ACCEPTED VERSION

Hung X. Nguyen, Bruce Northcote **User spectral efficiency: combining spectral efficiency with user experience**Proceedings of the 2016 IEEE International Conference on Communications (ICC):

Communications for all things, 2016 / pp.1-6

© 2016 IEEE

Published version at: http://dx.doi.org/10.1109/ICC.2016.7511204

PERMISSIONS

http://www.ieee.org/publications standards/publications/rights/rights policies.html

Authors and/or their employers shall have the right to post the accepted version of IEEE-copyrighted articles on their own personal servers or the servers of their institutions or employers without permission from IEEE, provided that the posted version includes a prominently displayed IEEE copyright notice (as shown in 8.1.9.B, above) and, when published, a full citation to the original IEEE publication, including a Digital Object Identifier (DOI). Authors shall not post the final, published versions of their articles.

23 January 2019

http://hdl.handle.net/2440/117346

User Spectral Efficiency: Combining Spectral Efficiency with User Experience

Hung X. Nguyen, Bruce Northcote
Teletraffic Research Centre, The University of Adelaide, Australia
Emails: {hung.nguyen, bruce.northcote}@adelaide.edu.au

Abstract—Electromagnetic spectrum is a scarce resource. Spectrum licensing is estimated to consume about 20% of cellular operators' capital expenditure (CAPEX). One important measure of spectrum use is "spectral efficiency" (SE), which is the amount of data bandwidth that a specific technology can extract from a certain amount of radio spectrum and is measured in bits per second per Hz (bps/Hz).

Using data from 4 different mobile operators, we show that spectral efficiency as the measure of bits per second per Hz correlates poorly with users' performance on cellular networks. Applying spectral efficiency directly in network performance monitoring or in network planning can provide misleading diagnostics and poorly targeted expansion plans.

We propose a new spectral efficiency metric that combines the raw bit per second per Hz measurement with user perceived performance. We show that this new metric correlates well with measured user throughput and is superior metric for network planning and performance monitoring in real networks. We present a number of applications of our new metric on network management and capacity planning tasks. Our implementation of this approach has been used in operational networks for over 8 years and has provided sound basis for CAPEX decision making.

Index Terms—Quality of Experience, Capacity Planning, Spectral Efficiency, Key Performance Indicators

I. INTRODUCTION

Electromagnetic spectrum is a scarce resource. Spectrum licensing is estimated to consume about 20% of cellular operators' capital expenditure (CAPEX). One important measure of spectrum use is "spectral efficiency" (SE), which is the amount of data bandwidth that a specific technology can extract from a certain amount of radio spectrum and is measured in bits per second per Hz (bps/Hz). Knowing how efficiently different technologies use spectrum, network operators can plan their network expansion optimally and policy makers can allocate spectrum more wisely.

Measuring spectral efficiency should not be simply taking the number of transmitted bits divided that by the available spectrum [6]. Today's wireless technologies adapt to the radio environment. With a good quality signal, the radio link can use high-order modulation and less forward error correction. Spectral efficiency at a given cell-carrier is, therefore, a function of its radio environment, its configuration and its traffic distributions. As these change, the spectral efficiency values also change. A single spectral efficiency value for a given cell-carrier is often used for capacity planning and is determined using either the average or a percentile value of the instantaneous spectral efficiency values.

Contrary to previous technologies, modern cellular networks (Wideband Code Division Multiple Access (WCDMA), Long-Term Evolution (LTE), and their extensions) are designed to maximize user experience using intelligent schedulers to allocate as much of the available resources as possible to service user flows. Earlier technologies, like GSM, serviced users by allocating to them fixed numbers of timeslots, analogous to the allocation of circuits in wireline telephony. Using this analogy, dimensioning the capacity in those early cellular networks was feasible with techniques adopted from circuit-based network dimensioning, such as the use of the well known Erlang-B formula. It also allowed more simplistic capacity planning to be done using the SE metric.

