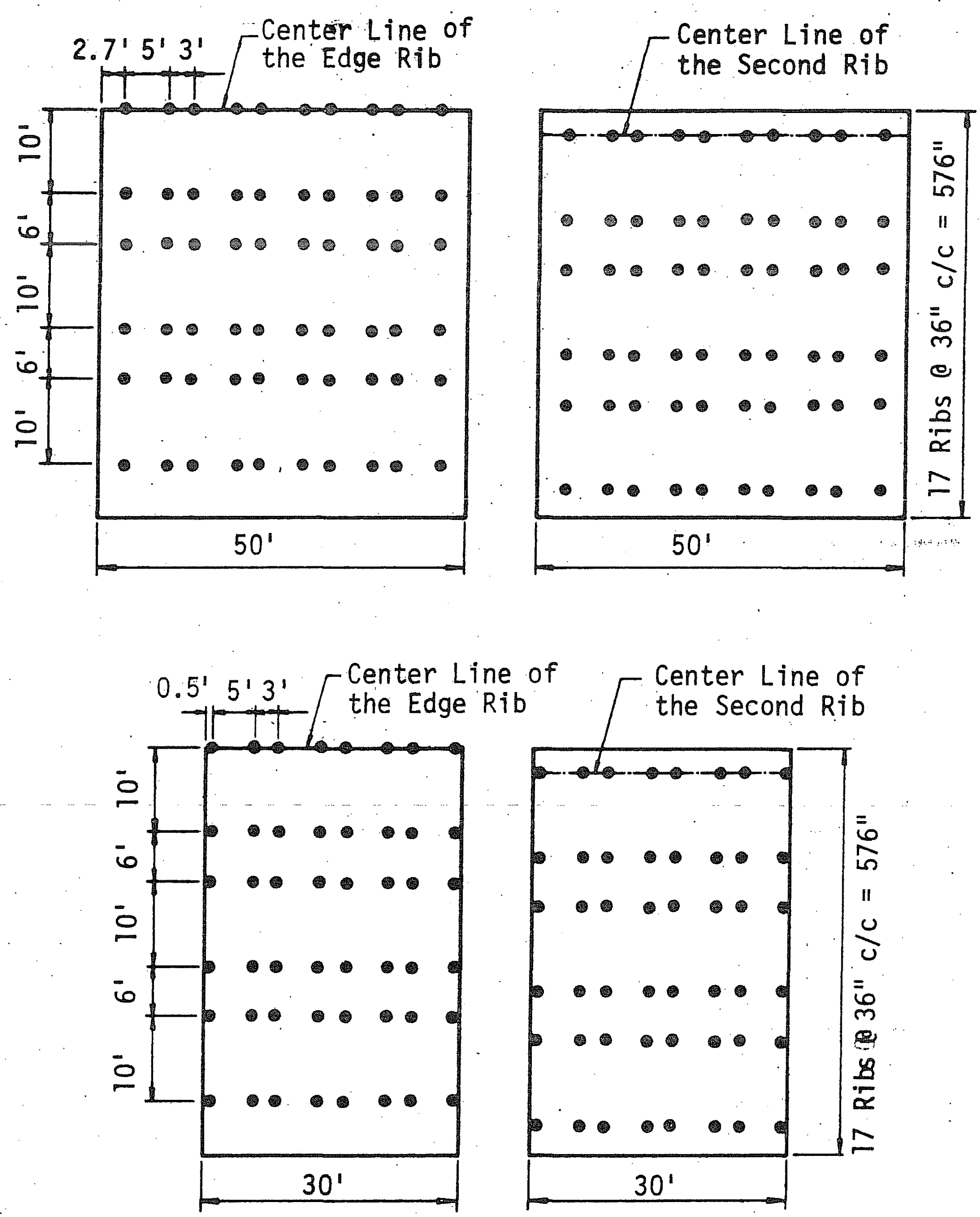


FIG. 3.30 VARIATIONS OF MAXIMUM TORSION AT THE SUPPORT WITH H AND b/a FOR ONE-SPAN RIBBED SLABS UNDER SINGLE LOADS ON THE EDGE AND CENTER RIBS



(a) Multiple Loads for Maximum Moment on One-Span Ribbed Slabs; for Two-Span, These are the Load Patterns on One-Span for Maximum Positive Moment, and on Both Spans the Patterns are for Maximum Negative Moment

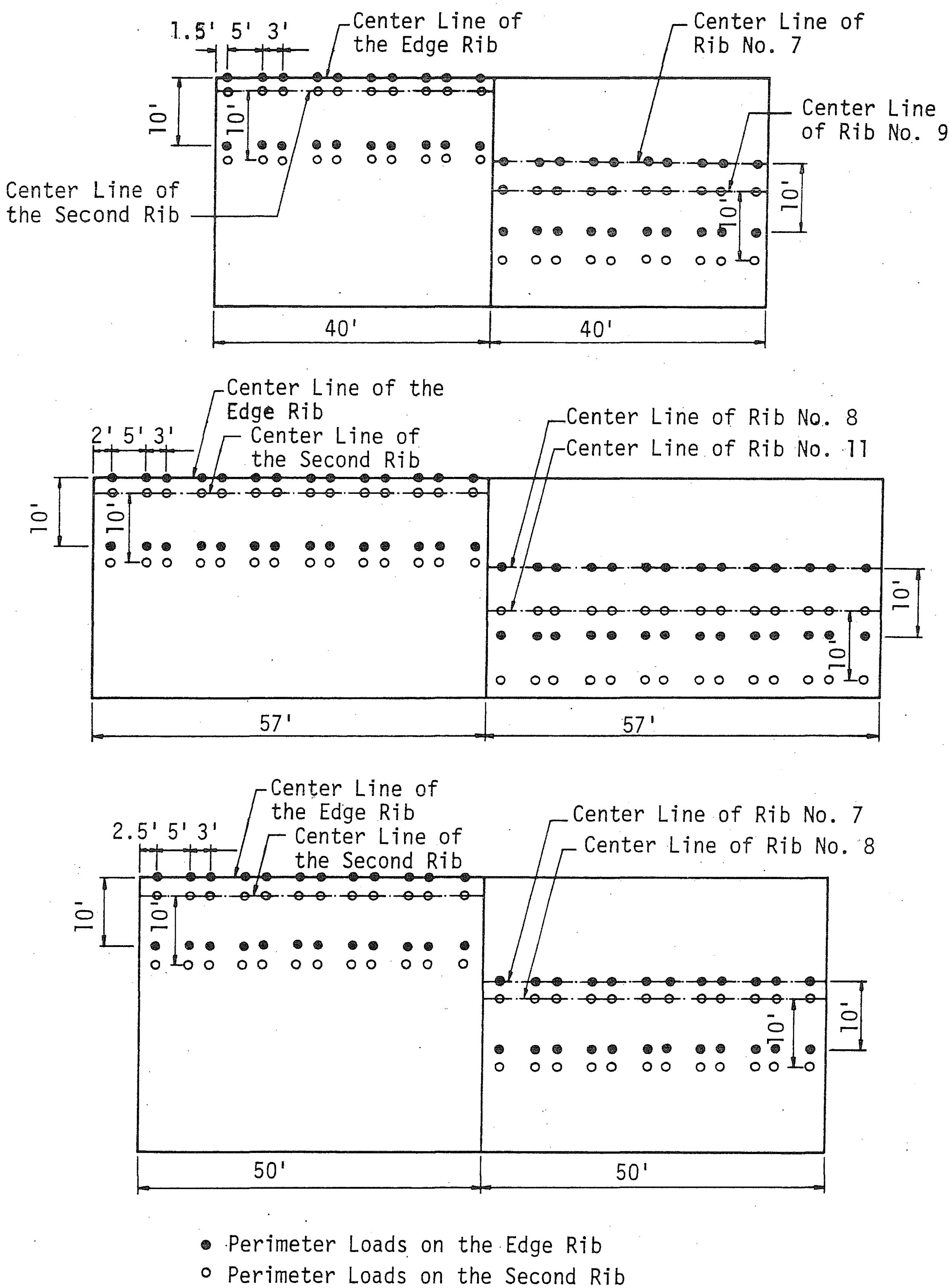
FIG. 4.1 LOADING SYSTEMS FOR MAXIMUM BENDING MOMENTS, SHEARS, AND TORSIONAL MOMENTS



Perimeter Loads on the Edge Rib

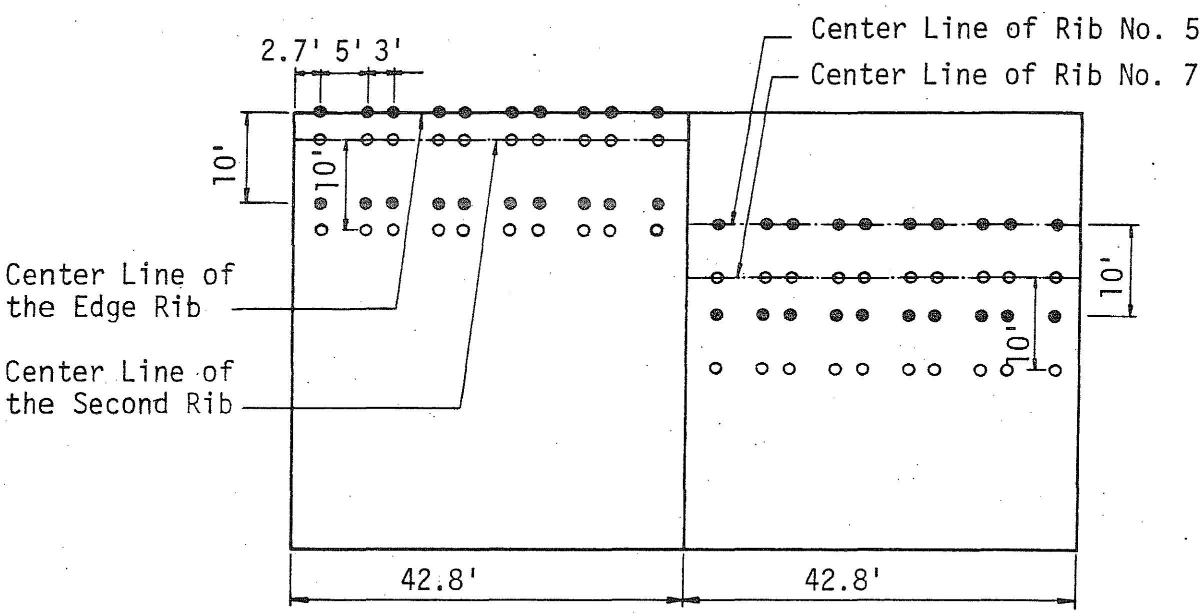
Perimeter Loads on the Second Rib

FIG. 4.1 (CONT.)



(b) Multiple Loads for Maximum Torsion on Two-Span Ribbed Slabs, and the Left Span Taken Separately Shows the Load Patterns for Maximum Torsion on One-Span Structures

FIG. 4.1 (CONT.)


Center Line of Rib No. 5
Center Line of Rib No. 7
2.7' 5' 3'
10'
10'
Center Line of the Edge Rib
Center Line of the Second Rib
10'
10'
42.8'
42.8'

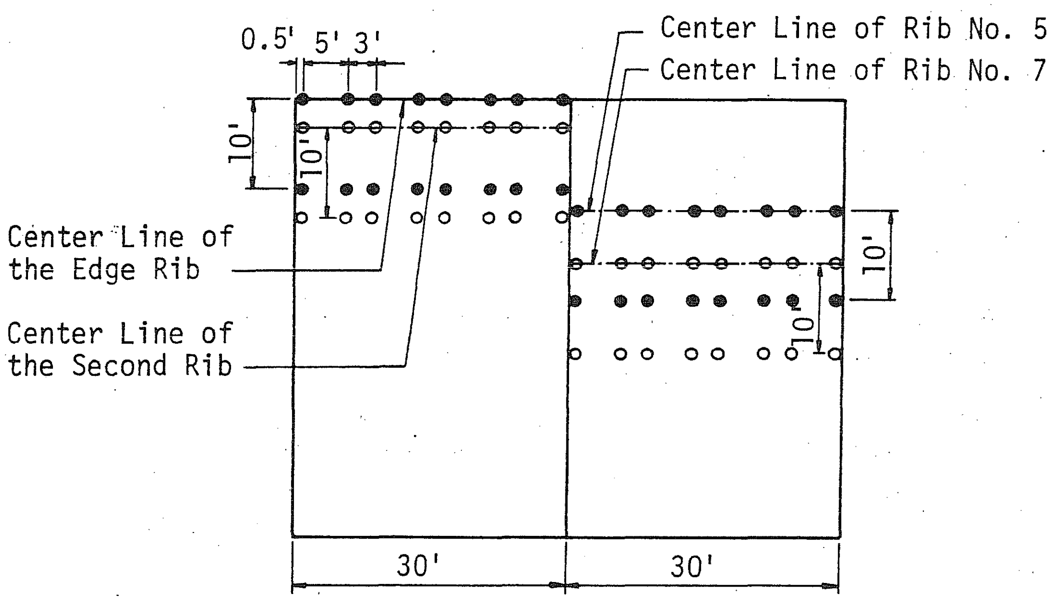

Center Line of Rib No. 5
Center Line of Rib No. 7
0.5' 5' 3'
10'
10'
Center Line of the Edge Rib
Center Line of the Second Rib
10'
10'
30'
30'

FIG. 4.1 (CONT.)

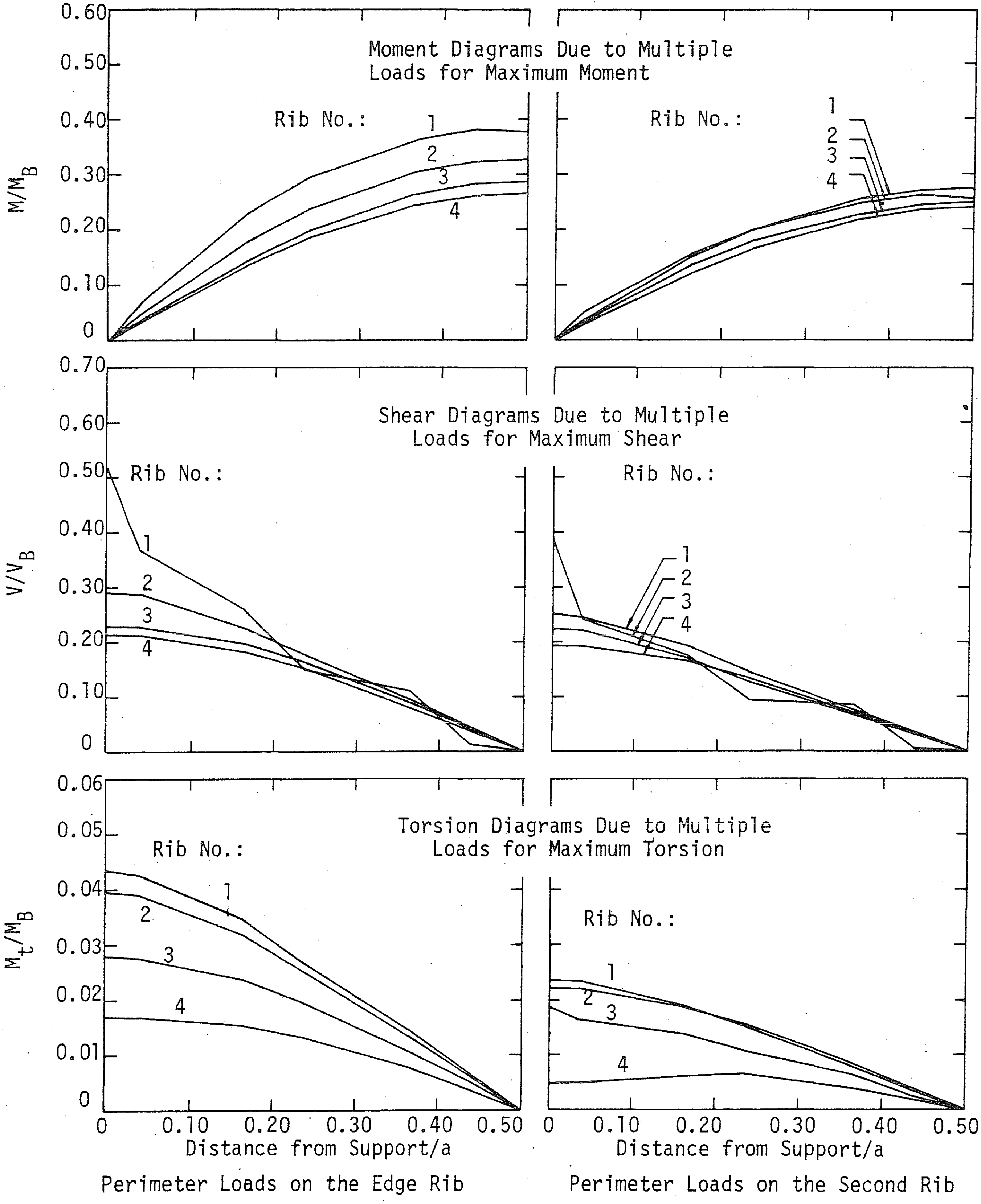
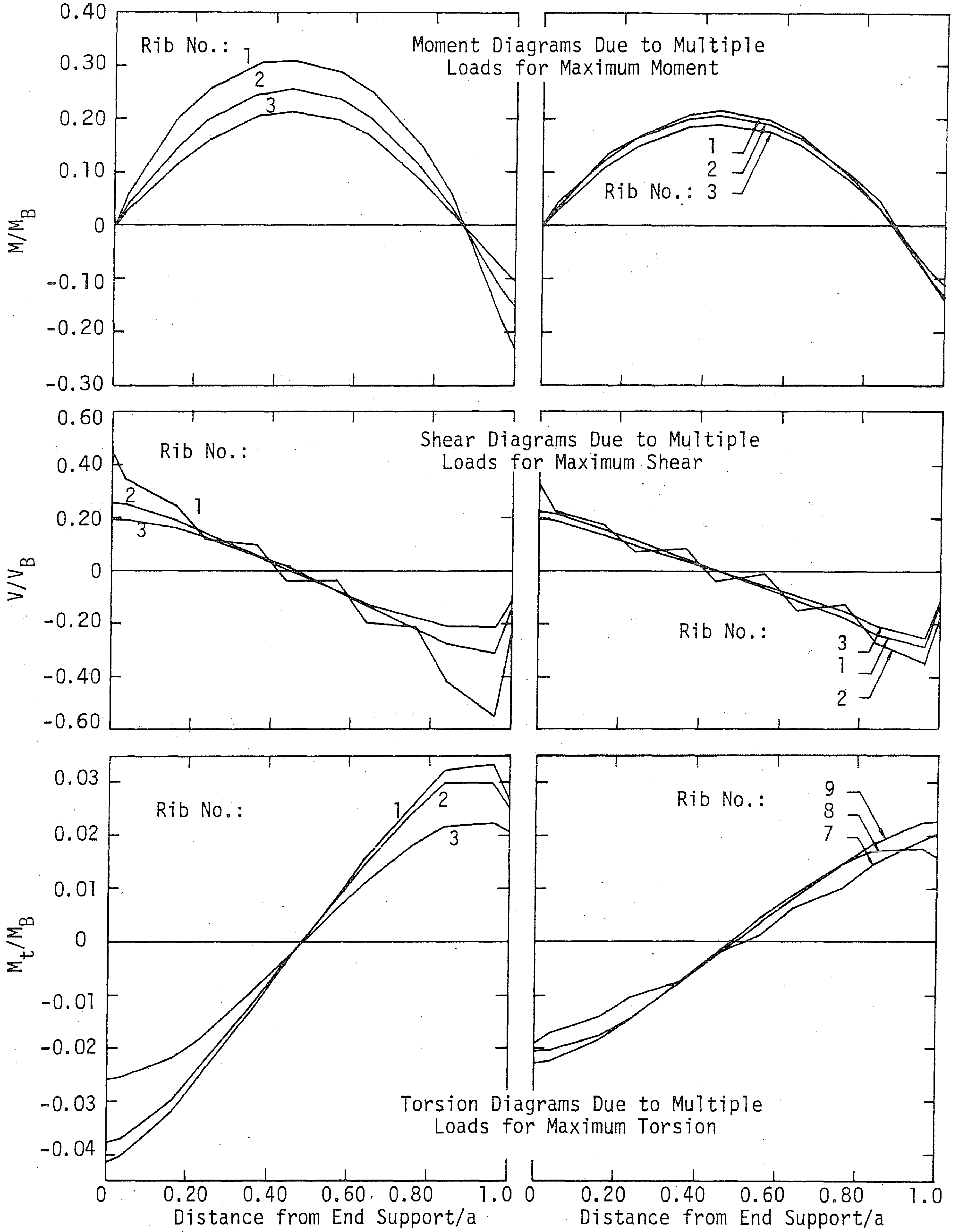


