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On the Simplified Modelling of Front Shapes of Fatigue Cracks

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Abstract. A direct three-dimensional (3D) finite element modelling of fatigue crack growth in structural components still represents a formidable task due to a complex singular behaviour of the stress field along the crack front as well as strong non-linearities associated with material plasticity and the change of contact conditions between crack faces during the loading cycle. The complexity of the 3D numerical modelling of fatigue crack growth largely motivates the development of simplified approaches. This paper describes several possible approaches for the evaluation of front shapes of fatigue cracks. These approaches are based on the (1) elimination of the corner singularity effect, (2) predictions based on the first-order plate theory, (3) the equivalent thickness concept, and (4) the Iso-K criterion. This paper briefly outlines these simplified approaches and presents some theoretical predictions for the case of through-the-thickness cracks propagating in plates under quasi-steady-state conditions. The theoretical predictions are also compared with experimental observations.

Introduction

The evaluation of fatigue failure of structural components is of permanent and primary interest for engineers. Hence, significant research effort has been directed towards the development of fatigue crack growth models over the past four decades. In particular, numerous early publications were dedicated to the study of the fatigue crack closure concept, which was first introduced by Elber [1] to explain the experimentally-observed features of fatigue crack growth in aluminum alloys. The number of publications grew rapidly since his pioneering study, reaching a maximum around 1970. It is now commonly accepted that the contributions of various mechanisms of crack closure, specifically the plasticity-induced closure, are significant, particularly at the near threshold fatigue crack growth, in retardation effects associated with overloads and acceleration of crack growth rates of physically short cracks [2]. According to this approach, the shape evolution is governed by the effective stress intensity factor (ΔK_{eff}), which is defined as:

$$\Delta K_{eff} = K_{max} - K_{op} = U\Delta K = U(K_{max} - K_{min}) \qquad (1)$$

where K_{max} and K_{min} are the maximum and minimum values respectively and U is the normalised load ratio parameter (or the normalised effective stress intensity factors) which is often used to describe the effects of loading and plate geometry on crack closure. K_{op} is the crack opening stress intensity factor under cyclic loading conditions.

Prior to 1970, the plasticity and crack closure mechanisms were intensively investigated for two-dimensional (2D) geometries utilising both plane strain and plane stress simplifications. With advances in numerical modelling and the increase in computational power, it became possible to study more realistic three-dimensional (3D) geometries as well as investigate the various near crack front 3D effects. A number of finite element (FE) models have been developed in the past to evaluate the effective stress intensity factor, ΔK_{eff}, and normalised load ratio parameter, U, for various geometries and loading conditions. However, these methods are difficult to implement in fatigue analysis due to convergence and repeatability issues. One of the reasons behind the difficulties in modelling plasticity and contact nonlinearities is the complex 3D singular stress fields, specifically near the vertex (corner) points.

In 3D problems the order of the singularity at the intersection of the crack front with the free surface depends on the Poisson's ratio and intersection angle. From energy considerations, it follows that shape of the fatigue crack front must evolve to preserve the inverse square root singular behaviour along the entire crack front. Therefore the fatigue crack has to intersect the free surface at a critical angle, β_c, which is a function of Poison's ratio, ν. Several experimental studies, specifically for quasi-brittle materials, have confirmed this prediction for mode I fatigue cracks. Other studies have indicated that the effect of 3D corner singularity might not be very significant in the presence of a sufficiently large crack front process zone. This is because the 3D corner singularity effect is a point effect and is quite localised. The experimental results for surface fatigue cracks in round bars show that the fatigue front preserves a semi-elliptical shape rather than the critical angle [3].

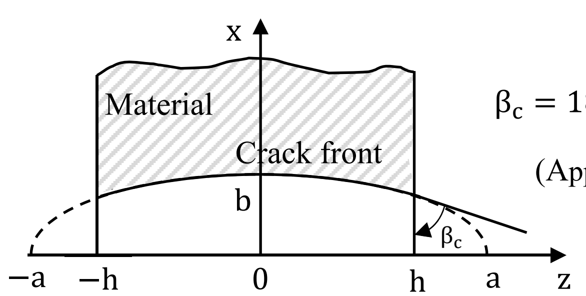
In this paper, we briefly outline four simplified approaches for the prediction of front shapes of fatigue cracks. We also describe the application of these approaches to through-the-thickness cracks as well as a comparison with experimental data.