The ability to deliver consistently good user experience, as measured against various Key Performance Indicators (KPIs), has been identified worldwide as one of the most important factors for attracting and retaining customers. Despite this, the traditional SE metric only measures how much data can be expected to be carried over a given cell-carrier. By itself, the SE metric provides no insight into user experience. For example it is not enough to fully capture the interactive downlink throughput that a user would get when being served by the cell-carrier. That is, the capacity planning method of only using SE does not provision the network to meet any given set of user experience target KPIs. These KPIs, like interactive downlink throughput and service blocking rates, are functions of both the cell-carrier's capacity and traffic demand. A cell-carrier with high SE does not guarantee that user experience KPIs will be met and vice versa.

We first show in this paper, using data from 4 different mobile carriers, that the SE metric correlates poorly with users' experience on cellular networks. Consequently, the SE metric can provide misleading results for network performance monitoring, capacity management, or in network planning if it is intended to account for user experience.

We then proceed to develop a new SE metric that combines the raw SE bps/Hz measurement with a headroom capacity metric that has been derived based on user perceived performance. We show that this modified SE metric correlates well with measured user experience and is particularly well suited for network planning, capacity management and trouble shooting. Our user centric metric has been used in live capacity planning and network monitoring of one of the world's most profitable (on a per-subscriber basis) mobile operators.

The rest of the paper is organized as follows. In Section II,

we discuss the related work. In Section III, we explore the suitability of using Spectral Efficiency for capacity planning in real world networks. We present our new spectral efficiency metric in Section IV and its applications in Section V. Finally, we conclude the paper in Section VI.

II. RELATED WORK

Wireless communication technologies have evolved around the quest to increase the amount of data that can be transmitted over a given amount of electro-magnetic spectrum. In 1948, Shannon [7] established that there is a hard limit to how much data can be transmitted in a given spectral bandwidth, commonly referred to as the Shannon limit,

$$\lim_{B \to \infty} \log_2 \left(1 + \frac{P}{BN_0} \right) = \frac{P}{N_0} \log_2 e \tag{1}$$

where P is the received power and N_0 is the one-sided noise spectral level. Since capacity is monotonically increasing with bandwidth B, the right-hand side of (1) is the maximum rate achievable with power P.

Verdu [8] provides a review of theoretical bounds on SE, defined as the net data rate in bits per second (bps) divided by the bandwidth in Hertz, in wideband communication systems. Several thousand papers have explored different techniques that can be used to push system SE towards these theoretical limits. Many of these techniques have been implemented in commercial WCDMA and LTE networks. A recent survey of these techniques is provided in [4]. A comparison study of SE as measured in real networks is given in [3].

We concentrate on the SE that is achieved in today's cellular networks in this paper. SE in a cellular system usually is expressed in the format "bits per second per Hertz," (bps/Hz) and is computed for each cell-carrier using the following formula:

$$SE = \frac{Throughput}{Spectrum},$$
 (2)

where Throughput is the number of bits per second in the downlink of the cell-carrier and Spectrum is the electromagnetic spectral bandwidth (in Hertz) available at the cell.

Spectral efficiency (SE) as defined above provides a useful tool to compare different technologies and to evaluate the amount of data that can be carried in a cell-carrier. Recently, Rysavy [6] pointed out that even though the SE bps/Hz metric is extremely useful when comparing technologies, by itself it does not fully capture how efficiently spectrum is used. For example, it does not address how spectrum can be reused across space. Rysavy then explored several different factors that need to be taken into account when studying the SE of a network, including variations in network and traffic conditions, technologies such as coding and error correction methods, the number of subscribers and cell density.

Of particular relevance to this work is the recommendation that SE should be assessed jointly with users' experience. Note that even though this concept is mentioned as being critical for evaluating SE in [6], no formal technical definition has been given. In this work, we formalize the idea of integrating users' experience with spectral efficiency. We provide a unified metric, *User Spectral Efficiency (USE)*, that is derived directly from existing network measurements.