FIG. 4.2 BENDING MOMENT, SHEAR, AND TORSIONAL MOMENT DIAGRAMS OF A ONE-SPAN RIBBED SLAB DUE TO MULTIPLE LOADS



Perimeter Loads on the Edge Rib   Perimeter Loads on the Second Rib

FIG. 4.3 BENDING MOMENT, SHEAR, AND TORSIONAL MOMENT DIAGRAMS OF A TWO-SPAN RIBBED SLAB DUE TO MULTIPLE LOADS

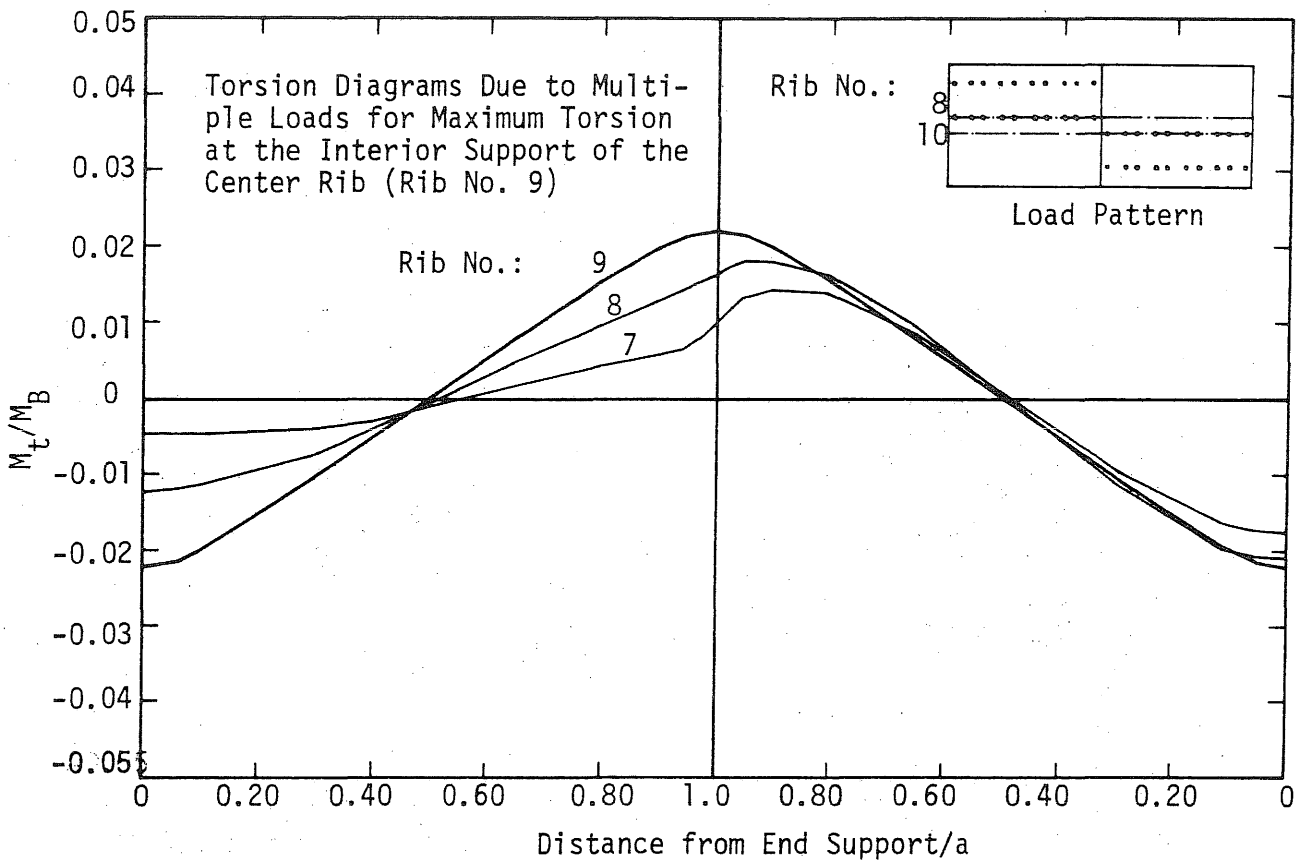

Torsion Diagrams Due to Multiple Loads for Maximum Torsion at the Interior Support of the Center Rib (Rib No. 9)
Rib No.:
8
10
Load Pattern
Rib No.: 9
8
7
M_t/M_B
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
0.20
0.40
0.60
0.80
1.0
0.80
0.60
0.40
0.20
0
Distance from End Support/a

FIG. 4.3 (CONT.)

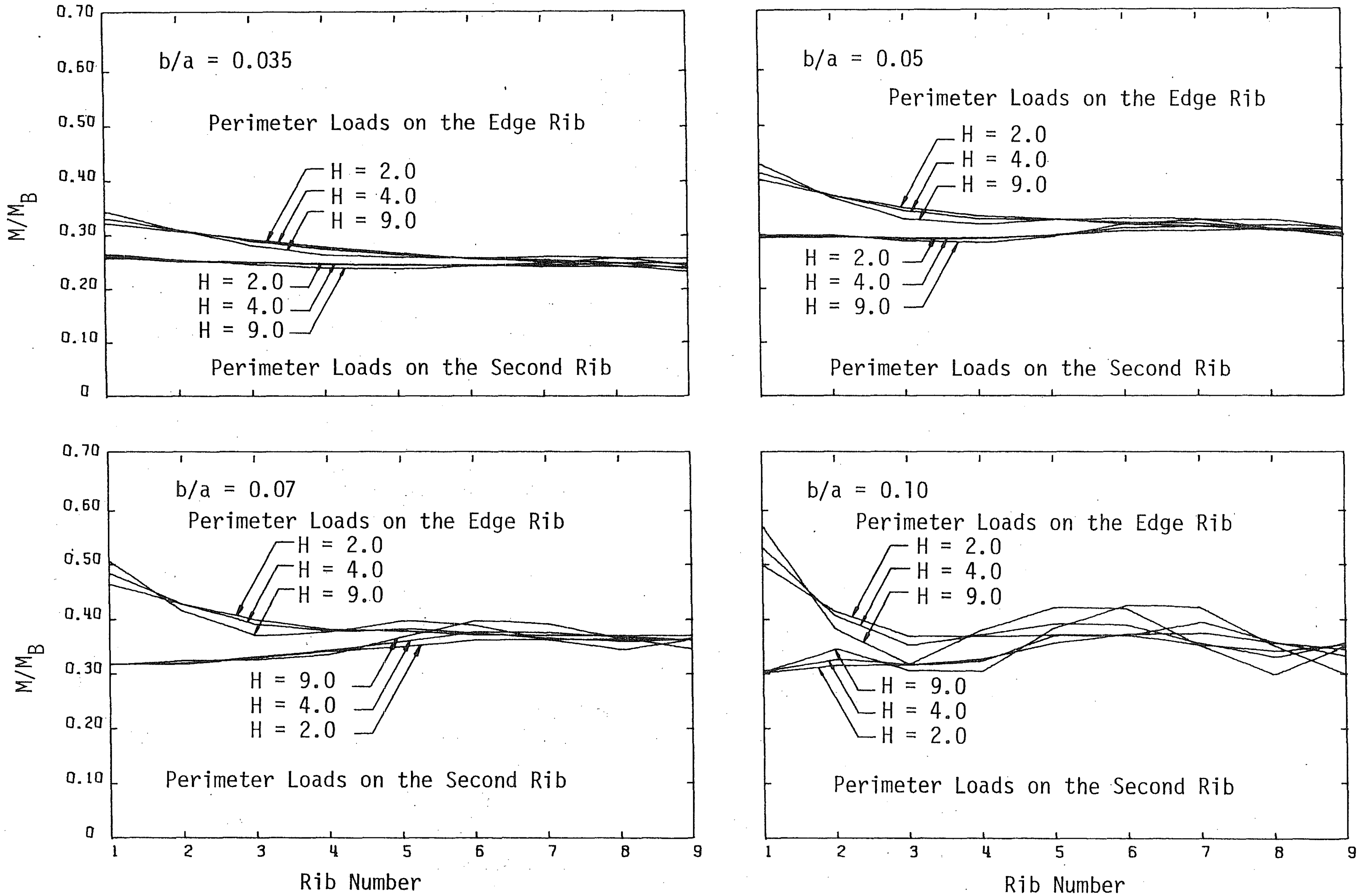


FIG. 4.4 MOMENT DISTRIBUTIONS AT MIDSPAN OF ONE-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM MOMENT

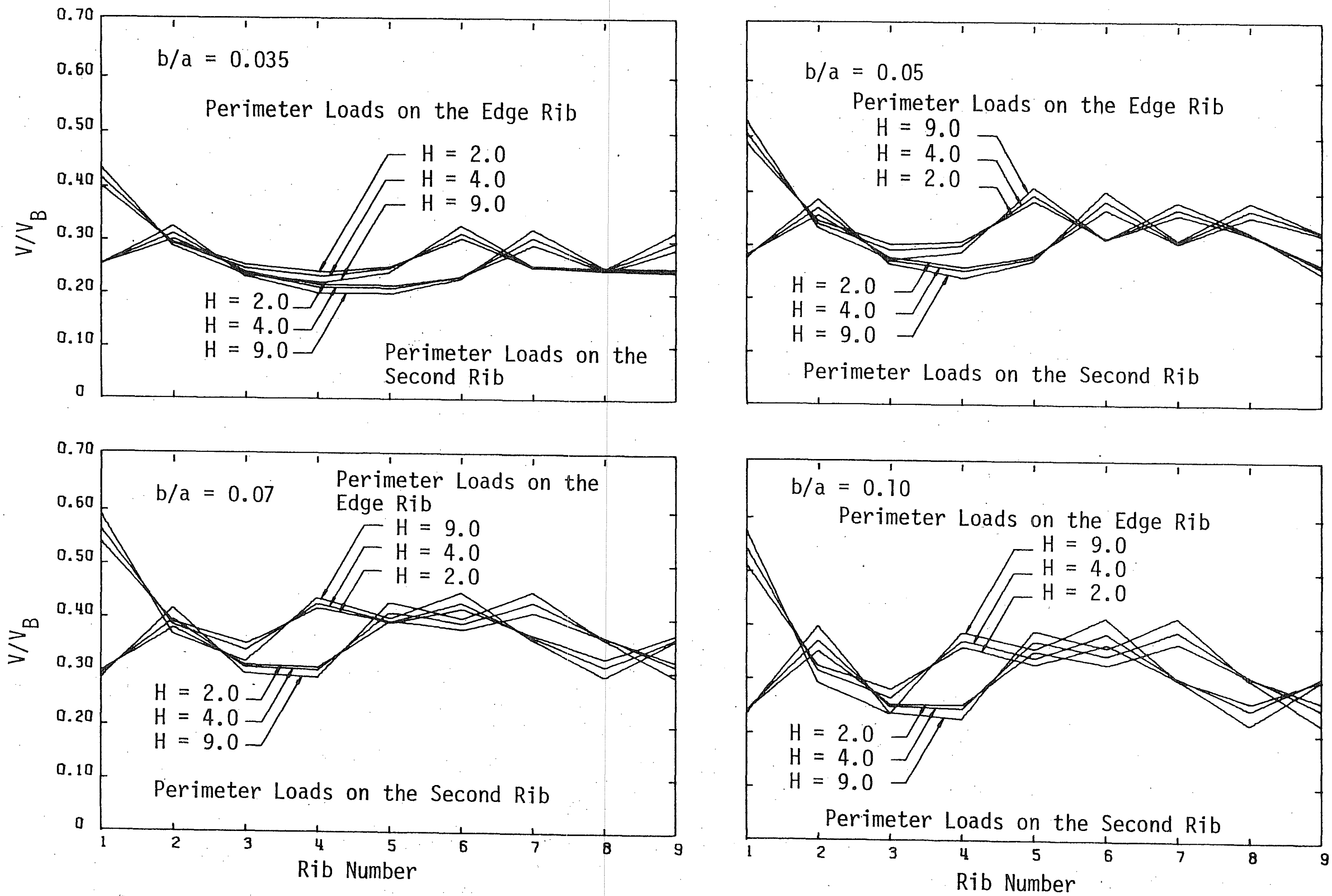


FIG. 4.5 SHEAR DISTRIBUTIONS AT THE SUPPORT OF ONE-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM MOMENT

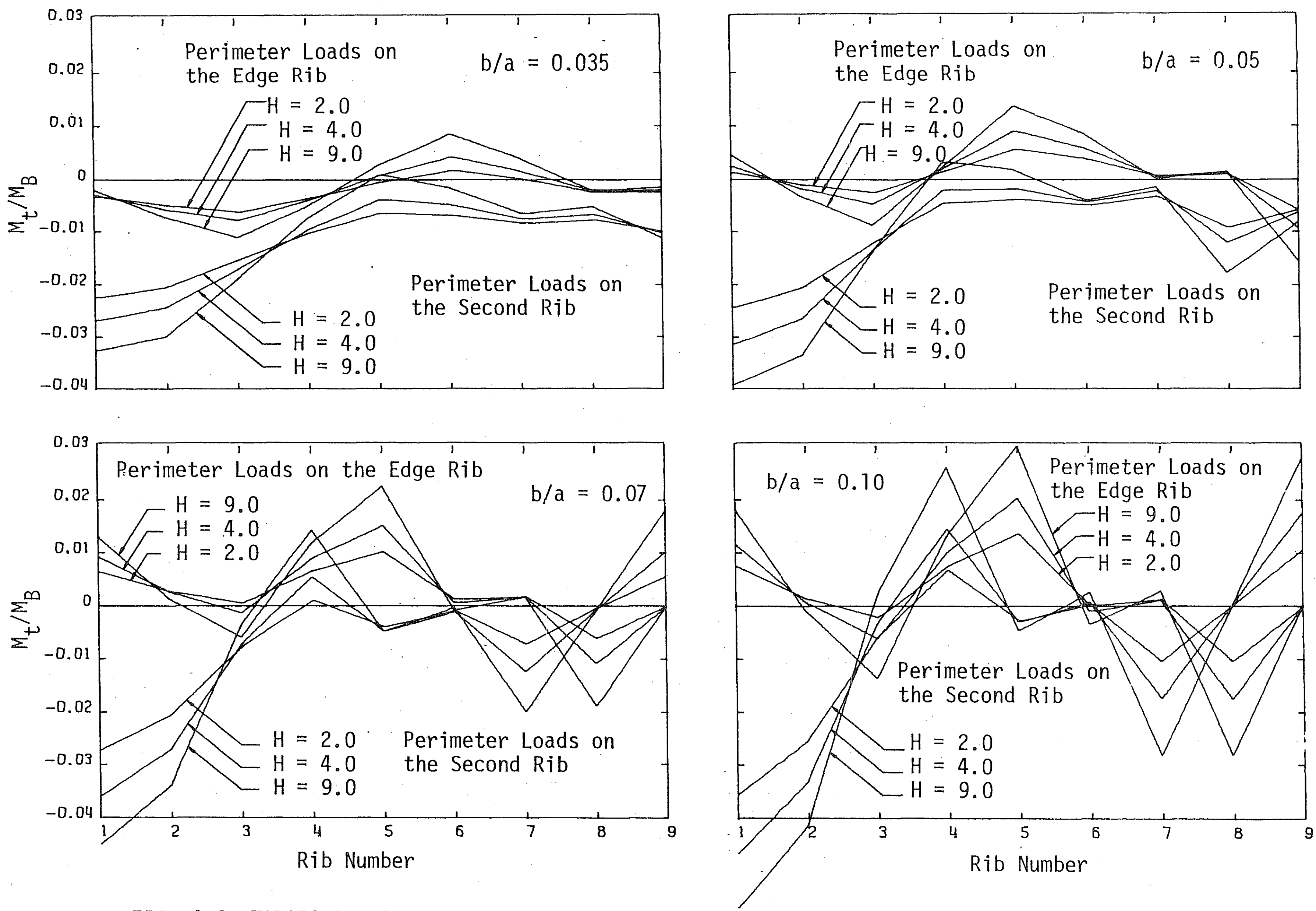


FIG. 4.6 TORSIONAL MOMENT DISTRIBUTIONS AT THE SUPPORT OF ONE-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM MOMENT

