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A STATISTICAL ESTIMATE OF RELATIVE DISTRIBUTION OF EXTREME SHEAR IN A TALL BUILDING SUBJECTED TO RANDOM EARTHQUAKE SHOCKS

By
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Technical Report
to
OFFICE OF NAVAL RESEARCH
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UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

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1. Introduction

On account of the irregularity of the ground motions in an earthquake and of their consequences on structures, a statistical approach has been suggested for analyzing the earthquake response of structures, even though the process is known mathematically as "non-stationary." In the present paper the method of random walks is used to obtain a frequency distribution function for the story shear developed at different levels of a tall building simulated by a tall uniform shear beam. Later a normal distribution is suggested for transforming the discrete distribution to a continuous one.

The lacking of a statistical analysis of the ground motion in an earthquake prevents the direct assessment of the base shear acting on the foundation of a building; however, the relative shear at different stories can be estimated. This information is believed to be helpful in achieving a unified aseismic design of tall buildings. The relative distribution of shear is found to be parabolic with respect to the height of the structure, which agrees with an early observation.[1]

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(1) Emilio Rosenblueth, "A Basis for Aseismic Design of Structures." Ph.D. Thesis, University of Illinois, 1951.

2. Uniform Shear Beam As A Model

For analyzing the dynamic response of a tall building, a tall uniform shear beam has been assumed as an idealized model. One obvious advantage of the simulation is the simple equation of motion to be dealt with:[2]

$$\alpha y_{xx} = \beta y_{tt} \tag{2-1}$$

where

α = stiffness per unit height

β = mass per unit height

t = time

y = lateral deflection

x = elevation above ground

and the subscripts x or t indicate differentiation with respect to the variable indicated. The boundary conditions are that y at x = 0 should be equal to the ground motion at any time, and y_x, a measure of shear, should be zero, at the top of the beam.

Equation (2-1), known as a one dimensional wave equation, has been solved explicitly as a sum of two parts

$$y = F_1\ (x + Vt) + F_2\ (x - Vt)$$

where $V = \sqrt{\alpha/\beta}$ is the velocity of propagation of a disturbance along the beam. The second part represents a forward wave starting at the base and propagating toward the top, and the first part represents a wave

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(2) Westergaard, H. M., "Earthquake-Shock Transmission In Tall Buildings," Eng. News-Record, 111, 30 Nov. 1933, p. 654-656.

moving from the top to the base, or in most cases, a reflected wave. Thus, the solution, at any time, for any point on the beam, can be obtained if proper steps are taken in ascertaining the value for the functions, $F(x \pm Vt)$, from the given initial conditions.

If a unit pulse is applied at the base, then because of the lateral displacement of the beam, a shear is set up in the beam with a magnitude equal to $\sqrt{\alpha \beta}$, until a reflected wave coming down from the top of the beam nullifies the shear. Nevertheless, the stage of zero shear remains only until a second shear wave coming up from the base, in order to fulfill the boundary condition at the base, leaves a shear acting opposite to the direction of the first shear. As this process proceeds indefinitely, the shear at any point on the beam changes its direction alternately but with an intermittent lull of a certain length of time between the changes.

Figure 1 schematically illustrates the propagation of shear at different heights. The length of lull is obviously linearly proportional to the height of the point under consideration.

3. A Random Walk Problem

For making a statistical estimate, the ground motion of an earthquake which shocks the foundation of the structure is assumed to involve a large number of random pulses each with equal order of magnitude. As a result, the state of shear in the structure will be the net effect of the shear waves due to the random pulses, traveling from the base to the top, and reflecting back to the base. When the pulses are traveling along the beam, this bears some resemblance to a problem of a random walk in which a particle may move forward or backward, or may stand still on a line, according to specified probabilities. The distance transversed after a large number of moves may represent the state of shear at certain points of the beam, the base of which has been subjected to a large number of shock pulses.