III. UNDERSTANDING SPECTRAL EFFICIENCY IN REAL WORLD NETWORKS

Most cellular operators regularly obtain measurements from their network equipment for on-going performance monitoring and for network planning and expansion. Most common measurements include connection drop rates, user data throughput and spectral efficiency.

We have analysed live data from 4 different mobile operators (one Tier-1 carrier in North America, one operator in Latin America, one leading operator in North Africa, and one Tier-1 operator in Australia) to understand SE in real world networks, especially the relationship between SE and user experience in terms of drop rates and throughputs. In this section, we analyze one month's worth of data for approximately 2000 WCDMA cells (in about 2 Radio Network Controller (RNCs)) and 2000 LTE cells in each operator.

Figure 1 below shows the spectral efficiency (SE) versus the average per user downlink throughput across LTE cell-carriers with 10MHz bandwidth using recent data (2015) from the 4 operators. For all networks that we analyze, SE correlates very poorly with per user throughput. The correlation coefficient is only 0.1165. More specifically, many cells with SE around 3 bps/Hz have low per user throughput of below 5 Mbps; whereas many cells with SE around 0.5 bps/Hz achieve high per user throughput of up to 20Mbps.

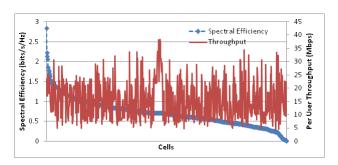


Fig. 1. Average spectral efficiency (= data rate/spectrum) versus average per user downlink throughput for LTE cells with 10MHz spectrum. The correlation coefficient of the two data series is 0.1165

Similar observations can be made for the WCDMA network shown in Figure 2 for WCDMA data in the 4 networks that we analyze. For clarity, we choose scatter plot format instead of plotting the two data series separately as in Figure 1.

We see a remarkable similarity in the two figures across all the operators and across different technologies (LTE and WCDMA): SE is a very poor indicator of user performance. The main reasons for the poor correlation between SE and user experience are:

 SE is computed using the cell carrier throughput regardless of the number of users; whereas per user throughput

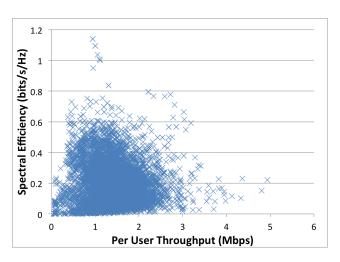


Fig. 2. Average spectral efficiency (= data rate/spectrum) versus average per user downlink throughput for WCDMA cells with 5 MHz spectrum. The correlation coefficient of the two data series is -0.1

depends strongly on the number of users competing for the same spectrum.

• SE is not simply the product of the number of users and the per user throughput. It depends strongly on the radio conditions that each user experiences, the traffic that each user generates, and the air interface and baseband resources available at each cell-carrier and base station.

We also observe poor correlation between SE and drop rates. Across all of our operator networks, SE for LTE is consistently much higher than for WCDMA. From these observations, we can conclude that even though SE can provide a reliable comparison between technologies, it is not suitable for daily network planning or continuous performance monitoring. For these purposes, a new metric is needed!

Spectral efficiency per number of users would be an appropriate metric to compare and analyze the performance of different cell carriers covering the same geographic area. For example, using this metric one can say that cell carrier A is more efficient than cell carrier B because A uses 5 MHz to service 50 users whereas B uses 10 MHz to service 50 users in the same geographical location.

However, an SE per number of users metric does not consider the users' quality of experience (QoE). If cell A has network congestion with poor QoE then A is not more efficient but may just need more spectrum, or there may be a need for an additional cell-carrier to improve QoE.

The example above highlights the fact that when doing capacity planning (either in spectrum allocation or network expansion), the users' QoE (as measured against a set of target per user experience KPIs) needs to be considered jointly with SE as fundamental inputs.

The key question is how to use available measurements from the network to produce a meaningful SE metric that can be used in network capacity management tools, a topic that we will discuss in the following section.