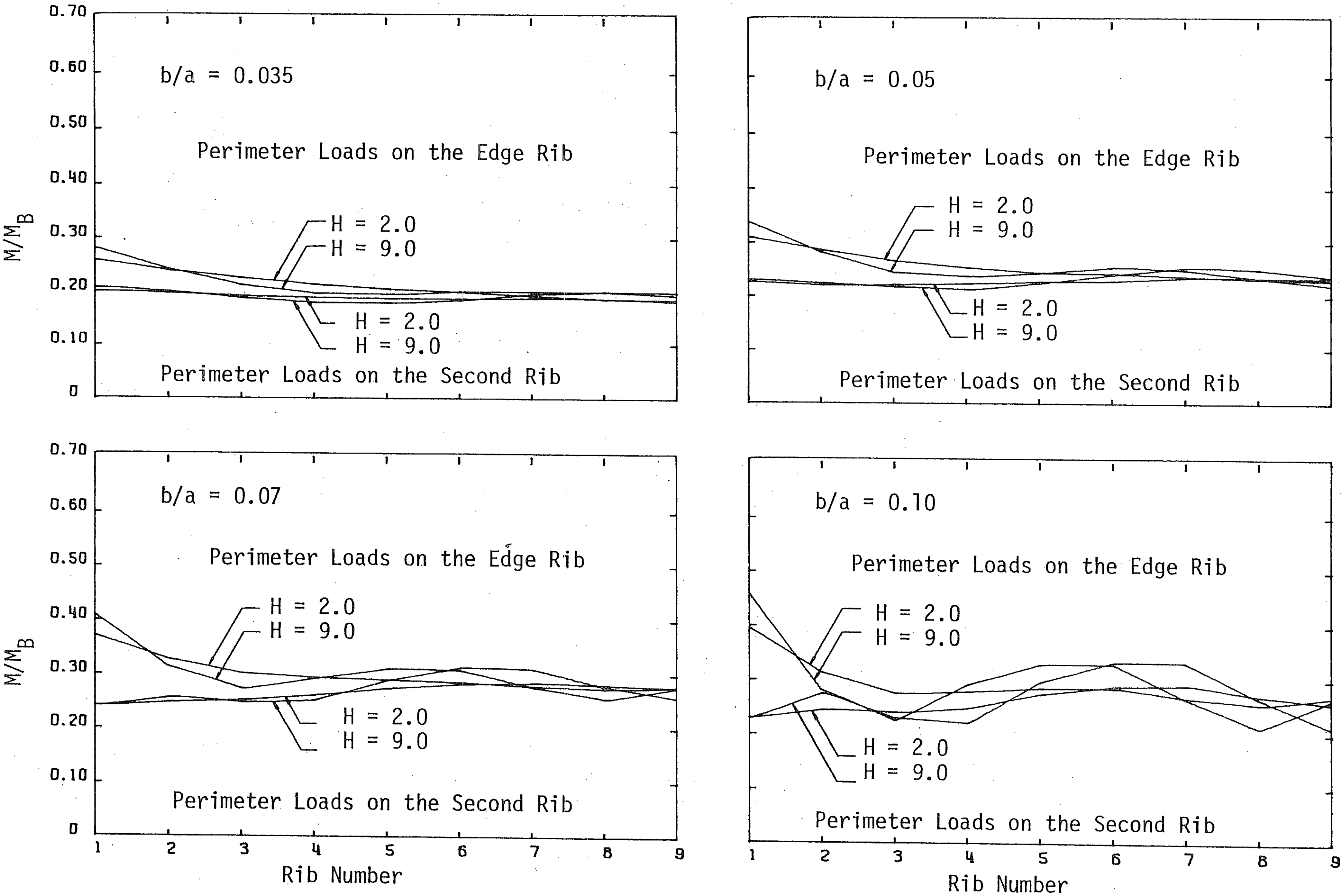


FIG. 4.7 POSITIVE MOMENT DISTRIBUTIONS AT THE 0.44 SPAN LOCATION OF TWO-RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM POSITIVE MOMENT

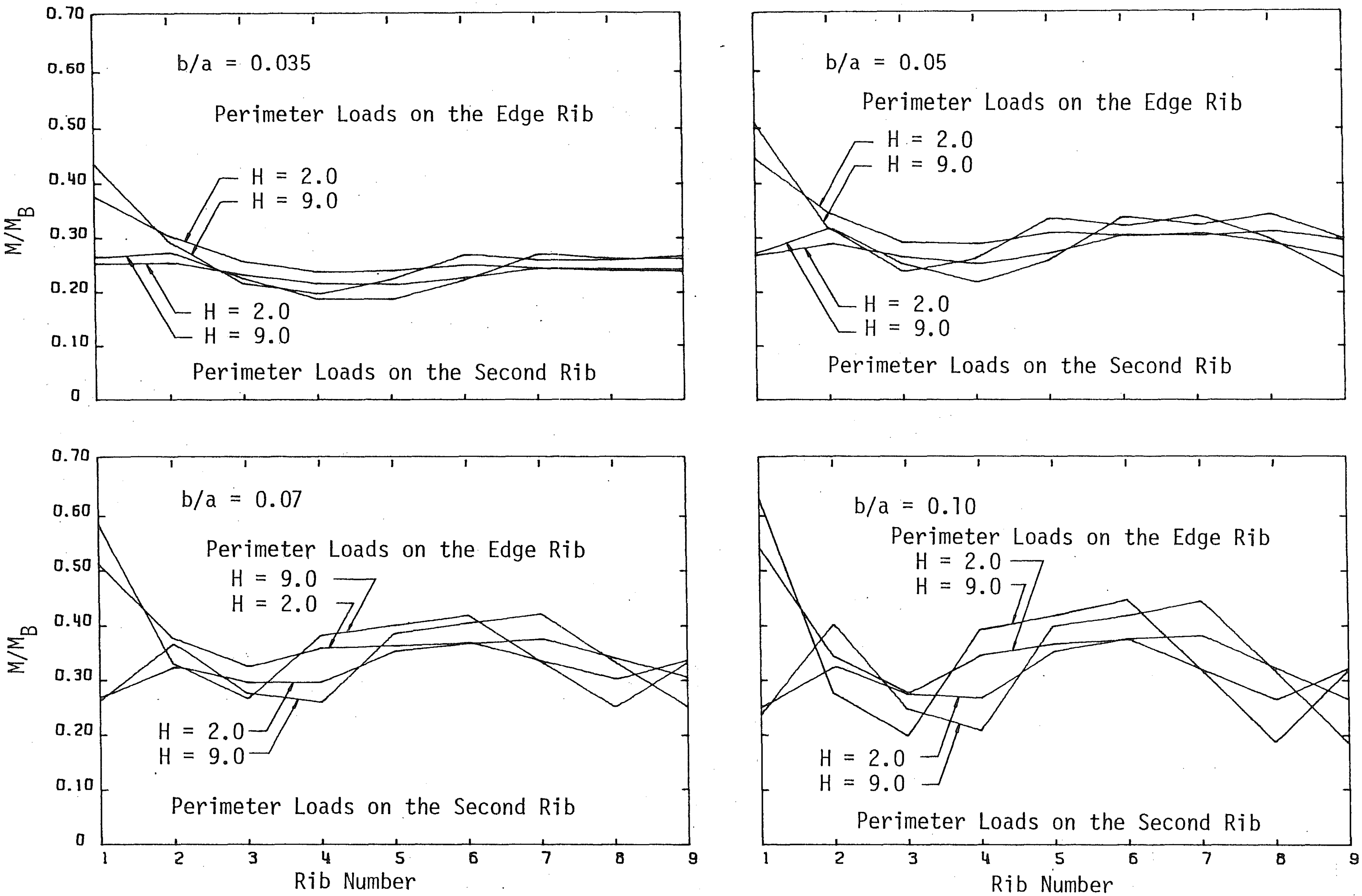


FIG. 4.8 NEGATIVE MOMENT DISTRIBUTIONS AT THE INTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM NEGATIVE MOMENT

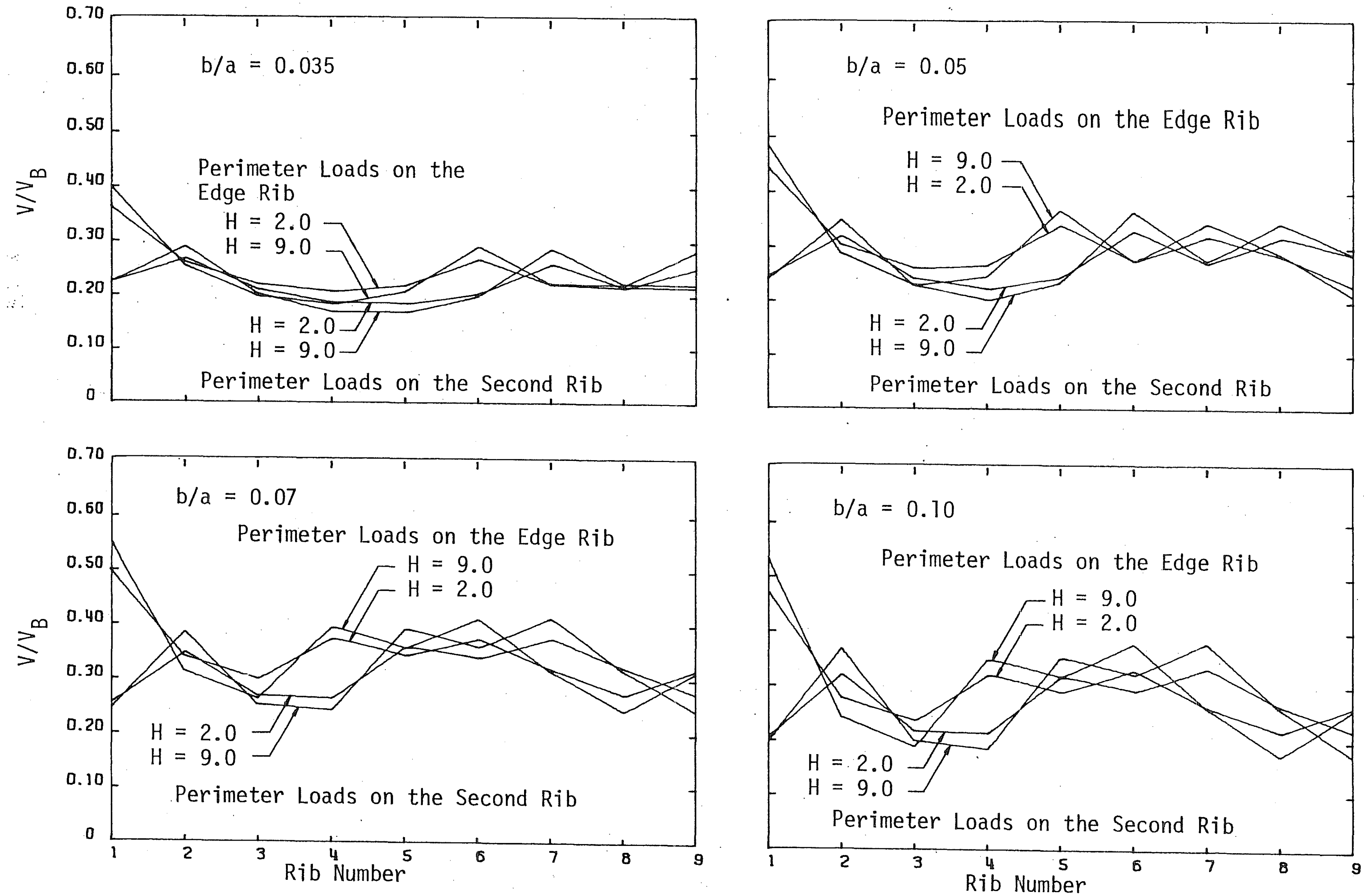


FIG. 4.9 SHEAR DISTRIBUTIONS AT THE EXTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM POSITIVE MOMENT

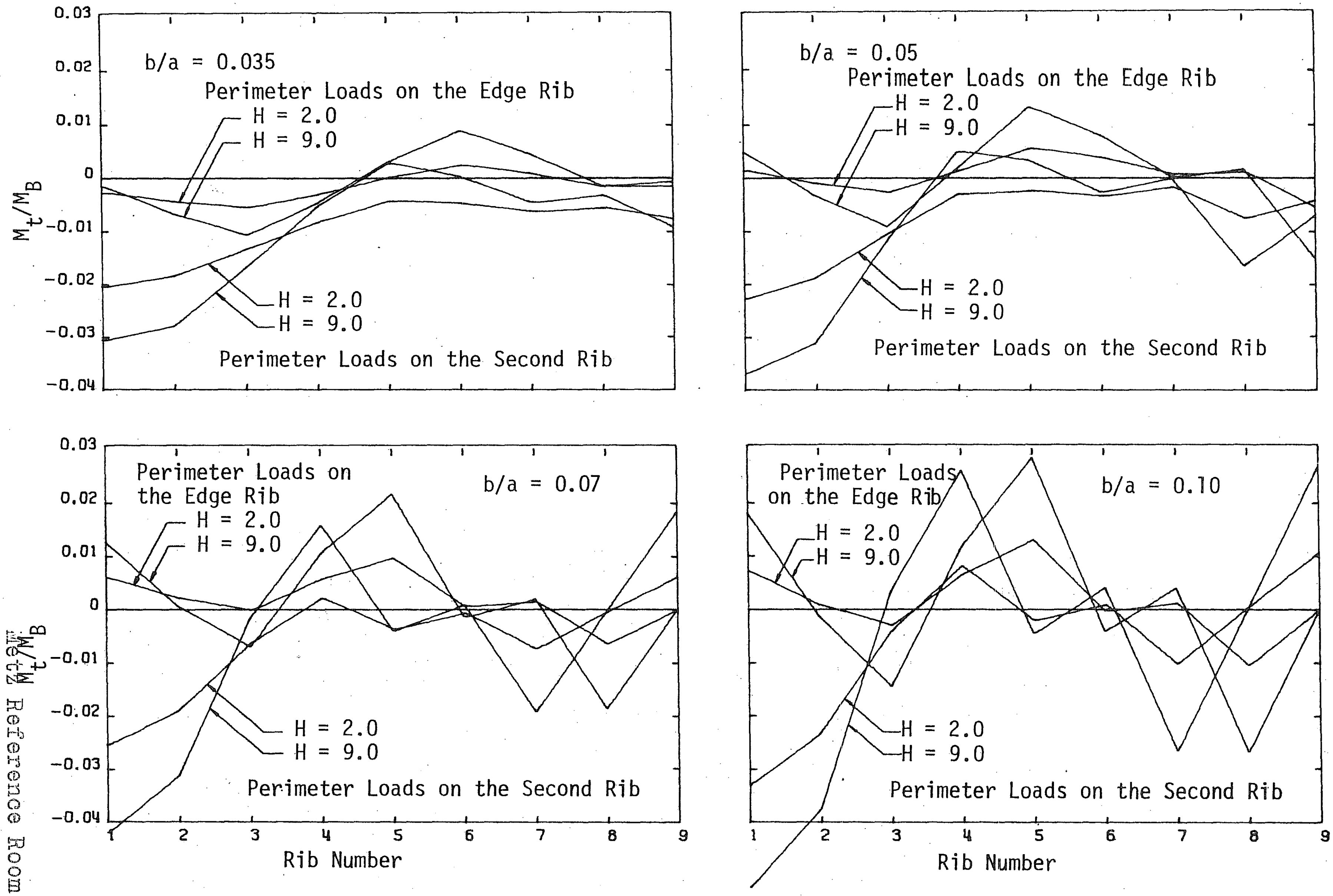


FIG. 4.10 TORSIONAL MOMENT DISTRIBUTIONS AT THE EXTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM POSITIVE MOMENT