Figure 2 shows a two-dimensional plan on which the particle moves. In each trial it must move upward one unit, but it may also move one unit either to the right, or to the left. This means the particle may move diagonally upward to the right or left, or it may move vertically one unit, but it can never move downward. The probability p_1 that the particle may take a move to the right represents the chance, at any time during the traveling of a single shear wave, of obtaining a positive shear at the observed point; p_2, for a move to the left, a negative shear; and q, for a straight vertical move, no shear at the observed point. Therefore, in view of Fig. 1, and because the durations for positive and negative shears are equal in one cycle, one finds the result:

$$p_1 = p_2 = \frac{\text{duration of positive shear in one cycle}}{\text{period of one cycle}} = p$$

and

$$q = 1 - 2p$$

Obviously in the case of a uniform shear beam, p is zero at the top of the beam, and 1/2 at the base, and varies linearly throughout the height of the structure.

After N number of moves the position of the particle will be N units above the initial horizontal axis, and M units to the right. The probability that the particle will move M units after making N trials, then, represents the chance that the observed point will get a shear of magnitude M after the foundation has been shocked by N pulses. For evaluating the probability, assume that among the N moves, there are i units to the right; j, to the left; and k, vertical moves. Then the probability for the occurrence of such an event is $p^i p^j q^k$. Since there is no restriction imposed on the order of occurrence of the individual events, the compound probability should be the sum of those of the individual trials, namely,

$$P(M,N) = \sum \frac{N!}{i!j!k!} \; p^{i+j} q^k \tag{3-1}$$

subject to the conditions that

$$N = i + j + k \tag{3-2}$$

$$M = i - j \tag{3-3}$$

This is a multi-normal distribution. It can be shown that its asymptotic form will tend to be normal, in view of the Central Limit Theorem.

Making use of Eqs. (3-2) and (3-3), one may reduce Eq. (3-1) to

$$P(M,N) = \sum \frac{N!}{k! \; (\frac{N+M-k}{2})! \; (\frac{N-M-k}{2})!} \cdot p^{N-k} q^k \tag{3-4}$$

in which k takes on all integral values from 0 to (N-M)/2, if N and M are both even; but it will be 1, 3, 5,.....N-M if N is even, and M odd. Equation (3-4) is not a series but a polynomial. For the sake of studying the probability an asymptotic form would be easy to deal with. By factorizing $p^N N!$ one obtains another expression for the probability P(M,N).

for M odd

$$P(M,N) = \sum^{\frac{N-|M|}{2}} \frac{N!\ p^{N}}{(2r)!\ (\frac{N-M}{2} - r)!\ (\frac{N+M}{2} - r)!} \left(\frac{q}{p}\right)^{2r} ; \qquad (3\text{-}5a)$$

for M even

$$P(M,N) = \sum_{k=1,3,5..}^{N-M} \frac{N!\ p^{N}}{k!\ (\frac{N+M-k}{2})!\ (\frac{N-M-k}{2})!} \cdot \left(\frac{q}{p}\right)^{k} . \qquad (3\text{-}5b)$$

Rearranging the expression shows that the polynomial can be represented by a hypergeometric function F:

$$P(M,N) = \frac{p^{N} N!}{\frac{N+M}{2}!\ \frac{N-M}{2}!}\ F\left[-\frac{N+M}{2}, -\frac{N-M}{2}; \frac{1}{2}; \left(\frac{q}{2p}\right)^{2}\right], \ m = \text{even}$$

$$(3\text{-}6)$$

$$= \frac{N!\ qp^{N-1}}{(\frac{N+M-1}{2})!\ (\frac{N-M-1}{2})!}\ F\left[-\frac{N-1+M}{2}; -\frac{N-1-M}{2}; \frac{3}{2}; \left(\frac{q}{2p}\right)^{2}\right], \ m = \text{odd}$$