IV. USER SPECTRAL EFFICIENCY

We want to develop a metric that fully captures how efficiently a given amount of spectrum is used in each cell carrier to service end users. We are constrained by the measurements that are readily available in today's cellular networks across different vendors.

Working with the operators, we have identified the following performance counters and configuration parameters that can be extracted from the cell carriers for most vendor equipments (including Ericsson, Huawei and ZTE)

- Air interface resources: spectrum, available codes and power;
- Traffic: number of users, traffic payload, and measured transmission time:
- Radio conditions: CQI distribution and uplink interference:
- Algorithms: Scheduling algorithms, Call Admission Control (CAC) algorithms and available modulation and error correction schemes; and
- User experience KPIs: such as maximum acceptable call drop rates, minimum data throughput, interactive throughput, etc.

Our aim is to develop a metric that summarises how well the spectrum is used to meet the targeted user KPIs for each cell carrier from the above inputs, with the goal of using this metric in network management tasks.

We will break the model into two smaller parts: (1) modelling user experience in WCDMA/LTE networks and (2) combining the user experience model with the SE metric.

A. Modelling User Experience: Cell Headroom

To evaluate the capability of a cell in meeting the users' experience KPIs, we use the model in [5] for WCDMA networks and its LTE extension (currently undergoing patent protection).

The main idea of this model is to estimate spare capacity, where capacity is defined as the maximum user traffic load that can be carried and still meet a predetermined set of target user experience KPIs. More specifically, using the input from **Air interface resources**, **Radio conditions**, **Algorithms** and **User experience KPIs**, the model determines the "capacity envelope" (or "admission region") which is the set of all combinations of traffic of different bearer types that the cell carrier can service simultaneously whilst meeting the target user experience KPIs. See the shaded (light green region) in Figure 3 for a simplified example of the capacity region with only two types of traffic (voice and HSPA data).

Given any operating **Traffic** point (current load) the model will calculate the "headroom metric" that is used to indicate the state of the system in relation to the capacity envelope surface. Headroom is the percentage of maximum user traffic load that remains beyond the current operating load, in the direction of current traffic growth as shown in Figure 3. By implication, if the current operating load already exceeds the maximal load for which user experience targets can be

maintained, then the resultant headroom metric is a negative quantity.

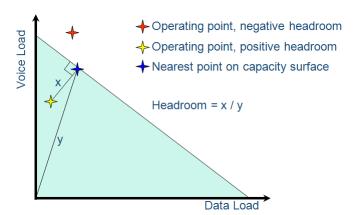


Fig. 3. Illustrative example of the headroom metric. The capacity region is the set of traffic load points where user KPIs are reached. Load points outside the capacity region have negative headrooms. Load points inside the capacity region have positive headroom. The actual headroom number is normalised with respect to the point closest to it on the admission region surface.

Headroom defined in this manner is an effective measure of spare capacity. For a given cell-carrier it is a single metric that summarises how the cell configuration, radio conditions, and traffic affect its ability to meet targeted user experience KPIs. A large headroom indicates that the cell-carrier can take in a lot more user traffic and still meet all the target KPIs. A small or negative headroom indicates that the cell-carrier is struggling to sustain the target user experience.

1) WCDMA Headroom Model: The current WCDMA model supports different services on different Radio Access Bearers (RABs) and Signalling Radio Bearers (SRBs) including constant bit rate bearers as defined in 3GPP Release 99 (R99) [1] and the High Speed Packet Access (HSPA) bearers in Release 7 [2] that is more spectrally efficient than the R99 bearers. HSPA bearers are subject to less stringent CAC than the R99 bearers, thereby allowing connectivity for more simultaneous data users. The scheduling mechanism of a given cell carrier shares any residual resources (those remaining after the R99 bearers have been allocated their requisite resources) amongst the set of active HSPA subscribers to provide each with a variable bit rate service.