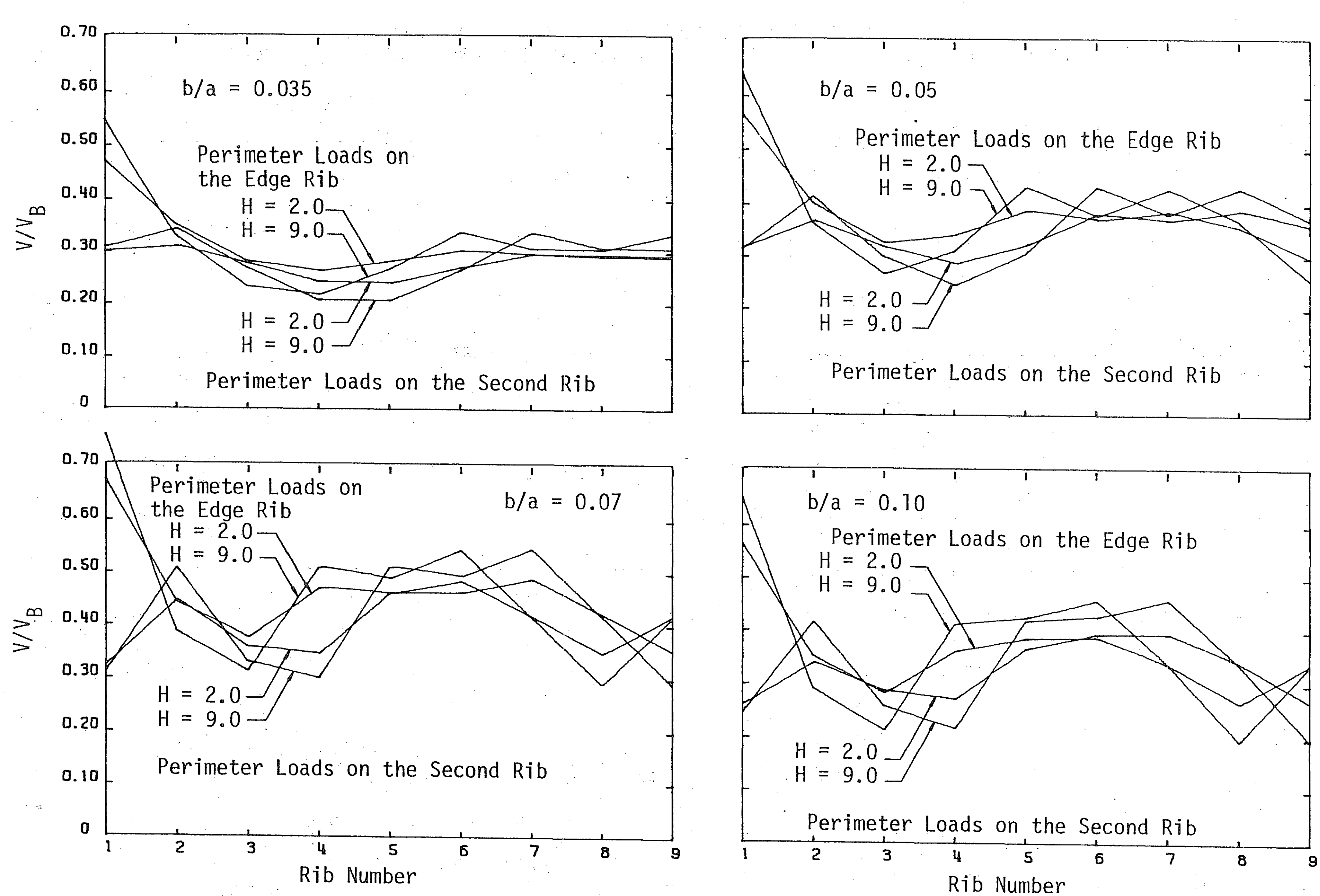


FIG. 4.11 SHEAR DISTRIBUTIONS AT THE 0.95 SPAN LOCATION OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM POSITIVE MOMENT

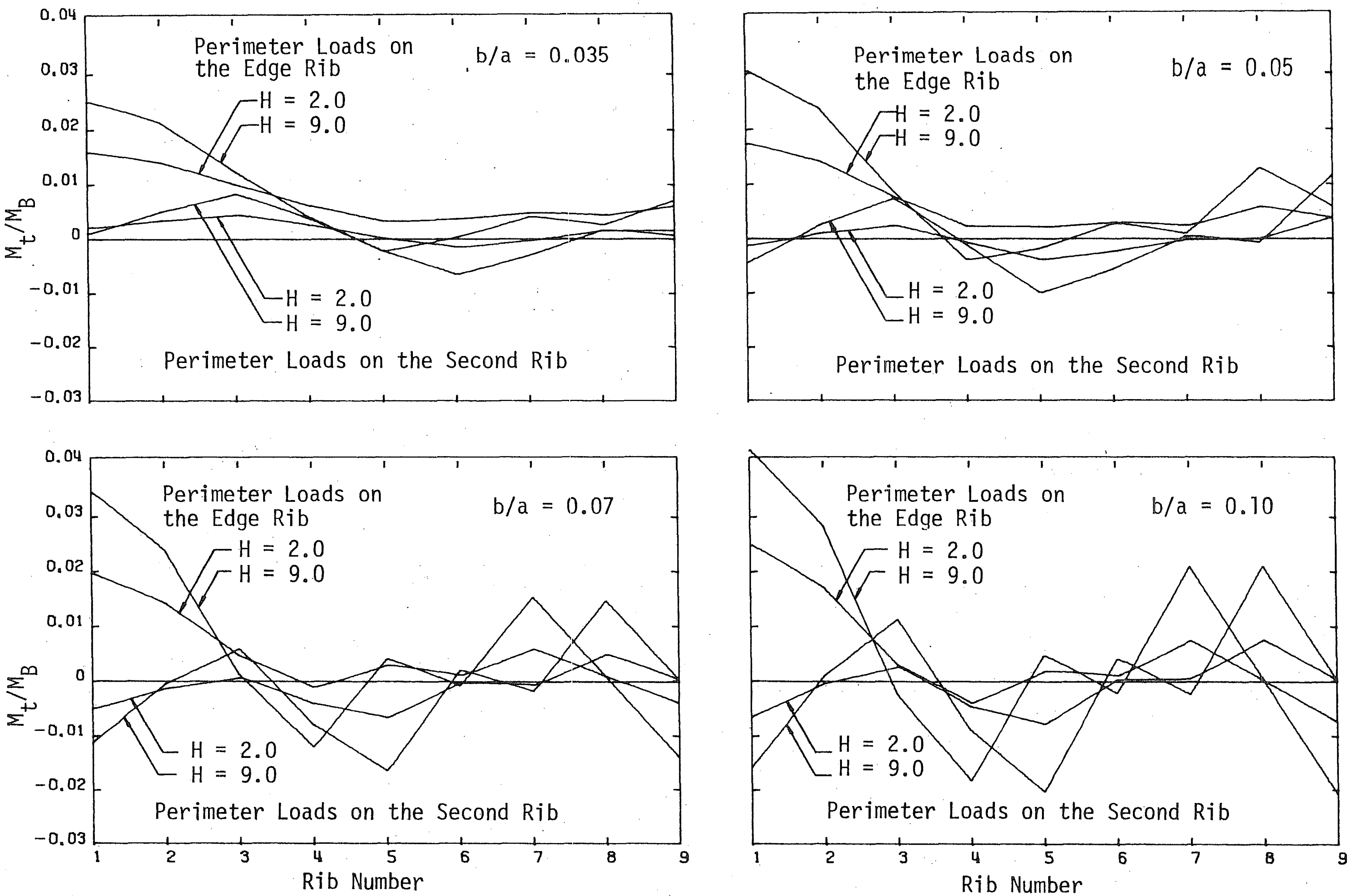


FIG. 4.12 TORSIONAL MOMENT DISTRIBUTIONS AT THE 0.95 SPAN LOCATIONS OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM POSITIVE MOMENT

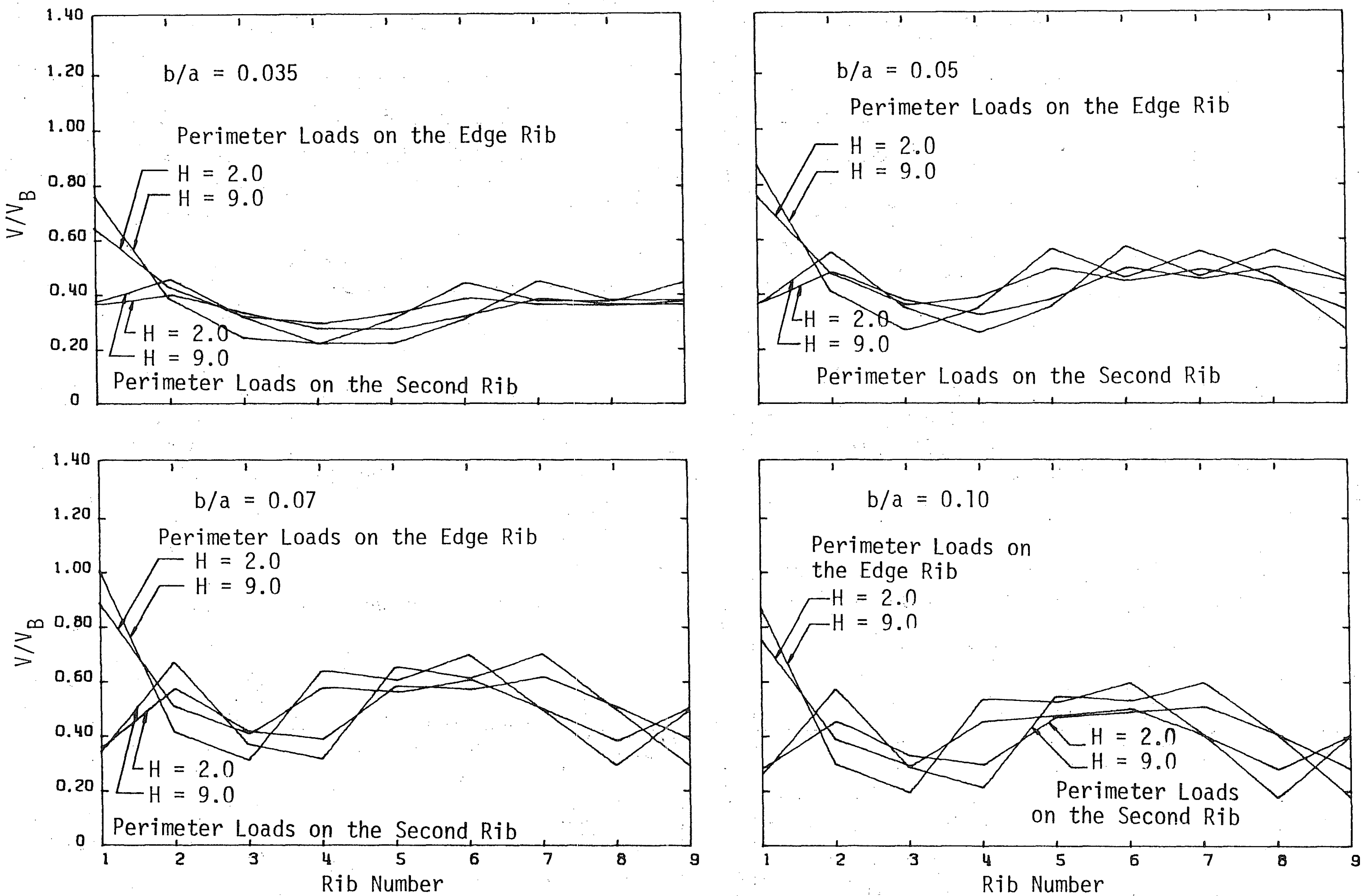


FIG. 4.13 SHEAR DISTRIBUTIONS AT THE 0.95 SPAN LOCATION OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM NEGATIVE MOMENT

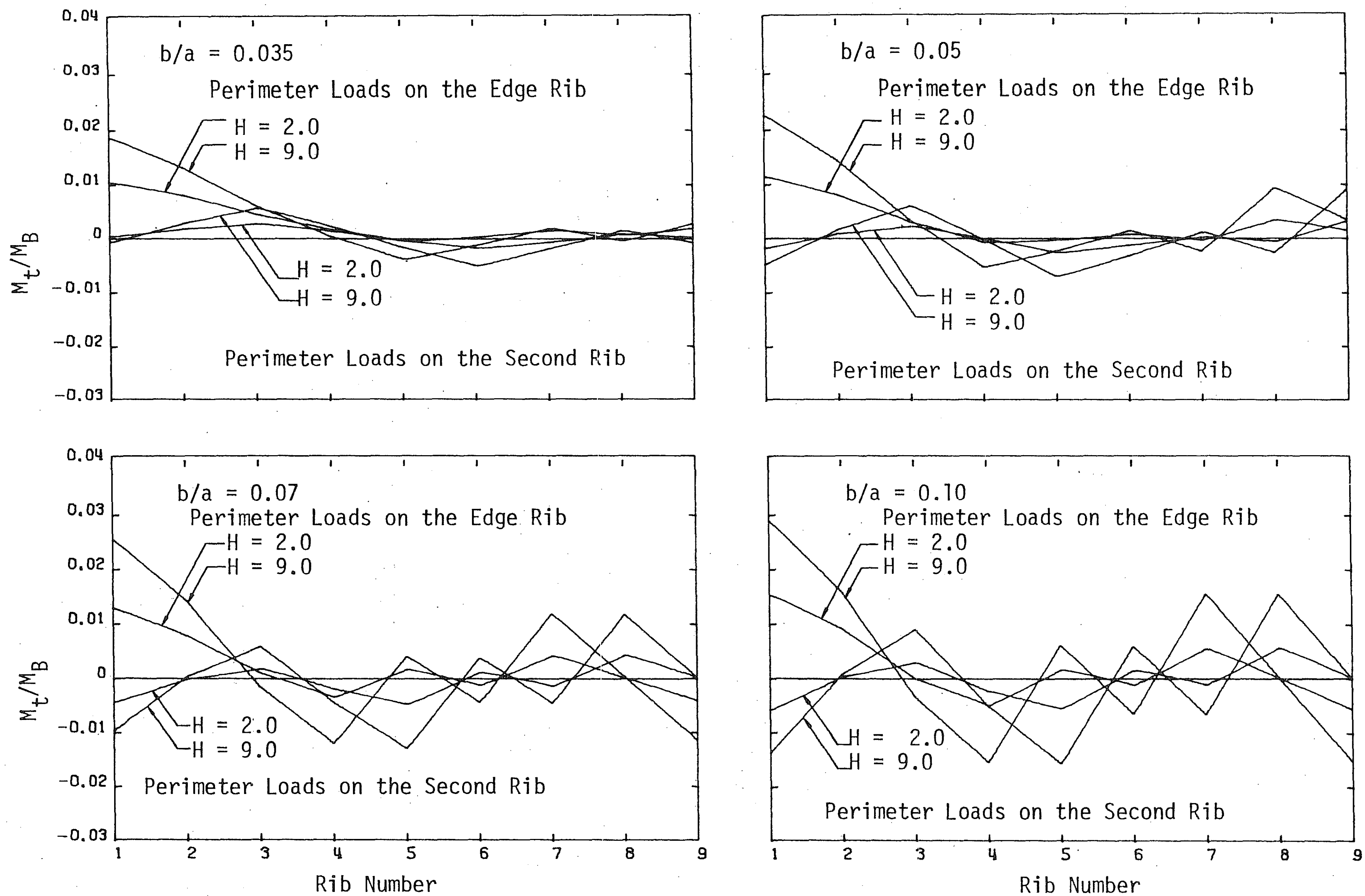
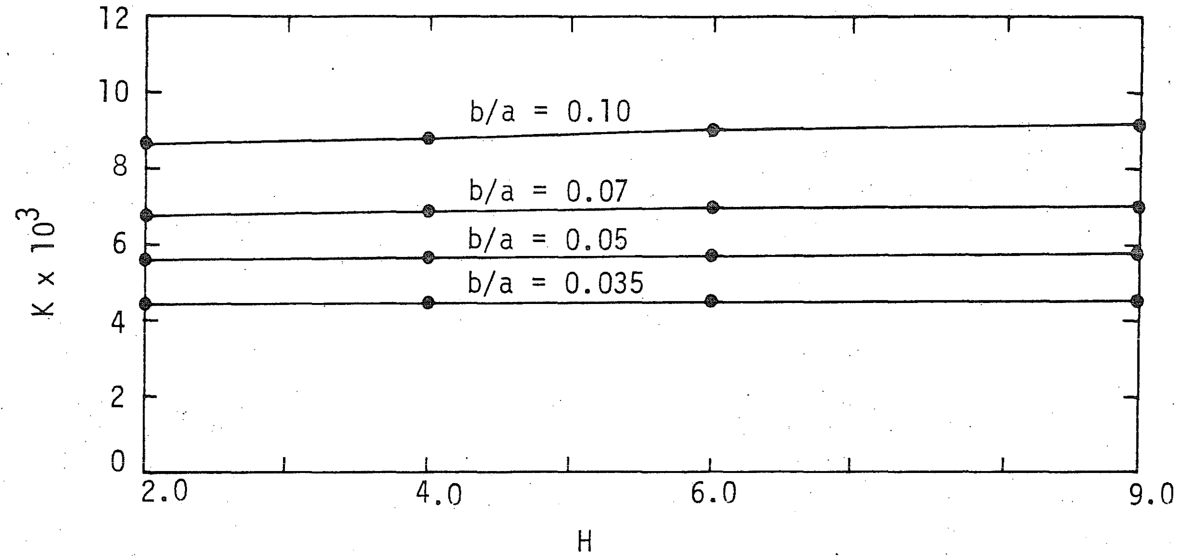


FIG. 4.14 TORSIONAL MOMENT DISTRIBUTIONS AT THE 0.95 SPAN LOCATION OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM NEGATIVE MOMENT