Methods for Evaluating the Front Shapes of Fatigue Cracks

In this Section we briefly describe four simplified approaches for evaluating the front shapes of fatigue cracks propagating in plates under quasi-steady state conditions. These approaches are based on (1) the elimination of the corner singularity effect; (2) predictions based on the first-order plate theory; (3) incorporation of plasticity-induced fatigue crack closure effect using the equivalent thickness concept; and (4) the Iso-K concept.

Approach Based on the Elimination of Corner Singularity Effect

This approach is based on the so-called stress singularity matching. In accordance with this assumption, the evolution of the crack front occurs in a manner that all points over the crack front (including the corner points) have the same inverse square root singularity of the stress field. This assumption implies that the angle, β, is the same during the crack front evaluation and equal to the critical angle, $\beta = \beta_c$, (see Fig. 1) at the condition of the steady-state propagation [4]. The critical angle is a function of Poisson's ratio only; for example, for $\nu = 0.3$, $\beta_c \approx 100.40$. It is interesting to note that in accordance to the experimental study by Heyder et al. [5], in structures with flat free surfaces, such as beams of rectangular or trapezoidal cross-sections, the fatigue crack front appears to follow the stress singularity matching assumption; however, it is generally not supported by experimental observations for structures with curved surfaces such as round bars [6].



$$\beta_c = 180 + \operatorname{atan}\left(\frac{2-\nu}{\nu}\right)$$

(Approximation by Pook [4])

Fig. 1. Critical angle, β_c, in the case of a through-the-thickness crack propagating in a plate

An application of this approach to a steady state fatigue crack propagation requires the fulfilment of two conditions: (1) stress singularity matching (or $\beta = \beta_c$ at the intersection with the free boundary) and (2) the same value of the stress intensity factor along the crack front (Iso-K approach). The practical realisation of this approach can be based on a minimisation of the stress intensity factor variation along different front shapes, which can be described by a multi-parametric equation.

First-Order Plate Theory Predictions

Another approach for the front shape evaluation is based on first-order theory predictions. This simplified theory is a natural extension of the classical plane stress/plane strain theories. The first-order plate theory explicitly incorporates the plate thickness and the transverse stress components into the governing equations, which retain the simplicity of 2D models. Based on this theory, and utilising Budiansky-Hutchinson crack closure model [7], Codrington and Kotousov [8] provided the following solution for the normalised load ratio, U, in the case of the small-scale plasticity:

$$U(R, \eta) = a(\eta) + b(\eta)R + c(\eta)R^2 \tag{2}$$

where R is the load ratio; a, b and c are fitting functions given by the following equations:

$$a(\eta) = 0.446 + 0.266 \cdot e^{-0.41\eta};\ b(\eta) = 0.373 + 0.354 \cdot e^{-0.235\eta};\ c(\eta) = 0.2 - 0.667 \cdot e^{-0.515\eta} \tag{3}$$

where $\eta = K_{max}/(h\sqrt{\sigma_f})$ is a dimensionless parameter, K_{max} is the maximum stress intensity factor, h is the half-plate thickness, and σ_f is the flow stress.

These equations correctly recover the limiting cases of very thin and very thick plates, when $\eta \to \infty$ or $\eta \to 0$, respectively. The details of the derivation of these equations can be found in the original paper [8]. The application of this solution to the evaluation of the front shape of through-the-thickness cracks can be found in He et al. [9], and will not be repeated here due to page restrictions.

Equivalent Thickness Concept

Several researches suggested a concept which simplifies the evaluation of the plastic constraint effect on the plasticity-induced crack closure [10,11]. For example, based on an extensive 3D elasto-plastic FE analysis for through-the-thickness cracks, She et al. [12] proposed to define the equivalent thickness for arbitrary point, P, located on the crack front, see Fig. 2, as follows:

$$B_{eq} = h - z^2/h \tag{4}$$

where z is the distance from the mid-plane and h is still the half-thickness of the plate.