The model allows for all services (voice, video, Circuit-based internet access and High Speed Packet Access) and setup types (Handover, Non-handover (new call setups)). It also supports different KPIs against which the capacity envelop is determined, including service availability (percentage of time each service can be accessed) and HSPA throughput in the uplink and downlink directions.

2) LTE Headroom Model: We have extended the WCDMA headroom idea in [5] to model LTE networks. The LTE model follows the same approach and is undergoing patent protection. The LTE model supports both data traffic and VoLTE as described in the 3GPP Release 12 [2]. Our LTE model supports QCI differentiation for the data bearers (when

there are multiple data QCIs) and allows for different per-QCI target user experience KPIs (blocking, interactive throughput, latency).

The models underlying the calculation of the headroom metric have been validated in operational WCDMA and LTE networks over more than 8 and 2 years respectively. The validation of the WCDMA model was described in details in [5].

B. USE: Combining User Performance with SE

The headroom metric quantifies how far a cell-carrier's available resources are from meeting targeted user experience KPIs. Spectrum is the most important of all the resources available at the cell-carrier.

We first explore in this section the relationship between SE and Headroom in our operator networks. In Figure 4 we plot SE versus headroom for the cells in the 4 operators. Note that the headroom number combines all the users performance metrics, not just per user throughput. Confirming the poor correlation between SE and user throughput in Figure 1 and Figure 2, there is an almost inverse relationship between SE and headroom in Figure 4. More specifically, in our datasets most cells with negative headroom are highly loaded cells. Even though these cells struggle to carry the user traffic, they still have very high SE simply because they manage to push through a large amount of data (at very low rates). Contrarily, cells with low load will have high headroom and low SE because there is very little traffic. From our dataset, almost 90% of cells with low measured SE (below average SE of the network) have low traffic load. These cells do not have any issue with their spectrum usage. They just simply do not have the traffic load to make the best use of the available spectrum. SE therefore is a very poor indicator of the cell performance and should not be used in network management and planning tasks.

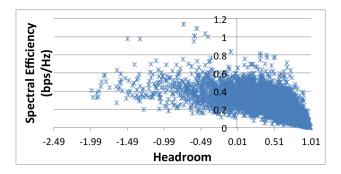


Fig. 4. Spectral efficiency versus Headroom for WCDMA cells

To explicitly quantify the efficiency of spectrum usage at the cell-carrier to meet user experience KPIs, we scale the SE metrics by the headroom to obtain a "User Spectral Efficiency (USE)" metric:

$$USE = \frac{SE}{1 - Headroom},$$
 (3)

where SE is defined in (2) and Headroom is defined as in the previous section. USE has the same units as SE of bps/Hz and is a measure of how effectively the available spectrum is being used at the cell carrier. The main difference between USE and SE is that USE adjusts SE by taking into account the actual cell ability to meet user KPIs.

Figure 5 below shows the correlation between our new metric USE and the average per user downlink throughput across LTE cells with 10MHz bandwidth in our operator networks. USE correlates strongly with user throughput, especially for cells with high per user throughput. The correlation coefficient is a robust 0.7889. Indeed, the two data series almost overlap.

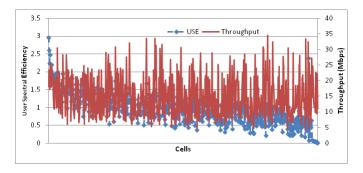


Fig. 5. User spectral efficiency versus Average per user downlink throughput for LTE cells with 10MHz spectrum. The correlation coefficient of the two data series is 0.7889

A similar observation also applies to WCDMA networks in Figure 6 with a correlation coefficient of 0.7. Again we use scatter plot for clarity.