By one of the transformation rules it has been found that

$$F\left[-\frac{N+M}{2}, -\frac{N-M}{2}; \frac{1}{2}; \left(\frac{q}{2p}\right)^{2}\right] = \left[1 - \left(\frac{q}{2p}\right)^{2}\right]^{\frac{N-M}{2}} F\left[1 + \frac{N+M}{2}; -\frac{N-M}{2}; \frac{1}{2}; \frac{\left(\frac{q}{2p}\right)^{2}}{\left(\frac{q}{2p}\right)^{2} - 1}\right]$$

and

$$F\left[-\frac{N-1+M}{2}, -\frac{N-1-M}{2}; \frac{3}{2}; \left(\frac{q}{2p}\right)^2\right] = \left[1-\left(\frac{q}{2p}\right)^2\right]^{-\frac{N-M}{2}} F\left[1+\frac{N+M}{2}, -\frac{N-1-M}{2}; \frac{3}{2}; \frac{\left(\frac{q}{2p}\right)^2}{\left(\frac{q}{2p}\right)^2 - 1}\right] \qquad (3\text{-}7)$$

The right-hand members can be recognized as the standard form of Legendre Polynomials.

4. Asymptotic Expression of the Distribution Functions.

The Legendre polynomials of higher orders are not easy to work with when they involve large numbers of terms as in the present case. An asymptotic expression will be useful in interpreting the result without sacrificing accuracy, as N takes on large values.

It is known that the distribution function will tend to be normal as N takes on large values. This has been clearly shown, for $p = 1/2$, in Chandrasekhar's paper.(3) For illustrative purposes it is shown herein also for $p = 1/4$ and $q = 1/2$. For M an even number,

$$P(M,N) = \frac{\left(\frac{1}{2}\right)^N N!}{\left(\frac{N+M}{2}\right)!\left(\frac{N-M}{2}\right)!} F\left(-\frac{N+M}{2}, -\frac{N-M}{2}; \frac{1}{2}; 1\right) \qquad (4\text{-}1)$$

(3) Chandrasekhar, S., "Stochastic Problems in Physics and Astronomy," Reviews of Modern Physics, Vol. 15, No. 1. January 1943.

Since $q/2p = 1$, one could make use of the following identity that

$$F\left(-\frac{N+M}{2}, -\frac{N-M}{2}; \frac{1}{2}; 1\right) = \frac{\Gamma(\frac{1}{2})\,\Gamma(\frac{1}{2}+N)}{\Gamma(\frac{1}{2}+\frac{N+M}{2})\,\Gamma(\frac{1}{2}+\frac{N-M}{2})} \qquad (4\text{-}2)$$

Then, by means of Sterling's formula,

$$\ln \Gamma(z) = (z - \frac{1}{2}) \ln (z - 1) - z + 1 + \frac{\ln 2\pi}{2} + 0\left(\frac{1}{z^2}\right) \qquad (4\text{-}3)$$

a simple asymptotic form is derived for $P(M,N)$

$$P(M,N) \sim \frac{1}{\sqrt{N\pi}} \exp\left(-\frac{M^2}{N}\right) \qquad (4\text{-}4)$$

when $N >> 1$.

With the same manipulation one will get the identical result when M takes on odd values, as one would expect.

Equation (4-4) shows that the distribution function is an even function in M. It follows directly that the mean is zero. This agrees perfectly with physical intuition that the probability of getting a positive shear should be the same as that of getting a negative shear of the same order of magnitude.

Nevertheless, a comparison between Eq. (4-4) and Eq. (12) in Chandrasekhar's paper shows that not only the derived distribution functions are of normal type, but also the ratio of the variances is 1/2. This implies that the ratio of the standard deviations is $1/\sqrt{2}$. Therefore, the comparison of standard deviations which will give a direct appraisal of expected extreme shear at different story heights may be achieved by comparing the distribution functions when M is taken as zero. This simplifies the algebraic manipulations to the extent that Watson's asymptotic formula[4] can be utilized; otherwise a revision of the formula which is beyond the scope of the present study would have to be made.