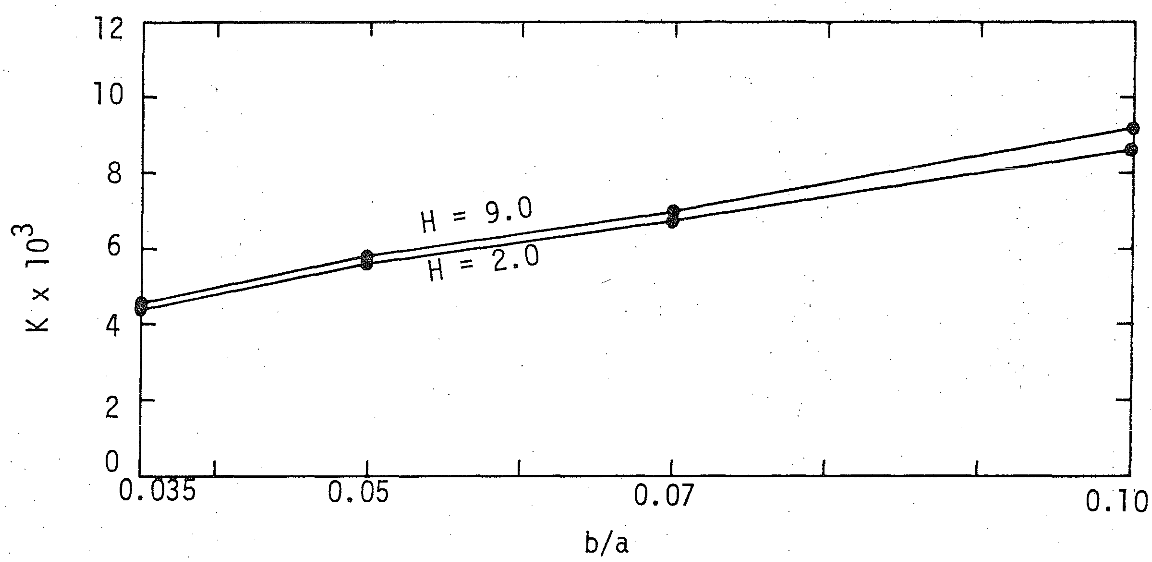


FIG. 4.15 RELATIONSHIPS OF THE MAXIMUM DEFLECTION COEFFICIENT WITH H AND b/a

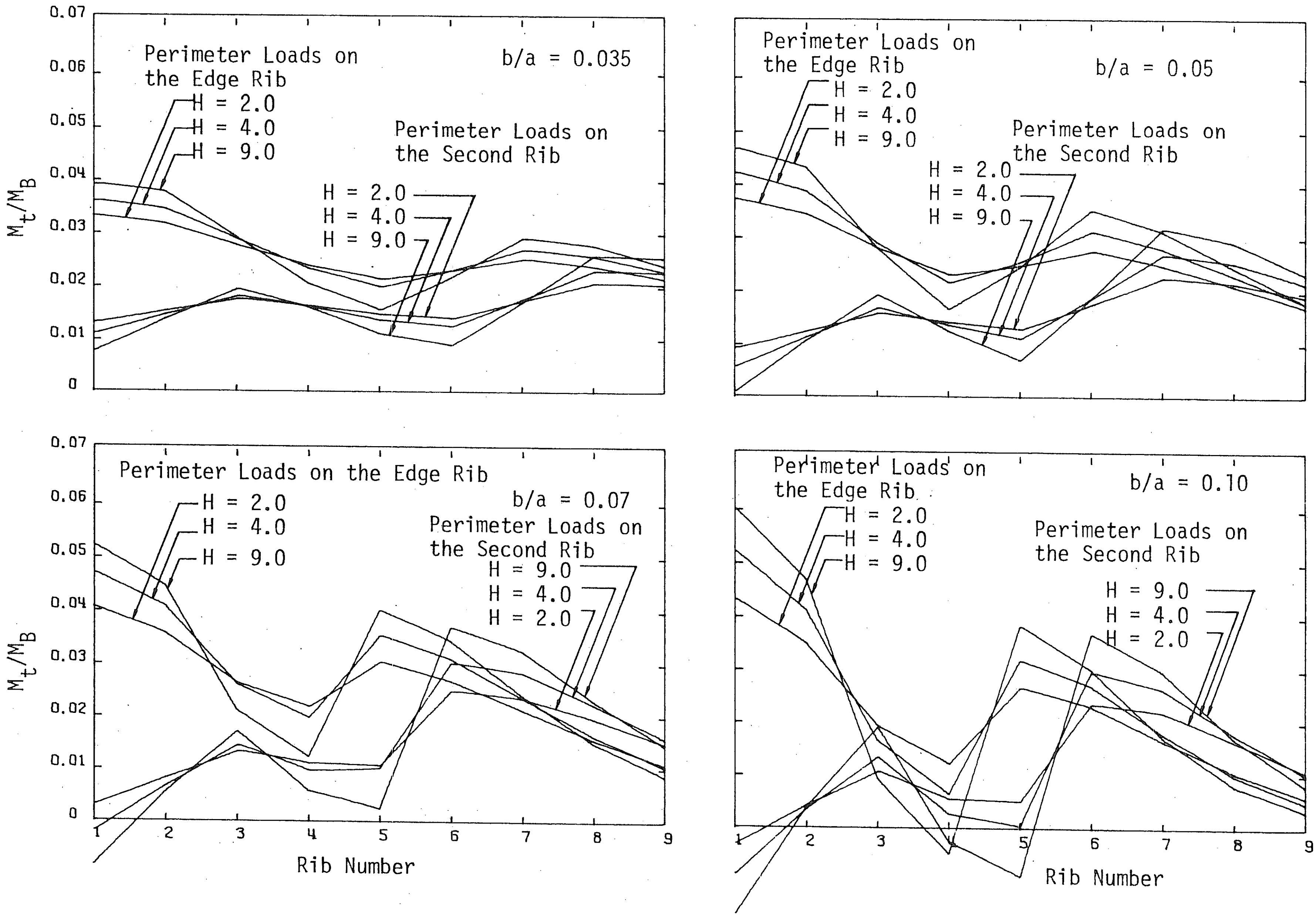


FIG. 4.16 TORSIONAL MOMENT DISTRIBUTIONS AT THE SUPPORT OF ONE-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM TORSION

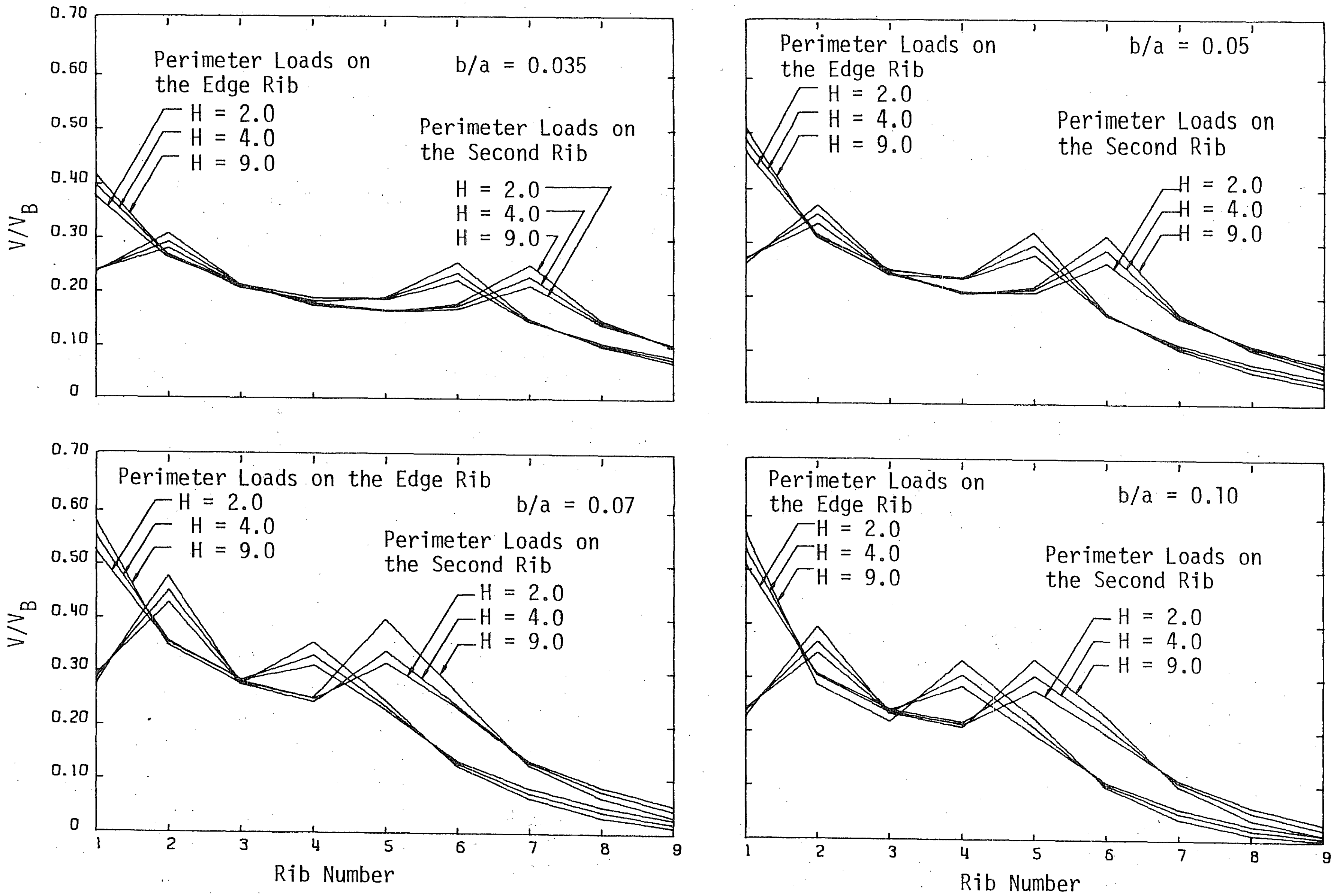


FIG. 4.17 SHEAR DISTRIBUTIONS AT THE SUPPORT OF ONE-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM TORSION

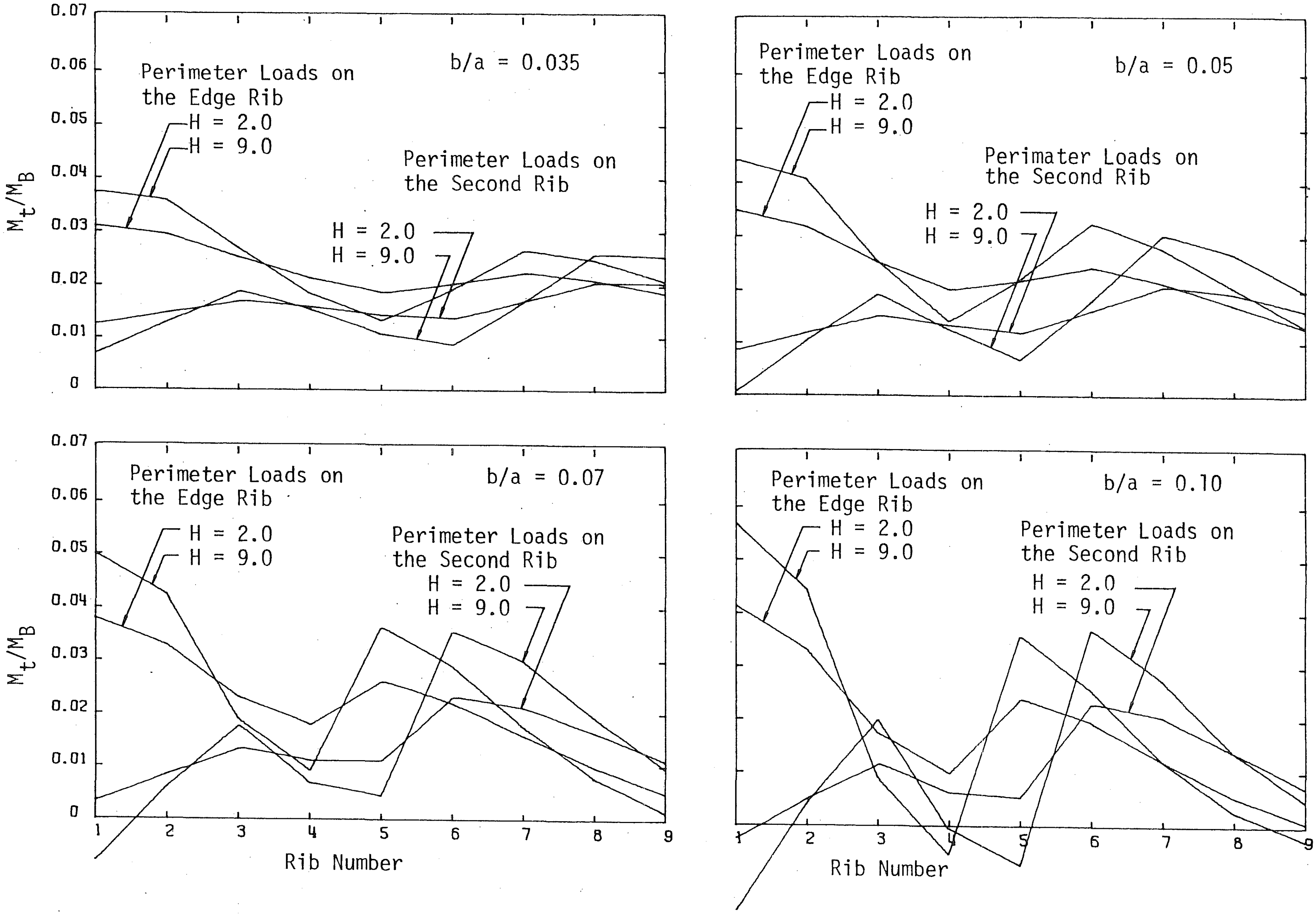


FIG. 4.18 TORSIONAL MOMENT DISTRIBUTIONS AT THE EXTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM TORSION

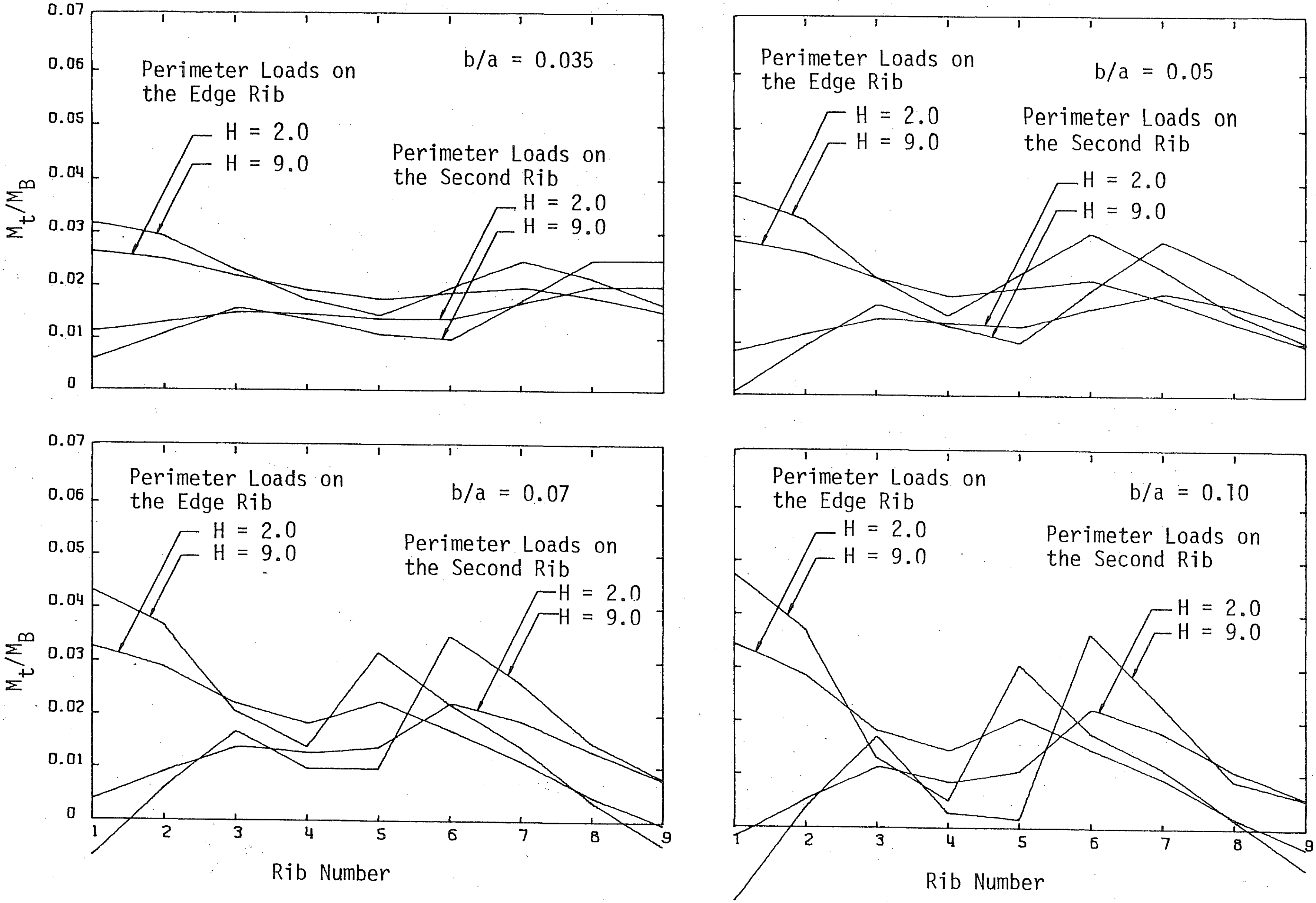


FIG. 4.19 TORSIONAL MOMENT DISTRIBUTIONS AT THE 0.95 SPAN LOCATION OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM TORSION