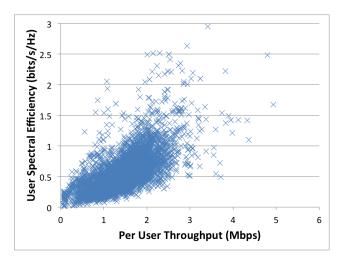


Fig. 6. User spectral efficiency versus Average per user downlink throughput for WCDMA cells with 5 MHz spectrum. The correlation coefficient of the two data series is 0.7

In both cases, USE correlates with the per user throughput very well. For example, most cells with low USE do indeed have either high call drop rates or low user data throughput. This is not a surprise as USE has incorporated user KPIs (through the headroom metric) in its calculation. We now have a metric that correlates well with user experience for use in spectrum analyses.

V. APPLICATIONS OF USE

The most valuable contribution of USE is its application to key network management tasks. We highlight a number of these in this section. USE has been used in large scale mobile networks in Australia and Asia, providing significant OPEX and CAPEX savings and excellent network performance for the operators.

A. Performance Forecasting

The first important application of USE is to forecast future performance for each cell-carrier. The validated headroom models can be re-exercised on forecasted loads (generated using any given traffic forecast algorithm) to obtain a set of forecast headroom values calculated for incremental dates into the future. These forecast headroom values then allow the USE metric to be predicted into the future. This becomes particularly useful when considering strategies for spectrum repurposing, small cell and hetnet deployment, WiFi offload, and even participation in spectrum auctions. Armed with a USE metric that correlates with end user experience and a forecasting capability, a network operator has a strategic advantage in planning strategy compared to any other competitors which rely on the outdated SE metric approach.

In Figure 7, we show an example of the forecasting power of USE for one LTE cell-carrier. The payload forecast (dashed blue line) is the actual traffic forecast from the operator. The capacity (red line) is determined by dividing the spectrum bandwidth by the user spectral efficiency of the cell. In this particular cell, the spectrum and USE are predicted to remain constant, the cell capacity remains constant. The point where the two lines intercept is the date when the cell no longer has enough spectral capacity to meet user KPIs for the forecast traffic. In this case, the cell will experience performance

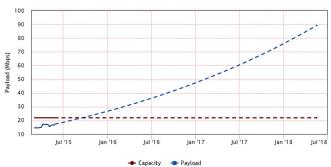


Fig. 7. Applying USE in network performance forecasting. When the payload exceeds the capacity, the cell will have performance issues.

B. Spectrum Planning

In addition to predicting when a cell runs out of spectrum our capacity management and planning solution can also accurately compute the required spectrum for a given WCDMA or LTE cell at any given time in the future. The required spectrum for a cell at a time t is computed using the following formula:

$$AdditionalBW = \frac{Load(t)}{USE(t)} - CurrentBW, \tag{4}$$

ForecastDate	CurrentCapacitySufficient	AdditionalBWRequired		
1/06/2015	Y	0		
1/12/2015	N	5		
1/06/2016	N	5		
1/12/2016	N	10		
1/06/2017	N	10		
1/12/2017	N	15		
" TARIEI '				

AN EXAMPLE OF THE CAPACITY PLANNING METHODOLOGY WITH USE. Y =YES, N=No. THE BANDWIDTH (BW) UNIT IS MHZ.

where Load (t) is the predicted traffic load at time t, USE(t) is the USE number computed for that predicted load, and CurrentBW is the spectral Bandwidth currently available at the cell-carrier.

Table I below shows the spectrum planning results for the cell-carrier in Figure 7. Using the equation in (4) we can determine the amount of extra bandwidth that the cell-carrier needs to support customer KPIs at any point in time.

Note that a similar simple planning method would be to scale up spectrum requirements as a ratio between the data payload and the SE. As we showed in Figure 1 and Figure 2, the correlation between SE and user performance is very poor. Methods that rely on SE therefore cannot be used to plan network expansion based on meeting targeted user KPIs.

C. What-if Analyses

USE can also be used in what-if analyses for spectrum refarming and data offloading. In these applications, the operator may wish to shift some 3G spectrum to support rapidly growing LTE traffic, or off-load some LTE traffic to WiFi. The frequently asked questions in these cases are "how much spectrum or traffic should be shifted from one technology to another?"