Upon taking $M = 0$, the first of Eq. (3-6) is then reduced to

$$P(0,N) = \frac{p^N N!}{[(\frac{N}{2})!\ ]^2} \; F\left[-\frac{N}{2},\ -\frac{N}{2}\ ;\ \frac{1}{2}\ ;\ (\frac{q}{2p})^2\right] \tag{4-5}$$

For the stories below the midheight of the building, in which $q < 2p$, the following transformation rule for the hypergeometrical series is valid.

$$F\left[-\frac{N}{2},\ -\frac{N}{2}\ ;\ \frac{1}{2}\ ;\ (\frac{q}{2p})^2\right] = \left[1 - (\frac{q}{2p})^2\right]^{\frac{N}{2}} F\left[\frac{1+N}{2},\ -\frac{N}{2}\ ;\ \frac{1}{2}\ ;\ \frac{(\frac{q}{2p})^2}{(\frac{q}{2p})^2 - 1}\right]$$

By means of the Watson and Sterling formulas, on taking

$$\mu = \frac{1 + (\frac{q}{2p})^2}{1 - (\frac{q}{2p})^2} = \cosh \xi$$

one can arrive at the following simplified version

$$F\left(\frac{N+1}{2},\ -\frac{N}{2}\ ;\ \frac{1}{2}\ ;\ \frac{1-\mu}{2}\right) = \frac{(1+\frac{N}{2})\ \ (\frac{1}{2})}{\pi\ \ (\frac{1+N}{2})\ \sqrt{2}}\ (1 + e^{-\xi})\left[\frac{e^{\frac{N\xi}{2}}\,\Gamma(\frac{1}{2})}{(\frac{N}{2})^{1/2}} + \frac{e^{-\frac{N+1}{2}}\,\Gamma(\frac{1}{2})}{(\frac{N}{2})^{1/2}} + O\left(\frac{1}{N}\right)\right] \simeq \frac{1}{\sqrt{2}} \cdot \frac{e^{(N+\frac{1}{2})\xi/2} + e^{-(N+\frac{1}{2})\xi/2}}{\sqrt{2\cosh \xi/2}} = \frac{\cosh\,(N+\frac{1}{2})\xi/2}{\sqrt{\cosh \xi/2}} \tag{4-6}$$

(4) Hobson, Earnest Williams, Theory of Ellipsoidal Harmonics and Spherical Harmonics, p. 307, Formula B.

Denoting

$\sigma_{1/2}$ = standard deviation for $p = 1/2$

σ = standard deviation for $0 < p < 1/2$

one obtains immediately the ratio of the standard deviations as

$$D = \frac{\sigma}{\sigma_{\frac{1}{2}}} = \left(\frac{2p}{\cosh\frac{\xi}{2}}\right)^{-N} \frac{\sqrt{\cosh \xi/2}}{\cosh\left(N+\frac{1}{2}\right)\frac{\xi}{2}}$$

$$\simeq \left(\frac{4p}{1+e^{-\xi}}\right)^{N} \frac{2\sqrt{\cosh \xi/2}}{e^{\xi/4}} = \frac{2\sqrt{\cosh \xi/2}}{e^{\xi/4}} = \sqrt{2p} \qquad (4\text{-}7)$$

The same result can be obtained for the probability with which the particle walking randomly makes an odd number of horizontal units after N moves. Since the reduction of a distribution function to its asymptotic form is essentially a mathematical manipulation, setting $M = 0$ in the second of Eqs. (3-6), which is only valid for M odd, is legitimate.

Similarly, for the distribution function of the probable shear in the upper half of the building, with $1/2 \leq q \leq 1$ and $1/4 \geq p \geq 0$ a like expression is derived:

$$P(0,N) = \sum_{r=0}^{N/2} \frac{N!}{(2r)!\left[\left(\frac{N}{2}-r\right)!\right]^2} p^{N-2r} q^{2r} = q^N F\left[-\frac{N}{2}, -\frac{N-1}{2}; 1; \left(\frac{2p}{q}\right)^2\right] \qquad (4\text{-}8)$$