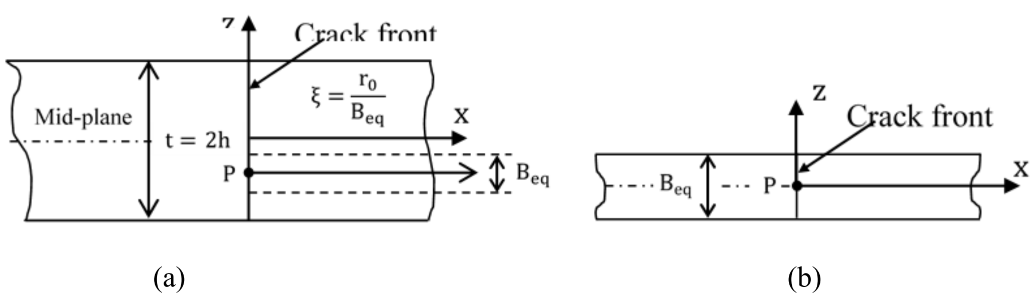


Fig. 2. Schematic illustration of the equivalent thickness method in the through-the-thickness cracks

The normalized load ratio is defined as:

$$U = \frac{\sqrt[3]{\kappa}}{1 - R} \tag{5}$$

where κ (see Eq. (6)) is a function of R and a global constraint factor, α_g.

$$\kappa = \frac{(1 - R^2)^2(1 + 10.34R^2)}{\left[1 + 1.67R^{1.61} + \frac{1}{0.15\pi^2\alpha_g}\right]^{4.6}} \tag{6}$$

The global constraint factor is a thickness $t(\xi)$, and Poisson's ratio (ν) dependence parameter:

$$\alpha_g = \frac{1 + t(\xi)}{1 - 2\nu + t(\xi)} \quad (7)$$

The normalized load ratio increases with an increase in the constraint factor at the constant applied stress ratio. In the last equation, t can be calculated from the following equation:

$$t(\xi) = 0.2088(\frac{r_0}{B_{eq}})^{0.5} + 1.5046(\frac{r_0}{B_{eq}}) \quad (8)$$

with the plastic zone size, r_0, as a function of flow stress, σ_f, defined as:

$$r_0 = \frac{\pi}{16}\left(\frac{K_{max}}{\sigma_f}\right)^2 \quad (9)$$

The practical realisation of this approach is normally accomplished by a simple crack advance scheme, in which each point along the crack front moves in accordance with the effective stress intensity factor range, ΔK_{eff}, see Eq. (1), with U provided by relationships (5) – (9).

A cracked plate under plane stress undergoes a change to plane strain behaviour near the crack tip. The radial position where the plane stress to plane strain transition takes place strongly depends on the position in the thickness direction. The degree of plane strain is essentially zero at distances from the tip greater than approximately five times of thickness. The effect of the distance from the crack tip on the evaluation of stress intensity factor is carefully considered in order to improve the accuracy of the numerical simulations. To simulate applied mode I loading, the displacement boundary conditions constant through the thickness were applied to the plate surface of the finite element model outside the 3D region per the William's solution [13].

Iso-K approach

In accordance with the Iso-K approach, the steady state fatigue crack propagation requires the uniform distribution of the local stress intensity factor range along the crack front. The stress intensity factor range can be evaluated numerically using 3D linear-elastic FEA.

The practical realisation of this approach for a steady-state propagation of through-the-thickness cracks in plates can involve the evaluation of the stress intensity factor for two characteristic points: at the middle, $z = 0$ and at the surface, $z = h$ for a two-parametric set of equations representing the front shapes, e.g. elliptical shapes. Further, the higher value of the crack closure at the free surface may be incorporated into the theoretical predictions using various empirical equations for crack closure proposed in the past. The steady-state crack growth requires the same fatigue crack growth rate, or

$$\frac{da}{dN} = C_S(\Delta K_S)^n = \frac{db}{dN} = C(\Delta K_M)^n \quad (10)$$