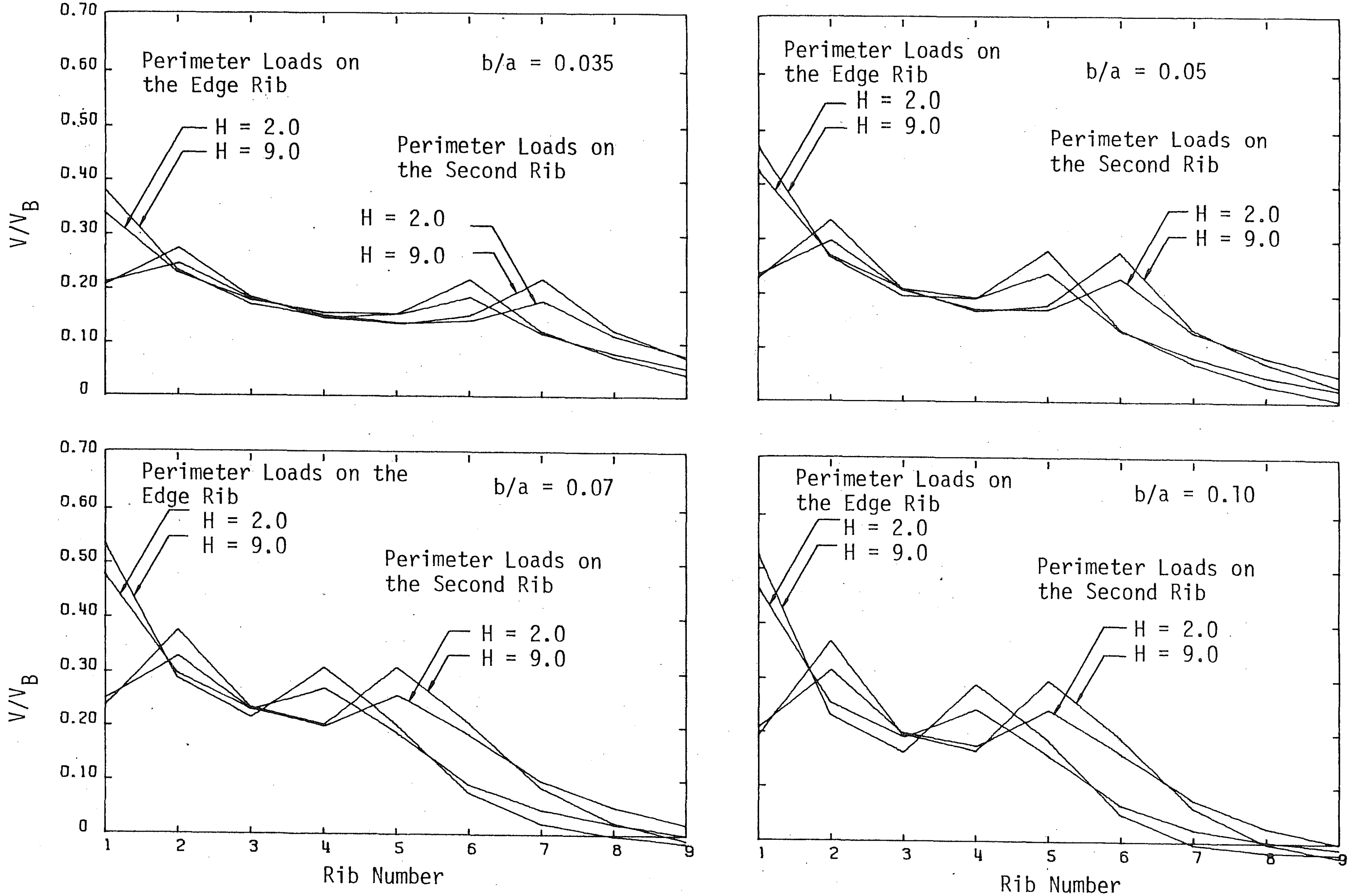


FIG. 4.20 SHEAR DISTRIBUTIONS AT THE EXTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM TORSION

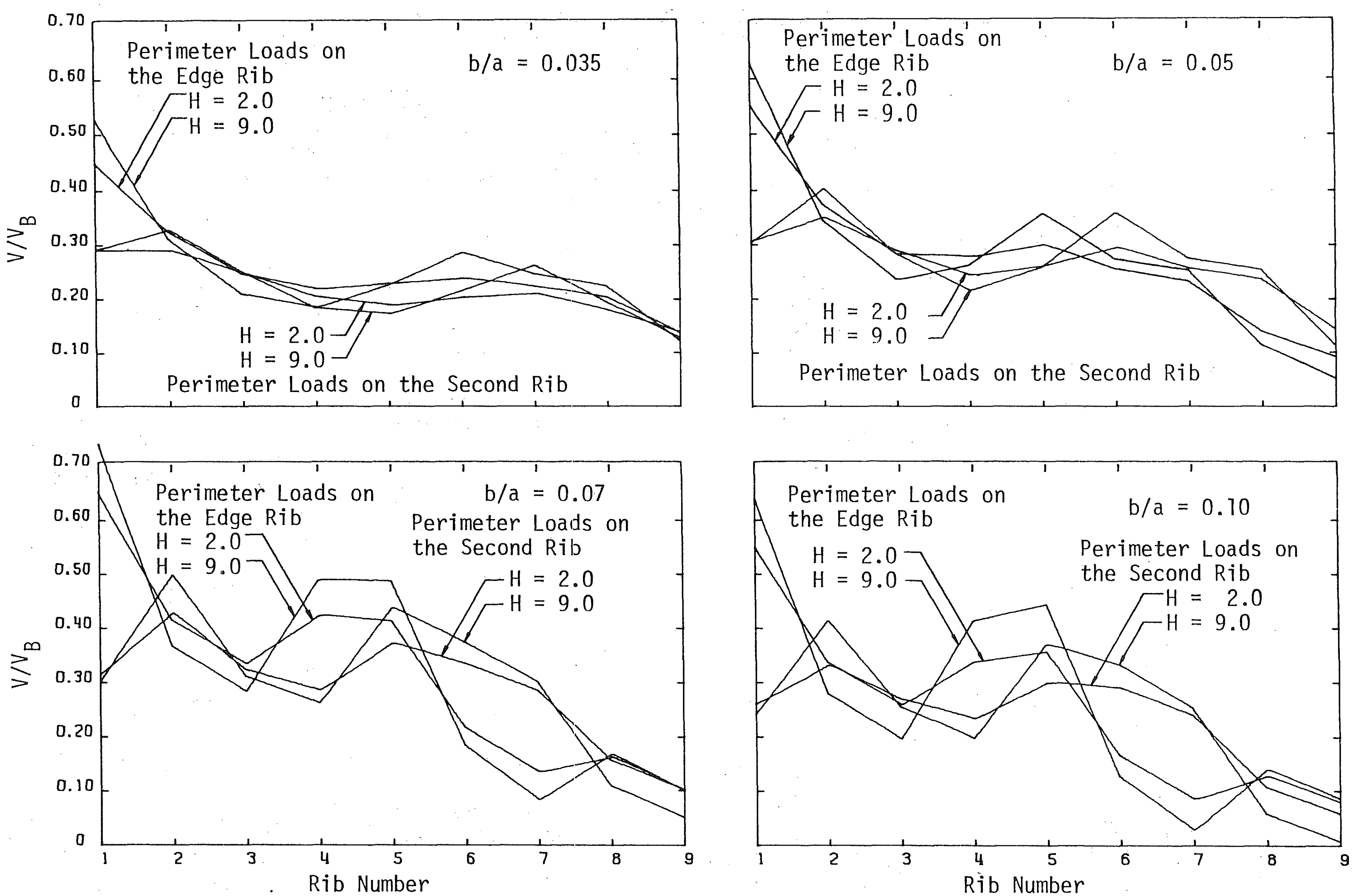


FIG. 4.21 SHEAR DISTRIBUTIONS AT THE 0.95 SPAN LOCATION OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM TORSION

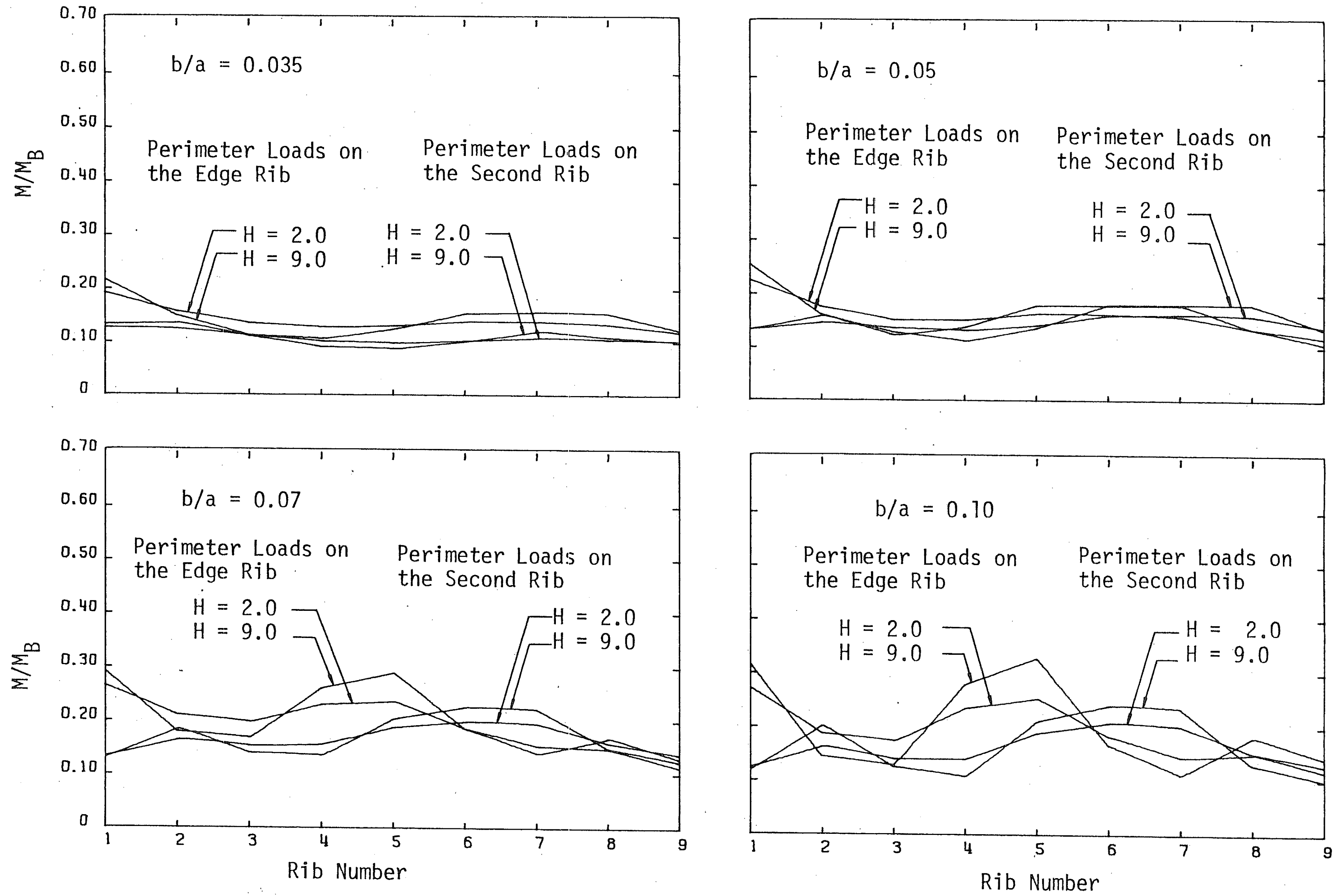
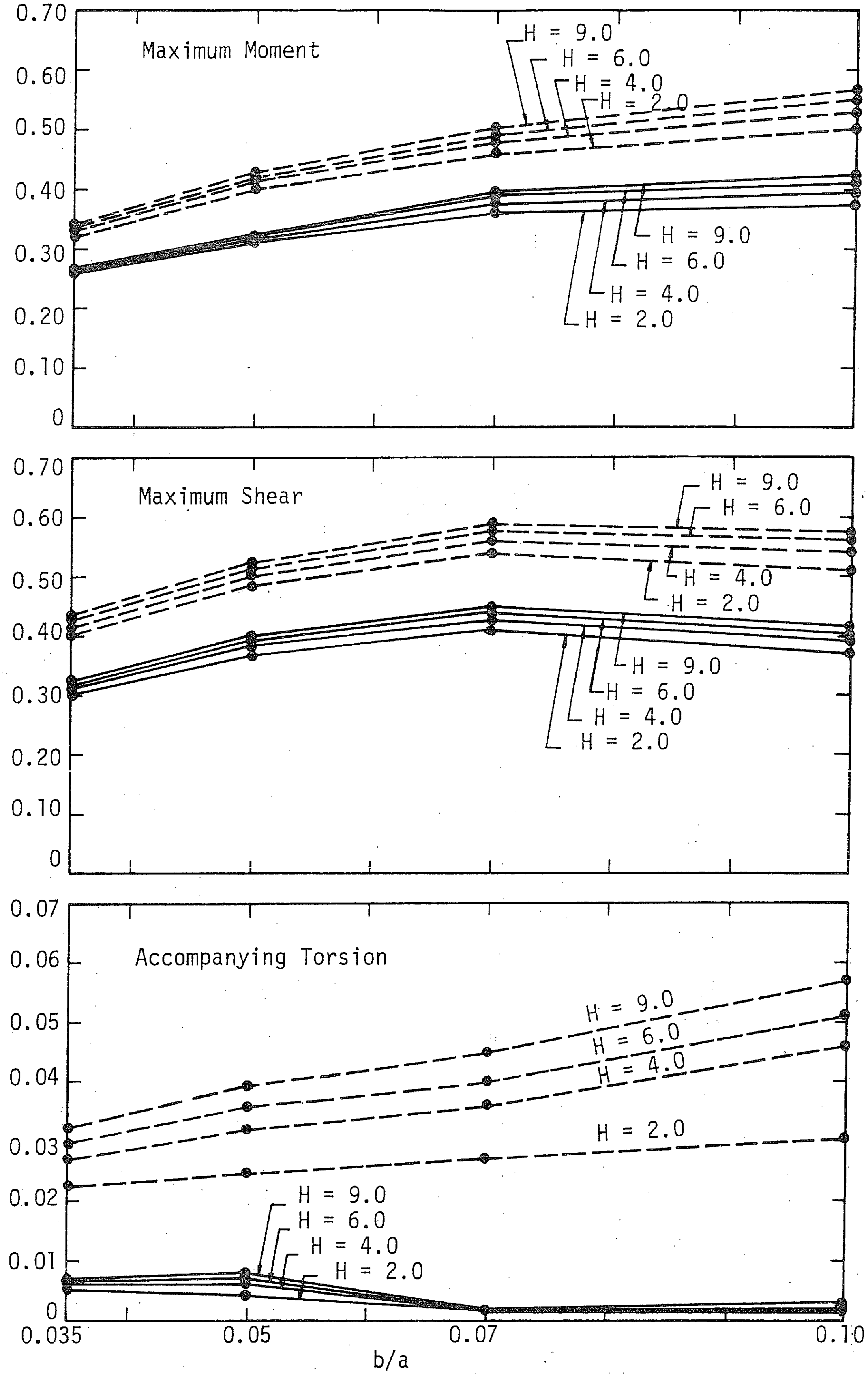


FIG. 4.22 NEGATIVE MOMENT DISTRIBUTIONS AT THE INTERIOR SUPPORT OF TWO-SPAN RIBBED SLABS DUE TO MULTIPLE LOADS FOR MAXIMUM TORSION



— — Perimeter Loads on the Edge Rib
——— Perimeter Loads on the Second Rib

FIG. 4.23 RELATIONSHIPS OF MAXIMUM MOMENT, MAXIMUM SHEAR AND THE ACCOMPANYING TORSION, WITH H AND b/a FOR ONE-SPAN RIBBED SLABS UNDER MULTIPLE LOADS FOR MAXIMUM MOMENT