This form is used in order that the argument of the hypergeometric function will not become infinitely large. Upon similar manipulation, using the Sterling and Watson formulas, and an asymptotic expression for the Legendre polynomial, it is found that

$$F\left[-\frac{N}{2}, -\frac{N-1}{2}; 1; (\frac{2p}{q})^2\right] = \left[1 - (\frac{2p}{q})^2\right]^{\frac{N-1}{2}} F(1 + \frac{N}{2}, \frac{1-N}{2}; 1; \frac{1-\mu}{2}) \quad (4-9)$$

where

$$\mu = \cosh \zeta = \frac{1 + (\frac{2p}{q})^2}{1 - (\frac{2p}{q})^2}$$

Then

$$F\left[1 + \frac{N}{2}, \frac{1-N}{2}; 1; \frac{1-\mu}{2}\right] = \frac{(\frac{N+1}{2})\sqrt{2}}{\pi\ (1 + \frac{N}{2})} \cdot \frac{(1 - e^{-\zeta})^{-1/2}}{1 + e^{-\zeta}} \left[\frac{e^{\frac{N-1}{2} - \zeta}\ \Gamma(\frac{1}{2})}{(\frac{N}{2})!} + \right.$$

$$\left. + \frac{e^{\pi i/2}\ e^{-(1 + N/2)\zeta}\ \Gamma(\frac{1}{2})}{(\frac{N}{2})^{1/2}} + O(\frac{1}{N})\right]$$

$$\simeq \frac{e^{N\zeta/2}}{\sqrt{N\pi}} \sqrt{\frac{2}{\sinh(1 + e^{-\zeta})}} \quad (4-10)$$

The above expression agrees with the following observations, (1) as $2p/q$ becomes very small or nearly vanishes, the distribution function should reduce to the δ-function; hence F becomes infinitely large; (2) when $2p/q$ is taken as 1, one obtains Eq. (4-4) since

$$(\frac{2}{1 + \cosh \zeta})^{(N-1)/2} \frac{e^{N\zeta/2}}{\sqrt{N\pi}} \sqrt{\frac{2}{\sinh(1 + e^{-\zeta})}} \simeq \frac{2^N}{\sqrt{N\pi}}$$

The ratio of standard deviations, D, is again found to be

$$D = \frac{\sqrt{(1 + e^{-\zeta}) \sinh \zeta}}{2 \cosh \frac{\zeta}{2}} = \sqrt{2p} \qquad (4\text{-}11)$$

in perfect agreement with Eq. (4-7).

5. Estimate of Extreme Shear

In practical design of buildings, the designer usually chooses the section of the members of the structure to meet the most probable maximum load under different combinations of loading conditions. But for economical reasons, a maximum but improbable loading is usually ignored or considered with certain reservations. The concept of using factors of safety in designing is, in effect, a probabilistic approach. The expected loading is so proportioned that weight factors are assigned to different loads in a way similar to the computation of expectation of a random variable, in which probabilities serve the function of weight factors. Therefore the extreme shear at different floor levels and its distribution, become the center of interest.

The previous analysis has shown that as N becomes large the distribution function of the probability that the random walker moves M units after N moves, tends to be normal, and the expected value of M is zero since the walk is symmetric. The implication of the results indicates that the dynamic shear, a random variable in the present analysis, at different heights of the building possesses a normal distribution with a zero mean, hence the dispersion of the trials will be of interest.

For a random variable x with a normal distribution, the distribution of its extreme value in a population of n samples has been derived,[5] from which the expectation E(x) and variance V(x) of the extreme value can be calculated

$$E(x) = M + \sigma \left[\sqrt{2\ln n} - \frac{\ln\ln n + \ln 4\pi + 2C}{2\sqrt{2\ln n}} + O\left(\frac{1}{\ln n}\right) \right]$$

$$V(x) = \frac{\sigma^2}{2\ln n} \left(\frac{\pi}{6}\right)^2 + O\left(\frac{1}{\ln^2 n}\right) \qquad (5\text{-}1)$$

where

n is the number of samples

m is the mean of the random variable

σ is the standard deviation

C is the Euler Constant.

For a distribution with zero mean, the expected extreme shear is, therefore, directly proportional to the σ of the parental normal distribution.

Since a statistical analysis of the ground motion is not available, the expected extreme base shear (p = 1/2) can not be evaluated. However, the ratio D in Eq. (4-7) can reveal, nonetheless, that the ratio of expected extreme shear at different heights to the maximum base shear varies parabolically with the height. Based on this theory a unified aseismic design may be achieved if the distribution of maximum dynamic shear at different story heights is assumed to vary parabolically.