With a methodology based on USE, answers to the above what-if questions are simply obtained by re-computing the predicted cell-carrier capacity by dividing the available spectrum with USE. Figure 8 presents the graphical user interface we have developed in a solution exercising the USE methodology to enable such a "what-if" analysis. In the figure, 10MHz of spectrum on 1900 MHz frequency band is removed from WCDMA and reused for LTE.

Figure 9 shows the results of such experiment on the cell-carrier performance. The run-out capacity of the cell is now pushed out further by 6 months. The operator then has a choice of either acquiring more spectrum for the cell or applying a spectrum refarming solution.

VI. CONCLUSION

We have presented USE, a modified spectral efficiency metric for cellular networks. Using data from 4 different mobile operators from different parts of the world, we have shown that USE is a much more accurate indicator of cell-carrier user performance than the traditional SE metric. By taking user experience as a key input in the metric, USE is suitable for network management tasks including network forecast and planning that aim at providing consistent user experience

Payload Forecast Adjustment		Current Resources	
Inflation Factor (% CAGR) 1	20	Number of W850 Carriers (5MHz/Carrier)	1
Oompound Annual Growth Rate about Ericsson Mobility Report, June 2015 pabove flat (zero linear) growth		Number of W1900 Carriers (5MHz/Carrier)	2
LTE Proportion of Payload		LTE Bandwidth (MHz)	20
Current LTE Proportion (%)	80.81	Resource Adjustment	
Growth Factor (% yoy)	15	Add LTE Bandwidth (MHz)	10
Maximum LTE Proportion	100	Remove W850 Carriers	0
Run Out Dates		Remove W1900 Carriers	2
Run Out Date WCDMA	1/06/2015		
Run Out Date LTE	18/01/2016		
Run Out Date Combined	18/01/2016		

Fig. 8. Graphical user interface for performing what-if analyses.



Fig. 9. A what-if analysis for shifting 3G spectrum to 4G cell-carriers covering the same sector. Using this what-if analysis, the operator can experiment with different refarming options and observe the effects they have on the cell-carrier capacity.

against a target set of KPIs. USE has been used in operational networks and has shown great results. Networks that deploy USE have been consistently assessed as being amongst the most profitable and best performing mobile networks.

REFERENCES

- 3GPP. Technical Specifications and Technical Reports for a GERANbased 3GPP system. TS 01.01, 3rd Generation Partnership Project (3GPP), Dec 2007.
- [2] 3GPP. Technical Specifications and Technical Reports for a UTRAN-based 3GPP system. TS 21.101, 3rd Generation Partnership Project (3GPP), Jun 2008.
- [3] Yuehong Gao, Xin Zhang, Yuming Jiang, and Jeong woo Cho. System spectral efficiency and stability of 3g networks: A comparative study. In *Communications*, 2009. ICC '09. IEEE International Conference on, pages 1–6, June 2009.
- [4] Jingon Joung, Chin Keong Ho, K. Adachi, and Sumei Sun. A survey on power-amplifier-centric techniques for spectrum- and energy-efficient wireless communications. *Communications Surveys Tutorials, IEEE*, 17(1):315–333, Firstquarter 2015.
- [5] B.S. Northcote and N.A. Tompson. Dimensioning telstra's wcdma (3g) network. In Computer Aided Modeling, Analysis and Design of Communication Links and Networks (CAMAD), 2010 15th IEEE International Workshop on, pages 81–85, Dec 2010.
- [6] P. Rysavy. Challenges and Considerations in Defining Spectrum Efficiency. Proceedings of the IEEE, 102(3):386–392, Mar 2014.
- [7] Claude E. Shannon. A Mathematical Theory of Communication. The Bell System Technical Journal, 27(3):379–423, 1948.
- [8] S. Verdu. Spectral efficiency in the wideband regime. *Information Theory*, IEEE Transactions on, 48(6):1319–1343, Jun 2002.