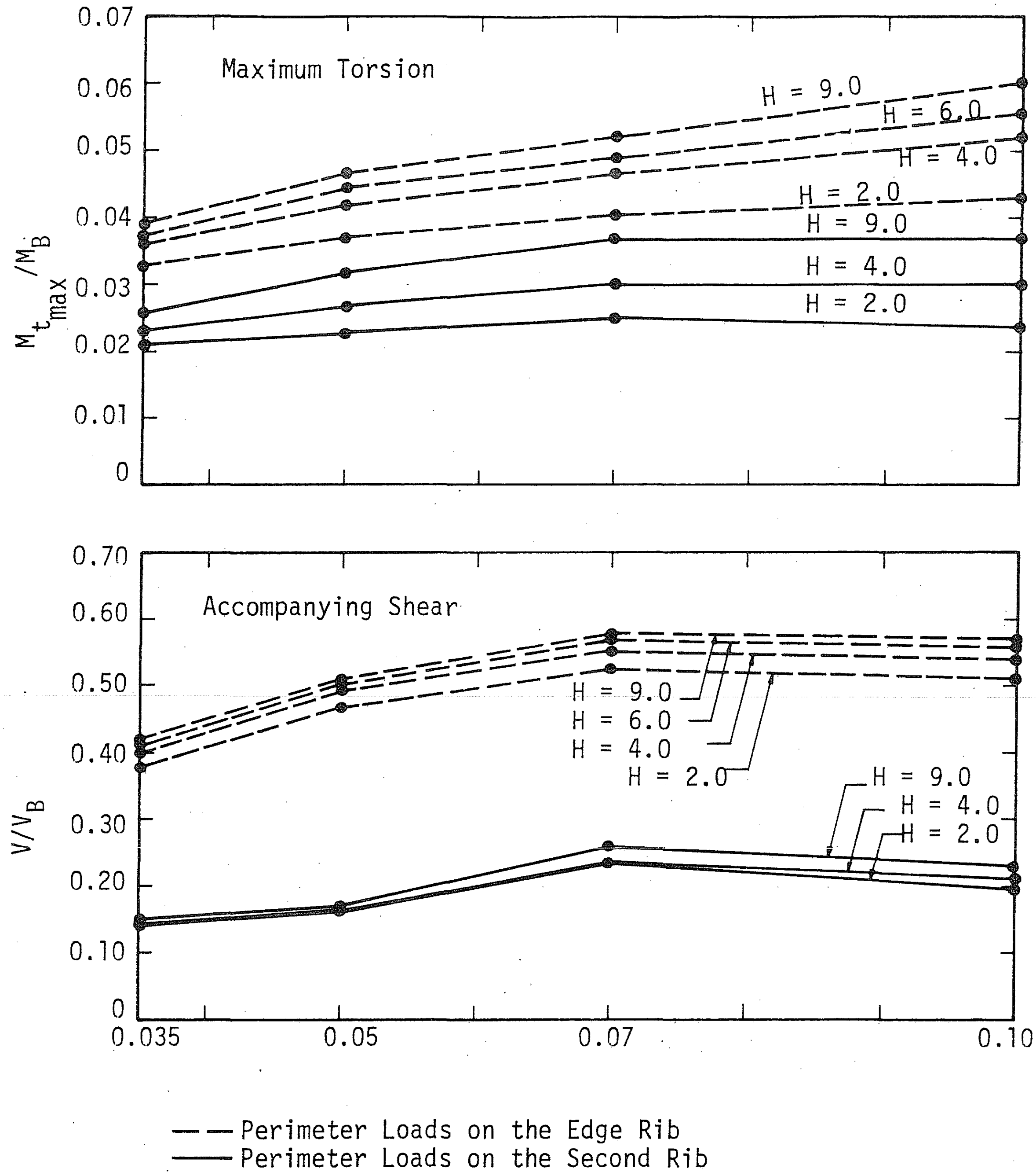


FIG. 4.24 RELATIONS OF MAXIMUM TORSIONAL MOMENT AND THE ACCOMPANYING SHEAR WITH H AND b/a FOR ONE-SPAN RIBBED SLABS UNDER MULTIPLE LOADS FOR MAXIMUM TORSION

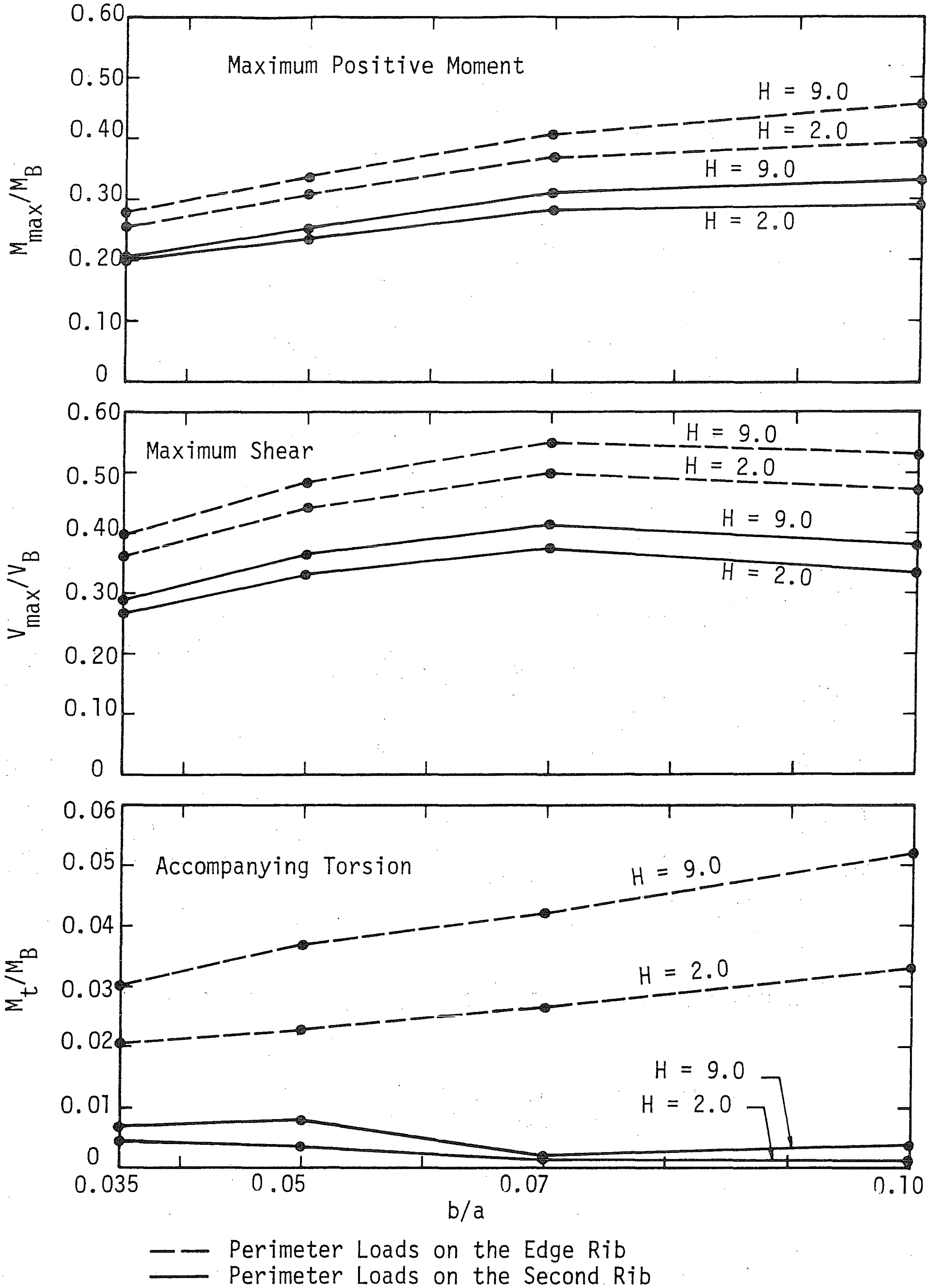


FIG. 4.25 RELATIONSHIPS OF MAXIMUM POSITIVE MOMENT AT THE 0.44 SPAN LOCATION, MAXIMUM SHEAR AND THE ACCOMPANYING TORSION AT THE EXTERIOR SUPPORT WITH H AND b/a FOR TWO-SPAN RIBBED SLABS UNDER MULTIPLE LOADS FOR MAXIMUM POSITIVE MOMENT

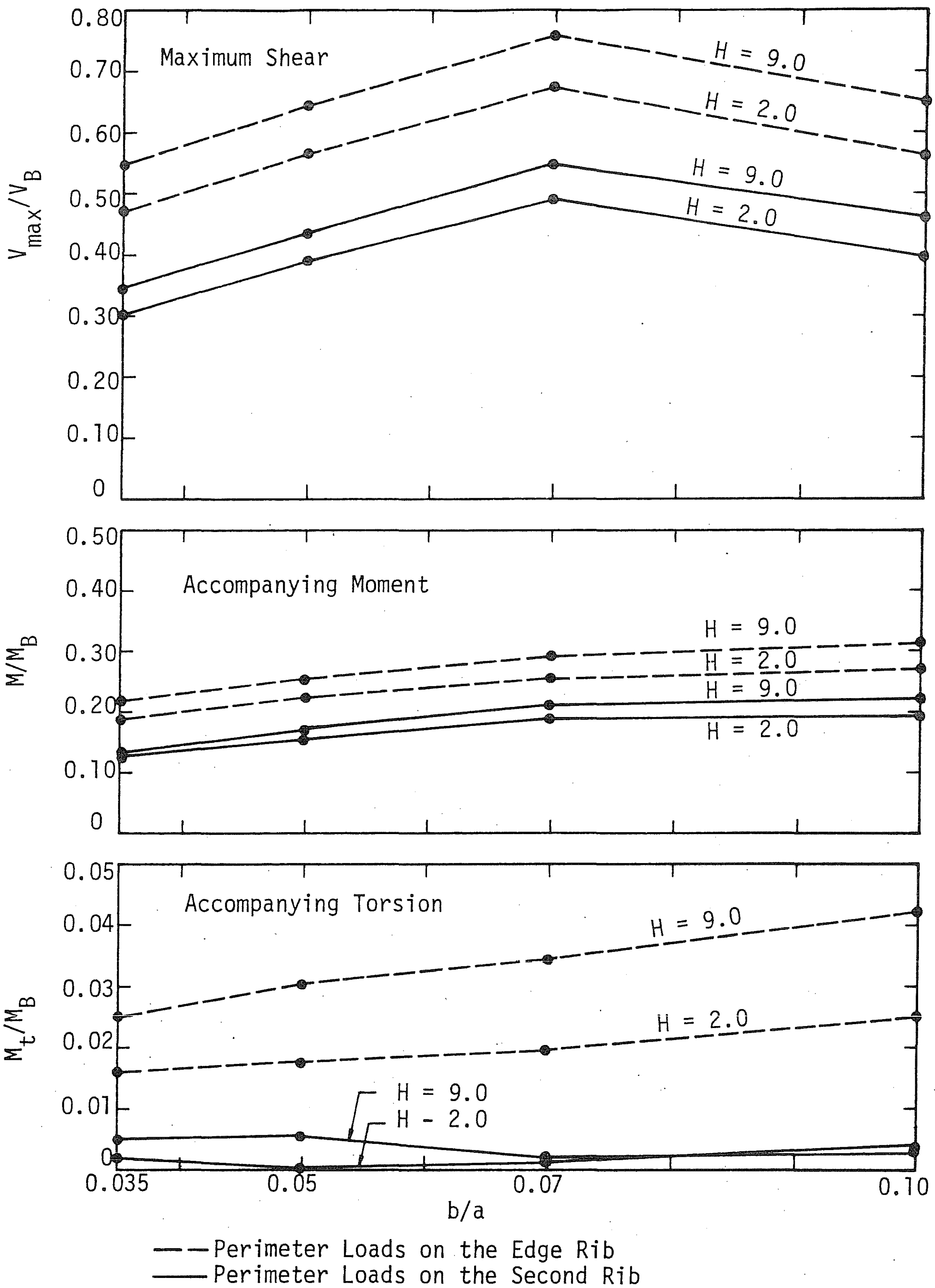


FIG. 4.26 RELATIONSHIPS OF MAXIMUM SHEAR AND THE ACCOMPANYING TORSION AT THE 0.95 SPAN LOCATION AND NEGATIVE MOMENT AT THE INTERIOR SUPPORT WITH H AND b/a FOR TWO-SPAN RIBBED SLABS UNDER MULTIPLE LOADS FOR MAXIMUM POSITIVE MOMENT

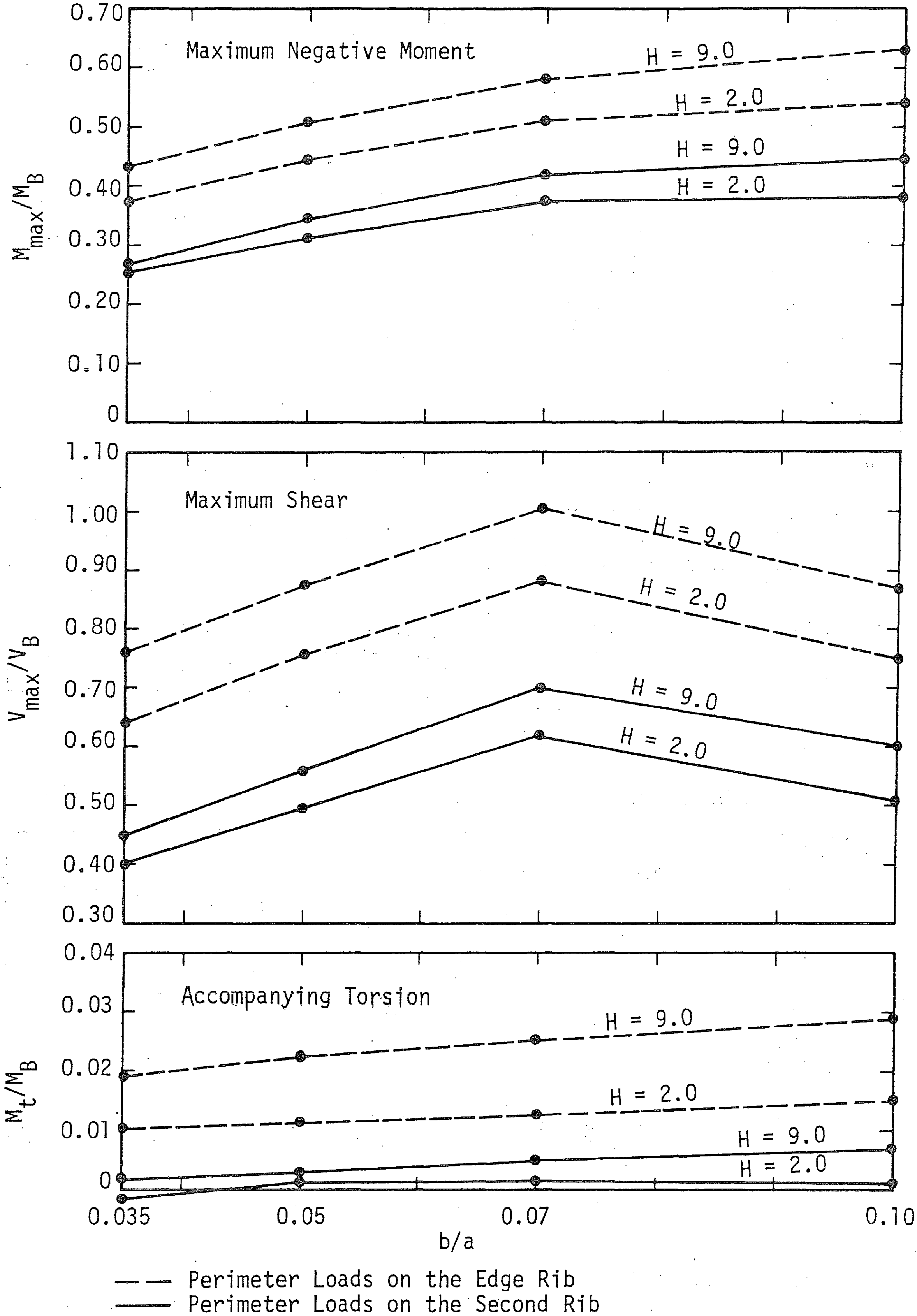


FIG. 4.27 RELATIONSHIPS OF MAXIMUM NEGATIVE MOMENT AT THE INTERIOR SUPPORT, MAXIMUM SHEAR AND THE ACCOMPANYING TORSION OF THE 0.95 SPAN LOCATION WITH H AND b/a FOR TWO-SPAN RIBBED SLABS UNDER MULTIPLE LOADS FOR MAXIMUM TORSION

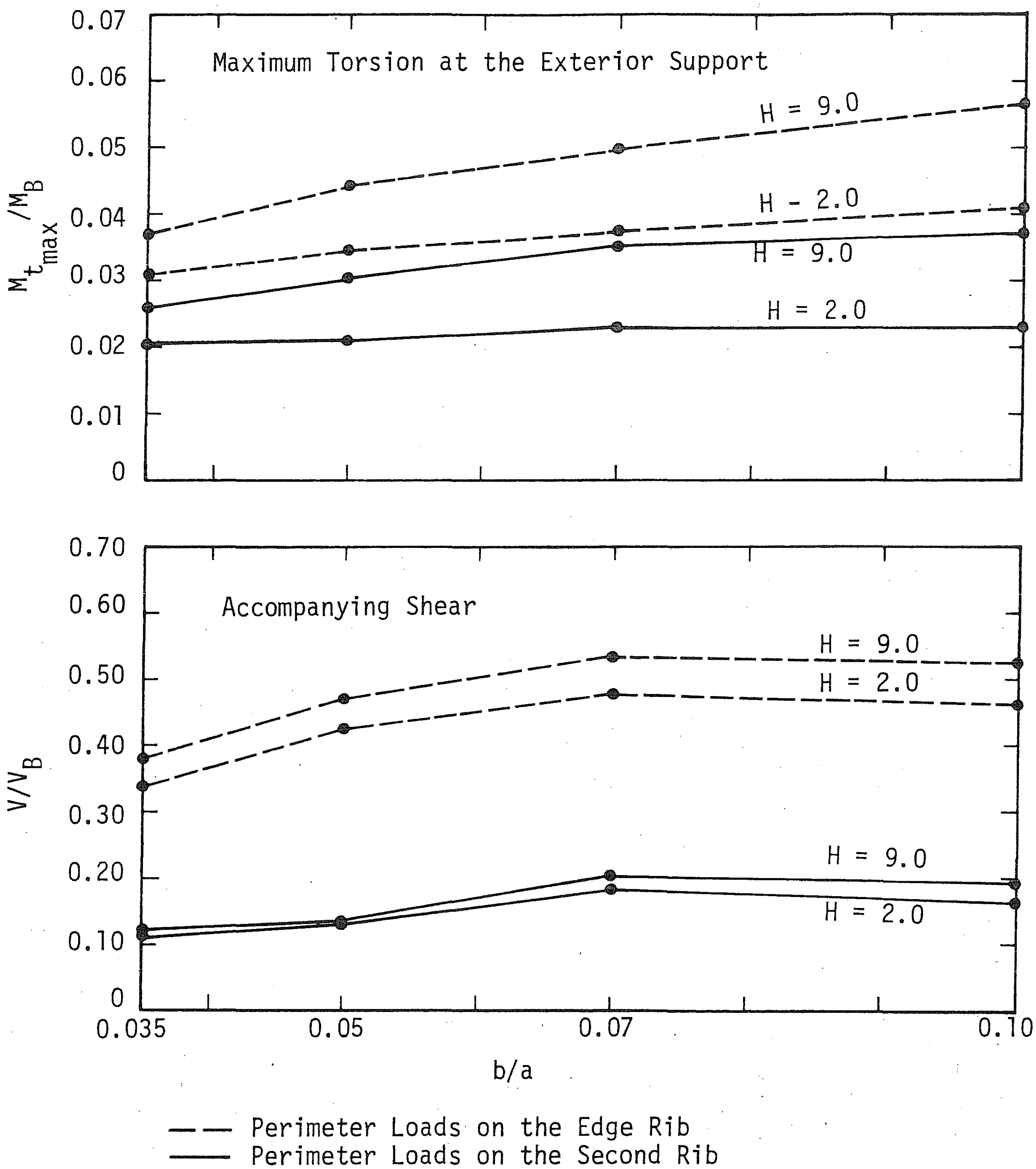


FIG. 4.28 RELATIONSHIPS OF MAXIMUM TORSIONAL MOMENT AND THE ACCOMPANYING SHEAR AT THE EXTERIOR SUPPORT WITH H AND b/a FOR TWO-SPAN RIBBED SLABS UNDER MULTIPLE LOADS FOR MAXIMUM TORSION