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(5) Cramer, H.,"Mathematical Methods of Statistics," Princeton University Press, 1951, pp. 375-7.

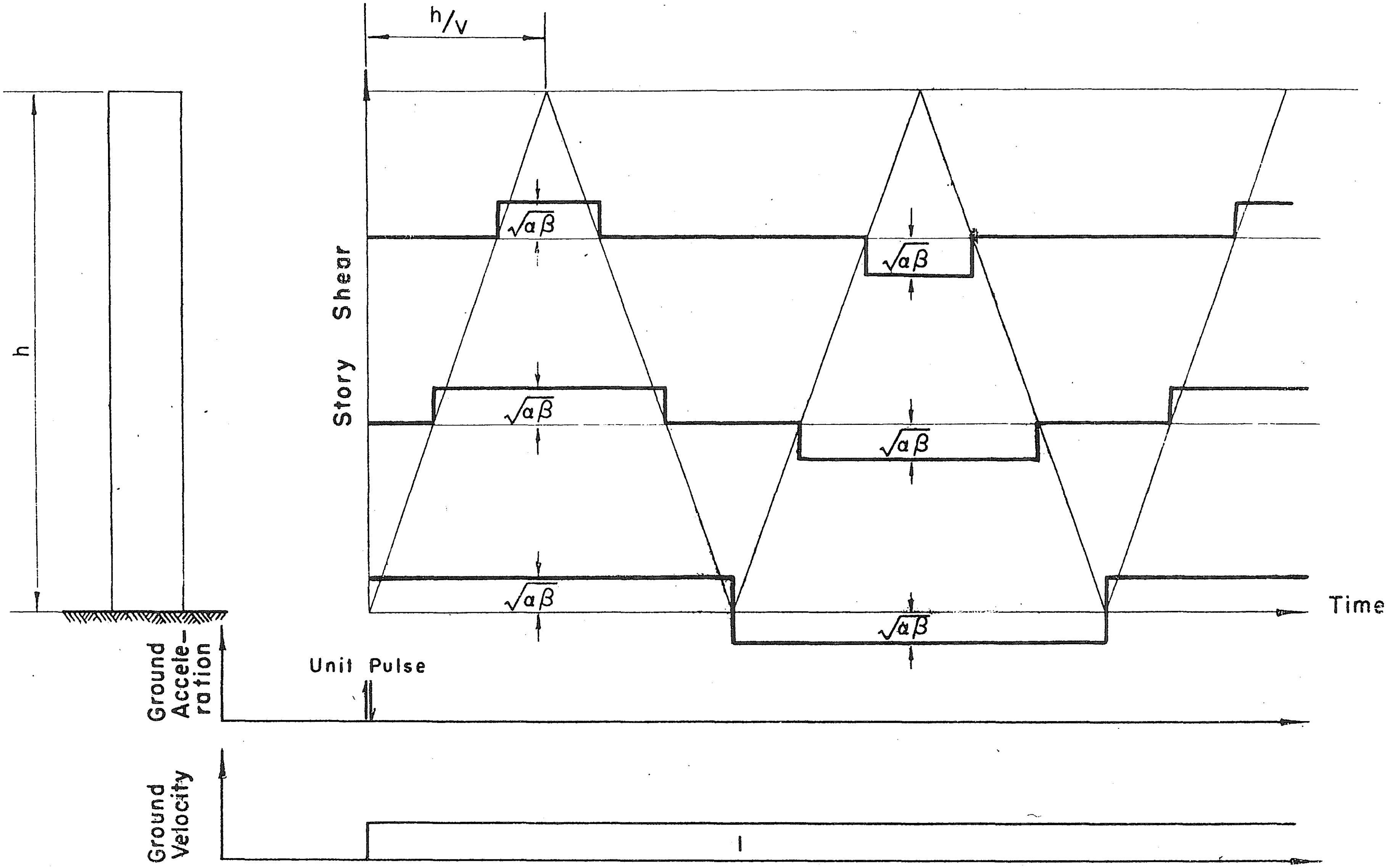