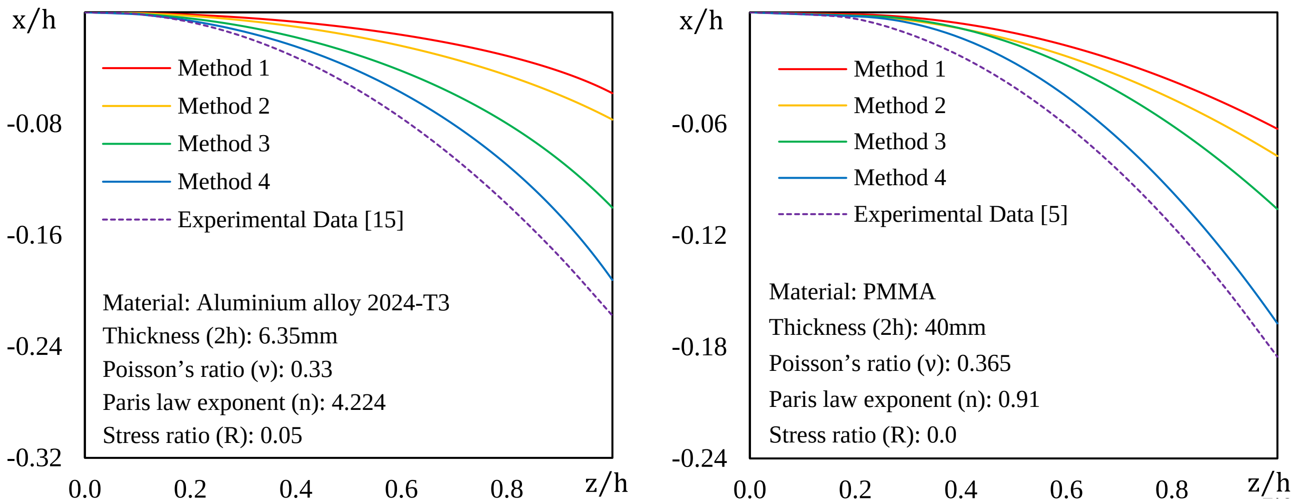
where ΔK_S and ΔK_M are the stress intensity factor ranges at the surface and the mid-thickness points of the crack front; C and n are Paris constants, which can be obtained experimentally for different materials. Newman and Raju proposed the following relationship between the Paris coefficients at the surface and deepest points for the plate components with a semi-circular crack under pure tension loading [14]:

$$C_S = 0.9^n C \quad (11)$$

In this study we also utilized a coefficient of 0.8 for both selected materials to get a better agreement with experimental data.

Comparison of Different Approaches

The proposed approaches for the evaluation of the steady-state crack front shapes were compared against experimental studies [5,15]. In these studies, the centre-cracked panels were made of 2024-T3 aluminum alloy and Polymethylmethacrylate (PMMA) with a thickness of 6.35mm and 40mm, respectively.



(a) (b)

Fig. 3. Comparison between the predicated crack shapes and experimental data for the specimens made of a) 2024-T3 aluminium alloy, and b) Polymethyl methacrylate (PMMA)

The advantage of PMMA material is its transparency which enabled an in-situ evaluation of the crack front shape. For the aluminum alloy specimens such an evaluation was done using benchmarking technique and post-mortem analysis of fracture surfaces. Both specimens were subjected to constant amplitude fatigue loading. The fatigue cracks were grown over a sufficiently large distance from the initial notch to ensure the quasi-steady state conditions of propagation.

As it follows from the analysis of Fig.3, the simplified approaches work a bit better for the quasi-brittle material (PMMA); and the Iso-K approach provides the best correlation with the experimental results. Unfortunately, none of the approaches is capable to accurately describe the front shape of fatigue cracks. This can be explained by the complexity of the crack closure phenomenon, which currently represents one of the major challenges in 3D Fracture Mechanics.

Conclusion

The capability of several simplified approaches for the evaluation of the shape of fatigue crack fronts has been studied using experimental results for a steady-state propagation of fatigue through-the-thickness cracks in different materials. It is demonstrated that none of the approaches is capable to accurately describe the shape of the fatigue cracks. An empirically introduced crack closure equation allows for a better matching of the theoretical and experimental predictions. The outcomes of this work and the comparison justify a need of further research in this area.

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