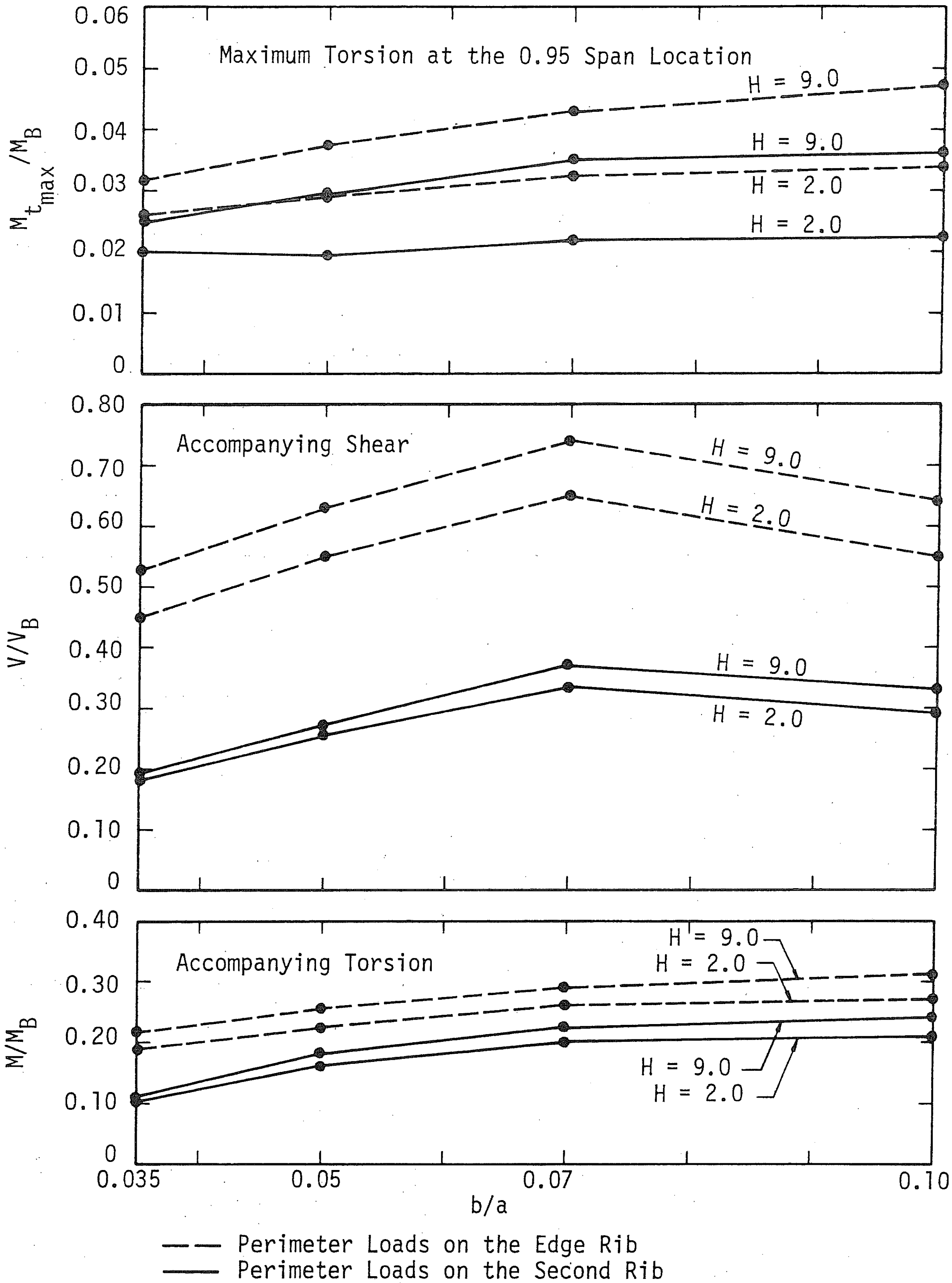


FIG. 4.29 RELATIONSHIPS OF MAXIMUM TORSIONAL MOMENT AND THE ACCOMPANYING SHEAR AT THE 0.95 SPAN LOCATION AND NEGATIVE MOMENT AT THE INTERIOR SUPPORT WITH H AND b/a FOR TWO-SPAN RIBBED SLABS UNDER MULTIPLE LOADS FOR MAXIMUM TORSION

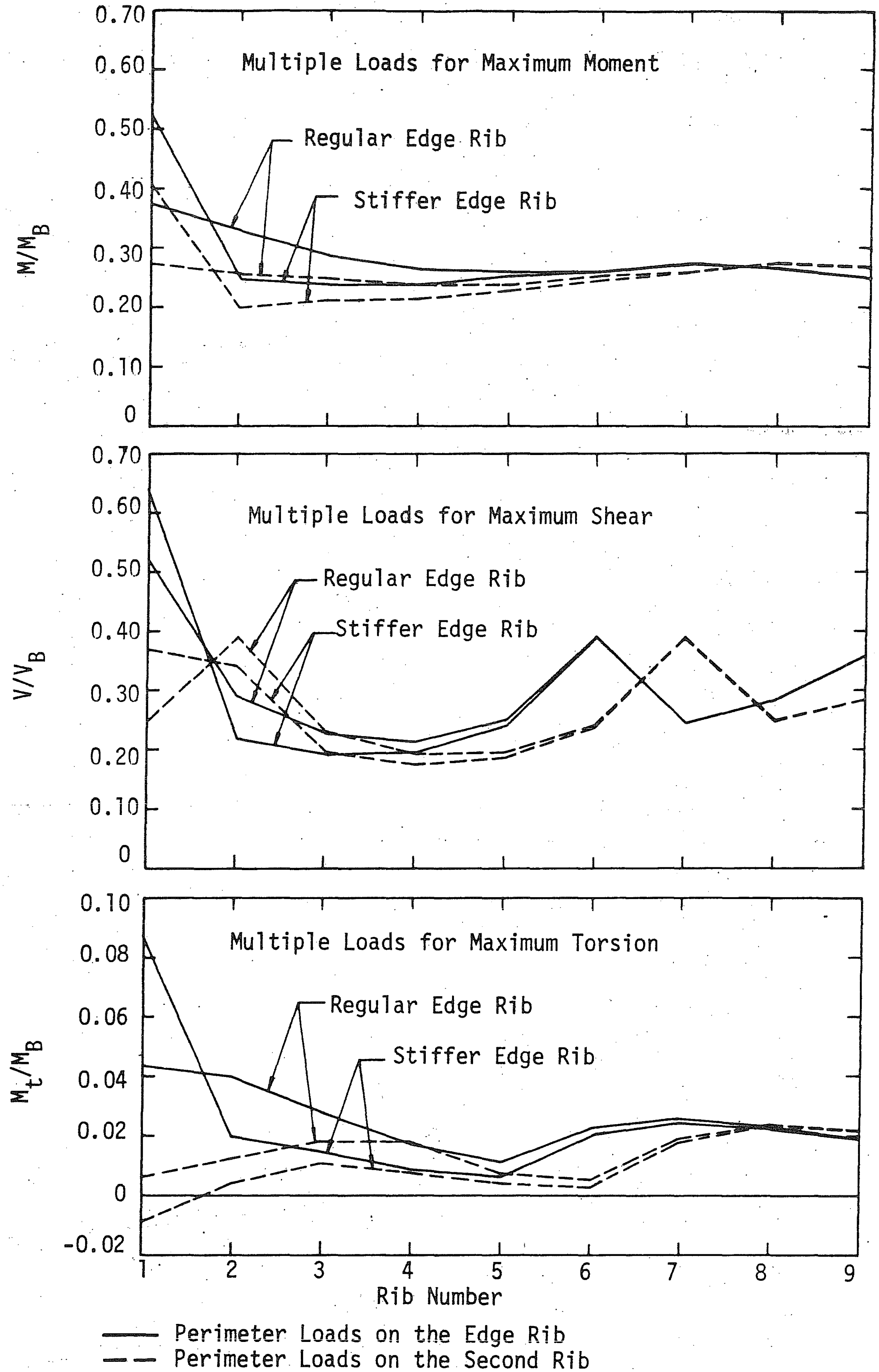


FIG. 4.30 EFFECTS OF THE STIFFENED EDGE RIB ON MOMENT, SHEAR AND TORSIONAL MOMENT DUE TO MULTIPLE LOADS

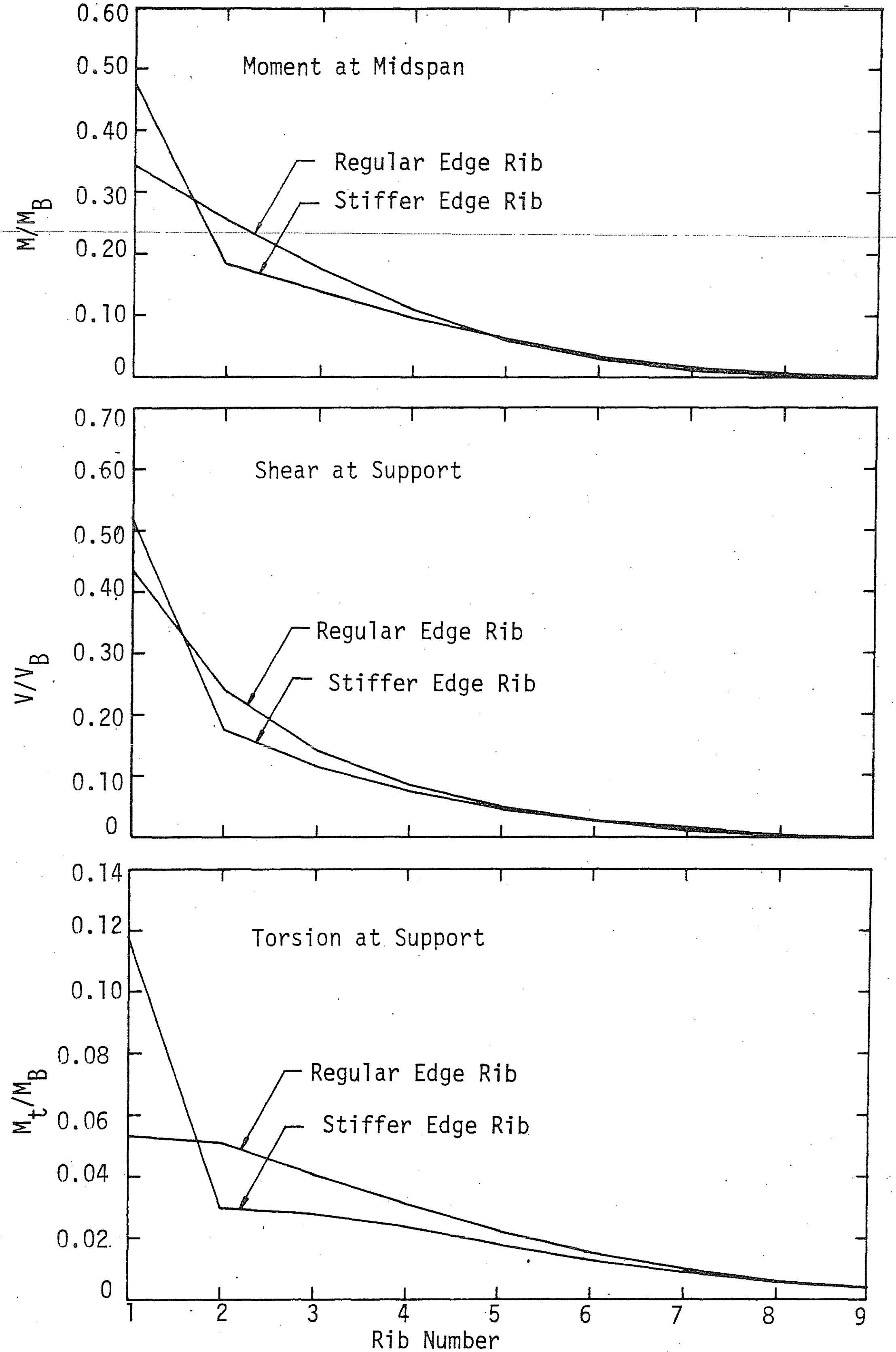


FIG. 4.31 EFFECTS OF THE STIFFENED EDGE RIB ON MOMENT, SHEAR, AND TORSIONAL MOMENT DUE TO A LINE LOAD ON THE EDGE RIB

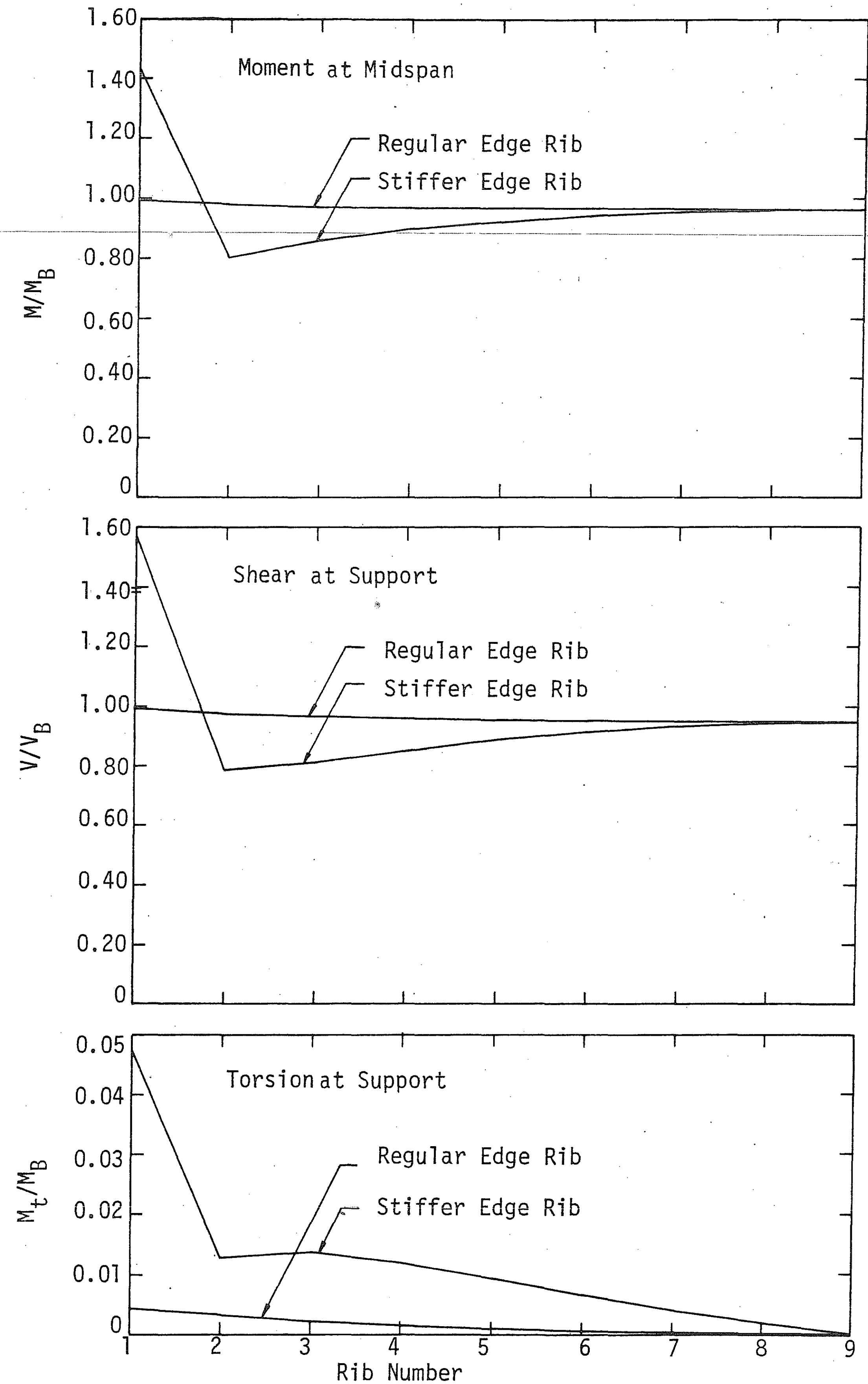


FIG. 4.32 MOMENTS, SHEARS, AND TORSIONS DUE TO A MIDSPAN LINE LOAD