Fig. I Responses of Uniform Shear-Beam to Unit Pulse

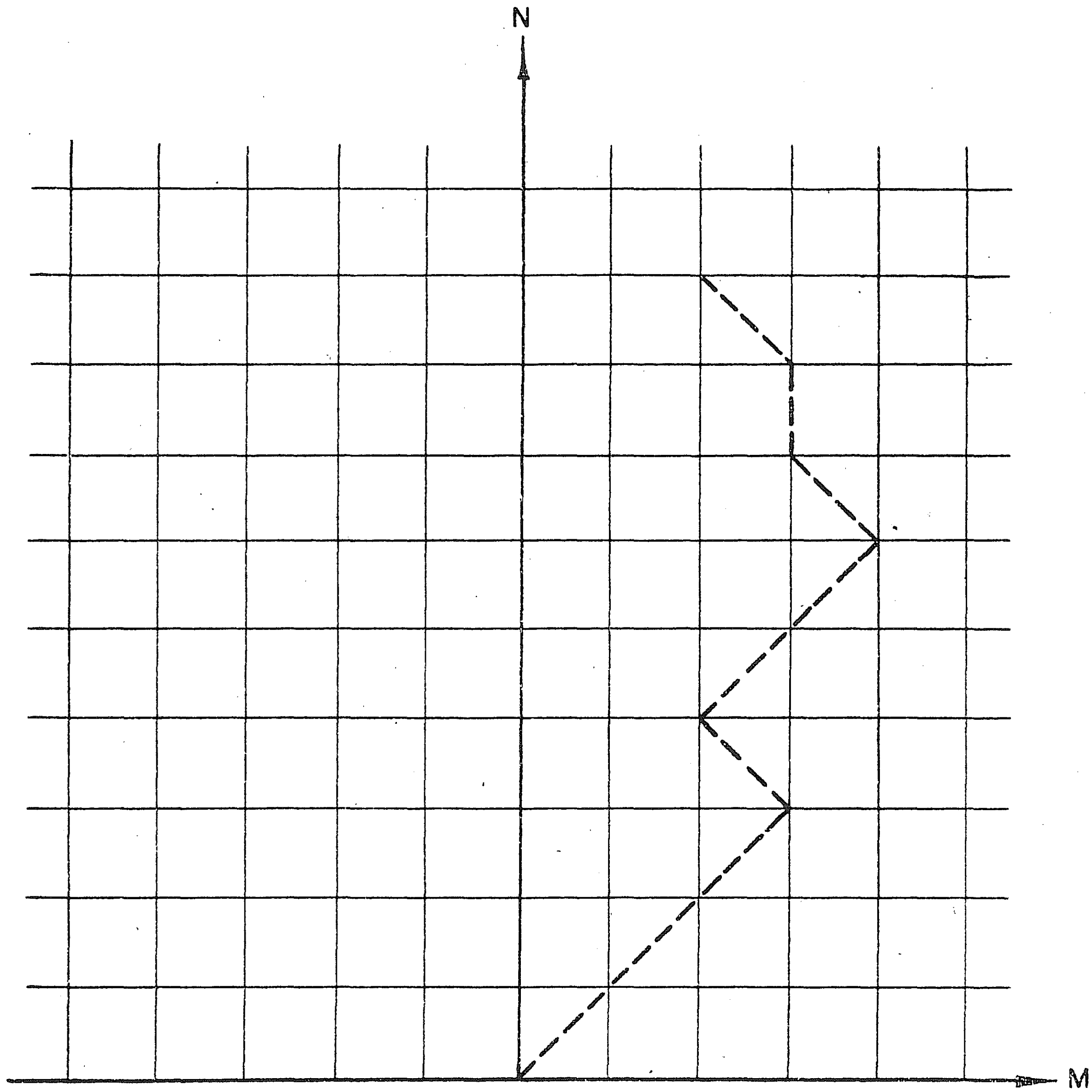

Fig. 2 Plan of Random Walk

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